# Interstrip resistance measurement 

## Technical Note

## by A.Chilingarov (Lancaster University)

In ATLAS SCT community two methods of interstrip resistance measurements are used: a) measuring the resistance between two strips and comparing it with a separately measured strip-to bias-rail resistance and b) applying DC voltage to one strip and measuring the current flowing to another strip. The method a) will further be referred to as Resistance Method and method b) as Induced Current Method.

## 1. Basic relations

Consider a semi-infinite chain of bias, $R_{b}$, and interstrip, $R_{i s}$, resistors as shown in Fig.1. Assuming that all $R_{b}$ are equal and the same is true for $R_{i s}$ one can find an equivalent resistance $R_{e q}$ of the chain presented in Fig.1.


Fig.1. Circuit diagram for $R_{e q}$
As shown in Appendix A $R_{e q}=b R_{b}$ where

$$
b=\frac{x+\sqrt{x^{2}+4 x}}{2}
$$

and $x=R_{i s} / R_{b}$ is the parameter quantifying the interstrip isolation. Obviously $b>x$. For $x \rightarrow 0 b \rightarrow \sqrt{x}$ and $R_{e q} \rightarrow \sqrt{R_{b} R_{i s}}$. For $x \rightarrow \infty b \rightarrow x$ and $R_{e q} \rightarrow R_{i s}$.

In both methods it is necessary to measure $R_{0}$ - the resistance between an individual strip and the bias rail. As demonstrated in Appendix A

$$
R_{0}=R_{b} \frac{b}{b+2}<R_{b}
$$

When $x \rightarrow 0 \quad R_{0} \rightarrow \frac{\sqrt{R_{b} R_{i s}}}{2}=\frac{R_{e q}}{2}$, while for $x \rightarrow \infty(\mathrm{~b} \rightarrow \infty) R_{0} \rightarrow R_{b}$.

## 2. Resistance Method

Typically the resistance is measured between two adjacent strips. However in some situations the access is possible only to every second strip. Therefore this case is also considered.

### 2.1 Neighbour strips

The resistance $R_{1}$ between two adjacent strips can be expressed as (see Appendix B)

$$
R_{1}=R_{i s} \frac{2}{b+2}<R_{i s}
$$

For $x \rightarrow 0(b \rightarrow 0) R_{1} \rightarrow R_{i s}$ while for $x \rightarrow \infty(b \rightarrow x) R_{1} \rightarrow 2 R_{i s} / x=2 R_{b}$. Note an interesting relation

$$
\frac{R_{0}}{R_{b}}+\frac{R_{1}}{R_{i s}}=1
$$

The experimentally measured parameters $R_{0}$ and $R_{1}$ allow finding $R_{b}$ and $R_{i s}$. It is useful to introduce parameter $\rho_{1}=2 R_{0} / R_{1}$. Then as shown in Appendix B

$$
\begin{gathered}
\rho_{1}=\frac{b}{x}>1 ; x=\frac{1}{\rho_{1}\left(\rho_{1}-1\right)} \\
R_{b}=R_{0}\left(2 \rho_{1}-1\right) \\
R_{i s}=R_{1} \frac{2 \rho_{1}-1}{2\left(\rho_{1}-1\right)}
\end{gathered}
$$

When $x \rightarrow \infty(b \rightarrow x) \rho_{1} \rightarrow 1, R_{b} \rightarrow R_{0}$ and $R_{i s} \rightarrow \infty$. In this situation $\varepsilon_{1}=\rho_{1}-1$ is close to $1 / x$. It is the experimentally achievable accuracy in $\varepsilon_{1}$ that limits the maximum reliably measurable $R_{i s}$. If e.g. the minimum reliably measurable $\varepsilon_{1}$ is estimated to be 0.05 then the maximum measurable $x=R_{i s} / R_{b}$ is $\sim 20$.

