# An optimization framework for the development of efficient one-way car-sharing systems

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**Abstract**

Electric vehicle sharing systems have been introduced to a number of cities around the world as a means of increasing mobility, reducing congestion, and pollution. Electric vehicle sharing systems can offer one or two-way services. One-way systems provide more flexibility to users since they can be dropped-off at any stations; however their modeling involves a number of complexities arising from the need to relocate vehicles accumulated at certain stations. The planning of one-way electric vehicle-sharing systems involves a host of strongly interacting decisions regarding the number, size and location of stations, as well as fleet size.

In this paper we develop and solve a multi-objective MILP formulation for planning one-way vehicle-sharing systems taking into account vehicle relocation and electric vehicle charging requirements. For real world problems the size of the problem becomes intractable due to the extremely large number of relocation variables. In order to cope with this problem we introduce an aggregate model using the concept of the virtual hub. This transformation allows the solution of the problem with a branch and bound approach, while the error introduced is less than 2%.

The proposed solution generates the efficient frontier and allows decision makers to examine the trade-off between operator's revenues and users' net benefits. The capabilities of the proposed approach are demonstrated on a large scale real world problem with available data from Nice, France. Extensive sensitivity analysis was performed by varying demand and station accessibility distance. The results provide useful insights regarding the efficient planning of one-way electric vehicle sharing systems.

*Keywords:* one-way car-sharing, multi-objective optimization, location modeling, vehicle relocation

## 1. Introduction

Car-sharing (also known as shared-use vehicle) systems have attracted considerable attention with multiple implementations worldwide [1] due to their potential to improve mobility and sustainability [2]. These systems provide benefits both to their users and the society as a whole . Reduced personal transportation cost and mobility enhancement have been cited as the two most notable benefits to individual users. Societal benefits include the reduction of parking space requirements, congestion reduction, provision of affordable mobility to economically disadvantaged groups [3, 1, 4, 2]. In cases of electric shared vehicle (many examples in European cities) systems, they can also provide significant reductions in emissions.

The attractiveness of car-sharing systems is determined by the level of service offered and the cost associated with the use of the system. The level of service is influenced by the accessibility of vehicle stations by the potential users, i.e. (i) the distance between user's origin and destination from pick-up and drop-off vehicle stations respectively, and (ii) the availability of vehicles at stations. On the other hand, station number and size, as well as fleet size and availability of vehicles, at the "right time" at the "right station", influence the cost of establishing and operating a car-sharing system.

The car-sharing systems can be classified into flexible "one-way" and the more restricted "two-way" types, according to whether the users should return the rented vehicle at a different or at the location they picked it up. The problem of ensuring vehicle availability becomes more prominent when the vehicles can be rented and used on a one-way basis. The one-way operation of the vehicles coupled with the imbalance of demand for cars, both at the origin of the trip (pick-up station) and at the destination (drop-off station), may result to a situation where the vehicles are accumulated to stations where they are not needed, while at the same time there is vehicle shortage at the stations where more vehicles are needed.

Vehicle relocation, i.e. transfer of vehicles from stations with

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high vehicle accumulation to stations where shortage is experienced, is a technique that has been proposed to improve the performance of one-way car-sharing systems (e.g. [5, 6]). The lack of efficient vehicle relocation coupled with the need to guarantee a given level of vehicle availability may lead to an unnecessary increase of the fleet size and vehicle underutilization. The efficient and cost-effective strategic planning, and the operation of one-way car-sharing systems require models that will determine the number and location of the service stations, the fleet size, and the dynamic allocation of vehicles to stations optimally. These models should assist decision makers to strike an optimum balance between the level of service offered and the total cost (including vehicle relocation costs) for implementing and operating the car-sharing system.

However, the literature currently lacks a model that can consider simultaneously decisions related to the determination of station location, size and number, and fleet size, while taking into account the dynamics of vehicle relocation and balancing. Existing models [7, 8] either look at station locations without due consideration to vehicle relocation decisions [7], or consider station locations assuming that only the demand in the catchment area of opened stations needs to be serviced [8]. In the case where vehicle relocation is modeled [8], the relocation of the vehicles and the associated costs are considered only at the end of the operating period (usually a day), and therefore they are influencing the fleet size.

The objective of this paper is twofold: (i) to develop and solve a mathematical model for determining the optimum fleet size, and the number and location of the required stations of one-way car-sharing systems by taking into account the dynamic repositioning (relocation) of vehicles, and (ii) to apply the proposed model for planning and operating a one-way electrical car-sharing system in the city of Nice, France.

The remainder of this paper is organized as follows. Section 2 provides an overview of previous related work and further elaborates on the arguments justifying the need for the proposed model, Section 3 presents the formulation and the solution approach of the proposed model, Section 4 describes the application of the proposed model for planning and operating a one-way electrical car-sharing system in Nice, France while Section 5 discusses the research conclusions and provides recommendations for future research.

## 2. Previous Related Research

Models related to the planning and operation of car-sharing systems can be classified into the following two broad categories: i) models addressing strategic planning decisions, and ii) models supporting operational decisions.

### 2.1. Models for Strategic Planning Decisions

Strategic planning decisions seek to determine the number, size and location of stations, and the number of the vehicles that should be assigned to each station, in order to optimize a measure or a combination of measures of system performance. Station location models have been developed to locate bicycle stations [7] and car stations [8]. Although the focus of our work is on electrical car-sharing systems, we also review models that address the station location of shared-use bicycles.

The problem of locating stations for shared-use bicycles has been studied recently [7]. This paper presents a model for determining the number and location of bicycle stations and the structure of the network of bicycle paths that should be developed to connect the bicycle stations. The problem is formulated as a non-linear integer model. The objective function used expresses the total yearly cost encountered by the operator and the users. A small scale example was used to illustrate the model and a branch and bound algorithm was used to solve it. This model does not consider the daily variation of demand and the problems arising from the dynamic accumulation/shortage of bicycles due to the variation of demand in time and space.

The optimization of car depot locations and the definition of the number of parking spaces (size of the depot) for each depot has been also addressed [8]. The number of parking spaces at each depot is determined by the maximum number of cars that are allocated to each station throughout an operating day. Vehicle relocation (and the associated relocation cost) is considered only at the end of the entire operating period (i.e. day). Thus, this model does not treat explicitly the imbalance created by the one-way operation and therefore it does not rebalance the vehicles at the end of each operating sub-interval (e.g. hour). This model assumes that the vehicle imbalance problem is bypassed through the optimum depot location and size. The objective function of the model seeks to maximize the profit of the operating agency and takes into account the depreciation, maintenance and relocation (at the end of the operating period) costs of the vehicles, the maintenance cost of the depots, and the revenues generated by the system operations. This model makes the assumption that only trips associated with open stations need to be served. Thus, the demand (trips) that falls outside the catchment area of open stations associated with the stations that are not open is ignored. As a consequence this model does not consider the access and egress cost of the potential users to/from the candidate station locations. A direct implication of this assumption is that the proposed model cannot be used to study the trade-off between station accessibility cost and system benefits. Finally, this model does not consider the dynamic relocation of vehicles throughout the operating period. The proposed model was used to analyze a case study in Lisbon and an optimizer based on branch-and-cut algorithms was used to solve the problem.

The dynamic allocation of vehicles among the stations of a car-sharing system to maximize profit has been modeled in [4]. The fleet size, the location of stations, and the demand for trips for a given planning horizon are known in advance. Penalties associated with unserved trip requests are not considered. A multistage stochastic linear model with recourse has been proposed to address this problem. A stochastic optimization method based on Monte Carlo simulation was used to solve the proposed model [4]. This model considers only the vehicle relocation decisions. Furthermore, vehicle relocation is performed at the end of the day.

The problem of determining the fleet size and the distribution of vehicles among the stations of a car-sharing system was studied in relation to the Personal Intelligent City Accessible Vehicles (PICAVs). This system uses a homogeneous fleet of eco-friendly vehicles and allows one-way trips [9]. The stations are parking lots that offer vehicle recharging services and are located at inter-modal transfer points and near major attraction sites within a pedestrian area. The number, location and capacity of stations are not determined by the model, hence constitute inputs to the simulated annealing process. To cope with the imbalance of vehicle accumulation of the one-way system, this model introduces the concept of supervisor. The task of the supervisor is to direct users that are flexible in returning the car to alternative stations, as to achieve a balanced operation and fulfill a maximum waiting time constraint. The objective function of this model includes the minimization of the daily system and user costs subject to a maximum waiting time constraint. The value of the objective function of the model was estimated through micro-simulation. A simulated annealing approach was used for determining the fleet size and for allocating vehicles among system stations.

Models for evaluating the performance of a network of car-sharing stations has been introduced in the literature [10, 11]. This problem arises when the demand for car-sharing services changes (increases) and as a consequence the network of stations should be adapted to serve better the emerging demand profile. In response to this need a decision support tool was developed which allows decision makers to simulate alternative strategies leading to different network configurations. Such strategies include opening and/or closing stations, and increasing the capacity of stations. This tool is based on discrete event simulation and seeks to maximize the satisfaction level of the users and to minimize the number of cars used [10]. This model does not address vehicle relocation as it is based on a system that does not allow one-way use of vehicles. Performance analysis for shared-use vehicles systems has been proposed in the literature using a closed queuing network model [11]. In this approach, both exact and approximate solution methods are proposed to evaluate the bike sharing system Vélib operating in Paris, France with over 20000 bicycles and 1500 locations.

### 2.2. Operational Decisions

A major decision associated with the operation of one-way car-sharing systems is how to relocate vehicles. The vehicle relocation problem arises from the imbalanced accumulation of vehicles at stations when the car-sharing system allows their one-way use. Different strategies and models have been proposed in the literature to cope with the vehicle relocation problem.

The relocation of shared vehicles can be realized by using operating staff [5] or it can be user based [12]. Two user-based relocation strategies namely, trip-joining and trip-splitting have been proposed [12]. The trip-joining strategy is used when two users have common pick-up and drop-off stations and there is a shortage of vehicles at the pick-up station. In this case, the users are asked to share the ride. The trip-splitting strategy is used when there is a surplus of vehicles at the pick-up station and there are users that are traveling as a group. Under this condition, the users are asked to use separate vehicles when there is a shortage of vehicles at their destination [12]. The trip-joining and the trip-splitting strategies have been analyzed using data collected from a car-sharing system operated at a university and through simulation. The results of the simulation model suggest that the need for vehicle relocations can be decreased by 42% by using these strategies [12]. User based relocation can be partially achieved by introducing different pricing policies for movements that create high system imbalances [13].

