Beamforming Optimization in Energy Harvesting Cooperative Full-Duplex Networks with Self-Energy Recycling Protocol

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Abstract

This paper considers the problem of beamforming optimization in an amplify-and-forward (AF) relaying cooperative network, in which the relay node harvests the energy from the radio-frequency (RF) signal. Based on the self-energy recycling relay protocol, we study the beamforming optimization problem. The formulated problem aims to maximize the achievable rate subject to the available transmitted power at the relay node. We develop a semidefinite programming (SDP) relaxation method to solve the proposed problem. We also use SDP and one-dimension (1-D) optimization to solve the beamforming optimization based on a time-switching relaying protocol as a benchmark. The simulation results are presented to verify that the self-energy recycling protocol achieves a significant rate gain compared to the time-switching relaying protocol and the power-splitting relaying protocol.

I. Introduction

Recently, there is a promising wireless energy harvesting technology emerging and it is considered as a more effective way to prolong the lifetime for the communication system [1]–[3]. Compared to the wireless energy harvesting, the traditional power supplies are more inconvenient or more costly to recharge or replace in the practical circumstance [1]. For example, replacing batteries or recharging to medical devices implanted under patients' skin is very challenging. However, the emerging energy harvesting technology could provide another way to solve this difficulty. The meaning of energy harvesting is wirelessly recharging batteries from the external power source. Some studies pointed out that the external power source can be solar, wind and thermoelectric power [4]–[7]. Besides, wireless energy harvesting via the radio-frequency (RF) signal has drawn a lot of attention [8]. But in the practical implementation of wireless energy harvesting, simultaneously decoding information and harvesting energy is unpractical due to the circuit limitation. Hence the authors in [3] proposed a potential structure for the receiver which harvests energy and detects information separately according a time switching relaying (TSR) protocol or a power splitting relaying (PSR) protocol. Based on the energy harvesting protocols, there are some works considering energy harvesting problems in the point-to-point communication network [1], [8]–[10]. However, for wireless relaying networks, the relay node may had limited battery supply and thus, it needs to be powered by the external energy sources [7], [11]. Hence energy harvesting is also a promising technology for the wireless cooperative network. Some works have studied energy harvesting protocols in the wireless cooperative

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network [12]–[15]. In [12], the authors studied the outage probability and the throughput in an amplify-to-forward (AF) cooperative network with an energy harvesting relay node. The authors in [16] proposed a model of the energy harvesting cooperative networks with source-to-destination pairs and one energy harvesting relay node. The difference of some power allocation strategies has also been considered in [16].

The above works considered energy harvesting problem in the half-duplex cooperative network model, however some works focused on using energy harvesting in the full-duplex model. The authors in [17], [18] studied the full-duplex wireless powered network with the time switching protocol. Due to the full-duplex structure, the node could transmit energy and receive information simultaneously. The authors in [19] proposed a self-energy recycling protocol. In [19], a full-duplex energy harvesting relay node is equipped with two groups of antenna. In the first transmission phase, the relay node uses its receiving antenna to receive the information from the source node. In the second transmission phase, the relay node uses its receiving antenna to collect the power transmitted from the source node and uses its transmission antennas to send the information to the destination node. During the second phase, the energy harvesting relay not only collects the power form the source node, but also recycles part of its transmitted power from its loop-back channel. The authors in [19] set up this self-energy recycling protocol in a MISO relaying channel.

Motivated by this, we study the beamforming optimization problem based on the self-energy recycling relaying protocol in a wireless cooperative network. We modeled a wireless energy harvesting cooperative network with a self-energy recycling relay. The relay node is equipped two groups of antenna, so it is capable of collecting the information or energy and relaying the information simultaneously. In this model, we formulated the beamforming optimization problem for maximizing the achievable rate subject the available transmitted power at the relay node and we used the semidefinite programming (SDP) relaxation approach to solve it. In order to highlight the advantage of the self-energy recycling scheme, we also provided the solution for the beamforming optimization problem with the time-switching relaying protocol and the power-splitting relaying protocol. Simulation results are provided to verify that the self-energy recycling relaying protocol achieves an obvious rate gain compared to TSR or PSR. The trade-off between the achievable rate and system parameters is also analyzed.

The rest of this paper is organized as follows. Section II presents a general system model of an energy harvesting wireless cooperative network. Section III presents the details for the transmission model based on the self-energy recycling relaying protocol, formulates the beamforming optimization problem and proposes an optimal solution. Section IV solves a beamforming optimization problem based on the time-switching relaying protocol as a benchmark scheme. The numerical results are provided in V and section VI concludes this paper.

$Notations$: The bold uppercase and lowercase letters are denoted as matrices and vectors, respectively. $\mathcal{E}(\cdot)$ is the expectation over the random variables within the bracket. $(\cdot)^*$, $(\cdot)^T$ and $(\cdot)^H$ are denoted as conjugate, transpose and conjugate and transpose, respectively. $\mathbf{I}_N$ denotes $N \times N$ identity matrix. $Tr(\cdot)$ is the trace of a matrix. vec is the matrix vectorization. $\otimes$ represents the Kronecker operator. $|x|$ denotes the absolution value of a scalar. $||\mathbf{x}||^2$ is the Frobenius norm. $\mathbb{C}^{M \times N}$ denotes the space of $M \times N$ matrices with complex entries.