### 2.2 Next neighbour strips

As in the previous section introduce $\rho_{2}=2 R_{0} / R_{2}$ where $R_{2}$ is the resistance between two next neighbour strips. As shown in Appendix B

$$
\begin{gathered}
\rho_{2}=\frac{(b+1)^{2}}{b(b+2)}>1 ; x=\frac{\left(\sqrt{\rho_{2}}-\sqrt{\rho_{2}-1}\right)^{2}}{\sqrt{\rho_{2}\left(\rho_{2}-1\right)}} \\
R_{b}=R_{0}\left(\sqrt{\rho_{2}}+\sqrt{\left(\rho_{2}-1\right)}\right)^{2}
\end{gathered}
$$

$$
R_{i s}=\frac{R_{2}}{2} \sqrt{\frac{\rho_{2}}{\rho_{2}-1}}
$$

When $x \rightarrow \infty(b \rightarrow x) \rho_{2} \rightarrow 1, R_{b} \rightarrow R_{0}$ and $R_{i s} \rightarrow \infty$. In this situation $\varepsilon_{2}=\rho_{2}-1$ is close to $1 / x^{2}$. Again the accuracy in $\varepsilon_{2}$ limits the maximum reliably measurable $R_{i s}$. For minimum reliably measurable $\varepsilon_{2}=0.05$ the maximum measurable $x=R_{i S} / R_{b}$ is 2.8 i.e. $\sim 7$ times smaller than for the same accuracy in $\varepsilon_{1}$. Thus the $R_{i s}$ reconstruction ability for measurements with next neighbours is significantly worse than that for the adjacent strips.

Fig. 2 shows the $\varepsilon_{1}$ and $\varepsilon_{2}$ as a function of $x$. The lines are $1 / x$ and $1 / x^{2}$ dependences.


Fig.2. Deviation of $\rho_{1}$ and $\rho_{2}$ from unity vs. $x$ (see text for further details)

Sometimes a more pragmatic approach is used when only "pad-pad" conductivity is measured as a function of bias and the voltage, above which this conductivity exceeds its plateau value by less than some fraction (e.g. $\delta<10 \%$ ), is considered to be the "strip isolation" voltage. Note that without any additional information this approach doesn't tell quantitatively how good the isolation is. If a usual assumption is made that the plateau value is equal to $1 /\left(2 R_{0}\right)$ then for the adjacent strips $\delta$ coincides with $\varepsilon_{1}$ and the plot in Fig. 2 (or corresponding formula) allow an estimate of the lower limit for $x$
(e.g. for $\delta<0.1 x>9$ ). The pragmatism here is in assuming the pad-pad conductivity plateau value to be one half of the conductivity between the pad and the bias rail without actually measuring the latter.

## 3. Induced Current Method

When potential $V_{0}$ is applied between a strip and the bias rail it induces a current flowing via $R_{i s}$ to the neighbouring strips. This current can be measured either directly or indirectly by connecting an ammeter or voltmeter parallel to the bias resistor, $R_{b}$, of the investigated strip. To work well the first approach requires the ammeter internal resistance to be much smaller than $R_{b}$ while the second one requires the voltmeter internal resistance to be much larger than $R_{b}$. The latter condition can be easily satisfied because for modern voltmeters $R_{\text {int }} \sim 10 \mathrm{G} \Omega$. For the ammeters however an ability to measure the currents below nA level is usually accompanied by an internal resistance being of an order of $\mathrm{M} \Omega$. Thus the applicability range of the current measurement approach is narrower than that of the voltage one and because of this the former will not be considered further in this Note.

### 3.1 Neighbour strips

Typical approach is to connect between a strip and the bias rail a source-meter unit (SMU) providing potential $V_{0}$ varying by a few volts around zero and to measure the current flowing out of the SMU, which allows $R_{0}$ measurement. Simultaneously a voltmeter connected between a neighbour strip and the bias rail measures the potential $V_{1}$ as a function of $V_{0}$. Ideally the induced potential $V_{1}$ should be simply proportional to $V_{0}$ and their ratio would characterise the inter-strip isolation. In practice however the slope $S_{1}=\mathrm{d} V_{1} / \mathrm{d} V_{0}$ is used instead of $V_{1} / V_{0}$.

