Accepted Manuscript

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Manthos Delis , Maria Iosifidi , Mike G. Tsionas

Please cite this article as: Manthos Delis , Maria Iosifidi , Mike G. Tsionas , Endogenous Bank Risk and Efficiency, *European Journal of Operational Research* (2016), doi: 10.1016/j.ejor.2016.12.024

Highlights

- We develop a model to incorporate bank risk within a model of frontier efficiency.
- We model bank risk from the variance of profits or returns.
- We estimate our model using panel data for U.S. banks and Bayesian techniques.
- Excluding risk from the efficiency model significantly biases efficiency estimates.
- There is a negative risk-efficiency nexus with causality running both ways.

Endogenous Bank Risk and Efficiency

Manthos Delis
Department of Finance and Accounting, Surrey Business School, University of Surrey
Guildford, Surrey, GU2 7XH, UK
Email: m.delis@surrey.ac.uk
Telephone number: +44 (0)1483 68 6332

Maria Iosifidi
Department of Finance and Accounting, Surrey Business School, University of Surrey
Guildford, Surrey, GU2 7XH, UK
Email: m.iosifidi@surrey.ac.uk

Mike G. Tsionas
Lancaster University Management School
Lancaster, LA1 4YX, UK
Email: m.tsionas@lancaster.ac.uk

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Abstract

We develop a framework to incorporate bank risk, as measured from the variance of profits or returns, within a model of frontier efficiency. Our framework follows the premise that risk is endogenously related to efficiency. We estimate our model using panel data for U.S. banks and Bayesian techniques. We show that excluding risk from the efficiency model significantly biases the efficiency estimates and the ranking of banks according to their competitive advantage. We also demonstrate that there is a negative risk-efficiency nexus with causality running both ways, while our estimates of risk are fully consistent with the developments in the banking industry over the period 1976-2014.

Keywords: OR in banking; Stochastic frontier; Endogenous risk; Risk-efficiency relationship; Bayesian methods

1. Introduction

Is there a causal relationship between the riskiness and efficiency of banks? And if yes what is the direction of causality? The goal of this paper is to develop a framework that incorporates risk as an endogenous variable into an empirical model of operational efficiency. We use the well-established stochastic frontier approach (SFA), under which the stochastic term of a production, cost, profit, or return function is decomposed into an efficiency-related component and the remainder disturbance.[1] We show that our framework allows the robust estimation of bank efficiency and risk, as well as the identification of the potency and direction in the risk-efficiency nexus.

There are two novel elements of our framework. First, in line with the standard finance literature (e.g., Markowitz, 1952, and many others henceforth), we model risk as the variability of the difference between actual and expected profits (returns). In doing this, we abstain from including specific variables reflecting only certain aspects of risk as inputs/outputs in the production of banking services and allow risk to be estimated in a more thorough way. Second, and quite important, we allow our risk and efficiency estimates to be endogenous to each other. This is an essential improvement of efficiency models given the well-established theoretical proposition that bank risk-taking is endogenous to the characteristics of the specific banking firm, including managerial decision-making, and does not solely emerge out of context (Hughes, 1999; Danielsson and Shin, 2003).

Theoretically, causality between risk and efficiency can run both ways. On the one hand, bank managers seek new risky projects to innovate and differentiate from rival banks, as well as manage existing risk through diversification with the ultimate goal being to

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[1] A competitive approach to the stochastic frontier modeling framework is the use of linear programming techniques, mainly Data Envelopment Analysis (DEA). Although DEA methods have advanced to incorporate a stochastic structure, which was the traditional drawback relative to the SFA, the inclusion of risk in DEA remains quite difficult given the requirement to incorporate stochastic assumptions with respect to the volatility of returns. We do, however, propose in the conclusions some possible extensions to incorporate stochastic risk within a DEA framework. For a review in using stochastic DEA methods, see Olesen and Petersen (2016), and for DEA methods in banking, see Fethi and Pasiouras (2010).

increase their revenue (Sharpe, 1964). In contrast, banks pursuing projects with very high risk, given inputs and outputs of production, would experience increased risk and inability to differentiate so as to achieve maximum returns. Thus, we expect both positive and negative forces defining a relation running from risk to profit (return) efficiency.

Conversely, under the impulse of the prospect and the behavioral theories, banks with relatively low efficiency levels are likely to take very high risks. This can be due to effort to catch up with rival banks, slow learning or ineffective risk management, and adaptation (Fiordelisi et al., 2011). For the same reasons, banks with efficient risk management are likely to exhibit good performance as well as low risk-taking. In a similar fashion, successful managers seek new operational structures and technologies to improve operational efficiency in the cost side and contain everyday operational risk. Based on the above arguments, a potential negative relation between risk and return, i.e. the Bowman (1980) paradox, can also be studied as a risk-efficiency nexus.

The above theoretical arguments imply that *bank efficiency and risk are endogenously determined*. We note that most of the empirical efficiency literature does not include a risk component and, when it does, *this risk component is neither formally modelled from the variance of profits nor is endogenously determined by efficiency* (e.g., Dong et al., 2016; references therein). We empirically demonstrate that failing to do so leads to biases in the estimates of efficiency and the competitive advantage of banks versus one another.

In this paper, we extend the SFA by formally building and estimating a four-equation vector autoregression (VAR) model. The first equation retains the standard profit or return function of the SFA, while the second is a stochastic equation differentiating between actual and expected profits (or returns), the variance of which is our measure for risk. The third equation models the contemporaneous level of risk as a function of lagged efficiency (and other determinants) and the fourth equation models efficiency as a function of lagged risk

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(and other determinants). Given that our model relies on quite a few latent variables (including risk and efficiency), we use Bayesian estimation methods organized around Markov Chain Monte Carlo (MCMC).

We estimate our model using a panel of large U.S. banks that fully compete on a national scale over a period of almost 40 years (1976-2014). The banking industry is ideal for our setting, given the special role of risk, the extensive literature on SFA in banking (e.g., Berger, 2007; references therein), and the availability of longitudal data over a large time span. However, our model can be applied to any other industry for which relevant data are available.

Our results suggest that the average level of estimated efficiency of banks is significantly lower (inefficiency is higher) when we incorporate risk into the SFA. Also, the ranking of banks based on their level of efficiency, and thus the identification of their relative competitive advantage, is also significantly different from the model without risk or from the model where risk is exogenous. Thus, the failure to include risk into the SFA and/or treating risk as an endogenous variable results in erroneous inference about both the absolute level of efficiency and the relative competitive advantage of banks.

Equally important, our findings demonstrate a strong *negative* relationship between risk and efficiency running both ways (from the previous period's risk to current levels of efficiency and vice versa). In this respect, our findings are consistent with the literature following the Bowman (1980) paradox. Our findings are also consistent with the implications of banking literature on important matters, such as the identification of risky periods closely following the historical episodes of financial turmoil and the positive effect of bank capital on risk.

Our results further motivate our paper from a managerial perspective. To the extent that the findings generalize to other industries, the estimation of efficiency while formally

incorporating risk as an endogenous variable can better inform firms that compete domestically or internationally about their competitive advantages and their sources. The example of the comparison of the U.S. and Japanese automotive industries by Chen et al. (2015) is particularly apt, as the risky strategic decisions of managers should offer an explanation for the divergence in the efficiency of the two industries. This line of research should also have important implications for recent endeavors to measure the efficiency of the public sector (e.g., Doumpos and Cohen, 2013; Haelermans and Ruggiero, 2013; Galariotis et al., 2016), energy sector (e.g., Fragkiadakis et al. (2016), as well as for cases in which some of the outputs impose negative social externalities (e.g., Chen and Delmas, 2012).

Our work is also related to a strand of finance literature suggesting that any measurement of risk should consider its endogeneity (Danielsson and Shin, 2003; Brunnermeir and Sannikov, 2014; Delis et al., 2015). This literature stresses that the consideration of risk as exogenous within any market or industry and across different measures (e.g., from simple accounting ratios, the net present value calculated by the discounted cash flow method or economic value added, and/or value-at-risk models) produces erroneous estimates and inferences.

The remainder of the paper is organized as follows. Section 2 presents and further motivates our model. Section 3 discusses the empirical application to the banking industry and presents the empirical results. Section 4 concludes the paper.

2. The framework

2.1. Profit and return efficiency

The estimation of firms' operational efficiency is a very popular practice in the operations research and economics literatures (e.g., Lozano-Vivas and Pasiouras, 2010; Kumbhakar and Lovell, 2000). The merit of frontier efficiency measures compared to the traditional

accounting-based measures of firm performance is that the former can identify the competitive advantage of a firm *vis-à-vis* its competitors (see the extensive discussion by Chen et al., 2015). In turn, the robust identification of competitive advantage and its sources (from e.g., superior cost management or profitable innovation) has unique implications for managerial efficiency, goals, and strategies.

The most comprehensive measures of frontier efficiency, and the ones used here, are based on profit or return on investment (also called return to outlay). The reason is that these measures incorporate both revenue effects (of producing at inefficient levels) and cost effects (of producing using an inefficient input mix). Under the assumption that firms maximize profits given a production set, the objective function of the firm is:

$$\Pi(y,w) = \max_{(x,p)}: \{p'y - w'x\}. \tag{1}$$

In (1), Π is the profit level of a firm at a specific point in time. In turn, x and y denote vectors of inputs and outputs of production, with their prices being the vectors w and p, respectively. This is the alternative profit function (e.g., Lozano-Vivas, 1997), which assumes that firms maximize profits by adjusting output prices and input quantities (as opposed to output and input quantities under the standard profit function). The main reason that this function has become popular in empirical research is the relative lack of information on output prices, especially for multi-output firms. However, an important merit of this approach is that it also allows for the estimation of profit (or return) functions in industries that deviate from perfect competition (Berger and Mester, 1997; Lozano-Vivas and Pasiouras, 2010; Lozano-Vivas, 1997).

