International trade and the division of labour*

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1 Introduction

"I always say there are two and a half theories of trade".

Paul Krugman to Peter Neary, as quoted in Neary (2010).

There are three main approaches to theoretical modelling of international trade: the approach based on comparative advantage and perfect competition from Ricardo to Heckscher and Ohlin, the approach based on monopolistic competition as in Krugman (1979, 1980), and the approach based on oligopoly as initially developed by Brander (1981)[1]. This paper develops a new model of international trade which takes a different approach to the preceding literature, by focussing on the division of labour as the reason for international trade. The role of the division of labour in raising per capita incomes has been recognised since at least Adam Smith (1776). The model we develop is based on trade in tasks, and shares features of both the comparative advantage and monopolistic competition approaches. From the comparative advantage literature, it uses a perfectly competitive market structure in order to isolate the effect of the division of labour; from the monopolistic competition literature, countries are ex ante identical to each other, so there is no comparative advantage reason for international trade. Grossman and Rossi-Hansberg (2008) make the point that the distinction between trade in tasks and trade in intermediate goods is largely semantic; we use the former term to be consistent with them. This type of trade constitutes over half of total goods trade, as documented by Miroudot et al (2009) and Sturgeon and Memedovic (2010).

In our model, the division of labour is limited by both the extent of the market, as in Smith (1776), and by coordination costs, as in Becker and Murphy (1992). International trade eliminates the duplication of coordination costs across countries, which encourages greater division of labour, and hence higher levels of output and welfare. Thus, similarly to models of trade based on monopolistic competition, we endogenise the number of tasks produced; however, this is done under perfectly competitive markets. Because countries are assumed to have identical technologies in producing tasks and there is no way of identifying individual tasks, the direction of trade is indeterminate; however, the volume of trade is determinate, and depends on the relative sizes of the trading partners.

[1] The oligopolistic approach is what Neary (2010) refers to as the half theory of trade, since it is not as widely used as the other two approaches, despite the efforts of Neary (2009).

Having established the main features of the model with one final good and one factor of production, we then proceed to extend the model to two final goods and two factors of production, similarly to Krugman's (1981) extension of his earlier (Krugman 1980) model. This enables us to consider the distributional implications of the model. With more than one factor of production, there are now two sources of the gains from trade: the division of labour as in the one factor model, and relative endowment differences as in the Heckscher-Ohlin model. As a result, it is possible, if the two countries are not too dissimilar from each other in their relative endowments, that both factors of production gain from trade. This is similar to the result in Krugman (1981), and is in contrast to the traditional Heckscher-Ohlin result, where the scarce factor always loses from trade.

Both the monopolistic competition model and our model may be viewed as different approaches to analysing the implications of the international division of labour. The key distinction between the two models is in the underlying economic mechanism. In the monopolistic competition model, the basic tradeoff is between love-for-variety, which encourages the production of more varieties, and increasing returns to scale, which limits the number of varieties. In our model, there is the same love-for-variety, but what restricts the number of varieties, is a combination of coordination costs and market size. By no means do we suggest that increasing returns to scale is not an important part of the explanation for international trade, but the alternative mechanism we propose may be viewed as a complementary explanation.

1.1 Related Literature

There is of course a large literature on offshoring and the international fragmentation of production. Many of the key issues and research have been ably summarised in Jones (2000) and Feenstra (2010). For instance, Feenstra and Hanson (1999) show that international outsourcing can explain 15 percent of the increase in the relative wage of nonproduction workers in the US between 1979 and 1990. Hummels et al (2001) document that growth in vertical specialisation accounts for 30 percent of the growth in total exports across a sample of 14 countries. Yi (2003) develops a dynamic Ricardian model of trade with vertical specialisation which can explain a large part of the growth of world trade since 1945. Costinot et al (2013) develop a model of vertical specialisation in which technological change may have a different impact on countries depending on where they are located along the value chain. Johnson and Noguera (2012) calculate the value added content of bilateral trade, using bilateral trade data, and data from input-output tables. They find that the ratio of value added to gross exports is about 70% for the average country, but there is large variation between countries.

The model we develop is based on that developed by Becker and Murphy (1992) in a closed economy setting, in which the extent of the division of labour is limited by the cost of coordinating inputs. This extends Adam Smith's original insight that it is the size of the market which constrains the extent of the division of labour. We extend a version of the Becker and Murphy (1992) model to the open economy, and allow for substitutability between tasks (in their formal model, Becker and Murphy assume a Leontief production function). The latter extension affects the quantitative but not the qualitative results of the model[2]. Also closely related is the model developed by Francois (1990a, 1990b), which makes use of the production function developed by Edwards and Starr (1987) to develop a model of international trade in which scale economies arise from producer services in a monopolistic competition model.

Another closely related paper is Grossman and Rossi-Hansberg (2008), who develop a model of task offshoring in which the gains from offshoring can be decomposed into three effects: a productivity effect, a relative-price effect, and a labour-supply effect. Their model is based on differences in factor endowments and factor intensities across sectors, hence has quite different foundations from the one-sector model we develop in Sections 2 and 3. Nevertheless, we are able to identify the analogue to the labour-supply effect. In our model, this occurs because with international trade, each country produces fewer tasks, so workers who used to be employed in tasks which are no longer being produced, will be re-employed in other tasks, at a larger scale, and hence benefit from higher wages as a result of the division of labour. This is the opposite to what Grossman and Rossi-Hansberg obtain, because they implicitly assume diminishing marginal product of labour, so the redistribution of labour results in lower wages. When we extend the model to two sectors, we are able to obtain results which are similar to their productivity and relative-price effects; these are discussed in Section 4. Also, we endogenise the number of tasks and the labour used in each task, whereas Grossman and Rossi-Hansberg (2008) assume a (fixed) unit interval of tasks, each produced with the same amount of labour.

