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Push or Pull? Performance Pay, Incentives, and Information

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Push or Pull? Performance Pay, Incentives, and Information *

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Abstract

We study a principal-agent model wherein the agent is better informed of the prospects of the project, and the project requires both an observable and unobservable input. We show (1) Performance pay may not be optimal, even if output is the only informative signal of an essential input; (2) Total surplus tends to be higher if one input is unobservable than if both inputs are observable; and (3) Bunching may arise amongst low and intermediate types. We explore the implications for push and pull programs used to encourage R&D activity, but our results have applications beyond this context.

KEYWORDS: Pay for Performance, Moral Hazard, Adverse Selection, Observable Action, Principal-Agent Problem, Grants, Prizes
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1 Introduction

To what extent should incentives be tied to performance? This question is relevant in a number of areas, including worker compensation – where it relates to the debate on salaries vs. piece rates (see, e.g. Lazear, 1986, 2000) – and innovation incentives, where it pertains to the efficacy of "push" and "pull" programs (see, e.g., Kremer, 2002). Push programs, such as research grants, or R&D tax credits, subsidize research input; payments are not contingent on results. Pull programs, such as innovation prizes, or patent buyouts, directly tie rewards to research output.

Adverse selection (AS) and moral hazard (MH) are inherent challenges in the provision of incentives. Given these problems, Kremer raises the concerns that push programs may reward researcher's unlikely to succeed, and provide weak incentives for unobservable inputs. Indeed, the literature on MH stresses the importance of performance pay. In the canonical MH model,[1] the agent's effort is unobservable by the principal, but output, a noisy signal of effort, is observable. In that model, compensation must be at least partially tied to output to provide an incentive for greater effort. More generally, Hölmstrom's (1979) "informativeness principle" implies that it is valuable for the principal to condition rewards on any signal – including output – that provides additional information about the agent's effort.

Despite these concerns, "low-powered" incentive schemes, in which compensation is weakly, or *not at all* tied to performance, are commonly used in practice. In this paper, we offer a new justification for the use of such schemes. We show that when AS and MH interact, and at least *some* of the agent's actions are observable, performance pay may not be optimal, even if output is the only informative signal of an essential input.

For concreteness, we present our model in the context of R&D funding, but our results are relevant in other contexts. We study a principal-agent model wherein a risk-neutral funder (he; the principal) incentivizes a risk-neutral researcher (she; the agent) to undertake an R&D project, which may yield a

[1] See, e.g., Grossman and Hart (1983); or Bolton and Dewatripont (2005) ch. 4.

new technology. The likelihood of success depends on the researcher's privately known type, and two essential and complementary inputs – "investment" and "effort". Investment is observable by the funder; effort is not.[2] If she succeeds, the researcher earns a profit by marketing the technology, but this incentive, alone, may be insufficient to elicit R&D activity.[3] To ensure the researcher has an incentive to truthfully reveal the project's outcome, the funder is subject to a "free-disposal" constraint, stipulating that the reward for success is no less than the reward for failure.

The contracts offered by the funder consist of a transfer independent of performance – a "grant" – and a payment for success – a "prize". The prize and grant may depend on investment, but neither can depend on effort. We show (i) Performance pay may not be optimal; (ii) Total surplus tends to be higher when effort is unobservable, than when it is observable; and (iii) Bunching may arise amongst low and intermediate types.[4]

We briefly describe the intuition for our results. (i) The prize creates a strong incentive for effort, but generates costly information rent for the researcher due to AS. As a result of this tradeoff, the prize may be zero for all, or a subset of types. In these cases, the researcher's compensation is completely independent of her performance. Effort is induced indirectly, through the grant. The grant is used to encourage greater investment, which increases the productivity of effort. The researcher's product-market profit may then provide a sufficient incentive for effort.

(ii) If both inputs are observable, then investment is distorted below the first-best to limit the researcher's information rent. When effort is unobservable, a larger investment and/or a prize may be necessary to induce effort. But it is advantageous for the funder to raise investment closer to the first-best. Doing so increases the researcher's rent, but increases total surplus, which

[2] Throughout this analysis, we use the terms "observable", "contractible", and "verifiable" interchangeably.

[3] In a more general setting, this incentive might reflect the prospect of a future outside job opportunity, or some intrinsic motivation.

[4] Bunching refers to a situation where the principal offers the same contract to multiple different types.

partially offsets the additional cost to the funder. A prize, in contrast, simply transfers surplus from the funder to the researcher. For this reason, investment and total surplus are (weakly) greater when effort is unobservable than when it is observable.

(iii) Bunching may arise amongst low and intermediate types due to a tension between rent extraction, effort inducement, and the second-order incentive compatibility constraint. As this is a more technical point, we defer a discussion of this finding to Section 4.4.

The main contributions of our analysis are twofold. First, we add to the literature on contracting. Following the tradition of the canonical MH model, most mixed models assume a production process that depends only on unobservable actions.[5] But this premise is rather extreme. The mental or physical effort of an agent may be prohibitively costly to monitor, or even quantify. But investments by a firm in large-scale capital, or the time a worker spends at work, are quantifiable, and likely easier to verify. We add to this literature by providing a full characterization of the optimal incentive scheme in a setting with AS, and inputs that consist of both an observable and unobservable component. We show that the partial observability of these inputs has dramatic consequences for the structure of optimal incentives, and interesting welfare implications.

If only output is observable then rewards cannot depend on actions(s), and performance pay is essential. A payment received independent of performance affects the agent's overall utility, but will not generate greater marginal incentives. This need not be the case when actions are partially observable. If, for instance, a researcher's investment in capital is observable, then rewards can be directly tied to this input, and greater investment can be encouraged. But it may be that the researcher's effort is more productive when she has better equipment with which to work. If so, then as long as there is *some* benefit to success, greater investment increases the marginal returns to effort, and thus,

[5]Early examples in the literature include Sappington (1982) and Picard (1987). Studies closer to our analysis include Laffont (1995), Lewis and Sappington (2000a,b) Ollier and Thomas (2013), and Gottlieb and Moreira (2015). There are, of course, notable exceptions, which will be discussed in Section 2.

effort is encouraged without tying rewards to performance.[6] This observation relates closely to findings by Hölmstrom and Milgrom (1991). When the agent undertakes multiple tasks and efforts are complements, the authors show that a stronger incentive for effort on one task, simultaneously induces a higher effort on some other task.

Hölmstrom and Milgrom provide an alternative explanation for rewards independent of performance. Their finding relies on actions being substitutes,[7] and the lack of an informative signal associated with some action. Our result relies on a complementarity between actions, the presence of an observable action, and the interaction between AS and MH. Additionally, the "fixed wage" in Hölmstrom and Milgrom's model is a transfer independent of *any* signal received by the principal, while the grant depends on the observable action, but is independent of output. This grant structure better captures the design of many incentive schemes used in practice. The U.S. R&D tax credit system, for example, rewards firms independently of performance, but the value of the credit is directly tied to R&D investment. Similarly, an hourly worker's wage may not be contemporaneously tied to performance, but she is only paid for the time she spends at work.

Meng and Tian (2012) study a multitasking model with AS and MH, and provide conditions under which lower-powered incentives arise, as compared to pure MH. We provide conditions under which optimal incentives are completely independent of performance. When the principal faces a multidimensional AS problem, Meng and Tian also show that optimal compensation may be independent of some performance measures. But their result differs fundamentally from our's. In their model the agent undertakes multiple tasks, each of which contribute to the principal's payoff. For those tasks where the performance measures are ignored, the agent exerts no effort. Thus, Meng and Tian's result explains why an agent may be lead to specialize on certain tasks. In our model

[6]In the context of worker compensation, it might be that the (unobservable) mental effort the worker devotes to her job is more productive when she spends more time at work.

[7]The so-called "effort substitution problem". In this case, a stronger incentive for effort on one task reduces the agent's effort on the other task. See also Laffont and Tirole (1993, Ch. 4).

there is a single task that requires multiple inputs. We show why incentives might be independent of the outcome of that task, even if this is the only verifiable signal associated with an essential input.

Our results also shed new light on the welfare implications of AS and MH. As this literature is quite extensive, we provide only a brief summary here; a more thorough overview may be found in Laffont and Martimort (2009, Ch. 7). In some models, adding MH creates no further welfare losses, as compared to pure AS.[8] In many other settings, the AS and MH problems exacerbate one another, leading to greater welfare losses than pure AS or pure MH. Laffont and Martimort emphasize that this is a quite common feature of models, such as our's, where "MH follows AS".[9] Basov and Bardsley (2005) present an example that follows a similar setup to our model, but without an observable action. They show that the first-best can be achieved under pure AS or pure MH, but distortions arise in the combined case.[10] In contrast, we show that there may be welfare gains to adding MH (relative to pure AS) in these models when actions are partially observable.

Laffont and Martimort discuss another class of models – those where AS follows MH[11] – in which adding MH may improve welfare, relative to pure AS. In these models, with pure AS distortions arise to limit the agent's rent. But when MH is added, it is this rent at the AS stage that motivates greater effort at the MH stage. So, under AS and MH, the principal may reduce distortions in order to raise the agent's rent, and encourage greater effort. In our model MH follows AS, so it is not the rent captured at the AS stage that motivates greater effort. Rather, the agent's optimal effort depends directly on her investment; to elicit effort under AS and MH, investment is raised closer to the socially optimal level.

Our second contribution is to the literature on innovation incentives. Many

[8]See, e.g., Laffont and Tirole (1986); Picard (1987); Guesnerie et al. (1989); Caillaud et al. (1992). Basov and Bardsley (2005) show that independence between the agent's type, and the noise in the production function is the basic assumption that lead to these findings.

[9]That is, the agent learns her type prior to choosing her unobservable action.

[10]See, also, Lewis and Sappington (2000b) and Gottlieb and Moreira (2015).

[11]In these models, the agent chooses an unobservable effort that stochastically determines her privately observed type.

studies have explored the optimal design of pull programs (e.g., optimal patent design),[12] or compared the performance of different pull programs (e.g., prizes vs. intellectual property).[13] Fewer studies have examined the optimal design of push programs, or attempted to justify their use, taking MH into account. Our results are useful in both respects. Further, we connect our results to the U.S. R&D tax credit system, and comment on its design.

Maurer and Scotchmer (2003) argue that repeated interaction between grantees and grantors resolves the MH problem. Our explanation for how a push program might overcome MH complements their's, as it is relevant in a static setting. This is important because some push programs, such as R&D tax credits, do not condition eligibility for rewards based on past performance. Fu et al. (2012) show that grants may be useful for facilitating greater competition in a research contest where researchers have asymmetric capital endowments. We abstract from the effects of competition, in order to focus on the role of information.

A number of other explanations for the emergence of low-powered incentive schemes have been posited in other contexts. Baker (1992) shows that performance pay may be muted if output is not contractible, and weakly correlated with the available performance measure. Baker's result may be important in settings where "success" and "failure" are difficult to define. Risk-sharing is also an important consideration. Pay-for-performance schemes place considerable risk on the agent when outcomes are uncertain. Push programs may serve as a means of risk sharing.[14] We abstract from such considerations, as both the researcher and funder are risk neutral. Still, in the canonical MH model, risk aversion, alone, cannot explain the emergence of a push program, where rewards are independent of performance. Prendergast (2002) shows that output-based pay may be beneficial if the principal is uncertain of the "correct" action an agent should take, while input-based pay is more relevant in less uncertain environments. But Prendergast allows costly monitoring of

[12]See Hall (2007) for an overview.

[13]See Maurer and Scotchmer (2003) for an overview.

[14]See, e.g. Prendergast (1999, 2002) for a more in-depth discussion of risk and incentives.

effort; we stick closer to the traditional MH paradigm, and assume effort is prohibitively costly to monitor.

2 Additional Related Literature

Others have examined the role of observable actions in MH models. Zhao (2008) studies a multitasking model where some actions are observable and shows, in stark contrast to our findings, that optimal compensation depends solely on output signals. But in Zhao's model, efforts and outcomes are independent, and the observable actions are not verifiable, which places restrictions on feasible contracts. In a setup similar to Zhao, Chen (2010) shows that if the observable actions are verifiable, then optimal compensation depends on both the observable actions and output signals. Chen (2012) generalizes this finding, allowing multiple agents and complementarities between efforts; Chen's results are consistent with our findings in a pure MH setting. Crucially, neither Zhao nor Chen incorporate AS.

Other models have allowed for AS and both observable and unobservable actions. For instance, in a large class of models following Laffont and Tirole (1986), the agent exerts unobservable effort to reduce marginal cost, and then chooses an observable level of production. But these models are quite distinct from our setup. First, many of these models involve "false moral hazard" (see Laffont and Martimort, 2009, ch. 7). Second, in these models, the technology through which the agent reduces cost depends only on an unobservable action. It is not until after the MH problem is resolved, that the observable action is chosen. In our model, the production process depends on both types of actions, and they are chosen simultaneously. In a quite general framework with AS and observable/unobservable actions, Caillaud et al. (1992) provide conditions under which a mechanism can be implemented via a menu of linear contracts. Meng and Tian's framework can accommodate observable actions, but they focus on the case where actions are all unobservable; none of their results explicitly depend on the existence of an observable action.

Lewis and Sappington (2000b) (henceforth, LS) study an environment with

AS, MH, and a direct contractible input from the principal. LS's framework differs from ours in several important respects. First, in LS there are multiple agents (at least two).[15] Second, each agent in LS's model faces a wealth constraint, which translates to a capital constraint in our model. In particular, if the agent has zero "wealth", the grant must reimburse the full cost of investment. We don't impose a capital constraint; this is important, as we show that the optimal grant typically does not fully reimburse investment. Third, our model explicitly takes into account an incentive outside the principal-agent relationship that can motivate effort.

3 The Model

3.1 The Primitives

We study a principal-agent model between a funder and a researcher. The researcher undertakes a single R&D project, which may or may not lead to the development of a new technology. The inputs are investment, $x \in \mathbb{R}_+$, and effort, $y \in \{0, 1\}$; where, $y = 1$ indicates working hard and $y = 0$ denotes shirking. Investment is observable by the funder; effort is not.

The researcher's type, θ, is a random variable drawn from a continuous distribution according to CDF, F, corresponding (smooth) PDF, f, with support, $\Theta = [\underline{\theta}, \overline{\theta}] \subset [0, 1]$, where $\overline{\theta} > \underline{\theta} > 0$. The researcher knows the true θ while the funder knows only its distribution. θ may capture some privately known characteristic of the project and/or the researcher's innate ability. Let $h(\theta) = \frac{1-F(\theta)}{f(\theta)}$ denote the inverse hazard rate; assume, for all $\theta \in \Theta$, $h'(\theta) < 0$ and $f(\theta) > 0$.

Given x, y, and θ, the probability of success is $\theta p(x, y)$. The function, $p : \mathbb{R}^2_+ \to [0, 1]$, is increasing in each argument, and twice continuously differentiable, with $p(0, \cdot) = p(\cdot, 0) = 0$, and for all $x, y > 0$, $p_{12}(x, y) > 0$. That is, investment and effort are both essential for success, and they are comple-

[15] This provides the principal with an additional instrument that we do not consider; namely, the probability with which the project is allocated to any one particular agent.

ments. We let $\rho(x)$ denote $p(x, 1)$, and we may then write, $\theta p(x, y) = \theta y \rho(x)$. We assume ρ is strictly increasing, but there are diminishing marginal returns to investment: $\rho' > 0$ and $\rho'' < 0$. It is useful to point out that ρ' contains some information about the complementarity between x and y. Specifically, the stronger is the complementarity between x and y, the larger will be ρ'.

