

# Data Based Mechanistic modelling optimal utilisation of rain gauge data for rainfall-riverflow modelling of sparsely gauged tropical basin in Ghana

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## Abstract

Data-Based Mechanistic (DBM) modelling is a Transfer Function (TF) modelling approach, whereby the data defines the model. The DBM approach, unlike physics-based distributed and conceptual models that fit existing laws to data-series, uses the data to identify the model structure in an objective statistical manner. The approach is parsimonious, in that it requires few spatially-distributed data and is, therefore, suitable for data limited regions like West Africa. Multiple Input Single Output (MISO) rainfall to riverflow modelling approach is the utilization of multiple rainfall time-series as separate input in parallel into a model to simulate a single riverflow time-series in a large scale. The approach is capable of simulating the effects of each rain gauge on a lumped riverflow response.

Within this paper we present the application of DBM-MISO modelling approach to 20778 km<sup>2</sup> humid tropical rain forest basin in Ghana. The approach makes use of the Bedford Ouse modelling technique to evaluate the non-linear behaviour of the catchment with the input of the model integrated in different ways including into new single-input time-series for subsequent Single Input Single Output (SISO) modelling. The identified MISO models were able to improve the efficiency and understanding of the rainfall-riverflow behaviour within the study catchment. The paper illustrates the potential benefits of the methodology in modelling large catchments with sparse network of rainfall stations.

**Keywords:** Ghana, DBM model, Rainfall, MISO, Transfer function,

## 1. Introduction

Spatial variation in rainfall distribution and scale has been found to influence riverflow generation characteristics (Klemes, 1983; Sivapalan *et al.*, 1987; Wood *et al.*, 1988; Shah *et al.*, 1996) and usually Thiessen Polygon method (Mutreja, 1984; Linsley *et al.*, 1988; Ward and Robinson, 1990; Shaw, 1994) is used to evaluate catchment rainfall input into hydrological models. However, this method requires an adequate network of rain gauges which is difficult to come by in developing countries. Generally the approach is unsuitable for mountainous catchments because of orographic effect which is not accounted for by the areal coefficients (Mutreja, 1984; Shaw, 1994).

The use of spatial averaged rainfall input in hydrological models result in errors in the model output as pointed out by Shah *et al.* (1996). They recommended that to predict a good riverflow output of a model at least one rain gauge should be located within an area of 10.55 km<sup>2</sup>. Again, it is very difficult to find this density of rainfall network in developing countries.

In modelling of rainfall-riverflow, the areal average rainfall per time step is often used as the sole input into the model (Tabrizi *et al.*, 1998; Chappell *et al.*, 2004a; Vongtanaboon, 2004). In some applications of this so called 'Single Input Single Output' (SISO) approach, rainfall from a single station is used as input into the model (e.g. see: Young *et al.*, 1997; Mwakalila *et al.*, 2001). The SISO approach usually results in models that are parsimonious requiring only a few well defined model parameters (e.g. see: Young *et al.*, 1997; Young and Beven, 1994; Chappell *et al.*, 1999; Chappell *et al.*, 2004a).

In the Multiple Input Single Output (MISO) rainfall to riverflow modelling methodology, rainfall occurring in each sub area of a catchment is used as a separate input in parallel to simulate a riverflow time-series observed at the large scale (Kothyari and Singh, 1999). Consequently, the approach is able to explicitly simulate the effects

of each rain gauge on a lumped riverflow response. In other MISO models the riverflows of sub-catchments are used as the inputs into the model of riverflow for a large river. This, MISO modelling is called 'flow-flow modelling' or 'riverflow routing' (see: Lees, 2000). Other MISO models incorporate both upstream riverflows and rainfall as inputs into the model (Tabrizi *et al.*, 1998; Lekkas and Onof, 2006). Further discussion of the application of the MISO concept in the modelling of hydrological systems have been presented by Liang (1988), Liang and Nash (1988), Cluckie *et al.* (1990), Kachroo and Liang (1992), and Liang *et al.* (1994). The MISO concept has been applied successfully in riverflow forecasting within large catchments (Natale and Todini, 1976; Huthman and Wilke, 1982; Yazigil *et al.*, 1982; Liang and Nash, 1988; Liang *et al.*, 1992; Papamichail and Papazafiriou, 1992; Kothyari and Singh, 1999; Tabrizi *et al.*, 1998; Lees, 2000; Yawson *et al.*, 2005, 2006).

The Data Based Mechanistic (DBM) modelling approach is a Transfer Function (TF) modelling technique which does not make prior assumptions about the complex hydrological processes operating within a catchment (Young and Beven, 1994; Young *et al.*, 1997; Young, 1998, 2001, 2005; Chappell *et al.*, 1999, 2001; Lees, 2000; Mwakalila *et al.*, 2001; Romanowicz *et al.*, 2006; Vigiak *et al.*, 2006). The approach, unlike physics-based (Beven *et al.*, 1987; Calver and Wood, 1995; Refsgaard and Storm, 1995; Refsgaard *et al.*, 1999) and conceptual modelling techniques (Blackie, 1979; Refsgaard *et al.*, 1995; Yawson *et al.*, 2005), which fit data to preconceived hydrological ideas allows the data to speak for itself (i.e. the data defines the model). It identifies the nature and structure of the model directly from the observed (hydrological) data series in an objective manner, using powerful statistical identification and estimation methods. The technique identifies a range of models, often incorporating Transfer Functions (TF), Time-Variable Parameters (TVPs) and non-linear dynamics, which are capable of simulating the hydrologic response of the catchment efficiently and without over-parameterisation. The statistically acceptable model which has the most acceptable physical interpretation is then accepted (Young and Beven, 1994; Young *et al.*, 1997; Chappell *et al.*, 1999).

Generally, MISO approach is a 'low cost' technique for the modelling of rainfall to riverflow in catchments with sparse network of rainfall stations as highlighted by Kothyari and Singh (1999). Thus, it can make use of fewer rainfall stations available within a catchment as input into the model. Within Ghana and Africa as a whole the dearth of meteorological and hydrological data is very common as pointed out by Giles (2005), Weston and Steven (2005) and Yawson *et al.* (2005). This calls for the application of modelling approaches such as the DBM MISO approach (Young *et al.*, 1997; Young, 1998) in rainfall to riverflow modelling of large catchments. The DBM MISO approach is parsimonious, in that it requires little or no internal catchment characterisation. It is, therefore, particularly suited to a data limited region like West Africa where dense and distributed rainfall monitoring is difficult and expensive to maintain. The DBM MISO approach has been applied successfully by Lees (2000) in flood routing along the River Trent (UK) and Lekkas and Onof (2006) to model flows from River Ali Efenti in Central Greece.

The aim of this study is to use Data-Based Mechanistic modelling approach (DBM: Young and Beven, 1994; Young *et al.*, 1997; Young, 2001; Lees, 2000; Ampadu *et al.*, 2013a; 2013b) to evaluate different rainfall time series combinations derived from individual raingauges in simulating the flows of 20,778 km<sup>2</sup> River Pra catchment gauged at Twifo Praso, Ghana in West Africa. The ability of a particular combinations of rainfall time-series to simulate the riverflow is a measure of the representativeness of those rainfall dynamics that are important at the whole basin scale (Eagleson, 1967). The study specifically has the objective of a) the examination of the applicability of the DBM-MISO methodology to a large catchment with tropical rainfall regime b) the identification of a parsimonious mathematical relationship between the catchment riverflow and the multiple rainfall inputs c) the identification of the effects of non-linearity on the riverflow generation process by converting the linear MISO transfer function models (b) into equivalent SISO models, based on the relative steady state gains of the rainfall inputs following Lees (2000) and d) the comparison of the performance of the non-linear DBM-MISO models (c) to various input scenarios of the DBM-SISO models

