Generalised t-V model in one dimension

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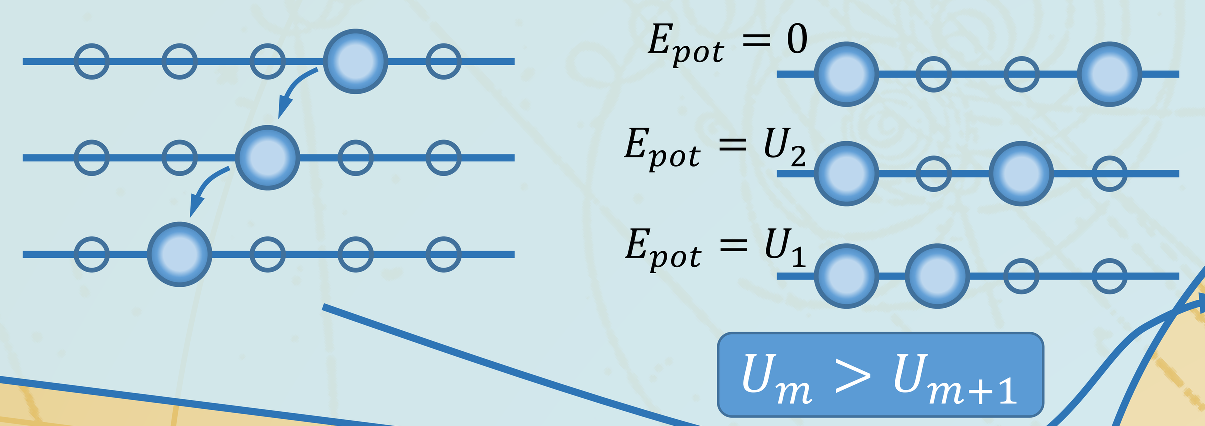
1 Introduction

The generalised t-V model [2] of fermions distributed on a chain of L sites:

$$\hat{H} = \underbrace{-t\sum_{i=1}^{L}\left(\hat{c}_i^\dagger \hat{c}_{i+1} + \text{h.c.}\right)} + \underbrace{\sum_{i=1}^{L}\sum_{m=1}^{p} U_m \hat{n}_i \hat{n}_{i+m}}$$

