Elsevier Editorial System(tm) for Coastal Engineering
Manuscript Draft

Manuscript Number: CENG-D-07-00160R1

Title: Non-Linear Transfer Function Modelling of Beach Morphology at Duck, North Carolina

Article Type: Research Paper

Section/Category:

Keywords: Beach; Volumes; Duck; Nonlinear Transfer Functions; Forecasting

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Manuscript Region of Origin:

Abstract: This paper presents a simple nonlinear data-based modelling approach for predicting the beach profile volume at Duck, North Carolina, USA. The state-dependent parameter form of the general transfer function (SDP TF) model is used to describe nonlinearity influencing these morphological data in two case examples. Case 1 investigates the nonlinearity associated with the dependency of wave forcing on the preceding beach volume. Case 2 investigates the ability to model the variables within the well known diffusion equation for beach volume using this data-based approach. The results of this study show that the SDP TF approach can be used successfully to develop statistically robust models for describing nonlinearity in beach morphological systems. Furthermore, these models are shown to predict the beach volumes over both short (1 month ahead) and long (2 years ahead) time periods, and thus show great potential for practical applications in coastal zone management and engineering.

Revision Notes for: CENG-D-07-00160

"Non-Linear Transfer Function Modelling of Beach Morphology at Duck, North Carolina" submitted by Y. Gunawardena, S. Ilic, H. N. Southgate and R. Romanowicz

Dear Prof. Burcharth

We thank you and the two reviewers for their constructive comments and suggestions. We have fully taken on board all suggested revisions, we believe these will further improve the manuscript. The following addresses all comments made by the two reviewers in detail. Please note that the revised figures are enclosed within the manuscript as these were generated using Matlab^{TM}, and hence could not be saved separately as image files.

Best regards

yohama

Comments from "Reviewer 1"

Specific Comments

C1. "C*larify position of profiles along coast (including position of Pier).*"

We welcome the suggestion of the reviewer. This has been clarified by including a location map (Figure 1) of the beach profiles (Profiles 58, 62, 188 and 190) and the FRF pier at Duck.

C2. *"Clear explanation of the 'volume' discussed in the paper"*

We agree with the reviewer and apologise for the confusion. We have clarified that the beach profile volume per unit metre of shoreline were computed by integrating the beach profile data between 75 m and 700 m, which extend well beyond the average position of the depth of closure at Duck (latter is at the 4 m depth at ~410 m cross-shore) to the 7 m depth (page 10).

C3. "*Applying Eq.(6) and a straight and parallel coastline is present, no interaction between the profiles should occur. That calls for equal 'volumes' in the 3 profiles; but the 'volume' depends [see also ii)] e.g. on the position of the reference line. Please clarify this point*."

Here, is our explanation for this comment, which is summarised in the revised paper (page 10). Although the bathymetric contours at Duck are parallel at times, this is not always the case (Miller et al., 1983; Plant et al., 1996; Miller and Dean, 2007) because of longshore differences in the morphology of profiles on either side of the pier. As seen in Figure 2 of the revised manuscript, alongshore volume differences between northerly and southerly profiles occur, particularly post 1991. Reversed migration directions of the nearshore sand bars on either side of the FRF pier (Plant et al., 1999), longshore rhythmic sand bars and sand waves (Lippmann and Holman, 1990; Plant and Holman, 1996; Miller and Dean, 2007a,b), and extended periods of shore-oblique waves generating unidirectional longshore sediment transport (Miller et al., 1983; Keen et al., 2003) have been observed to contribute to the alongshore variability at Duck. Furthermore, as the beach profile data used in the present study extend beyond the average position of the depth of closure, gains/losses in beach volume (Figure 2) are expected to be predominantly influenced by longshore processes as cross-shore processes would mainly influence the redistribution of sediment within the profile. This is further verified in Case 1, where it is found that the volumes are best correlated with the longshore component of the wave energy flux. Hence, the application of Equation 8 in the revised manuscript (i.e. Equation 6 in the original manuscript) to model the volume interaction between profiles (Case 2) is valid.

C4. "*… make clear what is meant with the 'normalised wave energy (U)'*."

