Economics Working Paper Series

2013/002

Technical Appendix to: A New Look at Variation in Employment Growth in Canada: The Role of Industry, Provincial, National and External Factors.

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Technical Appendix to: A New Look at Variation in Employment Growth in Canada: The Role of Industry, Provincial, National and External Factors

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February 1, 2013

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1 Priors, Posteriors and MCMC algorithms

The models listed in Table 1 of the main paper can each be estimated using MCMC. Depending on the assumptions of a model, we can use a selection of the Gibbs blocks described in the rest of the section to form a full conditional Gibbs sampler that is suitable for its estimation.

The following notations are used in various Gibbs blocks when necessary:

- Y_{ipt} denotes the growth rate of industry i in province p at time t;
- ε_{ipt} denotes the error term in the equation associated with Y_{ipt};
- h_{ip} denotes the precision of the *i.i.d.* errors in the equation associated with Y_{ipt};
- λ_{ip}, $\beta_{0,ip}$, and $\beta_{1,ip}$ are the constant and the coefficients of lagged US growth rates in the equation associated with Y_{ipt};
- $\Lambda_{ip,k}$ denotes the element at the n^{th} row and k^{th} column of Λ, note that the n^{th} row is directly associated with Y_{ipt};
- f_t^N, f_{pt}^P and f_{it}^I denote the national factor, the provincial factor for province p, and the industrial factor for industry i at time t;
- γ_{ip}^N, γ_{ip}^P, and γ_{ip}^I are the factor loadings for f_t^N, f_{pt}^P and f_{it}^I, respectively, in the equation associated with Y_{ipt};
- Y_t^i, Y_t^p, and Y_t^N denote the weighted averaged growth rates for industry i, province p, and the whole country;
- Λ_{ip}^i, Λ_{ip}^p, and Λ_{ip}^N denote the coefficients of lagged Y_t^i, Y_t^p, and Y_t^N, respectively, in the equation associated with Y_{ipt}.

1.1 Conditional Posteriors for λ, Λ, β_0, and β_1

1.1.1 Models without lagged ε_t

1.1.1.1 Lagged Y_ts do not enter the model

Let $Y^{\dagger}_{ipt} = Y_{ipt} - (\gamma^I_{ip} f^I_{it} + \gamma^P_{ip} f^P_{pt} + \gamma^N_{ip} f^N_t)$, we have

$$Y^{\dagger}_{ipt} = X^{\dagger}_t b^{\dagger}_{ip} + \varepsilon_{ipt}, \tag{1}$$

where $b^{\dagger}_{ip} = (\lambda_{ip}, \beta_{0,ip}, \beta_{1,ip})'$ and $X^{\dagger}_t = (1, DGDP^{US}_t, DGDP^{US}_{t-1})$.

Let $Y^{\dagger}_{ip} = (Y^{\dagger}_{ip,1}, Y^{\dagger}_{ip,2}, ..., Y^{\dagger}_{ip,T})'$, $E_{ip} = (\varepsilon_{ip,1}, \varepsilon_{ip,2}, ..., \varepsilon_{ip,T})$, and $X^{\dagger}$ be a $T \times 3$ matrix with the t^{th} row given by $X^{\dagger'}_t$. In matrix form, we rewrite equation (1) as

$$Y^{\dagger}_{ip} = X^{\dagger} b^{\dagger}_{ip} + E_{ip} \tag{2}$$

where $var(E_{ip}) = h^{-1}_{ip} I_T$.

We elicit Normal prior for $b^{\dagger}_{ip}$ as:

$$b^{\dagger}_{ip} \sim N(\underline{b}^{\dagger}, \underline{V}^{\dagger}_b)$$

for $i = 1, 2, ..., I$ and $p = 1, 2, ..., P$.

The conditional posterior for $b^{\dagger}_{ip}$ thus takes the following form

$$b^{\dagger}_{ip} | Y, \Sigma, F, \Gamma \sim N(\overline{b}^{\dagger}_{ip}, \overline{V}^{\dagger}_{b_{ip}})$$

where

$$\overline{V}^{\dagger}_{b_{ip}} = (\underline{V}^{\dagger^{-1}}_b + h_{ip} X^{\dagger'} X^{\dagger^{-1}})$$

$$\overline{b}^{\dagger}_{ip} = \overline{V}^{\dagger}_{b_{ip}} (\underline{V}^{\dagger^{-1}}_b \underline{b}^{\dagger} + h_{ip} X^{\dagger'} Y^{\dagger}_{ip})$$

.

In empirical work, we set $\underline{b}^{\dagger} = 0$, $\underline{V}_b^{\dagger} = \begin{pmatrix} 10 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

1.1.1.2 Λ Restricted* = only the weighted averaged lagged Y_ts enter the model

Let $Y_{ipt}^{\dagger} = Y_{ipt} - (\gamma_{ip}^{I} f_{it}^{I} + \gamma_{ip}^{P} f_{pt}^{P} + \gamma_{ip}^{N} f_t^{N})$, we have

$$Y_{ipt}^{\dagger} = X_t^{\dagger} b_{ip}^{\dagger} + \varepsilon_{ipt}, \tag{3}$$

where $b_{ip}^{\dagger} = (\lambda_{ip}, \beta_{0,ip}, \beta_{1,ip}, \Lambda_{ip}^{I}, \Lambda_{ip}^{P}, \Lambda_{ip}^{N})'$ and $X_t^{\dagger} = (1, DGDP_t^{US}, DGDP_{t-1}^{US}, Y_{t-1}^{i}, Y_{t-1}^{p}, Y_{t-1}^{N})$.

Let $Y_{ip}^{\dagger} = (Y_{ip,1}^{\dagger}, Y_{ip,2}^{\dagger}, ..., Y_{ip,T}^{\dagger})'$, $E_{ip} = (\varepsilon_{ip,1}, \varepsilon_{ip,2}, ..., \varepsilon_{ip,T})$, and $X^{\dagger}$ be a $T \times 6$ matrix with the t^{th} row given by $X_t^{\dagger'}$. In matrix form, we rewrite equation (3) as

$$Y_{ip}^{\dagger} = X^{\dagger} b_{ip}^{\dagger} + E_{ip} \tag{4}$$

where $var(E_{ip}) = h_{ip}^{-1} I_T$.

We elicit Normal prior for $b_{ip}^{\dagger}$ as:

$$b_{ip}^{\dagger} \sim N(\underline{b}^{\dagger}, \underline{V}_b^{\dagger})$$

for $i = 1, 2, ..., I$ and $p = 1, 2, ..., P$.

The conditional posterior for $b_{ip}^{\dagger}$ thus takes the following form

$$b_{ip}^{\dagger} | Y, \Sigma, F, \Gamma \sim N(\overline{b}_{ip}^{\dagger}, \overline{V}_{b_{ip}}^{\dagger})$$

where

$$\overline{V}_{b_{ip}}^{\dagger} = (\underline{V}_b^{\dagger^{-1}} + h_{ip} X^{\dagger'} X^{\dagger^{-1}})$$

$$\overline{b}_{ip}^{\dagger} = \overline{V}_{b_{ip}}^{\dagger} (\underline{V}_b^{\dagger^{-1}} \underline{b}^{\dagger} + h_{ip} X^{\dagger'} Y_{ip}^{\dagger})$$

In empirical work, we set $\underline{b}^{\dagger} = 0$, $\underline{V}_b^{\dagger} = 10 * I_6$.

1.1.1.3 Λ Restricted** = only own lag coefficients are non-zero

Let $Y_{ipt}^{\dagger} = Y_{ipt} - (\gamma_{ip}^{I} f_{it}^{I} + \gamma_{ip}^{P} f_{pt}^{P} + \gamma_{ip}^{N} f_{t}^{N})$, we have

$$Y_{ipt}^{\dagger} = X_t^{\dagger} b_{ip}^{\dagger} + \varepsilon_{ipt}, \tag{5}$$

where $b_{ip}^{\dagger} = (\lambda_{ip}, \Lambda_{ip,ip}, \beta_{0,ip}, \beta_{1,ip})'$ and $X_t^{\dagger} = (1, Y_{ip,t-1}, DGDP_t^{US}, DGDP_{t-1}^{US})$.

Let $Y_{ip}^{\dagger} = (Y_{ip1}^{\dagger}, Y_{ip2}^{\dagger}, ..., Y_{ipT}^{\dagger})'$, $E_{ip} = (\varepsilon_{ip1}, \varepsilon_{ip2}, ..., \varepsilon_{ipT})$, and $X^{\dagger}$ be a $T \times 4$ matrix with the tth row given by $X_t^{\dagger}$. In matrix form, we rewrite equation (5) as

$$Y_{ip}^{\dagger} = X^{\dagger} b_{ip}^{\dagger} + E_{ip} \tag{6}$$

where $var(E_{ip}) = h_{ip}^{-1} I_T$.

We elicit Normal prior for $b_{ip}^{\dagger}$ as:

$$b_{ip}^{\dagger} \sim N(\underline{b}^{\dagger}, \underline{V}_b^{\dagger}).$$

The conditional posterior for $b_{ip}^{\dagger}$ thus takes the following form

$$b_{ip}^{\dagger} | Y, \Sigma, F, \Gamma \sim N(\overline{b}_{ip}^{\dagger}, \overline{V}_{b_{ip}}^{\dagger})$$

where

$$\overline{V}_{b_{ip}}^{\dagger} = (\underline{V}_b^{\dagger^{-1}} + h_{ip} X^{\dagger'} X^{\dagger^{-1}})$$

$$\overline{b}_{ip}^{\dagger} = \overline{V}_{b_{ip}}^{\dagger} (\underline{V}_b^{\dagger^{-1}} \underline{b}^{\dagger} + h_{ip} X^{\dagger'} Y_{ip}^{\dagger})$$

In empirical work, we set $\underline{b}^{\dagger} = 0$, $\underline{V}_b^{\dagger} = \begin{pmatrix} 10 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$.

1.1.1.4 Λ Unrestricted

Let $Y_{ipt}^{\dagger}$ be the difference between Y_{ipt} and $(\gamma_{ip}^{I} f_{it}^{I} + \gamma_{ip}^{P} f_{pt}^{P} + \gamma_{ip}^{N} f_{t}^{N})$. We have

$$Y_{ipt}^{\dagger} = X_t^{\dagger} b_{ip}^{\dagger} + \varepsilon_{ipt}, \tag{7}$$

where $b_{ip}^{\dagger} = (\lambda_{ip}, \Lambda_{ip,1}, \Lambda_{ip,2}, ..., \Lambda_{ip,N}, \beta_{0,ip}, \beta_{1,ip})'$ and $X_t^{\dagger} = (1, Y_{11,t-1}, Y_{12,t-1}, ...,$ $+ Y_{IP,t-1}, DGDP_t^{US}, DGDP_{t-1}^{US})$.

