Non-Hermitian Photonics: Guest Editorial

Liang Feng[1], Li Ge[2,3], Ming-Hui Lu[4], Henning Schomerus[5]
[1]*Department of Materials Science and Engineering, University of Pennsylvania, Philadelphia, PA 19104, USA*
[2]*Department of Physics and Astronomy, College of Staten Island, CUNY, Staten Island, New York 10314, USA*
[3]*The Graduate Center, CUNY, New York, New York 10016, USA*
[4]*National Laboratory of Solid State Microstructures and Department of Materials Science and Engineering, Nanjing University, Nanjing, 210093, China*
[5]*Department of Physics, Lancaster University, Lancaster, LA1 4YB, United Kingdom*

Based on their expansive flexibility of controlling the propagation, coupling, and confinement of light, photonic systems are a prime physical platform to realize the full potential of devices whose functionality is enhanced by symmetry and topology. Most notably, the capability to manipulate photonic eigenstates through optical gain and loss and non-reciprocal couplings provides a powerful toolbox for tailoring non-Hermitian Hamiltonians[1,2] to study symmetry paradigms that go beyond conventional condensed matter systems—marking the birth of non-Hermitian photonics.[3] Rapid developments of these concepts over the past decade have already empowered us to achieve complex optical responses and unique light-matter interactions, in both classical and quantum domains. Despite the similarities shared with electron-based quantum systems, non-Hermitian photonics is rooted in the fundamentals of electromagnetics and governed by Bosonic statistics. The direct inclusion of optical non-Hermiticity in first-principle electromagnetic designs leads to new types of light-matter interaction and enriches topological physics that are beyond the fermionic symmetry paradigms, leading to a multitude of phenomena that have no counterpart in condensed matter and passive photonic systems. This special issue covers the most exciting developments in the field that enhance our fundamental understanding of quantum and optical physics and facilitate technological breakthroughs for photonic applications.

Below we briefly introduce the articles included in this Special Topic, categories by their main focuses.

1. Non-Hermitian properties related to exceptional points

The field of non-Hermitian photonics emerged from the first experimental demonstration of parity-time (PT) symmetry in a physical system, revealing phase transition from PT symmetry to PT breaking with the phase transition point, also known as an exceptional point (EP) where multiple eigenstates coalesce in a non-Hermitian system. However, the creation of EP is not confined to PT symmetry, which carries even richer non-Hermitian physics. Meng et al. review recent progress in generating and utilizing EPs on a variety of platforms.[4] This is based on advances in designing photonic systems such as waveguides, photonic crystals, and Fabry–Pérot resonators, and is supplemented by analogous plasmonic systems. Five different aspects are identified to set operations at EPs apart from other scenarios. Looking ahead to further developments in these fields, they identify nonlinearities as an additional handle to induce functionality into these systems.

From a practical perspective, it is necessary to develop a convenient means to design, observe, and characterize EPs. Shadrina et al. study a grating composed of infinitely long silicon rods and found the existence of EPs.[5] Their parameter space consists of the scales of the rectangular cross-section of these rods, their spacings, and the wave vector along the rods. They identify their EPs using the bifurcation of the transmission spectrum, which is possible thanks to the small imaginary parts of these EPs. They complete their study by presenting a generic coupled mode theory to elucidate this effect, and they show that structural fluctuations of the grating do not destroy the EPs but make their observations more challenging, due to the inhomogeneous broadening of the transmission peaks.

To address similar challenges in observing and characterizing EPs, Nyuyen et al. report a technique to observe an EP in photonic crystals using a single-shot measurement of the reflective spectrum.[6] Their approach utilizes cross-polarization to effectively suppress the background signal, which allows them to directly observe EPs in momentum-resolved resonant scattering. Their experimental findings match numerical simulations and analytical models, validating their approach.

Moreover, the intriguing physics associated with EPs shows promising photonic applications. Encircling an EP can be realized in dual-coupled waveguide systems where parameter tuning circles around the EP in the parameter space. Wang et al. introduce subwavelength gratings (SWGs) in the system to minimize crosstalk and broaden operation bandwidth by adjusting the refractive index, reducing mode mismatch, and weakening wavelength dependence on the EP path.[7] This advancement supports efficient chiral transmission devices in photonic integrated circuits, opening avenues in optical communication and topological quantum computing. To achieve enhanced sensing by EPs, Jiang et al. further expand the concept of an exceptional surface (ES) using a fiber ring cavity and exploit it to demonstrate enhanced displacement sensing.[8] This enhanced sensitivity at an EP is both a nice feature and a challenge, making the preparation of a system at an EP nontrivial. An ES is a surface in a two- or higher-dimensional parameter space, where each point on it is an EP. If the sensing signal comes from a dimension perpendicular to this surface, then the ES provides the same sensitivity as an EP on it, and it also provides some protection against errors produced in the fabrication and setup. While this scheme has been demonstrated, the authors provide a different platform to realize it, and their ES is in the two-dimensional parameter space of the resonant frequency of the fiber ring cavity and its decay rate, which are independent of the conditions required by the EP.

