Ensemble Prospectism*

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Abstract

Incomplete preferences displaying 'mildly sweetened' structure are common, yet theoretically problematic. This paper examines the properties of the rankings induced by the set of all coherent completions of the mildly sweetened partial preference structure. Building on these properties, I propose an ensemble-based refinement of Hare's (Analysis 70:237–247, 2010) prospectism criterion for rational choice when preferences are incomplete. Importantly, this ensemble-based refinement is immune to Peterson's (Theory & Decision 78:451–456, 2015) weak money pump argument. Hence, ensemble prospectism ensures outcome rationality. Furthermore, by recognizing the structural isomorphism between mildly sweetened preference structures and Cover's splitting rule in Blackwell's Pick the Largest Number problem (Ann Math Stat 22:393–399, 1951), ensemble prospectism can be shown to yield better-than-even odds of selecting the ex-post higher-utility option – despite the absence of all-things-considered preferences ex ante.

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1 Introduction

Along with reflexivity and transitivity, completeness is widely and routinely invoked as a necessary condition of preference-relation rationality. Yet far from being fundamental to rationality, the completeness axiom's function is to ensure tractability. Moreover, its descriptive and normative validity is questioned, even among theorists.

Hare (2010) introduces an approach to decision making under incomplete preferences labelled *prospectism*. Informally, an action is permissible under prospectism if and only if, for some particular completion of negatively transitive preferences which respects existing partial preference structure, no alternative action yields higher utility. Prospectism thus offers guidance for action when a decision maker is confronted by the 'mildly sweetened' configuration of incomplete preferences where strict preference holds between the mildly sweetened option A^+ and its unsweetened form A ($A^+ \succ A$) but no preference holds between options A^+ and B, nor between options A and B. Importantly however, the permissible choice is not identified uniquely.

Peterson (2015) shows that this non-uniqueness characteristic renders the decision maker susceptible to a weak money pump in which she is permitted to make a series of choices that lead to a sure loss of utility. Peterson (2015) argues that this possibility – and the decision maker's advance knowledge that she may make such a utility-diminishing sequence of choices – precludes prospectism from being a plausible theory of rational decision making. "A possible conclusion is that the very idea of trying to find decision rules for decision makers who lack complete preferences ... is problematic (Peterson, 2015)."

Although the problem is difficult, it is not insoluble. Moreover, it is soluble precisely by building upon Hare's (2010) central concept of a 'coherent completion' of preferences consistent with existing partial preference structure. Just as refinements of game-theoretic solution concepts narrow the number of candidate solutions and eliminate implausible equilibria, here we develop a refinement of prospectism that addresses the multiplicity of permissible choices under prospectism. This refinement is based on the entire ensemble of coherent completions of preferences consistent with existing partial preference structure.

We characterize the properties of the set of rankings induced by the ensemble of coherent completions of the mildly sweetened partial preference structure. The strict preference $A^+ \succ A$ places a constraint on the number and nature of possible coherent completions. The ensemble of coherent completions inherits this structure, which is reflected in the associated ensemble of rankings.

The results obtained here are facilitated by two crucial insights.

First, that the rankings ensemble is equivalent to a truncated variant of the set of inequalities exploited in Cover's 'splitting rule' that delivers better-than-even odds of selecting the larger (unknown) number in the problem known as Pick the Largest Number (Blackwell, 1951; Cover, 1987; Samet et al., 2004). We show that the Cover-rule solution applies equally to the problem in which a decision maker must choose between options whose *utilities* are unknown ex ante.

