Winner's Curse and Entry in Highway Procurement*

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Abstract

In procurement auctions, there are situations where a bidder's cost is uncertain at the time of bidding, leading to a "winner's curse." We use bridgework data from the State of Oklahoma and an empirical auction model to explore whether the winner's curse also affects entry, which can have serious implications for procurement costs and efficiency. We find that the winner's curse generally reduces entry in Oklahoma by reducing bidder markups conditional on participating. We then investigate various entry policies—including taxes, subsidies, and entry rights auctions.

Keywords: Winner's curse, endogenous entry, procurement auctions.

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1 Introduction

Public procurement spending is vast, accounting for just under 30 percent of government expenditures in OECD countries and roughly 13 percent of their GDP in 2019 (OECD, 2019). Public procurement is also the main mechanism by which governments build, repair, and maintain their infrastructure—making it a vital part of many nations.

On procurement projects, governments typically select firms through a first-price sealed-bid auction procedure as a means of obtaining the lowest possible price. In many situations, though, the full cost of completing a project is unknown at the time of bidding, requiring participating firms to estimate it given their knowledge (or "signal") of the required work. By relying on these cost estimates, firms face a "winner's curse" when bidding, in the sense that the winning firm may have an overly optimistic signal of the true cost. In equilibrium, contractors will compensate by bidding less aggressively, and there is a small literature investigating the consequences of the winner's curse on bidding in procurement contracting (see Hong and Shum (2002) and Somaini (2020)).

Another decision firms make, and one of the main focuses of this article, is whether to even participate in an auction, given the costly process of acquiring a signal. If this entry decision is influenced in part by a project's expected profitability, then the winner's curse has the potential to affect both bidding and the number of participants—which has implications for efficiency and procurement costs.

We contribute to the literature by extending analyses of the winner's curse in highway contracting to include a costly entry stage. In a related article, Compiani et al. (2020) establish non-parametric identification results in an auction model with entry and the potential for a winner's curse, except their entry model is a reduced form that can be rationalized by an entry game. Our study differs from Compiani et al. (2020) in that it is parametric and estimates the entry-stage parameters explicitly. These distinctions allows us to explore counterfactual entry policies, certain matters related to auction efficiency, and how the winner's curse affects entry.

We find that the winner's curse can have a marked impact on the number of entrants. The main intuition is that the winner's curse distorts equilibrium markups at the bid phase, thereby affecting a project's expected profits. Because anticipated profits are a factor in a contractor's entry decision, the winner's curse can also influence the total number of entrants, depending on the entry cost.

We then investigate how both the winner's curse and entry affect auction outcomes in an empirical setting by estimating an auction model with endogenous entry. When estimated on bridgework data from the State of Oklahoma, we find that cost uncertainty—a key factor in whether there is a winner's curse—is substantial, accounting for 20.6 percent of the variance in log signals. Counterfactual simulations that remove this uncertainty (and therefore remove the winner's curse) suggest that entry would have been 4.6 percent higher and low bids 4.4 percent lower had there been no winner's curse on a typical bridge project. Furthermore, auctions would have been more efficient had there been no winner's curse, as social costs—our measure of efficiency—would decrease by 3.5 percent.

Because the winner's curse becomes more severe as the number of bidders increases, governments might have an incentive to reduce competition. As such, we use our estimates to evaluate the impact of various entry policies on auction performance. On a project representative of the typical bridge contract, we find that Oklahoma could have used entry taxes to lower procurement costs by 2.6 percent relative to the status quo. Alternatively, Oklahoma could have regulated entry through an entry rights auction, which uses an initial auction to allocate entry spots in a secondary auction that awards the contract. By limiting the number of entrants to three, Oklahoma could have lowered procurement costs by 3.4 percent and increased efficiency relative to letting contractors enter freely. These results illustrate the possibility of improving auction outcomes through intervention and complement existing empirical work such as Bhattacharya et al. (2014), who find that entry rights auctions decrease procurement costs and increase efficiency significantly.

Our article connects to earlier studies in the winner's curse literature. Capen et al. (1971) first note the existence of the winner's curse in off-shore oil drilling, followed by an extensive study in Hendricks and Porter (1988). Hendricks et al. (2003) document off-shore oil bidding behavior consistent with a winner's curse in equilibrium. Similarly, Bukhchandani and Huang (1989) examine the existence of the winner's curse in treasury bill auctions with resale, and Ashenfelter and Genesove (1992) investigate the winner's curse in real estate auctions.

In highway procurement, Hong and Shum (2002) find that the winner's curse can lead to higher equilibrium bids as competition increases. Our study borrows their auction setup but extends it to include a costly entry stage. This extension is made possible through the data we have on the set of potential bidders, allowing us to directly model entry rather than imputing it from an unobserved reserve price as they did. Somaini (2020) estimates an auction model with interdependent costs and notes that policies restricting the number of participants reduce the severity of the winner's curse but not enough to lower overall procurement costs. We expand on his participation analysis by endogenizing it.

To that end, our study also relates to the literature on entry in auctions. We model entry following Levin and Smith (1994), assuming potential bidders have no information on their values before entering. Milgrom (2004) shows that when potential bidders have no pre-entry knowledge of their values, efficiency increases when the auctioneer selects a fixed number of firms. Our entry model is similar to Krasnokutskaya and Seim (2011), Li and Zheng (2009), and Li et al. (2015). Other entry models in the literature include Samuelson (1985) type entry models, which assume potential bidders observe their values before entering; Gentry and Li

(2014) and Bhattacharya et al. (2014) consider selective entry models, where bidders base entry on a signal of their value.

The remainder of this article proceeds as follows. Section 2 discusses the highway procurement process in Oklahoma as a backdrop for our empirical analysis. Section 3 outlines the auction model with entry and a potential winner's curse, and Section 4 flexibly parameterizes it. Section 5 outlines how we estimate the model with the data; we present those results in Section 6. Section 7 considers counterfactual scenarios and government interventions. Section 8 concludes.

2 Highway Procurement in Oklahoma

This section describes the procurement process in the State of Oklahoma and motivates several of our modeling assumptions. The Oklahoma Department of Transportation (ODOT) auctions construction projects monthly via sealed-bid auction, where the lowest overall bid wins the contract. ODOT announces upcoming construction projects at least 28 days before letting. During that time, any firm interested in submitting a bid on a project must first obtain that project's plans. Once acquired, the contractor becomes a "plan holder," and this plan holder information is publicly available before bid opening—meaning that bidders can identify their possible competitors. Given that plan holder status is required, public, and known in advance, we use it to determine the set of potential bidders.

The construction plans contain a detailed project description, such as a state engineer's construction specification, the required materials, an engineer's estimated quantity of materials, the funding status (either federal or state), and the project's location. However, the plans do not provide any information on the state engineers' cost on individual items or the overall engineer's cost estimate (ECE). In our analysis, we will follow the procurement literature in other states not releasing the cost estimate before letting (see Jofre-Bonet and Pesendorfer (2003) and Rosa (2019)) in using it to proxy for unobserved parts of the plan specifications.

Oklahoma allows plan holders to submit bids until a preset deadline, two and a half hours before letting. During our sample period, between January 1997 and October 1999, Oklahoma did not yet authorize a now common practice of online bidding. Based on our observations of the bid-letting process and discussions with practitioners and ODOT procurement officials at the time, we found that almost all bidders were present at bid letting and that mail-in bids were rare. As a result, we will assume that bidders observe the number of bidders before submitting bids.

After letting, ODOT procurement officials and state engineers inspect all submitted bids for inconsistencies or unbalanced bids and announce the winners. At this stage, ODOT releases further information, such as the value of all submitted bids, the identities of bidders, and the

ECE. In our empirical analysis, we will use this information along with the plan holders to estimate a contractor's entry cost and expected project cost.

Our analysis will focus on bridge projects to mitigate potential asymmetries that can arise in asphalt procurement.[1] De Silva et al. (2008) argue that there is the potential for substantial cost uncertainty in bridgework, which can create a winner's curse. They note that soil conditions at a site may not be fully known until excavation work begins and that repairs may not be fully understood until firms undertake some demolition work.

Although there are procedures in place to amend awarded contracts via renegotiation, ODOT typically negotiates only when it agrees that there is a change in a project's scope of required work or a need for individual work item adjustments. We abstract away from this renegotiation process but note that disagreements can arise in practice, resulting in contractors incurring extra costs. Even in cases where there is an agreement, ODOT uses prices from the original contract on any adjusted items and Blue Book estimates for any new items. A misestimated cost would then carry over into the renegotiated contract, and Blue Book estimates for new work may not align with actual costs—creating uncertainty. In the empirical model, we allow for uncertainty and investigate how it affects bidding and entry. For an analysis of the renegotiation process, see the work by Bajari et al. (2014).

3 Equilibrium in Common Value Auctions with Entry

In this section, we develop a theoretical model of bidding with endogenous entry that will guide the remainder of our empirical analysis. By endogenizing entry, we allow potential bidders to choose whether to submit bids, which reflects a plan holder's choice to bid in Oklahoma. These entry choices then determine the number of bidders, which will affect any bid adjustments induced by the winner's curse.

3.1 Environment

Formally, we assume N risk-neutral potential bidders are eligible to bid on a construction project offered for letting via a first-price sealed-bid procurement auction. Potential bidder i has an entry cost of D_i, representing the costly process of acquiring a signal. We assume that D_i is private information and is independently and identically distributed on some positive interval according to the distribution function, H.

Each potential bidder decides whether to participate in the auction, thereby becoming an actual bidder. They make this decision by weighing their entry-cost realization, d_i (of D_i),

[1] Somaini (2020) suggests that firms pre-qualified for hot-mix asphalt work may benefit from being closer to a project's location, whereas our model will abstract away from such asymmetries. Nevertheless, we discuss the possibility of asymmetries in our setting later in section 6.

against their expected profits from bidding. After the entry phase, a total of $n \leq N$ actual bidders submit bids, and we assume that participants observe n before bidding.

Each actual bidder has a cost of project completion (or project cost), C_i, that is unobserved when bidding. Instead, actual bidders observe a private signal, X_i, of their project cost and use it in determining their bids.

As is standard in the auction literature, we assume that the joint distribution of signals and project costs, $\mathcal{F}_n(c_1, \dots, c_n, x_1, \dots, x_n)$, is affiliated,[2] meaning that costs and signals exhibit some form of positive dependence. We also assume that the joint distribution is symmetric so that $\mathcal{F}_n$ is exchangeable on the indices $1, \dots, n$. These assumptions will generate unique equilibrium bidding functions, where each bidder follows the same monotonic bid strategy. As a matter of notation, we use the n subscript to denote dependence of functions on the number of entrants.

3.2 Equilibrium Bidding

We begin by characterizing equilibrium bidding, which follows the analyses in Milgrom and Weber (1982) and Hong and Shum (2002). At this stage, contractors observe the realization of their signal, x, and the number of actual bidders but—as is typical in highway construction—do not observe their project costs directly. As such, contractors form an expectation of their project costs, and to shed light on that expectation, we define the following function:

$$c_n(x, y_1) = E[C_i \mid X_i = x, Y_1 = y_1].$$

In words, this function is a contractor's expected project cost conditional on their signal being x and the lowest of the $n-1$ competing signals, Y_1, being y_1.

A bidder's objective is to maximize their expected profit conditional on winning. If every other actual bidder follows an increasing and differentiable strategy β_n, contractor i would then choose a bid, b, to maximize

$$\pi_n(b, x) = E[(b - c_n(X_i, Y_1))1\{b < \beta_n(Y_1)\} \mid X_i = x],$$

where $1\{\cdot\}$ is an indicator function that takes on a value of one if the term inside the brackets is true and zero otherwise.