Let $Y_{ip}^{\dagger} = (Y_{ip1}^{\dagger}, Y_{ip2}^{\dagger}, ..., Y_{ipT}^{\dagger})'$, $E_{ip} = (\varepsilon_{ip1}, \varepsilon_{ip2}, ..., \varepsilon_{ipT})$, and $X^{\dagger}$ be a $T \times (IP+3)$ matrix with the t^{th} row given by $X_t^{\dagger}$. In matrix form, we rewrite equation (7) as

$$Y_{ip}^{\dagger} = X^{\dagger} b_{ip}^{\dagger} + E_{ip} \tag{8}$$

where $var(E_{ip}) = h_{ip}^{-1} I_T$

Because in equation (8) the number of parameters is greater than the number of observations, we elicit the Minnesota prior for $b_{ip}^{\dagger}$ as:

$$b_{ip}^{\dagger} \sim N(\underline{b}^{\dagger}, \underline{V}_b^{\dagger}).$$

Note that $\underline{V}_b^{\dagger}$ is a diagonal matrix with the diagonal elements given by

$$\underline{V}_{b_{j,j}}^{\dagger} = \begin{cases} \pi_1, & \text{for parameter on own lag;} \\ \pi_2 \sigma_{ip}/\sigma_{-ip}, & \text{for parameters on other lags;} \\ \pi_3 \sigma_{ip}, & \text{for parameters on exogeneous/deterministic variables.} \end{cases}$$

where σ_{ip} is the standard OLS estimate of the error variance in the equation associated with Y_{ipt}, and σ_{-ip} is the standard OLS estimate of the error variance in the equation associated with the growth rate that is not Y_{ipt}.[1]

The conditional posterior for $b^{\dagger}_{ip}$ thus takes the following form

$$b^{\dagger}_{ip}|Y,\Sigma,F,\Gamma \sim N(\overline{b}^{\dagger}_{ip},\overline{V}^{\dagger}_{b_{ip}})$$

where

$$\overline{V}^{\dagger}_{b_{ip}} = (\underline{V}^{\dagger^{-1}}_{b} + h_{ip}X^{\dagger^{'}}X^{\dagger^{-1}})$$

$$\overline{b}^{\dagger}_{ip} = \overline{V}^{\dagger}_{b_{ip}}(\underline{V}^{\dagger^{-1}}_{b}\underline{b}^{\dagger} + h_{ip}X^{\dagger^{'}}Y^{\dagger}_{ip})$$

In empirical work, we set $\pi_1 = 0.05$, $\pi_2 = 0.005$, and $\pi_3 = 1000$.

1.1.2 Models with lagged ε_t

1.1.2.1 Υ Diagonal

Here we assume that $\varepsilon_{ip,t}$ follows a stationary AR(1) process:

$$\varepsilon_{ip,t} = \rho_{ip}\varepsilon_{ip,t-1} + \epsilon_{ip,t}$$

where ϵ_t is *i.i.d.* $N(0,1/h_{ip})$.

Let $Y^{\dagger}_{ip,t} = (1-\rho_{ip}L)[Y_{ip,t} - (\gamma^{I}_{ip}f^{I}_{it} + \gamma^{P}_{ip}f^{P}_{pt} + \gamma^{N}_{ip}f^{N}_{t})]$, we have

$$Y^{\dagger}_{ip,t} = X^{\dagger}_{t}b^{\dagger}_{ip} + \epsilon_{ip,t}, \tag{9}$$

where $b^{\dagger}_{ip} = (\lambda_{ip},\beta_{0,ip},\beta_{1,ip})^{'}$ and $X^{\dagger}_{t} = (1-\rho_{ip}L)(1, DGDP^{US}_{t}, DGDP^{US}_{t-1})$. Note that L is the lag operator.

[1]Various Bayesian priors can be used to shrink the parameter space: the traditional Minnesota prior of Doan et al (1984) and Litterman (1986) and its natural variants (e.g. Kadiyala and Karlsson, 1997; Banbura et al, 2010), the stochastic search variable selection (SSVS) prior of George, Sun and Ni (2008), the family of SSVS plus Minnesota priors of Koop (2011), Lasso of Park and Casella (2008), and the double adaptive elastic-net Lasso of Gefang (2012). The traditional Minnesota prior is popular because its computational cost is low.

Let $Y_{ip}^{\dagger} = (Y_{ip,1}^{\dagger}, Y_{ip,2}^{\dagger}, ..., Y_{ip,T}^{\dagger})'$, $E_{ip} = (\varepsilon_{ip,1}, \varepsilon_{ip,2}, ..., \varepsilon_{ip,T})$, and $X^{\dagger}$ be a $T \times 3$ matrix with the t^{th} row given by $X_t^{\dagger}$. In matrix form, we rewrite equation (9) as

$$Y_{ip}^{\dagger} = X^{\dagger} b_{ip}^{\dagger} + E_{ip} \tag{10}$$

where $var(E_{ip}) = h_{ip}^{-1} I_T$, with I_T be the $T \times T$ identity matrix.

We elicit Normal prior for $b_{ip}^{\dagger}$ as:

$$b_{ip}^{\dagger} \sim N(\underline{b}^{\dagger}, \underline{V}_b^{\dagger})$$

for $i = 1, 2, ..., I$ and $p = 1, 2, ..., P$.

The conditional posterior for $b_{ip}^{\dagger}$ thus takes the following form

$$b_{ip}^{\dagger} | Y, \Sigma, F, \Gamma \sim N(\overline{b}_{ip}^{\dagger}, \overline{V}_{b_{ip}}^{\dagger})$$

where

$$\overline{V}_{b_{ip}}^{\dagger} = (\underline{V}_b^{\dagger^{-1}} + h_{ip} X^{\dagger'} X^{\dagger^{-1}})$$

$$\overline{b}_{ip}^{\dagger} = \overline{V}_{b_{ip}}^{\dagger} (\underline{V}_b^{\dagger^{-1}} \underline{b}^{\dagger} + h_{ip} X^{\dagger'} Y_{ip}^{\dagger})$$

In empirical work, we set $\underline{b}^{\dagger} = 0$ and $\underline{V}_b^{\dagger} = \begin{pmatrix} 10 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

1.1.2.2 Υ Unrestricted

The $N \times 1$ vector ε_t is assumed to follow a stationary VAR(1) process:

$$\varepsilon_t = \Upsilon \varepsilon_{t-1} + \epsilon_t$$

where ϵ_t is *i.i.d.* $N(0, \Sigma)$. Σ is a $N \times N$ diagonal matrix with the diagonal element that corresponds to ε_{ipt} given by $1/h_{ip}$.

Let the $Y_t^\dagger = Y_t - (\gamma^I f_t^I + \gamma^P f_t^P + \gamma^N f_t^N)$, we have

$$Y_t^\dagger = BX_t^\dagger + \varepsilon_t, \tag{11}$$

where B is an $N \times 3$ coefficients matrix with the row associated with $Y_{ip,t}$ given by $(\lambda_{ip}, \beta_{0,ip}, \beta_{1,ip})$, $X_t^\dagger = (1, DGDP_t^{US}, DGDP_{t-1}^{US})'$.

Let $Y^\flat = (Y_1^\dagger, Y_2^\dagger, ..., Y_T^\dagger)' - (Y_0^\dagger, Y_1^\dagger, ..., Y_{T-1}^\dagger)'\Upsilon$, $X^\flat = I_N \otimes X_a^\dagger - \Upsilon' \otimes X_b^\dagger$, where $X_a^\dagger$ be a $T \times 3$ matrix with the t^{th} row given by $X_t^{\dagger'}$, $X_b^\dagger$ be a $T \times 3$ matrix with the t^{th} row given by$X_t^{\dagger'}$, and $E = (\epsilon_1', \epsilon_2', ..., \epsilon_T')$. Then we have

$$y^\flat = X^\flat b + e \tag{12}$$

where $y^\flat = vec(Y^\flat)$, $b = vec(B)$, and $e = vec(E)$. Note that $var(e) = I_T \otimes \Sigma$. For notational convenience, we use Ξ to denote $var(e)$.

We elicit Normal prior for b as:

$$b \sim N(\underline{b}, \underline{V}_b)$$

The conditional posterior for b thus takes the following form

$$b|Y, \Sigma, F, \Gamma \sim N(\overline{b}, \overline{V}_b)$$

where

$$\overline{V}_b = (\underline{V}_b^{-1} + X^{\flat'} \Xi^{-1} X^{\flat^{-1}})^{-1}$$

$$\overline{b} = \overline{V}_b(\underline{V}_b \underline{b} + X^{\flat'} \Xi^{-1} y^\flat)$$

In empirical work, we set $\underline{b}^\dagger = 0$ and $\underline{V}_b^\dagger = \begin{pmatrix} 10 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

1.2 Conditional Posteriors for γ^I, γ^P, and γ^N

1.2.1 Models without lagged ε_t

Conditional on λ, β_0, β_1, and the factors, we can estimate the elements in γ^I, γ^P, and γ^N equation by equation. Let $Y^{\ddagger}_{ipt} = Y_{ipt} - (\lambda_{ip} + \beta_{ip,0} DGDP^{US}_t + \beta_{ip,1} DGDP^{US}_{t-1})$, we have

$$Y^{\ddagger}_{ipt} = X^{\ddagger}_t b^{\ddagger}_{ip} + \varepsilon_{ipt}, \tag{13}$$

where $b^{\ddagger}_{ip} = (\gamma^I_{ip}, \gamma^P_{ip}, \gamma^N_{ip})'$ and $X^{\ddagger}_t = (f^I_{it}, f^P_{pt}, f^N_t)$.

Let $Y^{\ddagger}_{ip} = (Y^{\ddagger}_{ip1}, Y^{\ddagger}_{ip2}, ..., Y^{\ddagger}_{ipT})'$, $E_{ip} = (\varepsilon_{ip1}, \varepsilon_{ip2}, ..., \varepsilon_{ipT})$, and $X^{\ddagger}$ be a $T \times 3$ matrix with the t^{th} row given by $X^{\ddagger}_t$. In matrix form, we rewrite equation (13) as

$$Y^{\ddagger}_{ip} = X^{\ddagger} b^{\ddagger}_{ip} + E_{ip} \tag{14}$$

where $var(E_{ip}) = h^{-1}_{ip} I_T$.

We elicit Normal prior for $b^{\ddagger}_{ip}$ as:

$$b^{\ddagger}_{ip} \sim N(\underline{b}^{\ddagger}, \underline{V}^{\ddagger}_b)$$

for $i = 1, 2, ..., I$ and $p = 1, 2, ..., P$.