The role of the division of labour in international trade has been developed especially by Ethier (1979, 1982a). In the earlier paper, the distinction is not made between external and internal scale economies, while the later paper is explicit in its use of both

[2] To be precise, allowing for substitutability between tasks reduces the overall gains from trade, but increases the gap between winners and losers from trade, relative to the Leontief production function.

internal and external scale economies. Swanson (1999) develops a different model of the division of labour under perfect competition, in which a larger market enables greater specialisation and hence higher skill levels and output per worker via the endogenous development of comparative advantage. More recently, Chaney and Ossa (2013) open up the black box of the production function in the Krugman (1979) model of monopolistic competition, modelling the production process as a series of stages produced by teams. More closely related is Soo (2017), who develops a model of international trade based on the division of labour and comparative advantage in a perfectly competitive framework. Unlike Soo (2017), who makes use of comparative advantage to pin down the structure of production, in the present paper we focus on the cost of coordinating inputs that limits the extent of the division of labour.

In order to close the model, we assume that the production of tasks takes place under what Ethier (1979) refers to as national scale economies which are external to the firm. This is the same assumption as in most related work in this area, for instance Markusen and Melvin (1981) and Panagariya (1981). Helpman (1984) provides an insightful survey of this literature, while Grossman and Rossi-Hansberg (2010) offer a recent treatment. Throughout the paper we focus on efficient allocations, which are those that enable the replication of the integrated equilibrium (see Krugman, 1987). This enables us to sidestep the fact that models with external scale economies exhibit multiple, inefficient and possibly unstable equilibria.

The next section outlines the main building blocks of the model, and the autarkic equilibrium. Section 3 discusses the implications and patterns of international trade. Section 4 extends the model to two factors of production, while Section 5 provides some concluding comments.

2 The model

The model is set up with two countries, $i \in H, F$ for Home and Foreign, although the solution method allows for easy extension to many countries. There is a single final good which is used in consumption. Let the representative consumer's utility function be:

$$U_i = C_i^{\theta}, \qquad 0 < \theta < 1. \tag{1}$$

All markets are perfectly competitive, and all firms are identical to each other. There are many possible tasks, $j = 1, \ldots, N$. Production of the final good requires the combination of tasks using the following constant elasticity of substitution (CES) production function:

$$Q_i = \left[\sum_{j=1}^{n_H+n_F} x_{ij}^{\sigma}\right]^{1/\sigma}, \qquad -\infty < \sigma < 1. \tag{2}$$

Where n_i is the number of tasks actually performed in each country, and x_{ij} is the quantity of each task j used in country i. The parameter σ measures the degree of substitutability between tasks; if σ is close to 1, tasks become better substitutes for each other, whereas if σ is close to $-\infty$, love-for-variety increases (note the case of the Leontief production function as in Becker and Murphy (1992) implies $\sigma = -\infty$). That is, the CES production function exhibits gains from the division of labour: the more the production process is divided into different tasks, the larger the output of the final good[3]. Thus, firms will, in the absence of coordination costs, want to divide the production process into as many tasks as possible; it is the coordination cost that constrains the division of labour[4].

In assembling the final good from the tasks, there is a coordination cost that depends on the number of tasks used in the production process:

$$c_i = \psi n_i^{\rho}, \qquad \rho > 1. \tag{3}$$

The assembly process uses real resources in the sense that final output is reduced by the assembly cost (analogously to the "iceberg" trade costs in other papers). This cost is assumed to be shared by all firms producing the final good, and may be thought of as the cost of maintaining a production network; the more tasks there are, the more difficult and expensive it becomes to coordinate all the tasks. As we will see below, the restriction that $\rho > 1$ implies that the coordination cost is convex in n, and will play an important role in ensuring that a larger country not only has a larger number of tasks, but also that each task is produced at a larger scale.

Labour is the only factor of production, and each country has an endowment of L_i units of labour. Tasks are produced using labour with a production function that exhibits external scale economies which are national in nature (Ethier, 1979). That is, output of a task j in country i depends on employment in that task in country i:

$$q_{ij} = \left(\alpha l_{ij}\right)^{\gamma}, \qquad \alpha < 1, \quad \gamma > 1. \tag{4}$$

[3] The model is isomorphic to one in which consumers consume tasks directly. However, while it may be reasonable to assume division of labour in the production process, it is more difficult to justify on the consumption side.

[4] From the discussion, it should be clear that we have a flexible definition of what constitutes a task. For instance, if the final good is a car, then a coarse division of labour may involve dividing the car into the engine, chassis, and tyres. A finer division of labour may further divide the engine into many components, and so on.

Where $\alpha < 1$ is labour productivity, and $\gamma > 1$ indicates external scale economies; output increases more than proportionally to labour inputs. There are two reasons for assuming external scale economies. The first, technical reason, is that it enables us to pin down the number of tasks actually produced; if constant returns to scale were assumed, each final good firm could in principle demand a different set of tasks.

A second reason for assuming external scale economies which are national in nature is that with international trade, the efficient, integrated equilibrium implies that production of each task will occur in only one country. As a result, international trade leads to a saving in the coordination cost of assembling the final good from tasks; these savings would not materialise if each task is produced in more than one country. This is discussed in Section 3.2 below. External scale economies which are national (as opposed to international) in nature may be justified by appeal to Marshall's localised external economies (see Krugman, 1991). Such localised economies may lead to the formation of industrial clusters (Porter, 1990). In the context of the present model, it is helpful to think of each task as being produced by many perfectly competitive firms in the same location, because of the external scale economies. Different tasks may be produced in the same or in different locations.