We welcome the suggestion of the reviewer and apologise for not having defined this earlier. U in the original manuscript was a normalised wave energy term (given as $U = H^2 \times sin\theta$ for conditions when θ was larger and smaller than 0^o relative to the FRF pier; and $U = H^2$ for $\theta=0^o$, i.e. shore-normal wave directions) that was used in the companion paper (Gunawardena et al., 2008). This variable was used (by Gunawardena et al. (2008)) as the wave forcing term or TF model input to develop a linear TF model for predicting the beach volume; the results of which were compared with those of the nonlinear SDP TF model (which also used U) in Case 1 of the original manuscript. However, following recommendations made by the reviewers to the companion paper (i.e. Gunawardena et al., 2008), further tests were carried out to investigate the linear TF relation between other wave forcing variables (e.g. monthly average cross-shore and longshore components of the wave energy flux [as defined in Komar (1998)], as well as the sum of these 2 components) and the beach volumes. As expected, these tests showed that the beach volume was best correlated to the monthly average longshore component of the wave energy flux. This variable was therefore used in linear TF models presented in the companion paper. Hence, for consistency of the comparison with these linear TF model results, the latter wave forcing variable (defined as P in the revised manuscript) was also used for the SDP TF modelling in Case 1 of the present study. In the revised manuscript, we clarify that the monthly average alongshore component of the wave energy flux (P) is defined by (as in Komar (1998)):

$$P = \left(\frac{1}{8}\rho g H_b^2\right) C_g \sin\theta_b \cos\theta_b$$

where: ρ is density of water; g is acceleration due to gravity; H_b, θ_b, C_g are the significant wave height, wave approach angle and wave group velocity under breaking-wave conditions, which were computed using a linear wave transformation model (Gunawardena, 2008). The values of P (units of m3/s) computed are normalised by the ${}^{1}\!/_{8}\,\rho g$ term (pages 10 and 11).

<u>Minor Comments on Text</u>

The reviewer recommended improvements to the text in some places. These recommendations were pointed out in a copy of the original manuscript, which was posted to us. In general, we agree with these minor comments and have revised the manuscript's text accordingly. These include:

C1. "*Page 1 in title: "Non-linear …" should be Nonlinear*"

The text has been revised such that "nonlinear…" is used throughout.

C2. "*Page 1 in Abstract: meaning of SDP TF*"

The text was slightly modified to define the meaning of SDP TF (i.e. state-dependent parameter transfer function).

C3. "*Page 11, line 21: '… (erosion or accretion) …' is the wrong indication: better: 'eroded or accreted', as for e.g. erosion takes place during 1994-1999. In that period the SDP are far from constant.*"

We agree with the reviewer, and have taken this into consideration while interpreting the new results of Case 1 (i.e. the results of Case 1 are different to those in the original manuscript due to the use of the monthly average longshore component of the wave energy flux *P*) on pages 12 to 14.

C4. *"Page 12: Lines 22-23: 'The SDPs estimated were then used to calculate the effective proportion of the wave forcing input, which directly influences the beach profile volume.' Quite unclear what happens here, is the effective input = wave forcing input x SDP?"*

We apologise that this was not clear. In the revised manuscript, we clarify this point, i.e. effective input = original input variable × SDP (page 8).

C5. "*Page 13: Typo:* $\lfloor f\{V_{k-1}\}\cdot U \rfloor$ *should be* $\lfloor f\{V_{k-1}\}\cdot U_k \rfloor$"

This term has been corrected accordingly to $\lfloor f\{V_{k-1}\}\cdot P_k \rfloor$ as shown in page 13.

C6. "*Page 15: 'That is, in the case of the linear model, the beach volume lags behind the wave forcing input U (original data) by 8 months.' Is this discussed in companion paper?*"

Yes, the time-lags identified using the linear TF modelling approach are discussed in detail in this companion paper (Gunawardena et al., 2008) as well as in the lead author's PhD thesis (Gunawardena, 2008), which has now been examined and approved.

