Reduction of the value of information sharing as demand becomes strongly auto-correlated

Abstract

Information sharing has been identified, in the academic literature, as one of the most important levers to mitigate the bullwhip effect in supply chains. A highly-cited article on the bullwhip effect has claimed that the percentage inventory reduction resulting from information sharing in a two level supply chain, when the downstream demand is autoregressive of order one, is an increasing function of the autoregressive parameter of the demand. In this paper we show that this is true only for a certain range of the autoregressive parameter and there is a maximum value beyond which the bullwhip ratio at the upstream stage is reduced and the percentage inventory reduction resulting from information sharing decreases towards zero. We also show that this maximum value of the autoregressive parameter can be as high as 0.7 which represents a common value that may be encountered in many practical contexts. This means that large benefits of information sharing cannot be assumed for those Stock Keeping Units (SKUs) with highly positively auto-correlated demand. Instead, equally careful analysis is needed for these items as for those SKUs with less strongly auto-correlated demand.

Keywords: Supply chain, bullwhip effect, information sharing, autoregressive demand

1. Introduction

The business environment is becoming more competitive and uncertain. Organisational strategies for survival, sustainability and growth have moved away from just focussing on the company itself, and towards focussing on the whole supply chain. Sharing of customer demand information across the supply chain members is crucial for this broader focus (Agrawal et al, 2009). When the customer demand is not shared, the ordering process from downstream to upstream members of the supply chains results in amplification of the variability of the demand. This amplification of the demand variability is known as the Bullwhip Effect (Lee et al, 2000; Luong, 2007) and the bullwhip ratio is a critical metric for measuring information distortion in supply chains (Dejonckheere et al, 2003; Dejonckheere et al, 2004). The pervasive nature of the Bullwhip Effect has led to it being termed as the “First law of Supply Chain Dynamics” by Kouvelis et al. (2006) and it has been viewed as one of the most important research areas in the field of Operational Research (Fildes et al, 2008).

Information sharing has been identified as one of the most important means for the reduction of the Bullwhip Effect. Various practices such as Vendor Managed Inventory (VMI), Efficient Consumer Response (ECR), Continuous Replenishment (CR), and Electronic Data Interchange (EDI) are being used in industry to improve information visibility and to lead to further collaborations among supply chain partners (Yao and Dresner, 2008; Cannella and Ciancimino, 2010; Dong et al., 2014). For example, empirical research into 54 manufacturers in the Food and Consumer Package Goods (F&CPG) industry has shown that the highest profit margin companies are not simply exchanging information but using information as a vehicle for supply chain collaborations (Kulp et al, 2004). Similarly, case studies reported by Ganeshan (2001), Yu et al (2001), Zhou and Benton (2007) and Hosoda et al (2008) demonstrate the value of higher-level collaboration and partnerships in improving supply chain performance. Enhancements in technology such as Radio Frequency Identification (RFID) and inter-organisational Enterprise Resource Planning (ERP) help in the development of such collaborations (Lee and Whang, 2000; Disney et al, 2004; Machuca and Barajas, 2004).

An important practical problem facing organisations contemplating information sharing is the assessment of the benefit of doing so. Any financial benefit, through reduced stock-holdings or back-orders and improved shelf availability, will need to be compared with the costs of investing in new information systems and processes (Disney et al, 2008; Disney and Lambrecht, 2008).

Such organisations would find that the academic literature gives contradictory advice on the value of information sharing. Some authors contend that substantial benefits may be attained, whilst others argue that information sharing is un-necessary, because demand at the downstream partner can be inferred even if it is not shared. The reasons for these contradictory findings lie in the assumptions adopted by the authors.

In one of the seminal papers on this issue, Lee et al. (2000) assess the value of information sharing for an autoregressive process of order one [AR (1)] by assuming that the demand process and parameters are known to the supply chain partners. The paper concludes that information sharing in supply chains is valuable in terms of reductions in inventory holdings and cost when demand is positively auto-correlated. They also claim that the benefit of information sharing increases as demand becomes more strongly (positively) auto-correlated.

Other authors have challenged the findings of Lee et al (2000), arguing that demand can be inferred due to the presence of a mathematical relationship between demands and orders. These papers will be reviewed in Section 2. If their arguments are accepted, then there is no benefit of information sharing. However, some of the assumptions underpinning demand inference are questionable. If these assumptions are dropped, then there is a benefit of information sharing, and it becomes necessary to quantify that benefit.

In this paper, close attention will be paid to the form of the functions evaluating the benefit of information sharing for an AR(1) demand process. It will be shown that the financial benefit, at the Stock Keeping Unit (SKU) level, depends on the autoregressive parameter, with the benefit declining past a certain 'critical value' of that parameter. This means that the overall benefit is highly dependent on the distribution of auto-regressive parameters across all of those SKUs showing AR(1) demand patterns. If there is a preponderance of items with strongly positively auto-correlated demand patterns, then a careful analysis is needed to ensure that the financial benefit outweighs the significant costs involved in investing in information sharing.