Let $G_n(y_1 \mid x)$ be the distribution of Y_1 given $X_i = x$, and let $g_n(y_1 \mid x)$ be its associated

[2]The formal definition of affiliation is as follows. Let $\boldsymbol{z}$ and $\boldsymbol{z'}$ be points in $\mathbb{R}^n$. A distribution function $f : [\underline{z}, \overline{z}]^n \to \mathbb{R}_+$ is affiliated if $f(\boldsymbol{z})f(\boldsymbol{z'}) \leq f(\boldsymbol{z} \vee \boldsymbol{z'})f(\boldsymbol{z} \wedge \boldsymbol{z'})$, where $\boldsymbol{z} \vee \boldsymbol{z'} = (\max\{z_1, z_1'\}, \dots, \max\{z_n, z_n'\})$ and $\boldsymbol{z} \wedge \boldsymbol{z'} = (\min\{z_1, z_1'\}, \dots, \min\{z_n, z_n'\})$.

density. The first-order condition of the firm's bidding problem is then

$$\beta_n(x) = \frac{\beta_n'(x)\,(1 - G_n(x \mid x))}{g_n(x \mid x)} + c_n(x, x). \tag{1}$$

Note that a contractor that receives the highest signal, $\overline{x}$, cannot bid over $c_n(\overline{x}, \overline{x})$ or else they could lower their bid and obtain a positive expected profit. Thus, another condition of the firm's bidding problem is the boundary condition $\beta_n(\overline{x}) = c_n(\overline{x}, \overline{x})$.

As shown in Milgrom and Weber (1982) and Hong and Shum (2002), the solution to the differential equation in (1) is the equilibrium bid function,

$$\beta_n(x) = \underbrace{c_n(x, x)}_{\text{expected cost}} + \underbrace{\int_x^{\overline{x}} \left\{ \exp\left[- \int_x^y \frac{g_n(t \mid t)}{(1 - G_n(t \mid t))} dt \right] \right\} c_n'(y, y) dy}_{\text{markup}}. \tag{2}$$

Observe that equilibrium bids can be interpreted as a strategic markup over expected costs. The expected cost is given by the term $c_n(x, x)$ and maps a contractor's realized signal into an expectation for their latent project cost. This term relates to the winner's curse: when signals are noisy, more competition leads to an upward adjustment in a contractor's estimate to account for the increased possibility of having an overly optimistic signal conditional on winning. Hong and Shum (2002) refer to this phenomenon as the *winner's curse effect*.

The second component of equilibrium bidding is the markup term. Notice that when contractors use noisy signals to form estimates of their costs, there is a distortion in equilibrium markups. Indeed, with no cost uncertainty and therefore no winner's curse (i.e., $x = c_i$), $c_n(y, y) = y$, $c_n'(y, y) = 1$, and markups correspond to the ones obtained under affiliated private values. The winner's curse will generally alter these markups, which will affect entry decisions.

Intuitively, by changing the expected cost function, the winner's curse also changes the expected cost distribution. If this distribution is relatively more (less) dispersed than the underlying project cost distribution, contractors obtain higher (lower) information rents and thus higher (lower) markups in equilibrium.

3.3 Equilibrium Entry

At the entry stage, we assume an equilibrium in pure strategies, where potential bidders contrast their entry costs against their ex-ante expected profits. Here, potential bidders take expectations over the number of bidders and the possible signal realizations. Potential bidders adopt a threshold strategy, entering when their ex-ante profits exceed their entry costs.

Observe that the model abstracts away from selective entry, a scenario where contractors with lower project costs are more likely to enter. We opt for this approach because incorporating selective entry into a model with potentially common values is challenging, and to our knowledge, the literature has yet to establish identification results in those environments. Although it is difficult to ascertain precisely how selection and common values would jointly affect contractor behavior without explicitly modeling it, Gentry and Li (2014), Gentry et al. (2017), and Bhattacharya et al. (2014) provide insights on the implications of selective entry when values are private, rather than common.

To formalize our entry equilibrium, we let $\pi_n(x) = \pi_n(\beta_n(x), x)$. If p is a potential bidder's belief about the probability other firms will enter, then the expected profit before entry is given by

$$\overline{\pi}(p) = \sum_{n=1}^{N} \left(\int_{\underline{x}}^{\overline{x}} \pi_n(x) f(x) dx \right) \Pr(n \mid N),$$

where

$$\Pr(n \mid N) = \binom{N-1}{n-1} p^{n-1} (1-p)^{N-n}$$

is the probability that n out of the N potential bidders participate.

In equilibrium, a contractor with entry cost d_i enters whenever $d_i \leq \overline{\pi}(p)$. Furthermore, beliefs must be consistent with actual entry probabilities so that p solves

$$p = H(\overline{\pi}(p)). \tag{3}$$

This expression equates a contractor's belief that a competitor will enter to the probability that competitor entry costs are less than ex-ante profits.

To prevent situations where only one firm competes, we assume the government acts as a competitor whenever there is entry. In our empirical setting, Oklahoma has the right to reject all bids if they are unusually high, and we use this assumption to capture that bid-rejection power.[3] We also note that the model may have multiple entry equilibria.[4] In our application, we use simulations to verify that our entry equilibria are unique.

4 A Parameterized Model

To bring the model to the data, we make several parametric assumptions on the model's primitives. This section outlines those assumptions.

[3] Li and Zheng (2009) make a similar assumption in their study of highway mowing contracts in Texas.

[4] For example, assume the project cost distribution is degenerate at some constant, k. Then, $c_n(x, y) = E[C_i \mid X_i = x, Y_1 = y] = k$ for all x and y. Moreover, $c'_n(x, x) = 0$, so markups are zero. If $H(d_i)$ is degenerate at 0, then all firms bid k irrespective of their signal and are indifferent between entering and not entering.

4.1 Project-Cost and Signal Parameterization

We begin with the project-cost and signal parameterization, which follows the Wilson (1998) log-additive framework also used in Hong and Shum (2002). Observe that firms may have an expertise over their competitors on a particular construction project, which would be a component of their costs specific to them. Firms may also need to employ similar subcontractors and suppliers, possibly leading to a shared component in their costs.[5] To accommodate these possibilities, we assume that a contractor's project cost can have a private cost component, A_i, and a common cost component, V. Because they correspond to a project's actual cost, both components are unobserved throughout the entire bidding process, but their distributions are known at the entry stage. For simplicity, we assume A_i and V are independent and take the project cost as the product of the two: $C_i = A_i \times V$.

Following the Wilson (1998) formulation, we assume A_i and V are lognormal. With this specification, it will be convenient to work in logs, so letting $\widetilde{A}_i := \log(A_i)$ and $\widetilde{V} := \log(V)$, we have

$$\widetilde{V} = m + \epsilon_v \sim \mathcal{N}(m, \sigma_v^2)$$

and

$$\widetilde{A}_i = \overline{a} + \epsilon_{a,i} \sim \mathcal{N}(\overline{a}, \sigma_a^2),$$

where $\mathcal{N}$ denotes the normal distribution.

Signals are a noisy indicator of a contractor's project cost. In modeling this element, we assume that signals are the product of project costs and noise: $X_i = C_i \times E_i$, with $E_i = \exp(\sigma_e \xi_i)$ and $\xi_i \sim \mathcal{N}(0, 1)$. Setting $\widetilde{C}_i := \log(C_i)$ and $\widetilde{X}_i := \log(X_i)$, we have that, conditional on the realization of $\tilde{C}_i$,

$$\widetilde{X}_i = \tilde{c}_i + \epsilon_{e,i} \sim \mathcal{N}(\tilde{c}_i, \sigma_e^2),$$

where $\tilde{c}_i$ is the realized $\tilde{C}_i$. Thus, σ_e describes the degree of project-cost uncertainty.

An appealing feature of this setup is that it nests several informational paradigms within the auction literature. If $\sigma_a > 0$, $\sigma_v = 0$, and $\sigma_e > 0$, the auction reduces down to one with independent private values and uncertainty over project costs. Similarly, if $\sigma_a > 0$, $\sigma_v > 0$, and $\sigma_e = 0$, the model becomes one with affiliated private values. Common values require an unknown but common component on which bidders have private information, or in our parameterization, $\sigma_v > 0$ and $\sigma_e > 0$. A pure common value model occurs when $\sigma_a = 0$, $\sigma_v > 0$, and $\sigma_e > 0$. Thus—as pointed out by Hong and Shum (2002)—it is the relative value of the σs that dictate the relative importance of a bidder's common and private elements in their project costs.

[5] Rosa (2019) notes that there is substantial overlap in prices on items and tasks for highway construction projects in New Mexico.

In the same vein, increasing the relative value of σ_e will have implications for markups, although its overall effect is ambiguous. On the one hand, increasing σ_e leads to a more dispersed distribution of contractor private information, potentially increasing information rents. On the other hand, signals become less informative at higher σ_e values. In an extreme case—as σ_e approaches infinity—signals are so uninformative that contractors fail to extract any information rents. Thus, markups can either increase or decrease with a change in σ_e.

Observe that m and $\overline{a}$ are not separately identified here, as they always appear together as a sum. Therefore, we can identify only their sum, μ, where $\mu := m + \overline{a}$. This aspect of the model is not as limiting as it may seem, as models with the same μ that differ in m and $\overline{a}$ are indeed economically equivalent.

4.2 Entry Cost Parameterization

Next, we parameterize the entry-cost distribution. We assume that the log of a contractor's entry cost is distributed normally. Thus, $D_i \sim \mathcal{LN}(\mu_d, \sigma_d^2)$, where $\mathcal{LN}$ denotes the lognormal distribution. This parameterization ensures that entry costs are positive and does well in matching average participation in the data.

5 The Empirical Model and Estimation

In estimating the model, we must distinguish changes in bidding and entry that come from the uncertainty parameters from similar changes that can arise from differences in project characteristics. In this section, we do so through an empirical model. After explaining that model, we lay out our estimation approach and describe the variation in the data that identifies the model.

We assume contractors observe a realized set of characteristics $(\mathbf{z}_t, \mathbf{w}_t)$ before deciding whether to enter auction t. The vectors $\mathbf{z}_t$ and $\mathbf{w}_t$ contain project characteristics observed by both the potential bidders and researchers; the characteristics in $\mathbf{z}_t$ influence project costs, whereas the characteristics in $\mathbf{w}_t$ affect entry costs. In our setting, $\mathbf{z}_t$ and $\mathbf{w}_t$ can and do overlap[6] and are often referred to as observed heterogeneity.

Unlike Compiani et al. (2020) and Haile et al. (2003), we do not include unobserved characteristics affecting project costs, which the literature refers to as unobserved heterogeneity. In their settings, unobserved heterogeneity requires an exclusion restriction: one of the observables must affect entry but can otherwise be excluded from the auction model. Our formulation does not require such a restriction.

[6] Indeed, $\mathbf{w}_t$ is a subset of $\mathbf{z}_t$ in our data, but it need not be to estimate the model.

5.1 Homogenized Auctions and Scaled Entry Costs

To account for observed auction characteristics in the empirical model, we use a homogenization procedure similar to Compiani et al. (2020), Athey and Haile (2007), and Haile et al. (2003). The key insight behind this approach is that, under some independence assumptions, multiplicative separability in project costs implies multiplicative separability in bids. When that multiplicative term is a function of the project characteristics, one can calculate bids in an arbitrary auction by solving one "homogenized" bid function.

We implement this approach by assuming project costs are multiplicatively separable in project characteristics. Thus,

$$C_{it} = \Gamma(\mathbf{z}_t) C^0_{it},$$

where the index function, Γ, is parameterized as

$$\Gamma(\mathbf{z}_t) = \exp(\mathbf{z}_t' \gamma).$$

Our project-cost parameterization implies a similar form for the signals:

$$X_{it} = \Gamma(\mathbf{z}_t) X^0_{it}.$$

Assuming C^0_{it} and X^0_{it} are independent of $\mathbf{Z}_t$ given the realized number of bidders, n_t, one can show that equilibrium bids are also multiplicatively separable (Haile et al., 2003). Namely,

$$\beta_{n_t}(x_{it}; \mathbf{z}_t) = \Gamma(\mathbf{z}_t) \beta^0_{n_t}(x^0_{it}; \mathbf{z}^0),$$

where x^0_{it} is the realization of X^0_{it} and $\mathbf{z}^0$ is a vector of project characteristics such that $\Gamma(\mathbf{z}^0) = 1$. We refer to, $\beta^0_{n_t}$, C^0_{it}, and X^0_{it} as the homogenized bid, homogenized project cost, and homogenized signal, respectively.