If the researcher invests x and chooses effort, y, she incurs a total cost of $x+cy$. If the project succeeds she earns a profit in the product market of $\pi > 0$, and the funder captures a benefit, $W > 0$. Otherwise both receive nothing. W might represent, for example, the consumer surplus associated with the technology. Absent intervention, a type-θ researcher's expected payoff is,

$$\Pi(x, y, \theta) = \theta y \rho(x) \pi - x - cy.$$

We let $(\overline{x}(\theta), \overline{y}(\theta)) = \arg\max_{x \geq 0, y \in \{0,1\}} \Pi(x, y, \theta)$ denote the researcher's optimal no-intervention investment and effort; and we let $\overline{\Pi}(\theta) = \Pi(\overline{x}(\theta), \overline{y}(\theta), \theta)$ denote her maximized profit. We assume throughout that if the researcher is indifferent between working hard and shirking, she will choose to work; together with the strict concavity of ρ, this means $\overline{x}(\theta)$ and $\overline{y}(\theta)$ are unique. Note that π may or may not provide a sufficient incentive to elicit R&D activity (i.e., it may be that $\overline{x}(\theta), \overline{y}(\theta), \overline{\Pi}(\theta) > 0$ or $\overline{x}(\theta) = \overline{y}(\theta) = \overline{\Pi}(\theta) = 0$).

3.2 Feasible Contracts and the Funder's Problem

The funder designs contracts to motivate greater R&D activity from the researcher. A contract specifies a grant, $g \in \mathbb{R}$, a prize, $v \in \mathbb{R}$, and an investment, $x \in \mathbb{R}_+$. The grant is received by the researcher independent of success or failure, while the prize is a received only if the project succeeds.[16]

The outcome of the project is initially observed only by the researcher, who reports the result to the funder. If she reports success, the outcome is

[16]Note that $g < 0$ or $v < 0$ represent transfers from the researcher to the funder. Furthermore, note that it is without loss of generality that we focus on grants and prizes. In general, an optimal mechanism will specify a transfer, t_s, in the event of success, and a transfer, t_f, in the event of failure. However, as both parties are risk neutral, this is equivalent to specifying a grant, $g \equiv t_f$, and a prize, $v \equiv t_s - t_f$.

verifiable at zero cost. But, we assume that the researcher may shroud her success from the funder, if it is in her interest to do so. To avoid creating this incentive, we follow Innes (1990), and more recently, Poblete and Spulber (2012), and impose a free-disposal constraint, which requires that the reward for success is no less than the reward for failure, i.e., $v \geq 0$.[17]

We focus, without loss of generality, on "investment-forcing contracts", where the researcher is only eligible for the grant/prize if she follows through on the agreed-upon investment. Note that if the researcher deviates from the agreed-upon investment, x', and instead chooses $x \neq x'$, and effort, y, then her payoff is $\Pi(x, y, \theta) \leq \overline{\Pi}(\theta)$. So, as long as her payoff when she follows through on x' exceeds $\overline{\Pi}(\theta)$, it is never optimal to deviate from this investment.

By the Revelation Principle, it suffices to consider direct mechanisms. The funder commits to a menu of contracts, $m = \{v(\theta), g(\theta), x(\theta)\}_{\theta\in\Theta}$, where $v : \Theta \to \mathbb{R}_+$ denotes a prize schedule, $g : \Theta \to \mathbb{R}$ denotes a grant schedule, and $x : \Theta \to \mathbb{R}_+$ denotes an investment schedule. Throughout this analysis we restrict attention to continuous, and piecewise differentiable prize/grant/investment schedules. The researcher observes the menu, and if she participates, reports her type, $\hat{\theta}$, to the funder. The funder then specifies an investment level, $x(\hat{\theta})$, a prize, $v(\hat{\theta})$, and a grant, $g(\hat{\theta})$, according to the menu. If the researcher does not participate, she earns $\overline{\Pi}(\theta)$.

After the contract is formed, the researcher chooses investment and effort, the outcome of the project is realized, and transfers are made accordingly.

For a particular contract, $\{v, g, x\}$,[18] the researcher's payoff is,

$$y[\theta\rho(x)(v + \pi) - c] - x + g$$

Let $y^*(v, x, \theta)$ denote the researcher's optimal effort choice: $y^*(v, x, \theta) = \arg\max_{y\in\{0,1\}}\{y[\theta\rho(x)(v + \pi) - c] - x + g\}$. It holds,

[17] Note that when the free-disposal constraint is satisfied and the researcher behaves optimally, the environment is equivalent to one in which the funder may observe the outcome of the project directly. We therefore do not explicitly model the outcome-report game.

[18] Where it does not cause confusion, we will liberally abuse notation, and sometimes let $v \in \mathbb{R}_+, g \in \mathbb{R}$, and $x \in \mathbb{R}_+$ denote particular prize, grant, and investment amounts, respectively.

$$y^*(v, x, \theta) = 1 \iff \theta\rho(x)(v + \pi) - c \geq 0. \tag{1}$$

When it is clear, we suppress the arguments of the researcher's optimal choice of effort, and simply write y^*. For a given menu, $m = \{v(\theta), g(\theta), x(\theta)\}_{\theta\in\Theta}$, the payoff to a researcher of type θ who reports $\hat{\theta}$ is,

$$u(\hat{\theta}|\theta) = \theta y^* \rho(x(\hat{\theta}))\big[v(\hat{\theta}) + \pi\big] - x(\hat{\theta}) - cy^* + g(\hat{\theta}).$$

We let $u(\theta) \equiv u(\theta|\theta)$. If the researcher reports her type truthfully, the funder's payoff is,

$$\phi(m) = \int_{\underline{\theta}}^{\overline{\theta}} \Big[\theta y^* \rho(x(\theta))\,[W - v(\theta)] - g(\theta)\Big] f(\theta) d\theta$$

If one interprets W as the consumer surplus associated with the innovation, then the funder's payoff can be interpreted as expected consumer surplus, less the expected cost of funding.[19]

For a given θ, x, and y, let total (or social) surplus, $S(x, y, \theta)$, be defined as the sum of the researcher's and funder's payoffs:

$$S(x, y, \theta) = \theta y \rho(x)(W + \pi) - x - cy$$

Replacing $g(\theta)$ by $u(\theta)$ we can write the funder's payoff as follows:

$$\phi(m) = \int_{\underline{\theta}}^{\overline{\theta}} \Big[S(x(\theta), y^*, \theta) - u(\theta)\Big] f(\theta) d\theta \tag{2}$$

Equation (2) makes clear that the funder's payoff is expected total surplus less the expected payoff of the researcher. The funder's problem is then,[20]

[19] In this R&D context, the funder might also value the profit to the researcher. The qualitative nature of our conclusions generalize to such an environment, provided there is some social cost to raising funds, as in Laffont and Tirole (1986, 1993), or the funder values firm profits less than consumer welfare. The important point is that transfers from the funder to the researcher are costly to the funder.

[20] Note that $y^*(\cdot)$ is a single-valued function, and so we do not need to include an effort recommendation as part of the funder's strategy.

$$\max_m \phi(m)$$

$$\begin{aligned}
\text{s.t. for all } \ \theta, \hat{\theta} \in \Theta : \\
u(\theta) &\geq \overline{\Pi}(\theta) && \text{(IR)} \\
u(\theta) &\geq u(\hat{\theta}|\theta) && \text{(IC)} \\
x(\theta) &\geq 0; \ v(\theta) \geq 0
\end{aligned}$$

The first constraint is individual rationality (IR), the second is incentive compatibility (IC), the third gives the non-negativity constraint on investment, and the free-disposal constraint.

We will assume throughout that W is sufficiently large such that the funder would like to induce effort from a researcher of any type. To ensure that this is optimal for the researcher, using (1), we impose the constraint, $\theta\rho(x(\theta))(v(\theta)+\pi) - c \geq 0$, on the funder's problem. Under this constraint, it holds that $y^*(v(\theta), x(\theta), \theta) = 1$ for all θ. Then, using standard techniques (see, e.g., Laffont and Tirole, 1993, pp. 64 and 121), it can be shown that IC is satisfied if and only if, for all $\theta \in \Theta$:

$$u'(\theta) = \rho(x(\theta))(v(\theta) + \pi) \tag{IC-F}$$

$$\frac{d}{d\theta}\big[\rho(x(\theta))(v(\theta) + \pi)\big] = \rho'(x(\theta))(v(\theta) + \pi)x'(\theta) + \rho(x(\theta))v'(\theta) \geq 0 \tag{IC-S}$$

$$\theta\rho(x(\theta))(v(\theta) + \pi) \geq c \tag{IC-E}$$

Where (IC-F) and (IC-S) give, respectively, the first and second order conditions for the researcher's type-report problem; (IC-E) ensures that it is optimal for the researcher to exert effort, when she reports her type truthfully.

As stated, the IC and IR constraints do not rule out the possibility of a profitable deviation where the researcher misreports her type, *and* shirks on effort. Appendix A shows that the IC and IR constraints provided above are sufficient to rule out such a profitable deviation if $x(\theta) \geq \overline{x}(\theta)$ or $\overline{\Pi}(\underline{\theta}) = 0$.

In many contexts, it seems reasonable to think that the principal would like to induce greater activity on the project than what the agent would otherwise choose. One might expect this to be the case in the context of R&D, since the social value of an innovation often far exceeds the private value to the innovator (see, e.g., Hall et al., 2009). In this regard, it is natural to assume that W is large, relative to π. Specifically, we assume for all $\theta \in \Theta$,

$$W > \frac{h(\theta)}{\theta}\pi \tag{A1}$$

It will be seen that Assumption (A1) is sufficient to ensure that, in equilibrium, the funder's desired level of investment exceeds $\overline{x}(\cdot)$.

Assumption (A1) is also useful for dealing with the possibility of "countervailing incentives",[21] which may arise when the agent's outside option is type dependent (as it is in our model). These issues are explored by Lewis and Sappington (1989),[22] but under (A1), the issue does not arise in our model. The following lemma proves useful in establishing this fact.

Lemma 1. *Let $\{v(\theta), g(\theta), x(\theta)\}_{\theta\in\Theta}$ satisfy (IC-F), free disposal: $v(\cdot) \geq 0$, and suppose $x(\theta) \geq \overline{x}(\theta)$ for all θ. If $u(\underline{\theta}) \geq \overline{\Pi}(\underline{\theta})$, then $u(\theta) \geq \overline{\Pi}(\theta)$ for all θ.*

The significance of Lemma 1 is that IC and free-disposal are sufficient to ensure that IR is satisfied so long as (i) IR is satisfied for the lowest type, and (ii) the funder desires an investment level greater than what the researcher would otherwise choose. We will show that when W is sufficiently large, and (A1) holds, point (ii) is satisfied. So, IR is satisfied if $u(\underline{\theta}) \geq \overline{\Pi}(\underline{\theta})$ (and IC is satisfied). In this case, the IR constraint resembles the usual one that is independent of the agent's type. Note that as the funder's payoff is strictly decreasing in $u(\cdot)$, this IR constraint binds at the optimum: $u(\underline{\theta}) = \overline{\Pi}(\underline{\theta})$.

[21] Many AS models are structured in such a way that the agent has a systematic incentive to either under or over report her type. Countervailing incentives refers to a situation where some types have an incentive to under report, while others have an incentive to over report.

[22] See also, Maggi and Rodriguez-Clare (1995), and Jullien (2000).

For convenience, we assume $\overline{\Pi}(\underline{\theta}) = 0$; but it is straightforward to generalize our results to the case where $\overline{\Pi}(\underline{\theta}) > 0$, so long as (A1) is satisfied. Integrating both sides of (IC-F), and setting $u(\underline{\theta}) = \overline{\Pi}(\underline{\theta}) = 0$, we obtain,

$$u(\theta) = \int_{\underline{\theta}}^{\theta} \rho(x(t))(v(t) + \pi)dt \tag{3}$$

Taking expectations (with respect to θ) over both sides of (3), and integrating the RHS by parts yields,

$$\int_{\underline{\theta}}^{\overline{\theta}} u(\theta)f(\theta)d\theta = \int_{\underline{\theta}}^{\overline{\theta}} \rho(x(\theta))(v(\theta) + \pi)h(\theta)f(\theta)d\theta.$$

Substituting the expression above into (2), we obtain the following relaxed version of the funder's problem,[23]

$$\max_{x(\cdot),v(\cdot)} \left\{ \int_{\underline{\theta}}^{\overline{\theta}} \big[\theta\rho(x(\theta))[W + \pi] - x(\theta) - c - \rho(x(\theta))(v(\theta) + \pi)h(\theta)\big] f(\theta)d\theta \right\} \quad [P]$$

Subject to (IC-S), (IC-E), $x(\theta) \geq 0$, and free-disposal, $v(\theta) \geq 0$.

4 Results

This section provides a full characterization of the optimal funding contracts. We first study three benchmark settings: complete information, pure MH, and pure AS. Throughout the analysis we use lower-case letters (x, v, g, etc.) to denote arbitrary investments, prizes, grants, etc. and upper-case letters (X, V, G, etc.) to denote optimal solutions. We will say that a funding contract is a *pure grant* if $v = 0$ and $g > 0$, and we analogously define a *pure prize*.

[23] This problem is a relaxation of the funder's true problem, as it does not fully incorporate the IR constraint. Moreover, this problem does not explicitly rule out simultaneous profitable deviations by the researcher in both her type report, and effort choice. But by Lemma 1 IR is satisfied if the solution to [P], $X(\cdot)$, satisfies $X(\cdot) \geq \overline{x}(\cdot)$. Moreover, as we assume $\overline{\Pi}(\underline{\theta}) = 0$, the analysis in Appendix A ensures that simultaneous deviations in type report and effort choice are not profitable.

We will say that a funding contract is a *hybrid* if $v > 0$ and $g > 0$. Finally, define a type-θ's information rent as $u(\theta) - \overline{\Pi}(\theta)$. But note that as her rent is strictly increasing in $u(\cdot)$ (for a fixed $\overline{\Pi}(\theta)$), we will often just refer to $u(\cdot)$ when discussing the researcher's rent.

4.1 Complete Information

With complete information, the funder observes the true θ, and investment and effort are both observable. Given $\theta \in \Theta$, the funder offers a forcing contract, $m = \{v, g, x, y\}$, stipulating both investment and effort to solve,

$$\max_m \{\theta y \rho(x)(W + \pi) - x - cy - u(\theta)\} \text{ s.t.}$$
$$u(\theta) \geq \overline{\Pi}(\theta) \text{ and } v \geq 0$$

We let $(X_{FB}(\theta), Y_{FB}(\theta))$ denote the *first-best* investment and effort level, which solve the problem above. Since the funder's payoff is decreasing in $u(\theta)$, the IR constraint, $u(\theta) \geq \overline{\Pi}(\theta)$, binds at the optimum. Then, straightforward maximization yields, for W sufficiently large, $Y_{FB}(\theta) = 1$, and $X_{FB}(\theta) > 0$ given by the solution to the following first-order condition:

$$\theta \rho'(X_{FB}(\theta))(W + \pi) = 1 \tag{FB}$$

Note that $(X_{FB}(\theta), Y_{FB}(\theta))$ maximize total surplus at θ: The LHS of (FB) is the marginal social gain from investment, while the RHS is the marginal social cost of investment. Applying the implicit function theorem to (FB), the concavity of ρ implies $X'_{FB}(\theta) > 0$.

With complete information, there are many ways the funder can induce the researcher to take the first-best investment/effort levels. He offers a contract specifying, $X_{FB}(\theta)$ and $Y_{FB}(\theta)$, and calculates the prize/grant combination, $V(\theta) \geq 0$ and $G(\theta)$, that leaves the researcher with zero rent:

$$u(\theta) = \theta \rho(X_{FB}(\theta))(V(\theta) + \pi) - X_{FB}(\theta) - c + G(\theta) = \overline{\Pi}(\theta)$$

The expression above leaves open the possibility of a pure prize, a pure grant, or a hybrid scheme.

4.2 Pure Moral Hazard

This section studies the case of pure MH: Assume effort is unobservable by the funder, but he observes θ. Given θ, the funder offers a contract, $m = \{v, g, x\}$, to solve,

$$\max_{m} \{\theta\rho(x)(W+\pi) - x - c - u(\theta)\} \text{ s.t.}$$
$$u(\theta) \geq \overline{\Pi}(\theta), \;\; \theta\rho(x)(v+\pi) - c \geq 0, \;\; \text{and} \;\; v \geq 0$$

The distinction between the funder's problem with pure MH, and and complete-information, is the (IC-E) constraint: $\theta\rho(x)(v+\pi) - c \geq 0$, in the problem above. This reflects the fact that a choice of $y = 1$ must be optimal for the researcher. We now show that under pure MH the optimal means of funding is, in general, a pure prize scheme.