The remainder of the paper is structured as follows. We start in Section 2 by reviewing the literature dealing with the issue of information sharing in supply chains. Section 3 provides a high level but self-contained summary of the results presented by Lee et al. (2000), followed in Section 4 by our theoretical analysis and the implications of this work. Our conclusions, for theory and practice, are presented in Section 5. The detailed derivations related to the analysis are presented in the Appendices at the end of the paper.

2. Literature review

Although various papers clearly show the benefits of information sharing in the reduction of the Bullwhip Effect (e.g. Barlas and Gunduz, 2011), there is very little evidence on the direct financial impact of doing so. Hence, analysis of the inventory cost and service benefits, compared with the costs of investing in new information systems and processes, should be deepened (Disney et al, 2008; Disney and Lambrecht, 2008).

The order-up-to-level inventory policy has been identified as one the causes of the bullwhip effect. A stream of research has looked at the effect of other inventory ordering policies as a lever to reduce the bullwhip effect (Cachon, 1999; Holland and Sodhi, 2004; Noblesse et al, 2014) and also to demonstrate the benefits of information sharing (Cannella et al, 2011; Ciancimino et al, 2012; Cannella, 2014). Many of the research papers have considered simple supply chains (i.e. single supplier and single retailer). However, the literature has been extended to more complex supply chain models to show the value of information sharing, e.g. multiple retailers (Cachon and Fisher, 2000; Raghunathan, 2003), multiple echelons (Cheng and Wu, 2005; Trapero et al, 2012; Najafi and Zanjirani Farahani, 2014), revenue-sharing models (Zhang and Chen, 2013), VMI (Yu et al, 2002) and divergent supply chains (Dominquez et al, 2014).

It should be noted that there is no consensus in the academic literature on the value of information sharing within supply chains for order-up-to-level inventory policies. Two main streams of literature have been developed during the last fifteen years, providing contradictory advice on the value obtained from sharing information. The first stream has been developed based on the seminal paper of Lee et al (2000) where the authors considered an autoregressive process of order one [AR (1)] and, by assuming that the demand process and parameters are known to the supply chain partners, they assessed the benefit of sharing the downstream demand values in terms of reductions in the bullwhip effect and the inventory cost. Many other papers have been built upon the study of Lee et al (2000) by assuming other demand processes. Among others, Ali et al (2012) conducted research on MA(1) and ARMA(1,1) demand processes in conjunction with a minimum mean square error forecasting method (which is the same forecasting method as that assumed by Lee et al (2000)). Babai et al (2013) have extended this work by considering a non-stationary ARIMA(0,1,1) process and a single exponential smoothing forecasting method. (Readers who are interested in these ARIMA-type demand processes and forecasting methods are referred to Box et al (1976).) This stream of literature provides conclusions on the value of information sharing in supply chains in terms of reductions in inventory holdings and cost.

In the second stream of literature, the authors argue that there is no benefit of sharing the information because demand at the downstream partner can be inferred even if it is not shared. Raghunathan (2001) considers an autoregressive process of order one [AR (1)] and by using the assumption of demand process and parameters being known to all supply chain partners, he shows that downstream demand can be inferred due to the presence of a mathematical relationship between demands and orders. (At this point we should mention that although there are some differences in the assumptions made by Raghunathan (2001) and Lee et al (2000). For example, the former study utilises the history of orders whereas the analysis conducted in the latter is based solely on the last order. However, it is only the common fundamental assumption of the demand process and its parameters being known upstream in the supply chain that has implications for the arguments raised in our paper, and thus it is the only one considered in detail.) Zhang (2004) and Gaur et al (2005) have extended these results to ARMA(p,q) demand processes and Gilbert (2005) generalised the findings further to ARIMA(p,d,q) demand processes. This stream of literature has the common conclusion that sharing demand information is not beneficial when the demand process and parameters are known to the supply chain partners since the demand can be inferred. If these arguments are accepted, Lee et al's quantification of inventory and cost benefits is no longer relevant.

More recently, Ali and Boylan (2011, 2012) questioned whether companies would share demand process and parameters but not the demand itself. In a real world situation, the same information systems infrastructure is needed for sharing, at Stock Keeping Unit (SKU) level, the parameters and the demand itself. Therefore, the option of sharing only demand processes and parameters (for each SKU) is artificial. Sharing such detailed information is only possible if the systems infrastructure is in place to share the demand values too. Thus, it is highly unlikely that the supply chain links will invest in an information sharing mechanism just to share the information on demand process and parameters and not the actual value of demand itself. Indeed, to the best of our knowledge, no case-studies have yet been published of any organisations that have adopted a demand inference approach based on sharing only demand processes and parameters

Ali and Boylan (2011) showed that, under more general ARIMA(p,d,q) processes and forecasting methods, if demand processes and parameters are not shared, then inferring demand is not feasible, and information sharing is valuable. Hence, Lee et al's quantification of the benefits of information sharing is important for a full financial appraisal. However, although this quantification of the value of information sharing, under the assumption of unknown demand process and parameters, is necessary, it has been claimed that the on-hand inventory reduction resulting from the forecast information sharing may be substantial for highly auto-correlated demands. This claim is challenged in this paper where our objective is to show that there is little value of information sharing for highly auto-correlated demands.