II. System Model

The wireless cooperative network considered here includes one source-destination pair and one energy harvesting relay. The source node and the destination node are equipped one antenna respectively. The relay node has two groups of the RF chains, one is used for the information transmission while the other one is used for receiving. The number of transmission antennas N_t and the number of receiving antennas N_r are same, i.e. $N_t = N_r = N$. We assume that there is no direct link between the source node and the destination node, i.e. the source node intends to transmit message to the destination node with the assist of the relay node. Channels are modeled as quasi-static block fading channels. The perfect channel state information is available at the relay node.

The energy harvesting relay is solely powered by the source. It can harvest energy from the source node and utilize the energy to relay the source information. This assumption is used in [12]. The battery capacity of the energy harvesting relay is assumed as infinite. The amplify-and-forward (AF) scheme is employed in the cooperative network. A new energy harvesting protocol with self-energy recycling relay is considered in this paper. The detailed analysis based on this protocol is given in the following sections. We also assume that the relay node itself has an initial power to support its circuitry power consumption, however the relay node does not use its own power to relay the information. Therefore the relay node needs to be wirelessly powered by the source node.

III. Beamforming with Self-Energy Recycling Relay

In this section, we study the relay beamforming optimization problem in self-energy recycling relay networks. The transmission model is given as follows.

A. Relay Protocol and Transmission Model

The whole transmission process is operated in the block time, denoted by T. Without loss of generality, T is normalized to be unity. The information transmission process is split into two phases. In the first phase, the source transmits information to the relay for $T/2$ time and relay uses its receiving antennas to receive information. In the second phase, the source transmits RF signals to power the energy harvesting relay and the relay sends the amplified information to the destination. Recall that the energy harvesting relay is equipped with one group of transmission antenna and one group of receiving antenna, therefore it is capable of relaying the information and collecting energy simultaneously.

The energy harvesting relay first receives the information from the source with its receiving antennas. The received signal at the relay can be expressed as

$$y_r^1 = \sqrt{P_s}\mathbf{h_r} x_s + \mathbf{n_{r1}}, \tag{1}$$

where P_s is the transmitted power, $\mathbf{h_r} \in \mathbb{C}^{N\times 1}$ is the channel vector from the source to receiving antennas of the relay, and x_s is the transmitted data from the source node with $\mathcal{E}(|x_s|^2) = 1$ and $\mathbf{n_{r1}}$ is the additive complex Gaussian noise vector at the relay node following $\mathcal{CN}(\mathbf{0}, \sigma_{r1}^2 \mathbf{I}_N)$.

Since the energy harvesting relay is operated with the AF protocol, upon receiving the signal, the relay processes the signal by relay precoding. The signal transmitted by the relay node is given by

$$x_r = \sqrt{P_s}\mathbf{W}\mathbf{h_r} x_s + \mathbf{W}\mathbf{n_{r1}}, \tag{2}$$

where $\mathbf{W} \in \mathbb{C}^{\mathbf{N}\times\mathbf{N}}$ is the precoding matrix. The power for the transmitted signal from the relay is

$$P_r = P_s||\mathbf{W}\mathbf{h_r}||^2 + \sigma_{r1}^2||\mathbf{W}||^2, \tag{3}$$

In the second phase, the relay uses its transmission antennas to relay the information to the destination node. The received signal at the destination is given by

$$y_d = \sqrt{P_s}\mathbf{h_d}^H\mathbf{W}\mathbf{h_r}x_s + \mathbf{h_d}^H\mathbf{W}\mathbf{n_{r1}} + n_d, \tag{4}$$

where $\mathbf{h_d} \in \mathbb{C}^{N\times 1}$ is the channel vector between the transmission antennas of the relay and the destination node and n_d is additive Gaussian noise at the destination following $\mathcal{CN}(0, \sigma_d^2)$. Concurrently, the energy harvesting relay is wireless powered by the source node with dedicated energy-bearing signal. The received signal at the relay is

$$\begin{aligned} y_r^2 &= \sqrt{P_s}\mathbf{h_r}x_e + \mathbf{H_{rr}}\mathbf{x_r} + \mathbf{n_{r2}}, \\ &= \sqrt{P_s}\mathbf{h_r}x_e + \sqrt{P_s}\mathbf{H_{rr}}\mathbf{W}\mathbf{h_r}x_s + \mathbf{H_{rr}}\mathbf{W}\mathbf{n_{r1}} + \mathbf{n_{r2}}. \end{aligned} \tag{5}$$

where x_e is the transmitted signal from the source node with $\mathcal{E}(|x_e|^2) = 1$, $\mathbf{H_{rr}} \in \mathbb{C}^{\mathbf{N}\times\mathbf{N}}$ is the channel matrix of the loop channel at the relay node and $\mathbf{n_{r2}}$ is the received additive complex Gaussian noise vector following $\mathcal{CN}(\mathbf{0}, \sigma_{r2}^2\mathbf{I}_N)$. The relay node not only collects energy from the source node, but also recycles part of its transmitted power due to its two groups of antennas being activated at the same time. Unlike the other full-duplex relaying sudies, the relay employs interference cancellation techniques to eliminate the loop-back interference signal. In the energy harvesting cooperative network, the loop-back signal can be reused at the relay as the transmitted power. The amount of the harvested energy is given by

$$\begin{aligned} E = \frac{\eta T}{2}(&P_s||\mathbf{h_r}||^2 + P_s||\mathbf{H_{rr}}\mathbf{W}\mathbf{h_r}||^2 + \sigma_{r1}^2||\mathbf{H_{rr}}\mathbf{W}||^2 \\ &+\sigma_{r2}^2), \end{aligned} \tag{6}$$

where η is the energy conversion efficiency coefficient at the relay. Then the total available transmit power at the relay node is $\frac{E}{T/2}$ which can be expressed as

$$\begin{aligned} P_r^{max} = \eta(&P_s||\mathbf{h_r}||^2 + P_s||\mathbf{H_{rr}}\mathbf{W}\mathbf{h_r}||^2 + \sigma_{r1}^2||\mathbf{H_{rr}}\mathbf{W}||^2 \\ &+\sigma_{r2}^2), \end{aligned} \tag{7}$$