Proposition 1.
In the model with pure MH, there exists an optimal means of funding that is a pure prize; moreover, the funder attains the first best: $X(\theta) = X_{FB}(\theta)$, $G(\theta) = 0$ *and* $V(\theta) > 0$ *satisfies,*

$$u(\theta) = \theta\rho(X_{FB}(\theta))(V(\theta)+\pi) - X_{FB}(\theta) - c = \overline{\Pi}(\theta).$$

Under pure MH, there always exists an optimal means of funding that is a pure prize scheme. Intuitively, by only rewarding success, the prize creates a stronger incentive for unobservable effort than does a grant. In fact, the researcher's effort choice is completely independent of the grant.

Even so, it is important to emphasize that a grant *can* be used to encourage effort. The key point is that the researcher's effort depends both on the prize and on investment. As investment is observable, the funder may condition the grant on this variable,[24] and in this way, elicit greater investment. Then, greater investment increases the returns to effort, and through this complementarity, effort can be encouraged. We stress this point in our next proposition, which shows that there may be an optimal means of funding that is a pure grant. Before doing so, we introduce a key piece of notation.

Definition 1. *If* $\lim_{x\to\infty} \theta\rho(x)\pi > c$*, then let* $x_m(\theta)$ *satisfy,*

$$\theta\rho(x_m(\theta))\pi = c.$$

$x_m(\theta)$ is the smallest investment necessary to induce effort from a researcher of type θ if the prize is zero. As $\rho(\cdot)$ is strictly increasing, this implies $x_m'(\cdot) < 0$. We will say that an investment, x, is *sufficient to induce effort at* θ if $x \geq x_m(\theta)$; i.e., if it is optimal for the researcher to exert effort when the prize is zero: $y^*(0, x, \theta) = 1$.[25]

For a fixed θ, $x_m(\theta)$ provides a useful summary of the strength (or severity) of the MH problem, as well as the complementarity between investment and effort. If the researcher's effort cost is high, relative to her product market profit – i.e., $\frac{c}{\pi}$ is large – then a greater incentive is necessary to induce effort. When this is the case, we say that the MH problem is more severe. It is straightforward to show that, for any θ, $x_m(\theta)$ is strictly increasing in this ratio. Moreover, if the complementarity between investment and effort is weak, then the channel through which investment induces effort breaks down, and a higher investment is needed to induce effort. So, the weaker the complementarity between the inputs, the higher is $x_m(\cdot)$, ceteris paribus.[26]

The next proposition provides conditions under which a pure grant is optimal in the model with pure MH.

[24] Recall from Section 3 that we focus, WLOG, on a particular form of dependence, wherein the researcher only receives the grant if she follows through on the agreed upon level of investment. But in general, there are many ways in which this dependence can be modeled.

[25] If $\lim_{x\to\infty} \theta\rho(x)\pi < c$, then $x_m(\theta)$ is not well-defined. In this case, no investment is sufficient to induce effort, and a prize is necessary to elicit effort.

[26] Consider two probability of success functions, $\theta p(x, y)$ and $\theta\tilde{p}(x, y)$. Suppose, $\tilde{p}_{12} \geq p_{12}$ for all (x, y). It is straightforward to show that for any θ and $x \geq 0$, $\theta\tilde{\rho}(x) \geq \theta\rho(x)$. This implies $\overline{x}_{\tilde{p}}(\theta) \leq \overline{x}_p(\theta)$, where $\overline{x}_{\tilde{p}}$ corresponds to $\tilde{p}$, and $\overline{x}_p$ corresponds to p.

Proposition 2.
In the model with pure MH, if the researcher is of type θ, there exists an optimal means of funding that is a pure grant if and only if $x_m(\theta) \leq X_{FB}(\theta)$.

Through the grant, the funder induces the researcher to invest $X_{FB}(\theta)$. But when $X_{FB}(\theta)$ is sufficient to induce effort, the MH problem is overcome, without the need for a prize. The key condition of Proposition 2 is thus, $x_m(\theta) \leq X_{FB}(\theta)$.

4.3 Pure Adverse Selection

This section studies pure AS: Assume that both effort and investment are observable by the funder, but θ is only observed by the researcher. The funder offers a menu of contracts, $m = \{v(\theta), g(\theta), x(\theta), y(\theta)\}_{\theta\in\Theta}$, stipulating both investment and effort. The funder's problem is exactly as in [P] (see Section Section 3), but without (IC-E), since effort is contractible. Our next result characterizes the optimal funding scheme under pure AS.

Proposition 3. *In the model with pure AS, the optimal means of funding is a pure grant for all types: $V(\theta) = 0$ and $G(\theta) > 0$ for all θ. Moreover,*

(1) Investment is distorted below the first-best: For $\theta < \overline{\theta}$, $X(\theta) < X_{FB}(\theta)$; but there is "efficiency at the top": $X(\overline{\theta}) = X_{FB}(\overline{\theta})$. Specifically, for all θ, $X(\theta)$ satisfies:

$$\theta\rho'(X(\theta))(W + \pi) = 1 + h(\theta)\rho'(X(\theta))\pi \tag{4}$$

(2) The grant only partially reimburses expenditures: $G(\theta) < X(\theta) + c$ and $0 < G'(\theta) < X'(\theta)$ for all θ.

Proposition 3 shows that a pure grant scheme is optimal under pure AS; moreover, investment is distorted below the first-best, and the grant partially reimburses costs. To understand why the optimal prize is zero, consider a two-type version of the model: $\Theta = \{\underline{\theta}, \overline{\theta}\}$, where $\overline{\theta} > \underline{\theta}$. IC dictates, $u(\overline{\theta}) \geq u(\underline{\theta}|\overline{\theta})$;

but, in equilibrium, this constraint binds. Let $\underline{x}$ and $\underline{v}$ denote the investment and prize (respectively) offered to the low type, then one can show,

$$u(\overline{\theta}) = u(\underline{\theta}|\overline{\theta}) = (\overline{\theta} - \underline{\theta})\rho(\underline{x})(\underline{v} + \pi) > 0 \tag{5}$$

From (5) it is clear that $u(\overline{\theta})$ is strictly increasing in $\underline{v}$, but it does not depend on the grant offered to the low type. Intuitively, if the high type imitates the low type, the high type is more likely to succeed, and therefore more likely to receive the prize, $\underline{v}$, than the low type would be. Therefore, the expected value of the prize intended for the low type, $\theta\rho(\underline{x})\underline{v}$, is greater for the high type than the low type. To prevent under-reporting, the high type must be offered a rent to compensate her for this fact. A grant, in contrast, is received independently of success or failure, so its expected value is the same for both types. For this reason, the prize is a more expensive means of funding than the grant. As both inputs are observable, the funder induces effort/investment via the cheaper grant scheme.

Also from (5) it is clear that the high-type's information rent is increasing in $\underline{x}$. To limit the information rent of higher types, investment is distorted below the first best for all types below the highest type. The optimal investment schedule balances the trade-off between rent-extraction and efficiency: The LHS of (4) is the marginal social benefit of investment; the RHS is the marginal social cost plus the marginal information rent cost to the funder.

Although this efficiency/rent extraction trade-off is standard in AS models, we highlight the role played by π and the free-disposal constraint in our model. For simplicity, in the discussion that follows, suppose $\overline{\Pi}(\theta) = 0$ for all θ. If we relaxed the free-disposal constraint, or set $\pi = 0$, then the funder could appropriate all of the researcher's rent, and attain the first-best by setting $v(\cdot) = -\pi$, and $g(\cdot) = x(\cdot) = X_{FB}(\cdot)$ (see, e.g., Lewis and Sappington, 2000b; Bolton and Dewatripont, 2005). But $\pi > 0$, combined with free disposal, implies that the researcher must capture at least π in the event of success. This leaves an inappropriable rent for the researcher, and leads to the downward distortion in investment.

As another consequence of $\pi > 0$ (and free-disposal), Proposition 3 shows that the grant offers less than full cost reimbursement ($G < X + c$), and the cost borne by the researcher, $X+c-G$, increases in type (since $G' < X'$). This structure ensures that, only a researcher that is sufficiently likely to succeed, is willing to receive a large grant. A grant that fully reimburses investment would lead the researcher to always behave as if she is of type $\overline{\theta}$ – the type receiving the greatest investment recommendation – in order to have the greatest chance of success (and receiving π).

4.4 Mixed Case: Adverse Selection and Moral Hazard

We now explore the general case of AS and MH. The funder's (relaxed) problem is given by [P] in Section 3. Let $X_{AS}(\cdot)$ and $G_{AS}(\cdot)$ denote the optimal investment and grant schedules characterized in Proposition 3 under pure AS.[27] Recall, that $X_{AS}(\cdot)$ balances the tradeoff between efficiency and rent extraction. An investment above $X_{AS}(\theta)$, or a prize greater than zero, generates excessively high information rent for the researcher. It is useful to bear in mind that when MH is also a relevant concern, the funder would like to keep investment as close as possible to $X_{AS}(\cdot)$, and the prize as small as possible, subject to the constraint that the researcher exerts effort.

We are now ready to state our main results. It will be seen that the structure of the optimal scheme under AS and MH depends critically on $x_m(\cdot)$.

Proposition 4. *If $x_m(\underline{\theta}) \leq X_{FB}(\underline{\theta})$ then the optimal means of funding is a pure grant for all types: $V(\theta) = 0$ and $G(\theta) > 0$ for all θ. Moreover,*

(1) If $x_m(\underline{\theta}) \leq X_{AS}(\underline{\theta})$, then for all θ: $X(\theta) = X_{AS}(\theta)$, and $G(\theta) = G_{AS}(\theta)$.

(2) If $X_{AS}(\underline{\theta}) < x_m(\underline{\theta}) \leq X_{FB}(\underline{\theta})$, then there exists $\theta' \in (\underline{\theta}, \overline{\theta})$ such that

- *(i) For $\theta \in [\underline{\theta}, \theta']$ there is bunching: $X(\theta) = G(\theta) = x_m(\underline{\theta})$.*
- *(ii) For $\theta \in (\theta', \overline{\theta}]$: $X(\theta) = X_{AS}(\theta)$, $G(\theta) < X(\theta)$ and $0 < G'(\theta) < X'(\theta)$.*

[27] $G_{AS}(\cdot)$ is fully characterized in the proof of Proposition 3.

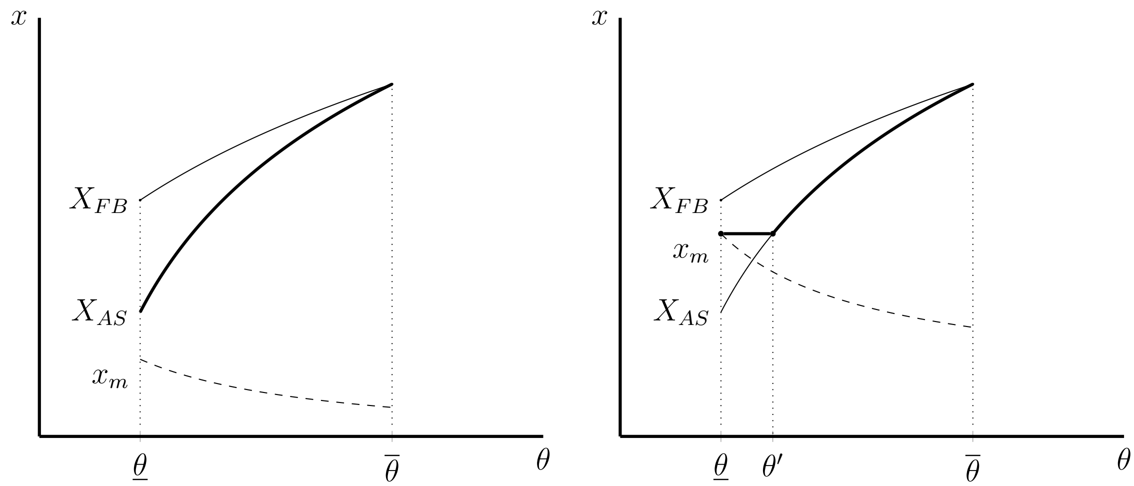

Figure 1: Investment schedules (in bold) for the cases covered in Proposition 4. Case (1) is shown in the left panel, and case (2) in the right panel.

Proposition 4 reveals the circumstances in which optimal funding takes the form of a pure grant scheme, despite the MH problem. The optimal investment schedules for the two cases covered by Proposition 4 are shown in Figure 1.

Under the hypothesis of Proposition 4(1), $X_{AS}(\theta)$ is sufficient to induce effort at each θ, i.e. $x_m(\theta) \leq X_{AS}(\theta)$ for all θ. In this case, the MH problem is completely resolved – without a prize – at no additional cost to the funder.

In the case covered by Proposition 4(2) there is an interval of low types such that for each θ in this interval, $X_{AS}(\theta)$ is not sufficient to induce effort. For these types, the funder must raise investment above $X_{AS}(\cdot)$, and/or offer a prize to elicit effort. Either way, greater rent will be generated for higher types. But there is an advantage to encouraging effort through greater investment. To see why, consider the problem of encouraging effort from a researcher of type $\underline{\theta}$. Under the hypothesis of Proposition 4(2), $X_{FB}(\underline{\theta})$ is sufficient to induce effort at $\underline{\theta}$, i.e., $x_m(\underline{\theta}) \leq X_{FB}(\underline{\theta})$. As total surplus at any θ is strictly increasing in x for $x < X_{FB}(\theta)$, the funder can raise the type-$\underline{\theta}$'s investment up to $x_m(\underline{\theta})$, which induces effort and increases total surplus. This increase in total surplus partially offsets the additional information rent cost to the funder. A prize, in contrast, does not affect total surplus, but simply transfers surplus from the funder to the researcher. For this reason, effort is encouraged through increased investment, incentivized by a grant.

The discussion above suggests that whenever $X_{AS}(\theta) < x_m(\theta) < X_{FB}(\theta)$ then it should be optimal for the funder to offer no prize, and specify the investment, $x_m(\theta)$, to induce effort. However, $x_m(\cdot)$ is strictly decreasing, and (IC-S) dictates that the investment schedule be non-decreasing when the prize is zero. Therefore, this investment schedule cannot be implemented over an interval of types, and bunching arises amongst low types.

The next result shows that if $x_m(\underline{\theta})$ is larger than in Proposition 4, then a hybrid scheme is used for some types.

Proposition 5. *If $X_{FB}(\underline{\theta}) < x_m(\underline{\theta}) < X_{FB}(\overline{\theta})$ then the optimal means of funding is a hybrid for sufficiently low types, and a pure grant for sufficiently high types. That is, $G(\theta) > 0$ for all θ. While there is some $\theta' \in (\underline{\theta}, \overline{\theta})$ such that if $\theta < \theta'$ then $V(\theta) > 0$, and if $\theta \geq \theta'$ then $V(\theta) = 0$. Moreover, there exists $\theta'' \in (\theta', \overline{\theta})$ such that,*

(1) For $\theta \in [\underline{\theta}, \theta')$ investment is equal to the first-best, and is fully reimbursed by the grant: $X(\theta) = G(\theta) = X_{FB}(\theta)$.

(2) For $\theta \in [\theta', \theta'']$ there is bunching: $X(\theta) = G(\theta) = X_{FB}(\theta')$.

(3) For $\theta \in (\theta'', \overline{\theta}]$: $X(\theta) = X_{AS}(\theta)$, $0 < G(\theta) < X(\theta)$, and $0 < G'(\theta) < X'(\theta)$.

The left panel of Figure 2 illustrates the prize and investment schedules for the case covered by Proposition 5. We first provide the intuition for the optimal hybrid scheme in the range of low types, $[\underline{\theta}, \theta']$, given in Part (1).

