Fault diagnosis in multi-machine power systems using the Derivative-free nonlinear Kalman Filter

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***Abstract*—In this paper a new approach to parametric change detection and failure diagnosis for interconnected power units is proposed. The method is based on a new nonlinear filtering scheme under the name Derivative-free nonlinear Kalman Filter and on statistical processing of the obtained state estimates, according to the properties of the χ^2 distribution. To apply this fault diagnosis method, first it is shown that the dynamic model of the distributed interconnected power generators is a differentially flat one. Next, by exploiting differential flatness properties a change of variables (diffeomorphism) is applied to the power system, which enables also to solve the associated state estimation (filtering) problem. Additionally, statistical processing is performed for the obtained residuals, that is for the differences between the state vector of the monitored power system and the state vector provided by the aforementioned filter when the latter makes use of a fault-free model. It is shown, that the suitably weighted square of the residuals' vector follows the χ^2 statistical distribution. This property allows to use confidence intervals and to define thresholds that demonstrate whether the distributed power system functions as its fault-free model or whether parametric changes have taken place in it and thus a fault indication should be given. It is also shown that the proposed statistical criterion enables fault isolation to be performed, that is to find out the specific power generators within the distributed power system which have exhibited a failure. The efficiency of the proposed filtering method for condition monitoring in distributed power systems is confirmed through simulation experiments.**

***Index Terms*—distributed power systems, multi-machine power systems, condition monitoring, fault diagnosis, Derivative-free nonlinear Kalman Filter, χ^2 statistical change detection test.**

I. INTRODUCTION

Energy needs grow in relentless manner worldwide. As new power generation units are installed and as distributed power generation sources get interconnected, the dynamics of the electric power generation, transmission and distribution grid becomes more complicated [1-3]. The monitoring of its condition becomes an elaborated task that can be accomplished only with the use of advanced fault diagnosis tools and methods [4-7]. To this end, in this article a new statistical fault diagnosis method is proposed for detecting and isolating failures in distributed and interconnected power generators. As it has been shown in several studies, by applying fault detection tests based on the χ^2 distribution it can be concluded if the structure remains healthy and if the nominal parameter values for its model still hold. Otherwise, a failure can be detected [8-10]. Moreover by applying the χ^2 tests in subsections of the monitored system, the faulty components of it can be also isolated [11-12].

The proposed fault diagnosis method makes use of the differential flatness properties of the distributed power generation system, that is of the ability to express its dynamics in compact form through a key subset of its state variables, named as flat output of the system [13-16]. Actually, differential flatness theory enables to perform a global linearization on the dynamic model of the monitored system and to transform it to the so-called canonical form [14-17]. By proving that the model of the distributed power generators is a differentially flat one, the solution of the associated filtering (state estimation problem) becomes possible, using a new nonlinear filtering method known as Derivative-free nonlinear Kalman Filter. The method consists of (i) a nonlinear transformation that enables to rewrite the system's dynamics into the canonical (Brunovsky) form, (ii) application of the Kalman Filter recursion on the linearized equivalent model, (iii) an inverse transformation, based again on differential flatness theory that allows to obtain estimates of the state variables of the initial nonlinear model [18-19]. Using a dynamic model of the fault-free power system (that is a model that retains the nominal values of the generators' parameters), the filter provides finally estimates of state vector elements of the distributed power units which cannot be directly measured.

The dynamic behavior of the distributed power generators is recorded through suitable sensors (in the form of a sensors network deployed at specific measurement points) and is compared against the response generated by the aforementioned Kalman Filter under the assumption of a damage-free model. By comparing the two signals, residuals sequences are generated. The processing of the residuals with the use of statistical decision making criteria provides an indication about the existence of parametric changes (damages) in the power system, which otherwise could not have been detected. It is shown, that the suitably weighted square of the residuals' vector follows the χ^2 statistical distribution [20-22]. This property allows to use confidence intervals and to define thresholds that demonstrate whether the distributed power system functions as its fault-free model or whether parametric changes have taken place in it and thus a fault indication should be given [23-24]. It is also shown that the proposed statistical criterion enables fault isolation to be performed, that is to find out the specific power generators within the distributed power system which have exhibited a failure.