We make a similar multiplicative separability assumption for entry costs but allow for heterogeneity in project characteristics $\mathbf{w}_t$. Namely,

$$D_{it} = \Gamma(\mathbf{z}_t) D^0_{it},$$

where $D^0_{it} \sim \mathcal{LN}(\mathbf{w}_t' \alpha, \sigma^2_d)$. By including $\mathbf{w}_t$, we add flexibility in how project characteristics affect entry. This addition means that the entry cost D^0_{it} is not fully homogenized, as it will depend on $\mathbf{w}_t$. Thus, we refer to D^0_{it} as the scaled entry cost, and our estimates will be in terms of these scaled costs.

In estimation, our homogenization and scaling assumptions will allow us to solve a homogenized auction with various scaled entry costs rather than solving a separate auction model for every variable in $\mathbf{z}_t$. Indeed, one can interpret the signal, project cost, and entry cost parameters outlined in Section 4 as coming from a homogenized auction with a (fixed) scaled entry cost distribution. We can expand those parameters to any arbitrary auction by multiplying homogenized bids and scaled entry costs (adjusted for $\mathbf{w}_t$) by the index function. That is, we can net out $\Gamma(\mathbf{z}_t)$ once we estimate it when solving the model.

5.2 Estimation Method

Estimation begins with the index function parameters: γ. Given the function's exponential form, we can obtain consistent estimates of these parameters by regressing log bids on observables. The γ estimates are then the parameters from the regression

$$\log(b_{it}) = \zeta(n_t) + \mathbf{z}_t'\gamma + \epsilon_{it}, \tag{4}$$

where $\zeta(n_t)$ are intercepts for the number of actual bidders.

We estimate the remaining parameters, $\Theta = (\sigma_a, \sigma_v, \sigma_e, \sigma_d, \alpha, \mu)$, through simulated method of moments (SMM). This technique aims to select structural estimates that minimize a weighted distance from data moments, $\check{m}$, to model moments generated from S simulations of the data, $\hat{m}_S(\Theta)$. Taking Λ as the weight matrix, the SMM estimator is formally

$$\widehat{\Theta} = \underset{\Theta}{\operatorname{argmin}}\ [\check{m} - \hat{m}_S(\Theta)]'\Lambda[\check{m} - \hat{m}_S(\Theta)].$$

Because SMM relies on simulated model moments and the weighting matrix, implementing SMM requires us to simulate the data, select the relevant moments, and choose an appropriate Λ. In simulating the data, we use the current values of σ_a, σ_v, and σ_e to calculate *homogenized* bid functions for every possible number of actual bidders. Then, we use the homogenized bids to calculate expected profits before entry. These profits depend on the number of potential bidders and entry-cost shifters, which are the project characteristics in $\mathbf{w}_t$ changing the scaled entry cost distribution. From there, we draw scaled entry costs for each potential bidder. Potential bidders with expected profits exceeding their scaled entry costs enter. Finally, we draw signals for all entrants and use them and the homogenized bid functions to generate simulated homogenized bids. In practice, we simulate each auction in the data 50 times, meaning that $S = 50$. We relegate a more detailed explanation of our simulation routine to the appendix.

In choosing the moments, we aim to select features of the data that provide information on the model's primitives. Thus, our first set of moments are the mean, standard deviation, and bias-corrected skewness of the aggregate homogenized bid distribution. These moments have information on the overall distribution of bids and will help pin down the σ parameters.

Note that bids should depend on the number of bidders and that this dependence can be nonlinear and increasing if there is a winner's curse. To allow for this possibility in the data, our next set of moments are conditional homogenized bid moments, with conditioning on the number of actual bidders. These moments are coefficients from an OLS regression of homogenized bids on a quadratic polynomial in the number of entrants and will again help pin down the σs. Also note that, depending on the relative value of σ_v, there can be bid correlations within auctions. We include as a moment the intraclass correlation coefficient—which captures that potential data feature.

The remaining moments pertain to entry, which depends on the number of potential entrants and entry-cost shifters. Thus, our last set of moments contain the standard deviation of aggregate entry decisions as well as coefficients from an OLS regression of entry on the number of potential bidders and entry-cost shifters. In total, there are 11 simulated moments that we match to equivalent data moments.

The last component of SMM is the weight matrix. In our application, we use the optimal weight matrix proposed in Gourieroux et al. (1993). This matrix minimizes the standard errors, which we calculate analytically.

Observe that our moment-based estimator differs from the quantile one used in Hong and Shum (2002). One advantage to using moments is that it allows us to target within auction bid correlations, which helps us identify the σ_v parameter. Another advantage comes from estimating the entry parameters. Because entry is a discrete choice, a quantile estimator would require us to estimate the conditional quantiles of a discrete outcome variable. These problems have a non-smooth objective function and non-standard convergence rate absent some artificially imposed data smoothing; see Manski (1975), Manski (1985), and Machado and Silva (2005). Our approach avoids these complications.

5.3 Identification

We conclude this section with a heuristic discussion of how the empirical model's parameters are identified from the data. Observe that our model is in a class of first-price sealed-bid auction models with affiliated values and endogenous entry. Compiani et al. (2020) provide conditions under which these types of models with unobserved characteristics are non-parametrically identified in the standard high-price auction. However, our model differs from theirs in that it has more project-cost components: Compiani et al. (2020) identify what we call expected costs as a whole, whereas we consider expected costs as a function of a private component, a common component, and an uncertainty component. This breakdown requires more structure, which we impose through our parametric assumptions. As a result, our identification argument is most closely related to the one made in Hong and Shum (2002).

The parameters of our model are the index function parameters (γ), the signal parameters

(μ, σ_a, σ_v, and σ_e), and the entry cost parameters (α and σ_d). In the data, bids tend to vary with different observed auction characteristics, conditional on the number of actual bidders. Given our separability assumption, this variation pins down the index function parameters, γ.

The remaining bid variation—both unconditional and conditional on the number of entrants—and within-auction bid correlations pin down the signal parameters. The unconditional bid distribution pins down μ and total signal variation, which is a function of σ_a, σ_v, and σ_e. The within-auction bid correlations pin down the relative value of σ_v, which controls the degree to which signals and project costs are affiliated. The bid variation conditional on the number of entrants pins down the relative magnitudes of the remaining σs. Indeed, if σ_a is relatively more important, then bids will decrease more rapidly in the number of entrants, as the auction will resemble a standard private-value auction. If σ_v and σ_e are relatively more important, then bids may eventually increase in the number of entrants due to the winner's curse in common value auctions.

Finally, we observe the probability a contractor enters (in equilibrium) given the number of potential bidders and scaled entry cost shifters $\mathbf{w}_t$. For a given $\mathbf{w}_t$, entry probability variation in potential bidders and the entry condition in equation (3) pins down scaled entry costs. That is, we can calculate expected profits before entry with the bid parameters and observed entry probabilities. The scaled entry cost parameters then require equation (3) to hold for any number of potential bidders.[7]

6 Empirical Analysis

We now turn to our analysis of the Oklahoma bridgework data, which we particularize in this section. Our analysis begins with a summary of various data statistics relevant to our empirical model. Next, we implement our empirical methods to obtain estimates of the primitives underlying the bidding and entry observed in the data. We then verify that our estimates fit key data moments well, including ones not targeted in estimation.

We perform these exercises for two distinct specifications: one that permits common values and another that assumes values are private. Doing so enables us to comment on various data features that would be difficult to explain without allowing for a common-value component. We end with a discussion of additional modeling considerations.

[7]We provide a supplemental simulation study illustrating how different parameter values affect bidding and entry in the Online Appendix.

Table 1: Summary Statistics for Bridge Projects

Variables	Mean/count	Standard deviation	Percentiles 25	50	75
Number of projects	448				
Number of plans held	2,667				
Number of bids	1,583				
Average bid (in $ millions)	0.537	1.561	0.151	0.245	0.362
ECE (in $ millions)	0.602	1.944	0.120	0.216	0.331
Average number of planholders	5.962	2.585	4.000	6.000	7.000
Average number of bidders	3.530	2.002	2.000	3.000	5.000
Federal project	0.745	0.436	0.000	1.000	1.000
Seasonally unadjusted unemployment rate (%)	4.049	0.563	3.800	4.100	4.400
Three-month average of the real volume of projects	0.985	0.267	0.736	0.947	1.168
Three-month average of the number of building permits	1.058	0.183	0.921	1.104	1.169

Note: The unemployment rate is the monthly state-level unemployment rate (in percentages) in Oklahoma from the US Bureau of Labor Statistics. The three-month average of the real volume of projects variable measures the three-month moving average of the real volume of projects for Oklahoma relative to the average real volume of all projects. The three-month average of the number of building permits variable measures the three-month moving average of the number of monthly building permits for Oklahoma relative to the average number of monthly permits. The data come from the US Bureau of Economic Analysis.

6.1 Data and Summary Statistics

We use data on all bridge construction and maintenance projects auctioned by ODOT from January 1997 to October 1999. Our data include information on the firms that purchase plans for a project, a set of project and market characteristics, and the identity and bids of all bidders on a project—including the low bid.

Table 1 reports our summary statistics. We observe 448 separate bridge contracts during our sample period, with 2,667 held plans leading to 1,583 submitted bids. These statistics imply that roughly 59.4 percent of all plan holders become bidders. At the project level, there are an average of approximately 6.0 plan holders and 3.5 bidders.

Our project characteristics include the engineer's estimate and whether a project is funded federally (as opposed to by the state). The average value of the engineer's estimate is about $0.62 million and has a standard deviation of $1.9 million. Over 74 percent of bridge projects in our data are federally funded.

We include additional market information that can potentially affect a contractor's cost, similar to De Silva et al. (2008). To account for employment factors, we report the seasonally unadjusted monthly state-level unemployment rate for Oklahoma, which we collect from the US Bureau of Labor Statistics. Then, we create a variable capturing the three-month average of the real volume of projects,[8]and we use it to assess ODOT-related market conditions. Finally, we generate a three-month average of the number of building permits for Oklahoma, which we

Table 2: Probability of bidding

Variable	Probability of bidding
Log of ECE	-0.036***
	(0.010)
Number of plan holders	-0.016***
	(0.005)
Federal projects	0.137***
	(0.028)
Market controls	Yes
Division effects	Yes
Quarter effects	Yes
Year effects	Yes
Observations	2,667
Wald χ^2	121.41

Robust standard errors clustered by auctions are in parentheses.
*** p<0.01, ** p<0.05, * p<0.1. Marginal effects are reported. Market controls include unemployment rate, three-month average of the real volume of projects, and relative three-month average of the building permits.

use to measure non-ODOT market conditions. We construct these three-month averages with data from the US Bureau of Economic Analysis.

6.2 Descriptive Analysis

As a first pass at exploring how bidding and participation vary under different competitive circumstances, we use the Oklahoma bridge data to conduct a series of descriptive regression analyses. Our first analysis concerns entry, for which we run a probit regression on whether a plan holder becomes an actual bidder. We include project, market, competition, division, and time controls to account for observed differences between contracts at the time of entry. Table 2 presents the regression estimates.

Our probit analysis indicates a negative correlation between a bridge project's size (as represented by the engineer's estimate) and participation, meaning that contractors are less likely to participate on larger projects. Furthermore, the negative coefficient on plan holders suggests that more potential competition is correlated with a lower probability of participating in an auction. This result is consistent with a contractor's strategic response to facing increased potential competition when more actual bidders reduces markups.