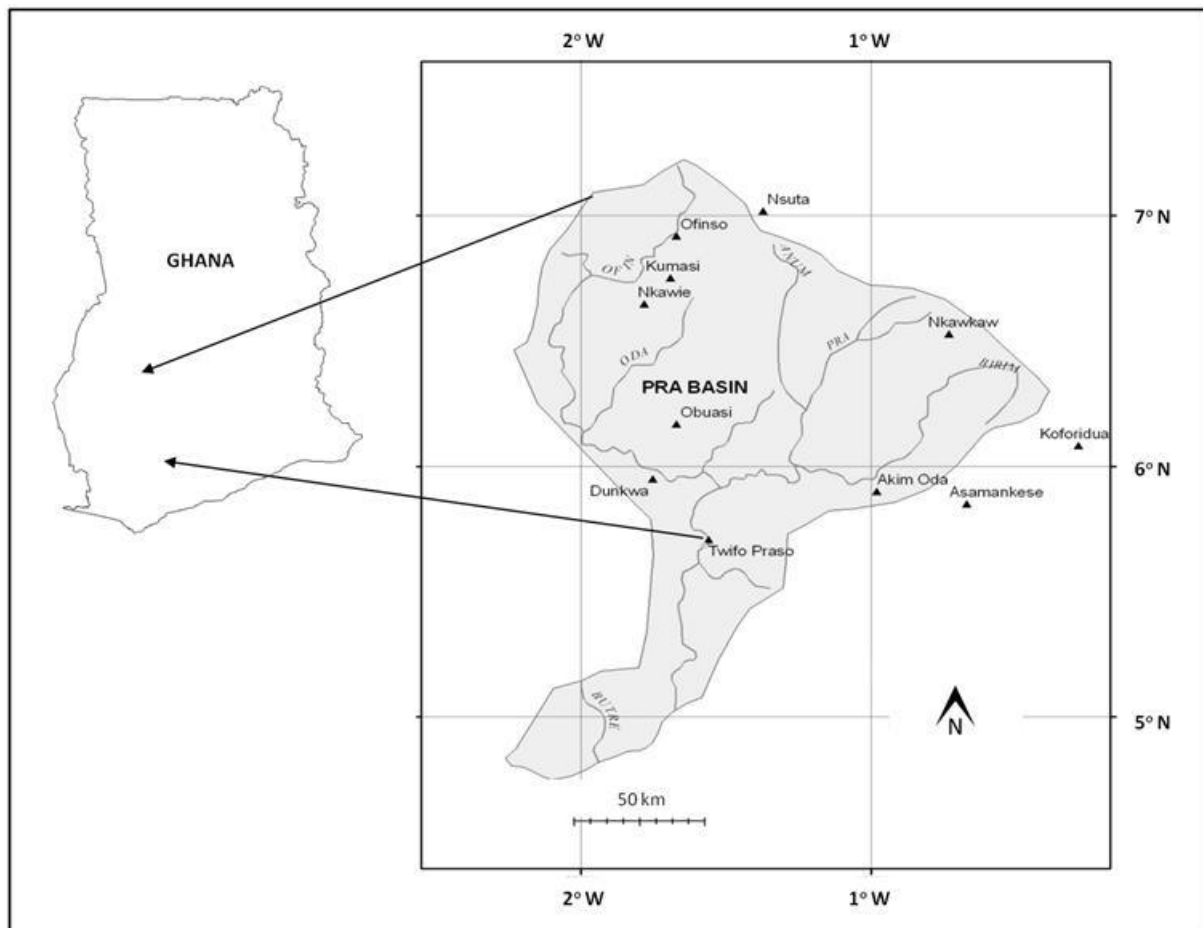
## 2. Materials and Methods

### 2.1 Study basin and time series data available.

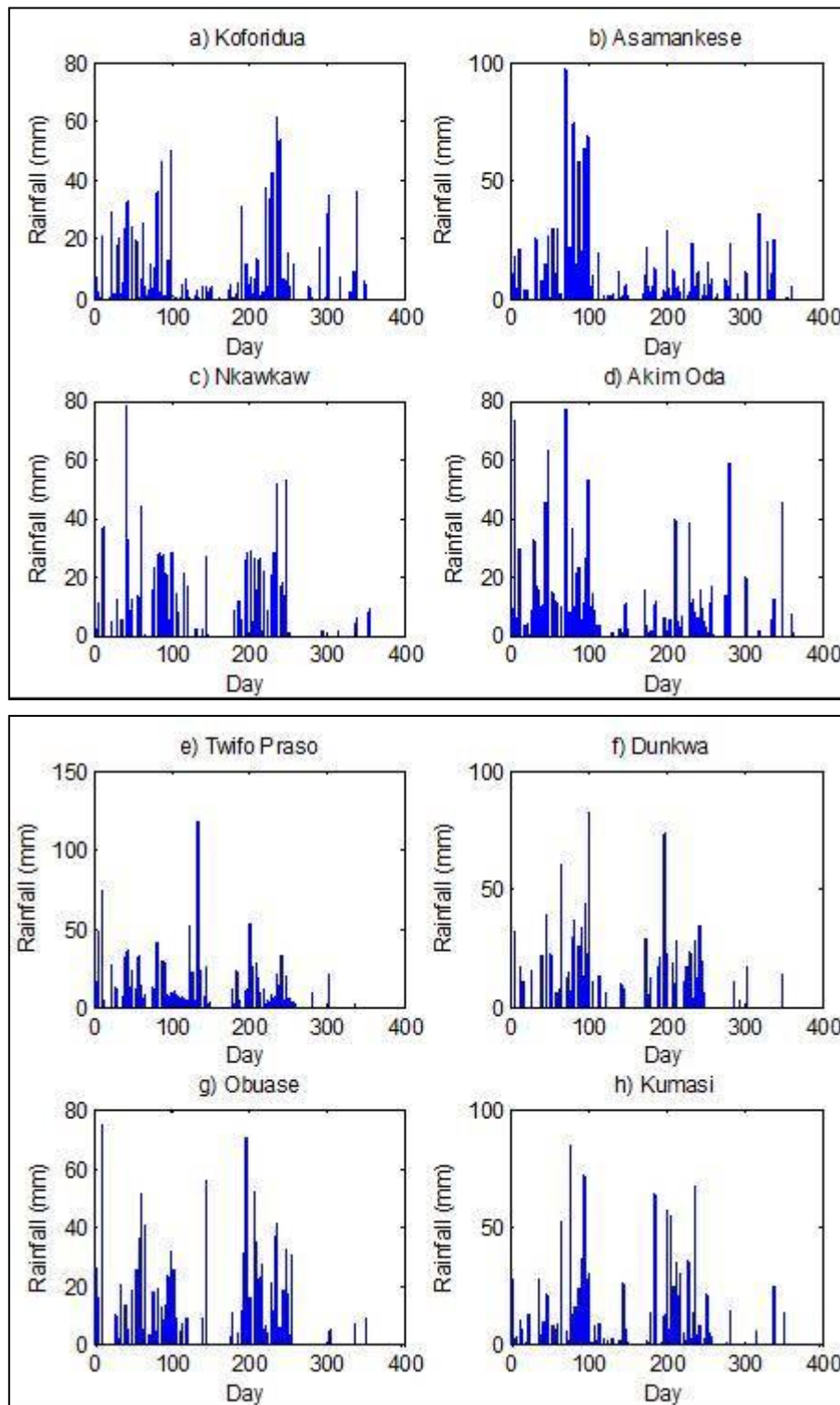
The 20,778 km<sup>2</sup> River Pra catchment gauged at Twifo Praso (Fig. 1) lies within southern Ghana and has a humid tropical climate. A large part is under agricultural activities, especially oil palm plantation (Gyasi, 1996). This region of Ghana experiences two distinct wet seasons (one in March-July and one in September-November), as a result of north-south oscillations of the Inter Tropical Convergence Zone, ITCZ (Ojo, 1977; Acheampong, 1982; Opoku-Ankomah and Cordery, 1994; Nicholson, 2009).

The basin is underlain by Precambrian igneous and metamorphic rocks (i.e., gneiss, phyllite, schist, migmatite, granite-gneiss and quartzite) of the Birimian and Tarkwaian formations (Boakye and Tumbuto, 2006), and contains the uplands of the Kwahu Plateau in the East and the Akan lowlands in the Southwest. For the period 1961-1990 the mean annual rainfall isohyets across the Pra Basin range from 1200 to 1800 mm yr<sup>-1</sup> (Boakye and Tumbuto, 2006). The mean annual rainfall at the Kumasi rain gauge in the north-western quadrant of the basin (Fig. 1) was 1564 mm yr<sup>-1</sup> over the period 1950-70 and 1274 mm yr<sup>-1</sup> over the period 1971-91 (Opoku-Ankomah and Amisigo, 1998).

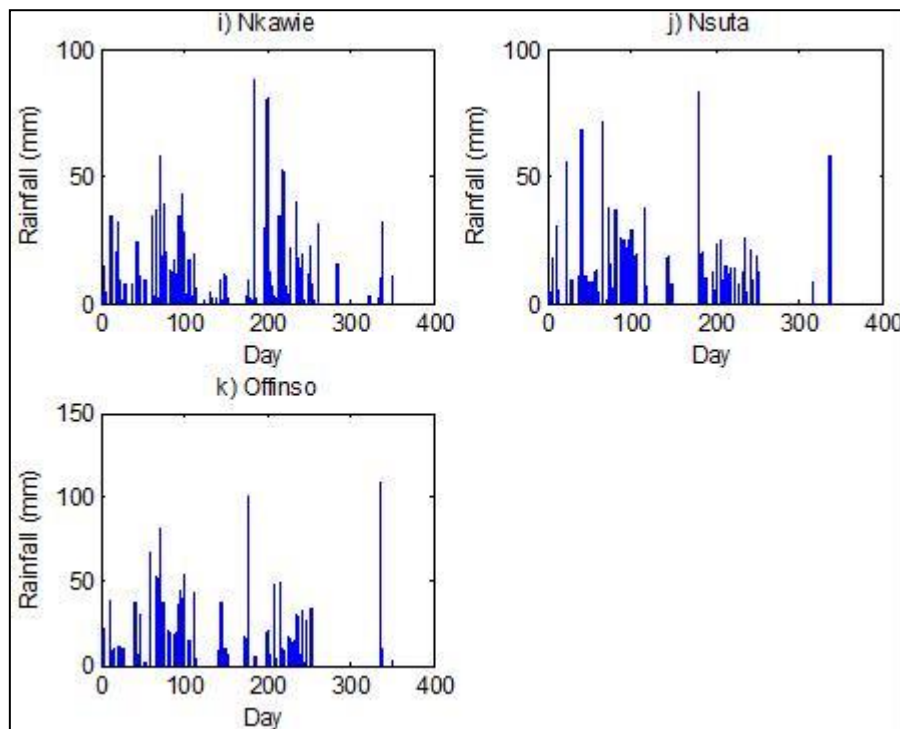
For this study we use daily rainfall in millimetres and Twifo Praso discharge in cumecs which were converted to millimetres per day for the 1978 water year (i.e., 1<sup>st</sup> March, 1978 to 28<sup>th</sup> February, 1979) which were obtained from the Meteorological Services Department (MSD) and Hydrological Services Department (HSD) of Ghana, respectively. Data from 11 rain gauges located either within the Pra basin or within 20 km of its boundary were used (Fig. 1). Figs. 2 and 3 show the distribution of rainfall at the selected stations whilst Fig. 4 shows riverflows at Twifo Praso during the 1978 water year. The Figures show that there is a marked spatial variability in the occurrence of rainfall within the catchment and also the bimodal rainfall regime in the basin is evidenced in the riverflows as depicted by the discharge hydrograph (Fig 4).