The conditional posterior for $b^{\ddagger}_{ip}$ thus takes the following form

$$b^{\ddagger}_{ip} | Y, \Sigma, F, \Gamma \sim N(\overline{b}^{\ddagger}_{ip}, \overline{V}^{\ddagger}_{b_{ip}})$$

where

$$\overline{V}^{\ddagger}_{b_{ip}} = (\underline{V}^{\ddagger^{-1}}_b + h_{ip} X^{\ddagger'} X^{\ddagger^{-1}})$$

$$\overline{b}^{\ddagger}_{ip} = \overline{V}^{\ddagger}_{b_{ip}} (\underline{V}^{\ddagger^{-1}}_b \underline{b}^{\ddagger} + h_{ip} X^{\ddagger'} Y^{\ddagger}_{ip})$$

In empirical work, we set $\underline{b}^{\ddagger} = 0$ and $\underline{V}_b^{\ddagger} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

1.2.2 Models with lagged ε_t

1.2.2.1 Υ Diagonal

Here we assume that $\varepsilon_{ip,t}$ follows a stationary AR(1) process:

$$\varepsilon_{ip,t} = \rho_{ip}\varepsilon_{ip,t-1} + \epsilon_{ip,t}$$

where ϵ_t is *i.i.d.* $N(0, 1/h_{ip})$. Let $Y_{ip,t}^{\ddagger} = (1-\rho_{ip}L)[Y_{ip,t} - (\lambda_{ip} + \beta_{ip,0}DGDP_t^{US} + \beta_{ip,1}DGDP_{t-1}^{US})]$, we have

$$Y_{ipt}^{\ddagger} = X_t^{\ddagger} b_{ip}^{\ddagger} + \epsilon_{ipt}, \tag{15}$$

where $b_{ip}^{\ddagger} = (\gamma_{ip}^I, \gamma_{ip}^P, \gamma_{ip}^N)'$ and $X_t^{\ddagger} = (1 - \rho_{ip}L)(f_{it}^I, f_{pt}^P, f_t^N)$.

Let $Y_{ip}^{\ddagger} = (Y_{ip,1}^{\ddagger}, Y_{ip,2}^{\ddagger}, ..., Y_{ip,T}^{\ddagger})'$, $\epsilon_{ip} = (\epsilon_{ip,1}, \epsilon_{ip,2}, ..., \epsilon_{ip,T})$, and $X^{\ddagger}$ be a $T \times 3$ matrix with the t^{th} row given by $X_t^{\ddagger'}$. In matrix form, we rewrite equation (15) as

$$Y_{ip}^{\ddagger} = X^{\ddagger} b_{ip}^{\ddagger} + \epsilon_{ip} \tag{16}$$

where $var(\epsilon_{ip}) = h_{ip}^{-1} I_T$

We elicit Normal prior for $b_{ip}^{\ddagger}$ as:

$$b_{ip}^{\ddagger} \sim N(\underline{b}^{\ddagger}, \underline{V}_b^{\ddagger})$$

for $i = 1, 2, ..., I$ and $p = 1, 2, ..., P$.

The conditional posterior for $b^{\ddagger}_{ip}$ thus takes the following form

$$b^{\ddagger}_{ip}|Y,\Sigma,F,\Gamma \sim N(\overline{b}^{\ddagger}_{ip},\overline{V}^{\ddagger}_{b_{ip}})$$

where

$$\overline{V}^{\ddagger}_{b_{ip}} = (\underline{V}^{\ddagger^{-1}}_{b} + h_{ip}X^{\ddagger^{'}}X^{\ddagger^{-1}})$$

$$\overline{b}^{\ddagger}_{ip} = \overline{V}^{\ddagger}_{b_{ip}}(\underline{V}^{\ddagger^{-1}}_{b}\underline{b}^{\ddagger} + h_{ip}X^{\ddagger^{'}}Y^{\ddagger}_{ip})$$

In empirical work, we set $\underline{b}^{\ddagger} = 0$ and $\underline{V}^{\ddagger}_{b} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$.

1.2.2.2 Υ Unrestricted

Here ε_ts are assumed to follow a stationary VAR(1) process:

$$\varepsilon_t = \Upsilon\varepsilon_{t-1} + \epsilon_t$$

where ϵ_t is *i.i.d.* $N(0,\Sigma)$. Σ is a diagonal matrix with the diagonal element that corresponds to ε_{ipt} given by $1/h_{ip}$.

First let $Y^{\ddagger}_t = Y_t - (\lambda + \beta_0 DGDP^{US}_t + \beta_1 DGDP^{US}_{t-1})$, $Y^{\ddagger}_a = (Y^{\ddagger}_1, Y^{\ddagger}_2, ..., Y^{\ddagger}_T)^{'}$, and $Y^{\ddagger}_b = (Y^{\ddagger}_0, Y^{\ddagger}_1, ..., Y^{\ddagger}_{T-1})^{'}$. We use $Y^{\ddagger}$ to denote the difference between $Y^{\ddagger}_a$ and $Y^{\ddagger}_b\Upsilon$. Next let $X^{\ddagger}_{ip,t} = (f^I_{it}, f^P_{pt}, f^N_t)$, we construct two $T \times 3N$ matrices: $X^{\ddagger}_a$ with the t^{th} row given by $(X^{\ddagger}_{1,t}, X^{\ddagger}_{2,t}, ..., X^{\ddagger}_{IP,t})$, and $X^{\ddagger}_b$ with the t^{th} row given by $(X^{\ddagger}_{1,t-1}, X^{\ddagger}_{2,t-1}, ..., X^{\ddagger}_{IP,t-1})$. Finally we have $X^{\ddagger} = I_N \otimes X^{\ddagger}_a - \Upsilon^{'} \otimes X^{\ddagger}_b$.

Now the model can be written as

$$y^{\ddagger} = X^{\ddagger}b^{\ddagger} + e, \tag{17}$$

where $y^{\ddagger} = vec(Y^{\ddagger})$, $b^{\ddagger} = vec(B^{\ddagger})$, and $B^{\ddagger}$ be the $3 \times N$ coefficient matrix. In $B^{\ddagger}$, the column that associated with the equation of Y_{ipt} is given by $(\gamma^I_{ip}, \gamma^P_{ip}, \gamma^N_{ip})'$.

We elicit Normal prior for $b^\ddagger$ as:

$$b^\ddagger \sim N(\underline{b}, \underline{V}_b)$$

The conditional posterior for $b^\ddagger$ thus takes the following form

$$b^\ddagger | Y, \Sigma, F, \Gamma, \Upsilon \sim N(\overline{b}^\ddagger, \overline{V}^\ddagger_b)$$

where

$$\overline{V}^\ddagger_b = (\underline{V}^{\ddagger^{-1}}_b + X^{\ddagger'} \Xi^{-1} X^\ddagger)^{-1}$$

$$\overline{b}^\ddagger = \overline{V}^\ddagger_b (\underline{V}^{\ddagger^{-1}}_b \underline{b}^\ddagger + X^{\ddagger'} \Xi^{-1} y^\ddagger)$$

where $\Xi = var(e)$.

In empirical work, we set $\underline{b}^\ddagger = 0$ and $\underline{V}^\ddagger_b = I_{3*N}$.

1.3 Conditional Posteriors for the Factors

Let $Y^\natural_t$ be the difference between Y_t and the sum of $\lambda + \beta_0 DGDP^{US}_t + \beta_1 DGDP^{US}_{t-1}$ and any other deterministic terms that associated with lagged Y_t, if the latter exist, we have

$$Y^\natural_t = \Gamma F_t + \varepsilon_t, \tag{18}$$

where $F_t = (f^I_t f^P_t f^N_t)$, and $var(\varepsilon_t) = \Omega$. When ε_ts are assumed to be *i.i.d.*, we have $\Omega = \Sigma$; when ε_{ipt}s are assumed follow an AR(1) process, $\Omega_{ip,ip} = \frac{1}{h_{ip}(1-\rho^2_{ip})}$; when ε_{ipt}s are assumed follow an VAR(1) process with Υ unrestricted, $vec(\Omega) = vec[(I_{N^2} - \Upsilon \otimes \Upsilon) vec(\Sigma)]$. Γ is the matrix of factor loadings. The dimension of F_t and Γ are $(I + P + 1) \times 1$ and $N \times (I + P + 1)$, respectively.

1.3.1 Static Factor Models

For static factor models, as shown in Lopes and West (2004), the conditional posteriors for F can be factored into independent normal distributions for F_t,

$$F_t|\Sigma, \lambda, \beta_0, \beta_1, \gamma^I, \gamma^P, \gamma^N \sim N[(I_{IP} + \Gamma^{'}\Sigma^{-1}\Gamma)^{-1}\Gamma^{'}\Sigma^{-1}Y_t^{\natural}, (I_{IP} + \Gamma^{'}\Omega^{-1}\Gamma)^{-1}]$$

1.3.2 Dynamic Factor Models

For dynamic factor models, we can rewrite the model into a state-space form, where the measurement equation is equation (18) , and the transition equation is the following:

$$F_t = \Phi F_t + \nu_t, \tag{19}$$

where $var(\nu_t) = I_{I+P+1}$.

Let $\widetilde{Y}_t^{\natural} = (Y_1^{\natural}, Y_2^{\natural}, ..., Y_t^{\natural})^{'}$. Following Kim and Nelson (1999, Ch. 8), conditional on Φ and Γ, we can draw the latent factors in the following steps.

First run Kalman filter to calculate $F_{t|t} = E(F_t|\widetilde{Y}_t^{\natural})$ and $P_{t|t} = Cov(F_t|\widetilde{Y}_t^{\natural})$ for $t = 1, 2, ..., T$:

$$F_{t|t-1} = \Phi F_{t-1}$$

$$P_{t|t-1} = \Phi P_{t|t-1}\Phi^{'} + I_{I+P+1}$$

$$F_{t|t} = F_{t|t-1} + P_{t|t-1}\Gamma^{'}(\Gamma P_{t|t-1}\Gamma^{'} + \Omega)^{-1}(Y_t^{\natural} - \Gamma F_{t|t-1})$$

$$P_{t|t} = P_{t|t-1} - P_{t|t-1}\Gamma^{'}(\Gamma P_{t|t-1}\Gamma^{'} + \Omega)^{-1}\Gamma P_{t|t-1}$$

Next, we draw F_T based on the last iteration of the Kalman filter:

$$F_T|\widetilde{Y}_T^{\natural} \sim N(F_{T|T}, P_{T|T})$$

Then we derive $F_{t|\widetilde{Y}_T^\natural}$ backward for $t = T-1, T-2, ..., 1$:

$$F_t|Y_t^\natural, F_{t+1} \sim N(F_{t|t,F_{t+1}}, P_{t|t,F_{t+1}})$$

where

$$F_{t|t,F_{t+1}} = F_{t|t} + P_{t|t}\Phi^{'}\{\Phi P_{t|t}\Phi^{'} + I_3\}^{-1}(F_{t+1} - \Phi F_{t|t})$$

$$P_{t|t,F_{t+1}} = P_{t|t} - P_{t|t}\Phi^{'}\{\Phi P_{t|t}\Phi^{'} + I_3\}^{-1}\Phi P_{t|t}$$

1.4 Conditional Posteriors for h_{ip}

We set Gamma prior for h_{ip} as $G(\underline{s}^{-2}, \underline{\nu})$. Let ϵ_{ipt} be the *i.i.d.* error term in the equation associated with Y_{ipt} and $\epsilon_{ip} = (\epsilon_{ip,1}, \epsilon_{ip,2}, ..., \epsilon_{ip,T})$. It can be verified that the conditional posterior for h_{ip} is Gamma

$$h_{ip}|Y, F, \Gamma, \lambda, \beta_0, \beta_1 \sim G(\overline{s}^{-2}, \overline{\nu})$$

where

$$\overline{\nu} = T + \underline{\nu}$$

$$\overline{s}^{-2} = \frac{\epsilon_{ip}^{'}\epsilon_{ip} + \underline{\nu s^2}}{\overline{\nu}}$$

Note that $\epsilon_t = \varepsilon_t$ if ε_t is *i.i.d.*, $\epsilon_{ipt} = \varepsilon_{ipt} - \rho_{ip}\varepsilon_{ip,t-1}$ if ε_{ipt} follows an AR(1) process, and $\epsilon_t = \varepsilon_t - \Upsilon\varepsilon_{t-1}$ if ε_t follows a VAR(1) process.