At the bidding stage, the winner's curse can potentially undo the conventional wisdom that increasing the number of bidders reduces bids. To explore this possibility in the data,

[8] We construct the real volume of projects variable by adding the engineer's estimate across projects up for bid in a month for Oklahoma and deflating its value by the PPI. We then divide it by the average of the real volume for Oklahoma to calculate the relative real volume.

Table 3: Regression results for bids

Variable	Log bid
Log of ECE	0.965***
	(0.013)
Number of bidders	-0.002
	(0.006)
Federal projects	0.031
	(0.028)
Market controls	Yes
Division effects	Yes
Quarter effects	Yes
Year effects	Yes
Observations	1,583
R-squared	0.998

Robust standard errors clustered by auctions are in parentheses.
*** p<0.01, ** p<0.05, * p<0.1. Marginal effects are reported. Market controls include unemployment rate, three-month average of the real volume of projects, and relative three-month average of the building permits.

we conduct a descriptive regression analysis relating log-bids to the number of actual bidders and various contract characteristics. Table 3 presents our findings. As is expected in bridge procurement, log-bids are positively correlated with the engineer's estimate, suggesting that larger projects may have larger expected costs to complete. Curiously, though, there is an economically and statistically insignificant correlation between log-bids and actual bidders. This finding is consistent with a winner's curse and provides suggestive evidence for our empirical setup.

Although suggestive, we acknowledge that there is still some ambiguity in our bid results. Pinkse and Tan (2005) show theoretically that a similar pattern between bids and the number of bidders can arise when project costs are affiliated but known. Because contractors observe their costs directly in that setup, there would be no winner's curse, but bids would not be decreasing in actual bidders due to an *affiliation effect*. To assess whether the observed pattern is due to a winner's curse or an affiliation effect, we rely on estimates from our structural model, which nests the two competing explanations. We also performed a semiparametric test, in the spirit of Haile et al. (2003). That test provides additional evidence for common values in our data and is available in Appendix B.

6.3 Estimation Results

Our first set of model estimates pertain to the index function and are in Table 4. The index function—and therefore bids and project costs—tends to increase in the engineer's estimate and

Table 4: Parameter Estimates for the Index Function

Variable	Coefficient	Standard error
Log of ECE	0.964	0.009
Federal projects	0.029	0.020
Unemployment rate	-0.012	0.032
Three month average of the real volume of projects	-0.035	0.031
Relative three month average of the building permits	-0.078	0.062
Division effects	Yes	
Quarter effects	Yes	
Year effects	Yes	
Observations	1,583	

on federal projects and decrease in the market variables. De Silva et al. (2008) observe similar patterns for contractor bidding on bridge projects in their data.

Table 5: Signal and Entry Cost Distribution Estimates

	Common Values		Private Values ($\sigma_e = 0$)	
	Coefficient	Standard error	Coefficient	Standard error
Signal parameters				
μ	0.3638	0.0144	0.3054	0.0060
σ_a	0.2041	0.0270	0.3317	0.0191
σ_v	0.2313	0.0068	0.3003	0.0291
σ_e	0.1573	0.0270		
Entry parameters				
Constant	-4.5022	0.0359	-3.7117	0.0143
Log of ECE	0.1505	0.0018	0.0995	0.0010
σ_d	0.6974	0.1370	0.6939	0.0151

Note: Parameter estimates for the homogenized signal and scaled entry cost distribution. For an auction with observed characteristics $\mathbf{z}_t$, scaled entry costs are multiplied by $\hat{\Gamma}(\mathbf{z}_t)$ to get actual entry costs.

Table 5 reports our next set of model estimates: the signal and scaled entry cost parameters. We estimate these parameters for the full model and an affiliated private value specification that imposes the restriction $\sigma_e = 0$. These secondary estimates allow us to compare the two models' performance, although we will focus our discussion mainly on the common value results. Observe that both specifications use the same index function estimates because our homogenization procedure is not contingent on whether values are private or common.

We find that all signal parameters are statistically significant individually across specifications.[9] Observing that common values require $\sigma_v > 0$ and $\sigma_e > 0$, the significance of both

[9]Because the signal parameters must be greater than or equal to zero, this statement applies to a one-sided t-test for each parameter value.

parameters in our primary, common value specification suggests there is statistical evidence for an environment with a winner's curse. Collectively, our signal parameters imply that 20.6 percent of the variance in log signals is due to cost uncertainty when we allow for common values, highlighting its relevance in our sample relative to the other sources of uncertainty. Our σ estimates are smaller than many of the ones estimated in Hong and Shum (2002), though, which is likely because we have more controls for observed heterogeneity available in our data.

The entry parameters have the expected signs and suggest that average entry costs increase with larger projects.[10] Using the median-sized project as a benchmark, our common value estimates indicate that the average entry cost of a bridge project with the median engineer's estimate is 5.1 percent of the estimate. This value is slightly higher than the mean entry costs of 1.5 percent of the estimate estimated by Bhattacharya et al. (2014) for similar bridge contracts in Oklahoma but is close to other entry costs estimated in the literature (e.g., Krasnokutskaya and Seim (2011) who estimate that entry costs range from 2.2 to 3.9 percent of the engineer's estimate and Bajari et al. (2010) who find that entry costs are around 4.5 percent of the estimate).

6.4 Model Fit

The outcomes from our assumed parameterization can clash with the data outcomes if our parametric assumptions are unreasonable. Thus, we verify the sensibility of our assumptions by assessing the model's fit in instances where values are private or common.

We begin by comparing the actual and predicted homogenized bid distributions, bearing in mind that our log-additive assumption on signals will dictate the predicted bid distribution's shape. Figure 1 plots our findings. Visually, the distributions appear similar, although both models have trouble accounting for several outlier homogenized bids in the data. Focusing on the left tail, both models face difficulties replicating some lower homogenized bids, even though the private value model performs slightly better on this metric. Despite these limitations, we will show later that the models predict average low bids well.

Next, we explore whether specific bid and entry patterns predicted by the models are close to the ones that arise in the actual data. We first consider the relationship between bids and the number of actual bidders, a pattern shaped by the signal assumptions. We also consider the relationship between the number of actual and potential bidders, a relation generated by the entry cost parameterization and contractor expected profits. Figure 2 plots our results.

The average bid results (in the left panel of Figure 2) highlight the limitations of using a private value model. Even with the possibility of an affiliation effect, the private value model struggles to replicate the flatness in homogenized bids. In contrast, the common value model

[10]Under our parameterization, the log mean of the entry cost distribution is $\mathbf{z}_t'\gamma + \mathbf{w}_t'\alpha$. Thus, larger projects will have higher average entry costs whenever the sum of the two log ECE parameters is positive.

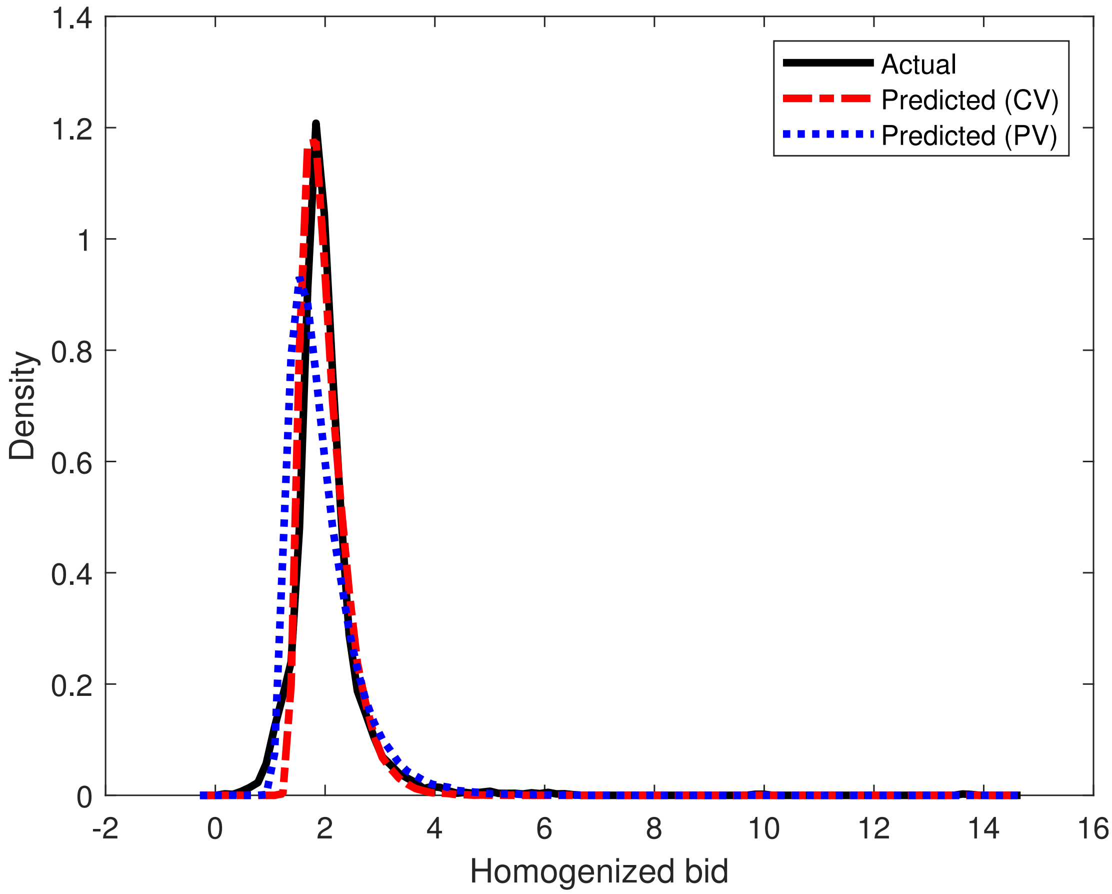

Figure 1: Actual vs. Predicted Homogenized Bid Distribution

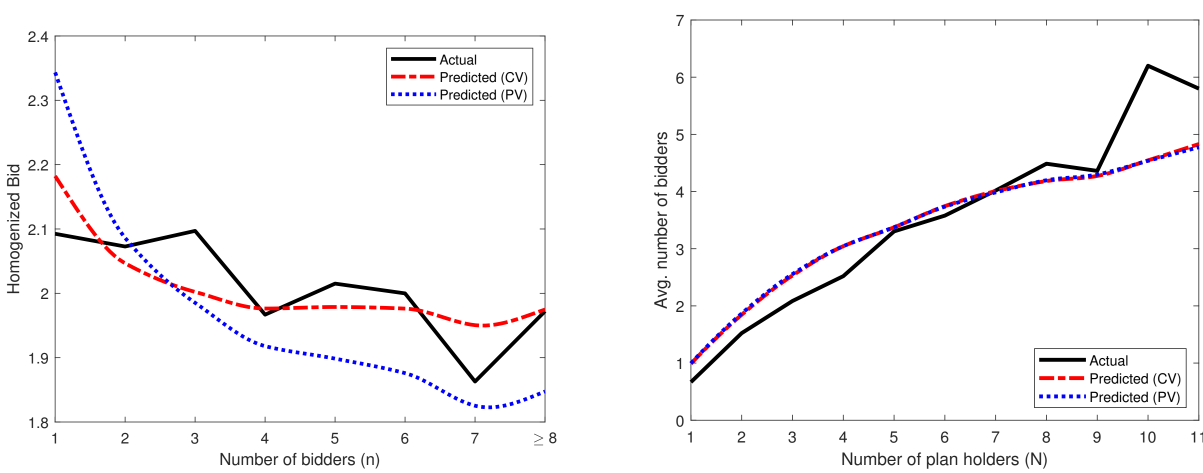

Figure 2: Actual vs. Predicted Bids and Entrants

fits the homogenized bids well because it adds a winner's curse effect that increases bids in the number of bidders. Both models can fit the observed entry patterns (in the right panel of Figure 2) well—suggesting that the entry cost parameterization is flexible enough to account for various entry patterns, even when ex-ante profits across models differ.