According to the aforementioned signal model, the receive signal-to-noise ratio (SNR) at the destination node is

$$SNR_d = \frac{P_s|\mathbf{h_d}^H\mathbf{W}\mathbf{h_r}|^2}{\sigma_{r1}^2||\mathbf{h_d}^H\mathbf{W}||^2 + \sigma_d^2}, \tag{8}$$

The achievable rate can be presented as

$$R = \frac{1}{2}\log_2\left(1 + SNR_d\right), \tag{9}$$

B. Problem Formulation and Beamforming Design

In this section, we consider the beamforming optimization problem for the considered network. We formulate the problem to maximize the achievable rate subject to the transmitted power constraint at the relay node. Particularly, the addressed optimization problem is formulated as follows:

$$\begin{aligned} &\max_{\mathbf{W}} \ R \\ &\quad s.t. \quad P_r \leq P_r^{max}, \end{aligned} \tag{10}$$

It is obvious that the problem stated above is not a standard convex problem. Then, we proposed an optimal solution based on SDR. By using the monotonicity, the original problem can be expressed as

$$\begin{aligned} &\max_{\mathbf{W}} \ \frac{P_s|\mathbf{h_d}^H\mathbf{W}\mathbf{h_r}|^2}{\sigma_{r1}^2||\mathbf{h_d}^H\mathbf{W}||^2+\sigma_d^2} \\ &s.t. \quad P_s||\mathbf{W}\mathbf{h_r}||^2+\sigma_{r1}^2||\mathbf{W}||^2 \\ &\qquad \leq \eta(P_s||\mathbf{h_r}||^2+P_s||\mathbf{H_{rr}}\mathbf{W}\mathbf{h_r}||^2 \\ &\qquad +\sigma_{r1}^2||\mathbf{H_{rr}}\mathbf{W}||^2+\sigma_{r2}^2, \end{aligned} \tag{11}$$

The beamforming matrix in our considered network can be decomposed as $\mathbf{W} = \mathbf{w_t}\mathbf{w_r}^H$. $\mathbf{w_t} \in \mathbb{C}^{\mathbf{N}\times\mathbf{1}}$ is the transmission beamforming vector at the relay node. $\mathbf{w_r} \in \mathbb{C}^{N\times 1}$ is the receiving beamforming vector with $||\mathbf{w_r}|| = 1$. We further choose the structure of the receiving beamforming as $\mathbf{w_r} = \frac{\mathbf{h_r}}{||\mathbf{h_r}||}$, this maximum ratio combining (MRC) structure can strengthen the received signal or power and it is widely used [20], [21]. Therefore, the problem can be further expressed as

$$\begin{aligned} &\max_{\mathbf{w_t}} \ \frac{P_s|\mathbf{h_d}^H\mathbf{w_t}|^2|\mathbf{w_r}^H\mathbf{h_r}|^2}{\sigma_{r1}^2|\mathbf{h_d}^H\mathbf{w_t}|^2+\sigma_d^2} \\ &s.t. \quad P_s||\mathbf{w_t}||^2|\mathbf{w_r}^H\mathbf{h_r}|^2+\sigma_{r1}^2||\mathbf{w_t}||^2 \\ &\qquad \leq \eta(P_s||\mathbf{h_r}||^2+P_s||\mathbf{H_{rr}}\mathbf{w_t}||^2|\mathbf{w_r}^H\mathbf{h_r}|^2 \\ &\qquad +\sigma_{r1}^2||\mathbf{H_{rr}}\mathbf{w_t}||^2+\sigma_{r2}^2), \end{aligned} \tag{12}$$

The above problem has one variable $\mathbf{w_t}$. Here we can find that the objective function in the above problem is an increasing function in $|\mathbf{h_d}^H\mathbf{w_t}|^2$. By using the monotonicity again, the problem can be simplified as

$$\begin{aligned} &\max_{\mathbf{w_t}} \ |\mathbf{h_d}^H\mathbf{w_t}|^2 \\ &s.t. \quad (||\mathbf{w_t}||^2-||\mathbf{H_{rr}}\mathbf{w_t}||^2\eta)(P_s|\mathbf{w_r}^H\mathbf{h_r}|^2+\sigma_{r1}^2) \\ &\qquad \leq \eta P_s||\mathbf{h_r}||^2+\eta\sigma_{r2}^2, \end{aligned} \tag{13}$$

The transformed problem is a quadratic form and we employ semidefinite programming relaxation to solve it. Define a new variable $\mathbf{X} = \mathbf{w_t}\mathbf{w_t}^H$, the optimization problem can be reformulate as

$$\begin{aligned} &\max_{\mathbf{X}\succeq\mathbf{0}} \ Tr(\mathbf{Q_1}X) \\ &s.t. \quad Rank(\mathbf{X}) = 1, \\ &\qquad Tr(\mathbf{Q_2}X) \leq \frac{\eta P_s||\mathbf{h_r}||^2+\eta\sigma_{r2}^2}{P_s|\mathbf{w_r}^H\mathbf{h_r}|^2+\sigma_{r1}^2}, \end{aligned} \tag{14}$$

where $\mathbf{Q_1} = \mathbf{h_d}\mathbf{h_d}^H$, $\mathbf{Q_2} = \mathbf{I} - \eta\mathbf{H_{rr}}^H\mathbf{H_{rr}}$.