C7. "*In Fig. 6, the forecast U data have been used. What happens if real U data are used? Can probably be found in Figure 4 but at quite a different scale.*"

We have modified Figure 8 of the revised manuscript (Figure 4 in the original manuscript) to now compare the beach volumes forecast using the SDP TF model identified for Case 1 and the linear TF model in Gunawardena et al. (2008) using forecast data for P, as well as the model validation of the SDP TF model using known P data for (2-year long period) 2001-2003. As discussed on page 16, although both linear and nonlinear models capture the overall long-term trend in the data well, the nonlinear model performs best as it forecasts some of the shorter-term variability in the volumes (like in the case of the model validation) and consequently has smaller RMS errors (Table 1).

C8. "pg 23: *Define* α_* *in Equation 14*"

We apologise for this. We now clarify on page 22 that α_* is the wave approach angle in the water depth, h_* (= 8 m depth).

C9. "pg 23: *Values of s = 2 are too small, suggest a value of 2.65*"

We changed the value of the sediment specific gravity (*s*) to 2.65 as suggested by the reviewer. However, this does not significantly change the values of the longshore sediment diffusivity term, *G* or the TF model results.

C10. "*pg 25: typo: 'that' should be 'than'*"

This was corrected.

C11. "*pg 27: In validation period (5 years), the volume on Profile 62 is estimated with the help of measured volumes on Profiles 58 and 188. So the method can be used to fill out some missing data in one profile using actual data from adjacent profiles*".

We appreciate this suggestion by the reviewer and have included it on page 28 using the following text: "The model could also be used for data-processing to fill gaps in volume data of a given profile by considering those of adjacent profiles".

C12. "*Typos in Reference list"*

These have all been corrected.

Comments from "Reviewer 2"

General Comment:

C1. "*The paper is potentially useful as a description of a new application of a well established model-identification approach. It is mainly clear but wordy and too reiterative, and would benefit from shortening. Examples are p.25, the last few lines of p.26 and first few of p.27, and the first few of p.28. I recommend revision of the whole paper, aiming for conciseness.*"

We welcome the suggestion of the reviewer and have edited the text in the above specified pages and lines, as well as the rest of the document, to avoid repetition and make the revised manuscript shorter and more concise. Despite the added explanations included in response to both reviewers' comments/suggestions, the word count of the revised manuscript is reduced to 7,925 (compared to the 8,180 word count of the original manuscript).

Technical Comments:

C1. (a) "*On p.11, referring to Fig. 1, the SDP (not SDPs) is said to "vary at an approximately constant rate" at beach volumes below about 2250 m^3/m. In fact it is initially flat then goes smoothly negative before returning to near zero. Whether the near-zero value is credible and whether the deviation to about -50 corresponds to a significant amount of non-linearity is not examined. If it does not, it needs to be established whether the subsequent change to about -180 implies significant non-linearity…."*

We welcome the reviewer's comment and suggestion. We apologise for the confusion in our interpretation of the results. In general, the R_T^2 value associated with the SDP relationship reflects on how well-defined this relationship is. The R_T^2 value of 0.3 corresponding to the estimated SDP relationship in Figure 1 of the original manuscript indicated that this nonlinearity was relatively well-defined. The strength of the SDP nonlinearity is reflected by the degree of variation of the SDP gain with respect to the state variable. The SDP relationship in Figure 1 (of the original manuscript) indicated the presence of nonlinearity (as shown by the variation of the SDP gain with volume), and was further interpreted to indicate stronger nonlinearity for profile volumes > 2250 m^3 because the slope of this relationship increased (negatively) for these high volumes showing increased variability of the SDP gain. During lower volume conditions, although some variation in the SDP gain with volume was observed, the results were interpreted to indicate weaker nonlinearity or approximately linear behaviour. Nevertheless, in predicting the beach volumes, the SDP TF model incorporates all conditions of the SDP relationship (whether strongly or mildly nonlinear). This is in fact a key advantage of the SDP TF modelling approach as it enables prediction of the beach response when there is no clear separation between conditions or timescales for linear and nonlinear behaviour.

