Incomplete Information, Proportional Representation and Strategic Voting

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Abstract

We introduce incomplete information to a multiparty election under proportional representation: each voter knows her preferences and votes strategically to maximize her payoffs, but is uncertain about the number and the preferences of the other voters. Parties are assumed to be purely office motivated and, hence, the resulting governments are always minimum winning. In this framework we prove a) generic existence of equilibria where only two parties receive a positive fraction of the votes and therefore lead to single-party governments and b) generic inexistence of equilibria that lead to coalition governments. That is, contrary to common wisdom, a proportional rule is found not to promote sincere voting and to be favorable towards single-party governments. The existence of two-party equilibria that lead to single-party governments is robust to parties having ideological concerns.

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1 Introduction

In his seminal work, Duverger (1954) formulated the following sociological laws: (1) a majority vote on one ballot is conducive to a two-party system, known as *Duverger's law* (Duverger, 1954, p. 217), (2) proportional representation (PR) is conducive to a multiparty system, known as *Duverger's hypothesis* (Duverger, 1954, p. 239). Despite the formal theoretical confirmation of Duverger's law by a class of models of *incomplete information* analyzing first-past-the-post (FPTP) elections (Fey, 1997; Myerson and Weber, 1993; Palfrey, 1989), there is no consensus on the theoretical validity of Duverger's hypothesis. In PR elections, a variety of spatial models sharing in common the feature of *complete information* provide contradicting results. While for example a class of very insightful but also very heterogeneous approaches prove the viability of many parties (Iaryczower and Mattozzi, 2013; Morelli, 2004; Baron and Diermeier, 2001; Austen-Smith and Banks, 1988),[1] another strand of the literature concludes that PR elections are favorable to two-party systems (De Sinopoli and Iannantuoni, 2007; Gerber and Ortuño-Ortín, 1998).[2]

The cardinality of the set of viable parties is very important as it closely relates to the nature of the formed governments under a PR rule: two-party systems lead to single-party governments while multi-party systems more often lead to coalition governments. In this paper, and given that in large elections the exact number of parties' supporters may not be commonly known, we introduce *incomplete information* in PR elections and

[1]While for example, Baron and Diermeier (2001); Austen-Smith and Banks (1988) focus on some bargaining process where further assumptions regarding the coalition formation process are necessary, Iaryczower and Mattozzi (2013) assume that each party forms a single-party government with a probability equal to its vote share.

[2]In a unidimensional policy space and under complete information voters strategically vote only for the two extreme parties. Centrist voters strategically vote only for the extreme parties since such strategic vote moves the implemented policy closer to their ideal point than a sincere vote for a centrist party. The main reason behind such strategic action is that implemented policies are a linear combination of the policies announced by the parties, weighted by parties' vote shares. With a similar argument De Sinopoli and Iannantuoni (2008) show that only extreme parties (but maybe more than two) obtain a positive vote share. Matakos et al. (2013) incorporate in such model different levels of the electoral rule disproportionality and show how it affects the number of competing parties. For two-party systems under alternative voting procedures such as the Borda count and approval voting see Dellis (2013); Dellis et al. (2011).

we show a) generic existence of equilibria where only two parties receive votes and hence lead to single-party governments[3] - and b) generic inexistence of equilibria which lead to coalition governments. That is, contrary to common belief, we argue that PR does not generically promote sincere voting nor is it favorable towards coalition governments.

Our modeling approach is in the framework of Bouton (2013); Fey (1997); Myerson and Weber (1993); Palfrey (1989). We consider a two stage game: in stage one, a multi-party election takes place under proportional representation and parties' vote shares are revealed. In stage two a government is formed. Borrowing the definitions of Le Breton et al. (2008) we require that the formed government is a) *winning* (the government is supported by a majority in the parliament), b) *stable* (there is no other winning coalition such that all parties that would participate in would be better off), and c) *vote-share consistent* (each coalition partner obtains office rents and influences the policy proportionally to its vote-share contribution). These three intuitive conditions broadly describe how proportional systems function. Proportional representation goes hand in hand with parliamentarism, and governments are, with few exceptions, supported by a majority of the parliament.[4] Governments are by definition stable (at least for a brief period of time that follows their formation), while the distribution of power and influence among the coalition partners is consistent with their vote shares.[5]

The formed government in stage two depends on parties' preferences: in our benchmark model parties are purely office motivated and care only about the share of government portfolios they control. In such an environment, if no party can form a single-party

[3]Even though two-party equilibria in PR elections may seem surprising at a first glance, historically, two-party systems contradicting Duverger's hypothesis were established in Germany, Austria, Ireland, and Australia (Lijphart, 1994; Riker, 1982). Currently, Malta provides an example of a two-party system in PR elections.

[4]Exceptions of countries traditionally governed by minority governments are Denmark and Norway. For a formal analysis of the emergence of minority governments see Kalandrakis (2015); Diermeier and Merlo (2000); Laver and Shepsle (1990); Austen-Smith and Banks (1990).

[5]We explicitly require that *Gamson's Law* holds in our setup. Gamson (1961) claimed that each party belonging to a coalition government obtains a share of portfolios proportional to the seats that each partner contributed to the coalition. This law has some empirical support (Warwick and Druckman, 2001; Browne and Franklin, 1973) and as Laver (1998) claims Gamson's law is associated with *"one of the highest non-trivial R-squared figures in political science"*. For recent theoretical advances see for example Le Breton et al. (2008); Carroll and Cox (2007); Fréchette et al. (2005).

government then the formed coalition government is minimal winning. As we show in section 3, non-minimal winning governments may be formed when we allow parties to also have policy motives - modeled by preferences on coalition partners.

In stage one, voters strategically vote for one of the parties maximizing their expected utility once the government is formed. Since we analyze PR elections, on one hand, one's vote may increase a party's weight in the formed coalition. On the other hand, a single vote may be pivotal in more than one dimensions: it may create or deter a single-party government, but it may also determine the coalition partners. Hence, under a PR rule the set of eventualities where one vote may affect the outcome of an election is larger than under a FPTP rule. That makes the analysis of strategic voting under PR systems not only interesting but also hard in terms of identifying all possible equilibria of the game.

We obtain most of our results in the context of purely office motivated parties. We first show the generic existence of two-party equilibria that lead to single-party governments. Such result is in line with the theoretical predictions regarding both FPTP (Fey, 1997; Palfrey, 1989) and two-round runoff systems (Bouton and Gratton, 2015; Bouton, 2013; Cox, 1997). We then obtain a uniqueness result regarding the type of formed governments by showing that when parties are office motivated only single-party governments are formed. Among others, this result allows us to argue that sincere voting is not a generic equilibrium of the game. Finally, we extend our model and show that the generic existence of two-party equilibria is robust to parties exhibiting any, possibly heterogenous, degree of mixture between office and ideology motivation.

In contrast to earlier papers supporting the existence of two-party equilibria, we assume that voters' utility depends on the formed government rather than on the electoral outcome. We further differ from these approaches by assuming that a voter never receives high utility when a coalition government is formed by parties she ranks low in her preferences. In particular, we assume that a voter can never be indifferent between a coalition government formed by her two least preferred parties and a single party government by her favorite party. Given that this assumption cuts the described channel of flows of votes from the "centrist" party to the extreme ones (see footnote 3), it promotes sincere voting, and thus works *against* two-party equilibria. Hence, our results further reinforce the relevance of the demonstrated two-party equilibria in PR elections (De Sinopoli and Iannantuoni, 2007; Gerber and Ortuño-Ortín, 1998). Another assumption selected in a way that works *against* two-party equilibria is the absence of a minimum vote-share threshold requirement for a party's representation in the parliament. A single vote may be enough for a party to obtain representation in the parliament and possibly become part of a coalition government. In that sense, we do not eliminate voters' incentives to cast a vote for a party that is expected to receive very few or even no votes.

When it comes to population uncertainty typically such assumption is made for technical reasons and tractability of the analysis while the main results hold for a deterministic number of voters (see for example Bouton 2013; Bouton and Castanheira 2012; McLennan 2011). In our model, population uncertainty has substantial implications and the existence of two-party equilibria in PR elections. If the number of voters were deterministic then two-party equilibria would exist conditional on the number of voters being even. Our general result thus crucially depends on one specific dimension of population uncertainty; on the uncertain parity of the cardinality of the set of voters. Given that such uncertainty is perfectly natural in large elections, where PR systems are employed, our results are of independent interest as far as the application of Poisson games in concerned.

In section 2, we present our benchmark model of purely office motivated parties and present most of our results. In section 3 we extend our model and show that our two-party equilibria existence result holds even if parties have some ideological concerns. In section 4 we conclude. All proofs can be found in the Appendix.

2 The Benchmark Model

We model PR elections as a two stage game. In the first stage, k voters vote for one of the *three office motivated parties* A, B, and C (our main results extend to any number of parties). In the second stage, and once parties' number of votes are revealed (V_A, V_B, V_C) a single-party or coalition government is formed. After stage two, parties' and voters' payoffs are realized.

We assume that the number of voters is a Poisson random variable k with mean n. We focus on large elections, that is when n is large.[6] Formally,

$$k \sim P(n) \Longleftrightarrow \Pr(k = m) = e^{-n}\tfrac{n^m}{m!}$$

Each voter has a strict preference ranking over the three parties that depends on her type. There are six types of voters denoted by $t \in T = \{t_{ABC}, t_{ACB}, t_{BAC}, t_{BCA}, t_{CAB}, t_{CBA}\}$. A t_{ABC} type denotes a voter who has a strict preference ordering for A over B, and for B over C. The six types of voters exhaust the possible strict preference rankings over the three parties. Each voter is assigned a type t by i.i.d. draws. The probability that a randomly drawn voter is assigned type t is $p(t)$, with $\sum_{t \in T} p(t) = 1$ and $p(t) > 0$ for all $t \in T$. These probabilities are common knowledge.