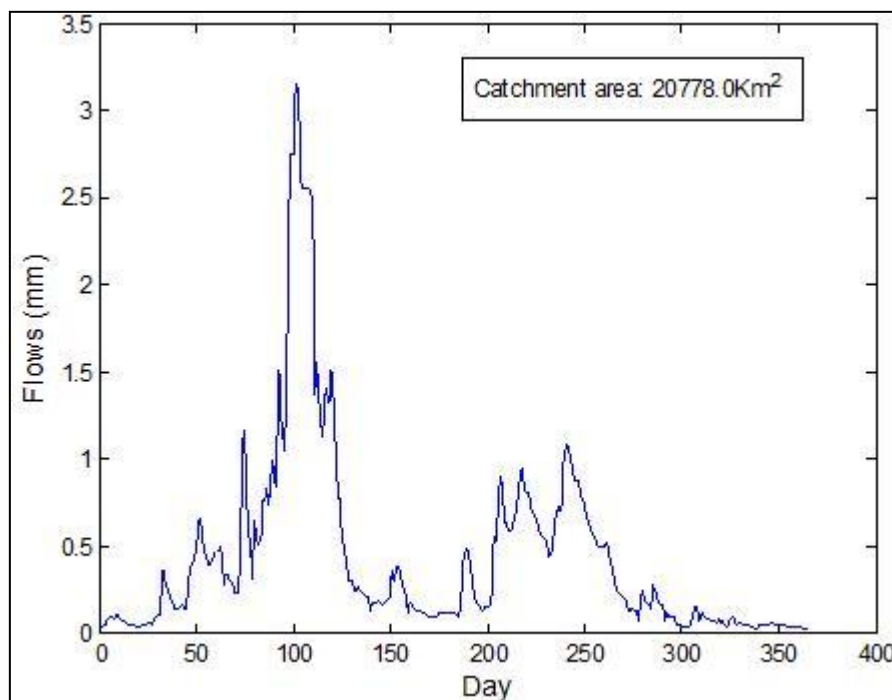
**Figure 1.** The River Pra basin showing the location of the Twifo Praso gauging station and rainfall stations in the Twifo Praso catchment for the DBM transfer function multiple input single output (MISO) rainfall-riverflow modelling.



**Figure 2.** Rainfall distribution in the Twifo Praso catchment during the 1978 water year (i.e. from 1st March, 1978 to 28th February, 1979) showing variation of rainfall with time at Koforidua, Asamankese, Nkawkaw, Akim Oda, Twifo Praso, Dunkwa, Obuase and Kumasi.



**Figure 3** Rainfall distribution in the Twifo Praso catchment during the 1978 water year (i.e. from 1st March, 1978 to 28th February, 1979) showing variation of rainfall with time at Nkawie, Nsuta and Ofinso.



**Figure 4.** Flows of River Pra at Twifo Praso during the 1978 water year (i.e. from 1st March, 1978 to 28th February, 1979) showing the variation of flow with time.

## 2.2 DBM-MISO and SISO approaches

The formulation and the procedures for the application of the DBM-MISO transfer function rainfall to riverflow modelling are similar to the DBM SISO approach (Young *et al.*, 1997; Young and Beven, 1994; Chappell *et al.*, 1999; Mwakalila *et al.*, 2001) but with some additional aspects (Young *et al.*, 1997; Lees, 2000).

### 2.2.1 MISO and SISO linear transfer function models

The general form of linear TF multiple-input-single-output (DBM-MISO) model for hydrological context is given in Young *et al.* (1997), Young (1998) and Lees (2000) as:

$$Q_m(t) = \sum_{i=1}^R \frac{B_i(z^{-1})}{A(z^{-1})} U_i(t - \delta_i) + \varepsilon_m(t) \quad (1a)$$

whilst the single- input-single-output (DBM-SISO) model is given as:

$$Q_s(t) = \frac{B(z^{-1})}{A(z^{-1})} U(t - \delta) + \varepsilon_s(t) \quad (1b)$$

where the transfer function polynomials are defined as:

$$A(z^{-1}) = 1 + a_1 z^{-1} + a_2 z^{-2} + \dots + a_n z^{-N} \quad (2)$$

$$B_i(z^{-1}) = b_{0,i} + b_{1,i} z^{-1} + b_{2,i} z^{-2} + \dots + b_{M,i} z^{-M,i} \quad (3a)$$

$$B(z^{-1}) = b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_M z^{-M} \quad (3b)$$

where  $Q_m$  and  $Q_s$  are the observed riverflows for MISO and SISO models respectively,  $U_i$  is the input rainfall at station  $i$ ,  $R$  is the number of inputs in parallel (different rainfall time series within the catchment),  $U$  is the 'effective rainfall' (for the SISO model),  $z$  is the backward shift operator (i.e.  $z^{-k}u(t) = u(t - k)$ ),  $\delta$  is the pure time delay between rainfall and initial river response. Term  $B_i$  is the system gain (polynomial) or water balance parameter at each input (rainfall station) and for the SISO model,  $B$  is the gain (polynomial) parameter which scales the difference in total volumes of input and output, and  $A$  is the recession (polynomial) parameter related to the residence time of water. Term  $N$  represents the number of denominator polynomial  $A$  parameters (order of  $A(z^{-1})$ ) and  $M_i$  is the number of numerator polynomial  $B_i$  parameters at each input  $i$  and for a SISO model  $M$  is the number of numerator polynomial  $B$  parameters. The residual noise term,  $\varepsilon_m$  and  $\varepsilon_s$  for MISO and SISO models, respectively are defined as:

$$\varepsilon_m(t) = Q_m(t) - X_m(t) \quad (4a)$$

$$\varepsilon_s(t) = Q_s(t) - X_s(t) \quad (4b)$$

where  $X_m$  and  $X_s$  are the model output for the MISO and SISO models, respectively defined as:

$$X_m(t) = \sum_{i=1}^R \frac{\widehat{B}_i(z^{-1})}{\widehat{A}(z^{-1})} U_i(t - \delta_i) \quad (5a)$$

$$X_s(t) = \frac{\widehat{B}(z^{-1})}{\widehat{A}(z^{-1})} U(t - \delta) \quad (5b)$$

where  $\widehat{A}(z^{-1})$ ,  $\widehat{B}_i(z^{-1})$  and  $\widehat{B}(z^{-1})$  are polynomial in  $z^{-1}$  with respective coefficients being the estimates of the parameters in Eqs. (2), (3a) and (3b), respectively. The residual or noise term accounts for all the riverflow not explained by  $X_m$  and  $X_s$  and includes factors such as modelling error, noise in rainfall and riverflow data as a



result of measuring errors e.g. faulty recording instruments, approximations made in the calibration of rating curves and the effects of unobserved inputs (Young, 2003).

### 2.2.2 SISO non-linear transfer function model

The riverflow generation process is inherently non-linear process due to the effects of varying subsurface moisture (FAO, 1981; Young and Beven, 1994; Chappell *et al.*, 1999). Within the SISO DBM methodology the Bedford Ouse Sub-Model (BOSM) (Young, 2001; Chappell *et al.*, 2004b, 2006) was used to model the non-linear component of the rainfall-riverflow generation process. The general form of the model is given in Chappell *et al.* (2004b, 2006) as:

$$U(t) = R(t)\theta_u(t) \quad (6)$$

$$\theta_u(t) = \theta_u(t-1) + \frac{1}{\tau_u} \{R(t) - \theta_u(t-1)\} \quad (7)$$

where  $U(t)$  is the effective rainfall (mm);  $R(t)$  is the average (gross) rainfall (mm);  $\theta_u(t-1)$  is the storage variable (probably shallow unsaturated zone) at the previous time step (mm);  $\tau_u$  is the dimensionless non-linearity term for the whole catchment response. The non-linearity term ( $\tau_u$ ) is obtained by an iterative process applied to the BOSM and the transfer function expressions with the objective function set at a higher coefficient of determination ( $R_t^2$ ) (Young, 2001; Lees, 2000) and a minimum Young Information Criterion (YIC) (Young and Beven, 1994; Lees, 2000; Young, 1998, 2001, 2003) with  $\theta_u$  initially set as zero. The IHACRES model (Jakeman *et al.*, 1990; Jakeman and Hornberger, 1993) has also been used in the modelling of non-linear behaviour in the rainfall-riverflow process (e.g. see: Post and Jakeman, 1996; Sefton and Howarth, 1998; Young, 2001). This model is an extension of the BOSM approach, which includes temperature effects.

### 2.2.3 MISO non-linear transfer function model

The identification of the effects of non-linearity on the riverflow generation within the DBM MISO model was investigated by converting the MISO transfer function model (Eq. (1)) into an equivalent SISO model based on the relative steady-state gains of each of the rainfall inputs. Lees (2000) and Lekkas and Onof (2006) have applied this approach successfully using State Dependent Parameter (SDP) analysis (Young and Beven, 1994; Young, 2001, 2003, 2006) to investigate the non-linear behaviour of the River Trent and River Ali Efenti catchments in UK and Greece, respectively. In this study, the Bedford Ouse Sub-Model (BOSM) (Chappell *et al.*, 2004b, 2006) was used to investigate the effects of non-linearity on the riverflow generation process in the Twifo Praso catchment.