In Section 3.1 (pages 12-13) the revised manuscript, we have clarified the above on the basis of the new SDP relationship estimated between the longshore component of the wave energy flux (*P*) and beach volume (now Figure 3). The corresponding R_T^2 value of 0.5 indicates that the SDP relationship estimated in the revised paper is well defined. The general variability of the SDP gain over the whole range of volumes considered indicates nonlinearity under all morphological conditions represented by these beach volumes. However, the variability of the SDP gain increases at volumes > 2270 m^3 (as illustrated by the increased negative gradient of the SDP relationship in Figure 3), which suggests that the nonlinearity becomes stronger when the profile volume are high. This is also shown in Figure 4, where the largest magnitudes of the SDP gain correspond to the 1990-1997 period when the beach volume was high (> 2270 m^3). By contrast, the SDP gain values corresponding to the 1985-1990 and 1997-2001 periods, which are associated with lower volumes (< 2270 m^3), are low and show less variation with beach volume in Figure 3. Thus, these results indicate that the strength of the nonlinearity varies depending on the morphological state (increased/reduced volume) of the profile (discussed in detail on pages 12-14).

C1. (b) " *…We do not know at this point what the form of the model is and how U enters it…"*

We appreciate the reviewers comment for clarity in defining the model explored in Case 1, and therefore this is explained in the revised manuscript on page 11. In Case 1, the SDP TF modelling approach was adopted to investigate the state-dependency of the longshore component of the wave energy flux P (i.e. model input) on the preceding beach volume (i.e. state variable). The TF model parameters associated with P at a given time-step were made to vary with time as a function of the beach volume at the previous time-step. The equation below (i.e. Equation 5 in the revised manuscript) shows the SDP TF model form used for this analysis. The SDP relationship illustrated in Figure 3 of the revised manuscript is estimated on the basis of this equation.

$$V_k = \frac{B(V_{k-1}, z^{-1})}{A(z^{-1})} P_{k-\delta}$$

where: V is the monthly beach volume per unit of shoreline (*m^3*) (model output); k is the kth discrete-time sample (*months*); P is the monthly average longshore component of the wave energy flux (*m^3/s*) (model input); δ represents the pure time delay ($\delta\Delta t$ time units) associated with P; $B(V_{k-1}, z^{-1})$ is the numerator TF polynomial comprising the SDP gain that varies with time as a function of V_{k-1}, which is the state variable; $A(z^{-1})$ is the denominator TF polynomial comprising constant parameters.

C1. (c) "*We do not even know what U is. The introduction of U on p. 10 says that "beach volume was best related to ...contributions of the normalised wave energy (U) (defined using...)". That sentence is impossible to construe. Has U got two components? How is U related to wave height and direction? Is U daily, monthly ..?*"
Again, we agree with the reviewer and apologise for any confusion. As discussed previously, the wave forcing term *U* that was used in the original manuscript has been replaced with *P,* which is the monthly average longshore component of the wave energy flux (defined above). The computation of *P* is explained in detail in the revised manuscript on pages 10 and 11.

C2. "*The last complete sentence on p.17 is vague and is contradicted by Fig. 6. Is the comment meant to refer to Fig. 4, and what fluctuations are being referred to?*"
In the revised manuscript we clarify that, although both linear TF and nonlinear SDP TF models predict the long-term volume trend well, the latter model also predict the short-term volume fluctuations occurring about this long-term trend, particularly during periods of high beach volume. This is seen in Figure 6 and 8 of the revised manuscript and is now discussed in detail on pages 13 to 17.

C3. "*Keeping the autoregressive (AR) parameter constant prevents the model from tracking changes in dominant time constant and may instead cause spurious variation* *in the numerator parameter. The erosion and deposition mechanisms discussed suggest variations in speed of response which demand a varying AR parameter. They also suggest differences in dynamics according to the direction of change, not apparently considered in devising the model or interpreting the results.*”

This is a fair comment. However, variation of both the numerator and denominator parameters of the TF model makes the model unstable, resulting in poor estimation of the model parameters. Hence, in the present study, the parameters of only the numerator TF polynomial are made state-dependent and time-varying for representing the input nonlinearities associated with P and beach volume in Case 1, and the volumes of Profiles 58 and 188 and the longshore diffusivity G in Case 2. Nonlinearity in natural systems has been predominantly studied in the past in the form of ‘input nonlinearities’ as it enables reliable estimation of the model parameters (e.g. Lees (2000) and Young (2000)), and for the same reason this approach was adopted in this study. Furthermore, we consider the results obtained using this approach satisfactory because (as discussed in detail on pages 12-13 of the revised manuscript), the estimated SDP gain does in fact effectively describe variation in the speed of the profile’s volume response as well as the direction of volume change (i.e. accretion or erosion). This is illustrated in Figure 4 of the revised manuscript, where it is seen that the time-variation of the magnitude of the SDP gain imitates that of the beach volume. For example, the long-term increase in volume between 1990 - 1994 corresponds to a similar long-term increase in the SDP gain magnitude, while the more rapid decrease in volume between 1995 - 1997 is associated with a similarly rapid reduction in the SDP gain magnitude. In addition, an increase or decrease in the SDP gain magnitude is predominantly associated with increases (accretion) and decreases (erosion), respectively, in beach volume. In the revised paper (page 18), we mention that further research may be carried out to investigate if having state-dependent parameters for the denominator TF polynomial instead alters these results.

