

Essays on Monetary Policy

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To my family.

Declaration

I hereby declare that this thesis is my own work and that it has not been submitted for any other degree.

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Abstract

This thesis consists of three essays which aim to evaluate the role played by monetary policy in economic outcomes. The first two essays investigate the properties of the historical conduct of monetary policy in the United Kingdom and the United States, respectively, and justify how these properties are related to economic performance. The third essay analyzes the impact of changes in the volatility of monetary policy shocks on the economy using a Dynamic Stochastic General Equilibrium (DSGE) model with financial frictions.

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Chapter 1

Introduction

Monetary policy has been shown to have short-run non-neutral effects on the real economy (see, e.g, the vast literature on New Keynesian economics, as discussed at length in Woodford, 2003) and, for this reason, studying its practice has been increasingly attracting the interests of both policymakers and academics. This thesis comprises three essays which aim to evaluate the role played by monetary policy in economic outcomes. Specifically, Chapters 2 and 3 investigate the features of the historical conduct of monetary policy in the United Kingdom and the United States, respectively, and discuss how these features are related to economic performance. Chapter 4 develops a DSGE model with financial frictions to analyze the impact of changes in the volatility of monetary policy shocks on the economy.

Following the introduction, Chapter 2 investigates U.K. monetary policy under the inflation targeting regime, which was introduced in October 1992, with the aim of explaining the low and stable rates of inflation observed in this regime. The model specifications are based on a Taylor rule in which the interest rate is assumed to respond symmetrically to the deviation of inflation from its target and the deviation of output from the potential level. A variety of Taylor-rule-type reaction functions are taken into consideration including backward-, contemporaneous-, and forward-looking models in order to seek the one explaining best the movement of interest rate. While most of studies on U.K. monetary policy use ex-post data, this work relies on real-time data for analysis. As argued by Orphanides (2001), using ex-post data might mislead the description of past policy and conceal the behavior proposed

by the information available to central bankers in real time. While estimating backward-looking rules is straightforward, the estimation of contemporaneous- and forward-looking rules are problematic due to lack of current and future data in real time. To deal with this issue, the study employs the two-step strategy introduced by Nikolsko-Rzhevskyy (2011). The first step is to construct the forecasts required. The second step is to use those forecasts for the estimation. Moreover, in order to obtain the estimates which are robust to outliers, the study introduces the impulse-indicator saturation (IIS), which is proposed by Hendry (1999), to the standard Taylor rules. Three main findings are obtained. First, the robust characteristics of monetary policy under inflation targeting are forward-looking and raising the interest rate by more than one-to-one to changes in inflation, thus satisfying the Taylor principle. Second, the granting of operational independence to the Bank of England in 1997 appears to have led to a stronger response to inflation. Third, dealing with outliers is important in the evaluation of monetary policy. Failing to do so can result in an improper interpretation that the post-1992 response to inflation was weak, below unity, perhaps not satisfying the Taylor principle. These results are therefore in line with the view that monetary policy has contributed to stabilize inflation in the U.K.

Chapter 3 turns the focus to U.S. monetary policy. Specifically, it investigates how the conduct of monetary policy has changed since the late 1960s. An important contribution of this chapter is to simultaneously take into account the four issues highlighted as important in modeling monetary policy: the type of time-variation in policy parameters, the treatment of heteroscedasticity, the real-time nature of data, and the role of asymmetric preferences. To the best of our knowledge, this is the first study that allows for all four features simultaneously. The empirical model is built on the derived optimal rule from the formal monetary policy design problem in which central bankers show asymmetric loss function as described in Nobay and Peel (2003). Following Boivin (2006), in this empirical model, parameters are allowed to be time-varying to capture potential changes in the conduct of policy. The issue of heteroscedasticity, which is emphasized by Sims and Zha (2006), is dealt with by letting the standard deviation of policy shocks to follow a stochastic volatility process. The model is then written in a non-linear state-space form which is estimated with real-time data

using particle filtering. Our results suggest that the conduct of U.S. monetary policy experienced considerable changes at the mid and late 1970s, and the early 1990s. The timing of the changes are consistent with the view that monetary variables impact on economic performance.

A key finding of Chapter 3 is that the volatility of monetary policy shocks and, hence, the uncertainty of monetary policy have changed overtime. Such a result is also supported by the vast literature on macroeconomic volatility (for instance, Fernández-Villaverde et al., 2010a; Justiniano and Primiceri, 2008). Nonetheless, only a few studies have analyzed the impact of changes in the volatility of monetary policy on real activity. Shedding light on this issue is the goal of Chapter 4. An important contribution of the chapter is to investigate how financial frictions influence the transmission of monetary volatility shocks. To do so, we develop a standard DSGE model, similar to Smets and Wouters (2007), but incorporate financial frictions à la Bernanke et al. (1999) and introduce stochastic volatility to monetary policy innovations (and to other structural shocks to capture aggregate dynamics). As with other DSGE models, it is required to solve the model before estimation. However, a solution to the first-order approximation is certainty-equivalent, which implies that there is no role for volatility shocks. To this end, the model is solved to a higher-order approximation which results in a non-linear state-space model. The state-space model is, in turn, estimated with U.S. data using maximum likelihood in which the value of likelihood is calculated by a sequential Monte Carlo method. Our results show that, first, the model captures aggregate dynamics fairly well. Second, an increase in monetary volatility shock leads to a fall in economic activity. Finally, financial frictions amplify and propagate the transmission of monetary volatility shocks to the economy via the financial accelerator mechanism.

Chapter 2

U.K. Monetary Policy under Inflation Targeting

2.1 Introduction

Low and stable rates of inflation have been observed in the U.K. since the early 1990s. This experience of price stability has been mainly documented as a result of improvements in the conduct of monetary policy associated with the adoption of inflation targeting in 1992. Notably, Nelson (2000) estimates U.K. monetary policy reaction functions with ex-post data using the split-sample approach and find that the post-1992 inflation could be characterized by a forward-looking rule with inflation coefficient being above unity. Therefore, when inflation increases, monetary policy raises the real interest rate, leading to a reduction in inflationary pressures. This feature is known as the Taylor principle (Woodford, 2001). Meanwhile, the 1972 – 1976 period of extremely high inflation is captured by a near-zero response of nominal interest rate to inflation. Also based on ex-post data, Cukierman and Muscatelli (2008) and Martin and Milas (2004) affirm the anti-inflationary stance of monetary policy under the inflation targeting regime.

However, according to Orphanides (2001), using ex-post data might mislead the description of past policy and conceal the behavior proposed by the information available to central bankers in real time. The author therefore argues that it is essential to take the real-time na-

ture of data into consideration when investigating historical episodes of monetary policy. On the basis of this rationale, we rely on real-time data for our analysis on U.K. monetary policy under inflation targeting. A variety of Taylor rules are considered including backward-, contemporaneous-, and forward-looking models. While estimating the backward-looking model is straightforward, complications arise in the estimation of the other two types of models because of lack of the contemporaneous- and forward-looking data. For the estimation of U.S. monetary policy rules, Orphanides (2002) uses forecasts from the Greenbook which is prepared by Federal Reserve Board staff for the Federal Open Market Committee before every regularly scheduled meeting. For the U.K. economy, we notice that the Bank of England (henceforth BoE) has produced quarterly forecasts of inflation since 1993. However, these forecasts appear to reflect the (expected) effect of changes in policy, instead of the cause of changes. For example, in August 2006, the interest rate was raised to 4.75 percent from 4.5 percent in July and inflation projections in the August 2006 *Inflation Report* were based on the value of 4.75, therefore less likely to justify the increase of the interest rate. In Appendix A.1, we estimate Taylor rules with the BoE's forecasts and find that the responses to inflation are wrongly signed. To deal with this data-related issue, we follow the two-step strategy introduced by Nikolsko-Rzhevskyy (2011). The first step is to construct the forecasts required. The second step is to use the constructed forecasts to estimate the contemporaneous- and forward-looking monetary policy reaction functions. Using the two-step approach is appealing for two reasons. First, it matches with the real-time nature of data. Second, it bypasses the problem of endogeneity, therefore does not require the use of instruments given that finding instruments might be problematic (Nikolsko-Rzhevskyy, 2011).

In a standard Taylor rule, the interest rate is characterized by a linear function of a constant intercept, the deviation of inflation from its target and the deviation of output from the potential level. However, policy makers in fact may desire to deviate from such a rule at some points in time, say, to moderate the economy under unfavorable global conditions or to respond sporadically to some other variables besides inflation and the output gap, such as

exchange rates, credit growth, or asset prices.¹ These deviations can be considered as outliers. To obtain reliable estimates of coefficients of interest, such as the response of monetary policy to inflation, we need to take the issue of outliers into consideration when evaluating monetary policy. One proposal is to add dummy variables over the periods of deviations; however, this approach requires prior knowledge on the timing of deviations which are too diversified in the real world to identify properly. Another proposal is to incorporate other variables in addition to inflation and the output gap into the reaction function. However, unless policy makers take those variables as their policy's objectives, it is unwise to expand the framework to include all of them because the inclusion could mislead the estimates of interest.² In addition, it is hard to address what "other" variables are.

In order to deal with the above issue, we employ the impulse-indicator saturation (henceforth IIS) approach which is introduced by Hendry (1999). Specifically, impulse indicators, one for every observation, are embedded into the standard Taylor-rule type model to create a new model that produces robust estimates to outliers (see, Johansen and Nielsen, 2009; Santos et al., 2008). In this framework, the number of variables (N) equals the number of observations (T) plus four (an intercept and coefficients on inflation, output gap, and the lag of interest rate). Because the proposed model involves more variables than observations, it cannot be estimated by customary econometric methods. We instead use Autometrics (Doornik, 2009) which is a method that handles the $N > T$ problem by implementing a mixture of expanding and contracting searches in order to seek the indicators relevant at a selected significance level α . The method is analogous to using dummies but not requiring advanced knowledge about break points.

Our study therefore has two main contributions. First, it enriches the literature on U.K. monetary policy in terms of type of data (real time data) and methodology (two-step estimation strategy). Second, it considers the issue of outliers carefully, which has been disregarded in previous studies, and conducts policy evaluations based on the estimates which

¹For example, Kharel et al. (2010) investigate the response of monetary policy to exchange rate fluctuations and Chadha et al. (2004) consider monetary policy reactions to asset prices and exchange rates.

²For the 1992Q4-2007Q4 period, our model detects only 6 outliers, suggesting that monetary policy mainly respond to inflation and output gap. Nevertheless, we show that it is important to deal with these outliers.

are robust to that problem. When it comes to the results, we find that forward-looking rules capture the post-1992 interest rate movements better than backward- and contemporaneous-looking rules in both the models with and without IIS. Diagnostic tests, however, reject the validity of the models without IIS; whereas, forward-looking models with IIS pass all misspecification tests. More importantly, failing to do so misleads the features of U.K. monetary policy under inflation targeting. The long-run response of interest rate to inflation is smaller than unity in the models without IIS, suggesting that the post-1992 response to inflation was not satisfying the Taylor principle. In contrast, the response to inflation was larger than unity in forward-looking models with IIS. Such a difference indicates the importance of dealing with outliers in evaluating the conduct of monetary policy. Moreover, we argue that monetary policy appears to have responded stronger to inflation since the granting of operational independence in 1997. Finally, based on these results we provide some possible explanations for the stability of inflation observed under the inflation targeting regime.

The remaining study is structured as follows. The next section provides an overview of Taylor-rule based model specifications and describes data for the estimation. Section 3 is about inflation forecasts. Section 4 presents the results. The last section concludes.

2.2 Taylor Rules and Data

2.2.1 Taylor Rule Specifications

The original Taylor (1993) rule has the following simple formula

$$r_t = c + \phi_\pi \pi_t + \phi_x x_t, \quad (2.1)$$

where r_t is the nominal interest rate, π_t is the inflation rate, and x_t is the output gap. For the U.S. economy in the 1984-1992 period, Taylor (1993) specifies that $c = 1$, $\phi_\pi = 1.5$, and $\phi_x = 0.5$ which implies that the federal funds rate increased by 1.5 percent for 1 percent positive deviation of inflation from the target and 0.5 percent for an increase by 1 percent in the output gap.

Clarida et al. (2000) modify (2.1) to capture the forward-looking behavior and the gradual adjustment of the interest rate. The modified Taylor rule has the following form

$$M_1 : \quad r_t = c + \rho r_{t-1} + \phi_\pi E_t \pi_{t+h} + \phi_x E_t x_{t+q} + \varepsilon_t, \quad (2.2)$$

in which the interest rate responds to the expected changes of inflation and output gap at the $t + p$ and $t + q$ period by ϕ_π and ϕ_x , respectively; ρ is the smoothing parameter; and ε_t is the policy shock which is assumed to have mean zero and variance σ^2 . This type of rule reflects the ‘leaning against the wind’ view in macroeconomic management.

The specification in (2.2) also nests the backward-looking (such as when $h = -1$ and $q = -1$) and contemporaneous-looking rules (by setting $h = 0$ and $q = 0$). Regarding the optimal choice of lead structure in the policy rule, it is suggested to be no further than one year for inflation or beyond the current quarter for output gap (Taylor and Williams, 2011). Therefore, the maximum values of h and q are set to be 4 and 0, respectively. In addition, we assume that the expected output gap at time t is equal to the output gap at time $t - 1$ observed at time t , or $E_t x_t = x_{t-1|t}$. This assumption is backed by the fact that the output gap is significantly inert. Alternatively, the reader can analyze our models as backward-looking on the output gap. This group of models without IIS is termed M_1 . We estimate M_1 for $h = -1, 0, 1, 2, 3, 4$ and $q = -1$.

We generate a group of models with IIS called M_2 by placing impulse indicators into (2.2) as follows

$$M_2 : \quad r_t = c + \rho r_{t-1} + \phi_\pi E_t \pi_{t+h} + \phi_x E_t x_{t+q} + \sum_{i=1}^T \beta_i 1_{i=t} + \varepsilon_t, \quad (2.3)$$

in which $1_{i=t}$ is the impulse indicator which has the value of 1 for every $i = t$ and 0 elsewhere, other notations are remained as in M_1 . As it can be seen, M_2 has T parameters of indicators, one for each observation, and four other parameters, including an intercept and coefficients on inflation, output gap, and the lag of interest rate. M_2 involves more variables than observations ($T + 4$ against T), thus cannot be estimated by customary econometric methods. We instead use Autometrics (Doornik, 2009) which is a method that handles the

$N > T$ problem by using a mixture of expanding and contracting searches in order to seek the indicators relevant.

2.2.2 Data

The estimation is executed by using the following data:

- *Interest rate*: The actual interest rate that is the instrument of the Bank of England has varied three times since 1992, including the *minimum band 1 dealing rate* (August 1981- April 1997), the *repo rate* (May 1997 - July 2006) and the *official bank rate* (since August 2007). Meanwhile, the treasury bill rate has moved very closely with these actual rates over time and is available for the period considered; thus, we follow (Nelson, 2000) to use the treasury bill rate as a proxy of the policy rate. The end-of-quarter series is collected from the International Financial Statistics (IFS) for the 1992Q4-2012Q1 period.
- *Output gap*: The quarterly *output gap* is defined as the deviation of the (log) *real Gross Domestic Product (GDP)* from its Hodrick-Prescott (HP) trend. The quarterly real GDP data is obtained from the real-time GDP database of the Office for National Statistics (ONS) and seasonally adjusted. It covers the vintages from 1992Q4 to 2012Q1. Moreover, we use the real-time quadratic output gap for robustness checks.
- *Inflation rate*: The *inflation rate* is the annual percentage change of the quarterly Retail Price Index (RPI) published by the ONS. We use both quarterly and monthly frequency. The former is used in backward-looking models and covers the period from 1992Q3 to 2011Q4; whereas the latter spans from 1988M1 to 2012M2 and is used to make forecasts of inflation. Besides, eight monthly price index series, which are the components of the RPI, including the price indexes of food, alcohol and tobacco, petrol and oil other goods, rent, utilities, shop services, and non-shop services, are also utilized to make inflation forecasts based on VAR models. The sample for these components is from 1988M1 to 2012M2.

2.3 Inflation Forecasts

This section presents how we construct the expected values of inflation in order to estimate contemporaneous- and forward-looking Taylor rules. The inflation forecasting strategy is similar to Hendry and Hubrich (2011) who execute 1-month, 6-month and 12-month ahead forecasts of U.S. inflation from 1970M1 to 2004M12. Specifically, they consider three types of models to forecast an aggregate like inflation:

1. Models that use only the past information of the aggregate. For example, simple autoregressive (AR) models;
2. Models that aggregate subcomponent forecasts to obtain aggregate forecasts, which is also known as indirect forecasts. For instance, vector autoregression models for all disaggregate components but the aggregate;
3. Models that incorporate disaggregate information directly into the aggregate model. For instance, vector autoregression models combining the aggregate and all disaggregate factors ($VAR^{agg,sub}$) or a set of selected disaggregate ones.

Hendry and Hubrich (2011) find that the univariate model using only the historical information of the aggregate dominates other forecasting models in terms of the Root Mean Square Forecast Error (RMSFE) criterion. In what follows, the study applies the Hendry and Hubrich (2011)'s approaches to the U.K data. However, we only consider the univariate model and the $VAR^{agg,sub}$ model because the indirect forecasting strategy requires the weights of subcomponents in aggregation which are not available. In addition, we incorporate the impulse indicator saturations to avoid systematic forecast failure (Castle et al., 2013, 2009) and use Autometrics to select the forecasting model from a general unrestricted model (Doornik, 2009).

2.3.1 Empirical Forecasting Specifications

It is known that the inflation rate of the reference month is released with one month delay. In other words, at time t (month), only the inflation rate of time $t - 1$ and earlier are observed.

The study aims to forecast the inflation at time t (nowcasting) and the next 12 months ($t + 12$), using the dynamic (ex-ante) forecasting method. As it is aforementioned, the two models considered are: [A] -the univariate model and [B] -the VAR model combining the aggregate and all disaggregate factors. The general unrestricted model specifications at each forecast origin are as follows

A - The univariate model

$$\pi_t = c + \sum_{k=1}^{13} \alpha_k \pi_{t-k} + \sum_{i=1}^T \beta_i 1_{i=t} + \varepsilon_t,$$

in which $t = 1, \dots, T$; $1_{i=t}$ is the impulse indicator saturation which has the value of 1 for every $i = t$ and 0 elsewhere; $\varepsilon_t = IID\{0, \sigma^2\}$.

B - The VAR model

$$p_t = \mathbf{c} + \sum_{k=1}^{13} \mathbf{A}_k p_{t-k} + \sum_{i=1}^T \mathbf{b}_i 1_{i=t} + u_t,$$

in which $p_t = [\pi, \pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7, \pi_8]_t'$ is the 9×1 vector of aggregate inflation and its disaggregate components;³ \mathbf{c} and \mathbf{A}_k are the constant vector and the coefficient matrix, respectively; \mathbf{b}_i is the 9×1 coefficient vector relating to the indicators; $t = 1, \dots, T$; $1_{i=t}$ is the impulse indicator which has the value of 1 for every $i = t$ and 0 elsewhere; and the innovation process u_t is a zero-mean white noise process with a time-invariant positive-definite variance-covariance matrix Σ .

2.3.2 Forecasting Performance Comparison

We compare forecasting performance between the univariate and the VAR models for the 1999Q1-2011Q2 period. The forecasts are assumed to be executed at the last month of every quarter; therefore, there are 49 forecast origins in total. At every forecast origin t , we conduct thirteen year-on-year monthly inflation forecasts from t to $t + 12$. We rely on the RMFSE measure to compare the forecasting performance between the two models.

³ $\pi, \pi_1, \pi_2, \pi_3, \pi_4, \pi_5, \pi_6, \pi_7, \pi_8$ are the annual percentage changes of the monthly RPI (π) and its eight components, including food (π_1), alcohol and tobacco (π_2), petrol and oil (π_3), other goods (π_4), rent (π_5), utilities (π_6), shop services (π_7) and non-shop services (π_8).

Table 2.1 Root Mean Square Forecast Error

Model	1999-2011	1999-2004	2005-2011
AR	1.23	0.73	1.69
$VAR^{agg,sub}$	1.58	1.25	1.88

Table 2.2 Other Measures of Forecast Error

Model	MAE	MAPE
AR	0.96	33.53
$VAR^{agg,sub}$	1.27	55.20

Notes: This table shows two other measures of forecast errors: Mean absolute error (MAE) and mean absolute percentage error (MAPE).

Table 2.1 shows the RMSFE of each model. The results indicate that it is hard for the VAR model to defeat the univariate model in forecasting U.K. inflation. This result remains if we compare forecasting performance in sub-periods: 1999Q1-2004Q4 and 2005Q5-2011Q2 as shown in the same table. Such a result is in line with what Stock and Watson (2007) and Hendry and Hubrich (2011) find for the U.S. economy. Similarly, Castle et al. (2013) documents that inflation forecasts using AR-type models have a lower RMSFE than those with variables, factors, or both. Furthermore, according to Stock and Watson (2010), in terms of forecasting inflation, simple univariate models are competitive with models using explanatory variables.

In addition to the RMSFE, we consider two other measures of forecast error: Mean absolute error (MAE) and mean absolute percentage error (MAPE) and present the results in Table 2.2. They confirm that the univariate model is more successful in forecasting inflation in the U.K. For this reason, we use the inflation forecasts from the univariate model to estimate contemporaneous- and forward-looking reaction functions of monetary policy.

2.3.3 Expected Quarterly Inflation Rates

To illustrate the process of constructing the quarterly series of $E_t \pi_{t+h}$ for $h = 0$ to $h = 4$, we use the forecasts obtained at the 1992M12 forecast origin as an example. It should be noted that the inflation rate of 1992M10 and 1992M11 are observed at 1992M12. At that origin, there are 13 forecasts carried out including the year-on-year inflation of that month, 1992M12, and the next 12 months from 1993M1 to 1993M12. We choose the (nowcasted) forecasted inflation rate of 1992M12 to be the expected contemporaneous inflation rate of 1992Q4, $E_t \pi_t$. Meanwhile, the expected one-quarter-ahead inflation rate, $E_t \pi_{t+1}$, equals the forecasted inflation rate of 1993M3. The expected two-, three-, and four-quarter-ahead inflation rates are computed in a similar way.

2.4 Results

In order to highlight the important characteristics of monetary policy under the inflation targeting regime, we exclude the recent crisis period from the sample in the first step. Nonetheless, given the flexibility of the IIS method, we later extend the analysis to include the post-2007 sample in the estimation. This is executed as a robustness check to the method used and the results obtained. We also investigate if the conduct of monetary policy was different before and after the granting of operational independence to the BoE. Furthermore, based on the empirical results, we provide explanations for the observed stability in the post-1992 period.

2.4.1 Taylor Rules without IIS

Table 2.3 presents the estimates of the models without IIS, known as M_1 , for the 1992Q4-2007Q4 period. The inflation coefficient ϕ_π is found to be positive and statistically significant only in the models whose policy horizons are from $h = 1$ to $h = 4$ or forward-looking models. The output gap coefficient is positive and significant regardless of the type of policy rule. Based on the Schwarz criterion (SC), we find that the worst fitting model is the one

responding to the past inflation rate. On the other hand, forward-looking rules dominate the others in terms of fitness. Especially, the forward-looking interest rate rule with 3-quarter-ahead expected inflation, $E_t \pi_{t+3}$, seems to describe best the evolution of interest rate. The standard error (*SEE*) and the residual sum of squares (*RSS*) also suggest similar results.

Table 2.3 Taylor Rule Estimates without IIS: 1992Q4-2007Q4, HP Output gap

	$h = -1$	$h = 0$	$h = 1$	$h = 2$	$h = 3$	$h = 4$
Constant c	0.92*	0.75*	0.66**	0.62**	0.60**	0.63*
	[0.25]	[0.25]	[0.25]	[0.24]	[0.24]	[0.24]
Smoothing ρ	0.81*	0.80*	0.81*	0.80*	0.80*	0.79*
	[0.05]	[0.05]	[0.04]	[0.04]	[0.04]	[0.04]
Inflation ϕ_π	-0.002	0.09	0.12*	0.14*	0.17*	0.17*
	[0.05]	[0.05]	[0.04]	[0.04]	[0.05]	[0.05]
Output Gap ϕ_x	0.24*	0.21*	0.19*	0.19*	0.19*	0.19*
	[0.05]	[0.05]	[0.05]	[0.05]	[0.05]	[0.05]
LR-Inflation $\gamma_\pi = \frac{\phi_\pi}{1-\rho}$	-0.01	0.44	0.60**	0.71*	0.81*	0.82*
	[0.28]	[0.25]	[0.25]	[0.26]	[0.27]	[0.27]
LR-Output Gap $\gamma_x = \frac{\phi_x}{1-\rho}$	1.30*	1.05*	0.98*	0.96*	0.94*	0.93*
	[0.35]	[0.28]	[0.27]	[0.25]	[0.24]	[0.23]
<i>Adj.R</i> ²	0.90	0.90	0.91	0.91	0.92	0.91
<i>SEE</i>	0.31	0.30	0.29	0.28	0.28	0.28
<i>RSS</i>	5.27	4.97	4.67	4.47	4.29	4.33
<i>LL</i>	-12.17	-10.44	-8.52	-7.24	-5.97	-6.29
<i>SC</i>	0.68	0.62	0.56	0.51	0.47	0.48
<i>AR</i> (F_{ar})	4.67*	4.39*	3.71*	3.42**	3.16**	3.19**
<i>ARCH</i> (F_{arch})	1.06	1.36	1.02	1.00	0.98	0.87
Normality ($\chi^2(2)$)	0.39	1.16	4.54	5.08	5.42	6.06**
Hetero (F_{het})	2.39**	1.68	0.82	0.69	0.75	0.97

Notes: The regression equation is

$$M_1 : r_t = c + \rho r_{t-1} + \phi_\pi E_t \pi_{t+h} + \phi_x E_t x_{t+q} + \varepsilon_t,$$

for $t = 1993Q1, \dots, 2007Q4$, $h = -1, 0, 1, 2, 3, 4$ and $q = -1$. *LR*: long run, *SEE*: standard error of the regression, *RSS*: residual sum of squares, *LL*: log-likelihood, and *SC*: Schwarz criterion. The columns correspond to different values of h . Standard errors are given in [.]. * $p < 0.01$ and ** $p < 0.05$. The autocorrelation test is the *F*-test suggested by Harvey (1990), normality test of Doornik and Hansen (2008), unconditional homoscedasticity test of White (1980) and *ARCH* (Autoregressive conditional heteroscedasticity) test of Engle et al. (1985).

The long-run responses of interest rate to inflation in all models are below unity, there-

Table 2.4 Taylor Rule Estimates with IIS: 1992Q4-2007Q4, HP Output gap

	$h = -1$	$h = 0$	$h = 1$	$h = 2$	$h = 3$	$h = 4$
Constant c	0.97* [0.18]	0.74* [0.19]	0.41** [0.17]	0.37** [0.18]	0.36** [0.17]	0.38** [0.17]
Smoothing ρ	0.75* [0.04]	0.77* [0.03]	0.82* [0.03]	0.82* [0.03]	0.82* [0.03]	0.81* [0.03]
Inflation ϕ_π	0.08** [0.04]	0.13* [0.03]	0.19* [0.03]	0.21* [0.03]	0.23* [0.03]	0.24* [0.04]
Output Gap ϕ_x	0.22* [0.04]	0.19* [0.04]	0.15* [0.03]	0.16* [0.04]	0.16* [0.03]	0.16* [0.03]
LR-Inflation $\gamma_\pi = \frac{\phi_\pi}{1-\rho}$	0.34** [0.14]	0.58* [0.16]	1.03* [0.23]	1.16* [0.26]	1.24* [0.26]	1.27* [0.26]
LR-Output Gap $\gamma_x = \frac{\phi_x}{1-\rho}$	0.88* [0.15]	0.83** [0.16]	0.84* [0.18]	0.86* [0.19]	0.87* [0.18]	0.86* [0.18]
$Adj.R^2$	0.95	0.96	0.96	0.96	0.96	0.96
SEE	0.20	0.20	0.19	0.20	0.19	0.19
RSS	1.91	1.86	1.69	1.96	1.84	1.87
LL	18.35	19.03	21.90	17.46	19.48	18.97
SC	0.34	0.25	0.09	0.10	0.03	0.05
$AR (F_{ar})$	2.53	3.51*	1.37	0.76	0.80	0.55
$ARCH (F_{arch})$	1.67	1.55	0.77	1.08	0.47	0.53
Normality ($\chi^2(2)$)	0.46	0.61	0.70	1.79	0.36	0.06
Hetero (F_{het})	1.69	1.67	0.71	0.24	0.57	0.54
NIS	10	9	8	6	6	6

Notes: The regression equation is

$$M_2: \quad r_t = c + \rho r_{t-1} + \phi_\pi E_t \pi_{t+h} + \phi_x E_t x_{t+q} + \sum_{i=1}^T \beta_i 1_{i=t} + \varepsilon_t,$$

for $t = 1993Q1, \dots, 2007Q4$, $h = -1, 0, 1, 2, 3, 4$ and $q = -1$. *NIS*: numbers of IIS retained. See footnotes in Table 2.3 for more explanations.

fore not satisfying the Taylor principle. Although such a result is quite different from the one documented in Nelson (2000), it is not unprecedented in the context of real-time data. For instance, Mihailov (2006) obtains a corresponding value of 0.81 for the 1992Q4 – 1997Q1 period and 0.5 for the 1997Q2 – 2001Q4 period. However, the mis-specification tests in Table 2.3 reject the validity of the models without IIS. In the following part, we argue that the results obtained from the M_1 -type models are likely misled by outliers and, thus, do not

reflect properly the properties of U.K. monetary policy under inflation targeting.

