Lancaster University
MANAGEMENT SCHOOL

Economics Working Paper Series

2013/007

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Abstract

This paper develops a many-good, many-country model of international trade which combines comparative advantage and external scale economies. It is shown that the gains from external scale economies outweigh those from comparative advantage as the number of goods increases. Small countries gain more than large countries from trade, because large countries are more similar to the rest of the world than small countries. Small countries are also more specialised in production than large countries, despite the presence of external scale economies.

Keywords: External scale economies; Comparative advantage; gains from trade.
JEL Classification Numbers: F11.

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1. Introduction

International trade theory has not emphasised the gains from external scale economies, but instead focussed on the gains from comparative advantage as developed by Ricardo (1817), or increasing returns to scale as in Helpman and Krugman (1985). There are important exceptions to this rule: Graham (1923), Young (1928) and Ohlin (1933) discussed the implications of international trade in the presence of increasing returns. More recent developments include Melvin (1969), Ethier (1982), Wong (2001), Ethier and Ruffin (2009) and Grossman and Rossi-Hansberg (2010); Choi and Yu (2003) provide a survey of this literature.

This paper develops a simple model of international trade with many goods and many countries based on both external scale economies and Ricardian comparative advantage (modelled as productivity differences across countries). The model extends that developed in Ruffin (2009) by incorporating external scale economies. We show that external scale economies play a bigger role in the gains from trade than comparative advantage as the number of goods and countries rises. A second contribution of the paper is to extend the basic model to allow for countries of different size. This extension enables us to decouple the number of countries from the number of goods. We show that larger countries are more diversified in production than smaller countries in free trade, despite the presence of external scale economies. This is in accord with the empirical findings of Hummels and Klenow (2005). The many-good, many-country structure of the model distinguishes the current paper from other recent contributions such as Ethier and Ruffin (2009) and Nordas (2000) who consider a two-country, two-good model, or Grossman and Rossi-Hansberg (2010) who develop a many-good, two-country model[1].

The global economy has expanded rapidly in the past two decades. This can be seen from the fall of the Berlin Wall in 1989 and the eventual integration of the former Soviet bloc into the global trading system, the opening up of many economies to international trade (of which China and India are but the prime examples), as well as the formation and expansion of the World Trade Organisation (WTO). At the same time, new information technology products have expanded the range of goods available for consumption, and have made the

[1] In our model, as in Grossman and Rossi-Hansberg (2010), the pathologies involving losses from international trade in the presence of external scale economies do not occur in equilibrium (see Graham, 1923, Ethier, 1982).

fragmentation, specialisation and coordination of production easier than ever (see Jones, 2000). The increasing number of goods and countries and the growing importance of international trade are features of the global economy which our model is well-placed to analyse, albeit in a stylised, simplified form.

The next section outlines the model assuming all countries are small. Section 3 considers the implications when there are small and large countries. Section 4 discusses trade volumes while Section 5 concludes.

2. The model

In this section we assume that all countries are the same size. There are n countries and n goods, $1, \ldots, n$. Each homogeneous good is produced under perfect competition by many small firms. Each country has 1 unit of labour, which is perfectly mobile across goods but perfectly immobile across countries. Country j has a comparative advantage in good j; the production functions take the following form in country j:

$$q_j = L_j^{\beta} \qquad q_i = (\alpha L_i)^{\beta} \qquad i \neq j \qquad \beta > 1 \qquad \alpha < 1 \qquad (1)$$

That $\beta > 1$ signifies that the production technology exhibits external scale economies: the more that is produced, the lower the average cost. Similarly, $\alpha < 1$ signifies that country j has a comparative *dis*advantage in goods i (equivalently, a comparative advantage in good j). More formally, for any single atomistic firm k in industry j in country j, its production function is given by $q_{kj} = L_{kj} L_j^{\beta-1}$, where the firm takes the industry labour input L_j as exogenous. As argued in Helpman and Krugman (1985), in equilibrium firms will set prices equal to average cost.