For rainfall inputs ( $U_R$ ) into the model (i.e. Eq. (1)), the resultant linear model relating the rainfall inputs to the riverflow  $Q(t)$  is given as:

$$Q_m(t) = \frac{1}{A(z^{-1})} [B_1(z^{-1})U_1(t - \delta_1) + B_2(z^{-1})U_2(t - \delta_2) + \dots + B_R(z^{-1})U_R(t - \delta_R)] + \varepsilon_t \quad (8)$$

when all  $B_i$  and  $A$  are first order, then  $B_1, B_2, \dots$  and  $B_R$  are the gain parameters due to the rainfall inputs  $U_1, U_2, \dots$  and  $U_R$  respectively,  $A$  is the recession parameters,  $\delta_1, \delta_2, \dots$ , and  $\delta_R$  are the delays on the respective inputs. The steady state gain (SSG) due to each rainfall input is calculated by setting  $z^{-1} = 1$  (Lees, 2000) in the transfer function model (Eq. (8)). The SSG due to each input are defined as:

$$SSG_1 = \frac{\sum B_1}{\sum A} \quad ; \quad SSG_2 = \frac{\sum B_2}{\sum A} ; \dots ; \quad SSG_R = \frac{\sum B_R}{\sum A} \quad (9)$$

The relative steady state gains (RSSGs) are obtained by dividing each of the SSGs by the minimum of the SSGs (*minSSG*) (see: Lees, 2000)

$$RSSG_1 = \frac{SSG_1}{\min SSG}; RSSG_2 = \frac{SSG_2}{\min SSG}; \dots; RSSG_R = \frac{SSG_R}{\min SSG} \quad (10)$$

Therefore, the new equivalent single input rainfall ( $U_{EQR}$ ) based on the  $RSSG$ s from which the effects of the presence of non-linearities could be investigated is given by:

$$U_{EQR}(t) = U_1(t - \delta_1)(RSSG_1) + U_2(t - \delta_2)(RSSG_2) + \dots + U_R(t - \delta_R)(RSSG_R) \quad (11)$$

The model is now converted to an equivalent SISO model incorporating variation in the rainfall inputs into the model through their relative steady state gains ( $RSSG$ s). This technique contrasts with the Thiessen Polygon approach which weights raingauge totals by the surrounding representative areas. As pointed out by Cluckie *et al.* (1990), Kothyari and Singh (1999) and Lees (2000), the characteristics of riverflow response can be influenced by the spatial variation of the rainfall input into the model. The non-linear behaviour in the rainfall and riverflow process was modelled by the application of the Bedford Ouse Sub-Model (BOSM) as in the SISO approach. Recalling Eqs. (6) and (7) the model is given as (Chappell *et al.*, 2004b, 2006):

$$U_{EFF}(t) = U_{EQR}(t)\theta_u(t) \quad (12)$$

where

$$\theta_u(t) = \theta_u(t-1) + \frac{1}{\tau_u} \{U_{EQR}(t) - \theta_u(t-1)\} \quad (13)$$

where  $U_{EFF}(t)$  is the effective rainfall (mm);  $U_{EQR}$  is the equivalent rainfall (mm) from Eq. (13);  $\theta_u(t-1)$  is the unsaturated zone storage variable at the previous time step (mm) with the first value of  $\theta_u$  set as zero;  $\tau_u$  is the dimensionless non-linearity term for the whole catchment response.

#### 2.2.4 Normalisation of ‘effective rainfall’ produced by non-linear sub-models

In order to maintain mass balance following the non-linear transform, the effective rainfall from the BOSM non-linear rainfall filter is normalised by the observed catchment average rainfall  $R$ . For the SISO model the normalised effective rainfall  $Ue$  is given in Chappell *et al.* (1999) as:

$$Ue(t) = U(t) \left( \frac{\sum R(t)}{\sum U(t)} \right) \quad (14)$$

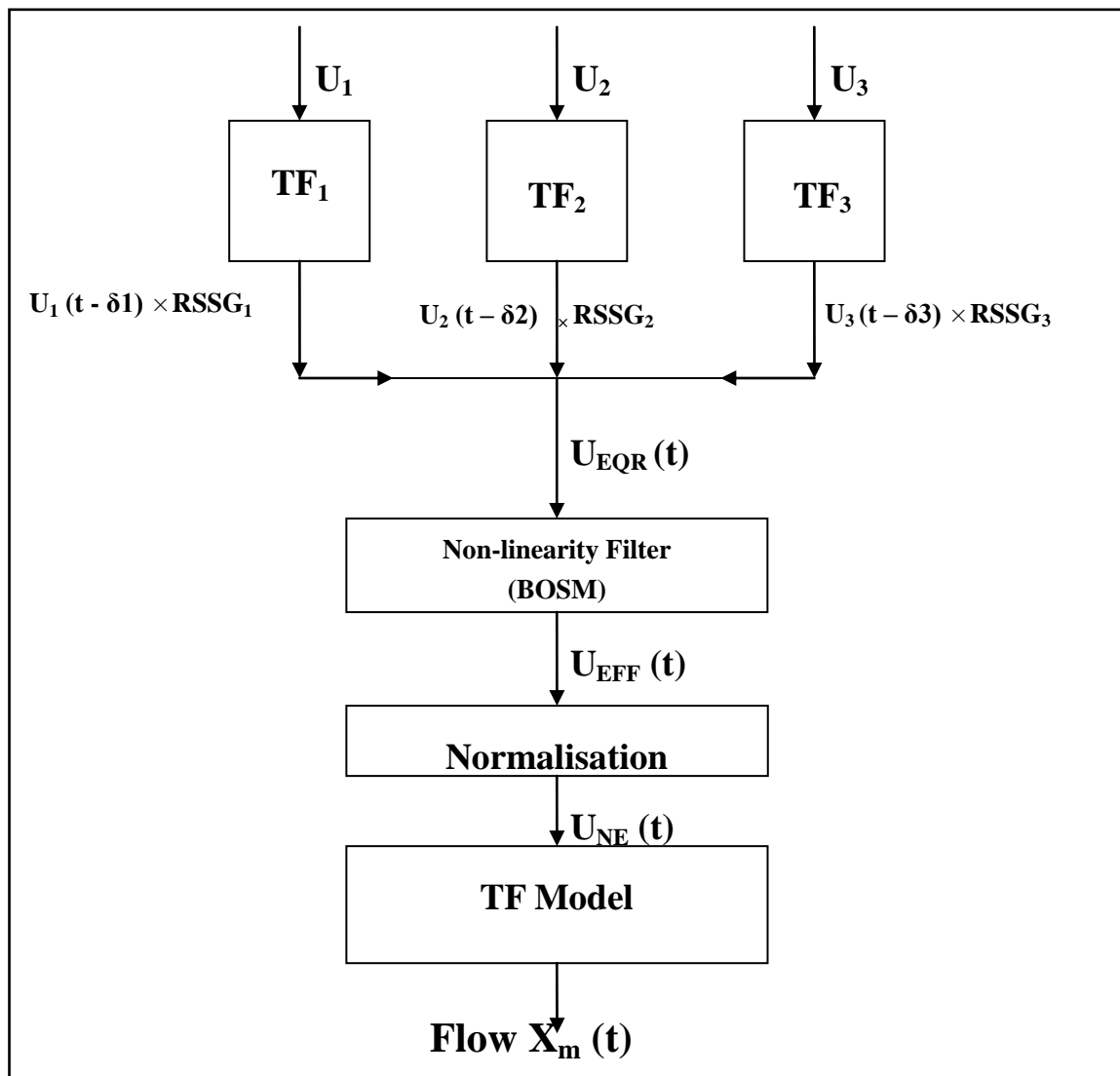
and similarly for a MISO model the normalised effective rainfall  $U_{NE}$  is given as:

$$U_{NE}(t) = U_{EFF}(t) \left( \frac{\sum U_{EQR}(t)}{\sum U_{EFF}(t)} \right) \quad (15)$$

The normalised effective rainfall is then used as input into the DBM MISO and SISO models and utilising the Simplified Recursive Instrumental Variable (SRIV) algorithm, which is so fundamental to the DBM toolbox (Young, 1985, 1991; Taylor *et al.*, 2007), a range of transfer function models were identified. Using the Young Information Criteria (YIC; Young and Beven, 1994; Young *et al.*, 1997; Young, 2001) and coefficient of determination ( $R_t^2$ ), (Lees, 2000; Young, 2001) the DBM model that explained the data well and had an acceptable physical interpretation was selected.

A flow chart of the DBM-MISO approach with three rainfall stations  $U_1$ ,  $U_2$  and  $U_3$  as inputs in parallel as an illustration of the approach is shown in Fig. 5.





**Figure 5.** A flow chart showing the modelling procedure of the DBM-MISO approach using three rainfall stations,  $U_1$ ,  $U_2$  and  $U_3$  as inputs in parallel into the model.  $TF_1$ ,  $TF_2$  and  $TF_3$  are linear TF models of inputs  $U_1$ ,  $U_2$  and  $U_3$  respectively and  $X_m$  is the model output (Flows).

### 2.3 Rainfall integration methods evaluated

The aim of the methodology adopted in this study is to identify models using multiple rainfall inputs (DBM-MISO models) integrated in different ways including into several new single-input time-series for subsequent SISO modelling. This should improve our understanding of the relationship between spatially distributed rainfall and the lumped output, and the best method of identifying the optimal riverflow simulation efficiency with the fewest parameters. Six separate methodologies were used to derive the optimal input time-series, which were all transformed using the BOSM non-linear filters (i.e. Eqs. (6) and (7) for DBM-SISO modelling and Eqs. (12) and (13) for DBM-MISO modelling).

The methods tested were:

1. SISO model using data from all the eleven rain gauges, in the 20,778 km<sup>2</sup> River Pra catchment gauged at Twifo Praso averaged arithmetically (see: Mutreja, 1984; Linsley *et al.*, 1988; Shaw, 1994). The arithmetic averaging is usually good for moderately flat catchments with uniformly distributed network of rain gauges where there is less spatial variability in rainfall depths (Mutreja, 1984; Linsley *et al.*, 1988; Shaw 1994). However, this approach is applied to test its suitability for the catchment. The method is called 1-ARTH.