2.4.2 Taylor Rules with IIS

To obtain the results which are robust to outliers, we estimate the Taylor rules with impulse indicators for the same sample, 1992Q4 – 2007Q4, and present the results in Table 2.4. We again find that forward-looking rules fit the data better than the other rules based on the value of SC . Unlike the previous case without IIS, forward-looking rules with IIS pass all mis-specification tests. More importantly, the responses to inflation in these rules satisfy the Taylor principle. All estimated coefficients are also statistically significant regardless of policy horizons. It appears that the three-period-ahead forward-looking rule is the best explaining model of interest rate, which is similar to the previous case. In this rule, the interest rate increases by 1.24 and 0.87 for an unexpected increase by 1 percent in inflation and the output gap, respectively.

By comparing M_2 with M_1 for every corresponding pair of h , we see that SC favors M_2 to M_1 , therefore, suggesting that M_2 fits the data better. More importantly, the above analysis shows that the interpretation of historical policy decisions can be misleading because of not taking outliers into consideration.

2.4.3 Quadratic Output Gap

As a robustness check, we replace the HP output gap by the quadratic gap and re-estimate the models with and without IIS. Table 2.5 presents the results from the models without IIS (M_1). The long-run responses of interest rate to inflation are again below unity. However, diagnostic tests reject the validity of these models.

We re-estimate the reaction functions but embed them with IIS (M_2). The results in Table 2.6 show that forward-looking rules capture the dynamics of interest rate better than backward- and contemporaneous-looking rules. Moreover, in the best fitting rule, which is the forward-looking rule with four-quarter-ahead expected inflation, the long-run response to inflation is 1.24 which is similar to the estimate with IIS using the HP output gap. Mean-

Table 2.5 Taylor Rule Estimates without IIS: 1992Q4-2007Q4, Quadratic Output gap

	$h = -1$	$h = 0$	$h = 1$	$h = 2$	$h = 3$	$h = 4$
Constant c	1.26*	1.03*	0.88*	0.83**	0.80**	0.80**
	[0.33]	[0.33]	[0.33]	[0.33]	[0.33]	[0.34]
Smoothing ρ	0.75*	0.74*	0.76*	0.77*	0.76*	0.77*
	[0.06]	[0.06]	[0.06]	[0.06]	[0.06]	[0.06]
Inflation ϕ_π	0.02	0.11**	0.13*	0.15*	0.17*	0.17*
	[0.05]	[0.05]	[0.05]	[0.05]	[0.05]	[0.06]
Output Gap ϕ_x	0.09*	0.08*	0.07*	0.07*	0.06*	0.06**
	[0.02]	[0.02]	[0.02]	[0.02]	[0.02]	[0.02]
LR-Inflation $\gamma_\pi = \frac{\phi_\pi}{1-\rho}$	0.07	0.43**	0.56**	0.63**	0.72**	0.72**
	[0.21]	[0.21]	[0.24]	[0.26]	[0.28]	[0.31]
LR-Output Gap $\gamma_x = \frac{\phi_x}{1-\rho}$	0.36*	0.31*	0.29*	0.28*	0.27*	0.26*
	[0.08]	[0.07]	[0.07]	[0.07]	[0.07]	[0.08]
$Adj.R^2$	0.88	0.89	0.90	0.90	0.90	0.90
SEE	0.32	0.31	0.30	0.30	0.29	0.30
RSS	5.86	5.40	5.10	4.98	4.87	5.07
LL	-15.37	-12.88	-11.17	-10.49	-9.82	-11.03
SC	0.79	0.70	0.65	0.62	0.60	0.64
$AR (F_{ar})$	7.47*	6.34*	5.23*	5.16*	5.10*	5.61*
$ARCH (F_{arch})$	1.76	1.21	0.59	0.63	0.66	0.67
Normality ($\chi^2(2)$)	6.83**	4.77	7.15**	6.91**	7.39**	7.91**
Hetero (F_{het})	1.38	1.23	0.77	0.77	0.79	0.88

Notes: The regression equation:

$$M_1 : \quad r_t = c + \rho r_{t-1} + \phi_\pi E_t \pi_{t+h} + \phi_x E_t x_{t+q} + \varepsilon_t$$

for $t = 1993Q1, \dots, 2007Q3$, $h = -1, 0, 1, 2, 3, 4$ and $q = -1$. See footnotes in Table 2.3 for more explanations.

while, the long-run output coefficient is equal to 0.20. The models with IIS are also shown to fit the data better than those without IIS. The results confirm the importance of dealing with outliers when studying historical policy decisions.

In summary, our findings are robust to different measures of real activity. Comparing the two best-fitting rules: one with the HP output gap and the other with the quadratic output gap, it appears that the former captures the movement of interest rate slightly better than the latter in terms of SC .⁴ We therefore use the model with the HP output gap in the following

⁴The model with HP output gap also retains fewer number of indicators, which implies that its explanatory variables can explain more the evolution of interest rate than those in the other model.

Table 2.6 Taylor Rule Estimates with IIS: 1992Q4-2007Q4, Quadratic Output gap

	$h = -1$	$h = 0$	$h = 1$	$h = 2$	$h = 3$	$h = 4$
Constant c	1.17*	0.89*	0.78*	0.49**	0.44	0.42
	[0.27]	[0.28]	[0.27]	[0.23]	[0.22]	[0.22]
Smoothing ρ	0.78*	0.79*	0.79*	0.82*	0.82*	0.82*
	[0.05]	[0.05]	[0.05]	[0.04]	[0.04]	[0.04]
Inflation ϕ_π	-0.003	0.09**	0.13*	0.19*	0.21*	0.22*
	[0.04]	[0.04]	[0.04]	[0.03]	[0.04]	[0.04]
Output Gap ϕ_x	0.09*	0.08*	0.07*	0.05*	0.05*	0.04**
	[0.02]	[0.02]	[0.02]	[0.02]	[0.02]	[0.02]
LR-Inflation $\gamma_\pi = \frac{\phi_\pi}{1-\rho}$	-0.02	0.41	0.63*	1.04*	1.17*	1.24*
	[0.20]	[0.22]	[0.23]	[0.30]	[0.32]	[0.35]
LR-Output Gap $\gamma_x = \frac{\phi_x}{1-\rho}$	0.43*	0.38*	0.32*	0.27*	0.26*	0.20*
	[0.08]	[0.08]	[0.07]	[0.07]	[0.06]	[0.07]
<i>Adj.R</i> ²	0.93	0.92	0.94	0.96	0.96	0.97
<i>SEE</i>	0.26	0.26	0.24	0.19	0.19	0.17
<i>RSS</i>	3.58	3.76	3.11	1.81	1.69	1.40
<i>LL</i>	-0.56	-2.04	3.66	19.89	22.01	27.59
<i>SC</i>	0.50	0.48	0.36	0.16	0.09	0.04
<i>AR (F_{ar})</i>	3.65**	3.70**	3.21**	0.43	0.45	1.26
<i>ARCH (F_{arch})</i>	0.27	1.17	1.69	0.57	0.45	1.07
Normality ($\chi^2(2)$)	1.98	0.51	0.05	1.80	1.46	1.93
Hetero (<i>F_{het}</i>)	0.64	0.86	1.59	1.75	2.13	1.05
NIIS	3	2	3	8	8	10

Notes: The regression equation:

$$M_2: \quad r_t = c + \rho r_{t-1} + \phi_\pi E_t \pi_{t+h} + \phi_x E_t x_{t+q} + \sum_{i=1}^T \beta_i 1_{i=t} + \varepsilon_t$$

for $t = 1993Q1, \dots, 2012Q1$, $h = -1, 0, 1, 2, 3, 4$ and $q = -1$. See footnotes in Table 2.4 for more explanations.

analysis.

2.4.4 Monetary Policy after the Operational Independence in 1997

The operational independence was granted to the Bank of England in May 1997. In this context, it is interesting to investigate if monetary policy was different between the pre- and post-1997 periods. To address this question, we conduct two exercises. First, we divide the sample 1992Q4-2007Q4 into two sub-samples 1992Q4-1997Q2 and 1997Q3-2007Q4,

estimate each sub-sample and then compare their results. Second, we employ the recursive estimation to the 1992Q4-2007Q4 sample.

Table 2.7 Taylor Rule Estimates with IIS for Sub-Samples

	1993Q1 – 1997Q2	1997Q3 – 2007Q4
Constant c	1.13** [0.48]	0.24 [0.31]
Smoothing ρ	0.68* [0.08]	0.85* [0.04]
Inflation ϕ_π	0.23* [0.07]	0.22* [0.06]
Output Gap ϕ_x	0.15** [0.05]	0.14 [0.12]
LR-Inflation $\gamma_\pi = \frac{\phi_\pi}{1-\rho}$	0.70** [0.26]	1.43** [0.62]
LR-Output Gap $\gamma_x = \frac{\phi_x}{1-\rho}$	0.49** [0.22]	0.91 [0.67]
$Adj.R^2$	0.88	0.96
SEE	0.19	0.21
RSS	0.45	1.54
LL	7.75	9.89
SC	-0.06	0.24
$AR (F_{ar})$	0.55	0.46
$ARCH (F_{arch})$	1.14	0.69
Normality ($\chi^2(2)$)	0.47	0.84
Hetero (F_{het})	0.87	1.34
NIIIS	1	4

Notes: The regression equation:

$$M_2 : \quad r_t = c + \rho r_{t-1} + \phi_\pi E_t \pi_{t+h} + \phi_x E_t x_{t+q} + \sum_{i=1}^T \beta_i 1_{i=t} + \varepsilon_t$$

for $h = 3$ and $q = -1$. See footnotes in Table 2.4 for more explanations.

Table 2.7 presents the results of the first exercise. Interestingly, the short-run responses of interest rate to inflation and the output gap are similar between the two sub-samples: 0.22 and 0.15, respectively. However, the smoothing parameter increases to 0.85 after the operational independence from 0.68 in the pre-independence. Consequently, the long-run responses to inflation and the output gap rise in the post-independence. For an increase by

1 percent in inflation, the long-run response of interest rate increased from only 0.7 percent in the pre-1997Q2 period to 1.43 percent after that. The long-run response to the output gap also doubles, but not statistically significant in the post-1997Q2 sub-sample.

Regarding the second exercise, Figure 2.1 plots the recursive estimate of the short-run inflation coefficient with the 1992Q4-1995Q2 sample used for the initial estimation. It appears that the response to inflation increased consistently from 0.13 in 1995Q3 to 0.28 in 1997Q3, then stayed stable around that level for a relatively long period to the mid-2000s before declining slightly. On the contrary, the short-run output gap coefficient reduced from 0.2 in 1995Q3 to 0.13 in 1997Q3 and remained stable at that level (Figure 2.2). Regarding the smoothing coefficient, it fell to the lowest level of 0.68 in 1997Q3 from 0.78 in 1995Q3, but went up gradually to 0.81 in 2007Q4 (Figure 2.3). It is more intuitive to look at the long-run responses to inflation and the output gap which are presented in Figure 2.4. The former has been above unity since 1997Q4, a few months after the operational independence, and stayed around 1.2 until the end of the sample. Meanwhile, the long-run output gap coefficient declined from 1.0 in 1995Q3 to 0.35 in 2000Q1, but went up continually to 0.9 by 2007Q4.

Based on these results, it is fair to say that monetary policy has responded stronger to inflation since the granting of operational independence to the Bank of England. This finding is in line with Adam et al. (2005).

2.4.5 Estimation Including the Recent Crisis Period

In the previous analysis we exclude the recent crisis period, the post-2007 sample, in order to highlight the important characteristics of U.K. monetary policy under inflation targeting. In this section, we examine the effects of taking this period into account. Table 2.8 presents the estimates using the 1992Q4-2012Q1 sample. As it is shown, most of quarters in the post-2007 sample are detected as outliers. Specifically, among 14 indicators retained, there are 11 indicators belong to the 2007Q4-2012Q1 period.

The model performs well and passes all mis-specification tests. All estimates are statistically significant in both the short run and long run. Most importantly, the results affirm that

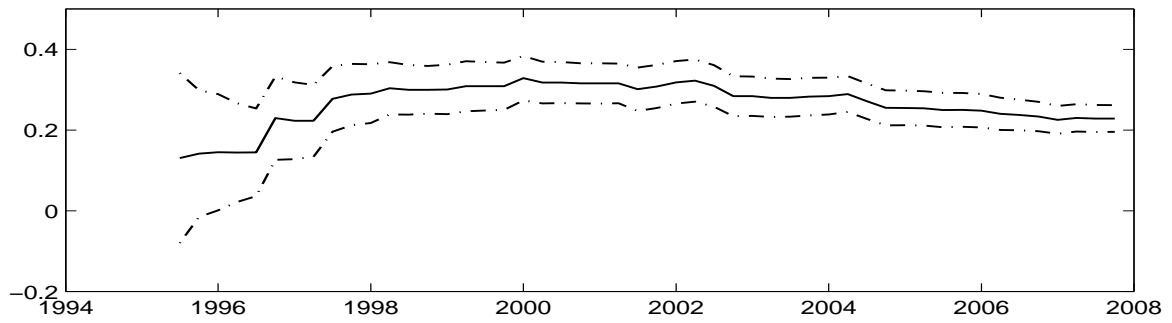


Fig. 2.1 Short-Run Response to Inflation (+/- 1SD)

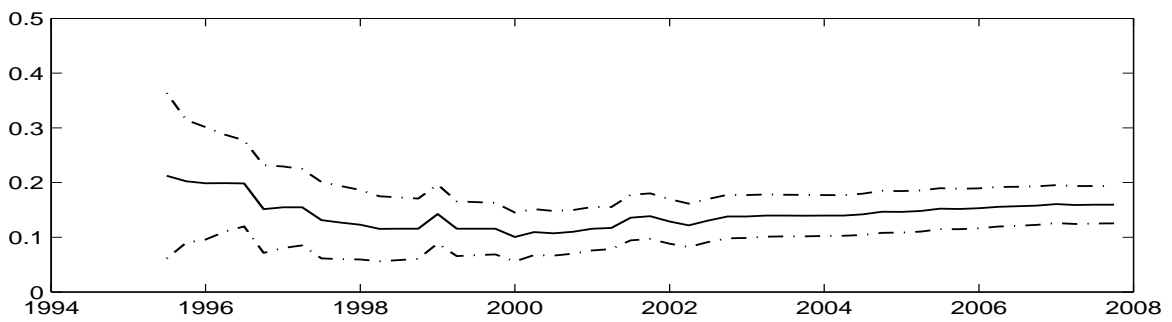


Fig. 2.2 Short-run Response to Output Gap (+/- 1SD)



Fig. 2.3 Smoothing Coefficient (+/- 1SD)

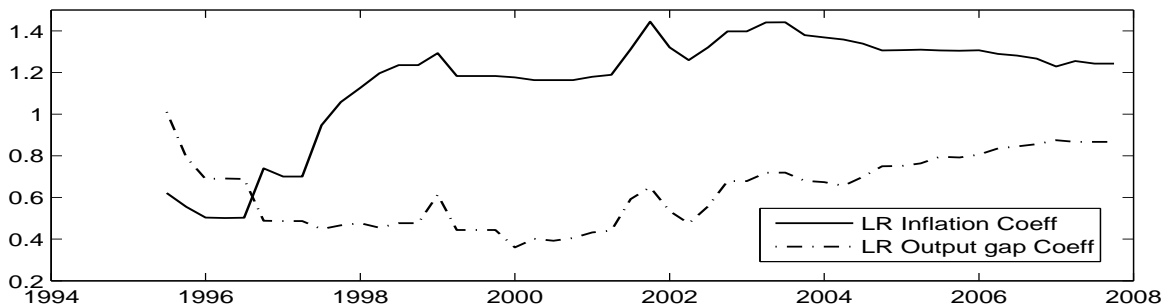


Fig. 2.4 Long-run Responses to Inflation and Output Gap (+/- 1SD)

the long-run response to inflation is above unity, thus satisfying the Taylor principle. This finding therefore corroborates our belief in the method applied and the results obtained.

Table 2.8 Taylor Rule Estimates with IIS for the 1993-2012 Period

Constant c	0.19** [0.14]
Smoothing ρ	0.88* [0.03]
Inflation ϕ_π	0.16* [0.03]
Output Gap ϕ_x	0.17* [0.03]
LR-Inflation $\gamma_\pi = \frac{\phi_\pi}{1-\rho}$	1.33* [0.36]
LR-Output Gap $\gamma_x = \frac{\phi_x}{1-\rho}$	1.39* [0.20]
$Adj.R^2$	0.99
SEE	0.22
RSS	2.84
LL	17.82
SC	0.55
$AR (F_{ar})$	1.33
$ARCH (F_{arch})$	0.47
Normality ($\chi^2(2)$)	0.62
Hetero (F_{het})	0.54
NIIIS	14
	1998Q4, 1999Q3-Q4, 2007Q4 2008Q1, 2008Q3-Q4, 2009Q2 2010Q4, 2011Q1-Q4, 2012Q1

Notes: The regression equation:

$$M_2 : r_t = c + \rho r_{t-1} + \phi_\pi E_t \pi_{t+h} + \phi_x E_t x_{t+q} + \sum_{i=1}^T \beta_i 1_{i=t} + \varepsilon_t$$

for $t = 1993Q1, \dots, 2012Q1$, $h = 3$, and $q = -1$. See footnotes in Table 2.4 for more explanations.

2.4.6 Stability of the Post-1992 Inflation - An Empirical Evaluation

We discuss some possible explanations for the stability of the post-1992 inflation. First, according to Clarida et al. (1998), the magnitude of the long-run inflation parameter γ_π is key to evaluate a central bank's policy rule. If $\gamma_\pi > 1$, when inflation rises, the real interest rate increases, thus slowing the economy and reducing inflationary pressures. On the contrary, with $\gamma_\pi < 1$, though the central bank raises the nominal rate to respond to an unexpected rise in inflation, it is not increased sufficiently to prevent the real rate from declining; thus the self-fulfilling bursts of inflation and output may be possible. Consequently, an effective policy rule should have $\gamma_\pi > 1$. As it is regarded above, such a property is called the Taylor principle. Clearly, the forward-looking rules with IIS satisfy this principle. Second, the forward-looking rules appear to capture the movement of the interest rate better than the others suggesting that the post-1992 monetary policy "leans against the wind". This is another important characteristic of an effective monetary policy as regarded by Taylor and Williams (2011). Third, the smoothing parameter is found to be large and significant which helps monetary authorities avoid the loss of credibility from sudden large policy reversals (Clarida et al., 1998). Last but not least, the granting of instrument independence to the Bank of England led to a stronger response to inflation which likely reflects the determination to anchor inflation expectations, accordingly, contributing to keep inflation stable.

2.5 Conclusion

The study analyzed U.K. monetary policy under inflation targeting based on a real-time Taylor-rule framework. It considered a wide range of rules including backward, contemporaneous, and forward-looking policy functions in order to address what rule describes best the post-1992 conduct of monetary policy. Some important results were established. First, we show the importance of dealing with outliers. Failing to do so misleads the features of U.K. monetary policy under inflation targeting. Second, the robust characteristics of monetary policy during this regime are forward-looking and rising the interest rate more than unity to movements in inflation. Third, monetary policy apparently responds stronger to

inflation after the operational independence. Based on these features of monetary policy, the documented price stability under inflation targeting could be explained. Nonetheless, the current study did not take into account some other issues such as the contemporaneous or forward-looking behavior of interest rate to the output gap or the role of asymmetric preferences in policymaking. We hope to solve these shortcomings in future investigations.

Chapter 3

Modeling Changes in U.S. Monetary Policy

3.1 Introduction

In a seminal paper, Clarida et al. (2000) show that U.S. monetary policy has been subject to important changes in the postwar period. Specifically, the authors find that the pre-Volcker monetary policy was greatly accommodative with the interest rate being raised less than the increase in expected inflation, perhaps not satisfying the Taylor principle. The basic logic behind this principle is that when inflation increases, monetary policy needs to raise the real interest rate in order to slow the economy and reduce inflationary pressures (Taylor and Williams, 2011). On the contrary, during the Volcker-Greenspan era, the anti-inflationary stance of monetary policy was strong with an increase in expected inflation being associated with a larger increase in the nominal interest rate.

The Clarida-Galí-Gertler results have been criticized in four main dimensions. The first dimension is that a single break may not be a proper characterization of the evolution of monetary policy. Boivin (2006) estimates a policy rule with time-varying parameters and finds that the response to inflation was strong until the mid-1970s, became weak in the remainder of the 1970s, and then rebounded to be strong in the early 1980s. Cogley and Sargent (2005, 2001) obtain similar results.

Regarding the second dimension, there have been arguments that it might not be monetary policy but luck that changed between the pre- and post-Volcker periods. Sims and Zha (2006), for instance, examine several multivariate regime-switching models for the U.S. economy and show that the best fit is the one allowing time variation in disturbance variances only. The authors reckon that the observed changes of monetary policy before and after 1979 can be attributed to the presence of heteroscedasticity.

The third dimension is about the type of data used: real-time data, i.e. the data available to policymakers when making decisions, *versus* ex-post data. Orphanides (2001) argues that using the latter misleads the properties of the historical conduct of monetary policy. Orphanides (2002) estimates forward-looking Taylor rules, similar to those considered by Clarida et al. (2000), with real-time data on inflation and unemployment and finds that monetary policy during the 1970s was essentially similar with the period after that.

Finally, a number of studies have argued that monetary policy may respond asymmetrically to changes in the state of the economy due to asymmetric preferences of the central bank with respect to inflation and/or real activity (see, e.g., Cukierman and Gerlach, 2003; Nobay and Peel, 2003; Ruge-Murcia, 2003). Notably, Dolado et al. (2004) show that the conduct of monetary policy in the Volcker-Greenspan regime could be characterized by an asymmetric rule that induced deflationary bias. However, this does not hold for the Burns-Miller regime. Contrary to Clarida et al. (2000), the authors do not find evidence that the response of interest rate to inflation in the post-Volcker period was larger than unity.

The above studies highlight four issues that are apparently crucial in evaluating the evolution of U.S. monetary policy: (i) the type of time-variation in policy parameters, (ii) the treatment of heteroscedasticity, (iii) the real-time nature of data, and (iv) the role of asymmetric preferences. This work, for the first time in the literature, takes all these issues simultaneously into account. To do so, we first derive a model specification as the discretionary outcome of the formal monetary policy design problem in which the central bank displays asymmetric preferences. The coefficients of the derived policy rule are allowed to vary over time, as in Boivin (2006) and Kim and Nelson (2006), in order to capture potential changes in policy parameters. Moreover, we deal with heteroscedasticity by assuming

the standard deviation of monetary policy innovations to follow stochastic volatility processes similar to Stock and Watson (2007) and Justiniano and Primiceri (2008). Finally, following Orphanides (2001), we use real-time Greenbook forecasts for estimation. The resulting econometric model is both time-varying and nonlinear with respect to parameters. Therefore, the popular Kalman filter can not be utilized. To overcome this issue, we apply a novel approach, which has gained popularity in economics and econometrics recently, called particle filtering (see, Creal, 2012; Doucet et al., 2001; Gordon et al., 1993).

Our findings suggest substantial changes in the response to inflation and real activity as well as in the Fed's preferences. Regarding the pre-Volcker period, the response to inflation was not uniformly weak as typically assumed. It violated the Taylor principle only in the second half of the 1970s, but not before that. This result is in line with Cogley and Sargent (2005, 2001) and Boivin (2006). Moreover, we find that the pre-Volcker monetary policy behaved as if possessing asymmetric preferences that could have caused inflationary bias in the conduct of monetary policy. This evidence could help explain the great inflation during this decade.

On the other hand, monetary policy in the post-Volcker era appears to have responded symmetrically to inflation and real activity, thus less likely creating the type of inflationary bias as before. Surico (2007) investigates changes in the Fed's preferences before and after 1979 and obtains similar results. Nonetheless, a single-regime symmetric policy rule is not a proper characterization for the post-Volcker monetary policy because there are considerable differences in the response of interest rate to inflation and real activity between the 1980s and the 1990s and thereafter period. In the former, monetary policy responded strongly to inflation but weakly to real activity, as in line with Kim and Nelson (2006) and Clarida et al. (2000), which implies a concentration of the Fed on stabilizing inflation. In contrast, once inflation has been stabilized, the Fed has paid more attention to stabilizing real activity, while hardly responding to inflation, since the early 1990s, which is broadly similar to Martin and Milas (2010) and Fernández-Villaverde et al. (2010*b*, 2015) in terms of the response to inflation and Kim and Nelson (2006) and Blinder and Reis (2005) in terms of the response to real activity. Overall, our findings suggest that the conduct of U.S. monetary

policy could have experienced important changes at the mid and late 1970s, and the early 1990s.

The study proceeds as follows. Section 3.2 discusses the formal monetary policy design problem in which the central bank's loss function is asymmetric with respect to inflation. Section 3.3 describes the empirical model with time-varying parameters and stochastic volatility. The same section deals with how to apply particle filtering to estimate the model. Section 3.4 presents the data, the empirical results, a number of robustness checks, and a discussion on the evolution of monetary policy through the lens of our model. Section 3.5 concludes.

3.2 The Theoretical Model

The central bank chooses the interest rate to minimize the present discounted value of its asymmetric loss function. This policy action is subject to the constraints imposed by the structure of the economy, which includes two components: the forward-looking Phillips curve and the IS curve.

3.2.1 The Loss Function

The loss function $L(\cdot)$ of the central bank is assumed to depend on the inflation gap, which is the difference between inflation and its target $\pi_t - \pi^*$, the output gap y_t , which is the gap between the actual output and the potential one, and the interest rate gap, which is the distance between the interest rate level and the target $i_t - i^*$ (Surico, 2007; Tillmann, 2011). Formally, the loss function takes the following form

$$L_t = \frac{e^{\alpha(\pi_t - \pi^*)} - \alpha(\pi_t - \pi^*) - 1}{\alpha^2} + \frac{\mu}{2}(y_t)^2 + \frac{\gamma}{2}(i_t - i^*)^2, \quad (3.1)$$

where α captures the asymmetry in the loss function with respect to inflation, μ and γ are parameters representing the central bank's preferences towards the output gap and the deviation of the interest rate from its target. The preference to inflation is normalized to

one. This specification of the loss function is called the linex function proposed by Varian (1975) and introduced to the optimal monetary policy literature by Nobay and Peel (2003). The loss function (3.1) differs from the conventional quadratic set-up in the way it deals

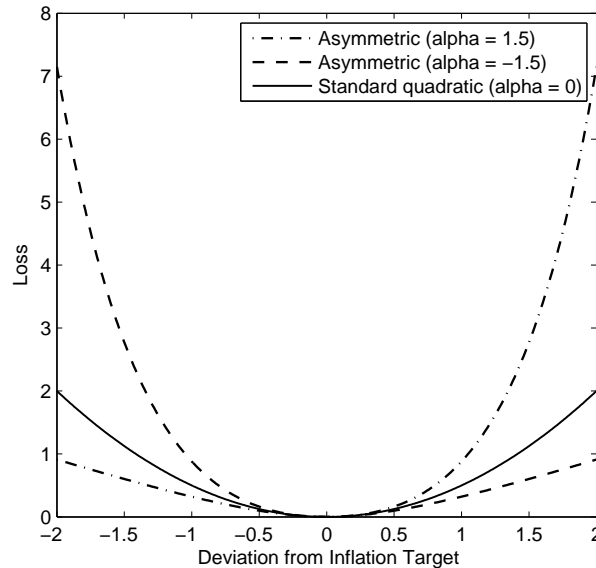


Fig. 3.1 Preference over Inflation Stabilization

with inflation deviations. For $\alpha > 0$, when inflation is above the target, the exponential term of the loss function $e^{\alpha(\pi_t - \pi^*)}$ dominates the linear term $\alpha(\pi_t - \pi^*)$, so the value of the loss function rises exponentially. However, if inflation is below the target, the linear term dominates the exponential term and the value of the loss function increases linearly. This implies that, for $\alpha > 0$, positive deviations of inflation relative to the target are more costly than negative deviations. In this case, the central bank can be said to have deflationary bias. In contrast, for $\alpha < 0$, negative deviations cause a greater loss than positive deviations. This case reflects the view of the central bank that deflation is more costly than inflation. Therefore, the central bank can be characterized as possessing inflationary bias. Another interesting feature is that when α approaches zero, the loss function becomes the common quadratic form. The quadratic loss function is thus a special case of the loss function (3.1). Figure 3.1 illustrates the above cases.