In empirical work, we set $\underline{s}^{-2} = 0.001$ and $\underline{\nu} = 1$.

1.5 Conditional Posteriors for Φ

1.5.1 Φ Unrestricted

Conditional on the $I+P+1$ factors, Φ can be estimated equation by equation. Using $f_{j,t}$ to denote the j^{th} element in F_t, we have

$$f_{j,t} = \phi_{j,1} f_{1,t} + \phi_{j,2} f_{2,t} + ... + \phi_{j,I+P+1} f_{I+P+1,t} + \nu_{j,t}, \tag{20}$$

where $var(\nu_{j,t}) = 1$.

Let $f_j = (f_{j,t}, f_{j,2}, ..., f_{j,T})^{'}$, $U_j = (\nu_{j,1}, \nu_{j,2}, ..., \nu_{j,T})$, $X^{\S}$ be a $T \times (P+I+1)$ matrix with the t^{th} row given by $(f_{1,t}, f_{2,t}, ..., f_{1+P+I,t})$, and $b_j^{\S}$ be the transpose of the j^{th} row in Φ. In matrix form, we rewrite equation (20) as

$$f_j = X^{\S} b_j^{\S} + U_j \tag{21}$$

where $var(U_j) = I_T$.

We elicit Minnesota prior for $b_j^{\S}$ as:

$$b_j^{\S} \sim N(\underline{b}^{\S}, \underline{V}_b^{\S})$$

for $j = 1, 2, ..., 1+I+P$. Note that $\underline{V}_b^{\S}$ is a diagonal matrix with the diagonal elements given by

$$\underline{V}_{b_{j,j}}^{\S} = \begin{cases} \pi_4, & \text{for parameter on own lag;} \\ \pi_5 & \text{for parameters on other lags} \end{cases}$$

The conditional posterior for $b_j^{\S}$ thus takes the following form

$$b_j^{\S} | Y, \Sigma, F, \lambda, \Lambda, \beta_0, \beta_1 \sim N(\overline{b}_j^{\S}, \overline{V}_{b_j}^{\S})$$

where

$$\overline{V}^{\S}_{b_j} = (\underline{V}^{\S^{-1}}_{b} + X^{\S^{'}} X^{\S^{-1}})$$

$$\overline{b}^{\S}_{j} = \overline{V}^{\S}_{b_j} (\underline{V}^{\S^{-1}}_{b} \underline{b}^{\S} + X^{\S^{'}} f_j)$$

In empirical work, we set $\underline{b}^{\S} = 0$, $\pi_4 = 0.05$, and $\pi_5 = 0.005$.

1.5.2 Φ Diagonal

Conditional on the $I + P + 1$ factors, Φ can be estimated equation by equation. Using $f_{j,t}$ to denote the j^{th} element in F_t, we have

$$f_{j,t} = \phi_{j,j} f_{j,t-1} + \nu_{j,t}, \tag{22}$$

Let $f_j = (f_{j,t}, f_{j,2}, ..., f_{j,T})^{'}$, $U_j = (\nu_{j,1}, \nu_{j,2}, ..., \nu_{j,T})$, $X^{\S} = (f_{j,-1}, f_{j,1}, ..., f_{j,T-1})^{'}$, and $b^{\S}_{j}$ be $\phi_{j,j}$. In matrix form, we rewrite equation (22) as

$$f_j = X^{\S} b^{\S}_{j} + U_j \tag{23}$$

where $var(U_j) = I_T$.

We elicit Normal prior for $b^{\S}_{j}$ as:

$$b^{\S}_{j} \sim N(\underline{b}^{\S}, \underline{V}^{\S}_{b})$$

for $j = 1, 2, ..., 1 + I + P$.

The conditional posterior for $b^{\S}_{j}$ thus takes the following form

$$b^{\S}_{j} | Y, \Sigma, F, \lambda, \Lambda, \beta_0, \beta_1 \sim N(\overline{b}^{\S}_{j}, \overline{V}^{\S}_{b_j})$$

where

$$\overline{V}^{\S}_{b_j} = (\underline{V}^{\S^{-1}}_{b} + X^{\S^{'}} X^{\S^{-1}})$$

$$\overline{b}_j^{\S} = \overline{V}_{b_j}^{\S}(\underline{V}_b^{\S^{-1}}\underline{b}^{\S} + X^{\S^{'}} f_j)$$

In empirical work, we set $\underline{b}^{\S} = 0$ and $\underline{V}_b^{\S} = 1$.

1.6 Conditional Posteriors for Υ

1.6.1 Υ Diagonal

Υ is a diagonal matrix with the diagonal elements given by ρ_{ip}, Conditional on the factors and the rest of the coefficients, we have $\varepsilon_{ip,t} = Y_{ip,t} - (\lambda_{ip} + \beta_{ip,0}DGDP_t^{US} + \beta_{ip,1}DGDP_{t-1}^{US} + \gamma_{ip}^I f_{it}^I + \gamma_{ip}^P f_{pt}^P + \gamma_{ip}^N f_t^N)$ Let $e_{ip}^{\sharp} = (\varepsilon_{ip,1}, \varepsilon_{ip,2}, ..., \varepsilon_{ip,T})$, and $x_{ip}^{\sharp} = (\varepsilon_{ip,0}, \varepsilon_{ip,1}, ..., \varepsilon_{ip,T-1})$. We have

$$e_{ip}^{\sharp} = \rho_{ip} x_{ip}^{\sharp} + \epsilon_{ip}$$

where $var(\epsilon_{ip}) = 1/h_{ip}I_T$.

We elicit Normal prior for ρ_{ip} as:

$$\rho_{ip} \sim N(\underline{\rho}, \underline{V}_{\rho}).$$

The conditional posterior for ρ_{ip} thus takes the following form

$$\rho_{ip}|Y, \Sigma, F, \Gamma \sim N(\overline{\rho}_{ip}, \overline{V}_{\rho_{ip}})$$

where

$$\overline{V}_{\rho_{ip}} = (\underline{V}_{\rho}^{-1} + h_{ip} x_{ip}^{\sharp^{'}} x_{ip}^{\sharp})^{-1}$$

$$\overline{\rho}_{ip} = \overline{V}_{\rho_{ip}}(\underline{V}_{\rho}^{-1}\underline{\rho} + h_{ip} x_{ip}^{\sharp^{'}} e_{ip}^{\sharp})$$

In empirical work, we set $\underline{\rho} = 0$ and $\underline{V}_{\rho} = 1$. To ensure the error terms are stationary, we draw the posteriors from a Truncated Normal.

1.6.2 Υ Unrestricted

Conditional on the coefficients and factors, $\varepsilon_t = Y_t - (\lambda + \beta_0 DGDP_t^{US} + \beta_1 DGDP_{t-1}^{US} + \gamma^I f_t^I + \gamma^P f_t^P + \gamma^N f_t^N)$. It is assumed that $\varepsilon_t = \Upsilon \varepsilon_{t-1} + \epsilon_t$. Let $E^\sharp = (\varepsilon_1, \varepsilon_2, ..., \varepsilon_T)^{'}$, $X^\sharp = (\varepsilon_0, \varepsilon_1, ..., \varepsilon_{T-1})^{'}$, in matrix form, we have

$$E^\sharp = X^\sharp \Upsilon + E, \tag{24}$$

Let $e_{ip}^\sharp$ be the column of error terms in $E^\sharp$ that associated with the i^{th} industry in the p^{th} province, $b_{ip}^\sharp$ is the row vector in Υ that associated with $e_{ip}^\sharp$, we have

$$e^\sharp = X^\sharp b_{ip}^\sharp + e_{ip}, \tag{25}$$

where $var(e_{ip}) = 1/h_{ip} I_T$. Because in equation (25) the number of parameters is greater than the number of observations, we elicit Minnesota prior for $b_{ip}^\sharp$ as:

$$b_{ip}^\sharp \sim N(\underline{b}^\sharp, \underline{V}_b^\sharp)$$

Note that $\underline{V}_b^\sharp$ is a diagonal matrix with the diagonal elements given by

$$\underline{V}_{b_{j,j}}^\dagger = \begin{cases} \pi_6, & \text{for parameter on own lag;} \\ \pi_7 \delta_{ip}/\delta_{-ip} & \text{for parameters on other lags} \end{cases}$$

The conditional posterior for $b_{ip}^\sharp$ thus takes the following form

$$b_{ip}^\sharp | Y, \Sigma, F, \Gamma, \lambda, \beta_0, \beta_1 \sim N(\overline{b}_{ip}^\sharp, \overline{V}_{b_{ip}}^\sharp)$$

where

$$\overline{V}_{b_{ip}}^\sharp = (\underline{V}_b^{\sharp^{-1}} + h_{ip} X^{\sharp^{'}} X^{\sharp^{-1}})$$

$$\overline{b}_{ip}^\sharp = \overline{V}_{b_{ip}}^\sharp (\underline{V}_b^{\sharp^{-1}} \underline{b}^\sharp + h_{ip} X^{\sharp^{'}} e_{ip}^\dagger)$$

In empirical work, we set tighter priors for $b_{ip}^{\sharp}$ to ensure stationary. In particular, we set $\underline{b}^{\sharp} = 0$, $\pi_6 = 0.01$, and $\pi_7 = 0.001$.

2 Variance Decompositions

In this section we provide details for variance decompositions.

2.1 US Growth Rate

Throughout, we assume the exogenous US growth follows an AR(2) process as in Altonji and Ham (1990):

$$DGDP_t^{US} = \alpha_0 + \alpha_1 DGDP_{t-1}^{US} + \alpha_2 DGDP_{t-2}^{US} + \mu_t \tag{26}$$

where μ_t is *i.i.d.* $N(0, \sigma_{us}^2)$.

Let u be the expected value for $DGDP_t^{US}$ at steady-state. In VAR form, we have

$$\begin{pmatrix} DGDP_t^{US} - u \\ DGDP_{t-1}^{US} - u \end{pmatrix} = \begin{pmatrix} \alpha_1 & \alpha_2 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} DGDP_{t-1}^{US} - u \\ DGDP_{t-2}^{US} - u \end{pmatrix} + \begin{pmatrix} \mu_t \\ 0 \end{pmatrix} \tag{27}$$

Hence, the s period ahead forecast errors for $DGDP_t^{US}$ is

$$\psi_{s-1}\mu_{t+1} + \psi_{s-2}\mu_{t+2} + ... + \psi_1\mu_{t+s-1} + \mu_{t+s} \tag{28}$$

where ψ_h is the $(1,1)^{th}$ element in matrix $\begin{pmatrix} \alpha_1 & \alpha_2 \\ 1 & 0 \end{pmatrix}^h$.