Under the hypothesis of Proposition 5, there is a range of low types such that for each type θ in this range, neither $X_{FB}(\theta)$, nor $X_{AS}(\theta)$, are sufficient to induce effort, i.e., $x_m(\theta) > X_{FB}(\theta) > X_{AS}(\theta)$. Consider the problem of eliciting effort from the type $\underline{\theta}$. As in the case of Proposition 4(2), it is feasible to elicit effort from this type through increased investment (i.e., set $v(\underline{\theta}) = 0$ and set $x(\underline{\theta}) = x_m(\underline{\theta})$). However, it is never optimal to increase investment

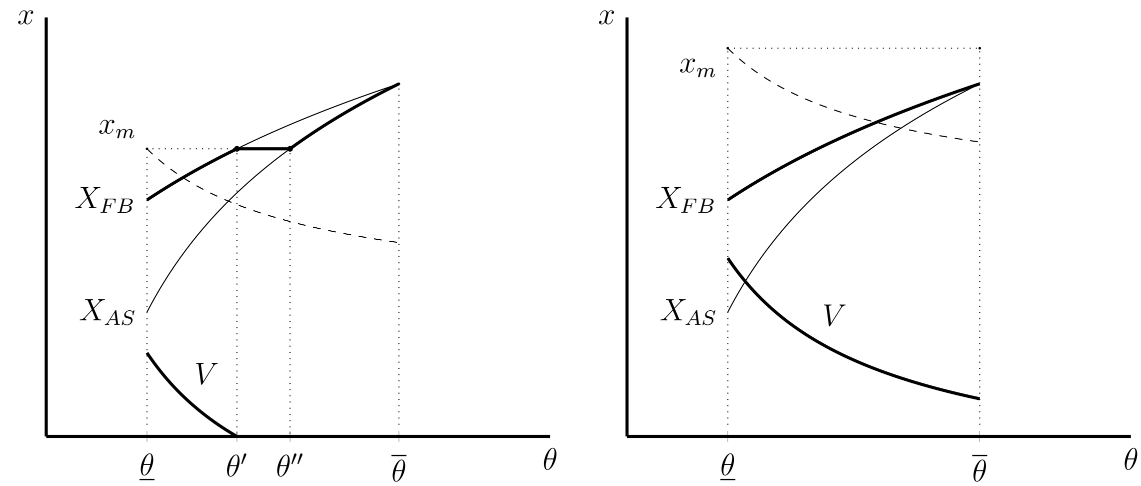

Figure 2: Investment and prize schedules (in bold) for the cases covered in Proposition 5 (left panel) and Proposition 6 (right panel).

beyond first-best, as total surplus at any θ is strictly decreasing in x for $x > X_{FB}(\theta)$. Instead, the funder sets investment equal to the first-best, $X(\underline{\theta}) = X_{FB}(\underline{\theta})$, and offers a prize, $V(\underline{\theta}) > 0$, to induce effort.

To limit the researcher's rent, the funder would like to keep the prize small. The smallest prize capable of eliciting effort from all types leaves (IC-E) binding at each θ: $\theta\rho(x(\theta))(v(\theta) + \pi) = c$. But for (IC-E) to bind over an interval of types, the term, $\rho(x(\cdot))(v(\cdot)+\pi)$, must be strictly decreasing, which violates (IC-S). As a result, (IC-E) binds only at $\underline{\theta}$, and whenever $V(\theta) > 0$, (IC-S) binds. To keep the prize as small as possible, a grant is used to fully offset the cost of investment. The optimal prize schedule, $V(\cdot)$, is the smallest prize consistent with IC that is capable of inducing effort from any type.

A researcher of type $\theta \in [\underline{\theta}, \theta']$ is indifferent between reporting her type truthfully, or any other $\hat{\theta} \in [\underline{\theta}, \theta']$. If, for instance, the researcher over reports, she receives a larger investment, which increases the likelihood of success, but she receives a smaller prize. In equilibrium, these two effects exactly offset.

To explain the bunching of intermediate types described in Proposition 5(2), first recall that when $v(\cdot) = 0$, (IC-S) requires that $x(\cdot)$ is non-decreasing. Examining Figure 2, it can be seen that for each type $\theta \in (\theta', \theta'')$, it must hold that $x(\theta) \geq X_{FB}(\theta') > X_{AS}(\theta)$. For each type in this interval, the funder would like to reduce investment closer to $\max\{X_{AS}(\theta), x_m(\theta)\}$, which is not feasible. Therefore, (IC-S) binds, and bunching arises.

Our final result in this section reveals conditions under which the optimal means of funding is a hybrid scheme for all types.

Proposition 6. *If $X_{FB}(\overline{\theta}) \leq x_m(\underline{\theta})$ then the optimal means of funding is a hybrid for all types. Moreover, investment is equal to the first-best, and is fully reimbursed by the grant: $V(\theta) > 0$ and $X(\theta) = G(\theta) = X_{FB}(\theta)$ for all $\theta \in \Theta$.*[28]

The intuition for the funding scheme outlined in Proposition 6 is similar to the intuition behind the hybrid scheme offered to low types in Proposition 5. The difference here is that the prize required to induce effort from the lowest type is large enough that, when combined with the bound on the slope of $V(\cdot)$ provided by (IC-S), the prize is strictly positive for all types.

If one compares the optimal investment schedule under pure AS, $X_{AS}(\cdot)$, with the optimal investment schedules characterized in Propositions 4-6, it is clear that for any θ, $X_{AS}(\theta) \leq X(\theta) \leq X_{FB}(\theta)$. Thus total surplus is (weakly) higher at each θ when effort is unobservable, than when it is observable. The following corollary formalizes this observation.

Corollary 1. *When the funder faces an AS problem, equilibrium total surplus is (weakly) higher at each θ when effort is unobservable, than when it is observable.*

As mentioned in the introduction, in mixed models where the agent's only action (effort) is unobservable and chosen after learning her type, effort tends to be distorted below the first-best to a greater extent than under pure AS. Corollary 1 shows that this is not the case when investment is observable. To understand the role played by the observable action, note that when actions are all unobservable, typically, the instruments that overcome MH also generate significant rent for the agent (due to AS), and do not directly contribute to

[28]Proposition 6 also applies when $\lim_{x\to\infty} \underline{\theta}\rho(x)\pi < c$, which means $x_m(\underline{\theta})$ is not well defined.

total surplus. For instance, if investment were unobservable in our model then greater investment/effort must be encouraged through a prize. The prize generates significant rent for the researcher; to limit these rents, weaker incentives are provided, as compared to pure AS. In addition, the prize does not contribute to total surplus. As a result, total surplus is distorted further below the first-best, as compared to pure AS. In contrast, when investment is observable, it can be incentivized through the grant, and greater effort can be encouraged through the complementarity between the inputs. The grant is much more effective in limiting the researcher's rent, and investment contributes directly to total surplus.

Let us make one remark as regards Corollary 1. We have assumed throughout that the funder finds it optimal to elicit effort from a researcher of any type. This assumption is more stringent in the model with combined AS and MH than with pure AS.[29] If we relax this assumption, the welfare comparison becomes less clear. Nevertheless, Corollary 1 provides a useful benchmark for comparing welfare when the social value of the project is sufficiently large.

Finally, Propositions 4 and 5 reveal that bunching may be a prominent feature of the optimal incentive scheme in our setting. In pure AS models, bunching is often avoided by assuming a monotone hazard rate (or inverse hazard rate) on the distribution over types. When bunching is not ruled out by this distributional assumption, frequently cited reasons for it to occur are countervailing incentives due to type-dependent outside options (see, e.g. Lewis and Sappington, 1989; Maggi and Rodriguez-Clare, 1995; Jullien, 2000), or "non-responsiveness" (see, e.g. Guesnerie and Laffont, 1984).[30] In models that combine AS and MH, bunching may arise for other reasons. Ollier and Thomas (2013) show that an ex post participation constraint may give rise to countervailing incentives, and bunching may occur for this reason. Gottlieb and Moreira (2015) show that bunching is a quite robust feature of binary

[29] i.e., If we relax this assumption, there may be instances where it is not optimal to elicit effort from some low types under AS and MH, but where it is optimal to elicit effort from all types with pure AS.

[30] Non-responsiveness refers to a situation in which the first-best allocation is not implementable.

outcome models with AS, MH and limited liability. In our model, bunching does not arise for any of the reasons previously described; but rather, it is due to the conflict between rent extraction, effort inducement, and the second-order incentive compatibility constraint.

5 Discussion: The Use and Design of Push Programs

When are push programs useful?

Proposition 4 shows that a pure grant scheme is optimal (for all types) in our model if $x_m(\underline{\theta}) \leq X_{FB}(\underline{\theta})$. This condition provides insights into the circumstances that a push program may be useful in practice. In particular, we conclude that a push program may be more relevant: (1) When AS is an issue; (2) For a more profitable project (i.e., π is large); (3) For a researcher with less valuable alternative endeavors, to which she can devote her time (i.e., the opportunity cost of effort, c, is small); (4) If there is a strong complementarity between capital and labor (i.e., $p_{12}(x, y)$ is large); and (5) For a project with a high social value (i.e., $W + \pi$ is large).

Point (1) gives rise to the trade-off between a push and pull program. When AS is an issue, a pull program, while more effective in motivating unobservable effort, is a more expensive means of funding than a push program. Points (2)-(4) imply that the MH problem is not too severe, and that (less easily observed) labor inputs (effort) can be encouraged through greater capital investments (which may be easier to verify). The intuition behind point (5) is the following:[31] When MH is a concern, the funder must make the project worth the researcher's time. This may be done either through a prize, or if capital and labor are complements, through greater investment (incentivized via a grant). For a project with a high social value, the funder is willing to finance a greater investment, as doing so increases total surplus.

[31] Mathematically, point (5) holds since $X_{FB}(\theta)$ is positively related to both W and π, while $x_m(\theta)$ does not depend on W, and is negatively related to π.

These observations may help to shed light on patterns of funding observed in practice. Let us provide one concrete example. The Bill and Melinda Gates Foundation (GF) provides incentives for researchers to undertake projects related to the development of pharmaceuticals used to treat/prevent certain diseases prevalent in the developing world. McCoy et al. (2009) offer a detailed report on the funding pattern of GF between 1998-2007. Over this period, it is reported that GF issued $8.95 billion in grants for global health. Of these funds, almost 37% were allocated to non governmental or non-profit research organizations, while less than 1% were awarded to for-profit firms.

Our results may help explain why GF uses a push program, and the distinction it makes between non-profits and for-profits. First, AS is likely an issue, as expert researchers probably have better information than GF about the prospects of drug development. Second, for-profits and non-profits may differ in their natural motivations to undertake these projects. Due to a number of market failures, the profitability of these projects is low.[32] A for-profit firm, if motivated primarily by monetary incentives, would have little incentive to devote resources to these projects. A non-profit – setup *specifically* to undertake these projects[33] – likely has other motivations (perhaps non-monetary). Third, for-profits and non-profits may differ in the opportunity cost of devoting their time to these ventures: For-profits likely have other, more profitable ventures available, while this may be less of an issue for non-profits.[34] Finally, these projects are also of tremendous social value, as health and economic productivity are intimately linked (see, e.g., Bleakley, 2010). Under these conditions, our model predicts that a push program may be optimal for motivating a non-profit, but it is less likely the case for a for-profit.

[32]See Kremer (2002) and Glennerster et al. (2006) for an overview of the issues

[33]The Program for Appropriate Technology in Health (PATH) is one example of this type of non-profit (see, `http://www.path.org/about/index.php`). According to McCoy et al. (2009), PATH was awarded $949 million in grants from GF between 1998-2007.

[34]Some of these organizations, such as the Medicines for Malaria Venture, are setup primarily to conduct R&D related to one particular condition; this organization received a $115 million grant from GF in 2009 (`http://www.mmv.org/newsroom/press-releases/mmv-receives-115-million-gates-foundation`).

Design of push programs: R&D tax credits and matching grants

Our results also provide insights into the optimal design of push programs. One such program is the R&D tax credit system in the U.S. The U.S. Congress estimated that this system cost the federal government \$6.9 billion in lost tax revenue 2013 (Hemel and Ouellette, 2013). In its simplest form, firms are awarded with a tax credit worth 20% of qualifying expenditures above some base amount. We show that the pure grant scheme characterized in Proposition 4(1) bears some semblance to this system.

Proposition 7. *Under the hypotheses of Proposition 4(1), the optimal funding scheme can be implemented via a menu of linear contracts,* $\{b(\theta), r(\theta)\}_{\theta \in \Theta}$*. For each* $\theta \in \Theta$*, the grant to the researcher takes the form,*

$$\tilde{G}(x, \theta) = \begin{cases} 0 & x \leq b(\theta) \\ r(\theta)(x - b(\theta)) & x \geq b(\theta) \end{cases}$$

where $b'(\cdot) > 0$*,* $r(\cdot) \in (0, 1)$*,* $r'(\cdot) > 0$*, and if* c *is sufficiently small,* $b(\cdot) > 0$*.*

Proposition 7 reveals that the pure grant scheme can be implemented via a menu of linear contracts, $\{b(\cdot), r(\cdot)\}$, each of which specifies a base amount, $b(\theta)$, and a reimbursement rate, $r(\theta)$. If the researcher selects $\{b(\theta), r(\theta)\}$, she is reimbursed nothing for each dollar she invests up to $b(\theta)$, and she is reimbursed at the rate of $r(\theta)$ per dollar she invests above $b(\theta)$. Higher types select higher base levels, and receive a greater rate of reimbursement.

Under the current system of R&D tax credits, a firm's base amount is set according to past R&D expenditures. The logic is that the government would only like to reward firms for investment above and beyond what it would otherwise choose.[35] Proposition 7 provides an alternative means to the same end. Under our system, the researcher is free to choose her base amount, but she faces a trade-off between the base and the reimbursement rate.

Note that for a given base amount, b, and investment, $x > b$, the marginal value of an increase in the reimbursement rate is $x - b$. Therefore, the greater

[35] See Hemel and Ouellette for a comprehensive overview of the R&D tax credit system.

the firm's investment, the higher is the marginal value of an increase in the reimbursement rate. For this reason, a particularly productive firm (i.e., a high θ) that would like to invest more, is willing to accept a higher base, in exchange for a higher reimbursement rate. In contrast to the current system, our system contemporaneously links the base amount to the firm's desired investment (rather than relying on past behavior), and provides stronger marginal incentives to more productive firms.

As mentioned in the introduction, one concern with the use of push programs is that they may pay for research that is unlikely to succeed. Propositions 3 and 4(1) shed light on this very issue. An important feature of the optimal funding scheme in these cases is that, while higher types receive larger grants, they are expected to bear a greater cost. In this way, only a researcher that is sufficiently likely to succeed is willing to receive a large grant. This feature of the funding scheme resembles a matching grant, which requires expenditures from the recipient in excess of the grant. Matching grants, and other cost-sharing programs, are commonly used by federal agencies in the U.S.[36] Our results suggest that such schemes may be particularly effective in dealing with AS.

Maurer and Scotchmer (2003) also point out that a matching grant can be an effective screening device in the presence of AS. Our results reveal the conditions under which this is in fact the optimal means of screening in a setting where MH is also relevant. Cost sharing policies have been advocated in other contexts for dealing with AS and MH. Laffont and Tirole (1986), for example, emphasize cost sharing as a way to elicit greater greater effort devoted to cost reduction, while still limiting the firm's rent.

Capital constraints

One potentially important consideration for the form of funding, from which our model abstracts, is a capital constraint. Some push programs, such as research grants, provide upfront funding. It might be argued that this is nec-

[36]See `http://www.grants.gov` for a comprehensive list of matching grants currently offered by federal agencies.

essary when the researcher has limited access to capital. As Scotchmer (2004, Ch. 8) also points out, this explanation is not satisfactory, as an appropriately designed pull program should be capable of attracting funding from financiers. Indeed, this is precisely the logic behind the "Pay for Success" model run by the U.S. Department of Labor (DOL).[37] It is also worth pointing out that other push programs, such as R&D tax credits, do not provide funding upfront.

Even so, while we do not include a capital constraint, our results may be useful for understanding the issue. Rather than an explicit inability to raise capital, one could alternatively imagine settings where (1) the socially-optimal level of investment is large; (2) the researcher has a strong incentive to devote her time and energy to a project; but (3) is unwilling to raise the necessary capital, given her costs and benefits. In our model, this translates to a setting where X_{FB} is large, $\frac{c}{\pi}$ is small, but the marginal cost of investment (normalized to 1) is large, relative to π. While a pull program *could* be used to encourage greater investment, our results imply that a push program may be optimal under these conditions.