3.2.2 The Structure of the Economy

The structure of the economy imposes constraints to the policy action. This structure includes the following two components

$$\pi_t = \beta E_t \pi_{t+1} + \kappa y_t + \varepsilon_t^s, \quad (3.2)$$

$$y_t = E_t y_{t+1} - \varphi (i_t - E_t \pi_{t+1}) + \varepsilon_t^d, \quad (3.3)$$

which represent the equilibrium conditions of the standard New Keynesian model. The reader is referred to Woodford (2003, Chapter 3) and Galí (2008, Chapter 3) for a complete derivation. Equation (3.2) is called the forward-looking Phillips curve. It is built from the Calvo-type staggered nominal price setting in which only a fraction of firms are allowed to reset their prices in any given period, whereas the others are constrained to keep their prevailing prices. When a firm has a chance to re-optimize, it chooses the price that maximizes the present discounted value of its profits subject to constraints on the frequency of future price changes. Because of common marginal costs, all firms that change their prices choose the same price. Inflation thus relies entirely on current and expected future economic conditions. Specifically, it relates to the current output gap and expected inflation. Regarding the equation's parameters, $\beta < 1$ is the discount factor and κ is the slope coefficient of the Phillips curve.

The second equation (3.3) is the log-linearized consumption Euler equation which is derived from the household's optimal consumption decision and the market clearing condition. This equation shows that the current output gap depends on the expected future output gap and the real interest rate. A higher level of expected future output leads to a greater level of current output because of the consumption smoothing behavior, whereas a higher real interest rate lowers the current output owing to the intertemporal substitution of consumption. The interest elasticity φ corresponds to the intertemporal elasticity of substitution. Finally, ε_t^s and ε_t^d are cost and demand disturbances.

3.2.3 Asymmetric Policy Rule

Central bankers conduct monetary policy to minimize the expected value of a loss criterion of the form

$$W = E_t \sum_{i=1}^{\infty} \beta^i L_{t+i}, \quad (3.4)$$

subject to the forward-looking Phillips curve (3.2) and the IS curve (3.3).

It is assumed that central bankers are unable to make any kind of commitment over the course of future monetary policy. Instead, they take private sector expectations as given and execute policy under discretion. According to Clarida et al. (1999), this scenario seems to accord best with reality. Given this assumption, the Lagrangian of the policy problem is written as follows

$$\begin{aligned} \text{Min}_{\pi_t, y_t, i_t} E_t \left\{ \frac{e^{\alpha(\pi_t - \pi^*)} - \alpha(\pi_t - \pi^*) - 1}{\alpha^2} + \frac{\mu}{2}(y_t)^2 + \frac{\gamma}{2}(i_t - i^*)^2 \right. \\ \left. - \phi_t^\pi (\pi_t - \kappa y_t - \varepsilon_t^s) - \phi_t^y (y_t + \varphi i_t - \varepsilon_t^d) \right\}, \end{aligned}$$

where ϕ_t^π and ϕ_t^y are the Lagrange multipliers. It is straightforward to obtain the following first-order conditions

$$\begin{aligned} E_t(\mu y_t + \phi_t^\pi \kappa - \phi_t^y) &= 0, \\ E_t[\gamma(i_t - i^*) - \phi_t^y \varphi] &= 0, \\ E_t \left\{ \frac{e^{\alpha(\pi_t - \pi^*)} - 1}{\alpha} - \phi_t^\pi \right\} &= 0. \end{aligned}$$

Using these conditions, we obtain

$$E_t \left\{ \frac{e^{\alpha(\pi_t - \pi^*)} - 1}{\alpha} + \frac{\mu}{\kappa} y_t - \frac{\gamma}{\varphi \kappa} (i_t - i^*) \right\} = 0. \quad (3.5)$$

Based on (3.5), the central bank sets the interest rate according to

$$i_t = i^* + E_t \left\{ \frac{e^{\alpha(\pi_t - \pi^*)} - 1}{\alpha} \frac{\varphi \kappa}{\gamma} + \frac{\mu \varphi}{\gamma} y_t \right\},$$

and the above expression can be approximated as

$$\begin{aligned} i_t &= i^* + E_t \left\{ (\pi_t - \pi^*) \frac{\varphi \kappa}{\gamma} + (\pi_t - \pi^*)^2 \frac{\alpha \varphi \kappa}{2\gamma} + \frac{\mu \varphi}{\gamma} y_t \right\} \\ &= i^* + \frac{\varphi \kappa}{\gamma} E_t(\pi_t - \pi^*) + \frac{\alpha \varphi \kappa}{2\gamma} E_t(\pi_t - \pi^*)^2 + \frac{\mu \varphi}{\gamma} E_t y_t. \end{aligned} \quad (3.6)$$

The expectations operator in (3.6) implies that the policy action is taken before the realization of inflation and the output gap. Therefore, the central bank chooses the interest rate at time t based on its expectations on the relevant variables conditional on the information available at that period. Let $\pi_{t|t}$, $\sigma_{\pi_t|t}^2$, $y_{t|t}$ be the nowcasts of inflation, the variance of inflation, and the output gap, respectively. Equation (3.6) can be re-written as

$$i_t = a_0 + a_1 \pi_{t|t} + a_2 \sigma_{\pi_t|t}^2 + a_3 y_{t|t}. \quad (3.7)$$

where $a_0 = i^* - \frac{\varphi \kappa}{\gamma} \pi^*$, $a_1 = \frac{\varphi \kappa}{\gamma}$, $a_2 = \frac{\alpha \varphi \kappa}{2\gamma}$, and $a_3 = \frac{\mu \varphi}{\gamma}$. Note that when α approaches zero, so does a_2 , the reaction function (3.7) collapses to a standard interest rate rule in which the interest rate responds symmetrically to the deviations of inflation and output from their targets. Equation (3.7) therefore nests the symmetric form as a special case. In the next section, we describe how to fit (3.7) to data.

3.3 The Empirical Model

The empirical counterpart of the theoretical model (3.7) is written as follows

$$i_t = \rho_t i_{t-1} + (1 - \rho_t)(a_{0,t} + a_{1,t} \pi_{t|t} + a_{2,t} \sigma_{\pi_t|t}^2 + a_{3,t} y_{t|t}) + \exp(a_{4,t}) \varepsilon_t, \quad (3.8)$$

$$\rho_t = \frac{1}{1 + \exp(-a_{5,t})}, \quad (3.9)$$

$$a_{k,t} = a_{k,t-1} + \exp(\sigma_{a_k}) \varepsilon_{a_k,t}, \quad k = 0, 1, \dots, 5, \quad (3.10)$$

where $\varepsilon_t \sim i.i.d.N(0, 1)$ and $\varepsilon_{a_k,t} \sim i.i.d.N(0, 1)$ for $k = 0, 1, \dots, 5$. The innovations, ε_t and $\varepsilon_{a_k,t}$ for $k = 0, 1, \dots, 5$, are independent.

This empirical model deals with the four issues raised in the literature on modeling monetary policy. First, its specification takes into consideration the asymmetric issue in monetary policy. Second, parameters are allowed to vary over time to capture potential changes in the conduct of monetary policy. Third, the issue of heteroscedasticity is treated by modeling the standard deviation of monetary policy innovations by a stochastic volatility process. Finally, the model is fitted with real-time data, as will be discussed below. It is also worth noting other features of the model. Following Clarida et al. (1999), the lag of interest rate is included as an explanatory variable to capture the observed smoothing of interest rate. Moreover, the smoothing parameter ρ_t is constrained to be positive but smaller than unity and then transformed to the real line by the logit transformation as in line with Kim and Nelson (2006). For the time-variation of parameters, it is assumed to follow random walk dynamics similar to Cogley and Sargent (2005) and Boivin (2006), among many others.

Substituting (3.9) into (3.8) yields

$$i_t = \frac{1}{1 + \exp(-a_{5,t})} i_{t-1} + \frac{\exp(-a_{5,t})}{1 + \exp(-a_{5,t})} (a_{0,t} + a_{1,t} \pi_t | t + a_{2,t} \sigma_{\pi_t}^2 + a_{3,t} y_t | t) + \exp(a_{4,t}) \varepsilon_t, \quad (3.11)$$

The combination of (3.10) and (3.11) generates a state-space system. In this system, the state model (3.10) describes the evolution of the state vector $\mathbf{x}_t = [a_{0,t}, a_{1,t}, a_{2,t}, a_{3,t}, a_{4,t}, a_{5,t}]'$ and the measurement model (3.11) relates the noisy measurement i_t to the state. In order to facilitate the analysis, the state-space system is written in its probabilistic form as follows

$$\mathbf{x}_t = h(\mathbf{x}_{t-1}, \mathbf{w}_t; \boldsymbol{\varpi}), \quad (3.12)$$

$$i_t = g(\mathbf{x}_t, \varepsilon_t; \boldsymbol{\varpi}, \mathbf{A}_t), \quad (3.13)$$

where $\mathbf{w}_t = [\varepsilon_{0,t}, \varepsilon_{1,t}, \varepsilon_{2,t}, \varepsilon_{3,t}, \varepsilon_{4,t}, \varepsilon_{5,t}]'$ is the vector of state noises, which has a multivariate

normal distribution with zero mean and identity covariance matrix, $\boldsymbol{\omega} = [\sigma_{a_0}, \sigma_{a_1}, \sigma_{a_2}, \sigma_{a_3}, \sigma_{a_4}, \sigma_{a_5}]'$ presents the set of time-invariant parameters, and $\mathbf{A}_t = [i_{t-1}, \pi_{t|t}, y_{t|t}, \sigma_{\pi_t}^2]$ includes observed inputs. To ease notation, in what follows we drop \mathbf{A}_t without any loss of generality. The functions $h(\cdot)$ and $g(\cdot)$ come from the equations that characterize the behavior of the model, which are (3.10) and (3.11), respectively.

We aim to estimate the evolution of state variables given the sequence of received measurement. In order to do so, it is required to construct the posterior probability density function of the state vector. However, the constraint on the smoothing parameter and the stochastic volatility of monetary policy shocks generate nonlinearities in the system, preventing us from using the well-known Kalman filter and, thus, completing the estimation. To deal with the nonlinearities, we apply the approach called the particle filter, which is proposed by Gordon et al. (1993). The key idea of particle filtering is to represent the required posterior density function by a set of random samples with associated weights and to compute estimates based on these samples and weights (Ristic et al., 2004).

Before we go into the details of the algorithm, let us introduce $X_t = \{\mathbf{x}_j, j = 0, \dots, t\}$ and $I_t = \{i_j, j = 0, \dots, t\}$ which represent the sequences of all states and available measurements, respectively, up to time t . The joint posterior density at time t is denoted by $p(X_t|I_t)$ and its marginal is $p(\mathbf{x}_t|I_t)$. Let $\{X_t^k, \omega_t^k\}_{k=1}^N$ denote a random measure that describes the joint posterior $p(X_t|I_t)$ where $\{X_t^k, k = 1, \dots, N\}$ is a set of support points with associated weights $\{\omega_t^k, k = 1, \dots, N\}$. The weights are normalized by dividing each by their sum. Thus, the joint posterior distribution at t can be approximated by

$$p(X_t|I_t) \approx \sum_{k=1}^N \omega_t^k \delta(X_t - X_t^k), \quad (3.14)$$

where $\delta(\cdot)$ is the Dirac delta measure. The normalized weights ω_t^k are chosen by applying the principle of importance sampling in which X_t^k is drawn from an importance density $q(X_t|I_t)$

$$\omega_t^k \propto \frac{p(X_t^k|I_t)}{q(X_t^k|I_t)}. \quad (3.15)$$

If the importance density is chosen so that it can be factorized by

$$q(X_t|I_t) \triangleq q(\mathbf{x}_t|X_{t-1}, I_t)q(X_{t-1}|I_{t-1}), \quad (3.16)$$

then the samples $X_t^k \sim q(X_t|I_t)$ can be achieved by augmenting each of the existing samples $X_{t-1}^k \sim q(X_{t-1}|I_{t-1})$ with the new state $\mathbf{x}_t^k \sim q(\mathbf{x}_t|X_{t-1}, I_t)$. At time step t when a measurement i_t becomes available, the posterior density $p(X_t|I_t)$ can be updated from $p(X_{t-1}|I_{t-1})$ by

$$\begin{aligned} p(X_t|I_t) &= \frac{p(i_t|X_t, I_{t-1})p(X_t|I_{t-1})}{p(i_t|I_{t-1})} \\ &= \frac{p(i_t|X_t, I_{t-1})p(\mathbf{x}_t|X_{t-1}, I_{t-1})p(X_{t-1}|I_{t-1})}{p(i_t|I_{t-1})} \\ &= \frac{p(i_t|\mathbf{x}_t)p(\mathbf{x}_t|\mathbf{x}_{t-1})p(X_{t-1}|I_{t-1})}{p(i_t|I_{t-1})} \\ &\propto p(i_t|\mathbf{x}_t)p(\mathbf{x}_t|\mathbf{x}_{t-1})p(X_{t-1}|I_{t-1}). \end{aligned} \quad (3.17)$$

Substituting (3.16) and (3.17) into (3.15) yields the weight update equation

$$\begin{aligned} \omega_t^k &\propto \frac{p(i_t|\mathbf{x}_t^k)p(\mathbf{x}_t^k|\mathbf{x}_{t-1}^k)p(X_{t-1}^k|I_{t-1})}{q(\mathbf{x}_t^k|X_{t-1}^k, I_t)q(X_{t-1}^k|I_{t-1})} \\ &= \omega_{t-1}^k \frac{p(i_t|\mathbf{x}_t^k)p(\mathbf{x}_t^k|\mathbf{x}_{t-1}^k)}{q(\mathbf{x}_t^k|X_{t-1}^k, I_t)}. \end{aligned}$$

Moreover, by choosing the importance density to depend only on \mathbf{x}_{t-1} and i_t , the weights are given by

$$\omega_t^k \propto \omega_{t-1}^k \frac{p(i_t|\mathbf{x}_t^k)p(\mathbf{x}_t^k|\mathbf{x}_{t-1}^k)}{q(\mathbf{x}_t^k|\mathbf{x}_{t-1}^k, i_t)}.$$

In this study, we use the bootstrap filtering proposed by Gordon et al. (1993) in which $q(\mathbf{x}_t|\mathbf{x}_{t-1}, \mathbf{z}_t) = p(\mathbf{x}_t|\mathbf{x}_{t-1})$. Therefore,

$$\omega_t^k \propto \omega_{t-1}^k p(i_t|\mathbf{x}_t^k).$$

Given these weights, the marginal posterior density $p(\mathbf{x}_t|I_t)$ can be approximated as

$$p(\mathbf{x}_t|I_t) \approx \sum_{k=1}^N \omega_t^k \delta(\mathbf{x}_t - \mathbf{x}_{t-1}^k). \quad (3.18)$$

As $N \rightarrow \infty$ the approximation (3.18) approaches the true marginal posterior density $p(\mathbf{x}_t|I_t)$ (Ristic et al., 2004). Based on this posterior density, we estimate the state vector as its conditional mean.

It is however worth emphasizing that, given the importance function of the form (3.16), the variance of importance weights can only increase over time, thus leading to the degeneracy problem (Ristic et al., 2004). Therefore, resampling that replaces the samples with low importance weights by those with high importance weights is required. There are many different resampling schemes, which can be referred to Doucet and Johansen (2009), but we use the systematic resampling method because it is easy to apply and outperforms other resampling schemes in most cases (Doucet and Johansen, 2009).

Moreover, a by-product of the particle filter is that the likelihood can be approximated by using the weights ω_t^i

$$p(I_T; \boldsymbol{\varpi}) \approx \prod_{i=1}^T \left(\sum_{i=1}^N \omega_t^i \right). \quad (3.19)$$

Appendix B.1 presents the details of this approximation. Once the likelihood is evaluated, we can use the maximum likelihood approach to estimate $\boldsymbol{\varpi}$, the vector of time-invariant parameters.

Note that particle filtering generates an approximation to the likelihood function that is not differentiable with respect to the parameters because of the inherent discreteness of the resampling step. Therefore, Newton's type algorithms, based on derivatives, are not applicable. Nguyen (2015) and van Binsbergen et al. (2012) deal with this issue by using the covariance matrix adaption evolutionary strategy (CMA-ES) because this optimization algorithm is designed to cope with objective functions that are non-linear, non-convex, rugged, multimodal as well as with those having other difficult conditions (Hansen, 2011). Following these papers, we employ the CMA-ES to obtain the maximum-likelihood estimates of $\boldsymbol{\varpi}$.

3.4 Data and Empirical Results

3.4.1 Data

The estimation of state-space system described by (3.10) and (3.11) require data for the nominal interest rate, expected inflation, the expected variance of inflation, and the expected output gap. For the nominal interest rate i_t , we use the effective federal funds rate extracted from the FRED economic data. For the expected value of inflation $\pi_{t|t}$, we use the Greenbook forecasts of the current-quarter annualized percentage change in the GNP or GDP deflator. The expected variance of inflation $\sigma_{\pi_t|t}^2$ is not available, we discuss the construction of this series in Section 3.4.2. For the expected output gap $y_{t|t}$, we proxy it by the unemployment gap for two reasons. First, because of repeated changes in the base year, no consistent time series of predicted real GDP or GNP can be derived from the Greenbook over the sample (Boivin, 2006). Second, maximum employment is one of the objectives of monetary policy clearly written in the Federal Act, thus the unemployment rate should be taken directly into the policy function. We define the unemployment gap as the difference between the natural rate of unemployment and the forecasted unemployment rate, therefore the sign of the unemployment gap is consistent with that of the conventionally-defined output gap. While the forecast of contemporaneous unemployment rate is collected directly from the Greenbook, the natural rate of unemployment is measured by a 5-year moving average of unemployment rate as in Bernanke and Boivin (2003). We name this proxy the 5-year moving average unemployment gap. Because of uncertainties in the real-time measures of real activity, we investigate the robustness of the baseline results to different measures of real activity, including the expected output gap per se.

The sample of the above data is from 1965Q4 to 2007Q4. The starting point coincides with the first period that predictions were recorded in the Greenbook, while the ending point is the latest period available. Note that the Greenbook forecasts are made publicly available with a 5-year lag. We also use monthly data of inflation and unemployment from 1948M1 to 2007M9 for the constructions of the expected variance of inflation and the expected variance of output gap, which we describe next.

3.4.2 Expected Variance of Inflation

Bollerslev (1986) and Dolado et al. (2004) obtain the conditional variance of inflation with ex-post revised data by estimating inflation dynamics using a GARCH(1,1) model. Because of the real-time data issue, i.e. inflation at time t can be only observed with a lag, we modify this GARCH procedure in two ways. First, we estimate a specification of inflation dynamics with GARCH(1,1) errors using information available at that time. Second, based on the estimated GARCH process, we forecast the contemporaneous variance of inflation. This procedure is conducted recursively from 1965Q4 to 2007Q4.

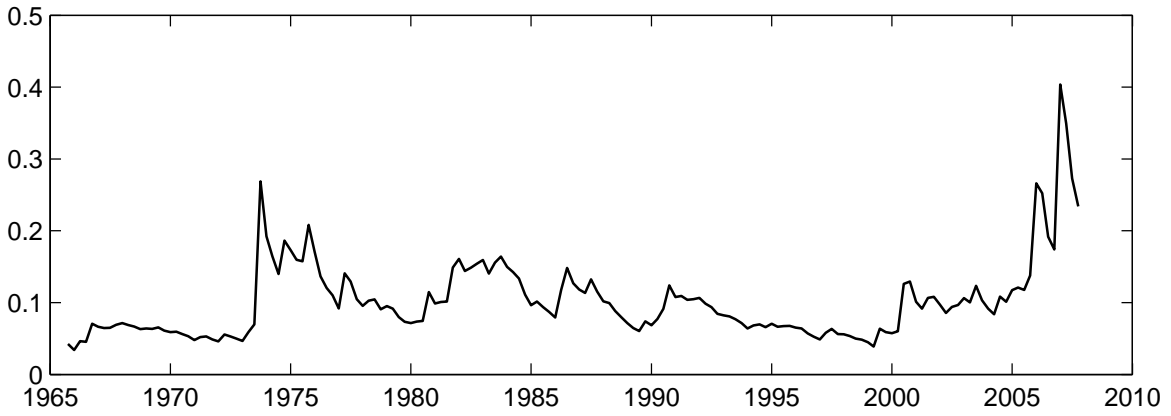


Fig. 3.2 Expected Variance of Inflation

Notes: The expected variance of inflation is computed by applying the four-step procedure described in Section 3.4.2, with the number of lags of inflation set equal to three and the output gap y_t proxied by the five-year moving average gap in Equation (3.20).

We use the following specification of inflation dynamics

$$\pi_t = c + \sum_{i=1}^n \beta_i \pi_{t-i} + \zeta y_t + \varepsilon_t, \quad (3.20)$$

$$\varepsilon_t = \sigma_{\pi,t} z_t, \quad (3.21)$$

$$\sigma_{\pi,t}^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{\pi,t-1}^2. \quad (3.22)$$

Equation (3.20) is derived from (3.2) by substituting $E_t \pi_{t+1}$ by a linear combination of the

lags of inflation. According to Rudebusch (2001), Equation (3.20) is fairly successful in capturing the dynamics of inflation in the U.S. economy. This specification is also similar to the one used in Bollerslev (1986) and Dolado et al. (2004).

As mentioned above, we proxy y_t by the unemployment gap. Because data on inflation and unemployment are available at the monthly frequency, we utilize this advantage to increase the number of observations, which most likely benefits GARCH-based estimations. We assume that the variance of inflation of quarter t is nowcasted using the information available at the first month of that quarter. This assumption is in line with the Greenbook forecasts because they are often published by the end of the first month or the middle of the second month of a quarter.

We detail the procedure to forecast the contemporaneous variance of inflation series as follows:

Step 0, Initiation: We start with the 1965Q4 period, set $i \rightsquigarrow 1965Q4$.

Step 1, Estimation: Let I_i be the information set at time i which includes monthly inflation and the output gap (proxied by the unemployment gap) from 1948M1 to the last month of the quarter $i - 1$. Given I_i , Equation (3.20) is estimated with GARCH(1,1) errors.

Step 2, Forecast: Based on the estimated-GARCH process, we forecast the conditional variances of inflation for the three months of quarter i . Take the average of those forecasts and save it as $\sigma_{\pi_i|I_i}^2$.

Step 3, Termination: If $i \neq 2007Q4$, move to the next period $i = i + 1$ and follow step 1. Otherwise, the procedure stops and we collect the expected variance of inflation $\sigma_{\pi_i|I_i}^2$ for $i = 1965Q4, \dots, 2007Q4$.

Figure 3.2 present one of the measures of the expected variance of inflation estimated by applying the above four-step procedure with three lags of inflation ($n = 3$) and the output gap (y_t) proxied by the 5-year moving average unemployment gap. Overall, we observe that the expected variance of inflation has changed considerably over time. Specifically, it was around 0.05 in the late 1960s and the first half of the 1970s, then increased substantially in

Table 3.1 Summary Statistics for Forecasts of Inflation Variance: 1965Q4-2007Q4

	\mathbf{M}_0	\mathbf{M}_1	\mathbf{M}_2	\mathbf{M}_3	\mathbf{M}_4	\mathbf{M}_5
Means	0.102	0.100	0.104	0.102	0.099	0.098
Standard deviations	0.055	0.056	0.059	0.058	0.051	0.052
Correlation matrix						
\mathbf{M}_0	1.000	0.994	0.993	0.991	0.992	0.988
\mathbf{M}_1	0.994	1.000	0.984	0.995	0.987	0.995
\mathbf{M}_2	0.993	0.985	1.000	0.993	0.978	0.973
\mathbf{M}_3	0.991	0.995	0.993	1.00	0.978	0.984
\mathbf{M}_4	0.992	0.987	0.978	0.978	1.000	0.994
\mathbf{M}_5	0.988	0.995	0.973	0.984	0.994	1.000

Notes: The table reports the means, the standard deviations and the correlation matrix of different estimates of the expected variance of inflation by applying the four-step procedure outlined in Section 3.4.2. The measures are different in terms of the number of lags of inflation (n) and the measure of output gap (y_t) used in Equation (3.20). The measure \mathbf{M}_0 is associated with three lags of inflation $n = 3$ and the output gap y_t proxied by the five-year moving average unemployment gap. For \mathbf{M}_1 , $n = 6$ and y_t proxied by the five-year moving average unemployment gap. For \mathbf{M}_2 , $n = 3$ and y_t proxied by the historical average unemployment gap. For \mathbf{M}_3 , $n = 6$ and y_t proxied by the historical average unemployment gap. For \mathbf{M}_4 , $n = 3$ and y_t proxied by the three-year moving average unemployment gap. Finally, for \mathbf{M}_5 , $n = 6$ and y_t proxied by the three-year moving average unemployment gap.

the mid-1970s. It reduced gradually to the level of 0.05 by 2000s, but rose again after that and reached a peak in 2007Q1.

We also consider alternative proxies of the output gap and different numbers of lags of inflation used in Equation (3.20) and summarize all the results in Table 3.1. As it can be seen, these forecasts are very similar, thus corroborating our constructed series.¹ For the estimation of the baseline model, we use the measure depicted in Figure 3.2.

3.4.3 Results for the Baseline Model

We proceed to analyze the estimation results, with a particular interest in the responses of interest rate to real activity, inflation, and the variance of inflation.² Figure 3.3 reports the response to real activity. We observe that the response was positive and statistically

¹The results remain similar if we substitute the real activity variable y_t in (3.20) by a linear combination of its lags.

²The estimates of time-invariant parameters are presented in Appendix B.2.

significant in the 1970s in line with Clarida et al. (2000). In the 1980s, it decreased and became insignificant, particularly in the second half of the decade. In the early 1990s, the response to real activity experienced a substantial increase from just above 1.0 in 1990 to approximately 2.0 from 1995 onwards, which implies that the Fed paid more attention to real activity since then. This result is in line with Blinder and Reis (2005) who find that monetary policy under the Greenspan regime responded stronger to unemployment than the policy under the Volcker period. Kim and Nelson (2006) also obtain similar results when estimating a time-varying parameter model using ex-post data.

The response to inflation is depicted in Figure 3.4. The pre-1979 response to inflation was not uniformly weak as typically assumed. It was above unity until 1975, then decreased considerably and went below unity, particularly between 1976 and 1978. This property of the pre-1979 monetary policy is surprisingly consistent with Boivin (2006) and Cogley and Sargent (2005, 2001). Based on narrative evidence, Romer and Romer (1989) also point out that the actual commitment of the Fed to combat inflation appeared to have been weak in the 1976-1978 period. In 1979Q3, Paul Volcker was appointed Chairman of the Fed, the response to inflation became strong, which is in line with the findings of Clarida et al. (2000) and Romer and Romer (1989). However, the main shift under the Volcker's tenure seems to have happened during the 1981-1982 period as documented by Boivin (2006). The inflation coefficient was mainly above unity to the early 1990s, then fell and has become not significant by the mid-1990s. This finding could be surprising given the fact that inflation has continued staying at low and stable levels in the 1990s and thereafter. In Section 3.4.5, we provide a discussion about this point. For the moment, we just note that the finding of a weak response to inflation in the post-1990 period is not unprecedented in the literature. Martin and Milas (2010) test the opportunistic approach in the U.S. and obtain a similar result. Fernández-Villaverde et al. (2010*b*, 2015) estimate a DSGE model with drifting parameters and stochastic volatility for the U.S. economy and find that the post-1990 response to inflation was weak.