Observe that both specifications predict higher homogenized bids when only one contractor participates. A potential explanation for this discrepancy is that Oklahoma utilizes a secret reserve price in evaluating bids, which can be somewhat discretionary in practice. The common value specification performs better on this front, indicating that we can reasonably approximate a contractor's behavior as the only bidder by allowing for common values and the government as an extra competitor.

We continue our fit analysis in Table 6 by comparing several data moments to their corresponding model moments. Both models fit various aspects of the data well, although they have difficulty matching the skewness moment. This lack of fit is driven by the outlier bids present in the data and the sensitivity of these higher-order moments to extreme observations.[11]

Both models have a reasonable fit to moments unused in estimation, including the average low bids. However, the common value model differentiates itself by having a better fit to the conditional bid moments, as indicated by the OLS coefficients and consistent with the results illustrated in Figure 2. Going forward, we will focus primarily on common values as we consider various counterfactual scenarios.

6.5 Discussion

Our analysis considers many features of bridge procurement in Oklahoma—including costly endogenous entry; common, affiliated, and private value auction paradigms; and observed auction heterogeneity. In other environments, there may be additional concerns beyond those considered in our study. Before moving further, we pause to discuss those additional considerations.

Unobserved Heterogeneity

In some situations, there might be aspects of an auction that are unobserved by researchers but observed by firms. When present, this issue is known as unobserved auction heterogeneity. Because it would otherwise be attributed to contractor uncertainty, unobserved heterogeneity can potentially lead to implausibly high markups—especially in construction contracting.

In the bridge data, we find that our observables explain a substantial part of the bid variation. Indeed, our descriptive bid regression in Table 3 has an R^2 of 0.99, largely due to our

[11] Predicted skewness is more similar to the data when we remove right-tail outliers by using homogenized bids below the 95^{th} percentile.

Table 6: Model Fit

	Actual	Predicted (CV)	Predicted (PV)
Moments used in estimation			
Avg. homogenized bid	2.012	1.992	1.945
S.D. homogenized bid	0.677	0.419	0.620
Skewness homogenized bid	5.625	1.414	1.832
Intraclass correlation coefficient	0.376	0.373	0.383
S.D. entry	0.491	0.491	0.491
OLS coefficents: homogenized bids			
Constant	2.212	2.162	2.347
Number of bidders	-0.070	-0.067	-0.150
Number of bidders squared	0.005	0.006	0.011
OLS coefficents: prob. of bidding			
Constant	0.992	1.227	1.080
Log of ECE	-0.024	-0.028	-0.015
Number of plan holders	-0.014	-0.040	-0.042
Moments not used in estimation			
Avg. homogenized low bid	1.750	1.776	1.657
Avg. homogenized bid ($n \leq 3$)	2.088	2.024	2.030
Avg. homogenized bid ($n > 3$)	1.980	1.976	1.898
Avg. number of bidders	3.522	3.533	3.533
Avg. number of bidders ($N \leq 6$)	2.801	3.092	3.095
Avg. number of bidders ($N > 6$)	4.747	4.283	4.278

Note: Table compares the actual data moments to the moments predicted by the common value (CV) and private value (PV) models.

observation of the engineer's estimate.[12] Furthermore, our parameter estimates generate expected markups that are in line with—and in some cases, lower than—other studies in the literature that explicitly account for unobserved heterogeneity. For example, Krasnokutskaya (2011) estimates mean markups at 8.4 percent, whereas average markups in our model are 6.1 percent for a median-sized project. We also estimated a model specification that assumes log-normal unobserved heterogeneity. The model produces economically small and statistically insignificant unobserved heterogeneity estimates, reinforcing the robustness of our analysis to these unobserved factors. Together, these results suggest that unobserved heterogeneity may not be a first-order concern in our analysis.

[12]Note that an R^2 this high means that—due to bids on larger projects—the residual variation is proportionally much lower than the bid variation, rather than the residual variation being almost nonexistent. Indeed, homogenized bids are based on residual variation and have a distribution illustrated by Figure 1.

Asymmetry

The model assumes symmetry, meaning that contractors on the same project draw signals from the same distribution. In some procurement settings, there may be asymmetries in contractors' signals, which can occur if the distance between a firm's headquarters and the project systematically affect project costs (see Somaini (2020) and Li and Zhang (2015)). To explore its role in our data, we experimented with including two measures of asymmetry in our index specification: backlog and distance to project. Both variables have parameter estimates that are small and statistically insignificant, so our symmetry assumption appears to be within reason.

Signal Acquisition Effort

Our analysis treats the signal distribution as fixed conditional on project characteristics. Persico (2000) shows that when a signal's accuracy is tied to an endogenous effort choice, contractors have different incentives to acquire better signals when the auction format changes. Therefore, our counterfactual section will keep the first-price sealed bidding format fixed. If effort (and thus accuracy) is costly, decided at entry, and increases in the expected number of entrants when there is endogenous entry—our counterfactual analysis might be affected. For example, subsidy policies may not expand the number of entrants as much as our model would suggest, and bidders might have less project cost uncertainty conditional on entering. As such, readers should interpret our results with this caveat in mind.

7 Counterfactual Analysis

Now, we use the estimated common value model to conduct counterfactual auction simulations. Our first set of counterfactuals highlight the winner's curse's role in Oklahoma's bridge projects. We then consider the effects of various government interventions, including entry subsidies (or taxes) and entry rights auctions.

7.1 The Role of Project Cost Uncertainty on Entry Incentives

We begin with a firm's incentive to participate in an auction and how project cost uncertainty and the winner's curse shapes that decision. In performing this analysis, we vary the value of σ_e from twice its estimated value to zero. Recall that by removing all project cost uncertainty, the model reduces down to an affiliated private values model with no winner's curse. Therefore, one can interpret results from counterfactuals with $\sigma_e = 0$ as having no winner's curse.

Because bridge projects are heterogeneous, we center this analysis on a single, representative project. We construct this project using the median engineer's estimate together with the median value of the index function and the median number of potential bidders. In the Oklahoma data, our representative project has an engineer's estimate of $\$216,237.85$ and a total of six potential entrants.

We report two outcomes relevant to participation: the probability of entry occurring in equilibrium and the ex-ante expected profits conditional on entry. In each counterfactual, we simulate the representative bridge project 1,000 times—varying the number of potential entrants, N, and the uncertainty parameter, σ_e, in each batch of simulations.

Table 7: Counterfactual Analysis of Entry Incentives

	Potential Entrants (N)					
	1	2	3	4	5	6
Expected profit ($1000s)						
Baseline	43.737	24.071	17.637	14.519	12.646	11.386
$2 \times \sigma_e$	28.390	17.884	14.037	12.035	10.761	9.868
No cost uncertainty	54.422	27.783	19.557	15.725	13.504	12.042
%Δ over baseline						
$2 \times \sigma_e$	-35.090	-25.702	-20.410	-17.103	-14.904	-13.331
No cost uncertainty	24.430	15.421	10.892	8.308	6.788	5.763
Entry probability						
Baseline	0.990	0.928	0.845	0.770	0.705	0.651
$2 \times \sigma_e$	0.955	0.850	0.755	0.680	0.621	0.573
No cost uncertainty	0.996	0.952	0.878	0.803	0.737	0.681
%Δ over baseline						
$2 \times \sigma_e$	-3.479	-8.421	-10.724	-11.601	-11.943	-12.040
No cost uncertainty	0.602	2.602	3.860	4.324	4.483	4.486

Note: Expected profits and entry probabilities for the representative project with various potential entrants. Representative project has an ECE of $216,237.85. Baseline uses the estimated parameters. No cost uncertainty sets $\sigma_e = 0$. The %Δ symbol refers to percent changes over the baseline.

Table 7 reports findings from three counterfactual scenarios. As we increase σ_e, we find that expected profits and entry probabilities decrease. These findings suggest that cost uncertainty generally leads to a downward distortion in markups at the bidding stage, which is then reflected in expected profits at the entry stage. By contrasting the baseline case with six potential entrants against the one with no cost uncertainty, we find that expected profits would have been 5.8 percent higher without a winner's curse. Thus, the winner's curse disincentivizes participation on the representative bridge project.

7.2 The Role of Project Cost Uncertainty on Auction Outcomes

Having established the role of project cost uncertainty and the winner's curse on entry incentives, we now move to an investigation of how these factors affect Oklahoma's bridge auction outcomes, again using the representative project. To separate project cost uncertainty and the winner's curse's effect from that of endogenous entry, we also simulate results under fixed entry, which holds entry fixed at the baseline level.

We record several outcomes in each simulated project. The first two are the bids and low bid submitted by entrants, and the next one is an indicator of whether a potential bidder decides to participate. Our last outcome is the social cost, which is the sum of the project cost of the winning bidder and all entry costs of the entrants when entry is endogenous. When entry is fixed, the social cost is just the project cost of the winner. We note that auctions with a lower social cost are considered more efficient, so this outcome sheds light on auction efficiency.

Some auctions may not attract any entrants, and the social cost of those projects—which may involve changing the project specifications and re-auctioning at a later date—are challenging to estimate. In these cases, we set the social cost to 1.25 times the engineer's cost estimate, an amount that regularly exceeds low bids in Oklahoma.

Table 8: Counterfactual Analysis of the Winner's Curse

	Avg. bid	Avg. low bid	Prop. entering	Avg. social cost
Endogenous entry ($M)				
Baseline	0.243	0.212	0.650	0.169
$2 \times \sigma_e$	0.242	0.214	0.570	0.185
No cost uncertainty	0.241	0.202	0.680	0.163
%Δ over baseline				
$2 \times \sigma_e$	-0.479	0.956	-12.327	9.529
No cost uncertainty	-0.702	-4.449	4.587	-3.490
Fixed entry ($M)				
Baseline	0.243	0.212	0.650	0.144
$2 \times \sigma_e$	0.243	0.211	0.650	0.163
No cost uncertainty	0.242	0.205	0.650	0.138
%Δ over baseline				
$2 \times \sigma_e$	-0.154	-0.263	0.000	13.389
No cost uncertainty	-0.325	-3.225	0.000	-4.481

Note: Counterfactual uncertainty simulations. Baseline uses the estimated parameters. No cost uncertainty sets $\sigma_e = 0$. The %Δ symbol refers to percent changes over the baseline.

Table 8 reports our results. We find that the winner's curse decreases contractor participation, as contractors have higher ex-ante expected profits when project cost uncertainty is less prevalent. We also find that average bids and average low bids are higher when there is project cost uncertainty relative to the no-uncertainty case, and that result holds irrespective of whether entry is fixed or endogenous. This finding suggests that the bid adjustment contractors make in equilibrium to account for the winner's curse generates higher costs for the government.

Furthermore, observe that average bids decrease for either high or low values of σ_e, regardless of whether entry is endogenous or fixed. This pattern highlights two different effects. On the one hand, there is the winner's curse effect, and on the other hand, there is the extent to which contractors can extract rents from their private information. As σ_e approaches zero, the winner's curse effect vanishes—which may lower bids. As σ_e approaches infinity, information rents approach zero because the signal becomes uninformative. Consequently, markups decrease, and bids may then decrease.

Table 8 also indicates that project cost uncertainty and the winner's curse increases average social costs, meaning that auctions are less efficient. To see why these results occur under fixed entry, we note that the lowest-signal firm will win the auction because bids increase in signals. Without project cost uncertainty, signals correspond to actual project costs, implying that the contractor with the lowest project cost will win and that social costs are minimized. Adding uncertainty can create a situation where a contractor has the lowest signal but not the lowest project cost, leading to an increase in social costs.