The objective function in (1) assumes that all firms produce at an optimal (ideal) profit frontier. Of course, this assumption is quite problematic because all firms produce with some level of inefficiency on either the cost or revenue side. To identify the level of

inefficiency, the usual practice (e.g., Kumbhakar and Lovell, 2000) is to estimate an equation of the form:

$$\log \Pi_{it} = x'_{it}\beta + v_{it} - u_{it}, \tag{2}$$

where x'_{it} is the vector of outputs and input prices of firm i at time t, β are technology parameters to be estimated, v_{it} is the stochastic disturbance, and $u_{it} \geq 0$ is a one-sided error term representing profit inefficiency. Profit efficiency can then be simply calculated as $\exp(-u_{it})$. This model is usually termed the SFA to firm efficiency measurement.

Instead of assuming that firms maximize profit, we can assume that firms maximize return on investments. This assumption might be more intuitive from a managerial viewpoint because managers are primarily interested not in the absolute level of profit but in the evaluation of the ability of their investments to generate profits. The formal model considers the maximization of the following objective value function:

$$V(p,w) = \max_{(x,y)}: \left\{ \frac{p'y}{w'x} \right\}, \tag{3}$$

where V = total revenue/ total cost and the rest are as in equation (1). As opposed to (1), equation (3) has the additional merit of being non-negative, which is important for the estimation because taking logs of profits in (2) can be problematic when firms are in fact realizing losses. Equation (3) is homogeneous of degree one in all prices, non-decreasing in p, and non-increasing in w. The equivalent of (2) as a return function is obtained simply by replacing Π with V in (2) and, for expositional brevity, we only provide the model for profits.

2.2. Endogenous risk in the model of efficiency

The basic SFA analyzed above does not consider the formal inclusion of a risk component. A number of studies, especially in the banking industry where risk deserves special attention (e.g., Mester, 1996; Hughes, 1999), recognize this omission and consider risk in the estimation of efficiency models via the inclusion of specific risk-related variables in the

objective function of the bank along with inputs, outputs, and associated prices. This practice is, of course, correct provided that one or a finite number of risk measures (e.g., capital, non-performing loans, etc.) fully control for the riskiness of the bank. Here, we instead propose the estimation of risk within the SFA from the variability of profits or returns, in a manner fully consistent with management, finance, and economics theory (since at least Markowitz, 1952).

Even more notably, this paper represents the first effort to consider the potential endogeneity between risk and efficiency within the well-established SFA. Theoretically, the relation between risk and efficiency in banking can go both ways. To make profits and create value for the banking firm, bank managers seek investments with the highest possible net present value and manage risk in their portfolios to achieve a maximum. Thus, both the level of risk and risk-diversification ability affect the future efficiency and performance of the bank and this is the essence of the well-established positive risk-returns relation (Sharpe, 1964). Further, banks seek to attract new customers and achieve monopolistically competitive profits through risky product innovation and differentiation from competition. Achieving these objectives given a fixed set of inputs and input prices, would also imply that these banks will appear more efficient in the future period.

A relation running from risk to efficiency can, however, be negative due to two main mechanisms. The first is the presence of a shock that increases bank risk exogenously (what Berger and DeYoung, 1997, refer to as the "bad luck hypothesis"). In this case, the increased risk emerges from increases in non-performing assets (loans and securities), the limited ability to securitize or sell these assets, and increases in adverse selection (concerning the screening of new projects) and moral hazard (monitoring of existing projects). Second, the prospect and behavioral theories posit that firms with low performance (low profit and return efficiency in our case) might seek higher risks in an effort to catch up with competition,

while firms with high performance exhibit low risks due to competent risk-management (e.g., Fiegenbaum and Thomas, 2004). More recently, Andersen and Bettis (2015) suggest that this negative relation can be due to "low-learning" or "mindless" behavior of firms. These theories emerged as the main explanations for the Bowman (1980) paradox (i.e., the negative risk-return nexus).

Causality, however, might run from efficiency to risk, with at least two theories pointing to such direction (Fiordelisi et al., 2011; Berger and De Young, 1997). First, the "bad management hypothesis" posits that bad monitoring and screening capabilities of inefficient banks, along with poor cost and asset management result in higher overall risk. On the same line, efficient bank managers seek new operational structures and technologies both in the asset (revenue) and the liability (cost) side of their balance sheet, and this leads to improved risk monitoring and lower probability of default. Second, the "cost skimping hypothesis" suggests that banks might achieve low costs by under-spending on loan underwriting and monitoring in the short run, and in the longer run this yields higher non-performing loans in particular and increased probability of bank default in general. Based on the above arguments, we expect that banks with higher efficiency exhibit lower overall risk.

To formally incorporate risk into the SFA, we augment (2) as follows:

$$\log \Pi_{it}^* = x'_{it}\beta + v_{it} - u_{it}, \tag{4}$$

$$\Pi_{it} = \Pi_{it}^* + \varepsilon_{it}. \tag{5}$$

In (5), the difference between Π and Π^* represents the deviation of the actual from expected profits ε of firms, where ε is distributed as $N(0, \sigma_{it}^2)$.[2] Using the implications of standard finance and management theory (e.g., Delis et al., 2015; and references therein), we define

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[2] The assumption of normal distribution can be criticized on the basis of skewness or kurtosis, as in the risk-return relation (e.g., Theodossiou and Savva, 2016). We show below that, at least in our empirical application, this does not cause considerable problems. However, we also highlight that the potential use of e.g. a skewed generalized distribution (Theodossiou, 1998) is a quite fruitful extension.

the variance of profits σ^2 as the risk of firm i at time t. By modeling return on investments instead of profit, we can alternatively obtain the equivalent variance of returns as our measure of risk.

This modelling choice assumes that risk (σ^2) is a latent variable and not an input or output of production as in the previous literature. This approach has three main advantages. First, the variability of profits or returns, as directly estimated from the profit or returns function, is a more thorough measure of total bank risk compared to specific accounting ratios entering the production function of the bank that separately reflect credit risk, liquidity risk, etc. in a non-exhaustive way. To this end, our approach is consistent with the modern portfolio theory (Markowitz, 1952; Boyd and Runkle, 1993) and its extensions in more thoroughly measuring total bank risk. Second, our framework allows the direct estimation of risk at each bank-year observation from the data and not from the volatility of profits or returns using backward information (lags) on these variables (e.g., Boyd and Runkle, 1993; Laeven and Levine, 2009). We view this as an important advantage, because using information from previous periods yields a measure of risk that is sensitive to the number of chosen periods and is problematic if data frequency is low (most studies in banking use annual data). Third, in line with our theoretical considerations on the endogenous relation between risk and efficiency, we show below that our framework can be extended to treating risk as an endogenous latent variable.

Specifically, we augment equations (4) and (5) with an equation modelling risk as an endogenous latent variable. Models of volatility come in a variety of forms (see, e.g., Chib et al., 2002), and here we use the following specification:[3]

$$\log \sigma_{it}^2 = \alpha_{o1} + \alpha_1 \log \sigma_{i,t-1}^2 + \gamma_1 \log \Pi_{i,t-1}^* + z'_{it}\delta_1 + \eta_1 \log u_{i,t-1} + c_{1,it},$$

(6)

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[3] We prefer the log specification for reasons of symmetry with the translog. Using a linear specification produces very similar results.

In (6), z' is a vector of other variables potentially affecting risk, c_1 is the stochastic disturbance, and $\alpha_{o1}, \alpha_1, \gamma_1, \delta_1,$ and η_1 are parameters to be estimated. Effectively, we model the variance of profits or returns, which follows a panel stochastic volatility structure and depends on past inefficiency.

The model of equations (4)-(6) is robust as long as the inefficiency component u is not affected by the risk component. To allow an examination of causality in the relationship between risk and efficiency running both ways, we model inefficiency as: [4]

$$\log u_{it} = \alpha_{o2} + \alpha_2 \log \sigma_{i,t-1}^2 + \gamma_2 \log \Pi_{i,t-1}^* + z'_{it}\delta_2 + \eta_2 \log u_{i,t-1} + c_{2,it}, \tag{7}$$

where $\alpha_{o2}, \alpha_2, \gamma_2, \delta_2,$ and η_2 are parameters to be estimated and c_2 is the disturbance. Equation (7) assumes that profit inefficiency is determined by the same variables as risk (although the vector z can contain different variables) and, importantly, the lagged risk itself. If $\alpha_2 \neq 0$, then risk has a systematic effect on profit inefficiency. Other determinants of inefficiency may have an effect through the parameters in δ_2, and persistence in inefficiency is allowed for by the parameter η_2. Both of these effects are crucial because they allow modelling inefficiency (much like risk in equation 6) as a function of a number of variables external to the managerial practices of firms, which is in line with a large literature on the determinants of efficiency (see e.g., Lozano-Vivas and Pasiouras, 2010). This model is essentially a VAR between the two latent variables (σ_{it}^2 and u_{it}).