Under perfect competition, normalising the wage rate to unity, and assuming average cost pricing (see Ethier, 1979), the zero profit condition implies that the price of each task is given by:

$$p_{ij} = \left(\alpha^{\gamma} l_{ij}^{\gamma-1}\right)^{-1}. \tag{5}$$

Since $\gamma > 1$, the larger the employment in sector j in country i, the lower the price; therefore it is more efficient for each task to be produced in only one country, as this maximises the scale of employment in that task in that country. Also, the higher is labour productivity α, the lower the price. Appendix A shows how equations (4) and (5) can be obtained from the production function for each perfectly competitive firm and the firm's profit-maximising condition, respectively.

2.1 Autarkic equilibrium

In autarky, all domestically produced tasks are used in producing the domestic final good. Since all task-producing firms have the same cost structure and tasks enter symmetrically into the production function of the final good[5], all tasks will in equilibrium be produced and consumed in equal quantities, so we have:

$$x_{ij} = q_{ij} = q_i = \left(\alpha l_{ij}\right)^{\gamma} = \left(\frac{\alpha L_i}{n_i}\right)^{\gamma}. \tag{6}$$

Substituting this into the production function (2) and subtracting the assembly cost (3) gives the production function for final goods net of assembly cost:

$$\widetilde{Q_i} = \left[n_i^{1-\gamma\sigma}(\alpha L_i)^{\gamma\sigma}\right]^{1/\sigma} - \psi n_i^{\rho}. \tag{7}$$

Each firm in the final good sector chooses the number of tasks to maximise profits. All firms are identical to each other, so total industry profits are:

$$\pi_i = P_i\widetilde{Q_i} - p_i n_i x_i, \tag{8}$$

where P_i is the price of the final good, and is taken as given by the perfectly competitive firms. Substituting from equations (5), (6) and (7), we can rewrite the profit function (8) as:

$$\pi_i = P_i\left[n_i^{\frac{1-\gamma\sigma}{\sigma}}(\alpha L_i)^{\gamma} - \psi n_i^{\rho}\right] - L_i. \tag{9}$$

Differentiating equation (9) with respect to n_i allows us to solve for the number of tasks produced in each economy (ignoring integer constraints)[6]:

$$n_i = \left[\frac{(\alpha L_i)^{\gamma}(1-\gamma\sigma)}{\sigma\psi\rho}\right]^{\frac{\sigma}{\sigma(\rho+\gamma)-1}}. \tag{10}$$

Equation (10) shows that $\gamma\sigma < 1$ is required to generate positive values of n_i. This implies that for the mechanism of the model to work, different tasks cannot be too substitutable for one another; there must be sufficient love-for-variety in tasks. Similarly, the external scale economies cannot be too strong. In principle, each final good producing firm could demand different tasks. However, because production of tasks occurs under external scale economies, the total number of tasks produced will be the minimum number that will satisfy equation (10). That is, all final good producers will use the same tasks. This is one of the roles played by the assumption of external scale economies.

Since from equation (6) $q_i = (\alpha L_i/n_i)^{\gamma}$, we also have:

$$q_i = (\alpha L_i)^{\frac{\gamma(\rho\sigma-1)}{\sigma(\rho+\gamma)-1}}\left(\frac{\sigma\psi\rho}{1-\gamma\sigma}\right)^{\frac{\gamma\sigma}{\sigma(\rho+\gamma)-1}}. \tag{11}$$

[5] This turns out to be a crucial assumption in simplifying the solution of the model. See also Soo (2017) for a comparison between the CES production function and the Leontief one, in a different setting.

[6] It can be verified, given the assumptions made on the parameter values, that $d^2\pi_i/dn_i^2 < 0$, so that equation (10) is indeed the profit-maximising expression for n_i.

Equation (11) shows that $dq_i/dL_i > 0$ provided $\rho\sigma > 1$. Similarly, from equation (10), as long as $\sigma(\rho + \gamma) > 1$, we have $dn_i/dL_i > 0$. That is, if these two conditions hold, then a larger country produces a larger number of distinct tasks, and produces each of these tasks at a larger scale. Following the terminology of the literature, a larger country expands both in terms of the intensive margin (more output of each task is produced) and in terms of the extensive margin (more tasks are produced). This gives similar results to Krugman (1979), and contrasts with the monopolistic competition literature based on the CES utility function (e.g. Krugman 1980), in which a larger country has a larger variety of goods, but not larger output in each sector. The extent of the division of labour depends on the size of the market as in Smith (1776), but also on the coordination cost as in Becker and Murphy (1992). All else equal, higher coordination costs (larger values of ψ and ρ) imply a smaller number of tasks, each produced at a larger scale.

We can also obtain the price of the final good. Setting the profit function (9) equal to zero and solving gives:

$$P_i = \frac{L_i}{n_i^{(1-\gamma\sigma)/\sigma}(\alpha L_i)^{\gamma} - \psi n_i^{\rho}}. \tag{12}$$

From equation (10) above, a country with a larger labour force will produce a larger number of tasks. This reduces the cost of production of the final good because of the division of labour, and hence reduces the price of the final good relative to tasks in equilibrium.

Substituting from the number of tasks (10) into the net production function for final goods (7) and then into the consumer's utility function (1), making use of $C_i = \widetilde{Q}_i/L_i$ gives autarkic utility as a function of the model's parameters:

$$U_i^A = \left\{\frac{1}{L_i}\left[n_i^{\frac{1-\gamma\sigma}{\sigma}}(\alpha L_i)^{\gamma} - \psi n_i^{\rho}\right]\right\}^{\theta} \tag{13}$$

Larger countries have a higher level of utility under autarky, since a larger economy enables greater division of labour: $dU_i^A/dL_i > 0$ (note that this is the case since n_i is also a function of L_i). It can also be shown that an innovation which reduces the cost of coordination (for instance, information technology) would raise utility, by encouraging greater division of labour; $dU_i^A/d\rho < 0$ and $dU_i^A/d\psi < 0$.