Shortest time, and inventory balancing strategies have been used [5] for staff based vehicle relocation. The shortest time strategy relocates cars from other stations to minimize the travel time needed for a staff member from his/her current location to the station where the car is available plus the travel time needed from the station that the car is available to the station where the car is needed. The inventory balancing strategy relocates cars from stations with over-accumulated vehicles to stations that experience vehicle shortages. Both strategies were tested through a simulation model which was validated using data from an operational car-sharing system [5].

Chance constraint modeling has been used to study fleet redistribution [14]. This model assumes that system configuration, current inventory of each station, costs and demand at each station are known in advance. The model aims to find the minimum cost fleet redistribution plan for the demand expected in the near future. The chance constrained model with reliability $p$ (CCM-$p$) is constructed and solved by utilizing a special technique involving $p$-efficient points (PEPs) [15]. The model is applied on the Intelligent Community Vehicle System in Singapore, a one-way system with 14 stations, 202 parking spaces and 94 vehicles.

In the literature, there are also other types of problems that share common structures with the one-way car-sharing problem. The multiple depot vehicle scheduling problem with time windows (MDVSPTW) is one of the examples [16]. In the MDVSPTW, each customer has a request of tight time windows with a precise start and end time of operations, and a fleet of vehicles serves these customers one at a time. Each vehicle in the fleet belongs to a depot and the vehicles have to return to their depot at the end of the service. The objective of the problem is to minimize the number of vehicles and empty trips.

The literature review revealed that existing modeling efforts make a sharp separation between strategic and tactical decisions. This means that strategic decision-making models do not integrate in their structure aspects of tactical and operational decisions (e.g. vehicle relocation, fleet size) that have a significant bearing on the cost and performance of the car-sharing system. On the other hand, operational models are focused on the detailed modeling of different types of relocation strategies, assuming that the location, number, and station and fleet size are exogenously defined.

In reality, strategic, tactical, and operational decisions are interweaved and therefore there is a strong interaction between the three decision making levels. Strategic decisions are primarily related to the definition of the location, number, and size of stations and interact with the tactical decision of fleet size

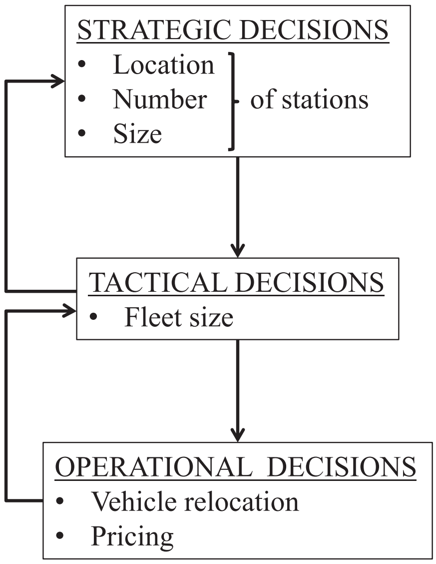



Figure 1: Relationship between strategic, tactical and operational decisions

determination. In turn the fleet size is affected by vehicle relocation which is an operational decision. Here it is important to stress the fact that both fleet size and vehicle relocation influence the strategic level decisions. The above discussion suggests that there is a need for a model that will be able to address the strategic and tactical decisions by taking into account (at a macroscopic level) the impact of vehicle relocation. Figure 1 illustrates these interactions. The above discussion suggests that there is a need for a model that will be able to address the strategic and tactical decisions by taking into account (at a macroscopic level) the impact of vehicle relocation. In what follows we are presenting such a model.

## 3. Model Description

The proposed model is motivated from the planning of electrical one-way car-sharing system. Shared-use electric cars are used to serve trips within a given geographical area. The system operates on the basis of reservations and therefore the origin-destination matrix for the planning period is known in advance. Stochastic and seasonal demand variations are also considered in the optimization process. In what follows we provide a description of the system in terms of its demand and supply characteristics.

### *3.1. System Characteristics*

**i. Vehicles:** A homogeneous fleet of electric cars is used to provide the services. Any type of trip request can be accommodated by any available car.

**ii. Stations:** Vehicles are picked-up and dropped-off at designated stations. Stations have the necessary infrastructure for parking and recharging the vehicles. Each station provides a specific number of parking places which defines the station size. Station size varies among stations and the size of each station determines its capacity.

**iii. Time Intervals:** An operating day is divided into time intervals (not necessarily equally long) and each operation (i.e. rental, relocation, charging) starts at the beginning and ends at the end of a time interval. The model assumes that demand is cyclic and it repeats itself on a daily basis for a given time horizon (e.g. season, day of the week) and the first time interval of a given day starts after the last time interval of the previous day (Figure 2).

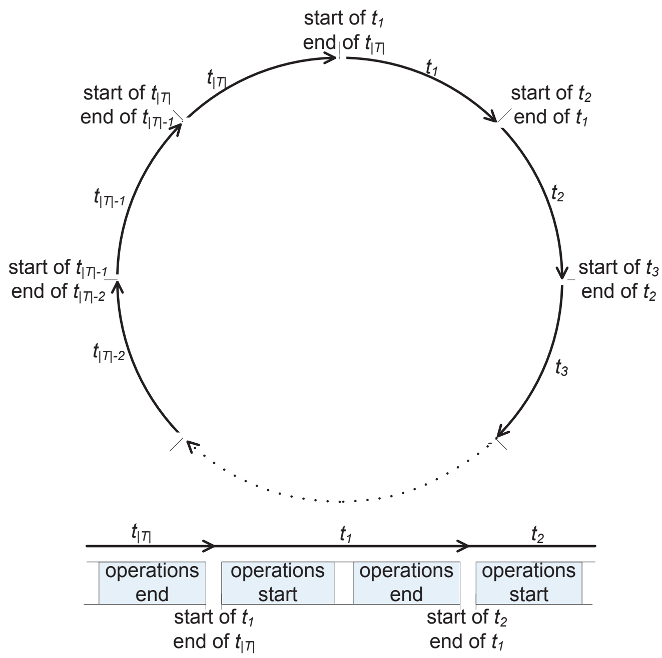



Figure 2: The relationship between time intervals and operations

**iv. Operations:** The system involves three types of operations: rental, relocation and charging.

**a. Rental:** The system operates on the basis of reservations and allows one-way rental of cars. Reservations are made in advance of the pick-up time. Origin and destination locations, and pick-up and drop-off times are also known. Cars are picked-up/dropped-off from/at a station that is accessible to the initial origin/destination location of the respective user at pre-specified (when reservation is made) periods. It is assumed that each rental starts at the beginning of a time interval and ends at the end of the same or a subsequent time interval (Figure 2).

**b. Relocation:** The system allows one way rental of cars. As a result, there might be accumulation and/or shortage of cars at stations. Relocation is used to rebalance the system resources, i.e. vehicles. Relocations can last more than one time interval (Figure 2). During relocation, the vehicle is not available with the exception of extremely closely located stations (i.e. less than 2kms) in which case rental and relocation can take place at the same time interval. The total time spend for relocation operations during a time interval cannot exceed the total available time of the staff assigned to a working shift.

**c. Charging:** The system modeled in this paper uses electric vehicles. In order to model the electric vehicles charging period, it is assumed that after a vehicle is returned from a rental operation, it has to stay in the station for a fixed period of time which represents the charging period of the vehicle.

**v. Working Shift:** A set of consecutive time intervals defines a working shift. Working shifts are used to model the personnel needed for relocation operations.

**vi. Centers:** In the model, centers represent demand points that can be served by the same set of (candidate) stations. To illustrate how the centers are defined we are using the example shown in Figure 3. Figure 3a depicts the origin and destination of demand and the station locations. Figure 3b shows the stations that are accessible by different origin and destination locations. Please note that more than one station may be accessible from a given origin/destination point. The origin/destination points that can access the same set of stations are clustered together and constitute a center. Figure 3c illustrates two centers (shaded areas) and trips (demand) associated with these centers. The grouping of demand into centers decreases the number of variables since the trips with the same origin and destination centers are grouped together and allows the solution of larger instances of problems. The distance between a center and a station is the average of all distances defined by the demand points of a given center and the associated station.

**vii. Demand:** Demand represents an aggregation of trip reservations (orders) of rentals that are associated with the same set of origin and destination centers and have common departure and arrival time intervals. In order to satisfy an "order" (i) a vehicle from a station that is accessible from the origin location (or equivalently center) at the beginning of the departure time interval, and (ii) a parking space at a station that is accessible from the destination location (or equivalently center) at the end of the arrival time interval have to be available. Note that, "orders" do not have to be assigned to the closest stations but to accessible ones.

**viii. Atoms:** An atom represents a small geographical area with known population. The atoms are used to model the population coverage of the car-sharing system. In our model, we assume that there is a maximum distance that determines if an atom is covered. Thus, if there is an open station closer than the predefined maximum value (coverage distance), the atom is covered.

**ix. Costs and Revenues:** The model includes two objective functions expressing the objectives of the users and the operator. The operator's benefits include vehicle rental revenues and subsidies, while costs include maintenance, operation and relocation of vehicles, and station opening costs. Users' net benefit is calculated as the difference between the utility gain in terms of monetary value, and the sum of vehicle rental and accessibility costs. In what follows (see items a to h below) we define all these terms.

