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On Forecasting Daily Stock Volatility: the Role of Intraday Information and Market Conditions

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On Forecasting Daily Stock Volatility: the Role of Intraday Information and Market Conditions

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Abstract

Several recent studies advocate the use of nonparametric estimators of daily price variability that exploit intraday information. This paper compares four such estimators, *realised volatility*, *realised range*, *realised power variation* and *realised bipower variation*, by examining their in-sample distributional properties and out-of-sample forecast ranking when the object of interest is the conventional conditional variance. The analysis is based on a 7-year sample of transaction prices for 14 NYSE stocks. The forecast race is conducted in a GARCH framework and relies on several loss functions. The realized range fares relatively well in the in-sample fit analysis, for instance, regarding the extent to which it brings normality in returns. However, overall the realised power variation provides the most accurate 1-day-ahead forecasts. Forecast combination of all four intraday measures produces the smallest forecast errors in about half of the sampled stocks. A market conditions analysis reveals that the additional use of intraday data on day $t-1$ to forecast volatility on day t is most advantageous when day t is a low volume or an up-market day. The results have implications for value-at-risk analysis.

Keywords: C53; C32; C14.

JEL Classification: Conditional variance; Quadratic variation; Nonparametric estimators; Intraday prices; Superior predictive ability.

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1 Introduction

Over the past decade there has been an enormous interest among academics and practitioners in modeling and forecasting the conditional variance of stock market returns. Volatility is a crucial concept for portfolio management, option pricing and financial market regulation, inter alios. One problematic issue is that, unlike prices or returns, the volatility process is unobserved even *ex post*. In a seminal paper, Andersen and Bollerslev (1998) focus on the problem of how the choice of proxy for the latent population measure of volatility can affect the quantitative assessment of volatility forecasting models. They illustrate that if the squared daily returns are used as proxy for the day's variance in the forecast evaluation, GARCH models do have very poor forecasting properties whereas using the sum of intraday squared returns which employs more information, the GARCH forecasts turn out to be far more accurate.[1] The rationale behind this is that the squared return is an extremely noisy (albeit unbiased) estimator of *ex post* volatility.

One related empirical question is how to obtain better daily volatility forecasts using intraday data. The literature branches in two broad directions. Several studies extend the daily GARCH model to incorporate the intraday information as an additional regressor. Instances include as augmentation variable the daily high-low price range (Parkinson, 1980; Taylor, 1987), the number of intraday price changes (Laux and Ng, 1993), daily trading volume (Bessembinder and Seguin, 1993), and the standard deviation of intraday returns (Taylor and Xu, 1997). Another group of studies focus on different ways of modeling directly the intraday data as a way of providing better out-of-sample forecasts of daily volatility. Two instances are Martens (2001) who models the intraday returns directly using GARCH models and Koopman et al. (2005) who compare daily GARCH models with ARFIMA and Unobserved Components models fitted to a *realised volatility*

[1] A risk measure developed in recent years is Value-at-Risk (VaR), a quantile of the conditional distribution of returns given past information, which gives the worst expected loss. According to the 2008 *European Investment Practices Survey*, the majority of asset managers use parametric VaR approaches derived from traditional location-scale models such as ARMA-GARCH which means that VaR calculations are based on forecasts of the old measure of investment risk, the conditional volatility. There is a close relation between GARCH(1,1) and the exponentially-weighted-average (of squared returns) historical volatility advocated by J.P. Morgan's Riskmetrics for daily and monthly VaR estimation. If normality is assumed, VaR adds no extra information over the old volatility measure.

measure based on the sum of 5-minute squared returns.

Aside from forecasting issues, much emphasis has been given in recent years to the use of nonparametric estimators of daily volatility that exploit intraday prices. The theoretical properties of these estimators have been investigated using advanced and novel asymptotic theory in stochastics and econometrics. For instance, the *realised variance* has been thoroughly studied by Barndorff-Nielsen and Shephard (BN-S, 2002a, 2002b). The sum of intraday high-low price ranges or *realised range* has been scrutinized by Christensen and Podolskij (2005) and Martens and van Dijk (2006). Two other intraday volatility estimators, introduced by BN-S (2004a, 2004b), are the *realised power variation*, based on summing powers of the intraday absolute returns, and the *realised bipower variation*, the sum of products of consecutive intraday absolute returns.

The paper complements the literature in several directions. First, it investigates the relative merit of the above nonparametric (intraday) volatility estimators from two perspectives. On the one hand, their in-sample distributional properties are compared, for instance, by gauging their efficiency, persistence and whether they can normalize the daily returns. In the Mixture of Distributions Hypothesis (MDH) literature, which builds on the tenet that volatility and trading volume are jointly driven by the latent information flow, the performance of volatility measures is typically assessed by the extent to which they bring return normality. On the other hand, we compare their ability to enhance the out-of-sample daily GARCH forecasts. For completeness, another updating variable is the daily volume computed by summing the number of shares traded over all intraday intervals. Following a large body of recent literature, the forecast race is based on 1-day-ahead predictions and the latent conditional variance is proxied by the 5-min realised variance.[2]

Second, given that market microstructure issues bedevil the above intraday volatility estimators in different ways, the paper addresses the question of whether forecast combining is fruitful. For this purpose, a rolling-window approach is adopted that allows for time-varying combination weights.

[2] Although regulators and fund managers might be mostly interested in longer horizons, derivative traders are interested in daily losses. One-day-ahead volatility forecasts are, for instance, relevant for VaR measurement since banks may wish to update their estimates of potential loss on a daily basis to determine capital requirements.

Third, an important question that has not been addressed as yet is whether the importance of updating daily conditional volatility (GARCH) models with intraday data depends on market conditions. This paper compares the forecast value of the four intraday volatility estimators during up- versus down-market days, and low- versus high-volume days. Finally, the study contributes to the existing literature by analysing 14 individual NYSE stocks whereas most related studies focus on FX data or stock market indices.

The sample spans 7 years of trading data over 02/01/97 to 31/12/03. The statistical properties of four nonparametric volatility estimators of daily price variation alongside the squared returns and the GARCH volatility are assessed according to criteria motivated by the MDH. The realised power variation and realised range are the top performers. Next, rolling out-of-sample forecasts are generated with a GARCH model augmented with either of the lagged nonparametric volatility measures or volume. Different forecast accuracy criteria are used which include asymmetric loss functions and the Mincer-Zarnowitz levels regression. Pairwise comparisons of forecast accuracy are conducted via the Diebold-Mariano (1995) test in the case of non-nested models and by the Harvey et al. (1998) encompassing test for nested models. The results reveal significant forecast gains from using intraday price information but not trading volume. GARCH updated with realised power variation is in the lead, followed closely by the realised range, and with realised bipower variation at the other extreme. Finally, a joint forecast comparison is performed using Hansen (2005) superior predictive ability test. For most stocks, the realised power variation is not beaten by any of the alterantive models. Combining the predictive information of the competing GARCH-augmented models is worthwhile. Finally, exploiting intraday returns at $t-1$ to forecast next day volatility is most fruitful when $t-1$ is a low-volume or up-market day.

The rest of the paper is organized as follows. Section 2 provides a review of the large literature on intraday volatility measuring which is by no means exhaustive. Section 3 presents the variable definitions and forecasting framework. Section 4 discusses the results and Section 5 concludes.

2 Background literature

The GARCH modeling framework introduced by Engle (1982) is still widely used to analyse the dynamics of daily return variation in all areas of finance by academics and practitioners alike. Several studies have documented that out-of sample regressions of squared returns on GARCH forecasts produce low R^2 statistics below 10% (see, inter alios, Franses and van Dijk, 1996, and Brooks, 1998). However, Andersen and Bollerslev (1998) for FX data and Blair et al. (2001) for stock indexes show that GARCH forecasts, when compared with the sum of intraday squared returns as the conditional volatility proxy are far more accurate with an R^2 of about 50%.

A weakness of GARCH models though is that the future variance of returns is cast as a polynomial of current and past squared returns. If on day $t-1$ the return is zero, the squared return at $t-1$ will also equal zero ignoring any within-day price fluctuations. One way forward is to augment the GARCH equation with variables that carry predictive power for future volatility. Lamoureux and Lastrapes (1990), Najand and Yung (1991), and Bessembinder and Seguin (1993) include contemporaneous volume in GARCH models and document an improvement in the in-sample fit. A problem with this approach is that volume cannot be assumed to be exogenous since, according to the MDH, volume and volatility are simultaneously influenced by the latent information arrival process. Brooks (1998) and Donaldson and Kamstra (2004) show that augmenting GARCH with lagged volume leads to no improvement in forecast performance. However, Donaldson and Kamstra (2004) show for the S&P100 index that trading volume has a switching role in forecasting. If volume on day $t-1$ is low relative to the recent past, then one-day-ahead ARCH forecasts are at least as effective as option implied volatilities (VIX). Conversely, if volume at $t-1$ is high, the best volatility forecast for day t can be obtained by placing more weight on market expectations.

With the increasing availability of high frequency data the research focus has shifted towards exploiting nonparametric estimators of daily volatility based on intraday returns. A large number of papers advocate the *realised variance* (RV) for the modeling and prediction of volatility of

FX returns (Taylor and Xu, 1997; Andersen and Bollerslev, 1998) and equity returns (Andersen, Bollerslev, Diebold and Ebens, ABDE, 2001). Luu and Martens (2002) find support for the assumptions underlying the MDH model when RV is used instead of daily squared returns. Pong et al. (2004) compare the forecasting ability of short memory (ARMA) and long memory (ARFIMA) models of RV, and the implied volatilities from OTC foreign currency options for horizons ranging from one day to 3 months. The models provide more accurate forecasts than the implied volatilities for short (one-day and one-week) horizons and this is attributed to the use of intraday returns rather than to the long memory specification. At the one- and 3-month horizons, the models of RV do not provide incremental information that is not already incorporated in the implied volatilities. Using an equity price index and two currencies, Galbraith and Kisinbay (2002) find that forecasts from AR fitted to daily RV outperform the forecasts from GARCH for a 1-day horizon whereas at 30 days the two methods become indistinguishable.

For the S&P100 index, Koopman et al. (2005) generate one-day-ahead forecasts from ARFIMA and Unobserved Components models fitted to RV, and from stochastic volatility (SV) and GARCH models fitted to daily returns and augmented with lagged RV and implied volatility.[3] Long memory models seem to provide the most accurate forecasts. Engle and Gallo (2006) develop a multiplicative-error model which combines several daily volatility indicators (absolute returns, squared high-low range and RV) and show that it forecasts quite well 1-month-ahead the VIX.

A second group of empirical studies advocate different nonparametric volatility estimators as an alternative or complement to the popular RV. Ghysels et al. (2006) introduce the MIDAS (MIxed DAta Sampling) regression approach and compare several daily volatility estimators based on FX data sampled at different intraday frequencies. They find that realised power variation (RPV) outperforms RV and that (intra-)daily absolute returns outperform, respectively, the corresponding squared returns. Using Yen/US$ and DM/US$ rates and the Spyder Exchange-Trade Fund that represents ownership in the S&P500 index, Liu and Maheu (2005) fit HAR-log and ARFIMA

[3] In the ARCH class of models, the expected volatility is parameterized as a function of past returns only. In contrast, the parameterized expectations in the SV class of models explicitly rely on latent state variables.

models to RV and augment them with lagged RPV and realized bipower variation. Only with RPV they find robust improvements in the 1-day-ahead forecasts of FX rates and the S&P500. For DM/US$ rates, the S&P500 and the 30-year US T-bond yield, Andersen et al. (2007) document that only the continuous part of the return process carries predictive power for future volatility.

3 Methodology

3.1 Population measures of volatility

In most of the volatility forecasting literature, the population measure of volatility is the *conditional variance.* Let r_t denote the daily stock return, its conditional variance is denoted $var(r_t|\mathcal{F}_{t-1}) \equiv \sigma_t^2$ where $\mathcal{F}_{t-1}$ is the sigma field containing all relevant information up to time $t-1$, which naturally refers to $r_{t-j}, j > 1$ but it may also include other variables. It is assumed that $E(r_t|F_{t-1}) \equiv E_{t-1}(r_t) = 0$ such that $\sigma_t^2 = E_{t-1}(r_t^2)$ is the object of interest.

But there are other possible population measures of variance. To define them, let the price process belong to the class of semimartingales with jumps. The dynamics of the log price change in continuous time can be characterized by the stochastic differential equation

$$dp(t) = \mu(t)dt + s(t)dW(t) + k(t)dq(t) \qquad 0 \leq t \leq T \tag{1}$$

where $\mu(t)$ denotes the drift term, $s(t)$ is the instantaneous or spot volatility process which is assumed to be stationary and independent of the standard Brownian motion $W(t)$, $dq(t)$ is a counting process with $dq(t) = 1$ if a jump occurs at time t and $k(t)$ is the jump size. Equation (1) embodies the intuitive idea that there are two types of randomness driving the stock returns. One is a Brownian motion generating the continuous sample path and small movements and the other consists of large but infrequent (discrete) jumps.

The *quadratic variation* (QV) or notional variability of the return process is defined as

$$QV_t = \int_{t-1}^{t} s^2(u)du + \sum_{t-1<j\leq t} k^2(j) = IV_t + J_t \tag{2}$$

where the first term is called *integrated variance* (IV_t) and corresponds to the continuous part of the log price process and the second term (J_t) reflects the contribution of the discrete jumps.[4] In a recent paper, BN-S (2004a) define the *integrated power variation* (IPV) of order z as

$$IPV_t(z) = \int_{t-1}^{t} s^z(u)du, \; 0 < z \leq 2 \tag{3}$$

which for $z = 2$ becomes the integrated variance.

In this paper, the population measure of interest is the conditional variance due to its prevalence in applied and theoretical forecasting work in the past two decades. Andersen et al. (2002) argue instead in favour of QV as the relevant notion of variability. However, both population measures of volatility are closely related since the conditional variance of future returns is the conditional expectation of QV as shown in BN-S (2002b). This result provides further theoretical underpinning for the widespread use in empirical finance of GARCH models.

3.2 Daily GARCH models and intraday updating variables

Let the conditional mean and conditional variance of daily returns be captured, respectively, by and ARMA(p,q) and GARCH(r,s) equation[5]

$$r_t = \theta_0 + \sum_{i=1}^{p} \theta_i r_{t-i} + \sum_{j=1}^{q} \lambda_j u_{t-j} + u_t, u_t|\mathcal{F}_{t-1} \sim iid(0, h_t) \tag{4a}$$

$$h_t = \omega + \sum_{i=1}^{r} \alpha_i u_{t-1}^2 + \sum_{j=1}^{s} \beta_j h_{t-j} \tag{4b}$$

where r_t are the daily returns and u_t^2 are the squared whitened returns. The lag orders of the (conditional) mean and variance equations will be appropriately selected so as to remove all the

[4] Most modern finance theory is based on semimartingales. If the return process is a semimartingale, then it has an associated QV_t process. The latter plays a central role in the option pricing literature. In particular, in the absence of jumps, QV_t equals the IV_t highlighted in the stochastic volatility models first proposed by Hull and White (1987) as an alternative to the classical Black-Scholes formulae for option pricing. However, there is an increasing body of empirical work in finance which concludes that continuous-time models must incorporate jumps or discontinuities in order to provide a satisfactory characterization of the daily return process.

[5] We do not consider asymmetric GARCH models because the asymmetric relation between price movements and volatility (e.g. rationalized as 'leverage effect') has been shown to be rather weak or absent in individual stock price series as compared to broad stock price index series (see, for instance, Kim and Kon, 1994; Tauchen et al., 1996).

return autocorrelation and volatility clustering. The Ljung-Box and ARCH LM tests, respectively, will be used for these purposes. The degree of volatility persistence is given by $\lambda = \Sigma\alpha_i + \Sigma\beta_j$.

The selected GARCH model for each stock is then augmented as follows

$$h_t = \omega + \sum_{i=1}^{r} \alpha_i u_{t-1}^2 + \sum_{j=1}^{s} \beta_j h_{t-j} + \gamma v_{t-1} \tag{5}$$

where v_{t-1} is a nonparametric estimator based on intraday prices at day $t-1$. In our case v_{t-1} is the realised variance (RV), realised range (RR), realised power variation (RPV), realised bipower variation (BPV) or trading volume (VOL). For this purpose, the time dimension is discretized and the daily time interval is divided into M equally-spaced subintervals of length δ. The price at the start of the jth intraday interval is computed as the average of the closing and opening prices of intervals $j-1$ and j, respectively. The jth intraday return (on day t) is computed as

$$r_{t,j} = 100 \left(\frac{\log(p_{t,j}^c) + \log(p_{t,j+1}^o)}{2} - \frac{\log(p_{t,j-1}^c) + \log(p_{t,j}^o)}{2} \right), j = 2, ..., M-1 \tag{6}$$

where each trading day [9:30am-4:00pm] amounts to a duration of $M \times \delta = 390$min, and $p_{t,j}^c$ ($p_{t,j}^o$) is the closing (opening) price of the jth intraday interval. For instance, $j = 2$ corresponds to 9:35am-9:40am. The extreme-interval returns are $r_{t,1} = 100 \left(\frac{\log(p_{t,1}^c)+\log(p_{t,2}^o)}{2} - \log(p_{t,1}^o) \right)$ and $r_{t,M} = 100 \left(\log(p_{t,M}^c) - \frac{\log(p_{t,M-1}^c)+\log(p_{t,M}^o)}{2} \right)$. For $\delta = 5\,$min, we have $M = 78$ intraday returns and one overnight return. However, a few trading days consist of $M < 78$ due to delayed openings and/or early closings of the NYSE. Overnight returns are not included due to the fact that the weight such a return should deserve is somewhat arbitrary as Hansen and Lunde (2006b) and Engle et al. (2006) argue. The usual logarithmic (or continuously compounded) daily returns used to estimate GARCH models amount to the aggregated intraday returns, $r_t = \sum_{j=1}^{M} r_{t,j} = \log(\frac{p_{t,M}^c}{p_{t,1}^o})$.

The most popular estimator, the *realised variance,* defined as the sum of intraday returns

$$RV_t = \sum_{j=1}^{M} r_{t,j}^2, \quad t = 1, 2, ..., T \tag{7}$$

converges in probability to the quadratic variation ($RV_t \underset{\rightarrow}{p} QV_t$) under suitable conditions as the intraday sampling frequency increases ($M \rightarrow \infty$) and so RV is a consistent estimator of QV (see ABDE, 2001; BN-S, 2002a,b). If $M = 1$, then RV becomes the noisy daily squared return (r_t^2).

The *realised range* estimator introduced by Christensen and Podolskij (2005) is a generalization of the range estimator of Parkinson (1980) and defined as

$$RR_t = \frac{1}{4\log 2}\left[\sum_{j=1}^{M} 100 \times \left(\log(p_{t,j}^h) - \log(p_{t,j}^l)\right)^2\right] \quad t = 1, 2, ..., T \tag{8}$$

where $\log(p_{t,j}^h)$ and $\log(\log p_{t,j}^l)$ are the high and low prices of the jth interval, and the scaling factor $4\log 2$ is a bias-correction for market microstructure effects given by the second moment of the range of a Brownian motion B_t, that is, $E(s_B^2) = 4\log 2$ where $s_B = \sup_{0\leq t,s\leq 1}(B_t - B_s)$.

BN-S (2002a) and Christensen and Podolskij (2005) show that RR is a more efficient estimator than RV. In an ideal world without market frictions (no bid-ask bounce, discontinuous trading or jumps) the asymptotic variance of the RR estimator is $0.4\int_{t-1}^{t}\sigma(u)^4 du$, where the integral is called integrated quarticity, which is 5 times smaller than the variance of RV at $2\int_{t-1}^{t}\sigma(u)^4 du$. Hence, theoretically the RR estimator is more efficient than other variance estimators based on squared returns. Christensen and Podolskij (2005) and Martens and van Dijk (2006) show that, in the absence of jumps, as $M \to \infty$ the realised range converges in probability to the quadratic variation ($RR_t \underset{\to}{p} QV_t$). This results does not hold, however, in a jump-difussion setting; Theorem 1 in Christensen-Podolskij establishes that with jumps, RR is not a consistent estimator of QV. For a DGP without jumps, Martens and van Dijk (2006) accommodate the bid-ask bounce in Monte Carlo simulations to show that: *i*) both RR and RV are upward biased but the former suffers more; *ii*) infrequent trading induces a downward bias in RR but not in RV.

Another estimator introduced by BN-S (2004a), the *realised power variation* of order z, is

$$RPV_t(z) = \mu_z^{-1}\delta^{1-z/2}\sum_{j=1}^{M}|r_{t,j}|^z, \ 0 < z < 2, \quad t = 1, 2, ..., T \tag{9}$$

where

$$\mu_z = E|\mu|^z = 2^{z/2}\frac{\Gamma(\frac{1}{2}(z+1))}{\Gamma(\frac{1}{2})}, \ \mu \sim N(0,1)$$

which for $z = 1$ becomes the realised absolute variation. BN-S (2004a) demonstrate its consistency by showing that as $M \to \infty$, it converges in probability to the integrated power variation, $RPV_t(z)$

$\xrightarrow{p} IPV_t(z)$, and so it is robust to jumps. Liu and Maheu (2005) study the 1-day-ahead forecasting properties of (9) for orders $z = \{0.25, 0.5, ..., 1.75\}$ and find that $0.5, 1$, and 1.5 yield the lowest RMSE. Absolute return measures are more persistent than the squared counterparts so RPV could outperform RV in forecasting financial risk. Also RPV may provide better predictions than RV when the sample period contains large jumps. Further discussion on the RPV estimator can be found in Ghysels et al. (2006) and Forsberg and Ghysels (2007).

In a similar fashion, BN-S (2004a) define the *realised bipower variation* estimator as

$$RBP_t = \mu_1^{-2} \sum_{j=2}^{M} |r_{t,j}|\,|r_{t,j-1}| \tag{10}$$

where $\mu_1 = E(|\mu|) = \sqrt{2}/\sqrt{\pi} \backsimeq 0.79788$ and $\mu \sim N(0,1)$. BN-S (2004a) show that as RBP converges in probability to the integrated variance ($RBP_t \xrightarrow{p} IV_t$) and so it is also immune to jumps. The result that $RV_t - RBP_t \xrightarrow{p} J_t$ where J_t is the jump component in (2) is exploited by BN-S (2006) alongside the joint asymptotic distribution of the two estimators (under the null of a continuous sample path) to develop a non-parametric test for jumps.

The asymptotics (as $M \to \infty$) of these nonparametric volatility estimators were derived under suitable theoretical conditions such as no market microstructure noise. Unfortunately, in real-world settings the semimartingale property of prices breaks down at ultra-high frequencies because the influence of market microstructure factors such as bid-ask bounce (Ross, 1984), screen fighting (Zhou, 1996), price discreteness and irregular trading become overwhelming. This means that, in practice, intraday measures of volatility calculated at very high frequencies become biased.[6]

As noted, we adopt $\delta = 5$ and the motivation for this choice is twofold. First, a 5-minute grid is short enough for the daily volatility dynamics to be picked up with reasonable accuracy and long enough for the adverse effects of market microstructure frictions not to be overwhelming.[7] Second,

[6] Several methods, mostly nonparametric, have been proposed to account for microstructure bias. Martens and Van Dijk (2006) suggest a bid-ask bias correction for the RR estimator, eq.(8), by scaling it with the ratio of the average level of the daily range and the average level of the RR over the previous q trading days. Adding autocovariances to the RV estimator, eq.(7), has been suggested as a way of mitigating bid-ask bounce biases (Barndorf-Nielsen et al., 2004; Hansen and Lunde, 2006b). Jungbacker and Koopman (2005) develop a parametric model-based approach that accounts for microstructure noise and intra-daily seasonality.

[7] ABDE (2001), BN-S (2002a,b), Taylor and Xu (1997) and Fleming and Paye (2006), inter alios, advocate this

it will enable meaningful comparisons with previous studies, most of which are based on 5-min data. Nevertheless, we check how sensitive the main results are to using 15- and 30-min grids.

Finally, the daily volume (VOL) measure adopted is the total number of shares traded each day computed as $VOL_t = \sum_{j=1}^{M} vol_{t,j}$, where $vol_{t,j}$ is the number of shares traded over the jth interval. This is the measure of volume used in Lamoureux and Lastrapes (1990).

3.3 Forecast evaluation and market conditions

The sample size is divided into an estimation period ($T - T_1$) of fixed length 1261 days, and a holdout or evaluation period (T_1) of 500 days. Hence, each model is estimated over an initial window, denoted $[1, t]$, and a 1-day-ahead ex post volatility forecast is generated. The window is rolled forward one day to $[2, t+1]$ to obtain the second forecast and so forth until 500 iterations.

The population volatility measure (σ_t^2) is the *conditional variance* and its proxy ($\tilde{\sigma}_t^2$) for forecast evaluation is the 5-min realized variance because it is an asymptotically conditionally unbiased estimator of the conditional variance — a further appealing property of the realized variance is that it converges in probability to the QV which plays a central role in the option pricing literature. The precision of model m forecasts, $\{h_{t,m}\}_{t=1}^{T_1}$, is gauged through several loss functions:

Mean absolute error	$MAE = \frac{1}{T_1} \sum_{t=1}^{T_1} \left\| \tilde{\sigma}_t^2 - h_{t,m} \right\|$
Mean squared error	$MSE = \frac{1}{T_1} \sum_{t=1}^{T_1} (\tilde{\sigma}_t^2 - h_{t,m})^2$
Heteroskedasticity-adjusted MAE	$HMAE = \frac{1}{T_1} \sum_{t=1}^{T_1} \left\| 1 - \tilde{\sigma}_t^{-2} h_{t,m} \right\|$
Heteroskedasticity-adjusted MSE	$HMSE = \frac{1}{T_1} \sum_{t=1}^{T_1} (1 - \tilde{\sigma}_t^{-2} h_{t,m})^2$
Adjusted mean absolute percentage error	$AMAPE = \frac{1}{T_1} \sum_{t=1}^{T_1} \left\| \frac{\tilde{\sigma}_t^2 - h_{t,m}}{\tilde{\sigma}_t^2 + h_{t,m}} \right\|$
Theil-U	$Theil\text{-}U = \sum_{t=1}^{T_1} (\tilde{\sigma}_t^2 - h_{t,m})^2 / \sum_{t=1}^{T_1} (\tilde{\sigma}_t^2 - h_{t,N}^2)^2$
Mean mixed error (U)	$MME(U) = \frac{1}{\#U} \sum I_U \cdot e_{t,m}^2 + \frac{1}{\#O} \sum I_O \cdot \|e_{t,m}\|$
Mean mixed error (O)	$MME(O) = \frac{1}{\#U} \sum I_U \cdot \|e_{t,m}\| + \frac{1}{\#O} \sum I_O \cdot e_{t,m}^2$
Logarithmic loss	$LL = \frac{1}{T_1} \sum_{t=1}^{T_1} (\ln \tilde{\sigma}_t^2 - \ln h_{t,m})^2$
Gaussian maximum likelihood error	$GMLE = \frac{1}{T_1} \sum_{t=1}^{T_1} \left(\ln h_{t,m} + \tilde{\sigma}_t^2 h_{t,m}^{-1} \right)$

In the MME(U) and MME(O) criteria, $e_{t,m} = \tilde{\sigma}_t^2 - h_{t,m}$ denotes the forecast error for model m. $\#U$ is the number of underpredictions and $I_U = 1$ if $e_{t,m} < 0$; likewise for $\#O$ and I_O.

grid also because daily returns standardized by 5-min realised volatility are approximately normal. In the forecasting literature, studies that use 1-, 5-, 15- and 30-min data report mixed results but overall they also tend to favour the 5-min sampling (Martens and van Dijk, 2006; Pong et al., 2004; Ghysels et al., 2006; Galbraith and Kisinbay, 2002).

MAE, MSE, HMAE, HMSE, AMAPE and Theil-U belong to the family of symmetric loss functions, in the sense that they equally penalize over- and under-predictions. The most widely adopted, MSE, proposed by Bollerslev et al. (1994) is based on a quadratic loss function and so it is particularly good where large forecast errors are disproportionately more worrisome than smaller errors. MAE is less sensitive to severe mispredictions than MSE whereas AMAPE, proposed by Makridakis (1993), is an interesting alternative in percentage. The heteroskedasticity-adjusted version of MSE and MAE, introduced by Bollerslev and Ghysels (1996), is used by Martens (2001) and Koopman et al. (2005) inter alios.[8] Theil-U is calculated as the ratio of MSE for the model at hand to the MSE of the naive model, typically a random-walk type model, $h_{t,N} = \tilde{\sigma}^2_{t-1}$.

A number of asymmetric loss functions have been employed in the volatility literature. Examples include the two mean mixed error statistics proposed by Brailsford and Faff (1996), MME(U) and MME(O), the logarithmic loss (LL) introduced by Pagan and Schwert (1990) and the Gaussian maximum likelihood error (GMLE) of Bollerslev et al. (1994) which corresponds to the loss function implied by a Gaussian likelihood. MME(U), LL and GMLE penalize under-predictions more heavily than over-predictions whereas MME(O) does the opposite. For instance, in option pricing it is well established that the higher the volatility the higher the value of the call option so the under-prediction (overprediction) of volatility is unattractive for the seller (buyer). In addition, we also utilize the R^2 of Mincer-Zarnowitz level regressions (MZ-R^2), also called unbiasedness-regressions in the literature, a measure of the informational content of the volatility forecasts.[9,10]

[8] The HMAE can also be referred to as mean absolute percentage error (MAPE) since it can be rewritten as $HMAE = \frac{1}{T_1}\sum_{t=1}^{T_1}\left|\frac{\sigma_t^2 - \hat{\sigma}^2_{t,m}}{\sigma_t^2}\right|$. Likewise, the HMSE is the mean squared percentage error (MSPE).