C4. “*It is remarkable that the exact antiphase behaviour of the parameters in Figs. 8 and 9 has not been noticed, and more remarkable as it is plain in Fig. 10 that they add to unity at every point in time, to a good approximation. This has implications which should be fully discussed.*”

We appreciate the reviewer’s comment and have expanded the discussion of the SDP gains illustrated in Figures 11, 12 and 13 of the revised manuscript (i.e. Figures 8, 9 and 10 in the original manuscript) to address these issues. The opposing patterns observed in the variability of the SDP gains in Figures 11 and 12 of the revised manuscript characterise the contrasting relationships between the volumes of Profile 62 and those of Profiles 58 and 188. That is, the positive SDP gain associated with Profile 58 indicates that the volumes of this profile are positively related to those of Profile 62 (i.e. volume increases on Profile 62 are linked with volume increases on Profile 58, and vice versa). By contrast, the SDP gain associated with Profile 188 is positive or negative at varied times, the latter which indicates that the volumes of this profile are negatively related with those of Profile 62 (i.e. volume increases on Profile 62 are linked with volume decreases on Profile 188 and vice versa) at particular periods of time. These contrasting relationships are further illustrated in Figure 10, where it is seen that increased volumes on Profiles 58 and 62 during certain periods coincide with significantly reduced volumes on Profile 188, and vice versa (e.g. Dec 1984 – Sep 1986, Dec 1986 – Feb 1987, 1989 – 1990, 1992 – 1997). This alongshore difference in the morphology of northerly and southerly profiles has also been observed by Miller and Dean (2007a) via the analysis of shoreline data at Duck. In the revised manuscript, we also clarify that the estimated SDP gains associated with Profiles 58 and 188 are normalised such that their summation at each k^{th} time-step is 1 while representing weights for relating the volumes of these two profiles to those of Profile 62. In general, the SDP gain associated with Profile 58 has larger magnitudes (closer to 1) than those estimated for Profile 188, indicating that the former has a more dominant influence in characterising the volume of Profile 62. This can be expected as Profile 62 is located at a closer proximity to Profile 58 (~ 100 m apart) than Profile 188 (~ 1000 m apart).

C5. "*We are not told anything about how to interpret the numerical value of the YIC, or why the SDP values are sorted by value of the state variable.*"

We appreciate the suggestion made by the reviewer. A discussion of the YIC and its interpretation are summarised in the revised manuscript on pages 8 and 9. Sorting the SDP values with respect to the state variable is generally done in order to ensure optimal estimation of these parameters using the SDP TF algorithms in CAPTAIN (Young, 1984; 2005; CRESS, 2004). However, since this is a technical detail related to the SDP TF model identification algorithms, it was omitted from the revised paper as we felt it is not essential to mention this.

Minor Comments

C1. "*p.4: how can you have "a ...relationship between a system input and an INDEPENDENT state variable"? Independent of what? Also on p.6, "completely independent" does not make sense.*"

We apologise for any confusion. By 'independent variable' we meant any system variable other than the input or output variables. We clarify this in the revised paper by saying: "… e.g. a state-dependent relationship between the input and any other system variable" (page 4).

C2. "*p.4: "physical dynamics" - are non-physical dynamics possible?*"

We've taken on board the reviewer's comment. The text was modified to state: "… physical mechanisms…" instead (page 4).

C3. "*It would help if you said on p. 6 that subscripts denote time (of regular samples).*"

The definition of *k* to denote the kth discrete-time sample is included in the revised manuscript (page 6).