The representative consumer's utility is a Cobb-Douglas function[2]:

$$U = \prod_{i=1}^{n} c_i^{\frac{1}{n}} \qquad (2)$$

In autarky, each country j will produce all goods, and given the Cobb-Douglas utility and free movement of labour across sectors, will devote $1/n$ labour to each good, producing $(1/n)^{\beta}$ units of good j and $(\alpha/n)^{\beta}$ units of all other goods i. Utility under autarky will be:

[2] In Ruffin (2009) the utility function includes an additional "love for variety" term so that total utility increases as the number of goods increases. This term cancels out in what follows so plays no role in the analysis, hence has been omitted for brevity.

$$U_A = \left(\frac{1}{n}\right)^{\frac{\beta}{n}} \left(\frac{\alpha}{n}\right)^{\frac{\beta(n-1)}{n}} = n^{-\beta}\alpha^{\beta\left(\frac{n-1}{n}\right)} \quad (3)$$

Under free trade, each country will specialise in the good in which it has a comparative advantage, and import the other $n-1$ goods from the other $n-1$ countries. Therefore, output of each good is equal to 1, so consumption of each good in each country is equal to $1/n$. Free trade utility is therefore:

$$U_T = n^{-1} \quad (4)$$

Define the gains from trade as the ratio between free trade and autarkic utility. The gains from trade are:

$$G_{TOTAL} = \frac{U_T}{U_A} = n^{\beta-1}\alpha^{\beta\left(\frac{1-n}{n}\right)} \quad (5)$$

Gains from trade arise since $G_{TOTAL} > 1$.

If $\beta = 1$, then the model reverts to that of Ruffin (2009), in which there are no external scale economies, only differences in productivity across countries. In this case, comparative advantage is the only reason for trade between countries. As a result,

$$G_{CA} = \alpha^{\frac{1-n}{n}} \quad (6)$$

so the gain from trade always increases in the number of goods (and countries) n, but at a decreasing rate.

If $\alpha = 1$, then the model only has external scale economies, and no comparative advantage. In this case, the gain from trade arises because countries will experience lower costs when they specialise. Hence:

$$G_{SCALE} = n^{\beta-1} \quad (7)$$

so that the gain from trade increases in n but also at a decreasing rate if $\beta < 2$.

We can decompose the total gains from trade into the component derived from comparative advantage alone, the component derived from external scale economies alone, and the component derived from the interaction between comparative advantage and external scale economies as follows:

$$G_{TOTAL} = G_{CA} \times G_{SCALE} \times \alpha^{\frac{(1-\beta)(n-1)}{n}} \quad (8)$$

Where the last term is the gain from the interaction between comparative advantage and external scale economies; it may be interpreted as the additional gain from having

comparative advantage in the presence of external scale economies, or vice versa. This term is always greater than 1; there is complementarity between external scale economies and comparative advantage. If either α or β is equal to 1, this interaction term vanishes[3]. Figure 1 shows the decomposition of the gains from trade as a function of n, drawn for $\alpha = 0.4$ and $\beta = 1.3$.

Figure 1 shows that the gains from external scale economies increase faster in n than do the gains from comparative advantage. This can also be seen by dividing equation (7) by equation (6). This gives:

$$\frac{G_{SCALE}}{G_{CA}} = n^{\beta-1}\alpha^{\frac{n-1}{n}} \tag{9}$$

This expression increases as n increases; that is, the gains from external scale economies become more important relative to the gains from comparative advantage as n increases, as shown in Figure 1.

Setting $G_{CA} = G_{SCALE}$ allows us to solve for the value of n for which the gains from comparative advantage are equal to the gains from external scale economies:

$$(n^*)^{\frac{n^*}{n^*-1}} = \alpha^{\frac{1}{1-\beta}} \tag{10}$$

From equations (9) and (10), values of n greater than n^* imply that the gains from external scale economies exceed those from comparative advantage. Hence we have:

Proposition 1: As the number of goods $n \to \infty$, the gains from trade predominantly result from external scale economies, not comparative advantage.

What Proposition 1 says is that as global markets become more integrated, and the number of goods and trading partners increases, external scale economies will become an increasingly important source of the gains from trade.

3. Large and small countries

Suppose now that there are n goods, and $n - m$ countries, where $m \leq n/2$ countries are large, in the sense that each large country has two regions, and the remaining $n - 2m$

[3] The complementarity between external scale economies and comparative advantage arises because of the way the production functions have been formulated in equation (1).

countries are small, having only one region each. Hence the total number of regions is the same as the total number of goods, n. As before, each region is assumed to have a comparative advantage in one good, so a large country has a comparative advantage in two goods.