[9] The MZ levels regression is $\tilde{\sigma}^2_t = a + bh_{t,m} + e_t, t = 1, ..., T_1$. Hence, h_t will be unbiased for the true variance σ^2_t if $a = 0$, $b = 1$ and $E(e_t) = 0$. The R^2 from this regression (called MZ-R^2) reflects the variance but not the bias-squared component of MSE, that is, it corrects for bias.

[10] Hansen and Lunde (2006a) study the distortion in model ranking from replacing $E[L(\sigma^2_t, h_{t,m}]$ by $E[L(\tilde{\sigma}^2_t, h_{t,m}]$. It is called *objective bias* to distinguish it from the *sampling error,* due to estimating $E[L(\tilde{\sigma}^2_t, h_{t,m}]$ by a sample average, which vanishes as T_1 increases. Taking the conditional variance as the latent volatility, σ^2_t, they derive a set of conditions under which the empirical model ranking obtained under $L(\tilde{\sigma}^2_t, h_{t,m})$ is consistent for the true model ranking under $L(\sigma^2_t, h_{t,m})$. The conditions are met by MSE, GMLE and the levels *MZ-*R^2. For a range of loss functions, Patton (2006) derives analytically the objective bias using the daily high-low range, daily square return and 5- and 30-min RV as conditional volatility proxies. They illustrate that the more precise the proxy, the less relevant the objective bias; in particular, the objective-bias, if any, is very small for the 5-min RV proxy.

The statistical significance of differences in forecast accuracy is assessed by means of the Diebold-Mariano (1995) test statistic for non-nested models and the Harvey et al. (1998) encompassing test for nested models. In the former, the null hypothesis is that of equal predictive accuracy of models A and B, that is, $H_0 : E(L_{t,A}) - E(L_{t,B}) = 0$, against the alternative $H_A : E(L_{t,A}) - E(L_{t,B}) \neq 0$. The DM test statistic is

$$DM = \frac{\bar{d}}{\sqrt{\hat{V}(d_t)/T_1}} \xrightarrow{p} N(0,1),$$

where $\bar{d} = \frac{1}{T_1}\sum_{t=1}^{T_1}(d_t - \bar{d})$, d_t is the loss differential and $\hat{V}(d_t)$ is a heteroskedasticiy and autocorrelation robust (HAC) estimator of the asymptotic variance of d_t.[11] The DM test can be employed under a variety of loss functions. For instance, $d_t^{MAE} = |\tilde{\sigma}_t^2 - h_{t,A}| - |\tilde{\sigma}_t^2 - h_{t,B}|$ and $d_t^{MZ\text{-}R^2} = \frac{(\hat{u}_t^A)^2 - (\hat{u}_t^B)^2}{T_1^{-1}\sum(\tilde{\sigma}_t^2 - \bar{\sigma}^2)^2}$ where $\hat{u}_t^A$ are the residuals of a regression of $\tilde{\sigma}_t^2$ on $h_{t,A}$; likewise for $\hat{u}_t^B$.

Let A denote a model which is nested in a larger model B. For nested models, the DM statistic is non-normal resulting in undersized tests with low power. In this paper, we deploy the Harvey et al. (1998) encompassing t-test (ENC-T) developed in the context of the MSE loss differential. Inferences are based on the critical values tabulated by Clark and McCracken (2001) for $(\pi, k_2) = (0.4, 1)$ where $\pi = \frac{T_1}{T - T_1}$ and k_2 is the number of excess parameters in model B. H_0 is as in the DM test, meaning here that the additional parameters in B do not help prediction (equal MSE) and H_A is that B has smaller MSE than A. Essentially, the ENC-T test statistic is a t-statistic for the covariance between $L_{t,A}$ and $L_{t,A} - L_{t,B}$ as follows

$$ENC - T = (T_1 - 1)^{1/2} \frac{\bar{C}}{\sqrt{T_1^{-1}\sum(C_t - \bar{C})^2}}$$

where $C_t = (L_{t,A} - L_{t,B}) \times L_{t,A} = (\tilde{\sigma}_t^2 - h_{t,A})^2 - (\tilde{\sigma}_t^2 - h_{t,A}) \times (\tilde{\sigma}_t^2 - h_{t,B})$.

When several forecasting models are compared through pairwise tests, data mining (snooping) may hinder the significance of the outcome. In contrast to the DM test of equal predictive ability,

[11] The DM statistic does not converge to a standard normal if the evaluation period grows at the same rate as the estimation period because the effect of parameter estimation error does not vanish; a HAC estimator that captures the contribution of parameter uncertainty is then required. However, the distortion from ignoring the latter depends on $\pi = \frac{T_1}{T - T_1}$ and it gets larger, the larger π is. In our analysis $\pi = 0.4$ which is not considered large and also the number of parameters to be estimated is relatively small so we do not account for parameter uncertainty.

the test of *superior predictive ability* (SPA) proposed by Hansen (2005) involves a composite hypothesis, thereby being less prone to data mining. The SPA test is designed to address the hypothesis H_0 :'any alternative forecast is not better than the benchmark' and it requires bootstrap critical values. Like the DM test, the SPA test can be conducted for any loss function.[12]

A motivation for combining forecasts from different models is that they are likely to capture distinct subtle aspects of the true volatility process, and the relative prominence of such aspects may vary over time. The four nonparametric volatility estimators considered suffer to different extents from market microstructure bias. Hence, it may pay to combine their information content while allowing for their relative role (weight) to time-vary through a rolling estimation approach (fixed window size at 1261 days) as follows. The combining weights for the t^{th} forecast, $\hat{\alpha}_0(t), ..., \hat{\alpha}_4(t)$, $t = 1, ..., T_1$ ($T_1 = 500$) are obtained by regressing the volatility proxy on the in-sample GARCH-RV, GARCH-RR, GARCH-RPV and GARCH-RBP fitted variances over the relevant window $[t-1-(1260), t-1]$. The t^{th} out-of-sample combined forecast is then computed as

$$h_{t,C} = \hat{\alpha}_0(t) + \hat{\alpha}_1(t)h_{t,RV} + \hat{\alpha}_2(t)h_{t,RR} + \hat{\alpha}_3(t)h_{t,RPV} + \hat{\alpha}_4(t)h_{t,RBP}, \quad t = 1, ..., 500$$

where $h_{t,RV}$ denotes the t^{th} out-of-sample forecast from the GARCH-RV model and so forth.

To the best of our knowledge, the issue of different market conditions or regime-switching in the context of volatility forecasting using intraday data has not been addressed. We compare the value-added of intraday information for one-day-ahead volatility forecasting during 'up-market' (U) versus 'down-market' (D) days, and 'high-volume' (H) versus 'low-volume' (L) days. For this purpose, we classify and average the forecast errors into those incurred during up-(high-) or down-market (low-volume) days. Our definition of up/down market days is a short-term one based on the moving average of the daily return over the most recent 5-day window. Since the goal is to forecast the volatility on day t, the switching variable is a one-day-lagged (t-1) indicator function

$$S_{t-1} = \begin{cases} 1 \quad if \quad \frac{1}{5}\sum_{i=1}^{5} r_{t-i} > 0 & (\textit{Up-market} \text{ day}) \\ 0 \quad else & (\textit{Down-market} \text{ day}) \end{cases} \tag{11}$$

[12] The SPA test is implemented in OxMetrics 5 using Peter Hansen's code which we gratefully acknowledge. We focus the analysis on the two predefined loss functions in the code, MSE and MAE.

which equals 1, signifying a positive direction of the market if the moving average of the daily returns over the most recent 5-day period is positive.

For the high- versus low-volume days comparison the short-term indicator function is

$$V_{t-1} = \begin{cases} 1 \quad if \quad VOL_{t-1} > \frac{1}{5}\sum_{t=1}^{5} VOL_{t-1-i} & (\textit{High-volume} \text{ day}) \\ 0 \quad else & (\textit{Low-volume} \text{ day}) \end{cases} \tag{12}$$

Two questions are asked: (a) Does the ranking of intraday augmentation measures differ over market conditions?, (b) Do the benefits from exploiting intraday data differ over market conditions?

4 Empirical Results

4.1 Data and distributional properties

The transaction price and number-of-shares traded data is from Tick Data.[13] The observations pertain to 14 stocks traded on the NYSE and span the period 02/01/97 to 31/12/03, a total of 1761 days. The stocks are American Express (AXP), AT&T (ATT), Boeing (BA), Caterpillar (CAT), DELL, General Electric (GE), General Motors (GM), JP Morgan (JP), KO (Coca-Cola), McDonald (MCD), Microsoft (MSFT), Procter & Gamble (PG), WAL-MART (WMT) and IBM.

Table 1 shows the distributional properties of daily returns and trading volume. For all stocks, the returns are non-normally distributed, particularly, in the form of excess kurtosis.[14]

[Table 1 around here]

The Ljung-Box (LB) statistic suggests no autocorrelation in daily returns for many stocks — the exceptions are ATT, DELL, GM, IBM PG, and WMT returns. By using mean volume as measure of trading activity, stocks can be ranked from more to less active as: MSFT, DELL, GE, IBM, JPM, WMT, AXP, MCD, KO, BA, GM, PG, ATT and CAT. Trading volume is stationary as borne out by the ADF test and the Robinson d statistic but the latter suggests long memory in

[13] *www.tickdata.com* provides high frequency data on a commercial basis for equity and commodity markets.

[14] Due to space constraints, an extensive collection of our empirical results are given in an Appendix which can be downloaded from *http://www.cass.city.ac.uk/faculty/a.fuertes*.

volume.[15] DELL's volume is the most persistent whereas the smallest persistence is shown by BA's volume. The assumption that volume follows a lognormal distribution was first advocated in Clark's (1973) MDH model and is still widely used. The Kolmogorov-Smirnov (KS) test suggests volume log-normality for half of the sampled stocks: BA, CAT, GE, GM, JPM, KO and MCD.[16]

Table 2 reports summary statistics for five distinct daily volatility estimators.

[Table 2 around here]

In line with the MDH theory, studies by Clark (1973), Tauchen and Pitts (1983), Richardson and Smith (1994) and Andersen and Bollerslev (1997a,b) document several interesting stylized facts about the unobserved, latent information flow driving the volatility process. These include small variation relative to its mean, lognormality, high persistence, correlation with volume and bringing normality in returns. We adopt these stylized facts as in-sample criteria to compare the estimators.

RV and RBP have approximately the same mean (e.g. for IBM, the mean RV and RBP are, respectively, 3.572 and 3.340). The mean of RR is generally smaller than that of RV with two exceptions only (DELL and MSFT). This is in line with the findings in Martens and van Dijk (2006) which illustrate that infrequent trading induces a downward bias in the RR, while it does not affect RV. The mean of RPV(for $z = 1.5$) is slightly higher than those of the other intraday-estimated volatility measures. But RPV is not in the same units as the other three measures, so any comparison of their moments has to be interpreted with caution.[17] Relative to its mean, RPV has generally the lowest dispersion (standard deviation) which suggests that it is the least

[15] A fractional integration parameter $0 < d < 0.5$ characterizes stationarity with long memory so that the auto-correlation function decays at a hyperbolic rate rather than exponentially as in short-memory ($d = 0$) processes.

[16] In contrast with the JB test that focuses on the skewness and kurtosis only, the KS test compares the cumulative distributions of the input data and the fitted distribution and so it has been shown to be more powerful that the JB test. The KS statistic is computed as $KS = max(D^+, D^-)$ with $D^+ = \max(\frac{t}{T} - F(VOL_t))$ and $D^- = \min(\frac{t}{T} - F(VOL_t)), t = 1, .., T$ where $F(\cdot)$ is the fitted lognormal distribution.

[17] The order chosen for the RPV is $z = 1.5$ throughout the paper. Building on the results in Liu and Maheu (2005), to make this choice we compare RPV(0.5), RPV(1) and RPV(1.5) according to their distributional properties, in-sample model-fit and out-of-sample forecasting properties. Firstly, daily returns standardized by RPV ($z = 0.5$) become normal at the 10%, 5% or 1% level in none of the stocks, 7 stocks ($z = 1$), and 9 stocks ($z = 1.5$). Second, the model fit of GARCH-RPV is clearly superior, according to the loglikelihood, AIC and SBC, for $z = 1.5$ also. Third, for the majority of stocks according to virtually all loss functions considered, the forecast errors of GARCH-RPV are smaller for $z = 1.5$. Detailed results can be found in Appendix Tables A and B.

noisy in the present context followed by RR. At the other extreme, the crude squared return has a StDev/Mean ratio about five times larger than RPV.

The ADF test suggests that all five unconditional volatility measures are stationary. But the degree of persistence (Robinson d) of the intraday-estimated measures is substantially higher than that of daily squared returns. RPV and RR are the most persistent (followed by RV and then RBP) and so they may provide a better signal for future volatility. All measures show positive skewness and large kurtosis with squared returns having the largest kurtosis. The KS test suggests that lognormality for the intraday measures but not for daily squared returns. This is in line with ABDE (2001) and ABDL (2001), inter alios, who show that the lognormal distribution provides a good fit for realised volatility. Figure 1 plots for the least traded (CAT) and most traded (MSFT) stock the different volatility estimators alongside volume (scaled by 10^7).

[Figure 1 around here]

Normality of returns is an assumption that underlies many financial theories, for instance, the Black-Scholes option pricing model and some VaR approaches. But daily stock returns are clearly non-normal. Several studies have recovered normality by subordinating returns to the 'financial clock' using RV as standardization variable (ABDE, 2001; ABDL, 2001; BN-S, 2002a; Bandi and Russell, 2006; and Areal and Taylor, 2001). On this basis, they conclude that RV reflects well the information flow to the market.[18] We standardize the daily returns by the various uncondi-

[18] Daily returns are non-normal because information is available to traders at a varying rate so the price process evolves at different rates during identical time intervals. When no information is available, trading is slow, and the price evolves slowly. When new information arrives, trading is brisk and the price process evolves much faster. The upshot is that the number of individual random effects added together to give the daily price change (return) is non-constant, rendering the Central Limit Theorem inapplicable. The theoretical motivation for expecting returns to be normal when standardized by volume starts from Clark (1973) and Monroe (1978). Clark argues that returns are non-normal when sampled at intervals which are equidistant in calendar time but are normal in the time scale of the latent trading activity (called 'financial time'), that is, when computed over intervals with equivalent trading activity. Clark further shows that trading volume can be taken as an instrument for the true operational time or 'imperfect clock' measuring the speed of evolution of the price change process. By standardizing returns with volume, in effect, the returns 'clock' is subordinated to that of volume. Monroe provides further justification by showing that any semi-martingale can be written as a time-changed Brownian motion. Monroe's result in essence tells us that there exists a time-filtration mechanism that can restore return normality. Andersen et al. (2005) show that using high-frequency data for the construction of the proxy for the 'financial time' (they sample at intervals of

tional (intraday) and conditional (GARCH) volatility measures and analyze the extent to which standardized returns become normal. Table 3 sets out the results.

[Table 3 around here]

The findings are consistent with the literature where it has been documented that historical GARCH (and SV) models do not adequately capture all the leptokurtosis in daily returns. The standardization by GARCH mitigates but does not eliminate all the factors that induce non-normality.[19] RR is the most successful in bringing normality (for all stocks except MSFT) followed by RPV, RBP and RV. The lognormality of the intraday-estimated variability measures together with their ability to bring normality of standardized returns provides support for Clark's (1973) contention that asset returns follow a normal-lognormal mixture in the context of cotton futures.

In the MDH literature, volume is taken as a proxy for the latent trading activity process (Clark, 1973). The MDH posits that volume and volatility are positively correlated because they are simultaneously influenced by the rate of news arrival. The extent of this correlation provides a further ranking for the volatility measures — Appendix C reports, for each stock, the pairwise correlations among volume and the nonparametric and GARCH volatility measures. The correlation of volume with each of the volatilities is positive but the nonparametric measures show higher correlation with volume than GARCH. The highest correlation occurs with RR and RPV at 43.4% and 42.04%, respectively, on average across stocks. The correlation between the four intraday measures is high (above 90%) but drops to about 50% between the intraday measures and GARCH.

To sum up, in the context of the MDH stylised facts, RPV and RR emerge as superior intraday-estimated measures of the daily price variation given that they display the longest memory, the smallest standard deviation relative to their mean, and the highest correlation with volume. RR fares slightly better than RPV, however, in bringing returns normality.

equivalent QV) will appproximately time-scale returns and render them i.i.d. Gaussian. Their approach deviates from the MDH literature in that there is no trading activity proxy involved.

[19] The GARCH model used, fitted to the daily returns, is a GARCH(1,1) for most stocks. However, in some cases higher orders are needed to absorb all the volatility clustering. The models are described in Section 4.2.

4.2 Model estimates and in-sample model fit ranking

The estimation results by QML over the entire sample for the least traded (CAT) and most traded (MSFT) stocks are presented in Table 4, and for the remaining stocks in Appendix Table D.

[Table 4 around here]

For all stocks, the lagged intraday volatility (v_{t-1}) is strongly significant at the 1% level but, in line with the literature, lagged volume is insignificant with the exception of CAT, DELL, KO and MSFT — the significance of lagged volume in a GARCH equation would provide support for a simple MDH version (Luu and Martens, 2002). A uniform result across stocks is that the inclusion of an intraday volatility measure in the GARCH equation results in a substantial reduction in volatility persistence which, according to the MDH argument, suggests that they capture well the news arrival process — the MDH posits that the volatility clustering is partly explained by time dependence in the public information flow. Moreover, the intraday-estimated volatilities turn the ARCH coefficients from strongly significant at the 1% level to either insignificant or marginally significant at the 10% level. This suggests that the predictive information on future daily volatility contained in RV, RR, RPV or RBP encompasses the information in daily squared returns.

In order to assess in-sample model fit, the log-likelihood (lnL), AIC and SBC values can be compared across models for a given stock since they all refer to the same dependent variable. The lnL of the GARCH models augmented by each of the four intraday volatilities are greater than those of GARCH for all stocks. But this is not the case for many stocks with the volume measure. Second, the GARCH models augmented with intraday volatility measures rank top also according to the AIC and SBC. For the least traded (CAT) stock, the dynamics of daily returns seems to be best captured by the GARCH-RPV model as suggested by the lnL (largest) and AIC (smallest) values. The model ranking according to lnL, AIC and SBC is GARCH-RPV, GARCH-RV, GARCH-RBP, GARCH-RR, GARCH-VOL and GARCH.

The volatility dynamics of the most traded (MSFT) stock is best captured by GARCH-RR according to the lnL and AIC. However, as for CAT, the least improvement is brought by GARCH-VOL with a relatively small increase in lnL and a fall in the AIC. Furthermore, according to SBC, the baseline GARCH is superior to GARCH-VOL. The model-fit ranking based on lnL and AIC is GARCH-RR, GARCH-RPV, GARCH-RV, GARCH-RBP, GARCH-VOL and GARCH and the only change in this ranking according to the SBC is in the relative fit of the last two models. The ranking for the other stocks (see Appendix D) suggests that overall the GARCH-RPV provides the best fit in 8 out of 14 stocks, and GARCH-RR, GARCH-RV and GARCH-RBP in 2 stocks each.

4.3 Out-of-sample forecast ranking

Figure 2 provides a bar-chart summary of the forecast 'horse race'. The bar length is the proportion of stocks for which a given model provides the most accurate forecasts.

[Figure 2 around here]

The RPV estimator brings the largest forecast gains for most stocks and loss functions, e.g. HMSE, MME(O) and LL are smallest for GARCH-RPV in 71.4%, 71.4% and 64.3% of the stocks, respectively. GARCH-RPV is the top performer according to (virtually) all forecast criteria in seven stocks: AXP, CAT, GE, IBM, JPM, ATT and WMT. Appendix E and F provide further details.

The forecast error measures and the MZ-R^2 are set out in Table 5. For each stock, the last row reports the improvement that the best augmented-GARCH brings versus the GARCH.

[Table 5 around here]

For all stocks and loss functions, the inclusion of a lagged intraday volatility measure in the GARCH equation notably improves the forecasts. In particular, for some stocks the MZ-R^2 of the best augmented-GARCH more than trebles that of GARCH. For instance, for the least traded CAT stock, the MZ-R^2 of GARCH is 13.99% whereas that of the best forecasts, GARCH-RPV, is 43.64%. The forecast enhancement of GARCH-RV versus GARCH is in line with the findings in Martens (2001) and Koopman et al. (2005) for FX rates and the S&P100 index, respectively, and with Grané and Veiga (2007) for four DJIA stocks, American Express, Coca-Cola, Disney and Pitzer. Adding lagged trading volume to GARCH does not bring forecast gains which is consistent with the results in Brooks (1998) and Donaldson and Kamstra (2004) for stock market indexes. The ENC-T test based on the MSE for the comparison of GARCH and best augmented-GARCH (nested models) suggests that forecast improvement is significant at the 1% level throughout. Moreover, there tend to be significant differences in forecast accuracy between the alternative augmented-GARCH (non-nested) models as suggested by the DM test for most loss functions.

Considering the 11 loss functions and the 14 stocks, a total of 154 pairwise combinations, in 55% of them the GARCH-RPV model is the top forecaster, followed by GARCH-RR (19%), GARCH-RV (14%) and GARCH-RBP (12%).[20] For the least traded (CAT) stock, GARCH-RPV leads for virtually all loss functions followed by GARCH-RV. Exceptions are the asymmetric MME(U) and MME(O) for which the minimum loss is achieved by GARCH-RV but is closely followed by GARCH-RPV. GARCH-VOL is ranked last. For the most traded (MSFT) stock, according to virtually all loss functions, the best forecasts are those from the GARCH-RR model followed by GARCH-RPV. The HMSE loss is an exception with a minimum that corresponds to GARCH-RPV.

DELL and MSFT show similar behavior in the sense that, virtually according to all loss functions, the GARCH-RR provides the best forecasts. This is in contrast with the earlier finding that DELL and MSFT are the two stocks for which the RR measure has most difficulty in bringing return normality (c.f. Table 3). This suggests that whether or not a given intraday volatility measure brings normality in daily returns may not necessarily tell us much regarding its forecasting power. Moreover, for the MSFT stock, RR is not lognormally distributed at the 5% level whereas RPV, RBP and RV are. Likewise for CAT stock, although RPV emerges as top forecaster, it is not lognormally distributed whereas RV, RR and RBP are. For IBM, unanimously across all loss

[20] Using 30-minute DM/US$ and Yen/US$ data, Martens (2001) shows that extending the daily GARCH model with the sum of intraday squared returns leads to similar improvement as modeling the intraday returns directly. Hence, our results indirectly suggest that extending the daily model with RPV is superior to the latter also.

functions, RPV is the best forecaster although, interestingly, it fails to bring normality in daily returns (c.f. Table 3). Moreover, for IBM the highest persistence is shown by RR ($d = 0.403$) followed by RPV ($d = 0.394$). Therefore, when scrutinizing the individual stocks, some mismatch is observed between the ranking from forecast and MDH-related criteria.[21]

Patton (2006) and Hansen and Lunde (2006a) show theoretically and via simulation that many criteria used in the literature are inconsistent when the evaluation is based on a volatility proxy (i.e. $\tilde{\sigma}_t^2$ instead of σ_t^2) so they may favour an inferior model with a probability that converges to one asymptotically as the holdout sample (T_1) increases. Exceptions are MSE, GMLE and the levels MZ-R^2. Patton shows that the more efficient (less noisy) the proxy, the smaller the degree of distortion in the ranking which depends also on the sampling frequency. Across various loss functions, he shows analytically that when 5-min RV is used as proxy almost all of the objective bias disappears. Interestingly, the model rankings are quite similar across loss functions in Table 5 but they become rather more unstable when the crude daily r_t^2 is used as volatility proxy (see Appendix H). For instance, for AXP virtually all loss functions chose the same (GARCH-RPV) model as best when the sum of 5-min squared returns is the volatility proxy in Table 5 whereas four models (GARCH, GARCH-RV, GARCH-RV and GARCH-RBP) emerge as best from one criteria or another when r_t^2 is used as proxy in Appendix H. Reassuringly, in a majority of cases the MSE, GMLE and the MZ-R^2 criteria tend to point to the same best model when the two different proxies are used — illustrative examples are AXP and MSFT for which GARCH-RPV and GARCH-RR, respectively, are selected according to all three criteria irrespective of the proxy.

[21] We investigated whether the forecast ranking of the nonparametric volatility estimators changes when they are based on 15- and 30-min data. Appendix G reports the results for the two least-traded (CAT, ATT) and the two most-traded stocks (MSFT, DELL). The top forecaster remains the RPV. As expected, the forecast losses (like-for-like models) rise as the sampling frequency decreases, in line with the results in Pong et al. (2004) for FX rates using 5- and 30-min frequencies. Futhermore, the value-added of the nonparametric volatilities is larger at the 5-min than at the 15- and 30-min. For instance, for CAT the forecast error reduction from GARCH to GARCH-RPV is 53.88%, 43.53% and 37.78%, respectively, at the 5-, 15- and 30-min frequencies. These findings corroborate that the 5-min sampling is more useful than 15- and 30-min from the viewpoint of predicting future daily volatility. The forecast comparison is conducted by proxying the forecast target (the conditional variance) by either the sum of 5-min squared returns or the sum of 15- (or 30-) min squared returns. The main finding is that the forecast errors of like-for-like models increase when the sampling frequency of the proxy decreases.

Table 5 also reports the average losses of the combined forecasts using the time-varying weighting approach described in Section 3.3 (denoted COMBINED). We also considered an equal-weights combining scheme (denoted COMB-EQW). Notwithstanding the high correlation between the GARCH-RV, GARCH-RR, GARCH-RPV and GARCH-RBP forecasts being combined (inefficiency of weight estimates), overall across stocks and loss functions the varying-weights approach gives better results than the equal-weights approach and so only the former are reported. It is tempting to attribute this to the fact that the regression-based combining approach accounts for bias through the intercept $\hat{\alpha}_0(t)$. But the bias corrected MZ-R^2 is only higher with the equal-weights approach for half of the stocks and there is evidence of forecast bias for all but 2 stocks (see Appendix I).[22]

The COMBINED model yields the smallest forecast error with virtually all loss functions for 4 stocks (CAT, DELL, GM and MSFT) and the gains relative to the best augmented-GARCH can be as much as 51%. For instance, the HMSE of the best augmented-GARCH is reduced by 45% for DELL, 51% for GM and 44% for MSFT. For 4 other stocks (AXP, GE, IBM and ATT) the COMBINED model is in the lead for at least half of the loss functions. The ENC-T test suggests that the MSE from COMBINED is significantly smaller than that from the best augmented-GARCH in 6 stocks. Hence, jointly exploiting the information from all four intraday measures can be fruitful which indirectly corroborates that they are affected differently by microstructure noise.

The results of the SPA test are set out in Table 6. The null hypothesis is that the GARCH-RPV model which was found to be in the lead for most stocks and across most forecast accuracy metrics (hence, taken as benchmark) is not worse than any of the alternative models. These are GARCH, GARCH-RV, GARCH-RR, GARCH-RBP, GARCH-VOL, COMBINED and COMB-EQW.

[Table 6 around here]

The SPA test p-values suggest that, for eight stocks, GARCH-RPV is not outperformed by any of the seven alternative models in terms of both the MSE and MAE losses. For four further stocks

[22] Appendix H further illustrates that: (a) the GARCH-RV, GARCH-RR, GARCH-RPV and GARCH-RBP forecasts suffer from upward (if any) biases, particularly the latter three, whereas GARCH-VOL is downward biased, (b) the equal-weight forecasts are upward biased but the estimated-weights combined forecasts are generally unbiased.

(CAT, DELL, GM, MSFT), GARCH-RPV is significantly beaten by the COMBINED model. For two stocks (ATT and IBM) the evidence is mixed with MSE and MAE favouring GARCH-RPV and the COMBINED model, respectively. The last column of the table summarises the SPA test results: GARCH-RPV significantly emerges most often as the model with superior predictive ability.

Tables 7 and 8 report the HMSE, AMAPE and LL criteria, respectively, for up- and down-market days, high- and low-volume days as defined in (11) and (12).[23] For each stock, the last row (Benefit %) reports the forecast error reduction that the best augmented-GARCH brings relative to GARCH. Italics are used to signify the regime in which the largest reduction is achieved.

[Table 7 around here]

The ranking of the augmented-GARCH models is virtually identical in both regimes and GARCH-RPV ranks top for most stocks. The forecast losses tend to be smaller for down-market days. This suggests that the daily stock volatility at day t is relatively more difficult to forecast when $t-1$ is an up-market day. In the light of this finding, it is not surprising to see that the largest benefits from exploiting intraday data in order to generate a day t volatility forecast tend to occur when $t-1$ represents an up-market day. For instance, for DELL the percentage reduction in the GARCH forecast errors is 5.59 (HMSE), 5.75 (AMAPE) and 10.30 (LL) over down-market days whereas it increases, respectively, to 37.90, 14.42 and 27.51 over up-market days.