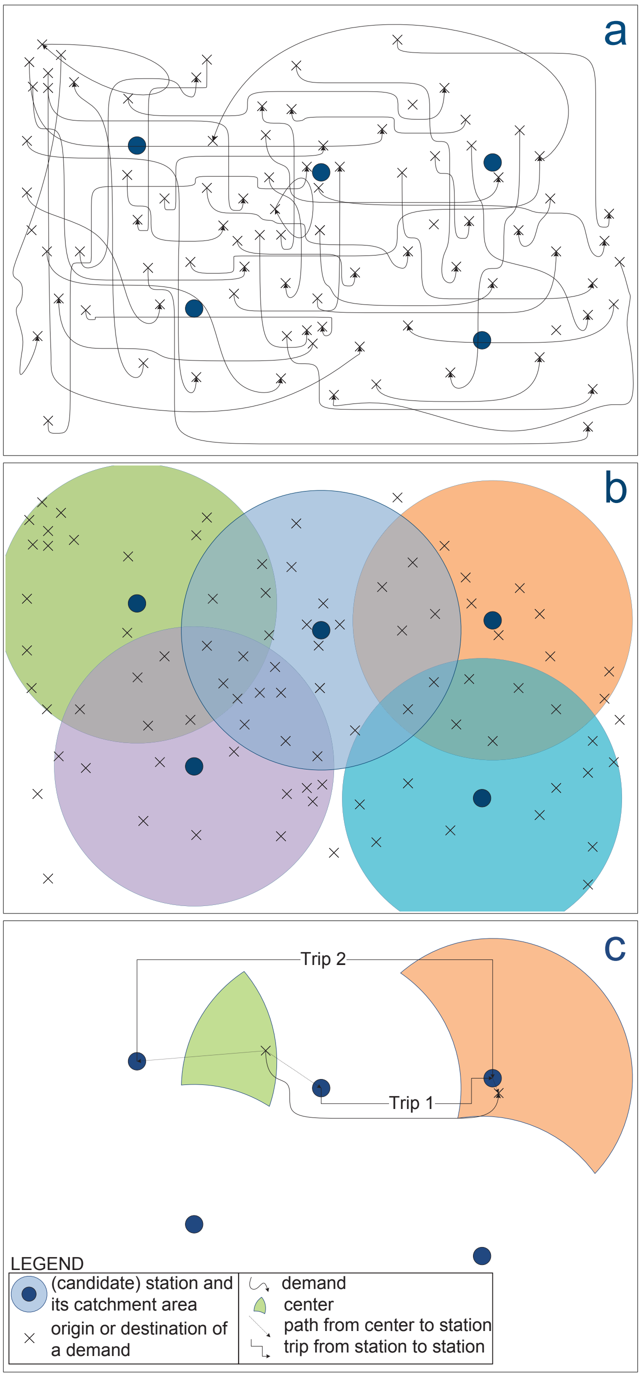



Figure 3: (a) Location of stations and historical trips generated between origins and destinations; (b) Origins and destinations are grouped according to the set of accessible (candidate) stations; (c) Based on this aggregation, a specific demand can be served in two different ways (trip 1 and 2)

**a. Vehicle Rental Cost:** The amount paid by the users to the operator to rent a vehicle expressed in €/unit time

**b. Subsidy:** It represents money paid directly to the operator, by public agencies, to cover revenue deficits per rental in €/unit time.

**c. Fixed Vehicle Cost:** The cost encountered by the operator expressed in €/day (e.g. depreciation, insurance)

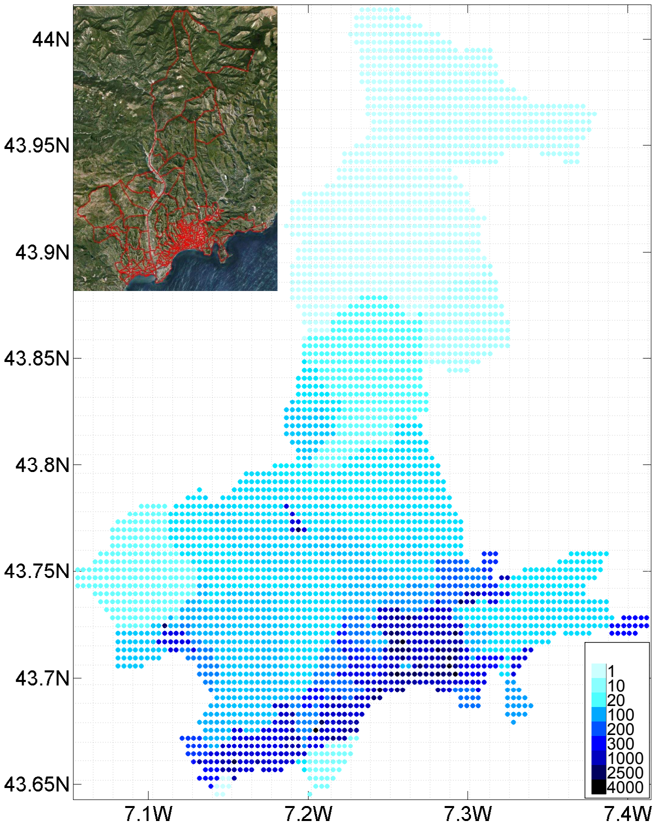



Figure 4: Atoms used in population coverage

**d. Variable Vehicle Cost:** The cost of the operator per km vehicle rented (e.g. cost of energy, maintenance cost due to wear-and-tear).

**e. Vehicle Relocation Cost:** The cost related to the relocation operations of the vehicles. It has two components: the relocation personnel cost (per shift) and the cost for driving vehicles between stations.

**f. Station Operating Cost:** The cost of operating a station. It is a function of the number of operating parking spaces.

**g. User Utility:** The monetary value of the utility gained by the users by each satisfied trip expressed in €/unit time.

**h. Accessibility Cost:** The monetary value of time of the users required to reach a station from their origin and from stations to their destination expressed in €/unit time.

**x. Scenarios:** Alternative scenarios are defined by varying the input parameters of the model (e.g. weekdays, weekends). Scenarios are used to obtain a more representative average system performance.

**xi. Scenario Groups:** The set of scenarios which addresses the same strategic decisions and parameters (e.g. number of vehicles, relocation personnel cost) belongs to the same scenario group. In order to account for daily variation within the same season (e.g. summer, autumn, winter), each season is set as a scenario group and more than one scenarios is generated according to day of the week (e.g. weekdays, weekends).

### *3.2. Mathematical Model*

In this part, we represent the mathematical structure of the proposed model. We first define the sets and indices used to describe the model as well as the functions, variables and parameters in Section 3.2.1. In Section 3.2.2, the detailed multi-objective mathematical model is given and its objective functions and constraints are described in details. The aggregate model and the rational for to have an aggregate model are presented in Section 3.2.3.

#### *3.2.1. Inputs*

**Sets and Indices:**

$i, k \in I$**:** center indices

$j, l \in J$**:** (candidate) station indices

$t, u, w \in T$**:** time interval indices

$f \in F$**:** working shift index

$a \in A$**:** atom index

$s \in S$**:** scenario index

$g \in G$**:** scenario group index

**Functions:**

**next**$(t, \#)$**:** time interval that is # intervals after time interval $t$

**cover**$(a)$**:** set of stations that are accessible from atom $a$

**btwn**$(t, u)$**:** set of time intervals from $t$ to $u$

**close** $(j)$**:** set of stations that relocation with station $j$ is possible during the same time interval

**Parameters:**

$\mathbf{SOC}_j$**:** cost for establishing station $j$

$\mathbf{PSC}_j$**:** cost per parking space available at station $j$

$\mathbf{VFC}^g$**:** fixed vehicle cost per vehicle-day in scenario group $g$

$\mathbf{VOC}_{jl}^{stu}$**:** operating cost of a vehicle rented at time interval $t$ from station $j$ to reach station $l$ at time interval $u$ in scenario $s$

$\mathbf{VRC}_{jl}^{gt}$**:** relocation cost of moving a vehicle from station $j$ to $l$ starting at time interval $t$ in scenario group $g$

$\mathbf{AC}_{ij}^{gt}/\overline{\mathbf{AC}}_{ij}^{gt}$**:** accessing/egressing cost from/to region $i$ to/from station $j$ at time interval $t$ in scenario group $g$

$\mathbf{RPC}_f^g$**:** cost of relocation personnel for working shift $f$ in scenario group $g$

**$RC_{jl}^{gtu}/SA_{jl}^{gtu}$:** rental charge/subsidy when a vehicle is rented at time interval $t$ from station $j$ to reach station $l$ at time time interval $u$ in scenario group $g$

**$UG_{jl}^{stu}$:** user utility when a vehicle is rented at time interval $t$ from station $j$ to reach station $l$ at time time interval $u$ in scenario $s$

**$CAP_j$:** maximum number of available parking spaces for station $j$

**COV:** minimum percentage of population need to be covered by open stations

**$PR_a$:** percent of population inhabiting in atom $a$

**$OD_{ik}^{stu}$:** number of orders starting at the beginning of time interval $t$ from center $j$ ending at the end of time interval $u$ at center $k$ for scenario $s$

**$RI_{jl}^{gt}$:** time intervals needed to relocate a vehicle from station $j$ to $l$ starting at the beginning of time interval $t$ in scenario group $g$

**$LRI_{jl}^{gt}$:** last time interval of relocation if a vehicle is relocated from station $j$ to $l$ starting at the beginning time interval $t$ in scenario group $g$

**$SI_f^g$:** time intervals included in working shift $f$ in scenario group $g$

**$RT_{jl}^{gt}$:** time spend to relocate a vehicle from station $j$ to $l$ at the beginning of time interval $t$ in scenario group $g$

**$WH^{gt}$:** total available working hours for a shift operating during time interval $t$ in scenario group $g$

**$SW^s$:** weight of the net benefit of scenario $s$ in the objective function

**$CT_{jl}^{stu}$:** charging periods of vehicles rented at time interval $t$ from station $j$ to reach station $l$ at time interval $u$ in scenario $s$

**N:** maximum number of open stations

$S(g)$**:** scenarios belonging to scenario group $g$

$G(s)$**:** scenario group of scenario $s$

**Variables:**

$x_j$**:** binary variable showing if (candidate) station $j$ is open or not

$n_j^{\star}$**:** number of parking spaces operating in station $j$

$n_j^{st}$**:** number of available vehicles in station $j$ at the beginning of time interval $t$ in scenario $s$

$y_{ikjl}^{stu}$**:** number of trip orders satisfied from center $i$ renting vehicle from station $j$ to make a trip at the beginning of time interval $t$ to reach center $k$ through station $l$ at the end of time interval $u$ in scenario $s$

$z_{jl}^{stu}$**:** number of vehicles rented from station $j$ at the beginning of time interval $t$ to reach station $l$ at the end of time interval $u$ in scenario $s$

$m_{ik}^{stu}$**:** number of unserved orders of $OD_{ik}^{stu}$

$v^g$**:** number of vehicles used in scenario group $g$

$c_a$**:** binary variable showing if atom $a$ is covered by a station or not

$p_{ij}^{st}/\overline{p}_{ij}^{st}$**:** number of cars rented/left from/to station $j$ at the beginning/end of time interval $t$ to/from center $i$ in scenario $s$

$q_j^{st}/\overline{q}_j^{st}$**:** number of vehicles rented/left from/to station $j$ at the beginning/end of time interval $t$ in scenario $s$

$h_f^g$**:** number of relocation personnel needed during shift $f$ in scenario group $g$

$b_t^s$**:** number of vehicles rented before time interval $t$ which are still rented during time interval $t$ in scenario $s$

$e_t^s$**:** number of vehicles being relocated during time interval $t$ for which their relocation started before $t$ in scenario $s$

$r_{jl}^{st}$**:** number of vehicles relocated from station $j$ to $l$ starting from the beginning of time interval $t$ in scenario $s$

*3.2.2. Detailed Model*

The problem formulation is described in equations 1-18. In order to facilitate the explanation of the model we introduced the following notation:

**i. $\mathbf{D}(i^t \rightarrow k^u\,[s])$:** The demand starting at the beginning of time interval $t$ from center $i$ to center $k$ ending at the end of time interval $u$ in scenario $s$.