We assume that there is a continuum of offices and that a government is a distribution of these offices among coalition partners that satisfies Gamson's law. Hence, each coalition partner's weight in the government is proportional to its vote share contribution (we call these governments *vote-share consistent*). To make notation simpler we henceforth denote by

$$G \in \mathcal{G} = \{\{A\}, \{B\}, \{C\}, \{A, B\}, \{A, C\}, \{B, C\}, \{A, B, C\}\}$$

any *vote-share consistent* government. Formally party $J \in \{A, B, C\}$ is represented in government G with weight $W_J^G = \frac{V_J}{\sum_{Q \in G} V_Q}$ if $J \in G$ and $W_J^G = 0$ if $J \notin G$. Hence, W_J^G

[6] In the proofs we use the assumption that $n \to \infty$ to simplify math. All the results still hold if we substitute "$n \to \infty$" with "n sufficiently large".

is simply the share of offices controlled by party J in a vote-share consistent government G.

The utility of a t-type voter if government G is formed is then given by

$$U_t(G) = \sum_{J\in\{A,B,C\}} W_J^G u_t(J)$$

Without loss of generality the values of $u_t(J)$ are normalized so that a t-type voter obtains utility $u_t(J) = 1$ for J being her first ranked party, $u_t(J) = 0$ for J being her second ranked party, and $u_t(J) = -1$ for party J being her third ranked party. Therefore, if party J forms a single-party government then a t-type voter obtains utility $U_t(J) = u_t(J)$. If a coalition government is formed then voters' utility depends on parties' relative weights in the coalition and the valuation that voters assign to each coalition partner.

Example 1. *Since formed governments are vote-share consistent the relative weights of parties A and B in a $G = \{A, B\}$ government are $W_A^{\{A,B\}} = V_A/(V_A + V_B)$ and $W_B^{\{A,B\}} = V_B/(V_A + V_B)$, while party C independently of its vote-share has zero weight in the government $W_C^{\{A,B\}} = 0$.*

Once the government is formed a t-type voter obtains utility

$$U_t(\{A,B\}) = W_A^{\{A,B\}} u_t(A) + W_B^{\{A,B\}} u_t(B) = \frac{V_A}{V_A + V_B} u_t(A) + \frac{V_B}{V_A + V_B} u_t(B)$$

Having assumed that $u_t(J) = 1$ for J being a t-type's first ranked party, $u_t(J) = 0$ for J being her second ranked party, and $u_t(J) = -1$ for party J being her third ranked party, then we can analytically compute as examples the utility of a t_{ABC}, and t_{ACB} voter:

$$\begin{aligned} U_{t_{ABC}}(\{A,B\}) &= \frac{V_A}{V_A + V_B} u_{t_{ABC}}(A) + \frac{V_B}{V_A + V_B} u_{t_{ABC}}(B) = \\ &= \frac{V_A}{V_A + V_B} * 1 + \frac{V_B}{V_A + V_B} * 0 = \frac{V_A}{V_A + V_B} \end{aligned}$$

$$
\begin{aligned}
U_{t_{ACB}}(\{A,B\}) &= \frac{V_A}{V_A+V_B} u_{t_{ACB}}(A) + \frac{V_B}{V_A+V_B} u_{t_{ACB}}(B) = \\
&= \frac{V_A}{V_A+V_B} * 1 + \frac{V_B}{V_A+V_B} * (-1) = \frac{V_A - V_B}{V_A+V_B}
\end{aligned}
$$

Naturally, it holds that $U_{t_{ABC}}(\{A,B\}) > U_{t_{ACB}}(\{A,B\})$.

Here it is important to mention that a) our assumptions regarding the numerical values of $u_t(J)$ are not important regarding our main results (single party governments, non-sincere voting equilibrium and the existence of two party equilibria), and b) our results still hold for any smooth monotonic transformation of the W_J^G's. Clearly if one were to characterize all possible equilibria of the game then varying the valuations could affect the exact equilibrium strategy profile. Nevertheless since a complete characterization is impossible in this setup the chosen values of $u_t(J)$ are a technical simplification in our effort to reveal important characteristics of the equilibrium outcomes. One could alternatively assume that voters derive different utility levels from their first, second and third ranked alternatives but this would imply a significant cost in terms of notation and readability without enhancing our results in any direction. The irrelevance of these assumptions will be evident in the discussion and formal proofs of our main results.

In our benchmark model parties are purely office motivated and aim at maximizing their weight in the government irrespectively of the identity of their coalition partners. Given any government G party J obtains payoffs

$$U_J(G) = W_J^G$$

In order for a government to be formed we require that the latter is *winning* and *stable*. If more than one government is winning and stable then we assume that they are formed with equal probability.

Definition 1 (*Winning*) A government G is *winning* if and only if $\sum_{J \in G} V_J >$

$\frac{1}{2}(V_A + V_B + V_C)$.

Definition 2 (*Stable*) A government G is *stable* if and only if there exists no other G' such that $\sum_{J \in G'} V_J > \frac{1}{2}(V_A + V_B + V_C)$ and $U_J(G') > U_J(G)$ for every $J \in G'$.

Going back to the first stage of the game we define voters' strategies. Strategies are defined for types and not for individuals. Let $\sigma_J(t)$ denote the probability that a t-type voter votes for party J. The strategy of t-type voters is respectively defined as the vector $\sigma(t) = (\sigma_A(t), \sigma_B(t), \sigma_C(t))$. A strategy profile is denoted by $\sigma = (\sigma(t_{ABC}), \sigma(t_{ACB}), \sigma(t_{BAC}), \sigma(t_{BCA}), \sigma(t_{CAB}), \sigma(t_{CBA}))$. The number of votes that party J receives is a Poisson random variable with parameter $v_J n$ (which coincides with the expected number of votes of party J) where $v_J = \sum_{t \in T} \sigma_J(t) p(t)$.

Parties' behavior in the second stage is unambiguous and the formed government is perfectly predictable. We therefore define an equilibrium of the game only in terms of voters' strategies in the first stage. Given the incomplete information setup regarding voters' types we consider Bayesian Nash Equilibria (BNE) in pure strategies.

2.1 Results

Given that parties are purely office motivated their behavior in the second stage is unambiguous and described by the following lemma:

Lemma 1. *If a party obtains the majority of the votes then it forms a single-party government. If no party obtains the majority of the votes and since all parties are purely office motivated then only the minimal winning coalition is a winning, stable, and vote-share consistent government.*

(All proofs can be found in the Appendix.)

If parties are purely office motivated and no party obtains a majority of votes, then parties form a minimal *winning* coalition letting aside the party that received the most votes (Riker, 1962). Under *vote-share consistency,* a minimal winning coalition guarantees the maximal weight for both coalition partners among all possible coalition governments and hence qualifies the *stability* requirement. If any of the two partners were to form a coalition with the excluded party this would result into lower weight in the formed government, and hence lower payoffs.

We are now ready to prove that proportional elections may be conducive to a two-party system. This result is in line with the strand of the literature where in PR systems only two parties obtain votes in equilibrium (De Sinopoli and Iannantuoni, 2007; Gerber and Ortuño-Ortín, 1998) and sustains the well documented two-party equilibria of FPTP (Fey, 1997; Palfrey, 1989) and two-round runoff systems (Bouton and Gratton, 2015; Bouton, 2013). To understand our result from a comparative perspective, let us comment on the different events that voters may be pivotal under alternative electoral rules.

In FPTP elections, the unique case in which a voter may be pivotal is to break (or create) a tie in favor of the winner of the election. Performing a strategic calculation, a strategic voter casts a vote based on the highest pivotal probability, so that her vote possibly determines the winner of the election. Since the highest pivotal probability is associated with the probability that any of the two "large" parties wins, any voter who is ideologically close to a third small party strategically votes the best of the two "large" parties, rather than wasting her vote by supporting her favorite losing party. With a relatively similar argument, two-round runoff elections may give rise to a hunt of a first round victory that also produces multiple two-party equilibria.[7]

Conversely, in PR elections, strategic incentives are present for a strategic voter not to abandon her favorite "small" party. In our setup a single vote guarantees representation in the parliament and thus the possibility that one's favorite party participates in the minimal winning coalition. The class of pivotal events is therefore richer in PR elections than in FPTP elections. First, and similar to FPTP elections, a voter may give one necessary vote to a party so that it forms a single-party government. Second, and in contrast to FPTP elections, a voter may give one extra (or one less) vote to one party so

[7]Notice that runoff elections may sustain sincere voting and hence a multiparty equilibrium (Bouton, 2013).

that she strategically affects a) the set of coalition partners and b) the weights of coalition partners.

Taking the above elements into consideration, voters in the first stage vote for the party that maximizes their expected utility. Given that voters know that parties are purely office motivate they can predict the possible (minimal winning coalition) government formed in stage two as a function of parties' vote shares.

Proposition 1. (Existence of two-party equilibria) *In large elections a two-party pure strategy equilibrium always exist.*

In order to illustrate the intuition behind this result consider the strategic considerations of a t_{ABC} voter ($A \succ B \succ C$) forming expectations that her favorite party A will not obtain any vote. Such strategic voter may be pivotal in the following three events: i) party B has one vote less than party C, ii) parties B and C tie, or iii) party B has one vote more than party C. Hence, this voter may create or break a tie between party B and C, and possibly determine the minimal winning coalition including her favorite party A. First, notice that in these three pivotal events a vote for party C is dominated by a vote for party B. Hence, given such expectations a vote for party C consists a dominated strategy.[8] In order to analyze the decision of such voter between a vote for party A or party B we summarize the three pivotal events in Table 1.

[8]Under PR elections, and in contrast to FPTP elections, a vote for the last ranked party may not always consist a dominated strategy.