Note that the market equilibrium as described above is efficient, since it yields the same outcome as would be obtained by a benevolent central planner, whose objective is to maximise the country's utility by choosing the optimal number of tasks to maximise net output. The reason for this is that the assumptions we have made above mean that firms internalise the effects of increasing numbers of tasks on their profits, as shown in equation (9). More tasks imply greater division of labour, but also higher coordination costs, and final goods firms take both effects into account when choosing the number of tasks.

3 International trade

In this section we allow for free international trade in both tasks and final goods between the two countries[7]. Similarly to Krugman (1979, 1980), international trade is equivalent to an increase in the size of the economy, since countries have identical technologies and there is only one factor of production. The crucial assumption here is that of national scale economies in the production of tasks. This means that, when international trade is allowed, the efficient allocation of resources (the integrated equilibrium) implies that each task is produced in only one country. As a result, since the two countries effectively become one market, the coordination cost is shared between the two countries. Following the same steps as for the autarkic equilibrium, the number of tasks that is consistent with profit maximisation by all final goods firms is:

$$n^T = \left\{\frac{[\alpha(L_H+L_F)]^\gamma(1-\gamma\sigma)}{\sigma\psi\rho}\right\}^{\frac{\sigma}{\sigma(\rho+\gamma)-1}}. \tag{14}$$

And the output of each task is:

$$q^T = [\alpha(L_H + L_F)]^{\frac{\gamma(\rho\sigma-1)}{\sigma(\rho+\gamma)-1}}\left(\frac{\sigma\psi\rho}{1-\gamma\sigma}\right)^{\frac{\gamma\sigma}{\sigma(\rho+\gamma)-1}}. \tag{15}$$

These expressions also indicate how the model can be extended to allow for many countries, and the implications of such an extension. Making the same assumptions on parameter values as in the previous sections, we can establish that:

$$n_H^A, n_F^A < n^T < n_H^A + n_F^A \tag{16}$$

and

$$q_H^A, q_F^A < q^T < q_H^A + q_F^A. \tag{17}$$

That is, the number of tasks and the output of each task both increase compared to the autarkic number and output of each task[8]. However, the increase is less than proportional to the expansion in market size resulting from trade liberalisation. International trade leads to an expansion on both the intensive and extensive margins.

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[7] It is also possible to consider trade frictions; this was done in a previous working paper version. Introducing trade frictions which are uniform across tasks yields few additional insights beyond the obvious ones, and has been omitted for brevity.

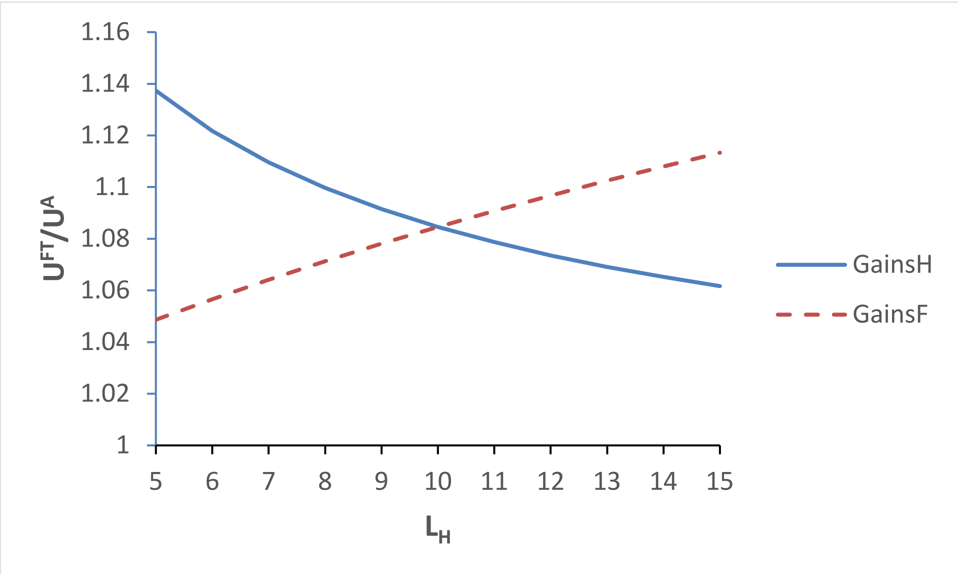
[8] We can also establish that $n_H^T < n_H^A$ and $n_F^T < n_F^A$; that is, each country produces fewer tasks in free trade than they do in autarky.

The representative consumer's utility with free international trade is given by:

$$U_i^T = \left\{\left(\frac{1}{L_H+L_F}\right)\left[(n^T)^{\frac{1-\gamma\sigma}{\sigma}}\left(\alpha(L_H+L_F)\right)^{\gamma} - (\psi(n^T)^{\rho})\right]\right\}^{\theta}. \tag{18}$$

It can be shown that $U_i^T > U_i^A$; that is, there are always gains from free international trade. These gains arise from the fact that international trade enables countries to avoid duplicating the coordination cost. Whereas in autarky the coordination cost is shared only by domestic firms, in international trade it is shared by both domestic and foreign firms. This cost saving enables firms to increase the division of labour, thus yielding a productivity gain in the output of the final consumption good[9]. Note also that since the free trade welfare is the same for all consumers in both countries whereas autarkic utility is higher in the larger country, we get the usual result that a smaller country gains more from trade than does a larger country.

Figure 1: The gains from trade for Home and Foreign.



Notes: Assumed parameter values: $L_F = 10, \alpha = 0.8, \gamma = 1.2, \rho = 1.8, \psi = 0.5, \sigma = 0.8, \theta = 0.5$.