C4. "*p.6: 3 lines before end, don't use "inputs" when you mean other things (outputs, etc.) too.*"

We agree with the reviewer. The text has been modified accordingly to clarify this. That is: "ξ_k is a noise term that accounts for uncertainty in the model (e.g. due to measurement noise, unmeasured inputs and uncertainty in the model)" (on page 6).

C5. "*2 lines after (3) it seems there are n+m+1 state variables, yet the dynamical order is only n: explain.*"

This was a typo, which has been corrected in the revised paper (pages 6 and 7) as shown below.

$$A\left(v_k, z^{-1}\right) = 1 + a_1(v_k)z^{-1} + a_2(v_k)z^{-2} + ... + a_n(v_k)z^{-n}$$

$$B\left(w_k, z^{-1}\right) = b_0(w_k) + b_1(w_k)z^{-1} + b_2(w_k)z^{-2} + ... + b_m(w_k)z^{-m}$$

where: $z^{-p}, p = 1,2...n(m)$ denotes the 'backward shift' time operator (defined as $z^{-n}y_k = y_{k-n}$); v_k and w_k are *the state variables*; $a_i, i = 1,2,...n$ and $b_j, j = 0,1,...m$

are the time-varying TF model parameters known as the *state-dependent parameters* (SDP), which vary as functions of the state variables. In the two above equations, there are only two state-variables (i.e. v and w) and the orders of the numerator and denominator TF polynomials are *m* and *n*, respectively.

C6. "*p.7, line 4 from end: "more efficient" - than what?*"
We apologise for the confusion. The text has been revised for clarification (on page 7).

C7. "*References to early papers on optimal smoothing (from the 1960's and 1970's) are needed on line 4 of p.8 to avoid the impression that FIS originated in 2000.*"
This comment is taken on board and references to earlier TF literature (Young, 1984; 1985) are included in the revised manuscript (page 8).

C8. "*p.8 middle "Data on" is redundant.*"
The word 'Data' was omitted from the relevant text.

C9. "*p.11, line 3 before sec. 3.1: in what sense is it "statistically optimal"?*"
We now clarify on page 8 that an optimal SDP TF model is identified and estimated on the basis of two statistical criteria namely, the coefficient of determination (R_T^2) and the Young Information Criterion (YIC). Hence, the words "statistically optimal" were omitted from the text.

C10 "*p.13, middle: explain "a [1 1 0] structure".*"
We clarify in the revised manuscript that a TF model comprising a [1 1 0] structure consists of a 1st order numerator TF polynomial, a 1st order denominator TF polynomial and a zero time-delay (page 13, an example of the general TF model structure is also explained on page 7).

C11. "*p.14, lines 3, 4 after (5): explain "a linear ... weights". Are you thinking of transforming the ARMA model into an MA model?*"
We apologise for any confusion. We clarify on page 14 that, Equation 7 in the revised manuscript (i.e. Equation 5 in the original manuscript) shows that successive beach volume differences at Duck are nonlinearly related to the longshore component of the

wave energy flux via time-varying state-dependent parameters, which characterise the nonlinear dependence of P on the preceding beach volume.

C12. "*p.14, middle: is the validation in simulation mode (as it should be), i.e. with V sub k-1 from the model, not measured?*"

Yes, model validation was carried out in simulation mode, where the volumes predicted by the SDP TF model at each time-step were used for the model validation at the next time-step. This is clarified in the revised manuscript (page 14).

C13. "*p.16, line 6 of sec. 3.3: U forecast from Sept. 2001 data?*"

We apologise for any confusion in the text. It is clarified (page 16) that P corresponding to a 2-year period between September 2001 and September 2003 is forecast using the Dynamic Harmonic Regression model on the basis of P data for the January 1985 - August 2001 period. The DHR model forecasts are generated by extrapolating the long-term trend and periodicity identified in the observed P data (corresponding to January 1985 - August 2001 period) into the 2-year period ahead.

C14. "*p.18, line 10 from end: "small"?*"

The text was reworded to state "low" instead (page 18).