Pivotal Event	Formed coalition, valuations and expected utility if she votes:	A	B
$\hat{V}_B = \hat{V}_C - 1$	$\hat{V}_B > 1$	$\{A,C\}(\frac{1-\hat{V}_C}{1+\hat{V}_C}), [\frac{1-\hat{V}_C}{1+\hat{V}_C}]$	$\{B,C\}(-1/2), [-1/2]$
	$\hat{V}_B = 1$	$\{A,C\}(-1/3), \{B,C\}(-2/3), [-1/2]$	$\{B,C\}(-1/2), [-1/2]$
	$\hat{V}_B = 0$	$\{A,C\}(0), [0]$	{B,C}(-1/2), [-1/2]
$\hat{V}_B = \hat{V}_C$	$\hat{V}_B = \hat{V}_C > 1$	$\{A,B\}(\frac{1}{1+\hat{V}_B}), \{A,C\}(\frac{1-\hat{V}_C}{1+\hat{V}_C}), [\frac{2-\hat{V}_B}{2+2\hat{V}_B}]$	$\{B\}(0), [0]$
	$\hat{V}_B = \hat{V}_C = 1$	$\{A,B\}(1/2), \{A,C\}(0), \{B,C\}(-1/2), [0]$	$\{B\}(0), [0]$
	$\hat{V}_B = \hat{V}_C = 0$	$A(1), [1]$	$\{B\}(0), [0]$
$\hat{V}_B = \hat{V}_C + 1$	$\hat{V}_C > 1$	$\{A,B\}(\frac{1}{1+\hat{V}_B}), [\frac{1}{1+\hat{V}_B}]$	$\{B\}(0), [0]$
	$\hat{V}_C = 1$	$\{A,B\}(1/3), \{B,C\}(-1/3), [0]$	$\{B\}(0), [0]$
	$\hat{V}_C = 0$	$\{A,B\}(1/2), [1/2]$	$\{B\}(0), [0]$

Table 1: *Strategic considerations of a t_{ABC} voter. $\hat{V}_J$ denotes the number of votes that party J receives by the rest of the population (i.e. by everybody except this voter). Governments appear in curly brackets, their valuations in parentheses and expected utilities in brackets.*

In all pivotal events a vote for B leads to a predictable government. If B had one vote less than C now the two parties tie and have to form a coalition government with equal weights. If the two parties were tying or when B had one vote more than C then a vote for B allows the latter form a single-party government. A vote for A leads to a predictable government when at least one of B and C parties is expected not to obtain any votes ($\hat{V}_B = 0$, $\hat{V}_C = 0$, or both) or when both parties receive votes by more than one voters and they do not tie ($\hat{V}_B = \hat{V}_C - 1$ and $\hat{V}_B > 1$ or $\hat{V}_B = \hat{V}_C + 1$ and $\hat{V}_C > 1$). In the remaining situations where all parties receive votes and after the voter's support for A at least two parties tie more than one minimal winning coalitions exist with one of them being randomly formed.

Let us first focus on "small" elections. If the t_{ABC} voter is the only voter participating in the election (i.e. $\hat{V}_B = \hat{V}_C = 0$) or there is at most one other voter ($\hat{V}_B = \hat{V}_C - 1$ and $\hat{V}_B = 0$ or $\hat{V}_B = \hat{V}_C + 1$ and $\hat{V}_C = 0$) then in all three pivotal events she is strictly better off voting her favorite party A rather than her second ranked party B. When both parties B and C receive votes and at least one of them has exactly one vote, a t_{ABC} voter is indifferent between supporting A or B for all such three pivotal events.

In "large" enough elections ($\hat{V}_B > 1$ and $\hat{V}_C > 1$) it turns out that the voter is better off supporting her second ranked party B. In such elections, and for all three pivotal events, a single vote for her favorite party A guarantees its representation in the minimal winning coalition. In such coalition A's actual weight depends on the number of voters participating in the election supporting its coalition partner. In general, if the number of voters participating in the election is large enough an $\{A, B\}$ (or $\{A, C\}$) government where A is supported by only one voter is almost identical to a $\{B\}$ (or $\{C\}$) single-party government given the minimal weight of A. Because of this, a t_{ABC} voter is willing to abandon her favorite party A that has almost zero influence in an $\{A, C\}$ government in the search for a balanced $\{B, C\}$ government as presented in the first pivotal event ($\hat{V}_B = \hat{V}_C - 1 > 1$) and is indifferent between a single-party $\{B\}$ government and an $\{A, B\}$ coalition as presented in the third pivotal event. Similarly, a single-party government by B as presented in the second pivotal event ($\hat{V}_B = \hat{V}_C$) is strictly preferred to an $\{A, C\}$ government and indifferent to an $\{A, B\}$ government (so is the case in the third pivotal event where $\hat{V}_B = \hat{V}_C + 1$ and $\hat{V}_C > 1$).

Notice now that all three pivotal events in large elections are practically equiprobable (Myerson, 2000). In the first and second events the utility difference by voting B rather than A is positive and non-degenerate while in the third event the utility difference is negative and converges to zero when $n \to +\infty$. Hence conditional on the expected number of voters being large a t_{ABC} voter strategically votes for party B. Similarly, we can argue that if a t_{ACB} voter expects that party A will not receive any other votes she also abandons the latter and votes for party C. As a consequence, in equilibrium, only parties B and C receive votes giving rise to a two-party equilibrium.[9]

[9]Notice that while the calculated expected utilities clearly depend on the numerical values assumed for the valuations voters attach to parties the argument is generalizable and rather depends on the pivotal probabilities.

Are these two-party equilibria the unique equilibria of the game? Unfortunately the complexity of the analyzed environment does not permit a complete characterization of all equilibria. Nevertheless we can obtain a uniqueness result pointing at the type of formed governments under PR elections.

Proposition 2. (single-party government) *In large elections and if parties are purely office motivated only single-party governments may be formed in a pure strategy equilibrium.*

We will provide the intuition by contradicting an equilibrium leading to a coalition government. By lemma 1 a coalition is formed when no party has a majority of the vote. Given that we focus on pure strategies it also holds that, generically, no ties among parties are sustained in equilibrium and no party can obtain exactly half of the votes. So as to demonstrate the intuition of our result, let without loss of generality the beliefs that are part of such a BNE be consistent with C being the party that obtains the most votes, followed by B and A respectively (i.e. $0 < v_A < v_B < v_C < 1/2$). Given parties' office motives the minimal winning coalition $\{A, B\}$ is expected to be formed. Now consider the strategic incentives of a voter that ranks party C above the two others. Her vote may increase the weight of a party in the coalition government and it may determine the coalition government that will be formed. The probability of the last event coincides with the probability that her vote determines the winner of the election and converges to the probability of two independent draws from distinct Poisson distributions taking the same value. Comparing the expected gain of a voter from voting the party she prefers between A and B (by increasing its weight in the coalition government) with the probability that her vote is pivotal in determining the coalition composition it turns out that the former tends to be infinitely larger compared to the latter. Hence this voter and all others ranking C first have incentives to abandon their favorite party C providing a contradiction of such profile being a BNE.

This line of reasoning also establishes that when no party is ranked first by an absolute majority, then *sincere voting cannot be an equilibrium* of the game. This is straightforwardly so because for such generic distributions of voters' types sincere voting would lead to a coalition government and, as the above proposition dictates, there are no such equilibria under a PR rule. This reading of our results directly opposes the conventional belief that PR systems provide incentives to the voters to truthfully reveal their preferences.

This second proposition essentially provides a generic characterization of the whole set of pure strategy equilibria while the first proposition guarantees that this set is never empty. Hence, our analysis has so far established a) that there always exists a single-party government equilibrium and it is such that only two parties receive votes (Proposition 1) and b) that single-party government equilibria are generically the unique pure strategy equilibria of the game (Proposition 2). But can there exist a single-party government equilibrium, pure or mixed, such that all three parties receive votes?[10] Are there mixed equilibria which lead to coalition governments? The first class of equilibria cannot be ruled out, but even if they exist, they are of the same kind as the generic two-party equilibria that we identified (single-party governments are formed). The second class of equilibria, though, can fortunately be ruled out following similar arguments as in the proof of Proposition 2. If the beliefs that are part of such a mixed BNE are consistent with a coalition government, say $\{A, B\}$, being formed with a probability that converges to one (i.e. $0 < v_A \leq v_B < v_C < 1/2$ or $0 < v_B < v_A < v_C < 1/2$),[11] then a voter that ranks party C above the two others will face the same dilemmas as detailed in the paragraph below Proposition 2 and will prefer to vote for the coalition partner she likes most (either A or B) rather than stick with her most preferred alternative, C. This shows that such beliefs cannot be part of a BNE. Similarly, beliefs that are consistent with a

[10]Notice that not in all cases where all three parties receive votes a coalition government has to be formed. For example, let party C be expected to secure an absolute majority and hence form a single-party government. Obviously, in this case the reasoning described before does not apply: Types of voters ranking party C first do not expect an $\{A, B\}$ government and hence need not consider that voting for C is a lost vote.

[11]Despite the fact that mixed equilibria with $v_C = \frac{1}{2}$ cannot be ruled out, their possible existence should crucially depend both on distributional assumptions and, perhaps more importantly, on the assumptions regarding voters' exact cardinal preferences. That is, even if they exist they may in no way represent a robust prediction of the model.

coalition government, say $\{A, B\}$ or $\{A, C\}$, each being formed with a probability that converges to one half (i.e. $0 < v_A < v_B = v_C < 1/2$), can not be part of a BNE either. In this instance, all voters prefer to vote either for B or for C so as to determine the minimal winning coalition and nobody wants to vote for A (a voter votes for B if she likes C more than B so as to induce $V_C < V_B$ and hence a minimal winning $\{A, C\}$ government, similarly a voter votes for C if she likes B more than C).[12] Overall, inexistence of such equilibria makes the model give a very clear prediction: proportional representation is favorable to single-party governments.