Figure 1 shows the gains from trade (the ratio of free trade to autarkic utility) for the two countries as a function of the Home country's size, holding the size of the Foreign country constant. As Home becomes larger, its gains from trade decrease, whereas Foreign's gains from trade increase. As discussed above, this can be explained by the fact that a smaller country experiences a larger increase in the size of the market with

[9] As discussed in the Introduction, this cost saving is different from that obtained by Grossman and Rossi-Hansberg (2008). There, the cost saving arises because offshoring of labour-intensive tasks leads to a greater-than-proportional expansion of labour-intensive sectors, hence an increase in the demand for labour and wages.

trade liberalisation, and hence experiences a larger increase in the number of tasks and the output of each task.

3.1 Trade patterns

The pattern of trade may be described as follows. There is no trade in the final good, since each country can assemble the final good using the same technology. All trade will be in tasks. Since production of each task exhibits national external scale economies and technologies are identical across countries, in the integrated equilibrium, each task will be produced in only one country. The number of tasks produced by each country will be proportional to its share of world labour supply: $L_i/(L_H + L_F)$. And since prices are the same across countries and preferences are homothetic, each country's demand for each task is proportional to its national income. Hence the volume of trade is equal to one country's national income times the other country's share of world labour (since we normalise the wage rate $w = 1$):

$$VT = \frac{L_H L_F}{L_H + L_F}. \tag{19}$$

This expression is identical to the expression for the volume of trade in Krugman (1979, 1980), and for the same reason: there are gains from increased variety (tasks in the present paper, final goods in Krugman 1979, 1980). The volume of trade is maximised for a given total size of the world economy when the two countries have the same size. The larger is a country's trading partner, the more varieties of tasks it will import from this trading partner. However, and again similarly to Krugman (1979, 1980), the direction of trade is indeterminate, since we do not know which country produces which task.

3.2 Alternative assumptions for the production of tasks

In developing the model, we have made use of the assumption that production of tasks takes place under conditions of national scale economies which are external to the firm. Because of the external scale economies, the fewest possible varieties of tasks are produced which is consistent with the equilibrium. Because the scale economies are national in nature, each task is produced in only one country in the integrated equilibrium, and this pins down the volume of trade. In this section we discuss the implications of making alternative assumptions for the production of tasks, whilst retaining the basic structure of the model[10].

[10] It is of course possible to make other assumptions about the production of tasks, for instance that it depends on comparative advantage. We leave such extensions to future work.

Perhaps the most natural alternative assumption to make on the production of tasks is to assume constant returns to scale. That is, let the output of each task be linearly related to the amount of labour used in its production:

$$q_{ij} = \alpha l_{ij}. \tag{20}$$

This of course is equivalent to setting $\gamma = 1$ in equation (4). Careful examination of the results in the previous sections will show that, apart from simplifying the expressions somewhat, all the main results remain valid.

However, the mechanism by which the model operates – that international trade allows for the production of each task to be concentrated and hence leads to gains from reduced coordination costs – does not operate in this case. Under constant returns to scale, with identical technologies across countries, the location of production of tasks does not matter. But if each task is produced in both countries, the duplication of coordination costs cannot be avoided. As a result, there would be no gains from trade! To generate gains from trade, what would be required is an additional assumption, that with international trade, each task is produced in only one country. This is satisfied by the assumption made in previous sections that tasks exhibit external scale economies, provided we focus on the case of the efficient, integrated equilibrium.

The other possible assumption to make about task production is that it takes place under *international* as opposed to *national* scale economies. Such an assumption may be justified on the basis of improved communications and transportation across countries (Ethier 1979). Thus, instead of equation (4), the output of a task j is now a function of the total labour used anywhere in the world in that task (note the omission of the country subscript i):

$$q_j = (\alpha l_j)^{\gamma}. \tag{21}$$

Under this assumption, once again the location of production of each task does not matter; a firm can produce an task anywhere in the world and still benefit from the international scale economies. Therefore, this leads to the same problem as faced by assuming constant returns to scale – that because production of tasks may be dispersed, the cost saving of removing the duplication of the production network does not materialise. Thus we can conclude that, for the fundamental mechanism of the model to work, external scale economies which are national in nature are essential.

4 Two factors of production and the distribution of income

In the previous sections the model was set up with only one factor of production, and one final good. This made the mechanism underlying the model more transparent, but at the same time limits the scope of the model. In this section we extend the model to introduce two different final goods and two factors of production, and explore the implications for trade and the gains from trade. Alternatively, the model in this section may be viewed as a special case of the factor endowments model of international trade, extended to include the division of labour introduced in the previous sections[11]. In the interests of simplicity, we focus on a special case of the model in which each final good is produced using a different set of tasks, which in turn are produced using only one of the two factors of production. The basic structure of the model bears close similarity to the analysis of the model of monopolistic competition with different factors of production in Krugman (1981).

Now there are two homogeneous final goods, 1 and 2. Suppose that utility takes the following Cobb-Douglas form:

$$U = C_1^{\theta} C_2^{1-\theta}, \qquad 0 < \theta < 1. \tag{22}$$

There are two types of labour, 1 and 2. Final good 1 is produced using tasks which are produced using type 1 labour, while final good 2 is produced using tasks which are produced using type 2 labour. The two sectors are assumed to share the same production technologies in both task and final goods stages. As a result of this structure, the production side of both final goods are decoupled from each other, and in each sector the autarkic and trading equilibria remain as in Sections 2 and 3 above[12].

To focus attention on the implications of relative factor endowments, we follow Krugman (1981) and let each country be endowed with the following amounts of the two types of labour:

$$L_{1H} = L_{2F} = 2 - z, \qquad L_{2H} = L_{1F} = z, \qquad 0 < z \leq 1. \tag{23}$$

Hence each country has a total of 2 units of labour, the two countries are symmetric in terms of their relative endowments, and the world has 2 units of each of the two

[11] A factor-endowments model of international trade with trade in intermediate but not final goods can be shown to be isomorphic to the standard model in which all trade is in final goods. See Jones and Neary (1984) for details.