Second, that ensemble-based methods – which we formalize through an ensemble voting procedure – constitute a suitable procedure for *rationally* selecting a choice option under incomplete preferences. Aggregation of individual preferences to inform collective decision making has been studied extensively, for centuries, in the social-choice literature. Here we draw on this literature in formalizing ensemble voting rules. Furthermore ensemble methods are used extensively in diverse areas where a single decision maker seeks to improve predictive or inferential performance by aggregating across diversiform models, hypotheses, or inferences. Ensemble methods are also used in areas where the information to be aggregated is supplied by diverse individuals or forecasters. Surowiecki (2004) provides numerous illustrations of, and a shortlist of necessary preconditions for, successful aggregation across individuals, now popularly known as 'the wisdom of crowds'.[1]

Ensemble prospectism, which implements a voting procedure, delivers a unique ranking that aggregates across the entire ensemble of rankings. Consequently ensemble prospectism is immune to weak money pumps. In turn, it also satisfies outcome rationality.[2] Moreover, due to the structure of rankings and relative option frequencies picked up in the ensemble voting procedure, by Cover's (1987) splitting rule ensemble prospectism achieves better-than-even odds of selecting the option with the larger ex-post utility, despite these utilities being unknown ex ante.

2 Incomplete preferences

2.1 Provenance

The completeness axiom simplifies rational choice theory, and it is present in the most widely known variants of revealed preference theory (Sen, 1973). Nevertheless completeness is neither a requirement for, nor an implication of, rationality or revealed preference. Within the specialist literature, a consistent view concerning the universal applicability of the completeness axiom prevails among a large fraction of the authors responsible for developing rational choice theory: that its appropriateness and convenience is dubious, that it is perhaps the most questionable of all the axioms of utility theory, that it is without adequate justification, and that it is without

[1] after the title of his best-selling book
[2] i.e. 'making choices that do not lower welfare' (Gilboa et al., 2010)

reason (see Table 1).

Although completeness, together with transitivity, is often advanced as a minimal requirement for consistency-based rationality – e.g. as in Savage's first postulate (Savage, 1954) – recent work has shown that completeness of psychological preferences is not a necessary requirement for rationality when rationality is conceived as 'outcome rationality' (Mandler, 2005) or 'making choices that do not lower welfare' (Gilboa et al., 2010). Only choices violating psychological preferences – whether complete or incomplete – would, under this view, be viewed as violations of rationality (Sen, 1973; Mandler, 2005).

Table 1: The completeness axiom's provenance among theorists.

von Neumann & Morgenstern (1944)	"It is very dubious, whether the realization of reality which treats this [completeness] postulate as a valid one, is appropriate or even convenient."
Herman Chernoff (1954)	"...this assumption is too much to make if one wants to determine whether a rational approach exists via the decision function formulation."
Robert Aumann (1962)	"Of all the axioms of utility theory, the completeness axiom is perhaps the most questionable.... ...we find it hard to accept even from the normative viewpoint."
Robert Sugden (1991)	"There is no adequate justification for the requirement that preferences are complete."
Michael Mandler (2004)	"...even for standard consumption goods there is no reason why agents should always be able to judge which bundles leave them better off."

2.2 Prospectism

Here we present *prospectism* following Hare (2010) except insofar as our context of application permits simplification. Notably, the choice options in the mildly sweetened partial preference structure – generically denoted $o_i \in O$ – are neither explicitly nor implicitly defined as being risky or uncertain. For this reason we collapse Hare's (2010) expected-utility formulation into an ordinal utility-under-certainty formulation. An incomplete strict partial order $\succ$ cannot be represented by a real-valued function in the usual manner.[3] However it is possible to associate a function $u^* : O \to \mathbb{R}$, unique up to an affine transformation, such that for any comparable pairs in $\succ$, $o_i \succ o_j$ implies $u^*(o_i) > u^*(o_j)$ for all $o_i, o_j \in O$. u^* constitutes a *coherent completion* of $\succ$ over the set of all choice options O. The real-valued function u^* imposes a complete order on O. There is more than one possible u^* consistent with a specific partial order

[3]Although Hare (2010) posits the existence of such a real-valued function, labelling it u, his subsequent development of prospectism does not crucially hinge upon u as distinct from $\succ$.