The mean squared error of 1 period ahead forecast for $DGDP_t^{US}$:

$$MSE(\widehat{DGDP}_{t+1}^{US}|t) = \sigma_{us}^2 \tag{29}$$

The mean squared error of s period ahead forecast for $DGDP_t^{US}$:

$$MSE(\widehat{DGDP}_{t+s}^{US}|t) = (\psi_{s-1}^2 + \psi_{s-2}^2 + ... + \psi_1^2 + 1)\sigma_{us}^2 \tag{30}$$

2.2 The Static Factor Model

The model takes the following form:

$$Y_t = \lambda + \beta_0 DGDP_t^{US} + \beta_1 DGDP_{t-1}^{US} + \gamma^I f_t^I + \gamma^P f_t^P + \gamma^N f_t^N + \varepsilon_t \tag{31}$$

where the variance for ε_{ipt} is assumed to be ϖ_{ip}^2.

The 1 period ahead forecast error for Y_{ipt} is $\beta_{ip,0}\mu_{t+1} + \gamma_{ip}^I f_{i,t+1}^I + \gamma_{ip}^P f_{p,t+1}^P + \gamma_{ip}^N f_{t+1}^N + \varepsilon_{ip,t+1}$. Thus, the mean squared error of 1 period ahead forecast is:

$$MSE(\widehat{Y}_{ip,t+1}|t) = \beta_{ip,0}^2\sigma_{us}^2 + (\gamma_{ip}^I)^2 + (\gamma_{ip}^P)^2 + (\gamma_{ip}^N)^2 + \varpi_{ip}^2 \tag{32}$$

The s period ahead forecast error for Y_{ipt} is $\beta_{ip,0}(\psi_{s-1}\mu_{t+1} + \psi_{s-2}\mu_{t+2} + ... + \psi_1\mu_{t+s-1} + \mu_{t+s}) + \beta_{ip,1}(\psi_{s-2}\mu_{t+1} + \psi_{s-3}\mu_{t+2} .... + \psi_1\mu_{t+s-2} + \mu_{t+s-1}) + \gamma_{ip}^I f_{i,t+s}^I + \gamma_{ip}^P f_{p,t+s}^P + \gamma_{ip}^N f_{t+s}^N + \varepsilon_{ip,t+s}$. Thus, the mean squared error of s period ahead forecast is following:

$$\begin{aligned} MSE(\widehat{Y}_{ip,t+s}|t) =& [\beta_{ip,0}^2(\psi_{s-1}^2 + \psi_{s-2}^2 + ... + \psi_1^2 + 1) + \beta_{ip,1}^2(\psi_{s-2}^2 + \psi_{s-3}^2 + ... + \psi_1^2 + 1)]\sigma_{us}^2 \\ &+ (\gamma_{ip}^I)^2 + (\gamma_{ip}^P)^2 + (\gamma_{ip}^N)^2 + \varpi_{ip}^2 \end{aligned} \tag{33}$$

2.3 VAR-Factor models 1, 2, and 7

The models share the following general form:

$$Y_t = \lambda + \Lambda Y_{t-1} + \beta_0 DGDP_t^{US} + \beta_1 DGDP_{t-1}^{US} + \gamma^I f_t^I + \gamma^P f_t^P + \gamma^N f_t^N + \varepsilon_t \tag{34}$$

where the variance for ε_{ipt} is ϖ_{ip}^2.

With some manipulation, we have the following VAR form:

$$\begin{pmatrix} Y_t \\ DGDP_t^{US} \\ DGDP_{t-1}^{US} \end{pmatrix} = \begin{pmatrix} \lambda + \beta_0\alpha_0 \\ \alpha_0 \\ 0 \end{pmatrix} + \begin{pmatrix} \Lambda & \beta_0\alpha_1 + \beta_1 & \beta_0\alpha_2 \\ 0 & \alpha_1 & \alpha_2 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} Y_{t-1} \\ DGDP_{t-1}^{US} \\ DGDP_{t-2}^{US} \end{pmatrix} + \begin{pmatrix} \beta_0\mu_t + \gamma^I f_t^I + \gamma^P f_t^P + \gamma^N f_t^N + \varepsilon_t \\ \mu_t \\ 0 \end{pmatrix} \tag{35}$$

Let $A = \begin{pmatrix} \Lambda & \beta_0\alpha_1 + \beta_1 & \beta_0\alpha_2 \\ 0 & \alpha_1 & \alpha_2 \\ 0 & 1 & 0 \end{pmatrix}$, and c be the expectation of $\begin{pmatrix} Y_t \\ DGDP_t^{US} \\ DGDP_{t-1}^{US} \end{pmatrix}$ at the steady state. We have

$$\left(\begin{pmatrix} Y_t \\ DGDP_t^{US} \\ DGDP_{t-1}^{US} \end{pmatrix} - c\right) = A\left(\begin{pmatrix} Y_{t-1} \\ DGDP_{t-1}^{US} \\ DGDP_{t-2}^{US} \end{pmatrix} - c\right) + \begin{pmatrix} \beta_0\mu_t + \gamma^I f_t^I + \gamma^P f_t^P + \gamma^N f_t^N + \varepsilon_t \\ \mu_t \\ 0 \end{pmatrix} \tag{36}$$

Thus, the s period ahead forecast error for $\begin{pmatrix} Y_{t+s} \\ DGDP_{t+s}^{US} \\ DGDP_{t+s-1}^{US} \end{pmatrix}$ is as following:

$$A^{s-1} \begin{pmatrix} \beta_0\mu_{t+1} + \gamma^I f_{t+1}^I + \gamma^P f_{t+1}^P + \gamma^N f_{t+1}^N + \varepsilon_{t+1} \\ \mu_{t+1} \\ 0 \end{pmatrix}$$

$$+A^{s-2}\begin{pmatrix} \beta_0\mu_{t+2}+\gamma^I f_t^I+\gamma^P f_{t+2}^P+\gamma^N f_{t+2}^N+\varepsilon_{t+2} \\ \mu_{t+2} \\ 0 \end{pmatrix}+...$$

$$+A\begin{pmatrix} \beta_0\mu_{t+s-1}+\gamma^I f_{t+s-1}^I+\gamma^P f_{t+s-1}^P+\gamma^N f_{t+s-1}^N+\varepsilon_{t+s-1} \\ \mu_{t+s-1} \\ 0 \end{pmatrix}$$

$$+\begin{pmatrix} \beta_0\mu_{t+s}+\gamma^I f_{t+s}^I+\gamma^P f_{t+s}^P+\gamma^N f_{t+s}^N+\varepsilon_{t+s} \\ \mu_{t+s} \\ 0 \end{pmatrix}$$

The mean squared error of 1 period ahead forecast for Y_{ipt}:

$$MSE(\widehat{Y}_{ip,t+1}|t)=\beta_{ip,0}^2\sigma_{us}^2+(\gamma_{ip}^I)^2+(\gamma_{ip}^P)^2+(\gamma_{ip}^N)^2+\varpi_{ip}^2 \qquad (37)$$

The mean squared error of s period ahead forecast for Y_{ipt} is the relevant diagonal element in:

$$A^{s-1}\Xi(A^{s-1})^{'}+A^{s-2}\Xi(A^{s-2})^{'}+...+A\Xi(A)^{'}+\Xi \qquad (38)$$

where $\Xi=\begin{pmatrix} \beta_0\sigma_{us}^2\beta_0^{'}+\gamma^I(\gamma^I)^{'}+\gamma^P(\gamma^P)^{'}+\gamma^N(\gamma^N)^{'}+\Sigma & \beta_0\sigma_{us}^2 & 0 \\ \beta_0^{'}\sigma_{us}^2 & \sigma_{us}^2 & 0 \\ 0 & 0 & 0 \end{pmatrix}$, and Σ is the diagonal error covariance matrix for ε_t. The $(ip,ip)^{th}$ element in Σ is ϖ_{ip}^2.

2.4 VAR-Factor models 3, 4, 5, 6, 8 and 9

The six models share the following general form:

$$Y_t=\lambda+\Lambda Y_{t-1}+\beta_0 DGDP_t^{US}+\beta_1 DGDP_{t-1}^{US}+\gamma^I f_t^I+\gamma^P f_t^P+\gamma^N f_t^N+\varepsilon_t \qquad (39)$$

$$f_t = \Phi f_{t-1} + v_t \tag{40}$$

where v_t is i.i.d. $N(0, I)$, and the variance for ε_{ipt} is ϖ_{ip}^2.

First we collect the equations together in a big VAR:

$$\begin{pmatrix} f_t \\ Y_t \\ DGDP_t^{US} \\ DGDP_{t-1}^{US} \end{pmatrix} = \begin{pmatrix} 0 \\ \lambda + \beta_0\alpha_0 \\ \alpha_0 \\ 0 \end{pmatrix} + \begin{pmatrix} \Phi & 0 & 0 & 0 \\ \Gamma\Phi & \Lambda & \beta_0\alpha_1 + \beta_1 & \beta_0\alpha_2 \\ 0 & 0 & \alpha_1 & \alpha_2 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} f_{t-1} \\ Y_{t-1} \\ DGDP_{t-1}^{US} \\ DGDP_{t-2}^{US} \end{pmatrix} + \begin{pmatrix} v_t \\ \beta_0\mu_t + \Gamma v_t + \varepsilon_t \\ \mu_t \\ 0 \end{pmatrix} \tag{41}$$

where Γ is a parameter matrix with γ^I, γ^P and γ^N appropriately stacked in.

Let $A = \begin{pmatrix} \Phi & 0 & 0 & 0 \\ \Gamma\Phi & \Lambda & \beta_0\alpha_1 + \beta_1 & \beta_0\alpha_2 \\ 0 & 0 & \alpha_1 & \alpha_2 \\ 0 & 0 & 1 & 0 \end{pmatrix}$, and the expected value for $\begin{pmatrix} f_t \\ Y_t \\ DGDP_t^{US} \\ DGDP_{t-1}^{US} \end{pmatrix}$

be c. We have the following form

$$\left(\begin{pmatrix} f_t \\ Y_t \\ DGDP_t^{US} \\ DGDP_{t-1}^{US} \end{pmatrix} - c\right) = A\left(\begin{pmatrix} f_{t-1} \\ Y_{t-1} \\ DGDP_{t-1}^{US} \\ DGDP_{t-2}^{US} \end{pmatrix} - c\right) + \begin{pmatrix} v_t \\ \beta_0\mu_t + \Gamma v_t + \varepsilon_t \\ \mu_t \\ 0 \end{pmatrix} \tag{42}$$

where the covariance matrix of $\begin{pmatrix} v_t \\ \beta_0\mu_t + \Gamma v_t + \varepsilon_t \\ \mu_t \\ 0 \end{pmatrix}$ is assumed to be Ξ.

