Published in *New Scientist* 9 January 1999, pp 36-39.

Glorious Noise

Random noise can be used to drive chemical reactions or stop a spacecraft spinning out of control – you just have to know how to tweak it. Peter McClintock and Dmitri Luchinsky tune in.

THERE'S no doubt it's had a bad press. For most of us, noise is what goes on next door when you're trying to sleep. It's that horrible hiss and crackle when an old recording ends or when you try to tune your short-wave radio into the BBC World Service when you're in the Brazilian jungle. But within that seemingly meaningless barrage of uncontrolled and usually disruptive rapid-fire shocks lies something far more interesting. If those shocks arrive in just the right sequence, interesting things happen. Without noise, ice would never grow on a lake in winter, nor would chemical reactions happen; noise drives our muscles to contract when they should and it helps our cells to pump crucial materials though their membranes. Life itself wouldn't exist without that messy stuff called noise.

But how does noise do its work? And if we understand it better, can we harness its power? After studying the problem for some years, we think we've found some answers. Unlike most forces, when noise makes significant things happen, it doesn't do so through a gradual accumulation of effects. Instead, noise does its work all at once in dramatic, exceptional events. To understand the workings of a noisy system, it seems, you can ignore most of what goes on, so long as you keep track of the rare events that really count.

Physicists' present fascination with noise stems from a discovery made in the 1980s. Suppose you send a weak periodic signal through a noisy black box, and look at the ratio of signal and noise strengths coming out – a measure of quality called the signal-to-noise ratio (SNR). For some black boxes, it turns out that adding noise at the input actually increases both the signal and the SNR at the output. Adding noise can, paradoxically, boost a signal's ability to get through the system ("Noises on", *New Scientist*, 1 June 1996, p 28).

Stochastic resonance, as this effect is called, has now been seen in black boxes ranging from the sensory neurons in crayfish tails to the microscopic ion channels that carry messages across cell membranes. In the case of crayfish, Frank Moss and his colleagues at the University of Missouri, St Louis, have shown that adding noisy, irregular currents in the water near the crayfish's tail increases its ability to detect regular, periodic fluid motions which might betray the presence of a predator.

How does this strange effect work? Imagine that the black box is a light switch, and the incoming signal is your finger, which tries to flick the switch on and off with a steady rhythm. The output signal is the light intensity of a lamp to which the switch is attached. If the signal of your finger is strong, then the output signal is also strong – the light flickers on and off in the same rhythm. But if you have a broken finger and can flick the switch only weakly, you might not be able to move it, in which case no signal will get through. Here is where noise can help.

Suppose that the switch is slightly noisy, and has a tendency to vibrate although not so vigorously that it would actually flip between on and off by itself. When your broken finger tries to flick this noisy switch, it will occasionally be reinforced by a fortuitous vibration acting in the same direction, and so push the switch over the hump (see Diagram). The noise increases the ability of your finger to make things happen so, at the lamp, there now appears a signal, not a perfectly regular signal, to be sure, but a signal nonetheless, which goes on and off in rough synchrony with your finger.

Breaking waves

Physicists now know that this two-state switching scenario is just one of many ways in which stochastic resonance can occur. In 1990, physicist Mark Dykman, then at the Institute for Semiconductors in Kiev,Ukraine, realised that stochastic resonance could be brought within the fold of traditional theoretical physics by using something called linear response theory. This provides a simple and general way of describing how a fluctuating system responds to a weak periodic driving force.Using Dykman's ideas, we showed that the conditions under which stochastic resonance arises should occur in just about every noisy nonlinear system you can imagine.

But stochastic resonance is just one of noise's odd effects. Another occurs in a device known as a stochastic ratchet. Suppose you have some particles trapped in a ratchet-shaped base – some gravel at rest in a strip of corrugated steel, for example, where the corrugations all lean in one direction like breaking waves. Here, it turns out that purely random jiggling of the base can cause the particles all to drift in just one direction. This seems to defy the second law of thermodynamics, since you can extract useful work from seemingly random noise. But it works.

The trick, as physicist Marcelo Magnasco of Rockefeller University in New York pointed out in 1993, is that the noise has to be "coloured". The archetype of noise – the kind considered originally by Einstein-is "white" noise, in which each little noisy shock is independent of its predecessors. White noise has no memory. In coloured noise, however, there is a kind of memory at work – a shock that pushes a bit of gravel to the right is more likely to be followed by another similar shock, rather than by one which pushes back to the left. Coloured noise is still random, and ultimately the bits of gravel get little kicks to the left and right in equal proportion. Yet the sequence can conspire with the ratchet to move the particles. The direction of the steady flow can even be changed without altering the ratchet merely by altering the colour of the noise – that is, by changing the pattern in which successive shocks tend to follow one another.

This mechanism probably lies at the root of how living cells move molecules around. They may also help engineers to build nanoscale motors, able to function in the micro-world where noise rules and conventional engineering techniques fail.

So physicists have come to understand several ways in which noise can be useful. But is there any way to get inside stochastic resonance, or the workings of a stochastic ratchet, and understand them in the same way that we understand, say, how a clock works, or a car? Think of the noisy switch. Starting from the off position, there are infinitely many possible paths that it could follow in executing a noisy transition from off to on. It could make the change quickly, or vibrate about in the off position for several minutes, and then suddenly hop over to on. Then again, it might just take a slow, gradual noisy walk over the hill between those states. The same goes for a piece of gravel moving from one dip in the corrugated steel to another.