3 Policy Motives

Aiming our analysis to be as general as possible, we now allow parties to be both office and ideology motivated. Parties not only aim at participating and maximizing their weight in the government *(office dimension)*, but also care for the ideological proximity of their coalition partner *(ideology dimension)*. Notice that for each party there exist two voter types who rank the latter first. For party A for example these are the t_{ABC} and t_{ACB} types. With no need to explicitly define the preference ordering over coalition partners, we assume that each party's J preference ordering aligns with the preference ranking of one of these two types.

Formally, let P_J denote the preference ordering of party J that coincides with that of type t. Given any government G party J obtains payoffs

$$U_J(G) = \lambda_J W_J^G + (1 - \lambda_J) U_t(G)$$

[12] The change in the weights of the coalition partners that one can induce by one's vote is bounded from above by $\frac{1}{(k+1)/2} = \frac{2}{k+1}$ (when k other voters participate then a coalition government has to receive at least $(k+1)/2$ votes). Hence, the expected change in a coalition's weights when voting for A is strictly smaller than $E(\frac{2}{k+1}) = 2\frac{e^n - 1}{ne^n}$ while the probability of being pivotal between B and C converges to $\frac{e^{-n(\sqrt{v_B} - \sqrt{v_C})^2}}{2\sqrt{\pi n \sqrt{v_B v_C}}} = \frac{1}{2\sqrt{\pi n v_B}}$ (see Myerson, 2000). We observe that $2\frac{e^n - 1}{ne^n} / \frac{1}{2\sqrt{\pi n v_B}} = 4\frac{(e^n - 1)\sqrt{\pi v_B}}{\sqrt{n}e^n}$ and that $\lim_{n \to +\infty} \frac{(e^n - 1)\sqrt{\pi v_B}}{\sqrt{n}e^n} = 0$. That is, the probability that one's vote is decisive in the choice of A's coalition partner becomes infinitely larger than the expected utility gain from increasing the weight of one's prefered party in a given coalition when the electorate becomes arbitrarily large.

where $\lambda_J \in [0, 1]$.

On the one hand, parties care for office (with relevant importance λ_J), and hence prefer to form a coalition with a weak partner in terms of vote share, so that they have a high weight in the coalition (W_J^G). On the other hand, parties care for the ideology of the government (with relevant importance $1 - \lambda_J$), and hence wish to form coalitions with ideologically close partners. Observe that i) parties may have different λ's and ii) that the two extreme cases of purely office motivated ($\lambda_J = 1$) and purely policy motivated parties ($\lambda_J = 0$) are incorporated in our analysis.

Regarding the second stage of the game we assume that a known to the voters *Government Formation Process* (GFP) takes place. Most papers studying proportional representation describe such process into detail (see for example Seidmann et al. 2007; Austen-Smith and Banks 1988). We refrain from doing so in order to guarantee the highest possible degree of generality. Given the electoral outcome $\{V_A, V_B, V_C\}$, parties' ideological and office-holding orientation $\{\lambda_A, \lambda_B, \lambda_C\}$, and parties' preference profile $\{P_A, P_B, P_C\}$ we allow any GFP that provides a government G that is *vote-share consistent, winning*, and *stable* (if meaningful).

A *winning* and *vote-share consistent* coalition government always exists for any possible electoral outcome and model parameters. If one party obtains more than half of the votes then it is straightforward that a single-party government is *winning*, and is by definition *vote-share consistent* since the winner of the election enjoys all rents. If no party obtains the majority of the votes then any two-party coalition is *winning*. *Vote-share consistency* can always be satisfied by splitting benefits proportionally to coalition partners' vote-share contributions. Hence there always exists at least one *winning* and *vote-share consistent* government.

Although a single-party government is always *stable* notice that if no party obtains the majority of the votes then a two-party *stable* government may not always exist (this is why we require a stable government only when meaningful). If for example party A prefers to form a coalition with B rather than C, and B prefers to form a coalition with A

rather than C, then the coalition government $\{A, B\}$ is formed and is *stable*. Conversely, in case of non-mutual agreements that may for example stem from a cycle in terms of preference orderings for coalition partners the notion of *stability* does not apply. For example, if party A prefers as a partner B to C, B prefers as a partner C to A, and C prefers as a partner B to A the stability condition need not be satisfied. In such cases we require that the GFP provides only a *winning* and *vote-share consistent* government according to some (undefined) rule known to the voters without imposing any further restrictions.[13]

Similarly, if more than one stable and winning coalition exist we assume that from the voters' point of view the selection of the coalition government by the GFP follows a stochastic process. That is, voters know that each stable and winning coalition is selected as the government coalition with some non-degenerate positive probability.[14] As it will be clear, the above inexplicit characterization of the GFP by two (or three) broad properties is enough to guarantee the existence of two-party equilibria. Henceforth, it is important to bear in mind that our GFP is a general government formation process known to the voters, and guarantees that for every electoral outcome a government is formed.

3.1 Two-party Equilibria

Although the main result of this section is related to our previous arguments some further lemmas are necessary.

Lemma 2. *For all values of $\lambda_J \in [0, 1]$, if party J obtains the majority of the votes then it forms a single-party government. If not all parties are purely office motivated then the government coalition may be non-minimal.*

[13]The description of how a government is formed when no stable coalitions exist is presented here only for completeness. Such description is irrelevant regarding our results since in all cases of interest there always exists at least one stable coalition.

[14]The probability with which the coalition government is selected out of a set a stable winning coalitions should obviously depend on parties' exact vote shares, on their *ideology-office holding* orientation and on their policy preferences. Since for our analysis an explicit formal structure of all the above is redundant, we refrain from formally defining properties of this probability. The only assumption we impose is that whenever the set of stable winning coalitions includes more than one element, then a non-degenerate positive probability is assigned to each of these elements.

The first part of this result is straightforward and is implied by the definition of parties' preferences and the assumed GFP. A single-party government guarantees all office and ideology rents for the winner of the election. Hence, a single-party government is the unique government that is *winning, stable* and *vote-share consistent* when a party obtains the majority of the votes.

In contrast to the case of purely office motivated parties a minimal winning coalition may not be *stable* when parties care for the ideological proximity of their coalition partners (Axelrod, 1970; De Swan, 1973). One of the two partners in the minimal winning coalition may be disposed to renounce some weight in the coalition (some office rents) in order to participate in a government with an ideologically close partner. The amount of office rents party J is willing to give away in order to collaborate with an ideologically close coalition partner depends on the relative importance parties attach to their office versus ideology motives (parameter λ_J). Intuitively, the more ideology motivated parties are (small values of λ_A, λ_B and λ_C), the higher is the probability that the winning coalition is non-minimal.

To illustrate the formal proof of the above result consider for example three parties competing in a large election obtaining approximately the same amount of votes. Let in particular one party receive one and two more votes than its two competitors respectively. If all parties are purely office motivated, then clearly the two parties that received the least votes form a minimal winning coalition. If parties are not purely office motivated, a minimal winning coalition may not be *stable* any longer. The party that received the second largest vote share may have incentives to form a coalition with the party that received the most votes if the latter is closer to its ideology than the party that received the least votes. In other words, the party that received the second largest vote share may opt for less weight in a coalition with an ideologically close partner rather than more weight in the minimal winning coalition. Next we show that these forces have their limitations: when two parties are equally large and the third one is tiny, then both office and policy motives make this third party be an attractive coalition partner.

Lemma 3. *Let the level of participation in the election be sufficiently high ($k > 7$). If the votes for the two parties that receive the most votes differ by at most one vote and the third party receives a single vote, then the latter is a member of at least one winning, vote-share consistent, and stable coalition.*

The above lemma provides a sufficient condition ($k > 7$) for the existence of a winning, vote-share consistent, and stable government in which a party that obtains a single vote participates.[15] Since the latter is not powerful in terms of vote share, both other parties prefer it as a coalition partner to the other strong party no matter how close it is to them ideologically. Moreover, when the two parties that receive the most votes tie, the party that receives a single vote determines the coalition government by choosing its coalition partner.

Building on the previous preliminary results, we can now show that proportional elections may be conducive to a two-party system even if parties have some ideological concerns. Remember that when voters calculate their expected utility they are aware of the GFP and can predict the possible coalition governments formed in stage two as a function of the electoral outcome.

Proposition 3. (Existence of two-party equilibria) *In large elections and for all values of $\lambda_A, \lambda_B, \lambda_C \in [0, 1]$ two-party equilibria exist.*

In order to illustrate the intuition of this result we follow similar arguments as in Proposition 1. Let us assume, without loss of generality, that party's A preferences coincide with those of a t_{ABC} type (A prefers as a coalition partner party B to party C). As before consider the strategic considerations of a t_{ABC} voter who forms expectations that party A will not obtain any vote, while the two other parties are expected to obtain a positive number of votes. Remember that the events in which this voter may be pivotal

[15] Given the numerical values we have attached to the valuations $u_t(J)$ the condition of Lemma 3 is satisfied if more than seven voters participate in the election ($k > 7$). This numerical condition would vary for different values of $u_t(J)$. However, different thresholds for k do not affect the intuition and results of neither Lemma 3 nor of the following proposition.

are the same as when parties are purely office motivated (parties B and C tie, or when one of the two parties has an advantage of one vote) and that a vote for party C is dominated by a vote for party B. We summarize the pivotal events, whether the condition of Lemma 3 is satisfied or not, and the sign of the corresponding utility difference from a vote for her second ranked party B rather than her first ranked party A in the following table.