2.3. Econometric estimation

We specify all profit and return functions (i.e., equation 4) using the translog, which is the most frequently employed functional form in the relevant literature due to its nice mathematical properties and flexibility (e.g., Pasiouras et al., 2009). The econometric estimation of our model is conducted with maximum likelihood Bayesian techniques and

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[4] The log formulation is used because u_{it} is one-sided and treating it as such in the context of a VAR is more demanding in the estimation of this function below.

associated inference organized around MCMC. To avoid overburdening the reader with technical estimation details, we move all these to the Appendix.[5]

Despite the complexity of the Bayesian methods relative to conventional maximum likelihood that is used in the SFA literature (Kumbhakar and Lovell, 2000), our approach is preferred for two main reasons. First, our model includes dynamic latent variables, a fact that renders estimation with the usual maximum likelihood techniques completely impractical (if not impossible). Second, taking logs of profits can be particularly problematic if the firm is actually realizing losses because the observations with losses will be dropped.[6] Note that for the profit efficiency model above, the negative-profits problem is overcome if we assume that profit expectations Π^* are always positive within the Bayesian framework and simulated with MCMC. If this assumption is considered too strong, then the return efficiency model is clearly the preferred one because V in equation (3) is always positive.

3. Risk and efficiency in the U.S. banking industry

Our model can be applied to any industry, given data availability. Here, we use data from the U.S. banking industry. The banking industry is an ideal setting given the importance of the risky decisions of banks for their efficiency and the endogenous determination of the two. For example, banks with very low or very high levels of risky loans will be inefficient in the sense that they do not produce the optimal level of risky loans. Similarly, banks that misprice the riskiness of loans will be inefficient on the revenue side of their profit or return function.

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[5] Estimations are conducted using Fortran with extensive use of the NAG compiler, and all modules are available to the readers in an online supplement and in the personal website of the corresponding author. We should note that the modelling framework presented in the Appendix is our own work and specific to the estimation of equations (4)-(6). Previous studies using Bayesian methods to estimate stochastic frontier models are quite frequent (e.g., Van den Broeck et al., 1994; Tsionas, 2002; Orea and Kumbhakar, 2004).

[6] The usual practice in the literature is to add to all observations a constant larger than the maximum value of losses in the sample, thereby rescaling all profits to be positive. However, this approach is subject to substantial criticism because it biases the standard error and the inefficiency component of the regression.

The banking industry is also ideal given the availability of data over a long time period, allowing the robust identification of the risk-efficiency nexus.

We collect accounting bank-year (end-of-year) data from the Reports of Condition and Income (Call Reports) over the period 1976-2014.[7] We consider a relatively homogenous panel of commercial banks by restricting our sample to only the large ones (top 10% in terms of total assets) that operate and compete nationally. Our final working sample after further cleaning the data to eliminate objectionable values (negative total assets) and banks that ceased operations (due to M&As, failures or liquidations) consists of 15,922 bank-year observations.

In Table 1, we report summary statistics for the variables used in the empirical analysis. To define the inputs and outputs for the banking production process we use the intermediation approach (Sealey and Lindley, 1977; Pasiouras et al., 2009), which considers the financial assets of banks as outputs of production and financial liabilities and physical factors as inputs. Specifically, as bank outputs, we use the logs of total loans (y_1) and total securities (y_2). The inputs and associated prices include the fixed assets and their price (expenditures on fixed assets divided by premises and fixed assets), labor and its unit price (personnel salaries divided by the number of full-time equivalent employees), and borrowed funds and their price (interest expenses on deposits and interest expenses on fed funds divided by the sum of total deposits and fed funds purchased). We also include in equations (4), (6), and (7) the ratio of equity capital to total assets (*EQ/TA*) to control for bank capitalization and a time trend.

[Insert Table 1 about here]

For log*V,* we use total bank revenue (total income before taxes) to total cost (total operating cost). For the logΠ^*, which needs to be positive, we use simulated data from the

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Bayesian MCMC technique (see Appendix). The issue here is to choose the correct values of Π^* in the conditioning. The problem occurs in many other cases with so-called random effect models or models with missing data, which makes the Bayesian framework an ideal setting. Given our data set (D) and parameter γ in equations (6) and (7), if we manage to draw from the posterior conditional distributions $(\gamma|\Pi^*, D)$ and $(\Pi^*|\gamma, D)$, then effectively, we have a set of (correlated) draws from the joint posterior $(\gamma, \Pi^*|D)$. By retaining the draws for γ, we have draws from the posterior marginal $(\gamma|D)$, which is the central object of interest.

3.1. Empirical results

Before we estimate our model, we use the standard SFA and translog alternative profit and return functions to estimate equation (2) without risk. We use the well-established method of Battese and Coelli (1995), which allows the inclusion of *EQ/TA* as a determinant of inefficiency (in the fashion that we also favor in our models). The results show that the average bank inefficiency from the profit model is 10.3%, distributed with a standard deviation of 0.018. The equivalent average efficiency estimated from the return model is 15.2%, distributed with a standard deviation of 0.027.[8] Also, the coefficient estimate on *EQ/TA* as determinant of inefficiency equals 0.142 (0.041) in the profit (return) model and in both cases is statistically significant at the 5% level. We use these findings as benchmark to infer the importance of modeling risk within the SFA.

Turning to our modelling framework, we first estimate a simple stochastic frontier model equations (4)-(6) using the profit and return models. In Table 2 we report the posterior mean and standard deviation from these estimations. For expositional brevity, we only report the results for (6) because the parameter estimates of the translog profit function are too numerous to report and do not provide any substantial insights. The posterior mean and

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[8] It is quite impractical to report the full set of results from the translog specification (a very large number of estimates from the main and interactive terms of the translog), which also do not offer any important intuition for our purposes. These results are thus available on request.

standard deviation represent the mean and the standard deviation of the densities of each parameter estimate conditional on the data. These results can be interpreted in a similar fashion to the coefficient estimates and standard errors of the conventional econometric models. We cannot, however, directly speak about the level of statistical significance of coefficient estimates and we abstain from doing so in the discussion of our results (Lee et al., 2003).

The model fits the data reasonably well, with most of the variables included as determinants of risk in equation (6) strongly explaining it. Also, all models easily pass the Jarque-Bera test for normality, indicating that the effect of skewness and kurtosis in the form considered by Theodossiou and Savva (2016) is less potent in our data set. This might be due to the fact that we allow for skewness in equation (6) by including log expected profits and covariates z_{it}. Also, the system of equations (6) and (7) incorporates u_{it} and $u_{i,t-1}$, which is part of the error term, and this further reduces the effect of skewness.

The mean level of inefficiency is 14.7% for the profit model and 17.1% for the return model. Compared to the model without risk, inefficiency is higher and has a wider variance.[9] Additionally, Spearman's rank correlation between the two models (the degree to which the ranking of banks from the two models is the same) is statistically significant but not quite high (0.13). These are first-hand evidence that a failure to include risk results in significant downward biases in the estimates of inefficiency and misleading guidance with respect to which bank has a competitive advantage over another.

[Insert Table 2 about here]

The parameter estimates reported in Table 2 and their signs are intuitive. A value of the parameter estimate on the lagged component of risk between zero and one implies that

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[9] We note that the variance of inefficiency is relatively small in the return model and very small in the profit model. This is a first indication that the return model yields more appealing results. However, the small variance comes from the use of a sample of very large banks. In companion research we use all banks (not only the limited sample for large banks) and the variance is almost 10 times as much. Thus, the result in the variance of the inefficiency component is sample-driven and concerns the similarity of the banks used in the sample.

the volatility of profits persists but will eventually return to its normal (average) level. A value on this term close to zero implies a high speed of adjustment, while a value close to one implies very slow adjustment. Given the posterior mean estimates of 0.214 and 0.121 for the profit and the return models, risk is only moderately persistent, all else being equal. Further, a 10% increase in profit (return) inefficiency in the previous period is associated with a 3.2% (5.1%) increase in the volatility of profits (returns). This is first-hand evidence for a negative nexus between inefficiency and risk (positive between efficiency and risk), which we analyze in more detail below.

Finally, the effect of the equity to assets ratio is not strongly related to the volatility of either profits or returns (low posterior mean relative to the posterior standard deviation). This contradicts the standard finding in the banking literature that banks with higher capital are more risky due to a moral hazard mechanism, which holds that increasing capital requirements constitute a means of reducing bank managers' perceptions of the risk of default and increase risk-taking (e.g., Jokipii and Milne, 2011; Delis et al., 2015).

The results from the estimation of equations (4)-(6) cannot infer anything substantial about the direction of causality from risk to efficiency because inefficiency is not endogenous to risk. To study the interplay between risk and efficiency we estimate the VAR model of equations (4)-(7) and report the results in Tables 3 and 4 for the profit and return models, respectively.

[Insert Table 3 about here]

A striking result is that the mean inefficiency level of banks further diverges between the profit and return models (11.7% and 18.5%, respectively) compared to the results in Table 2 and the model without risk. Intuitively, the mere fact that the VAR model has more structure, by imposing interdependence between risk and inefficiency, should in principle imply higher values of inefficiency (as inefficiency additionally depends on risk). This

provides a strong indication in favor of the results of the return model (the mean inefficiency in Table 3 is actually lower than the equivalent of Table 2) compared to the profit model. Further, the Spearman correlation coefficient between the VAR return model and the equivalent benchmark model without risk is as low as 0.10, which provides a strong indication of the importance of the inclusion of risk to robustly identify the competitive advantage of one bank *versus* another.

[Insert Table 4 about here]

Importantly, both the profit and the return models predict that the lagged level of the inefficiency component has a strong positive impact on the current level of risk and that the lagged level of risk has a strong positive impact on the current level of inefficiency. Thus, we find a negative risk-efficiency nexus running both ways. Our estimates are also economically significant. According to our preferred return model, a 10% increase in the lagged inefficiency leads to a 5.6% increase in σ^2. Also, a 10% increase in the lagged risk leads to a 3.1% increase in inefficiency. Thus, risk is more responsive to inefficiency than inefficiency is to risk.