3. Previous results on the value of information sharing

We recall in this section the main findings by Lee et al (2000) that constitute the focus of our analysis. We use in this paper the same notation (that follows) and assumptions that are considered by Lee et al (2000).

L : Manufacturer lead-time

l : Retailer lead-time

h : Unit inventory holding cost at the retailer

p : Unit inventory backordering cost at the retailer

H : Unit inventory holding cost at the manufacturer

P : Unit inventory backordering cost at the manufacturer

We consider a two stage supply chain (e.g. a retailer and a manufacturer) where the demand at the retailer at any time period *t*, denoted by D_t , follows an AR(1) process that is given by:

$$D_t = d + \rho D_{t-1} + \varepsilon_t \tag{1}$$

where $d > 0$ and ε_t is the noise term in the retailer's demand. The noise term is assumed to be a serially independent white noise process normally distributed with mean equal to 0 and variance equal to σ^2 . We assume that $-1 < \rho < 1$ but for the purpose of the analysis we focus, as in Lee et al (2000), on the case of ρ being in [0,1), thus ignoring the 'Anti-Bullwhip' region ($\rho < 0$). The demand is forecasted based on the minimum mean square error (MMSE) method. The inventory at each stage is controlled according to a periodic order-up-to (OUT) (T,S) policy, where T is the review period and S is the OUT level. (The OUT policy is very often used in supply chain to control material flow; see, e.g., Disney, 2008.)

Under the No-information sharing strategy, the manufacturer's total shipment quantity over the manufacturer lead-time is normally distributed with variance $V\sigma^2$ where

$$V = \frac{\left(1-\rho^{l+2}\right)^2 + \sum_{i=1}^{L}\left(1-\rho^{L+l+3-i}\right)^2 + \dfrac{\rho^2\left(1-\rho^{L+1}\right)^2\left(1-\rho^{l+1}\right)^2}{(1-\rho)^2}}{(1-\rho)^2}. \tag{2}$$

An approximation of the average on-hand inventory is given by $I = \dfrac{d}{2(1-\rho)} + k\sigma\sqrt{V}$ where $k = \Phi^{-1}(p/p+h)$ and $\Phi^{-1}(.)$ the inverse standard normal distribution.

Under the information sharing strategy, the manufacturer's total shipment quantity over the manufacturer lead-time is normally distributed with variance $V'\sigma^2$ where

$$V' = \frac{\left(1-\rho^{l+2}\right)^2 + \sum_{i=1}^{L}\left(1-\rho^{L+l+3-i}\right)^2}{(1-\rho)^2}. \tag{3}$$

An approximation of the average on-hand inventory is given by $I' = \dfrac{d}{2(1-\rho)} + K\sigma\sqrt{V'}$ where $K = \Phi^{-1}(P/P+H)$.

Assuming that $k = K$ as in Lee et al (2000), the percentage inventory reduction from information sharing is given by:

$$\Delta I = \frac{I - I'}{I} = \frac{(1 - \sqrt{V'} / \sqrt{V})}{\frac{d}{2K\sigma(1-\rho)\sqrt{V}} + 1} = \frac{(\sqrt{V} - \sqrt{V'})}{\frac{d}{2K\sigma(1-\rho)} + \sqrt{V}} \tag{4}$$

Lee et al (2000) claim in their Proposition 2 (page 633) that ΔI is increasing in ρ for any $\rho > 0$ and thus the percentage inventory reduction from information sharing is larger when ρ increases (i.e. when demand becomes more highly positively auto-correlated). We show in the following section that this is not true since this proposition holds only for a certain range of ρ in [0,1). Although Luong (2007) has shown the non-monotonicity of the bullwhip ratio at the retailer with respect to ρ, no results have been shown at the manufacturer, which would allow an analysis of the monotonicity of the value of information sharing between the retailer and the manufacturer. The analysis of the monotonicity of the bullwhip ratio at the manufacturer is provided in the following section.

4. New results on the bullwhip effect and the value of information sharing

4.1. Theoretical findings

In this section, we establish two new results, labelled as Proposition 1 and Proposition 2, which both point to the benefit of information sharing being a non-monotonic function of the auto-regressive parameter.