Table 8 suggests that the forecast ranking is almost identical over high- and low-volume days with the GARCH-RPV model having the smallest forecast errors.

[Table 8 around here]

Generally, the losses tend to be somewhat smaller in the high-volume regime suggesting that the conditional volatility on day t appears to be less forecastable if there was low volume on day $t-1$;

[23] Since one goal is to compare the forecast errors across market conditions, and down-market (high volume) days tend to be more volatile than up-market (low volume) days, measures such as MSE and GMLE can be misleading for this purpose. Unit-free measures such as the HMSE, AMAPE, LL and Theil-U will be more informative. The unreported (for space constaints) HMAE and Theil-U give similar results.

exceptions are IBM, MCD, MSFT and ATT. Consistent with the latter, the largest reduction in forecast errors from using intraday data (benefit %) is obtained for the low-volume days. For instance, for DELL the reduction in the GARCH forecast error by including RPV is 8.38% (HMSE), 3.76% (AMAPE) and 9.38% (LL) in the high-volume regime whereas it increases, respectively, to 39.68, 16.31 and 29.46 in the low-volume days. Another interesting case is GM (and to some extent BA) for which there is no forecast error reduction in high-volume days, but the reduction is 12.13% (HMSE), 3.68% (AMAPE) and 6.61% (LL) in low-volume days.

The upshot is that 1-day-ahead the conditional volatility is easier to forecast when the stock is underperforming ($t-1$ is a down-market day) and when trading volume is relatively high ($t-1$ is a high-volume day). Table 9 reports the average return, volatility and volume for days $t-1$ and t over different market conditions. As expected, the volatility is higher in down-market and high-volume days. Our findings are in line with the view that high volatility and volume arise largely from news arrival: when $t-1$ is a high volatility (volume) day more public information is available which, in turn, helps to forecast volatility at t.[24] The same rationale applies if $t-1$ is a down market day, because trading volume (and volatility) is higher in down versus up days. This effect is exacerbated because, as Admati and Pfleiderer (1988) show, trades from both informed and discretionary liquidity traders come in clusters, with both groups preferring to trade during 'thick' markets. This clustering of trades, when trading activity is already high, triggers the release of even more information. Moreover, high trading activity may to some extent mitigate the microstructure noise (e.g. infrequent-trading effects) and this could also explain why the augmented-GARCH models tend to produce better forecasts during high volume (volatility) days.

[Table 9 around here]

Table 9 also reports the average correlation between the 'true' volatility on days $t-1$ and t. The stronger correlations for down-market and high-volume days are in line with the finding of smaller

[24] Using firm-specific announcements data, Kalev et al. (2002) show that public information flows drive volatility and volume simultaneously, in line with the MDH. But the observed volume may also partly reflect liquidity pressures or the 'game' played by strategic traders with heterogenous information and the revision of dispersed beliefs.

forecast errors during such market conditions. Hence, the use of intraday data is more crucial when markets are relatively tranquil (low volatility), that is, on up-market and low-volume days.

5 Conclusions

How to forecast daily volatility is a challenging question because, unlike prices and volume, volatility is not directly observable. A recent literature focuses on exploiting the intraday variation and proposes several nonparametric estimators called *realised* variance, range, power variation and bipower variation. This paper compares these estimators on the basis both of their distributional properties and their ability to forecast one-day-ahead the conditional variance of returns. A GARCH framework is adopted as platform to compare their incremental predictive content which is not to suggest that GARCH is the 'best' framework for volatility prediction. For completeness, a volume measure of intraday trading activity is also included in the horse race.

The popular realized variance estimator is dominated by the realised power variation and the realised range. The realized range fares relatively well in the in-sample distributional analysis regarding the extent to which it brings normality in standardized returns. However, overall across stocks and loss functions the realised power variation appears to be the top performer for short term forecasts of one day. This means that, among the four nonparametric volatility estimators, the lattter enhances the GARCH forecasting ability the most. A rationale for this finding is that the realized power variation is not only immune to jumps, like the realized bipower variation, but it is also the most persistent and less noisy. Nevertheless, forecast combining appears worthwhile for about half of the stocks which indirectly corroborates that the four intraday-estimated volatility measures are impacted by microstructure noise in different ways. The additional use of intraday data on day $t-1$ to forecast volatility on day t is more advantageous when $t-1$ is an up-market or low volume day relative to the recent past. Since daily volatility forecasts are key inputs for VaR analysis, our findings may have important practical implications for this area of risk management.

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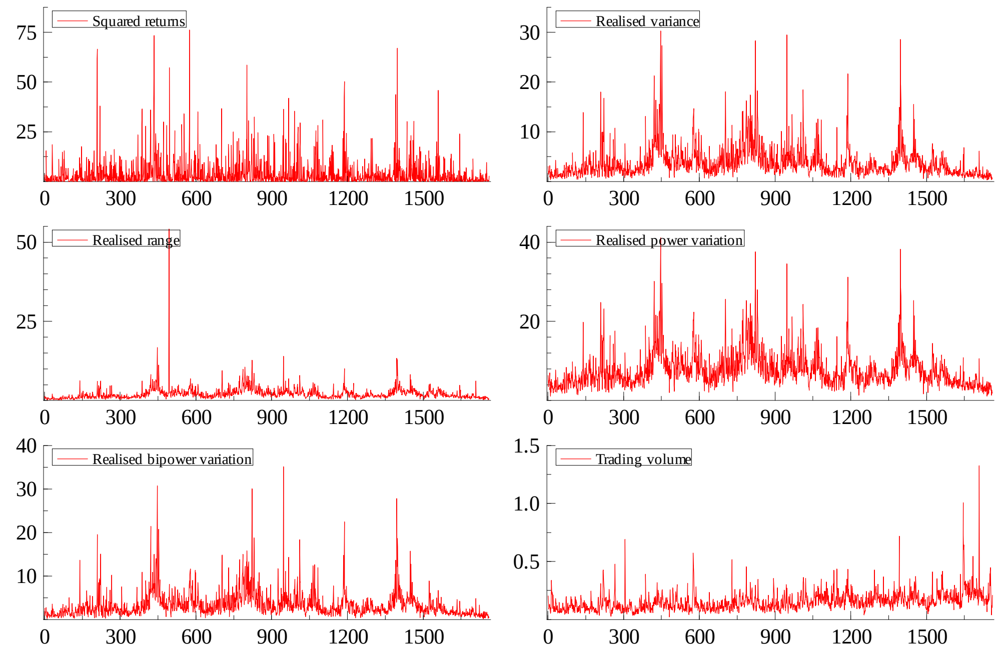
A. Least traded stock (CAT, Caterpillar)



B. Most traded stock (MSFT, Microsoft)

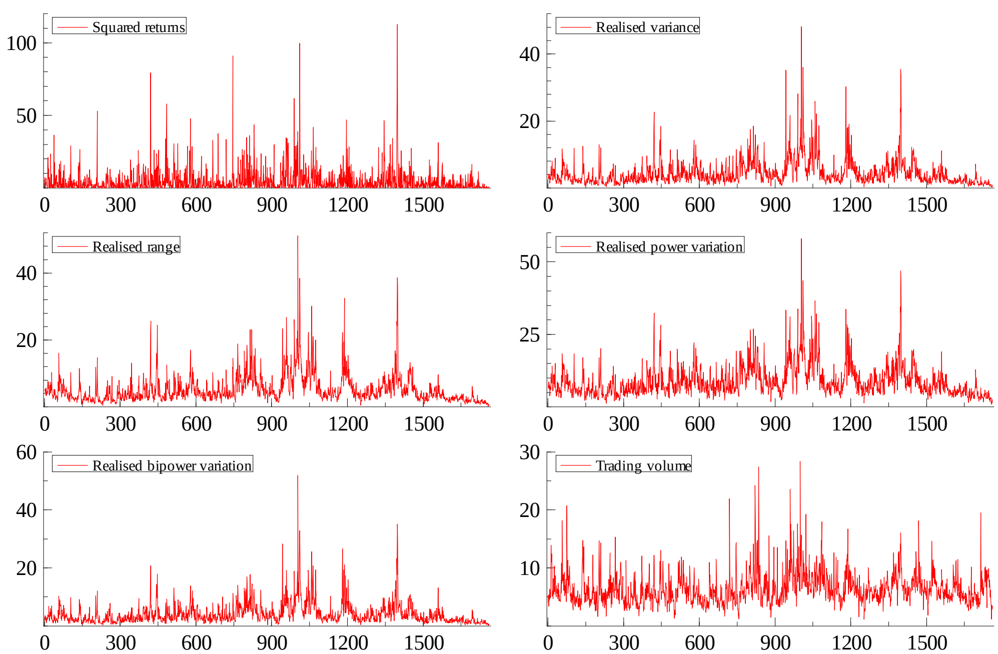


Fig. 1 Time series plots for daily volatility and volume

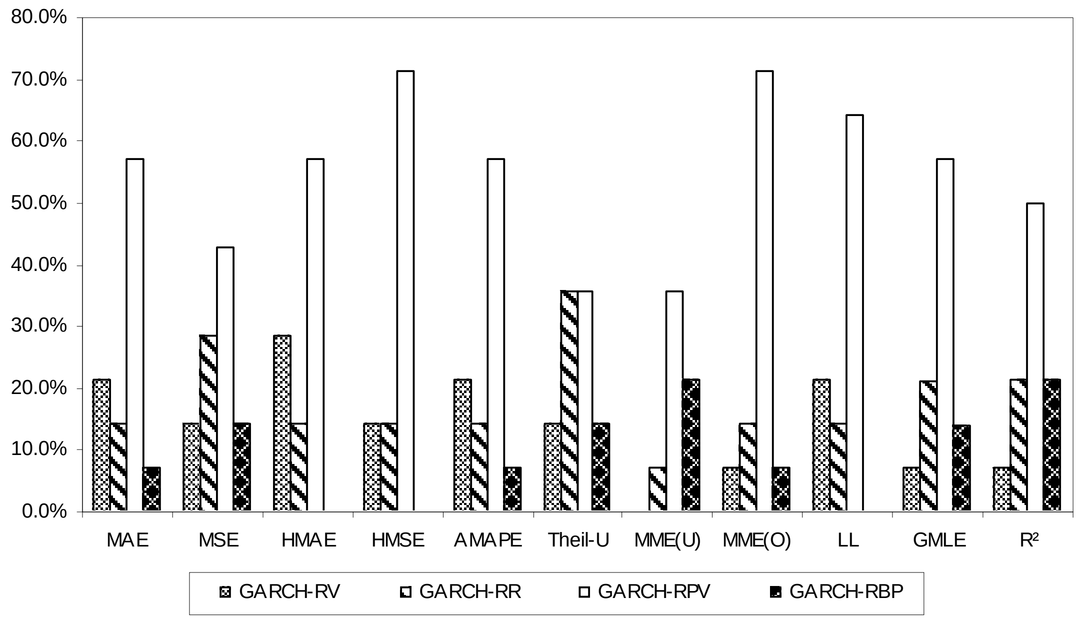


Fig. 2. Summary of forecast competition for the 14 individual NYSE stocks.

The figure plots the frequency that each model had the smallest out-of-sample loss in terms of several loss functions and the R^2 of the levels Mincer-Zarnowitz regression.

Table 1. Unconditional daily stock return and trading volume distribution

	ATT	**AXP**	**BA**	**CAT**	**DELL**	**GE**	**GM**	**IBM**	**JPM**	**KO**	**MCD**	**MSFT**	**PG**	**WMT**
							daily returns (%)							
Mean	-0.095	-0.001	0.012	-0.036	0.102	-0.015	-0.106	-0.012	-0.015	0.081	0.083	0.010	0.130	-0.003
StDev	2.268	2.165	1.985	1.998	2.993	1.866	1.886	1.927	2.353	1.653	1.784	2.184	1.635	2.005
Skewness	0.194	-0.026	0.044	0.132	0.218	0.269	0.142	0.172	0.960	0.027	-0.174	0.210	-0.062	0.130
Kurtosis	4.915	4.392	5.231	4.334	6.218	5.293	4.107	5.150	14.035	5.951	6.740	4.048	7.077	5.497
JB test	280.3	274.3	366.1	135.9	774.0	407.2	95.9	472.8	9207.1	639.2	1035.416	93.7	1220.3	462.8
(p-value)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
LB(10) test	***22.189***	**13.276**	**12.340**	**10.545**	25.033	**12.894**	***18.668***	32.738	**15.675**	**15.413**	**6.822**	**12.626**	34.987	***21.456***
(p-value)	***(0.014)***	**(0.209)**	**(0.263)**	**(0.394)**	(0.005)	**(0.230)**	***(0.045)***	(0.000)	**(0.109)**	**(0.118)**	**(0.742)**	**(0.245)**	(0.000)	***(0.018)***
							daily trading volume ($VOL \times 10^{-7}$)							
Mean	0.247	0.438	0.317	0.155	3.950	1.480	0.293	0.714	0.695	0.368	0.419	6.394	0.266	0.608
StDev	2.233	0.215	0.179	0.084	3.086	0.667	0.184	0.343	0.350	0.170	0.246	2.743	0.176	0.252
Skewness	7.775	2.309	3.815	3.570	2.182	2.006	2.191	2.722	2.398	1.842	3.027	2.032	9.788	1.752
Kurtosis	128.364	14.241	29.778	34.609	9.507	10.277	10.172	18.462	15.756	8.883	19.028	11.460	189.924	8.817
ADF test	-4.77(13)	-8.81(6)	-14.06(3)	-6.96(11)	-4.76(24)	-5.99(12)	-3.83(21)	-6.31(19)	-6.21(10)	-8.13(8)	-4.27(23)	-6.00(18)	-8.22(9)	-4.59(19)
Robinson d	0.385	0.337	0.230	0.293	0.434	0.371	0.407	0.340	0.371	0.325	0.330	0.316	0.309	0.327
KS test	0.0407	0.0457	**0.0304**	***0.0379***	0.0574	**0.0198**	**0.0277**	0.0458	**0.0187**	**0.0223**	**0.0191**	0.0546	0.0407	0.0441
(p-value)	(0.006)	(0.001)	**(0.075)**	***(0.012)***	(0.000)	**(0.489)**	**(0.132)**	(0.001)	**(0.564)**	**(0.337)**	**(0.533)**	(0.000)	(0.006)	(0.002)

JB denotes the Jarque-Bera test statistic for the null hypothesis of normality. LB(p) denotes the Ljung-Box Q-statistic Q-statistic for the null of no autocorrelation up to a maximum lag of p days. ADF is the augmented Dickey-Fuller statistic for the null of a unit root with 5% and 1% critical values of -2.862 and -3.433, respectively. The numbers in parenthesis indicate the truncation lag chosen based on the AIC. Robinson d is the fractional differencing parameter based on Robinson's (1995) estimator. KS denotes the Kolmogorov-Smirnov statistic for the null of log-normality. Bold and bold italics denote insignificant, respectively, at the 5% (or 10%) and the 1% level.

Table 2. Unconditional daily stock volatility distributions

	ATT	**AXP**	**BA**	**CAT**	**DELL**	**GE**	**GM**	**IBM**	**JPM**	**KO**	**MCD**	**MSFT**	**PG**	**WMT**
Squared returns(r_t^2)														
Mean	5.150	4.683	3.940	3.991	8.965	3.481	3.567	3.714	5.535	2.740	3.189	4.765	2.690	4.021
StDev	10.141	9.289	8.108	7.279	20.529	7.206	6.251	7.890	19.972	6.093	7.610	8.328	6.597	8.522
StDev/Mean	1.969	1.983	2.057	1.823	2.289	2.070	1.752	2.124	3.608	2.223	2.386	1.747	2.452	2.119
Skewness	5.434	5.820	5.568	4.352	9.916	7.059	4.100	8.178	20.758	8.601	9.084	5.035	8.003	6.898
Kurtosis	46.480	56.130	47.390	29.840	160.129	82.450	27.220	113.350	586.580	124.030	121.660	44.530	92.770	87.457
ADF	-10.27(8)	-5.64(23)	-24.83(1)	-8.76(12)	-10.81(8)	-6.72(18)	-7.07(15)	-12.76(6)	-31.48(0)	-10.07(9)	-35.30(0)	-17.51(3)	-5.95(23)	-6.19(18)
Robinson d	0.180	0.269	0.139	0.175	0.197	0.086	0.294	0.152	0.080	0.174	0.096	0.129	0.163	0.167
KS	0.0591	0.0731	0.0479	0.0684	0.053	0.0696	0.0647	0.0777	0.0653	0.066	0.0509	0.08	0.0631	0.0634
(p-value)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
Realised variance(RV)														
Mean	4.506	4.673	4.078	3.782	8.170	3.648	3.021	3.572	5.622	2.836	3.550	4.441	2.905	4.140
StDev	4.356	5.065	3.950	3.176	7.689	3.776	2.950	3.663	7.717	2.417	3.337	3.892	3.080	4.298
StDev/Mean	0.966	1.083	0.968	0.839	0.941	1.035	0.976	1.025	1.372	0.852	0.940	0.876	1.470	1.038
Skewness	4.190	4.673	6.773	3.054	3.570	4.629	4.145	6.442	8.931	3.206	4.511	3.486	5.152	6.340
Kurtosis	31.610	32.030	105.240	17.800	25.870	38.220	33.070	92.930	136.670	19.070	35.830	24.560	49.260	90.930
ADF	-4.73(18)	-5.20(17)	-8.63(7)	-4.68(18)	-3.75(24)	-5.48(20)	-6.70(7)	-5.04(18)	-5.31(20)	-4.85(21)	-5.68(15)	-5.60(16)	-4.49(21)	-5.72(13)
Robinson d	0.345	0.358	0.337	0.409	0.373	0.330	0.385	0.371	0.318	0.368	0.329	0.387	0.358	0.350
KS	**0.0181**	**0.0253**	**0.0199**	**0.0203**	**0.0284**	***0.0387***	***0.0331***	**0.0211**	**0.0234**	**0.0265**	**0.0319**	**0.0226**	**0.0215**	**0.0225**
(p-value)	**(0.606)**	**(0.207))**	**(0..481)**	**(0.455)**	**(0.114)**	**(0.010)**	***(0.042)***	**(0.408)**	**(0.284)**	**(0.168)**	**(0.055)**	**(0.323)**	**(0.382)**	**(0.327)**
Realised range(RR)														
Mean	3.014	2.859	2.748	2.131	10.028	2.741	1.745	2.525	3.839	1.889	2.554	5.243	1.871	2.648
StDev	2.656	2.711	2.284	2.014	9.300	2.345	1.624	2.119	4.408	1.363	2.124	4.285	1.697	2.137
StDev/Mean	0.881	0.948	0.831	0.945	0.927	0.855	0.930	0.383	1.148	0.721	0.831	0.817	0.907	0.807
Skewness	3.671	3.239	3.509	11.102	3.830	3.765	3.357	3.557	7.050	2.624	3.662	3.144	4.556	2.872
Kurtosis	26.610	20.020	28.630	259.890	33.470	31.970	20.650	30.240	90.890	14.690	25.890	20.260	43.510	18.090
ADF	-4.60(18)	-4.66(17)	-7.01(7)	-5.28(13)	-3.43(24)	-4.85(20)	-5.65(9)	-4.36(18)	-4.82(23)	-4.53(15)	-5.91(11)	-4.88(18)	-4.31(20)	-4.95(12)
Robinson d	0.370	0.404	0.391	0.381	0.396	0.382	0.426	0.406	0.350	0.388	0.364	0.400	0.382	0.397
KS	**0.0148**	**0.0239**	**0.0218**	**0.0199**	**0.0136**	**0.0302**	0.0394	**0.0211**	**0.0145**	**0.0202**	**0.0126**	***0.0348***	**0.0316**	**0.0135**
(p-value)	**(0.827)**	**(0.265)**	**(0.363)**	**(0.479)**	**(0.898)**	**(0.079)**	(0.008)	**(0.403)**	**(0.850)**	**(0.462)**	**(0.938)**	***(0.028)***	**(0.058)**	**(0.901)**
Realised power variation(RPV)														
Mean	8.680	8.863	8.111	7.468	13.888	7.573	6.356	7.408	10.112	6.315	7.315	8.887	6.280	8.121
StDev	5.679	6.351	5.064	4.471	9.158	5.145	4.172	4.770	8.256	3.741	4.486	5.408	4.293	5.491
StDev/Mean	0.654	0.716	0.624	0.598	0.659	0.679	0.656	0.643	0.816	0.592	0.613	0.608	0.683	0.676
Skewness	2.832	2.659	3.130	2.161	2.396	2.978	2.469	2.367	4.875	2.293	2.895	2.252	2.856	2.832
Kurtosis	17.017	14.370	24.786	10.819	13.350	17.554	12.818	13.373	47.943	11.694	17.751	12.366	17.600	20.182
ADF	-4.42(18)	-4.71(17)	-6.79(10)	-4.73(12)	-3.53(24)	-4.98(20)	-5.87(9)	-4.37(18)	-4.66(20)	-4.21(18)	-5.71(12)	-5.25(16)	-5.34(12)	-4.93(13)
Robinson d	0.370	0.389	0.377	0.421	0.392	0.363	0.409	0.394	0.380	0.385	0.365	0.392	0.396	0.393
KS	**0.0156**	**0.0245**	**0.0233**	0.0430	**0.0117**	***0.0333***	**0.0302**	**0.0204**	**0.0219**	**0.0220**	**0.0246**	**0.0222**	**0.0295**	**0.0167**
(p-value)	**(0.779)**	**(0.238)**	**(0.286)**	(0.003)	**(0.968)**	***(0.040)***	**(0.078)**	**(0.449)**	**(0.358)**	**(0.354)**	**(0.232)**	**(0.343)**	**(0.091)**	**(0.706)**
Realised bipower variation(RBP)														
Mean	4.090	4.292	3.702	3.401	7.652	3.416	2.734	3.340	5.175	2.598	3.252	4.146	2.676	3.724
StDev	4.252	4.732	3.775	3.048	7.480	3.710	2.783	3.210	7.024	2.369	3.263	3.753	2.879	3.888
StDev/Mean	1.039	1.102	1.019	0.896	0.977	1.086	1.0179	0.961	1.357	0.911	1.003	0.905	1.075	1.044
Skewness	4.506	3.931	7.403	3.529	3.656	4.945	3.818	6.442	7.799	3.549	4.798	3.658	5.253	4.961
Kurtosis	34.320	25.60	124.150	23.790	25.350	42.870	25.730	92.930	101.450	22.800	40.060	28.230	51.290	51.810
ADF	-4.79(18)	-6.46(11)	-8.67(7)	-4.71(19)	-3.82(24)	-5.55(24)	-6.69(7)	-7.00(7)	-4.98(21)	-5.00(21)	-6.33(12)	-6.96(6)	-4.42(21)	-5.01(17)
Robinson d	0.340	0.364	0.336	0.402	0.364	0.327	0.387	0.383	0.331	0.363	0.328	0.387	0.350	0.361
KS	**0.0180**	**0.0267**	**0.0170**	**0.0226**	**0.0225**	***0.0359***	**0.0262**	**0.0188**	**0.0229**	**0.0290**	**0.0233**	**0.0240**	**0.0221**	**0.0177**
(p-value)	**(0.613)**	**(0.159)**	**(0.683)**	**(0.324)**	**(0.329)**	***(0.021)***	**(0.174)**	**(0.551)**	**(0.309)**	**(0.102)**	**(0.288)**	**(0.259)**	**(0.351)**	**(0.634)**

See note in Table 1. The daily RV, RR, RPV and RBP are based on prices sampled at the 5-min frequency. RPV is computed for z=1.5.

Table 3. Normality tests for daily returns and standardized daily returns

	ATT	**AXP**	**BA**	**CAT**	**DELL**	**GE**	**GM**	**IBM**	**JPM**	**KO**	**MCD**	**MSFT**	**PG**	**WMT**
								r_t						
JB	280.28	274.28	366.12	135.87	774.04	407.18	95.88	472.83	9207.09	639.23	1035.42	93.68	1220.25	462.83
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
								$r_t/\sqrt{\mathbf{GARCH}_t}$						
JB	214.92	76.34	175.91	67.52	246.85	146.56	19.89	105.05	268.85	169.67	931.94	43.82	313.67	53.33
p-value	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
								$r_t/\sqrt{\mathbf{RV}_t}$						
JB	***6.81***	12.80	**4.43**	**4.65**	12.25	14.66	12.24	15.42	***8.94***	**4.42**	**2.52**	22.43	**4.94**	**5.66**
p-value	**(0.033)**	(0.000)	**(0.109)**	**(0.097)**	(0.002)	(0.000)	(0.002)	(0.000)	**(0.011)**	**(0.109)**	**(0.283)**	(0.000)	**(0.084)**	**(0.058)**
								$r_t/\sqrt{\mathbf{RR}_t}$						
JB	**2.16**	***6.34***	**0.05**	**1.45**	***10.19***	***8.49***	**2.59**	**3.04**	**1.40**	**4.34**	**3.89**	22.82	**1.23**	**5.36**
p-value	**(0.339)**	***(0.042)***	**(0.977)**	**(0.485)**	***(0.010)***	***(0.014)***	**(0.274)**	**(0.219)**	**(0.497)**	**(0.111)**	**(0.143)**	(0.000)	**(0.540)**	**(0.069)**
								$r_t/\sqrt{\mathbf{RPV}_t}$						
JB	**5.68**	9.35	**1.10**	**3.34**	***6.79***	13.23	12.73	9.36	***7.37***	**2.81**	**0.29**	19.61	**0.41**	**2.87**
p-value	**(0.058)**	(0.009)	**(0.576)**	**(0.188)**	***(0.033)***	(0.001)	(0.001)	(0.009)	***(0.025)***	**(0.244)**	**(0.867)**	(0.000)	**(0.816)**	**(0.283)**
								$r_t/\sqrt{\mathbf{RBP}_t}$						
JB	**4.71**	10.72	**3.31**	**3.69**	11.21	17.35	10.84	12.14	***6.10***	***7.36***	**1.38**	21.51	**2.87**	**4.26**
p-value	**(0.095)**	(0.005)	**(0.191)**	**(0.158)**	(0.004)	(0.000)	(0.004)	(0.002)	***(0.047)***	***(0.023)***	**(0.501)**	(0.000)	**(0.241)**	**(0.118)**

RV, RR, RBP and RPV are calculated using prices sampled at the 5-min frequency. RPV is calculated using power order $p = 1.5$. JB is the Jarque-Bera test. Bold and bold italics indicate that the normality null cannot be rejected at the 5% (or 10%) and the 1% level, respectively
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Table 4. GARCH estimation results from 02/01/97 to 31/12/03

	CAT (least traded stock)						**MSFT (most traded stock)**					
	GARCH	with RV	with RR	with RPV	with RBP	with VOL	GARCH	with RV	with RR	with RPV	with RBP	with VOL
θ_0	0.0200	-0.0084	-0.0146	-0.0077	-0.0087	0.0260	0.0213	-0.0095	-0.0087	-0.0074	-0.0128	0.0251
	(0.0437)	*(0.0439)*	*(0.0444)*	*(0.0438)*	*(0.0444)*	*(0.0450)*	*(0.0477)*	*(0.0461)*	*(0.0465)*	*(0.0472)*	*(0.0473)*	*(0.0484)*
ω	0.0366	**0.3260**	**0.6074**	0.0903	**0.3092**	**0.0894**	**0.2384**	**0.5931**	**0.5774**	0.0298	**0.5970**	0.2866
	(0.0231)	*(0.1426)*	*(0.2241)*	*(0.1151)*	*(0.1329)*	*(0.0329)*	*(0.0897)*	*(0.1852)*	*(0.2149)*	*(0.1656)*	*(0.1792)*	*(0.2080)*
α_1	**0.1108**	0.0559	**0.0788**	0.0577	0.0645	**0.1004**	**0.1015**	0.0278	0.0170	0.0302	0.0426	**0.1379**
	(0.0401)	*(0.0412)*	*(0.0420)*	*(0.0421)*	*(0.0431)*	*(0.0220)*	*(0.0217)*	*(0.0301)*	*(0.0265)*	*(0.0258)*	*(0.0259)*	*(0.0262)*
α_2	**-0.0844**	-0.0472	-0.0380	-0.0543	-0.0544	**-0.0768**						
	(0.0398)	*(0.0358)*	*(0.0342)*	*(0.0351)*	*(0.0375)*	*(0.0227)*						
β_1	**0.9646**	**0.6891**	**0.5760**	**0.6871**	**0.7261**	**0.9656**	**0.8507**	**0.3730**	**0.2258**	**0.4006**	**0.4028**	**0.5585**
	(0.0124)	*(0.0996)*	*(0.1250)*	*(0.1033)*	*(0.0888)*	*(0.0094)*	*(0.0343)*	*(0.1111)*	*(0.0975)*	*(0.0775)*	*(0.0778)*	*0.0625*
γ		**0.2350**	**0.4317**	**0.1494**	**0.2211**	**-2.97e-08**		**0.5176**	**0.5889**	**0.3013**	**0.5026**	**1.83e-08**
		(0.0823)	*(0.1498)*	*(0.0523)*	*(0.0790)*	*(1.05e-08)*		*(0.1108)*	*(0.0811)*	*(0.0431)*	*(0.0760)*	*(4.55e-09)*
lnL	-3670.44	-3643.614	-3650.790	-3641.110	-3645.447	-3663.152	-3813.564	-3762.681	-3753.741	-3761.323	-3763.916	-3810.397
AIC	4.1743	4.1473	4.1554	4.1444	4.1494	4.1695	4.3357	4.2815	4.2713	4.2799	4.2829	4.3357
SBC	4.1898	4.1659	4.1741	4.1631	4.1680	4.1882	4.3481	4.2970	4.2868	4.2955	4.2984	4.3512
$LB(10)$	0.463	0.231	0.264	0.230	0.206	0.563	0.549	0.450	0.372	0.453	0.451	0.491
$ARCH(10)$	0.488	0.481	0.460	0.478	0.508	0.451	0.342	0.714	0.454	0.660	0.651	0.525
Ranking	6	2	4	1	3	5	5 or 6	3	1	2	4	6 or 5

The reported estimation results are for the conditional mean equation $r_t = \theta_0 + u_t, u_t|\mathcal{F}_{t-1} \sim iid(0, h_t)$, and conditional variance equation $h_t = \omega + \alpha_1 u_{t-1}^2 + ... + \alpha_r u_{t-r}^2 + \beta_1 h_{t-1} + ... + \beta_s h_{t-s} + \gamma v_{t-1}$. $LB(10)$ is the Ljung-Box test for autocorrelation in u_t. $ARCH(10)$ is Engle's ARCH LM test. The intraday sampling frequency for RV, RR, RPV, RBP is 5 min. Bold denotes significance at the 1%, 5% or 10% level. Bollerslev-Wooldridge s.e. are reported in parentheses. The degree of volatility persistence is given by $\lambda = \sum_{i=1}^{r} \alpha_i + \sum_{j=1}^{s} \beta_j$. Rankings 1 and 6 mean top and bottom, respectively, according to in-sample model fit.