Our finding on the negative risk-efficiency nexus is in line with the Bowman (1980) paradox and consistent with the findings of e.g. Andersen and Bettis (2015). By distinguishing between the upside and the downside variance of profits, we further note that most of the negative risk-efficiency nexus is attributable to the downside variance (approximately 86% of the estimate), which is risk stemming from decreasing profits. Moreover, our sample consists of large banks and, thus, our result is in line with the literature suggesting a bell-shaped effect of bank size on performance (e.g., Avramidis et al., 2016). The premise of this literature is that banks experience diseconomies of scale or scope after reaching a very large size and this negatively affects their performance. In a nutshell, our empirical findings are mostly consistent with the prospect and behavioral theories of the risk-

return nexus, suggesting that inefficient banks either take on very high levels of risk in an effort to catch up with more efficient rival banks or their inherent inefficiency (due to e.g. large size in our sample) leads to risk mismanagement. Either way, the result is an inefficiently high level of risk, which will in turn further increase inefficiency in the following period.

Another interesting finding is the positive nexus between equity capital and both risk and inefficiency in the VAR return model, which we did not identify in the results of Table 2. The positive effect of capital on risk is consistent with the prediction of moral hazard theory, suggesting that the managers of well-capitalized banks seek higher levels of risk in search for yield (Jokipii and Milne, 2011; Delis et al., 2015). This is intuitive because banks facing higher capital requirements are forced to hold more capital on their balance sheets instead of using capital for more productive and profitable purposes.

Perhaps more important, Figure 1 shows the time evolution (annual mean estimates) of the downside variance of profits and returns. The findings are consistent with the history of financial turbulence in the U.S. The first small peak is around the early 1980s recession. Subsequently, our preferred return model indicates a short-lived peak around the Black Monday stock market crash in 1987, with risk remaining relatively high around the credit crunch in the early 1990s. The model also identifies the increased volatility around the Russian debt crisis, the dot-com bubble, and September 11 in the late 1990s and early 2000s. Finally, the annual mean estimates of the return model capture the substantial increase in bank risk related to the subprime crisis as early as 2005. In fact, the level of risk during the subprime crisis was by far the highest in our 1975-2014 sample period, a result in line with the perceptions of the depth of the subprime crisis and the toll that it took on the U.S. economy in general and banks in particular.

[Insert Figure 1 about here]

We intentionally keep the analysis presented in the paper short to highlight our baseline results. However, we did conduct a series of sensitivity tests on the VAR model. First, we add more variables as z in equations (6) and (7). We experiment with more than 20 variables characterizing bank size, liquidity, earnings management, and the competitive environment of banks. Second, we experiment with the Fourier and Leontief flexible functional forms for equation (4). The results from the above exercises (reported in Appendix C) are strikingly similar with those in Tables 3 and 4. Third, we add more lags for risk and inefficiency in equations (6) and (7). We find that this increases computational burden, whereas most of the higher lags do not seem to be important determinants of risk and inefficiency.

4. Conclusions and managerial implications

We build and estimate a stochastic frontier model that formally includes risk. The novel features of our model are that (i) risk is estimated from the variance of profits or returns and (ii) risk and efficiency are both endogenously determined as a function of each other. We apply the model to the U.S. banking sector (large banks) over a period of 40 years.

Our results have a number of managerial implications. First, we demonstrate that efficiency models that do not include a measure of risk, as estimated from the volatility of returns, produce a significant downward bias in the average level of inefficiency and a bias in the ranking of banks based on their efficiency levels. The latter is particularly important for correctly identifying the ranking of banks by their competitive advantage.

Second, there is a clear trade-off between risk and efficiency that is primarily attributable to banks with decreasing levels of efficiency (downside variance of risk). This trade-off goes both ways, with larger levels of inefficiency in the previous period yielding higher risk in the current period (which is the most potent effect) and higher risk in the

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previous period yielding higher inefficiency in the current period. Managers should be well aware of this trade-off, especially when they observe a downsizing of their efficiency levels, to avoid further future efficiency losses.

Third, the efficiency and risk estimates are intuitive and theoretically apt when risk and inefficiency are endogenous with respect to one another. For example, our risk estimates stemming from the return model are fully in line with the episodes of adverse developments in the U.S. banking sector over the last 40 years. These estimates even show how much higher was the average realized riskiness of the banking sector before and after the subprime crisis compared to other systemic events in the 1980s and 1990s.

An obvious extension of our work is to apply our model to other industries and compare the findings with those provided in our analysis of the banking sector. We predict that the findings will be very similar, provided that the sample includes homogenous and directly competitive firms. When working in this direction, we expect to encounter data sets where the normality assumption in the distribution of risk does not hold. Thus, it is imperative to extend our model in the direction proposed by Theodossiou and Savva (2016) and use alternative distributions such as the skewed generalized. Finally, our research opens up the possibility to introduce risk within a stochastic DEA model (e.g., Simar and Zelenyuk, 2011). This can be done in a multi-stage setting, including a non-parametric stochastic frontier framework that incorporates risk as per our analysis, a data generating process to transform data, and a DEA procedure on the transformed data. As our paper already covers considerable ground, we leave these proposals as a *desideratum* for future research.

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Appendix A. Markov Chain Monte Carlo methods

Part A

In this appendix, we specify the technical details of the MCMC around the basic risk and efficiency model. Consider the model

$$\log \Pi^*_{it} = x'_{it}\beta + v_{it} - u_{it} \tag{A.1}$$

where $\Pi_{it} = \Pi^*_{it} + \varepsilon_{it}$, with $\varepsilon_{it}|\sigma_{it} \sim N(0, \sigma^2_{it})$. The specification of the volatility equation is:

$$\log \sigma^2_{it} = \alpha_0 + \alpha \log \sigma^2_{i,t-1} + \gamma Q_{i,t-1} + z'_{it}\delta + w_{it}$$

where $w_{it} \sim iidN(0, \sigma^2_w)$, and $Q_{i,t-1}$ can be $\Pi^*_{i,t-1}, \Pi_{i,t-1}$ or $\log V_{i,t-1}$. Also, z_{it} is a vector of other predetermined variables.

The model can be written as:

$$\log \sigma^2_{it} = \alpha_0 + \alpha \log \sigma^2_{i,t-1} + \gamma \Pi^*_{i,t-1} + z'_{it}\delta + w_{it} \equiv \theta'_\sigma Q_{it} + w_{it} \tag{A.2}$$

or

$$\log \sigma^2_{it} = \alpha_0 + \alpha \log \sigma^2_{i,t-1} + z'_{it}\delta + w_{it} \tag{A.3}$$

Since $Q_{i,t-1}$ contains the observed variables $\Pi_{i,t-1}$ or $\log V_{i,t-1}$, these variables can be effectively treated as predetermined. The latent variables in this model are Π^*_{it}, u_{it}, and σ^2_{it}. The structural parameters are $\theta = (\beta, \sigma_v, \sigma_u, \alpha_0, \alpha, \gamma, \delta, \sigma_w)$.

Let $\theta_\sigma = (\alpha_0, \alpha, \gamma, \delta)$ be the parameter vector in the stochastic volatility equation. All error terms $(\varepsilon_{it}, w_{it}, v_{it}, u_{it})$ are assumed independent of x_{it} and z_{it}. Assuming $\theta_\sigma \in \mathcal{R}^{K_\sigma}$, we adopt the following priors:

$$\beta \sim \mathcal{N}_K(\bar{\beta}, \bar{V}_\beta) \tag{A.4}$$

$$\theta_\sigma \sim \mathcal{N}_{K_\sigma}(\bar{\theta}_\sigma, \bar{V}_{\theta_\sigma})$$

$$p(\sigma_j) \propto \sigma_j^{-(\bar{n}_j+1)}\exp(-\frac{\bar{q}_j}{2\sigma_j^2})$$

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where $j \in \{v, u, w\}$ and $\bar{\eta}_j, \bar{q}_j$ are prior hyper parameters. Moreover, $\bar{\beta}$ and $\bar{\theta}_\sigma$ denote the prior means of β and $\bar{\theta}_\sigma$, and $\bar{V}_\beta$ and $\bar{V}_{\theta_\sigma}$ denote the prior covariance matrices for these parameters.

Conditional posterior of β

Given Π_{it}^*, σ_w, and u_{it}, we can show that:

$$\beta | \Pi_{it}^*, \sigma_b, u_j, \Pi, X \sim N(\hat{\beta}, \hat{V}_\beta) \tag{A.5}$$

where $\hat{\beta} = \left(X'X + \sigma_N^2 \bar{V}_\beta^{-1}\right)^{-1} \left(X'\tilde{\Pi}^* + \sigma_N^2 \bar{V}_\beta^{-1} \bar{\beta}\right)$.

Conditional posterior of σ_v

Given Π_{it}^*, u_{it}, and β, we have that:

$$\frac{(\tilde{\Pi}^* - X\beta)'(\tilde{\Pi}^* - X\beta) + \bar{q}_N}{\sigma_v^2} | \Pi^*, u, \beta, \Pi, X \sim \chi^2(nT + \bar{\eta}_v) \tag{A.6}$$

Conditional posterior of u_{it}

Given $\Pi_{it}^*, \beta, \sigma_v$, the $u'_{it}s$ are conditionally independent in the posterior and we have:

$$u_{it} | \Pi^*, \beta, \sigma_v, \Pi, X \sim N_+\left(-\frac{r'_{it}\sigma_u^2}{\sigma_N^2 + \sigma_u^2}, \frac{\sigma_V^2 \sigma_u^2}{\sigma_V^2 + \sigma_u^2}\right) \tag{A.7}$$

where $r_{it} = \log \Pi_{it}^* - X'_{it}\beta$.