2. SISO model using data from all eleven rain gauges averaged by the Thiessen polygon method (Mutreja, 1984; Linsley *et al.*, 1988; Shaw, 1994). The Thiessen polygon approach like the 'Arithmetic average method' also normally requires an adequate network of rain gauges and it is generally unsuitable for mountainous catchments because of orographic effects, which is not accounted for by the areal coefficients (Mutreja, 1984; Shaw, 1994). Usually, the technique copes well with uneven distribution of rain gauges (Ward and Robinson, 1990). It is applied here to access its suitability in the efficient modelling of riverflow generation within the catchment. The method is called 2-THIESEN.
3. SISO modelling of each rainfall time series individually to find the rain gauge time-series producing the highest simulation efficiencies. This method is applied to find out the possibility of using a single rainfall station to model the riverflow of a large 20,778 km<sup>2</sup> tropical catchment and also determine which of the rainfall stations best links with the riverflow generation process within the catchment. The method is called 3-SINGLE.
4. DBM-MISO model with all eleven rain gauges to identify a MISO model comprising only rain gauges producing positive SSGs with the riverflow, and to use the positive SSGs to weight the relative contributions of each selected raingauge within a lumped rainfall estimate for use in a SISO model. This methodology is adopted to produce an efficient model for a catchment with sparsely network of rainfall stations by using all rainfall stations as input. The rain gauges producing negative steady state gains are neglected because they are deemed to be unrealistic in hydrological sense since adding them will reduce flow. The method is called 4-SSG POSITIVE.
5. Derive a DBM-MISO model based on those rain gauges producing a SISO linear rainfall-riverflow model with an  $R_t^2 \geq 0.6$ , and to use the SSGs of the higher efficiency models to weight the relative contribution of each selected gauge within a lumped rainfall estimate for a SISO model. This approach is used to select rainfall stations, which are closely linked to the riverflow generation process in the catchment and also limit the number of inputs to the model in order to avoid over- parameterisation of the resulting model. The method is called 5-RT60.
6. Take rain gauge time-series producing positive SSGs in a DBM-MISO model and combine in pairs and threes, to find which combination produces the highest efficiency MISO model, and the highest efficiency SISO model (when weighted by the RSSGs). The method is called 6-SSG POSITIVE AND PAIRS. This approach is to find which of the rainfall station combinations are well connected to the riverflow dynamics.

### 3. Results and Discussion

#### 3.1 Value of DBM-MISO technique rainfall-riverflow modelling

The application of the SISO modelling methods and the MISO approach to the daily rainfall and riverflow data of the 20,778 km<sup>2</sup> Twifo Praso catchment for the 1978 water year within the six separate methodologies resulted in efficient models for the simulating of riverflow from the rainfall data.

##### 3.1.1 DBM-SISO model of arithmetically averaged rainfall

For every daily time-step the rainfall data for the eleven stations in and around the Twifo Praso basin (Fig. 1) were integrated with an arithmetic average (Mutreja, 1984; Linsley *et al.*, 1988; Shaw 1994). The SISO model with the BOSM non-linear filter gave efficiency ( $R_t^2$ ) of 0.8487 and a YIC of -7.676 (Table 1; Fig. 6).

##### 3.1.2 DBM-SISO model of rainfall averaged using Thiessen Polygon method

For every daily time-step, the rainfall data for the eleven rain gauges in and around the Twifo Praso basin (Fig. 1) were integrated into a single value by weighting each gauge according to its representative area of the catchment, following the Thiessen Polygon method (Mutreja, 1984; Linsley *et al.*, 1988; Shaw 1994). The SISO model, again with a BOSM non-linear filter gives efficiency ( $R_t^2$ ) of 0.7794 and a YIC of -6.918 (Table 1; Fig. 6). Normally, the Thiessen Polygon method copes better with an uneven rain gauge distribution to give a more representative catchment-mean rainfall (Mutreja, 1984; Linsley *et al.*, 1988; Ward and Robinson, 1990; Shaw 1994), and expected to give a more efficient model. However, the model efficiency ( $R_t^2$ ) and parameter efficiency (YIC) is worse at 0.7794 and -6.918 respectively, (Table 1; Fig. 6). It may be that in the case of the Twifo Praso catchment the area weighting gives less emphasise to those rain gauge records which are more strongly linked with the Twifo Praso riverflow compared to the simple arithmetic averaging technique.

### 3.1.3 DBM-SISO model of each rain gauge time-series separately

To examine whether some rain gauge time-series are indeed more closely linked with the riverflow dynamics than others, the relationship between rainfall for individual stations to the riverflow generated at the 20,778 km<sup>2</sup> scale was modelled. Of the eleven rain gauge time-series examined, the Dunkwa records from 30 km North west of the Twifo Praso riverflow station gave the highest efficiency of 0.8465, with a YIC of -8.433 (Table 1). This single rainfall time series in this humid tropical catchment, despite the localised convective nature of the rainfall (Acheampong, 1982; van de Geissen *et al.*, 2001), was able to predict the riverflow almost as well as the arithmetic average of the rainfall from 11 rain gauges distributed throughout the catchment.

### 3.1.4 Modelling using rain gauges having DBM-MISO positive SSGs with riverflow

A MISO model using all the eleven rain gauges records as input was used to predict Twifo Praso riverflow. Several of the rainfall records produced negative steady state gains (i.e. adding the record decreased the simulated riverflow), which is considered as less physically/hydrologically realistic. Six of the rain gauges (i.e. those at Asamankese, Oda, Nkawkaw, Dunkwa, Kumasi and Ofinso) produced positive SSGs (Table 2). A further non-linear model with input based on the relative SSGs (i.e. RSSGs; see Eq. (11)) of these six rainfall records was run with a BOSM non-linear filter and produced an efficiency ( $R_t^2$ ) of 0.9006 and a YIC of -0.9047 (Table 1; Fig. 6). This model has a much higher efficiency than the SISO models based on single rain gauge records, arithmetically averaged records and Thiessen Polygon integrated records. The YIC was also better (i.e. more negative) indicating that the model was not over-parameterised, despite the increase in number to 14 parameters (i.e. 6 RSSGs, 3  $\delta$ s: 2 from two of the six inputs and 1 from the final model, 1  $\tau_u$ , 1  $\mathfrak{R}$ , 1 P, 1 TC and 1 SSG: Table 1). In an attempt to reduce the parameter numbers, a SISO model, where the six rainfall records are lumped, was identified. This model gave a slightly worse model ( $R_t^2$  0.8841) with a poorer YIC of -8.533 (Table 1; Fig. 6) despite the smaller number of parameters (i.e. 6 from 1  $\tau_u$ , 1  $\delta$ , 1  $\mathfrak{R}$ , 1 P, 1 TC and 1 SSG: Table 1).

**Table 1. Non-linear DBM model parameters identified for the Twifo Praso catchment (20778.0 km<sup>2</sup>) using BOSM as a non-linear filter with MISO model compared with six rainfall stations lumped (SISO), all the rainfall stations (11) lumped, Thiessen rainfall input (11 rain gauges: SISO) and the rain station with the highest SSG used as the only input (SISO) models. The rainfall stations are Asamankese, Nkawkaw, Oda, Dunkwa, Kumasi and Ofinso which were selected based on the **Positive SSG Criteria**.**

Parameter and Statistics	Type of input into the model				
	Positive SSGs based on six rain gauges (MISO)	Six rain gauges with positive SSGs lumped (SISO)	Eleven rain gauges lumped (SISO)	Thiessen input; 11 Rain gauges (SISO)	Rain gauge with the highest SSG as input i.e. Dunkwa (SISO)
eff. <sub>L</sub>	<b>0.8262</b>	<b>0.7060</b>	<b>0.6322</b>	<b>0.5841</b>	<b>0.7125</b>
eff. <sub>NL</sub>	<b>0.9006</b>	<b>0.8841</b>	<b>0.8487</b>	<b>0.7794</b>	<b>0.8465</b>
Model order	[1 1 2]*	[1 1 2]	[1 1 2]	[1 1 1]	[1 1 0]
YIC	<b>-9.047</b>	<b>-8.533</b>	<b>-7.676</b>	<b>-6.918</b>	<b>-8.433</b>
$\tau_u$	30	30	30	30	30
$\mathfrak{R}$	-0.9066	-0.9045	-0.8901	-0.9051	-0.9297
$\sigma(\mathfrak{R})$	0.0035	0.0042	0.0063	0.0066	0.0029
P	0.00056	0.0118	0.0143	0.0114	0.0101
$\sigma(P)$	0.00002	0.0005	0.0008	0.0008	0.0004
TC (days)	10.197	9.9632	8.5923	10.0316	13.7241
SSG	0.006	0.1240	0.1301	0.1201	0.1434
No. of parameters	14	6	6	6	5

**NB:** eff.<sub>L</sub> and eff.<sub>NL</sub>: Nash and Sutcliffe (1970) efficiency  $R_t^2$  for linear and non-linear model; Model order: [No. of denominators, numerators, pure time delays]; YIC: Young Information Criterion;  $\mathfrak{R}$ : recession parameter;  $\sigma(\mathfrak{R})$ : standard deviation of recession parameter; P: production parameter;  $\sigma(P)$ : standard deviation of production parameter; TC: time constant; SSG: steady state gain of the transfer function;  $\tau_u$ : BOSM non-linearity term. \* Model order for the normalised effective rainfall input from

the equivalent rainfall with additional parameters (see: Eq. (13)).