Pivotal Event	**k > 7**	**Utility from a vote for B minus utility from a vote for A.**
$\hat{V}_B = \hat{V}_C - 1$	Yes	$+$
	No	$+ / -$
$\hat{V}_B = \hat{V}_C$	Yes	$-$
	No	$+ / -$
$\hat{V}_B = \hat{V}_C + 1$	Yes	$-$
	No	$+ / -$

Table 1: *Strategic considerations of a t_{ABC} voter. $\hat{V}_J$ denotes the number of votes that party J receives by the rest of the population (i.e. by everybody except this voter).*

The crucial and only pivotal event in which the utility from voting party B is guaranteed to be larger than the utility from voting party A is when party B is expected to have one vote less than party C conditional on the number of voters being large (condition of lemma 3 is satisfied). In this event, a vote for party B creates a tie between parties B and C. Thus, a balanced $\{B, C\}$ government is formed. Conversely, a vote for party A may result either to an unbalanced $\{B, C\}$ (where parties B and C have "almost" equal weights since party B has one less vote than party C) or to an $\{A, C\}$ coalition. If a $\{B, C\}$ coalition is to be formed then the voter is better off supporting party B rather than party A since it raises B's weight in such coalition. If an $\{A, C\}$ coalition is to be formed notice that in such coalition party C has much larger weight than party A. Given the large number of voters, the utility from an $\{A, C\}$ government where party A has tiny weight converges to the utility from a single-party $\{C\}$ government. Therefore, a

t_{ABC} voter significantly prefers a balanced $\{B, C\}$ coalition than an unbalanced $\{A, C\}$ coalition, whereas she only marginally prefers the balanced $\{B, C\}$ to the unbalanced $\{B, C\}$ coalition. Since both coalitions (the balanced $\{B, C\}$ or the unbalanced $\{A, C\}$) are formed with a non-degenerate probability the utility for a vote for B minus the utility for a vote for A is positive and non-degenerate for any $k > 7$. Hence, conditional on the election being large and that party B is expected to have one vote less than party C a t_{ABC} voter is significantly better off by voting her second ranked party B than her first ranked party A.

Remaining in large elections ($k > 7$), the other two pivotal events are situations in which party B is expected to tie or to have one vote more than party C. In both events, a vote for party B allows B to form a single-party government. Conversely, a vote for A leads to an $\{A, B\}$ government and hence larger utility for a t_{ABC} voter than a single party $\{B\}$ government. Similar to our previous argument notice that, since we refer to large elections, the weight of party A in the $\{A, B\}$ coalition is tiny, and thus the utility difference between an $\{A, B\}$ coalition or a single party $\{B\}$ government converges to zero.

Remember that all three pivotal events in large elections are practically equiprobable (Myerson, 2000). Given that in the first event the utility difference is positive and non-degenerate while in the two other events the utility difference is negative and converges to zero, we conclude that conditional on the number of voters being large a t_{ABC} voter strategically votes for party B.

Now, if only few voters participate in the election (condition of Lemma 3 is not satisfied), in all three pivotal situations a t_{ABC} voter could be either better off or worse off by voting her first ranked party A compared to party B (this would depend on the exact values of each λ_J and k). For example, given the small number of the total votes cast, a single vote for party A could possibly translate to an important weight in the formed government and thus strong incentives for such voter not to abandon her first ranked party A. Nevertheless, when the voter takes into consideration the pivotal probabilities it turns out that the pivotal events conditional on few voters participating in the election do not affect her decision. In particular, the following formal argument is crucial: As n becomes large the probability that $k \leq 7$ converges to zero and also becomes infinitely smaller than the probability of the event $\{k > 7 \text{ and } \hat{V}_B = \hat{V}_C + \eta \text{ where } \eta \in \{-1, 0, 1\}\}$ despite the probability of the latter event also converging to zero. Therefore, the utility differences for $k \leq 7$ are insignificant as far as the voters' decision is concerned.

Remember that conditional on the election being large $(k > 7)$ a t_{ABC} voter strategically votes party B in the effort to create a tie between parties B and C. For the same reasoning if a t_{ACB} voter expects that party A will not receive any other votes while parties B and C will obtain a positive number of votes then a t_{ACB} voter abandons party A and votes for party C. Therefore, in equilibrium, only parties B and C receive votes. Hence, two-party equilibria exist as both types who rank party A first (t_{ABC} and t_{ACB}) aim at creating a tie between parties B and C and thus a $\{B, C\}$ government. Notice that such tie is meaningful only if the number of voters k is even. This is possible in our model since we assume a random number of voters. Had we assumed a deterministic number of voters then our main result would not any longer hold if it was known that the number of voters is odd.

Without entering into details, notice that two-party equilibria exist no matter how many parties compete in the election. The reasoning just described remains unaffected; if a voter expects that in equilibrium only two parties receive votes by the rest of the voters, then in the effort to create the "desired" tie she strategically votes the party she ranks higher between these two parties. As far as stability is concerned, notice that the two-party equilibria that we proved to exist are pure strategy equilibria. This implies that voters are, generically, not indifferent between two distinct available actions. Since in Poisson games the probabilities of all relevant eventualities are continuous in parties' expected vote shares, we must have that for any n there should exist $\mathring{\varepsilon} > 0$ such that whenever party A is expected to receive a vote share $\varepsilon \in (0, \mathring{\varepsilon})$ then all voters still strictly prefer to vote for either B or C. That is, our equilibrium satisfies the notion of $\varepsilon-$stability used in similar models (see Bouton 2013 for a more detailed discussion of this property).

4 Concluding Remarks

We introduce incomplete information in PR elections. Given the absence of consensus in the literature regarding the number of surviving parties, our results further suggest that PR elections may be conducive to a two-party system. Even though the strategic incentives in FPTP systems to ultimately vote one of the two "large" parties are unambiguously more direct than in PR systems, we show that the latter may as well have a devastating effect towards all but two parties. Hence, despite PR systems being often considered to represent voters' preferences better than FPTP systems and promote sincere voting our results show that if information is incomplete this is may not be true.

Since this paper consists a first attempt to analyze incomplete information in PR elections we considered a simple and intuitive setup comparable to the ones focusing on FPTP and two-round runoff systems (Bouton, 2013; Fey, 1997; Myerson and Weber, 1993; Palfrey, 1989). When it comes to the details regarding PR systems, in order to obtain robust results, we structured the model so that two-party equilibria are not promoted. First, we did not require a minimum threshold such that a party is represented in the parliament. Second, we assumed that for any voter a coalition government between two low ranked parties can never be as near as good as a single-party government by her highest ranked party.

Despite our formal analysis focusing on a three party election our results naturally extend to a more general setup with any finite number of parties. As long as voters have strict preferences over parties, no matter how many parties compete in the election, two-party equilibria exist. Finally, we consider our analysis of interest regarding the application of Poisson games. Assuming population uncertainty we provide a result of two-party equilibria that vanishes once it is certain that the number of voters is odd. Given that our analysis focuses in large elections, considering a random number of voters as introduced by Myerson (2000) seems the appropriate methodology to proceed.

5 Appendix (Proofs)

Lemma 1 If party J obtains the majority of the vote (that is $V_J > \frac{1}{2}k$) then by forming a single-party government $G = J$ where $W_J^G = 1$ it obtains payoff $U_J(J) = 1$. For any other coalition government $G \neq J$ then it is straightforward that $U_J(G) < U_J(J)$ since it holds that $W_J^G < 1$. If no party has a majority of votes then party J maximizes its utility by forming a coalition government with Q if and only if $V_Q \leq V_R$.[16] It is obvious that if all parties tie then any coalition is minimum winning. If at least one party's number of votes is different from the ones of the other parties then *a)* in case all number of votes are different the two bigger parties want to form a coalition with the smallest one and the smallest one wants to form a coalition with the second smallest one *b)* if two parties tie and receive less votes than the other party then they wish to form a coalition with each other and *c)* if two parties tie and receive more votes than the other party then both want to form a coalition with the smallest party while the smallest party is indifferent. In all three cases only a minimal winning coalition is *stable, winning* and *vote-share consistent.*QED

Proposition 1 A BNE consists of strategies and beliefs. We will prove that if the expected vote share of A is zero and the expected votes shares of B and C are strictly positive then a t_{ABC} will vote for B (equivalently one can show that a t_{ACB} voter will vote for C). Consider the first pivotal event $\hat{V}_B = \hat{V}_C - 1$ as presented in Table 1, where $\hat{V}_B$ and $\hat{V}_C$ denote the number of votes that B and C receive by the rest of the population. If $\hat{V}_B = 0$ then the voter is strictly better off voting A. If $\hat{V}_B = 1$ then the voter is indifferent between voting A and B. If $\hat{V}_B > 1$ then the voter is better off voting B for a large enough number of voters. Notice that $prob(\hat{V}_B = 0|\hat{V}_B = \hat{V}_C - 1) \to 0$, $prob(\hat{V}_B = 1|\hat{V}_B = \hat{V}_C - 1) \to 0$ and $prob(\hat{V}_B > 1|\hat{V}_B = \hat{V}_C - 1) \to 1$ as $n \to \infty$ and that utility differences are bounded from above and below (conditional on $\hat{V}_B = \hat{V}_C - 1$ our voter's choice will affect her utility by at most $\frac{1}{2}$ independently of how many voters vote). Therefore, conditional on $\hat{V}_B = \hat{V}_C - 1$ the voter decides which party to support

[16] Where J, Q and R are distinct elements of $\{A, B, C\}$.

based on the most probable event in large elections (i.e. $\hat{V}_B > 1$) and votes B (i.e. the expected utility difference by voting B rather than A is equal to $\frac{1}{2}$). Exactly the same argument applies for the second pivotal event where $\hat{V}_B = \hat{V}_C$. Consider now the third pivotal event $\hat{V}_B = \hat{V}_C + 1$. If $\hat{V}_C = 0$ then the voter is strictly better off voting A. If $\hat{V}_C = 1$ then the voter is indifferent between voting A and B. If $\hat{V}_B > 1$ then for a large enough number of voters the voter is essentially indifferent between voting A and B. Given the same argument regarding pivotal probabilities, conditional on $\hat{V}_B = \hat{V}_C + 1$ the expected utility difference by voting B rather than A is negative but converges to zero as $n \to \infty$. Notice now that all three pivotal events are equiprobable (Myerson, 2000). Given the positive and non-degenerate utility difference from the first two events a t_{ABC} will vote for B (equivalently a t_{ACB} voter will vote for C). Hence, beliefs $\{v_A = 0, v_B = p(t_{BAC}) + p(t_{BCA}) + p(t_{ABC}), v_C = p(t_{CAB}) + p(t_{CBA}) + p(t_{ACB})\}$ and the strategy profile that is consistent with these beliefs form a BNE. QED