6 Comparative Statics

This section compares the performance of grant and prize based funding when the strength of the MH problem increases. We then explore the relationship between the profitability of the project, and the optimal funding scheme.

From our results in Section 4.4, one may be tempted to draw the general conclusion that as the strength of the MH problem increases, a prize becomes

[37] According to DOL (`https://www.doleta.gov/workforce_innovation/success.cfm`):

> Under the Pay for Success model, a government agency commits funds to pay for a specific outcome that is achieved within a given timeframe. The financial capital to cover the operating costs of achieving the outcome is provided by independent investors. In return for accepting the risks of funding the project, the investors may expect a return on their investment if the project is successful; however, payment of the committed funds by the government agency is contingent on the validated achievement of results.

a relatively more attractive means of funding. As it happens, this conclusion is not precisely accurate in the context of our model. In fact, when the MH problem is weak, an increase in its severity (as measured by an increase in c) renders prizes, in some sense, *less* attractive to the funder.

In order to facilitate a comparison of grant and prize-based funding, suppose that the funder, for whatever reason, uses a pure prize scheme (i.e. $g(\cdot) \equiv 0$). Let $\phi_p(c)$ denote the funder's optimal payoff when he encourages R&D activity using a pure prize, when the cost of effort is c. Let $\phi_g(c)$ denote the funder's optimal payoff when he uses a pure grant scheme (i.e. $v(\cdot) \equiv 0$). Let $D(c) \equiv \phi_g(c) - \phi_p(c)$ denote the difference between the funder's optimal pure-grant and pure-prize payoffs. Define the function $\tilde{h}$ as follows:

$$\tilde{h}(\theta) = \frac{\int_{\theta}^{\overline{\theta}} t f(t)\, dt}{\theta^2 f(\theta)}$$

For our next result, we assume that $\tilde{h}'(\cdot) < 0$. Similar to the decreasing inverse hazard rate condition, this condition ensures full separation of types when the funder offers only a prize. This assumption is satisfied, for example, by a uniform distribution. We also assume that the researcher's outside option is zero for each type: $\overline{\Pi}(\theta) = 0$ for all θ. Neither of these assumptions is necessary for the next result, but we impose them for ease of exposition.

Proposition 8. *Assume $\tilde{h}'(\cdot) < 0$, and $\overline{\Pi}(\theta) = 0$ for all θ. Under the hypotheses of Proposition 4(1), the difference between the funder's optimal pure-grant and pure-prize payoffs, $D(\cdot)$, is strictly positive, and strictly increasing in c.*

Proposition 8 shows that, in some sense, when the MH problem is weak, pure grant funding becomes relatively more attractive as the strength of the MH problem increases. To convey the intuition, suppose the effort cost increases slightly from c' to $c'' = c' + \Delta$, but assume the hypotheses of Proposition 4(1) are satisfied, for both costs. By Proposition 4(1), it follows that the optimal means of funding is the pure-grant scheme. Thus, $D(\cdot) > 0$.

If the funder uses a pure grant, then following increase in c, the grant of the lowest type must increase by Δ in order to maintain IR. To maintain IC, the grant then increases by Δ for each type $\theta > \underline{\theta}$. Optimal investment, and the slope of the grant schedule, are unchanged. So following the cost increase, the researcher's rent is unchanged, and the the funder's payoff decreases by Δ.

If the funder uses a pure-prize, the expected value of the prize offered to the lowest type increases by Δ to maintain IR. However, when the prize offered to the lowest type increases, this creates a stronger incentive for higher types to underreport, and generates a greater rent for these types. To maintain IC, the expected value of the prize offered to all higher types increases by more than Δ. That is, the slope of the prize schedule increases. Therefore, the researcher's expected rent increases, and the funder's payoff decreases by more than Δ.

We now explore the comparative statics with respect to the profitability of the project, π. Since the optimal investment schedule under AS and MH is closely related to $X_{AS}(\cdot)$ and $X_{FB}(\cdot)$, we provide our comparative statics results with respect to these two functions. In what follows, we let ϕ^* denote the funder's ex-ante equilibrium expected payoff.

Proposition 9.

(i) For each $\theta \in \Theta$, $\frac{\partial X_{FB}(\theta)}{\partial \pi} > 0$

(ii) $\frac{\partial X_{AS}(\theta)}{\partial \pi} > 0$ *if and only if* $\theta > h(\theta)$

(iii) $\frac{\partial \phi^*}{\partial \pi} > 0$

An increase in π increases the total surplus generated in the event of success, and so it is intuitive that $X_{FB}(\theta)$ is strictly increasing in π. But there are two competing forces acting on $X_{AS}(\theta)$: On the one hand, the funder may want to increase $X_{AS}(\theta)$ due to the increase in total surplus generated in the event of success. On the other hand, an increase in π increases the marginal cost of investment to the funder, due to the increased cost of maintaining IC.[38] When $\theta > h(\theta)$, the total surplus effect dominates the IC effect, and $X_{AS}(\theta)$ increases in π (vice-versa when $\theta < h(\theta)$). Since $\overline{\theta} > 0 = h(\overline{\theta})$, investment

[38]See discussion following Proposition 3

increases in π for sufficiently high types. Finally, part (*iii*) of Proposition 9 reveals that the funder is always better off (on average) when π increases.

Interestingly, the researcher's equilibrium payoff may be either positively or negatively related to π. The fact that the researcher's payoff may be decreasing in the profitability of the project seems somewhat counterintuitive. To convey the idea, first note that the rent of a type-θ researcher is positively related to both π, and the investment of types just below θ. When an increase in π leads the funder to reduce investment of some low types, the reduction in the researcher's available rent may outweigh the gain in the available rent caused by the increase in π. The following example illustrates; for the purposes of the example, we abstract away from MH, and set $c = 0$.

Example 1. *Let $c = 0$, $\rho(x) = 1 - \exp(-x)$, and $\theta \sim U\left[\frac{1}{4}, 1\right]$. Using Proposition 4, $V(\theta) = 0$, $X(\theta) = \log(\theta(W + \pi) - h(\theta)\pi)$, where $h(\theta) = 1 - \theta$. To calculate the researcher's equilibrium payoff, $u^*(\theta)$, we plug $X(\cdot)$ into (3). Suppose $W = 6$ and consider a slight increase in π from $\pi = 1$ to $\pi = 1.05$. Following the increase in π, the equilibrium investment and payoff of a type just above $\underline{\theta}$ both decrease. For a researcher of type $\theta = .26$, for example, $X(\theta)$ decreases from about $.07696$ to $.05449$ and $u^*(\theta)$ decreases from about $.00038$ to $.00015$. The equilibrium investment and payoff of a high type both increase following the increase in π. Setting $\theta = .8$, for example, $X(\theta)$ increases from about 1.6864 to 1.69194 and $u^*(\theta)$ increases from about $.3392$ to $.35489$.*

One may also wonder whether more profitable projects should receive smaller or greater rewards from the funder. When $V(\theta) > 0$, it holds, $G(\theta) = X_{FB}(\theta)$ and (as shown in the proof of Propositions 5 and 6) $V(\theta) = \frac{c}{\underline{\theta}\rho(X_{FB}(\theta))} - \pi$. Following an increase in π, $X_{FB}(\theta)$ increases, so $V(\theta)$ decreases, and $G(\theta)$ increases.

When the funder uses a pure grant scheme, the impact of a change in π on the grant is unclear. To simplify the following discussion, assume that $\overline{\Pi}(\theta) = 0$ for all θ, so IR just requires $u(\theta) \geq 0$. The grant serves two purposes: It is necessary to satisfy IR, and it is used to reward higher types with information rent (to maintain IC). Following an increase in π, there are two competing forces on the grant related to IR. There is a direct effect: The project becomes more profitable, and hence, a smaller grant is required to ensure participation. But there is also an indirect effect: An increase in π may lead the funder to either increase or decrease investment (by Proposition 9(ii)). Ceteris paribus, an increase (decrease) in investment means a larger (smaller) grant is necessary to satisfy IR.

Similarly, following an increase in π there is a direct effect on IC: Ceteris paribus, an increase in π generates greater information rent for the researcher, and a larger grant is used to maintain IC. But there is also an indirect effect since investment around some θ may increase or decrease. All else equal, an increase (decrease) in investment for types just below θ, increases (decreases) the rent of the type θ, and increases (decreases) the size of the grant necessary to maintain IC. The net effect of a change in π on the grant depends on the balance of these forces and is, in general, ambiguous.

7 Conclusion

In this paper we fully characterized the optimal contracts in a setting where the inputs to production consist of both an observable and unobservable component, and the agent holds private information regarding the prospects of the project. We provided conditions under which pay-for-performance may not be optimal, and used our findings to shed light on push programs used in practice to encourage R&D. Although we focus on policy implications related to R&D funding, our model is useful for understanding the emergence of low-powered incentive schemes in many other contexts, e.g., worker compensation.

In addition, we provided a novel explanation for the emergence of bunching in contracts, and shed new light on the welfare implications of AS and MH. In particular, we showed that when AS and MH interact, total surplus tends to be higher than in a pure AS setting, which contrasts the typical finding in models where the agent's action is chosen after learning her type.

References

Baker, G. P. (1992). Incentive Contracts and Performance Measurement. *Journal of Political Economy*, pages 598 – 614.

Basov, S. and Bardsley, P. (2005). A General Model of Coexisting Hidden Action and Hidden Information. Working paper No. 958, Department of Economics, University of Melbourne.

Bleakley, H. (2010). Malaria Eradication in the Americas: A Retrospective Analysis of Childhood Exposure. *American Economic Journal: Applied Economics*, 2(2):1–45.

Bolton, P. and Dewatripont, M. (2005). *Contract Theory*. MIT press.

Caillaud, B., Guesnerie, R., and Rey, P. (1992). Noisy Observation in Adverse Selection Models. *The Review of Economic Studies*, 59(3):595–615.

Chen, B. (2010). All-or-Nothing Monitoring: Comment. *The American Economic Review*, 100(1):625–627.

Chen, B. (2012). All-or-nothing payments. *Journal of Mathematical Economics*, 48(3):133–142.

Fu, Q., Lu, J., and Lu, Y. (2012). Incentivizing R&D: Prize or Subsidies? *International Journal of Industrial Organization*, 30(1):67–79.

Glennerster, R., Kremer, M., and Williams, H. (2006). Creating Markets for Vaccines. *Innovations*, 1(1):67–79.

Gottlieb, D. and Moreira, H. (2015). Simple Contracts with Adverse Selection and Moral Hazard. The Wharton School Research Paper No. 78.

Grossman, S. J. and Hart, O. D. (1983). An Analysis of the Principal-Agent Problem. *Econometrica*, pages 7–45.

Guesnerie, R. and Laffont, J. J. (1984). A Complete Solution to a Class of Principal-Agent Problems with an Application to the Control of a Self-Managed Firm. *Journal of Public Economics*, 25(3):329–369.

Guesnerie, R., Picard, P., and Rey, P. (1989). Adverse selection and Moral Hazard with Risk Neutral Agents. *European Economic Review*, 33(4):807–823.

Hall, B. H. (2007). Patents and patent policy. *Oxford Review of Economic Policy*, 23(4):568–587.

Hall, B. H., Mairesse, J., and Mohnen, P. (2009). Measuring the Returns to R&D. National Bureau of Economic Research w15622.

Hemel, D. J. and Ouellette, L. L. (2013). Beyond the Patents-Prizes Debate. *Texas Law Review*, 92(2):303–382.

Hölmstrom, B. (1979). Moral Hazard and Observability. *The Bell Journal of Economics*, pages 74–91.

Hölmstrom, B. and Milgrom, P. (1991). Multitask Principal-Agent Analyses: Incentive Contracts, Asset Ownership, and Job Design. *Journal of Law*, 7:24–52.

Innes, R. D. (1990). Limited Liability and Incentive Contracting with Ex-Ante Action Choices. *Journal of Economic Theory*, 52(1):45–67.

Jullien, B. (2000). Participation Constraints in Adverse Selection Models. *Journal of Economic Theory*, 93(1):1–47.

Kremer, M. (2002). Pharmaceuticals and the Developing World. *The Journal of Economic Perspectives*, 16(4):67–90.

Laffont, J. J. (1995). Regulation, Moral Hazard and Insurance of Environmental Risks. *Journal of Public Economics*, 58(3):319–336.

Laffont, J. J. and Martimort, D. (2009). *The Theory of Incentives: The Principal-Agent Model*. Princeton University Press.

Laffont, J. J. and Tirole, J. (1986). Using Cost Observation to Regulate Firms. *The Journal of Political Economy*, pages 614–641.

Laffont, J. J. and Tirole, J. (1993). *A Theory of Incentives in Procurement and Regulation*. Cambridge, Mass: MIT Press.

Lazear, E. P. (1986). Salaries and Piece Rates. *Journal of Business*, pages 405–431.

Lazear, E. P. (2000). The Power of Incentives. *American Economic Review*, 90(2):410–414.

Lewis, T. R. and Sappington, D. (1989). Countervailing Incentives in Agency Problems. *Journal of Economic Theory*, 49(2):294–313.

Lewis, T. R. and Sappington, D. E. (2000a). Contracting With Wealth-Constrained Agents. *International Economic Review*, 41(3):743–767.

Lewis, T. R. and Sappington, D. E. (2000b). Motivating Wealth-Constrained Actors. *American Economic Review*, 90(4):944–960.

Maggi, G. and Rodriguez-Clare, A. (1995). On Countervailing Incentives. *Journal of Economic Theory*, 66(1):238–263.

Maurer, S. M. and Scotchmer, S. (2003). Procuring Knowledge. National Bureau of Economic Research w9903.

McCoy, D., Kembhavi, G., Patel, J., and Luintel, A. (2009). The Bill & Melinda Gates Foundation's Grant-Making Programme for Global Health. *The Lancet*, 373(9675):1645–1653.

Meng, D. and Tian, G. (2012). Multi-task Incentive Contract and Performance Measurement with Multidimensional Types. *Games and Economic Behavior*, 77(1):1–45.

Ollier, S. and Thomas, L. (2013). Ex Post Participation Constraint in a Principal-Agent Model with Adverse Selection and Moral Hazard. *Journal of Economic Theory*, 148(6):2383–2403.

Picard, P. (1987). On the Design of Incentive Schemes Under Moral Hazard and Adverse Selection. *Journal of Public Economics*, 33(3):305–331.

Poblete, J. and Spulber, D. (2012). The Form of Incentive Contracts: Agency with Moral hazard, Risk Neutrality, and Limited liability. *The RAND Journal of Economics*, 43(2):215–234.

Prendergast, C. (1999). The Provision of Incentives in Firms. *Journal of economic literature*, 37(1):7–63.

Prendergast, C. (2002). The Tenuous Trade-off between Risk and Incentives. *Journal of Political Economy*, 110(5):1071–1102.

Sappington, D. (1982). Optimal Regulation of Research and Development Under Imperfect Information. *The Bell Journal of Economics*, pages 354–368.

Scotchmer, S. (2004). *Innovation and Incentives*. MIT press.

Zhao, R. R. (2008). All-or-Nothing Monitoring. *The American Economic Review*, 98(4):1619–1628.

Appendices

A Further Analysis of IC

In this appendix, we show that under free-disposal, and the IR/IC constraints given in Section 3.2, it is never optimal for the researcher to choose zero effort when $\overline{\Pi}(\underline{\theta}) = 0$ or $x(\underline{\theta}) \geq \overline{x}(\underline{\theta})$. Regardless of her type, if the researcher reports θ and shirks on effort, the project fails with certainty and her payoff is $g(\theta) - x(\theta)$. So, if $x(\theta) \geq g(\theta)$ for all θ, then the researcher's payoff if she shirks on effort is non-positive, and IR would ensure that this is suboptimal.