$\succ$. Let $\mathcal{L}(O, \succ) = \mathcal{U}^*_{O,\succ}$ denote the *set of all coherent completions* of $\succ$, the existence of which is guaranteed by Szpilrajn's (1930) theorem.[4] In turn, the original partial order may be recovered as the intersection of all coherent completions:

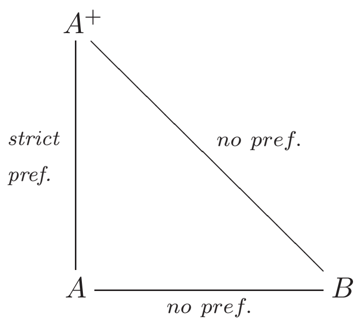
$$\succ \;=\; \bigcap_{u^* \in \mathcal{U}^*_{O,\succ}} u^* \;. \tag{2.1}$$

Definition 2.1 (Prospectism). It is permissible to choose an option iff, *for some utility function* u^* that represents a coherent completion of (partial, incomplete) preferences $\succ$, no other option yields greater u^*-utility.

2.3 'Mildly sweetened' partial preference structure

Consider the choice options $o_i, o_j, o_k \in O$ $(i \neq j \neq k)$, for which we also have the more intuitive labels $o_i = A^+$, $o_j = A$, and $o_k = B$. The absence of all-things-considered preferences is sometimes illustrated as *insensitivity to mild sweetening.* That is, knowing that a decision maker has no preference between A and B, even the introduction of a discrete improvement over A – a mild sweetening of A, which we label A^+, and for which $A \prec A^+$ is apparent – nevertheless does not suffice for preferences of any sort to materialize between A^+ and B. In other words, the absence of preference between A and B, sometimes described as incommensurability, is so fundamental that it is robust to the mild sweetening of A to A^+. The mildly sweetened partial-preference structure is illustrated in Figure 1.

Figure 1: Mild sweetening.



2.4 Weak money pump

Peterson (2015) considers preference structures that are insensitive to mild sweetening (see Section 2.3 above). As the definition of prospectism stipulates the *permissibility* of choosing in

[4] The symbol $\mathcal{L}(\cdot,\cdot)$ refers to the set of all linear orders, i.e. the set of all complete orders, which in keeping with Hare's (2010) terminology we refer to as the set of all 'coherent completions'.

accordance with *any* coherent completion u^* of the underlying incomplete preferences, the weak money pump is simple to construct.

At time t_1 a decision maker with preferences illustrated in Figure 1 is offered a swap in which she may give up A^+ in exchange for B. Although she has no preferences between A^+ and B, there is a coherent completion of her preferences u^* in which $B \succ^* A^+$. Hence, according to the definition of prospectism, she is permitted to swap A^+ for B at time t_1, although she is under no obligation to do so. At the later date t_2 she faces a choice situation in which she is offered a swap in which she may give up B in exchange for A. Again, as she has no preferences between B and A, she is not compelled by her preferences to choose either. But among the coherent completions of her incomplete preferences there is a u^* – different from that which permitted choice at time t_1 – in which $A \succ^* B$. Hence, according to prospectism, she is permitted to swap B for A. At a yet-later date t_3 she faces a choice situation in which she is asked to pay a small amount of money in order to swap A in exchange for A^+. As she does have explicit preference for the latter $A \prec A^+$, she increases her utility by accepting this swap – as long as the cost of doing so is sufficiently small. Hence, it is also permissible under prospectism.

Taken together, this sequence of trades – each of which is permissible under prospectism – leads to a sure monetary loss. The decision maker is not obligated by prospectism to participate in these trades, but prospectism explicitly permits this sure-monetary-loss sequence. Peterson (2015) asks the question, "How could rationality permit you to carry out a sequence of acts that you know in advance will lead to a sure loss?" His answer is that a rational theory cannot permit such a sure-loss-making sequence, and hence prospectism should be rejected as a theory of rationality.

3 Cover's random-threshold rule

3.1 Pick the largest number

Consider the following problem, a variant of which was originally introduced by David Blackwell (1951).