Table 5. Out-of-sample forecast evaluation

Stock	Model	MAE	MSE	HMAE	HMSE	AMAPE	Theil-U	MME(U)	MME(O)	LL	GMLE	MZ-R^2
		Forecast accuracy measures										
	GARCH	3.0166	22.4841***	0.9492	2.2441	0.2831	0.8428	59.3852	13.6460	0.5699	2.5595	28.8653
	GARCH-RV	2.6179***	20.3382*	0.7629***	1.3708**	0.2442***	0.7624*	48.8266	13.5681***	0.4401	2.5101***	36.9995**
	GARCH-RR	2.8303***	21.3482**	0.8636***	1.7020***	0.2626***	0.8002**	52.8509**	15.6177***	0.4941***	2.5215***	35.8314*
ATT	GARCH-RPV	**2.5203**	**18.8451**	**0.7224**	**1.1681**	**0.2390**	**0.7064**	49.3295	**11.0480**	**0.4118**	**2.5006**	**39.6859**
	GARCH-RBP	2.6583***	20.8671**	0.7779***	1.4241**	0.2467***	0.7822**	**46.8354**	15.0448***	0.4483***	2.5118***	37.2140**
	GARCH-VOL	5.2470***	47.8291***	2.2905***	17.3949***	0.3869***	1.7781***	68.8293**	47.6280***	1.3396***	2.7446***	2.5456***
	COMBINED	2.4400	20.0675	0.6141	0.8372	0.2223	0.7522	48.9171	10.8525	0.3564	2.4891	35.1258
	Benefit (%)	16.45	16.18	23.89	47.95	15.58	16.18	21.13	19.04	27.74	2.30	37.49
	GARCH	2.0818	16.4060***	0.8336	1.9394	0.2572	1.1495	49.3419	7.7150	0.4831	2.2114	29.0442
	GARCH-RV	1.7938***	10.6703**	0.6812***	1.0788**	0.2263	0.7476**	**28.7188**	7.0649	0.3497*	2.1425***	54.8918
	GARCH-RR	2.2248***	12.3740***	0.9791**	2.0343***	0.2794*	0.8668***	41.7935	9.6238***	0.5314***	2.1975***	53.2825
AXP	GARCH-RPV	**1.6658**	**10.3212**	**0.6106**	**0.8284**	**0.2113**	**0.7230**	29.6144	**5.6148**	**0.3079**	**2.1287**	**56.9665**
	GARCH-RBP	1.7594**	10.4086	0.6790**	1.0871**	0.2242**	0.7291	28.9881	6.8762	0.3487**	2.1413**	56.1080
	GARCH-VOL	2.3054**	18.2394***	1.5165**	38.9584***	0.2708***	1.2780**	49.7656*	10.3297*	0.7109**	2.2552**	22.2397**
	COMBINED	1.6789	10.3938	***0.5053***	***0.5119***	***0.2037***	***0.6963***	***22.4327***	6.2478	0.3227	2.2939	56.0890
	Benefit (%)	19.98	37.09	26.75	57.29	17.84	37.10	41.80	27.22	36.27	3.74	96.14
	GARCH	1.6993	7.8206***	0.4806	0.5322	0.1972	1.1670	15.2132	4.4878	0.2804	2.3894	35.4082
	GARCH-RV	**1.4898**	**5.4187**	**0.4530**	0.4560	**0.1790**	**0.8086**	**11.1988**	4.2482*	0.2251***	2.3501	54.4613**
	GARCH-RR	1.7047**	5.8298**	0.5712***	0.7002***	0.2010**	0.8698**	12.3551	5.7955***	0.2817***	2.3638***	53.5012*
BA	GARCH-RPV	1.5012	5.4405	0.4562	**0.4455**	0.1799	0.8118	11.6785	**4.0773**	**0.2241**	**2.3491**	**55.7621**
	GARCH-RBP	1.5287**	5.4633	0.4694***	0.4866**	0.1826**	0.8152	11.4482	4.5720***	0.2312**	2.3515	53.6656**
	GARCH-VOL	2.3346***	11.1985**	0.7861***	1.6217***	0.2577***	1.6705**	22.2042*	8.8998***	0.4785*	2.4588***	8.4811*
	COMBINED	1.7468	6.0563	0.5482	0.5976	0.1990	0.9036	11.5966	6.3922	0.2674	2.3611	53.7150
	Benefit (%)	12.33	30.71	5.74	16.29	9.23	30.71	26.39	9.15	20.08	1.69	57.48
	GARCH	1.7797	6.1406***	0.9471	1.8327	0.2811	1.2597	16.8100	5.7511	0.5316	2.0682	13.9861
	GARCH-RV	1.3050**	3.8605	0.6660**	0.9716**	0.2215*	0.7921	**11.2370**	**3.6301**	0.3350***	1.9917	42.6145*
	GARCH-RR	1.5758**	4.7520***	0.8491***	1.4886***	0.2580	0.9750***	14.7864**	4.8381***	0.4481**	2.0293*	37.0931**
CAT	GARCH-RPV	**1.2880**	**3.8056**	**0.6340**	**0.8451**	**0.2165**	**0.7809**	12.0083	3.6810	**0.3130**	**1.9850**	**43.6401**
	GARCH-RBP	1.3401	3.9384***	0.6808***	0.9656	0.2263***	0.8081***	12.1031**	3.8194**	0.3420	1.9945	42.4124**
	GARCH-VOL	2.2153***	8.9839***	1.2814***	4.6883***	0.3117***	1.8430***	19.9459**	9.2635**	0.7403**	2.1294***	4.9856*
	COMBINED	1.1176	***3.5523***	0.4617	0.5100	0.1813	0.7287	7.7120	3.2263	0.2373	1.9708	43.8049
	Benefit (%)	27.63	38.03	34.91	53.88	22.98	38.01	33.15	36.88	41.21	4.04	212.03
	GARCH	2.2559	10.2684***	0.6453	0.8808	0.2217	1.2250	17.7316	10.6282	0.3252	2.4342	42.3244
	GARCH-RV	2.3391***	9.7267***	0.6947***	0.9314***	0.2307***	1.1614***	19.2699**	10.7855***	0.3372***	2.4332***	51.5926
	GARCH-RR	**1.9811**	**8.0345**	**0.5513**	**0.6249**	**0.1983**	**0.9562**	16.5241	**8.1202**	**0.2569**	**2.4068**	**51.6494**
DELL	GARCH-RPV	2.3758***	9.9858***	0.6863***	0.9127***	0.2311***	1.1919***	**15.8615**	11.2197***	0.3393***	2.4371***	51.2849
	GARCH-RBP	2.3606***	10.0715***	0.7020***	0.9877***	0.2311***	1.2025***	18.3796*	11.0655***	0.3454***	2.4365***	49.6899***
	GARCH-VOL	7.3802***	119.4348***	2.1633***	13.8656***	0.3659***	14.2745***	16.1837	149.0813***	1.2698***	2.6959***	12.8040**
	COMBINED	1.6742	***6.9393***	0.4129	0.3455	0.1767	0.8275	11.7587	6.6457	0.2293	2.4282	54.1353
	Benefit (%)	41.80	21.76	14.57	29.05	10.55	21.94	10.55	23.60	21.00	1.13	22.06

The target is the daily conditional variance proxied by the sum of 5-min squared returns. Bold indicates the top performer. Asterisks denote that the forecasts of the model are significantly worse (DM test), * at 10% , ** 5%, *** or 1% level, than those of the top performer. The GARCH vs augmented-GARCH comparison is based on the ENC-T test for nested models developed for the MSE loss. Likewise for the pairwise comparison of intraday-volatility.agurmented GARCH and the COMBINED model. For the latter, italics bold (under MSE) indicates that it beats the best individual forecast at the 10%, 5% or 1% level.

Table 5. Out-of-sample forecast evaluation (*cont.*)

		Forecast accuracy measures										
Stock	**Model**	MAE	MSE	HMAE	HMSE	AMAPE	Theil-U	MME(U)	MME(O)	LL	GMLE	MZ-R^2
	GARCH	1.7272	10.1324***	0.6340	0.8646	0.2287	0.9502	29.0961	5.0890	0.3619	2.1341	41.1487
	GARCH-RV	1.5628***	7.9192	0.5758**	0.7745	0.2099	0.7426	19.9198*	5.1772**	0.3023	2.0969	52.5671*
	GARCH-RR	1.6990***	**7.6688**	0.7016***	1.0634	0.2327***	**0.7191**	23.0784**	5.9512***	0.3675***	2.1128**	**55.5787**
GE	GARCH-RPV	**1.4915**	7.7464	**0.5167**	**0.5886**	**0.1972**	0.7264	19.8920	**4.4162**	**0.2648**	**2.0835**	55.2760
	GARCH-RBP	1.5755***	7.9490	0.5757**	0.7466	0.2109	0.7455	**18.6959**	5.4607**	0.3007	2.0961	52.0659**
	GARCH-VOL	2.0702***	10.2605***	0.8652***	1.4644***	0.2701***	0.9615***	30.9637**	7.5387***	0.4812***	2.1544***	41.9056***
	COMBINED	1.5448	***7.3832***	0.4780	0.4221	0.2166	0.6840	13.0771	5.5854	0.3906	2.2940	55.6449
	Benefit (%)	13.65	23.55	18.50	31.92	13.77	24.32	35.74	13.22	26.83	2.37	35.07
	GARCH	1.4905	4.8797***	0.7436	1.3186	0.2366	0.7623	11.9501	4.9465	0.3872	1.9968	53.5156
	GARCH-RV	1.4402	4.4830	**0.7256**	1.2516	0.2332	0.7004	11.9056*	4.6104	**0.3757**	1.9934	57.3629*
	GARCH-RR	1.9465***	6.4279***	1.0046***	2.0327***	0.2880***	1.0043***	13.0465*	7.7755****	0.5385***	2.0432**	57.0323*
GM	GARCH-RPV	1.4508	**4.4050**	0.7296	**1.2294**	0.2350*	**0.6882**	11.9278	**4.4331**	0.3768	1.9942	58.8185
	GARCH-RBP	**1.4400**	4.4190	0.7320	1.3035*	**0.2325**	0.6903	**10.9091**	4.8677	0.3776**	**1.9925**	**59.0001**
	GARCH-VOL	1.5957**	5.8561***	0.7498	1.4247	0.2397	0.9150***	12.7515	5.8363***	0.4093	2.0122*	45.5787***
	COMBINED	1.2868	***4.1314***	0.5276	0.6028	0.2030	0.6457	8.8911	3.9220	0.2859	1.9846	58.4042
	Benefit (%)	3.39	9.73	2.42	6.76	1.73	9.72	8.71	10.38	2.97	0.215	10.25
	GARCH	1.4223	4.0962***	0.8016	1.1481	0.2601	1.3608	13.2574	4.3714	0.4228	1.8151	42.7414
	GARCH-RV	1.1013**	2.5202**	0.6505**	0.8326***	0.2199	0.8369**	6.4769	3.1139***	0.3195	1.7721**	63.9620**
	GARCH-RR	1.2717***	2.8058***	0.7823***	1.1345***	0.2483***	0.9315***	7.5023**	3.7029***	0.3981***	1.7983***	64.9736
IBM	GARCH-RPV	**1.0543**	**2.3986**	**0.5879**	**0.6620**	**0.2070**	**0.7965**	**6.4115**	**2.7681**	**0.2784**	**1.7580**	**65.9708**
	GARCH-RBP	1.1041***	2.6050**	0.6470*	0.8170*	0.2193	0.8651**	6.6680	3.0939***	0.3174*	1.7717**	62.7126*
	GARCH-VOL	4.7177***	27.0749***	3.7685***	31.2741***	0.5174***	8.9977***	19.8335**	30.7368***	2.1877***	2.2842***	3.4678**
	COMBINED	1.0223	***2.3060***	0.4975	0.4563	0.2139	0.7549	4.2532	2.8345	0.4336	2.1966	66.1373
	Benefit (%)	25.87	41.44	26.66	42.34	20.42	41.47	51.64	36.68	34.15	3.16	54.35
	GARCH	3.3925	57.9052***	0.7523	1.1999	0.2506	0.9217	**107.8462**	44.1741	0.4308	2.4915	51.6710
	GARCH-RV	2.5663	52.7961	0.4741**	0.4968**	0.1869*	0.8404	114.3802	23.3971*	0.2504	2.4202***	52.7068
	GARCH-RR	2.7198***	**49.2440**	0.5943***	0.6837***	0.2132*	**0.7822**	156.7707	16.4277*	0.3080***	2.4389***	59.1868
JPM	GARCH-RPV	**2.3625**	50.1563	**0.4191**	**0.3293**	**0.1777**	0.7968	118.1784	**9.3414**	**0.2216**	**2.4183**	**64.2387**
	GARCH-RBP	2.5247	50.3707	0.4675**	0.4917*	0.1847	0.8018	110.5105	20.1947*	0.2468	2.4188**	55.5118
	GARCH-VOL	4.1993***	63.0839***	1.2677***	10.7165***	0.3127**	1.0040	138.3577	47.5825**	0.7148***	2.5738***	51.2399**
	COMBINED	3.0559	49.9339	0.4982	0.4441	0.2380	0.6715	95.0156	20.3398	0.7358	4.8830	62.7711
	Benefit (%)	30.36	14.96	44.29	72.56	29.10	15.14	0.00	78.85	48.60	2.94	24.32
	GARCH	0.8576	2.7742***	0.4397	0.4438	0.1842	1.3925	6.0087	1.8582	0.2427	1.6508	44.5086
	GARCH-RV	**0.7269**	**1.6299**	**0.3880**	**0.3010**	**0.1646**	**0.8177**	**3.3919**	1.6673	**0.1834**	1.6147	**66.8741**
	GARCH-RR	0.8568*	1.7987*	0.4953***	0.4605***	0.1893**	0.9025*	4.1824*	2.0507*	0.2312***	1.6286***	64.8503
KO	GARCH-RPV	0.7508*	1.7307	0.3934	0.3043	0.1693	0.8665	3.6560	**1.6245**	0.2071	1.6518*	65.8549*
	GARCH-RBP	0.7483**	1.6682	0.4133***	0.3484***	0.1684**	0.8372	3.5920*	1.8148**	0.1903***	**1.6137**	66.0930
	GARCH-VOL	1.2095***	3.3809***	0.7807***	1.5950**	0.2419***	1.6859***	6.1372*	3.7124***	0.4425***	1.7046***	35.9107**
	COMBINED	0.8134	1.7910	0.4480	0.3778	0.1800	0.8986	4.0973	1.8946	0.2120	1.6271	64.4227
	Benefit (%)	15.24	41.25	11.76	32.18	10.64	41.28	43.55	12.58	24.43	2.25	50.25

Table 5. Out-of-sample forecast evaluation (*cont.*)

		Forecast accuracy measures										
Stock	**Model**	MAE	MSE	HMAE	HMSE	AMAPE	Theil-U	MME(U)	MME(O)	LL	GMLE	MZ-R^2
	GARCH	1.9435	17.3639***	0.5624	0.9484	0.2227	0.8972	36.6720	6.0531	0.4201	2.3551	7.9065
	GARCH-RV	1.8887***	14.6883*	0.5322***	0.7568***	0.2122***	0.7590	28.0535	8.0831	0.3630***	2.2988***	21.9645*
	GARCH-RR	1.8626***	**13.7368**	0.5460**	0.7525***	0.2088**	**0.7098**	30.6672**	6.8102***	0.3441***	**2.2724**	24.6457
MCD	GARCH-RPV	**1.7801**	13.8080	**0.4685**	**0.5537**	**0.2018**	0.7135	**27.9033**	**4.9403**	**0.3315**	2.2928***	**26.2765**
	GARCH-RBP	1.8483**	14.0844	0.5233***	0.7236***	0.2086**	0.7278	28.1466**	6.7899***	0.3525***	2.2915***	23.8496*
	GARCH-VOL	1.9901***	17.8297***	0.5564*	0.9086***	0.2294**	0.9213	33.4877	6.1521	0.4452	2.3895**	7.0180*
	COMBINED	1.8361	13.8876	0.5504	0.7843	0.2077	0.7176	35.2170	6.0494	0.3431	2.2726	23.3070
	Benefit (%)	8.41	20.89	16.70	41.62	9.38	20.89	23.91	18.38	21.09	3.51	232.34
	GARCH	1.9524	8.6105***	0.7942	1.2819	0.2515	1.5277	19.6059	8.4880	0.4235	2.2235	33.5199
	GARCH-RV	1.5284***	5.3117***	0.6150***	0.7773***	0.2106***	0.9427***	9.6405	6.0302***	0.3000***	2.1745***	57.2189**
	GARCH-RR	**1.4355**	**4.7795**	**0.5647**	**0.6499**	**0.2007**	**0.8481**	**9.5153**	**5.1823**	**0.2701**	**2.1650**	**59.9528**
MSFT	GARCH-RPV	1.5153***	5.1317***	0.5886***	0.6887**	0.2054***	0.9107***	10.3163	5.4635	0.2828***	2.1687**	58.1559**
	GARCH-RBP	1.5164***	5.3134**	0.6054**	0.7557***	0.2091***	0.9429**	10.0238*	5.8701***	0.2947***	2.1731***	56.5251*
	GARCH-VOL	3.4251***	18.6881***	1.5101***	5.1865***	0.3489***	3.3161***	22.8637*	21.1438***	0.8773***	2.3555***	16.8304**
	COMBINED	1.3098	***4.6077***	0.4259	0.3648	0.1766	0.8162	8.2106	4.4980	0.2176	2.1642	59.6248
	Benefit (%)	26.48	44.49	28.90	49.30	20.20	44.49	51.47	38.95	36.22	2.63	78.86
	GARCH	0.6066	0.9080***	0.5756	0.6819	0.2031	1.2261	2.2570	1.2715	0.2836	1.1973	54.4355
	GARCH-RV	**0.4621**	0.5863	**0.3645**	**0.2629**	**0.1572**	0.7918	1.2696	0.9195	**0.1653**	**1.1577**	68.7963
	GARCH-RR	0.5120**	0.7127**	0.4267*	0.3677***	0.1725**	0.9627**	1.6092*	1.0204*	0.2010	1.1719***	62.3984**
PG	GARCH-RPV	0.4681*	0.5720***	0.3716*	0.2734*	0.1637***	0.7724***	**1.1931**	**0.9138**	0.1855**	1.1739	69.5375**
	GARCH-RBP	0.4720*	**0.5600**	0.3874***	0.2965***	0.1614***	**0.7561**	1.2313**	0.9487*	0.1722***	1.1581	**70.3142**
	GARCH-VOL	4.3179***	22.7805***	4.5441***	36.3093***	0.5733***	30.6737***	5.3454***	24.8107***	2.6818***	1.8456***	18.1998*
	COMBINED	0.5323	0.6801	0.4435	0.3924	0.1907	0.9145	1.2516**	1.1585	0.2861	1.2643	66.6211**
	Benefit (%)	23.82	38.33	36.67	61.45	22.59	38.33	47.14	28.13	41.71	3.31	29.17
	GARCH	1.2801	6.0674***	0.6151	0.7723	0.2243	1.3870	16.6024	3.5017	0.3426	1.7990	39.8960
	GARCH-RV	1.0887**	4.1895	0.4930*	0.5157	0.1907***	0.9575	9.4215*	3.3163	0.2470*	1.7519**	57.9931
	GARCH-RR	1.1819**	4.4596**	0.5554***	0.6405*	0.2037*	1.0189**	10.7409*	3.6044*	0.2757***	1.7586***	56.1337
WMT	GARCH-RPV	**1.0543**	4.1195	**0.4562**	**0.4326**	**0.1835**	0.9417	**9.1475**	**2.9934**	**0.2295**	**1.7479**	58.5898
	GARCH-RBP	1.0659	**4.0894**	0.4933**	0.5289	0.1893**	**0.9347**	9.6257***	3.0971	0.2460*	1.7508	**58.9156**
	GARCH-VOL	1.2223**	6.2784**	0.5272*	0.5731**	0.2122***	1.4352*	14.6043*	3.0023	0.3137	1.8040***	38.7879**
	COMBINED	1.1002	4.2943	0.4746	0.4642	0.1927	0.9793	9.0753	3.3252	0.2832	1.8698	56.9535
	Benefit (%)	17.64	32.60	25.83	43.99	18.19	32.61	44.90	14.52	33.01	2.84	47.67

Table 6. Hansen's Superior Predictive Ability (SPA) test

	Benchmark model: GARCH-RPV						
		Alternative models				SPA	
Stock	**Loss**	**Best Performing**	***p*-val**	**Most Significant**	***p*-val**	***p*-val**	**Superior**
ATT	MAE	COMBINED	0.0060	COMBINED	0.0060	0.0060	COMBINED
	MSE	COMB-EQW	0.8990	COMBINED	0.7780	0.9970	GARCH-RPV
AXP	MAE	COMBINED	0.0200	COMBINED	0.0200	0.0200	COMBINED
	MSE	COMBINED	0.1520	COMBINED	0.1520	0.2350	GARCH-RPV
BA	MAE	GARCH-RV	0.2340	GARCH-RV	0.2340	0.3780	GARCH-RPV
	MSE	COMB-EQW	0.1710	COMB-EQW	0.1710	0.3350	GARCH-RPV
CAT	MAE	COMBINED	0.0000	COMBINED	0.0000	0.0000	COMBINED
	MSE	COMBINED	0.0010	COMBINED	0.0010	0.0020	COMBINED
DELL	MAE	COMBINED	0.0000	COMBINED	0.0000	0.0000	COMBINED
	MSE	COMBINED	0.0000	COMBINED	0.0000	0.0000	COMBINED
GE	MAE	COMBINED	0.8890	COMBINED	0.8890	0.8900	GARCH-RPV
	MSE	COMBINED	0.0880	COMBINED	0.0880	0.1590	GARCH-RPV
GM	MAE	COMBINED	0.0000	COMBINED	0.0000	0.0000	COMBINED
	MSE	COMBINED	0.0000	COMBINED	0.0000	0.0000	COMBINED
IBM	MAE	COMBINED	0.0230	COMBINED	0.0230	0.0230	COMBINED
	MSE	COMBINED	0.1030	COMBINED	0.1030	0.1960	GARCH-RPV
JPM	MAE	COMB-EQW	0.9510	GARCH-RBP	0.9020	0.9240	GARCH-RPV
	MSE	COMBINED	0.1840	COMBINED	0.1840	0.3240	GARCH-RPV
KO	MAE	GARCH-RV	0.0750	GARCH-RV	0.0750	0.1170	GARCH-RPV
	MSE	COMB-EQW	0.1810	COMB-EQW	0.1810	0.5020	GARCH-RPV
MCD	MAE	COMB-EQW	0.9400	COMBINED	0.9030	0.9960	GARCH-RPV
	MSE	GARCH-RR	0.4180	GARCH-RR	0.4180	0.9740	GARCH-RPV
MSFT	MAE	COMBINED	0.0000	COMBINED	0.0000	0.0000	COMBINED
	MSE	COMBINED	0.0060	COMBINED	0.0060	0.0090	COMBINED
PG	MAE	GARCH-RV	0.1320	GARCH-RV	0.1320	0.2780	GARCH-RPV
	MSE	GARCH-RPB	0.2610	GARCH-RPB	0.2610	0.6160	GARCH-RPV
WMT	MAE	GARCH-RPB	0.8070	GARCH-RPB	0.8070	0.8260	GARCH-RPV
	MSE	GARCH-RPB	0.3830	GARCH-RPB	0.3830	0.7950	GARCH-RPV

The table presents the consistent SPA test p-values for H_0: 'any alternative model is not better than the benchmark' based on B=1000 bootstrap replications and q=0.5 time-dependence parameter for the block bootstrap length. Best performer is the alternative model with minimum loss. Most significant is the alternative model giving the highest t-statistic in the pairwise comparisons with the benchmark. The p-values of the pairwise comparisons (H_0 :'best performer/most significant model is not better than benchmark') that do not control for the full set of alternatives are also reported. COMBINED is the combined model using time-varying weights as outlined in Section 3.3. COMB-EQW is the equal-weights combined model. The last column reports the model with superior predictive ability according to the SPA test.

Table 7. Forecast evaluation and market conditions: up-market vs down-market days

		Forecast accuracy measures					
		Up market			Down market		
Stock	**Model**	HMSE^U	AMAPE^U	LL^U	HMSE^D	AMAPE^D	LL^D
	GARCH-RV	1.3371	0.2362	0.4086	1.4154	0.2522	0.4701
ATT	GARCH-RR	1.7217	0.2532	0.4601	1.7095	0.2720	0.5272
	GARCH-RPV	**1.1766**	**0.2319**	**0.3831**	**1.1736**	**0.2462**	**0.4385**
	GARCH-RBP	1.3882	0.2370	0.4125	1.4695	0.2560	0.4814
	Benefit (%)	*49.36*	12.88	26.33	47.02	*17.66*	*29.03*
	GARCH-RV	1.5765	0.2394	0.4079	0.5877	0.2138	0.2934
AXP	GARCH-RR	2.8559	0.3006	0.6294	1.2300	0.2590	0.4368
	GARCH-RPV	**1.1585**	**0.2199**	**0.3498**	**0.5004**	**0.2027**	**0.2666**
	GARCH-RBP	1.6098	0.2365	0.4105	0.5689	0.2121	0.2881
	Benefit (%)	*61.55*	*18.24*	35.71	43.11	17.70	*37.38*
	GARCH-RV	0.5706	**0.1860**	0.2439	**0.3047**	**0.1700**	0.1990
BA	GARCH-RR	0.8777	0.2159	0.3202	0.4768	0.1834	0.2334
	GARCH-RPV	**0.5463**	0.1870	**0.2427**	0.3048	0.1705	**0.1972**
	GARCH-RBP	0.5953	0.1899	0.2513	0.3430	0.1733	0.2039
	Benefit (%)	*23.15*	6.51	14.62	-5.44	*12.78*	*27.15*
	GARCH-RV	1.1568	0.2379	0.3742	0.7687	0.2029	0.2921
CAT	GARCH-RR	1.7984	0.2838	0.5136	1.1500	0.2290	0.3766
	GARCH-RPV	**0.9993**	**0.2312**	**0.3459**	**0.6747**	**0.1997**	**0.2766**
	GARCH-RBP	1.1291	0.2431	0.3795	0.7867	0.2072	0.3008
	Benefit (%)	*56.00*	*24.31*	*42.60*	50.07	21.46	39.29
	GARCH-RV	1.1349	0.2483	0.3841	0.7107	0.2111	0.2866
DELL	GARCH-RR	**0.7562**	**0.2099**	**0.2846**	**0.4764**	**0.1839**	**0.2245**
	GARCH-RPV	1.1342	0.2505	0.3887	0.6689	0.2091	0.2848
	GARCH-RBP	1.2166	0.2510	0.3974	0.7366	0.2086	0.2880
	Benefit (%)	*37.90*	*14.42*	*27.51*	5.59	5.75	10.30
	GARCH-RV	0.7266	0.2165	0.3115	0.8220	0.2057	0.2982
GE	GARCH-RR	0.9816	0.2465	0.3854	1.1415	0.2233	0.3577
	GARCH-RPV	**0.4909**	**0.2016**	**0.2622**	**0.6706**	**0.1944**	**0.2690**
	GARCH-RBP	0.6961	0.2193	0.3148	0.7963	0.2056	0.2931
	Benefit (%)	*39.32*	*15.43*	*27.98*	27.02	12.69	26.17
	GARCH-RV	1.7644	**0.2494**	0.4439	0.8799	0.2218	0.3272
GM	GARCH-RR	2.7927	0.3074	0.6368	1.4871	0.2752	0.4698
	GARCH-RPV	**1.7034**	0.2509	**0.4426**	0.8864	0.2239	0.3300
	GARCH-RBP	1.8966	0.2525	0.4595	**0.8710**	**0.2181**	**0.3184**
	Benefit (%)	*18.23*	*5.81*	*10.47*	-14.92	-0.71	-2.82
	GARCH-RV	0.9333	0.2241	0.3351	0.7536	0.2162	0.3077
IBM	GARCH-RR	1.3039	0.2615	0.4356	0.9965	0.2362	0.3665
	GARCH-RPV	**0.7941**	**0.2129**	**0.3019**	**0.5523**	**0.2013**	**0.2587**
	GARCH-RBP	0.9498	0.2239	0.3379	0.7097	0.2150	0.3013
	Benefit (%)	*47.68*	*25.61*	*40.16*	34.59	15.67	27.52

The up- and down-market classification is for day $t-1$ and the forecast is for day t. Bold denotes the best forecasting model. Benefit (%) indicates the percentage forecast error reduction that the best forecasting model brings relative to the baseline GARCH. Italics in the last row (Benefit %) denotes the regime where the largest forecast error reduction is achieved.