Under endogenous entry, social costs also include the entry costs of all participants, and there are two competing effects coming from project cost uncertainty. The first effect is that higher uncertainty reduces the number of contractors who enter and pay the entry cost, decreasing social costs as a result. The second effect is that the reduced participation also decreases the number of project-cost draws in the bidding stage. This reduction together with the possibility that the lowest-cost firm may not even win increases the final social cost. The overall increase with higher uncertainty implies that the second effect dominates. All in all, these results point to a winner's curse in Oklahoma bridge projects that increases bids, reduces entry, and lowers efficiency.

7.3 Entry Taxes and Subsidies

Given contractor cost structure and the winner's curse, there are various entry policies ODOT could have considered implementing on its bridge projects—whether it be to increase participation, lower procurement costs, or increase efficiency. We now pivot to an analysis of these counterfactual entry policies, starting out with a lump-sum subsidy or tax paid upon entry. Because contract heterogeneity can change the impact of any entry policy, we again focus on the representative bridge project here and throughout the remainder of this section.

In simulating this counterfactual, we note that any non-zero subsidy or tax would change the entry cost distribution, making it distinct from our lognormal specification. As a result, we obtain kernel density approximations implied by our interventions and use them as our counterfactual entry cost distributions.[13] In addition to our previous outcomes, we also include the average number of entrants and the average procurement cost, which is the average of the low bid plus any subsidy payments (or tax charges) when there are entrants and assumed to be 1.25 times the engineer's estimate absent entry.[14]

Table 9: Subsidies

	Subsidy (% of ECE)				
	-5	-2.5	0	2.5	5
Avg. low bid ($M)	0.231	0.222	0.212	0.205	0.202
Avg. procurement cost ($M)	0.208	0.207	0.212	0.231	0.259
Avg. social cost ($M)	0.175	0.169	0.169	0.175	0.182
Avg. number of entrants	2.348	3.029	3.876	4.687	5.274
%Δ over no subsidy					
Avg. low bid	8.830	4.840	0.000	-3.155	-4.871
Avg. procurement cost	-2.121	-2.643	0.000	8.700	21.910
Avg. social cost	3.542	0.187	0.000	3.278	7.714
Avg. number of entrants	-39.422	-21.852	0.000	20.924	36.068

Note: Counterfactual subsidy simulations for the representative bridge contract. Representative project has six potential entrants and an ECE of $216,237.85. Negative subsidies correspond to taxes.

Table 9 summarizes our results. We find that Oklahoma could have lowered procurement costs on the representative project by taxing entry at a rate of either 5.0 or 2.5 percent of the engineer's estimate. Subsidies would have resulted in payments to entering firms that would not be offset by having more competition. Relative to the no-intervention case, the tax performs better because the gain in tax revenues outweigh the minor increase in winning bids from having less entrants.

From an efficiency standpoint, no intervention is the most efficient of our simulated policies. This result implies that subsidies would have led to inefficiently high entry in Oklahoma and that taxes would have reduced entry inefficiently. Overall, the counterfactual simulations suggests that, although Oklahoma could have taxed entry to lower procurement costs, taxing or subsidizing entry has limited scope for improving efficiency.

7.4 Entry Rights Auction

Consistent with how Oklahoma auctions its construction projects in practice, we have focused our analysis on an environment with "free entry," meaning any bidder that finds entry profitable

[13]We use the actual entry costs in the calculation of social costs, though.

[14]For the entry subsidies and taxes we consider here, almost all simulated auctions have at least one entrant.

can enter. However, there are situations where governments may want to restrict the number of entrants—particularly if there is a winner's curse. Indeed, if the winner's curse effect is large, a cost-minimizing procurer would want to limit participation to lower costs. As such, we now consider a counterfactual policy in which Oklahoma regulates entry via entry rights auction.[15]

Entry Rights Auction Setup

In an entry rights auction, the government first holds an auction for the right to participate and then auctions the project through first-price sealed bidding. Let $\overline{n}$ denote the number of participants allowed in the secondary (first-price) auction. Viewing a spot in the secondary auction as an object and noting that each potential bidder desires only one spot, one can interpret the initial auction as a multi-unit auction with single-unit demand. Notice that $\overline{n}$ must be lower than the number of potential entrants for there to be competition in the initial round; thus, we assume $1 \leq \overline{n} < N$.

We assume bidders submit first-round bids after observing only their entry cost and that Oklahoma uses a uniform price auction—where the winning bidders pay the highest losing bid. In a uniform price auction with single-unit demand, these bids correspond to the bidders' "values"—namely, the value of competing in a secondary auction against $\overline{n}$ bidders after paying the entry cost. Let $\varphi_{\overline{n}}$ denote the equilibrium bid function in the initial auction when there are $\overline{n}$ spots available. Bids are then characterized by the following equation:

$$\varphi_{\overline{n}}(d) = \int_{\underline{x}}^{\overline{x}} \pi_{\overline{n}}(x) f(x) dx - d,$$

where $\pi_{\overline{n}}(x) = \pi_{\overline{n}}(\beta_{\overline{n}}(x), x)$ is the expected profit in an auction with $\overline{n}$ participants and d is the entry cost.

There are many ways Oklahoma could auction these $\overline{n}$ spots, aside from a uniform price auction. For instance, Oklahoma could use a discriminatory auction where all winners pay their bid. We use the uniform price auction for analytical convenience—although the two methods would lead to the same allocations and be revenue equivalent in our context because of the single-unit demand.[16]

We note that first-round bids can be positive or negative (or zero) depending on d. When the highest losing bid is positive, bidders pay a fee to bid in the second round. A negative highest losing bid would result in a subsidy (or negative fee).

Once selected, the first round's winning bidders pay the entry fee and incur their entry

[15]This exercise complements existing studies on the use of entry rights auctions to regulate entry, including Bhattacharya et al. (2014), Ye (2007), and Sweeting and Bhattacharya (2015).

[16]Indeed, Krishna (2003) shows that any two auctions with the same allocation rule have the same expected payments and are therefore revenue equivalent. Allocations are the same because bids are monotonic in a bidder's private information in both formats, and the government selects the highest $\overline{n}$ bids.

cost to learn their signal, x. Because entry costs and signals are independent, the second-round bid function does not change from its characterization in equation (2). The winning bidder is then the lowest of the $\overline{n}$ second-round bids. Observe that we maintain our earlier assumption that the government acts as a competing bidder so that an auction with $\overline{n} = 1$ will not have unreasonably high bids.

Table 10: Entry Rights Auctions

	Entrants ($\overline{n}$)				
	1	2	3	4	5
Avg. entry fee ($1000s)	6.317	4.840	2.185	-2.179	-11.700
Avg. low bid ($M)	0.269	0.233	0.218	0.209	0.202
Avg. procurement cost ($M)	0.231	0.204	0.205	0.222	0.273
Avg. social cost ($M)	0.192	0.170	0.167	0.170	0.180
%Δ over free entry					
Avg. low bid	26.862	9.971	2.840	-1.686	-4.547
Avg. procurement cost	8.905	-3.799	-3.419	4.392	28.441
Avg. social cost	13.697	0.577	-0.940	0.600	6.688

Note: Counterfactual simulations for the representative bridge contract using an entry rights auction to select entrants. Representative project has six potential entrants and an ECE of $216,237.85.

Counterfactual Results

Table 10 reports average outcomes from 1,000 counterfactual entry rights auction simulations of the representative bridge project, repeated for a range of different restrictions on entry. For every simulation, we compute the entry fee arising in the initial uniform-price auction, the second round's low bid, and the procurement cost—where the procurement cost is the low bid less all paid entry fees. We compare these values against a baseline of free entry in the bottom panel of the table.

We find that procurement costs first decrease and then increase in the number of bidders allowed to participate, even though the low bid decreases in $\overline{n}$. This pattern indicates a trade-off between collecting entry fees and lowering the winning bid. On the one hand, raising the number of second-round bidders lowers the winning bid. On the other hand, it also lowers the highest losing bid in the first round because there are more first-round winners. An entry rights auction with $\overline{n} = 2$ generates the lowest procurement cost and saves money over a baseline of free entry.

An entry rights auction can also be more efficient, although at a higher $\overline{n}$ than the one that minimizes procurement costs. In the representative project, setting $\overline{n} = 3$ both improves efficiency and lowers procurement costs relative to free entry, whereas all other entry restrictions would be less efficient. With an average of 3.9 bidders entering in the baseline, this result implies that constraining entry generates a more efficient auction than having slightly more bidders participate on average through free entry.[17]

Our results are comparable to the ones obtained by Bhattacharya et al. (2014). Both studies find that two bidders will generally minimize procurement costs, although there are a small set of parameter values in Bhattacharya et al. (2014) where three bidders are optimal. Our results differ slightly in magnitude. Relative to their baseline values, we have a more considerable reduction in procurement costs under the cost-minimizing $\overline{n}$ (3.8 percent versus 2.4 percent) but a smaller decrease in social costs under our efficiency maximizing $\overline{n}$ (0.9 percent versus 2.4 percent).[18]

There are many possible explanations for these differences. For procurement costs, there is an additional benefit to reducing the number of entrants in our model from reducing the winner's curse effect. Because this effect increases bids in the number of entrants, reducing entry may generate more savings for the government relative to an auction without such an effect. For social costs, there is a selection effect in Bhattacharya et al. (2014), whereby low-cost firms are more likely to enter. Because an entry rights auction can prevent less efficient firms from entering in their model, there can be more gains in efficiency by controlling entry relative to a model without selection. Despite these differences, both studies show that there is room to reduce procurement costs and increase efficiency by using entry rights auctions.

8 Conclusion

When project costs are uncertain, contractors must construct signal-based estimates of them before bidding. That process creates a winner's curse at the bidding stage and can affect the number of bidders who find participation profitable. In this article, we explore whether and how the winner's curse affects entry and bidding decisions on bridge projects from Oklahoma, using a parameterized auction model.

We find that cost uncertainty—and thus, the winner's curse—plays a significant role in Oklahoma. Our parameter estimates imply that about 20.6 percent of the variance in log signals is due to uncertainty in contractor project costs. We explore these implications further in a counterfactual analysis, using the estimated parameters to simulate auctions without uncertainty. We find that the entrant proportion would increase by 4.6 percent had there been no winner's curse and that the lowered cost uncertainty would cause auctions to be more efficient and less costly for the government. Simulations of various entry policies reveal that Oklahoma could

[17] We also performed a sensitivity analysis by re-simulating the entry rights auction for $\sigma_e = 0$ and $\sigma_e = 2 \times \hat{\sigma}_e$, in case signal accuracy changes. We found that Oklahoma can still reduce procurement costs by allowing only two to three bidders, but efficiency is generally higher when $\sigma_e = 0$ and lower when $\sigma_e = 2 \times \hat{\sigma}_e$.

[18] Bhattacharya et al. (2014) find that the $\overline{n}$ that minimizes procurement costs is usually the same as the one that minimizes social costs. When they differ, the difference in social costs is small.

have lowered its procurement costs through taxing entry and both lowered procurement costs and improved auction efficiency through using entry rights auctions.

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A Model Simulation

To simulate from the model, we must evaluate the optimal bid function and determine contractor entry decisions given a candidate parameter vector. This section outlines how we simulate those choices. For notational convenience, we focus on a single homogenized auction—where $X_i = X_i^0$, $\mathbf{z} = \mathbf{z}^0$, and $\Gamma(\mathbf{z}^0) = 1$. To compute bids in an arbitrary auction t with characteristics $\boldsymbol{z}_t$, we multiply homogenized bids by $\Gamma(\mathbf{z}_t)$.

A.1 Simulating Optimal Bids

As mentioned in section 3.2, the bid function consists of a markup and an expected cost conditional on winning. Because markups depend on expected costs, we begin by deriving an approximation for the expected costs.

A.1.1 Approximating Expected Costs

We approximate expected costs through the algorithm proposed by Hong and Shum (2002), which we describe here for completeness. Suppose n contractors enter an auction. We aim to approximate a contractor's expected cost conditional on winning: $c_n(x, x) = E[C_i \mid X_i = x, Y_1 = x]$.