As shown in Appendix C the slope dependence on $x$ is very simply expressed via $b$ :

$$
S_{1}=\frac{1}{1+b}
$$

For $x \rightarrow 0(b \rightarrow 0) S_{1} \rightarrow 1$ while for $x \rightarrow \infty(b \rightarrow x) S_{1} \rightarrow 1 / x \rightarrow 0$. Experimentally measured parameters $R_{0}$ and $S_{1}$ allow reconstruction of the parameters in question: $R_{b}$ and $R_{i s}$. As demonstrated in Appendix C

$$
\begin{gathered}
R_{b}=R_{0} \frac{1+S_{1}}{1-S_{1}} \\
R_{i s}=R_{0}\left(\frac{1}{S_{1}}-S_{1}\right) .
\end{gathered}
$$

In a typical situation when $x \rightarrow \infty, S_{1} \rightarrow 0$ one gets $R_{b} \rightarrow R_{0}$ and $R_{i s} \rightarrow R_{0} / S_{1}=R_{0}\left(\mathrm{~d} V_{0} / \mathrm{d} V_{1}\right)$. A minimum detectable slope $S_{1}$ defines the maximum measurable $R_{i s}$. For a proven to be detectable $S_{1}$ of $\sim 10^{-6}$ the limit for $R_{i s}$ is $\sim 10^{6} R_{0}$, which for a typical $R_{0} \sim 1 \mathrm{M} \Omega$ corresponds to $R_{i s} \sim 1000 \mathrm{G} \Omega$.

### 3.2 Next neighbour strips

As shown in Appendix C the slope of the voltage induced on the next neighbour strip $S_{2}=\mathrm{d} V_{2} / \mathrm{d} V_{0}$ is related to $S_{1}$ in a very simple way: $S_{2}=S_{1}{ }^{2}$. Therefore the reconstruction formulae become

$$
\begin{gathered}
R_{b}=R_{0} \frac{1+\sqrt{S_{2}}}{1-\sqrt{S_{2}}} \\
R_{i s}=R_{0}\left(\frac{1}{\sqrt{S_{2}}}-\sqrt{S_{2}}\right) .
\end{gathered}
$$

For $x \rightarrow \infty S_{2} \rightarrow 0$ as $1 / x^{2}$. Therefore for the same limit of measurable $S_{2} \sim 10^{-6}$ the limit for the $R_{i s}$ is $\sim 10^{3} R_{0}$, which for a typical $R_{0} \sim 1 \mathrm{M} \Omega$ corresponds to $R_{i s} \sim 1 \mathrm{G} \Omega$.


Fig.3. $S_{1}$ and $S_{2}$ vs. $x$.

Fig. 3 shows $S_{1}$ and $S_{2}$ as a function of $x$ together with $1 / x$ and $1 / x^{2}$ lines. Similarly to the Resistance Method the measurements with neighbour strip have higher sensitivity compared to that for the next neighbour strip. But even for the latter the sensitivity is much higher than what can be achieved by the Resistance Method.
3.3 The effects of non-zero resistance to the ground Another limit for the maximum measurable $R_{i s}$ appears from a non-zero resistance to the ground $R_{g}$. As demonstrated in Appendix C in a typical situation $R_{g} \ll R_{b} \ll R_{i s}$ the $R_{i s}$ calculated from the measured $S_{1}$ represents the actual $R_{i s}$ in parallel with the effective parasitic resistance $R_{p}=R_{b}{ }^{2} / R_{g}$. For a typical $R_{0} \sim 1 \mathrm{M} \Omega$ even $R_{g} \sim 1 \Omega$ results in $R_{p} \sim 1000 \mathrm{G} \Omega$, which sets a practical limit for the sensitivity of the induced voltage method.

As explained in Appendix C a non-zero $R_{g}$ results in an offset to the slope, which is the same for the neighbour and the next neighbour strips. Comparison of $S_{1}$ and $S_{2}$ measured under the same conditions allows decoupling of the effects related to $R_{i s}$ and $R_{g}$. An example of such analysis is presented in Fig.4.