Problem 1 (Pick the largest number). Two slips of paper $i = (1, 2)$, each inscribed with a real number $x_i \in \mathbb{R}$, lie face down. One of the two slips is chosen at random, and the number x_i on it is shown to you. You have to guess whether your number is the larger of the two. How can you guarantee that the probability of your guess being correct is strictly greater than one-half?

Solution (Cover, 1987) The idea is to pick a random *splitting number* T according to a density $f(t)$, $f(t) > 0$, for $t \in (-\infty, \infty)$. If the number in hand is less than the realization t, decide that it is the smaller; if greater than t, decide that it is the larger.

Regardless of the specific form of the density $f(t)$, its cdf $F(t)$ is monotonically strictly increasing in t, given that $f(t) > 0 \quad \forall t \in (-\infty, \infty)$. Following Cover's random-threshold rule, the Guesser concludes that the unobserved number is larger than the observed number x_i according to a monotonically strictly decreasing function of the observed number x_i, as $P(t - x_i > 0) = 1 - F(x_i)$.

Let Nature determine the numbers (x_1, x_2) as the realization of a pair of random variables (X_1, X_2), the support of each being the entire real line $\mathbb{R}$. Nothing further is stipulated regarding the specific distributions of X_1 and X_2. They may be viewed as arbitrary. It follows that $P(X_1 \neq X_2) = 1$, whereby it must hold that either $x_1 < x_2$ or $x_1 > x_2$. The following claims (notation adapted) are proven by Samet et al. (2004).

Claim 1. *If the Guesser plays an arbitrary threshold strategy against any realization* (x_1, x_2)*, then she*

- *wins with probability* $1/2$ *when either* $x_1, x_2 < t$ *or* $x_1, x_2 > t$*;*
- *wins for sure when either* $x_1 < t \leq x_2$ *or* $x_2 < t \leq x_1$*.*

Consider the random-threshold strategy T such that $P(t \in (x_1, x_2]) > 0$ whenever $x_1 < x_2$ and similarly $P(t \in (x_2, x_1]) > 0$ whenever $x_2 < x_1$.

Claim 2. *The strategy* T *guarantees that the Guesser wins with probability higher than* $1/2$ *against any realization* (x_1, x_2)*.*

Let us examine the $x_1 < x_2$ case first. Here the realization t splits the two numbers $x_1 < t < x_2$ with probability $P(t \in (x_1, x_2]) > 0$, whereby the Guesser correctly opts for the unobserved number. If the realization t does not split the two numbers, the Guesser succeeds with probability $1/2$. Overall, then, the Guesser succeeds with probability $1/2 + P(t \in (x_1, x_2]) > 1/2$. Similarly in the $x_1 > x_2$ case the Guesser succeeds with probability $1/2 + P(t \in (x_2, x_1]) > 1/2$. Together, these two cases are exhaustive. Either way, the Guesser's random-threshold splitting strategy succeeds with probability greater than $1/2$.

3.2 Application to mildly sweetened preference structure

The problem of choosing between A^+ and B in the mildly sweetened partial preference structure is formally equivalent to a truncated variant of the problem of sticking or switching in Pick The Largest Number.

Recall that in Pick The Largest Number, the Guesser has to choose between two initially unknown real numbers $x_1, x_2 \in \mathbb{R}$, one of which is revealed to be greater-than or less-than the realization of a third real-valued $t \in \mathbb{R}$ random-threshold splitting number T. In the event that the realized value of the third real number t is less than x_1, it is possible to conclude that $P(x_1 > x_2) > 1/2$ by an mount $1/2 \geq P(t \in (x_2, x_1]) > 0$.

Similarly, the decision maker's task in the mildly sweetened preference structure is to choose between two options yielding unknown real-valued utility $u^*(A^+), u^*(B) \in \mathbb{R}$, the utility of one of which (A^+) is revealed to be greater than the otherwise-unknown real-valued utility of a third choice option (A); that is, $u^*(A^+) > u^*(A) \;\; \forall\, u^*(\cdot) \in \mathcal{U}_{O,\succ}$. If in addition $u^*(B) > u^*(A)$, then $P(u^*(A^+) > u^*(B)) = 1/2$. If however $P(u^*(A^+) > u^*(A) > u^*(B)) > 0$ then $P(u^*(A^+) > u^*(B)) > 1/2$.