Makris investigates the phenomenon of transient amplification in non-Hermitian photonic systems.[9] The transient amplification arises from mode non-orthogonality and occurs even if all states in the system ultimately decay. The author considers how this effect can be characterized and maximized in waveguide lattices with a few embedded gain channels. The conclusion is that the maximal gain is achieved when the gain channels are placed at the edges. This is backed up by an analysis of the pseudospectrum, which delivers upper and lower bounds on the power amplification factor, and of the singular value spectrum, which delivers detailed quantitative insights for further experimental verifications.

We further note that EPs exist not only in an effective Hamiltonian but also in other non-Hermitian operators, such as a scattering matrix. Loran and Mostafazadeh utilize non-Hermitian concepts to design photonic structures with broad-band directional invisibility in two and three dimensions.[10] This significantly extends the understanding of this effect from earlier findings, which were confined to one dimension. The authors surmount the challenges of inverse scattering theory by identifying settings in which the Born approximation is exact and apply their method both to scalar and electromagnetic waves. Their findings can be directly transferred to conditions for the permittivity and permeability tensors. Note that in all these non-Hermitian designs the sophisticated control of gain-loss interplay to maximize optical gain is the key.

2. Coupled optical elements and modes in non-Hermitian systems

A large number of experimental demonstrations of non-Hermitian photonics to date have focused on the coupling of individual resonators on an active semiconductor platform. In this scenario, it is critical to both optimize the properties and individual resonators in the linear regime and understand the effects of gain dynamics from active semiconductors in the nonlinear regime. Rodemund et al. describe how to enhance non-Hermitian properties of coupled dielectric microdisks by combining shape asymmetry and coupling orientation.[11] This configuration allows them to induce mode-dependent chirality of the coupled cavity modes, which leaves distinct signatures in the far field. These features can be understood semiclassically, which the authors explain by adopting a phase space representation. Furthermore, they condense the key mechanisms into an efficient coupled-mode description. This work adds to the tools to design dielectric systems with favorable properties, such as those desired for sensing and laser applications.

Matogawa et al. provide an advanced description of non-linear laser dynamics in non-orthogonal chiral modes.[12] This description is based on the Maxwell–Bloch equations, which captures the nonlinear dynamics in the gain medium. This results in a consistent and predictive semiclassical Lamb laser theory of the non-Hermitian chiral modes that is amendable to tools from nonlinear systems theory. In particular, the authors show that the chirality has no effect on the intensity of the clockwise (CW) and counterclockwise (CCW) lasing modes, but impacts the basin of attraction of these modes, to the extent that close to an EP only the basis of attraction of the mode with the preferred chiral survives. The analytical and conceptual results are corroborated numerically by finite-difference time-domain computations.

Xian et al. study the dynamics of breathing solitons near EPs within non-Hermitian systems incorporating gain nonlinearity.[13] In these systems, gain nonlinearity weakens changes in mode intensity, while EPs enhance the localization of the relative phase in probability distribution. Experiments and simulations with breathing soliton lasers reveal that both the breathing frequency and relative phase vary over time near EP, with the relative phase showing localized distribution.

Moreno et al. describe a technique to simplify the study of quantum dynamics of coupled damped oscillators using quantum state diffusion.[14] The latter unravels the Lindblad equation to a stochastic Schrödinger equation, and their technique introduces a non-Hermitian Hamiltonian in this equation, which can be diagonalized in the so-called eigenmode oscillators (EOs) basis away from an EP. As a result, each product state, once put in the EO basis, can be evolved using the diffusion of each EO individually, at the cost that their noises are now correlated. They compare their results to exact numerical solutions of the density matrix propagation for two coupled oscillators, which show evidence that such an approach can be reliably applied to larger systems as well, potentially even in cases where the Markov approximation, and the Lindblad equation, fail.