Conditional posterior of θ_σ

Given σ_{it}^2 and σ_w^2 we have:

$$\theta_\sigma |. \sim \mathcal{N}_{K_\sigma}(\hat{\theta}_\sigma, \hat{V}_{\theta_\sigma}) \tag{A.8}$$

where $\hat{\theta}_\sigma = \left(X'X + \sigma_\omega^2 \bar{V}_{\theta_\sigma}^{-1}\right)\left(X'X \log \theta^2 + \sigma_\omega^2 V_{\theta_\sigma}^{-1} \bar{\theta}_\sigma\right)$ and $\hat{V}_{\theta_\sigma} = \sigma_u^2 \left(X'X + \sigma_\omega^2 \bar{V}_{\theta_\sigma}^{-1}\right)^{-1}$.

Conditional posterior of σ_v^2

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We can show that

$$\frac{(\log \sigma^2 - Q'\theta_\sigma)'(\log \sigma^2 - Q'\theta_\sigma) + \bar{q}_w}{\sigma_w^2}|. \sim \chi^2(nT + \bar{n}_w) \tag{A.9}$$

where $Q = [Q_{it}; i = 1, \dots, n, t = 1, \dots, T]$. Drawing random numbers from (A.5)-(A.9) is straightforward. To draw from the posterior conditional distributions of Π_{it}^* and σ_{it}^2 is more involved. For this purpose let us consider the full kernel distribution:

$$p(\theta, \Pi^*, \log \sigma^2, u | \Pi, X) \propto \sigma_v^{-nT} \sigma_u^{-nT} \sigma_w^{-nT} e^{-\frac{1}{2\sigma_v^2}\sum_{i=1}^n \sum_{t=1}^T (\log \Pi_{it}^* + u_{it} - x'_{it}\beta)^2} \tag{A.10}$$

$$e^{-\sum_{i=1}^n \sum_{t=1}^T \log \Pi_{it}^*} e^{-\frac{1}{2\sigma_{it}^2}\sum_{i=1}^n \sum_{t=1}^T (\Pi_{it} - \Pi_{it}^*)^2}$$

where the terms $e^{-\sum_{i=1}^n \sum_{t=1}^T \log \Pi_{it}^*}$ and $e^{-\sum_{i=1}^n \sum_{t=1}^T \log \sigma_{it}^2}$ come from the Jacobian of transformation.

Conditional posterior of Π_{it}^*

It is convenient to define $h_{it} = log\Pi_{it}^*$, so that from (A.10) we can obtain the following conditional posterior:

$$p(h_{it}|.) \propto e^{-\frac{1}{2\sigma_v^2}(h_{it} + r_{it})^2 - h_{it} - \frac{1}{2\sigma_{it}^2}\left(\Pi_{it} + e^{h_{it}}\right)^2} \tag{A.11}$$

where $r_{it} = u_{it} - x'_{it}\beta$. The distribution does not belong to any known family. Since the tail behavior is determined by the first two terms in the exponential, which lead to a normal distribution if we employ completion of square, we use the following strategy.

Prepare a draw $h_{it}^* \sim \mathcal{N}(-(r_{it} + \sigma_v^2), \sigma_v^2)$. If the current draw is h_{it}^o we accept the proposed draw with a Metropolis-Hastings probability given by:

$$min\left\{1, \frac{e^{-\frac{1}{2\sigma_{it}^2}\left(\Pi_{it} + e^{h_{it}^*}\right)^2}}{e^{-\frac{1}{2\sigma_{it}^2}\left(\Pi_{it} + e^{h_{it}^o}\right)^2}}\right\} \tag{A.12}$$

A useful result is that if $\Pi_{it} < 0$ then (A.10) is always log-concave and the derivatives of the conditional log-posterior are:

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$$\frac{d}{dh}\log p(h|.) = -\frac{1}{\sigma_v^2}(h + r_{it}) - 1 + \frac{1}{2\sigma_{it}^2}(\Pi_{it} + e^h)e^h$$

$$\frac{d^2}{dh^2}\log p(h|.) = -\frac{1}{\sigma_v^2} + \frac{1}{\sigma_{it}^2}(\Pi_{it} + 2e^h)e^h$$

For the cases where $\Pi_{it} < 0$, log-concavity may be used to craft a better proposed distribution $h_{it}^* \sim \mathcal{N}(\hat{h}_{it}, \sigma_*^2)$, where $\frac{d}{dh}\log p(\hat{h}_{it}|.) = 0$ and $\frac{1}{2\sigma_*^2} = -\frac{d^2}{dh^2}\log p(h|.)$. Since in our sample the cases with negative profit are relatively few, we did not pursue further this approach, although it could prove essential in terms of numerical efficiency in other data sets, where losses were exclusively observed.

Conditional posterior of σ_{it}^2

From (A.10) and using $h_{it} = \log \sigma_{it}^2$, we obtain the following conditional posterior distribution:

$$p(h_{it}|.) \propto e^{-\frac{1}{2\sigma_w^2}\left(h_{it} - a_0 - ah_{i,t-1} - z_{it}'\delta\right)^2 - h_{it}} e^{-\frac{1}{2\sigma_w^2}\left(h_{it} - a_0 - ah_{i,t} - z_{i,t+1}'\delta\right)^2} e^{-\frac{q_{it}}{2\exp(h_{it})}} \qquad (A.13)$$

where $q_{it} = (\Pi_{it} - \Pi_{it}^*)^2$. The first two derivatives of the log-posterior in (A.11) are as follows:

$$\frac{d}{dh}\log p(h|.) = -\frac{1}{\sigma_w^2}\{B_{it} - A_{it} + (1-a)h\} + \frac{q_{it}}{2}e^{-h} - 1$$

$$\frac{d^2}{dh^2}\log p(h|.) = -\frac{1}{\sigma_w^2}(1-a) - \frac{q_{it}}{2}e^{-h}$$

Thus, the distribution is log-concave for $a < 1$ where:

$$A_{it} = a_0 + ah_{i,t-1} + z_{it}'$$

$$B_{it} = h_{i,t+1} - a_0 + z_{i,t+1}'\delta$$

Our strategy is to draw a proposal $h_{it}^* \sim \mathcal{N}(\hat{h}_{it}, \sigma_*^2)$, where $\frac{d}{dh} \log p(\hat{h}_{it}|.) = 0$ and $\frac{1}{2\sigma_*^2} = -\frac{d^2}{dh^2} \log p(\hat{h}_{it}|.)$. If the current draw is h_{it}^o, then the proposal is accepted using the Metropolis-Hastings probability:

$$\min\left\{1, \frac{p(h_{it}^*|.)/f_N(h_{it}^*; \hat{h}_{it}, \sigma_*^2)}{p(h_{it}^*|.)/f_N(h_{it}^o; \hat{h}_{it}, \sigma_*^2)}\right\} \tag{A.14}$$

where $f_N(h; \hat{h}_{it}, \sigma_*^2)$ denotes the density of a univariate normal distribution with mean $\hat{h}_{it}$ and variance σ_*^2.

Note

Under (A.2), drawing Π_{it}^* as in (A.11) with using (A.12) is no longer valid. The correct conditional posterior distribution in this case is given by:

$$p(h_{it}|.) \propto e^{-\frac{1}{2\sigma_v^2}(h_{it}+r_{it})^2 - h_{it} - \frac{1}{2\sigma_{it}^2}\left(\Pi_{it}-e^{h_{it}}\right)^2} e^{-\frac{1}{2\sigma_{it}^2}\left(\Pi_{it}-e^{h_{it}}\right)^2} e^{-\frac{1}{2\sigma_W^2}(G_{it}-\gamma e^{h_{it}})^2} \tag{A.15}$$

where $G_{it} = \log \sigma_{i,t+1}^2 - a_0 - a \log \sigma_{it}^2 - z_{it}'\delta h_{it} = \log \Pi_{it}^*$. In (A.15) the first two terms of the exponential still determine the tail behavior. Therefore, we can still use the proposal in (A.11), albeit the Metropolis-Hastings acceptance probability in (A.12) has to be modified as follows:

$$\min\left\{1, \frac{e^{-\frac{1}{2\sigma_{it}^2}\left(\Pi_{it}-e^{h_{it}^*}\right)^2 - \frac{1}{2\sigma_W^2}(G_{it}-\gamma e^{h_{it}^*})^2}}{e^{-\frac{1}{2\sigma_{it}^2}\left(\Pi_{it}-e^{h_{it}^o}\right)^2 - \frac{1}{2\sigma_W^2}(G_{it}-\gamma e^{h_{it}^o})^2}}\right\} \tag{A.16}$$

Part B

In the presence of systematic expectations, the second equation in (A.1) is modified as:

$$\Pi_{it} = \Pi_{it}^* + \varepsilon_{it} + I_{it}\xi_{it}^+ + J_{it}\xi_{it}^- \tag{A.17}$$

where $\xi_{it}^+ \sim N(0, \sigma_{\xi+}^2)$, $\xi_{it}^- \sim N(0, \sigma_{\xi-}^2)$,

$$I_{it}^* = z_{it}'\zeta_1 - \varepsilon_{1,it} \tag{A.18}$$

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$$J_{it}^* = z_{it}'\zeta_2 - \varepsilon_{2,it}$$

and

$$\begin{bmatrix} \varepsilon_{1,it} \\ \varepsilon_{2,it} \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix} = \Sigma\right) \tag{A.19}$$

with $I_{it} = 1 \Leftrightarrow I_{it}^* \geq 0, J_{it} = 1 \Leftrightarrow J_{it}^* \geq 0$.