Let $\mathbf{X} = (X_1, X_2, \ldots, X_n)$ be the vector of all cost signals and $\mathbf{x} = (x_1, x_2, \ldots, x_n)$ be their realizations. By applying the law of iterated expectations, we can approximate $c_n(x, x)$ by taking the expectation of $E[C_i \mid \mathbf{X} = \mathbf{x}]$ over a region where the lowest two signals are x and the remaining signals are greater than x.

An Expression for $E[C_i \mid \mathbf{X} = \mathbf{x}]$

Our first task is then to derive an expression for $E[C_i \mid \mathbf{X} = \mathbf{x}]$. Because log-signals are normally distributed, much of our derivations will be in logs. If tildes denote a variable's log value, then we can express expected costs in terms of their logged components using the formula

$$E[C_i \mid \mathbf{X} = \mathbf{x}] = \exp\Big(E[\widetilde{C}_i \mid \widetilde{\mathbf{X}} = \tilde{\mathbf{x}}] + \frac{1}{2}\text{Var}[\widetilde{C}_i \mid \widetilde{\mathbf{X}} = \tilde{\mathbf{x}}]\Big).$$

We now need to find expressions for $E[\widetilde{C}_i \mid \widetilde{\mathbf{X}} = \tilde{\mathbf{x}}]$ and $\text{Var}[\widetilde{C}_i \mid \widetilde{\mathbf{X}} = \tilde{\mathbf{x}}]$. Let $\boldsymbol{\mu}$ be a column vector of length n, with each entry being μ. The joint distribution of contractor i's cost and all signals is then

$$\begin{bmatrix} \widetilde{C}_i \\ \widetilde{\mathbf{X}} \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \mu \\ \boldsymbol{\mu} \end{bmatrix}, \begin{bmatrix} \sigma_c^2 & \boldsymbol{\Sigma_{cx}} \\ \boldsymbol{\Sigma_{xc}} & \boldsymbol{\Sigma_{xx}} \end{bmatrix}\right),$$

where $\sigma_c^2 = \sigma_a^2 + \sigma_v^2$, $\boldsymbol{\Sigma_{xx}}$ is the covariance matrix of $\widetilde{\mathbf{X}}$, and $\boldsymbol{\Sigma_{cx}}$ is the covariance matrix of $\widetilde{\mathbf{X}}$ and $\widetilde{C}_i$ with transpose $\boldsymbol{\Sigma_{xc}}$. [19]

The equations for the conditional mean and variance are then

$$\begin{aligned} E[\widetilde{C}_i \mid \widetilde{\mathbf{X}} = \tilde{\mathbf{x}}] &= \mu + \boldsymbol{\Sigma_{xc}}\boldsymbol{\Sigma_{xx}^{-1}}(\tilde{\mathbf{x}} - \boldsymbol{\mu}) \\ \mathrm{Var}[\widetilde{C}_i \mid \widetilde{\mathbf{X}} = \tilde{\mathbf{x}}] &= \sigma_c^2 - \boldsymbol{\Sigma_{xc}}\boldsymbol{\Sigma_{xx}^{-1}}\boldsymbol{\Sigma_{cx}}. \end{aligned}$$

Taking Expectations of $E[C_i \mid \mathbf{X} = \mathbf{x}]$

We now need to draw signals consistent with contractor i winning the contract on the margin and derive the corresponding probability weights. Because costs are symmetric, we focus on contractor 1 and have contractor 2 draw the lowest competing signal without loss of generality.

Let $\widetilde{\boldsymbol{X}}_{\boldsymbol{i}} = (\widetilde{X}_1, \widetilde{X}_2, \ldots, \widetilde{X}_i)$ and $\widetilde{\boldsymbol{X}}_{\boldsymbol{-i}} = (\widetilde{X}_{i+1}, \widetilde{X}_{i+2}, \ldots, \widetilde{X}_n)$. If the lowest log-winning signal is $\tilde{x}$, we need draws from $\widetilde{\boldsymbol{X}}_{\boldsymbol{-2}} \mid \widetilde{\boldsymbol{X}}_{\boldsymbol{2}}$, such that $\tilde{x} \leq \widetilde{\boldsymbol{X}}_{\boldsymbol{-2}}$ and $\widetilde{\boldsymbol{X}}_{\boldsymbol{2}} = \tilde{x}$.

Let $\boldsymbol{\mu_{-i}}$ and $\boldsymbol{\mu_i}$ be column vectors of μ with lengths $n - i$ and i, respectively. To find $\widetilde{\boldsymbol{X}}_{\boldsymbol{-2}} \mid \widetilde{\boldsymbol{X}}_{\boldsymbol{2}}$, note that

$$\begin{bmatrix} \widetilde{\boldsymbol{X}}_{\boldsymbol{-2}} \\ \widetilde{\boldsymbol{X}}_{\boldsymbol{2}} \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} \boldsymbol{\mu_{-2}} \\ \boldsymbol{\mu_2} \end{bmatrix}, \begin{bmatrix} \boldsymbol{\Sigma_{-2}} & \boldsymbol{\Sigma_{-2,2}} \\ \boldsymbol{\Sigma_{2,-2}} & \boldsymbol{\Sigma_2} \end{bmatrix} \right),$$

where $\boldsymbol{\Sigma_{-2,2}}$, $\boldsymbol{\Sigma_{2,-2}}$, $\boldsymbol{\Sigma_2}$, and $\boldsymbol{\Sigma_{-2}}$ are the covariance matrices.

Let $\tilde{\boldsymbol{x}}_{\boldsymbol{i}}$ be a column vector of length i with elements $\tilde{x}$. Applying the formula for a conditional normal distribution, we find that $\widetilde{\boldsymbol{X}}_{\boldsymbol{-2}} \mid \widetilde{\boldsymbol{X}}_{\boldsymbol{2}}$ is normal with the following mean and variance:

$$\begin{aligned} E[\widetilde{\boldsymbol{X}}_{\boldsymbol{-2}} \mid \widetilde{\boldsymbol{X}}_{\boldsymbol{2}} = \tilde{\boldsymbol{x}}_{\boldsymbol{2}}] &= \boldsymbol{\mu_{-2}} + \boldsymbol{\Sigma_{-2,2}}\boldsymbol{\Sigma_2^{-1}}(\tilde{\boldsymbol{x}}_{\boldsymbol{2}} - \boldsymbol{\mu_2}) \\ \mathrm{Var}[\widetilde{\boldsymbol{X}}_{\boldsymbol{-2}} \mid \widetilde{\boldsymbol{X}}_{\boldsymbol{2}} = \tilde{\boldsymbol{x}}_{\boldsymbol{2}}] &= \boldsymbol{\Sigma_{-2}} - \boldsymbol{\Sigma_{-2,2}}\boldsymbol{\Sigma_2^{-1}}\boldsymbol{\Sigma_{2,-2}}. \end{aligned}$$

We use the GHK algorithm from Hajivassiliou and Ruud (1994) to draw signals from this distribution given the truncation $\tilde{x} \leq \widetilde{\boldsymbol{X}}_{\boldsymbol{-2}}$. This algorithm importance samples from the truncated region, generating sampling weights for each draw. We then use these weights as probabilities in a Monte Carlo integral to compute our conditional expected costs. In practice, our Monte Carlo integral uses 50 simulations.

[19] Given the signal and cost structure, the ith element of $\boldsymbol{\Sigma_{cx}}$ is σ_c^2 and the remaining elements are σ_v^2 because the covariance between $\widetilde{C}_i$ and $\widetilde{X}_j$ is σ_v^2 when $i \neq j$. For $\boldsymbol{\Sigma_{xx}}$, its diagonal elements are $\sigma_x^2 = \sigma_a^2 + \sigma_v^2 + \sigma_e^2$, and its off diagonal elements are σ_v^2.

A.1.2 Markup Computation

With our expected cost approximation completed, we now turn to the markup. The theoretical markup in the bid function is

$$\int_{x}^{\overline{x}} \left\{ \exp\left[-\int_{x}^{y} \frac{g_n(t \mid t)}{(1 - G_n(t \mid t))} dt \right] \right\} c_n'(y, y) dy. \tag{5}$$

To calculate this expression, we need the derivative of the expected cost function, $c_n'(x, x)$, and the distribution and density of the first order statistic given signal x—respectively, $G_n(y_1 \mid x)$ and $g_n(y_1 \mid x)$.

In computing the derivative, we solve $c_n(x, x)$ on a grid of 100 signal values and interpolate using Hermite splines. We use the derivative of this spline as our value for $c_n'(x, x)$.

To derive the distribution and density functions we note that $\widetilde{\mathbf{X}}_{-\mathbf{1}} \mid \widetilde{\mathbf{X}}_{\mathbf{1}} \sim \mathcal{N}(\overline{\boldsymbol{\mu}}, \overline{\boldsymbol{\Sigma}})$, where

$$\begin{aligned} \overline{\boldsymbol{\mu}} &= \boldsymbol{\mu}_{-\mathbf{1}} + \boldsymbol{\Sigma}_{-\mathbf{1},\mathbf{1}} \boldsymbol{\Sigma}_{\mathbf{1}}^{-\mathbf{1}} (\tilde{\boldsymbol{x}}_{\mathbf{1}} - \boldsymbol{\mu}_{\mathbf{1}}) \\ \overline{\boldsymbol{\Sigma}} &= \boldsymbol{\Sigma}_{-\mathbf{1}} - \boldsymbol{\Sigma}_{-\mathbf{1},\mathbf{1}} \boldsymbol{\Sigma}_{\mathbf{1}}^{-\mathbf{1}} \boldsymbol{\Sigma}_{\mathbf{1},-\mathbf{1}}. \end{aligned}$$

We standardize each $\widetilde{X}_i \in \widetilde{\mathbf{X}}_{-\mathbf{1}}$ using the transformation

$$\bar{\widetilde{X}}_i = \frac{\widetilde{X}_i - \overline{\mu}}{\overline{\sigma}},$$

where $\overline{\mu}$ is an element of $\overline{\boldsymbol{\mu}}$ and $\overline{\sigma}$ is the square root of a diagonal element in the matrix $\overline{\boldsymbol{\Sigma}}$.

Given our information structure, each standardized signal is equally correlated, in the sense that the off diagonal elements of $\overline{\boldsymbol{\Sigma}}$ are the same. For $i \neq j$, let $\rho = \mathrm{Cov}(\bar{\widetilde{X}}_i, \bar{\widetilde{X}}_j) = \mathrm{Cov}(\widetilde{X}_i, \widetilde{X}_j)/\overline{\sigma}^2$. According to Tong (1990), the distribution and density of the first order statistic for equally correlated and standardized normal random variables are

$$\begin{aligned} G_{\bar{\tilde{x}}}(\bar{\tilde{x}}) &= \int_{-\infty}^{\infty} F_{(1)}\left(\frac{\bar{\tilde{x}} + \sqrt{\rho} z}{\sqrt{1-\rho}}\right) \phi(z) dz \\ g_{\bar{\tilde{x}}}(\bar{\tilde{x}}) &= \int_{-\infty}^{\infty} \frac{1}{\sqrt{1-\rho}} f_{(1)}\left(\frac{\bar{\tilde{x}} + \sqrt{\rho} z}{\sqrt{1-\rho}}\right) \phi(z) dz, \end{aligned}$$

where

$$\begin{aligned} F_{(1)}(y) &= \sum_{j=1}^{n} \binom{n}{j} [\Phi(y)]^j [\Phi(-y)]^{n-j} \\ &= 1 - [\Phi(-y)]^n \\ f_{(1)}(y) &= n\Phi(-y)^{n-1} \phi(y). \end{aligned}$$

Given this distribution and our approximation for $c_n'(x, x)$, we solve for the markup in equation (5) via numerical integration. In estimation, we bound the highest signal, $\overline{x}$, below infinity to keep our algorithm numerically stable.