[12] Allowing for final goods to use a combination of the two types of tasks, or for tasks to use both types of labour, would enrich the model and may yield additional insights, but would also add considerable complexity. For instance, in the present formulation, there is no Rybczynski-type magnification effect of an increase in the endowment of one type of labour. However, we believe that the results for the distribution of the gains from trade discussed below are robust to this extension, and leave this extension to future work.

types of labour. That $z \leq 1$ implies that Home is relatively abundant in type 1 labour; the larger is z, the more similar the relative endowments of the two countries.

The interaction between the two final goods occurs on the demand side. From the consumer's maximisation problem, we have:

$$\frac{P_1}{P_2} = \frac{\theta}{1-\theta}\frac{C_2}{C_1}. \tag{24}$$

Making use of this and the market clearing condition shows that expenditure on each good is a constant fraction of total expenditure in the economy:

$$P_1 C_1 = \theta(P_1 C_1 + P_2 C_2), \qquad P_2 C_2 = (1-\theta)(P_1 C_1 + P_2 C_2). \tag{25}$$

Since each final good is produced using only one type of labour, the expenditure share of each final good is also the income share of the labour used in producing that good. In autarky, the representative consumer's utility in the Home country is, by analogy to equation (13):

$$U_H^A = \left(\frac{1}{2}\right)\left[n_{1H}^{\frac{1-\gamma\sigma}{\sigma}}\left((2-z)\alpha\right)^{\gamma} - \psi n_{1H}^{\rho}\right]^{\theta}\left[n_{2H}^{\frac{1-\gamma\sigma}{\sigma}}(z\alpha)^{\gamma} - \psi n_{2H}^{\rho}\right]^{1-\theta}. \tag{26}$$

And therefore the utility of each type of worker is proportional to the share of national income earned by that type of worker, divided by the share of that type of worker in Home's total labour force:

$$U_{1H}^A = \left(\frac{2\theta}{2-z}\right)U_H^A, \qquad U_{2H}^A = \left[\frac{2(1-\theta)}{z}\right]U_H^A. \tag{27}$$

For example, if $\theta = 0.5$, then since we assume $z \leq 1$, in the Home country, the relatively abundant type 1 workers have lower utility than the relatively scarce type 2 workers.

4.1 The gains from trade and the distribution of income

In free trade, since the world has identical endowments of the two types of labour, world market clearing implies:

$$\frac{P_1}{P_2} = \frac{\theta}{1-\theta}\frac{L_{2H}+L_{2F}}{L_{1H}+L_{1F}} = \frac{\theta}{1-\theta}. \tag{28}$$

Since each final good uses only one type of labour, factor price equalisation (FPE) is always achieved, since the FPE set spans the entire endowment space. Hence, $w_{1H} = w_{1F} = w_1$ and $w_{2H} = w_{2F} = w_2$. Given identical technologies for producing the final goods from tasks, and for producing tasks from each type of labour, we have, from equation (28):

$$\frac{w_1}{w_2} = \frac{p_{ij1}}{p_{ij2}} = \frac{P_1}{P_2} = \frac{\theta}{1-\theta}. \tag{29}$$

Hence relative wages depend only on the share of each final good in consumer expenditure. National income in the two countries is the sum of labour income, which, substituting from (29) and setting $w_2 = 1$ gives:

$$Y_H = w_1 L_{1H} + w_2 L_{2H} = \frac{\theta(2-z)+(1-\theta)z}{1-\theta}. \tag{30}$$

$$Y_F = w_1 L_{1F} + w_2 L_{2F} = \frac{\theta z+(1-\theta)(2-z)}{1-\theta}. \tag{31}$$

Since preferences are homothetic and prices are equalised under free trade, each country consumes the same proportion of the two goods, and the proportion is determined by the share of national income in world income. Hence, Home's consumer's utility under free trade is:

$$U_H^{FT} = \left[\frac{Y_H}{2(Y_H+Y_F)}\right]\left[n_{1T}^{\frac{1-\gamma\sigma}{\sigma}}(2\alpha)^{\gamma} - \psi n_{1T}^{\rho}\right]^{\theta}\left[n_{2T}^{\frac{1-\gamma\sigma}{\sigma}}(2\alpha)^{\gamma} - \psi n_{2T}^{\rho}\right]^{1-\theta}. \tag{32}$$

Each type of labour still gets a fraction of world income proportional to θ, but each country's share of this income is now proportional to the country's share of that type of labour, so utility of both types of labour in Home under free trade is:

$$U_{1H}^{FT} = \frac{\theta(2-z)}{2}\left[\frac{2(Y_H+Y_F)}{Y_H}\right]U_H^{FT} = \frac{2\theta(2-z)}{\theta(2-z)+(1-\theta)z}U_H^{FT}. \tag{33}$$