II. Dynamics of the interconnected power generators

A multi-machine power system with n machines (Fig. 1), in which the first machine is chosen as the reference machine can be described by the following nonlinear dynamic model

$$\begin{array}{c}\dot{\delta}_i=\omega_i-\omega_0 \\ \dot{\omega}_i=-\frac{D_i}{2J_i}(\omega_i-\omega_0)+\omega_0\frac{P_{m_i}}{2J_i}- \\ -\omega_0\frac{1}{2J_i}[G_{ii}{E_{qi}^{'}}^2+E_{qi}^{'}\sum_{j=1,j\neq i}^{n}E_{qj}^{'}G_{ij}sin(\delta_i-\delta_j-\alpha_{ij})] \\ \dot{E}_{q_i}^{'}=-\frac{1}{T_{d_i}^{'}}E_{q_i}^{'}+\frac{1}{T_{d_{oi}}}\frac{x_{d_i}-x_{d_i}^{'}}{x_{d_{\Sigma i}}^{'}}V_{s_i}cos(\Delta\delta_i)+\frac{1}{T_{d_{oi}}}E_{f\,i}\end{array} \tag{1}$$

In this model δ_i is the turn angle of the i-th generator's rotor, ω_i is the rotation speed of the i-th rotor with respect to synchronous reference frame, ω_0 is the synchronous speed of the generator, J_i is the moment of inertia of the i-th rotor, P_{e_i} is the active power of the i-th generator, P_{m_i} is the mechanical input torque to the i-th generator which is associated with the mechanical input power, D_i is the damping constant of the i-th generator, T_{e_i} is the electrical torque which is associated to the generated active power and G_{ij} are coefficients denoting coupling (power exchange) between the i-th and the j-th generator. Variable $E_{q_i}^{'}$ is the quadrature-axis transient voltage of the i-th generator (actually expressing magnetic flux),and E_{f_i} is the field's exhitation voltage. Moreover, the following variables are defined: $\Delta\delta_i=\delta_i-\delta_0$ and $\Delta\omega_i=\omega_i-\omega_0$ with ω_0 denoting the synchronous speed. Additionally, the electric torque P_{e_i} which is associated with the active power at the i-th generator is now given by

$$P_{e_i}=G_{ii}{E_{qi}^{'}}^2+E_{qi}^{'}\sum_{j=1,j\neq i}^{n}E_{qj}^{'}G_{ij}sin(\delta_i-\delta_j-\alpha_{ij}) \tag{2}$$

for $i=1,2\cdots,n$. For a power grid that consists of n generators the aggregate state vector comprises the state vectors of the local machines, i.e. $x=[x^1,x^2,\cdots,x^n]^T$, where $x^i=[x_1^i,x_2^i,x_3^i]^T$, with $x_1^i=\Delta\delta_i$, $x_2^i=\Delta\omega_i$ and $x_3^i=E_{qi}^{'}$ are the state variables for the $i-th$ machine and $i=1,2,\cdots,n$.

III. Differential flatness of the distributed synchronous generators' model

A. Differential flatness of the distributed power generators

It will be proven that the multi-machine power generation system is also a differentially flat one. As flat output of the

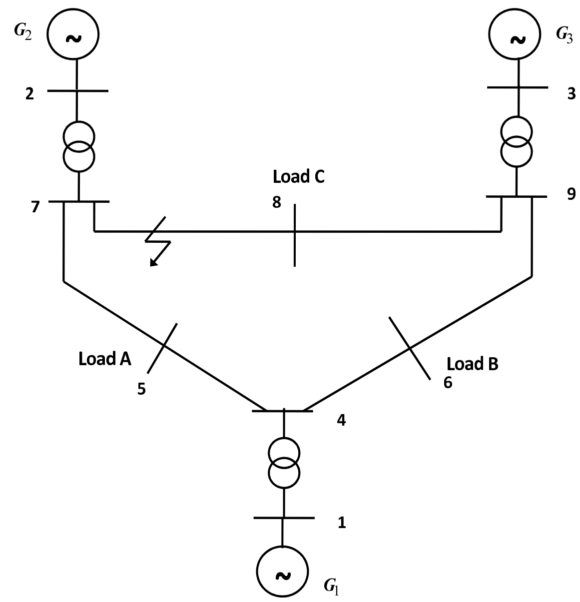


Fig. 1. A multi-machine (3-area) distributed power generation model

distributed power generation system, consisting of n PMSGs, the following vector is defined $y=[y_1^1,y_1^2,\cdots,y_1^n]$ or $y=\Delta\delta^1,\Delta\delta^2,\cdots,\Delta\delta^n$. For the n-machines power generation system it holds $x_1^1=y^1$, $x_1^2=y^2$, $x_1^3=y^3$, $\cdots$, $x_1^n=y^n$ and $x_2^1=\Delta\omega^1=\dot{y}^1$, $x_2^2=\Delta\omega^2=\dot{y}^2$, $x_2^3=\Delta\omega^3=\dot{y}^3$, $\cdots$, $x_2^n=\Delta\omega^n=\dot{y}^n$. Moreover, it holds

$$\begin{array}{c}\dot{x}_2^i=-\frac{D_i}{2J_i}x_2^i+\frac{\omega_0}{2J_i}P_{mi}- \\ -\frac{\omega_0}{2J_i}[G_{ii}{x_3^i}^2+x_3^i\sum_{j=1,j\neq i}^{n}[x_3^jG_{ij}sin(x_1^i-x_1^j-\alpha_{ij})]\end{array} \tag{3}$$

or, using the flat output variables

$$\begin{array}{c}\ddot{y}^i=-\frac{D_i}{2J_i}\dot{y}^i+\frac{\omega_0}{2J_i}P_{mi}- \\ -\frac{\omega_0}{2J_i}[G_{ii}{x_3^i}^2+x_3^i\sum_{j=1,j\neq i}^{n}[x_3^jG_{ij}sin(y^i-y^j-\alpha_{ij})]\end{array} \tag{4}$$