**ii. $\mathbf{T}(j^t \rightarrow l^u\,[s])$:** The trip starting from station $j$ to station $l$ from the beginning of time interval $t$ to time interval $u$ in scenario $s$.

**iii. $\mathbf{S}\left(i_j^t \rightarrow k_l^u\,[s]\right)$:** The demand that is assigned to a trip starting from center $i$ at the beginning of time interval $t$ by a vehicle from station $j$, ending at the end of time interval $u$ in center $k$ through station $l$ in scenario $s$.

**iv. $\mathbf{R}(j^t \rightarrow l\,[s])$:** The relocation starting from station $j$ at time interval $t$ to station $l$ in scenario $s$.

The first objective function (Equation 1) expresses the maximization of the net revenue for the operator. Net revenue is calculated as the difference between the sum of total rental revenue and subsidy minus station, vehicle and relocation costs. Note that all of the values in both objective functions except station opening cost are weighted analogous to the number of days (e.g. two for weekdays, five for weekends) of each scenario ($SW^s$). This is due to the fact that the location of the stations and the number of parking spaces are regarded as strategic decisions and therefore have to be the same in all scenarios. However the rest of the parameters are scenario (e.g. the number of vehicles)

$$
\begin{aligned}
\max \sum_{(s,j,l,t)} \mathrm{SW}^s & \left[ \overbrace{\sum_u \left(\mathrm{RC}^{stu}_{jl} + \mathrm{SA}^{stu}_{jl} - \mathrm{VOC}^{stu}_{jl}\right) z^{stu}_{jl}}^{\text{rental charge + subsidy - vh. operating costs}} - \overbrace{\mathrm{VRC}^{G(s)t}_{jl} r^{st}_{jl}}^{\text{vh. relocation cost}} \right] \\
& - \sum_g \sum_{s\in S(g)} \mathrm{SW}^s \left( \overbrace{\sum_f \mathrm{RPC}^g_f h^g_f}^{\text{personnel cost}} + \overbrace{\mathrm{VFC}^g v^g}^{\substack{\text{vehicle}\\ \text{maintenance cost}}} \right) - \overbrace{\sum_j \left(\mathrm{SOC}_j x_j + \mathrm{PSC}_j n^*_j\right)}^{\text{st. operating and parking costs}}
\end{aligned}
\tag{1}
$$

$$
\max \sum_s \mathrm{SW}^s \left[ \overbrace{\sum_{(j,l,t,u)} \left(\mathrm{UG}^{stu}_{jl} - \mathrm{RC}^{stu}_{jl}\right) z^{stu}_{jl}}^{\text{utility - rental charge}} - \overbrace{\sum_{(i,j,t)} \left(\mathrm{AC}^{G(s)t}_{ij} p^{st}_{ij} + \overline{\mathrm{AC}}^{G(s)t}_{ij} \overline{p}^{st}_{ij}\right)}^{\text{accessibility cost}} \right] \tag{2}
$$

$$
\text{s.t. } n^\star_j \leq \mathrm{CAP}_j x_j \quad \textbf{(a)} \qquad n^{st}_j \leq n^\star_j \quad \textbf{(b)} \qquad \sum_j x_j \leq \mathrm{N} \quad \textbf{(c)} \qquad \forall j \text{ and } \forall s, j, t \tag{3}
$$

$$
n^\star_j \geq x_j \quad \textbf{(a)} \qquad \sum_{(s,t)} n^{st}_j \geq x_j \quad \textbf{(b)} \qquad \forall j \tag{4}
$$

$$
c_a \leq \sum_{j\in \mathrm{cover}(a)} x_j \quad \textbf{(a)} \qquad \sum_a \mathrm{PR}_a c_a \geq \mathrm{COV} \quad \textbf{(b)} \qquad \forall a \tag{5}
$$

$$
\sum_{(j,l)} y^{stu}_{ikjl} + m^{stu}_{ik} = \mathrm{OD}^{stu}_{ik} \qquad \forall s, i, k, t, u \tag{6}
$$

$$
\sum_{(i,k)} y^{stu}_{ikjl} = z^{stu}_{jl} \qquad \forall s, j, l, t, u \tag{7}
$$

$$
\sum_{(k,l,u)} y^{stu}_{ikjl} = p^{st}_{ij} \quad \textbf{(a)} \qquad \sum_{(k,l,u)} y^{sut}_{kilj} = \overline{p}^{st}_{ij} \quad \textbf{(b)} \qquad \forall s, i, j, t \tag{8}
$$

$$
\sum_{(i,k,l,u)} y^{stu}_{ikjl} = q^{st}_j \quad \textbf{(a)} \qquad \sum_{(i,k,l,u)} y^{sut}_{iklj} = \overline{q}^{st}_j \quad \textbf{(b)} \qquad \forall s, j, t \tag{9}
$$

$$
q^{st}_j \leq n^{st}_j - \sum_l r^{st}_{jl} + \sum_{l\in \mathrm{close}(j)} r^{st}_{lj} \qquad \forall s, j, t \tag{10}
$$

$$
n^{st}_j - q^{st}_j + \overline{q}^{st}_j - \sum_l r^{st}_{jl} + \sum_{\substack{(l,u):\\ \mathrm{LRI}^{G(s)u}_{jl}=t}} r^{su}_{jl} = n_j^{\mathrm{snext}(t,1)} \qquad \forall s, j, t \tag{11}
$$

$$
\sum_{\substack{(j,l,u,w):\\ t\in \mathrm{btwn}(u,w)\setminus u}} z^{suw}_{jl} = b^{st} \quad \textbf{(a)} \qquad \sum_{\substack{(j,l,u):\\ t\in \mathrm{btwn}\left(u, \mathrm{LRI}^{G(s)t}_{jl}\right)\setminus u}} r^{su}_{jl} = e^{st} \quad \textbf{(b)} \qquad \forall s, t \tag{12}
$$

$$
\sum_j n^{st}_j + b^{st} + e^{st} \leq v^{G(s)} \qquad \forall s, t \tag{13}
$$

$$
\sum_{\substack{(j,l,u):\\ t\in \mathrm{RI}^{G(s)u}_{jl}}} \mathrm{RT}^{G(s)t}_{jl} r^{su}_{jl} \leq \mathrm{WH}^{G(s)t} h^{G(s)}_f \qquad \forall s, f, t \in \mathrm{SI}^{G(s)}_f \tag{14}
$$

$$
r^{st}_{jl} \leq n^\star_j \quad \textbf{(a)} \qquad r^{st}_{lj} \leq n^\star_j \quad \textbf{(b)} \qquad \forall s, j, l, t \tag{15}
$$

$$
n^{st}_j \geq \sum_{(s,j,l,u,w): t\in \mathrm{CT}^{suw}_{lj}} z^{suw}_{lj} \qquad \forall s, j, t \tag{16}
$$

$$
x_j, c_a \in \{0, 1\} \qquad \forall j, a \tag{17}
$$

$$
n^{st}_j, n^\star_j, p^{st}_{ij}, \overline{p}^{st}_{ij}, r^{st}_{jl}, m^{stu}_{ik}, z^{stu}_{jl}, y^{stu}_{ikjl}, q^{st}_j, \overline{q}^{st}_j, b^{st}, e^{st}, v^s, h^g_f \in \mathcal{N} \qquad \forall s, i, k, j, l, t, u, g, f \tag{18}
$$

specific. The net revenue for $\mathrm{T}(j^t \rightarrow l^u\,[s])$ of given type equals the rental charge per trip $\left(\mathrm{RC}^{stu}_{jl}\right)$ plus subsidy $\left(\mathrm{SA}^{stu}_{jl}\right)$ minus operating cost $\left(\mathrm{VOC}^{stu}_{jl}\right)$ times the number of trips of the same type served $\left(z^{stu}_{jl}\right)$.

The relocation cost has two components: (i) The vehicle cost related to the total km driven to relocate and (ii) the personnel cost associated with the labor cost of the personnel used to relocate the vehicles. The total vehicle relocation cost is equal to the expenses of all the relocations. The vehicle relocation cost for $\mathrm{R}(j^t \rightarrow l\,[s])$ is equal to the sum per relocation $\left(\mathrm{VRC}_{jl}^{G(s)t}\right)$ times the number of relocations $\left(r_{jl}^{st}\right)$. Similarly, the relocation personnel cost equals the sum of all personnel costs. The total personnel cost for shift $f$ in scenario group $g$ equals the unit personnel cost $\left(\mathrm{RPC}_f^g\right)$ times the number of staff hired for this shift $\left(h_f^g\right)$.

The fixed vehicle cost depends on the total number of vehicles operating in the system. For scenario $s$, this cost is equal to the product of the unit fixed vehicle cost ($\mathrm{VFC}^g$) and the number of vehicles in the system ($v^g$) in scenario group $g$. Note that, for scenarios belonging to the same (scenario) group, the number of vehicles is the same, since we regard the number of vehicles as a tactical decision.

The station operating and parking space costs are the costs dedicated to station operations. There is a fixed cost for operating a station $\left(\mathrm{SOC}_j\right)$ and a variable cost $\left(\mathrm{PSC}_j\right)$ for each parking space $\left(n_j^*\right)$ operating at given station $j$.

The second objective (Equation 2) expresses the maximization of the users' net benefit. $\mathrm{UG}_{jl}^{stu}$ can be defined as the monetary value (i.e. €) of the utility gain for each realized $\mathrm{T}(j^t \rightarrow l^u\,[s])$ of the same type. Similarly, the rental fee is the money paid to the operator for the rental of vehicles by the users $\left(\mathrm{REV}_{jl}^{stu}\right)$ and total rental charge equals the sum of them. The accessibility cost is the cost associated with the access or egress of a station from a center.