C15. "*p.18, last sentence: this is odd, as the beach is at a research facility and there are data for at least 25 years.*"

Although alongshore differences in beach morphology at Duck have been commented on by several authors, there is limited literature on the detailed study of the influences of wave direction and/or longshore sediment transport on morphology over long time periods (e.g. interannual or decades) since Miller et al. (1983). Since submitting the original manuscript, we came across the study by Miller and Dean (2007b), who analysed the correlation between wave direction (by considering the longshore component of the wave energy flux) and longshore shoreline variability. The latter is now referenced in the revised manuscript.

C16. "*Right-hand side of (12) needs normalizing.*"

The transfer function representation of the diffusion equation formulated by Equation 14 in the revised manuscript (i.e. Equation 12 in the original manuscript) has been normalised such that it can be compared with the SDP TF model (Equation 15 in revised manuscript) investigated in Case 2.

C17. “*The use of z-transform notation is unnecessary anywhere in the paper and should be avoided.*”

For consistency with past and present literature on TF models and the companion paper, the z^{-1} time operator (defined on page 7) is used within the manuscript for defining the SDP TF modelling approach. However, when discussing the model results, this operator is used to expand the model equations as shown by Equations 7 and 18 (on pages 14 and 25, respectively).

C18. “*The sentence "In this analysis..." on p.21 is contradicted by "Here, only..." on p.22.*”

We appreciate this comment by the reviewer. The text has been modified accordingly to improve clarity and conciseness (page 21).

C19. “*p.26, line 3 before (15): what errors? One s.d.? 2 s.d.? 6 s.d.?*”

The standard errors associated with these TF model parameters are defined as = 2 × standard deviation of the estimated parameters. This is clarified on page 25.

C20. “*Much of sec. 5 merely reiterates earlier comments. It should be much shortened.*”

We have taken on board the reviewer’s suggestion and made the conclusions shorter and more concise (Section 5).

C21. “*Do we really need all 15 of the references with Young as sole or first author?*”

The number of references in which Young is the first or sole author has been reduced by omitting 4 of these references. Only references that are directly relevant to the revised manuscript are included.

C22. “*If Fig. 2 is to appear in black and white, the bottom plot will need to be larger for legibility. Similarly Figs. 3, 4, 7, 11*”

The clarity of the figures has been improved. See Figures 4, 5, 6, 7, 8, 10 and 14 in the revised manuscript.

Typos and Minor Grammatical Errors

C1. "*pp.2, 3: Birkemeier*"

The spelling of 'Birkemeier' throughout the thesis has been checked and corrected.

C2. "*p.2 foot: linearly*"

This typo has been corrected (i.e. 'linearly' used instead of 'linear')

C3. "*p.6: delete "in" on 2nd line after (1)*"

The word 'in' has been deleted

C4. "*p.7 middle: italics n, m*"

'n' and 'm' are presented is italics (page 7).

C5. "*p.9: Hermite*"

Capital 'H' was used in the word 'Hermite' (page 9).

C6. "*p.9: do you mean + or - 6 standard errors, and is the standard error normalised by sqrt(number of samples) or not?*"

We have clarified that the 'standard errors' associated with the model parameters is = 2 × standard deviation of parameter estimates (page 9). Furthermore, in the present study, the maximum range of errors associated with the model parameters are represented by (±6 × standard errors) (Young *pers comm*) (page 9).

C7. "*p.9, line 5 from end: cover, encompass, as data is plural*"

The text has been revised accordingly (i.e. 'cover' is used instead of 'covers' and 'encompass' is used instead of 'encompasses') (page 9).

C8. "*p.10, line 7: replace 'in the' by 'by'.*"

The above typo is corrected as suggested (page 10).

C9. "*p.10, line 7 of sec. 3: replace "being...model" by "the model being linear"*"

The text has been reworded accordingly (i.e. "..the latter TF model being linear …" is used instead) (now on page 4).

C10. “*p.12, middle: replace "a relatively fewer number of" by "fewer"*”
The above suggestion has been made (page 13).

C11. “*p.15, middle: delete "relatively". Also on p.26, line 5.*”
The word ‘relatively’ has been omitted from the relevant text.

C12. “*p.19, line 3 before (6): one author.*”
This typo has been corrected (page 19).

C13. “*p.22, line 8: delete "on"*”
The word ‘on’ has been deleted.