The measurements were performed at room temperature and $\sim 40 \%$ relative humidity with non-irradiated $n$-in- $p$ microstrip sensor w27-bz1-p7 produced by Hamamatsu within the ATLAS Tracker Upgrade R\&D Program. The sensor has 104 strips with a pitch of $74.5 \mu \mathrm{~m}$ and a length of 8 mm . There is no p-spray or p-stop interstrip isolation in this sensor, which results in a relatively low $R_{i s}$ values even at quite high bias values. The sensor depletion voltage is 153 V . The bias voltage was changing downwards from 300 V after the sensor was kept at this bias for $\sim 3$ hours. Three consecutive strips 59,60 and 61 were used. At each bias value two separate $V_{0}$ scans were performed with the SMU connected either to strip 60 or 59 (with the connection to the other of these two strips floating) and the potential induced at the strip 61 was measured. In this way both slopes $S_{1}$ and $S_{2}$ were measured for each bias point.

Fig.4a shows $S_{1}$ and $S_{2}$ as a function of bias voltage. As expected $S_{2}$ is usually lower than $S_{1}$ but at $\mathrm{U}_{\text {bias }} \geq 200 \mathrm{~V}$ both slopes are very close and do not change with bias. This indicates that in this bias range both slopes are dominated by the $R_{g}$ contribution. For
comparison $S_{1}{ }^{2}$ is also shown outside the high bias range. This curve is in a reasonable agreement with $S_{2}$.


Fig.4a. Bias dependence of $S_{1}$ and $S_{2}$

The constant slope level observed at high bias was subtracted from $S_{1}, S_{2}$ and the $R_{i s}$ and $R_{b}$ were calculated using these corrected slopes. The results are presented in Figs. 4 b and 4 c respectively.


Fig.4b. The interstrip resistance calculated from the corrected $S_{1}$ and $S_{2}$

The $R_{i s}$ reconstructed from $S_{1}$ and $S_{2}$ agree quite well. Better sensitivity of the measurement with the neighbour strip allows a wider bias range of measurable $R_{i s}$.


Fig.4c. Measured, $R_{0}$, and reconstructed, $R_{b}$, resistances

At low bias the measured resistance $R_{0}$ is significantly lower than its plateau level corresponding to the bias resistor value. However the reconstructed $R_{b}$ remains approximately constant down to the lowest bias point. It is interesting that the bias resistance reconstruction works better for the next neighbour strip data.

Consistency between the results obtained from the measurements with neighbour and next neighbour strips validates the model used in the calculations.

## Appendix A. Basic relations details

Looking at the circuit presented in Fig. 1 one may notice that $R_{e q}$ can be presented as $R_{i s}$ plus $R_{b}$ parallel to $R_{e q}$ that results in the following equation ${ }^{*}$

$$
\begin{equation*}
R_{e q}=R_{i s}+\frac{R_{b} R_{e q}}{R_{b}+R_{e q}} \tag{A.1}
\end{equation*}
$$

Using parameters $x=R_{i s} / R_{b}$ and $b=R_{e q} / R_{b}$ the eq. (A.1) can be re-written as

$$
\begin{array}{ll} 
& b=x+\frac{b}{1+b} \\
\text { or } & b^{2}-x b-x=0 \\
\text { from which it follows: } & x=\frac{b^{2}}{1+b} \\
\text { and } & b=\frac{x+\sqrt{x^{2}+4 x}}{2} .
\end{array}
$$

The last is the result of solving eq. (A.3) vs. $b$ and keeping only the positive solution.