Wait long enough, and you'll see all these possibilities play out. There is no "one way" that noise can make things happen. But we have found that in practice, almost all of those infinitely many possible motions turns out to be irrelevant to significant events – only a tiny subset really matters. We have also found that by focusing on these special paths, we can understand the workings of noise with just a few simple equations. The important thing, then, is to identify the few pathways that matter.

The ideas and theoretical results leading to this view of noise have accumulated over decades, stretching back to Einstein and Boltzmann. But in the intervening years, nobody could see how to test things experimentally. The breakthrough came in 1992, when we worked out with Dykman, now at Michigan State University in East Lansing, and Vadim Smelyanskiy, who works at NASA, how to measure and interpret a new physical quantity, the forbiddingly named prehistory probability density.

The idea is to take some noisy system and monitor its fluctuations, waiting for something rare and dramatic to happen – a flip of the switch, for example. When it does, you examine the immediate prehistory of the system and record exactly how it happened. It's rather like recording, every time you spill the milk, a detailed history of what led up to the event. Doing this over and over, thousands of time, you accumulate information on the histories that are most likely to lead to it. This is the prehistory probability density.

Large fluctuations

Our first experiment in 1992 used electronic circuits from which it is easy to record detailed histories and get good data. We didn't drive our circuits – they were in simple equilibrium with their noisy surroundings. One circuit we studied was rather like a switch in that it could be in one of either two stable states. Thermal noise made it vibrate, leading to occasional "large fluctuation" events in which it moved a very long way from a stable state. The experiments verified our ideas: on almost every occasion, the events took place by way of a special pathway, predicted by theory, that mirrored the path by which the system relaxed back to its closest stable state.

Getting theoretical results for non-equilibrium systems is fundamentally more difficult because their fluctuations lack time symmetry. But these systems are among the most important for applications because they are so common, especially in biology. Theory suggested that they must display very complicated behaviour. To study this, we turned again to our switchlike circuit. The state of this circuit is given by a variable q, and its two stable states are $q = -1$ and $q = 1$. (You might think of these states as “off” and “on” for a switch.) We started with the circuit in the state at $= -1$, and then subjected it to random noise, as well as a regular, oscillating force. This force pushed the circuit out of equilibrium. We then waited for an unusual state to occur.

Once in a great while, we found it to be in the state $q = -0.63$, having wandered well away from the stable point $q = -1$. Every time we found it there, we immediately looked in detail at the events that brought it there. To make things simple, we always took the moment when the system arrived at $q = -0.63$ as the time $t = 0$.This was so we could compare a large number of case histories of the same special event, and not get confused by the different times when they occurred (see Diagram, below right).

In the diagram, the mountainous heap towards the bottom shows the resultant prehistory probability density. Where many histories pass through a certain point, the mountain is high. Where few histories pass, the mountain is low. The upper plane shows in red dots the location a mountain’s ridges, which first separate and then come together again, giving the mountain the appearance of a volcano. The two ridges correspond to the two special paths that the circuit is most likely to follow if it begins near $q = -1$ and later arrives at $q = -0.63$. In almost ali case, it is via these two pathways, and these these two pathways alone, that the system reaches this particular, unusual state. The upper plane also shows the pattern of special pathways (grey lines) predicted by mathematical theory, which seem to fit our data.

That’s great for us, but will this new understanding of the work habits of noise be of use? We think so. Instead of making our “special event” $q = -0.63$ for example, we could just as well have made it $q = 1$,in which case the special pathways would show the circuit’s preferred ways of switching from one stable state to the other. Since events like these typically underlie stochastic resonance or the workings of a stochastic ratchet, this theory should make it possible to understand these things in detail, even in complicated problems in the real world.

Making noise work

There may be other uses too. Suppose there is some event that you want to happen – a chemical reaction, for example, which is the key step in manufacturing an expensive drug. Random thermal noise will drive the reaction at a certain rate. But you might do better. If you want to speed things up, and follow the most energy-efficient approach, you might try to apply well-designed forces to push the molecules involved along one of the special paths most likely to lead to the reaction. By carefully measuring the chemical system, you should be able to work out the required forces directly. This is exactly what chemist Herschel Rabitz and his collaborators at Princeton University are doing, using lasers to do the “pushing” (see “No toil no trouble”, *New Scientist*, 18 July 1998, p33).

The new view on noise could also help in the world of far bigger things. Suppose, say, there is some event you don't want to happen. You might be a physicist who doesn't want the power in your laser to exceed some limit which would destroy it, or guiding a spacecraft or oil tanker, and want to keep it from being driven by random and uncontrollable forces into some dangerous predicament. In principle, you ought to be able to calculate the special paths along which such hazardous fluctuations would be bound to develop. When something ominous does start to happen, you can be ready to apply small corrective forces to steer away from trouble. This approach would be much more efficient and cheaper than applying correction forces all the time, or applying huge forces at the last moment.

So noise works in a relatively simple way, which we can understand, and even control. In future, we should be able to command noisy systems with the same ease as ordinary machinery. Far from being a disruptive nuisance, it looks as if noise could turn out to be a valuable ally.

Peter McClintock and **Dmitri Luchinsky** are physicists at Lancaster University

Further reading: "Analogue studies of non-linear systems" by Dmitri Luchinsky and others, *Reports on Progress in Physics*, vol **61**, p 889.

Figure caption

A finger that flicks a switch on and off (a) will cause a light to flash in synchrony. A weak finger, unable to push the switch over the barrier between on and off (b), won't cause any flashing. But if the switch is noisy and vibrates randomly, then when the weak finger has moved the switch part way over the barrier, the noise will occasionally contribute a kick that takes it over the top. The paths by which such "special events" take place can be shown graphically (right-see text for details) progress in Physics, vol 61, P 889