Proposition 2 Consider that there exists a BNE that leads to a coalition government with a non-degenerate probability. Since we focus on pure strategy equilibria, the beliefs which are part of such a BNE should be consistent with $0 < v_J < v_Q < v_R < 1/2$ for any generic distribution of players' types. Let us assume without loss of generality that $0 < v_A < v_B < v_C < 1/2$. In such case the probability with which the minimal winning coalition $\{A, B\}$ will be formed after the election converges to one as n becomes arbitrarily large. Let us study the problem that a t_{CAB} voter faces. Her vote may increase the weight of a party in a coalition government (such a change in the weight is bounded from below by $\frac{1}{k+1}$) or/and it may determine the coalition government that will be formed. Conditional on the outcome being such that a vote cannot affect the coalition government that will be formed, a t_{CAB} voter obviously prefers to vote for party A since the coalition government $\{A, B\}$ will be formed with infinitely larger probability than any other coalition government. Now, conditional on her vote being pivotal for the determination of the exact coalition (or single-party) government a t_{CAB} voter could be better off by not voting for party A. Since parties are purely office motivated, all such pivotal events are a subset of the cases in which the difference between the votes of (at least) two parties is not more than one vote plus the case in which a party lacks one vote to form a single-party government. Moreover, the probability of any such pivotal event must not exceed the probability of the most probable tie which is given by $\frac{e^{-n(\sqrt{\hat{\psi}}-\sqrt{\hat{\omega}})^2}}{2\sqrt{\pi n\sqrt{\hat{\psi}\hat{\omega}}}}$ (see Myerson, 2000) where $(\hat{\psi}, \hat{\omega}) = \arg\min\{(\sqrt{\psi} - \sqrt{\omega})^2$ such that ψ and ω take distinct values from the set $\{v_A, v_B, v_C, 1 - v_A, 1 - v_B, 1 - v_C\}\}$.

Then, to establish that a t_{CAB} voter would prefer to vote for A it is enough to show that

$$\lim_{n\to+\infty} \frac{E(\frac{1}{k+1}|\frac{k}{2}-1>\hat{V}_C>\max\{\hat{V}_A,\hat{V}_B\}+1)\times prob(\frac{k}{2}-1>\hat{V}_C>\max\{\hat{V}_A,\hat{V}_B\}+1)}{\frac{e^{-n(\sqrt{\hat{\psi}}-\sqrt{\hat{\omega}})^2}}{2\sqrt{\pi n\sqrt{\hat{\psi}\hat{\omega}}}}} = +\infty.$$

Given that $E(\frac{1}{k+1}|\frac{k}{2}-1 > \hat{V}_C > \max\{\hat{V}_A, \hat{V}_B\}+1) > \frac{1}{2}E(\frac{1}{k+1}|k \geq m) = \frac{1}{2}\frac{\Gamma(1+m)-\Gamma(1+m,n)}{n\Gamma(1+m)}$ where $\Gamma(z)$ is the Euler gamma function, $\Gamma(a, z)$ is the incomplete gamma function and m a sufficiently large integer, we can establish the above relationship if the following is true:

$$\lim_{n\to+\infty} \frac{prob(\frac{k}{2}-1>\hat{V}_C>\max\{\hat{V}_A,\hat{V}_B\}+1)}{\frac{2\Gamma(1+m)}{\Gamma(1+m)-\Gamma(1+m,n)}\frac{\sqrt{n}e^{-n(\sqrt{\hat{\psi}}-\sqrt{\hat{\omega}})^2}}{2\sqrt{\pi\sqrt{\hat{\psi}\hat{\omega}}}}} = +\infty.$$

Pure office motivation and $0 < v_A < v_B < v_C < 1/2$ suggest that $prob(\frac{k}{2} - 1 > \hat{V}_C > \max\{\hat{V}_A, \hat{V}_B\}+1) \to 1$ as $n \to +\infty$, m being positive suggests that $\lim_{n\to+\infty} \frac{2\Gamma(1+m)}{\Gamma(1+m)-\Gamma(1+m,n)} = 2$ and $(\sqrt{\hat{\psi}} - \sqrt{\hat{\omega}})^2 > 0$ suggests that $\lim_{n\to+\infty} \frac{\sqrt{n}e^{-n(\sqrt{\hat{\psi}}-\sqrt{\hat{\omega}})^2}}{2\sqrt{\pi\sqrt{\hat{\psi}\hat{\omega}}}} = 0$. Hence, the required relationship holds.

This proves that when a strategy profile generates expected vote shares $0 < v_A < v_B < v_C < 1/2$, then a t_{CAB} voter is strictly better off by voting for A than for C. Similarly one can show that a t_{CBA} voter is strictly better off by voting for B than for C and that C is not a best response to such expectations for any voter type. That is, beliefs which are consistent with $0 < v_A < v_B < v_C < 1/2$ cannot be part of a BNE since they lead to $\hat{V}_C = 0$ and, hence, to a voting behavior which is inconsistent with them. QED

Lemma 2 Similar to Lemma 1 if a party J obtains the majority of the votes then by performing a single-party government $G = J$ where $W_J^G = 1$ it maximizes its payoff $U_J(J) = 1$. To show that under policy motives the government coalition may be non-minimal let $\lambda_A = \lambda_B = \lambda_C = \lambda < 1$ and $\tilde{V}_A = \tilde{V}_C + \varepsilon = \tilde{V}_B + 2\varepsilon$ where $\tilde{V}_J = V_J/(V_A + V_B + V_C)$ denotes party's J vote share. Assume that the policy preferences of party A are compatible with those of a voter of type t_{ABC} and the policy preferences of party B are compatible with those of a voter of type t_{BAC}. Then due to both office seeking incentives and ideology proximity party A strictly prefers to form a coalition with B than with C. Now, B strictly prefers to form a coalition with A rather than with C if and only if $U_B(\{A, B\}) > U_B(\{B, C\})$. The latter holds if and only if:

$$
\begin{gathered}
\lambda \tfrac{V_B}{V_A+V_B} + (1-\lambda)[\tfrac{V_B}{V_A+V_B} u_{t_{BAC}}(B) + \tfrac{V_A}{V_A+V_B} u_{t_{BAC}}(A)] > \\
\lambda \tfrac{V_B}{V_B+V_C} + (1-\lambda)[\tfrac{V_B}{V_B+V_C} u_{t_{BAC}}(B) + \tfrac{V_C}{V_B+V_C} u_{t_{BAC}}(C)] \Longrightarrow \\
\lambda \tfrac{V_B}{V_A+V_B} + (1-\lambda)[\tfrac{V_B}{V_A+V_B}] > \lambda \tfrac{V_B}{V_B+V_C} + (1-\lambda)[\tfrac{V_B}{V_B+V_C} - \tfrac{V_C}{V_B+V_C}] \Longrightarrow \\
\tfrac{V_B}{V_A+V_B} > \tfrac{V_B}{V_B+V_C} - (1-\lambda)\tfrac{V_C}{V_B+V_C} \Longrightarrow \\
\tfrac{\tilde{V}_B}{2\tilde{V}_B+2\varepsilon} > \tfrac{\tilde{V}_B}{2\tilde{V}_B+\varepsilon} - (1-\lambda)\tfrac{\tilde{V}_B+\varepsilon}{2\tilde{V}_B+\varepsilon} \Longrightarrow \\
\tfrac{\tilde{V}_B}{2\tilde{V}_B+2\varepsilon} > \tfrac{\tilde{V}_B - \tilde{V}_B - \varepsilon + \lambda\tilde{V}_B + \lambda\varepsilon}{2\tilde{V}_B+\varepsilon} \Longrightarrow \\
\tfrac{\tilde{V}_B}{2\tilde{V}_B+2\varepsilon} > \tfrac{\lambda\tilde{V}_B + \varepsilon(1-\lambda)}{2\tilde{V}_B+\varepsilon}
\end{gathered}
$$

For small values of ε (that is for $\varepsilon \to 0$) the above becomes $\frac{1}{2} > \frac{\lambda}{2}$ and holds for any $\lambda \in [0, 1)$.QED

Lemma 3 Let $V_A = V_B + \eta > V_C = 1$, where $\eta \in \{-1, 0, 1\}$. Therefore, $\frac{k}{2} - 1 \leq V_A \leq \frac{k}{2} + 1$. Assume that $\eta = 1$ (the equivalent arguments holds for $\eta = -1$). In this case $V_A = \frac{k}{2}$ and the only winning coalition in which C participates is $\{A, C\}$. Thus, we are interested in finding for which values of k the $\{A, C\}$ coalition is also stable. Notice that $\{A, C\}$ is stable if and only if A prefers to form a coalition with C than with B. It is obvious that $\frac{V_A}{V_A+V_C} > \frac{V_A}{V_A+V_B}$, that is, if A is a purely office-motivated party it would prefer to form a coalition with C. To establish our result we need to find the condition that guarantees that $\frac{V_A}{V_A+V_C} u_{t_{ABC}}(A) + \frac{V_C}{V_A+V_C} u_{t_{ABC}}(C) > \frac{V_A}{V_A+V_B} u_{t_{ABC}}(A) + \frac{V_B}{V_A+V_B} u_{t_{ABC}}(B)$ as well. The latter inequality implies that even if party A is purely ideology motivated and even if it dislikes the policy of party C the most (party A has preferences compatible with a t_{ABC} voter), it would still prefer to form a coalition with C rather than with B. We know that $\frac{V_A}{V_A+V_C}u_{t_{ABC}}(A) + \frac{V_C}{V_A+V_C}u_{t_{ABC}}(C) = \frac{V_A-V_C}{V_A+V_C}$ and that $\frac{V_A}{V_A+V_B}u_{t_{ABC}}(A) + \frac{V_B}{V_A+V_B}u_{t_{ABC}}(B) = \frac{V_A}{V_A+V_B}$. We notice that $\frac{V_A-V_C}{V_A+V_C} = \frac{\frac{k}{2}-1}{\frac{k}{2}+1}$ and that $\frac{V_A}{V_A+V_B} = \frac{\frac{k}{2}}{k-1}$. Therefore, the required inequality holds if $\frac{\frac{k}{2}-1}{\frac{k}{2}+1} > \frac{\frac{k}{2}}{k-1}$ and since k is always a natural number, the later inequality holds if and only if $k > 7$. We replicate this exercise for $\eta = 0$ and we also find that $k > 7$ is a sufficient condition for the existence of a stable coalition in which C participates. The only difference of the $\eta = 0$ case compared to the cases in which $\eta \in \{-1, 1\}$ is that there are two winning coalitions in which C participates; $\{A, C\}$ and $\{B, C\}$. Since both A and B received exactly the same votes, party C will choose partner according to its ideological preferences (or it will choose a partner randomly if it is purely office-motivated) and $k > 7$ will guarantee that A (B) strictly prefers to be in a coalition with C rather than with B (A). QED