Regarding the response to inflation variance, Figure 3.5 shows that it was negative and statistically significant in the pre-1979 period, but not statistically different from zero in

the post-1979 period. These results therefore suggest that the Fed in the pre-1979 period behaved as if having asymmetric preferences in the sense that negative inflation gaps are considered to be more costly than positive gaps of the same absolute size. In other words, deflation was thought to be more costly than inflation. Meanwhile, the post-1979 monetary policy could be characterized by symmetric rules. Our results are broadly similar to Surico (2007) who investigates changes in asymmetric preferences of the Fed before and after 1979.

The smoothing parameter and the standard deviation of monetary policy shocks also vary over the sample as displayed in Figure 3.6. Regarding the former, it was large and stable from the mid-1980s to the mid-1990s, which suggests that the interest rate was persistent during these periods. The smoothing parameter then decreased and reached the bottom in the early 2000s, before returning to the high level of persistence at the end of the sample. Concerning the standard deviation of monetary policy shocks, it has become lower and more stable since the mid-1980s. This result therefore affirms the importance of taking heteroscedasticity into account as emphasized by Sims and Zha (2006).

3.4.4 Robustness Checks

We study the robustness of the results in the baseline model along two dimensions: alternative measures of real activity and another model specification with asymmetric preferences with respect to both inflation and output gap.

Real Activity Measures

In the baseline model, we proxy the output gap by the difference between a 5-year moving average of unemployment rate and the expected contemporaneous unemployment rate. Because there is no guarantee that this proxy corresponds to the real activity measure perceived by policymakers, it is important to investigate how robust the results are with respect to different measures of the output gap.

First, we consider two alternative measures of the natural rate of unemployment: a historical average and a 3-year moving average (Boivin, 2006), leading to the two different measures of unemployment gap. The estimates with these alternatives are presented in Fig-

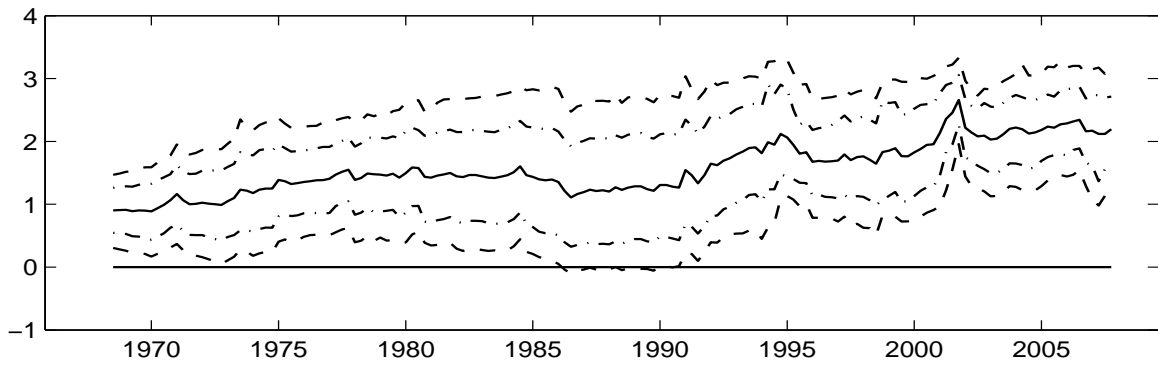


Fig. 3.3 Response to Real Activity

Note: Dashed lines are 68% and 90% percentile intervals.

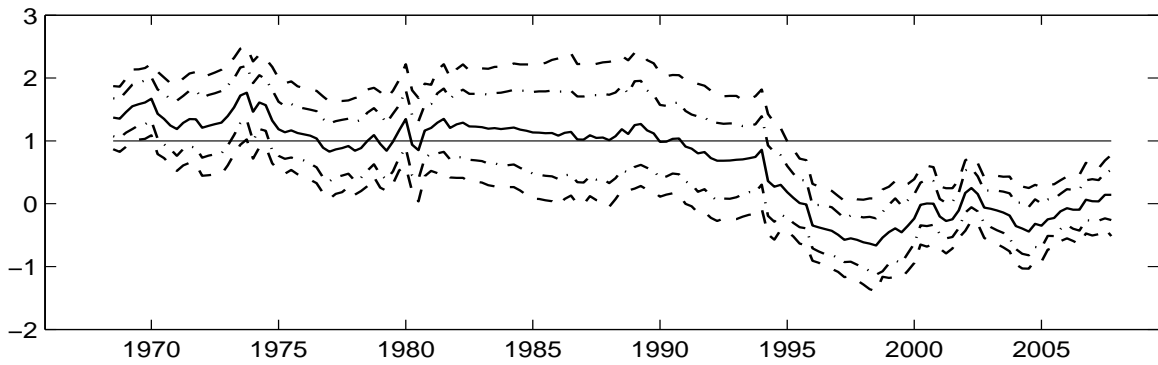


Fig. 3.4 Response to Inflation

Note: Dashed lines are 68% and 90% percentile intervals.

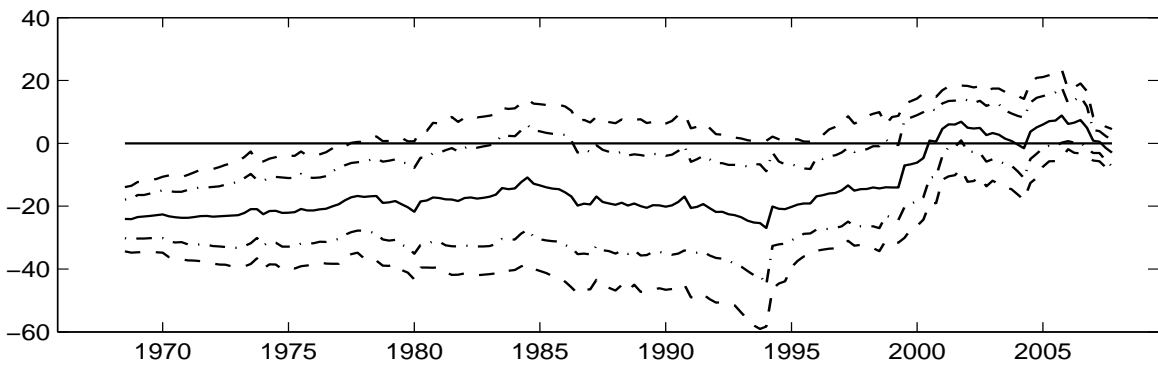
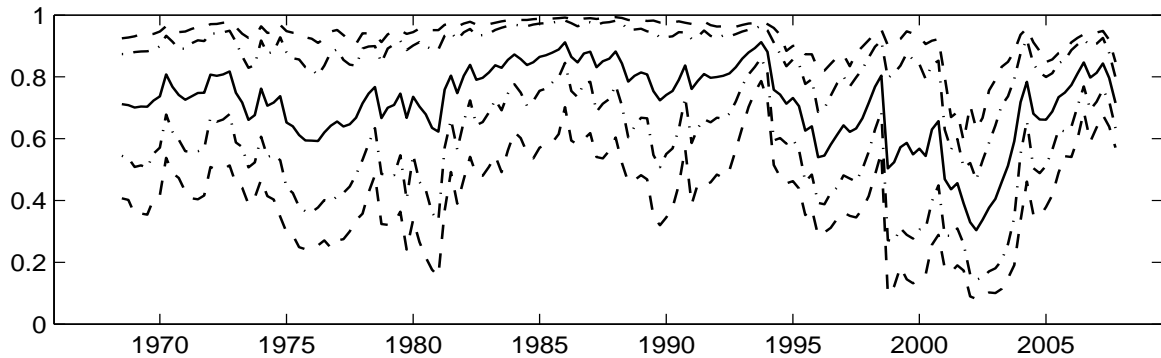
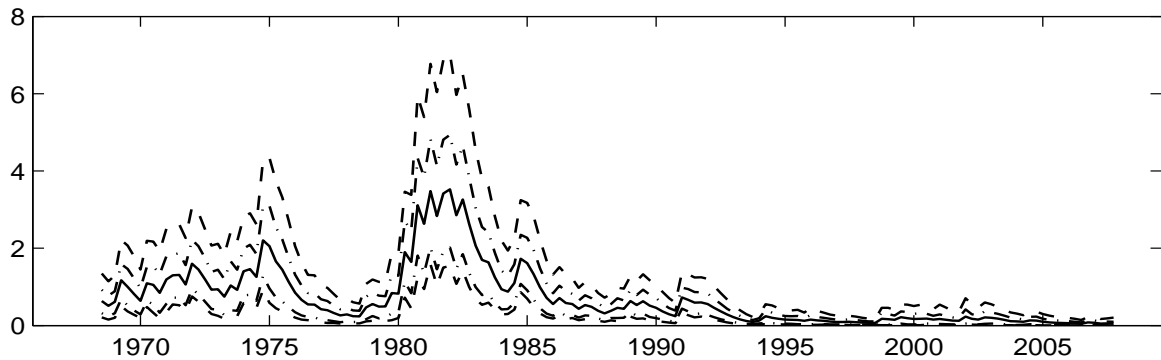


Fig. 3.5 Response to Inflation Variance

Note: Dashed lines are 68% and 90% percentile intervals.



(a) Time-varying Degree of Interest Rate Smoothing



(b) Stochastic Volatility

Fig. 3.6 Interest Rate Smoothing Degree and Stochastic Volatility

Note: Dashed lines are 68% and 90% percentile intervals.

ures 3.7 and 3.8, respectively. The response to inflation follows the similar path regardless of the measure used. In terms of the response to real activity, the one with the 3-year moving average unemployment gap was not distinguishable from zero in the 1980s. Meanwhile, the one with the historical average unemployment gap was statistically significant over the sample; although, it was only marginally significant in the second half of the 1980s. However, both cases affirm that the Fed has responded stronger to real activity since the early 1990s. For the coefficient on the inflation variance, it was negative and significant in the 1970s, then has become insignificant since the early 1980s. Overall, the results are similar to those in the baseline model.

We have so far considered the unemployment rate as a proxy for the output gap be-

cause, as mentioned previously, it is built from a time series that has a consistent definition throughout the sample. Though this is not the case for real GDP- or GNP-based measures. In this robustness check, we conduct the estimation with the output gap per se, which is the HP output gap, and then compare the results obtained with those in the baseline model. In order to do so, we need to construct the expected contemporaneous real-time HP output gap series. This step is described in Appendix B.3 and the constructed series is shown in Figure 3.9 together with the above measures of unemployment gap. As can be seen in the figure, the HP output gap series shows a similar trend with the measures based on unemployment. We then re-estimate the state-space system in Section 3.3 using this HP output gap series and display the results in Figure 3.10. The main finding is that the evolving path of the estimated responses using the HP output gap are similar with those documented in the baseline model. Thus, the results drawn are not sensitive to the output gap measure.

Asymmetric Preferences to Both Inflation and Output Gap

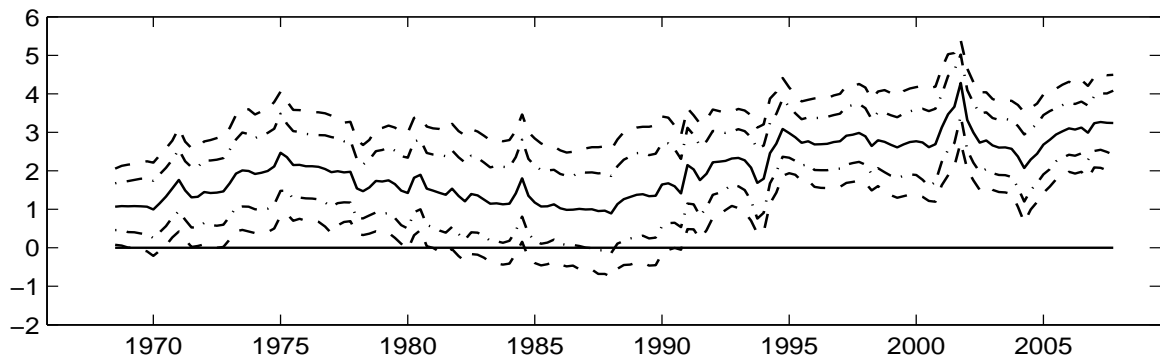
So far, we have considered asymmetric preferences with respect to inflation. However, the central bank may also react asymmetrically to real activity, for instance, a negative output gap may be considered more costly than a positive gap of the same absolute size. This kind of asymmetry is studied in Ruge-Murcia (2003) and Cukierman and Gerlach (2003). For this reason, we modify the loss function in Equation (3.1) to include a linex function of the output gap as follows

$$L_t = \frac{e^{\alpha(\pi_t - \pi^*)} - \alpha(\pi_t - \pi^*) - 1}{\alpha^2} + \mu \left(\frac{e^{\lambda y_t} - \lambda y_t - 1}{\lambda^2} \right) + \frac{\gamma}{2} (i_t - i^*)^2, \quad (3.23)$$

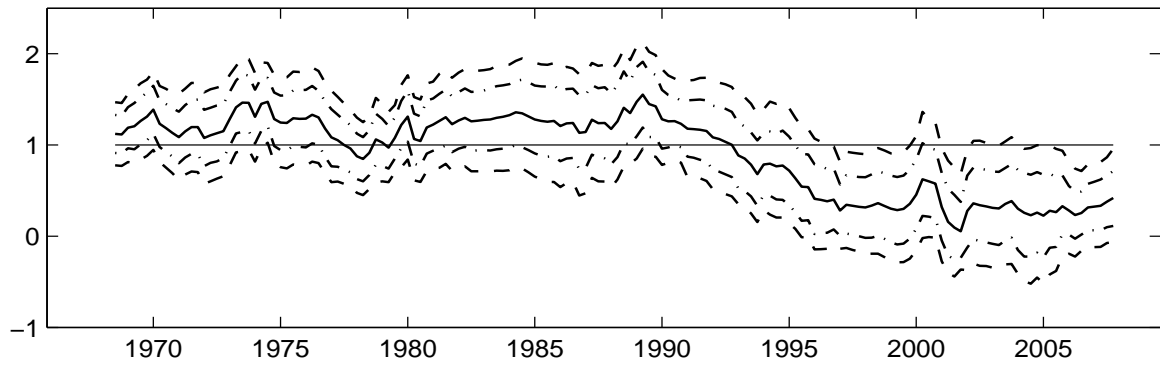
where λ captures the asymmetry in the loss function with respect to the output gap and other notations are the same as in (3.1).

By following the steps in Section 3.2.3, we derive a policy rule in which the interest rate responds to inflation, output gap, the variance of inflation and the variance of output gap.³

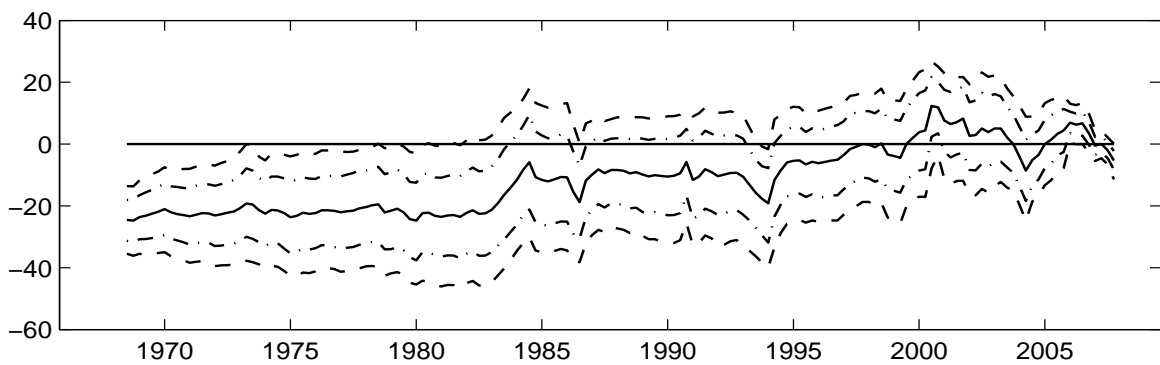
³Appendix B.4 presents the derivation in detail.



(a) Response to Real Activity



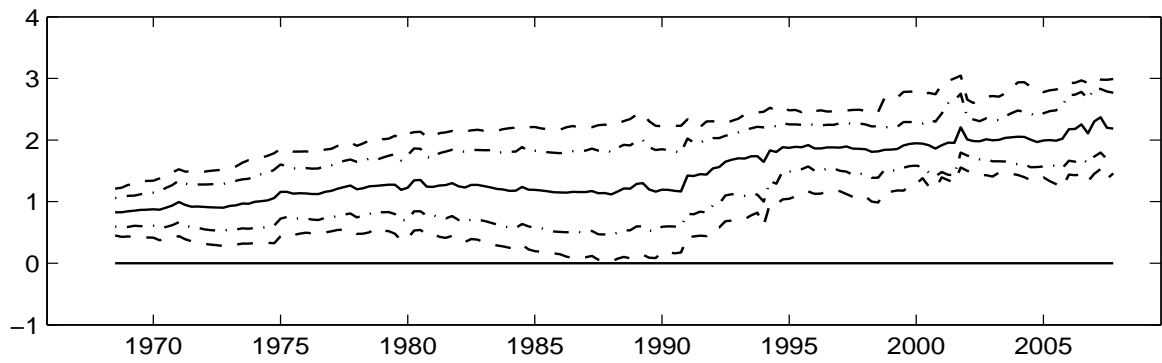
(b) Response to Inflation



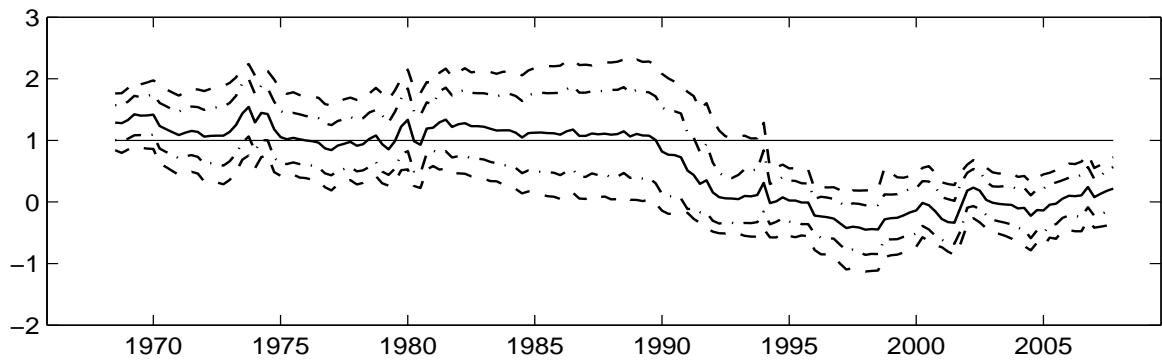
(c) Response to Inflation Variance

Fig. 3.7 Robustness Check with Three-Year Moving Average Unemployment Gap

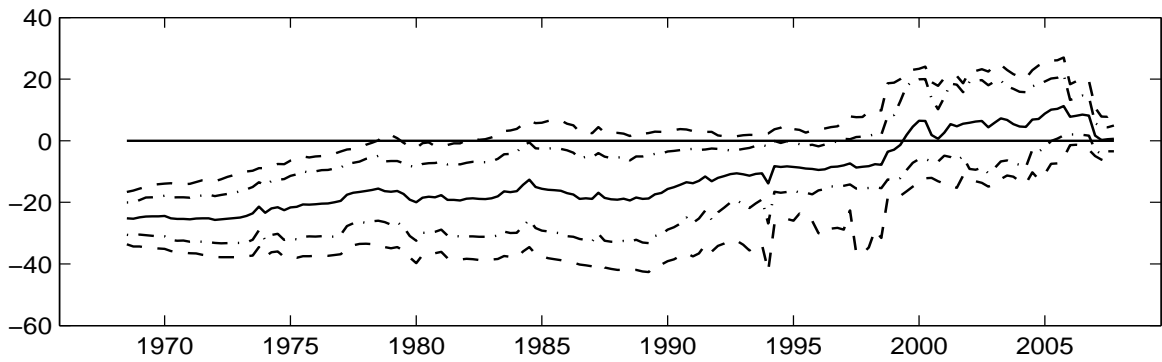
Note: Dashed lines are 68% and 90% percentile intervals.



(a) Response to Real Activity



(b) Response to Inflation



(c) Response to Inflation Variance

Fig. 3.8 Robustness Check with Historical Average Unemployment Gap

Note: Dashed lines are 68% and 90% percentile intervals.

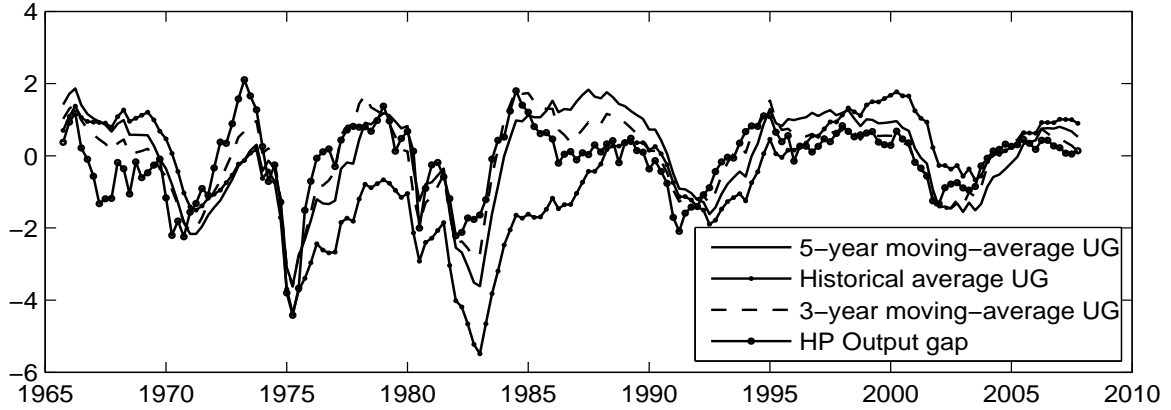


Fig. 3.9 Measures of Real Activity for the Contemporaneous Quarter.

The corresponding empirical model is then given by

$$i_t = \frac{1}{1 + \exp(-b_{6,t})} i_{t-1} + \frac{\exp(-b_{6,t})}{1 + \exp(-b_{6,t})} (b_{0,t} + b_{1,t} \pi_{t|t} + b_{2,t} \sigma_{\pi_t|t}^2 + b_{3,t} y_{t|t} + b_{4,t} \sigma_{y_t|t}^2) + \exp(b_{5,t}) \varepsilon_t \quad (3.24)$$

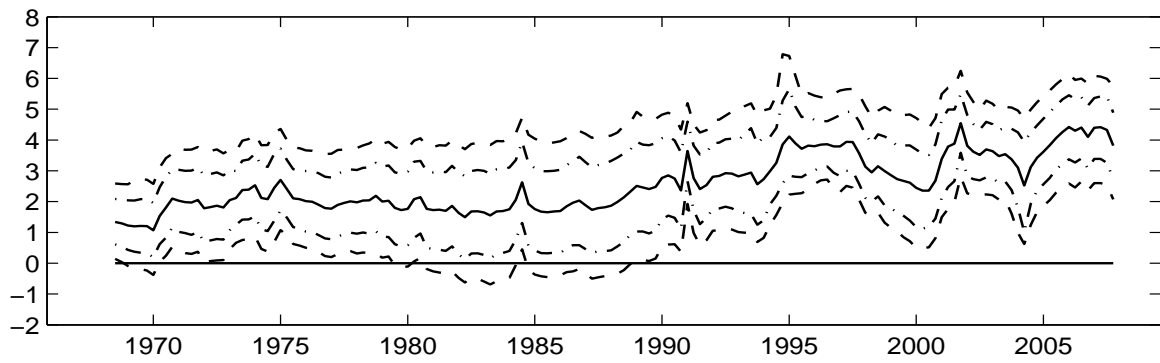
and the state vector becomes $\mathbf{x}_t = [b_{0,t}, b_{1,t}, b_{2,t}, b_{3,t}, b_{4,t}, b_{5,t}, b_{6,t}]'$.

The estimation of (3.24) requires the data of the expected variance of output gap which is not available. We describe the construction of this series in Appendix B.4.1. Other variables remain as in the baseline model.

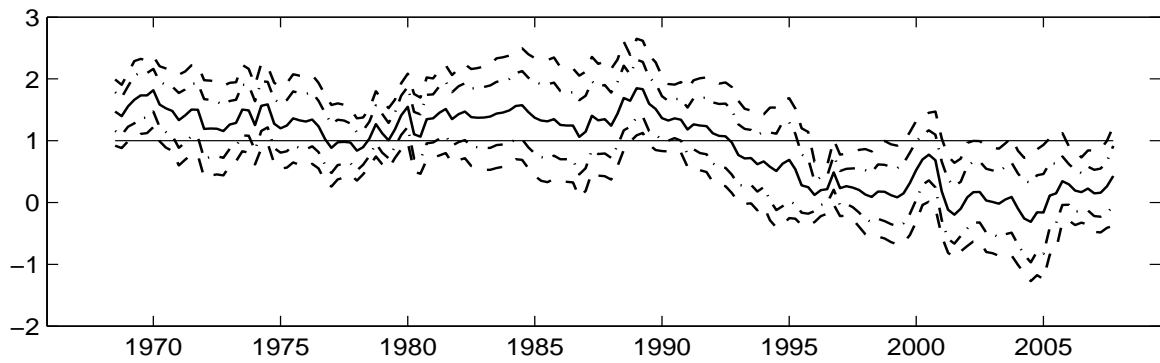
Figure 3.11 presents the time-varying parameter results. It is apparent that the response to inflation, output gap, and the variance of inflation are essentially similar with those in the baseline model. Meanwhile, the response to the variance of output gap is mostly insignificant over the sample. Overall, the results are again in line with those documented in the baseline model.

3.4.5 Discussion

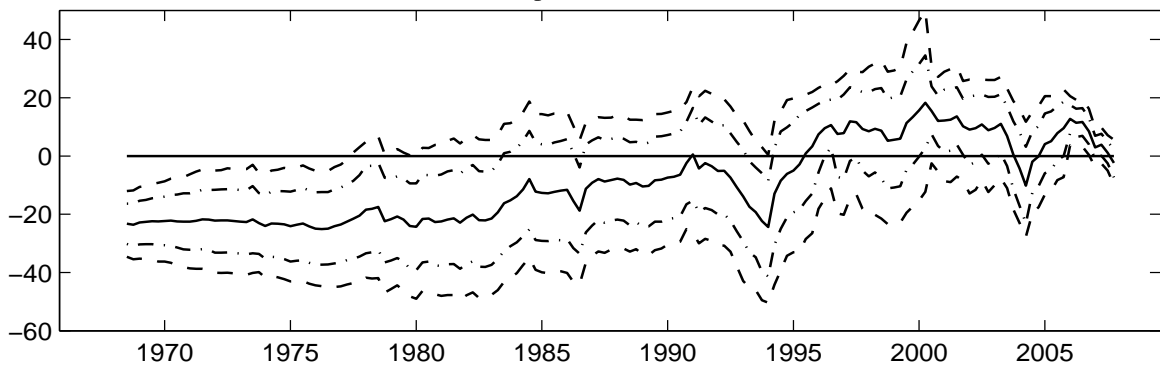
Our discussion focuses on two issues. First, based on the empirical results, we interpret how inflation stabilization was achieved. This is accomplished by addressing crucial differences in the conduct of monetary policy between the pre-Volcker period and the Volcker period. Second, we provide an interpretation of monetary policy in the post-1990 period that appears



(a) Response to Real Activity



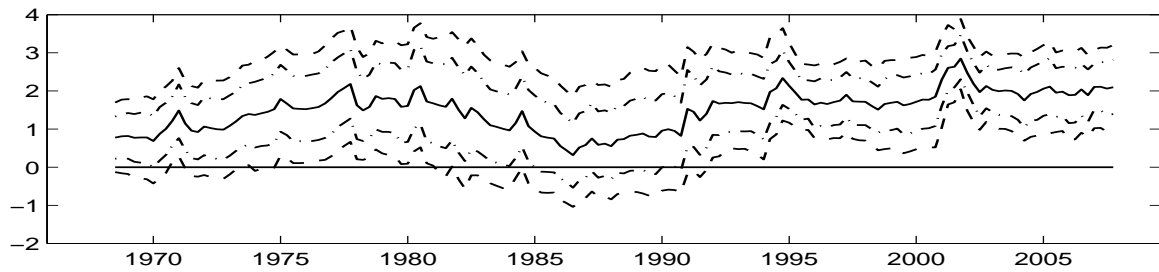
(b) Response to Inflation



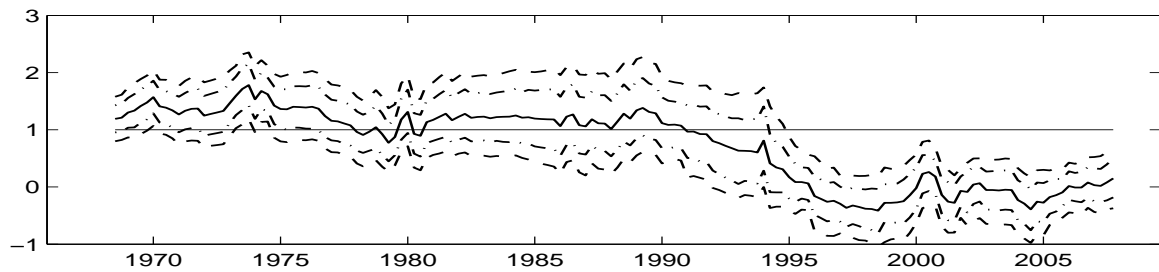
(c) Response to Inflation Variance

Fig. 3.10 Robustness Check with HP Output Gap

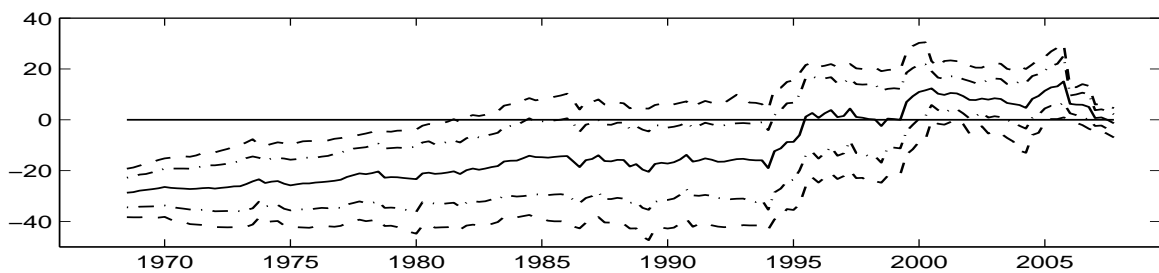
Note: Dashed lines are 68% and 90% percentile intervals.