We first provide the expression of the bullwhip ratio at the manufacturer that we denote by BE_m (Lee et al, 1997; Hosoda and Disney, 2006). We recall that the bullwhip effect here is expressed as the ratio of the variance of the orders to that of the downstream demand.

$$BE_m = \frac{\left(1 - \rho^{l+L+1}\right)^2 + \rho^2\left(1 - \rho^{l+L}\right)^2 - 2\rho^2\left(1 - \rho^{l+L+1}\right)\left(1 - \rho^{l+L}\right)}{\left(1 - \rho\right)^2} \tag{5}$$

Proposition 1 extends the results of Luong (2007) by analyzing the monotonicity of BE_m with respect to the autoregressive demand parameter ρ. The proof of Proposition 1 is given in *Appendix A*. We show through this proposition that the bullwhip ratio at the manufacturer is a non-monotonic function of ρ and there exists a value of the autoregressive demand parameter at which the maximum bullwhip ratio is reached and beyond which the bullwhip ratio decreases.

Proposition 1.

The bullwhip ratio at the manufacturer, BE_m*, is a non-monotonic function of* ρ*. Moreover, this function has a unique maximum in* [0,1).

The proof of Proposition 1 is given in *Appendix A*.

It should be noted that the result given by Proposition 1 has been illustrated graphically in an earlier investigation by Hosoda and Disney (2006) but since this issue was not the focus of that paper, there was no analysis or comments on the behaviour of the bullwhip ratio at the manufacturer when the autoregressive parameter varies in the range [0,1).

Proposition 2 analyzes the monotonicity of the percentage inventory reduction resulting from information sharing, ΔI , with respect to the autoregressive demand parameter, ρ . The proof of Proposition 2 is given in *Appendix B.* As the bullwhip ratio is a non-monotonic function of ρ , it is expected that the percentage inventory reduction function is also a non-monotonic function of the autoregressive parameter. This is confirmed by the following proposition. We show that the percentage inventory reduction function is also a non-monotonic function of the autoregressive parameter, which means that beyond a certain break point of the autoregressive parameter, the value of the information sharing decreases and tends towards zero.

Proposition 2.

The percentage inventory reduction ΔI *resulting from information sharing between the retailer and the manufacturer is a non-monotonic function of* ρ

The proof of Proposition 2 is given in *Appendix B.*

Note that in the proof of Proposition 2 in Lee et al. (2000), the authors claim (in the first line of page 643) that "*the last term inside the bracket for V in (3.9) is increasing in* ρ ". It is easy to show that this is not true as this term is a non-monotonic function of ρ in [0,1).

4.2. Numerical examples

The manufacturer bullwhip ratio results are shown in Figure 1 for l = 10, L = 5; l = 5, L = 5 and l = 1, L = 1.

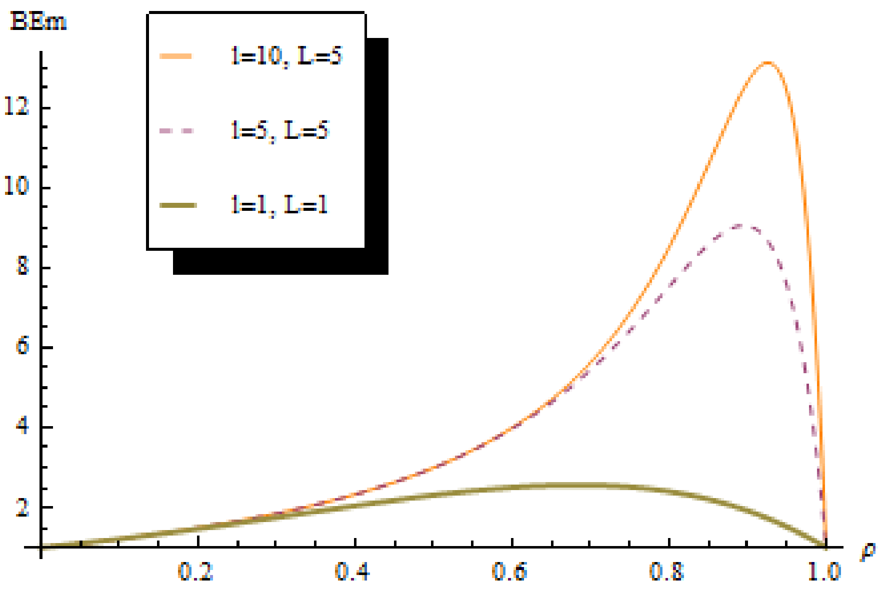

Figure 1: Bullwhip ratio at the manufacturer as a function of ρ

Figure 1 illustrates clearly the non-monotonicity of the bullwhip ratio function in [0,1). It also shows that the bullwhip ratio increases with the lead-time and the maximum value can be reached for values of ρ less than 0.9. Figure 1 illustrates, for example, that when the lead-time is equal to 1, the maximum bullwhip ratio at the manufacturer is reached for $\rho = 0.7$ and then decreases as the ρ value approaches unity. This finding has consequences for practical applications. Lee et al (2000) examined the weekly sales pattern of 165 products in US supermarkets and found the value of ρ ranging from 0.26 to 0.89. Similarly, other studies (Erkip et al, 1990; Lee et al, 1997) found that it is common to have positive auto-correlations and values as high as $\rho = 0.7$ in the high-tech and other consumer product industries. Ali et al (2012) found values of ρ ranging from 0.22 to 0.86 for products from a major European Supermarket located in Germany. Therefore, the fact that the bullwhip ratio declines after $\rho = 0.7$, for lead-times of one period, is not merely of theoretical interest.