Proposition 3 A BNE consists of strategies and beliefs. We will prove that if the expected vote share of A is zero and the expected vote shares of the other two parties are positive (formally, $v_A = 0$, $v_B > 0$ and $v_C > 0$ such that $v_A + v_B + v_C = 1$) then a t_{ABC} will vote for B (equivalently one can show that a t_{ACB} voter will vote for C). That is, we will show that beliefs $\{v_A = 0, v_B = p(t_{BAC}) + p(t_{BCA}) + p(t_{ABC}), v_C = p(t_{CAB}) + p(t_{CBA}) + p(t_{ACB})\}$ and the strategy profile which is consistent with these beliefs form a BNE. We know that $A's$ ideological preferences should be given by one of the strict orders $\{A \succ B \succ C\}$ or $\{A \succ C \succ B\}$ (assume without loss of generality that $A's$ preferences are given by the first linear order). For large polities ($n \to \infty$) the probability that there exists a positive measure of voters with preferences that are compatible with $A's$ ideological preferences converges to one. Let us investigate the problem that a voter i with such preferences faces in this case. If she expects that $v_A = 0$, $v_B > 0$ and $v_C > 0$ then the only possibility that her vote can define the outcome is when $\hat{V}_B = \hat{V}_C + \eta$ where $\eta \in \{-1, 0, 1\}$. Notice that *a)* in such eventualities voting for C is always a strictly dominated strategy and *b)* $\eta \in \{-1, 1\}$ implies that k is even and $\eta = 0$ implies that k is odd. We detail below the possible pivotal eventualities.

I) If $\eta = -1$ then voter i gets utility $-\frac{1}{2}$ if she votes for B (because B and C will form a coalition in which they will have equal weights). *a)* If $k > 7$ and our voter votes for A then both coalitions $\{A, C\}$ and $\{B, C\}$ are stable. In this case the coalition $\{A, C\}$ is formed with probability $0 << \xi << 1$ and the coalition $\{B, C\}$ is formed with probability $1 - \xi$. Voter i gets utility $-\frac{\hat{V}_C - 1}{1 + \hat{V}_C}$ if she votes for A and $\{A, C\}$ is formed (because the condition of Lemma 3 is satisfied and A and C can form a coalition in which they will have weights $\frac{1}{1+\hat{V}_C}$ and $\frac{\hat{V}_C}{1+\hat{V}_C}$ respectively) and utility strictly less than $-\frac{1}{2}$ if she votes for A and $\{B, C\}$ is formed. *b)* If $k \leq 7$ voter i gets utility of, *at most*, equal to 0 when the voter votes for A (and government $\{A, C\}$ is formed if $\hat{V}_A = 0, \hat{V}_B = 0, \hat{V}_C = 1$).

II) If $\eta = 0$ then voter i gets utility 0 if she votes for B (because B will form a single-party government) and if $k > 7$ voter i gets utility $\frac{1}{1+\hat{V}_B}$ if she votes for A. This is because the condition of Lemma 3 is satisfied and, thus, B will prefer to form a coalition with A rather than with C. Due to ideological proximity with B, A will prefer to form a coalition with B and, thus A and B will form a coalition in which they will have weights $\frac{1}{1+\hat{V}_B}$ and $\frac{\hat{V}_B}{1+\hat{V}_B}$ respectively. If $k \leq 7$ voter i gets utility of, *at most*, equal to 1 by voting A (and a single-party government $\{A\}$ is formed if $\hat{V}_A = 0, \hat{V}_B = 0, \hat{V}_C = 0$).

III) If $\eta = 1$ then voter i gets utility 0 if she votes for B (because B will form a single-party government) and if $k > 7$ voter i gets utility $\frac{1}{1+\hat{V}_B}$ if she votes for A. This is because the condition of Lemma 3 is satisfied and, thus, B will prefer to form a coalition with A rather than with C. Due to ideological proximity with B, A will prefer to form a coalition with B and, thus A and B will form a coalition in which they will have weights $\frac{1}{1+\hat{V}_B}$ and $\frac{\hat{V}_B}{1+\hat{V}_B}$ respectively. If $k \leq 7$ voter i gets utility of, *at most*, equal to $\frac{1}{2}$ by voting A (and government $\{A, B\}$ is formed if $\hat{V}_A = 0, \hat{V}_B = 1, \hat{V}_C = 0$).

Formally, voter i will vote for her second ranked candidate B if the following holds:

$$
\begin{aligned}
prob(\hat{V}_B &= \hat{V}_C-1 \text{ and } k>7)[-\frac{1}{2}-\xi(-E(\frac{\hat{V}_C-1}{1+\hat{V}_C}|\hat{V}_B=\hat{V}_C-1 \text{ and } k>7))-(1-\xi)(-\frac{1}{2})]+\\
+prob(\hat{V}_B &= \hat{V}_C-1 \text{ and } k\leq 7)[-\frac{1}{2}-0]+\\
+prob(\hat{V}_B &= \hat{V}_C \text{ and } k>7)[0-E(\frac{1}{1+\hat{V}_B}|\hat{V}_B=\hat{V}_C \text{ and } k>7)]+\\
+prob(\hat{V}_B &= \hat{V}_C \text{ and } k\leq 7)[0-1]+\\
+prob(\hat{V}_B &= \hat{V}_C+1 \text{ and } k>7)[0-E(\frac{1}{1+\hat{V}_B}|\hat{V}_B=\hat{V}_C+1 \text{ and } k>7)]+\\
+prob(\hat{V}_B &= \hat{V}_C+1 \text{ and } k\leq 7)[0-\frac{1}{2}]>0
\end{aligned}
$$

We observe that:

$prob(\hat{V}_B=\hat{V}_C-1 \text{ and } k\leq 7)=prob(\hat{V}_B=\hat{V}_C-1|k\leq 7)\times prob(k\leq 7)$,

$prob(\hat{V}_B=\hat{V}_C \text{ and } k\leq 7)=prob(\hat{V}_B=\hat{V}_C|k\leq 7)\times prob(k\leq 7)$ and

$prob(\hat{V}_B=\hat{V}_C+1 \text{ and } k\leq 7)=prob(\hat{V}_B=\hat{V}_C+1|k\leq 7)\times prob(k\leq 7)$.

It is obvious that all $prob(\hat{V}_B=\hat{V}_C-1|k\leq 7), prob(\hat{V}_B=\hat{V}_C|k\leq 7)$ and $prob(\hat{V}_B=\hat{V}_C+1|k\leq 7)$ take a positive value significantly higher than 0. On the other side we have that $prob(k\leq 7)=\sum_{m=0}^{7}\frac{n^m}{m!}e^{-n}$.

Notice that $prob(\hat{V}_B=\hat{V}_C-1)=prob(\hat{V}_B=\hat{V}_C-1 \text{ and } k>7)+prob(\hat{V}_B=\hat{V}_C-1 \text{ and } k\leq 7)$ and $prob(\hat{V}_B=\hat{V}_C-1 \text{ and } k\leq 7)=prob(\hat{V}_B=\hat{V}_C-1|k\leq 7)\times prob(k\leq 7)$. That is, $1=\frac{prob(\hat{V}_B=\hat{V}_C-1 \text{ and } k>7)}{prob(\hat{V}_B=\hat{V}_C-1)}+prob(\hat{V}_B=\hat{V}_C-1|k\leq 7)\times\frac{prob(k\leq 7)}{prob(\hat{V}_B=\hat{V}_C-1)}$.