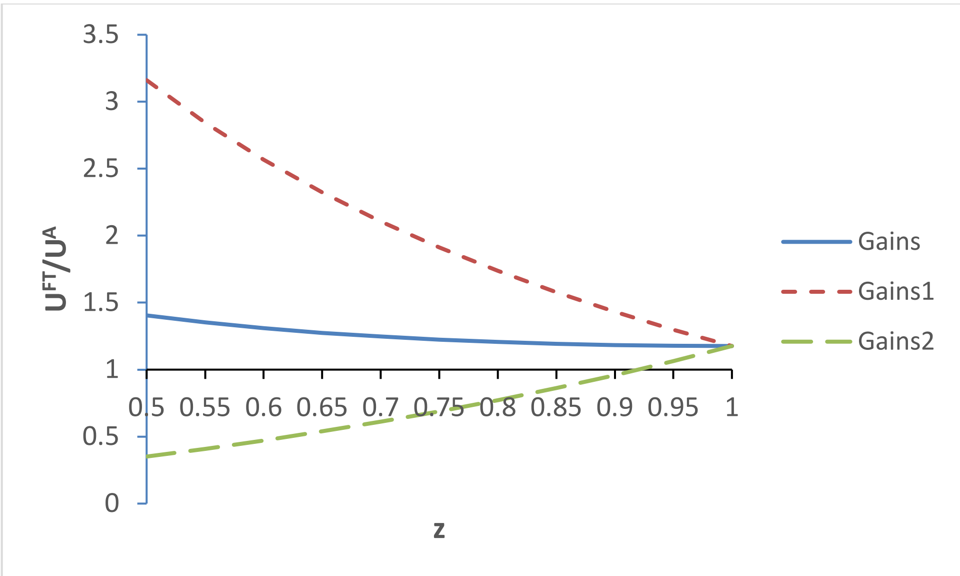
$$U_{2H}^{FT} = \frac{(1-\theta)z}{2}\left[\frac{2(Y_H+Y_F)}{Y_H}\right]U_H^{FT} = \frac{2(1-\theta)z}{\theta(2-z)+(1-\theta)z}U_H^{FT}. \tag{34}$$

The ratio of autarkic utility (26) and (27) to free trade utility (32), (33) and (34) shows whether the country and each type of labour experiences gains from trade. From the discussion in Sections 2 and 3, international trade leads to overall gains: $U_H^{FT} > U_H^A$. Similarly, since $[2\theta(2-z)]/[\theta(2-z)+(1-\theta)z] > 2\theta/(2-z)$, we have $U_{H1}^{FT} > U_{H1}^A$; the relatively abundant type 1 workers always gain from trade, and by a larger proportion than the overall gains. This of course implies that the relatively scarce type 2 workers gain less than the overall gains (or may actually lose). From equations (27) and (34), the condition for type 2 workers to gain from trade is $z^2 > [\theta(2-z)+(1-\theta)z](U_H^A/U_H^{FT})$; all else equal, this is more likely, the larger is z. That is, and similarly to Krugman (1981), the relatively scarce type 2 workers are more likely to gain from trade, the more similar are the two countries' relative endowments.

Figure 2 shows how the gains from trade vary with the relative endowment parameter z. When $z = 1$, the two countries have identical relative endowments. In this case, the only source of the gains from trade is the division of labour. The Home country experiences overall gains from trade, and both types of labour experience the same gain. As z decreases, the Home country becomes relatively more abundant in type 1 labour. Type 1 labour experiences greater gains from trade, while type 2 labour's gains decrease, until after a certain point, it starts to experience losses from trade.

Also, from Figure 2, the Home country's overall gains from trade increase as relative endowments become more different between the two countries (i.e. as z becomes smaller). Now there are two sources of the gains from trade: the division of labour, and comparative advantage in the form of differences in relative factor endowments. It is possible to decompose the total gains from trade into the component derived from the division of labour (where $z = 1$), and the component derived from relative endowment differences. For example, in Figure 2, when $z = 0.5$, the gains from the division of labour are equal to 17 percent, while the gains from relative endowment differences are equal to 20 percent, for an overall gain of 40 percent.

Figure 2: The gains from trade for the Home country in a two-factor model.



Notes: Assumed parameter values: $\alpha = 0.8, \gamma = 1.2, \rho = 1.8, \psi = 0.5, \sigma = 0.8, \theta = 0.5$.

Hence, returning to the effect of international trade on the relatively scarce labour (type 2 in the Home country), when relative endowments are sufficiently similar, the gain from the division of labour more than offsets the loss from being the relatively scarce factor of production (this loss arising from the standard factor-endowments fact that the scarce factor is relatively less scarce in the free trade equilibrium, and thus experiences a fall in its real return). However, when relative endowments are sufficiently different, the decrease in the real return to the scarce factor as a result of international trade more than offsets the gain from the division of labour, leading to an overall loss for the scarce factor.

We can draw a parallel between the results we obtain here, with the productivity and relative-price effects of Grossman and Rossi-Hansberg (2008). Suppose, as they do, that there is trade in tasks in sector 1, but not in sector 2. Then, there is a productivity effect of this trade, as the increased division of labour results in cost savings in the traded sector 1, leading to higher wages in sector 1 relative to sector 2, even if goods and task prices are held constant. At the same time, trade in sector 1 tasks has an ambiguous effect on the relative price of final good 1, in the type-1-labour-abundant Home. On the one hand, trade leads to greater division of labour, pushing down the relative price of final good 1. On the other hand, because Home is type-1-labour-abundant, there will be an increase in the relative price of good 1 with international trade[13]. As with the labour supply effect (discussed in the Introduction), the direction of these other effects depends on the precise assumptions made in the model.

4.2 Trade patterns

As in section 3.1 above, there is no trade in final goods, since assembly of each final good does not depend on its location, hence may be assumed to be assembled locally to consumption. In each sector, each country produces a number of tasks which is proportional to its endowment of the labour used in that sector. Hence Home will produce a fraction $(2-z)/2$ of the total number of type 1 tasks, and a fraction $z/2$ of the total number of type 2 tasks. Since preferences are homothetic, each country demands a fraction of each task which is proportional to its share of world income. Hence the value of the Home country's exports and imports of the two types of tasks are:

$$Export_{1H} = \left(\frac{2-z}{2}\right)\theta Y_F \qquad Export_{2H} = \left(\frac{z}{2}\right)(1-\theta)Y_F \tag{35}$$

$$Import_{1H} = \left(\frac{z}{2}\right)\theta Y_H \qquad Import_{2H} = \left(\frac{2-z}{2}\right)(1-\theta)Y_H \tag{36}$$