**Table 2.** Model order and steady state gains of the respective rainfall inputs from the DBM TF MISO linear modelling of flows of River Pra at Twifo Praso

Input	Rainfall Station	Model order	SSG	Remark
u1	Koforidua	[ 1 1 1 ]	-0.0225	Exclude from the model
<b>u2</b>	<b>Asamankese</b>	[ 1 1 0 ]	<b>0.0221</b>	<b>Include in the model</b>
<b>u3</b>	<b>Nkawkaw</b>	[ 1 1 1 ]	<b>0.0313</b>	<b>Include in the model</b>
<b>u4</b>	<b>Oda</b>	[ 1 1 0 ]	<b>0.0196</b>	<b>Include in the model</b>
u5	Twifo Praso	[ 1 1 0 ]	-0.0240	Exclude from the model
<b>u6</b>	<b>Dunkwa</b>	[ 1 1 0 ]	<b>0.1622</b>	<b>Include in the model</b>
u7	Obuasi	[ 1 1 1 ]	-0.0753	Exclude from the model
<b>u8</b>	<b>Kumasi</b>	[ 1 1 0 ]	<b>0.1254</b>	<b>Include in the model</b>
u9	Nkawie	[ 1 1 1 ]	-0.0528	Exclude from the model
u10	Nsuta	[ 1 1 0 ]	-0.0437	Exclude from the model
<b>u11</b>	<b>Ofinso</b>	[ 1 1 1 ]	<b>0.0177</b>	<b>Include in the model</b>

**Table 3.** Results of linear transfer function rainfall and riverflow modelling of flows of River Pra at Twifo Praso in the River Pra basin using individual rainfall stations in the catchment as input into the model (i.e. SISO approach)

Inputs	Rainfall Station	Model	$R_t^2$	YIC
1	Koforidua	[ 1 1 0 ]	0.3696	-4.921
<b>2</b>	<b>Asamankese</b>	<b>[ 3 1 0 ]</b>	<b>0.6969</b>	<b>-6.681</b>
3	Nkawkaw	[ 2 1 1 ]	0.3969	-6.749
4	Oda	[ 1 1 1 ]	0.4163	-5.005
5	Twifo Praso	[ 1 1 2 ]	0.1627	-4.666
<b>6</b>	<b>Dunkwa</b>	<b>[ 1 1 3 ]</b>	<b>0.7086</b>	<b>-7.047</b>
7	Obuase	[ 1 1 3 ]	0.2973	-5.094
<b>8</b>	<b>Kumasi</b>	<b>[ 1 1 3 ]</b>	<b>0.7348</b>	<b>-7.423</b>
9	Nkawie	[ 3 1 0 ]	0.5151	-6.653
10	Nsuta	[ 1 1 2 ]	0.5731	-5.637
11	Ofinso	[ 1 1 3 ]	0.5856	-6.161

**Table 4.** Non-linear DBM model parameters identified for the Twifo Praso catchment (20778.0 km<sup>2</sup>) using BOSM as a non-linear filter with MISO model compared with three selected rainfall stations lumped (SISO), all the rainfall stations (11) lumped (SISO), Thiessen rainfall input (11 rain gauges: SISO) and rain gauge with the highest linear efficiency as input models. The rainfall stations are Asamankese, Kumasi and Dunkwa which were selected based on the Efficiency Criteria.

Parameters and statistics	Type of input into the model				
	Three rain gauges as inputs (MISO)	Three rain gauges Lumped (SISO)	Eleven rain gauges lumped (SISO)	Thiessen input; 11 Rain gauges (SISO)	Rain station with the highest linear efficiency as input (SISO) i.e. Kumasi
eff <sub>L</sub>	<b>0.7985</b>	<b>0.8012</b>	<b>0.6322</b>	<b>0.5841</b>	<b>0.6801</b>
eff <sub>NL</sub>	<b>0.9131</b>	<b>0.9167</b>	<b>0.8487</b>	<b>0.7794</b>	<b>0.8369</b>
Model order	[1 1 1]*	[1 1 2]	[1 1 2]	[1 1 1]	[1 1 2]
YIC	<b>-9.328</b>	<b>-9.529</b>	<b>-7.676</b>	<b>-6.918</b>	<b>-8.292</b>
$\tau_u$	30	30	30	30	30
$\mathcal{R}$	-0.9083	-0.9173	-0.8901	-0.9051	-0.9252
$\sigma(\mathcal{R})$	0.0032	0.0027	0.0063	0.0066	0.0033
P	0.0032	0.0102	0.0143	0.0114	0.0087
$\sigma(P)$	0.0001	0.0003	0.0008	0.0008	0.0003
TC (days)	10.3974	11.5856	8.5923	10.0316	12.8636
SSG	0.0347	0.1231	0.1301	0.1201	0.1166
No. of parameters	11	6	6	6	6

**NB:** eff<sub>L</sub> and eff<sub>NL</sub>: Nash and Sutcliffe (1970) efficiency  $R_t^2$  for linear and non-linear model; Model order: [No. of denominators, numerators, pure time delays]; YIC: Young Information Criterion;  $\mathcal{R}$  : recession parameter;  $\sigma(\mathcal{R})$  ; standard deviation of recession parameter; P: production parameter;  $\sigma(P)$ : standard deviation of production parameter; TC: time constant; SSG: steady state gain of the transfer function;  $\tau_u$ : BOSM non-linearity term. \* Model order for the normalised effective rainfall input from the equivalent rainfall with additional parameters (see: Eq. (13)).

**Table 5.** The best six combinations of two and three rainfall stations from the six selected stations with positive SSGs.

Inputs	Rainfall stations	YIC	$R_t^2$
<b>uu1,uu5</b>	<b>Asamankese, Kumasi</b>	<b>-5.4019</b>	<b>0.7397</b>
uu2,uu4	Nkawkaw, Dunkwa	-4.9023	0.7322
uu2, uu3	Nkawkaw, Akim Oda	-4.8693	0.6841
uu1, uu4	Asamankese, Dunkwa	-4.7305	0.7499
uu1, uu3, uu5	Asamankese, Akim Oda, Kumasi	-4.7121	0.7488
uu1, uu3, uu4	Asamankese, Akim Oda, Dunkwa	-4.6267	0.7611

**Table 6.** First order non-linear DBM model parameters identified for the Twifo Praso catchment (20778.0 km<sup>2</sup>) using BOSM as a non-linear filter with MISO model compared with two selected rainfalls stations lumped (SISO), all the rainfall stations (11) lumped (SISO), Thiessen rainfall input (11 rain gauges: SISO) and rain gauge with the highest linear efficiency as input models. The rainfall stations are Asamankese and Kumasi which were selected based on the **Pairing Criteria**

Parameters and statistics	Type of input into the model				
	Two rain gauges as inputs (MISO)	Two rain gauges lumped (SISO)	Eleven rain gauges lumped (SISO)	Thiessen input; 11 rain gauges (SISO)	Rain station with the highest linear efficiency as input (SISO) i.e. Kumasi
<b>eff<sub>L</sub></b>	<b>0.8106</b>	<b>0.8009</b>	<b>0.6322</b>	<b>0.5841</b>	<b>0.6801</b>
<b>eff<sub>NL</sub></b>	<b>0.9200</b>	<b>0.8990</b>	<b>0.8487</b>	<b>0.7794</b>	<b>0.8369</b>
Model order	[1 1 0]*	[1 1 2]	[1 1 2]	[1 1 1]	[1 1 2]
<b>YIC</b>	<b>-9.680</b>	<b>-9.238</b>	<b>-7.676</b>	<b>-6.918</b>	<b>-8.292</b>
$\tau_u$	30	30	30	30	30
$\mathfrak{R}$	-0.9194	-0.9264	-0.8901	-0.9051	-0.9252
$\sigma(\mathfrak{R})$	0.0025	0.0025	0.0063	0.0066	0.0033
P	0.0039	0.0085	0.0143	0.0114	0.0087
$\sigma(P)$	0.0001	0.0003	0.0008	0.0008	0.0003
TC (days)	11.8973	13.0772	8.5923	10.0316	12.8636
SSG	0.0485	0.1154	0.1301	0.1201	0.1166
No. of parameters	9	6	6	6	6

**NB:** eff<sub>L</sub> and eff<sub>NL</sub>: Nash and Sutcliffe efficiency  $R_t^2$  for linear and non-linear model; Model order: [No. of denominators, numerators, pure time delays]; YIC: Young Information Criterion;  $\mathfrak{R}$ : recession parameter;  $\sigma(\mathfrak{R})$ : standard deviation of recession parameter; P: production parameter;  $\sigma(P)$ : standard deviation of production parameter; TC: time constant; SSG: steady state gain of the transfer function;  $\tau_u$ : BOSM non-linearity term. \* Model order for the normalised effective rainfall input from the equivalent rainfall with additional parameters (see: Eq. (13)).

### 3.1.5 Modelling using rain gauges with a linear DBM-SISO model efficiency ( $R_t^2$ ) of 60 per cent

Purely linear transfer functions between individual rain gauge records and the Twifo Praso riverflow were estimated (Table 3). Those models considered to be ‘behavioural’ by having an  $R_t^2$  of 60 per cent and above (Beven and Freer, 2001) were identified for subsequent incorporation into a MISO model. Only rainfall stations Asamankese ( $R_t^2$  of 0.6969 or 70%), Dunkwa (0.7086) and Kumasi (0.7348) met this criterion. A non-linear DBM-MISO model using these 3 stations as inputs produced an efficiency of 0.9131 and YIC of -9.328 (Table 4; Fig. 7). Thus, this model had a better efficiency ( $R_t^2$ ) and better YIC compared to the six rain gauges selected on the basis of DBM-MISO model positive SSGs (Table 1). By lumping the three rainfall records, a DBM-SISO non-linear model produced an even higher  $R_t^2$  of 0.9167 and even better YIC of -9.529 (Table 4; Fig. 7). Here parsimony helped to improve the model efficiency.

### 3.1.6 Pairs and threes of rain gauges having DBM-MISO positive SSGs with riverflow

One further MISO-based methodology was attempted to help improve interpretation and simulation efficiency. The MISO model with eleven rain gauge produced positive SSGs for only six rain gauges (Table 2). Pairs and threes of rain gauge records sampled from the six records with positive SSGs produced a range of simulation with linear efficiency, the best six being shown in Table 5. Non-linear MISO modelling using the Asamankese and Kumasi rainfall records (i.e. pairs with higher YIC and  $R_t^2$ ) gave an efficiency of 0.9200 and YIC of -9.680 (Table 6; Fig. 8).