The resistance $R_{0}$ between a strip and the bias rail consists of three resistors in parallel: bias resistor $R_{b}$ and two $R_{e q}$ i.e.

$$
\begin{equation*}
\frac{1}{R_{0}}=\frac{1}{R_{b}}+\frac{2}{R_{e q}} \tag{A.6}
\end{equation*}
$$

Using $R_{e q}=b R_{b}$ one gets from eq. (A.6)

$$
\begin{equation*}
R_{0}=R_{b} \frac{b}{b+2} \tag{A.7}
\end{equation*}
$$

## Appendix B. Resistance method calculations

a) Adjacent strips

An equivalent circuit diagram for measuring resistance $R_{1}$ between two adjacent strips is shown in Fig.B1. $R_{1}$ is the resistance between the points A and B and can be expressed as follows

$$
\begin{equation*}
\frac{1}{R_{1}}=\frac{1}{R_{i s}}+\frac{R_{b}+R_{e q}}{2 R_{b} R_{e q}} \tag{B.1}
\end{equation*}
$$

[^0]

Fig.B1. Equivalent circuit diagram for measuring $R_{1}$

Using parameters $x=R_{i s} / R_{b}$ and $b=R_{e q} / R_{b}$ the eq. (B.1) can be re-written as

$$
\begin{equation*}
\frac{R_{i s}}{R_{1}}=1+\frac{x(b+1)}{2 b} \tag{B.2}
\end{equation*}
$$

Expressing $x$ via $b$ using eq.(A.4) one gets from (B.2)

$$
\begin{equation*}
R_{1}=R_{i s} \frac{2}{b+2} \tag{B.3}
\end{equation*}
$$

Introduce parameter $\rho_{1}=2 R_{0} / R_{1}$. Using eqs. (A.7) and (B.3) one obtains

$$
\begin{equation*}
\rho_{1}=\frac{b R_{b}}{R_{i s}}=\frac{b}{x} \tag{B.4}
\end{equation*}
$$

Expressing $b$ via $x$ from eq.(A.5) and finding x from the resulting equation one gets

$$
\begin{equation*}
x=\frac{1}{\rho_{1}\left(\rho_{1}-1\right)} \tag{B.5}
\end{equation*}
$$

Combining eqs. (B.4) and (B.5) one can express $b$ vs. $\rho_{1}$ :

$$
\begin{equation*}
b=\frac{1}{\rho_{1}-1} \tag{B.6}
\end{equation*}
$$

Using eq. (B.6) one obtains from (A.7)

$$
\begin{equation*}
R_{b}=R_{0}\left(2 \rho_{1}-1\right) \tag{B.7}
\end{equation*}
$$

and from (B.3)

$$
\begin{equation*}
R_{i s}=R_{1} \frac{2 \rho_{1}-1}{2\left(\rho_{1}-1\right)} \tag{B.8}
\end{equation*}
$$

b) Next-neighbour strips

An equivalent circuit diagram for measuring resistance $R_{2}$ between two nextneighbour strips is shown in Fig.B2. Due to the symmetry there is no potential difference between the ends of the central bias resistor. Therefore it can be either removed or replaced by a short.


Fig.B2. Equivalent circuit diagram for measuring $R_{2}$

In both cases the resistance $R_{2}$ between the points A and B can be expressed as:

$$
\begin{equation*}
\frac{1}{R_{2}}=\frac{1}{2}\left(\frac{1}{R_{i s}}+\frac{1}{R_{b}}+\frac{1}{R_{e q}}\right) \tag{B.9}
\end{equation*}
$$

From (B.9) the parameter $\rho_{2}=2 R_{0} / R_{2}$ can be expressed as (using also eq. (A.7)):

$$
\begin{equation*}
\rho_{2}=\frac{R_{0}}{R_{b}}\left(1+\frac{R_{b}}{R_{i s}}+\frac{R_{b}}{R_{e q}}\right)=\frac{b}{b+2}\left(1+\frac{1}{x}+\frac{1}{b}\right) \tag{B.10}
\end{equation*}
$$

Using for $x$ its form of eq. (A.4) one finally obtains:

$$
\begin{equation*}
\rho_{2}=\frac{(b+1)^{2}}{b(b+2)} \tag{B.11}
\end{equation*}
$$

