Vibrating Grid as a Tool for Studying the Flow of Pure He II and its Transition to Turbulence

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Abstract. We report a detailed experimental study of the flow of isotopically-pure He II, generated by a vibrating grid. Our measurements span a wide range of temperatures ($50\,\text{mK} \leq T \leq 1.37\,\text{K}$) and pressures ($2\,\text{bar} \leq p \leq 15\,\text{bar}$). The response of the grid was found to be of a Lorentzian form up to a sharply-defined threshold value. This threshold value does not change appreciably with pressure; the form of the resonant response of the grid is qualitatively the same for all temperatures while the threshold value is a monotonically increasing function of temperature. We discuss the measured variation of the resonant frequency of the grid as a function of applied pressure (density) of He II and relate this to a hydrodynamic effective mass of the grid. These measurements extend our previously reported studies [Nichol *et al*, *Phys. Rev. E* **70**, 056307 (2004)] and form an integral part of a series of experiments aimed at providing a better understanding of classical and quantum turbulence.

The study of turbulence in quantum fluids [1] has attracted considerable attention, both from experimental and theoretical physicists. It has already been established that, although quantum turbulence (QT) involves a tangle of quantized vortex lines, the macroscopic flow of He II exhibits classical features such as the existence of an energy decay spectrum consistent with that proposed by Kolmogorov for turbulence in classical fluids [2].

We have recently demonstrated that a vibrating grid in He II can be used to study the transition to turbulence, in the zero temperature limit [3]. Here we present further measurements on the flow of He II induced by a vibrating grid (using a different grid but of the same specifications as that used in [3]), for a range of temperatures and pressures. Details of the experimental setup and procedure used can be found in [3].

The response of the grid was first studied in vacuum in order to establish that a constant temperature had been reached, as well as to compare the resonance characteristics with those obtained with the grid immersed in isotopically-pure He II. The resonance characteristic in vacuum was found to be of a Lorentzian shape, centered at a frequency of $1036.9(1)\,\text{Hz}$ and exhibiting a quality factor of $Q \sim 4000$ [4].

The resonance characteristics of the grid were also found to be Lorentzian in He II for low drive amplitude, but with the resonant frequency reduced by approximately $40\,\text{Hz}$—more about this below. When the response of the grid exceeds a pressure-independent threshold, however, the resonance curves broaden markedly, signaling the onset of nonlinearity and addi-
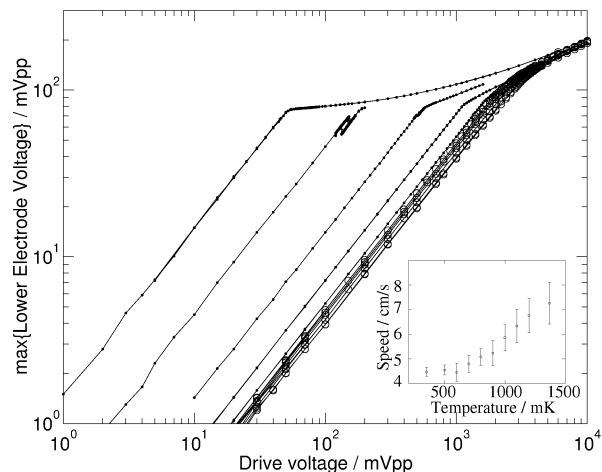

FIGURE 1. The variation of the response of the grid at resonance as a function of the driving voltage, for a range of temperatures at a pressure of 15 bar. Shown in the inset is the resonant response at the onset of dissipation, extracted from the data plotted in the main graph.

tional dissipation in the system. Shown in Fig. 1 is the variation of the response of the grid at resonance as a function of the applied driving voltage, for temperatures ranging from $350\,\text{mK}$ to $1370\,\text{mK}$. All the data were obtained at a pressure of 15 bar. It can readily be seen that, for low driving voltages, the response of the grid is proportional to the driving force.

In relation to the nonlinear regime, recall that the drag coefficient C_D is defined as [5]

$$C_D = \frac{F}{\frac{1}{2}\rho u^2 A},\qquad(1)$$

where F is the magnitude of the drag force on the grid, A is the area of the grid, ρ the density of He II and u is the grid speed which can be calculated [3] from the amplitude of the signal picked up on the lower electrode. It is also known that for steady turbulent flow of a classical fluid at very large Reynolds number, the drag coefficient is roughly constant and hence the grid speed should be proportional to the square root of the drag force. The data plotted in Fig. 1 indicate that this might be the case in the present system, but we are unable to provide more conclusive evidence as this would require the application of considerably larger drive voltages to the grid, with the risk of causing electrical breakdown and hence damage to the system. An unexpected and unexplained observation (Fig. 1 inset) is that the maximum grid speed at onset of dissipation is a monotonically increasing function of temperature. Further experiments are in progress in order to further explore this phenomenon.