A.2 Simulating Entry

Our markup calculation enables us to simulate equilibrium entry. Because we allow shifts in entry costs due to characteristics in $\mathbf{w}_t$, equilibrium entry beliefs and expected profits will depend on $\mathbf{w}_t$, even in a homogenized auction.

Let $p_N(\mathbf{w}_t)$ be the equilibrium entry beliefs for a homogenized auction with N plan holders and entry-cost shifters $\mathbf{w}_t$; then, $p_N(\mathbf{w}_t)$ solves

$$p_N(\mathbf{w}_t) = H(\overline{\pi}_N(p_N(\mathbf{w}_t)) \mid \mathbf{w}_t),$$

where $\overline{\pi}_N(p_N(\mathbf{w}_t))$ are the equilibrium expected profits given N plan holders in the homogenized auction. A firm will enter if $d^0_{it} \leq \overline{\pi}_N(p_N(\mathbf{w}_t))$.

B A Test for Common Values

This section tests for common values conditional on the number of bidders semiparametrically by adapting the Haile et al. (2003) framework to a procurement auction.

B.1 The Setup

Like Haile et al. (2003), our test involves flexibly estimating expected cost distributions given n with observed bids and then comparing the resultant distribution estimates. To that end, let $\breve{G}_n(y_1 \mid b)$ be the distribution function for the lowest competing equilibrium bid, with corresponding density $\breve{g}_n(y_1 \mid b)$.

Because bids strictly increase in signals, the following relationships must hold:

$$\begin{aligned} G_n(y_1 \mid x) &= \breve{G}_n(\beta_n(y_1) \mid \beta_n(x)) \\ g_n(y_1 \mid x) &= \breve{g}_n(\beta_n(y_1) \mid \beta_n(x))\beta_n'(y_1). \end{aligned} \tag{6}$$

By noting that $b_i = \beta_n(x_i)$ in equilibrium, one can insert the expressions in (6) into the first-order conditions in (1) to get

$$c_n(x_i, x_i) = b_i - \frac{1 - \breve{G}_n(b_i \mid b_i)}{\breve{g}_n(b_i \mid b_i)} := \Upsilon_n(b_i), \tag{7}$$

where $\Upsilon_n(b_i)$ is a contractor's expected cost expressed as a function of their bid.

Viewing expected costs as a random variable, define $F_{c,n}(c)$ as the marginal expected cost distribution given n. In a private-value setting, expected costs should be independent of n, whereas the winner's curse effect produces expected costs that increase in n in a common-value model. Statistically, this intuition means that,

$$F_{c,n}(c) = F_{c,n+1}(c) \quad \forall c, n \tag{8}$$

under private values, and

$$F_{c,n}(c) > F_{c,n+1}(c) \quad \forall c, n \tag{9}$$

for common values. We use these relationships as a basis for our analysis, testing the null of equality in (8) against the alternative of stochastic dominance in (9).

B.2 The Copula Representation

The expected cost formulation for $\Upsilon_n(b_i)$ requires estimates of $\breve{G}_n(b_i \mid b_i)$ and $\breve{g}_n(b_i \mid b_i)$. Because our data lack the observations necessary to reliably estimate these objects nonparametrically, we take a semiparametric, copula-based approach using homogenized bids.[20]

Under those considerations, let $F^0_{b,n}(b)$ be the marginal homogenized bid distribution when there are n bidders. We assume the joint homogenized bid distribution can be expressed by a copula function $\mathcal{C}$ so that

$$\Pr(B^0_1 < b_1, B^0_2 < b_2, \ldots, B^0_n < b_n) = \mathcal{C}[F^0_{b,n}(b_1), F^0_{b,n}(b_2), \ldots, F^0_{b,n}(b_n)].$$

Copulas are convenient given our data because they separate the dependence in bids from their marginal distributions, allowing us to estimate them separately.

Hubbard et al. (2012) derive expressions for the numerator and denominator in the fraction of equation (7) in terms of the survival copula, $\mathcal{S}$, where

$$\mathcal{S}[1 - F^0_{b,n}(b_1), 1 - F^0_{b,n}(b_2), \ldots, 1 - F^0_{b,n}(b_n)] = \Pr(B^0_1 > b_1, B^0_2 > b_2, \ldots, B^0_n > b_n).$$

Let $\breve{G}^0_n(b_i \mid b_i)$ denote the distribution function for the lowest competing homogenized bid, with density $\breve{g}^0_n(b_i \mid b_i)$. Focusing on bidder 1 without loss of generality, Hubbard et al. (2012) show that

$$\begin{aligned} 1 - \breve{G}^0_n(b_1 \mid b_1) &= \mathcal{S}_1[1 - F^0_{b,n}(b_1), 1 - F^0_{b,n}(b_1), \ldots, 1 - F^0_{b,n}(b_1)] \\ \breve{g}^0_n(b_1 \mid b_1) &= (n-1) f^0_{b,n}(b_1) \mathcal{S}_{12}[1 - F^0_{b,n}(b_1), 1 - F^0_{b,n}(b_1), \ldots, 1 - F^0_{b,n}(b_1)], \end{aligned}$$

[20] Recall that multiplying homogenized bids by the index function yields bids. Because bids are the sum of an expected cost and markup, multiplying homogenized expected costs by the index function yields expected costs.

where

$$\begin{aligned}\mathcal{S}[1-F^0_{b,n}(b_1), 1-F^0_{b,n}(b_2), \ldots, 1-F^0_{b,n}(b_n)] &= 1-\sum_{i=1}^{n}\mathcal{C}[F^0_{b,n}(b_i)] + \sum_{1\leq i\leq j\leq n-1}\mathcal{C}[F^0_{b,n}(b_i), F^0_{b,n}(b_j)] \\ &\quad - \cdots + (-1)^n\mathcal{C}[F^0_{b,n}(b_1), F^0_{b,n}(b_2), \ldots, F^0_{b,n}(b_n)],\end{aligned}$$

$\mathcal{S}_1$ is the partial derivative of $\mathcal{S}$ with respect to its first argument and $\mathcal{S}_{12}$ is the cross partial of $\mathcal{S}$ with respect to its first and second arguments.

B.3 Estimation

Estimating homogenized expected costs then reduces down to estimating the marginal homogenized bid distributions and the dependence implied by the copula function. We assume a Clayton copula, which has the form

$$\mathcal{C}[F^0_{b,n}(b_1), F^0_{b,n}(b_2), \ldots, F^0_{b,n}(b_n)] = \left(\sum_{i=1}^{n} F^0_{b,n}(b_i)^{-\theta} - n + 1\right)^{-\frac{1}{\theta}},$$

where $\theta \in [-1, \infty) \setminus \{0\}$ is the dependence parameter. Here, a positive θ value indicates that homogenized bids are affiliated, whereas homogenized bids approach independence as θ tends to zero.

After homogenizing the bids, we use a kernel density estimator with a normal kernel to estimate the marginal homogenized bid distributions for each n. Then, we estimate the dependence parameter, θ, via minimum distance. For this step, we note that Kendall's tau, a measure of dependence that we denote by τ, has the form $\tau = \frac{\theta}{\theta+2}$ for a Clayton copula. Thus, our θ estimate minimizes the distance between the sample pairwise Kendall's tau for homogenized bids and $\frac{\theta}{\theta+2}$.

Our estimation procedure produces a dependence parameter of $\hat{\theta} = 0.573$, with a standard error of 0.271. We then use equation (7) to obtain our homogenized expected cost estimates while maintaining our assumption that the government acts as an additional bidder. We finish by estimating a kernel density function for the homogenized expected costs, again using a normal kernel.

B.4 Testing

In testing, we consider homogenized expected costs in auctions with a "low" ($n \leq 3$), "medium" ($n \in \{4, 5\}$), and "high" ($n \in \{6, 7\}$) number of bidders. Including auctions with more than 7 bidders in the high n group does not change the statistical significance of our findings, but many of those auctions are limited in observations.

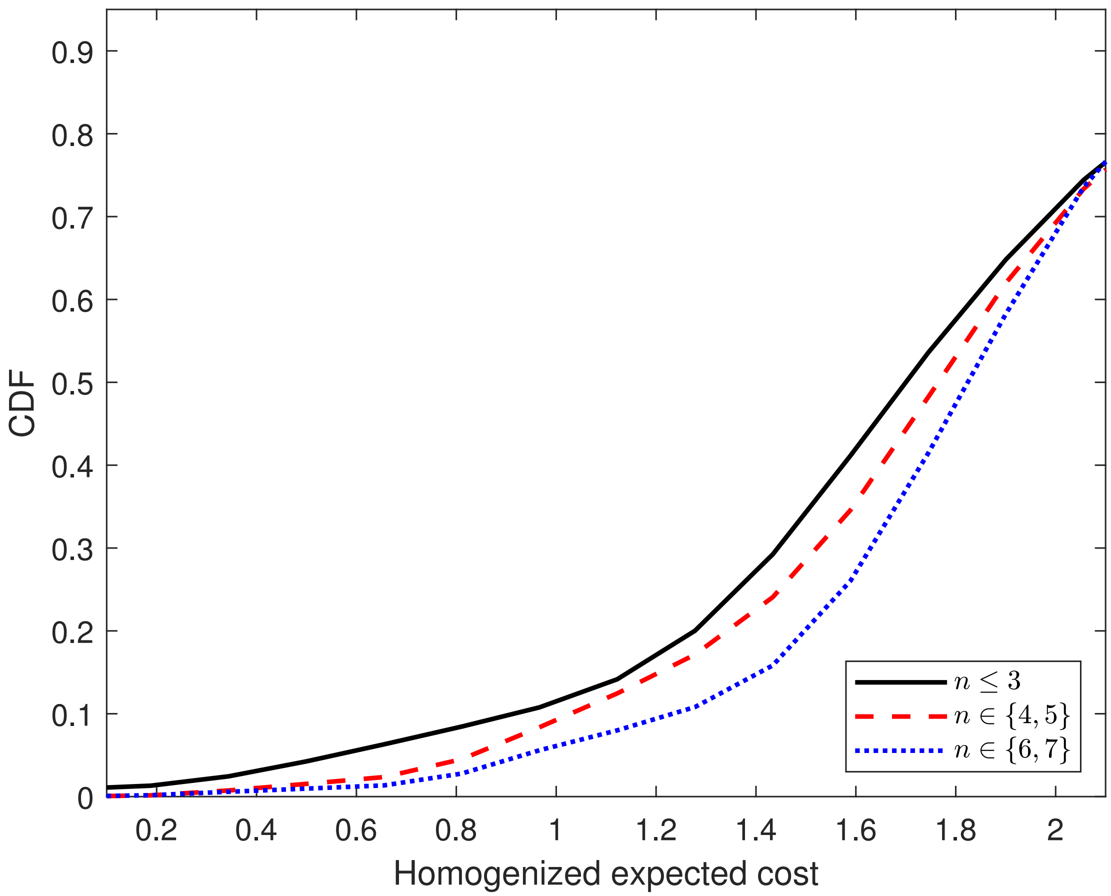

Figure 3: Estimated homogenized expected cost CDFs

We first plot the distribution functions for each group, which we display in Figure 3. Visually, homogenized expected costs appear to align with common values, as lower n distributions tend to be above higher n ones.

Table 11: Test for Common Values

	p-value
$n \leq 3$ vs $n \in \{4,5\}$	0.011
$n \in \{4,5\}$ vs $n \in \{6,7\}$	0.009

Note: Table reports p-values for a one-sided K-S test for the homogenized expected cost CDFs.

We test this pattern formally through a one-sided two-sample Kolmogorov-Smirnov test; Table 11 contains the resulting p-values. We find that the visual patterns hold statistically because we reject the equality of the various expected cost distributions. These results provide further motivation for our common-value model.