As follows from (B.11)

$$
\begin{equation*}
\rho_{2}-1=\frac{1}{b(b+2)} ; \frac{\rho_{2}}{\rho_{2}-1}=(b+1)^{2} ; b+1=\frac{\sqrt{\rho_{2}}}{\sqrt{\rho_{2}-1}} \tag{B.12}
\end{equation*}
$$

The last part can be transformed into

$$
\begin{equation*}
b^{2}=\frac{\left(\sqrt{\rho_{2}}-\sqrt{\rho_{2}-1}\right)^{2}}{\rho_{2}-1} \tag{B.13}
\end{equation*}
$$

Substituting in eq. (A.4) ( $b+1$ ) from (B.12) and $b^{2}$ from (B.13) one obtains

$$
\begin{equation*}
x=\frac{\left(\sqrt{\rho_{2}}-\sqrt{\rho_{2}-1}\right)^{2}}{\sqrt{\rho_{2}\left(\rho_{2}-1\right)}} \tag{B.14}
\end{equation*}
$$

To express $R_{b}$ via $R_{0}$ and $\rho_{2}$ one can re-write eq. (A.7)

$$
\begin{equation*}
R_{b}=R_{0} \frac{b+2}{b}=R_{0} \frac{b(b+2)}{b^{2}} \tag{B.15}
\end{equation*}
$$

Substituting $b(b+2)$ by $1 /\left(\rho_{2}-1\right)$ from eq. (B.12) and using $b^{2}$ from (B.13) one gets

$$
\begin{equation*}
R_{b}=\frac{R_{0}}{\left(\sqrt{\rho_{2}}-\sqrt{\rho_{2}-1}\right)^{2}}=R_{0}\left(\sqrt{\rho_{2}}+\sqrt{\rho_{2}-1}\right)^{2} \tag{B.16}
\end{equation*}
$$

To find $R_{i s}$ one can transform (B.16) as follows

$$
\begin{equation*}
R_{i s}=x R_{b}=\frac{x R_{0}}{\left(\sqrt{\rho_{2}}-\sqrt{\rho_{2}-1}\right)^{2}} \tag{B.17}
\end{equation*}
$$

Using the relation $R_{0}=\left(\rho_{2} R_{2}\right) / 2$ following from the $\rho_{2}$ definition and $x$ from (B.14) one obtains from (B.17)

$$
\begin{equation*}
R_{i s}=\frac{R_{2}}{2} \sqrt{\frac{\rho_{2}}{\rho_{2}-1}} \tag{B.18}
\end{equation*}
$$

## Appendix C. Induced voltage calculations

An equivalent circuit diagram for the $R_{i s}$ measurement using the voltage induced at the neighbour strip is presented in Fig.C1. The SMU is connected between the point marked $V_{0}$ and the ground while the induced voltage is measured between the point marked $V_{1}$ and the ground.


Fig.C1. Equivalent circuit diagram for measurement with the neighbour strip First let us consider a situation when the resistance to the ground, $\mathrm{R}_{\mathrm{g}}$, is zero.

## a) Zero $R_{g}$.

As follows from the diagram in Fig.C1 the induced voltage V1 can be expressed via applied voltage V0 as

$$
\begin{equation*}
V_{1}=V_{0} \frac{\frac{R_{b} R_{e q}}{R_{b}+R_{e q}}}{R_{i s}+\frac{R_{b} R_{e q}}{R_{b}+R_{e q}}}=V_{0} \frac{R_{b} R_{e q}}{R_{i s}\left(R_{b}+R_{e q}\right)+R_{b} R_{e q}} \tag{C.1}
\end{equation*}
$$

Using the relations $R_{i s}=x R_{b}, R_{e q}=b R_{b}$ and the eq. (A.4) one finds from (C.1)

$$
\begin{equation*}
S_{1}=\frac{V_{1}}{V_{0}}=\frac{1}{1+b} \tag{C.2}
\end{equation*}
$$