Constraints 3a, 3b restrict the number of parking spaces (station capacity constraint), and the number of available vehicles for each time interval and station. If a station is not open in a candidate station location, the station capacity is set to zero. If the station is open then there is an upper bound ($\mathrm{CAP}_j$) for its capacity. Constraint 3c limits the total number of operating stations. Constraints 4a and 4b require that if a station is open, at least one parking space and an operation (i.e. rental, relocation) from this station should be assigned as well. These constraints are essential in order to guarantee the coverage of the demand by an open station that has at least a capacity of one parking space. Constraints 5a and 5b are the atom coverage constraints, i.e. if an atom is covered or not, and population coverage constraints, i.e. the car-sharing system is accessible by a given percentage of the population, respectively. Constraints 6 ensure that the total number of orders is equal to the sum of the satisfied demand and unserved (lost) orders.

Constraints 7 postulate that the total number of $\mathrm{S}\left(i_j^t \rightarrow k_l^u\,[s]\right)$ over origin/destination center pairs $(i,k)$, is equal to $\mathrm{T}(j^t \rightarrow l^u\,[s])$. Constraints 8a indicate that the total number of $\mathrm{S}\left(i_j^t \rightarrow k_l^u\,[s]\right)$ from station $j$ is equal to the number of vehicles rented from station $j$ at the beginning of time interval $t$ to serve demand from centers $i$. Constraints 8b do the same as constraints 8a for the cars originating from center $i$ left at station $j$ at the end of period $t$. Constraints 9a and 9b are equivalent to Constraints 8a and 8b and ensure respectively the same conditions for the cars that are rented/left from/to a station $j$. Thus, Constraints 7, 8a, 8b, 9a and 9b establish the functional relationship between the variables $y$, and $z$, $p$ ($\overline{p}$) and $q$ ($\overline{q}$) respectively. Please note that, variables $z$ express car assignments independent of the center to which originate/end their movement, variables $p$ and $\overline{p}$ indicate customer movements from centers to stations and from stations to centers respectively, and variables $q$ and $\overline{q}$, signify the number of vehicles rented from and left to station respectively.

Constraints 10 require that the number of cars leaving a station (due to rental and relocation) at the beginning of interval $t$ cannot exceed the number of vehicles available at that stations at the same time interval. Constraints 11 are the "car conservation" constraints for each station.

Constraints 12 are used to establish the functional relationship between variables $b$, $e$, and $z$, $r$ respectively. Variables $b$ and $e$ are used in Constraints 13 to determine the total number of cars (fleet size) of the system. Constraints 14 are introduced to ensure the per shift availability of the workforce needed to perform car relocations.

Constraints 15a and 15b set an upper bound to relocation to from and relocation to of every station respectively. This upper bound equals to the number of operating parking spaces in related station. For a station which is not open, the number of relocations from and to this station are set to zero with the same constraints respectively.

Constraints 16 are restrictions specific to electric-car-sharing systems. These constraints force the vehicles to stay and be charged after each rental operations by keeping them in the station they arrived. The constraints requires that the number of vehicles in the station should be greater than or equal to the number of vehicles need charging.

### 3.2.3. *Aggregate Model*

In real life instances, the model described by equations 1-18 may result in problem sizes that are not possible to be efficiently solved. Although for most of the variables, we only generate those that have positive values and construct the corresponding constraints accordingly, we do not have this opportunity for the relocation variables $r_{jl}^{st}$. As the relocations can happen between any station pairs, we need to generate $|J|^2|S||T|$ number of variables which renders the case of Nice, France impossible to solve. An instance of 142 candidate stations, 12 scenarios and 15 time intervals needs more than 3.6 millions variables of type $r_{jl}^{st}$ only. In order to cope with this issue, we assume that the relocated vehicles are firstly accumulated in an imaginary hub and then distributed from that hub to the stations. For this issue, two new variables, $r_j^{st}$ and $\overline{r}_j^{st}$ are defined as that the number of vehicles relocated from/to station $j$ starting from the beginning/finishing at the end of time interval $t$ in scenario $s$. With this change, the number of variables of type $r$ decreases to $2|J||S||T|$ which means 51120 variables instead of over 3.6 millions.

In addition, we substitute the constraints 10, 11, 12b and 14 and 15, with the following constraints 21-28. Moreover, the vehicle relocation cost part of the operator's objective function (Equation 1) is replaced with Equation 20. Note that, parameters $\mathrm{LRI}_j^{gt}$, $\mathrm{VRC}_j^{gt}$ and $\mathrm{RT}_j^{gt}$ shows the last time interval, the ve-

$$\text{Equations } 2-9, 12\text{a}, 13, 16-18 \tag{19}$$

$$\max \sum_{(s,j,l,t)} \mathrm{SW}^s \left[\sum_u \left(\mathrm{RC}_{jl}^{stu} + \mathrm{SA}_{jl}^{stu} - \mathrm{VOC}_{jl}^{stu}\right) z_{jl}^{stu}\right] - \overbrace{\sum_{(s,j,t)} \mathrm{SW}^s \mathrm{VRC}_j^{G(s)t}\left(r_j^{st} + \bar{r}_j^{st}\right)}^{\text{new vh. relocation cost}}$$
$$- \sum_g \sum_{s\in S(g)} \mathrm{SW}^s \left(\sum_f \mathrm{RPC}_f^g h_f^g + \mathrm{VFC}^g v^g\right) - \sum_j \left(\mathrm{SOC}_j x_j + \mathrm{PSC}_j n_j^*\right) \tag{20}$$

$$q_j^{st} \leq n_j^{st} - r_j^{st} + \sum_{l\in \mathrm{close}(j)} r_l^{st} \qquad \forall s, j, t \tag{21}$$

$$q_j^{st} + \sum_{l\in \mathrm{close}(j)} q_l^{st} \leq n_j^{st} + \sum_{l\in \mathrm{close}(j)} n_l^{st} \qquad \forall s, j, t \tag{22}$$

$$n_j^{st} - q_j^{st} + \bar{q}_j^{st} - r_j^{st} + \sum_{(j,u):t=\mathrm{LRI}_j^{G(s)u}} \bar{r}_j^{su} = n_j^{\mathrm{snext}(t,1)} \qquad \forall s, j, t \tag{23}$$

$$\sum_{(j,u):t=\mathrm{LRI}_{jl}^{G(s)u}} r_j^{su} = \sum_j \bar{r}_j^{st} \qquad \forall s, j, t \tag{24}$$

$$r_j^{st} \leq n_j^\star \quad \textbf{(a)} \qquad \bar{r}_j^{st} \leq n_j^\star \quad \textbf{(b)} \qquad \forall s, j, t \tag{25}$$

$$\sum_{(j,u):t\in \mathrm{btwn}\left(u, \mathrm{LRI}_j^{G(s)t}\right)\setminus u} r_j^{su} + \bar{r}_j^{su} = e^{st} \qquad \forall s, t \tag{26}$$

$$\sum_{(j,l,u):t\in \mathrm{RI}_j^{G(s)u}} \mathrm{RT}_j^{G(s)t}\left(r_j^{su} + \bar{r}_j^{su}\right) \leq \mathrm{WH}^{G(s)t} h_f^{G(s)} \qquad \forall s, f, t \in \mathrm{SI}_f^{G(s)} \tag{27}$$

$$r_j^{st} \geq 0 \qquad \bar{r}_j^{st} \geq 0 \qquad \forall s, j, t \tag{28}$$

hicle relocation cost and time spent when a vehicle is relocated from/to station $j$ to/from hub starting in time interval $t$ in scenario group $g$ respectively. The aggregate model is expressed by equations 19-28.

Constraints 21 and 22 replace constraints 10. Constraints 21 postulate that the total number of $\mathrm{T}(j^t \rightarrow l^u\,[s])$ cannot be more than the number of available vehicles at the beginning of the time interval $t$; minus the number of relocations from station $j$; plus the number of relocations from the stations that are close enough to station $j$ to have relocations at the same time interval. Constraints 22 set an upper bound for each station group close enough to have relocations to the same station. For each set of stations, the total number of trips started from the corresponding set of stations cannot be more than the total number of available vehicles at these stations.

Constraints 23 replace constraints 11 of the first model. Constraints 24 require that the total number of relocations from stations to the imaginary hub ending in time interval $t$ should be equal to the number of relocations to the stations from the imaginary hub starting in time interval $t$. This is applicable for each time interval and scenario.

Constraints 25a and 25b replace constraints 15a and 15b. They set the number of relocations to the number of operating parking spaces. Constraints 26 and 27 work the same as constraints 12b and 14 respectively. The former constraints calculate the number of vehicles under relocation whereas the latter constraints decide on the manpower need for each time interval in each scenario.

## 4. Model Application

The model presented in Section 3.2.3 was applied to plan a one-way electrical car sharing system in Nice, France. The study area is 294.19km$^2$ and has a population 327188 inhabitants between ages 15-64 with a density 1112 persons/km$^2$. The area under consideration consists of 210 regions. The population of each region was obtained from 2009 census data. We assume that the population is uniformly distributed inside regions and calculate the population of each atom accordingly. The atoms and their population can be seen in Figure 4.

The whole model is implemented in C# .NET environment. IBM ILOG Cplex Version 12.2 with Concert Technology is used for solving MILPs. To cope with the enormous number of relocation variables, the aggregate model (Section 3.2.3) is used. For each station, half of the average distance of closest $n$ stations is calculated and regarded as the distance of the same station to the imaginary hub. This way, we tried to generate values that are close to real relocation distances. To further investigate the performance of the approximation, a simulation environment that compares average real and hub relocation distance for 1000 case is generated with different $n$ parameters. In Figure 5, the error for different $n$ values with the number of relocations from five to 20 is compared. We decided to use 20 for the value of $n$ since it has on average minimum error. In other words, when distance for relocation is calculated, the distance from a station to the hub is assumed half of the average distance of 20 closest (candidate) stations. Note that a relocation is composed of two legs in aggregate model: relocating vehicle from its old station to the imaginary hub and to its new destination from the hub. A similar approach is used for the second leg. The number of relocations per personnel has values between seven and 15 which results in error not more than 0.7km per relocation. Since distance per relocation observed is around 4kms and the total cost of relocation is not more than 10% of the objective function value of each case (see in figures 9 and 10), this relaxation might not create an error more than 2%. In order to deal with the extremely large size of the problem, we take advantage of the sparsity of the matrices of the variables and we do not generate the variables that have zero value. This decreases the number of variables of aggregate model from over $7.6 \times 10^{10}$ to less than $3.8 \times 10^{5}$.