Table 7. Forecast evaluation and market conditions: up- vs down-market days (*cont.*)

		Forecast accuracy measures					
		Up market			Down market		
Stock	**Model**	$HMSE^U$	$AMAPE^U$	LL^U	$HMSE^D$	$AMAPE^D$	LL^D
JPM	GARCH-RV	0.4908	0.1797	0.2214	0.5099	0.1953	0.2810
	GARCH-RR	0.6533	0.2116	0.2870	0.7197	0.2158	0.3304
	GARCH-RPV	**0.2927**	**0.1695**	**0.1886**	**0.3673**	**0.1873**	**0.2549**
	GARCH-RBP	0.5136	0.1785	0.2209	0.4786	0.1919	0.2745
	Benefit (%)	*80.09*	*35.87*	*58.64*	61.98	22.00	38.32
KO	GARCH-RV	0.3356	**0.1685**	**0.1908**	**0.2435**	**0.1577**	**0.1704**
	GARCH-RR	0.4895	0.1929	0.2400	0.4181	0.1836	0.2182
	GARCH-RPV	**0.3326**	0.1739	0.2277	0.2570	0.1626	0.1762
	GARCH-RBP	0.3895	0.1717	0.1984	0.2834	0.1623	0.1766
	Benefit (%)	28.46	4.53	16.02	*38.40*	*18.06*	*34.33*
	GARCH-RV	0.6195	0.2036	0.3431	0.9406	0.2241	0.3927
MCD	GARCH-RR	0.6494	0.2026	0.3304	0.8911	0.2177	0.3651
	GARCH-RPV	**0.4322**	**0.1975**	**0.3146**	**0.7127**	**0.2086**	**0.3567**
	GARCH-RBP	0.5575	0.2028	0.3303	0.9412	0.2168	0.3842
	Benefit (%)	29.09	5.64	9.38	*48.78*	*13.65*	*31.18*
	GARCH-RV	0.8588	0.2158	0.3137	0.7290	0.2085	0.2939
MSFT	GARCH-RR	**0.6800**	**0.2024**	**0.2755**	**0.6370**	**0.2009**	**0.2695**
	GARCH-RPV	0.7535	0.2099	0.2951	0.6511	0.2036	0.2774
	GARCH-RBP	0.8689	0.2138	0.3133	0.6838	0.2071	0.2849
	Benefit (%)	*52.06*	*22.50*	*38.35*	46.96	18.38	34.57
	GARCH-RV	**0.2634**	**0.1556**	**0.1627**	**0.2664**	**0.1612**	**0.1722**
PG	GARCH-RR	0.4085	0.1723	0.2070	0.3063	0.1742	0.1940
	GARCH-RPV	0.2733	0.1611	0.1808	0.2778	0.1694	0.1961
	GARCH-RBP	0.2965	0.1592	0.1697	0.3005	0.1661	0.1784
	Benefit (%)	*62.59*	*24.79*	*43.60*	59.40	18.44	38.27
	GARCH-RV	0.5214	0.1916	0.2355	0.5056	0.1896	0.2564
WMT	GARCH-RR	0.6356	0.2080	0.2738	0.6358	0.1987	0.2747
	GARCH-RPV	**0.4364**	**0.1831**	**0.2193**	**0.4281**	**0.1839**	**0.2391**
	GARCH-RBP	0.5556	0.1882	0.2371	0.5024	0.1903	0.2540
	Benefit (%)	*53.87*	*21.67*	*37.14*	31.15	15.13	29.71

Table 8. Forecast evaluation and market conditions: high volume vs low volume days

		Forecast accuracy measures					
		High-volume regime			Low-volume regime		
Stock	**Model**	HMSE^H	AMAPE^H	LL^H	HMSE^L	AMAPE^L	LL^L
	GARCH-RV	1.3273	0.2577	0.4662	1.4227	0.2356	0.4256
ATT	GARCH-RR	1.8214	0.2734	0.5240	1.6410	0.2565	0.4781
	GARCH-RPV	**1.1375**	**0.2512**	**0.4356**	**1.2052**	**0.2313**	**0.3980**
	GARCH-RBP	1.3726	0.2602	0.4753	1.4812	0.2381	0.4328
	Benefit (%)	23.62	10.67	17.36	*57.59*	*19.28*	*34.69*
	GARCH-RV	0.8144	0.2344	0.3456	1.3061	0.2208	0.3564
AXP	GARCH-RR	1.7002	0.2855	0.5268	2.3333	0.2757	0.5404
	GARCH-RPV	**0.6148**	**0.2156**	**0.2958**	**1.0078**	**0.2082**	**0.3194**
	GARCH-RBP	0.8121	0.2313	0.3455	1.3210	0.2190	0.3538
	Benefit (%)	53.96	*19.03*	*38.87*	*58.95*	17.22	34.57
	GARCH-RV	0.4284	**0.1814**	0.2190	0.4649	**0.1770**	0.2272
BA	GARCH-RR	0.6494	0.2070	0.2821	0.7289	0.1966	0.2796
	GARCH-RPV	**0.4148**	0.1824	**0.2174**	**0.4522**	0.1775	**0.2258**
	GARCH-RBP	0.4565	0.1866	0.2289	0.4982	0.1793	0.2306
	Benefit (%)	-19.46	7.02	14.65	*30.11*	*10.78*	*23.68*
	GARCH-RV	0.8655	0.2254	0.3234	1.0636	0.2190	0.3462
CAT	GARCH-RR	1.3967	0.2649	0.4394	1.5799	0.2539	0.4593
	GARCH-RPV	**0.7333**	**0.2237**	**0.3090**	**0.9375**	**0.2113**	**0.3176**
	GARCH-RBP	0.8283	0.2309	0.3296	1.0793	0.2231	0.3534
	Benefit (%)	49.81	21.01	36.98	*56.19*	*24.80*	*44.17*
	GARCH-RV	0.8471	0.2303	0.3322	1.0195	0.2321	0.3456
DELL	GARCH-RR	**0.5771**	**0.1970**	**0.2555**	**0.6703**	**0.1985**	**0.2582**
	GARCH-RPV	0.8208	0.2312	0.3381	1.0051	0.2317	0.3435
	GARCH-RBP	0.9178	0.2299	0.3443	1.0626	0.2329	0.3497
	Benefit (%)	8.38	3.76	9.38	*39.68*	*16.31*	*29.46*
	GARCH-RV	0.5550	0.2078	0.2939	0.9635	0.2122	0.3119
GE	GARCH-RR	0.8313	0.2299	0.3558	1.2681	0.2362	0.3814
	GARCH-RPV	**0.4492**	**0.1963**	**0.2679**	**0.7086**	**0.1983**	**0.2644**
	GARCH-RBP	0.5633	0.2128	0.3034	0.9064	0.2103	0.3016
	Benefit (%)	27.52	*15.04*	*27.27*	*34.12*	13.18	26.89
	GARCH-RV	1.1687	0.2261	**0.3465**	1.3335	**0.2394**	**0.4014**
GM	GARCH-RR	1.7939	0.2746	0.4821	2.2502	0.3001	0.5881
	GARCH-RPV	**1.1490**	0.2270	0.3469	**1.3085**	0.2419	0.4028
	GARCH-RBP	1.2192	**0.2241**	0.3469	1.3863	0.2394	0.4040
	Benefit (%)	-2.17	-0.45	-2.63	*12.13*	*3.68*	*6.61*
	GARCH-RV	0.9134	0.2215	0.3289	0.7810	0.2187	0.3145
IBM	GARCH-RR	1.2293	0.2455	0.4051	1.0733	0.2498	0.3942
	GARCH-RPV	**0.6869**	**0.2062**	**0.2793**	**0.6491**	**0.2071**	**0.2787**
	GARCH-RBP	0.8400	0.2177	0.3165	0.8090	0.2202	0.3202
	Benefit (%)	32.65	15.82	26.73	*48.72*	*24.18*	*39.46*

See note in Table 7.

Table 8. Forecast evaluation and market conditions: high vs low volume (*cont.*)

		Forecast accuracy measures					
		High volume			Low volume		
Stock	**Model**	HMSE^H	AMAPE^H	LL^H	HMSE^L	AMAPE^L	LL^L
JPM	GARCH-RV	0.4093	0.1885	0.2537	0.5723	0.1877	0.2518
	GARCH-RV	0.6559	0.2158	0.3086	0.7141	0.2126	0.3111
	GARCH-RPV	**0.3390**	0.1862	**0.2439**	0.3264	0.1734	0.2076
	GARCH-RBP	0.4020	**0.1854**	0.2482	0.5688	0.1861	0.2497
	Benefit (%)	62.82	22.95	38.66	*77.37*	*33.50*	*55.18*
KO	GARCH-RV	0.2859	**0.1655**	**0.1901**	**0.3066**	**0.1626**	**0.1759**
	GARCH-RR	0.4086	0.1861	0.2254	0.5009	0.1910	0.2351
	GARCH-RPV	**0.2835**	0.1676	0.1958	0.3153	0.1702	0.2148
	GARCH-RBP	0.3291	0.1699	0.1943	0.3587	0.1658	0.1851
	Benefit (%)	*34.66*	*12.32*	*28.32*	28.39	8.37	19.18
	GARCH-RV	1.0436	0.2341	0.4230	0.5718	0.1987	0.3267
MCD	GARCH-RR	0.8801	0.2217	0.3873	0.6744	0.2010	0.3182
	GARCH-RPV	**0.6893**	**0.2128**	**0.3591**	**0.4679**	**0.1959**	**0.3167**
	GARCH-RBP	0.9416	0.2265	0.4054	0.5840	0.1977	0.3204
	Benefit (%)	38.57	6.67	*22.15*	*44.75*	*11.50*	20.53
	GARCH-RV	0.7841	0.2138	0.3101	0.7844	0.2095	0.2958
MSFT	GARCH-RR	**0.6805**	**0.2070**	**0.2881**	**0.6350**	**0.1969**	**0.2589**
	GARCH-RPV	0.7343	0.2118	0.3009	0.6624	0.2015	0.2717
	GARCH-RBP	0.7753	0.2134	0.3076	0.7524	0.2069	0.2883
	Benefit (%)	34.33	14.38	27.52	*57.73*	*24.59*	*42.54*
	GARCH-RV	**0.2609**	**0.1634**	**0.1702**	**0.2686**	**0.1533**	**0.1636**
PG	GARCH-RR	0.3188	0.1716	0.1921	0.4138	0.1745	0.2111
	GARCH-RPV	0.2747	0.1656	0.1784	0.2763	0.1636	0.1942
	GARCH-RBP	0.2881	0.1658	0.1737	0.3074	0.1588	0.1729
	Benefit (%)	56.75	19.30	38.79	*64.52*	*24.90*	*43.78*
	GARCH-RV	0.4574	0.1910	0.2413	0.5610	0.1909	0.2514
WMT	GARCH-RR	0.5413	0.1976	0.2634	0.7157	0.2084	0.2842
	GARCH-RPV	**0.3556**	**0.1812**	**0.2244**	**0.4966**	**0.1860**	**0.2347**
	GARCH-RBP	0.4328	0.1838	0.2267	0.6083	0.1944	0.2625
	Benefit (%)	*53.42*	*20.73*	*36.21*	37.31	16.34	30.85

Table 9. Average daily statistics over different market conditions

	Regime classification of day $t-1$			
	Up market	Down market	High volume	Low volume
r_{t-1}	0.6480	-0.6511	0.0608	-0.0531
r_t	-0.0131	0.0112	0.0257	-0.0276
$\tilde{\sigma}^2_{t-1}$	3.3091	3.9892	4.3227	3.1242
$\tilde{\sigma}^2_t$	3.2125	4.0836	4.0673	3.3180
VOL_{t-1}	11,162,392	11,531,944	13,534,828	9,620,517
VOL_t	11,091,917	11,594,947	12,432,407	10,499,129
$\rho(\tilde{\sigma}^2_{t-1}, \tilde{\sigma}^2_t)$	0.6687	0.7034	0.7118	0.6633

r_t is the return on day t, $\tilde{\sigma}^2_t$ is the sum of 5-min squared returns, VOL_t is volume. The last row reports the correlation between the population variance (proxied by the sum of 5-min squared returns) on days t and $t-1$. The regime classification is based on (11) and (12).

APPENDIX TABLES

On Forecasting Daily Stock Volatility: the Role of Intraday Information and Market Conditions

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Table A. Returns normality using RPV(z) as standardization variable for orders $z = 0.5, 1$ and 1.5.

	ATT	**AXP**	**BA**	**CAT**	**DELL**	**GE**	**GM**	**IBM**	**JPM**	**KO**	**MCD**	**MSFT**	**PG**	**WMT**
							RPV(0.5)							
StDev/Mean	0.239	0.235	0.222	0.209	0.213	0.214	0.244	0.209	0.235	0.203	0.219	0.200	0.209	0.259
Skewness	0.478	0.589	0.544	0.356	0.700	0.805	0.501	0.461	0.920	0.595	0.440	0.579	0.627	-0.0004
Kurtosis	4.293	4.053	4.146	4.225	4.182	4.913	3.961	3.734	5.899	4.347	4.315	3.940	4.350	3.795
ADF	-3.868(16)	-3.763(16)	-5.495(10)	-4.427(12)	-4.157(17)	-5.428(9)	-4.869(9)	-3.702(18)	-4.591(9)	-3.801(18)	-4.256(14)	-4.767(15)	-5.148(9)	-3.932(12)
Robinson d	0.399	0.423	0.409	0.426	0.398	0.395	0.433	0.399	0.432	0.399	0.414	0.383	0.416	0.447
KS	0.0872	0.0497	0.0713	0.0770	***0.0347***	0.0612	0.0587	0.0910	0.0583	0.0730	0.0760	0.0424	0.0580	0.1130
(p-value)	(0.000)	(0.000)	(0.000)	(0.000)	***(0.028)***	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
JB$\left[\frac{r_t}{\sqrt{\mathbf{RPV0.5}}}\right]$	46.522	33.988	64.163	29.938	127.832	52.649	12.698	87.181	899.417	123.119	185.335	10.828	224.155	82.123
(p-value)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.004)	(0.000)	(0.000)
							RPV(1)							
StDev/Mean	0.417	0.449	0.393	0.377	0.422	0.418	0.424	0.406	0.475	0.380	0.384	0.390	0.420	0.433
Skewness	1.717	1.601	1.649	1.360	1.502	1.860	1.561	1.286	2.495	1.496	1.724	1.383	1.568	1.372
Kurtosis	8.754	7.476	8.422	6.537	7.058	8.954	7.010	5.686	15.969	6.949	9.000	6.513	7.708	6.785
ADF	-4.429(16)	-3.744(23)	-6.145(10)	-4.436(12)	-3.372(24)	-5.275(13)	-5.337(9)	-4.173(20)	-4.148(20)	-3.942(18)	-4.824(14)	-4.910(15)	-5.683(7)	-4.384(13)
Robinson d	0.385	0.410	0.398	0.428	0.403	0.384	0.422	0.402	0.417	0.397	0.390	0.391	0.417	0.420
KS	**0.0312**	**0.0310**	**0.0312**	0.0549	**0.0234**	**0.0278**	**0.0290**	***0.0341***	**0.0312**	**0.0228**	**0.0297**	**0.0207**	0.0574	0.0460
(p-value)	**(0.064)**	**(0.066)**	**(0.063)**	(0.000)	**(0.281)**	**(0.128)**	**(0.305)**	***(0.033)***	**(0.063)**	**(0.310)**	**(0.087)**	**(0.431)**	(0.000)	(0.000)
JB$\left[\frac{r_t}{\sqrt{\mathbf{RPV1}}}\right]$	***7.434***	**2.201**	**5.808**	**3.632**	15.268	9.517	***8.206***	***6.547***	67.949	18.924	26.819	10.542	20.769	***8.097***
(p-value)	***(0.024)***	**(0.332)**	**(0.054)**	**(0.162)**	(0.000)	(0.008)	***(0.016)***	***(0.037)***	(0.000)	(0.000)	(0.000)	(0.005)	(0.000)	***(0.017)***
							RPV(1.5)							
StDev/Mean	0.654	0.716	0.624	0.598	0.659	0.679	0.656	0.643	0.816	0.592	0.613	0.608	0.683	0.676
Skewness	2.832	2.659	3.130	2.161	2.396	2.978	2.469	2.367	4.875	2.293	2.895	2.252	2.856	2.832
Kurtosis	17.017	14.370	24.786	10.819	13.350	17.554	12.818	13.373	47.943	11.694	17.751	12.366	17.600	20.182
ADF	-4.418(18)	-4.71(17)	-6.79(10)	-4.73(12)	-3.53(24)	-4.98(20)	-5.87(9)	-4.37(18)	-4.66(20)	-4.21(18)	-5.71(12)	-5.25(16)	-5.34(12)	-4.93(13)
Robinson d	0.370	0.389	0.377	0.421	0.392	0.363	0.409	0.394	0.380	0.385	0.365	0.392	0.396	0.393
KS	**0.0156**	**0.0245**	**0.0233**	0.0430	**0.0117**	***0.0333***	**0.0302**	**0.0204**	**0.0219**	**0.0220**	**0.0246**	**0.0222**	**0.0295**	**0.0167**
(p-value)	**(0.779)**	**(0.238)**	**(0.286)**	(0.003)	**(0.968)**	***(0.040)***	**(0.078)**	**(0.449)**	**(0.358)**	**(0.354)**	**(0.232)**	**(0.343)**	**(0.091)**	**(0.706)**
JB$\left[\frac{r_t}{\sqrt{\mathbf{RPV1.5}}}\right]$	**5.679**	9.353	**1.103**	**3.341**	***6.790***	13.228	12.726	9.356	***7.373***	**2.813**	**0.285**	19.606	**0.406**	**2.868**
(p-value)	**(0.058)**	(0.009)	**(0.576)**	**(0.188)**	***(0.033)***	(0.001)	(0.001)	(0.009)	***(0.025)***	**(0.244)**	**(0.867)**	(0.000)	**(0.816)**	**(0.283)**

ADF is the augmented Dickey-Fuller test for the null hypothesis of unit root non-stationarity. KS is the Kolmogorov-Smirnov test for the null of lognormality. JB is the Jarque-Bera test statistic for the null of normality. Bold and bold italics indicate insignificant, respectively, at the 5% (or 10%) and the 1% level.

Table B. Out-of-sample forecast evaluation for GARCH augmented with RPV(z) for orders $z = 0.5, 1$ and 1.5

		Forecast accuracy measures										
Stock	**Model**	MAE	MSE	HMAE	HMSE	AMAPE	Theil-U	MME(U)	MME(O)	LL	GMLE	MZ-R^2
	GARCH-RPV(0.5)	2.6922***	19.8091**	0.8052***	1.4885***	0.2570***	0.7425**	61.6104*	10.8163*	0.4611***	2.5152***	37.5752**
ATT	GARCH-RPV(1)	**2.5193**	**18.7982**	**0.7147**	**1.1137**	0.2392	**0.7046**	54.4078*	**10.1453**	**0.4061**	**2.4984**	**39.9901**
	GARCH-RPV(1.5)	2.5203	18.8451	0.7224	1.1681	**0.2390**	0.7064	**49.3295**	11.0480	0.4118	2.5006	39.6859
	GARCH-RPV(0.5)	1.9031***	13.3308	0.6741**	0.9334***	0.2301***	0.9281	41.1595	6.4478**	0.3599	2.1713***	45.5534
AXP	GARCH-RPV(1)	1.7419*	11.5166	0.6185	**0.8158**	0.2159	0.8067	32.6573	5.7286	0.3202	2.1389*	53.4296
	GARCH-RPV(1.5)	**1.6658**	**10.3212**	**0.6106**	0.8284	**0.2113**	**0.7230**	**29.6144**	**5.6148**	**0.3079**	**2.1287**	**56.9665**
	GARCH-RPV(0.5)	1.7057***	6.8535	0.5463***	0.6574***	0.2009***	1.0226	16.6650	4.8532***	0.2808**	2.3698***	48.6243
BA	GARCH-RPV(1)	1.5553	5.8857	0.4775***	0.4871***	0.1849***	0.8782	13.9147	4.2290	0.2379*	2.3543**	54.7976
	GARCH-RPV(1.5)	**1.5012**	**5.4405**	**0.4562**	**0.4455**	**0.1799**	**0.8118**	**11.6785**	**4.0773**	**0.2241**	**2.3491**	**55.7621**
	GARCH-RPV(0.5)	1.6146***	4.8841***	0.8012*	1.2200***	0.2574***	1.0018***	17.0742	5.1888***	0.4248***	2.0339***	37.0818*
CAT	GARCH-RPV(1)	1.3397	3.9838***	0.6416***	**0.8370**	0.2204	0.8174***	12.7275	3.9214***	0.3185	1.9892*	42.3219
	GARCH-RPV(1.5)	**1.2880**	**3.8056**	**0.6340**	0.8451***	**0.2165**	**0.7809**	**12.0083**	**3.6810**	**0.3130**	**1.9850**	**43.6401**
	GARCH-RPV(0.5)	**2.3698**	10.1521**	**0.6572**	**0.8786**	**0.2273**	1.2090	**13.4427**	11.7246	**0.3361**	2.4414	50.8927
DELL	GARCH-RPV(1)	2.4231	10.0844	0.6911	0.9336	0.2339	1.2027	14.0315	11.5218	0.3491	2.4431	**51.6345**
	GARCH-RPV(1.5)	2.3758	**9.9858**	0.6863	0.9127	0.2311	**1.1919**	15.8615**	**11.2197**	0.3393	**2.4371**	51.2849
	GARCH-RPV(0.5)	1.5657*	8.9163*	0.5165	0.5802	0.1992	0.8310*	21.5653	4.1501	0.2736	2.1006*	52.7673*
GE	GARCH-RPV(1)	1.4929	8.1015	**0.4991**	**0.5423**	**0.1943**	0.7597	**19.6777**	**4.0366**	**0.2584**	**2.0827**	**55.6953**
	GARCH-RPV(1.5)	**1.4915**	**7.7464**	0.5167	0.5886	0.1972	**0.7264**	19.8920	4.4162*	0.2648	2.0835	55.2760
	GARCH-RPV(0.5)	1.6298***	4.9854***	0.8790**	1.7575***	0.2623***	0.7786***	15.7598	5.1713***	0.4657***	2.0211***	57.9520
GM	GARCH-RPV(1)	1.4870**	4.5363***	0.7574***	1.3009***	0.2406***	0.7087***	12.7873	4.4831***	0.3934***	1.9997***	58.7850
	GARCH-RPV(1.5)	**1.4508**	**4.4050**	**0.7296**	**1.2294**	**0.2350**	**0.6882**	**11.9278**	**4.4331**	**0.3768**	**1.9942**	**58.8185**

The forecast 'target' is the daily conditional variance proxied by the sum of 5-min squared returns. Bold indicates the top performer. Asterisks indicate that the forecasts of the model at hand are significantly worse than those of the top performer on the basis of a two-tailed Diebold-Mariano (1995) test. * significance at 10% level, ** at 5%, *** at 1%.

Table B. Out-of-sample forecast evaluation for GARCH augmented with RPV for orders $z = 0.5, 1$ and 1.5 (*cont.*)

		Forecast accuracy measures										
Stock	**Model**	MAE	MSE	HMAE	HMSE	AMAPE	Theil-U	MME(U)	MME(O)	LL	GMLE	MZ-R^2
	GARCH-RPV(0.5)	1.1997	2.9934***	0.6560***	0.7939***	0.2250	0.9936***	8.5427	3.2429***	0.3236	1.7767***	59.4714**
IBM	GARCH-RPV(1)	1.0745	2.5261**	**0.5866**	**0.6476**	0.2094	0.8387**	7.2959	2.8094	0.2846	1.7647	64.9428
	GARCH-RPV(1.5)	**1.0543**	**2.3986**	0.5879	0.6620	**0.2070**	**0.7965**	**6.4115**	**2.7681**	**0.2784**	**1.7580**	**65.9708**
	GARCH-RPV(0.5)	2.5648***	**46.5758**	0.4933**	0.4461***	0.1997***	**0.7398**	122.0684*	10.5476*	0.2694	2.4399***	64.8222
JPM	GARCH-RPV(1)	2.3834	49.8706	0.4220	**0.3215**	0.1837**	0.7938	120.3537*	**8.0829**	0.2357	2.4260**	67.1292
	GARCH-RPV(1.5)	**2.3625**	50.1563	**0.4191**	0.3293	**0.1777**	0.7968	**118.1784**	9.3414	**0.2216**	**2.4183**	**64.2387**
	GARCH-RPV(0.5)	0.8582***	2.2662*	0.4895**	0.5642*	0.1902**	1.1347*	5.3933	2.0387***	0.2798***	1.6863*	55.1443
KO	GARCH-RPV(1)	0.8058*	2.2244	0.4276	0.3883	0.1796**	1.1162	4.8500	1.7908	0.2434	1.6715**	55.4915
	GARCH-RPV(1.5)	**0.7508**	**1.7307**	**0.3934**	**0.3043**	**0.1693**	**0.8665**	**3.6560**	**1.6245**	**0.2071**	**1.6518**	**65.8549**
	GARCH-RPV(0.5)	1.8231***	15.2595	0.5091	0.6895***	0.2078***	0.7885	34.2881	4.8981**	0.3427	2.2870	18.2695
MCD	GARCH-RPV(1)	**1.7343**	14.2644	**0.4478**	**0.5106**	**0.1962**	0.7371	29.2425	**4.3361**	**0.3181**	**2.2861**	25.3341
	GARCH-RPV(1.5)	1.7801**	**13.8080**	0.4685***	0.5537***	0.2018***	**0.7135**	**27.9033**	4.9403**	0.3315	2.2928*	**26.2765**
	GARCH-RPV(0.5)	1.6276***	5.9401*	0.6230***	0.7512**	0.2158***	1.0537**	13.6694	5.7804**	0.3083	2.1808***	52.1821
MSFT	GARCH-RPV(1)	2.3792**	10.9789**	0.9057***	1.5939***	0.2717***	1.9482**	10.7763	13.5022***	0.4904***	2.2409***	53.2825***
	GARCH-RPV(1.5)	**1.5153**	**5.1317**	**0.5886**	**0.6887**	**0.2054**	**0.9107**	**10.3163**	**5.4635**	**0.2828**	**2.1687**	**58.1559**
	GARCH-RPV(0.5)	0.4950*	0.5966**	0.4360***	0.3539***	0.1741	0.8051**	1.5246	0.9430	0.2200***	1.2021	**70.6584**
PG	GARCH-RPV(1)	0.4922***	0.5830	0.4118***	0.3470***	0.1775***	0.7870	1.2446**	0.9564	0.2442**	1.2332*	69.1746
	GARCH-RPV(1.5)	**0.4681**	**0.5720**	**0.3716**	**0.2734**	**0.1637**	**0.7724**	**1.1931**	**0.9138**	**0.1855**	**1.1739**	69.5375
	GARCH-RPV(0.5)	1.1747***	5.1305*	0.5202***	0.5430*	0.2024***	1.1728*	12.7072*	3.2633	0.2809	1.7728***	48.7770*
WMT	GARCH-RPV(1)	1.0859*	4.4105	0.4751*	0.4655**	0.1895**	1.0082	10.4918	**3.0000**	0.2461	1.7562***	55.9227**
	GARCH-RPV(1.5)	**1.0543**	**4.1195**	**0.4562**	**0.4326**	**0.1835**	**0.9417**	**9.1475**	3.0023	**0.2295**	**1.7479**	**58.5898**

Table C. Correlation matrix between volume, nonparametric volatility and parametric (GARCH) volatility measures