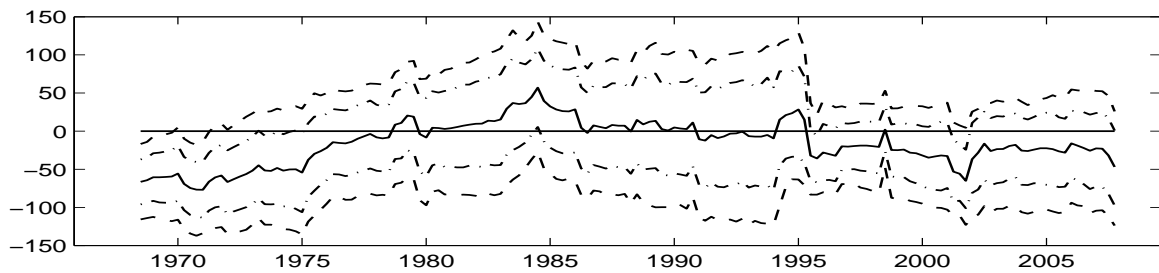
(a) Response to Real Activity



(b) Response to Inflation



(c) Response to Inflation Variance



(d) Response to Output Gap Variance

Fig. 3.11 Robustness Check with Asymmetric Preferences to Both Inflation and Output Gap

Note: Dashed lines are 68% and 90% percentile intervals.

to have responded weakly to inflation.

We begin with the issue on inflation stabilization. Our results suggest that this achievement can be attributed to both changes in the direct response to inflation, as suggested by Clarida et al. (1999, 2000), Cogley and Sargent (2005, 2001), and Boivin (2006), and changes in preferences, as put forward by Cukierman and Gerlach (2003), Dolado et al. (2004), and Surico (2007). Regarding the former, the response to inflation was weak in the second half of the 1970s, which likely contributed to the high level of inflation during this era. In contrast, the strong response to inflation in the 1980s played a role in bringing inflation down and keeping it stable. Nevertheless, this explanation may not be completely satisfactory because the confidence bounds of the estimated coefficient on expected inflation suggest that monetary policy between the pre-Volcker period and the Volcker years were not essentially different as indicated by the point estimates. Moreover, the pre-1975 inflation is hardly explained if we rely only on the direct response to inflation.

It appears that the Fed's preferences changed in the late 1970s as well. Prior to that, the Fed's behavior was seemingly asymmetric in the sense that negative inflation deviations from the target are considered to be more costly than positive ones of the same magnitude. This asymmetry led to inflationary bias in the conduct of monetary policy, which might have accounted for the great inflation during this decade. On the contrary, we do not find the evidence of such an inflationary bias from the early 1980s onwards.

If so, the question that arises is what made the Fed's preference shift. According to De Long (1997), the Fed in the 1970s did not have enough autonomy to control inflation. The author provides extensive narrative evidence about the influence of Nixon's administration on the Chairmanship of Burns at the Fed. Among those is the following conversation between Richard Nixon (the speaker) and Arthur Burns (the listener), reported in Ehrlichman (1982), on October 23 1969, after Nixon had announced his intention to nominate Burns to replace Martin as chairman of the Fed:

I know there's the myth of the autonomous Fed... [short laugh] and when you go up for confirmation some Senator may ask you about your friendship with the President. Appearances are going to be important, so you can call Ehrlichman

to get messages to me, and he'll call you (Ehrlichman, 1982, p.248-249 cited in De Long, 1997, p.263).

The Fed was therefore quite sensitive to the concerns of political authorities, who were not willing to accept the possibility of recession to lower inflation given the prevailing view of a permanent negative trade-off between unemployment and inflation at the time. Disinflation might have been thought to be more costly than inflation, so that it would be better not to reduce inflation or to do so substantially gradually (Taylor, 1997). However, given fear about the possibility of unanchored structure of expectations and the permanent double-digit inflation, fighting inflation by inducing a significant recession actually became the Fed's mandate in 1979 (De Long, 1997). This implies a greater independence of the Fed since then. In addition, Taylor (1992) argues that changes in the perceptions of how the economy works at the early 1980s, which rejects the trade-off view between unemployment and inflation, provided more impetus to curb inflation. For these reasons, the post-1980 monetary policy less likely created inflationary bias than it used to do in the 1970s.

We continue with the interpretation of the post-1990 conduct of monetary policy. In short, the inflation coefficient began to fall at the beginning of the 1990s, becoming statistically insignificant by the mid-1990s (Figure 3.4), while the response to real activity increased substantially. These features appear to be consistent with the opportunistic approach to disinflation in conducting monetary policy. The idea of this approach is that if inflation stays within a range around a target, the interest rate should not respond to inflation, but rather should wait for external circumstances to bring inflation back to the target. In this case, the focus is on stabilizing output (Orphanides and Wilcox, 2002).

Although the response to inflation variance has been insignificant since the early 1980s, its point estimate seems to be relevant to the opportunistic approach as well. Specifically, the estimate increased substantially from the mid-1990s and has been mainly above zero since the early 2000s (Figure 3.5) (with the HP output gap, Figure 3.10, it has even become positive since the mid-1990s). This finding is apparently consistent with an aspect of the opportunistic approach that the variance of shocks matter. If the variance of shocks were to increase, the speed of convergence to the long-run target would increase as well (Aksoy

et al., 2006).

The opportunistic approach is first described by President Boehne of the Federal Reserve Bank of Philadelphia during the FOMC meeting in December 1989:

Now, sooner or later, we will have a recession. I don't think anybody around the table wants a recession or is seeking one, but sooner or later we will have one. If in that recession we took advantage of the anti-inflation (impetus) and we got inflation down from 4 1/2 percent to 3 percent, and then in the next expansion we were able to keep inflation from accelerating, sooner or later there will be another recession out there. And so, if we could bring inflation down from cycle to cycle just as we let it build up from cycle to cycle, that would be considerable progress over what we've done in other periods in history (Federal Reserve Board, 1989, p.19).

Alan Blinder, a former Vice Chairman of the Fed, also testified this issue before the Senate committee in 1994:

If monetary policy is used to cut our losses on the inflation front when luck runs against us, and pocket the gains when good fortune runs our way, we can continue to chip away at the already low inflation rate (Blinder, 1994, p.4).

The timing of these statements coincides with changes in monetary policy in the post-1990 period, including the insignificant response of monetary policy to inflation. Regarding the empirical evidence, Martin and Milas (2010) also point out that expected inflation since the early 1990s seldom moved away from the zone of inaction.

The insignificant response to inflation can be also explained from the statistical perspective. If inflation was close to the implicit target in the post-1990 period as suggested by Martin and Milas (2010), it might be hard to identify the response of interest rate to inflation due to lack of variability.

3.5 Conclusion

This study analyzed how the conduct of U.S. monetary policy has changed since the late 1960s by accounting for the four issues that have been highlighted as important in the literature, including the type of time-variation in policy parameters, the treatment of heteroscedasticity, the real-time nature of data, and the role of asymmetric preferences. The findings show that monetary policy since the late 1960s has evolved richly in terms both of the response to inflation and real activity and of preferences. Specifically, the Fed behaved like having asymmetric preferences, which induced inflation bias, in the pre-Volcker period, but changed to symmetric preferences in the post-Volcker era. Regarding the response to inflation, it was strong in the first half of the 1970s and the 1980s, but weak elsewhere. Meanwhile, the response to real activity was seemingly weaker in the 1980s than in other periods. The properties and timings of these changes, which suggests a nontrivial role of monetary policy in economic performances, are highly consistent with the literature on the evolution of monetary policy.

Chapter 4

Financial Frictions and the Volatility of Monetary Policy Shocks

4.1 Introduction

The role of financial frictions in business cycles has been attracting the interest of both academics and policy makers, especially after the recent financial crisis. The seminal work of Bernanke et al. (1999) develops a framework combining nominal rigidities with an agency cost model and argues that endogenous developments in the credit market can significantly amplify and propagate shocks to the economy through the financial accelerator mechanism. The core of this mechanism lies at the negative relationship between the net worth of firms and the external premium demanded by lenders. With respect to monetary policy, their model shows that an unanticipated increase in the nominal interest rate decreases the demand for capital and therefore causes a fall in its price. The decline in the value of capital reduces entrepreneurs' net worth and thus leads to a higher external premium, which further lowers investment and output. Christensen and Dib (2008) and Christiano et al. (2010), among others, provide quantitative evidence to support the financial accelerator and assert that financial frictions play a significant role in transmitting monetary policy disturbances

to the real economy.¹

This work investigates further the interaction between financial frictions and monetary policy. However, our attention is directed to the impact of changes in the volatility of monetary policy on the economy instead of those in its level. Shifts in the volatility of monetary policy are important because they relate to monetary policy uncertainty which has been a pivotal theme in policy discussions, especially after the recent financial crisis. For example, hawks and doves at the Federal Reserve System have argued about the extent of quantitative easing and the appropriate monetary stance given opposing signals from core and headline inflation measures (Born and Pfeifer, 2014). Furthermore, an increasing number of studies (for instance, Fernández-Villaverde et al., 2010*a*; Justiniano and Primiceri, 2008; Mumtaz and Zanetti, 2013) have shown that the volatility of monetary policy shocks has changed substantially in the U.S. Specifically, it was large during the Great Inflation of the mid 1970s and early 1980s, became mild after the mid 1980s and increased significantly during the recent crisis (Mumtaz and Zanetti, 2013).

In order to model changes in the volatility of shocks, the literature has proposed three alternatives: stochastic volatility, GARCH, and Markov regime switching models. A detailed comparison between these approaches is reported in Fernández-Villaverde and Rubio-Ramírez (2010). We use the first method following most of the literature on macroeconomics and volatility (for example, Arellano et al., 2010; Born and Pfeifer, 2014; Cesa-Bianchi and Fernandez-Corugedo, 2014; Fernández-Villaverde et al., 2010*a*; Gilchrist et al., 2014; Justiniano and Primiceri, 2008). With this specification, there are two types of shocks relating to monetary policy: one affects the level of the interest rate (first moment shocks or structural shocks or level shocks) and the other affects the standard deviation of the interest rate (second moment shocks or volatility shocks). Note that we assume the nominal interest rate to be the only instrument of monetary policy, as opposed to a monetary supply aggregate, following Smets and Wouters (2007). This assumption appears to be reasonable to describe U.S. monetary policy (Clarida et al., 1999).

¹Christensen and Dib (2008) estimate a dynamic stochastic general equilibrium model with the Bernanke-Gertler-Gilchrist financial frictions for the U.S. economy, while Christiano et al. (2010) consider both the Euro Area and the U.S.

We incorporate the stochastic volatility of monetary policy into a sticky-price DSGE model embedded with the financial frictions à la Bernanke et al. (1999). We also allow time-variation in the standard deviations of other structural innovations, including those of government spending innovations, investment-specific technology innovations, and technology innovations, in order to capture aggregate dynamics. The diverse sources of volatility in our study are desirable as argued by the growing literature on the role of volatility in business fluctuations such as Sims and Zha (2006) and Justiniano and Primiceri (2008) among others. Moreover, Hamilton (2008) shows that even if the object of interest is in the conditional mean, correctly modeling time-varying volatility can still be quite important. Stochastic volatility has been mostly ignored in the literature on financial frictions though.

Our work is related to the studies on the aggregate effects of uncertainty. Although this strand has been rapidly growing since the recent financial crisis (for instance, Alexopoulos and Cohen, 2009; Bachmann and Bayer, 2011; Bloom, 2009; Bloom et al., 2012; Popescu and Smets, 2010), there are only few studies on the effects of policy uncertainty.² Mumtaz and Zanetti (2013) estimate an SVAR model for the U.S. economy and show that an increase in the volatility of monetary policy leads to a fall in output growth. The authors also calibrate a simple DSGE model enriched with the time-varying standard deviation of monetary policy shocks in order to match with and provide an interpretation of SVAR results. Born and Pfeifer (2014) consider both fiscal and monetary uncertainty in a DSGE model and conclude that policy risk has an adverse effect on output. This result is also supported by Fernández-Villaverde et al. (2013) and Fernández-Villaverde et al. (2011). The role of financial frictions are not considered in these models though.

The present study is one of the few that integrates volatility and financial frictions, which are the two important issues emerging from the crisis, into a united framework to analyze macroeconomic dynamics. We briefly review this branch as follows. Dorofeenko et al.

²The influential paper of Bloom (2009) shows that jumps in uncertainty in response to major economic and political shocks cause firms to pause their investment and hiring, leading to a fall in productivity growth and then in output and employment. Alexopoulos and Cohen (2009) and Bloom et al. (2012) affirm that an increase in the uncertainty results in a sharp drop and slow recovery in GDP. In contrast, Bachmann and Bayer (2011) argue that uncertainty is unlikely to be a major quantitative source of business cycles. Popescu and Smets (2010) report similar results with Bachmann and Bayer (2011).

(2008) extend the Carlstrom and Fuerst (1997) agency cost model to study the effect of the volatility of firm's idiosyncratic productivity shocks and show that an increase in the volatility leads to a fall in investment supply. Christiano et al. (2014) consider a so-called risk shock in an estimated DSGE model incorporating the Bernanke et al. (1999) financial frictions and find that an increase in this shock reduces consumption, investment, and output. Moreover, they argue that this shock plays the most important role in driving the U.S. business cycles over the 1985-2010 period. Arellano et al. (2010) build a model with heterogeneous firms and financial frictions and find that increases in uncertainty at the firm level cause a large increase in the dispersion of growth rates across firms and a contraction in economic activity. Gilchrist et al. (2014) consider a model with heterogeneous firms, partial investment, irreversible, nonconvex capital adjustment costs, and financial frictions in both the debt and equity markets. The authors document the negative effects of firm level uncertainty shocks on the economy and argue that credit spreads are an important channel through which uncertainty shocks affect the economy. Cesa-Bianchi and Fernandez-Corugedo (2014) investigate the impacts of two different types of uncertainty shocks: TFP and firm level uncertainty. They find that the latter has a greater impact on economic activity because it is greatly magnified by credit frictions. Finally, Bonciani and Van Rye (2013) consider the volatility of TFP and show that financial frictions amplify the effect of uncertainty on the economy.

Our work differs from the above papers in two important aspects. First, we are, to our best knowledge, the first to investigate the interaction between financial frictions and policy uncertainty. Second, the parameters of exogenous processes of volatility in our study are jointly estimated with other parameters of the model instead of being calibrated as common in this strand of the literature. Note that a few papers have used proxies for uncertainty shocks to estimate those parameters separately while calibrating other parameters of the model- an approach that differs from the one applied in this study.

Regarding the estimation, likelihood-based inference is a useful tool to take DSGE models to the data (An and Schorfheide, 2007). However, those models mostly do not imply a likelihood function that can be calculated numerically or analytically. Therefore, the model

must be solved before it can be estimated. Linear approximation methods are very popular because they result in a linear state-space representation of the model whose likelihood can be obtained by the Kalman filter (An and Schorfheide, 2007). Nevertheless, in a linearized version of our model, stochastic volatility would drop, canceling any possibility of studying its impacts on the real economy. We therefore have to solve the model to higher-order approximations. This solution however leads to a non-linear state-space representation so that the Kalman filter can not be utilized to evaluate the likelihood function. To overcome this issue, Fernández-Villaverde and Rubio-Ramírez (2007) propose to use the particle filter which performs sequential Monte Carlo estimation using a point mass representation of probability densities. Fernández-Villaverde et al. (2015) apply the method to estimate a DSGE model with stochastic volatility. Following these studies, we take advantages of the particle filter to evaluate the likelihood function in a maximum likelihood framework. We use U.S. data for the estimation.

The results first show that our model captures aggregate dynamics relatively well. Second, we find that an increase in the volatility of monetary policy shocks causes a contraction in consumption, investment, output and hours worked. The model is therefore successful in generating business-cycle co-movements among key macroeconomic variables, suggesting that monetary policy uncertainty might have played a certain role in business cycles (Basu and Bundick, 2012). Moreover, this contractionary effect resembles the findings of Mumtaz and Zanetti (2013) and Born and Pfeifer (2014). Most importantly, we argue that financial frictions amplify and propagate the transmission of volatility shocks to the economy through the financial accelerator mechanism. This finding is in line with Gilchrist et al. (2014) and Bonciani and Van Roye (2013).

The rest of the study is organized as follows. Section 4.2 presents the baseline DSGE model. Section 4.3 shows the state-space representation of the model. In section 4.4, we present the estimates of model parameters and of structural and volatility shocks. Section 4.5 analyzes impulse response functions. Finally, section 4.6 concludes.

4.2 The DSGE Model

Our model is a cashless-limited closed-economy New Keynesian DSGE model that incorporates the financial-accelerator mechanism proposed by Bernanke et al. (1999). In this small-sized model of the economy, there are five agents: households, capital producers, entrepreneurs, retailers and policy authorities. Households make decisions on consumption and hours worked to maximize their utilities subject to their intertemporal budget constraints. Capital producers transform the investment component of output into new capital goods which replace depreciated capital and add to capital stock. Entrepreneurs produce wholesale goods. They borrow from financial intermediates to cover for the difference between the expenditure on new capital and their net worth. Because of imperfect information between entrepreneurs and lenders, the former faces an external finance premium that rises when their leverage increases. This is how financial frictions are incorporated into the model. Retailers are introduced to motivate sticky prices. They buy the wholesale goods from the entrepreneurs, transform them into differentiated goods, and set prices in the Calvo type. Finally, authorities conduct both monetary and fiscal policy. The nominal interest rate, which is supposed to be the only tool of monetary policy, follows a Taylor rule that responds to the deviations of inflation and output from their steady states. Regarding fiscal policy, government spending is financed by lump-sum taxes.

Although our main interest is on monetary policy innovations, we include technology innovations, investment specific technology innovations, and government spending innovations into the model to capture aggregate dynamics. All standard deviations are assumed to be time-varying following an AR(1) process. Consequently, there are four structural shocks and four volatility ones brought into the model, which makes the number of shocks driving the economy eight.

4.2.1 Households

The representative household chooses consumption C_t , the amount of risk-less bonds B_{t+1} and hours worked h_t to maximize the following lifetime utility function

$$E_t \sum_{k=0}^{\infty} \beta^k \left(\log(C_{t+k} - \chi C_{t+k-1}) - \varpi \frac{h_{t+k}^{1+\vartheta}}{1+\vartheta} \right), \quad (4.1)$$

where $\beta \in (0, 1)$ is the discount factor, χ controls habit persistence, ϖ controls the level of labor supply, and ϑ is the inverse of the Frisch elasticity. Moreover, C_t is the consumption index given by

$$C_t = \left(\int_0^1 C_t(i)^{1-\frac{1}{\zeta}} di \right)^{\frac{\zeta}{\zeta-1}}, \quad (4.2)$$

where ζ is the elasticity of substitution and $C_t(i)$ represents the quantity of good i consumed by the household in period t . We assume the existence of a continuum of goods represented by the interval $[0, 1]$.

Maximization of (4.1) is subject to a sequence of flow budget constraints given by

$$\frac{\int_0^1 P_t(i) C_t(i) di}{P_t} + \frac{B_{t+1}}{P_t} \leq \frac{R_{n,t-1} B_t}{P_t} + \frac{W_t h_t}{P_t} + \text{Transfers} + \text{Profits}, \quad (4.3)$$

where $P_t(i)$ is the price of good i , B_{t+1} is the amount of risk-less bonds held between period t and period $t + 1$ which pay a nominal gross interest rate $R_{n,t}$ at maturity, and W_t is the wage rate. The household receives lump-sum transfers from the government and profits from firms. P_t is the aggregate price index given by

$$P_t = \left(\int_0^1 P_t(i)^{1-\zeta} di \right)^{\frac{1}{1-\zeta}}.$$

For each differentiated good i , the household must decide how to choose C_i to maximize (4.2) for any given level of expenditures $\int_0^1 P_t(i) C_t(i) di$. The first-order solution yields the

set of demand equations for consumption

$$C_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\zeta} C_t,$$

for all $i \in [0, 1]$. Thus,

$$\int_0^1 P_t(i) C_t(i) di = P_t C_t. \quad (4.4)$$

Substituting (4.4) into the budget constraint (4.3) results in

$$C_t + \frac{B_{t+1}}{P_t} \leq \frac{R_{n,t-1} B_t}{P_t} + \frac{W_t}{P_t} h_t + \text{Transfers} + \text{Profits}. \quad (4.5)$$

We then derive the first-order conditions for the household's problem as follows

$$\begin{aligned} \frac{1}{C_t - \chi C_{t-1}} - \beta E_t \left(\chi \frac{1}{C_{t+1} - \chi C_t} \right) &= \lambda_t, \\ \lambda_t &= \beta E_t \left(\lambda_{t+1} \frac{R_{n,t}}{\Pi_{t+1}} \right), \\ \varpi h_t^\vartheta &= \lambda_t \frac{W_t}{P_t}, \end{aligned}$$

where λ_t is the Lagrangian multiplier associated with the budget constraint in (4.5).

4.2.2 Capital Producers

Suppose that there is a single, representative, competitive capital producer who uses a portion of final goods purchased from retailers as investment goods I_t to produce capital goods. The production, which is subject to quadratic capital adjustment costs $S(\cdot)$ and an investment-specific technology shock κ_t , generates $e^{\kappa_t} (1 - S(\frac{I_t}{I_{t-1}})) I_t$ capital goods. These goods are sold at a real price Q_t per unit at the end of period t .

The adjustment cost function, similar to Smets and Wouters (2007) and Christiano et al. (2005), is specified as

$$S\left(\frac{I_t}{I_{t-1}}\right) = \phi_s \left(\frac{I_t}{I_{t-1}} - 1 \right)^2,$$

where ϕ_s is the adjustment parameter. Along the balanced growth path, $S(1) = S'(1) = 0$.

The investment specific technology shock is assumed to follow an AR(1) process

$$\kappa_t = \rho_\kappa \kappa_{t-1} + \sigma_\kappa e^{\sigma_{\kappa t}} \varepsilon_{\kappa t}, \quad \varepsilon_{\kappa t} \sim \mathcal{N}(0, 1),$$

where $\sigma_{\kappa t}$ is the time-variant component of the standard deviation of investment specific technology shock $\varepsilon_{\kappa t}$. Its evolution is given by

$$\sigma_{\kappa t} = \rho_{\sigma_\kappa} \sigma_{\kappa t-1} + \eta_\kappa u_{\kappa t}, \quad u_{\kappa t} \sim \mathcal{N}(0, 1).$$

The capital producer's optimization problem is to maximize its discounted profits with respect to I_t

$$E_t \sum_{k=0}^{\infty} \Lambda_{t,t+k} [Q_{t+k} e^{\kappa_{t+k}} (1 - S(X_{t+k})) I_{t+k} - I_{t+k}],$$

where $X_t = \frac{I_t}{I_{t-1}}$ and $\Lambda_{t,t+k}$ is the *real* stochastic discount factor over the interval $[t, t+k]$.

The first-order condition for this problem is

$$Q_t e^{\kappa_t} (1 - S(X_t) - X_t S'(X_t)) + E_t [\Lambda_{t,t+1} Q_{t+1} e^{\kappa_{t+1}} S'(X_{t+1}) X_{t+1}^2] = 1.$$

The produced capital goods combine with the existing capital stock to generate new capital goods. In other words, the capital accumulation process is described by

$$K_t = (1 - \delta) K_{t-1} + e^{\kappa_t} (1 - S(X_t)) I_t.$$

4.2.3 Entrepreneurs

Entrepreneurs manage the firms that produce the wholesale goods. This production uses labor and capital. While the former is supplied by both households and entrepreneurs, the latter is bought from capital producers. The entrepreneurs finance the expenditure on capital by entrepreneurial net worth (internal finance) and debts (external finance). In the latter, they face an external finance premium caused by the inability of lenders to monitor borrowers'

actions or to share borrowers' information. In this way, financial market imperfections are introduced into the model.

The premium relies on the balance-sheet condition of the entrepreneurs. When their net worth declines, internal sources of funds are limited, forcing them to seek external sources by borrowing. However, the deterioration of their balance sheets causes the potential divergence between them (the borrowers) and the lenders to be greater, leading to an increase in agency costs. Consequently, the cost of external finance is pushed up resulting in a contraction in investment spending and then output.

The entrepreneurs are risk neutral. They are endowed with h_t^e units of entrepreneurial labor at the nominal entrepreneurial wage W_t^e in order to start off. Moreover, each of them is assumed to survive until the next period with a probability σ_E . This is to assure that they do not accumulate enough funds to finance their expenditures on capital only with their net worth. New entrepreneurs are allowed to enter to replace those exiting.

Production. The wholesale goods are produced according to a constant-return-to-scale technology

$$Y_t^W = e^{a_t} A(H_t)^{1-\alpha} K_{t-1}^\alpha,$$

where K_{t-1} denotes the number of capital units, H_t is the labor input which is a composite of household labor h_t and entrepreneurial labor h_t^e , A is the level of technology which is normalized to one, and a_t is a shifter to the technology level which evolves as

$$a_t = \rho_a a_{t-1} + \sigma_a e^{\sigma_{at}} \varepsilon_{at}, \quad \varepsilon_{at} \sim \mathcal{N}(0, 1),$$

where σ_{at} is the time-variant component of the standard deviation of technology shock ε_{at} and it follows an AR(1) process

$$\sigma_{at} = \rho_{\sigma_a} \sigma_{at-1} + \eta_a u_{at}, \quad u_{at} \sim \mathcal{N}(0, 1).$$

For the labor input, h_t^e is assumed to be constant at one. In addition, the share of income going to the entrepreneurial labor is calibrated to be small (the order of 0.01), so that the

modification of the production function does not have substantial effects on the results. The labor input H_t is written as follows

$$H_t = h_t^\Omega (h_t^e)^{1-\Omega}.$$

The demand for household and entrepreneurial labor is obtained by equating the marginal product of each type of labor to its corresponding cost

$$\frac{P_t^W}{P_t} \frac{(1-\alpha)\Omega Y_t^W}{h_t} = \frac{W_t}{P_t}, \quad (4.6)$$

$$\frac{P_t^W}{P_t} \frac{(1-\alpha)(1-\Omega)Y_t^W}{h_t^e} = \frac{W_t^e}{P_t}. \quad (4.7)$$

Meanwhile, the demand for capital of the entrepreneurs is considered below with the occurrence of financial frictions.