We present in Figure 2 the percentage inventory reduction resulting from the information sharing between the retailer and the manufacturer. The results are given for the same values considered by Lee et al (2000). We consider d = 100, p = 50, h = 2, P = 25, H = 1, σ = 50, l = 10, L = 5 and we show ΔI as a function of ρ. Results for l = 5, L = 5 and l = 1, L = 1 are also provided in Figure 2.

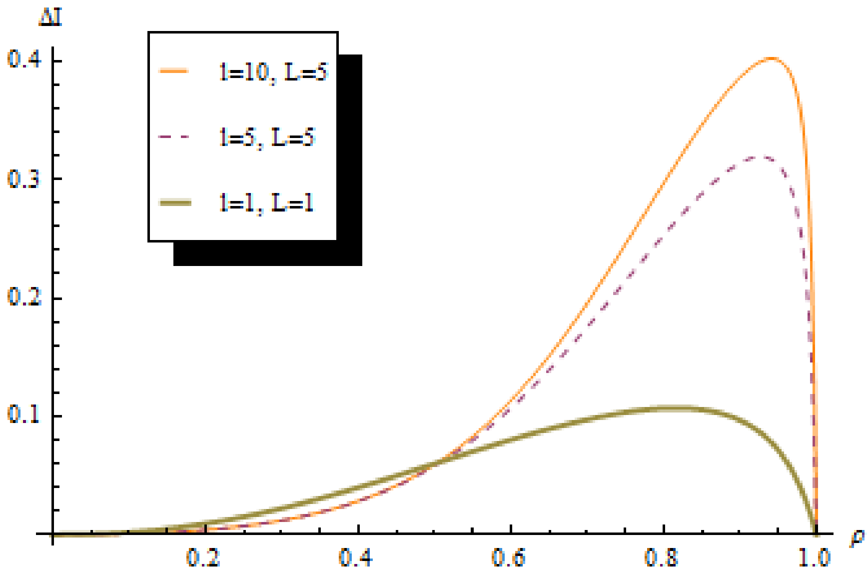

Figure 2: Percentage inventory reduction as a function of ρ

For the values l = 10 and L = 5, Figure 2 shows the same results presented by Lee et al. (2000) for $0 < \rho \leq 0.9$. However, by considering values of $\rho > 0.9$, it is clear that ΔI is not increasing in ρ, which confirms our theoretical findings. Figure 3 also shows that for shorter lead-times, the break point at which the percentage inventory reduction ΔI becomes a decreasing function of ρ can be reached for values of ρ less than 0.9. Figure 2 shows that when l = L = 1, the percentage inventory reduction becomes a decreasing function from ρ =0.82. As noted previously, such parameter values do arise in practice.

5. Conclusion and managerial implications

In this paper, we have analyzed a two stage supply chain where the downstream demand follows an AR(1) process that it is estimated based on a MMSE forecasting method. The inventory at each stage is controlled according to a periodic order-up-to (T,S) policy. Under the realistic assumption that the demand process and parameters are not known to the upstream supply chain stage, inferring demand is not feasible and sharing demand information is valuable. Lee et al (2000) claim that the value of demand information sharing, in terms of inventory reductions, is a monotonic function of the autoregressive parameter. This means that this value may be high, especially when the demand is highly auto-correlated over time.

We have shown analytically and confirmed through numerical experiments that both the bullwhip ratio at the manufacturer and the percentage inventory reduction resulting from information sharing between the retailer and the manufacturer are non-monotonic functions of the autoregressive demand parameter. These findings show that there is little value of demand information sharing for highly positively auto-correlated demand, which contradicts what has been claimed in the academic literature.

This is an important finding from a practitioner perspective since such demand patterns (i.e. patterns associated with high positive auto-correlation) are not atypical in many industrial contexts. Information sharing is generally regarded as a value-adding strategy in terms of inventory performance but the findings of this paper call for a reappraisal of the potential relevant benefits in the context of highly positive auto-correlated demand.

The results in this paper are relevant for any organization wishing to quantify the benefit of information sharing. If the organization has discounted the possibility of sharing demand processes and parameters, but not the demand itself, then quantification is highly relevant to their investment decisions. In conducting an evaluation of benefits, our results show that high benefits of information sharing cannot be assumed for those Stock Keeping Units with highly positively auto-correlated demand. Instead, equally careful analysis is needed for these items as for those Stock Keeping Units with less strongly auto-correlated demand.

References

Agrawal, S., Sengupta, R., Shanker, K. (2009). Impact of information sharing and lead time on bullwhip effect and on-hand inventory. *European Journal of Operational Research,* **192:** 576-593.