It can be seen that the rank-one constraint makes the problem still difficult to solve. Therefore we drop the rank-one constraint and a semi-definite programming (SDP) problem can be obtained as

$$\begin{aligned} &\max_{\mathbf{X}\succeq\mathbf{0}} Tr(\mathbf{Q_1}X) \\ &s.t. \;\; Tr(\mathbf{Q_2}X) \leq \frac{\eta P_s||\mathbf{h_r}||^2 + \eta\sigma_{r2}^2}{P_s|\mathbf{w_r}^H\mathbf{h_r}|^2 + \sigma_{r1}^2}, \end{aligned} \tag{15}$$

Then the problem is a standard SDP problem and one can efficiently find its global optimal solution via available solvers [22]. If the rank of $\mathbf{X}^\star$ is one, $\mathbf{w_t}$ is exactly computed via eigenvalue decomposition. Otherwise we can use the Shapiro-Barvinok-Pataki (SBP) rank reduction theorem which is obtained from the result in [23] to obtain the rank-one solution. The theorem is summarized as follows.

Theorem 1. *For a matrix $\mathbf{X}^\star$ which has a higher rank, the rank-one solution can be acquired via the follow procedure.*

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1. Decompose $\mathbf{X}^\star$ as $\mathbf{X}^\star = \mathbf{V}\mathbf{V}^H$ with $\mathbf{V} \in \mathbb{C}^{N\times r}$, where r is the rank of $\mathbf{X}^\star$.
2. Find a nonzero $r \times r$ Hermitian matrix $\mathbf{M}$ to satisfy the equations as follows $Tr(\mathbf{V}^H\mathbf{Q_2}\mathbf{V}\mathbf{M}) = 0$.
3. Evaluate all eigenvalues λ_1, $\lambda_2 \cdots \lambda_r$ for matrix $\mathbf{M}$ and define $|\lambda| = max\{|\lambda_1|, |\lambda_2| \cdots |\lambda_r|\}$.
4. Update matrix $\mathbf{X}^\star = \mathbf{V}(\mathbf{I}_r - \frac{1}{\lambda}\mathbf{M})\mathbf{V}^H$. If $\mathbf{X}^\star$ still has the higher rank, we repeat the step 1-3 until the rank of $\mathbf{X}^\star$ is one.

Proof. The proof is similar to the one in [23]. $\mathbf{M}$ has r^2 real elements. If $r^2 \geq 3$, we an always find a nonzero solution $\mathbf{M}$. It is can be found that the rank of $\mathbf{X}^\star$ is reduced at least one by performing one iteration. And the updated $\mathbf{X}^\star$ in step 4 satisfies equations in step 2, which is also a solution for the SDP problem, i.e. the updated $\mathbf{X}^\star$ can achieve the same value of objective function for our transformed problem but with the lower rank. We can finally acquire a rank-one solution by repeating the procedure. The proof is completed. ■

IV. Beamforming with Time-Switching Relaying Protocol

In this section, we study the beamforming optimization based on the time-switching relaying protocol as a benchmark. Compared with to TSR protocol, the self-energy recycling protocol does not need to allocate dedicated time slot for the energy transmission, thus has the potential to improve the throughput. The energy harvesting relay in TSR protocol works in half-duplex mode. The transmission model with the TSR protocol and the corresponding optimization problem are given as follows.

A. Time Switching-based Relay Protocol and Transmission Model

In the time-switching relaying protocol, the information transmission process is split into three phases. In the first phase, the source transmits RF signals to power the energy harvesting relay for αT, where T denotes the duration of one block and $0 \leq \alpha \leq 1$. During the rest of the block time $(1-\alpha)T$, the information is transmitted from the source to the relay and then the relay will use the harvested energy to deliver the source information the destination.

The energy harvesting relay harvests the energy in the first phase. The received signal at the relay can be expressed as follows

$$y_r = \sqrt{P_s}\mathbf{h_r}x_e + \mathbf{n_{r3}}, \tag{16}$$

where P_s is the transmitted power at the source node, $\mathbf{h_r} \in \mathbb{C}^{N\times 1}$ denotes the channel vector the between the source node and the relay node, x_e is the dedicated energy-bearing signal from the source node with $\mathcal{E}(|x_e|^2) = 1$ and $\mathbf{n_{r3}}$ is the additive complex Gaussian noise vector at the relay node following $\mathcal{CN}(\mathbf{0}, \sigma_{r3}^2\mathbf{I}_N)$.

The relay harvests energy from the RF signal sent by the source for α time, and the amount of the harvested energy is given by

$$E = \eta\alpha(P_s||\mathbf{h_r}||^2 + \sigma_{r3}^2). \tag{17}$$

where η is the energy conversion efficiency coefficient at the relay. Therefore, the total available transmitted power at the relay is then expressed as follows:

$$P_r^{max} = \frac{2\eta\alpha(P_s||\mathbf{h_r}||^2 + \sigma_{r3}^2)}{1-\alpha}. \tag{18}$$

After the energy transmission, the received information from the source node to the relay node can be expressed as

$$y_r = \sqrt{P_s}\mathbf{h_r}x_s + \mathbf{n_{r4}} + \mathbf{n_c}, \tag{19}$$

where x_s is the dedicated information-bearing signal from the source node with $\mathcal{E}(|x_s|^2) = 1$, $\mathbf{n_{r4}}$ is the additive complex Gaussian noise vector at the relay node following $\mathcal{CN}(\mathbf{0}, \sigma_{r4}^2\mathbf{I}_N)$ and $\mathbf{n_c}$ is the additive complex Gaussian noise vector due to the RF to baseband conversion following $\mathcal{CN}(\mathbf{0}, \sigma_c^2\mathbf{I}_N)$.