Financial frictions. At the end of time t , an entrepreneur borrows l_t equivalent to the difference between the expenditure on new capital $Q_t k_t$ and the net worth $n_{E,t}$

$$l_t = Q_t k_t - n_{E,t}.$$

The net worth accumulation $n_{E,t}$ is calculated by³

$$n_{E,t} = \psi_t R_{k,t} Q_{t-1} k_{t-1} - R_{l,t} l_{t-1},$$

where ψ_t is an idiosyncratic shock to the entrepreneur's return,⁴ $R_{l,t}$ is the real loan rate set at time $t-1$, and $R_{k,t}$ is the real return on capital computed by

$$R_{k,t} = \frac{\alpha \frac{P_t^W}{P_t} \frac{Y_t^W}{K_{t-1}} + (1-\delta)Q_t}{Q_{t-1}}.$$

³Lower case variables denote the representative entrepreneur, while upper case variables introduced later denote the aggregate.

⁴The shock at $t+1$ is revealed at the end of period t right before investment decisions are made.

Note that the idiosyncratic shock is the private information of the entrepreneur. We follow Bernanke et al. (1999) to assume that ψ_t is distributed log-normally with positive support and its standard deviation is time-invariant. The distribution of ψ_t hence can be written as follows

$$\log(\psi_t) \sim \mathcal{N}(-1/2\sigma_\psi^2, \sigma_\psi^2),$$

where σ_ψ is the standard deviation of the idiosyncratic shock ψ_t .

At time $t + 1$, if the net worth $n_{E,t+1}$ becomes negative, the entrepreneur is bankrupt. In other words, the default occurs if the idiosyncratic shock falls below the cut-off value $\bar{\psi}_{t+1}$ given by

$$\bar{\psi}_{t+1} = \frac{R_{l,t+1}l_t}{R_{k,t+1}Q_tk_t}. \quad (4.8)$$

Otherwise, the entrepreneur makes the full payment of her loans, $R_{l,t+1}l_t$, to the lender.

Let f_{ψ_t} and ψ_{min} be the density function and the lower bound of ψ_t , respectively. Then, the probability of default at time $t + 1$ is calculated by

$$F(\bar{\psi}_{t+1}) = \int_{\psi_{min}}^{\bar{\psi}_{t+1}} f(\psi) d\psi.$$

If default happens, the lender obtains the assets of the firm. However, it has to pay a proportion μ to observe the realized return. Therefore, the expected gross return on the loan of the lender is given by

$$E_t \left[(1 - F(\bar{\psi}_{t+1}))R_{l,t+1}l_t + (1 - \mu)R_{k,t+1}Q_tk_t \int_{\psi_{min}}^{\bar{\psi}_{t+1}} \psi f(\psi) d\psi \right].$$

Substituting $R_{l,t+1}l_t$ by $\bar{\psi}_{t+1}R_{k,t+1}Q_tk_t$ (see (4.8)) yields

$$E_t \left[R_{k,t+1}Q_tk_t (\bar{\psi}_{t+1}(1 - F(\bar{\psi}_{t+1})) + (1 - \mu) \int_{\psi_{min}}^{\bar{\psi}_{t+1}} \psi f(\psi) d\psi) \right]. \quad (4.9)$$

Define $\Gamma(\bar{\psi}_{t+1})$ as the share of entrepreneurial earnings accrued to the lender

$$\Gamma(\bar{\psi}_{t+1}) = \bar{\psi}_{t+1}(1 - F(\bar{\psi}_{t+1})) + G(\bar{\psi}_{t+1}), \quad (4.10)$$

where

$$G(\bar{\psi}_{t+1}) = \int_{\psi_{min}}^{\bar{\psi}_{t+1}} \psi f(\psi) d\psi. \quad (4.11)$$

For the optimal contract, the entrepreneur needs to find k_t and $\bar{\psi}_{t+1}$ to maximize her expected net earnings

$$E_t [(1 - \Gamma(\bar{\psi}_{t+1}))R_{k,t+1}Q_t k_t], \quad (4.12)$$

subject to

$$E_t [R_{k,t+1}Q_t k_t (\Gamma(\bar{\psi}_{t+1}) - \mu G(\bar{\psi}_{t+1}))] = E_t (R_{t+1}^{ex} l_t). \quad (4.13)$$

The constraint reflects the assumption that the lender is indifferent between the expected return from lending to the entrepreneur and the one from owning risk-free bonds. Then, using the Lagrange multiplier method, we obtain

$$E_t [R_{k,t+1}] = E_t [\iota(\bar{\psi}_{t+1}) R_{t+1}^{ex}],$$

where $\iota(\bar{\psi}_{t+1})$ is the *premium on external finance* given by

$$\iota(\bar{\psi}_{t+1}) = \frac{\Gamma'(\bar{\psi}_{t+1})}{(1 - \Gamma(\bar{\psi}_{t+1}))(\Gamma'(\bar{\psi}_{t+1}) - \mu G'(\bar{\psi}_{t+1})) + \Gamma'(\bar{\psi}_{t+1})(\Gamma(\bar{\psi}_{t+1}) - \mu G(\bar{\psi}_{t+1}))}. \quad (4.14)$$

For the calculation of $\Gamma(\cdot)$, $\Gamma'(\cdot)$, $G(\cdot)$ and $G'(\cdot)$, see Appendix C.1.

So far we have established the optimizing decision of a representative entrepreneur. We now assume that a fraction $1 - \sigma_E$ of entrepreneurs exits at the end of period $t - 1$ and they consume all their residual equities. Therefore, the aggregate net worth accumulating at the end of time t is calculated by

$$N_{E,t} = \sigma_E (1 - \Gamma(\bar{\psi}_t)) R_{k,t} Q_{t-1} K_{t-1} + \frac{W_{E,t}}{P_t}$$

and the consumption of the exiting entrepreneurs is

$$C_{E,t} = (1 - \sigma_E)(1 - \Gamma(\bar{\psi}_t)) R_{k,t} Q_{t-1} K_{t-1}.$$

4.2.4 Retailers

In order to motivate sticky prices, two additional ingredients are added to the model. First, the retail sector is assumed to be monopolistically competitive. Second, there are costs of adjusting nominal prices.

Optimal Price Setting. Retailers purchase the wholesale goods from the entrepreneurs and transform them into differentiated goods according to

$$Y_t = \frac{Y_t^W}{\Delta_t},$$

where $Y_t = (\int_0^1 Y_t(i)^{1-\frac{1}{\zeta}} di)^{\frac{\zeta}{\zeta-1}}$ and $\Delta_t = \int_0^1 (\frac{P_t(i)}{P_t})^{-\zeta} di$ is the price dispersion. The retailers then set prices to optimize their expected profits. The setting is however constrained by the so-called Calvo-typed price rigidity (Calvo, 1983). Specifically, each retailer can reoptimize her price in a given period with a constant probability $1 - \xi$. The law of large number suggests that a fraction $1 - \xi$ of firms re-optimize their prices at each period. The remaining retailers are assumed to adjust their prices based on the lagged inflation with a degree of indexation $\gamma \in [0, 1]$ in order to capture the inertia observed in the response of inflation to a monetary policy shock (Woodford, 2003).

Given a common real marginal cost MC_t to all retail firms, a new price $P_t^*(i)$ chosen by the retailer i in period t should maximize her discounted nominal profits given by

$$E_t \sum_{k=0}^{\infty} \xi^k D_{t,t+k} Y_{t+k}(i) \left[P_t^*(i) \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^\gamma - P_{t+k} MC_{t+k} \right],$$

subject to the sequence of demand constraints

$$Y_{t+k}(i) = \left(\frac{P_{t+k}(i)}{P_{t+k}} \right)^{-\zeta} Y_{t+k}.$$

Note that $D_{t,t+k} = \beta^k \frac{\lambda_{t+k}}{\lambda_t}$ is the *nominal* stochastic discount factor over the interval $[t, t+k]$ and $k = 0, 1, 2, \dots$

The first order condition associated with the above problem has the form

$$E_t \sum_{k=0}^{\infty} \xi^k D_{t,t+k} Y_{t+k}(i) \left[P_t^*(i) \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^\gamma - \mathcal{M} P_{t+k} MC_{t+k} \right] = 0, \quad (4.15)$$

where $\mathcal{M} \equiv \frac{\zeta}{\zeta-1}$ is the frictionless markup. We rearrange $D_{t,t+k} Y_{t+k}(i)$ as follows

$$\begin{aligned} D_{t,t+k} Y_{t+k}(i) &= \beta^k \frac{\lambda_{t+k}}{\lambda_t} \frac{P_t}{P_{t+k}} \left[\frac{P_t^*(i)}{P_{t+k}} \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^\gamma \right]^{-\zeta} Y_{t+k} \\ &= \beta^k \frac{\lambda_{t+k}}{\lambda_t} \frac{P_t}{P_{t+k}} \left(\frac{P_t^*(i)}{P_{t+k}} \right)^{-\zeta} \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{-\zeta \gamma} Y_{t+k} \\ &= \beta^k \frac{\lambda_{t+k}}{\lambda_t} \left(\frac{P_t^*(i)}{P_t} \right)^{-\zeta} \Pi_{t,t+k}^{\zeta-1} \Pi_{t-1,t+k-1}^{-\zeta \gamma} Y_{t+k}, \end{aligned}$$

where $\Pi_{t,t+k} = \frac{P_{t+k}}{P_t}$. By substituting this rearrangement into (4.15), then canceling out $\left(\frac{P_t^*(i)}{P_t} \right)^{-\zeta}$ and multiplying by $\frac{\lambda_t}{\bar{P}_t}$ in (4.15), we get

$$E_t \sum_{k=0}^{\infty} (\xi \beta)^k \lambda_{t+k} \Pi_{t,t+k}^{\zeta-1} \Pi_{t-1,t+k-1}^{-\zeta \gamma} Y_{t+k} \left[\frac{P_t^*(i)}{P_t} \Pi_{t-1,t+k-1}^\gamma - \mathcal{M} \Pi_{t,t+k} MC_{t+k} \right] = 0.$$

This is equivalent to

$$\frac{P_t^*(i)}{P_t} E_t \sum_{k=0}^{\infty} (\xi \beta)^k \lambda_{t+k} Y_{t+k} \bar{\Pi}_{t,t+k}^{\zeta-1} = \mathcal{M} E_t \sum_{k=0}^{\infty} (\xi \beta)^k \lambda_{t+k} Y_{t+k} \bar{\Pi}_{t,t+k}^\zeta MC_{t+k}, \quad (4.16)$$

where $\bar{\Pi}_t = \frac{\Pi_t}{\Pi_{t-1}^\gamma}$. We now define

$$H_t = E_t \sum_{k=0}^{\infty} (\xi \beta)^k \lambda_{t+k} Y_{t+k} \bar{\Pi}_{t,t+k}^{\zeta-1} \quad (4.17)$$

and

$$J_t = \mathcal{M} E_t \sum_{k=0}^{\infty} (\xi \beta)^k \lambda_{t+k} Y_{t+k} \bar{\Pi}_{t,t+k}^\zeta MC_{t+k}. \quad (4.18)$$

From (4.17) and (4.18), we derive

$$H_t - \xi \beta E_t [\bar{\Pi}_{t+1}^{\zeta-1} H_{t+1}] = \lambda_t Y_t$$

and

$$J_t - \xi \beta E_t [\bar{\Pi}_{t+1}^\zeta J_{t+1}] = \mathcal{M} \lambda_t MC_t Y_t.$$

Combining (4.16), (4.17), and (4.18) yields

$$\frac{P_t^*(i)}{P_t} = \frac{J_t}{H_t}. \quad (4.19)$$

Aggregate Price Level Dynamics. Equation (4.19) implies that all the retailers that are resetting their prices will choose an identical price which is P_t^* . The aggregate price level at time t therefore evolves according to

$$P_t = \left[(1 - \xi) P_t^{*1-\zeta} + \xi \left(P_{t-1} \left(\frac{P_{t-1}}{P_{t-2}} \right)^\gamma \right)^{1-\zeta} \right]^{\frac{1}{1-\zeta}}. \quad (4.20)$$

Dividing both side of (4.20) by P_t results in

$$1 = (1 - \xi) \left(\frac{J_t}{H_t} \right)^{1-\zeta} + \xi \bar{\Pi}_t^{\zeta-1}.$$

4.2.5 The Central Bank

The model is closed by the presence of a central bank that sets the nominal interest rate according to a Taylor-type rule

$$\frac{R_{n,t}}{R_n} = \left(\frac{R_{n,t-1}}{R_n} \right)^{\rho_r} \left(\left(\frac{\Pi_t}{\bar{\Pi}} \right)^{\theta_\pi} \left(\frac{Y_t}{\bar{Y}} \right)^{\theta_y} \right)^{(1-\rho_r)} e^{\sigma_m e^{\sigma_{mt}} \varepsilon_{mt}}, \quad \varepsilon_{mt} \sim \mathcal{N}(0, 1),$$

where $\varepsilon_{m,t}$ is the monetary policy innovation whose time-varying component of the standard deviation σ_{mt} evolves according to an AR(1) process

$$\sigma_{mt} = \rho_{\sigma_m} \sigma_{mt-1} + \eta_m u_{mt}, \quad u_{mt} \sim \mathcal{N}(0, 1).$$

In the Taylor rule, the first term on the right-hand-side $\frac{R_{n,t-1}}{R_n}$ represents the smoothing behavior of the central bank in setting the interest rate. The second term $\frac{\Pi_t}{\bar{\Pi}}$ denotes the deviation

of inflation from its steady level Π . The third term $\frac{Y_t}{Y}$ is the output gap which is the deviation of output from its balanced state Y .

4.2.6 Resource Constraint

The market for final goods clears in every period

$$Y_t = C_t + C_{E,t} + I_t + Ge^{g_t} + \mu G(\bar{\Psi}_t)R_{k,t}Q_{t-1}K_{t-1}.$$

In that the government spending is financed by lump-sum taxes on the basis of a balanced budget. Government spending, whose steady state is G , is influenced by an exogenous shock g_t following an AR(1) process

$$g_t = \rho_g g_{t-1} + \sigma_g e^{\sigma_{g_t}} \varepsilon_{g_t}, \quad \varepsilon_{g_t} \sim \mathcal{N}(0, 1),$$

where ε_{g_t} is the government spending shock. The time-varying component of the standard deviation is σ_{g_t} whose evolution is given by

$$\sigma_{g_t} = \rho_{\sigma_g} \sigma_{g_{t-1}} + \eta_g u_{g_t}, \quad u_{g_t} \sim \mathcal{N}(0, 1).$$

4.3 State-Space Representation

4.3.1 State Transition Equations

The optimal decisions of households, capital producers, entrepreneurs, and retailers, the Taylor rule and the resource constraint form a non-linear rational expectations system. This system can not be estimated by likelihood-based approaches directly because the system does not imply a likelihood function that can be calculated numerically or analytically (Fernández-Villaverde and Rubio-Ramírez, 2007). Therefore, we need to solve the model before estimating it.

As regarded in the introduction, the most popular method in the literature is lineariza-

tion because it leads to a linear state space representation of the model whose likelihood can be obtained by the Kalman filter (An and Schorfheide, 2007). However, linearization is certainty-equivalent, which means that all volatility shocks will be dropped out, therefore canceling any chance of analyzing their impacts on the economy. In a second-order approximation, volatility shocks enter as cross-products with the corresponding level shocks in the policy functions. In a third-order approximation, volatility shocks play a role by themselves, thus allowing us to calculate the impulse response functions to a monetary volatility shock, while holding constant its level shock. This feature makes the third-order approximation very attractive, but it comes with high computational costs in the estimation, given that the particle filter is employed to obtain the likelihood function (see the computational issues of particle filters in Fernández-Villaverde and Rubio-Ramírez, 2007). In contrast, although the second-order approximation does not allow us to investigate the independent effects of volatility shocks, it is sufficient to estimate the parameters of the model including those of stochastic processes, while having smaller computational costs than the third-order approximation does. Fernández-Villaverde and Rubio-Ramírez (2007) and Fernández-Villaverde et al. (2015) also estimate dynamic macroeconomic models with stochastic volatility based on their second-order approximations. Therefore, we first follow those papers to estimate a second-order approximation of our DSGE model. Given the estimates, we then solve the model to a third-order approximation and compute the impulse response functions to a monetary volatility shock. By using such a strategy, we can take advantages of each method.⁵

Let s_t be the vector of all variables of the model at time t with each variable expressed in terms of log deviation from its steady state. The system is driven by the vector of structural shocks $v_t = (\varepsilon_{\kappa t}, \varepsilon_{at}, \varepsilon_{gt}, \varepsilon_{mt})$ and by the vector of volatility shocks $w_t = (u_{\kappa t}, u_{at}, u_{gt}, u_{mt})$. The solution of the rational expectations system takes the form

$$\mathbf{s}_t = \Theta(\mathbf{s}_{t-1}, v_t, w_t; \Xi), \quad (4.21)$$

where Ξ is the vector of parameters in the model. Equation (4.21) represents the state

⁵We apply the pruning procedure to avoid the explosion which often happens to paths simulated by the higher-order approximated model. See Kim et al. (2008) and Andreasen et al. (2013) for more details.

transition equations in the state-space representation, which is non-linear.⁶ The following part describes the measurement equations.

4.3.2 Measurement Equations

We assume that the time period t corresponds to one quarter. For the estimation, we use four data series including the Hodrick-Prescott output gap per capita, the log difference of the GDP deflator, the federal funds rate, and the Moody's seasoned data corporate bond yields, which are denoted by OUT_t , INF_t , INR_t , and CBY_t , respectively. Details on the sources and constructions of these time series are documented in Appendix C.2. These series are connected to the model variables by

$$\begin{aligned} INF_t &= \hat{\Pi}_t + \sigma_{m\pi} \varepsilon_{m\pi,t}, \quad \varepsilon_{m\pi,t} \sim \mathcal{N}(0, 1), \\ OUT_t &= \hat{Y}_t + \sigma_{my} \varepsilon_{my,t}, \quad \varepsilon_{my,t} \sim \mathcal{N}(0, 1), \\ INR_t &= \hat{R}_{n,t} + \sigma_{mr_n} \varepsilon_{mr_n,t}, \quad \varepsilon_{mr_n,t} \sim \mathcal{N}(0, 1), \\ CBY_t &= \hat{R}_{k,t} + \hat{\Pi}_t + \sigma_{mr_k} \varepsilon_{mr_k,t}, \quad \varepsilon_{mr_k,t} \sim \mathcal{N}(0, 1), \end{aligned}$$

where $\varepsilon_{my,t}$, $\varepsilon_{m\pi,t}$, $\varepsilon_{mr_n,t}$, and $\varepsilon_{mr_k,t}$ are measurement errors and their standard deviations are σ_{my} , $\sigma_{m\pi}$, σ_{mr_n} and σ_{mr_k} , respectively. The notation “ $\hat{\cdot}$ ” above a variable denotes the log deviation of that variable from its steady state. These four measurement equations and the state transition equations in (4.21) establish the non-linear state-space representation of the model.

⁶For example, a second-order approximation is given by

$$\begin{aligned} \mathbf{s}_{j,t} &= C_j + \sum_{i=1}^J \Theta_{j,i}^{(s)} s_{i,t-1} + \sum_{l=1}^n \Theta_{j,l}^{(v)} v_{l,t} + \sum_{l=1}^n \Theta_{j,k}^{(v2)} v_{l,t}^2 + \sum_{i=1}^J \sum_{l=1}^J \Theta_{j,il}^{(ss)} s_{i,t-1} s_{l,t-1} \\ &+ \sum_{i=1}^J \sum_{l=1}^n \Theta_{j,il}^{(sv)} s_{i,t-1} v_{l,t} + \sum_{l=1}^n \Theta_{j,ll}^{(vw)} v_{l,t} w_{l,t}. \end{aligned}$$

4.4 Estimation

In order to estimate the non-linear state-space system described in the previous section, we follow Fernández-Villaverde and Rubio-Ramírez (2007) to use the particle filter to evaluate its likelihood function. Basically, the particle filter performs sequential Monte Carlo estimation using a point mass representation of probability densities to approximate the posterior density of the states and the likelihood function (see Appendix C.3 for the algorithm of the particle filter). As discussed above, we use the four quarterly U.S. time series for the estimation. Regarding the coverage of the sample, while including the post-2007 could be beneficial because of the increased observations, it would introduce extra problems originating from the recent crisis and its on-going consequences, among which is the zero-lower bound of the interest rate. Given the inherent complexity in the estimation of a higher-order approximated model, a more ‘safe and sound’ solution is to exclude the post-2007 period. Our sample therefore spans from 1959Q1 to 2007Q1. Advancing the model to include the recent crisis episode into consideration is a potential expansion of our work.

We summarize the procedure of the estimation in three steps. First, given the values of parameters, we solve the non-linear rational expectations system by performing a second-order perturbation around the deterministic steady states. Second, we construct the state-space representation of the model and apply the particle filter to evaluate its likelihood. Finally, we use an maximum-likelihood algorithm to estimate parameters.

We are aware that obtaining the MLE is complicated because the shape of the likelihood function may be rugged and multimodal. In addition, the use of optimization algorithms based on derivatives is not applicable because the particle filter generates an approximation to the likelihood function that is not differentiable with respect to parameters. Instead, we follow van Binsbergen et al. (2012) to use the covariance matrix adaption evolutionary strategy, whose aim is to cope with objective functions which are non-linear, non-convex, multimodal, as well as those with other difficult conditions, in order to obtain the maximum-likelihood estimates.

As customary when taking DSGE models to data, some parameters are fixed to values which are common in the existing literature or selected to satisfy some certain conditions in

the steady state (for instance, Fernández-Villaverde et al., 2009; Justiniano and Primiceri, 2008; Smets and Wouters, 2007). This helps to reduce the numbers of parameters required to estimate, therefore lessening the computational issues. We discuss those fixed parameters in subsection 4.4.1. The estimates of unknown parameters are presented in subsection 4.4.2. Given the values of all parameters, we perform particle filtering to compute the posterior densities of structural and volatility shocks and then estimate their means. Combining these estimates over the sample shows us the evolution of structural and volatility shocks. They are presented in subsection 4.4.3.

4.4.1 Fixed Parameters

The standard parameters calibrated includes $\{\beta, \zeta, \alpha, \Omega, \vartheta, \phi_s, \chi, \varpi, \delta\}$. The discount factor $\beta = 0.985$ is chosen to match the inverse of the average of risk-free rate observed in the U.S. The elasticity of substitution ζ is fixed at 10 which implies a 10% markup. The elasticity of capital to output $\alpha = 0.3$ reflects the share of national income that goes to capital. As mentioned previously, the share of income to entrepreneurial labor $(1 - \alpha)(1 - \Omega)$ is set to a very small number 0.01, which implies a value of 0.98 for Ω . The depreciation rate δ is assigned to 0.025, which is a common value in the literature of DSGE models on the U.S. economy. The inverse of the Frisch labor elasticity ϑ is set to 1.3 which pins down the Frisch elasticity to around 0.75 as suggested by Chetty et al. (2011). The adjustment cost $\phi_s = 4.5$ is similar to other estimates from DSGE models, for example, Fernández-Villaverde (2010). The habit persistence χ is set to 0.9 in order to reflect the observed sluggish response of consumption to shocks (Fernández-Villaverde et al., 2010a). The steady state government spending to GDP ratio G/Y is fixed at 0.2 to match the U.S. data on average. The parameter controlling the level of labor supply ϖ is calibrated in such a way that generates a steady state level of hours worked $h = 0.35$.

We also calibrate three non-standard financial parameters including $\{\mu, \sigma_\psi, \sigma_E\}$. They are chosen to imply the three following conditions in the steady state: (i) a probability of default equal to 3%, (ii) a credit spread of 66.5 basis points which is consistent with the data over the sample, and (iii) a ratio of capital to net worth QK/N of 2. Specifically, the

fraction of realized payoffs lost in bankruptcy μ is 0.0555, the existing rate of entrepreneurs σ_E is found to be 0.9708, and the steady state level of the variance of the idiosyncratic productivity variable σ_ψ is equal to 0.3388.

4.4.2 Parameter Estimates

Table 4.1 reports the estimates for the remaining 24 parameters. First, the degree of indexation γ is 0.2 implying a moderate inflation inertia. The price rigidity ξ is around 0.7 which suggests that the prices are reoptimized approximately once every three quarters. This value is common in the literature, see e.g., Smets and Wouters (2007). Regarding the estimates of policy parameters, the response to the deviation of inflation in the long run is about 1.560, which is close to the estimate of Christensen and Dib (2008) in a linearized DSGE model with financial frictions. In contrast, the interest rate does not appear to respond strongly to changes in the output gap. Given a 1% increase in the output gap, the interest rate only rises about 7 basis points. Smets and Wouters (2007) also document a weak response to the output gap (0.09). Finally, the interest rate shows a moderate inertia with the smoothing parameter ρ_r around 0.6.

Turning to the stochastic processes of structural shocks, they appear to be considerably persistent with estimated $AR(1)$ coefficients equal to 0.962, 0.978, and 0.959 for the investment-specific technology, technology, and government spending process, respectively. For the time-invariant component of the standard deviations of structural shocks, the government shock has the largest value of 0.04. The smallest figure is for the investment-specific shock, roughly 0.0006.

Regarding the stochastic volatility processes, the standard deviation of technology shock is the most persistent with an estimated $AR(1)$ coefficient of 0.980, followed closely by the coefficient of monetary shock, 0.971. The standard deviation of government shock is found to be fairly persistent with a coefficient of 0.624. Meanwhile, the corresponding value for the investment-specific technology shock is the least persistent, 0.424. For the standard deviations of volatility shocks, we find that those of investment-specific technology and government spending innovations are similar with a value of about 0.35 for each. Mean-

Table 4.1 Parameters' Estimates of the DSGE model

Parameter	Mean	S.E. ($\times 10^{-2}$)
Nominal rigidities parameters		
γ	0.203	0.308
ξ	0.708	0.162
Policy parameters		
ρ_r	0.597	0.189
θ_π	1.560	0.400
θ_y	0.069	0.114
Parameters of the stochastic process for structural shocks		
ρ_κ	0.962	0.101
ρ_a	0.978	0.092
ρ_g	0.959	0.159
σ_κ	0.060×10^{-2}	0.020
σ_a	0.881×10^{-2}	0.013
σ_m	0.188×10^{-2}	0.004
σ_g	4.130×10^{-2}	0.039
Parameters of the stochastic process for volatility shocks		
ρ_{σ_κ}	0.424	0.741
ρ_{σ_a}	0.980	0.685
ρ_{σ_m}	0.971	0.482
ρ_{σ_g}	0.624	0.651
η_κ	0.352	2.239
η_a	0.168	2.000
η_m	0.163	2.058
η_g	0.337	2.350
Parameters of measurement equations		
σ_{my}	0.338×10^{-2}	0.011
$\sigma_{m\pi}$	0.412×10^{-2}	0.012
σ_{mr_n}	0.154×10^{-2}	0.010
σ_{mr_k}	0.434×10^{-2}	0.012

Notes: The table shows the estimates of parameters in the baseline DSGE model in section 4.2.

while, the corresponding values of monetary and technology innovations are close to each other, 0.17.