	ATT						**AXP**						**BA**					
	Volume	RV	RR	RPV	RBP	GARCH	Volume	RV	RR	RPV	RBP	GARCH	Volume	RV	RR	RPV	RBP	GARCH
Volume	1.000						1.000						1.000					
RV	0.338	1.000					0.571	1.000					0.521	1.000				
RR	0.411	0.944	1.000				0.528	0.974	1.000				0.500	0.923	1.000			
RPV	0.308	0.981	0.938	1.000			0.593	0.973	0.951	1.000			0.523	0.968	0.943	1.000		
RBP	0.320	0.979	0.921	0.969	1.000		0.575	0.964	0.927	0.970	1.000		0.509	0.984	0.915	0.957	1.000	
GARCH	0.144	0.446	0.454	0.481	0.448	1.000	0.366	0.515	0.552	0.585	0.534	1.000	0.266	0.474	0.486	0.517	0.464	1.000

	CAT						**DELL**						**GE**					
	Volume	RV	RR	RPV	RBP	GARCH	Volume	RV	RR	RPV	RBP	GARCH	Volume	RV	RR	RPV	RBP	GARCH
Volume	1.000						1.000						1.000					
RV	0.281	1.000					0.302	1.000					0.555	1.000				
RR	0.230	0.752	1.000				0.297	0.961	1.000				0.572	0.947	1.000			
RPV	0.277	0.984	0.758	1.000			0.310	0.987	0.953	1.000			0.552	0.980	0.952	1.000		
RBP	0.277	0.977	0.739	0.968	1.000		0.306	0.983	0.946	0.976	1.000		0.551	0.987	0.934	0.972	1.000	
GARCH	0.083	0.520	0.417	0.533	0.495	1.000	0.246	0.567	0.612	0.601	0.556	1.000	0.332	0.473	0.528	0.525	0.471	1.000

	GM						**IBM**						**JPM**					
	Volume	RV	RR	RPV	RBP	GARCH	Volume	RV	RR	RPV	RBP	GARCH	Volume	RV	RR	RPV	RBP	GARCH
Volume	1.000						1.000						1.000					
RV	0.341	1.000					0.352	1.000					0.522	1.000				
RR	0.435	0.923	1.000				0.311	0.948	1.000				0.558	0.971	1.000			
RPV	0.372	0.972	0.933	1.000			0.386	0.949	0.956	1.000			0.545	0.967	0.968	1.000		
RBP	0.361	0.973	0.915	0.970	1.000		0.382	0.945	0.933	0.972	1.000		0.524	0.986	0.968	0.972	1.000	
GARCH	0.255	0.513	0.590	0.576	0.530	1.000	0.070	0.466	0.555	0.549	0.498	1.000	0.367	0.648	0.669	0.646	0.640	1.000

	KO						**MCD**						**MSFT**					
	Volume	RV	RR	RPV	RBP	GARCH	Volume	RV	RR	RPV	RBP	GARCH	Volume	RV	RR	RPV	RBP	GARCH
Volume	1.000						1.000						1.000					
RV	0.401	1.000					0.436	1.000					0.516	1.000				
RR	0.390	0.942	1.000				0.457	0.926	1.000				0.575	0.963	1.000			
RPV	0.395	0.986	0.950	1.000			0.416	0.975	0.931	1.000			0.526	0.983	0.961	1.000		
RBP	0.408	0.983	0.927	0.973	1.000		0.432	0.972	0.909	0.966	1.000		0.508	0.983	0.956	0.975	1.000	
GARCH	0.149	0.536	0.575	0.566	0.526	1.000	0.196	0.385	0.422	0.433	0.375	1.000	0.312	0.519	0.563	0.552	0.512	1.000

	PG						**WMT**					
	Volume	RV	RR	RPV	RBP	GARCH	Volume	RV	RR	RPV	RBP	GARCH
Volume	1.000						1.000					
RV	0.443	1.000					0.288	1.000				
RR	0.526	0.941	1.000				0.285	0.922	1.000			
RPV	0.396	0.973	0.945	1.000			0.286	0.966	0.955	1.000		
RBP	0.430	0.985	0.923	0.965	1.000		0.285	0.977	0.922	0.970	1.000	
GARCH	0.189	0.517	0.550	0.562	0.513	1.000	0.091	0.514	0.593	0.581	0.522	1.000

Table D. GARCH estimation results from 02/01/97 to 31/12/03

	AXP						**BA**					
	GARCH	with RV	with RR	with RPV	with RBP	with VOL	GARCH	with RV	with RR	with RPV	with RBP	with VOL
θ_0	0.0642	0.0116	0.0111	0.0137	0.0082	0.0685	0.0350	0.0215	0.0198	0.0232	0.0268	0.0338
	(0.0462)	*(0.0436)*	*(0.0453)*	*(0.0438)*	*(0.0437)*	*(0.0460)*	*(0.0433)*	*(0.0430)*	*(0.0433)*	*(0.0431)*	*(0.0430)*	*(0.0431)*
ω	**0.0801**	**0.3879**	**0.9926**	-0.0319	**0.3880**	0.1104	**0.1569**	**0.9189**	**0.9721**	0.4784	**0.9670**	0.1112
	(0.0364)	*(0.1244)*	*(0.2188)*	*(0.1129)*	*(0.1191)*	*(0.0757)*	*(0.0603)*	*(0.2688)*	*(0.2833)*	*(0.3237)*	*(0.2680)*	*(0.0862)*
α_1	**0.0722**	**-0.0480**	-0.0039	-0.0388	-0.0417	**0.0715**	**0.0737**	0.0107	**0.0703**	0.0187	0.0140	**0.0703**
	(0.0161)	*(0.0255)*	*(0.0276)*	*(0.0236)*	*(0.0238)*	*(0.0163)*	*(0.0190)*	*(0.0384)*	*(0.0406)*	*(0.0399)*	*(0.0361)*	*(0.0176)*
β_1	**0.9118**	**0.4404**	**0.2135**	**0.5070**	**0.5035**	**0.9181**	**0.8879**	**0.2350**	**0.2977**	**0.2370**	**0.2334**	**0.8795**
	(0.0189)	*(0.0833)*	*(0.1105)*	*(0.0943)*	*(0.0799)*	*(0.0165)*	*(0.0282)*	*(0.1063)*	*(0.1363)*	*(0.1210)*	*(0.1085)*	*(0.0303)*
γ		**0.5471**	**0.9554**	**0.2816**	**0.5147**	-1.27e-08		**0.5014**	**0.5462**	**0.2955**	**0.5393**	2.88e-08
		(0.0914)	*(0.1741)*	*(0.0579)*	*(0.0935)*	*(3.66e-08)*		*(0.0970)*	*(0.1503)*	*(0.0609)*	*(0.1011)*	*(6.04e-08)*
lnL	-3757.83	-3710.94	-3733.09	-3710.64	-3710.87	-3754.25	-3646.36	-3607.84	-3619.46	-3608.92	-3607.71	-3643.014
AIC	4.2724	4.2227	4.2478	4.2223	4.2226	4.2719	4.1458	4.1055	4.1187	4.1067	4.1055	4.1455
SBC	4.2848	4.2382	4.2634	4.2379	4.2381	4.2874	4.1582	4.1211	4.1343	4.1228	4.1209	4.1610
$LB(10)$	0.584	0.549	0.481	0.552	0.571	0.605	0.279	0.100	0.103	0.100	0.074	0.269
$ARCH(10)$	0.070	0.153	0.092	0.115	0.133	0.076	0.267	0.925	0.822	0.896	0.932	0.332
Ranking	6 or 5	3	4	1	2	5 or 6	6 or 5	1	4	3	2	5 or 6

The reported estimation results are for the conditional mean equation $r_t = \theta_0 + \theta_1 r_{t-1} + ... + \theta_p r_{t-p} + \lambda_1 u_{t-1} + ... + \lambda_p u_{t-q} + u_t$, with $u_t|\mathcal{F}_{t-1} \sim iid(0, h_t)$, and conditional variance equation $h_t = \omega + \alpha_1 u_{t-1}^2 + ... + \alpha_r u_{t-r}^2 + \beta_1 h_{t-1} + ... + \beta_s h_{t-s}$. $LB(10)$ is the Ljung-Box test for autocorrelation in u_t. $ARCH(10)$ is Engle's LM test for ARCH effects. The intraday sampling frequency for RV, RR, RPV, RBP is 5 minutes. Bold denotes significant. Bollerslev-Wooldridge standard errors are reported in parentheses. The degree of volatility persistence is given by $\lambda = \sum_{i=1}^{r} \alpha_i + \sum_{j=1}^{s} \beta_j$. Rankings 1 and 6 mean top and bottom, respectively, according to in-sample model fit. The sample size for ATT and PG is 1760 and 1759, instead of 1761, so for forecasting purposes the rolling window has fixed size 1260 and 1259, respectively.

Table D. GARCH estimation results from 02/01/97 to 31/12/03 (*cont.*)

	DELL						GE					
	GARCH	with RV	with RR	with RPV	with RBP	with VOL	GARCH	with RV	with RR	with RPV	with RBP	with VOL
θ_0	**0.2501**	**0.2510**	**0.2552**	**0.2581**	**0.2481**	0.2107	0.0160	-0.0277	-0.0196	-0.0250	-0.0240	0.0163
	(0.1143)	*(0.1111)*	*(0.1096)*	*(0.1111)*	*(0.1114)*	*(0.1375)*	*(0.0399)*	*(0.0393)*	*(0.0397)*	*(0.0392)*	*(0.0393)*	*(0.0398)*
θ_1	**-0.8965**	**-0.8825**	**-0.8967**	**-0.8831**	**-0.8846**	**-0.0449**						
	(0.1353)	*(0.1250)*	*(0.1212)*	*(0.1221)*	*(0.1195)*	*(0.0282)*						
θ_2	**-0.0836**	-0.0551	**-0.0675**	**-0.0620**	-0.0572	**-0.0922**						
	(0.0414)	*(0.0347)*	*(0.0347)*	*(0.0348)*	*(0.0349)*	*(0.0415)*						
θ_3	**-0.0471**	-0.0406	**-0.0449**	**-0.0464**	-0.0435	-0.0417						
	(0.0271)	*(0.0267)*	*(0.0262)*	*(0.0266)*	*(0.0270)*	*(0.0281)*						
λ_1	**0.8293**	**0.8365**	**0.8441**	**0.8355**	**0.8389**	**0.9975**						
	(0.1314)	*(0.1239)*	*(0.1194)*	*(0.1210)*	*(0.1184)*	*(0.0041)*						
ω	**0.0892**	**0.6938**	**0.6949**	**-0.4241**	**0.6549**	**6.7228**	**0.0967**	**0.2461**	**0.2600**	-0.0559	**0.2616**	**0.0960**
	(0.0364)	*(0.2821)*	*(0.2537)*	*(0.2500)*	*(0.2539)*	*(1.6851)*	*(0.0329)*	*(0.0985)*	*(0.1078)*	*(0.0936)*	*(0.0964)*	*(0.0488)*
α_1	**0.0833**	0.0009	-0.0061	0.0178	0.0172	0.0476	**0.0789**	0.0301	**0.0608**	0.0273	0.0375	**0.0721**
	(0.0209)	*(0.0287)*	*(0.0304)*	*(0.0286)*	*(0.0291)*	*(0.0432)*	*(0.0209)*	*(0.0274)*	*(0.0322)*	*(0.0273)*	*(0.0290)*	*(0.0199)*
β_1	**0.9111**	**0.3203**	**0.2371**	**0.4191**	**0.3859**	-0.1880	**0.8952**	**0.5575**	**0.5386**	**0.5591**	**0.5638**	**0.9110**
	(0.0197)	*(0.1041)*	*(0.1282)*	*(0.1144)*	*(0.1018)*	*(0.2243)*	*(0.0244)*	*(0.0806)*	*(0.1059)*	*(0.0870)*	*(0.0786)*	*(0.0220)*
γ		**0.6655**	**0.6262**	**0.3899**	**0.6246**	**8.62e-08**		**0.3369**	**0.4231**	**0.1982**	**0.3427**	-2.10e-09
		(0.1297)	*(0.1256)*	*(0.0917)*	*(0.1341)*	*(4.77e-08)*		*(0.0720)*	*(0.1116)*	*(0.0433)*	*(0.0716)*	*(4.86e-09)*
lnL	-4294.63	-4254.77	-4243.09	-4257.88	-4259.358	-4370.711	-3523.27	-3480.88	-3487.67	-3477.53	-3480.98	-3521.21
AIC	4.8949	4.8507	4.8374	4.8542	4.8559	4.9826	4.0060	3.9612	3.9689	3.9574	3.9613	4.0071
SBC	4.9198	4.8787	4.8654	4.8823	4.8839	5.0106	4.0184	3.9768	3.9845	3.9730	3.9769	4.0226
LB(10)	0.148	0.329	0.315	0.284	0.295	0.054	0.240	0.074	0.066	0.079	0.072	0.224
ARCH(10)	0.149	0.425	0.439	0.382	0.478	0.096	0.100	0.702	0.713	0.610	0.629	0.063
Ranking	5	2	1	3	4	6	5 or 6	2	4	1	3	6 or 5

Table D. GARCH estimation results from 02/01/97 to 31/12/03 (*cont.*)

	GM						**IBM**					
	GARCH	with RV	with RR	with RPV	with RBP	with VOL	GARCH	with RV	with RR	with RPV	with RBP	with VOL
θ_0	**-0.1559**	**-0.1729**	**-0.1192**	**-0.1748**	**-0.1764**	-0.1138	0.0665	-0.0295	-0.0137	-0.0244	-0.0266	-0.0130
	(*0.0732*)	(*0.0756*)	(*0.0493*)	(*0.0756*)	(*0.0754*)	(*0.0810*)	(*0.0740*)	(*0.0725*)	(*0.0737*)	(*0.0716*)	(*0.0720*)	(*0.0620*)
θ_1	**-0.8924**	**-0.8912**	**-0.2519**	**-0.8911**	**-0.8883**	-0.0382	**-0.9220**	**-0.8667**	**-0.8727**	**-0.8631**	**-0.8601**	-0.0736
	(*0.0638*)	(*0.0652*)	(*0.0894*)	(*0.0651*)	(*0.0673*)	(*0.2868*)	(*0.0450*)	(*0.0862*)	(*0.0816*)	(*0.0858*)	(*0.0897*)	(*2.3716*)
λ_1	**0.8720**	**0.8660**	**0.2290**	**0.8655**	**0.8634**	0.0050	**0.8927**	**0.8297**	**0.8358**	**0.8241**	**0.8221**	0.0050
	(*0.0685*)	(*0.0719*)	(*0.0932*)	(*0.0719*)	(*0.0740*)	(*0.2745*)	(*0.0518*)	(*0.0969*)	(*0.0923*)	(*0.0966*)	(*0.1007*)	(*2.4051*)
ω	**0.0852**	**0.2070**	**0.2814**	0.0176	**0.2205**	**3.5394**	**0.0560**	**0.2149**	**0.2871**	**-0.1331**	**0.2025**	**3.6959**
	(*0.0348*)	(*0.0884*)	(*0.1180*)	(*0.0876*)	(*0.0857*)	(*2.1325*)	(*0.0245*)	(*0.0851*)	(*0.1236*)	(*0.0809*)	(*0.0797*)	(*1.8552*)
α_1	**0.0725**	0.0206	**0.0502**	0.0217	0.0216	**0.1495**	**0.0784**	-0.0061	0.0416	-0.0046	-0.0035	0.1499
	(*0.0161*)	(*0.0210*)	(*0.0210*)	(*0.0211*)	(*0.0212*)	(*0.0363*)	(*0.0189*)	(*0.0277*)	(*0.0412*)	(*0.0322*)	(*0.0301*)	(*0.0168*)
β_1	**0.9042**	**0.7136**	**0.7038**	**0.7128**	**0.7176**	**0.5979**	**0.9088**	**0.5392**	**0.4708**	**0.5904**	**0.5627**	0.5999
	(*0.0230*)	(*0.0683*)	(*0.0841*)	(*0.0744*)	(*0.0652*)	(*0.2265*)	(*0.0202*)	(*0.0888*)	(*0.1073*)	(*0.0949*)	(*0.0864*)	(*0.5426*)
γ		**0.2419**	**0.3369**	**0.1431**	**0.2568**	-3.41e-07		**0.4363**	**0.6095**	**0.2249**	**0.4395**	-1.81e-07
		(*0.0660*)	(*0.1141*)	(*0.0426*)	(*0.0688*)	(*2.86e-07*)		(*0.0992*)	(*0.1594*)	(*0.0568*)	(*0.1014*)	(*4.80e-07*)
lnL	-3528.99	-3506.23	-3517.09	-3506.29	-3504.33	-3804.83	-3562.08	-3521.13	-3528.77	-3517.23	-3518.57	-3853.82
AIC	4.0170	3.9923	4.0046	3.9924	3.9901	4.3314	4.0546	4.0092	4.0179	4.0048	4.0063	4.3873
SBC	4.0357	4.0140	4.0264	4.0141	4.0119	4.3532	4.0733	4.0310	4.0397	4.0266	4.0281	4.4091
$LB(10)$	0.225	0.143	0.038	0.153	0.151	0.172	0.292	0.527	0.463	0.539	0.567	0.007
$ARCH(10)$	0.935	0.874	0.815	0.883	0.915	0.073	0.853	0.898	0.979	0.859	0.875	0.000
Ranking	5	2	4	3	1	6	5	3	4	1	2	6

Table D. GARCH estimation results from 02/01/97 to 31/12/03 (*cont.*)

	JPM						KO					
	GARCH	with RV	with RR	with RPV	with RBP	with VOL	GARCH	with RV	with RR	with RPV	with RBP	with VOL
θ_0	-0.0025	-0.0304	-0.0404	-0.0265	-0.0343	0.0055	**0.2543**	0.0070	0.0070	0.0071	0.0076	0.0816
	(0.0450)	*(0.0447)*	*(0.0453)*	*(0.0433)*	*(0.0445)*	*(0.0448)*	*(0.0637)*	*(0.0091)*	*(0.0100)*	*(0.0092)*	*(0.0105)*	*(0.1286)*
θ_1							**-0.8833**	**0.8982**	**0.8959**	**0.9010**	**0.8927**	-0.0074
							(0.0891)	*(0.0947)*	*(0.1073)*	*(0.0937)*	*(0.1087)*	*(1.4611)*
θ_2							0.0138	0.0296	0.0282	0.0279	0.0279	0.0007
							(0.0294)	*(0.0264)*	*(0.0264)*	*(0.0264)*	*(0.0264)*	*(0.0331)*
λ_1							**0.8663**	**-0.9157**	**-0.9136**	**-0.9168**	**-0.9098**	0.0050
							(0.0837)	*(0.0907)*	*(0.1036)*	*(0.0896)*	*(0.1055)*	*(1.4609)*
ω	**0.0668**	0.1019	**0.3544**	**-0.2228**	0.1183	**0.1030**	0.0057	0.0174	0.1459	**-0.1319**	0.0182	**2.7357**
	(0.0301)	*(0.0795)*	*(0.1724)*	*(0.0260)*	*(0.0811)*	*(0.0616)*	*(0.0056)*	*(0.0435)*	*(0.1047)*	*(0.0809)*	*(0.0344)*	*(0.3624)*
α_1	**0.1643**	0.0367	0.0304	0.0563	0.0380	**0.1645**	**0.1279**	**0.0637**	**0.0732**	**0.0646**	**0.0716**	**0.1199**
	(0.0653)	*(0.0499)*	*(0.0481)*	*(0.0594)*	*(0.0480)*	*(0.0658)*	*(0.0538)*	*(0.0336)*	*(0.0345)*	*(0.0337)*	*(0.0339)*	*(0.0613)*
α_2	-0.0984	-0.0223	-0.0133	-0.0546	-0.0263	-0.0969	-0.0157	-0.0073	0.0117	-0.0050	-0.0065	0.0399
	(0.0627)	*(0.0472)*	*(0.0366)*	*(0.0474)*	*(0.0438)*	*(0.0636)*	*(0.0553)*	*(0.0438)*	*(0.0399)*	*(0.0438)*	*(0.0448)*	*(0.0608)*
α_3							**-0.0812**	-0.0230	-0.0234	-0.0254	-0.0274	0.0399
							(0.0442)	*(0.0304)*	*(0.0230)*	*(0.0315)*	*(0.0330)*	*(0.0786)*
β_1	**0.9234**	**0.6398**	**0.4581**	**0.7191**	**0.6411**	**0.9238**	**0.9673**	**0.7038**	**0.4220**	**0.7000**	**0.7687**	**0.4799**
	(0.0150)	*(0.0718)*	*(0.1422)*	*(0.0639)*	*(0.0748)*	*(0.0143)*	*(0.0108)*	*(0.0828)*	*(0.1424)*	*(0.0879)*	*(0.0654)*	*(0.0819)*
γ		**0.3267**	**0.6354**	**0.1686**	**0.3527**	-6.54e-09		**0.2524**	**0.6685**	**0.1355**	**0.2023**	**-3.08e-07**
		(0.0730)	*(0.1829)*	*(0.0387)*	*(0.0822)*	*(1.66e-08)*		*(0.0771)*	*(0.1773)*	*(0.0411)*	*(0.0662)*	*(7.56e-08)*
lnL	-3827.16	-3783.81	-3781.37	-3778.06	-3780.40	-3822.70	-3269.57	-3245.90	-3253.73	-3245.79	-3250.14	-3429.85
AIC	4.3522	4.3066	4.3038	4.3000	4.3027	4.3508	3.7278	3.7020	3.7109	3.7019	3.7068	3.9111
SBC	4.3678	4.3253	4.3225	4.3187	4.3214	4.3695	3.7558	3.7331	3.7420	3.7330	3.7379	3.9423
$LB(10)$	0.249	0.307	0.289	0.273	0.302	0.273	0.183	0.035	0.031	0.038	0.035	0.038
$ARCH(10)$	0.561	0.509	0.429	0.559	0.586	0.615	0.298	0.477	0.166	0.4157	0.557	0.000
Ranking	5 or 6	4	3	1	2	6 or 5	5	2	4	1	3	6

Table D. GARCH estimation results from 02/01/97 to 31/12/03 (*cont.*)

	MCD						**PG (1759 obs.)**					
	GARCH	with RV	with RR	with RPV	with RBP	with VOL	GARCH	with RV	with RR	with RPV	with RBP	with VOL
θ_0	**0.0761**	**0.0858**	**0.0968**	**0.0870**	**0.0894**	0.0601	**0.1535**	**0.1348**	**0.1421**	**0.1404**	**0.1349**	**0.1445**
	(*0.0400*)	(*0.0389*)	(*0.0390*)	(*0.0390*)	(*0.0389*)	(*0.0413*)	(*0.0314*)	(*0.0309*)	(*0.0317*)	(*0.0314*)	(*0.0311*)	(*0.0598*)
ω	**0.0313**	**0.4662**	**0.8148**	0.1201	**0.5493**	**3.2460**	**-0.0789**	**-0.0624**	**-0.0605**	**-0.0637**	**-0.0640**	**-0.1049**
	(*0.0145*)	(*0.1775*)	(*0.2621*)	(*0.1837*)	(*0.1910*)	(*0.5337*)	(*0.0270*)	(*0.0260*)	(*0.0261*)	(*0.0263*)	(*0.0263*)	(*0.0633*)
α_1	**0.1588**	0.0693	0.0712	0.0707	0.0640	**0.1650**	0.0083	0.0554	0.0159	-0.1097	0.0628	2.6480
	(*0.0547*)	(*0.0500*)	(*0.0498*)	(*0.0499*)	(*0.0471*)	(*0.0542*)	(*0.0054*)	(*0.0503*)	(*0.0656*)	(*0.0941*)	(*0.0479*)	(*7.3021*)
α_2	**-0.1368**	-0.0545	-0.0289	-0.0598	-0.0463	**0.1313**	**0.0604**	0.0190	**0.0510**	**0.0389**	0.0312	0.1499
	(*0.0547*)	(*0.0420*)	(*0.0405*)	(*0.0396*)	(*0.0444*)	(*0.0259*)	(*0.0130*)	(*0.0304*)	(*0.0276*)	(*0.0323*)	(*0.0306*)	(*0.3984*)
β_1	**0.9687**	**0.4324**	0.1502	**0.4789**	**0.3960**	**-0.4529**	**0.9393**	**0.4123**	**0.4511**	**0.4213**	**0.4361**	0.5999
	(*0.0108*)	(*0.1310*)	(*0.1761*)	(*0.1477*)	(*0.1332*)	(*0.2017*)	(*0.0116*)	(*0.0799*)	(*0.0960*)	(*0.1001*)	(*0.0821*)	(*0.8816*)
γ		**0.3697**	**0.6953**	**0.2036**	**0.4136**	1.09e-07		**0.5001**	**0.6852**	**0.2317**	**0.5061**	-1.71e-07
		(*0.1133*)	(*0.1770*)	(*0.0704*)	(*0.1163*)	(*1.89e-07*)		(*0.0886*)	(*0.0159*)	(*0.0546*)	(*0.0957*)	(*1.65e-06*)
lnL	-3448.92	-3426.59	-3426.78	-3423.96	-3427.70	-3476.06	-3193.47	-3127.40	-3143.66	-3130.86	-3132.02	-3616.20
AIC	3.9227	3.9007	3.9009	3.8977	3.9019	3.9569	3.6388	3.5647	3.5833	3.5687	3.5699	4.1208
SBC	3.9382	3.9193	3.9195	3.9163	3.9206	3.9755	3.6543	3.5834	3.6019	3.5870	3.5887	4.1395
$LB(10)$	0.874	0.862	0.828	0.870	0.869	0.711	0.225	0.274	0.182	0.252	0.245	0.061
$ARCH(10)$	0.668	0.419	0.181	0.408	0.400	0.024	0.105	0.116	0.173	0.166	0.090	0.000
Ranking	5	2	3	1	4	6	5	1	4	2	3	6

Table D. GARCH estimation results from 02/01/97 to 31/12/03 (*cont.*)

	ATT (1760 obs.)						**WMT**					
	GARCH	with RV	with RR	with RPV	with RBP	with VOL	GARCH	with RV	with RR	with RPV	with RBP	with VOL
θ_0	**-0.2357**	-0.0032	-0.0031	-0.0031	**-0.0031**	**-0.2622**	-0.0083	-0.0459	-0.0362	-0.0417	-0.0389	-0.0042
	(*0.1100*)	(*0.0029*)	(*0.0029*)	(*0.0030*)	(*0.0029*)	(*0.1271*)	(*0.0398*)	(*0.0401*)	(*0.0401*)	(*0.0401*)	(*0.0400*)	(*0.0394*)
θ_1	**-0.2299**	**1.0831**	**1.0988**	**1.0879**	**1.0811**	**-0.6065**	**-0.0785**	**-0.0681**	**-0.0713**	**-0.0678**	**-0.0678**	**-0.0788**
	(*0.0269*)	(*0.0738*)	(*0.1899*)	(*0.1337*)	(*0.1302*)	(*0.0251*)	(*0.0273*)	(*0.0255*)	(*0.0266*)	(*0.0259*)	(*0.0257*)	(*0.0271*)
θ_2	**-0.9758**	**-0.1759**	-0.1945	-0.1785	-0.1746	**-0.9692**	**-0.0629**	**-0.0635**	**-0.0656**	**-0.0616**	**-0.0613**	**-0.0644**
	(*0.0094*)	(*0.0845*)	(*0.1878*)	(*0.1359*)	(*0.1359*)	(*0.0165*)	(*0.0248*)	(*0.0247*)	(*0.0247*)	(*0.0246*)	(*0.0246*)	(*0.0247*)
θ_3	0.0324	**-0.0638**	**0.0681**	**0.0615**	**0.0654**	0.0308						
	(*0.0264*)	(*0.0263*)	(*0.0262*)	(*0.0266*)	(*0.0265*)	(*0.0239*)						
λ_1	**0.2542**	**-1.0822**	**-1.0957**	**-1.0849**	**-1.0786**	**0.6305**						
	(*0.0058*)	(*0.0704*)	(*0.1874*)	(*0.1304*)	(*0.1267*)	(*0.0095*)						
λ_2	**0.9895**	**0.1260**	0.1381	0.1285	0.1216	**0.9839**						
	(*0.0055*)	(*0.0762*)	(*0.1843*)	(*0.1301*)	(*0.1260*)	(*0.0074*)						
ω	**0.1758**	**0.8059**	**1.1156**	**0.2872**	**0.8707**	**0.1716**	**0.0382**	**0.0986**	**0.0635**	**-0.0545**	**0.1196**	**0.1225**
	(*0.0598*)	(*0.2535*)	(*0.2275*)	(*0.1711*)	(*0.1878*)	(*0.0393*)	(*0.0165*)	(*0.0521*)	(*0.0502*)	(*0.0617*)	(*0.0519*)	(*0.0600*)
α_1	**0.0731**	**0.0598**	**0.0907**	**0.0583**	**0.0597**	**0.0723**	**0.0730**	**0.0266**	**0.0661**	**0.0371**	**0.0304**	**0.0753**
	(*0.0164*)	(*0.0353*)	(*0.0265*)	(*0.0232*)	(*0.0247*)	(*0.0101*)	(*0.0159*)	(*0.0201*)	(*0.0244*)	(*0.0217*)	(*0.0199*)	(*0.0172*)
β_1	**0.8933**	**0.3818**	**0.2733**	**0.3817**	**0.3582**	0.8988	**0.9194**	**0.7208**	**0.7512**	**0.7242**	**0.7155**	**0.9125**
	(*0.0238*)	(*0.1112*)	(*0.0823*)	(*0.0687*)	(*0.0732*)	(*0.0140*)	(*0.0158*)	(*0.0608*)	(*0.0618*)	(*0.0680*)	(*0.0609*)	(*0.0178*)
γ		**0.4526**	**0.6996**	**0.2911**	**0.5180**	-7.76e-09		**0.2244**	**0.2553**	**0.1229**	**0.2459**	-1.14e-08
		(*0.1066*)	(*0.1034*)	(*0.0393*)	(*0.0763*)	(*7.29e-09*)		(*0.0576*)	(*0.0815*)	(*0.0351*)	(*0.0617*)	(*1.75e-08*)
lnL	-3842.485	-3811.44	-3819.13	-3810.09	-3810.27	-3843.61	-3592.27	-3571.11	-3577.25	-3571.08	-3568.41	-3589.03
AIC	4.3846	4.3499	4.3587	4.3484	4.3486	4.3866	4.0913	4.0683	4.0753	4.0683	4.0653	4.0887
SBC	4.4127	4.3811	4.3899	4.3796	4.3798	4.4177	4.1099	4.0901	4.0971	4.0901	4.0871	4.1105
$LB(10)$	0.092	0.212	0.177	0.242	0.254	0.163	0.277	0.106	0.181	0.117	0.112	0.225
$ARCH(10)$	0.270	0.999	0.999	0.999	0.999	0.205	0.258	0.993	0.937	0.991	0.986	0.431
Ranking	5	3	4	1	2	6	6 or 5	3	4	2	1	5 or 6