Hence, the forecast error for $\begin{pmatrix} f_{t+s} \\ Y_{t+s} \\ DGDP_{t+s}^{US} \\ DGDP_{t+s-1}^{US} \end{pmatrix}$ at time $t+s$ is

$$A^{s-1}\begin{pmatrix} v_{t+1} \\ \beta_0\mu_{t+1}+\Gamma v_{t+1}+\varepsilon_{t+1} \\ \mu_{t+1} \\ 0 \end{pmatrix} + A^{s-2}\begin{pmatrix} v_{t+2} \\ \beta_0\mu_{t+2}+\Gamma v_{t+2}+\varepsilon_{t+2} \\ \mu_{t+2} \\ 0 \end{pmatrix} + ...$$
$$+ A\begin{pmatrix} v_{t+s-1} \\ \beta_0\mu_{t+s-1}+\Gamma v_{t+s-1}+\varepsilon_{t+s-1} \\ \mu_{t+s-1} \\ 0 \end{pmatrix} + \begin{pmatrix} v_{t+s} \\ \beta_0\mu_{t+s}+\Gamma v_{t+s}+\varepsilon_{t+s} \\ \mu_{t+s} \\ 0 \end{pmatrix} \tag{43}$$

The mean squared error of 1 period ahead forecast for Y_{ipt}:

$$MSE(\widehat{Y}_{ip,t+1}|t) = \beta_{ip,0}^2\sigma_{us}^2 + (\gamma_{ip}^I)^2 + (\gamma_{ip}^P)^2 + (\gamma_{ip}^N)^2 + \varpi_{ip}^2 \tag{44}$$

The mean squared error of s period ahead forecast for Y_{ipt} is the relevant diagonal element in:

$$A^{s-1}\Xi(A^{s-1})^{'} + A^{s-2}\Xi(A^{s-2})^{'} + ... + A\Xi(A)^{'} + \Xi \tag{45}$$

2.5 DFM1

The model takes the following form:

$$Y_t = \lambda + \beta_0 DGDP_t^{US} + \beta_1 DGDP_{t-1}^{US} + \gamma^I f_t^I + \gamma^J f_t^J + \gamma^P f_t^P + \gamma^N f_t^N + \varepsilon_t \tag{46}$$

$$\varepsilon_t = \Upsilon\varepsilon_{t-1} + \epsilon_t \tag{47}$$

where the variance of ϵ_t is assumed to be Σ, a diagonal matrix with appropriate ϖ_{ip}^2s as its diagonal elements.

The 1 period ahead forecast error for Y_{ipt} is as following:

$$\beta_{ip,0}\mu_{t+1} + \gamma_{ip}^I f_{i,t+1}^I + \gamma_{ip}^P f_{p,t+1}^P + \gamma_{ip}^N f_{t+1}^N + \epsilon_{ip,t+1} \tag{48}$$

Hence, the mean squared error of 1 period ahead forecast is

$$MSE(\widehat{Y}_{ip,t+1}|t) = \beta_{ip,0}^2\sigma_{us}^2 + (\gamma_{ip}^I)^2 + (\gamma_{ip}^P)^2 + (\gamma_{ip}^N)^2 + \varpi_{ip}^2. \tag{49}$$

The s period ahead forecast error for Y_{ipt} contains two parts. The first part is associated with the US growth rates and the factors:

$$\begin{aligned} &\beta_{ip,0}(\psi_{s-1}\mu_{t+1} + \psi_{s-2}\mu_{t+2} + ... + \psi_1\mu_{t+s-1} + \mu_{t+s}) \\ &+ \beta_{ip,1}(\psi_{s-2}\mu_{t+1} + \psi_{s-3}\mu_{t+2} .... + \psi_1\mu_{t+s-2} + \mu_{t+s-1}) \\ &+ \gamma_{ip}^I f_{i,t+s}^I + \gamma_{ip}^P f_{p,t+s}^P + \gamma_{ip}^N f_{t+s}^N \end{aligned} \tag{50}$$

The second part associated with the VAR(1) idiosyncratic error terms is the relevant element in $\epsilon_{t+s} + \Upsilon\epsilon_{t+s-1} + ... + \Upsilon^{s-1}\epsilon_{t+1}$.

Thus the mean squared error of s period ahead forecast for Y_{ipt} also contains two parts. The first part is $[\beta_{ip,0}^2(\psi_{s-1}^2 + \psi_{s-2}^2 + ... + \psi_1^2 + 1) + \beta_{ip,1}^2(\psi_{s-2}^2 + \psi_{s-3}^2 + ... + \psi_1^2 + 1)]\sigma_{us}^2 + (\gamma_{ip}^I)^2 + (\gamma_{ip}^P)^2 + (\gamma_{ip}^N)^2$; The second part is the relevant diagonal element in $\Sigma + \Upsilon\Sigma\Upsilon' + ... + \Upsilon^{s-1}\Sigma(\Upsilon^{s-1})'$.

2.6 DFM2

The model takes the following form:

$$Y_t = \lambda + \beta_0 DGDP_t^{US} + \beta_1 DGDP_{t-1}^{US} + \gamma^I f_t^I + \gamma^J f_t^J + \gamma^P f_t^P + \gamma^N f_t^N + \varepsilon_t \tag{51}$$

$$\varepsilon_{ipt} = \rho_{ip}\varepsilon_{ip(t-1)} + \nu_{ipt} \tag{52}$$

whereν_{ipt} are assumed to be i.i.d. $N(0, \varpi_{ip}^2)$.

The 1 period ahead forecast error for Y_{ipt}:

$$\beta_{ip,0}\mu_{t+1} + \gamma_{ip}^I f_{i,t+1}^I + \gamma_{ip}^P f_{p,t+1}^P + \gamma_{ip}^N f_{t+1}^N + \nu_{ip,t+1} \tag{53}$$

The mean squared error of 1 period ahead forecast:

$$MSE(\widehat{Y}_{ip,t+1}|t) = \beta_{ip,0}^2\sigma_{us}^2 + (\gamma_{ip}^I)^2 + (\gamma_{ip}^P)^2 + (\gamma_{ip}^N)^2 + \varpi_{ip}^2 \tag{54}$$

The s period ahead forecast error for Y_{ipt}:

$$\begin{aligned}
&\beta_{ip,0}(\psi_{s-1}\mu_{t+1} + \psi_{s-2}\mu_{t+2} + ... + \psi_1\mu_{t+s-1} + \mu_{t+s}) \\
&+ \beta_{ip,1}(\psi_{s-2}\mu_{t+1} + \psi_{s-3}\mu_{t+2}.... + \psi_1\mu_{t+s-2} + \mu_{t+s-1}) \\
&+ \gamma_{ip}^I f_{i,t+s}^I + \gamma_{ip}^P f_{p,t+s}^P + \gamma_{ip}^N f_{t+s}^N + \nu_{ip,t+s} + \rho\nu_{ip,t+s-1} + ... + \rho^{s-1}\nu_{ip,t+1}
\end{aligned} \tag{55}$$

The mean squared error of s period ahead forecast:

$$\begin{aligned}
MSE(\widehat{Y}_{ip,t+s}|t) =& [\beta_{ip,0}^2(\psi_{s-1}^2 + \psi_{s-2}^2 + ... + \psi_1^2 + 1) + \beta_{ip,1}^2(\psi_{s-2}^2 + \psi_{s-3}^2 + ... + \psi_1^2 + 1)]\sigma_{us}^2 \\
&+ (\gamma_{ip}^I)^2 + (\gamma_{ip}^P)^2 + (\gamma_{ip}^N)^2 + \frac{\varpi_{ip}^2(1-\rho^{2s})}{1-\rho^2}
\end{aligned} \tag{56}$$

2.7 DFM 3&5

The models share the following general form:

$$Y_t = \lambda + \beta_0 DGDP_t^{US} + \beta_1 DGDP_{t-1}^{US} + \gamma^I f_t^I + \gamma^J f_t^J + \gamma^P f_t^P + \gamma^N f_t^N + \varepsilon_t \tag{57}$$

$$\varepsilon_t = \Upsilon \varepsilon_{t-1} + \epsilon_t \tag{58}$$

where the variance of ϵ_t is assumed to be Σ.

$$f_t = \Phi f_{t-1} + v_t \tag{59}$$

where v_t is i.i.d. $N(0, I)$.

The 1 period ahead forecast error for Y_{ipt}:

$$\beta_{ip,0}\mu_{t+1} + \gamma_{ip}^I v_{i,t+1}^I + \gamma_{ip}^P v_{p,t+1}^P + \gamma_{ip}^N v_{t+1}^N + \epsilon_{ip,t+1} \tag{60}$$

The mean squared error of 1 period ahead forecast:

$$MSE(\widehat{Y}_{ip,t+1}|t) = \beta_{ip,0}^2 \sigma_{us}^2 + (\gamma_{ip}^I)^2 + (\gamma_{ip}^P)^2 + (\gamma_{ip}^N)^2 + \varpi_{ip}^2 \tag{61}$$

The s period ahead forecast error for Y_{ipt} contains three parts. The first part is associated with the US growth: $\beta_{ip,0}(\psi_{s-1}\mu_{t+1} + \psi_{s-2}\mu_{t+2} + ... + \psi_1\mu_{t+s-1} + \mu_{t+s}) + \beta_{ip,1}(\psi_{s-2}\mu_{t+1} + \psi_{s-3}\mu_{t+2} .... + \psi_1\mu_{t+s-2} + \mu_{t+s-1})$; The second part is the relevant element in $\Gamma(\Phi^{s-1}v_{t+1} + \Phi^{s-2}v_{t+2} + ... + \Phi v_{t+s-1} + v_{t+s})$; The third and the last part is the relevant element in $\epsilon_{t+s} + \Upsilon\epsilon_{t+s-1} + ... + \Upsilon^{s-1}\epsilon_{t+1}$.

As a result, the mean squared error for s period ahead forecast contains three parts too. The first part is $[\beta_{ip,0}^2(\psi_{s-1}^2 + \psi_{s-2}^2 + ... + \psi_1^2 + 1) + \beta_{ip,1}^2(\psi_{s-2}^2 + \psi_{s-3}^2 + ... + \psi_1^2 + 1)]\sigma_{us}^2$; The second part is the relevant diagonal element in $\Gamma[\Phi^{s-1}(\Phi^{s-1})' + \Phi^{s-2}(\Phi^{s-2})' + ... + \Phi\Phi' + I]\Gamma'$; The third part is the relevant diagonal element in $\Sigma + \Upsilon\Sigma\Upsilon' + ... + \Upsilon^{s-1}\Sigma(\Upsilon^{s-1})'$.