This is the highest efficiency achieved by any of the models attempted, and clearly matches the peak flow and recession characteristics of the Twifo Praso riverflow hydrograph (Fig. 8a) better than any of the other models (Figs. 6-8). Interestingly, the highest non-linear model using a single rainfall time-series, used the Dunkwa



records (Table 1:  $R_t^2$  0.8465; YIC -8.433) rather than those of Asamankese or Kumasi. Thus, the two raingauges located in the East and North of the Twifo Praso basin best characterised the riverflow dynamics.

The non-linear MISO model using Asamankese and Kumasi rainfall records is expressed as:

$$X_m(t) = \frac{0.0039(0.0001)}{1 - 0.9194(0.0025)z^{-1}} U_{NE}(t) \quad (16)$$

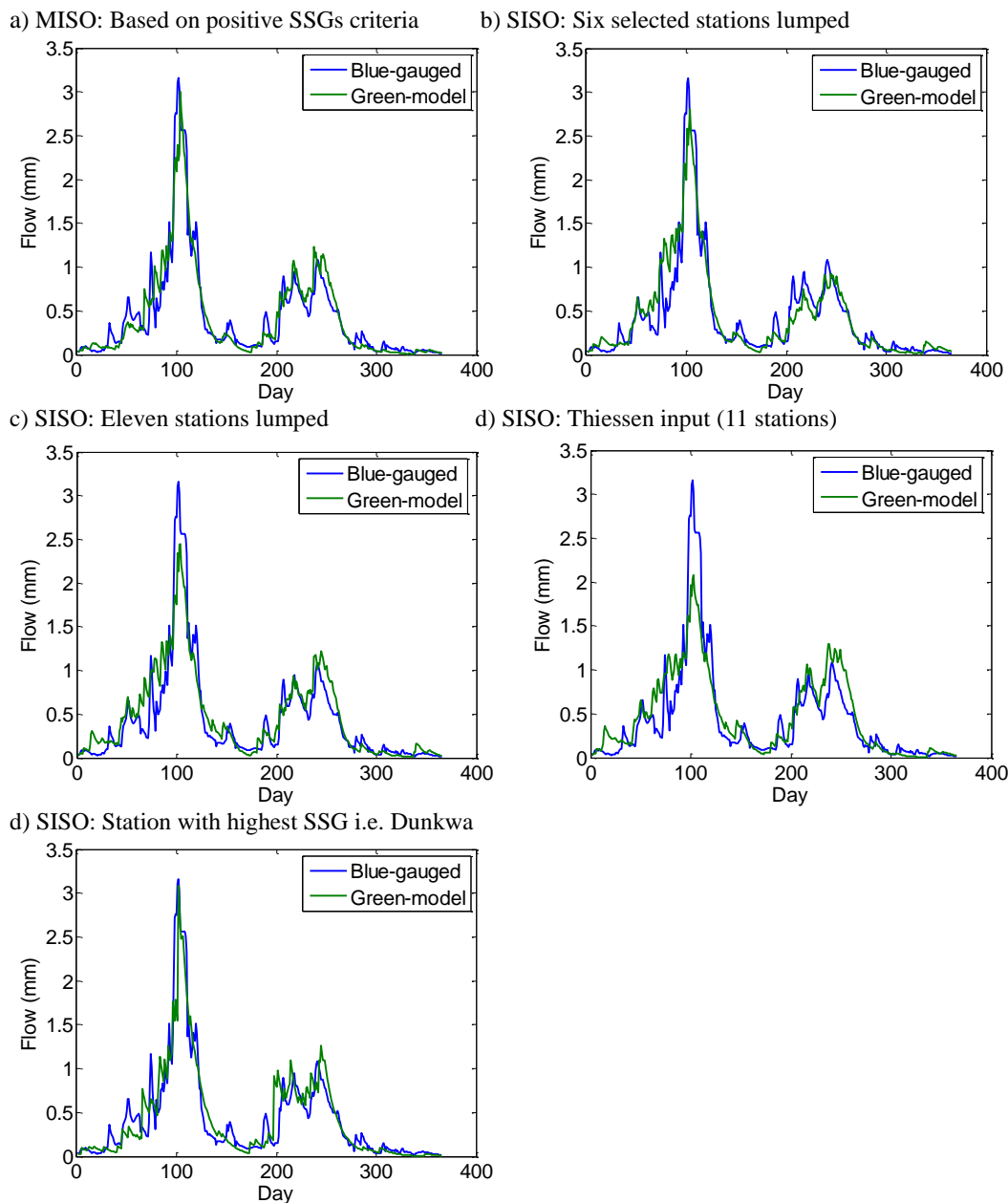
where  $U_{NE}$  is the normalised effective rainfall (from Eq. (15)), with no pure initial time delay and the standard errors on the parameters shown in the parenthesis.

Lumping the two rain gauges records into a SISO non-linear model, did, however reduce the efficiency to 0.8990 and YIC to -9.238 (Table 6), suggesting that the two separate rainfall inputs (Asamankese and Kumasi with 9 model parameters) were the most parsimonious DBM-MISO model for the simulation of the Twifo Praso riverflow. The time constant (TC) and the steady state gain (SSG) of the non-linear DBM-MISO model using the Asamankese and Kumasi records were 12 days and 0.05, respectively (Table 6). The TCs estimated for all the models are similar except the SSGs which that of the MISO model is low (Table 6). This is possible because the modelled amount of the effective rainfall entering the system is very high because of the  $RSSGs$  (see: Eq. (11)) which were used to transform the MISO model to SISO type.

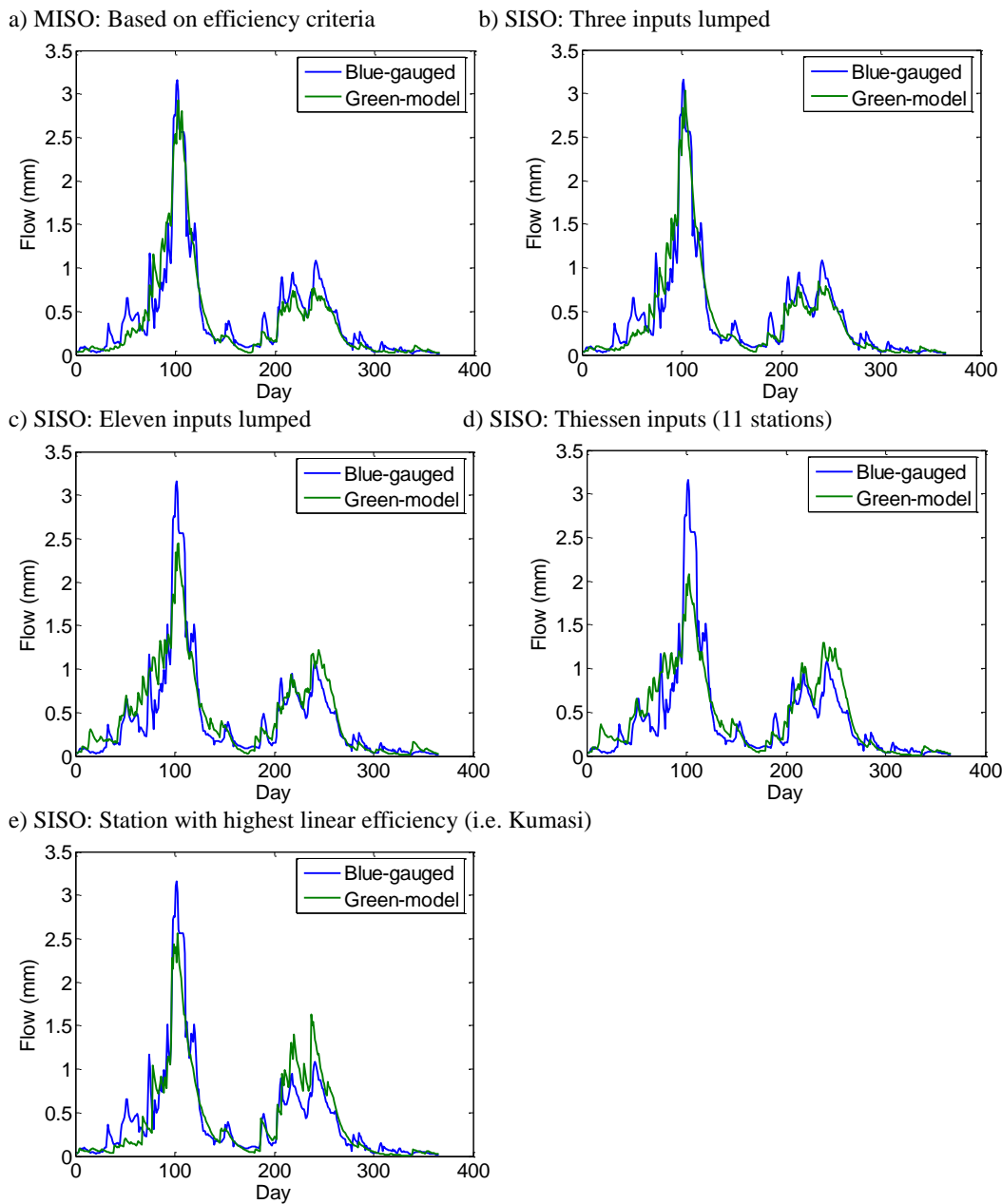
### 3.2 Optimal model performance

The DBM-MISO approach based on the selection of inputs using the ‘Pairs’ and ‘Threes’ criterion performed better than the ‘Efficiency’ criteria and the ‘Positive SSG’ criteria and all the scenario inputs of the SISO approach in terms of both  $R_t^2$  and YIC (i.e. 0.9200 and -9.680; Table 6). The model (i.e. the ‘Pair’ criterion model) predicted the peak and recession flows of the observed riverflows of Twifo Praso better than any of the other models by using only two rain gauges located at Kumasi and Asamankese (Fig. 8a).