The external mechanical torque P_{mi} is considered to be a piecewise constant variable. For $i=1,2,\cdots,n$ one obtains n equations of the form of Eq. (4), with unknowns the state variables x_3^i, $i=1,2,\cdots,n$. By solving this system of equations with respect to x_3^i, $i=1,2,\cdots,n$ one arrives at defining the state variables x_3^i as functions of the elements of the flat outputs vector y^i, $i=1,2,\cdots,n$ and of their derivatives. Thus one has $x_3^i=f_{x_3}(y^1,y^2,\cdots,y^n)$. Additionally, from the relation $\dot{E}_{q_i}=-\frac{1}{T_{d_i}}E_{q_i}^{'}+\frac{1}{T_{d_{oi}}}\frac{x_{d_i}-x_{d_i}^{'}}{x_{d_{\Sigma_i}}}V_{s_i}cos(\Delta\delta_i)+\frac{1}{T_{d_{oi}}}E_{f_i}$ and knowing that the state variables x_1^i, x_2^i, x_3^i, $i=1,2,\cdots,n$ can be written as functions of the flat output and its derivatives, one can solve with respect to the control input u_i thus showing that all control inputs u_i, $i=1,2,\cdots,n$ can be written as function of the flat output and its derivatives.

B. Linearized model of the multi-generator system

By deriving the expression about $\ddot{y}^i$ once more with respect to time one obtains

$$y^{(3)^i} = a^i(x) + b_1^i(x)g_1u_1 + b_2^i(x)g_2u_2 + b_3^i(x)g_3u_3 \tag{5}$$

By defining $z_1^i = y$, $z_2^i = \dot{y}$ and $z_3^i = \ddot{y}$ and by considering additive disturbances one arrives at a description of the form $\dot{z}_3^i = a^i(x) + b_1^i(x)g_1u_1 + b_2^i(x)g_2u_2 + b_3^i(x)g_3u_3 + \tilde{d}^i$, where for a power generation with $n = 3$ machines, and considering for instance $i = 1,\ j = 2, 3$ one has

$a^i = (\frac{D_i}{2J_i})^2 x_2^i + \frac{D_i\omega_0}{(2J_i)^2}[G_{ii}{x_3^i}^2 + x_3^i\sum_{j=1,j\neq i}^n x_3^j G_{ij} sin(x_1^i - x_1^j - \alpha_{ij})] - \frac{\omega_0}{2J_i}[G_{ii}x_3^i + \sum_{j=1,j\neq i}^n x_3^j G_{ij} sin(x_1^i - x_1^j - \alpha_{ij})(-\frac{1}{T'_{d_i}}x_3^i + (\frac{1}{T_{doi}}\frac{x_{d_i} - x'_{d_i}}{x_{d\Sigma_i}'}V_{s_i}cos(x_1^i))] - \frac{\omega_0}{2J_i}x_3^i\sum_{j=1,j\neq i}^n G_{ij} sin(x_1^i - x_1^j - \alpha_{ij})(-\frac{1}{T'_{d_i}}x_3^i + (\frac{1}{T_{doi}}\frac{x_{d_i} - x'_{d_i}}{x_{d\Sigma_i}'}V_{s_i}cos(x_1^i)) - \frac{\omega_0}{2J_i}x_3^i\sum_{j=1,j\neq i}^n x_3^j G_{ij} cos(x_1^i - x_1^j - \alpha_{ij})x_2^i \frac{\omega_0}{2J_i}x_3^i\sum_{j=1,j\neq i}^n x_3^j G_{ij} cos(x_1^i - x_1^j - \alpha_{ij})x_2^j$

while for functions b_1^i, b_2^i and b_3^i it holds $b_1^i = -\frac{\omega_0}{2J_i}[2G_{ii}x_3^i + \sum_{j=1,j\neq i}^n x_3^j G_{ij} sin(x_1^i - x_1^j - \alpha_{ij})]\frac{1}{T_{doi}}$, $b_2^i = -\frac{\omega_0}{2J_i}G_{i2} sin(x_1^i - x_1^2 - \alpha_{i2})\frac{1}{T_{do2}}$ and $b_3^i = -\frac{\omega_0}{2J_i}G_{i3} sin(x_1^i - x_1^3 - \alpha_{i3})\frac{1}{T_{do3}}$.