First, assume that labour cannot move between the two regions of a large country, but there is free movement of goods within a country. This corresponds to the idea of a lumpy country as proposed by Courant and Deardorff (1992). In this case, the representative consumer's utility under autarky in the large country is (keeping in mind that the large country has two goods in which it has a comparative advantage):

$$U_A^{LARGE} = \left(\frac{1}{n}\right)^{\frac{2\beta}{n}} \left(\frac{\alpha}{n}\right)^{\frac{\beta(n-2)}{n}} = n^{-\beta}\alpha^{\beta\left(\frac{n-2}{n}\right)} \tag{11}$$

In free trade every region specialises in the good in which it has a comparative advantage, so free trade utility for the large country is the same as for a small country:

$$U_{FT}^{LARGE} = n^{-1} \tag{12}$$

The gains from trade for a large country are therefore:

$$G^{LARGE} = \left(\frac{U_{FT}^{LARGE}}{U_A^{LARGE}}\right) = n^{\beta-1}\alpha^{\beta\left(\frac{2-n}{n}\right)} \tag{13}$$

Comparing the gains from trade for a large country with that of a small country from the previous section gives:

$$\frac{G^{LARGE}}{G^{SMALL}} = \alpha^{\frac{\beta}{n}} \leq 1 \tag{14}$$

This gives:

Proposition 2: Under the assumptions of the model, the gains from trade are never larger for a large country than for a small country.

Intuitively, in the model, since in autarky regions in a country trade between themselves, a large country already has a larger market than a small country, so the process of international trade does not enlarge the large country's market by as much as it does the small country's market. This of course is equivalent to saying that the vector of autarkic prices in the large country is more similar to the free trade price vector than in the small country (Ethier, 2009). As $n \to \infty, \frac{G^{LARGE}}{G^{SMALL}} \to 1$; a large country gains as much as a small country when there are many goods and countries, since a large country will be "small" relative to the size of the

world market. If $\alpha = 1$ (no comparative advantage), then a large country gains as much from international trade as does a small country. In this case, a large country experiences no advantage relative to a small country in autarky.

Next, consider the case when there is free movement of both goods and labour between the two regions of a large country. Assume that labour productivity is location-specific, in the sense that if labour moves to a region, it adopts the productivity of that region. The question arises of whether or not it makes sense for the large country to specialise in only one of the two goods in which it has a comparative advantage in free trade (in autarky this possibility is ruled out by the Cobb-Douglas utility function). For this to be possible, the welfare under free trade must exceed that in equation (12), where each region of the large country specialises in the good in which it has a comparative advantage. To give the best possible outcome in this case, suppose that each of the m large countries produce only one good in free trade. The $(n-2m)$ small countries will produce the $(n-2m)$ goods in which they have a comparative advantage in, as well as the m goods which the large countries no longer produce. Assuming for simplicity that goods are infinitesimally divisible, each small country will produce $1+\left(\frac{m}{n-2m}\right)$ goods. That is, the representative consumer's utility under free trade would be[4]:

$$U_{FT}^{*} = \left(\frac{2^{\beta}}{n}\right)^{\frac{m}{n}}\left[\left(\frac{n-2m}{n-m}\right)^{\beta}\left(\frac{\alpha}{n}\right)\right]^{\frac{m}{n}}\left[\left(\frac{n-2m}{n-m}\right)^{\beta}\left(\frac{1}{n}\right)\right]^{\frac{n-2m}{n}} \quad (15a)$$

$$= n^{-1}\left(2^{\beta}\alpha\right)^{\frac{m}{n}}\left[\frac{n-2m}{n-m}\right]^{\frac{\beta(n-m)}{n}} \quad (15b)$$

Where the first term in equation (15a) is the utility from the m goods produced by the m large countries, the second term is the utility from the goods in which the $n-2m$ small countries do not have a comparative advantage in, and the third term is the utility from the goods in which the small countries have a comparative advantage in. It can be shown that $\frac{U_{FT}^{*}}{U_{FT}^{LARGE}} \leq 1$; that is, large countries can never gain more from completely specialising in only one of the two goods in which they have a comparative advantage. This gives:

Proposition 3: Under the assumptions of the model, a large country will be more diversified in production than a small country under free trade.

[4] Equation (15a) holds because the Cobb-Douglas utility function means that all consumers have the same income level under free trade.