Since the energy harvesting relay is operated with the AF protocol, the signal transmitted by the relay node can be written as follows:

$$x_r = \sqrt{P_s}\mathbf{W}\mathbf{h_r}x_s + \mathbf{W}\mathbf{n_{r4}} + \mathbf{W}\mathbf{n_c}. \tag{20}$$

where $\mathbf{W} \in \mathbb{C}^{\mathbf{N}\times\mathbf{N}}$ is the precoding matrix.

The relay transmission power is given by

$$P_r = P_s||\mathbf{W}\mathbf{h_r}||^2 + \sigma_{r4}^2||\mathbf{W}||^2 + \sigma_c^2||\mathbf{W}||^2. \tag{21}$$

Then the received signal at the destination node is given by

$$y_d = \sqrt{P_s}\mathbf{h_d}^H\mathbf{W}\mathbf{h_r}x_s + \mathbf{h_d}^H\mathbf{W}\mathbf{n_{r4}} + \mathbf{h_d}^H\mathbf{W}\mathbf{n_c} + n_d. \tag{22}$$

where $\mathbf{h_d} \in \mathbb{C}^{N\times 1}$ denotes the channel vector between the relay node and the destination node and n_d is the additive Gaussian noise at the destination following $\mathcal{CN}(0, \sigma_d^2)$.

According to the described signal model, the received signal-to-noise ratio (SNR) at the destination node is

$$SNR_d = \frac{P_s|\mathbf{h_d}^H\mathbf{W}\mathbf{h_r}|^2}{\sigma_{r4}^2||\mathbf{h_d}^H\mathbf{W}||^2 + \sigma_c^2||\mathbf{h_d}^H\mathbf{W}||^2 + \sigma_d^2}, \tag{23}$$

The achievable rate can be expressed as

$$R = \frac{1-\alpha}{2}\log_2\left(1+SNR_d\right),$$

B. Problem Formulation and Beamforming Design

We formulate the beamforming optimization problem of maximizing the achievable rate subject to the transmitted power constraint at the relay node. The optimization problem is formulated as follows:

$$\begin{aligned} &\underset{\mathbf{W},\alpha}{max}\ R \\ &s.t.\quad P_r \le P_r^{max}, \end{aligned} \tag{24}$$

This problem is in a form similar to the one proposed in the previous section, and is also not a standard convex problem. By using the monotonicity, the problem can be expressed by

$$\begin{aligned} &\underset{\mathbf{W},\alpha}{max}\ \frac{P_s|\mathbf{h_d}^H\mathbf{W}\mathbf{h_r}|^2}{\sigma_{r4}^2||\mathbf{h_d}^H\mathbf{W}||^2+\sigma_c^2||\mathbf{h_d}^H\mathbf{W}||^2+\sigma_d^2} \\ &s.t.\quad P_s||\mathbf{W}\mathbf{h_r}||^2+\sigma_{r4}^2||\mathbf{W}||^2+\sigma_c^2||\mathbf{W}||^2 \\ &\qquad \le \frac{2\eta\alpha(P_s||\mathbf{h_r}||^2+\sigma_{r3}^2)}{1-\alpha}, \end{aligned} \tag{25}$$

For the problem above, we first transform the objective function as

$$\begin{aligned} SNR_d &= \frac{P_s|\mathbf{h_d}^H\mathbf{W}\mathbf{h_r}|^2}{\sigma_{r4}^2||\mathbf{h_d}^H\mathbf{W}||^2+\sigma_c^2||\mathbf{h_d}^H\mathbf{W}||^2+\sigma_d^2}, \\ &= \frac{P_s\mathbf{w^H}\mathbf{h}\mathbf{h^H}\mathbf{w}}{\sigma_{r4}^2\mathbf{w^H}\mathbf{H_1}\mathbf{H_1^H}\mathbf{w}+\sigma_c^2\mathbf{w^H}\mathbf{H_1}\mathbf{H_1^H}\mathbf{w}+\sigma_d^2}, \end{aligned} \tag{26}$$

where $\mathbf{w} = vec(\mathbf{W})$, $\mathbf{h} = \mathbf{h_r^*}\otimes\mathbf{h_d}$ and $\mathbf{H_1} = \mathbf{I}\otimes\mathbf{h_d}$. The above transform is obtained by using the rule as follows [24]

$$vec(\mathbf{ABC}) = (\mathbf{C}^T\otimes\mathbf{A})vec(\mathbf{B}), \tag{27}$$

The transmitted power constraint can be rewritten as

$$\begin{aligned} P_r &= P_s||\mathbf{W}\mathbf{h_r}||^2+\sigma_{r4}^2||\mathbf{W}||^2+\sigma_c^2||\mathbf{W}||^2, \\ &= P_s\mathbf{w^H}\mathbf{H_2}\mathbf{H_2^H}\mathbf{w}+\sigma_{r4}^2\mathbf{w^H}\mathbf{w}+\sigma_c^2\mathbf{w^H}\mathbf{w}, \end{aligned} \tag{28}$$

where $\mathbf{H_2} = \mathbf{h_r^*}\otimes\mathbf{I}$.