Proof. Although the magnitude of the attribute increment(s) between A and A^+ is (are) known, the associated increment in utility is not known, due to the underlying incompleteness of preferences. If the utility difference can take *any* strictly positive (but arbitrarily small) value $u^*(A^+) - u^*(A) \in \mathbb{R}_{++}$, then $P(u^*(A^+) > u^*(A) > u^*(B)) > 0$ no matter how close the spacing $u^*(A^+) - u^*(B) \in \mathbb{R}_{++}$, and thus $P(u^*(A^+) > u^*(B)) > 1/2$. Let us assert the contrary, that is $u^*(A^+) - u^*(A) \in \{\mathbb{R}_{++} \setminus D\}$ where $D \subset \mathbb{R}_{++}$ and $D \neq \varnothing$. However if $D \neq \varnothing$ we would know the lower bound of the marginal utility (utilities) of the attribute(s) upon which A^+ and A differ, thereby contradicting the absence of all-things-considered preferences.

4 Ensemble prospectism

Section 3.2 shows that choosing A^+ over B is not only consistent with *some* coherent completion of $\succ$, u^*, but that it is better than outcome rational to do so. Meanwhile, Peterson's (2015) weak money pump argument shows that Hare's (2010) prospectism criterion does not ensure outcome rationality on its own. Nevertheless, prospectism can be reconciled with outcome rationality by bringing more of the information contained in the set of coherent completions $\mathcal{U}^*_{O,\succ}$ to bear upon the choice between A^+ and B. Specifically, this can be accomplished with the *ensemble* of rankings consistent with $\mathcal{U}^*_{O,\succ}$.

4.1 Preliminaries

Knowing only that $A^+ \succ A$, three strict rank orderings are possible, and these correspond to the set of rank-order equivalence classes within the set of all coherent completions of $\succ$, $\mathcal{U}^*_{O,\succ}$.

Label this set S (the ensemble of possible rank orderings):

$$S = \{(A^+, B, A), (B, A^+, A), (A^+, A, B)\} \tag{4.1}$$

Properties 4.1 (Ensemble of rankings).

(i) The subset of first-ranked options is $S_1 = \{A^+, B, A^+\}$, and

$$\frac{|S_1|_{A^+}}{|S_1|} = 2/3 \ , \qquad \frac{|S_1|_B}{|S_1|} = 1/3 \ .$$

(ii) The subset of first- or second-ranked options is $S_{1,2} = \{A^+, B, B, A^+, A^+, A\}$, and

$$\frac{|S_{1,2}|_{A^+}}{|S_{1,2}|} = 1/2 \ , \qquad \frac{|S_{1,2}|_B}{|S_{1,2}|} = 1/3 \ , \qquad \frac{|S_{1,2}|_A}{|S_{1,2}|} = 1/6 \ .$$

(iii) The subset of last-ranked options is $S_3 = \{A, A, B\}$, and

$$\frac{|S_3|_{A^+}}{|S_3|} = 0 \ , \qquad \frac{|S_3|_B}{|S_3|} = 1/3 \ , \qquad \frac{|S_3|_A}{|S_3|} = 2/3 \ .$$

(iv) Two rankings form a mutually off-setting symmetrical pair insofar as A^+ and B are ranked above A: $S_{\text{sym}} = \{(A^+, B, A), (B, A^+, A))\}$. There is one symmetry-breaking ranking $S_{\text{n-sym}} = \{(A^+, A, B)\}$ in which A splits A^+ and B.

In each of the ranking properties (i)–(iii), A^+ dominates B by relative frequency.[5] This follows as a direct consequence of the symmetry-breaking ranking $S_{\text{n-sym}} = \{(A^+, A, B)\}$, which is detailed in property (iv). This ranking is precisely the splitting ranking that Section 3.2 proves to occur with strictly positive probability. This symmetry-breaking ranking plays a pivotal role in the ensemble-based approach, to which we now turn.