We have also measured the resonant frequency as a function of pressure p and hence density ρ of He II. Before discussing the results, we note that any submerged body moving in a fluid acquires an effective mass equal to its "bare" mass plus a quantity Δm proportional to the mass of fluid displaced [5]. We may thus write $\Delta m = \beta V \rho(p)$, where β is a mass enhancement factor, V is the volume occupied by the body and $\rho(p)$ is the density of the fluid at pressure p. The factor β can be calculated exactly for bodies of certain geometrical shape; for instance, it can be shown that $\beta = \frac{1}{2}$ for a sphere, $\beta = 1$ for an infinite circular cylinder and $\beta = 3$ for an elliptical cylinder of axes ratio 3, oscillating in the direction of its minor axis. For the present experimental system, it is straightforward to show [4] that

$$\frac{1}{f_1^2(p)} \approx \frac{1}{f_0^2}\left(\frac{\beta}{\rho_{Ni}}\rho(p) + 1\right),\qquad(2)$$

where f_1 (f_0) refers to the resonant frequency of the nickel grid in He II (vacuum) and ρ_{Ni} is the density [6] of the grid. Shown in Fig. 2 is a plot of $1/f_1^2$ against ρ within the linear regime of weak forcing in the temperature-independent range near $T \sim 20$ mK. Also shown are two straight-line fits to the data: the continuous line fit was obtained by taking all data points into account whereas the dashed line fit was calculated after excluding the point for which $\rho = 0$ (vacuum). Shown in the inset is a magnified view of the region around the points for which $\rho \neq 0$, for clarity. Note that the continuous line fit gives $\beta \approx 4.4(2)$ and the corresponding value for the dashed line fit is $\beta \approx 3.1(1)$. It can be seen
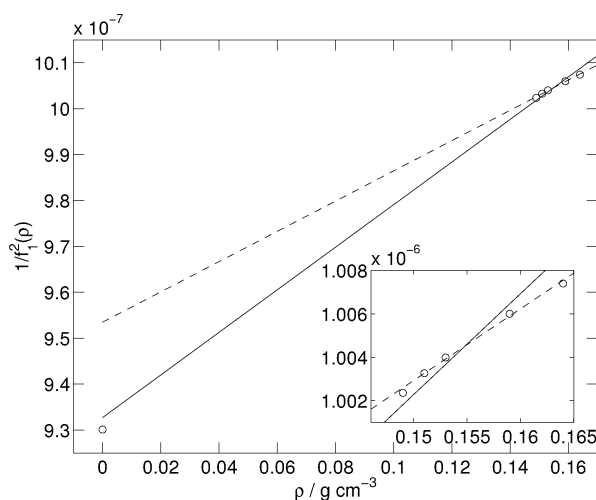

FIGURE 2. The measured resonance frequency of the grid as a function of the density of He II; the value of the density was extracted from the applied pressure using the data provided by Brooks & Donnelly [6]. The dashed line is a straight-line fit to the points excluding the one at zero density; the continuous line is a similar fit taking into account all data points. Note that the errorbars are too small to show on this scale.

that the data points of Fig. 2 do not all lie on a straight line, as one might expect from Eq. (2). Instead, the points for which $\rho \neq 0$ do appear to form an approximately straight line; if this is indeed the case, then one might conclude that the vibrating grid exhibits an additional effective mass renormalization which cannot be accounted for by the above analysis. We conjecture that this additional effective mass enhancement might be due to an effective "boundary layer" formed by vortex loops pinned to the vibrating grid.

REFERENCES

1. Vinen, W. F., and Niemela, J. J., *J. Low Temp. Phys.*, **128**, 167–231 (2002).
2. Skrbek, L., and Stalp, S., *Phys. Fluids*, **12**, 1997–2019 (2000).
3. Nichol, H. A., Skrbek, L., Hendry, P. C., and McClintock, P. V. E., *Phys. Rev. E*, **70**, 056307 (2004).
4. Charalambous, D., Skrbek, L., Hendry, P. C., McClintock, P. V. E., and Vinen, W. F., submitted to *Phys. Rev. E* (2005).
5. Landau, L. D., and Lifshitz, E. M., *Fluid Mechanics*, Butterworth-Heinemann, Oxford, 1984, 2nd edn.
6. Brooks, J. S., and Donnelly, R. J., *J. Phys. Chem. Ref. Data*, **6**, 51 (1977).