Ali, M.M., Boylan, J.E. (2011). Feasibility principles for downstream demand inference in supply chains. *Journal of the Operational Research Society*, **62**: 474-482.

Ali, M.M., Boylan, J.E. (2012). On the effect of non-optimal forecasting methods on supply chain downstream demand. *IMA Journal of Management Mathematics*, **23**: 81-98.

Ali, M.M., Boylan, J.E., Syntetos, A.A. (2012). Forecast errors and inventory performance under forecast information sharing. *International Journal of Forecasting*, **28**: 830-841.

Babai, M.Z., Ali, M.M., Boylan, J.E., Syntetos, A.A., (2013). Forecasting and inventory performance in a two-stage supply chain with ARIMA (0,1,1) demand: Theory and empirical analysis. *International Journal of Production Economics*, **157**:212-219.

Barlas, Y., Gunduz, B. (2011). Demand forecasting and sharing strategies to reduce fluctuations and the bullwhip effect in supply chains. *Journal of the Operational Research Society*, **62**: 458-473.

Box, G.E.P., Jenkins, G.M. (1976). *Time series analysis: forecasting and control*, 2nd ed., Holden Day: San Francisco.

Cachon, G.P. (1999). Managing supply chain demand variability with scheduled ordering policies. *Management Science*, **45**: 843-856.

Cachon, G.P., Fisher, M. (2000). Supply chain inventory management and the value of shared information. *Management Science,* **46:** 1032-1048.

Cannella, S. (2014). Order-up-to policies in information exchange supply chains. *Applied Mathematical Modelling*, **38**: 5553-5561.

Cannella, S., Ciancimino, E. (2010). On the bullwhip avoidance phase: supply chain collaboration and order smoothing. *International Journal of Production Research*, **48**: 6739-6776.

Cannella, S., Ciancimino, E., Framinan, J.M. (2011).Inventory policies and information sharing in multi-echelon supply chains. *Production Planning & Control*, **22**: 649-659.

Cheng, T.C.E., Wu, Y. (2005). The impact of information sharing in a two-level supply chain with multiple retailers. *The Journal of the Operational Research Society*, **56**: 1159-1165.

Ciancimino, E., Cannella, S., Bruccoleri, M., Framinan, J.M. (2012). On the bullwhip avoidance phase: the synchronised supply chain. *European Journal of Operational Research*, **221**: 49-63.

Dejonckheere, J., Disney, S.M., Lambrecht, M. R., Towill, D.R. (2003). Measuring and avoiding the bullwhip effect: a control theoretic approach. *European Journal of Operational Research*, **147:** 567-590.

Dejonckheere, J., Disney, S.M., Lambrecht, M. R., Towill, D.R. (2004). The impact of information enrichment on the bullwhip effect in supply chains: a control engineering perspective. *European Journal of Operational Research*, **153**: 727-750.

Disney, S.M. (2008). Supply chain aperiodicity, bullwhip and stability analysis with Jury's inners. *IMA Journal of Management Mathematics*, **19**: 101-116.

Disney, S.M., Lambrecht, M., Towill, D.R., Van de Velde, W. (2008).The value of coordination in a two-echelon supply chain. *IIE Transaction*, **40**: 341-355.

Disney, S.M., Lambrecht, M. (2008). On replenishment rules, forecasting, and the bullwhip effect in supply chains. Foundations and Trends in Technology, *Information and Operations Management*, **2**: 1-80.

Disney, S.M., Naim, M.M., Potter, A. (2004). Assessing the impact of e-business on supply chain dynamics. *International Journal of Production Economics*, **89:** 109-118.

Domínguez, R., Cannella, S., Framinan, J.M. (2014). On bullwhip-limiting strategies in divergent supply chain networks. *Computers & Industrial Engineering*, **73**: 85-95.

Dong, Y., Dresner, M., Yao, Y. (2014). Beyond information sharing: an empirical analysis of vendor-managed inventory. *Production and Operations Management*, **23**: 817-828.

Gaur, V, Giloni, A., Seshadri, S. (2005). Information sharing in a supply chain under ARMA demand. *Management* Science, **51**: 961-969.

Gilbert, K. (2005). An ARIMA supply chain model, *Management Science*, **51**: 305-310.

Holland, W., Sodhi, M.S. (2004).Quantifying the effect of batch size and order errors on the bullwhip effect using simulation, *International Journal of Logistics Research and Applications*,**7**: 251-261.

Hosoda, T., Disney, S.M. (2006). On variance amplification in a three-echelon supply chain with minimum mean square error forecasting. *OMEGA: International Journal of Management Science*, **34**: 344-358.

Hosoda, T., Naim, M.M., Disney, S.M., Potter, A. (2008). Is there a benefit to sharing market sales information? Linking theory and practice. *Computers and Industrial Engineering*, **54**: 315-326.

Kulp, S.C., Lee, H.L., Ofek, E. (2004). Manufacturer benefits from information integration with retail customers. *Management Science*, **50**: 431-444.