Trade is balanced; total exports equal total imports. The total value of exports depends on relative endowments and consumer preferences:

$$TE = \left(\frac{1}{2}\right)[\theta(2-z) + z(1-\theta)]Y_F = \frac{[\theta(2-z)+z(1-\theta)][\theta z+(1-\theta)(2-z)]}{2(1-\theta)} \tag{37}$$

Note that, provided $\theta \neq 0.5$, $dTE/dz > 0$; the more similar are the two countries in their relative endowments, the larger will be the total volume of trade between them. Trade may be divided into the component which is inter-industry in nature (exporting

[13] In the type-1-labour-scarce Foreign, there is no ambiguity in the relative price effect: trade in sector 1 tasks leads to a fall in the relative price of final good 1, because of both the increased division of labour, and the increased relative abundance of final good 1.

type 1 tasks in exchange for type 2 tasks), and the component which is intra-industry in nature (simultaneously exporting and importing the same type of task). An index of intra-industry trade is given by the Grubel-Lloyd (1975) index, defined for each sector as:

$$GL = 1 - \frac{|Exports - Imports|}{Exports + Imports} \tag{38}$$

Larger values of this index imply greater intra-industry trade as a fraction of total trade. Substituting from equations (35) and (36), we get:

$$GL = \frac{2zY_H}{(2-z)Y_F + zY_H} = \frac{2z[\theta(2-z)+(1-\theta)z]}{(2-z)[\theta z+(1-\theta)(2-z)]+z[\theta(2-z)+(1-\theta)z]} \tag{39}$$

That is, the Grubel-Lloyd index of intra-industry trade depends on consumer preferences and relative endowments. It can be shown that $dGL/dz > 0$; the more similar are the two countries in their relative endowments, the greater the share of trade which is intra-industry in nature. Note that if $\theta = 0.5$ as in Krugman (1981), then we get exactly the same results as Krugman does: total exports will be equal to $0.5 \times Y_F$, and the Grubel-Lloyd index of intra-industry trade will be equal to z, the measure of similarity in relative endowments.

5 Conclusions

This paper develops a simple model of international trade based only on the division of labour; there is no comparative advantage or imperfect competition. Firms assemble final goods from tasks, and there are gains to having a larger variety of tasks. The extent of the division of labour is limited by the cost of coordinating tasks and the size of the market. International trade eliminates the duplication of coordination costs, resulting in an increased variety of tasks, greater division of labour, and hence to gains from trade. Extending the basic one-factor model to two factors of production, we obtain the additional result that, if relative endowments are sufficiently similar between the two countries, then both factors of production will benefit from trade. This is in contrast with the traditional factor endowments model, in which the scarce factor of production always loses from trade, and arises because, when the basic model is combined with the factor endowments model, there are now two sources of the gains from trade: from the division of labour, and from comparative advantage.

The model represents an alternative treatment to the issue of scale economies and the division of labour in international trade to the now-conventional monopolistic competition approach pioneered by Krugman (1979, 1980) among others. Some of the results we obtain are similar to those in Krugman (1979), and contrast with those in Krugman (1980): international trade leads to an increase in both the number of tasks, and the scale of production of each task. Similarly, when extending the basic model to two factors of production, we obtain results which are similar to those obtained in Krugman's (1981) extension of the monopolistic competition model to more than one factor of production. In the conventional approach, there are scale economies which are internal to the firm; as a result, there are only a limited number of firms in the market, and each firm is associated with a different variety of the good. There, the division of labour occurs *across* firms. In the current model, because we assume atomistic perfectly competitive firms, it is possible that these "firms" are in fact part of the same organisation, and therefore the division of labour may occur *within* firms. As in Grossman and Rossi-Hansberg (2008), we do not consider the organisational structure of the firm as in Antras (2003), but it is a potential avenue for future research.

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Appendix A: Further details of the production function for tasks

First we establish that the production function for a task given in equation (4) can be derived from the production function of each firm producing that task (see Panagariya, 1981). The production function for a firm k producing task j in country i depends on the total output of that task:

$$q_{ijk} = \alpha q_{ij}^{\delta} l_{ijk}, \qquad 0 < \delta < 1. \tag{A1}$$

That $\delta > 0$ implies that firm k's output is higher, the larger is total output of task j. Total output of task j is:

$$q_{ij} = \sum_k q_{ijk} = \alpha q_{ij}^{\delta} l_{ij} = \left(\alpha l_{ij}\right)^{\frac{1}{1-\delta}} = \left(\alpha l_{ij}\right)^{\gamma} \tag{A2}$$

Where $\gamma = 1/(1-\delta) > 1$.

Next, we solve for the prices of tasks. Under perfect competition, each firm employs labour so that the value marginal product of labour is equal to the wage rate. Differentiating equation (A1) with respect to l_{ijk} gives:

$$MPL_{ijk} = \frac{dq_{ijk}}{dl_{ijk}} = \alpha q_{ij}^{\delta} \tag{A3}$$

Hence, we have:

$$w = p_{ij} \alpha q_{ij}^{\delta} \tag{A4}$$

Setting the wage rate equal to unity, we can solve for the price of each task as:

$$p_{ij} = \left(\alpha q_{ij}^{\delta}\right)^{-1} = \left[\alpha^{\gamma} l_{ij}^{\gamma-1}\right]^{-1} \tag{A5}$$

Which is equation (5) in the text. These results hold in both autarky and international trade, with the only difference being that the labour used in each task, l_{ij}, differs between autarky and international trade. Note that equation (A5) also shows that, although each firm practices marginal cost pricing, at the industry level, average cost pricing is being practiced; average cost for the industry is (substituting from equation (A2)) $wl_{ij}/q_{ij} = \left[\alpha^{\gamma} l_{ij}^{\gamma-1}\right]^{-1}$.