By Myerson (2000) we know that $prob(\hat{V}_B=\hat{V}_C-1)\simeq\frac{e^{n(2\sqrt{v_Bv_C}-v_B-v_C)}}{2\sqrt{\pi n\sqrt{v_Bv_C}}}$ for $n\to\infty$. So it must be the case that for $n\to\infty$, $prob(k\leq 7)$ becomes infinitely smaller than $prob(\hat{V}_B=\hat{V}_C-1)$. This is due to the fact that $\sum_{m=0}^{7}\frac{n^m}{m!}e^{-n}<\frac{e^{n(2\sqrt{v_Bv_C}-v_B-v_C)}}{2\sqrt{\pi n\sqrt{v_Bv_C}}}$ is equivalent to $\frac{2\sqrt{\pi n\sqrt{v_Bv_C}}\sum_{m=0}^{7}\frac{n^m}{m!}}{e^{n(2\sqrt{v_Bv_C}-v_B-v_C)+n}}<1$ and $2\sqrt{\pi n\sqrt{v_Bv_C}}\sum_{m=0}^{7}\frac{n^m}{m!}$ increases in a polynomial rate while $e^{n(2\sqrt{v_Bv_C}-v_B-v_C)+n}$ increases in an exponential rate ($n(2\sqrt{v_Bv_C}-v_B-v_C)+n$ is always positive for any $v_B>0$ and $v_C>0$). That is, $\frac{2\sqrt{\pi n\sqrt{v_Bv_C}}\sum_{m=0}^{7}\frac{n^m}{m!}}{e^{n(2\sqrt{v_Bv_C}-v_B-v_C)+n}}\to 0$ and therefore $\frac{prob(k\leq 7)}{prob(\hat{V}_B=\hat{V}_C-1)}\to 0$. This implies that for $n\to\infty$ we have $1=\frac{prob(\hat{V}_B=\hat{V}_C-1 \text{ and } k>7)}{prob(\hat{V}_B=\hat{V}_C-1)}+$

$prob(\hat{V}_B = \hat{V}_C - 1|k \leq 7) \times 0$ which is equivalent to $\frac{prob(\hat{V}_B=\hat{V}_C-1 \ and \ k>7)}{prob(\hat{V}_B=\hat{V}_C-1)} \rightarrow 1$ and to $prob(\hat{V}_B = \hat{V}_C - 1 \ and \ k > 7) \rightarrow prob(\hat{V}_B = \hat{V}_C - 1)$.

With the same logic we can demonstrate that $prob(\hat{V}_B = \hat{V}_C \ and \ k > 7) \rightarrow prob(\hat{V}_B = \hat{V}_C)$ and that $prob(\hat{V}_B = \hat{V}_C + 1 \ and \ k > 7) \rightarrow prob(\hat{V}_B = \hat{V}_C + 1)$.

By this last observation and the offset theorem of Myerson (2000) we have that $\frac{prob(\hat{V}_B=\hat{V}_C \ and \ k>7)}{prob(\hat{V}_B=\hat{V}_C-1 \ and \ k>7)} \rightarrow \sqrt{\frac{v_B}{v_C}}$ and $\frac{prob(\hat{V}_B=\hat{V}_C+1 \ and \ k>7)}{prob(\hat{V}_B=\hat{V}_C-1 \ and \ k>7)} \rightarrow \frac{v_B}{v_C}$.

All these imply that if we divide our relevant inequality with $prob(\hat{V}_B = \hat{V}_C - 1 \ and \ k > 7)$ we should get:

$$
\begin{aligned}
& \xi[-\frac{1}{2} - (-E(\frac{\hat{V}_C - 1}{1 + \hat{V}_C}|\hat{V}_B = \hat{V}_C - 1 \ and \ k > 7))] + \\
+prob(\hat{V}_B &= \hat{V}_C - 1|k \leq 7)\frac{prob(k \leq 7)}{prob(\hat{V}_B = \hat{V}_C - 1 \ and \ k > 7)}[-\frac{1}{2} - 0] + \\
& +\sqrt{\frac{v_B}{v_C}}[0 - E(\frac{1}{1 + \hat{V}_B}|\hat{V}_B = \hat{V}_C \ and \ k > 7)] + \\
+prob(\hat{V}_B &= \hat{V}_C|k \leq 7)\frac{prob(k \leq 7)}{prob(\hat{V}_B = \hat{V}_C - 1 \ and \ k > 7)}[0 - 1] + \\
& +\frac{v_B}{v_C}[0 - E(\frac{1}{1 + \hat{V}_B}|\hat{V}_B = \hat{V}_C + 1 \ and \ k > 7)] + \\
+prob(\hat{V}_B &= \hat{V}_C|k \leq 7)\frac{prob(k \leq 7)}{prob(\hat{V}_B = \hat{V}_C - 1 \ and \ k > 7)}[0 - \frac{1}{2}] > 0
\end{aligned}
$$

Since

$$E(\tfrac{\hat{V}_C-1}{1+\hat{V}_C}|\hat{V}_B = \hat{V}_C - 1 \ and \ k > 7) = 1 - E(\tfrac{2}{1+\hat{V}_C}|\hat{V}_B = \hat{V}_C - 1 \ and \ k > 7) \rightarrow 1$$

and

$$E(\tfrac{1}{1+\hat{V}_B}|\hat{V}_B = \hat{V}_C \ and \ k > 7) \rightarrow E(\tfrac{1}{1+\hat{V}_B}|\hat{V}_B = \hat{V}_C + 1 \ and \ k > 7) \rightarrow 0$$

the above inequality becomes $\frac{\xi}{2} > 0$ and always holds. That is, when $v_A = 0$, $v_B > 0$ and $v_C > 0$ then a t_{ABC} will vote for B (equivalently a t_{ACB} voter will vote for C). Therefore, beliefs $\{v_A = 0, v_B = p(t_{BAC}) + p(t_{BCA}) + p(t_{ABC}), v_C = p(t_{CAB}) + p(t_{CBA}) + p(t_{ACB})\}$ and the strategy profile which is consistent with these beliefs form a BNE. QED

References

Austen-Smith, D. and J. Banks (1988): "Elections, coalitions, and legislative outcomes," *American Political Science Review*, 82, 405–422.

——— (1990): "Stable governments and the allocation of policy portfolios," *American Political Science Review*, 84, 891–906.

Axelrod, R. (1970): *Conflict of interest*, Markham Pub. Co.

Baron, D. and D. Diermeier (2001): "Elections, governments, and parliaments in proportional representation systems," *Quarterly Journal of Economics*, 116, 933–967.

Bouton, L. (2013): "A theory of strategic voting in runoff elections," *American Economic Review*, 103, 1248–1288.

Bouton, L. and M. Castanheira (2012): "One person, many votes: Divided majority and information aggregation," *Econometrica*, 80, 43–87.

Bouton, L. and G. Gratton (2015): "Majority runoff elections: Strategic voting and Duverger's hypothesis," *Theoretical Economics*, 10, 283–314.

Browne, E. and M. Franklin (1973): "Aspects of coalition payoffs in European parliamentary democracies," *American Political Science Review*, 67, 453–469.

Carroll, R. and G. Cox (2007): "The Logic of Gamson's Law: Pre-election Coalitions and Portfolio Allocations," *American Journal of Political Science*, 51, 300–313.

Cox, G. (1997): *Making votes count: strategic coordination in the world's electoral systems*, vol. 7, Cambridge Univ Press.

De Sinopoli, F. and G. Iannantuoni (2007): "A spatial voting model where proportional rule leads to two-party equilibria," *International Journal of Game Theory*, 35, 267–286.

——— (2008): "Extreme voting under proportional representation: the multidimensional case," *Social Choice and Welfare*, 30, 401–417.

De Swan, A. (1973): *Coalition theories and cabinet formations*, Elsevier.

Dellis, A. (2013): "The two-party system under alternative voting procedures," *Social Choice and Welfare*, 40, 263–284.

Dellis, A., S. D'Evelyn, and K. Sherstyuk (2011): "Multiple votes, ballot truncation and the two-party system: an experiment," *Social Choice and Welfare*, 37, 171–200.

Diermeier, D. and A. Merlo (2000): "Government turnover in parliamentary democracies," *Journal of Economic Theory*, 94, 46–79.

Duverger, M. (1954): *Political Parties Their Organization and Activity in the Modern State*, Trans. Barbara and Robert North. New York: Wiley.

Fey, M. (1997): "Stability and coordination in Duverger's law: A formal model of preelection polls and strategic voting," *American Political Science Review*, 91, 135–147.

Fréchette, G., J. Kagel, and M. Morelli (2005): "Gamson's Law versus non-cooperative bargaining theory," *Games and Economic Behavior*, 51, 365–390.

Gamson, W. (1961): "A theory of coalition formation," *American Sociological Review*, 26, 373–382.

Gerber, A. and I. Ortuño-Ortín (1998): "Political compromise and endogenous formation of coalitions," *Social Choice and Welfare*, 15, 445–454.

Iaryczower, M. and A. Mattozzi (2013): "On the nature of competition in alternative electoral systems," *Journal of Politics*, 75, 743–756.

Kalandrakis, T. (2015): "A theory of minority and majority governments," *Political Science Research and Methods*, 3, 309–328.

Laver, M. (1998): "Models of government formation," *Annual Review of Political Science*, 1, 1–25.

Laver, M. and K. Shepsle (1990): "Coalitions and cabinet government," *American Political Science Review*, 84, 873–890.

Le Breton, M., I. Ortuño-Ortín, and S. Weber (2008): "Gamson's law and hedonic games," *Social Choice and Welfare*, 30, 57–67.

Lijphart, A. (1994): *Electoral systems and party systems: A study of twenty-seven democracies, 1945-1990*, vol. 18, Oxford.

Matakos, K., O. Troumpounis, and D. Xefteris (2013): "Electoral rule disproportionality and platform polarization," *EPSA 2013 Annual General Conference Paper 809*.

McLennan, A. (2011): "Manipulation in elections with uncertain preferences," *Journal of Mathematical Economics*, 47, 370–375.

Morelli, M. (2004): "Party formation and policy outcomes under different electoral systems," *Review of Economic Studies*, 71, 829–853.

Myerson, R. (2000): "Large Poisson games," *Journal of Economic Theory*, 94, 7–45.

Myerson, R. and R. Weber (1993): "A theory of voting equilibria," *American Political Science Review*, 87, 102–114.

Palfrey, T. (1989): "A mathematical proof of Duverger's law," *Models of Strategic Choice in Politics*, 69–92.

Riker, W. (1962): *The theory of political coalitions*, Yale University Press New Haven.

——— (1982): "The two-party system and Duverger's Law: An essay on the history of political science," *American Political Science Review*, 76, 753–766.

Seidmann, D., E. Winter, and E. Pavlov (2007): "The formateurs' role in government formation," *Economic Theory*, 31, 427–445.

Warwick, P. and J. Druckman (2001): "Portfolio salience and the proportionality of payoffs in coalition governments," *British Journal of Political Science*, 31, 627–649.