Expressing from (C.2) $b$ via $S_{1}$ one gets

$$
\begin{equation*}
b=\frac{1-S_{1}}{S_{1}} . \tag{C.3}
\end{equation*}
$$

Substituting $b$ in eq. (A.7) by its expression from (C.3) one obtains

$$
\begin{equation*}
R_{b}=R_{0} \frac{1+S_{1}}{1-S_{1}} \tag{C.4}
\end{equation*}
$$

Substituting $b$ in eq. (A.4) by its expression from (C.3) one obtains

$$
\begin{equation*}
x=\frac{\left(1-S_{1}\right)^{2}}{S_{1}} . \tag{C.5}
\end{equation*}
$$

Using the relation $R_{i s}=x R_{b}$ and eqs. (C.4), (C.5) one gets

$$
\begin{equation*}
R_{i s}=R_{0}\left(\frac{1}{S_{1}}-S_{1}\right) \tag{C.6}
\end{equation*}
$$

Typically the parameter $R_{0}$ is measured with a good accuracy while $S_{1}$ (especially when it is very small) has a significant relative error $\sigma\left(S_{1}\right) / S_{1}$. Using (C.6) one can calculate the uncertainty in $R_{i s}$ due to the error in $S_{1}$

$$
\begin{equation*}
\sigma\left(R_{i s}\right)=R_{0} \frac{\sigma\left(S_{1}\right)}{S_{1}}\left(\frac{1}{S_{1}}+S_{1}\right) \tag{C.7}
\end{equation*}
$$

For measurement with the next neighbour strip the diagram shown in Fig.C1 can also be used but using potential $V_{1}$ instead of $V_{0}$ and $V_{2}$ instead of $V_{1}$. Obviously one gets in this case

$$
\begin{equation*}
\frac{V_{2}}{V_{1}}=\frac{V_{1}}{V_{0}} \rightarrow S_{2}=\frac{V_{2}}{V_{0}}=\left(\frac{V_{1}}{V_{0}}\right)^{2}=S_{1}^{2} \rightarrow S_{1}=\sqrt{S_{2}} . \tag{C.8}
\end{equation*}
$$

Therefore for the measurements with next neighbour strips the eqs. (C.4) and (C.6) can be written as

$$
\begin{align*}
& R_{b}=R_{0} \frac{1+\sqrt{S_{2}}}{1-\sqrt{S_{2}}}  \tag{C.9}\\
& R_{i s}=R_{0}\left(\frac{1}{\sqrt{S_{2}}}-\sqrt{S_{2}}\right) \tag{C.10}
\end{align*}
$$

while the eq. (C.7) is transformed to

$$
\begin{equation*}
\sigma\left(R_{i s}\right)=\frac{R_{0}}{2} \frac{\sigma\left(S_{2}\right)}{S_{2}}\left(\frac{1}{\sqrt{S_{2}}}+\sqrt{S_{2}}\right) \tag{C.11}
\end{equation*}
$$

## b) Effects of non-zero $R_{g}$.

As can be seen from the circuit diagram presented in Fig.C1 a non-zero resistor $R_{g}$ results in a voltage drop on it

$$
\begin{equation*}
V_{g}=V_{0} \frac{R_{g}}{R_{0}+R_{g}}, \tag{C.12}
\end{equation*}
$$

which adds up to $V_{1}$ or $V_{2}$ that would be measured with $R_{g}=0$. In other words the measured slopes $S_{1}, S_{2}$ will include an additional component

$$
\begin{equation*}
S_{g}=\frac{V_{g}}{V_{0}}=\frac{R_{g}}{R_{0}+R_{g}}, \tag{C.13}
\end{equation*}
$$

which is the same for neighbour and the next neighbour strips. The $S_{g}$ may be measured e.g. as the slope in the situation $S_{1}=S_{2}$. It then can be subtracted from the slopes measured under other conditions thus suppressing the effects from non-zero $R_{g}$.

To verify this model a special measurement was made with $1 \mathrm{k} \Omega$ resistor inserted between the bias rail and the ground. The measurements were performed with an SCT End-Cap sensor w31-225 at $\mathrm{U}_{\text {bias }}=50 \mathrm{~V}$. Fig.C2 summarises the results.