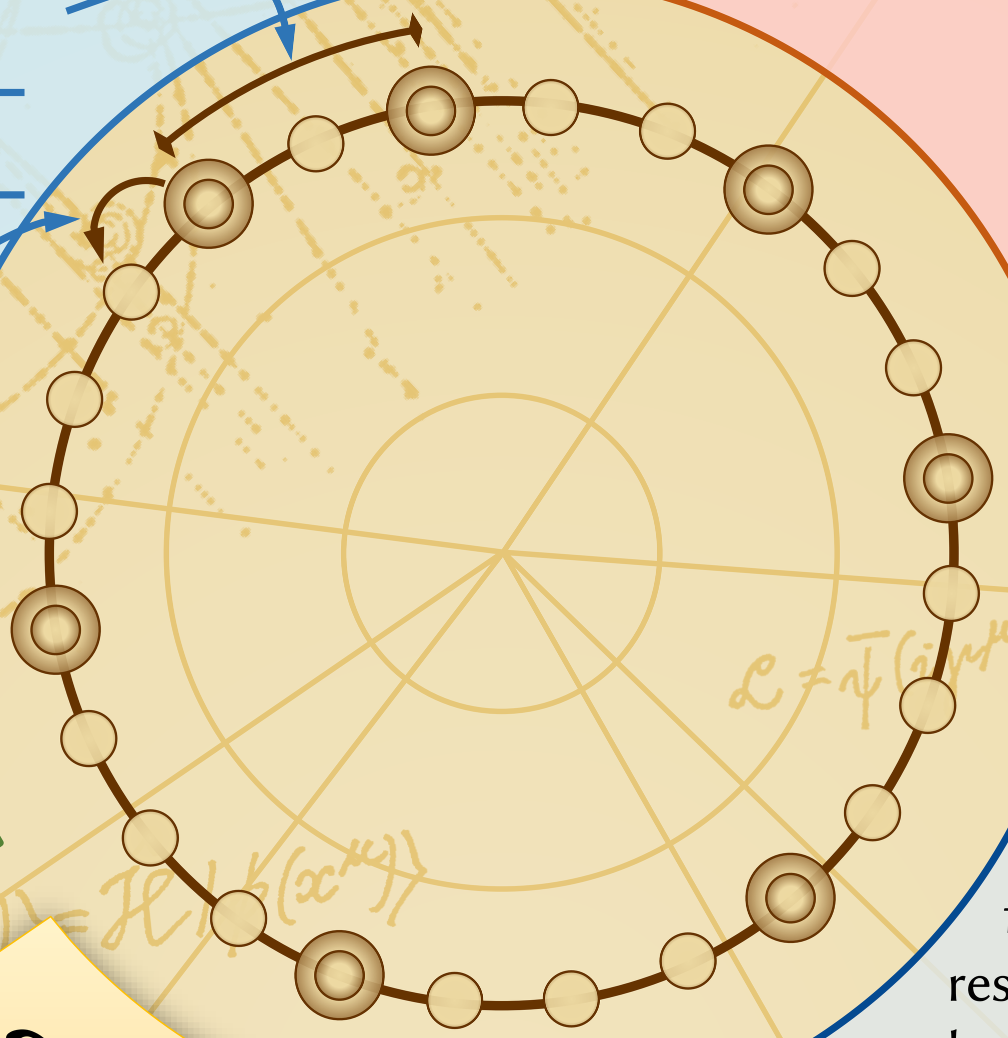
Kinetic energy, i.e. the hopping term, is much smaller than the potential: $t \ll U_m$.

Potential energy makes sure the particles are not closer than p sites: otherwise energy cost is U_m. Example for $p = 2$:



2 The Objective

Spacing: $p > 1$ Non-integrable	Spacing: $p = 1$ Integrable
Solved only in the first order perturbation [1,2]. Using **strong coupling expansion**, we will try to approximate the analytical solutions to a very high order.	Solved by using Bethe ansatz approach [3].