Table E. Summary of forecast competition

	Stock													
Criteria	**ATT**	**AXP**	**BA**	**CAT**	**DELL**	**GE**	**GM**	**IBM**	**JPM**	**KO**	**MCD**	**MSFT**	**PG**	**WMT**
MAE	RPV	RPV	RV	RPV	RR	RPV	RBP	RPV	RPV	RV	RPV	RR	RV	RPV
MSE	RPV	RPV	RV	RPV	RR	RR	RPV	RPV	RR	RV	RR	RR	RBP	RPV
MAPE	RPV	RPV	RV	RPV	RR	RPV	RV	RPV	RPV	RV	RPV	RR	RV	RPV
MSPE	RPV	RPV	RPV	RPV	RR	RPV	RPV	RPV	RPV	RV	RPV	RR	RV	RPV
HMAE	RPV	RPV	RPV	RPV	RR	RPV	RBP	RPV	RBP	RBP	RR	RR	RBP	RBP
HMSE	RR	RPV	RR	RPV	RR	RR	RBP	RPV	RR	RBP	RR	RPV	RBP	RR
MME(U)	RBP	RV	RV	RV	RPV	RBP	RBP	RPV	RBP	RV	RPV	RR	RPV	RPV
MME(O)	RPV	RPV	RPV	RV	RR	RPV	RPV	RPV	RPV	RPV	RPV	RR	RPV	RPV
LL	RPV	RPV	RPV	RPV	RR	RPV	RV	RPV	RPV	RV	RV	RR	RV	RPV
QLE	RPV	RPV	RPV	RPV	RR	RPV	RBP	RPV	RPV	RBP	RR	RR	RV	RPV
$\mathbf{MZ - R^2}$	RPV	RPV	RPV	RPV	RR	RR	RBP	RPV	RPV	RV	RPV	RR	RBP	RBP

	Criteria										
Model	MAE	MSE	HMAE	HMSE	AMAPE	Theil-U	MME(U)	MME(O)	LL	GMLE	MZ-R^2
GARCH	0%	0%	0%	0%	0%	0%	7.1%	0%	0%	0%	0%
GARCH-RV	21.4%	14.3%	28.6%	14.3%	21.4%	14.3%	28.1]%	7.1%	21.4%	7%	7.1%
GARCH-RR	14.3%	28.6%	14.3%	14.3%	14.3%	**35.7**%	7.1%	14.3%	14.3%	21%	21.4%
GARCH-RPV	**57.1**%	**42.9**%	**57.1**%	**71.4**%	**57.1**%	**35.7**%	**35.7**%	**71.4**%	**64.3**%	**57**%	**50.0**%
GARCH-RBP	7.1%	14.3%	0%	0%	7.1%	14.3%	21.4%	7.1%	0%	14%	21.4%
GARCH-VOL	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%	0%

The entries in the bottom panel are the frequency that each model had the smallest out-of-sample loss or largest MZ–R^2. For instance, the entry 57.1% under MAE indicates that GARCH augmented with RPV gave the smallest MAE in 8 out of 14 stocks. For each criteria, the model that emerges most often as top performer is indicated in bold.

Table F. Out-of-sample forecast model ranking based on 500 one-day-ahead projections.

Stock	Model	Forecast accuracy measures										
		MAE	MSE	HMAE	HMSE	AMAPE	Theil-U	MME(U)	MME(O)	LL	GMLE	MZ-R^2
	GARCH	6	6	6	6	6	6	6	4	6	6	6
	GARCH-RV	3	2	3	3	3	3	2	3	3	3	3
	GARCH-RR	5	5	5	5	5	5	5	6	5	5	4
ATT	GARCH-RPV	2	1	2	2	2	1	4	2	2	2	1
	GARCH-RBP	4	4	4	4	4	4	1	5	4	4	2
	GARCH-VOL	7	7	7	7	7	7	7	7	7	7	7
	COMBINED	1	3	1	1	1	2	3	1	1	1	5
	GARCH	5	6	5	5	5	6	6	5	5	5	6
	GARCH-RV	4	4	4	3	4	4	2	4	4	3	4
	GARCH-RR	6	5	6	6	6	5	5	6	6	4	5
AXP	GARCH-RPV	1	1	2	2	2	2	4	1	1	1	1
	GARCH-RBP	3	3	3	4	3	3	3	3	3	2	3
	GARCH-VOL	7	7	7	7	7	7	7	7	7	6	7
	COMBINED	2	2	1	1	1	1	1	2	2	7	2
	GARCH	4	6	4	4	4	6	6	3	5	6	6
	GARCH-RV	1	1	1	2	1	1	1	2	2	2	2
	GARCH-RR	5	4	6	6	5	4	5	5	6	4	4
BA	GARCH-RPV	2	2	2	1	2	2	3	1	1	1	1
	GARCH-RBP	3	3	3	3	3	3	2	4	3	3	3
	GARCH-VOL	7	7	7	7	6	7	7	7	7	7	7
	COMBINED	6	5	5	5	4	5	4	6	4	5	5
	GARCH	6	6	6	6	6	6	6	6	6	6	6
	GARCH-RV	3	3	3	4	3	3	2	2	3	3	3
	GARCH-RR	5	5	5	5	5	5	5	5	5	5	5
CAT	GARCH-RPV	2	2	2	2	2	2	3	3	2	2	2
	GARCH-RBP	4	4	4	3	4	4	4	4	4	4	4
	GARCH-VOL	7	7	7	7	7	7	7	7	7	7	7
	COMBINED	1	1	1	1	1	1	1	1	1	1	1
	GARCH	3	6	3	3	3	6	5	3	3	4	6
	GARCH-RV	4	3	5	5	4	3	7	4	4	3	3
	GARCH-RR	2	2	2	2	2	2	4	2	2	1	2
DELL	GARCH-RPV	6	4	4	4	6	4	2	6	5	6	4
	GARCH-RBP	5	5	6	6	5	5	6	5	6	5	5
	GARCH-VOL	7	7	7	7	7	7	3	7	7	7	7
	COMBINED	1	1	1	1	1	1	1	1	1	2	1
	GARCH	6	6	5	5	5	6	6	2	4	5	7
	GARCH-RV	3	4	4	4	2	4	4	3	3	3	4
	GARCH-RR	5	2	6	6	6	2	5	6	5	4	2
GE	GARCH-RPV	1	3	2	2	1	3	3	1	1	1	3
	GARCH-RBP	4	5	3	3	3	5	2	4	2	2	5
	GARCH-VOL	7	7	7	7	7	7	7	7	7	6	6
	COMBINED	2	1	1	1	4	1	1	5	6	7	1

Number 1 to 7 stands for the rank of the models according to their forecast performance, 1 denotes the best and 7 the worst.

Table F. Model ranking by forecast accuracy based on 500 out-of-sample one-day-ahead projections (*cont*).

		Forecast accuracy measures										
Stock	**Model**	MAE	MSE	HMAE	HMSE	AMAPE	Theil-U	MME(U)	MME(O)	LL	GMLE	MZ-R^2
	GARCH	5	5	5	5	5	5	5	5	5	5	5
	GARCH-RV	3	4	2	3	3	4	3	3	2	3	4
	GARCH-RR	7	7	7	7	7	7	7	7	7	7	5
GM	GARCH-RPV	4	2	3	2	4	2	4	2	3	4	2
	GARCH-RBP	2	3	4	4	2	3	2	4	4	2	1
	GARCH-VOL	6	6	6	6	6	6	6	6	6	6	7
	COMBINED	1	1	1	1	1	1	1	1	1	1	3
	GARCH	6	6	6	6	6	6	6	6	5	5	6
	GARCH-RV	3	3	4	4	4	3	3	4	3	3	4
	GARCH-RR	5	5	5	5	5	5	5	5	4	4	3
IBM	GARCH-RPV	2	2	2	2	1	2	2	1	1	1	2
	GARCH-RBP	4	4	3	3	3	4	4	3	2	2	5
	GARCH-VOL	7	7	7	7	7	7	7	7	7	7	7
	COMBINED	1	1	1	1	2	1	1	2	6	6	1
	GARCH	6	6	6	6	6	6	2	6	5	5	6
	GARCH-RV	3	5	3	4	3	5	4	5	3	3	5
	GARCH-RR	4	1	5	5	4	2	7	2	4	4	3
JPM	GARCH-RPV	1	3	1	1	1	3	5	1	1	1	1
	GARCH-RBP	2	4	2	3	2	4	3	3	2	2	4
	GARCH-VOL	7	7	7	7	7	7	6	7	6	6	7
	COMBINED	5	2	4	2	5	1	1	4	7	7	2
	GARCH	6	6	4	5	5	6	6	4	6	5	6
	GARCH-RV	1	1	1	1	1	1	1	2	1	2	1
	GARCH-RR	5	5	6	6	6	5	5	6	5	4	4
KO	GARCH-RPV	3	3	2	2	3	3	3	1	3	6	3
	GARCH-RBP	2	2	3	3	2	2	2	3	2	1	2
	GARCH-VOL	7	7	7	7	7	7	7	7	7	7	7
	COMBINED	4	4	5	4	4	4	4	5	4	3	5
	GARCH	6	6	7	7	6	6	7	3	6	6	6
	GARCH-RV	5	5	3	4	5	5	2	7	5	5	5
	GARCH-RR	4	1	5	3	4	1	4	6	3	1	2
MCD	GARCH-RPV	1	2	1	1	1	2	1	1	1	4	1
	GARCH-RBP	3	4	2	2	3	4	3	5	4	3	3
	GARCH-VOL	7	7	6	6	7	7	5	4	7	7	7
	COMBINED	2	3	4	5	2	3	6	2	2	2	4

Table F. Model ranking by forecast accuracy based on 500 out-of-sample one-day-ahead projections (*cont*).

Stock	**Model**	**Forecast accuracy measures**										
		MAE	MSE	HMAE	HMSE	AMAPE	Theil-U	MME(U)	MME(O)	LL	GMLE	MZ-R^2
MSFT	GARCH	6	6	6	6	6	6	6	6	6	6	6
	GARCH-RV	5	4	5	5	5	4	3	5	5	5	4
	GARCH-RR	2	2	2	2	2	2	2	2	2	2	1
	GARCH-RPV	3	3	3	3	3	3	5	3	3	3	3
	GARCH-RBP	4	5	4	4	4	5	4	4	4	4	5
	GARCH-VOL	7	7	7	7	7	7	7	7	7	7	7
	COMBINED	1	1	1	1	1	1	1	1	1	1	2
PG	GARCH	6	6	6	6	6	6	6	6	5	5	6
	GARCH-RV	1	3	1	1	1	3	4	2	1	1	3
	GARCH-RR	4	5	4	4	4	5	5	4	4	3	5
	GARCH-RPV	2	2	2	2	3	2	1	1	3	4	2
	GARCH-RBP	3	1	3	3	2	1	2	3	2	2	1
	GARCH-VOL	7	7	7	7	7	7	7	7	7	7	7
	COMBINED	5	4	5	5	5	4	3	5	6	6	4
WMT	GARCH	7	6	7	7	7	6	7	6	7	5	6
	GARCH-RV	4	3	3	3	3	3	3	4	3	3	3
	GARCH-RR	5	5	6	6	5	5	5	7	4	4	5
	GARCH-RPV	2	2	1	1	1	2	2	1	1	1	2
	GARCH-RBP	3	1	3	4	2	1	4	3	2	2	1
	GARCH-VOL	6	7	5	5	6	7	6	2	6	6	7
	COMBINED	1	4	2	2	4	4	1	5	5	7	4

Table G. Out-of-sample forecast evaluation: 15min- and 30min-based nonparametric volatility estimators

		Forecast accuracy measures									
Stock	**Model**	MAE	MSE	HMAE	HMSE	AMAPE	Theil-U	MME(U)	MME(O)	LL	GMLE
		15min-based estimators (volatility proxy: sum of 15min squared returns)									
	GARCH	1.8452	6.1452	1.0725	2.6368	0.2930	0.9699	16.2367	6.3406	0.6232	2.0727
	GARCH-RV	1.4895	4.5613	0.8488	1.7772	0.2512	0.7199	12.1175	4.4958	0.4711	2.0188
	GARCH-RR	1.5945	4.7522	0.9262	2.0242	0.2653	0.7500	12.0351	5.0393	0.5135	2.0297
CAT	GARCH-RPV	**1.4475**	**4.2501**	**0.7881**	**1.4891**	**0.2439**	**0.6707**	**10.4625**	**4.3105**	**0.4288**	**2.0052**
	GARCH-RBP	1.4760	4.3446	0.8266	1.6234	0.2499	0.6857	10.7070	4.3567	0.4538	2.0126
	GARCH-VOL	2.3252	9.0130	1.4416	6.2737	0.3297	1.4225	19.1205	9.9885	0.8400	2.1336
	Benefit (%)	21.58	30.84	26.52	43.53	16.76	30.84	35.56	32.02	31.19	3.26
	GARCH	2.2002	10.5039	1.1857	3.6009	0.3030	1.1680	25.0551	10.1579	0.6828	2.1647
	GARCH-RV	1.7947	7.2930	0.9021	2.0557	0.2622	0.8111	16.2483	6.9558	0.4980	2.1042
	GARCH-RR	1.8134	7.1194	0.9143	2.0353	0.2652	0.7918	16.7347	7.1029	0.4984	2.1014
MSFT	GARCH-RPV	1.7444	**6.7196**	**0.8434**	**1.6794**	**0.2551**	**0.7473**	16.3173	**6.3318**	**0.4566**	**2.0894**
	GARCH-RBP	**1.7291**	6.7928	0.8549	1.8312	0.2554	0.7555	**16.1334**	6.5660	0.4651	2.0922
	GARCH-VOL	3.6935	21.3694	2.0386	9.7979	0.3978	2.3763	33.2459	23.4384	1.2074	2.3086
	Benefit (%)	21.41	36.03	28.86	53.36	15.81	36.02	35.61	37.67	33.13	3.48
		30min-based estimators (volatility proxy: sum of 30min squared returns)									
	GARCH	2.1018	8.7908	1.2671	4.0111	0.3202	0.7841	24.3994	7.5443	0.7711	2.1185
	GARCH-RV	1.8281	7.1560	1.0595	2.9427	0.2904	0.6379	18.8075	5.9769	0.6291	2.0688
	GARCH-RR	1.8580	7.1819	1.0976	3.1335	0.2938	0.6403	19.8798	6.2230	0.6467	2.0710
CAT	GARCH-RPV	**1.7829**	**7.0134**	**0.9924**	**2.6561**	**0.2829**	**0.6250**	**17.9472**	**5.6571**	**0.5885**	**2.0596**
	GARCH-RBP	1.8408	7.2984	1.0803	3.2576	0.2918	0.6506	19.8291	6.0669	0.6416	2.0741
	GARCH-VOL	2.5615	11.6055	1.6429	8.0610	0.3538	1.0352	26.0878	11.3587	0.9825	2.1753
	Benefit (%)	15.17	20.22	21.68	33.78	11.65	20.29	26.44	25.03	23.68	2.78
	GARCH	2.3166	11.2329	1.4395	5.9148	0.3261	0.9939	27.4053	10.8263	0.8435	2.1550
	GARCH-RV	1.9299	8.3523	1.0841	3.3056	0.2849	0.7394	18.5342	7.7908	0.6230	2.0887
	GARCH-RR	1.9724	8.1171	1.1455	3.6760	0.2934	0.7184	18.6099	7.9044	0.6507	2.0920
MSFT	GARCH-RPV	**1.8926**	**7.6060**	**1.0737**	**3.2706**	**0.2828**	**0.6733**	**16.9271**	**7.1985**	**0.6108**	**2.0811**
	GARCH-RBP	1.9519	8.1193	1.1403	3.7236	0.2903	0.7187	17.4053	8.0002	0.6503	2.0915
	GARCH-VOL	3.7655	22.1015	2.3966	16.9093	0.4142	1.9560	34.4497	24.1989	1.3966	2.2982
	Benefit (%)	18.30	32.29	25.41	44.70	13.28	32.26	38.23	33.51	27.59	3.43

The variable of interest is the daily conditional variance of returns proxied by the sum of 15-min or 30-min squared returns.

Table G. Out-of-sample forecast evaluation: 15min- and 30min-based nonparametric volatility estimators (*cont.*)

		Forecast accuracy measures									
Stock	**Model**	MAE	MSE	HMAE	HMSE	AMAPE	Theil-U	MME(U)	MME(O)	LL	GMLE
		15min-based estimators (volatility proxy: sum of 15min squared returns)									
	GARCH	2.6086	12.4931	0.9836	2.3056	0.2796	1.0357	20.7275	13.5062	0.5475	2.3486
	GARCH-RV	**2.4917**	11.5091	**0.9393**	**1.9903**	**0.2723**	0.9558	22.3038	**12.2819**	**0.5077**	**2.3322**
	GARCH-RR	2.5436	**11.2070**	0.9616	2.0502	0.2767	**0.9275**	19.0468	12.2937	0.5179	2.3343
DELL	GARCH-RPV	2.5737	11.4780	0.9467	1.9951	0.2766	0.9528	**17.2454**	12.9164	0.5152	2.3361
	GARCH-RBP	2.6245	11.6859	0.9991	2.2590	0.2829	0.9697	21.9929	12.7462	0.5437	2.3428
	GARCH-VOL	7.8289	128.9082	2.8343	26.0628	0.4148	10.7154	19.7739	153.2335	1.6134	2.6239
	Benefit (%)	4.48	10.29	4.50	13.68	2.61	10.45	16.80	9.06	7.27	0.70
	GARCH	3.3104	33.8337	1.0834	2.9916	0.3027	0.7227	105.7357	15.8139	0.6843	2.5746
	GARCH-RV	3.1754	35.1483	0.9818	2.7708	0.2801	0.7510	**85.4884**	21.2511	0.5987	**2.5344**
	GARCH-RR	3.1872	33.0419	0.9987	2.4946	0.2862	0.7060	92.6409	18.9587	0.6051	2.5371
ATT	GARCH-RPV	**3.0149**	**31.8107**	**0.9152**	**2.0591**	**0.2763**	**0.6796**	96.9213	**14.3782**	0.6303	3.1826
	GARCH-RBP	3.1489	34.8024	0.9548	2.3815	0.2795	0.7435	93.9180	19.9500	**0.5872**	2.5350
	GARCH-VOL	5.4347	56.9724	2.4617	20.3447	0.3998	1.2631	119.6941	47.4503	1.4362	2.7406
	Benefit (%)	8.93	5.98	15.53	31.17	8.72	5.96	19.15	9.08	14.19	1.56
		30min-based estimators (volatility proxy: sum of 30min squared returns)									
	GARCH	2.7127	14.9369	1.2721	5.7954	0.3031	0.9047	24.8385	15.2411	0.7192	2.3214
	GARCH-RV	**2.5075**	**13.3291**	**1.1292**	**4.2759**	**0.2844**	**0.8085**	25.8539	**13.0193**	**0.6263**	**2.2901**
	GARCH-RR	2.7015	13.8750	1.2870	5.3847	0.3069	0.8390	29.7938	13.7262	0.7171	2.3161
DELL	GARCH-RPV	2.6760	13.9793	1.2207	4.9289	0.2967	0.8472	25.6367	14.1998	0.6836	2.3071
	GARCH-RBP	2.6221	13.7961	1.2083	5.1431	0.2938	0.8363	26.0665	13.9405	0.6684	2.3007
	GARCH-VOL	7.9859	131.4992	3.3477	54.4920	0.4351	7.9865	**19.7363**	156.1476	1.8092	2.5935
	Benefit (%)	7.56	10.76	11.23	26.22	6.17	10.63	20.54	14.58	12.92	1.35
	GARCH	3.8085	51.6052	1.4584	9.0917	0.3365	0.6502	159.1720	18.4110	0.9237	2.6306
	GARCH-RV	3.8992	55.1674	1.4571	9.5978	0.3262	0.6950	141.9845	28.0203	0.8945	2.6090
	GARCH-RR	3.7863	52.4398	1.3579	**6.5849**	0.3248	0.6607	**140.3081**	24.1156	0.8518	**2.5945**
ATT	GARCH-RPV	**3.6855**	**50.3509**	**1.3560**	7.9681	**0.3218**	**0.6343**	151.0111	**18.3004**	**0.8460**	2.5956
	GARCH-RBP	3.7915	52.0651	1.3712	7.9464	0.3265	0.6559	145.2212	21.0086	0.8664	2.6087
	GARCH-VOL	5.9068	73.7424	3.0960	46.7766	0.4265	0.9360	191.0647	50.8446	1.7061	2.7800
	Benefit (%)	3.23	2.43	7.02	27.57	4.37	2.45	11.85	0.60	8.41	1.37

The variable of interest is the daily conditional variance of returns proxied by the sum of 15-min or 30-min squared returns.

Table G. Out-of-sample forecast evaluation: 15min- and 30min-based nonparametric volatility estimators (*cont.*)

Stock	Model	MAE	MSE	HMAE	HMSE	AMAPE	Theil-U	MME(U)	MME(O)	LL	GMLE
		15min-based estimators (volatility proxy: sum of 5-min squared returns)									
	GARCH	1.7797	6.1406	0.9471	1.8327	0.2811	1.2597	16.8100	5.7511	0.5316	2.0682
	GARCH-RV	1.3622	4.2293	0.7154	1.1267	0.2305	0.8676	12.9636	3.8652	0.3709	2.0055
	GARCH-RR	1.4853	4.3960	0.7883	1.2985	0.2467	0.9020	13.9171	4.4382	0.4089	2.0160
CAT	GARCH-RPV	**1.3049**	**3.8927**	**0.6565**	**0.9312**	**0.2206**	**0.7987**	**11.9828**	**3.6529**	**0.3282**	**1.9898**
	GARCH-RBP	1.3471	4.0403	0.6945	1.0274	0.2284	0.8290	12.5185	3.7621	0.3536	1.9987
	GARCH-VOL	2.2153	8.9839	1.2814	4.6883	0.3117	1.8430	19.9459	9.2635	0.7403	2.1294
	Benefit (%)	26.68	36.61	30.68	49.19	21.52	36.60	28.72	36.48	38.26	3.79
	GARCH	1.9524	8.6105	0.7942	1.2819	0.2515	1.5277	19.6059	8.4880	0.4235	2.2235
	GARCH-RV	1.4887	5.2056	0.5732	0.6848	0.2035	0.9239	10.2429	5.3254	0.2806	2.1710
	GARCH-RR	1.4584	**4.7250**	0.5803	0.6797	0.2038	**0.8384**	**9.0520**	5.2240	0.2779	2.1672
MSFT	GARCH-RPV	**1.4567**	5.1700	**0.5355**	**0.5675**	**0.1964**	0.9175	11.4970	**4.7962**	**0.2549**	**2.1612**
	GARCH-RBP	1.4634	5.0944	0.5494	0.6072	0.2008	0.9041	10.2542	5.0741	0.2629	2.1646
	GARCH-VOL	3.4251	18.6881	1.5101	5.1865	0.3489	3.3161	22.8637	21.1438	0.8773	2.3555
	Benefit (%)	25.39	45.13	32.57	55.73	21.91	45.12	53.83	43.49	39.81	2.80
		30min-based estimators (volatility proxy: sum of 5-min squared returns)									
	GARCH	1.7797	6.1406	0.9471	1.8327	0.2811	1.2597	16.8100	5.7511	0.5316	2.0682
	GARCH-RV	1.3741	4.1549	0.7227	1.1579	0.2320	0.8525	12.2521	3.9134	0.3728	2.0045
	GARCH-RR	1.4301	4.2282	0.7596	1.2166	0.2406	0.8675	13.2305	4.1403	0.3922	2.0106
CAT	GARCH-RPV	**1.3140**	**4.0064**	**0.6641**	**0.9694**	**0.2220**	**0.8221**	**11.7884**	**3.6603**	**0.3377**	**1.9940**
	GARCH-RBP	1.4202	4.3551	0.7464	1.2231	0.2383	0.8936	12.1667	4.0697	0.3891	2.0113
	GARCH-VOL	2.2153	8.9839	1.2814	4.6883	0.3117	1.8430	19.9459	9.2635	0.7403	2.1294
	Benefit (%)	26.17	34.76	29.88	47.11	21.74	34.74	29.87	36.35	36.47	3.59
	GARCH	1.9524	8.6105	0.7942	1.2819	0.2515	1.5277	19.6059	8.4880	0.4235	2.2235
	GARCH-RV	1.5328	5.6871	0.5599	0.6354	0.2029	1.0093	11.9556	5.4297	0.2709	2.1682
	GARCH-RR	**1.4675**	**4.6440**	0.5842	0.6858	0.2052	**0.8241**	**9.7384**	4.9778	0.2803	2.1681
MSFT	GARCH-RPV	1.4884	5.4398	**0.5470**	**0.5986**	**0.1990**	0.9655	12.3397	**4.9458**	**0.2625**	**2.1639**
	GARCH-RBP	1.5240	5.2796	0.5841	0.7019	0.2059	0.9370	10.5467	5.4810	0.2835	2.1704
	GARCH-VOL	3.4251	18.6881	1.5101	5.1865	0.3489	3.3161	22.8637	21.1438	0.8773	2.3555
	Benefit (%)	24.84	46.07	31.13	53.30	20.87	46.06	50.33	41.73	38.02	2.68

The variable of interest is the daily conditional variance of returns proxied by the sum of 5-min squared returns.