Define a new variable $\mathbf{X} = \mathbf{w}\mathbf{w}^H$, the optimization problem can be reformulate as

$$\begin{aligned} &\underset{\mathbf{X}\succeq\mathbf{0},\alpha}{max}\ \frac{Tr(\mathbf{Q_1}\mathbf{X})}{Tr(\mathbf{Q_2}\mathbf{X})+\sigma_d^2} \\ &s.t.\quad Tr(\mathbf{Q_3}\mathbf{X}) \le \frac{2\eta\alpha(P_s||\mathbf{h_r}||^2+\sigma_{r3}^2)}{1-\alpha}, \\ &\qquad 0\le\alpha\le 1, \\ &\qquad Rank(\mathbf{X}) = 1, \end{aligned} \tag{29}$$

where $\mathbf{Q_1} = \mathbf{P_s}\mathbf{h}\mathbf{h^H}$, $\mathbf{Q_2} = (\sigma_{\mathbf{r4}}^{\mathbf{2}}+\sigma_{\mathbf{c}}^{\mathbf{2}})\mathbf{H_1}\mathbf{H_1^H}$ and $\mathbf{Q_3} = \mathbf{P_s}\mathbf{H_2}\mathbf{H_2^H}+\sigma_{\mathbf{r4}}^{\mathbf{2}}\mathbf{I}+\sigma_{\mathbf{c}}^{\mathbf{2}}\mathbf{I}$.

It is worthy to point out that it is still challenging to solve this problem directly, mainly due to the rank one constraint and the time switching coefficient in the problem, which renders the optimization problem non-convex. In order to tackle these difficulties, we can drop the rank one constraint. Then we can perform one-dimension (1-D) optimization with respect to the time-switching coefficient. Specifically, for a given time-switching coefficient, the optimal relay beamforming matrix can be efficiently obtained. We perform a full search with respect to the time-switching coefficient, then solve the problem with all possible value of the time-switching coefficient. In practice, we can discretize the range of the time-switching coefficient into $M \gg 1$ equally spaced intervals with an interval width of $1/M$ for facilitating the full search. After we obtain all corresponding achieved system performances, we can select the best performance among those possible choices of the time-switching coefficient. If the value of α is set, the problem can be treated as a quasi-convex SDP problem after that we drop the rank one constraint. Instead of employing the bisection search approach, we use the Charnes-Cooper transformation [25] to solve it. Specifically, we define a new variable $t = \frac{1}{Tr(\mathbf{Q_2X})+\sigma_d^2}$ and let $\widetilde{\mathbf{X}} = t\mathbf{X}$. The problem can be recast as follows

$$\begin{aligned} &\max_{\widetilde{\mathbf{X}}\succeq \mathbf{0}, \mathbf{t}\geq \mathbf{0}} \; Tr(\mathbf{Q_1}\widetilde{\mathbf{X}}) \\ &s.t. \;\; Tr(\mathbf{Q_2}\widetilde{\mathbf{X}}) + \sigma_d^2 t = 1, \\ &Tr(\mathbf{Q_3}\widetilde{\mathbf{X}}) \leq \frac{2t\eta\alpha(P_s||\mathbf{h_r}||^2 + \sigma_{r3}^2)}{1-\alpha}, \end{aligned} \tag{30}$$

Then the problem is solvable like the problem with the self-energy recycling relaying protocol. The optimal solution is denoted by $\widetilde{\mathbf{X}}^\star, t^\star$. Then the solution of $\mathbf{X}$ denoted by $\mathbf{X}^\star$ is obtained by $\mathbf{X}^\star = \frac{\widetilde{\mathbf{X}}^\star}{t^\star}$. If the rank of $\widetilde{\mathbf{X}}^\star$ is not one, we can also use the Shapiro-Barvinok-Pataki (SBP) rank reduction theorem to obtain the rank-one solution.

It is necessary to mention that we can also use power-splitting relaying protocol as another benchmark scheme compared with the self-energy recycling scheme. The power-splitting relaying protocol can be referred as [12]. Our method to optimize the beamforming with the time-switching relaying protocol is easily extended to the same optimization problem based on the power-splitting scheme. Hence we omit its detailed procedure here, and we provide its simulation result in the next section.

V. Numerical Results

In this section, simulation results are provided to evaluate the performance of the proposed beamforming optimization solution. The number of antenna is 3. We use the TGn path loss model [26]. The loop channel path loss is -15dB [27]. The distance between each node is set as 10m. The system bandwidth is 200 kHz. The carrier frequency is 470 MHz which is accorded with the IEEE 802.11af Wi-Fi parameters [28]. The noise variance is assumed as $\sigma_{r1}^2 = \sigma_{r2}^2 = \sigma_{r3}^2 = \sigma_{r4}^2 = \sigma_c^2 = \sigma_d^2 = -25$ dBm. The energy conversion efficient is 0.8. Simulation results were averaged over 1000 independent trials.