With grounded bias rail the slope $\mathrm{d} V_{1} / \mathrm{d} V_{0}$ of the voltage induced at the neighbour strip was found to be $4.7 \pm 0.2 \mu \mathrm{~V} / \mathrm{V}$, as can be seen from the experimental data in Fig.C2a.


Fig.C2a. $V_{1}$ vs. $V_{0}$ for grounded bias rail

When $1.0 \mathrm{k} \Omega$ resistor was inserted between the bias rail and the ground, the slope increased to $869 \pm 5 \mu$ V/V as shown in Fig.C 2 b .


Fig.C2b. $V_{1}$ vs. $V_{0}$ for $1 \mathrm{k} \Omega$ resistor between the bias rail and the ground

In the first case $R_{0}$ was measured to be $1156 \mathrm{k} \Omega$ while in the second it increased to $1157 \mathrm{k} \Omega$ due to the additional $1 \mathrm{k} \Omega$ resistor. The slope $S_{1}$ due to $1 \mathrm{k} \Omega$ resistor calculated from the eq. (C.13) is $S_{g}=1 \mathrm{k} \Omega / 1157 \mathrm{k} \Omega=864.310^{-6}=864.3 \mu \mathrm{~V} / \mathrm{V}$. Adding it to the initial slope $S_{1}=4.7 \mu \mathrm{~V} / \mathrm{V}$ one obtains for the second measurement $869 \mu \mathrm{~V} / \mathrm{V}$ in a perfect agreement with the results presented in Fig.C2b that validates the model.

For further discussion let's restrict ourselves to a typical in practice situation $R_{g} \ll R_{0}$. Then the usual gradient $\mathrm{d} V_{0} / \mathrm{d} I_{0}$ still correctly measures $R_{0}$ and the additional slope $S_{g}=R_{g} / R_{0} \ll 1$. Let us now consider only the situation when the real slopes $S_{1}, S_{2}$ are comparable with $S_{g}$ and therefore the effects of $R_{g}$ are essential. The measured slopes $S_{1}{ }^{\text {meas }}, S_{2}{ }^{\text {meas }}$ are then also $\ll 1$. For the measurements with the neighbour strip we then obtain from eq. (C.6) for the measured interstrip resistance

$$
\begin{equation*}
\frac{1}{R_{i s}^{\text {meas }}}=\frac{1}{R_{0}} S_{1}^{\text {meas }}=\frac{1}{R_{0}}\left(S_{1}+S_{g}\right)=\frac{1}{R_{i s}}+\frac{R_{g}}{R_{0}^{2}} . \tag{C.14}
\end{equation*}
$$

As follows from (C.14) the measured $R_{i s}$ is equal to the actual $R_{i s}$ with an effective parasitic resistance $R_{p}=R_{0}{ }^{2} / R_{g}$ connected in parallel and restricting the measurable $R_{i s}$. For typical values of $R_{0}=1 \mathrm{M} \Omega$ and $R_{g}=1 \Omega R_{p}=1000 \mathrm{G} \Omega$.

For the measurements with next neighbour strip one can obtain from eq. (C.10)

$$
\begin{equation*}
\frac{1}{\left(R_{i s}^{\text {meas }}\right)^{2}}=\frac{1}{R_{0}^{2}} S_{2}^{\text {meas }}=\frac{1}{R_{0}^{2}}\left(S_{2}+S_{g}\right)=\frac{1}{R_{i s}^{2}}+\frac{R_{g}}{R_{0}^{3}} . \tag{C.15}
\end{equation*}
$$

Thus for the next neighbour the measured $R_{i s}$ is limited by an effective parasitic resistance $R_{p}=R_{0} \sqrt{\frac{R_{0}}{R_{g}}}$. For typical values of $R_{0}=1 \mathrm{M} \Omega$ and $R_{g}=1 \Omega R_{p}=1 \mathrm{G} \Omega$.


[^0]:    * I am indebted to Nobu Unno for the idea of this calculation. - A.C.