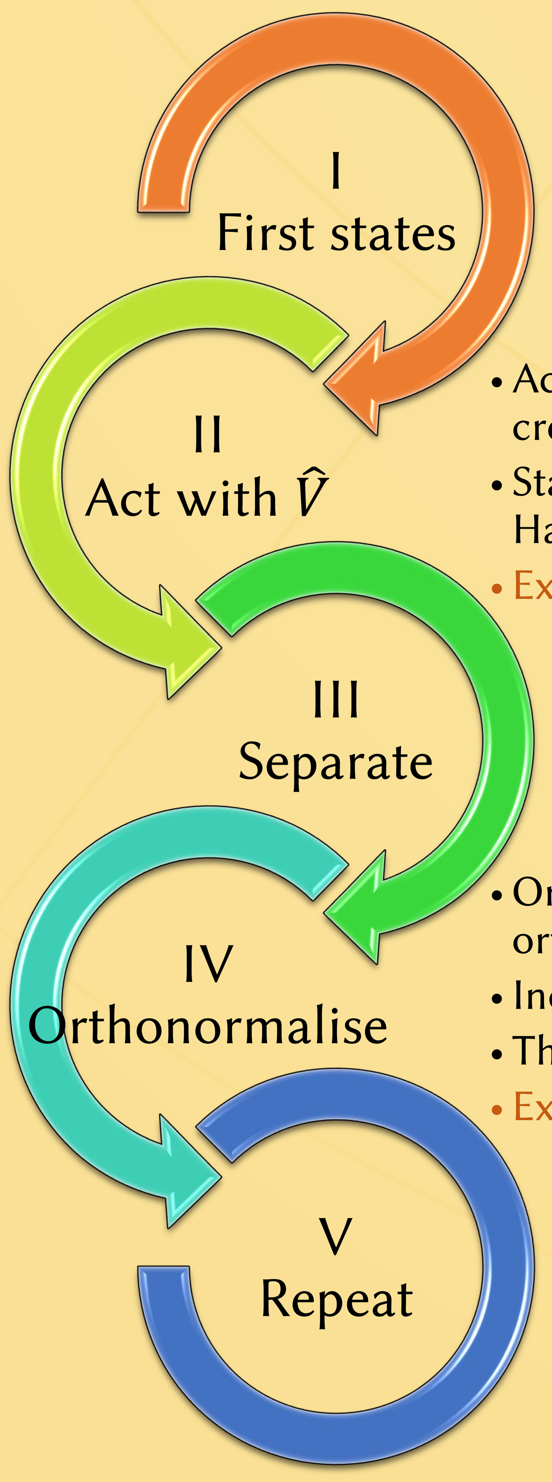


3 Strong Coupling Expansion

The method starts similarly to the perturbation theory. Assume:

$$\hat{H} = \hat{H}_0 + \lambda \hat{V}$$

where $\lambda \ll 1$, so we can treat $\hat{V}$ as perturbation. Eigenstates $|\alpha_n\rangle$ of $\hat{H}_0$ are known. Now, we want to create a **new truncated basis of** $\hat{H}$ using $|\alpha_n\rangle$.



- Include in your basis **the desired subspace of unperturbed states** that you want to approximate.
- They are of step "0" in SCE.
- Example: Ising state $|\downarrow\uparrow\uparrow\downarrow\uparrow\downarrow\rangle$.

- Act with $\hat{V}$ on states from previous SCE step ("n"), creating set of states S.
- States in S are linear combinations of the unperturbed Hamiltonian eigenstates.
- Example: $|\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\rangle + |\downarrow\uparrow\downarrow\uparrow\uparrow\downarrow\rangle + |\downarrow\uparrow\uparrow\uparrow\downarrow\downarrow\rangle + |\downarrow\uparrow\uparrow\downarrow\downarrow\uparrow\rangle$.

- Separate every state in S according to their unperturbed energy.
- Example: $|\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\rangle$
  $|\downarrow\uparrow\downarrow\uparrow\uparrow\downarrow\rangle + |\downarrow\uparrow\uparrow\uparrow\downarrow\downarrow\rangle + |\downarrow\uparrow\uparrow\downarrow\downarrow\uparrow\rangle$

- Orthonormalise the states in set S, so they would be orthonormal to each other and the basis.
- Include them in the basis.
- They are of step "n+1" in SCE.
- Example: The basis is now: $|\downarrow\uparrow\uparrow\downarrow\uparrow\downarrow\rangle$, $|\uparrow\downarrow\uparrow\downarrow\uparrow\downarrow\rangle$,
  $1/\sqrt{3}\ \left(|\downarrow\uparrow\downarrow\uparrow\uparrow\downarrow\rangle + |\downarrow\uparrow\uparrow\uparrow\downarrow\downarrow\rangle + |\downarrow\uparrow\uparrow\downarrow\downarrow\uparrow\rangle\right)$

- Repeat from II until you achieve desired SCE step.

With every SCE step we are increasing the accuracy by two orders in λ.

All the information about **the desired states** (e.g. ground states) will be encoded in the truncated $\hat{H}$ in the new basis [4,5].

4 Using SCE on the Model

Depending on fermion density $Q = N/L$ we have different phases:

Critical density $Q_C = \frac{q}{p+1}$; $q = 1, \ldots, p$	Away from critical density
• Mott insulator • Simple unperturbed ground state	• Luttinger liquid • Highly degenerate ground state of $\hat{H}_0$.