Conditionally on $\xi_{it} = (\xi_{it}^+, \xi_{it}^-)$ and (I_{it}, J_{it}), we can still draw Π_{it}^* using (A.11) and (A.12) or (A.15) and (A.16) with minimal modifications. For example, (A.15) changes to:

$$p(h_{it}|.) \propto e^{-\frac{1}{2\sigma_v^2}(h_{it}+r_{it})^2 - h_{it} - \frac{1}{2\sigma_{it}^2}(\Pi_{it} - e^{h_{it}} - \mu_{it})^2} e^{-\frac{1}{2\sigma_W^2}(G_{it} - \gamma e^{h_{it}})^2} \tag{A.20}$$

where $\mu_{it} = I_{it}\xi_{it}^+ + J_{it}\xi_{it}^-$, and (A.16) is modified in the obvious way. The other posterior conditional distributions remain the same.

Conditional posterior of ξ_{it}

Given $I_{it}, J_{it}, \Pi_{it}^*$, and σ_{it}^2, from (A.17) we have:

$$\xi_{it}^+ \sim N_+\left(\frac{R_{it}^+\sigma_{\xi+}^2}{\sigma_{\xi+}^2 + \sigma_{it}^2}, \frac{\sigma_{\xi+}^2 + \sigma_{it}^2}{\sigma_{\xi+}^2 + \sigma_{it}^2}\right) \tag{A.21}$$

$$\xi_{it}^- \sim N_+\left(\frac{R_{it}^-\sigma_{\xi-}^2}{\sigma_{\xi-}^2 + \sigma_{it}^2}, \frac{\sigma_{\xi-}^2 - \sigma_{it}^2}{\sigma_{\xi-}^2 + \sigma_{it}^2}\right) \tag{A.22}$$

if $J_{it} = 1$ and zero otherwise, where $R_{it}^+ = \Pi_{it} - \Pi_{it}^* - I_{it}\xi_{it}^+$. In (A.21) and (A.22) the ξ_{it}^+ and ξ_{it}^- components are drawn conditional on each other.

Conditional posterior of I_{it}^* and J_{it}^*

Given parameter vectors I_1 and I_2 in (A.18) and Σ in (A.19) (that is given ρ), it is clear that:

$$\begin{bmatrix} I_{it}^* \\ J_{it}^* \end{bmatrix} \sim N\left(\begin{bmatrix} z_{it}'\zeta_1 \\ z_{it}'\zeta_2 \end{bmatrix}, \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}\right) \tag{A.23}$$

subject to $I_{it}^* \geq 0$ if $I_{it} = 1$ and $J_{it}^* \geq 0$ if $J_{it} = 1$. The requirement is equivalent to drawing from a bivariate truncated normal distribution from (A.23). The joint distribution is proportional to:

$$e^{-\frac{1}{2(1-\rho^2)}\{v_1^2 + v_2^2 - 2\rho v_1 v_2\}}$$

where $v_1 = I_{it}^* - z_{it}'\zeta_1$, $v_2 = J_{it}^* - z_{it}'\zeta_2$. Conditionally on J_{it}^* we have:

$$I_{it}^*|J_{it}^* \sim \mathcal{N}(z_{it}'\zeta_1 + \rho(J_{it}^* - z_{it}'\zeta_2), 1 - \rho^2) \tag{A.24}$$

$$J_{it}^*|I_{it}^* \sim \mathcal{N}(z_{it}'\zeta_2 + \rho(I_{it}^* - z_{it}'\zeta_1), 1 - \rho^2)$$

subject to

$$I_{it}^* \geq 0 \text{ if } I_{it} = 1, I_{it}^* < 0, \text{ otherwise}$$

$$J_{it}^* \geq 0 \text{ if } J_{it} = 1, J_{it}^* < 0, \text{ otherwise}$$

Drawing random numbers from truncated normal distributions (in the positive or negative directions) is straightforward. We use acceptance sampling from an exponential distribution whose parameter is optimized so that the acceptance rate is maximized (Tsionas, 2000, 2002).

Conditional posterior of $\zeta = (\zeta_1, \zeta_2)$

We adopt the priors:

$$\zeta_i \sim N(\bar{\zeta}_i, \bar{V}_i) \tag{A.25}$$

To derive the required conditional posterior distributions we proceed as follows. Standard results for Bayesian multivariate regression yield the following, based on Zellner (1971, pp. 241-242). Defining:

$\mathbb{Y}^* = \begin{bmatrix} I^* \\ J^* \end{bmatrix}$ and $\mathbb{Z} = \begin{bmatrix} Z & \\ & Z \end{bmatrix}$, where Z is the matrix of regressors z_{it}, we have:

$$\mathbb{Y}^* = \mathbb{Z}\zeta + \mathbb{U} \tag{A.26}$$

where $E(\mathbb{U}\mathbb{U}') = \Sigma \otimes I_{nT}$, $\Sigma = \begin{bmatrix} 1 & \rho \\ \rho & 1 \end{bmatrix}$, from which we obtain:

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$$\zeta|. \sim \mathcal{N}(\hat{\zeta}, \hat{V}) \tag{A.27}$$

where

$$\hat{\zeta} = [\mathbb{Z}'(\Sigma^{-1} \otimes I_{nt})\mathbb{Z} + \bar{V}^{-1}]^{-1} \times [\mathbb{Z}'(\Sigma^{-1} \otimes I_{nt})\mathbb{Y} + \bar{V}^{-1}\bar{\zeta}] \tag{A.28}$$

$$\hat{V} = [\mathbb{Z}'(\Sigma^{-1} \otimes I_{nt})\mathbb{Z} + \bar{V}^{-1}]^{-1}$$

$$\zeta = \begin{bmatrix} \zeta_1 \\ \zeta_2 \end{bmatrix}, \hat{\zeta} = \begin{bmatrix} \hat{\zeta}_1 \\ \hat{\zeta}_2 \end{bmatrix}, \hat{V} = \begin{bmatrix} \hat{V}_1 & \\ & \hat{V}_2 \end{bmatrix}$$

where Σ and Σ^{-1} have a particularly simple form.

Conditional posterior of ρ

This is given by:

$$p(\rho|.) \propto (1 - \rho^2)^{-\frac{nt}{2}} e^{-\frac{1}{2(1-\rho^2)}[Q - 2S_\rho]}, -1 < \rho < 1 \tag{A.29}$$

where $Q = \sum_{i=1}^{n} \sum_{t=1}^{T}(v_{1,it}^2 + v_{2,it}^2)$, $S = \sum_{i=1}^{n} \sum_{t=1}^{T} v_{1,it} v_{2,it}$, $v_{1,it} = I_{it}^* - z_{it}'\zeta_1$, and

$v_{2,it} = J_{it}^* - z_{it}'\zeta_2$. The maximum $\hat{\rho}$ of (A.29) can be located by standard univariate

optimization techniques, under the assumption that the prior is:

$$p(\rho) \propto const. 1_{(-1,1)}(\rho) \tag{A.30}$$

Although this is not essential, if the maximum is interior we can compute $\sigma_*^2 = -\frac{1}{\frac{d^2}{dp^2} \log p(e|.)}$

and propose a draw $\rho^* \sim N(\hat{h}, \sigma_*^2)$ restricted to the interval (-1, 1). Relative to the existing

draw ρ^0, the new draw is accepted with Metropolis-Hastings probability:

$$\min\left\{1, \frac{p(\rho^*|.)/f_N(\rho^*; \hat{\rho}, \sigma_*^2)}{p(\rho^0|.)/f_N(\rho^0; \hat{\rho}, \sigma_0^2)}\right\} \tag{A.31}$$

We have not encountered cases where the maximum of (A.29) occurs on the boundary.

Part C

In this part we take posterior analysis in the class of models defined by VAR:

$$\log \sigma_{it}^2 = a_{01} + a_{11} \log \sigma_{i,t-1}^2 + a_{21} \log u_{i,t-1} + \gamma_1 \Pi_{i,t-1}^* + z_{it}'\delta_1 + w_{1,it} \tag{A.32}$$

$$\log u_{it} = a_{02} + a_{12} \log \sigma_{i,t-1}^2 + a_{22} \log u_{i,t-1} + \gamma_2 \Pi_{i,t-1}^* + z_{it}' \delta_2 + w_{2,it}$$

where $w_{it} = \begin{bmatrix} w_{1,it} \\ w_{2,it} \end{bmatrix} \sim \mathcal{N}(0, \Omega)$. For all observations we can write the panel VAR in (A.32) as:

$$\log \sigma^2 = a_{01} + a_{11} \log \sigma_{(-1)}^2 + a_{12} \log u_{(-1)} + \gamma_1 \Pi_{(-1)}^* + z\delta_1 + w_1 \tag{A.33}$$

$$\log u = a_{02} + a_{12} \log \sigma_{(-1)}^2 + a_{22} \log u_{(-1)} + \gamma_2 \Pi_{(-1)}^* + z\delta_2 + w_2$$

which is in the form of multivariate regression as in (A.26) and therefore coefficients a_{0j}, a_{ij}, γ_i, and δ_i can be drawn following the procedure in (A.27) and (A.28).

The conditional posterior distribution of Ω is given by:

$$p(\Omega^{-1}|.) \propto |\Omega^{-1}|^{nt - \frac{3}{2}} \exp(-\frac{1}{2} tr \mathbb{A}^* \Omega^{-1}) \tag{A.34}$$

where $\mathbb{A}^* = \mathbb{W}'\mathbb{W}$, $\mathbb{V}^* = \begin{bmatrix} \log \sigma^2 \\ \log u^2 \end{bmatrix}$, $\mathbb{W} = \mathbb{V}^* - \mathbb{Z}^* \theta_{VAR}$, assuming (A.33) is written in the general form:

$$\mathbb{V}^* = \mathbb{Z}^* \theta_{VAR} + W \tag{A.35}$$

The prior $p(\Omega^{-1}) \propto |\Omega^{-1}|^{-3/2}$ is assumed. Drawing the exponential components ξ_{it}^+ and ξ_{it}^-, as in Section B, can be performed without modifications. Drawing the remaining latent variables is changed as follows.