First, we will show $x(\underline{\theta}) \geq g(\underline{\theta})$ if $\overline{\Pi}(\underline{\theta}) = 0$ or $x(\underline{\theta}) \geq \overline{x}(\underline{\theta})$. We then show $x'(\theta) \geq g'(\theta)$ for all θ. Initially, suppose $\overline{\Pi}(\underline{\theta}) = 0$. Then, $u(\underline{\theta}) = \overline{\Pi}(\underline{\theta}) = 0$ means,

$$x(\underline{\theta}) - g(\underline{\theta}) = \underline{\theta}\rho(x(\underline{\theta}))(v(\underline{\theta}) + \pi) - c \tag{6}$$

By (IC-E) the RHS of (6) is non negative, and hence $x(\underline{\theta}) \geq g(\underline{\theta})$. Next, suppose $\overline{\Pi}(\underline{\theta}) > 0$ and $x(\underline{\theta}) \geq \overline{x}(\underline{\theta})$. It must be that $\overline{y}(\underline{\theta}) = 1$, and $\overline{x}(\underline{\theta}) > 0$. After some re-arranging and simplification, $u(\underline{\theta}) = \overline{\Pi}(\underline{\theta})$ means,

$$x(\underline{\theta}) - g(\underline{\theta}) = \underline{\theta}[\rho(x(\underline{\theta}))(v(\underline{\theta}) + \pi) - \rho(\overline{x}(\underline{\theta}))\pi] + \overline{x}(\underline{\theta})$$

Since $x(\underline{\theta}) \geq \overline{x}(\underline{\theta})$ (by assumption), and $v(\underline{\theta}) \geq 0$ (by free-disposal), the term in square brackets is non-negative. Thus, the RHS of the expression above is strictly positive. This establishes that $x(\underline{\theta}) \geq g(\underline{\theta})$ if $\overline{\Pi}(\underline{\theta}) = 0$ or $x(\underline{\theta}) \geq \overline{x}(\underline{\theta})$. Next, we show $x'(\theta) \geq g'(\theta)$ for all θ. Fix $\theta \in \Theta$. Using the definition of $u(\theta)$, (IC-F) can be written:

$$x'(\theta) - g'(\theta) = \theta\left[\rho'(x(\theta))(v(\theta) + \pi)x'(\theta) + \rho(x(\theta))v'(\theta)\right] \tag{7}$$

By (IC-S) the RHS of (7) is non-negative, and hence $x'(\theta) \geq g'(\theta)$; combined with $x(\underline{\theta}) \geq g(\underline{\theta})$ this means that for each $\theta \in \Theta$, $x(\theta) \geq g(\theta)$.

B Proofs

Proof of Lemma 1

By assumption, $u(\underline{\theta}) \geq \overline{\Pi}(\underline{\theta})$; so, the result follows if $u'(\theta) \geq \overline{\Pi}'(\theta)$ for all θ. Fix $\theta \in \Theta$. (IC-F) implies, $u'(\theta) = \rho(x(\theta))(v(\theta) + \pi)$. By the envelope theorem, $\overline{\Pi}'(\theta) = \overline{y}(\theta)\rho(\overline{x}(\theta))\pi \leq \rho(\overline{x}(\theta))\pi$. But since $v(\theta) \geq 0$, $x(\theta) \geq \overline{x}(\theta)$, and ρ is strictly increasing, it holds, $\rho(x(\theta))(v(\theta) + \pi) \geq \rho(\overline{x}(\theta))\pi$. Hence, $u'(\theta) \geq \overline{\Pi}'(\theta)$. $\square$

Proof of Proposition 1

Set $X = X_{FB}(\theta)$, $G = 0$, and let V satisfy $\theta\rho(X)V = \overline{\Pi}(\theta) - \Pi(X, 1, \theta)$. As $X_{FB}(\theta) > \overline{x}(\theta)$, it holds that $\overline{\Pi}(\theta) - \Pi(X, 1, \theta) > 0$; hence $V > 0$. Note, moreover, that $\theta\rho(X)(V + \pi) - c = X > 0$, which ensures $y^*(V, X, \theta) = 1$. Finally, by construction, $u(\theta) = \overline{\Pi}(\theta)$, so IR is satisfied. This contract induces the first-best investment/effort levels, and sets $u(\theta) = \overline{\Pi}(\theta)$. Therefore, the funder's payoff is equal to the first-best payoff, and the specified contract is an optimal contract. □

Proof of Proposition 2

Set $X = X_{FB}(\theta)$, and $V = 0$. Let $G = \overline{\Pi}(\theta) - \Pi(X, 1, \theta) > 0$. Then, see that $y^*(0, X, \theta) = 1$ if and only if $X_{FB}(\theta) \geq x_m(\theta)$. So, this contract satisfies IR (by construction), and induces the first-best investment/effort levels if and only if $X_{FB}(\theta) \geq x_m(\theta)$.

Proof of Proposition 3

The funder's relaxed problem is,

$$\max_{x(\cdot),v(\cdot)} \left\{ \int_{\underline{\theta}}^{\overline{\theta}} \big[\theta\rho(x(\theta))[W + \pi] - x(\theta) - c - \rho(x(\theta))(v(\theta) + \pi)h(\theta)\big] f(\theta)d\theta \right\}$$

Subject to (IC-S) and $v \geq 0$. As explained in Section 3 (see footnote 23), the problem above is a relaxation of the funder's problem, as we have not fully incorporated IR, or ruled out the possibility of simultaneous profitable deviations in both type and effort. However, as the problem incorporates (IC-F), and sets $u(\underline{\theta}) = \overline{\Pi}(\theta)$, by Lemma 1, IR is satisfied so long as $X(\theta) \geq \overline{x}(\theta)$ for all θ. We show that this is the case under (A1). Note, moreover, that by the analysis in Appendix A, shirking on effort is not profitable for the researcher, regardless of her type report, since $\overline{\Pi}(\underline{\theta}) = 0$.

For the moment, we further relax the problem above and ignore (IC-S); we will then verify that it is satisfied. Examining the funder's objective, it is clear that his payoff is strictly decreasing in $v(\cdot)$. Therefore, for all types, the funder sets $V(\theta) = 0$. Setting $V(\theta) = 0$, pointwise maximization of the maximand of the funder's problem with respect to x, yields the pointwise first-order condition,

$$\theta\rho'(X(\theta))(W + \pi) - 1 - \rho'(X(\theta))\pi h(\theta) = 0 \tag{8}$$

Fix $\theta < \overline{\theta}$. Since $h(\theta) > 0$, (8) implies $\theta\rho(X(\theta))(W + \pi) - 1 > 0$; by concavity of ρ, and the definition of $X_{FB}(\theta)$, this means $X(\theta) < X_{FB}(\theta)$. Moreover, since $h(\overline{\theta}) = 0$, (8) implies $X(\overline{\theta}) = X_{FB}(\overline{\theta})$.

We now show that $X(\theta) \geq \overline{x}(\theta)$, for all θ. Fix $\theta \in \Theta$. Clearly, $X(\theta) > \overline{x}(\theta)$ if $\overline{x}(\theta) = 0$. So, suppose $\overline{x}(\theta) > 0$. In this case, $\overline{x}(\theta)$ is the unique solution to the first-order condition, $\theta\rho'(\overline{x}(\theta))\pi - 1 = 0$. Re-arranging (8), we may write:

$$\theta\rho'(X(\theta))\pi - 1 = \rho'(X(\theta))[\pi h(\theta) - \theta W] < 0 \tag{9}$$

Where the inequality follows by (A1). Hence, $\theta\rho'(X(\theta))\pi - 1 < 0$. Concavity of ρ and the definition of $\overline{x}(\theta)$ imply, $X(\theta) > \overline{x}(\theta)$.

Next, we show that (IC-S) is satisfied. When $V(\theta) = 0$ for all θ, (IC-S) simply requires $X'(\theta) \geq 0$. Differentiating (8) with respect to θ and re-arranging yields,

$$X'(\theta) = -\frac{\rho'(X(\theta))\big[W + \pi - \pi h'(\theta)\big]}{\rho''(X(\theta))\big[\theta(W + \pi) - \pi h(\theta)\big]}$$

As $\rho'(\cdot) > 0$ and $h'(\cdot) < 0$ (by assumption), the numerator is strictly positive. Moreover, since $\rho'' < 0$, and since (A1) implies $\theta(W+\pi)-\pi h(\theta) > 0$, the denominator is strictly negative. Hence, $X'(\cdot) > 0$ and (IC-S) is satisfied. This establishes part (1). To establish part (2), note that equation (6) in Appendix A implies $X(\underline{\theta}) + c - G(\underline{\theta}) > 0$. Next, fix θ. Since $V(\theta) = 0$ equation (7) can be written,

$$G'(\theta) = X'(\theta)\left[1 - \theta\rho'(X(\theta))\pi\right] > 0$$

The inequality above holds since $X'(\theta) > 0$, and by (9) the term in square brackets on the RHS is strictly positive. Moreover, the term in square brackets is also clearly less than 1; this means $G'(\theta) < X'(\theta)$. This establishes the proposition. Note that equations (6) and (7) provide a full characterization of the optimal grant schedule.

$\square$

Proof of Propositions 4 - 6

In this section, we provide proofs of Propositions 4-6. First, we state and prove the following lemma.

Lemma B.1. *(IC-S) and (IC-E) imply that for all* $\theta \in \Theta$, $v(\theta) \geq \frac{c}{\underline{\theta}\rho(x(\theta))} - \pi$.

Proof. Evaluating (IC-E) at $\underline{\theta}$ yields, $\underline{\theta}\rho(x(\underline{\theta}))(v(\underline{\theta}) + \pi) \geq c$. But (IC-S) implies $\rho(x(\cdot))(v(\cdot) + \pi)$ is non-decreasing, and this means, for each $\theta \geq \underline{\theta}$, $\underline{\theta}\rho(x(\theta))(v(\theta) + \pi) \geq c$. Re-arranging this expression yields the desired result.

$\square$

We now derive properties of the solution to a relaxed problem through a series of lemmas. We then show that the solution to the relaxed problem solves $[P]$. Recall, however, that $[P]$ itself is a relaxation of the funder's problem. To ensure IR is satisfied, we must still check that the solution to $[P]$ satisfies, $X(\theta) \geq \overline{x}(\theta)$ (see footnote 23 and the proof of Proposition 3). But see that each investment schedule given in Propositions 4-6 has the property that, for all θ, $X(\theta) \geq X_{AS}(\theta) > \overline{x}(\theta)$, where the final inequality is shown in the proof of Proposition 3. Thus, once we establish that the solutions given in Propositions 4-6, solve $[P]$, we know that IR is satisfied.

Now consider the following relaxation of $[P]$:

$$\max_{x(\cdot),v(\cdot)} \left\{ \int_{\underline{\theta}}^{\overline{\theta}} \left[\theta\rho(x(\theta))[W + \pi] - x(\theta) - c - \rho(x(\theta))(v(\theta) + \pi)h(\theta)\right] f(\theta) d\theta \right\} \quad [P']$$

s.t.

(i) $v(\theta) \geq \frac{c}{\underline{\theta}\rho(x(\theta))} - \pi$

(ii) $v(\theta) \geq 0$

$[P']$ is a relaxation of $[P]$ since by Lemma B.1, (i) is implied by (IC-S) and (IC-E). To establish that the solution to $[P']$ also solves $[P]$, we must only check that (IC-S) is satisfied at the solution to $[P']$; (IC-E) is implied by (i). Setup the Lagrangian for the problem $[P']$:

$$L = \left\{ \theta\rho(x(\theta))[W+\pi] - x(\theta) - c - \rho(x(\theta))(v(\theta)+\pi)h(\theta) \right\} f(\theta)$$

$$+ \mu_1(\theta) \left[\underline{\theta}\rho(x(\theta))(v(\theta)+\pi) - c \right] f(\theta) + \mu_2(\theta)v(\theta)f(\theta)$$

Where μ_1 and μ_2 are the Lagrange multipliers for the constraints given in (i) and (ii), respectively. For each $\theta \in \Theta$, the (point-wise) first-order conditions are:

$$\frac{\partial L}{\partial v} = -\rho(X(\theta))h(\theta) + \mu_1(\theta)\underline{\theta}\rho(X(\theta)) + \mu_2(\theta) = 0 \tag{10}$$

$$\frac{\partial L}{\partial x} = \theta\rho'(X(\theta))(W+\pi) - 1 - \rho'(X(\theta))(V(\theta)+\pi)(h(\theta) - \mu_1(\theta)\underline{\theta} = 0 \tag{11}$$

And the complementary slackness conditions:

$$\mu_1(\theta) \left[\underline{\theta}\rho(X(\theta))(V(\theta)+\pi) - c \right] = 0; \quad \mu_1(\theta) \geq 0; \quad \underline{\theta}\rho(X(\theta))(V(\theta)+\pi) - c \geq 0 \tag{12}$$

$$\mu_2(\theta)V(\theta) = 0; \quad \mu_2(\theta) \geq 0; \quad V(\theta) \geq 0 \tag{13}$$

Lemma B.2. *At the solution to $[P']$: If $V(\theta) > 0$ then $V(\theta) = \frac{c}{\underline{\theta}\rho(X(\theta))} - \pi$, and $X(\theta) = X_{FB}(\theta)$. Moreover, if for some $\tilde{\theta} \in \Theta$, $\underline{\theta}\rho(X_{FB}(\tilde{\theta}))\pi \geq c$ then $V(\theta) = 0$ for all $\theta \geq \tilde{\theta}$.*

Proof. For some $\theta < \overline{\theta}$ suppose $V(\theta) > 0$. Then by (13), $\mu_2(\theta) = 0$; (10) then gives $\mu_1(\theta) = \frac{h(\theta)}{\underline{\theta}}$. Plugging $\mu_1(\theta) = \frac{h(\theta)}{\underline{\theta}}$ into (12) yields, $V(\theta) = \frac{c}{\underline{\theta}\rho(X(\theta))} - \pi$. Then, plugging $\mu_1(\theta) = \frac{h(\theta)}{\underline{\theta}}$ and $V(\theta) = \frac{c}{\underline{\theta}\rho(X(\theta))} - \pi$ into (11) yields, $\theta\rho'(X(\theta))(W + \pi) - 1 = 0 \iff X(\theta) = X_{FB}(\theta)$.

By (13), $V(\theta) \geq 0$. We have shown that if $V(\theta) > 0$ then $V(\theta) = \frac{c}{\underline{\theta}\rho(X(\theta))} - \pi$. So, if there exists $\tilde{\theta}$ such that $\frac{c}{\underline{\theta}\rho(X_{FB}(\tilde{\theta}))} - \pi \leq 0$ (equivalently, $\underline{\theta}\rho(X_{FB}(\tilde{\theta}))\pi \geq c$), then it must be that $V(\tilde{\theta}) = 0$. Since $X_{FB}(\cdot)$ is strictly increasing, for any $\theta > \tilde{\theta}$: $\frac{c}{\underline{\theta}\rho(X_{FB}(\tilde{\theta}))} - \pi \leq 0 \implies \frac{c}{\underline{\theta}\rho(X_{FB}(\theta))} - \pi < 0 \implies V(\theta) = 0$. □

Lemma B.3. *At the solution to [P′]: If $\underline{\theta}\rho(X_{AS}(\tilde{\theta}))\pi \geq c$ for some $\tilde{\theta} \in \Theta$ then for any $\theta > \tilde{\theta}$: $V(\theta) = 0$, $\mu_1(\theta) = 0$, and $X(\theta) = X_{AS}(\theta)$.*

Proof. Suppose $\underline{\theta}\rho(X_{AS}(\tilde{\theta}))\pi \geq c$ for some $\tilde{\theta} \in \Theta$. Then since $X_{FB}(\cdot) \geq X_{AS}(\cdot)$ this means $\underline{\theta}\rho(X_{AS}(\tilde{\theta}))\pi \geq c \implies \underline{\theta}\rho(X_{FB}(\tilde{\theta}))\pi \geq c$, and by Lemma B.2, $V(\theta) = 0$ for all $\theta \geq \tilde{\theta}$.