Using SCE for **near-critical densities**, the Hamiltonian is small enough to calculate approximate solution to a very high precision.

Example: $p = 3$, $Q = 1/4$, step "2" in SCE:

$$\hat{H} = \begin{pmatrix} \cdot & -\sqrt{L/2}\ t & \cdot & \cdot & \cdot \\ -\sqrt{L/2}\ t & U_3 & -\sqrt{3}t & -2t & -\sqrt{L-10}t \\ \cdot & -\sqrt{3}t & U_2 & \cdot & \cdot \\ \cdot & -2t & \cdot & U_3 & \cdot \\ \cdot & -\sqrt{L-10}t & \cdot & \cdot & 2U_3 \end{pmatrix}$$

This simple 5x5 Hamiltonian gives the ground state energy of the system up to order $(t/U_3)^5$.

Results 5

Below we present results for a system with $p = 3$. Similar results have been obtained for $p = 1$ (integrable) and $p = 2$ systems. $Q_C = 1/4$. This is step "3" in SCE.

Ground state energy:

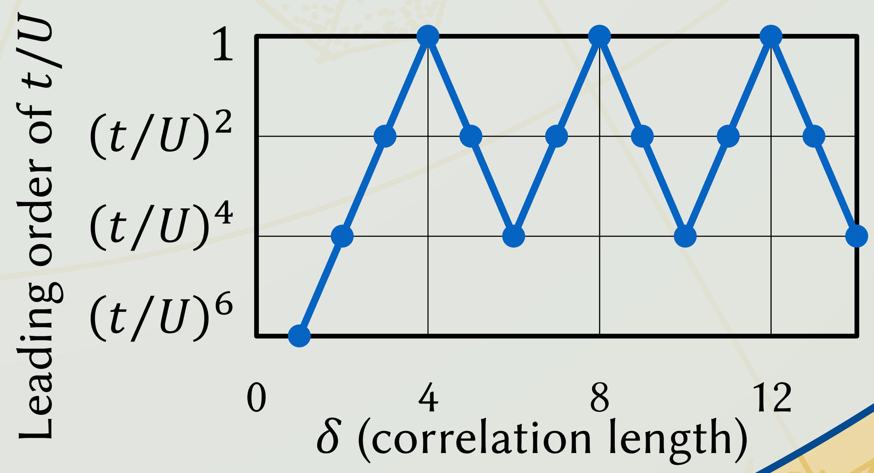
$$E_0 = -\frac{L}{2U_3}t^2 + \left(\frac{L}{2U_3^3} - \frac{3L}{2U_2U_3^2}\right)t^4 + \left(\frac{4L}{U_2U_3^4} - \frac{17L}{4U_2^2U_3^3} - \frac{5L}{2U_2^3U_3^2} - \frac{5L}{U_1U_2^2U_3^2}\right)t^6 + O\left(\frac{t^8}{U^7}\right)$$

Current density:

$$\frac{J}{-i} = \frac{L}{U_3}t^2 + L\left(\frac{2}{U_3^3} - \frac{6}{U_2U_3^2}\right)t^4 + L\left(-\frac{15}{U_2^3U_3^2} - \frac{51}{2U_2^2U_3^3} + \frac{24}{U_2U_3^4} - \frac{30}{U_1U_2^2U_3^2}\right)t^6 + O\left(\frac{t^8}{U^7}\right)$$

Density-density correlations:

$\langle \hat{n}_i \hat{n}_{i+\delta} \rangle$ were also obtained. Leading order is cyclic in δ, which is consistent with expectations.



Obtained accuracy was $O(t^6) \div O(t^8)$.

6 Conclusions & Outlook

Summary:

- ❖ High precision results for both **integrable and non-integrable models** in Mott insulating phases.
- ❖ Results are fully consistent with other works [1,2,3].

Further work:

- Densities not exactly equal to Q_C
- Phase transition investigation
- More observables
- Temperature dependence
- Time dependence

References

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