Finally, we find that the standard deviations of measurement noises are small, suggesting that the model captures the aggregate dynamics relatively well. To corroborate the statement, we plot the actual data and the data generated by the model (filtered states) in

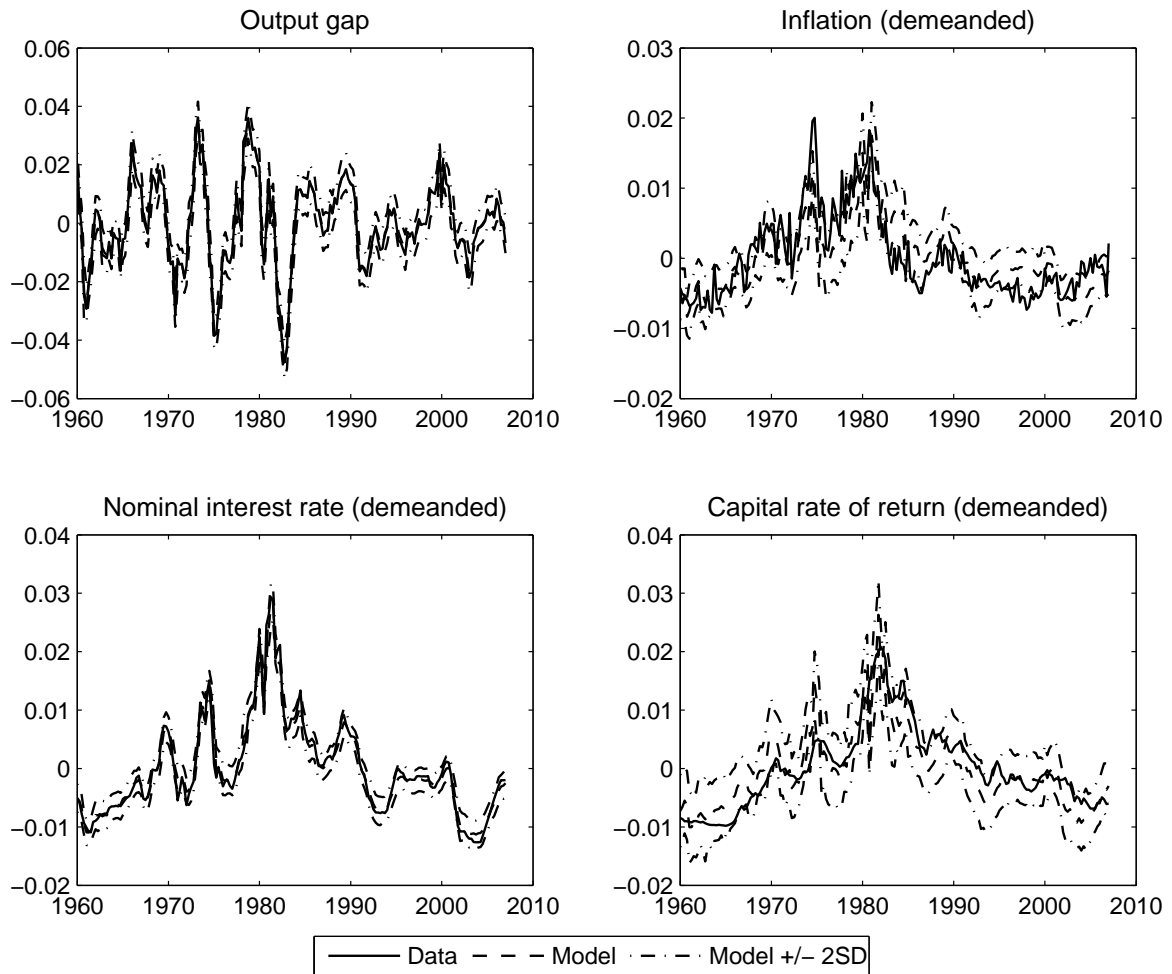


Fig. 4.1 Model vs. Data

Notes: The graphs show the data generated by the DSGE model with 2 SD bounds and the actual data.

Figure 4.1. The top left graph shows that the model captures much of the dynamics of the real output gap *per capita*. The model value has a correlation of 99% with the data. The standard deviation of the former and the latter are of equal magnitude, 0.015. The top right plot displays the actual data and the generated data for inflation. The correlation between them is 80% and their standard deviations are similar around 0.005. The bottom left graph depicts the actual observation and the one created by the model for the nominal interest rate, it appears that the model replicates the data very well with a correlation of 99% and a standard deviation of 0.008 for each. Finally, the actual data and the value produced by

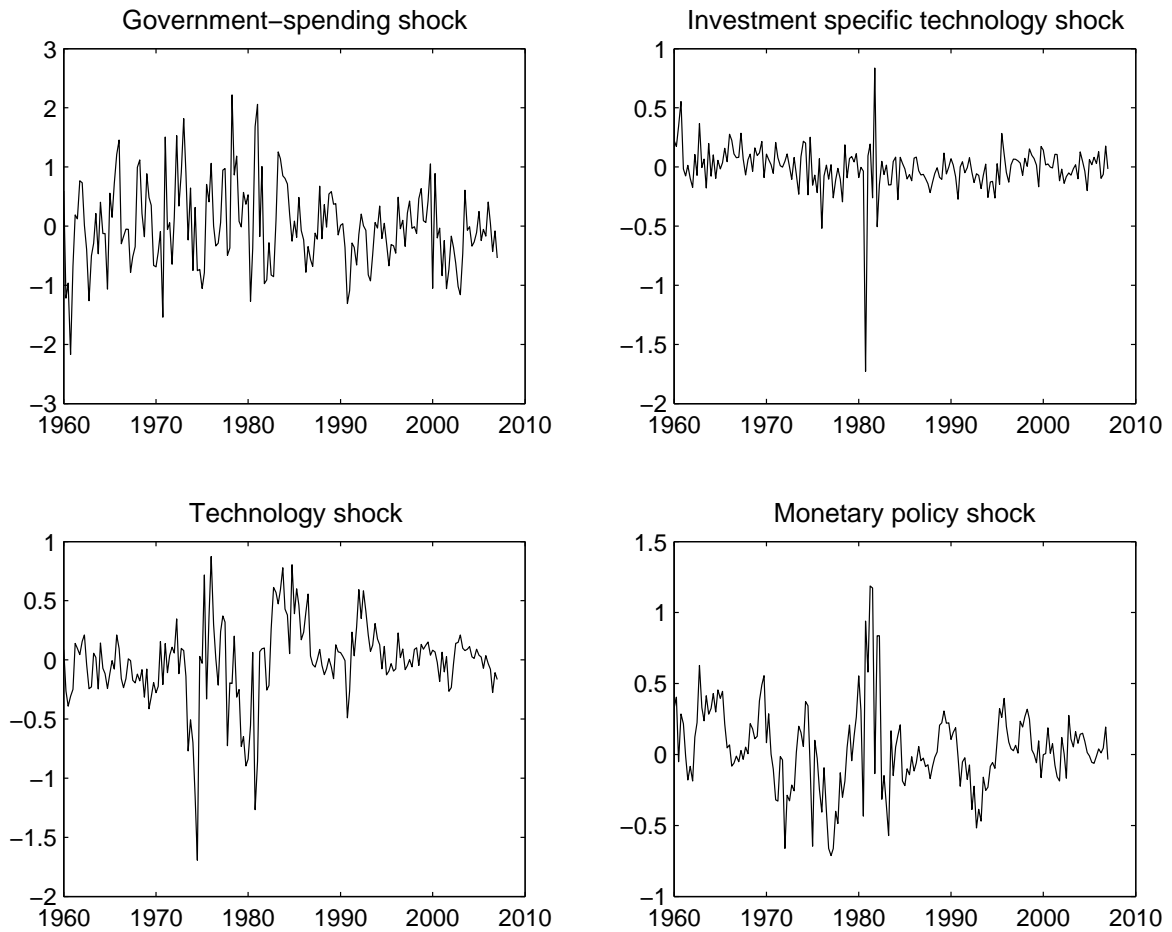


Fig. 4.2 Structural Shocks

Notes: The graphs present the estimates of structural shocks. They are obtained by performing particle filtering to compute the posterior densities of the shocks given the values of parameters. Combining all the estimates of their means over the sample provides us the measures of the shocks.

the model for the nominal rate of return on capital are displayed in the bottom right graph. Their correlation is 88% and they have similar standard deviations of 0.007. Based on these evidence, we conclude that the model is fairly successful in characterizing the properties of the economy.

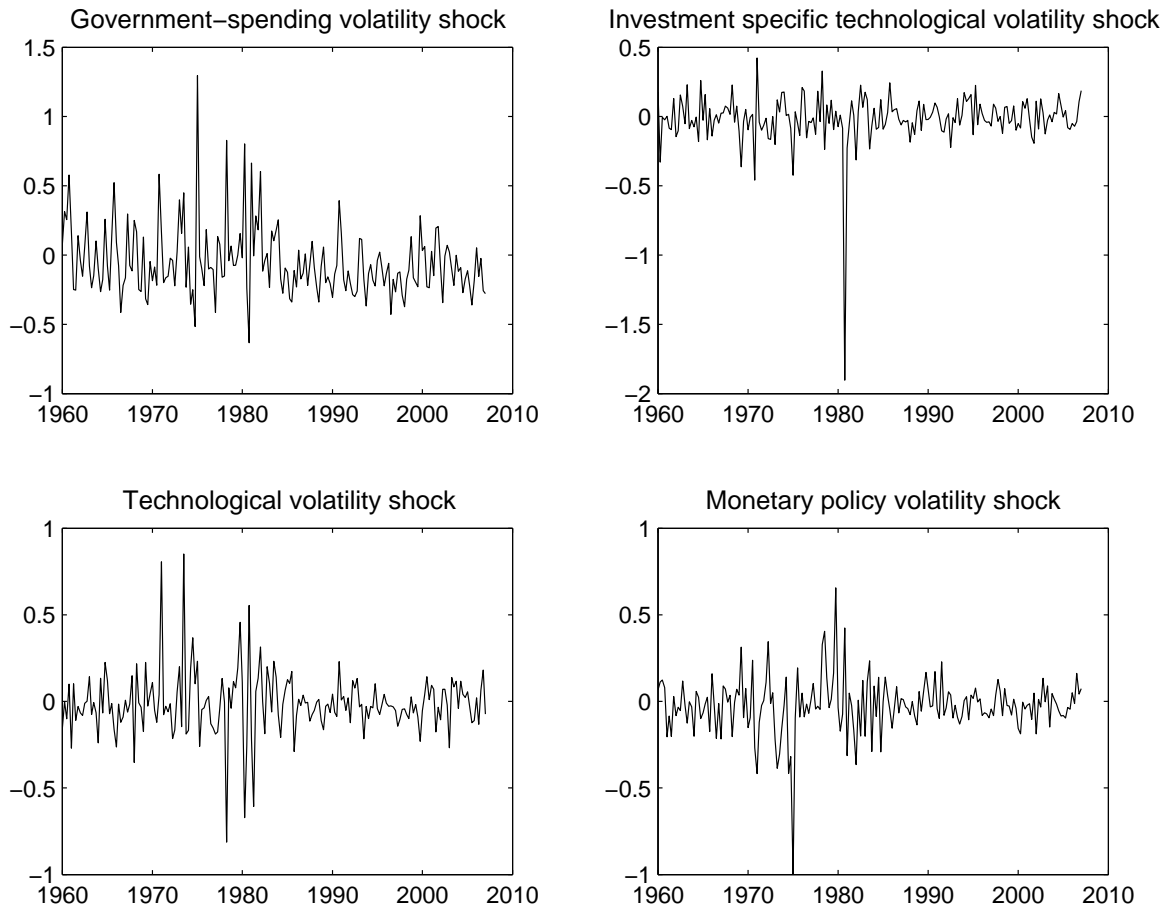


Fig. 4.3 Volatility Shocks

Notes: The graphs present the estimates of volatility shocks. They are obtained by performing particle filtering to compute the posterior densities of the shocks given the values of parameters. Combining all the estimates of their means over the sample provides us the measures of the shocks.

4.4.3 The Evolution of Structural and Volatility Shocks

In this subsection, we present the estimates of the structural shocks and the volatility shocks of the model. This exercise has been done in models without financial frictions, for instance Fernández-Villaverde et al. (2010a) and Justiniano and Primiceri (2008).

Figure 4.2 reports the evolution of structural shocks (ε_{mt} , ε_{at} , ε_{gt} , and ε_{kt}). The figure shows that our model is successful in capturing striking features documented in the literature. First, there are two clear drops in the technology shocks in 1972 – 1974 and

1980 – 1981 and one substantial reduction in the investment-specific technology shocks in 1980 which are likely the consequences of the oil price shocks. Second, regarding the monetary policy shocks, our model shows large fluctuations in the first half of the 1980s which might be caused by fast changes in the policy by the Fed chairman Paul Volcker.

The volatility shocks are plotted in Figure 4.3. One common feature is that the shocks were higher in the 1970s and early half of the 1980s than in other periods. This result therefore asserts Blanchard and Simon (2001)'s observation that volatility had fallen in the 20th century with a temporal and surprising rise in the 1970s. Especially, the volatility shocks have substantially declined since the middle of the 1980s, around 1984. McConnell and Perez-Quiros (2000) and Kim and Nelson (1999) also document a decline in the volatility of U.S real GDP growth around this point in time. Stock and Watson (2002) consider 1984 as the start of the 'Great Moderation' period in the U.S economy. Our results therefore suggest that the fall in the magnitude of shocks might have contributed to the stability during the Great Moderation period in the U.S., in accordance with Born and Pfeifer (2014) and Justiniano and Primiceri (2008).

4.5 Impulse Response Functions

This section analyzes the impulse response functions (IRFs) generated by our model to a 1 S.D. monetary volatility shock u_{mt} . There are two issues deserving discussion. First, recall that in the second order approximation the volatility shocks enter policy functions in the cross-product with the corresponding level shocks, e.g. $u_{mt}\varepsilon_{mt}$. This connection prevents us from disentangling the impact of volatility shocks on the economy. To overcome this issue, we solve the model to the third-order approximation, given the parameters estimated in the previous section, because at that order volatility shocks play a role by themselves, therefore allowing us to compute the IRFs to a second-moment shock of monetary policy while keeping its level shock unchanged.

The second issue is that the higher-order approximation of the model not only results in a nonlinear environment, which makes the computation of IRFs somewhat complicated,

but also makes the simulated paths of states and controls in the model move away from their state values. To deal with these issues, we follow the process proposed by Fernández-Villaverde et al. (2011) which calculates the IRFs as percentage deviations from their ergodic means rather than their steady states. This process includes four following steps.

1. The model is simulated for 2096 periods. The first 2000 periods are disregarded as a burn-in.
2. We calculate the mean for each variable based on the last 96 periods. Adding more periods does not essentially affect the mean.
3. Starting from the mean and in the absence of shocks, we hit the model with a one-standard-deviation second-moment shock of monetary policy u_{mt} .
4. The impulse responses are defined as percentage deviations from the variables' means.

Figure 4.4 plots the IRFs to a positive 1 S.D. monetary volatility shock. This shock causes a prolonged contraction in economic activity: output, consumption, investment, real wages and hours worked fall. Our model is therefore successful in generating business-cycle co-movements in response to changes in the uncertainty of monetary policy. This feature is an important prerequisite for any shock that seeks to explain business cycles because those co-movements are observed in the data (see Basu and Bundick, 2012; Cesa-Bianchi and Fernandez-Corugedo, 2014).

The principal transmission mechanism for monetary volatility shocks is in line with Basu and Bundick (2012). The uncertainty causes households to consume less, save more, and supply more hours worked for any given wage (precautionary behavior). The increased labor supply decreases wages, leading to a fall in marginal cost. The decline in marginal cost raises markups because prices adjust slowly due to price rigidity. Consequently, the demand for household labor falls, which lowers the real wage earned by the representative household. Moreover, the decrease in labor demand reduces investment in capital stock by entrepreneurs. Financial frictions amplify and propagate the decrease in investment via the financial accelerator mechanism as will be analyzed below. The increase in inflation can

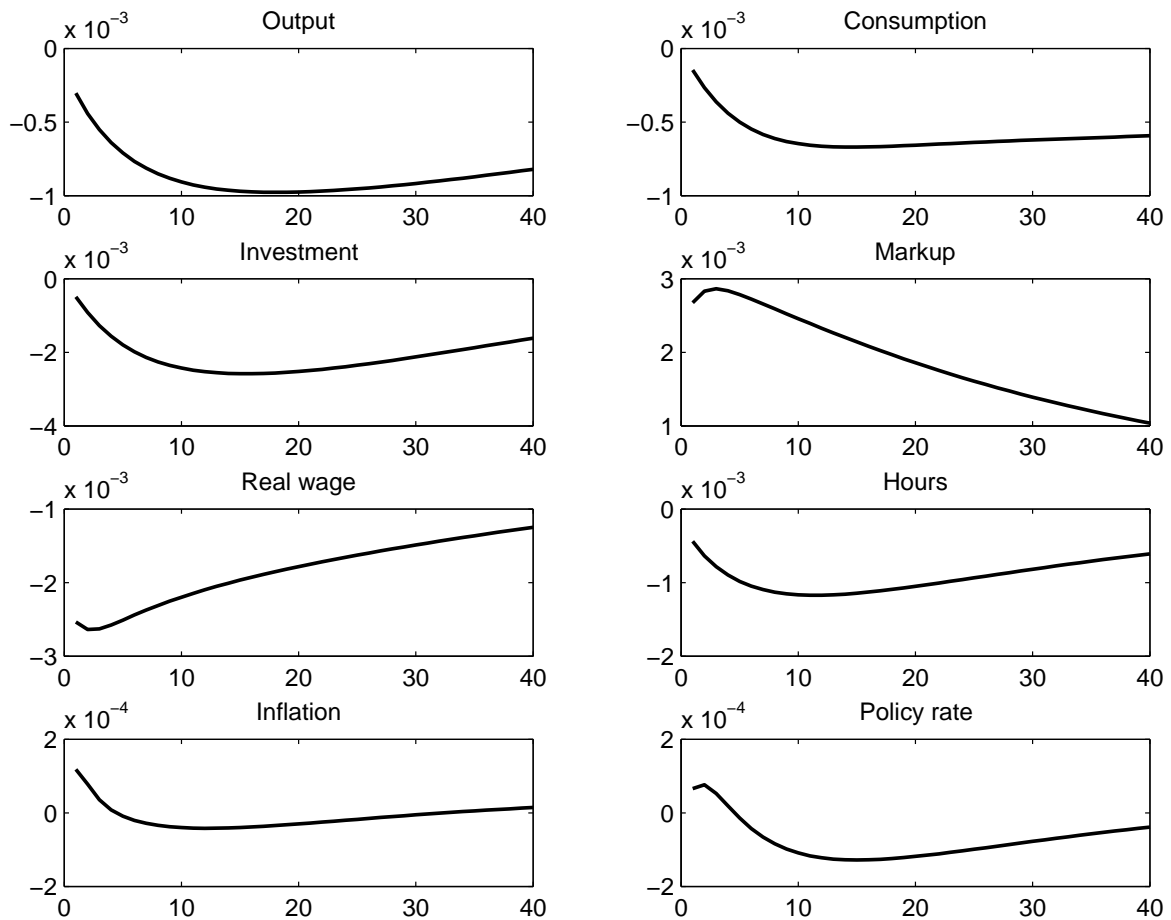


Fig. 4.4 IRFs to a 1 S.D. Monetary Volatility Shock

Notes: The graphs are expressed as percentage changes from their ergodic means.

be explained as a supply-shock-alike effect of the uncertainty because it lowers labor and capital demand. Policy rate, which follows a Taylor rule, rises in response to the increase in inflation. Then both inflation and the interest rate fall because of the contraction of economic activity.

In order to investigate the role of financial frictions, we compare the IRFs to a monetary volatility shock generated by the baseline model with those created by two counterfactual models: one with a smaller level of financial frictions and the other with a more pronounced level of financial frictions. These alternative cases are formed by modifying the value of the monitoring cost parameter μ . The idea is that monitoring cost introduces a wedge in the lender's zero profit condition. Therefore, if the monitoring cost is higher, they require

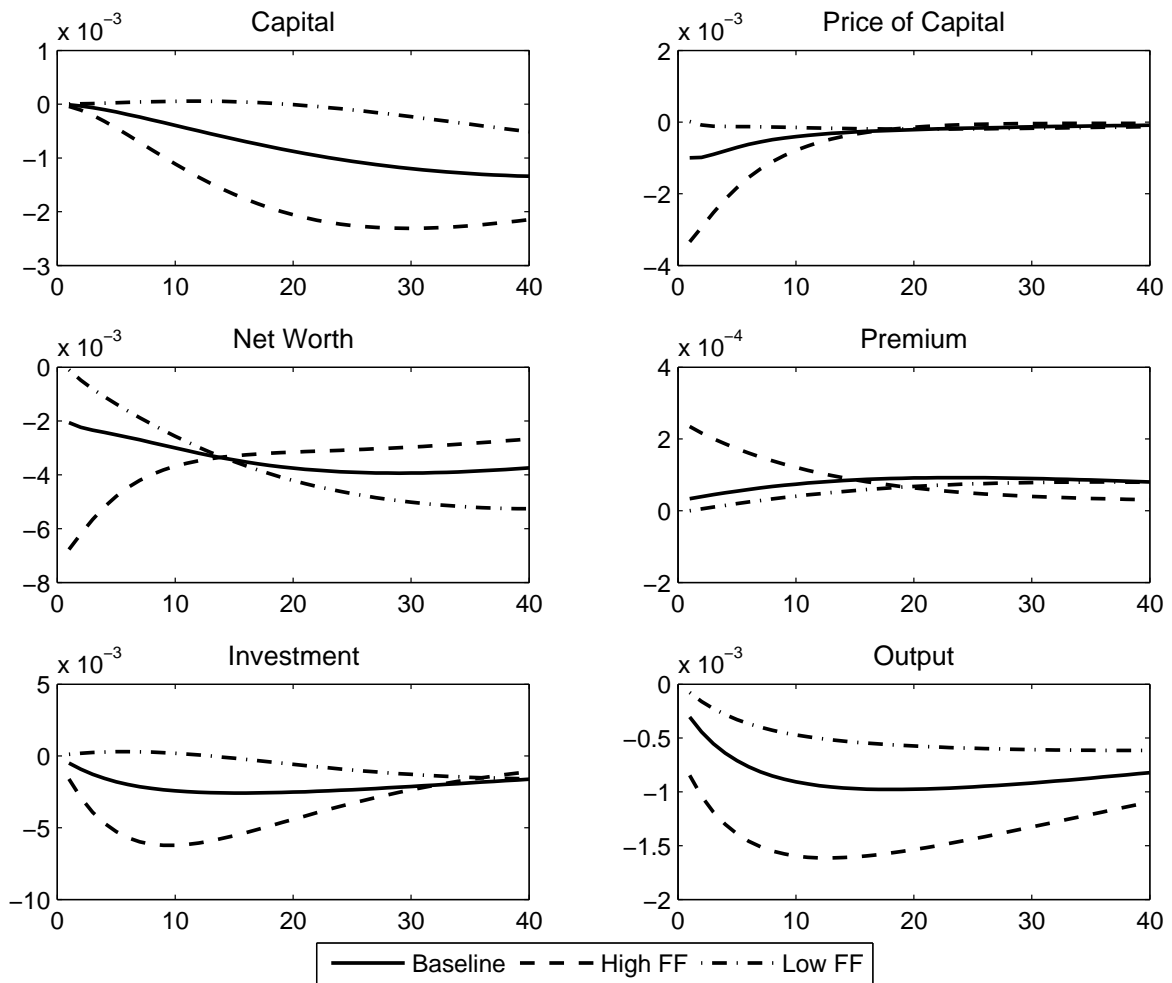


Fig. 4.5 The Effect of Financial Frictions

Notes: The graphs present the IRFs to a 1 S.D. monetary volatility shock, expressed as percentage changes from their ergodic means with three different levels of financial frictions. The level of financial frictions in the baseline model: $\mu_{Base} = 0.055$. For low level of financial frictions: $\mu_{Low} = \frac{1}{2}\mu_{Base}$. For high level of financial frictions: $\mu_{High} = 2\mu_{Base}$.

a higher return from lending, which in turn causes a greater external premium or, in other words, a more pronounced level of financial frictions. This intuition is captured by Equation (4.14). Figure 4.5 presents the IRFs generated by the baseline and the counterfactual models together. The financial accelerator mechanism is prominent. The decline in capital demand caused by increased markups leads to a fall in its price, therefore decreasing firms' net worth. The fall in the net worth increases the external premium required by lenders, thus forcing

down investment and output. More importantly, we note that a higher level of financial frictions lead to a greater premium, which decreases investment further. A kind of multiplier effect arises, since the fall in investment lowers the price of capital and net worth, therefore pushing down investment to a greater extent. Consequently, the decline in output is larger when financial frictions are more pronounced and vice versa.

We conduct a number of experiments to check the robustness of the above results. These experiments are described in Table 4.2. In the first robustness experiment (RE), the inverse of the Frisch labor elasticity ϑ is increased, which therefore makes labor supply less flexible in response to shocks. In the next three experiments, we decrease the values of Calvo parameter prices ξ , capital adjustment costs ϕ_s , and consumption habits λ , thus reducing the persistence in the model. The last experiment is related to the counteracting reaction of monetary policy in which we shut off the response of interest rate to output gap and considerably increase the smoothing parameter.

Table 4.2 Robustness Experiments (RE)

Parameter	Descriptions	Baseline	RE I	RE II	RE III	RE IV	RE V
ϑ	Inverse of the Frisch elasticity	1.3	10	*	*	*	*
ξ	Calvo parameter prices	0.708	*	0.6	*	*	*
ϕ_s	Capital adjustment costs	4.5	*	*	0.5	*	*
λ	Consumption habits	0.9	*	*	*	0.6	*
θ_y	Taylor rule output gap	0.069	*	*	*	*	0
ρ_r	Interest smoothing	0.597	*	*	*	*	0.98

Notes: “*” means that the value of relevant parameter in the experimental model is the same with the one in the baseline model.

Figure 4.6 presents the impulse response functions of output to a 1 S.D. monetary volatility shock generated by these experimental models with respect to different levels of financial frictions. Similar to the results documented above, we find in all experiments that output falls persistently after the shock and that the more pronounced financial frictions the larger the decline of output. Our findings are therefore robust to these experiments. Another feature of Figure 4.6, which may be worth noting, is that the reductions of output in the robustness experiments are mostly larger than that in the baseline model. This is somewhat expected because the dampening general equilibrium effects and the counteracting reaction

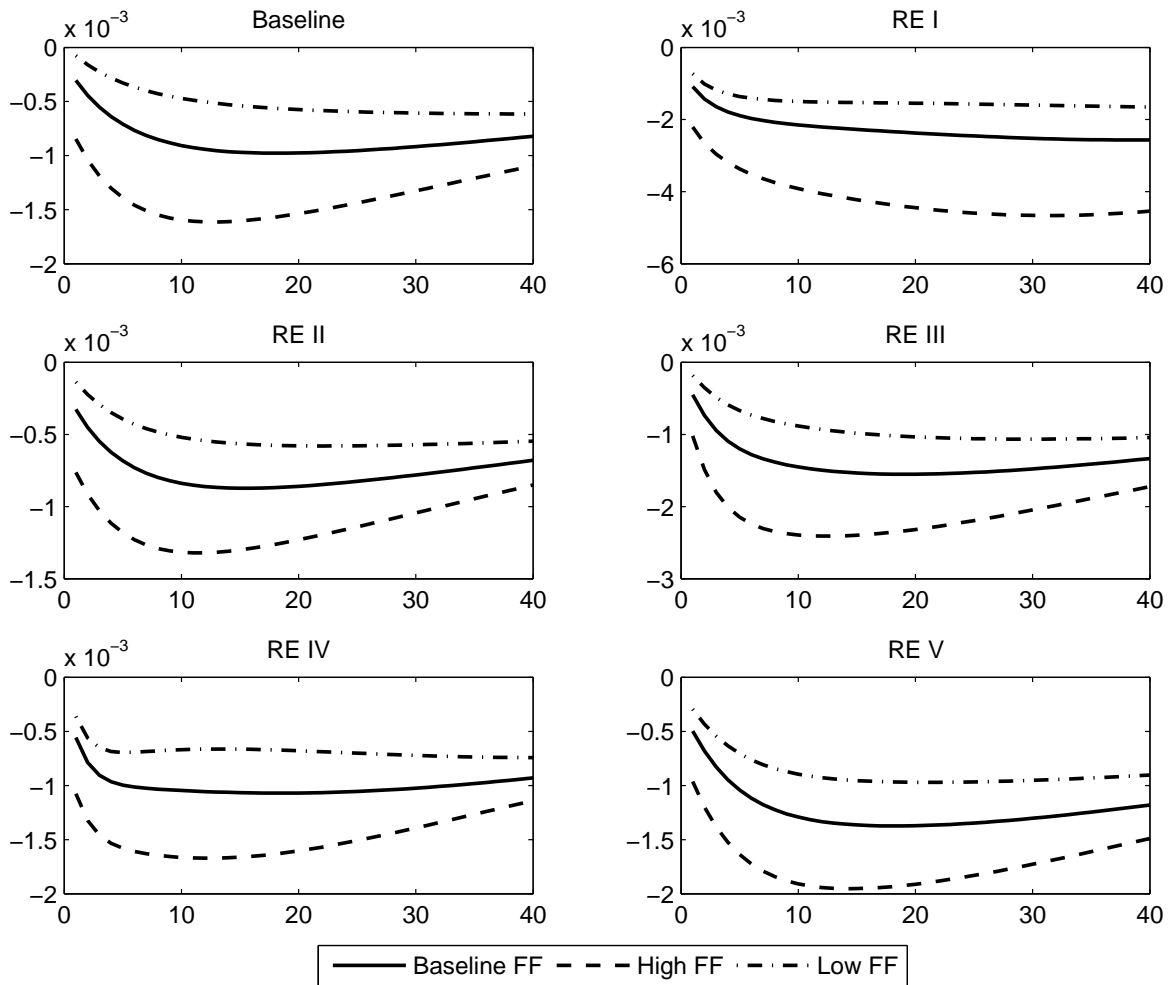


Fig. 4.6 Robustness Experiments

Notes: The graphs present the IRFs of output to a 1 S.D. monetary volatility shock, expressed as percentage changes from their ergodic means. The level of financial frictions in the baseline model: $\mu_{Base} = 0.055$. For low level of financial frictions: $\mu_{Low} = \frac{1}{2}\mu_{Base}$. For high level of financial frictions: $\mu_{High} = 2\mu_{Base}$.

are limited in the experiments.