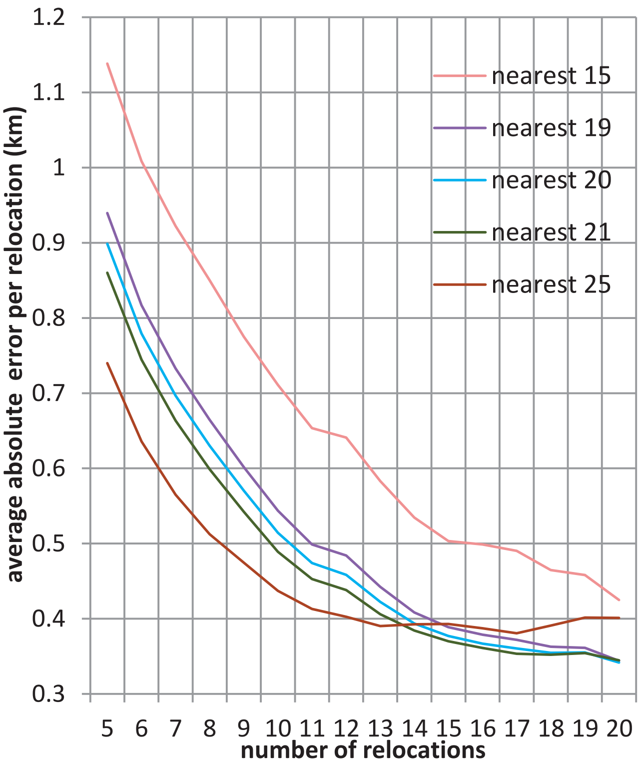



Figure 5: Average absolute error of imaginary hub usage in relocation for different number of relocations. Different $n$ values are compared in order to find the most suitable value for our case.

To guarantee generation of feasible solutions in reasonable time, extra cuts are generated with CPLEX. The runs are taken on a computer with 3.00 Ghz Intel Core 2 Quad CPU and 8 GB of RAM working with Microsoft Windows 7 environment. All runs are realized as single threaded programs and every run is terminated when either they reach 2% optimality gap or 9 hours run time. Most of the runs that are represented here were terminated in less than three hours and all of the runs had an optimality gap less than 8%.

The pseudocode for the entire algorithm can be seen in Figure 6 where $w_{\text{operator}}$ and $w_{\text{users}}$ stands for weights of operator and users benefit respectively. The terms *superior* and *inferior* used in finding candidate station section refers to superiority and inferiority in coverage respectively. If a candidate station covers one more origin or destination location in addition to another candidate station's covered locations, the former candidate station is superior to the latter and latter is inferior to the former.

The current system operating in Nice is a two-way car-sharing system (no need for relocation operations). However, the proposed model deals with the case of one-way car-sharing, which makes the implementation more demanding. Therefore, there was a need to convert the existing two-way car-sharing data into one-way. This conversion was achieved by looking at the current database and creating one-way data by splitting the trips into more one-way legs when the idle time of the rented vehicle at the same location was exceeding one hour. We use the origin and destination locations of the real demand in two steps. First, we solve a *maximal set covering problem* [17] to identify the candidate locations for the aggregate model. For each location, the locations that are accessible (the distance between two points is less than the maximum accessibility distance) are found. Then a maximal set covering problem is solved. In addition to existing 42 stations, the model was forced to choose 100 new candidate locations for the stations. Second, we group the locations into centers. This grouping was done according to the (existing or candidate) stations that are accessible to them. The locations with the same accessible stations were assigned to the same centers. The accessibility distance between a center and a station is calculated by taking the average of the distance between the elements of the center and the station (Figure 3). The graph showing the locations of the origin and destination of the trips (crosses), the operating (blue) and candidate (red, gray and black) stations' locations (dots) and their catchment areas (circles with the same colors) can be seen in Figure 7 in which $x$-axis shows the longitude and $y$-axis shows the latitude values. Note that, the covered origin and destination locations have dark gray color and each grid is a square with sides of 1 km.

After solving set covering problems, the set of candidate locations for the aggregate model defined in Section 3.2.3 is produced. The aggregate model is solved with different weights (of users' and operator's benefit) in order to generate an efficient frontier for the given case. A total of 12 different scenarios of three seasons (summer, autumn, winter) for two different day groups (weekdays, weekends) were selected. It was also assumed that the number of operating vehicles and relocation personnel for the same season is the same. This is because the fleet and crew size decisions are considered tactical and do not change within the same season. Each scenario was constructed by using two days of the real demand of the same day group in the same season. The capacity of each station was set to five vehicles and the model was asked to choose 28 more stations (from a set of 100 candidates) in addition to already operating 42 stations. Each day was divided into 15 time intervals with approximately the same demand ("orders"). Each vehicle has to be charged for two hours after rental. The values for some of the other parameters applied in the model are presented in

1. Data reading and parameter creation
   (a) Read population data and create atoms
   (b) Read historical demand data
   (c) Conversion from two-way demand data to one-way
      i. For each historical demand datum
         A. If waiting time is greater than predefined value
            - Split the historical demand and create two new demands
   (d) Time Interval Selection
      i. Set working shifts
      ii. Find time intervals consistent with working shifts that minimizes the variation of demand count in each time interval
2. Finding candidate locations
   (a) Finding all candidate locations for set covering model
      i. Iterate over all demand origin and destination locations
         A. If current location is *superior* to any previously added location
            - Remove previously added *inferior* solution
         B. If current location is not inferior to any previously added location
            - Add current location to locations for candidate locations for set covering model
   (b) Finding candidate locations for aggregate model
      i. Set $i = N$
      ii. While $|J| <$ maximum number of stations
         - Run *maximal set covering problem* [17] with number of sets = $i$
         - Add candidate locations found in the solution to $|J|$
         - Set $i \leftarrow i + 1$
3. Mathematical model
   (a) Select predefined number of days from historical demand and create scenarios
   (b) Set values of $w_{\text{operator}} > 0$ and $w_{\text{users}} > 0$
   (c) Variable creation
      i. Create variables $c_a$, $x_j$, $n_j$, $n_j^\star$ and $b_t^s$
      ii. For each $\mathrm{D}(i^t \to k^u\,[s])$
         - Create (or increment upper bound if already created) variables $y_{ikjl}^{stu}$, $m_{ik}^{stu}$ and $z_{jl}^{stu}$
         - Create (or increment upper bound if already created) variables $p_{ij}^{st}$, $\overline{p}_{ij}^{st}$, $q_j^{st}$ and $\overline{q}_j^{st}$
      iii. Create variables $r_{jl}^{st}$, $\overline{r}_{jl}^{st}$ and $e_t^s$ (if $\neq 0$)
      iv. Create variables $h_f^g$ and $v^g$
   (d) Constraint creation
      i. For each $j \in J$: Create constraints 3a, 4a and 4b
      ii. For each $(s, j, t)$: Create constraint 3b
      iii. For each $a \in A$: Create constraint 5a
      iv. Create constraints 3c and 5b
      v. For each $\mathrm{D}(i^t \to k^u\,[s])$: Create constraints 6-9
      vi. For each $(s, t)$: Create constraints 12a, 13 and 26
      vii. For each $(s, j, t)$: Create constraints 16, 21-25
   (e) Create objective function with the multiplication of $w_{\text{operator}}$ and $w_{\text{users}}$, and 20 and 2 respectively
   (f) Solve the model and, calculate net users' and operator's benefit

Figure 6: Pseudocode of the entire algorithm with the weights $w_{\text{operator}}$ and $w_{\text{users}}$ for the users' and operator's benefit respectively

Table 1.

Using the parameters presented in Table 1, we solved the model and we generated the efficient frontier provided in Figure 8. The selected candidate stations can also be seen in Figure 7. The candidate stations shown with red color are the candidates that are not selected, the ones with gray and black are the stations selected at least once. The intensity of the color given to the selected candidate stations increases as the frequency of their appearance in the efficient frontier increases. For instance, black means the candidate station appears in all the efficient solutions whereas the lightest gray suggests that it appeared in only one of them.

As it can be seen in Figure 8, although the part of the data used to create the efficient frontier composed of 24 of the 244 days, selected candidate stations manage to cover on average more than 88.6% of the whole demand. This value climbed to 89.8% if the station locations are selected with a maximal coverage problem over the entire set of origins and destinations. For instance, there is an accumulation of demand around the coordinates 43.73N-7.187E and the model selects to operate a station there in all efficient solutions.

As expected, the operator should sacrifice some of its net revenue in order to improve total users' benefit and vice versa. Although the revenue and subsidy from the served demand is higher when more demand is served, the rate of increase of the operational costs (e.g. vehicle operating cost, relocation cost) is higher than the rate of increase of the associated benefits. Both the number of vehicles in the system and the increase of relocation operations decrease the utilization of the vehicles.