In Fig. 1, the achievable rates achieved by different energy harvesting schemes versus the transmitted power of the source are plotted. The result for the self-energy recycling protocol is demonstrated and the results for the time-switching relaying scheme and the power-splitting relaying scheme are also shown as benchmarks. The results for PSR and TSR schemes are obtained with their optimal energy harvesting coefficient. It can be seen that curves of achievable rate for these energy harvesting schemes are monotonically non-decreasing functions of the transmitted power because the higher transmitted power results in more available power at the relay for the relay-destination transmission. The self-energy recycling scheme outperforms the TSR scheme and PSR scheme. This is because that the full-duplex structure relay used in the self-energy recycling relaying protocol not only harvests the power transmitted form the source node, but also recycles the part of transmitted power at the relay node. Compared to TSR scheme and PSR scheme, the self-energy harvesting scheme achieves an obvious rate gain. The self-energy recycling protocol has a better performance than the other energy harvesting protocols.

In Fig. 2, the performances of the self-energy recycling protocol with different loop channel path loss are presented. From the figure, we can see that the system performance with the path loss -15dB is better than that with the path loss -5dB or -25dB. This is due to that if the path loss is smaller, the energy harvesting relay cannot recycle more power in the second transmission phase, which leads to the available transmitted power at the relay node is smaller. However, if the loop channel path loss is too large, it will deteriorate the system performance, even though the relay can harvest more power.

In Fig. 3, we present the achievable rate versus the transmitted power for the self-energy recycling relaying protocol with different system parameters. It can be seen from the figure that the achievable rate improves with the increased number of the antenna and is degraded with the increase of the distance between each node. This is attributed to the fact that the relay could exploit the array gain to achieve better performance with more antennas and the longer distance makes channel attenuation larger in turns resulting in the worse performance of relay networks.

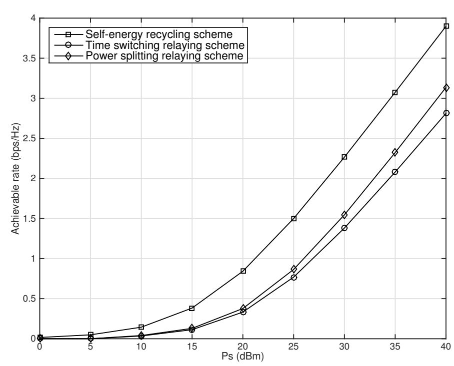


Fig. 1. Achievable rate versus transmitted power at the source node for different energy harvesting protocols.

VI. Conclusions

In this paper, we formulated the beamforming optimization problem in an AF relay network based on the self-energy recycling relaying protocol. We proposed a SDP-based solution to obtain the optimal solution for the proposed problem. In order to demonstrate the advantage of the self-energy recycling relaying protocol, we also used the SDP relaxation and 1-D optimization to solve the beamforming optimization problem with the time-switching relaying protocol and the power-splitting relaying protocol. Simulation results illustrated that the self-

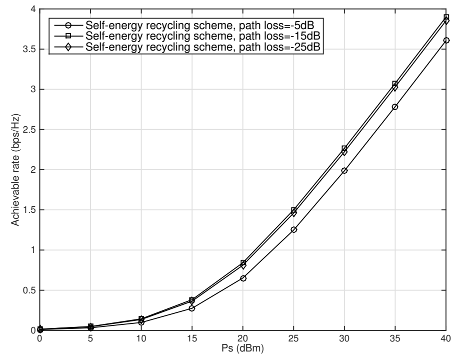
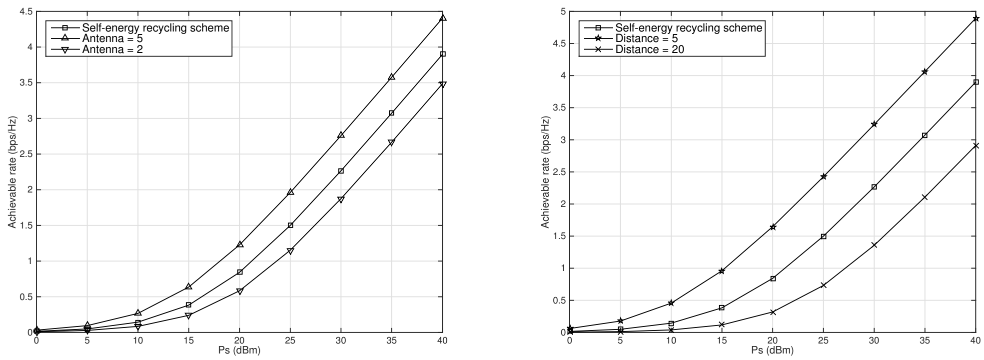


Fig. 2. Achievable rate versus transmitted power at the source node for the self-energy recycling relaying protocol with different loop channel path loss.



(a) Performance comparison of the self-energy recycling protocol with different numbers of antenna.

(b) Performance comparison of the self-energy recycling protocol with different distances.

Fig. 3. Achievable rate versus transmitted power at the source node for the self-energy recycling relaying protocol with different parameters.

energy recycling relaying protocol could improve the achievable rate compared to TSR scheme and PSR scheme. The trade-off between the achievable rate and system parameters is also provided.

REFERENCES

[1] R. Zhang and C. K. Ho, "MIMO broadcasting for simultaneous wireless information and power transfer," *IEEE Trans. Wireless Commun.*, vol. 12, no. 5, pp. 1989–2001, May 2013.

[2] L. Liu, R. Zhang, and K. C. Chua, "Wireless information and power transfer: A dynamic power splitting approach," *IEEE Trans. Commun.*, to appear in 2013 (available on-line at arXiv:1302.0585).