Finally for the additional input term one has $\tilde{d}^i = -\frac{D_i\omega_0}{2{J_i}^2}P_m^i + \frac{\omega_0}{2J_i}\dot{P}_m^i$. In this term $\tilde{d}^i$ one can one also include external disturbance inputs which are exerted on each generator's model and which stand for faults. Thus, one has the following description of the dynamics of the i-th power generator

$$\begin{aligned} \dot{z}_1^i &= z_2^i \\ \dot{z}_2^i &= z_3^i \\ \dot{z}_3^i &= a^i(x) + {b_1}^i g_1u_1 + {b_2}^i g_2u_2 + {b_3}^i g_3u_3 + \tilde{d}^i \end{aligned} \tag{6}$$

For the complete system of the 3 generators one has

$$\begin{aligned} \dot{z}_3^1 &= a^1(x) + {b_1}^1 g_1u_1 + {b_2}^1 g_2u_2 + {b_3}^1 g_3u_3 + \tilde{d}^1 \\ \dot{z}_3^2 &= a^2(x) + {b_1}^2 g_1u_1 + {b_2}^2 g_2u_2 + {b_3}^2 g_3u_3 + \tilde{d}^2 \\ \dot{z}_3^3 &= a^3(x) + {b_1}^3 g_1u_1 + {b_2}^3 g_2u_2 + {b_3}^3 g_3u_3 + \tilde{d}^3 \end{aligned} \tag{7}$$

or in matrix form

$$\dot{z}_3 = f_a(x) + Mu + \tilde{d} \tag{8}$$

where $z_3 = [z_3^1, z_3^2, z_3^3]^T$, $u = [u_1, u_2, u_3]^T$ and $\tilde{d} = [\tilde{d}_1, \tilde{d}_2, \tilde{d}_3]^T$ while

$$f_a(x) = \begin{pmatrix} a^1(x) \\ a^2(x) \\ a^3(x) \end{pmatrix}, \quad M = \begin{pmatrix} {b_1}^1 g_1 & {b_2}^1 g_2 & {b_3}^1 g_3 \\ {b_1}^2 g_1 & {b_2}^2 g_2 & {b_3}^2 g_3 \\ {b_1}^3 g_1 & {b_2}^3 g_2 & {b_3}^3 g_3 \end{pmatrix} \tag{9}$$

Setting $v = f_a(x) + Mu + \tilde{d}$, one obtains again the linear canonical form for the i-th generator given by

$$\begin{pmatrix} \dot{z}_1^i \\ \dot{z}_2^i \\ \dot{z}_3^i \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} z_1^i \\ z_2^i \\ z_3^i \end{pmatrix} + \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} (v^i + \tilde{d}^i) \tag{10}$$

In this manner the initial nonlinear power system is transformed into three decoupled linear subsystems which are in the canonical Brunovksy form. For each one of these subsystems the appropriate control law is

$$v^i = z_d^{(3)^i} - k_3(\ddot{z}^i - \ddot{z}_d^i) - k_2(\dot{z}^i - \dot{z}_d^i) - k_1(z^i - z_d^i) - \tilde{d}^i \tag{11}$$

IV. Statistical fault detection

A. Fault detection

For the linearized equivalent model of the power system that was described above Kalman Filtering is applied. This is known as Derivative-free nonlinear Kalman Filter because it solves the problem of nonlinear state estimation without the need to compute Jacobian matrices and partial derivatives [19]. The residuals' sequence, that is the differences between the real output of the monitored multi-machine power system and the one estimated by the Kalman Filter (Fig. 2) is a discrete error process e_k with dimension $m \times 1$ (here $m = N$). Actually, it is a zero-mean Gaussian white-noise process with covariance given by E_k. A conclusion can be stated based on a measure of certainty that the parameters of the dynamic model of the multi-machine power system remain unchanged. To this end, the following *normalized error square* (NES) is defined [19]

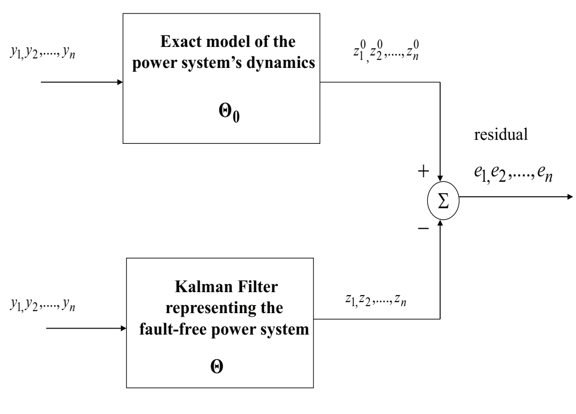


Fig. 2. Residuals' generation for the distributed power system, with the use of the Kalman Filtering

$$\epsilon_k = e_k^T E_k^{-1} e_k \tag{12}$$

The normalized error square follows a χ^2 distribution. An appropriate test for the normalized error sum is to numerically show that the following condition is met within a level of confidence (according to the properties of the χ^2 distribution)