Next, we show $\theta > \tilde{\theta} \implies \mu_1(\theta) = 0$. Contrary to the proposition, suppose there exists a non-empty interval $I \subset (\tilde{\theta}, \overline{\theta}]$ such that $\theta \in I \implies \mu_1(\theta) > 0$. Fix $\theta \in I$. By (12), $\underline{\theta}\rho(X(\theta))\pi = c$, which means $X(\theta) = x_m(\underline{\theta})$. Plugging $V(\theta) = 0$ and $X(\theta) = x_m(\underline{\theta})$ into (11) yields,

$$\theta\rho'(x_m(\underline{\theta}))(W + \pi) - 1 - \rho'(x_m(\underline{\theta}))\pi h(\theta) = -\mu_1(\theta)\underline{\theta} < 0.$$

By concavity of ρ, the expression above implies $x_m(\underline{\theta}) > X_{AS}(\theta)$, which implies $\underline{\theta}\rho(x_m(\underline{\theta}))\pi > \underline{\theta}\rho(X_{AS}(\theta))\pi > c$, which contradicts the definition of $x_m(\underline{\theta})$. Note that the final inequality holds since $X_{AS}(\cdot)$ is strictly increasing, $\theta > \tilde{\theta}$, and by assumption, $\underline{\theta}\rho(X_{AS}(\tilde{\theta}))\pi \geq c$. So, it must be that $\mu_1(\theta) = 0$. Plugging $\mu_1(\theta) = V(\theta) = 0$ into (11) yields $\theta\rho'(X(\theta))(W + \pi) - 1 - \rho'(X(\theta))h(\theta)\pi = 0 \iff X(\theta) = X_{AS}(\theta)$.

□

Lemma B.4. *At the solution to [P′]: If for some $\tilde{\theta} \in \Theta$, $\underline{\theta}\rho(X_{FB}(\tilde{\theta}))\pi \leq c$, then for all $\theta < \tilde{\theta}$: $\mu_1(\theta) > 0$, $V(\theta) = \frac{c}{\underline{\theta}\rho(X(\theta))} - \pi > 0$, and $X(\theta) = X_{FB}(\theta)$.*

Proof. Suppose there exists $\tilde{\theta}$ such that, $\underline{\theta}\rho(X_{FB}(\tilde{\theta}))\pi \leq c$. We first show that $\mu_1(\theta) > 0$ for $\theta < \tilde{\theta}$. Contrary to the proposition, suppose there exists a non-empty interval $I \subset [\underline{\theta}, \tilde{\theta})$ such that $\theta \in I \implies \mu_1(\theta) = 0$. Fix $\theta \in I$. By (10): $\mu_2(\theta) = \rho(X(\theta))h(\theta) > 0 \implies V(\theta) = 0$. Plugging $V(\theta) = \mu_1(\theta) = 0$ into (11) yields $X(\theta) = X_{AS}(\theta)$. But since $X_{AS}(\theta) < X_{FB}(\tilde{\theta})$ this means $\underline{\theta}\rho(X_{AS}(\theta))\pi < \underline{\theta}\rho(X_{FB}(\tilde{\theta}))\pi \leq c$, which violates (12). Thus, $\mu_1(\theta) > 0$ for all $\theta \in [\underline{\theta}, \tilde{\theta})$.

To complete the proof, Lemma B.2 implies that it is sufficient to show $V(\theta) > 0$ for all $\theta \in [\underline{\theta}, \tilde{\theta})$. Suppose to the contrary there exists a non-empty interval $I \subset [\underline{\theta}, \tilde{\theta})$ such that $\theta \in I \implies V(\theta) = 0$. Fix $\theta \in I$. We've already established $\mu_1(\theta) > 0$, and so (12) implies $\underline{\theta}\rho(X(\theta))\pi = c$, which means $X(\theta) = x_m(\underline{\theta})$. By assumption, $\underline{\theta}\rho(X_{FB}(\tilde{\theta}))\pi \leq c \iff x_m(\underline{\theta}) \geq X_{FB}(\tilde{\theta})$. Since $\theta < \tilde{\theta}$ and $X_{FB}(\cdot)$ is strictly increasing, we have the following string of inequalities:

$$x_m(\underline{\theta}) \geq X_{FB}(\tilde{\theta}) > X_{FB}(\theta) \tag{14}$$

Since $\mu_2(\theta) \geq 0$, (10) implies $h(\theta) - \mu_1(\theta)\underline{\theta} \geq 0$. It then follows from (11):

$$\theta\rho'(x_m(\underline{\theta}))(W + \pi) - 1 = \rho'(x_m(\underline{\theta}))(V(\theta) + \pi)(h(\theta) - \mu_1(\theta)\underline{\theta}) \geq 0$$

By concavity of ρ, the expression above implies $x_m(\underline{\theta}) \leq X_{FB}(\theta)$, which contradicts (14). Thus, $V(\theta) > 0$ for $\theta \in [\underline{\theta}, \tilde{\theta})$.

□

We are now ready to prove Propositions 4-6.

Proof of Proposition 4(1).

As an immediate consequence of Lemma B.3, the investment/prize schedules given in the proposition solve the problem $[P']$. To establish that the solution to $[P']$ also solves $[P]$, we must show (IC-S) is satisfied. Note that as $V(\theta) = 0$ and $X'_{AS}(\cdot) > 0$, (IC-S) is satisfied. To establish the stated properties of the grant schedule, we refer the reader to the proof of Proposition 3, as the proof here is nearly identical.

$\square$

Proof of Proposition 4(2).

We first show that the solution given in the proposition solves the relaxed problem, $[P']$. By assumption, $X_{FB}(\underline{\theta}) > x_m(\underline{\theta})$, which means $\underline{\theta}\rho(X_{FB}(\underline{\theta}))\pi > c$ and so Lemma B.2 implies $V(\theta) = 0$ for all $\theta \in \Theta$. Equation (12) then implies $X(\theta) \geq x_m(\underline{\theta})$.

Next, the hypotheses of the proposition implies $X_{AS}(\underline{\theta}) < x_m(\underline{\theta}) < X_{AS}(\overline{\theta})$; by continuity of $X_{AS}(\cdot)$, there exists $\theta' \in (\underline{\theta}, \overline{\theta})$ such that $X_{AS}(\theta') = x_m(\underline{\theta})$. Fix $\theta \in [\underline{\theta}, \theta')$. As already shown, $V(\theta) = 0$; if it were the case that $\mu_1(\theta) = 0$ then (11) implies $X(\theta) = X_{AS}(\theta)$. However, since $X_{AS}(\theta) < x_m(\underline{\theta})$ this contradicts (12). So, it must be that $\mu_1(\theta) > 0$, and hence $\underline{\theta}\rho(X(\theta))\pi = c \iff X(\theta) = x_m(\underline{\theta})$. Finally, at θ' we have $\underline{\theta}\rho(X_{AS}(\theta'))\pi = c$, and so by Lemma B.3, $X(\theta) = X_{AS}(\theta)$ for all $\theta \in [\theta', \overline{\theta}]$.

This establishes that the investment/prize schedules given in the proposition solve the problem $[P']$. It remains to be shown that these schedules also solve $[P]$, and that the grant schedule satisfies the stated properties. These proofs follow along similar lines as the proof of Cases *ii* and *iii* in the proof of Proposition 5 (Part II), and so we omit these here.

Proof of Proposition 5.

Part I. We first show that the solution given in the proposition solves the relaxed problem, $[P']$. Since $X_{FB}(\underline{\theta}) < x_m(\underline{\theta}) < X_{FB}(\overline{\theta})$ (by assumption), continuity of $X_{FB}(\cdot)$ implies that there exists $\theta' \in (\underline{\theta}, \overline{\theta})$ such that $X_{FB}(\theta') = x_m(\underline{\theta})$; equivalently, $\underline{\theta}\rho(X_{FB}(\theta'))\pi = c$. Lemma B.4 implies that for all $\theta \in [\underline{\theta}, \theta')$, $V(\theta) = \frac{c}{\underline{\theta}\rho(X(\theta))} - \pi > 0$ and $X(\theta) = X_{FB}(\theta)$. Moreover, Lemma B.2 implies that for all $\theta \in (\theta', \overline{\theta}]$, $V(\theta) = 0$.

Next, since $X_{AS}(\theta') < X_{FB}(\theta') = x_m(\underline{\theta})$ and $X_{AS}(\overline{\theta}) = X_{FB}(\overline{\theta}) > x_m(\underline{\theta})$ continuity of X_{AS} implies $X_{AS}(\theta'') = x_m(\underline{\theta})$ for some $\theta'' \in (\theta', \overline{\theta})$; equivalently, $\underline{\theta}\rho(X_{AS}(\theta''))\pi = c$. Lemma B.3 then implies $X(\theta) = X_{AS}(\theta)$ for $\theta \in (\theta'', \overline{\theta}]$.

It remains to be shown that $X(\theta) = x_m(\underline{\theta})$ for $\theta \in [\theta', \theta'']$. Contrary to the proposition, suppose there exists a nonempty interval, $I \subset [\theta', \theta'']$ such that $\theta \in I \implies X(\theta) \neq x_m(\underline{\theta})$. Fix $\theta \in I$. Since $V(\theta) = 0$ and $X(\theta) \neq x_m(\underline{\theta})$, (12) implies $X(\theta) > x_m(\underline{\theta})$, and $\mu_1(\theta) = 0$. We then have the following string of inequalities:

$$X(\theta) > x_m(\underline{\theta}) = X_{FB}(\theta') > X_{FB}(\theta) > X_{AS}(\theta) \tag{15}$$

Plugging $\mu_1(\theta) = 0$ into (11) yields $X(\theta) = X_{AS}(\theta)$, which contradicts (15). Hence, $X(\theta) = x_m(\underline{\theta})$ for all $\theta \in [\theta', \theta'']$. This establishes that the solution given in the proposition solves $[P']$.

Part II. To complete the proof of Proposition 5, we show that the investment and grant schedules have the stated properties. We must also show that the solution to $[P']$ solves $[P]$; to do so, we must show that (IC-S) is satisfied. We will proceed in three cases.

Case i**:** $\theta \in [\underline{\theta}, \theta')$

In this range, the solution to $[P']$ yields: $X(\theta) = X_{FB}(\theta)$ and $V(\theta) = \frac{c}{\underline{\theta}\rho(X(\theta))} - \pi$. It can be verified that,

$$\rho'(X(\theta))(V(\theta) + \pi)X'(\theta) + \rho(X(\theta))V'(\theta) = 0$$

And hence (IC-S) is satisfied. It can also be verified that the solution to $[P']$ gives $u(\theta) = c\left(\frac{\theta - \underline{\theta}}{\underline{\theta}}\right)$. Hence,

$$\begin{aligned} G(\theta) &\equiv u(\theta) - \theta\rho(X(\theta))(V(\theta) + \pi) + X(\theta) + c \\ &= c\left(\frac{\theta - \underline{\theta}}{\underline{\theta}}\right) - \theta\frac{c}{\underline{\theta}} + X(\theta) + c \\ &= X(\theta) \end{aligned}$$

Case ii**:** $\theta \in [\theta', \theta'']$

In this range, the solution to $[P']$ yields a constant investment and prize schedule: $X(\theta) = x_m(\underline{\theta})$ and $V(\theta) = 0$; so clearly (IC-S) is satisfied. It may then be verified that $u(\theta) = c\left(\frac{\theta - \underline{\theta}}{\underline{\theta}}\right)$, and a similar string of inequalities as presented in Case 1 reveals $G(\theta) = X(\theta)$.

Case ***iii*****:** $\theta \in (\theta'', \overline{\theta}]$

In this range the solution to $[P']$ yields the investment and prize schedules: $X(\cdot) = X_{AS}(\cdot)$ and $V(\cdot) = 0$. When $V'(\theta) = 0$, (IC-S) holds if and only if X is non-decreasing; since $X'_{AS}(\cdot) > 0$, (IC-S) is satisfied. Next, see that

$$u(\theta) = \int_{\underline{\theta}}^{\theta} \rho(X(t))(V(t) + \pi)\, dt = c\left(\frac{\theta'' - \underline{\theta}}{\underline{\theta}}\right) + \int_{\theta''}^{\theta} \rho(X(t))\pi\, dt$$

So,

$$G(\theta) \equiv u(\theta) - \theta\rho(X(\theta))(V(\theta) + \pi) + X(\theta) + c$$

$$= c\left(\frac{\theta'' - \underline{\theta}}{\underline{\theta}}\right) + \int_{\theta''}^{\theta} \rho(X(t))\pi\, dt - \theta\rho(X(\theta))\pi + X(\theta) + c$$

Differentiating both sides of the expression above, we find $G'(\theta) = X'(\theta)\left[1 - \theta\rho(X(\theta))\pi\right]$. Since $X'_{AS}(\theta) > 0$, and since $X_{AS}(\theta) > \overline{x}(\theta)$ implies $1 - \theta\rho(X(\theta))\pi > 0$, it holds that $G'(\theta) > 0$. We can also write: $G'(\theta) - X'(\theta) = -\theta\rho(X(\theta))X'(\theta) < 0$. Since $G(\theta'') = X(\theta'')$ and $G'(\theta) < X'(\theta)$ for $\theta > \theta''$, this means $X(\theta) > G(\theta)$ for $\theta > \theta''$.

Proof of Proposition 6

As an immediate consequence of Lemma B.4, the investment/prize schedules given in the proposition solve the problem $[P']$. It remains to be shown that these schedules also solve $[P]$, and that the grant schedule satisfies the stated properties. These proofs follow along similar lines as the proof of Case i in the proof of Proposition 5 (Part II), and so we omit these here.

□

Proof of Proposition 7

We first show that the investment schedule $X_{AS}(\theta)$ can be implemented via a menu of linear grant contracts $\{a(\theta), r(\theta)\}_{\theta\in\Theta}$, where the grant is then given by,

$$\tilde{G}(x,\theta) = a(\theta) + r(\theta)x$$

Define

$$a(\theta) \equiv G(\theta) - r(\theta)X_{AS}(\theta)$$

Where $G(\cdot)$ is the optimal grant schedule given in Proposition 4(1), and fully characterized in Proposition 3. Define $r(\theta)$ as follows:

$$r(\theta) \equiv 1 - \theta\rho'(X_{AS}(\theta))\pi$$

As $\rho' > 0$, it is clear that $r(\cdot) < 1$; moreover, $X_{AS}(\theta) > \overline{x}(\theta)$ implies $r(\cdot) > 0$. Let $\tilde{u}(x,\hat{\theta}|\theta)$ denote the payoff to a type θ researcher who chooses the contract $\{a(\hat{\theta}), r(\hat{\theta})\}$, and invests x:

$$\tilde{u}(x,\hat{\theta}|\theta) = \theta\rho(x)\pi - x - c + a(\hat{\theta}) + r(\hat{\theta})x$$

The researcher solves, $\max_{x,\hat{\theta}}\{\tilde{u}(x,\hat{\theta}|\theta)\}$. Let (x^*, θ^*) denote the solution to the first-order conditions of the researcher's problem. Using the definitions of $a(\cdot)$ and $r(\cdot)$, the first-order condition with respect to x can be expressed,

$$\frac{\partial\tilde{u}(x,\hat{\theta}|\theta)}{\partial x}|_{(x,\hat{\theta})=(x^*,\theta^*)} = \theta\rho'(x^*)\pi - \theta^*\rho'(X_{AS}(\theta^*))\pi = 0$$

The expression above implies $\theta\rho'(x^*) = \theta^*\rho'(X_{AS}(\theta^*))$. Next, using the definitions of $a(\cdot)$ and $r(\cdot)$, the first-order condition with respect to $\hat{\theta}$ can be written (after some simplification),

$$\frac{\partial \tilde{u}(x,\hat{\theta}|\theta)}{\partial \hat{\theta}}|_{(x,\hat{\theta})=(x^*,\theta^*)} = G'(\theta^*) - r(\theta)X'_{AS}(\theta^*) + r'(\theta^*)(x^* - X_{AS}(\theta^*)) = 0$$

By definition of r and G, $G'(\theta^*) - X'_{AS}(\theta^*)r(\theta) = G'(\theta^*) - X'_{AS}(\theta^*)[1 - \theta^*\rho'(X_{AS}(\theta^*))\pi] = 0$. Hence, the first-order condition with respect to $\hat{\theta}$ simplifies to,

$$\frac{\partial \tilde{u}(x,\hat{\theta}|\theta)}{\partial \hat{\theta}}|_{(x,\hat{\theta})=(x^*,\theta^*)} = (x^* - X_{AS}(\theta^*))r'(\theta^*) = 0$$

So long as $r'(\cdot) \neq 0$, the expression above then implies $x^* = X_{AS}(\theta^*)$. We will now show that $r'(\theta) > 0$. $X_{AS}(\theta)$ is defined:

$$\theta\rho'(X_{AS}(\theta))(W+\pi) - 1 - h(\theta)\rho'(X_{AS}(\theta))\pi = 0$$

Differentiating the expression above with respect to θ, we can write,

$$\frac{W+\pi}{\pi}\left[\rho'(X_{AS}(\theta))\pi + \theta\rho''(X_{AS}(\theta))X'_{AS}(\theta)\pi\right] = \pi\left[h'(\theta)\rho'(X_{AS}(\theta)) + h(\theta)\rho''(X_{AS}(\theta))X'_{AS}(\theta)\right]$$

Using the definition of $r(\cdot)$, the expression above yields,

$$r'(\theta) = -\frac{\pi^2}{W+\pi}\left[h'(\theta)\rho'(X_{AS}(\theta)) + h(\theta)\rho''(X_{AS}(\theta))X'_{AS}(\theta)\right]$$

Note that as $h'(\cdot) < 0$, $X'_{AS}(\cdot) > 0$, $\rho'(\cdot) > 0$, and $\rho''(\cdot) < 0$, the term in square brackets is strictly negative. Hence, for all θ, $r'(\theta) > 0$. Then, the first-order condition with respect to $\hat{\theta}$ implies, $x^* = X_{AS}(\theta^*)$. But, as we've already shown, $\theta\rho'(x^*) = \theta^*\rho'(X_{AS}(\theta^*))$. Substituting $x^* = X_{AS}(\theta^*)$, we obtain $\theta\rho'(X_{AS}(\theta^*)) = \theta^*\rho'(X_{AS}(\theta^*)) \iff \theta = \theta^*$.