[3] X. Zhou, R. Zhang, and C. K. Ho, "Wireless information and power transfer: Architecture design and rate-energy tradeoff," *IEEE Transactions on Communications*, (submitted) Available: http://arxiv.org/abs/1205.0618.

[4] K. Huang and V. K. N. Lau, "Enabling wireless power transfer in cellular networks: architecture, modeling and deployment," *IEEE J. Sel. Areas Commum.*, 2012. Available at http://arxiv.org/abs/1207.5640.

[5] V. Raghunathan, S. Ganeriwal, and M. Srivastava, "Emerging techniques for long lived wireless sensor networks," *IEEE Commum. Mag.*, vol. 44, no. 4, pp. 108–114, Apr. 2006.

[6] J. A. Paradiso and T. Starner, "Energy scavenging for mobile and wireless electronics," *IEEE Trans. Pervasive Comput.*, vol. 4, no. 1, p. 1827, Jan. 2005.

[7] B. Medepally and N. B. Mehta, “Voluntary energy harvesting relays and selection in cooperative wireless networks,” *IEEE Trans. Wireless Commun.*, vol. 9, no. 11, pp. 3543–3553, Nov. 2010.

[8] L. R. Varshney, “Transporting information and energy simultaneously,” in *Proc. IEEE Int. Symp. Inf. Theory (ISIT)*, Toronto, Canada, Jul. 2008.

[9] L. Liu, R. Zhang, and K. C. Chua, “Wireless information transfer with opportunistic energy harvesting,” *IEEE Trans. Wireless Commun.*, 2012. Available at http://arxiv.org/abs/1204.2035.

[10] Z. Xiang and M. Tao, “Robust beamforming for wireless information and power transmission,” *IEEE Wireless Commun. Lett.*, vol. 1, no. 4, pp. 372–375, 2012.

[11] P. T. Venkata, S. N. A. U. Nambi, R. V. Prasad, and I. Niemegeers, “Bond graph modeling for energy-harvesting wireless sensor networks,” *IEEE Computer*, vol. 45, no. 9, pp. 31–38, Sep. 2012.

[12] A. A. Nasir, X. Zhou, S. Durrani, and R. A. Kennedy, “Relaying protocols for wireless energy harvesting and information processing,” *IEEE Trans. Wireless Commun.*, vol. 12, pp. 3622–3636, 2013.

[13] Z. Ding and H. V. Poor, “Cooperative energy harvesting networks with spatially random users,” *IEEE Signal Process. Lett.*, vol. 20, no. 12, pp. 1211–1214, Dec. 2013.

[14] ——, “Energy harvesting cooperative networks: Is the max-min criterion still diversity-optimal?” to appear in 2014 (available on-line at http://arxiv.org/pdf/1403.0354.pdf).

[15] Z. Ding, I. Krikidis, B. Sharif, and H. V. Poor, “Wireless information and power transfer in cooperative networks with spatially random relays,” *IEEE Trans. Wireless Commun.*, vol. PP, no. 99, p. 1, Mar. 2014.

[16] Z. Ding, S. M. Perlaza, I. Esnaola, and H. V. Poor, “Power allocation strategies in energy harvesting wireless cooperative networks,” *IEEE Trans. Wireless Commun.*, submitted (available at http://arxiv.org/abs/1307.1630).

[17] H. Ju and R. Zhang, “Optimal resource allocation in full-duplex wireless powered communication network,” *IEEE Trans. Commun.*, Available at http://arxiv.org/abs/1403.2580.

[18] C. Zhong, H. A. Suraweera, G. Zheng, I. Krikidis, and Z. Zhang, “Wireless information and power transfer with full duplex relaying,” *IEEE Trans. Commun.*, Available at http://arxiv.org/abs/1409.3904.

[19] Y. Zeng and R. Zhang, “Full-duplex wireless-powered relay with self-energy recycling,” *IEEE Wireless Commun. Letters*, 2015.

[20] D. W. K. Ng, E. Lo, and R. Schober, “Energy-efficient resource allocation in ofdma systems with large numbers of base station antennas,” *IEEE Trans Wireless Commun.Commun.*, vol. 11, pp. 3292–3304, Sep. 2012.

[21] T. Marzetta, “Noncooperative cellular wireless with unlimited numbers of base station antennas,” *IEEE Trans. Wireless Commun.*, vol. 9, pp. 3590–3600, Nov. 2010.

[22] M. Grant and S. Boyd, “Cvx: Matlab software for disciplined convex programming,” Jun. 2012. Available at http://cvxr.com.cvx.

[23] Y. Huang and D. P. Palomar, “Rank-constrained separable semidefinite programming with applications to optimal beamforming,” *IEEE Trans. Signal Process.*, vol. 58, no. 2, pp. 664–678, 2010.

[24] R. A. Horn and C. R. Johnson, *Matrix Analysis*. Cambridge University Press, 1985.

[25] A. Charnes and W. W. Cooper, “Prgramming with linear fractional functions,” *Naval Res. Logist. Quart.*, vol. 9, pp. 181–186, 1962.

[26] “Tgn channel models,” *IEEE 802.1103/940r4, Tech. Rep.*, May 2004.

[27] H. G. Schantz, “Near field propagation law and a novel fundamental limit to antenna gain versus size,” *IEEE Antennas and Propag. Society Int. Symposium*, 2005.

[28] H. S. Chen and W. Gao, “Mac and phy proposal for 802.11af,” Tech. Rep., Feb. Available at http://mentor.ieee.org/802.11/dcn/10/11-10-0258-00-00af-mac-and-phy-proposal-for-802-11af.pdf.