Lee, H.L., Padmanabhan, V., Whang, S. (1997). Information distortion in a supply chain: the bullwhip effect. *Management Science*, **43**: 546-558.

Lee, H.L., Whang, S. (2000). Information sharing in a supply chain. *International Journal of Manufacturing Technology and Management*, **1:** 79-93.

Lee, H.L., So, K.C., Tang, C.S. (2000). The value of information sharing in a two-level supply chain. *Management Science*, **46**: 626-643.

Luong, H.T. (2007). Measure of bullwhip effect in supply chains with autoregressive demand process, *European Journal of Operational Research*, **180**: 1086–1097.

Machuca, J.A.D., Barajas, R.P. (2004). The impact of electronic data interchange on reducing bullwhip effect and supply chain inventory costs. *Transportation Research Part E: Logistics and Transportation Review*, **40**: 209-228.

Najafi, M., Zanjirani Farahani, R. (2014). New forecasting insights on the bullwhip effect in a supply chain. *IMA Journal of Management Mathematics*, **25**: 259-286.

Noblesse, A.M., Boute, R.N., Lambrecht, M.R., van Houdt, B. (2014). Characterizing order processes of continuous review (s; S) and (r; nQ) policies. *European Journal of Operational Research*, **236**: 534-547.

Raghunathan, S. (2001). Information sharing in a supply chain: a note on its value when demand is non-stationary. *Management* Science, **47**: 605-610.

Raghunathan, S. (2003).Impact of demand correlation on the value of and incentives for information sharing in a supply chain. *European Journal of Operational Research*, **146**: 634-649.

Trapero, J.R., Kourentzes, N., Fildes, R. (2012). Impact of information exchange on supplier forecasting performance.*Omega*, **40**: 738-747.

Yao, Y., Dresner, M. (2008), The inventory value of information sharing, continuous replenishment and vendor-managed inventory. *Transportation Research Part E*, **44**: 361-378.

Yu, Z., Yan, H., Cheng, T.C.E. (2001). Benefits of information sharing with supply chain partnerships. *Industrial Management & Data Systems*,**101**: 114-121.

Yu, Z., Yan, H., Cheng, T.C.E. (2002). Modelling the benefits of information sharing-based partnerships in a two-level supply chain. *Journal of the Operational Research* Society, **53**: 436-446.

Zhang, X. (2004). Evolution of ARMA demand in supply chains. *Manufacturing and Service Operations Management*, **6**: 195-198.

Zhang, X. (2004). Impact of forecasting methods on the bullwhip effect. *International Journal of Production Economics*, **88**: 15-27.

Zhang, J., Chen, J. (2013).Coordination of information sharing in a supply chain. *International Journal of Production Economics*, **143**: 178-187.

Zhou, H., Benton Jr, W. (2007). Supply chain practice and information sharing. *Journal of Operations Management*, **25**:1348-1365.

Appendix A. Proof of Proposition 1

The bullwhip effect at the manufacturer is given by:

$$BE_m = \frac{\left(1-\rho^{l+L+1}\right)^2 + \rho^2\left(1-\rho^{l+L}\right)^2 - 2\rho^2\left(1-\rho^{l+L+1}\right)\left(1-\rho^{l+L}\right)}{(1-\rho)^2}$$

Let $BE_m = \frac{F(\rho)}{G(\rho)}$ where

$$F(\rho) = \left(1-\rho^{l+L+1}\right)^2 + \rho^2\left(1-\rho^{l+L}\right)^2 - 2\rho^2\left(1-\rho^{l+L+1}\right)\left(1-\rho^{l+L}\right)$$

and $G(\rho) = (1-\rho)^2$.

As the functions $F(\rho)$ and $G(\rho)$ are differentiable on [0,1] and based on l'Hôpital's Theorem, we have $\lim_{\rho\to 1} BE_m = \lim_{\rho\to 1} \frac{F(\rho)}{G(\rho)} = \lim_{\rho\to 1} \frac{F'(\rho)}{G'(\rho)}$

where $F'(\rho)$ and $G'(\rho)$ are the first derivatives of $F(\rho)$ and $G(\rho)$ with respect to ρ.

As $F'(\rho)$ and $G'(\rho)$ are polynomial functions of ρ so they are also differentiable on [0,1], Consequently, based on l'Hôpital's Theorem, we have:

$$\lim_{\rho\to 1} BE_m = \lim_{\rho\to 1} \frac{F(\rho)}{G(\rho)} = \lim_{\rho\to 1} \frac{F'(\rho)}{G'(\rho)} = \lim_{\rho\to 1} \frac{F''(\rho)}{G''(\rho)}$$

where $F''(\rho)$ and $G''(\rho)$ are the second derivatives of $F(\rho)$ and $G(\rho)$ with respect to ρ.