2.8 DFM 4&6

The models share the following general form:

$$Y_t = \lambda + \beta_0 DGDP_t^{US} + \beta_1 DGDP_{t-1}^{US} + \gamma^I f_t^I + \gamma^J f_t^J + \gamma^P f_t^P + \gamma^N f_t^N + \varepsilon_t \quad (62)$$

$$\varepsilon_{ipt} = \rho_{ip}\varepsilon_{ip(t-1)} + \nu_{ipt} \quad (63)$$

whereν_{ipt} are assumed to be i.i.d. $N(0, \varpi_{ip}^2)$.

$$f_t = \Phi f_{t-1} + v_t \quad (64)$$

where v_t is i.i.d. $N(0, I)$.

The 1 period ahead forecast error for Y_{ipt}:

$$\beta_{ip,0}\mu_{t+1} + \gamma_{ip}^I v_{i,t+1}^I + \gamma_{ip}^P v_{p,t+1}^P + \gamma_{ip}^N v_{t+1}^N + \nu_{ip,t+1} \quad (65)$$

The mean squared error of 1 period ahead forecast:

$$MSE(\widehat{Y}_{ip,t+1}|t) = \beta_{ip,0}^2 \sigma_{us}^2 + (\gamma_{ip}^I)^2 + (\gamma_{ip}^P)^2 + (\gamma_{ip}^N)^2 + \varpi_{ip}^2 \quad (66)$$

The s period ahead forecast error for Y_{ipt} contains two parts. The first part is $\beta_{ip,0}(\psi_{s-1}\mu_{t+1} + \psi_{s-2}\mu_{t+2} + ... + \psi_1\mu_{t+s-1} + \mu_{t+s}) + \beta_{ip,1}(\psi_{s-2}\mu_{t+1} + \psi_{s-3}\mu_{t+2} .... + \psi_1\mu_{t+s-2} + \mu_{t+s-1}) + \nu_{ip,t+s} + \rho\nu_{ip,t+s-1} + ... + \rho^{s-1}\nu_{ip,t+1}$; The second part is the relevant element in $\Gamma(\Phi^{s-1}v_{t+1} + \Phi^{s-2}v_{t+2} + ... + \Phi v_{t+s-1} + v_{t+s})$.

Thus, the mean squared error for s period ahead forecast for Y_{ipt} contains two parts too. The first part is $[\beta_{ip,0}^2(\psi_{s-1}^2 + \psi_{s-2}^2 + ... + \psi_1^2 + 1) + \beta_{ip,1}^2(\psi_{s-2}^2 + \psi_{s-3}^2 + ... + \psi_1^2 + 1)]\sigma_{us}^2 + (\gamma_{ip}^I)^2 + (\gamma_{ip}^P)^2 + (\gamma_{ip}^N)^2 + \frac{\varpi_{ip}^2(1-\rho^{2s})}{1-\rho^2}$; The second part is the relevant diagonal element in $\Gamma[\Phi^{s-1}(\Phi^{s-1})^{'} + \Phi^{s-2}(\Phi^{s-2})^{'} + ... + \Phi\Phi^{'} + I]\Gamma^{'}$

2.9 DFM7&8

The two models share the following general form:

$$Y_t = \lambda + \beta_0 DGDP_t^{US} + \beta_1 DGDP_{t-1}^{US} + \gamma^I f_t^I + \gamma^J f_t^J + \gamma^P f_t^P + \gamma^N f_t^N + \varepsilon_t \quad (67)$$

where the variance of ε_{ipt} is ϖ_{ip}^2.

$$f_t = \Phi f_{t-1} + v_t \quad (68)$$

where v_t is i.i.d. $N(0, I)$.

The 1 period ahead forecast error for Y_{ipt}:

$$\beta_{ip,0}\mu_{t+1} + \gamma_{ip}^I v_{i,t+1}^I + \gamma_{ip}^P v_{p,t+1}^P + \gamma_{ip}^N v_{t+1}^N + \varepsilon_{ip,t+1} \quad (69)$$

The mean squared error of 1 period ahead forecast:

$$MSE(\widehat{Y}_{ip,t+1}|t) = \beta_{ip,0}^2 \sigma_{us}^2 + (\gamma_{ip}^I)^2 + (\gamma_{ip}^P)^2 + (\gamma_{ip}^N)^2 + \varpi_{ip}^2 \quad (70)$$

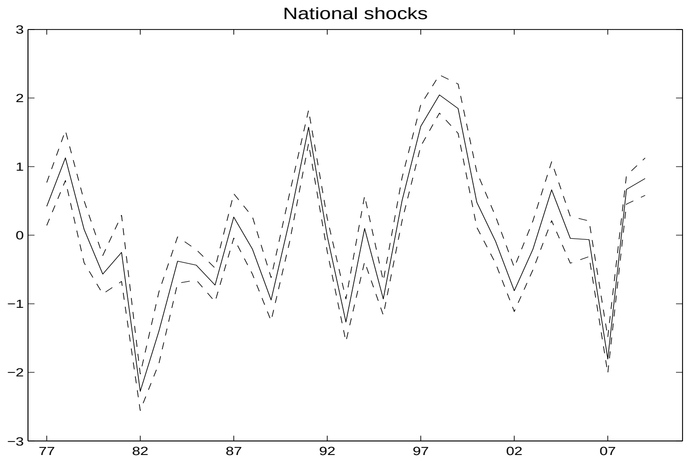
The s period ahead forecast error for Y_{ipt} contains two parts. The first part is $\beta_{ip,0}(\psi_{s-1}\mu_{t+1} + \psi_{s-2}\mu_{t+2} + ... + \psi_1\mu_{t+s-1} + \mu_{t+s}) + \beta_{ip,1}(\psi_{s-2}\mu_{t+1} + \psi_{s-3}\mu_{t+2} .... + \psi_1\mu_{t+s-2} + \mu_{t+s-1}) + \varepsilon_{ip,t+s}$; The second part is the relevant element in $\Gamma(\Phi^{s-1}v_{t+1} + \Phi^{s-2}v_{t+2} + ... + \Phi v_{t+s-1} + v_{t+s})$.

Thus, the mean squared error for s period ahead forecast for Y_{ipt} contains two parts too. The first part is $[\beta_{ip,0}^2(\psi_{s-1}^2 + \psi_{s-2}^2 + ... + \psi_1^2 + 1) + \beta_{ip,1}^2(\psi_{s-2}^2 + \psi_{s-3}^2 + ... + \psi_1^2 + 1)]\sigma_{us}^2 + (\gamma_{ip}^I)^2 + (\gamma_{ip}^P)^2 + (\gamma_{ip}^N)^2 + \varpi_{ip}^2$; The second part is the relevant diagonal element in $\Gamma[\Phi^{s-1}(\Phi^{s-1})^{'} + \Phi^{s-2}(\Phi^{s-2})^{'} + ... + \Phi\Phi^{'} + I]\Gamma^{'}$.

3 The Factors

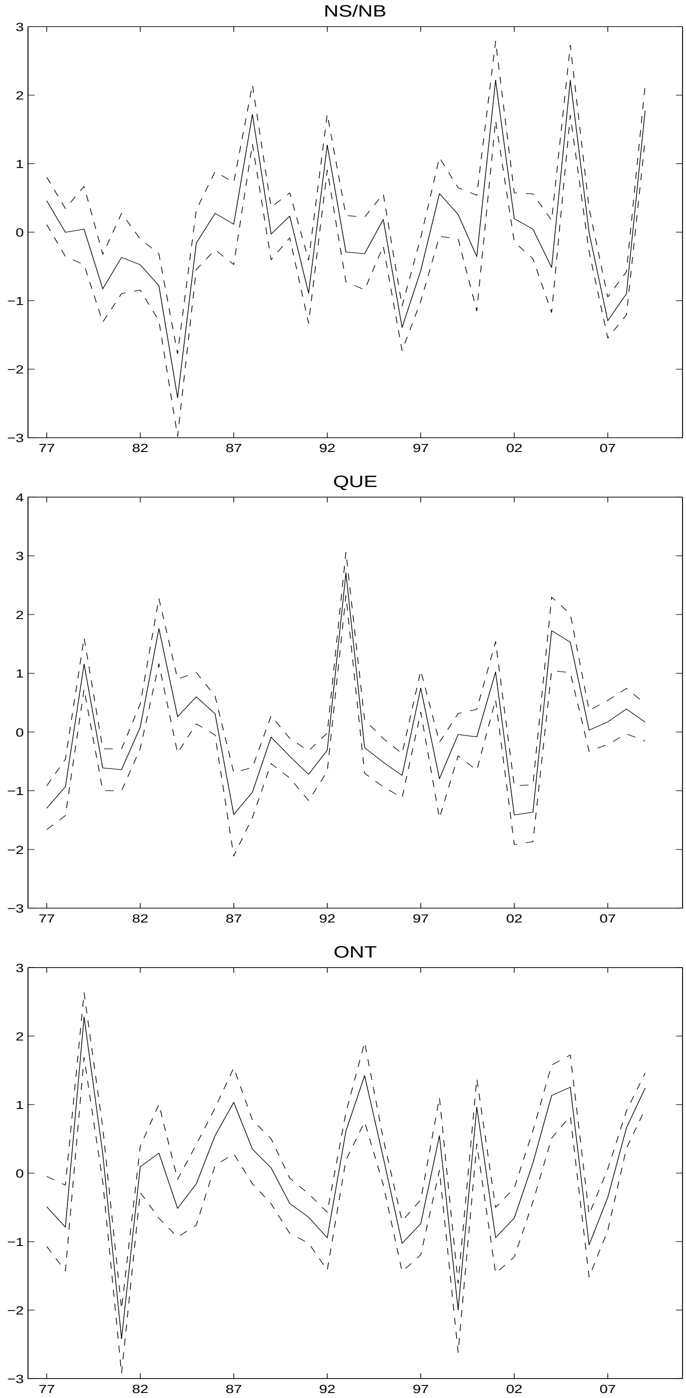
In this section, we plot the factors for the most preferred model. Discussions on these figures are provided in the main paper.