The nine parameters estimated for the technique (i.e. ‘Pairs’ criterion) is comparable to the six parameters estimated for the scenario inputs of the SISO approaches. This demonstrates the potential of DBM-MISO technique to model large catchments in the tropics without over-parameterisation (i.e. using few parameters). Kothiyari and Singh (1999) report of the successful modelling of rainfall to riverflows of the 17,157 km<sup>2</sup> Narmada catchment in India by using MISO approach. Yawson *et al.*



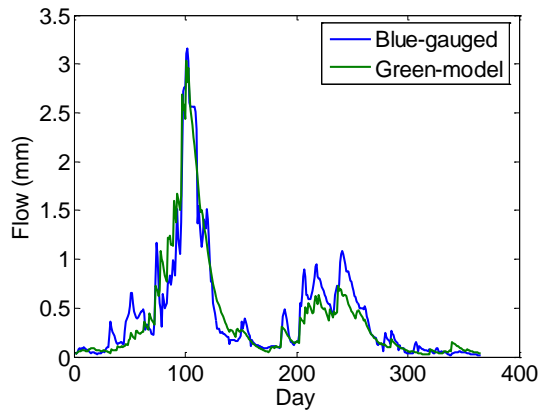
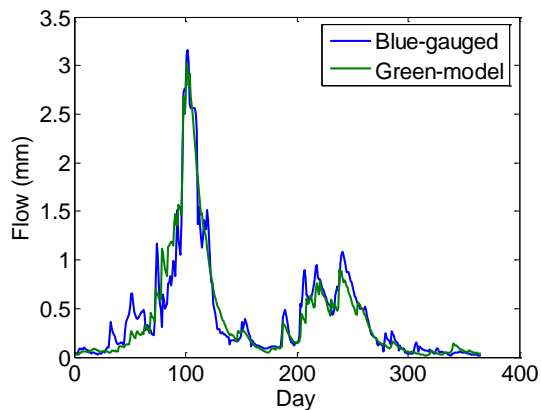
**Figure 6** Flows predicted by non-linear transfer function model (green) against observed flows (blue). a) MISO approach based on positive steady state gains criteria, b) SISO; six inputs lumped (i.e. Asamankese, Nkawkaw, Akim Oda, Dunkwa, Kumasi and Ofinso), c) SISO; all stations lumped, d) SISO; Thiessen average input (11 stations) and e) SISO; station with the highest SSG as input (i.e. Dunkwa) showing the models ability to capture the dynamics of the rainfall riverflow generating mechanism in the Twifo Praso catchment, during the 1978 water year (i.e. 1st March, 1978-28th February, 1978).



**Figure 7.** Flows predicted by non-linear transfer function model (green) against observed flows (blue). a) MISO approach based on efficiency criteria, b) SISO; three inputs lumped (i.e. Kumasi, Asamankese and Dunkwa), c) SISO; all stations lumped, d) SISO; Thiessen average input (11 stations) and e) SISO; station with the highest linear efficiency as input (i.e. Kumasi) showing the models ability to capture the dynamics of the rainfall riverflow generating mechanism in the Twifo Praso catchment, during the 1978 water year (i.e. 1st March, 1978-28th February, 1978).

a) MISO: Based on pairing criteria

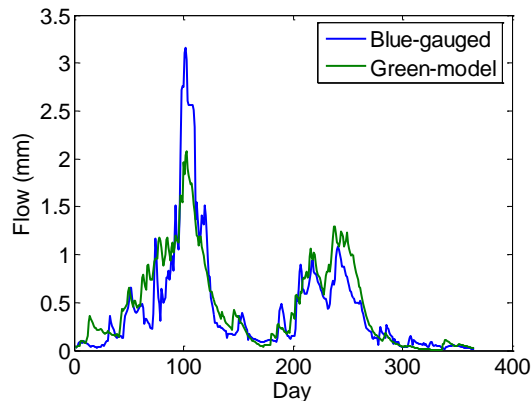
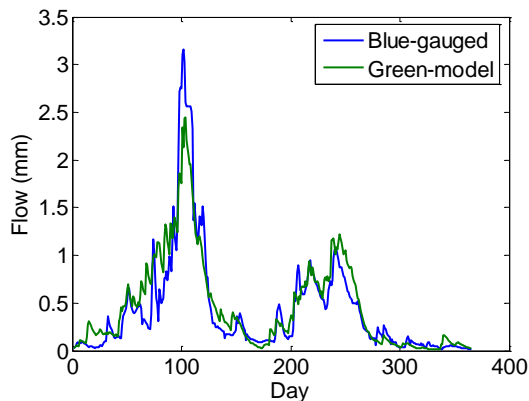
b) SISO: Two stations lumped



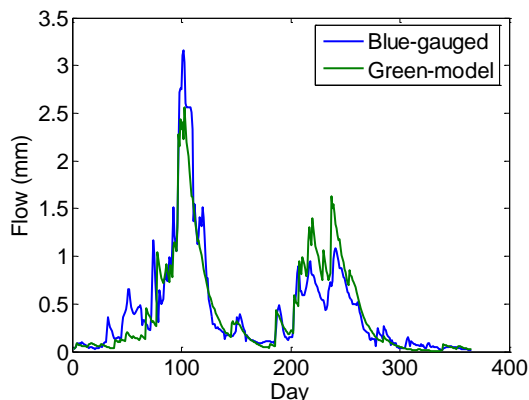
c)

SISO: Eleven inputs lumped

d) SISO: Thiessen inputs (11 stations)



e) SISO: Station with highest linear efficiency (i.e. Kumasi)



**Figure 8.** Flows predicted by non-linear transfer function model (green) against observed flows (blue). a) MISO approach based on pairing criteria, b) SISO; two inputs lumped (i.e. Asamankese and Kumasi), c) SISO; all stations lumped, d) SISO; Thiessen average input (11 stations) and e) SISO; station with the highest efficiency as input (i.e. Kumasi) showing the models ability to capture the dynamics of the rainfall riverflow generating mechanism in the Twifo Praso catchment, during the 1978 water year (i.e. 1st March, 1978 - 28th February, 1978)

(2005) have used MISO approach to model the flows of 33,066 km<sup>2</sup> Kilombero basin in Tanzania. This study and these applications of MISO approach suggest that the technique should be considered for the modelling of rainfall to riverflows in large catchments in the tropics, particularly, in catchments with sparse network of rainfall stations, using the DBM-MISO approach. The DBM-MISO technique presented here provides a parsimonious approach with degree of freedom in the choice of input data in addition to the objective and statistical manner the models are identified. Generally, the MISO approach is a ‘low cost’ technique for the modelling of rainfall to riverflow in catchments with sparse network of rainfall stations as highlighted by Kothiyari and Singh (1999).

However, the excellent performance of individual stations used alone as input into the model need consideration. For instance, the performance of Dunkwa (Table 1) which was also the station with the highest SSG when all the rainfall stations were used as inputs (Table 2) has revealed that, in a large catchment with sparse network of rainfall stations like the Twifo Praso catchment, it is possible one of the rainfall stations may be capable of modelling the flows, despite the convective nature of the rainfall distribution in the area (Acheampong, 1982; van de Geissen *et al.*, 2001). The input scenarios of the SISO approach also indicate that in SISO rainfall-riverflow modelling which involves the lumping of rainfall stations, it may be possible to improve the performance of the model if some of the stations are excluded from the lumping process.

#### 4. Conclusions

The application of the DBM-MISO technique (Young *et al.*, 1997; Young, 1998), including the conversion of the inputs into SISO type using the relative SSGs of the inputs in order to facilitate ease of the investigation of non-linear behaviour (Lees, 2000; Lekkas and Onof, 2006) in Ghana, is probably the first of its kind in the tropics. The approach was able to model the riverflows of the Twifo Praso catchment effectively. The MISO approach uses separate rainfall in parallel as input into the model and integrates spatial variation in the rainfall into the model (Kothiyari and Singh, 1999). The application of the DBM TF MISO approach to the 20778 km<sup>2</sup> Twifo Praso catchment and the analysis of the results yielded the following four conclusions:

1. The DBM-MISO approach based on the pairing criteria using Kumasi and Asamankese raingauge data as inputs required only 9 parameters and captured most of the dynamics of the rainfall-riverflow generating mechanism in the Twifo Praso catchment more efficiently, as compared to the other modelling approaches. This shows the potential of the DBM-MISO technique to model flows of large catchments in the tropics with only a few parameters.
2. The performance of the DBM approach using some of the rainfall stations alone as input (SISO) was excellent. Notably the rainfall station at Dunkwa alone gave an  $R_t^2$  of 0.8465 and YIC of -8.433. This demonstrates that large catchments like the 20,778 km<sup>2</sup> Twifo Praso catchment with a sparse network of rainfall stations could be modelled by using only one of the rainfall stations as input into the model.
3. The scenario modelling based on the SISO approach has revealed that the performance of rainfall-riverflow modelling based on the lumping (i.e. averaging) of rainfall stations could be enhanced by excluding some of the stations from the lumping process. This approach may be suitable for catchments with sparse network of rainfall stations where lumping in a SISO approach may not give a representative value.
4. The DBM-MISO rainfall-river flow modelling technique and the scenario inputs of SISO methodology, which have evolved out of this study, are innovative approaches for the advancement of the DBM technique and hydrological studies in Ghana and the tropics as a whole. The approach is recommended for application in other catchments in the country and the tropics to ascertain its versatility.

#### Acknowledgements

The authors wish to thank the Meteorological Service Agency and the Hydrological Service Department in Accra, Ghana for the rainfall and riverflow data, respectively.

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