4.2 Ensemble approach

Ensemble methods are used extensively in diverse areas in which either (a) multiple individuals' beliefs, forecasts, or preferences are aggregated for collective action or (b) a single decision-making entity develops multiple different explanations, hypotheses, or models that it aggregates for the purpose of making a specific inference. The term 'ensemble method' is used in machine

[5] In (iii), pertaining to last-ranked options, better options have lower relative frequencies. The anti-plurality voting procedure, discussed in Section 4.2, assigns a single point to each option ranked higher than the lowest-ranked option. In this sense, the relative frequencies in (iii) pick up performance in the anti-plurality voting procedure.

learning and artificial intelligence, while the same collection of aggregation techniques – by voting or by averaging – are referred to by a variety of labels across the different areas in which they are employed.

As in the contexts of artificial intelligence, multiple forecasters, and Bayesian model uncertainty, the problem of choosing an option under incomplete preferences can be partitioned into two steps, the first of which applies Epicurus' (c. 342–270 BCE) Principle of Multiple Explanations: *If more than one theory is consistent with the observations, keep all theories.*[6] Construction of the set of all coherent completions of preferences $U^*_{O,\succ}$ is the corresponding step in prospectism. It is in the second step where the presently proposed 'ensemble prospectism' departs from Hare's (2010) seminal formulation. In a manner consistent with ensemble methods more generally, ensemble prospectism utilizes the full range of coherent completions of the decision maker's incomplete, partial preferences as enumerated in $U^*_{O,\succ}$, rather than any (arbitrary) individual element of this set, to guide choice. The task of rationally guiding choice in the context of incomplete preferences can be viewed as a problem of inference about the decision-maker's underlying, as-yet unknown preferences. Due to this close parallelism, ensemble methods are not only a convenient source of apposite terminology, but also a source of apposite solutions.

Because of the sparseness of information in this context, determining tight bounds on the prior over the ensemble of possible rank orderings is problematic. Approaches based on Bayesian Model Averaging – or, equally, weighted voting procedures – are therefore not suitable in the present context. This leaves a large collection of potentially applicable unweighted voting procedures (Brams and Fishburn, 2002; Brandt et al., 2013). The problem is simplified from that typically addressed in social choice however, because strategic voting is not a consideration within the present context.

The second step can be formalized as the application of a voting rule to the ensemble of rankings $S^*_{O,\succ}$ associated with[7] the set of all coherent extensions of $\succ$, $U^*_{O,\succ}$. Given the number of choice options $|O|$ and the restriction(s) imposed by the strict preferences (if any) contained in $\succ$, the number of different rankings present in the ensemble is given by $n = |S^*_{O,\succ}|$. The set of all non-empty subsets of the choice options may be written as $\mathcal{F}(O)$, which we will see below may be understood as the set of feasible voting-rule outcomes. We specialize the definition of a voting rule to the ensemble of rankings as follows.

Definition 4.1. An ensemble voting rule is a function $f : S^*_{O,\succ} \to \mathcal{F}(O)$.

A voluminous literature has developed a veritable zoo of voting rules, and a decision maker

[6]Epicurus' letter to Pythocles: events "have multiple causes of coming into being and a multiple prediction of what exists, in agreement with the perceptions."

[7]ordinally equivalent to

might in principle employ any one of a large number of different voting rules. For ensemble prospectism to be immune to weak money pump arguments, the ensemble voting rule must satisfy additional requirements. First, the decision maker must not have the possibility to switch from one voting rule to another within a time frame that would make it possible for the different properties of distinct voting rules to introduce decision-sequence-level intransitivity. Second, the voting rule must not be a member of the class of stochastic (or lottery) voting rules, as such rules could also introduce decision-sequence-level intransitivity. Third – and for the very same reason – the voting procedure should be *resolute*, yielding a unique winner, or else if it is not resolute, and yields a set of co-winners, the tie-breaking rule applied must not be random. In the definition below, the second and third requirements above are jointly invoked with the 'completely non-stochastic' qualification.