$$E\{\epsilon_k\} = m \tag{13}$$

This can be succeeded using statistical hypothesis tests, which are associated with confidence intervals. A 95% confidence interval is frequently applied, which is specified using $100(1-a)$ with $a = 0.05$. Actually, a two-sided probability region is considered cutting-off two end tails of 2.5% each. For M runs the normalized error square that is obtained is given by

$$\bar{\epsilon}_k = \frac{1}{M}\sum_{i=1}^{M}\epsilon_k(i) = \frac{1}{M}\sum_{i=1}^{M}e_k^T(i)E_k^{-1}(i)e_k(i) \tag{14}$$

where ϵ_i stands for the i-th run at time t_k. Then $M\bar{\epsilon}_k$ will follow a χ^2 density with Mm degrees of freedom. This condition can be checked using a χ^2 test. The hypothesis holds, if the condition $\bar{\epsilon}_k \in [\zeta_1, \zeta_2]$ is satisfied, where ζ_1 and ζ_2 are derived from the tail probabilities of the χ^2 density. For example, for $m = 20$ and $M = 100$ one has $\chi^2_{Mm}(0.025) = 1878$ and $\chi^2_{Mm}(0.975) = 2126$. Using that $M = 100$ one obtains $\zeta_1 = \chi^2_{Mm}(0.025)/M = 18.78$ and $\zeta_2 = \chi^2_{Mm}(0.975)/M = 21.26$.

B. Fault isolation

By applying the statistical test into the n individual generators of the multi-machine power system it is also possible to find out the specific generators within the distributed power generation model that has been subjected to a fault [19],[24]. In the case of a single fault one has to carry out n χ^2 statistical change detection tests, where each test is applied to the subset that comprises generators $i-1$, i and $i+1$, $i = 1, 2, \cdots, n$. Actually, out of the n χ^2 statistical change detection tests, the one that exhibit the highest score (or equivalently indicates the largest parameter deviation from the nominal value) are those that identify the generator that has been subjected to failure (the faulty components for this generator are the parameters of its mechanical or electrical model). In the case of multiple faults one can identify the subset of generators that have been subjected to parametric change by applying the χ^2 statistical change detection test according to a combinatorial sequence. This means that

$$\binom{n}{k} = \tfrac{n}{k!(n-k)!} \tag{15}$$

tests have to take place, for all generators' clusters in the monitored power system, that finally comprise n, $n-1$, $n-2$, $\cdots$, 2, 1 generators. Again the χ^2 tests that give the highest scores indicate the generators which are most likely to have been subjected to damage.

V. Disturbances estimation with the Derivative-free nonlinear Kalman Filter

Kalman Filtering applied on the previously described linearized equivalent model of the distributed power system, is known as Derivative-free nonlinear Kalman Filter [18-19]. This form of the Kalman Filter is not only a method for performing fault diagnosis in the distributed power system, but is also a tool for estimating the perturbation terms that affect this power system. It is considered that the multi-machine power system's dynamics is affected by additive input disturbances:

$$\begin{array}{l}
\dot{x}_{1,1} = x_{1,2} \quad \dot{x}_{1,2} = x_{1,3} \quad \dot{x}_{1,3} = v_1 + \tilde{d}_1 \\
\dot{x}_{2,1} = x_{2,2} \quad \dot{x}_{2,2} = x_{2,3} \quad \dot{x}_{2,3} = v_2 + \tilde{d}_2 \\
\cdots \quad \cdots \\
\dot{x}_{i,1} = x_{i,2} \quad \dot{x}_{i,2} = x_{i,3} \quad \dot{x}_{i,3} = v_i + \tilde{d}_i \\
\dot{x}_{i+1,1} = x_{i+1,2} \quad \dot{x}_{i+1,2} = x_{i+1,3} \quad \dot{x}_{i+1,3} = v_{i+1} + \tilde{d}_{i+1} \\
\cdots \quad \cdots \\
\dot{x}_{n-1,1} = x_{n-1,2} \quad \dot{x}_{n-1,2} = x_{n-1,3} \quad \dot{x}_{n-1,3} = v_{n-1} + \tilde{d}_{n-1} \\
\dot{x}_{n,1} = x_{n,2} \quad \dot{x}_{n,2} = x_{n,3} \quad \dot{x}_{n,3} = v_n + \tilde{d}_n
\end{array} \tag{16}$$