Another interesting result is associated with the selection of

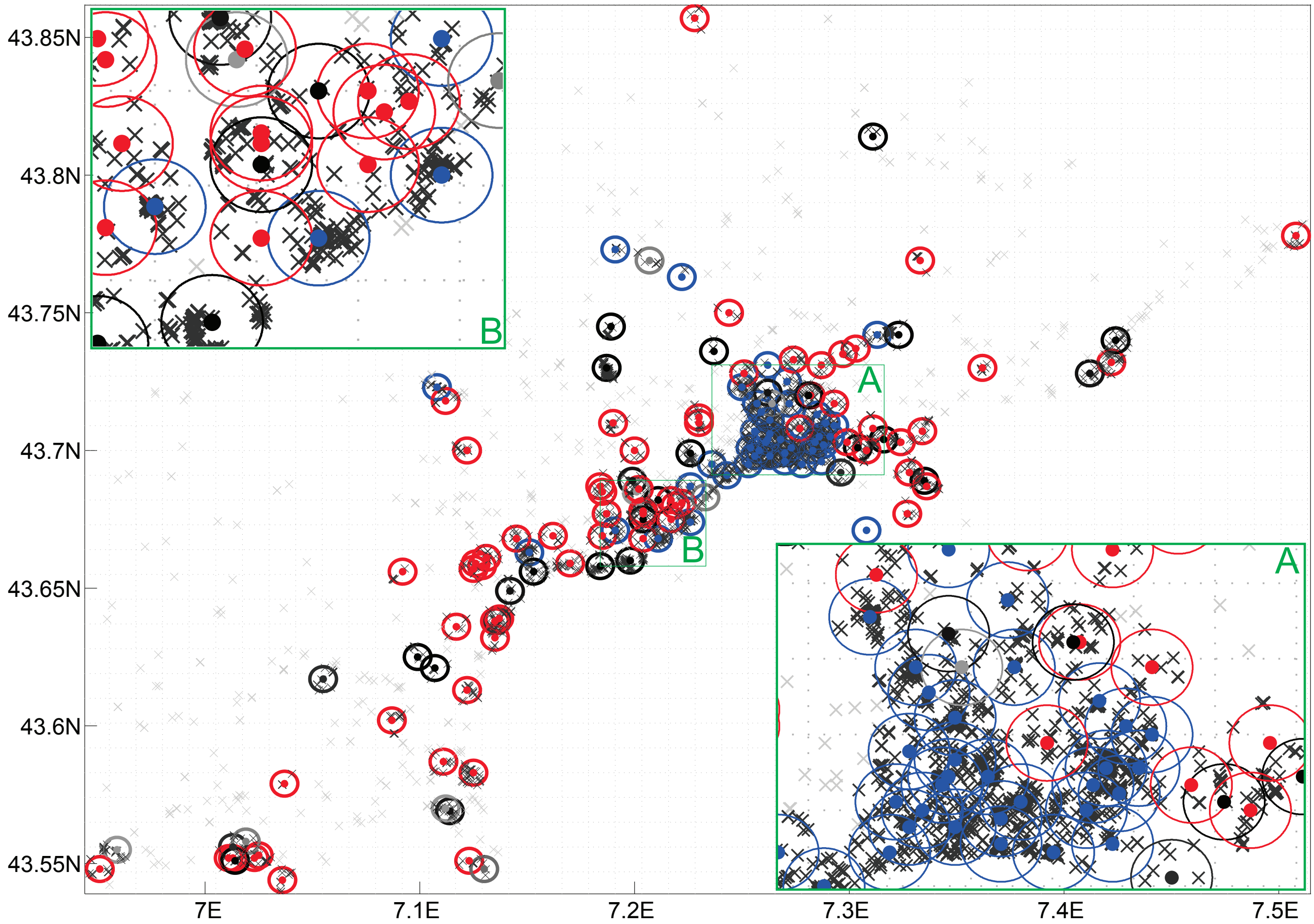



Figure 7: The origin and destinations of the divided trips, the operating (blue) and candidate (gray, black and red) stations and their catchment areas

| | |
|---|---|
| fix vehicle cost (€/day): | 20 |
| vehicle operating cost (€/km): | 0.01 |
| average number of trips per scenario: | 79.25 |
| average trip length (km): | 30 |
| max accessibility distance (km): | 0.5 |
| minimum coverage: | 20% |
| subsidization (€/int): | 5 |
| revenue per time interval (€/int): | 8 |
| accessibility cost (€/km): | 5 |
| utility (€/time int.): | 20 |
| relocation speed (km/h): | 30 |
| relocation personnel cost (€/hour): | 16 |

Table 1: Some values of the parameters used in the model

common stations in determining the efficient frontier. It is observed that (in addition to 42 already operating stations) all eight efficient solutions select stations among a set of 36 candidate locations. More specifically, 19 of these stations appear in all solutions, five in seven solutions and four in six solutions. This result suggests that from station location point of view, the efficient station locations are not in conflict when considering the user and the operator objectives and the solution is robust. Since there is no conflict in station locations, these 28 stations are assumed to be operating stations in addition to already operating 42 stations in the further analysis.

After deciding about the number and location of the stations (strategic decision), we perform further analysis in order to explore if different demand levels and different coverage distances influence the solution.

Firstly, we examine the effect of demand by using five different levels and equal weight for the users' and the operator's objectives. The results of these runs are demonstrated in Figure 9. In Figure 9, there are two sets of bar charts for each level of demand. These bar charts correspond to different number of available vehicles, fixed vs. relaxed. Fixed is referred to the number of vehicles identified in the baseline scenario while in the relaxed case this constraint is not applicable anymore. Moving from left to right we generate for both cases (fixed and relaxed), alternative demand levels by increasing the baseline demand by 50% up to the level of 200%. Please note, in order to eliminate bias, different days are considered in determining the demand. In the bottom of the graph, the total number of trip requests, the number of unsatisfied demand and their percentage can also be seen for the related cases.

For the relaxed case, the operator's benefits are increasing

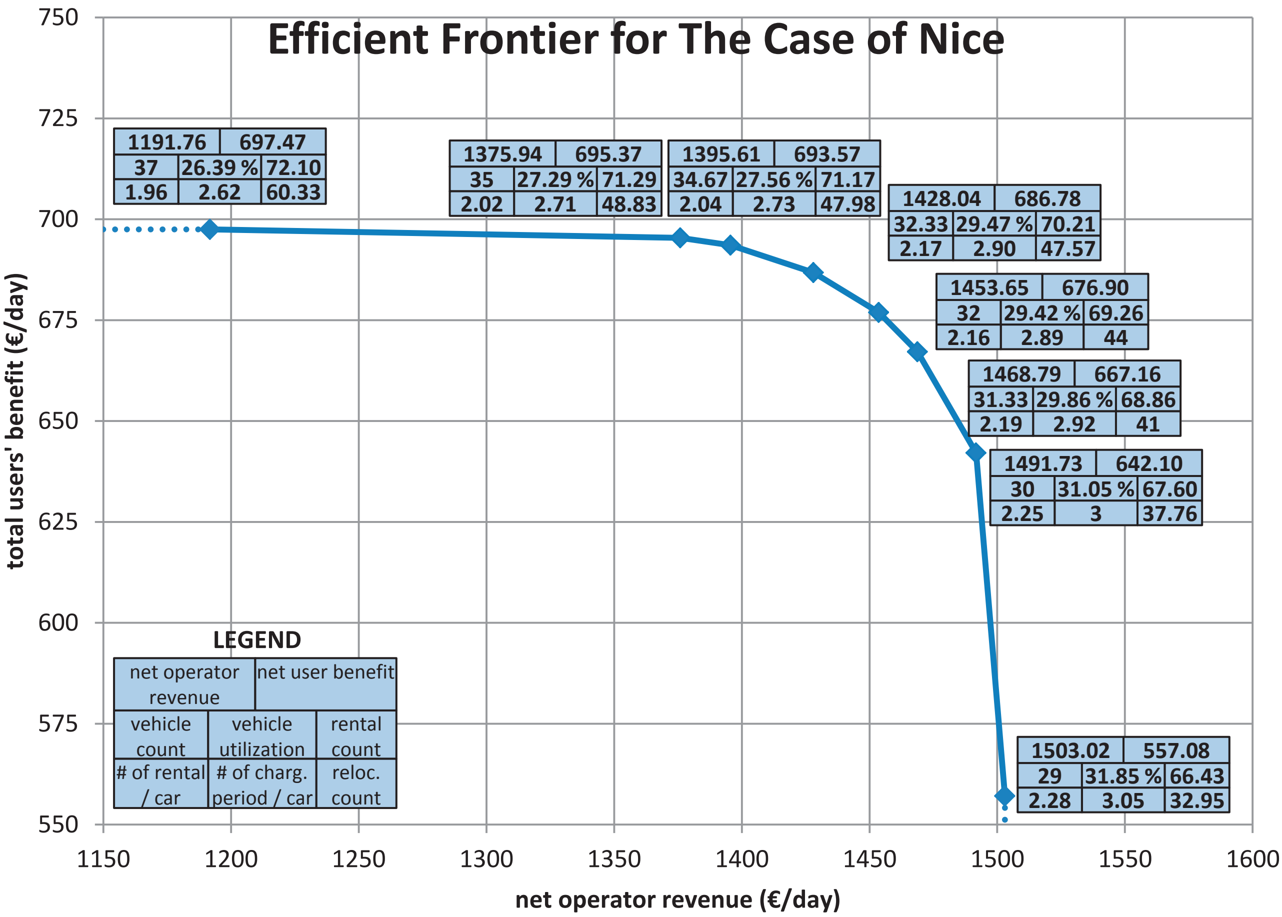


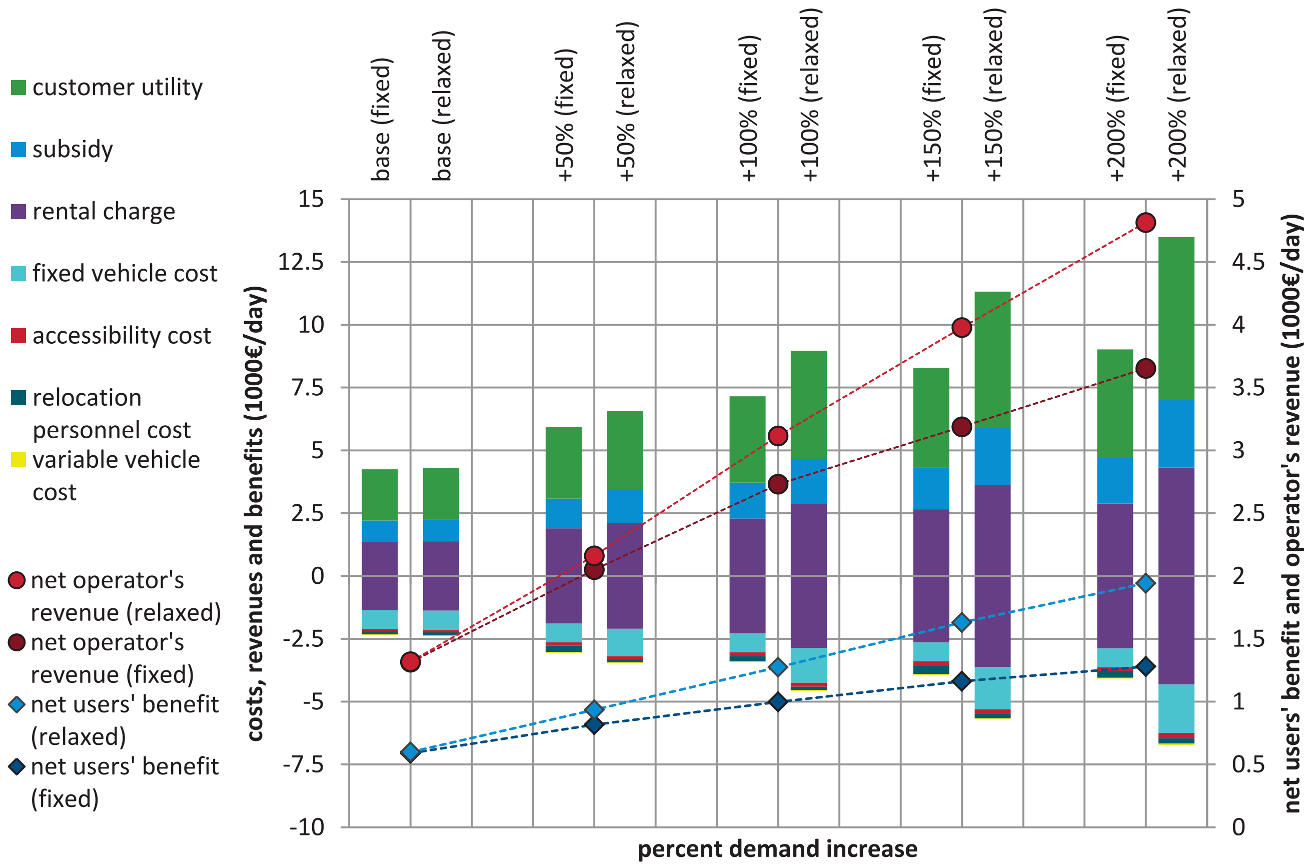

Figure 8: The efficient frontier for the case of Nice, France.

faster than the users' benefits for increasing levels of demand. In the fixed case we observe the same pattern as well. As the demand increases net benefits are increasing as the model can select to serve the most profitable customers to serve from a larger pool of candidate customers. In the fixed case, the slope of users' and operator's benefits curves decreases as the demand increases. This is because of the limitation on the number of vehicles.