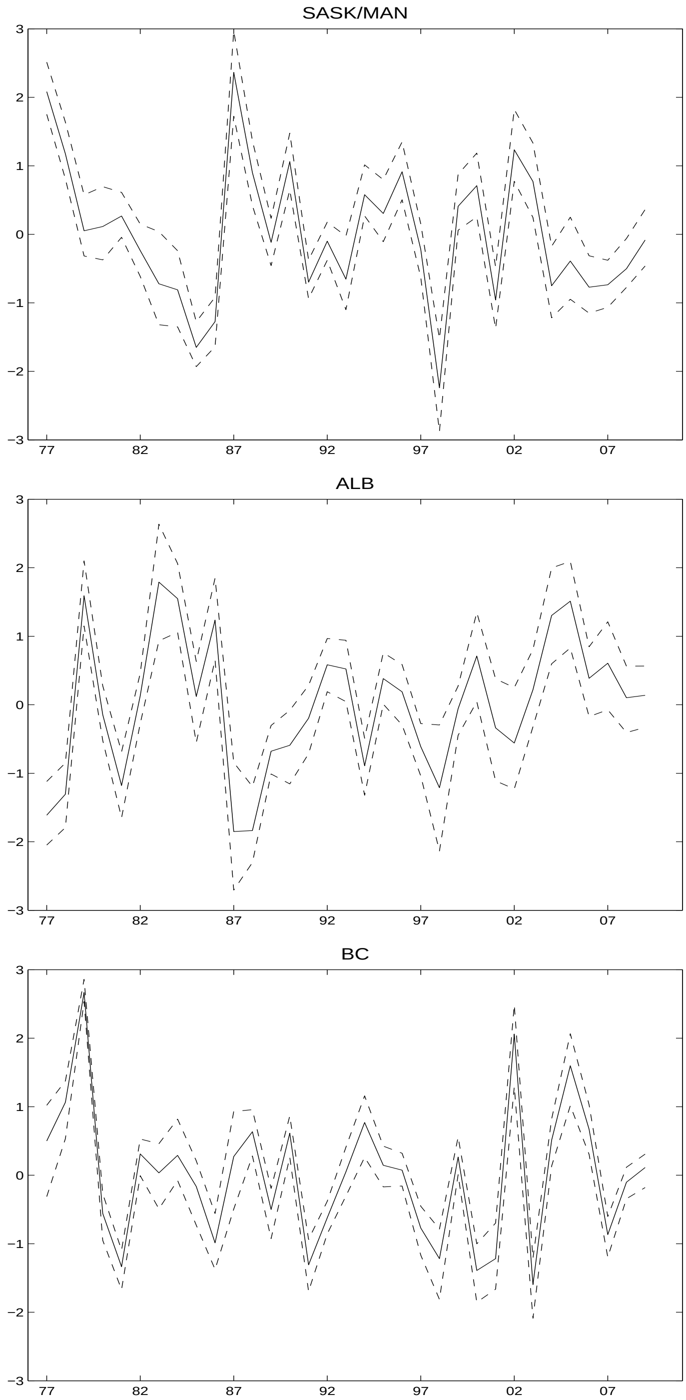
Figure 1: National Factor



Note: 33-66% quantile bands are in dash lines.

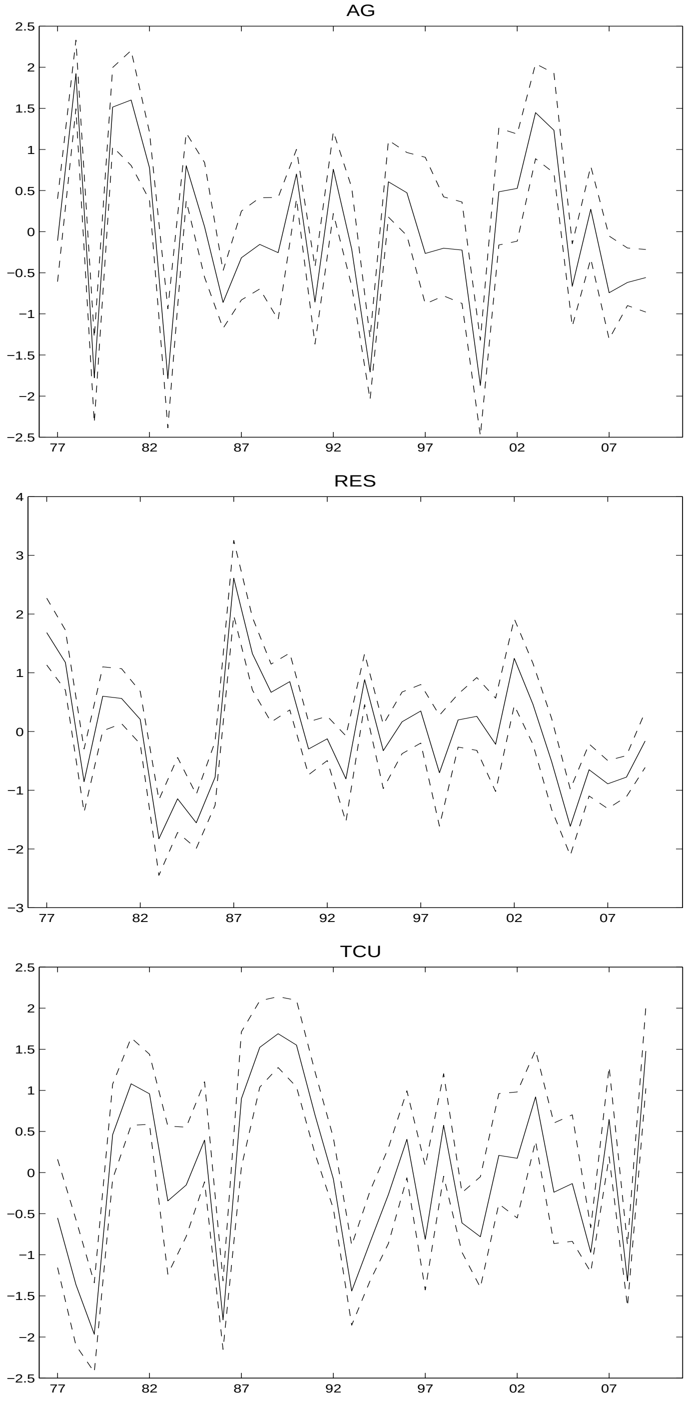
Figure 2: Province Factors

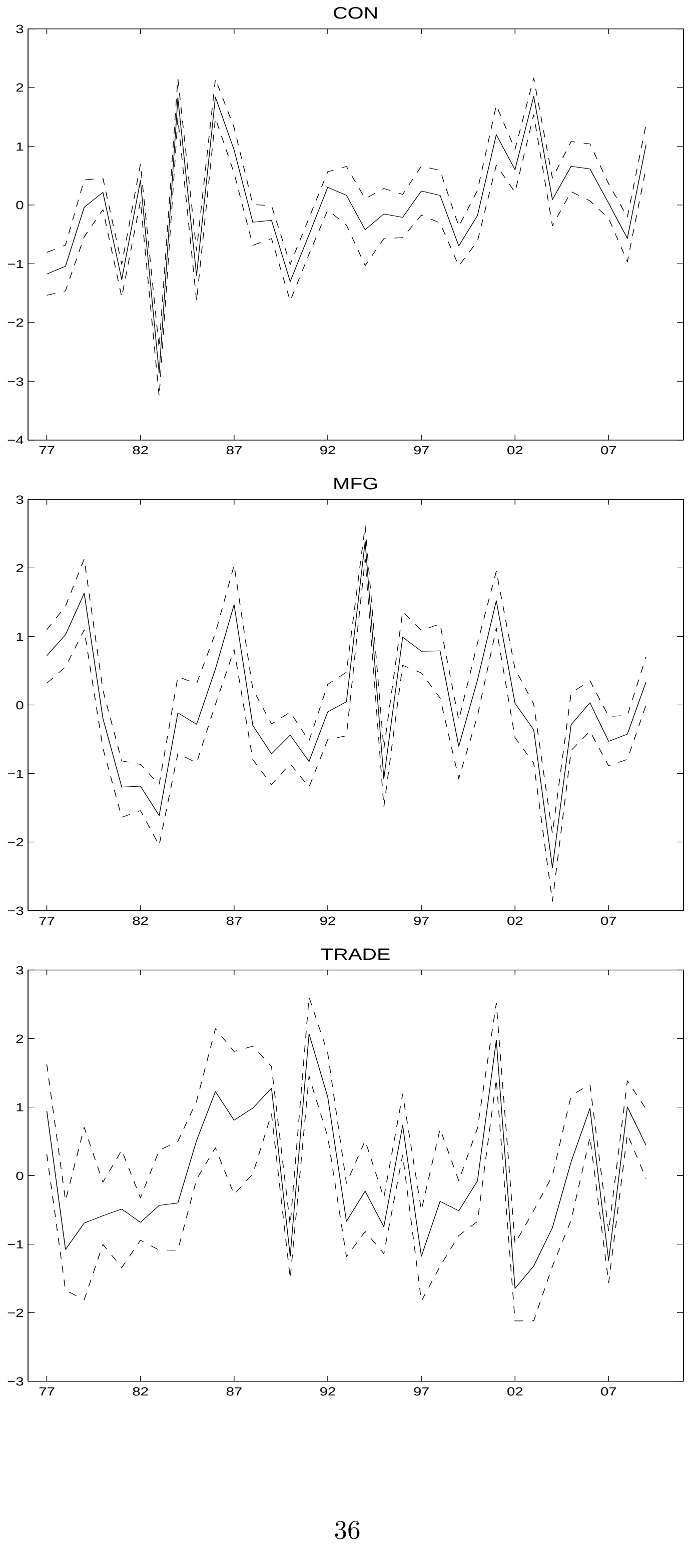


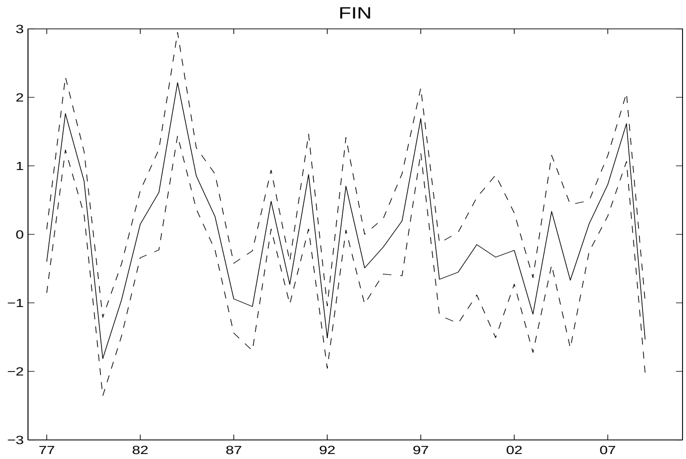


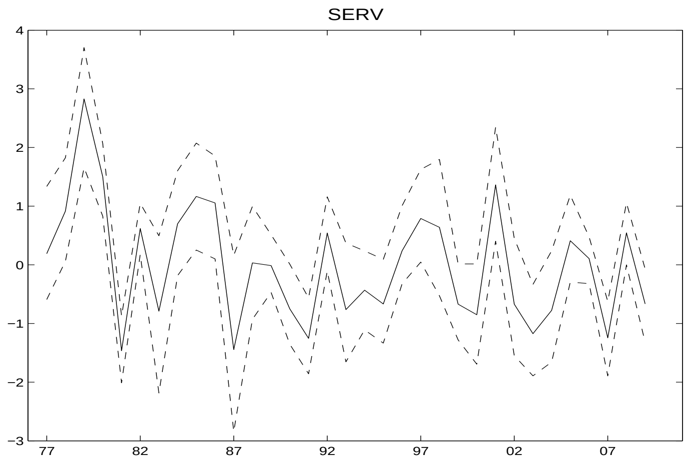
Note: 33-66% quantile bands are in dash lines.

Figure 3: Industry Factors




CON
3
2
1
0
−1
−2
−3
−4
77
82
87
92
97
02
07
MFG
3
2
1
0
−1
−2
−3
77
82
87
92
97
02
07
TRADE
3
2
1
0
−1
−2
−3
77
82
87
92
97
02
07





Note: 33-66% quantile bands are in dash lines.

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