It is considered that the dynamics of each perturbation term is described by its n-th order derivative, that is $\tilde{d}^{(n)} = f_d(t)$, and of the associated initial conditions. However, the reconstruction of the signals $\tilde{d}_i$ $i = 1, \cdots, n$ will be performed with the use of Kalman Filtering, and the convergence of the latter estimation method does not depend on initial conditions. Therefore, initial conditions are finally unnecessary for estimating the disturbance terms' evolution in time. According to the above and without loss of generality it is assumed that $\tilde{d}^{(n)} = f_d(t)$ with $n = 3$. Next, the state vector of the system is extended by introducing as additional state variables the disturbance terms and their derivatives

$$\begin{array}{l}
z_{1,1} = x_{1,1} \quad z_{1,2} = x_{1,2} \quad z_{1,3} = x_{1,3} \\
z_{2,1} = x_{2,1} \quad z_{2,2} = x_{2,2} \quad z_{2,3} = x_{2,3} \\
\cdots \quad \cdots \quad \cdots \quad \cdots \\
z_{i,1} = x_{i,1} \quad z_{i,2} = x_{i,2} \quad z_{i,3} = x_{i,2} \\
z_{i+1,1} = x_{i+1,1} \quad z_{i+1,2} = x_{i+1,2} \quad z_{i+1,3} = x_{i+1,3} \\
\cdots \quad \cdots \quad \cdots \quad \cdots \\
z_{n-1,1} = x_{n-1,1} \quad z_{n-1,2} = x_{n-1,2} \quad z_{n-1,3} = x_{n-1,3} \\
z_{n,1} = x_{n,1} \quad z_{n,2} = x_{n,2} \quad z_{n,3} = x_{n,3} \\
\\
z_{n+1,1} = \tilde{d}_1 \quad z_{n+1,2} = \dot{\tilde{d}}_1 \quad z_{n+1,3} = \ddot{\tilde{d}}_1 \\
z_{n+2,1} = \tilde{d}_2 \quad z_{n+2,2} = \dot{\tilde{d}}_2 \quad z_{n+2,3} = \ddot{\tilde{d}}_2 \\
\cdots \quad \cdots \quad \cdots \quad \cdots \\
z_{n+i+1,1} = \tilde{d}_i \quad z_{n+i+1,2} = \dot{\tilde{d}}_i \quad z_{n+i+1,3} = \ddot{\tilde{d}}_i \\
z_{n+i+2,1} = \tilde{d}_{i+1} \quad z_{n+i+2,2} = \dot{\tilde{d}}_{i+1} \quad z_{n+i+2,3} = \ddot{\tilde{d}}_{i+1} \\
\cdots \quad \cdots \quad \cdots \quad \cdots \\
z_{2n-1,1} = \tilde{d}_{n-1} \quad z_{2n-1,2} = \dot{\tilde{d}}_{n-1} \quad z_{2n-1,3} = \ddot{\tilde{d}}_{n-1} \\
z_{2n,1} = \tilde{d}_n \quad z_{2n,2} = \dot{\tilde{d}}_n \quad z_{2n,3} = \ddot{\tilde{d}}_n
\end{array} \tag{17}$$

For the extended state-space description of the system new matrices A, B and C are formulated, comprising a double number of rows comparing to its initial description. However, even in this extended state-space form the system remains observable. For example, in the case of a model of three interconnected power generators of Fig. 1, the extended state-space description of the system and the system's extended state vector are

$$\begin{aligned} z_e = \ & [z_{1,1}, z_{1,2}, z_{1,3}, z_{2,1}, z_{2,2}, z_{2,3}, z_{3,1}, z_{3,2}, z_{3,3}, \\ & [z_{4,1}, z_{4,2}, z_{4,3}, z_{5,1}, z_{5,2}, z_{5,3}, z_{6,1}, z_{6,2}, z_{6,3}]^T \end{aligned}$$

The measurable state variables are $z_{1,1}$, $z_{2,1}$ and $z_{3,1}$. By denoting the extended state vector as $z_e \in R^{18\times 1}$ (and after omitting the disturbance functions $f_{d,i},\ i = 1, \cdots, 3$ from the control vector) one has the linear state-space equation in the form

$$\begin{aligned} \dot{z}_e &= A_e z_e + B_e v \\ z_e^{meas} &= C_e z_e \end{aligned} \tag{18}$$

where

$$A_e = \begin{pmatrix}
0&1&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&1&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&1&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&1&0&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&1&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0
\end{pmatrix} \quad B_e = \begin{pmatrix}
0&0&0\\
0&0&0\\
1&0&0\\
0&0&0\\
0&0&0\\
0&1&0\\
0&0&0\\
0&0&0\\
0&0&1\\
0&0&0\\
0&0&0\\
0&0&0\\
0&0&0\\
0&0&0\\
0&0&0\\
0&0&0\\
0&0&0\\
0&0&0
\end{pmatrix} \tag{19}$$

$$C_e = \begin{pmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0
\end{pmatrix} \tag{20}$$

A state estimator for the system of Eq. (18) has the form

$$\begin{aligned} \dot{\hat{z}} &= A_e \hat{z} + B_e v + K(z^{meas} - \hat{z}^{meas}) \\ \hat{z}^{meas} &= C_e \hat{z} \end{aligned} \tag{21}$$

The computation of the estimator's gain K can be performed with the use of the Kalman Filter recursion, which consists of two stages: the *measurement update* and the *time update*

measurement update:

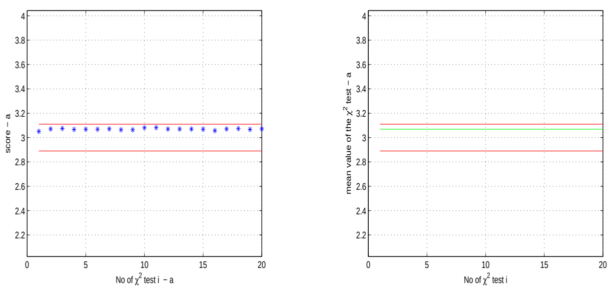
$$\begin{aligned} K &= P^- C_{e,d}^T [C_{e,d} P^- C_{e,d}^T + R]^{-1} \\ \hat{z}_e &= \hat{z}_e^- + K_f(z^{meas} - \hat{z}^{meas}) \\ P(k) &= P^-(k) - K(k) C_{e,d} P^-(k) \end{aligned} \tag{22}$$

time update:

$$\begin{aligned} P^-(k+1) &= A_{e,d} P(k) A_{e,d}^T + Q \\ \hat{z}(k+1) &= A_{e,d} \hat{z}_e(k) + B_{e,d} v(k) \end{aligned} \tag{23}$$

VI. Simulation tests

The performance of the proposed fault diagnosis scheme for distributed power generators was tested through simulation experiments. For the case of faults appearing in the individual power generators the obtained results are depicted in Fig. 3 to Fig. 5. It can be noticed that the proposed diagnosis test that is based on Kalman Filtering and on the statistical properties of the χ^2 distribution, achieved detection and isolation of failures taking place at the individual generators. It is noted that comparing to the RMSE (root mean sqaure error) index, the proposed fault diagnosis method which is based on the statistical properties of the χ^2 distribution is much more efficient. Actually, for the additive faults described above, the RMSE index was of the order of 10^{-5}. This RMSE indication misleads to the conclusion that the generators function properly, although faults have affected them.

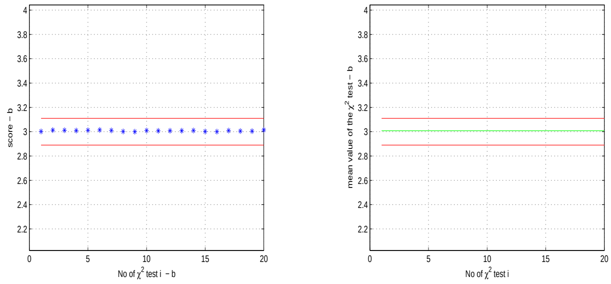


(a) **(b)**

Fig. 3. Fault at power Generator 3: (a) consecutive χ^2 tests performed at Generator 1, (b) the mean value (green line) of the χ^2 tests performed at Generator 1 remains within the thresholds (red lines) indicating healthy condition

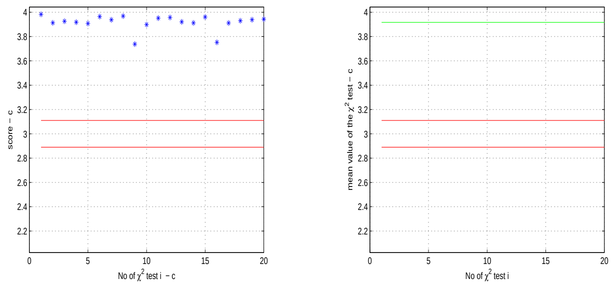
VII. Conclusions

A method for fault diagnosis in distributed and interconnected power generators has been developed. The method is based on Kalman Filtering and on the statistical properties of the χ^2 distribution. To apply this fault diagnosis method, first it was shown that the dynamic model of the distributed interconnected power generators is a differentially flat one. Next, by exploiting differential flatness properties a change of variables (diffeomorphism) was applied to the power system, which enabled also to solve the associated state estimation (filtering) problem. The new filtering technique consists of (i) a change variables (diffeomorphism) which results



(a) **(b)**

Fig. 4. Fault at power Generator 3: (a) consecutive χ^2 tests performed at Generator 3, (b) the mean value (green line) of the χ^2 tests performed at Generator 3 exceeds the thresholds (red lines) indicating healthy condition



(a) **(b)**

Fig. 5. Fault at power Generator 3: (a) consecutive χ^2 tests performed at Generator 3, (b) the mean value (green line) of the χ^2 tests performed at Generator 3 exceeds the thresholds (red lines) indicating healthy condition

into a linearized equivalent model for the power system, (ii) application of the Kalman Filter recursion, and (iii) an inverse transformation based again on differential flatness theory which permits to obtain state estimates for the initial nonlinear model.