Conditional posterior of σ_{it}^2

$$p(h_{it}|.) \propto e^{-\frac{1}{2\sigma_v^2}(h_{it} + r_{it})^2 - h_{it} - \frac{1}{2\sigma_{it}^2}\left(\Pi_{it} - e^{h_{it}} - \mu_{it}\right)^2} \tag{A.36}$$

$$e^{-\frac{1}{2\sigma_w^2}(G_{it} - \gamma e^{h_{it}})^2} e^{-\frac{1}{2}(\mathbb{V}_{it}^* - \mathbb{Z}_{it}^* \theta_{VAR})'(\mathbb{V}_{it}^* - \mathbb{Z}_{it}^* \theta_{VAR})}$$

where $\mathbb{V}_{it}^*$ and $\mathbb{Z}_{it}^*$ are the typical components of $\mathbb{V}^*$ and $\mathbb{Z}^*$ defined in (A.34) and (A.35).

Notably: $\mathbb{V}^* = \begin{bmatrix} \log \sigma_{it}^2 \\ \log u_{it} \end{bmatrix}$, $\mathbb{Z}^* = \begin{bmatrix} 1 & \log \sigma_{i,t-1}^2, \log u_{i,t-1}, ... \\ 1 & \log \sigma_{i,t-1}^2, \log u_{i,t-1}, ... \end{bmatrix}$ and $h_{it} \equiv \log \sigma_{it}^2$. Our strategy is

to locate the maximum and Hessian of (A.36) and propose a normal draw, which is accepted using the analogous necessary modification of (A.14).

Conditional posterior of u_{it}

This is given by

$$p(u_{it}) \propto e^{-\frac{1}{2\sigma_v^2}\left(\log \Pi_{it}^* - x_{it}'\beta + u_{it}\right)^2 - \frac{1}{2\sigma_u^2}u_{it}^2 - \log u_{it}} \tag{A.37}$$

$$e^{-\frac{1}{2}(\mathbb{V}_{it}^* - \mathbb{Z}_{it}^*\theta_{VAR})'\Omega^{-1}(\mathbb{V}_{it}^* - \mathbb{Z}_{it}^*\theta_{VAR})} e^{-\frac{1}{2}(\mathbb{V}_{i,t+1}^* - \mathbb{Z}_{i,t+1}^*\theta_{VAR})'\Omega^{-1}(\mathbb{V}_{i,t+1}^* - \mathbb{Z}_{i,t+1}^*\theta_{VAR})}$$

where u_{it} appears in both $\mathbb{V}_{it}^*$ and $\mathbb{Z}_{i,t+1}^*$. We use again a Metropolis-Hastings step and the model and Hessian of (A.37) as in the previous case of σ_{it}^2.

Conditional posterior of Π_{it}^*

Relative to (A.20) we now have the extra term:

$$e^{-\frac{1}{2}(\mathbb{V}_{it}^* - \mathbb{Z}_{it}^*\theta_{VAR})'\Omega^{-1}(\mathbb{V}_{it}^* - \mathbb{Z}_{it}^*\theta_{VAR})} \tag{A.38}$$

and Π_{it}^* appears in $\mathbb{Z}_{i,t+1}^*$. The most convenient in our case is to propose a draw as in (A.20) and use (A.38) or the Metropolis-Hastings acceptance probability relative to the existing draw $\Pi_{it}^{*(0)}$. However, a more efficient algorithm resulted by combining (A.20) and (A.38), is to find the mode and Hessian and proceed using an overall Metropolis-Hastings step.

Other numerical details

Our MCMC relies heavily on locating the mode and second derivative for a particular conditional posterior, denoted generically by $f(\theta)$. In all cases, we use a quasi-Newton algorithm with numerical derivatives to perform the computation. This is done during the transient or burn-in phase, which is tested for convergence using Geweke's (1992) diagnostics.

After the burn-in, to minimize computational costs we use one Gauss-Newton iteration away from the existing draw. In the case of the VAR model in (A.32) we find it more effective to maximize jointly with respect to $(\log \sigma_{it}^2, \log u_{it}, \log \Pi_{it}^*)$ and form a joint proposal to substitute univariate proposals from (A.36)-(A.38). Relative to the other schemes, this resulted in faster convergence and lower autocorrelations at lag 50. Replacing in this case the Gauss-Newton iteration with an algorithm that relies on full convergence did not provide significantly different performance and the results were mixed when the procedures were applied to subsets of the original data set.

Appendix B. Computation of Bayes factors

In the computation of Bayes factors the role of marginal likelihood is critical. If $p(\theta|Y)$ is a kernel posterior distribution, then the marginal likelihood is $\mathcal{M}(Y) = \int_\Theta p(\theta|Y)d\theta$, where $\theta \in \Theta \subseteq \Re^p$ is a structural parameter vector. For a number of models whose posterior distributions are $p_m(\theta|Y), m = 1, \ldots, M$, the marginal likelihoods are $\mathcal{M}_m(Y) = \int_\Theta p_m(\theta|Y)d\theta$ and the Bayes factors against, say, model 1 are given as:

$$BF_m = \frac{\mathcal{M}_m(Y)}{\mathcal{M}_1(Y)}, m = 2, \ldots, M$$

In this paper we use two ways to compute Bayes factors. First, following Verdinelli and Wasserman (1995), given a model whose posterior distribution is $p(\alpha, \beta|Y)$, a restricted model corresponding to $\alpha = 0$ can be evaluated based on the Bayes factor given by $BF = \frac{\int p(\alpha=0|\beta,Y)d\beta}{p(\alpha=0)}$, where the denominator provides the value of the prior distribution of α and the numerator can be computed as $\int p(\alpha = 0, \beta|Y)d\beta \simeq S^{-1} \sum_{s=1}^S p(\alpha = 0|\beta^{(s)}, Y)$ given the MCMC draws $\{\beta^{(s)}, s = 1, \ldots, S\}$. This approach can be used to test stochastic volatility models against their EGARCH counterparts by testing $\sigma_w = 0$.

This approach cannot be used always for non-nested models.[10] In general, the kernel posterior distribution has the form $p(\theta, \Lambda|Y)$, where $\theta \in \Theta \subseteq \Re^p$ is the structural parameter vector and Λ denotes the latent variables, like the collection of σ_{it}^2 and u_{it}. If all normalizing constants are included in the components of the kernel posterior, then we have the factorization:

$$\mathcal{M}(Y) = \int \int p(\theta, \Lambda|Y)d\theta d\Lambda = \int \int p(\Lambda|\theta, Y)p(\theta|Y)d\theta d\Lambda$$

Often the conditional distributions $p(\Lambda|\theta, Y)$ have a simple form and their normalizing constants are available in closed form. From MCMC we have a sequence of draws $\{\theta^{(s)}, \Lambda^{(s)}, s = 1, \dots, S\}$, which converges in distribution to the posterior whose kernel is given by $p(\theta, \Lambda|Y)$. Therefore, $\{\theta^{(s)}\} \to_D p(\theta|Y)$. From these results we can approximate the marginal likelihood as follows:

$$\mathcal{M}(Y) \simeq S^{-1} \sum_{s=1}^{S} \int p(\Lambda|\theta^{(s)}, Y)d\Lambda$$

Since the inner integral is not available in closed form, the marginal likelihood is approximated as:

$$\mathcal{M}(Y) \simeq S^{-1} \sum_{s=1}^{S} p(\Lambda^*|\theta^{(s)}, Y)$$

where $\Lambda^* = S^{-1} \sum_{s=1}^{S} \Lambda^{(s)}$ is the posterior mean of the latent variables, a point of high posterior probability mass. This approach makes it possible to approximate marginal likelihoods and Bayes factors easily and without large computational costs.

Appendix C. Results from additional sensitivity tests

In the tables of this Appendix, we report the results from additional sensitivity tests on the VAR model. In Tables C1 and C2, we include more variables (measured at the bank-year

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[10] For a general discussion see DiCiccio et al. (1997).

level) in the vector z in equations (6) and (7). Specifically, we use *size* (measured by the natural logarithm of total assets), *liquidity* (the ratio of liquid assets to total assets), *provisions* (the ratio of loan-loss provisions to total loans) and *market share* (the ratio of a bank's assets to the total bank assets in a given state). In Tables C3 and C4, we use the Fourier functional form

Table C1
Empirical results from the VAR model with additional variables
included as z in equations (6) and (7): Profit function

	$log\sigma_{it}^2$		$logu_{it}$	
	posterior mean	posterior s.d.	posterior mean	posterior s.d.
$log\sigma_{i,t-1}^2$	0.542	0.020	0.211	0.010
$logu_{i,t-1}$	0.302	0.112	0.212	0.002
$log\Pi_{i,t-1}^*$	0.148	0.020	0.106	0.122
Time trend	0.012	0.005	-0.030	0.014
EQ/TA	0.068	0.039	0.169	0.002
Size	0.105	0.091	0.014	0.011
Liquidity	-0.047	0.010	0.125	0.048
Provisions	0.081	0.008	0.018	0.003
Market share	0.040	0.059	-0.022	0.038
u	0.115	0.046		

Notes: The table reports the results (posterior mean and standard deviation) obtained from the estimation of equations (4)-(7) using Bayesian maximum likelihood and Markov Chain Monte Carlo. We report only the results from the determinants of stochastic volatility σ and inefficiency u, i.e. equations (6)-(7). We estimate a profit function and the functional form for (4) is the translog.