What Proposition 3 says is that, as far as the gains from trade are concerned, it never makes sense for a large country to be completely specialised, despite the presence of external scale economies. This is in accord with Hummels and Klenow (2005) who find empirically that the exports of large countries are more diversified than those of small countries. Intuitively, although specialising results in lower production cost in the specialised good, this is more than offset by the increased cost resulting from production of other goods using both smaller scale and less efficient technologies. The greatest welfare can be achieved when countries produce goods symmetrically in accordance with their comparative advantage.[5]

From equation (13), it can be seen that the gain from trade for large countries does not depend on the number of large countries m. However, the number of large countries does enter into the gain from trade for the world as a whole. When all countries are small, the global gains from trade are:

$$G^{GLOBAL} = n \times G^{SMALL} \tag{16}$$

On the other hand, when there are m large countries and $n - 2m$ small countries, the global gains from trade are:

$$G^{GLOBAL} = [(n - 2m) \times G^{SMALL}] + [2m \times G^{LARGE}] \tag{17a}$$

$$= [n \times G^{SMALL}] + [2m \times (G^{LARGE} - G^{SMALL})] \tag{17b}$$

$$= n \times G^{SMALL} \times \left[1 + 2\left(\alpha^{\beta/n} - 1\right)\left(\frac{m}{n}\right)\right] \tag{17c}$$

It can be shown that $\frac{\partial G^{GLOBAL}}{\partial m} < 0$ (since $G^{LARGE} - G^{SMALL} < 0$), and $\frac{\partial G^{GLOBAL}}{\partial n} > 0$. That is, for the world as a whole, the gains from trade decrease as the number of large countries m increases, independently of the number of goods, and increase as the number of goods n increases, independently of the number of large countries. Note also that, if both m and n increase at a constant rate so that m/n is constant, then equation (17c) shows that the global gains from trade will increase. That is, if globalisation leads to an increase in the total number of countries and goods, then the gains from trade increase if the proportion of large to small countries remains constant.

[5] Proposition 3 also holds when there are no external economies of scale ($\beta = 1$), as in Ruffin (2009). Propositions 1, 2 and 3 also generalise to more diverse country sizes than those considered here.

4. The volume of trade

Consider first the case where all countries are small. Each country will export the good in which it is specialised, and will import the remaining $n-1$ goods. The volume of international trade is the same as in Ruffin (2009). Since there are n countries, each producing one unit of one good in the free trade equilibrium, the volume of trade is $n-1$, so the share of trade in world GDP is $(n-1)/n$, which is increasing in the number of goods and countries n. Hence as globalisation proceeds, international trade as a share of world GDP also increases.

If there are m large countries, then the $n-2m$ small countries will each export a fraction $(n-1)/n$ of its output, while the m large countries will each export a fraction $(n-2)/n$ of its output. The volume of trade will be:

$$VT^{SOME\ LARGE} = (n-2m)\left(\frac{n-1}{n}\right) + 2m\left(\frac{n-2}{n}\right) = n - 1 - \frac{2m}{n} \tag{18}$$

Hence the share of trade in world output when there are m large countries will be:

$$S^{SOME\ LARGE} = \frac{n-1}{n} - \frac{2m}{n^2} \tag{19}$$

Comparing the volume of trade when all countries are small with when there are some large countries shows that the volume of trade is larger when all countries are small; the difference $2m/n^2$ is decreasing in the number of goods n and increasing in the number of large countries m. The intuition for this result is that large countries trade less than small countries as a share of their GDP, since some of the trade is carried out between the regions of the large country and therefore does not count towards international trade flows.

5. Conclusions

As the number of goods in the global economy increases, consumers become better off. But what is the source of this gain? Trade economists have provided several possible explanations: comparative advantage, increasing productivity through specialisation, increased product variety, and more recently, gains from firm selection and economies of scope from multi-product firms. In this paper, we develop a model of international trade to compare the contributions of comparative advantage and external scale economies to the gains from trade. In the model, as the number of goods and countries increases, the gains

from specialisation due to external scale economies outweigh the gains from comparative advantage. The model also shows that large countries will be less specialised than small countries, even in the presence of external scale economies. As the number of goods rises exogenously, countries become more specialised in production relative to consumption, and international trade as a share of world output increases.

It is possible to extend the model developed here in several directions. The most obvious extensions are to introduce trade costs and a home bias in demand, and to allow for a greater variety of country sizes. None of these extensions would qualitatively change any of the main results. More challenging would be to unpack the sources of external scale economies; this path has been left for future work.

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Figure 1: Decomposition of the gains from trade.