Next, statistical processing was performed for the obtained residuals, that is for the differences between the state vector of the monitored power system and the state vector provided by the aforementioned filter when the latter makes use of a fault-free model. It was shown, that the suitably weighted square of the residuals' vector follows the χ^2 statistical distribution. This property allows to use confidence intervals and to define thresholds that demonstrate whether the distributed power system functions as its fault-free model or whether parametric changes have taken place in it and thus a fault indication should be given. It was also shown that the proposed statistical criterion enables fault isolation to be performed, that is to find out the specific power generators within the distributed power system which have exhibited a failure.

References

[1] G.G. Rigatos, P. Siano and A. Piccolo, A neural network-based approach for early detection of cascading events in electric power systems, IET Journal on Generation Transmission and Distribution, vol.3, no. 7, pp. 650-665, 2009.

[2] P. Kundur, *Power System Stability and Control*, New York: McGraw-Hill, 1994.

[3] J.C. Tan, P.A. Crossley, and P.G. McLaren, Application of a wide area backup protection expert system to prevent cascading outages, *IEEE Transactions on Power Delivery*, vol. 17, no.2, pp. 375-380, 2002.

[4] N. Kakimoto, M. Sugumi, T. Makino, and K. Tomiyama, Monitoring of inter-area oscillation mode by synchronized phasor measurement, *IEEE Transactions on Power Systems*, vol. 21, no. 1, pp. 260-268, 2006.

[5] N. Tleis, Power system modelling and fault analysis: Theory and practice, Elsevier, 2008.

[6] G. Rigatos, P. Siano and N. Zervos, Sensorless control of distributed power generators with the Derivative-free nonlinear Kalman Filter, IEEE Transactions on Industrial Electronics, vol. 61, no. 11, pp. 6369-6382, 2014

[7] N. Yadaiah and N. Venkata Ramara, Linearization of multi-machine power system: Modelling and control - A survey, *Electrical Power and Energy Systems*, Elsevier, vol. 29, pp. 297-311, 2007.

[8] M. Basseville and I. Nikiforov, Detection of Abrupt changes, Prentice Hall, 1993.

[9] G. Rigatos and Q. Zhang, Fuzzy model validation using the local statistical approach, Fuzzy Sets and Systems, Elsevier, vol. 60, no.7, pp. 882-904, 2009.

[10] M. Basseville, A. Benveniste and Q.Zhang, Towards the handling of uncertainties in statistical FDI, 18th IFAC World Congress Milano, Italy, 2011

[11] Q. Zhang and M. Basseville, Statistical fault detection and isolation for linear time-varying systems, in Proc. 16th IFAC Conference on Systems Identification, Brussels, Belgium, July 2012.

[12] G. Rigatos and P. Siano, Fuzzy Kalman Filter Validation Using the Local Statistical Approach, Journal of Intelligent Industrial Systems, Springer, vol. 1, no.1, pp. 15-28, 2015

[13] H. Sira-Ramirez and S. Agrawal, Differentially Flat Systems, Marcel Dekker, New York, 2004.

[14] J. Rudolph, Flatness Based Control of Distributed Parameter Systems, Examples and Computer Exercises from Various Technological Domains, Shaker Verlag, Aachen, 2003.

[15] J.Lévine, Analysis and Control of Nonlinear Systems: A flatness-based approach, Springer 2009.

[16] L. Menhour, B. d'Andréa-Novel, M. Fliess and H. Mounier, Coupled nonlinear vehicle control: Flatness-based setting with algebraic estimation techniques, Control Engineering Practice, Elsevier, vol. 22, pp. 135146, 2014

[17] S. Bououden, D. Boutat, G. Zheng, J.P. Barbot and F. Kratz, A triangular canonical form for a class of 0-flat nonlinear systems, International Journal of Control, Taylor and Francis, vol. 84, no. 2, pp. 261-269, 2011.

[18] G.G. Rigatos, Modelling and control for intelligent industrial systems: adaptive algorithms in robotcs and industrial engineering, Springer, 2011.

[19] G. Rigatos, Nonlinear control and filtering using differential flatness approaches: applications to electromechanicsl systems, Springer, 2015

[20] R.G. Gibbs, New Kalman filter and smoother consistency tests, Automatica, Elsevier, vol. 49, no. 10, pp. 3141-3144, 2013.

[21] A. Zolghadri, An Algorithm for Real-Time Failure Detection in Kalman Filters, IEEE Transactions on Automatic Control, vol. 41, no. 10, pp. 1537 - 1539, 1996.

[22] J.L. Crassidis and J.L. Junkins, Optimal estimation of dynamic systems (2nd Edition), CRC Press, 2012.

[23] J. Liu, D. Li, and Z. Xiong, Research on an improved residual Chi-square fault detection method for federated unscented Kalman filter, Chinese Journal of Scientific Instrument, vol. 30, no. 12, pp. 2568-2573, 2009.

[24] G. Rigatos and E. Soldatos, Structural condition monitoring with the use of the Derivative-free nonlinear Kalman Filter, In: Handbook on Research in Robotics and Mechatronics (M. Habib Editor), IGI Publications, 2014.