This is an expected result since the model does not penalizes unsatisfied demand while at the same time increases the value of the objective function from the satisfied demand. The increase of demand lets the model to choose from a larger set and both the fixed and relaxed models benefit from it and increase their objectives. Note that, this increase of demand results to a higher density of demand, a fact that gives more flexibility to the model to select customers leading to improved objective function values. For the 50% increased demand, the benefit lost for both the operator and the users are almost impreceptible. However, the difference between the relaxed and fixed cases becomes significant with a demand increase of 100%.

Another important finding is the relationship of costs, benefits and revenues. Since the rental fee is a cost for the users and a benefit for the operator, it has no effect in our objective function since equal weights are used for the users' and the operator's objectives. The subsidy and the users utility are the only two values contributing to the increase of the value of the objective function and force the model to serve more orders. The most significant cost is the fixed vehicle cost. The rest of the costs (fuel cost, relocation personnel cost and accessibility cost) are almost insignificant as compared to fixed vehicle cost. The fuel cost is low because the system is operating with electric vehicles and the cost of fuel is 0.01 €/km. In the calculation of the relocation personnel count, it is observed that it is also insignificant to the net income of the operator. In the most congested system not more than three relocation personnels (equivalent to 24 hours of relocation personnel) are used which makes a cost of €384. This finding shows that relocation is not an operation significantly increasing the cost of operators in reality. The accessibility cost is not significant because both the accessibility cost per km (5 €/km) and maximum accessibility distance (0.5 km) are insignificant compared to other costs (e.g. utility: 20 €/time int., relocation personnel cost: 16 €/hour).

Another important finding related to the change in the percentage of unserved requests. Although the number of unsatisfied demand is increasing with the increase of the total number of trips as expected, in the "relaxed" cases the percentage of lost




| | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| average number of lost demand | 9.8 | 8.8 | | 24.0 | 9.5 | | 49.7 | 11.0 | | 73.8 | 11.6 | | 104.8 | 14.7 |
| average number of served demand | 61.9 | 62.9 | | 81.0 | 95.5 | | 91.0 | 129.7 | | 101.4 | 163.6 | | 105.8 | 195.9 |
| average number of demand | 71.7 | | | 105.0 | | | 140.7 | | | 175.1 | | | 210.6 | |
| average lost demand ratio (%) | 13.7 | 12.3 | | 22.8 | 9.0 | | 35.3 | 7.8 | | 42.1 | 6.6 | | 49.8 | 7.0 |

Figure 9: The costs, benefits and revenues with the increased demand

demand is decreasing. This may be due to the fact that the cost of unserved demand due to shortage of vehicles is less than the cost of acquiring extra vehicles to serve the lost demand. From a detailed observation of the results, it can be inferred that the concentration of demand during specific intervals at specific geographical locations is high. During these intervals the model prefers not to serve additional "orders", since the cost is more than the benefit.

The effect of maximum accessibility distance was also investigated for two different levels of demand (e.g. base and +200%). Six different accessibility distances from 500 to 1000m in every 100m intervals were tested. The graph showing the components of the objective function value can be seen in Figure 10.

In both graphs, it can be seen that the maximum accessibility distance does not affect the net users' benefit while operator's revenue is improved 1-4% for each distance increment defining the accessibility. There is also a slight increase both in the total satisfied demand and the ratio of the satisfied demand to total demand. The former is because of the increase in the covered demand while the latter is the consequence of the flexibility introduced to the system. The average number of serving stations for the covered origin or destination points increases from 2.306 to 6.329. This results to an expanded feasible region for the model and consequently leads to an improvement of the objective function value. The same argument holds for the decrease of the unserved demand.

A detailed look to the effect of accessibility distance shows the importance of the accessibility to the stations. In our model, the effect of other public transportation systems to accessibility distance is not taken into consideration. It is assumed that the users can reach stations that are close enough to walk. However, the increase in the accessibility distance tremendously increase the flexibility and efficiency in the system. It is obvious that accessibility highly affects the service quality and efficiency of the system. This underlines the nature of the car sharing systems that work as systems complimentary to public transportation, which contribute to the increase of the overall mobility.

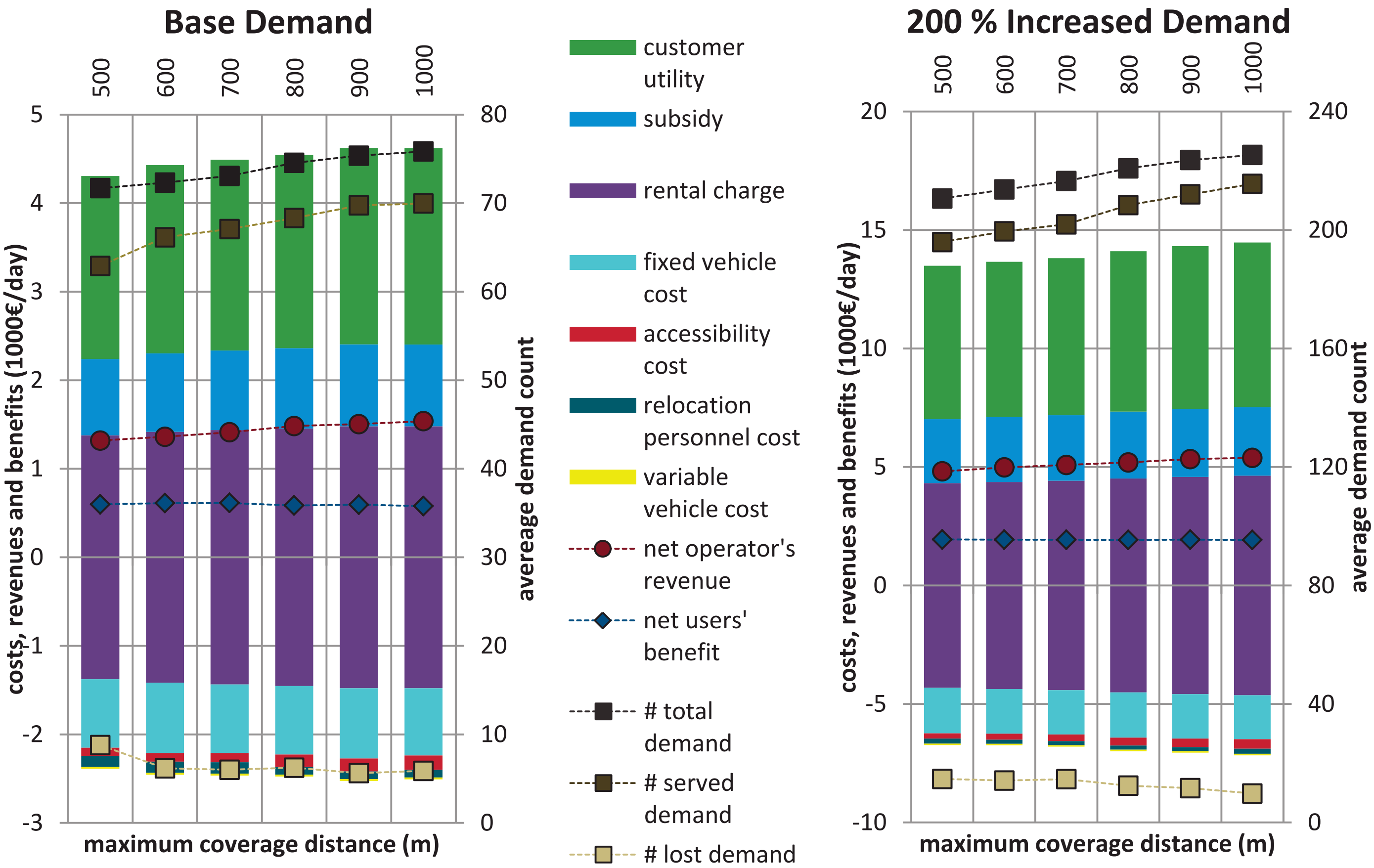



Figure 10: The costs, benefits, revenues and demand for different maximum accessibility distances

## 5. Concluding Remarks

A multi-objective model for supporting strategic and tactical planning decisions for car-sharing systems with a model that maximizes the users' and the operators benefits separately was developed and tested in a large scale real world setting. The model considers simultaneously the net benefits of both the operator and the users. The proposed model closes a gap in the existing literature by considering simultaneously decisions associated with the allocation of strategic assets, i.e. stations and vehicles of car-sharing systems and the allocation of personnel for relocation operations (tactical decision). The model provides the decision makers with ample opportunities to perform sensitivity analysis for the relevant model parameters, a feature particularly useful for cost values that are difficult to establish empirically (e.g. utility gain of satisfied customers, population coverage, station accessibility cost). Furthermore, the multi-objective nature of the model allows the decision maker to examine the trade-off between operator's profit and users' level of service. This last feature is of particular importance if we consider that car-sharing systems are subsidized with public funds. The results obtained from the application of the model to a case resembling real world decision making requirements, provides the decision maker with useful information regarding the system performance.

Although the model provides satisfactory results for the case under consideration, it should be pointed out that the results are dependent on the model parameters used and cannot be directly generalized. The value of the research presented herein stems from the innovative model proposed and its use for supporting strategic and tactical decision for car sharing systems.

Research work under way involves the integration of the proposed model with a simulation model that will provide a more realistic representation of the relocation operation costs by looking on operational decisions. Modeling the operational problem and assigning the vehicle rosters while taking their electrical charge level into consideration is another future work directions. A field implementation of the proposed framework for one-way car sharing is under preparation.

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