Definition 4.2 (Ensemble prospectism). It is permissible to choose an option iff it is identified as the winner by the decision-maker's completely non-stochastic ensemble voting rule f applied to the ensemble of rankings $S^*_{O,\succ}$ associated with the set of all coherent extensions of $\succ$, $U^*_{O,\succ}$.

Many voting rules satisfy these requirements, including e.g. simple and common (positional) scoring rules such as the plurality rule, the anti-plurality rule, and Borda's rule. Among all positional scoring procedures, Borda's rule is least susceptible to paradoxes and other pathological behavior, including being the only scoring rule that will never award a Condorcet winner the lowest cumulative score (Brandt et al., 2013). Where Borda's rule and the other positional procedures do fall short is in susceptibility to strategic manipulation – which, fortunately, is not a consideration in the present context, just as it is not a consideration in contexts where Borda's rule has been used to combine inferences from diverse methods to improve inferential performance (Marbach et al., 2012). Under the Borda rule each of the n rankings present in $S^*_{O,\succ}$ awards a score (or 'points') from a maximum of $|O| - 1$ to the highest-ranked option, through to 0 for the lowest-ranked option $(|O|-1, |O|-2, ..., 1, 0)$. Each option's *Borda count* is the sum of its scores across all n ranking profiles, and the Borda rule chooses the option with the highest Borda count. Indeed all $|O|$ options may be ordered (completely and transitively) by their respective Borda counts, which here we may denote $\succ_{BC}$.

In the example below we revisit the ensemble of possible rank orderings S detailed in equation (4.1). We present its ensemble-prospectism solution, operationalized for illustrative purposes with a 'Borda' ensemble-voting rule.

Example 4.1 (Ensemble-prospectism solution to mildly sweetened partial preferences). Operationalization of ensemble prospectism with the Borda rule yields the unique, complete, and transitive ordering $\succ_{BC} = (A^+ \succ B \succ A)$ (see Borda scores in Table 2). The Borda-count ranking $\succ_{BC}$ here is consistent with the relative-ranking properties (i)–(iii) set out in Section 4.1 above. Moreover, the Borda-count ranking $\succ_{BC}$ here is also consistent with the recommendation derived from Cover's (1987) splitting-number solution in Section 3.2.

Table 2: Application of Borda rule to mildly sweetened partial preferences.

ranking	A^+	B	A
A^+, B, A	2	1	0
B, A^+, A	1	2	0
A^+, A, B	2	0	1
Borda count	5	3	1

5 Conclusions

This paper develops a refinement of prospectism that reconciles Hare's (2010) theory of prospectism with Peterson's (2015) weak money pump. The refinement restricts the range of prospectism-consistent choices by grafting ensemble methodology upon Hare's (2010) set of all coherent completions of the incomplete preference relation. The resulting ensemble-prospectism refinement is not susceptible to weak money pump arguments. By precluding choices that would lower individual welfare, ensemble prospectism ensures outcome rationality.

Yet there is also a sense in which the form of rationality embodied in ensemble prospectism *dominates* outcome rationality. Ensemble voting rules not only ensure that individual welfare is not lowered, but also ensure strictly greater-than-even odds of selecting the best option – despite the absence of all-things-considered preferences. This surprising, counter-intuitive finding is underpinned by the structural isomorphism between the problem of choice under incomplete preferences – particularly under 'mildly sweetened' incomplete preferences – and the Guesser's problem in the game Pick the Largest Number. Whereas Cover's random-splitting-number solution ensures greater-than-even odds of guessing the larger (unknown) number in the latter problem, ensemble voting rules ensure greater-than-even odds of selecting the higher (unknown) utility option in the former problem. The symmetry-breaking ranking, which is pivotal to the ensemble-voting result in the mildly sweetened preference structure, corresponds precisely to the splitting-number case which is pivotal to achieving greater-than-even odds in Pick the Largest Number.

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