For a type-θ researcher, we have shown that $(X_{AS}(\theta), \theta)$ is the unique solution to the first-order conditions; it still remains to be shown that IR is satisfied: $\tilde{u}(X_{AS}(\theta), \theta|\theta) \geq \overline{\Pi}(\theta)$, and the the second-order condition is satisfied. To check IR, it is straightforward to show that the researcher's optimal payoff, $\tilde{u}(X_{AS}(\theta), \theta|\theta)$ is equal to her equilibrium payoff under the conditions of Proposition 4(1). Thus, IR is satisfied. Moreover, it is straightforward to show that the researcher's second-order conditions are satisfied at $(x, \hat{\theta}) = (X_{AS}(\theta), \theta)$. Hence, $(X_{AS}(\theta), \theta)$ is the unique solution to the researcher's problem.

We now show that for c sufficiently small, $a(\theta) < 0$ for all θ. To see this, first note that by definition, $G(\underline{\theta})$ satisfies,

$$G(\underline{\theta}) - X_{AS}(\underline{\theta}) = -\underline{\theta}\rho(X_{AS}(\underline{\theta}))\pi + c \tag{16}$$

Using (16) and the definition of $a(\underline{\theta})$, we obtain

$$a(\underline{\theta}) = \underline{\theta}\pi \left[-\rho(X_{AS}(\underline{\theta})) + X_{AS}(\underline{\theta})\rho'(X_{AS}(\underline{\theta}))\right] + c$$

Concavity of ρ implies that the term in square brackets is strictly negative. Hence, if c is sufficiently small, $a(\underline{\theta}) < 0$. We now show that $a'(\theta) < 0$:

$$\begin{aligned} a'(\theta) &= G'(\theta) - r(\theta)X'_{AS}(\theta) - r'(\theta)X_{AS}(\theta) \\ &= -r'(\theta)X_{AS}(\theta) \\ &< 0 \end{aligned}$$

The first equality follows by definition of $a(\cdot)$. The second equality follows since, as we've already shown, $G'(\theta) = r(\theta)X'_{AS}(\theta)$, and the inequality follows since $r'(\theta) > 0$.

Now, note that $\tilde{G}(X_{AS}(\theta), \theta) = G(\theta) > 0$; as it is never optimal for the researcher to report her type and invest in such a way that $\tilde{G}(x, \theta) < 0$, we may consider a truncated grant function, which can also implement the same allocation:

$$\tilde{G}(x, \theta) = \begin{cases} 0, & a(\theta) + r(\theta)x \leq 0 \\ a(\theta) + r(\theta)x, & a(\theta) + r(\theta)x \geq 0 \end{cases}$$

Letting $b(\theta) = -\frac{a(\theta)}{r(\theta)}$, we can equivalently write,

$$\tilde{G}(x,\theta) = \begin{cases} 0 & x \leq b(\theta) \\ r(\theta)(x - b(\theta)) & x \geq b(\theta) \end{cases}$$

Using the definitions of $a(\theta)$ and $r(\theta)$ along with the fact that $G'(\theta) = r(\theta)X'_{AS}(\theta)$, it is straightforward to show that $b'(\theta) > 0$. Moreover, as already shown, $r(\theta) > 0$ and for c sufficiently small, $a(\theta) < 0$; hence, if c is small, $b(\theta) > 0$.

□

Proof of Proposition 8

We must first characterize the optimal menu of contracts when the funder offers only a prize to all types: $g(\theta) \equiv 0$. The problem faced by the funder is the following:

$$\max_{x(\cdot),v(\cdot)} \int_{\underline{\theta}}^{\overline{\theta}} \theta\rho(x(\theta))(W - v(\theta))f(\theta)\, d\theta \tag{17}$$

Subject to $v(\theta) \geq 0$, IR and IC. Note that the IC constraints are given by (IC-F), (IC-S), and (IC-E). The IR constraint is, for all θ,

$$u(\theta) = \theta\rho(x(\theta))(v(\theta) + \pi) - c - x(\theta) \geq \overline{\Pi}(\theta) = 0$$

Note that as $\overline{\Pi}(\theta) = 0$ (by assumption), for any investment schedule, $x(\cdot)$, $\Pi(x(\theta), 1, \theta) \leq \overline{\Pi}(\theta) = 0$, where $\Pi(\cdot)$ is as defined in Section 3. Moreover, note that we can equivalently write the IR constraint, $\Pi(x(\theta), 1, \theta) + \rho(x(\theta))v(\theta) \geq 0$. Thus, IR implies $v(\theta) \geq 0$. Further, the IR constraint implies $\theta\rho(x(\theta))(v(\theta) + \pi) - c \geq x(\theta) \geq 0$, and hence (IC-E) is satisfied. Thus, the only relevant IC constraints are (IC-F) and (IC-S).

Also note that since the funder's payoff is decreasing in $u(\cdot)$, and (IC-F) implies $u'(\cdot) \geq 0$, optimality implies $u(\underline{\theta}) = 0$. Next, note that (IC-F) can also be written, $u_1(\hat{\theta}|\theta)|_{\hat{\theta}=\theta} = 0$:

$$\frac{d}{d\hat{\theta}}[\theta\rho(x(\hat{\theta}))(v(\hat{\theta}) + \pi) - x(\hat{\theta}) - c]|_{\hat{\theta}=\theta} = 0$$

Equivalently,

$$\frac{d}{d\theta}[\rho(x(\theta))(v(\theta)+\pi)] = \frac{x'(\theta)}{\theta}$$

The expression above implies that (IC-S) is satisfied if and only if $x'(\cdot) \geq 0$. Next, using the definition of $u(\theta)$, we can write:

$$v(\theta) \equiv \frac{u(\theta)+x(\theta)+c}{\theta\rho(x(\theta))} - \pi \tag{18}$$

Substituting (18) into (IC-F), we can express (IC-F) as follows:

$$u'(\theta) = \rho(x(\theta))(v(\theta)+\pi) = \frac{u(\theta)+x(\theta)+c}{\theta} \tag{IC-FP}$$

Solving the differential equation given by (IC-FP) with initial condition $u(\underline{\theta}) = 0$ we obtain,

$$u(\theta) = \theta \int_{\underline{\theta}}^{\theta} \frac{x(t)+c}{t^2}\, dt$$

Taking expectations over both sides of the expression above (w.r.t. θ),

$$\int_{\underline{\theta}}^{\overline{\theta}} u(\theta) f(\theta)\, d\theta = \int_{\underline{\theta}}^{\overline{\theta}} \left[\int_{\underline{\theta}}^{\theta} \frac{x(t)+c}{t^2}\, dt\right] \theta f(\theta)\, d\theta$$

Integrating the RHS by parts yields,

$$\int_{\underline{\theta}}^{\overline{\theta}} u(\theta) f(\theta)\, d\theta = \int_{\underline{\theta}}^{\overline{\theta}} \left(\frac{\int_{\theta}^{\overline{\theta}} t f(t)\, dt}{\theta^2}\right)(x(\theta)+c)\, d\theta = \int_{\underline{\theta}}^{\overline{\theta}} \tilde{h}(\theta)(x(\theta)+c) f(\theta)\, d\theta \tag{19}$$

Finally, replacing $v(\cdot)$ in the funder's problem by the expression given in (18), and replacing $u(\cdot)$ using (19), we can express the funder's problem as,

$$\max_{x(\cdot)} \int_{\underline{\theta}}^{\overline{\theta}} [\theta\rho(x(\theta))(W+\pi) - x(\theta) - c - \tilde{h}(\theta)(x(\theta)+c)] f(\theta)\, d\theta \tag{20}$$

Subject to $x'(\theta) \geq 0$. For the moment, we ignore this constraint; we will show that it is satisfied if $\tilde{h}'(\cdot) < 0$. Let $X_p(\cdot)$ denote the solution to the funder's problem. Pointwise maximization of the maximand in (20) yields the following first-order condition for each $\theta \in \Theta$:

$$\theta\rho'(X_p(\theta))(W+\pi) - 1 - \tilde{h}(\theta) = 0$$

Differentiating the first-order condition above with respect to θ, it is straightforward to show that if $\tilde{h}'(\cdot) < 0$ then $X_p'(\cdot) > 0$ and hence (IC-S) is satisfied.

We are now able to prove the proposition. Note that under the hypotheses of Proposition 4(1), the optimal grant-based funding scheme is in fact the optimal funding scheme. So, Proposition 4(1) implies $D(\cdot) > 0$, and,

$$\phi_g(c) = \int_{\underline{\theta}}^{\overline{\theta}} [\theta\rho(X_{AS}(\theta))(W+\pi) - X_{AS}(\theta) - c - \rho(X_{AS}(\theta))\pi h(\theta)]\, f(\theta)\, d\theta$$

As shown above,

$$\phi_p(c) = \int_{\underline{\theta}}^{\overline{\theta}} \left[\theta\rho(X_p(\theta))(W+\pi) - X_p(\theta) - c - \tilde{h}(\theta)(X_p(\theta)+c)\right] f(\theta)\, d\theta$$

By the envelope theorem,

$$D'(c) = \frac{\partial}{\partial c}\phi_g(c) - \frac{\partial}{\partial c}\phi_p(c) = \int_{\underline{\theta}}^{\overline{\theta}} -f(\theta)\, d\theta - \int_{\underline{\theta}}^{\overline{\theta}} [-f(\theta) - \tilde{h}(\theta)f(\theta)]\, d\theta = \int_{\underline{\theta}}^{\overline{\theta}} \tilde{h}(\theta)f(\theta)\, d\theta > 0$$

□

Proof of Proposition 9

First, applying the implicit function theorem on the definitions of $X_{FB}(\theta)$ and $X_{AS}(\theta)$, it is straightforward to show $\frac{\partial X_{FB}(\theta)}{\partial \pi} > 0$ and $\frac{\partial X_{AS}(\theta)}{\partial \pi} > 0 \iff \theta > h(\theta)$. To establish (iii), we apply the envelope theorem to the funder's relaxed problem, $[P']$, and obtain:

$$\frac{\partial \phi^*}{\partial \pi} = \int_{\underline{\theta}}^{\overline{\theta}} \left[\rho(X(\theta))(\theta - h(\theta)) + \mu_1(\theta)\underline{\theta}\rho(X(\theta)) \right] f(\theta)\, d\theta$$

Note that since $\mu_1(\theta) \geq 0$ for all θ, it holds:

$$\frac{\partial \phi^*}{\partial \pi} \geq \int_{\underline{\theta}}^{\overline{\theta}} \rho(X(\theta))[\theta - h(\theta)] f(\theta)\, d\theta \tag{21}$$

We will show that the RHS of (21) is strictly positive. Note that since $h'(\cdot) < 0$, the term, $[\theta - h(\theta)]$, is strictly increasing in θ with $\overline{\theta} - h(\overline{\theta}) = \overline{\theta} > 0$. So, there are two cases to consider: If $[\underline{\theta} - h(\underline{\theta})] > 0$ then $[\theta - h(\theta)] > 0$ for all θ, and part (iii) of the proposition follows immediately. The other possibility is that $[\theta - h(\theta)] < 0$ for $\theta \in [\underline{\theta}, \theta^*)$, and $[\theta - h(\theta)] > 0$ for $\theta \in (\theta^*, \overline{\theta})$, where $\theta^* \in (\underline{\theta}, \overline{\theta})$. Suppose this is the case. It then follows,

$$\begin{aligned}
\frac{\partial \phi^*}{\partial \pi} &\geq \int_{\underline{\theta}}^{\theta^*} \rho(X(\theta))(\theta - h(\theta)) f(\theta)\, d\theta + \int_{\theta^*}^{\overline{\theta}} \rho(X(\theta))(\theta - h(\theta)) f(\theta)\, d\theta \\
&> \int_{\underline{\theta}}^{\theta^*} \rho(X(\theta^*))(\theta - h(\theta)) f(\theta)\, d\theta + \int_{\theta^*}^{\overline{\theta}} \rho(X(\theta^*))(\theta - h(\theta)) f(\theta)\, d\theta \\
&= \rho(X(\theta^*)) \int_{\underline{\theta}}^{\overline{\theta}} (\theta - h(\theta)) f(\theta)\, d\theta
\end{aligned}$$

The strict inequality holds since $[\theta - h(\theta)] < 0$ for $\theta \in [\underline{\theta}, \theta^*]$, $[\theta - h(\theta)] > 0$ for $\theta \in [\theta^*, \overline{\theta}]$, and since $X(\cdot)$ is non decreasing, and strictly increasing for θ sufficiently close to $\overline{\theta}$. Therefore, the first term in the first line is no less than the corresponding term in the second line, while the second term is strictly greater than the corresponding term in the second line.

Next, note that

$$
\begin{aligned}
\int_{\underline{\theta}}^{\overline{\theta}} [\theta - h(\theta)] f(\theta)\, d(\theta) &= \int_{\underline{\theta}}^{\overline{\theta}} \theta f(\theta)\, d(\theta) - \int_{\underline{\theta}}^{\overline{\theta}} [1 - F(\theta)]\, d(\theta) \\
&= \int_{\underline{\theta}}^{\overline{\theta}} \theta f(\theta)\, d(\theta) - (\overline{\theta} - \underline{\theta}) + \int_{\underline{\theta}}^{\overline{\theta}} F(\theta)\, d(\theta) \\
&= \overline{\theta} - \int_{\underline{\theta}}^{\overline{\theta}} F(\theta)\, d(\theta) - (\overline{\theta} - \underline{\theta}) + \int_{\underline{\theta}}^{\overline{\theta}} F(\theta)\, d(\theta) \\
&= \underline{\theta} > 0
\end{aligned}
$$

Where the third line follows by integrating the first term in the second line by parts. Hence, $\frac{\partial \phi^*}{\partial \pi} > \rho(X_{AS}(\theta^*)) \int_{\underline{\theta}}^{\overline{\theta}} (\theta - h(\theta)) f(\theta)\, d\theta = \rho(X_{AS}(\theta^*)) \underline{\theta} > 0.$ $\square$