Table C2
Empirical results from the VAR model with additional variables
included as z in equations (6) and (7): Return function

	$log\sigma_{it}^2$		$logu_{it}$	
	posterior mean	posterior s.d.	posterior mean	posterior s.d.
$log\sigma_{i,t-1}^2$	0.419	0.019	0.294	0.015
$logu_{i,t-1}$	0.511	0.123	0.389	0.017
$log\Pi_{i,t-1}^*$	0.201	0.039	0.148	0.185
Time trend	0.008	0.009	-0.011	0.046
EQ/TA	0.095	0.015	0.082	0.021
Size	0.081	0.090	0.009	0.018
Liquidity	-0.035	0.027	0.111	0.061
Provisions	0.070	0.006	0.023	0.005
Market share	0.030	0.061	-0.016	0.090
u	0.180	0.041		

Notes: The table reports the results (posterior mean and standard deviation) obtained from the estimation of equations (4)-(7) using Bayesian maximum likelihood and Markov Chain Monte Carlo. We report only the results from the determinants of stochastic volatility σ and inefficiency u, i.e. equations (6)-(7). We estimate a return function and the functional form for (4) is the translog.

Table C3
Empirical results from the VAR model: Profit function (Fourier)

	$log\sigma_{it}^2$		$logu_{it}$	
	posterior mean	posterior s.d.	posterior mean	posterior s.d.
$log\sigma_{i,t-1}^2$	0.510	0.016	0.194	0.008
$logu_{i,t-1}$	0.320	0.129	0.183	0.002
$log\Pi_{i,t-1}^*$	0.103	0.021	0.093	0.140
Time trend	0.009	0.006	-0.027	0.016
EQ/TA	0.073	0.036	0.152	0.002
u	0.125	0.040		

Notes: The table reports the results (posterior mean and standard deviation) obtained from the estimation of equations (4)-(7) using Bayesian maximum likelihood and Markov Chain Monte Carlo. We report only the results from the determinants of stochastic volatility σ and inefficiency u, i.e. equations (6)-(7). We estimate a profit function and the functional form for (4) is the translog.

Table C4
Empirical results from the VAR model: Return function (Fourier)

	$log\sigma_{it}^2$		$logu_{it}$	
	posterior mean	posterior s.d.	posterior mean	posterior s.d.
$log\sigma_{i,t-1}^2$	0.487	0.014	0.301	0.009
$logu_{i,t-1}$	0.482	0.105	0.452	0.019
$log\Pi_{i,t-1}^*$	0.169	0.022	0.126	0.215
Time trend	0.006	0.010	-0.009	0.025
EQ/TA	0.089	0.010	0.082	0.020
u	0.188	0.044		

Notes: The table reports the results (posterior mean and standard deviation) obtained from the estimation of equations (4)-(7) using Bayesian maximum likelihood and Markov Chain Monte Carlo. We report only the results from the determinants of stochastic volatility σ and inefficiency u, i.e. equations (6)-(7). We estimate a return function and the functional form for (4) is the translog.

Table 1
Summary statistics

Variable	Mean	Median	Std. dev.	Min.	Max.
$\log\Pi$	9.16	9.01	1.31	1.09	15.57
$\log V$	0.21	0.20	0.30	-5.57	3.58
$\log w_1$	2.25	2.19	0.68	-0.17	5.56
$\log w_2$	2.63	2.58	0.61	0.69	4.54
$\log w_3$	1.10	1.17	0.43	-0.24	2.20
$\log y_1$	13.70	13.35	1.19	5.12	20.29
$\log y_2$	12.50	12.33	1.37	1.54	19.22
EQ/TA	11.71	11.36	1.15	7.92	18.44
Number of obs.	15,922	15,922	15,922	15,922	15,922

Notes: The table reports summary statistics for the main variables used in the empirical analysis. All variables are in natural logarithms. Π is profits before taxes; V is the ratio of total revenue to total cost; w_1 is the ratio of expenditures on fixed assets to premises and fixed assets; w_2 is the ratio of personnel salaries divided by the number of full-time equivalent employees; w_3 is interest expenses on deposits and interest expenses on fed funds divided by the sum of total deposits and fed funds purchased; y_1 is total loans; $y2$ is total securities; and EQ/TA is the ratio of total equity to total assets.

Table 2
Posterior results from the basic model for the volatility equation

	Profit function		Return function	
	Posterior mean	Posterior s.d.	Posterior mean	Posterior s.d.
$log\sigma^2_{i,t-1}$	0.214	0.098	0.121	0.015
$logu_{i,t-1}$	0.324	0.120	0.518	0.102
$log\Pi^*_{i,t-1}$	0.131	0.057	0.044	0.002
Time trend	0.007	0.007	0.004	0.013
EQ/TA	0.040	0.032	0.007	0.022
u	0.147	0.035	0.171	0.107
JB test	0.302		0.415	

Notes: The table reports the results (posterior mean and standard deviation) obtained from the estimation of equations (4)-(6) using Bayesian maximum likelihood and Markov Chain Monte Carlo. We report only the results from the determinants of stochastic volatility σ, i.e. equation (6). We estimate models by alternatively using profit and return functions. The functional form for (4) is the translog. The variables are defined in Table 1 and u represents inefficiency. JB test is the p-value of the Jarque-Bera tests for normality.

Table 3
Empirical results from the VAR model: Profit function

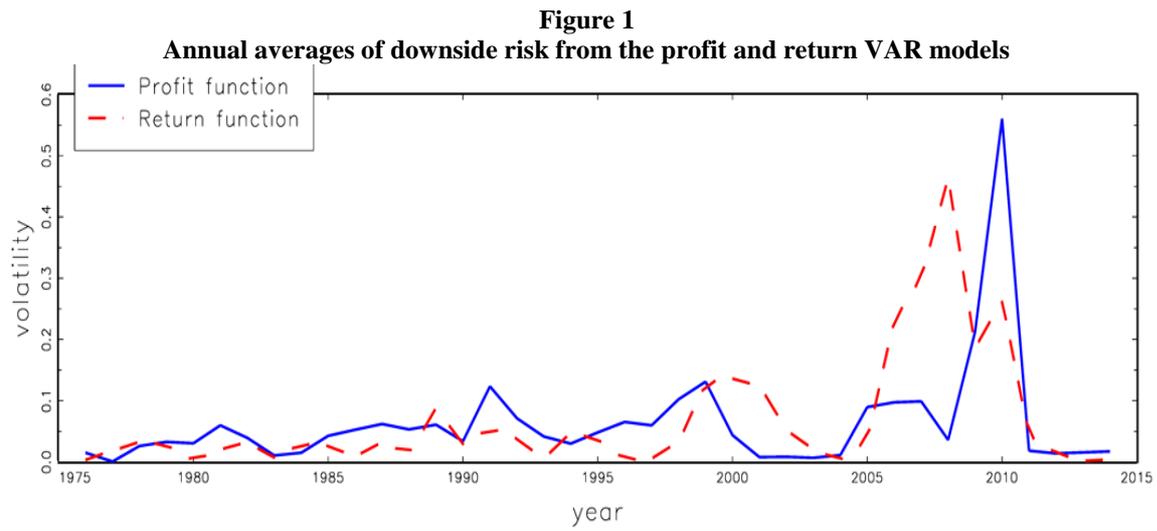|	$log\sigma_{it}^2$		$logu_{it}$	
	posterior mean	posterior s.d.	posterior mean	posterior s.d.
$log\sigma_{i,t-1}^2$	0.617	0.022	0.221	0.011
$logu_{i,t-1}$	0.315	0.116	0.225	0.002
$log\Pi_{i,t-1}^*$	0.151	0.026	0.117	0.116
Time trend	0.011	0.004	-0.032	0.012
EQ/TA	0.071	0.043	0.181	0.001
u	0.117	0.048		
JB test	0.364		0.259	

Notes: The table reports the results (posterior mean and standard deviation) obtained from the estimation of equations (4)-(7) using Bayesian maximum likelihood and Markov Chain Monte Carlo. We report only the results from the determinants of stochastic volatility σ and inefficiency u, i.e. equations (6)-(7). We estimate a profit function and the functional form for (4) is the translog. The variables are defined in Table 1 and u represents inefficiency. JB test is the p-value of the Jarque-Bera tests for normality.

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Table 4
Empirical results for the VAR model: Return function

	$log\sigma_{it}^2$		$logu_{it}$	
	posterior mean	posterior s.d.	posterior mean	posterior s.d.
$log\sigma_{i,t-1}^2$	0.453	0.014	0.315	0.017
$logu_{i,t-1}$	0.556	0.104	0.401	0.015
$log\Pi_{i,t-1}^*$	0.212	0.026	0.155	0.201
Time trend	0.009	0.007	-0.013	0.038
EQ/TA	0.102	0.021	0.077	0.025
u	0.185	0.124		
JB test	0.484		0.360	

Notes: The table reports the results (posterior mean and standard deviation) obtained from the estimation of equations (4)-(7) using Bayesian maximum likelihood and Markov Chain Monte Carlo. We report only the results from the determinants of stochastic volatility σ and inefficiency u, i.e. equations (6)-(7). We estimate a return function and the functional form for (4) is the translog. The variables are defined in Table 1 and u represents inefficiency. JB test is the p-value of the Jarque-Bera tests for normality.

Figure 1
Annual averages of downside risk from the profit and return VAR models

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