Table G. Out-of-sample forecast evaluation: 15min- and 30min-based nonparametric volatility estimators (*cont.*)

Stock	Model	MAE	MSE	HMAE	HMSE	AMAPE	Theil-U	MME(U)	MME(O)	LL	GMLE
		15min-based estimators (volatility proxy: sum of 5-min squared returns)									
DELL	GARCH	2.2559	10.2684	0.6453	0.8808	0.2217	1.2250	17.7316	10.6282	0.3252	2.4342
	GARCH-RV	**2.0969**	9.0972	**0.5927**	**0.6993**	**0.2086**	1.0869	19.9389	8.9669	**0.2824**	2.4169
	GARCH-RR	2.1242	**8.5166**	0.6087	0.7280	0.2122	**1.0142**	19.2102	**8.8219**	0.2871	**2.4168**
	GARCH-RPV	2.1791	9.2390	0.6080	0.7287	0.2143	1.1035	**16.9183**	9.4937	0.2961	2.4235
	GARCH-RBP	2.2276	9.4638	0.6456	0.8411	0.2199	1.1299	20.2923	9.4413	0.3137	2.4268
	GARCH-VOL	7.3802	119.4348	2.1633	13.8656	0.3659	14.2745	16.1837	149.0813	1.2698	2.6959
	Benefit (%)	7.05	17.06	8.15	20.61	5.91	17.21	4.59	17.00	13.16	0.71
ATT	GARCH	3.0166	22.4841	0.9492	2.2441	0.2831	0.8428	59.3852	13.6460	0.5700	2.5595
	GARCH-RV	2.8205	24.1470	0.8325	1.8335	0.2539	0.9051	52.0885	18.3332	0.4876	2.5234
	GARCH-RR	2.7574	21.6207	0.8328	1.6141	0.2559	0.8104	53.3509	15.5744	0.4785	2.5191
	GARCH-RPV	**2.6125**	**20.4166**	**0.7683**	**1.4210**	**0.2460**	**0.7653**	53.1982	**12.1212**	0.5065	3.0246
	GARCH-RBP	2.7368	23.7940	0.8041	1.5802	0.2496	0.8919	**49.7607**	17.9149	**0.4704**	**2.5181**
	GARCH-VOL	5.2470	47.8291	2.2905	17.3949	0.3869	1.7781	68.8293	47.6280	1.3396	2.7446
	Benefit (%)	13.40	9.20	19.06	36.68	13.10	9.20	16.21	11.17	17.47	1.62
		30min-based estimators (volatility proxy: sum of 5-min squared returns)									
DELL	GARCH	2.2559	10.2684	0.6453	0.8808	0.2217	1.2250	17.7316	10.6282	0.3252	2.4342
	GARCH-RV	**1.9557**	8.7888	**0.5299**	**0.5946**	**0.1928**	1.0500	18.2675	**8.3534**	**0.2467**	**2.4039**
	GARCH-RR	2.1427	**8.7482**	0.6265	0.7874	0.2147	**1.0418**	20.0996	8.9452	0.2990	2.4208
	GARCH-RPV	2.1359	9.5049	0.5963	0.7320	0.2097	1.1346	19.2488	9.2978	0.2902	2.4210
	GARCH-RBP	2.0574	9.1862	0.5805	0.7211	0.2029	1.0968	18.6672	9.0959	0.2770	2.4137
	GARCH-VOL	7.3802	119.4348	2.1633	13.8656	0.3659	14.2745	**16.1837**	149.0813	1.2698	2.6959
	Benefit (%)	13.31	14.80	17.88	32.49	13.04	14.96	8.73	21.40	24.14	1.24
ATT	GARCH	3.0166	22.4841	0.9492	2.2441	0.2831	0.8428	59.3852	13.6460	0.5700	2.5595
	GARCH-RV	3.0183	28.0137	0.9102	2.3990	0.2616	1.0500	**53.0276**	23.0714	0.5322	2.5366
	GARCH-RR	2.7987	23.6329	0.8345	1.7112	0.2550	0.8859	53.0910	17.6079	0.4807	2.5199
	GARCH-RPV	**2.7341**	**21.9999**	**0.8205**	**1.7018**	**0.2524**	**0.8246**	58.6334	**13.0288**	**0.4763**	**2.5191**
	GARCH-RBP	2.8518	23.9296	0.8437	1.7316	0.2595	0.8969	55.0704	16.1209	0.4983	2.5299
	GARCH-VOL	5.9068	73.7424	3.0960	46.7766	0.4265	0.9360	191.0647	50.8446	1.7061	2.7800
	Benefit (%)	9.36	2.16	13.56	24.17	10.84	2.16	10.71	4.52	16.44	1.58

The variable of interest is the daily conditional variance of returns proxied by the sum of 5-min squared returns.

Table H. Out-of-sample forecast evaluation (volatility proxy: daily squared returns)

		Forecast accuracy measures										
Stock	**Model**	MAE	MSE	HMAE	HMSE	AMAPE	Theil-U	MME(U)	MME(O)	LL	GMLE	MZ-R^2
	GARCH	6.3015	143.2639	47.6416	42264.45	0.5519	0.6041	451.9938	35.4842	5.9138	2.7865	6.2967
	GARCH-RV	6.2570	137.2443	44.5023	36712.07	0.5484	0.5787	412.6125	40.2616	5.6964	2.7332	10.2698
	GARCH-RR	6.4485	136.6274	49.1782	48408.86	0.5564	0.5761	415.2557	42.8201	5.8828	2.7294	10.4342
ATT	GARCH-RPV	**6.1900**	**135.9724**	**43.6922**	**36073.37**	**0.5478**	**0.5734**	**408.8060**	**35.4202**	**5.6404**	**2.7239**	**11.4699**
	GARCH-RBP	6.3242	136.1108	45.8696	40182.85	0.5503	0.5740	390.7512	43.0999	5.7511	2.7404	10.9893
	GARCH-VOL	8.2539	164.9580	87.0718	142865.7	0.5971	0.6937	521.9965	72.6693	7.5385	2.9141	0.2942
	GARCH	4.2236	62.6220	152.0397	708437.77	0.5607	0.5045	207.0013	**23.1858**	6.9852	2.1695	9.3050
	GARCH-RV	4.3229	59.4837	146.5489	748908.29	0.5580	0.4792	168.7590	30.9801	6.8241	2.1228	14.8017
	GARCH-RR	4.7157	62.2868	172.0750	969838.79	0.5757	0.5018	190.6222	35.0850	7.4536	2.1733	13.4228
AXP	GARCH-RPV	**4.2215**	**57.9405**	146.1013	722286.99	**0.5524**	**0.4668**	171.6658	25.6834	**6.7521**	**2.1137**	**16.4378**
	GARCH-RBP	4.3265	59.3417	**143.6928**	**664984.22**	0.5569	0.4780	**165.7089**	30.9186	6.8553	2.1169	15.0193
	GARCH-VOL	4.4330	64.3175	162.8977	751133.21	0.5670	0.5213	211.4856	25.7138	7.3231	2.2042	6.9269
	GARCH	**3.7664**	46.9967	**126.3685**	510163.41	**0.5387**	0.5086	139.6074	**17.5770**	**6.2591**	2.2319	2.8528
	GARCH-RV	3.8925	43.6839	126.7890	**483160.30**	0.5436	0.4728	**128.3642**	20.2798	6.3610	**2.1933**	9.7324
	GARCH-RR	4.2231	46.1312	151.7622	713233.95	0.5537	0.4994	137.7833	24.9299	6.8308	2.2354	7.9455
BA	GARCH-RPV	3.8773	**43.6494**	131.6863	536419.06	0.5427	**0.4725**	129.3183	19.4834	6.3954	2.1941	9.8115
	GARCH-RBP	3.9374	43.7249	131.1591	527675.68	0.5435	0.4733	128.8742	21.4027	6.4219	2.1941	**10.2259**
	GARCH-VOL	4.1422	49.6316	160.0658	832237.33	0.5508	0.5440	153.7675	21.6332	6.8062	2.2886	0.2075
	GARCH	3.6740	35.7996	109.6238	582801.57	0.5777	0.5014	124.4265	16.7594	6.9674	2.1395	0.3303
	GARCH-RV	3.3826	32.8225	**94.5286**	**460673.57**	0.5612	0.4625	101.0365	**14.5445**	6.4306	2.0484	5.6152
	GARCH-RR	3.6117	33.8518	108.6538	601315.58	0.5735	0.4741	107.3935	16.7388	6.8652	2.0884	4.4533
CAT	GARCH-RPV	**3.3814**	**32.7256**	95.7670	486729.88	**0.5594**	**0.4611**	**98.5982**	14.7122	**6.4044**	**2.0424**	**5.9447**
	GARCH-RBP	3.4355	33.2166	97.1943	488859.37	0.5651	0.4652	101.1889	14.9964	6.5304	2.0646	5.2179
	GARCH-VOL	3.9745	37.7798	120.8555	749389.70	0.5828	0.5323	129.8949	20.7515	7.2970	2.1616	0.1393
	GARCH	5.1937	67.4479	821.8808	270176727	0.5715	0.6620	213.1742	34.5635	6.7447	2.3699	11.1480
	GARCH-RV	5.2316	**61.1869**	865.5358	301926636	0.5787	**0.6007**	193.1281	35.1471	6.9237	2.3636	**21.7191**
	GARCH-RR	**4.9465**	61.6509	**760.5759**	**232057606**	**0.5686**	0.6047	198.1145	**30.3292**	**6.5638**	**2.3336**	18.6648
DELL	GARCH-RPV	5.3086	63.3740	916.4131	342778313	0.5768	0.6221	203.0319	36.9506	6.8708	2.3634	18.2591
	GARCH-RBP	5.2904	63.1545	860.7616	297818109	0.5788	0.6200	201.1269	36.1356	6.9330	2.3662	18.6351
	GARCH-VOL	9.5629	172.2279	2611.544	2936592881.1	0.6297	1.6921	**178.6569**	180.5703	8.7801	2.6278	4.5066

The variable of interest is the conditional variance proxied by the daily squared returns. Bold indicates the top performer. A small number of days where the return is identically zero were removed so that the HMAE and HMSE measures are feasible.

Table H. Out-of-sample forecast evaluation (*cont.*)

		Forecast accuracy measures										
Stock	**Model**	MAE	MSE	HMAE	HMSE	AMAPE	Theil-U	MME(U)	MME(O)	LL	GMLE	MZ-R^2
	GARCH	3.6844	53.3717	**120.7744**	**764071.55**	0.5428	0.5658	162.8653	**16.5029**	6.3283	2.1556	6.5737
	GARCH-RV	3.6484	50.8210	138.9576	1099405.11	0.5341	0.5388	**139.4726**	19.0491	6.2911	2.1305	11.0989
	GARCH-RR	3.8614	51.0415	153.7196	1254192.56	0.5454	0.5411	159.1601	21.7061	6.6511	2.1435	11.3313
GE	GARCH-RPV	**3.5933**	**50.4202**	132.2983	972602.75	**0.5304**	**0.5345**	140.0772	17.4284	**6.1808**	**2.1159**	**11.7674**
	GARCH-RBP	3.6537	50.9484	141.2382	1213632.99	0.5336	0.5401	142.6096	19.8378	6.2698	2.1235	11.0107
	GARCH-VOL	3.9810	52.7001	141.0581	985802.927	0.5514	0.5632	176.7315	20.6456	6.7812	2.1452	7.7415
	GARCH	3.9684	52.8355	109.9439	662736.66	0.5618	0.4996	165.3453	18.6361	5.9371	2.1529	11.2048
	GARCH-RV	**3.9081**	51.6850	109.6962	656431.25	0.5590	0.4888	159.7060	18.1794	**5.9025**	2.1407	13.4637
	GARCH-RR	4.2777	51.8981	126.4774	782194.14	0.5748	0.4918	166.8669	24.4755	6.4660	2.1689	13.4364
GM	GARCH-RPV	3.9175	**51.5031**	110.4242	652302.91	0.5587	0.4874	160.8661	**17.9612**	5.9270	2.1399	**14.0200**
	GARCH-RBP	3.9245	51.5386	109.1125	646788.61	0.5590	**0.4870**	**156.5503**	19.0658	5.9179	**2.1347**	13.4982
	GARCH-VOL	4.0204	54.7829	**108.1616**	**606431.05**	**0.5556**	0.5181	159.9171	19.8151	5.9167	2.1692	7.9442
	GARCH	3.0620	23.7474	237.2394	3387218.50	0.5553	0.4879	74.7933	12.7829	7.1502	1.9351	6.5930
	GARCH-RV	2.9230	22.5063	234.6603	3590150.61	0.5473	0.4624	59.8341	12.6139	6.9021	1.8866	11.2178
	GARCH-RR	3.0763	23.2811	254.4623	4124396.47	0.5548	0.4782	62.8026	14.1025	7.1840	1.9109	9.4134
IBM	GARCH-RPV	**2.8828**	**22.0646**	**223.2996**	**3335281.32**	**0.5403**	**0.4551**	**57.0409**	**11.9710**	**6.7477**	**1.8613**	**12.2873**
	GARCH-RBP	2.9148	22.3077	234.1494	3659748.99	0.5457	0.4583	59.4515	12.4653	6.8977	1.8783	11.9037
	GARCH-VOL	5.7060	44.1744	712.5044	40029810.2	0.6532	0.9114	92.6877	43.7519	10.7190	2.3148	0.3787
	GARCH	7.8624	976.0118	77.7196	189006.26	0.5601	0.7620	3568.01	130.594	6.0748	2.5818	4.0047
	GARCH-RV	7.1693	951.2205	61.6837	114180.91	0.5422	0.7382	**3116.35**	75.214	5.5637	2.4883	6.2094
	GARCH-RR	7.2597	**923.6852**	71.0999	166039.84	0.5536	**0.7154**	3678.03	62.254	5.8318	**2.4666**	**10.7986**
JPM	GARCH-RPV	**6.8718**	955.5976	**58.9987**	111677.16	**0.5353**	0.7401	3229.67	**50.300**	**5.3976**	2.4808	6.9296
	GARCH-RBP	7.1399	946.5913	59.8557	**106823.61**	0.5426	0.7346	3220.98	70.923	5.5340	2.4850	6.8068
	GARCH-VOL	8.5106	1009.632	97.8928	330405.37	0.5791	0.7882	4677.46	148.628	6.6931	2.6174	2.3410
	GARCH	1.9648	14.7101	112.9508	752211.10	0.5437	0.5580	45.0172	**6.7110**	**6.1692**	1.4766	5.3055
	GARCH-RV	**1.9376**	**12.5834**	112.1942	**618279.65**	**0.5381**	0.4820	**37.2845**	7.8332	6.2273	**1.4097**	**19.2942**
	GARCH-RR	2.1031	13.0529	120.5203	671379.99	0.5554	0.4951	39.9547	8.6239	6.6264	1.4615	17.5811
KO	GARCH-RPV	1.9541	13.0005	**110.8040**	619093.90	0.5402	0.4921	37.2868	7.4147	6.2309	1.4693	16.2764
	GARCH-RBP	1.9715	12.7058	116.9976	662016.59	0.5419	**0.4773**	36.9110	8.0570	6.3576	1.4187	18.1103
	GARCH-VOL	2.2836	15.1273	142.8468	1101796.5	0.5650	0.5730	49.1287	9.1407	6.9388	1.5393	7.3503

Table H. Out-of-sample forecast evaluation (*cont.*)

		Forecast accuracy measures										
Stock	**Model**	MAE	MSE	HMAE	HMSE	AMAPE	Theil-U	MME(U)	MME(O)	LL	GMLE	MZ-R^2
	GARCH	3.6427	65.0217	63.2475	77815.05	0.5743	0.5627	203.1470	15.6988	6.4481	2.1345	2.2658
	GARCH-RV	3.6133	61.4147	59.8748	67321.45	0.5703	0.5315	200.6780	17.5427	6.3970	2.1352	7.6232
	GARCH-RR	3.6786	61.2474	64.8739	77567.02	0.5720	0.5300	210.2118	17.7929	6.5808	2.1192	7.7964
MCD	GARCH-RPV	**3.4680**	60.7055	58.5449	66489.58	**0.5651**	0.5254	198.3034	**14.1434**	**6.2375**	**2.1180**	**9.1462**
	GARCH-RBP	3.5882	**60.2284**	60.4829	66855.22	0.5704	**0.5213**	202.9395	16.2182	6.4298	2.1322	8.6784
	GARCH-VOL	3.6114	65.4162	**58.3954**	**65767.87**	0.5752	0.5662	**193.6849**	15.2015	6.3078	2.1526	1.9301
	GARCH	4.1663	56.7666	671.3523	123936392.2	0.5411	0.7741	169.2647	21.8518	7.5329	2.2725	10.3233
	GARCH-RV	3.9652	49.8318	545.2763	76094529.2	0.5345	0.6797	140.2714	19.7766	7.1875	2.2137	22.6159
	GARCH-RR	**3.8762**	**49.3169**	562.2056	83165043.0	**0.5305**	**0.6726**	**139.4090**	**18.2833**	**7.0561**	**2.2005**	**24.1039**
MSFT	GARCH-RPV	3.9646	50.8537	539.1633	**73417781.6**	0.5332	0.6936	144.8472	19.3594	7.1493	2.2112	21.4083
	GARCH-RBP	3.9487	50.4070	**538.9716**	74714072.6	0.5337	0.6875	144.9946	19.4899	7.1615	2.2127	21.4688
	GARCH-VOL	5.2658	65.2167	1528.542	807474461.5	0.5731	0.8894	225.2962	37.9153	8.6198	2.3855	6.3562
	GARCH	1.4478	6.7761	199.0418	2256027.34	0.5663	0.5659	21.9860	4.5574	7.6119	1.0738	11.6802
	GARCH-RV	**1.3057**	**6.3706**	**167.2666**	**1578421.81**	**0.5439**	**0.5321**	18.7867	4.1107	6.9570	**1.0091**	14.9425
	GARCH-RR	1.3756	6.4948	179.8002	1831439.18	0.5556	0.5425	19.5979	4.2060	7.2005	1.0522	13.8502
PG	GARCH-RPV	1.3134	6.3776	171.5876	1721437.69	0.5448	0.5327	**17.5526**	**4.1098**	**6.9376**	1.0511	15.0010
	GARCH-RBP	1.3277	6.3753	176.5994	1738376.37	0.5470	0.5325	18.6951	4.2749	7.0866	1.0162	**15.3476**
	GARCH-VOL	4.8189	30.3664	612.4256	20516001.9	0.7317	2.5295	48.4577	32.3833	13.1176	1.7851	3.7033
	GARCH	2.4636	19.8197	119.5730	725093.66	0.5182	0.5374	54.2224	10.0575	5.5761	1.7712	7.8889
	GARCH-RV	2.4442	**18.2257**	109.6650	645158.83	0.5120	**0.4940**	**38.8236**	12.4767	5.3789	1.7475	**16.6228**
	GARCH-RR	2.5286	18.7082	115.8620	722656.25	0.5150	0.5069	42.5088	12.7480	5.5384	1.7627	15.2878
WMT	GARCH-RPV	2.4318	18.2421	**106.9418**	625260.68	0.5121	0.4945	39.7915	11.7612	5.3282	1.7605	15.9262
	GARCH-RBP	2.4480	18.4490	108.4303	606764.64	0.5129	0.5001	41.2725	12.1012	5.4136	**1.7471**	15.3357
	GARCH-VOL	**2.3630**	19.8178	108.0169	**593534.51**	**0.5117**	0.5373	51.1813	**8.9786**	**5.3131**	1.7777	7.2838

Table I. Out-of-sample Mincer-Zarnowitz levels regression (volatility proxy: sum of 5 minute squared returns)

	$\hat{a}$	$\hat{b}$	MZ-R^2	$\chi^2_{(2)}$	$\hat{a}$	$\hat{b}$	MZ-R^2	$\chi^2_{(2)}$	$\hat{a}$	$\hat{b}$	MZ-R^2	$\chi^2_{(2)}$	$\hat{a}$	$\hat{b}$	MZ-R^2	$\chi^2_{(2)}$
	ATT				**AXP**				**BA**				**CAT**			
GARCH	-0.129	0.909	28.87	5.28	-0.108	0.909	29.04	0.503	-0.103	1.129	35.41	4.098	-0.177	0.812*	13.99	26.43
	(0.838)	(0.161)		[0.071]	(0.518)	(0.161)		[0.777]	(0.396)	(0.114)		[0.129]	(0.371)	(0.111)		[0.000]
GARCH-RV	0.385	0.826	36.40	10.73	-0.285	0.955	54.89	17.005	-0.424*	1.106*	54.46	3.190	-1.026***	1.137	42.61	94.00
	(0.601)	(0.111)		[0.005]	(0.240)	(0.066)		[0.000]	(0.237)	(0.063)		[0.203]	(0.273)	(0.092)		[0.000]
GARCH-RR	0.124	0.818	35.83	24.00	-1.047***	0.961	53.28	120.23	-0.583**	0.997	53.50	44.33	-1.224***	1.074	37.09	164.9
	(0.851)	(0.148)		[0.000]	(0.236)	(0.059)		[0.000]	(0.274)	(0.066)		[0.000]	(0.266)	(0.083)		[0.000]
GARCH-RPV	-0.356	0.979	39.69	8.15	-1.104***	1.167	56.97	38.404	-0.990***	1.233***	55.76	16.36	-0.888***	1.090	43.64	112.3
	(0.608)	(0.119)		[0.017]	(0.388)	(0.112)		[0.000]	(0.267)	(0.073)		[0.000]	(0.279)	(0.096)		[0.000]
GARCH-RBP	0.629	0.773**	37.21	14.91	-0.389	0.971	56.11	23.988	-0.155	1.012	53.67	7.550	-0.980***	1.102	42.41	98.64
	(0.574)	(0.103)		[0.000]	(0.270)	(0.076)		[0.000]	(0.248)	(0.062)		[0.427]	(0.276)	(0.091)		[0.000]
GARCH-VOL	3.359**	0.221***	2.55	51.75	0.163	0.868	22.24	1.561	1.606***	0.610**	8.48	0.503	1.352***	0.360***	4.98	73.53
	(1.387)	(0.191)		[0.000]	(0.488)	(0.148)		[0.458]	(0.585)	(0.157)		[0.023]	(0.358)	(0.090)		[0.000]
COMBINED	0.139	0.949	35.13	0.361	-0.317	1.009	56.09	10.163	-0.123	0.885*	53.72	53.36	0.162	0.878	43.80	7.028
	(0.846)	(0.176)		[0.835]	(0.334)	(0.099)		[0.006]	(0.247)	(0.061)		[0.000]	(0.195)	(0.081)		[0.030]
COMB_EQW	0.258	0.851	37.90	10.52	-0.793***	1.028	56.43	59.242	-0.639***	1.106*	55.61	13.25	-1.090***	1.117	42.03	130.2
	(0.591)	(0.110)		[0.005]	(0.253)	(0.071)		[0.000]	(0.246)	(0.063)		[0.001]	(0.267)	(0.088)		[0.000]
	DELL				**GE**				**GM**				**IBM**			
GARCH	0.082	0.782**	42.32	44.43	-1.039**	1.310**	41.15	6.295	-0.537	0.990	56.52	43.04	-0.914***	1.093	42.74	64.12
	(0.450)	(0.101)		[0.000]	(0.422)	(0.148)		[0.043]	(0.413)	(0.135)		[0.000]	(0.320)	(0.134)		[0.000]
GARCH-RV	-0.521	0.834*	51.59	187.06	-0.382	1.097	52.57	1.750	-0.639***	1.022	57.36	83.64	-0.544***	1.037	63.96	76.66
	(0.468)	(0.086)		[0.000]	(0.311)	(0.096)		[0.417]	(0.246)	(0.079)		[0.000]	(0.156)	(0.059)		[0.000]
GARCH-RR	-0.360	0.895	51.65	66.78	-0.885***	1.089	55.58	38.43	-0.585**	0.823***	57.03	331.2	-0.845***	1.033	64.97	179.6
	(0.376)	(0.081)		[0.000]	(0.301)	(0.086)		[0.000]	(0.249)	(0.068)		[0.000]	(0.166)	(0.059)		[0.000]
GARCH-RPV	-0.352	0.803**	51.28	170.3	-0.795***	1.238**	55.28	7.770	-0.899***	1.082	58.82	124.9	-0.741	1.115*	65.97	111.8
	(0.472)	(0.090)		[0.000]	(0.295)	(0.099)		[0.021]	(0.244)	(0.041)		[0.000]	(0.160)	(0.066)		[0.777]
GARCH-RBP	-0.443	0.820**	49.69	150.1	-0.203	1.041	52.07	0.690	-0.474**	0.959	59.00	94.49	-0.526***	1.029	62.71	75.35
	(0.451)	(0.084)		[0.000]	(0.309)	(0.093)		[0.708]	(0.234)	(0.074)		[0.000]	(0.154)	(0.059)		[0.000]
GARCH-VOL	2.924***	0.148***	12.80	645.6	-1.728**	1.261***	41.91	28.29	-0.088	0.857	45.58	28.37	0.983**	0.238***	3.47	333.1
	(0.314)	(0.036)		[0.000]	(0.696)	(0.191)		[0.000]	(0.313)	(0.108)		[0.000]	(0.439)	(0.076)		[0.000]
COMBINED	0.419	0.862*	54.14	4.774	0.072	0.974	55.90	0.098	-0.093	0.960	58.40	11.24	-0.070	0.928	66.14	21.88
	(0.292)	(0.075)		[0.092]	(0.233)	(0.084)		[0.952]	(0.192)	(0.075)		[0.004]	(0.141)	(0.064)		[0.000]
COMB-EQW	-0.539	0.857*	52.26	150.6	-0.608**	1.125	54.46	7.741	-0.678***	0.971	58.57	167.4	-0.713***	1.068	65.35	132.9
	(0.442)	(0.085)		[0.000]	(0.296)	(0.091)		[0.021]	(0.239)	(0.074)		[0.000]	(0.150)	(0.058)		[0.000]

The numbers in parentheses are Newey-West h.a.c. standard errors. The * (10%), **(5%) and ***(1%) after each coefficient indicate, respectively, significant differences from a =0 and b =1 (Wald test). The $\chi^2_{(2)}$ column gives the point estimate and asymptotic p-value for H_0: a=0, b =1. A significantly negative a indicates upward bias and vice versa. COMBINED denotes the combined forecasts using the time-varying weighting approach based on rolling regressions as outlined in Section 3.3. COMB-EQW denotes the equal-weights combined forecasts.

Table I. Out-of-sample Mincer-Zarnowitz levels regression results (*cont.*)

	$\hat{a}$	$\hat{b}$	MZ-R^2	$\chi^2_{(2)}$	$\hat{a}$	$\hat{b}$	MZ-R^2	$\chi^2_{(2)}$	$\hat{a}$	$\hat{b}$	MZ-R^2	$\chi^2_{(2)}$	$\hat{a}$	$\hat{b}$	MZ-R^2	$\chi^2_{(2)}$
	JPM				**KO**				**MCD**				**MSFT**			
GARCH	0.661	0.795	51.67	1.804	-0.269	1.183	44.51	0.820	1.676**	0.694	7.907	8.56	0.128	0.784	33.52	19.12
	(0.886)	(0.187)		[0.406]	(0.337)	(0.210)		[0.664]	(0.731)	(0.210)		[0.014]	(0.642)	(0.158)		[0.000]
GARCH-RV	-0.188	1.077	52.71	0.706	-0.156	1.066	66.87	1.379	1.279**	0.754	21.96	11.25	-0.303	0.925	57.22	53.59
	(1.063)	(0.246)		[0.703]	(0.192)	(0.105)		[0.502]	(0.534)	(0.160)		[0.004]	(0.453)	(0.116)		[0.000]
GARCH-RR	-2.199*	1.320	59.19	1.529	-0.409*	1.056	64.85	39.57	0.631	0.888	24.64	3.74	-0.397	0.983	59.95	33.05
	(1.317)	(0.259)		[0.216]	(0.246)	(0.120)		[0.000]	(0.542)	(0.162)		[0.154]	(0.412)	(0.109)		[0.000]
GARCH-RPV	-2.607**	1.602**	64.24	4.551	-0.350	1.160	65.85	3.25	0.272	1.115	26.28	20.44	-0.680*	1.018	58.16	86.36
	(1.227)	(0.283)		[0.103]	(0.260)	(0.140)		[0.197]	(0.543)	(0.186)		[0.000]	(0.411)	(0.110)		[0.000]
GARCH-RBP	-0.539**	1.144	55.52	0.512	-0.184	1.044	66.09	7.009	0.971**	0.843	23.85	11.15	-0.284	0.933	56.53	39.53
	(1.092)	(0.253)		[0.774]	(0.200)	(0.105)		[0.030]	(0.467)	(0.144)		[0.004]	(0.447)	(0.117)		[0.000]
GARCH-VOL	0.097	0.757*	51.24	12.22	-0.067	0.858	35.91	16.27	1.925***	0.647*	7.02	11.99	0.683	0.479***	16.83	119.66
	(0.789)	(0.140)		[0.002]	(0.537)	(0.241)		[0.000]	(0.678)	(0.199)		[0.003]	(0.817)	(0.152)		[0.000]
COMBINED	-0.463	1.147	62.77	0.758	-0.391	1.086	64.42	29.97	0.168	0.996	23.31	1.195	-0.080	0.997	59.62	2.312
	(1.151)	(0.253)		[0.685]	(0.288)	(0.145)		[0.000]	(0.573)	(0.177)		[0.550]	(0.317)	(0.102)		[0.315]
COMB_EQW	-1.366	1.279	58.31	1.745	-0.328	1.103	67.38	13.34	0.714	0.917	24.93	10.08	-0.447	0.972	58.44	52.20
	(1.162)	(0.259)		[0.418]	(0.212)	(0.111)		[0.001]	(0.496)	(0.155)		[0.007]	(0.431)	(0.113)		[0.000]
	PG				**WMT**											
GARCH	-0.132	0.946	54.44	17.76	-0.554***	1.186***	39.90	8.286								
	(0.173)	(0.133)		[0.000]	(0.210)	(0.065)		[0.016]								
GARCH-RV	-0.005	1.017	68.80	0.401	-0.031	0.970	57.99	1.882								
	(0.078)	(0.070)		[0.818]	(0.306)	(0.140)		[0.390]								
GARCH-RR	-0.119	1.026	62.40	12.06	-0.277	0.986	56.13	17.70								
	(0.122)	(0.103)		[0.002]	(0.330)	(0.143)		[0.000]								
GARCH-RPV	0.025	0.986	69.54	0.350	-0.135	1.027	58.59	1.511								
	(0.076)	(0.071)		[0.839]	(0.311)	(0.147)		[0.470]								
GARCH-RBP	0.003	0.970	70.31	2.151	-0.143	1.019	58.92	2.128								
	(0.075)	(0.065)		[0.341]	(0.318)	(0.145)		[0.345]								
GARCH-VOL	0.017	0.249***	18.20	1732.4	-0.384	1.245	38.79	1.310								
	(0.232)	(0.054)		[0.000]	(0.515)	(0.265)		[0.519]								
COMBINED	0.107	0.852**	66.62	13.87	-0.070	0.985	56.95	2.237								
	(0.083)	(0.071)		[0.001]	(0.314)	(0.148)		[0.327]								
COMB-EQW	-0.053	1.109*	69.18	2.738	-0.173	1.009	58.43	5.108								
	(0.079)	(0.071)		[0.254]	(0.316)	(0.143)		[0.078]								