3. Topological non-Hermitian lattice models

Topological photonics, in addition to non-Hermitian photonics, is another recently emerging field in optics and photonics. While intrinsically different, their synergy leads to unique topologies that are inherent to non-Hermitian crystals. Different non-Hermitian topological lattice models are exemplified. Zhang et al. study a non-Hermitian extension of the breathing Kagome lattice,[15] with the latter displaying a topological transition when the inter-cell coupling becomes stronger than the intra-cell coupling. By including gain and loss strategically in a unit cell, they induce a topological phase by essentially decoupling the corners of the unit cell from the rest, even when the underlying Hermitian system is topologically trivial. Midya formulates the theory of topological braiding in non-Hermitian multiband lattice models.[16] The setting is the large scope of nontrivial braiding that arises from the complex energy spectrum, as well as the phase transitions that occur at EPs. The author demonstrates that nontrivial braiding can be obtained from purely scalar non-Hermiticity, corresponding to on-site gain and loss. This is elaborated for two specific examples, covering the Abelian braid group B_2 and the non-Abelian braid group B_3.

Zelenyanova and Bergholtz analyze one non-Hermitian extension of the 1D Su-Schrieffer-Heeger chain (SSH) and Rice-Mele models.[17] These two models, in the Hermitian case, both feature alternate strong and weak nearest neighbor couplings, with the Rice-Mele model also including a detuning between the two sublattices. The non-Hermitian extension the authors considered is non-reciprocal coupling, first introduced by Hatano and Nelson and studied extensively recently in the context of non-Hermitian skin modes. The imaginary gauge field due to the non-reciprocal coupling is known to delocalize (localize) the edge (bulk) modes in the underlying Hermitian model, and here the authors provide a different perspective using the biorthogonal polarization their group introduced previously.

Additionally, Jiang et al. combine the study of non-Hermitian skin effect and spin-orbital coupling.[18] Different from the previous studies of Z_2 non-Hermitian skin effect where each (pseudo)spin is subject to a different imaginary gauge field, their model takes elements from the SSH model, with alternate reciprocal and non-reciprocal couplings along two "rails" of each quasi-1D model. They calculate the spectrum of their system and characterize its topology using the generalized Brillouin zone. Although the realization of the non-Hermitian skin effect usually requires non-reciprocal couplings, which is challenging to achieve in many photonic systems, Jiang et al. meticulously design twisted optical waveguide arrays, where twisting introduces a geometrical phase in photon coupling, creating artificial gauge fields to achieve the non-Hermitian skin effect.[19] The study allows control over the localization strength and direction of skin modes by adjusting the twist, providing new ways to control light localization and transport in photonic systems.

Luo and Zhang investigate the effect of disordered gain and loss on topological insulators, leading to non-Hermitian versions of topological Anderson insulators.[20] Such systems carry topology in their localized eigenstates. To transfer the description to the non-Hermitian setting, the authors make use of the biorthogonal structure of the eigenmodes. They find a rich phase diagram, in which phase transitions are associated with a diverging biorthogonal localization length, and they identify the biorthogonal chiral displacement as a concrete observable that makes the topological quantum numbers experimentally accessible.

4. Non-Hermitian lattices with other notable properties

Several studies further showcase exotic features emerged from non-Hermitian crystals beyond their Hermitian counterparts. Lin et al. study a non-Hermitian lattice with two tunable flatbands.[21] One previously studied type of non-Hermitian flatband is defined by only requiring the real part of an energy band to be momentum independent. Here the authors used the other definition which is inherited from a Hermitian system, i.e., with an identical energy across the Brillouin zone, including both the real and imaginary parts of the band energy. Such a flatband features wave functions that can be decomposed using compact localized states, and the authors show that their flatbands can be embedded into dispersive bands and merge into a single one at an EP. Zhang et al. design a PT-symmetric Lieb lattice and study its multiple EPs and flatbands.[22] The Lieb lattice is a quasi-1D lattice where every other site is decorated with a single site on its side. It is one of the first few lattice models that were found to display a flatband, and it was also the first such lattice where PT-symmetry was introduced to study non-Hermitian behaviors. In this work, the authors report EPs in multiple bands and higher-order EPs and analyze their robustness against diagonal and off-diagonal disorders.

König et al. match different space groups in a non-Hermitian setting by extending the classification of non-Hermitian crystals to include the 17 wallpaper space groups in two dimensions.[23] They base their classification directly on the momentum space representation, which gives a clear meaning to symmetry operations. This allows them to identify symmetry classes in which nodal lines, EPs and chains are protected by symmetry, which also allows them to provide minimal models that realize these cases.

Other than individual and periodic photonic systems that are typically considered when designing non-Hermitian Hamiltonians, non-Hermitian systems with disorders are also investigated. Longhi addresses the interplay of scalar gain and loss with disorder, which is paramount for the manipulation of wave dynamics of reciprocal photonic systems.[24] By adopting two settings, that of a tight-binding waveguide and that of discrete-time quantum walks, the author describes how spatially distributed gain and loss can be used to control the localization of light. The study introduces concrete models that address the distinction between uncorrelated and incommensurate disorders. The findings amount to universal mechanisms of enhanced and suppressed wave spreading, which the author illustrates concretely in numerical simulations.