$$F''(\rho) = -2\begin{bmatrix} \rho + (1+l^2+L+2lL+L^2)\rho^{l+L-1} + 2\left(3+2l^2+5L+2L^2+l(5+4L)\right)\rho^{1+2l+2L} \\ -\left(6+l^2+5L+L^2+l(5+2L)\right)\rho^{1+l+L} - 2\left(1+2l^2+3L+2L^2+l(3+4L)\right)\rho^{2l+2L} \end{bmatrix}$$

$$G''(\rho) = -2$$

From the expressions of $F''(\rho)$ and $G''(\rho)$, $\lim_{\rho\to 1} BE_m = \lim_{\rho\to 1} \frac{F''(\rho)}{G''(\rho)} = 1$

Since $BE_m = 1$ for $\rho = 0$ and $\lim_{\rho\to 1} BE_m = 1$, and knowing that the function BE_m is continuous in [0,1), then, based on Rolle's Theorem, there exists a value ρ_o where $BE_m{}'(\rho_o) = 0$ which means that the function BE_m is non-monotonic in ρ. As we also know that $BE_m \geq 1$ for ρ in [0,1), there is at least one local maximum value of BE_m that is reached for ρ in [0,1).

We now show that there is a unique maximum of the BE_m function in [0,1).

The first derivative of BE_m with respect to ρ, denoted by $BE_m{}'$, is given by:

$$BE_m{}' = -2\left[\frac{-1+(1+l+L)\rho^{l+L}+2\rho^{1+l+L}+(1+2l+2L)\rho^{2+2l+2L}-(1+l+L)\rho^{2+l+L}-2(1+l+L)\rho^{1+2l+2L}}{(1-\rho)^2}\right]$$

Based on Descartes' Rule of Signs, the numerator polynomial function of $BE_m{}'$ has three changes of signs which means that it has at most 3 positive roots. As $\rho = 1$ is a root of this function, it means that the numerator polynomial function of $BE_m{}'$ (and consequently $BE_m{}'$) has at most two roots in [0,1). Moreover, by looking at the second derivative of BE_m with respect to ρ, denoted by $BE_m{}''$, we can easily show that: $\lim_{\rho\to 0} BE_m{}'' = 4$ and $\lim_{\rho\to 1} BE_m{}'' = -2\left(2l^3 + l^2(3+6L) + L(1+3L+2L^2) + l(1+6L+6L^2)\right) < 0$.

Consequently, based on Rolle's Theorem, we deduce that $BE_m{}'$ has at least one root in [0,1) and the number of roots is an odd number (as the gradient in 0 is positive and in 1 is negative). As the number of roots is at most equal to 2 in [0,1), it means that the numerator polynomial function of $BE_m{}'$ (and therefore $BE_m{}'$) has exactly only one root in [0,1). As the function $BE_m \geq 1$ for any ρ in [0,1), this means that the BE_m function has a unique maximum in [0,1). $\square$

Appendix B. Proof of Proposition 2

The percentage inventory reduction resulting from the information sharing between the downstream and upstream stage is given by:

$$\Delta I = \frac{(\sqrt{V} - \sqrt{V'})}{\frac{d}{2K\sigma(1-\rho)} + \sqrt{V}} = \frac{(\sqrt{(1-\rho)^2 V} - \sqrt{(1-\rho)^2 V'})}{\frac{d}{2K\sigma} + \sqrt{(1-\rho)^2 V}}$$

Let $\Delta I = \frac{N(\rho)}{M(\rho)}$

where $$N(\rho) = \sqrt{(1-\rho^{l+2})^2 + \sum_{i=1}^{L}(1-\rho^{L+l+3-i})^2 + \frac{\rho^2(1-\rho^{L+1})^2(1-\rho^{l+1})^2}{(1-\rho)^2}} - \sqrt{(1-\rho^{l+2})^2 + \sum_{i=1}^{L}(1-\rho^{L+l+3-i})^2}$$

and $$M(\rho) = \frac{d}{2K\sigma} + \sqrt{(1-\rho^{l+2})^2 + \sum_{i=1}^{L}(1-\rho^{L+l+3-i})^2 + \frac{\rho^2(1-\rho^{L+1})^2(1-\rho^{l+1})^2}{(1-\rho)^2}}$$

Since $\frac{\rho^2(1-\rho^{L+1})^2(1-\rho^{l+1})^2}{(1-\rho)^2} = \rho^2(1-\rho^{L+1})^2\left(\sum_{j=0}^{l}\rho^j\right)^2$ then

$$M(\rho) = \frac{d}{2K\sigma} + \sqrt{(1-\rho^{l+2})^2 + \sum_{i=1}^{L}(1-\rho^{L+l+3-i})^2 + \rho^2(1-\rho^{L+1})^2\left(\sum_{j=1}^{l}\rho^j\right)^2}$$

As $\frac{N(0)}{M(0)} = \frac{N(1)}{M(1)} = 0$, i.e. $\Delta I = 0$ for $\rho = 0$ and $\rho = 1$ and since ΔI is a continuous function in [0,1), this means that ΔI is a non-monotonic function of ρ in [0,1) which ends the proof of Proposition 2. □