Particularly, in the experiment with the monetary policy reaction function (denoted by 'RE V' in Figure 4.6), output drops by 0.5% on impact, falls as great as 1.5%, reaching to the lowest point, after 20 quarters, and then slowly returns to its mean. A similar result is documented by Born and Pfeifer (2014). In the baseline model, with a positive response to output gap and a moderate value of interest smoothing parameter, monetary policy is more aggressive and quicker to offset negative shocks, therefore mitigating the potential impacts

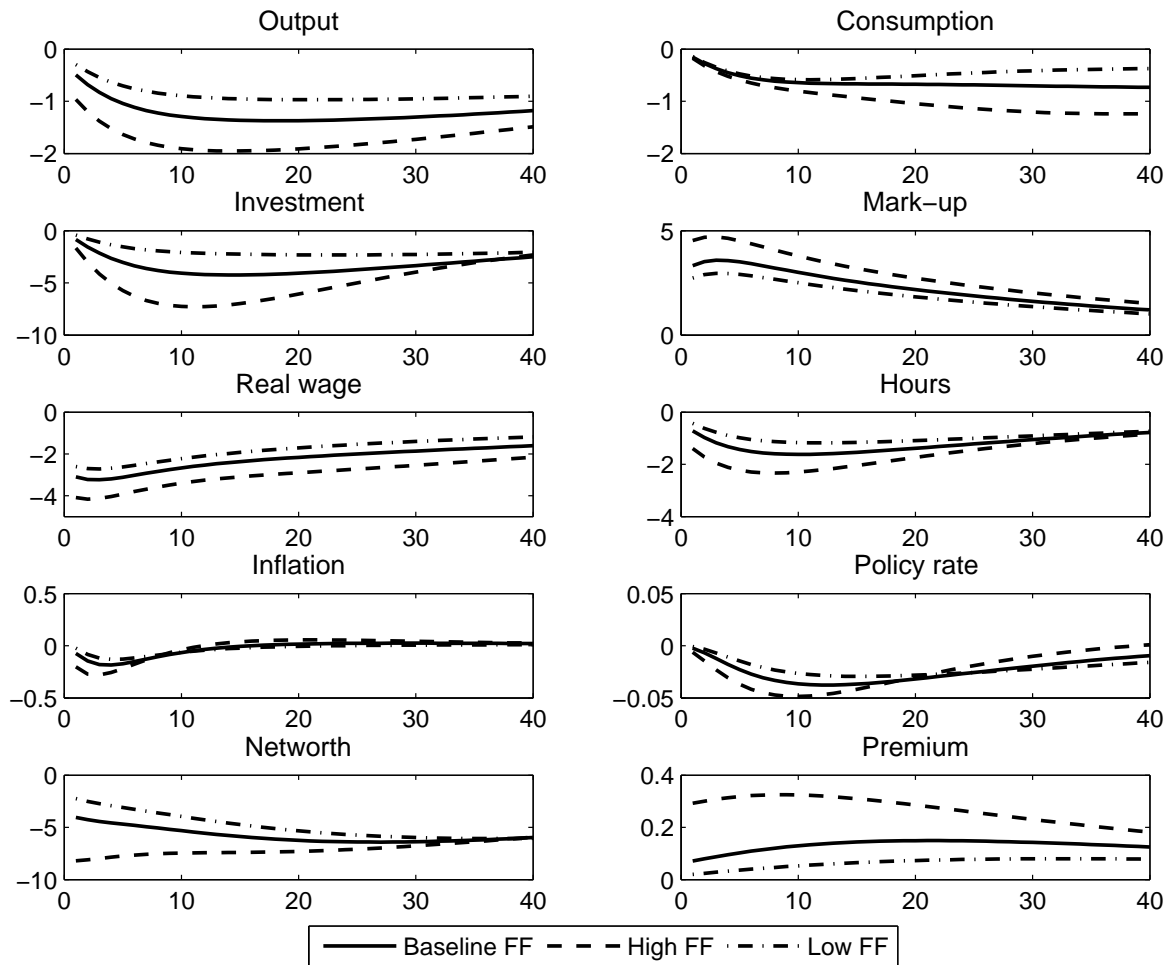


Fig. 4.7 Robustness Experiment V

Notes: The graphs present the IRFs to a 1 S.D. monetary volatility shock, expressed as percentage changes from their ergodic means. In this experiment, the response of interest rate to output gap is shut off and the smoothing parameter is increased considerably. The level of financial frictions in the baseline model: $\mu_{Base} = 0.055$. For low level of financial frictions: $\mu_{Low} = \frac{1}{2}\mu_{Base}$. For high level of financial frictions: $\mu_{High} = 2\mu_{Base}$.

of uncertainty. In the experiment V, we however force the response to output down to zero and give more weight to past interest rates. Hence, the current economic conditions affect the nominal interest rate less than its past values. Figure 4.7 plots the IRFs of output and other variables to a monetary volatility shock of this experiment. The transmission mechanism of the shock is similar to what we discussed in the baseline model with increased markups and greater premium. However, the sluggish response of monetary policy exacer-

bates the contraction. While inflation reduces because of the contraction, the sluggishness causes the nominal interest rate to fall much slower than the reduction in inflation, which lead to an increase in the real interest rate. Consequently, investment decreases further, which is again amplified by the existence of financial frictions in the model. Eventually, investment falls by more than 4% after 10 quarters, resulting in a substantial decline in output.

The finding that the more sluggish monetary policy the more substantial the effects of monetary volatility shocks on the economy might have an important implication regarding the zero-lower bound in the nominal interest rate, although our current model does not explicitly account for it. In such a situation, the nominal interest rate is likely independent to current conditions and substantially, if not completely, depends on its past values. This limits the ability of the nominal interest rate to mitigate negative shocks to the economy, which likely causes a greater contraction of economic activity. Basu and Bundick (2012) consider the uncertainty of TFP shocks and argue that the uncertainty has larger effects under the zero-lower bound. A similar result is documented by Fernández-Villaverde et al. (2013) who consider fiscal uncertainty.

4.6 Conclusion

The study attempted to investigate the role of financial frictions in the transmission of monetary volatility shocks on the economy. To do so, we employed the particle filter to estimate a non-linear DSGE model that incorporates the financial frictions à la Bernanke et al. (1999) and introduces stochastic volatility to shocks. The results show that our model captures aggregate dynamics relatively well. More importantly, we document that an increase in the volatility of monetary policy causes a contraction in economic activity: output, consumption, investment, hours worked, and real wages fall. The co-movement of these variables suggests that monetary volatility shocks may play a certain role in business fluctuations. Regarding the role of financial frictions, we find that financial frictions amplify and propagate the effects of monetary volatility shocks via the financial accelerator mechanism.

Our work does not examine the impact of monetary volatility shocks under environments in which there is a zero-lower bound in the nominal interest rate or unconventional monetary policies. Advancing the model to address these issues is an interesting and important expansion which we would like to consider in the future research.

Chapter 5

Concluding Remarks

This thesis developed three essays on monetary policy. Chapters 2 and 3 provided evidence on the role played by monetary policy in economic outcomes through the analysis of the historical conduct of monetary policy. Chapter 4 studied the impact of monetary uncertainty on the economy using a DSGE model with financial frictions.

In Chapter 2, we considered a variety of reaction functions in the context of real time data to analyze U.K. monetary policy under inflation targeting adopted in 1992. There were two important features regarding the estimation procedure. First, expected variables in contemporaneous- and forward-looking rules were forecasted before estimation. Second, we used the impulse indicator saturation approach to obtain estimates robust to outliers. We found that monetary policy after 1992 was forward-looking and satisfied the Taylor principle. Moreover, the response of monetary policy to inflation has been stronger since the granting of operational independence to the Bank of England in 1997. Importantly, we showed that failing to deal with outliers can lead to a distorted result that the post-1992 response to inflation was weak, perhaps not satisfying the Taylor principle.

Chapter 3 modeled changes in U.S. monetary policy by taking four issues that have been highlighted in the literature as crucial into consideration. These issues are: (i) the type of changes in policy parameters, (ii) the treatment of heteroscedasticity, (iii) the real-time nature of data, and (iv) the role of asymmetric preferences. The empirical model specification was built on the optimal interest rate rule derived from the formal monetary policy

design problem in which the loss function is asymmetric with respect to inflation. In this empirical model, we introduced time-varying parameters and dealt with heteroscedasticity in policy shocks via a stochastic volatility specification. The estimation was based on real-time data using particle filtering. The findings suggested that the conduct of U.S. monetary policy could have experienced important changes at the mid-1970s, the late 1970s, and the early 1990s. Therefore, a single division at the late 1970s as conventionally assumed might mislead the evaluation of monetary policy.

Finally, Chapter 4 investigated the impacts of the volatility of monetary policy on the economy in a DSGE model with financial frictions *à la* Bernanke, Gertler, and Gilchrist (1999). The model was estimated for the U.S. economy by maximum likelihood with the value of the likelihood approximated by particle filtering. The results show that, first, the model was fairly successful in capturing aggregate dynamics. Second, a positive monetary volatility shock was found to cause a contraction in economic activity. Finally, we demonstrate that financial frictions amplified and propagated the effects of the shock via the financial accelerator mechanism. Our results therefore contribute to further advance understanding of the role played by financial frictions in business fluctuations.

In future research, we would like to expand further the issues discussed in the above studies. The first direction is to investigate other sources of asymmetric monetary policy. Regarding the second direction, we would investigate the conduct of monetary policy in recent years associated with the financial crisis and its consequences. Last but not least, we would consider the impact of the volatility of monetary policy in different scenarios such as in a small open economy model or in a model with the zero-lower bound of interest rates.

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Appendix A

Appendix of Chapter 2

A.1 Estimation with BoE's Forecasts

This appendix presents the estimates of Taylor rules using BoE's inflation forecasts in two cases: with and without the impulse-indicator saturation. See explanations on IIS in the main text.

Table A.1 Taylor Rule Estimates without IIS: 1992Q4-2007Q4, using BoE's Inflation Forecasts and Real-time HP Output gap

	$h = 0$	$h = 1$	$h = 2$	$h = 3$	$h = 4$
Inflation ϕ_π	-0.07 [0.08]	-0.13 [0.08]	-0.16** [0.08]	-0.17** [0.08]	-0.17** [0.08]
Output Gap ϕ_x	0.24* [0.05]	0.24* [0.05]	0.25 [0.05]	0.26* [0.05]	0.26* [0.05]

Notes: The regression equation is

$$M_1 : r_t = c + \rho r_{t-1} + \phi_\pi E_t \pi_{t+h} + \phi_x E_t x_{t+q} + \varepsilon_t,$$

for $t = 1993Q1, \dots, 2007Q4$, $h = -1, 0, 1, 2, 3, 4$ and $q = -1$. The columns correspond to different values of h . Standard errors are given in [.]. * $p < 0.01$ and ** $p < 0.05$.

Table A.2 Taylor Rule Estimates with IIS: 1992Q4-2007Q4, using BoE's Inflation Forecasts and Real-time HP Output gap

	$h = 0$	$h = 1$	$h = 2$	$h = 3$	$h = 4$
Inflation ϕ_π	-0.08 [0.08]	-0.14 [0.07]	-0.16** [0.08]	-0.15** [0.08]	-0.16** [0.08]
Output Gap ϕ_x	0.21* [0.05]	0.22* [0.05]	0.23* [0.04]	0.24* [0.05]	0.24* [0.05]

Notes: The regression equation is

$$M_2: \quad r_t = c + \rho r_{t-1} + \phi_\pi E_t \pi_{t+h} + \phi_x E_t x_{t+q} + \sum_{i=1}^T \beta_i 1_{i=t} + \varepsilon_t$$

for $t = 1993Q1, \dots, 2007Q4$, $h = -1, 0, 1, 2, 3, 4$ and $q = -1$. The columns correspond to different values of h . Standard errors are given in [.]. * $p < 0.01$ and ** $p < 0.05$.

Appendix B

Appendix of Chapter 3

B.1 An Approximation for the Likelihood Value

The likelihood function is derived as follows

$$\begin{aligned} p(I_T; \boldsymbol{\omega}) &= \prod_{t=1}^T p(i_t | I_{t-1}; \boldsymbol{\omega}) \\ &= \prod_{t=1}^T \left(\int p(i_t | \mathbf{x}_t; \boldsymbol{\omega}) p(\mathbf{x}_t | I_{t-1}; \boldsymbol{\omega}) d\mathbf{x}_t \right) \\ &= \prod_{t=1}^T \left(\int \int p(i_t | \mathbf{x}_t; \boldsymbol{\omega}) p(\mathbf{x}_t | X_{t-1}; \boldsymbol{\omega}) p(X_{t-1} | I_{t-1}; \boldsymbol{\omega}) dX_{t-1} d\mathbf{x}_t \right) \\ &= \prod_{t=1}^T \left(\int \int p(i_t | \mathbf{x}_t; \boldsymbol{\omega}) p(\mathbf{x}_t | \mathbf{x}_{t-1}; \boldsymbol{\omega}) p(X_{t-1} | I_{t-1}; \boldsymbol{\omega}) dX_{t-1} d\mathbf{x}_t \right) \\ &= \prod_{t=1}^T \left(\int \frac{p(i_t | \mathbf{x}_t; \boldsymbol{\omega}) p(\mathbf{x}_t | \mathbf{x}_{t-1}; \boldsymbol{\omega})}{\pi(\mathbf{x}_t | X_{t-1}, I_t)} \pi(\mathbf{x}_t | X_{t-1}, I_t) p(X_{t-1} | I_{t-1}; \boldsymbol{\omega}) dX_{t-1} \right) \\ &\approx \prod_{t=1}^T \left(\frac{1}{N} \sum_{k=1}^N \frac{p(i_t | \mathbf{x}_t^k; \boldsymbol{\omega}) p(\mathbf{x}_t^k | \mathbf{x}_{t-1}^k; \boldsymbol{\omega})}{\pi(\mathbf{x}_t^k | X_{t-1}^k, I_t)} \right) \\ &= \prod_{t=1}^T \left(\sum_{k=1}^N \omega_t^{k-1} \frac{p(i_t | \mathbf{x}_t^k; \boldsymbol{\omega}) p(\mathbf{x}_t^k | \mathbf{x}_{t-1}^k; \boldsymbol{\omega})}{\pi(\mathbf{x}_t^k | X_{t-1}^k, I_t)} \right) \\ &= \prod_{t=1}^T \left(\sum_{k=1}^N \omega_t^k \right). \end{aligned}$$

In the above derivation, we used an assumption that $\pi(X_{t-1}|I_{t-1}) = \pi(X_{t-1}|I_t)$.

B.2 Estimates of Time-Invariant Parameters

The estimates of time-invariant parameters are presented in the following table

Means and Standard Deviations of Time-Invariant Parameters

Parameters	Means	Standard deviations
σ_{a_0}	-0.86	0.11
σ_{a_1}	-2.22	0.08
σ_{a_2}	0.63	0.11
σ_{a_3}	-2.23	0.14
σ_{a_4}	-1.10	0.07
σ_{a_5}	-1.28	0.08

Notes: The table presents the estimates of the time-invariant parameters of the state space system:

$$i_t = \frac{1}{1 + \exp(-a_{5,t})} i_{t-1} + \frac{\exp(-a_{5,t})}{1 + \exp(-a_{5,t})} (a_{0,t} + a_{1,t} \pi_{t|t} + a_{2,t} \sigma_{\pi_t|t}^2 + a_{3,t} y_{t|t}) + \exp(a_{4,t}) \varepsilon_t,$$

$$a_{k,t} = a_{k,t-1} + \exp(\sigma_{a_k}) \varepsilon_{a_{k,t}}, \quad k = 0, 1, \dots, 5.$$

B.3 Constructing the Contemporaneous Real-Time HP Output Gap Series

In order to construct the contemporaneous real-time HP output gap from 1965Q4 to 2007Q4, we use two data sets: the Greenbook projections and the real-time data set for macroeconomists. Both can be downloaded from the website of the Philadelphia Fed. For more explanations about real-time data, we refer the readers to Orphanides (2001).

The procedure to construct the contemporaneous real-time HP output gap for a given quarter i is described as follows:

Step 1: Collect the entire time-series history perceived at the quarter i vintage (the vintage is shown at the column header of the real-time data set) which includes the real output up to the previous quarter. Note that the real output of quarter i is not available to observe in that quarter. Denote this series by $X_{j:i-1|i} = [x_{j|i}, x_{j+1|i}, x_{j+2|i}, \dots, x_{i-1|i}]$ where j is the first quarter with data recorded in the vintage i data set and $x_{h|i}$ is the data of real output of the quarter h perceived at the vintage i .

Step 2: Use the Greenbook forecasts for the quarter-to-quarter growth in real GDP (with quarterized percentage points) to calculate the expected value of real GDP for the contemporaneous quarter $x_{i|i}$ from $x_{i-1|i}$. In order to reduce the end-of-sample issue of the HP filter, we also compute the expected value of real GDP in the following quarters when the forecasts of the growth rate for those quarters are available at that vintage.

Step 3: Combine these expected values with the historical series to generate the new series:

$$X_{j:i+k|i} = [x_{j|i}, x_{j+1|i}, x_{j+2|i}, \dots, x_{i-1|i}, x_{i|i}, \dots, x_{i+k|i}] \text{ where } 0 \leq k \leq 4.$$

Step 4: Apply the HP filter to the series $X_{j:i+k|i}$ to achieve the HP output gap series $X_{j:i+k|i}^* = [x_{j|i}^*, x_{j+1|i}^*, x_{j+2|i}^*, \dots, x_{i-1|i}^*, x_{i|i}^*, \dots, x_{i+k|i}^*]$. We then record $x_{i|i}^*$ as the contemporaneous real-time HP output gap at the quarter i .

B.4 Asymmetric Preferences to Both Inflation and Output Gap

The Lagrangian of the policy problem in this case is written as follows

$$\begin{aligned} \text{Min}_{\pi_t, y_t, i_t} E_t \left\{ \frac{e^{\alpha(\pi_t - \pi^*)} - \alpha(\pi_t - \pi^*) - 1}{\alpha^2} + \mu \left[\frac{e^{(\lambda y_t)} - \lambda y_t - 1}{\lambda^2} \right] + \frac{\gamma}{2} (i_t - i^*)^2 \right. \\ \left. - \phi_t^\pi (\pi_t - \kappa y_t - \varepsilon_t^s) - \phi_t^y (y_t + \phi i_t - \varepsilon_t^d) \right\}, \end{aligned} \quad (\text{B.1})$$

in which ϕ_t^π and ϕ_t^y are the Lagrange multipliers. It is straightforward to derive the first-order optimal conditions

$$\begin{aligned} E_t \left\{ \frac{e^{\alpha(\pi_t - \pi^*)} - 1}{\alpha} - \phi_t^\pi \right\} &= 0, \\ E_t (\gamma(i_t - i^*) - \phi_t^y \varphi) &= 0, \\ E_t \left\{ \mu \frac{e^{\lambda y_t} - 1}{\lambda} + \phi_t^\pi \kappa - \phi_t^y \right\} &= 0. \end{aligned}$$

Combine these conditions to eliminate the Lagrange multipliers

$$E_t \left\{ \frac{e^{\alpha(\pi_t - \pi^*)} - 1}{\alpha} \kappa + \frac{e^{\lambda y_t} - 1}{\lambda} \mu - \frac{\gamma}{\varphi} (i_t - i^*) \right\} = 0. \quad (\text{B.2})$$

Then the central bank sets the interest rate in order to respond to inflation and output deviations

$$i_t = i^* + E_t \left\{ \frac{e^{\alpha(\pi_t - \pi^*)} - 1}{\alpha} \frac{\kappa \varphi}{\gamma} + \frac{e^{\lambda y_t} - 1}{\lambda} \frac{\mu \varphi}{\gamma} \right\}. \quad (\text{B.3})$$

and the above expression can be approximated as

$$\begin{aligned} i_t &= i^* + E_t \left\{ \frac{\kappa \varphi}{\gamma} (\pi_t - \pi^*) + \frac{\kappa \varphi \alpha}{2\gamma} (\pi_t - \pi^*)^2 + \frac{\mu \varphi}{\gamma} y_t + \frac{\mu \varphi \lambda}{2\gamma} y_t^2 \right\} \\ &= i^* + \frac{\kappa \varphi}{\gamma} E_t (\pi_t - \pi^*) + \frac{\kappa \varphi \alpha}{2\gamma} E_t (\pi_t - \pi^*)^2 + \frac{\mu \varphi}{\gamma} E_t y_t + \frac{\mu \varphi \lambda}{2\gamma} E_t y_t^2 \\ &= \left(i^* - \frac{\kappa \varphi}{\gamma} \pi^* \right) + \frac{\kappa \varphi}{\gamma} \pi_{t|t} + \frac{\kappa \varphi \alpha}{2\gamma} \sigma_{\pi_t|t}^2 + \frac{\mu \varphi}{\gamma} y_{t|t} + \frac{\mu \varphi \lambda}{2\gamma} \sigma_{y_t|t}^2 \\ &= b_0 + b_1 \pi_{t|t} + b_2 \sigma_{\pi_t|t}^2 + b_3 y_{t|t} + b_4 \sigma_{y_t|t}^2, \end{aligned} \quad (\text{B.4})$$

where $b_0 = i^* - \frac{\kappa \varphi}{\gamma} \pi^*$, $b_1 = \frac{\kappa \varphi}{\gamma}$, $b_2 = \frac{\kappa \varphi \alpha}{2\gamma}$, $b_3 = \frac{\mu \varphi}{\gamma}$, $b_4 = \frac{\mu \varphi \lambda}{2\gamma}$ and $\sigma_{y_t|t}^2$ is the expected variance of unemployment gap conditional on the information available at time t . Other notations are as in the baseline model.

B.4.1 Expected Variance of Output Gap

The process of generating the expected variance of output gap series is similar to the one used to create the expected variance of inflation series in Section 3.4.2. The model specification used is given by

$$y_t = c + \sum_{i=1}^3 \beta_i y_{t-i} + \varphi i_t + \sum_{j=1}^3 \psi_j \pi_{t-j} + \varepsilon_t, \quad (\text{B.5})$$

where the output gap is proxied by the 5-year moving average gap. This specification is derived from the IS curve equation (3.3) by substituting the expectations by a linear combination of lags of inflation and the output gap.

Appendix C

Appendix of Chapter 4

C.1 Choice of Density Function for ψ_t

Then, one can draw that $E_t(\psi_{t+1}) = 1$. Some other outputs can be calculated including

$$F(\psi_t) = \Phi(z_t)$$

$$\begin{aligned} G(\psi_t) &= \int_0^{\psi_t} \psi f(\psi) d\psi \\ &= 1 - \int_{\psi_t}^{\infty} \psi f(\psi) d\psi \\ &= 1 - \Phi(\sigma_\psi e^{\sigma\psi_t} - z_t) \\ &= \Phi(z_t - \sigma_\psi e^{\sigma\psi_t}) \end{aligned}$$

$$\Gamma(\psi_t) = \psi_t(1 - \Phi(z_t)) + \Phi(z_t - \sigma_\psi e^{\sigma\psi_t})$$

$$G'(\psi_t) = \psi_t f(\psi_t)$$

$$\Gamma'(\psi_t) = 1 - F(\psi_t)$$

where $z_t = \left(\frac{\log(\psi_t) + 0.5\sigma_\psi^2 e^{2\sigma\psi_t}}{\sigma_\psi e^{\sigma\psi_t}} \right)$, $f(\psi)$ is the p.d.f of ψ , and $\Phi(\cdot)$ is the standard normal c.d.f.

C.2 Data Sources and Construction

The original time series' sources are summarized as follows

- RGDP: Real Domestic Product, Billions of chained (2005) dollars, Seasonally adjusted at annual rates, Bureau of Economic Analysis Table 1.1.6, line 1
- GDPDEF: Gross Domestic Product: Implicit Price Deflator (GDPDEF), Index 2009 = 100, Quarterly, Seasonally Adjusted, Federal Reserve Economic Data
- LNU00000000Q: Labor force status: Civilian noninstitutional population; Bureau of Labor Statistics
- LNS10000000Q: Labor force status: Civilian noninstitutional population; Bureau of Labor Statistics (Before 1976: LNU00000000Q)
- LNSindex: $LNS10000000Q(2005 : 2) = 1$
- FFR: Federal Funds Rate; Federal Reserve Bank of St. Louis
- BAA: Moody's seasoned Baa corporate bond yields; Federal Reserve Bank of St. Louis

The four observable data used in the estimation are constructed as below

- $ROUT_t = LN \left(\frac{RGDP_t}{LNSindex_t} \right)$
- $OUT_t = ROUT_t - \overline{ROUT}_t$ in which \overline{ROUT}_t is the potential output per capital filtered by the Hodrick-Prescott method.
- $INP_t = LN \left(\frac{GDPDEF_t}{GDPDEF_{t-1}} \right)_{demeaned}$
- $INR_t = FFR_t / 400_{demeaned}$
- $CBY_t = BAA_t / 400_{demeaned}$

C.3 Particle Filter Algorithm

The model considered above belongs to a larger class of non-linear and/or non-normal dynamic macroeconomic models which can be written generally in the following state-space system. First, the law of motion for the state vector \mathbf{x}_t is given by

$$\mathbf{x}_t = h(\mathbf{x}_{t-1}, \mathbf{w}_t; \Xi) \quad (\text{C.1})$$

where \mathbf{w}_t is a random vector of innovations, in our specific case \mathbf{w}_t includes structural and volatility shocks, with dimension n_w and Ξ is the vector of parameters of the model. Second, the set of observables denoted by \mathbf{z}_t are connected to the state variables \mathbf{x}_t by the measurement equation

$$\mathbf{z}_t = g(\mathbf{x}_t, \mathbf{v}_t; \Xi) \quad (\text{C.2})$$

where \mathbf{v}_t is a random vector of measurement errors. To be convenient, we assume independence between \mathbf{v}_t and \mathbf{w}_t . The functions h and g come from the equations that characterize the behavior of the model. The particle filter algorithm is presented below.

Particle Filter Algorithm

- *Initialization* $t = 0$

Draw N particles $\mathbf{x}_0^{(i)}$, $i = 1, 2, \dots, N$, from $p(\mathbf{x}_0; \Xi)$ and let $\pi_0^{(i)} = \frac{1}{N}$ for all i .

- *Propagation*

Draw N particles $\mathbf{x}_t^{(i)}$, $i = 1, 2, \dots, N$, from $p(\mathbf{x}_t | \hat{\mathbf{x}}_{t-1}^{(i)}; \Xi)$.

- *Importance weights*

Evaluate the importance weights $\pi_t^{(i)}$, $i = 1, 2, \dots, N$

$$\pi_t^{(i)} = \pi_{t-1}^{(i)} p(\mathbf{z}_t | \mathbf{x}_t^{(i)}; \Xi)$$

- *Log-Likelihood Contribution*

$$\log L_t = \log L_{t-1} + \log\left(\sum_{i=1}^N \pi_t^{(i)}\right)$$

- *Normalization*

Normalize the importance weights $\pi_t^{(i)}$, $i = 1, 2, \dots, N$

$$\tilde{\pi}_t^{(i)} = \frac{\pi_t^{(i)}}{\sum_{i=1}^N \pi_t^{(i)}}$$

- *Resampling step*

We use the systematic resampling algorithm to generate a new set of particles $\{\hat{\mathbf{x}}_t^{(j)}\}_{j=1}^N$ by resampling (with replacement) from the existing particles $\{\mathbf{x}_t^{(i)}\}_{i=1}^N$ with probability $\{\tilde{\pi}_t^{(i)}\}_{i=1}^N$.

- *Propagation*

Set $t = t + 1$ and go to Step 2: Propagation

Systematic Resampling Algorithm

- *Construction of the cumulative sum of weights (CSW)*

Let $c_1 = \tilde{\pi}_t^1$ and define $c_i = c_{i-1} + \tilde{\pi}_t^{(i)}$ for $i = 2, \dots, N$

- *Resampling step*

Generate a starting point from a uniform distribution: $u_1 \sim U[0, N^{-1}]$ and define $u_j = u_1 + N^{-1}(j-1)$ for $j = 2, \dots, N$. For each $j = 1, \dots, N$, find $i = 1, \dots, N$ to satisfy

$$c(i-1) \leq u(j) \leq c(i)$$

- Assign sample: $\mathbf{s}_k^{(j)*} = \mathbf{x}_k^{(i)}$

- Assign weight: $\pi_t^{(j)} = N^{-1}$