5. Diversified non-Hermitian platforms and technological applications

In addition to advancing fundamental studies of non-Hermitian physics, it is equally crucial to explore feasible designs, practical demonstrations, and real-world applications of non-Hermitian photonics, which hold the potential to revolutionize a wide range of technological fields. Gao et al. apply the non-Hermitian engineering to a pixelated metasurface.[25] They tailor the meta-atoms to have them operate at an EP, where the polarization states of the system collapse to a circular polarization state. Specifically, in this study the conversion from right-hand circular polarization to left-hand circular polarization is suppressed. The demonstrated polarization-sensitive metasurface enables accurate and efficient real-time full-Stokes detection and offers potential applications in areas such as polarization imaging and optical computing.

Bardonnet et al. consider the functionalization of Littrow grating structures by gain and loss.[26] These gratings play an important technological role in the design of resonant photonic structures, where they can be utilized to confine light by inducing dispersions exhibiting multiple slow light. Their work fuses technological bottom-up considerations based on the specific characteristics of device materials with the top-down power of emergent non-Hermitian concepts. As the authors show, the addition of gain and loss can improve selective absorption in CMOS pixels at technologically relevant wavelengths. Furthermore, they identify how gain and loss modify the flat bands that induce the targeted slow-light characteristics.

Wang et al. investigate the enhancement of light-matter interactions at the nanoscale through tunable flatband quasi-bound states in the continuum (quasi-BICs) in plasmonic systems.[27] By integrating monolayer graphene with a plasmonic metasurface, modulating the chemical potential of graphene leads to the control of the Q-factor of the quasi-BIC modes and their associated enhanced absorption in a broader spatial frequency range. These findings enable dynamic control over near-field enhancement, advancing applications that use tightly focused light.

Jian et al. propose an anti-parity-time symmetry in a non-Hermitian system, demonstrating a chirality reversal from resonance EP to absorption EP under external excitation. This phenomenon, where external input shapes the system's eigenstates, is analytically studied and experimentally demonstrated in a system with two coupled resonant coils.[28] These findings offer potential applications such as chiral antennas, polarization converters, and wireless communication tools.

Last but not least, non-Hermitian physics extends far beyond the realm of conventional optical and electromagnetic systems, offering a versatile framework that can be applied to a wide variety of systems and enabling new possibilities for innovation across diverse fields. Niu et al. report achieving anti-PT symmetry with four-wave mixing in hot Rubidium vapor.[29] It is an extension of the previous work in cold atoms, where two photons in a strong pump generate a pair of "probe" and Stokes photons with different frequencies. Just like in PT symmetric systems, anti-PT symmetry warrants two different regimes or "phases" across an EP. Here they manifest themselves as either the power oscillation between the probe and Stokes fields as they propagate in space or their simultaneous exponential growth. Such a behavior can be used to define a squeezing parameter defined by the relative variance of photon numbers in these two fields, and the authors show that this squeezing parameter displays a non-monotonic behavior in the exponentially growing regime, due to optical loss and imperfect detection efficiency.

Kim et al. extend the possibility of realizing rich non-Hermitian physics and EP to ion-cavity systems.[30] With the ions driven by a pump field, a three-level non-Hermitian Hamiltonian can be modeled, identifying a third-order EP at a specific balance of the pump laser's Rabi frequency and the atom-cavity coupling constant with the system's loss rates. The study also outlines feasible parameters for future experimental investigations. More naturally, with sound sharing myriad similar wave properties with light, the demonstrated non-Hermitian principles are now offering new concepts in acoustics, yielding advanced control over sound propagation.

Yue et al. propose a design for realizing an acoustic second-order nodal-loop semimetal by leveraging spinless time-reversal symmetry and tunable gauge fields, achieved by incorporating projective translation symmetry within a three-dimensional stacked acoustic graphitic lattice.[31] Here, non-Hermitian modulation is applied to control the topologically protected propagation of degenerate drumhead surface and hinge states, influenced by specific on-site gain and loss patterns. This method can be also extended to other classical wave systems, paving the way for functional topological devices operating outside optical and acoustical domains.

We hope that the twenty-eight articles mentioned above provide a glimpse into the fast-evolving and expanding scopes of non-Hermitian photonics. We thank all the authors who have contributed to this Special Topic, as well as the journal editors and staff who helped put this impressive collection together.

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