C14. “*p.23, line 2 after (14) move "group" to after "wave"*”
This typo has been corrected (i.e. ‘..wave group…’ is used) (page 22).

C15. “*p.24, line 5 from end: delete "number of"*”
These words have been omitted from the text (pages 23 and 24).

C16. “*p.25, line 2: a new paragraph at "As' would help.*”
The original text in Section 4.1 has been reworded and made shorter to improve clarity and conciseness. Paragraphs are introduced to improve readability as well.

C17. “*p.29, line 4: replace "than" by "to"*”
The reviewer’s suggestion was carried out (page 28).

C18. “*p.31, line 4: "comprising" is not the right word.*”
We agree with the reviewer. The word ‘exhibiting’ is used instead of ‘comprising’ (page 29).

C19. “*p.36: acute accents are missing from evolution, Journees, Energies*”
Acute accents are included on the specified words.

C20. “*Fig. 2 caption: 2001, not 1999.*”

The caption of Figure 4 in the revised paper (i.e. Figure 2 in original manuscript) was corrected as pointed out by the reviewer.

C21. “*Fig. 6 caption: past participle of "forecast" is "forecast"*”
This typo has been corrected in the caption of Figure 8 in the revised manuscript (i.e. Figure 6 in original manuscript).

* Manuscript

Nonlinear Transfer Function Modelling of Beach Morphology at Duck, North Carolina

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Abstract

This paper presents a simple nonlinear data-based modelling approach for predicting the beach profile volume at Duck, North Carolina, USA. The state-dependent parameter form of the general transfer function (SDP TF) model is used to describe nonlinearity influencing these morphological data in two case examples. Case 1 investigates the nonlinearity associated with the dependency of wave forcing on the preceding beach volume. Case 2 investigates the ability to model the variables within the well known diffusion equation for beach volume using this data-based approach. The results of this study show that the SDP TF approach can be used successfully to develop statistically robust models for describing nonlinearity in beach morphological systems. Furthermore, these models are shown to predict the beach volumes over both

short (1 month ahead) and long (2 years ahead) time periods, and thus show great potential for practical applications in coastal zone management and engineering.

Keywords: Beach, Volumes, Duck, Nonlinear Transfer Functions, Forecasting

1. Introduction

Prediction of beach morphology remains a challenge due to the complexity of the driving hydrodynamic and sediment transport processes operating on various temporal and spatial scales. Inherent interactions between evolving morphological features, such as sand bars, with external forcing factors often generate nonlinear feedback mechanisms. For example, nearshore sand bars have a significant influence on the location of wave breaking and the distribution of wave height in the surf zone (Thornton and Guza, 1983; Lippmann and Holman, 1990). Changes in the sand bar position or shape will alter wave breaking (and sediment transport) patterns across the profile. Consequently, as the beach profile responds to wave forcing, alterations in the sediment transport patterns may suppress or reinforce further morphological changes (Plant et al., 2001). Thus, the response of the beach profile can be nonlinearly related to external forcing conditions (e.g. Lippmann and Holman, 1990; Lippmann et al., 1993; Plant et al., 2001). However, previous studies have shown that the beach profile response can also be linearly related with forcing. For example, Kraus et al. (1991), Larson and Kraus (1992; 1994) and Larson et al. (2000) derived linear relationships between nearshore bar properties and the beach elevation, and various simple hydrodynamic variables. In another example, Lee et al. (1995; 1998) and Birkemeier et al. (1999) observed that the nearshore profile volume increased linearly in response to groups of high-energy storms. However, these typical linear responses do not occur consistently through time (Birkemeier et al., 1999; Larson et al., 2003). The latter authors concluded that the beach profile response may be linearly or nonlinearly related with forcing operating on varied spatial and temporal scales depending on the preceding profile configuration and the forcing conditions. Hence, in developing numerical models for predicting beach morphology, it is important to distinguish between the extent of linear and nonlinear behaviour that influences the system.

Process-based models have been used with varied success for predicting morphological changes (De Vriend, 1991). Hence, alternative modelling approaches, of which an important category encompasses data-based or data-driven models, have been adopted in various situations. Data-based models are statistical models whose entire structure and associated parameters are determined directly from the objective analysis of observational data. Prior assumptions on the governing processes are kept to a minimum. Several linear (examples mentioned in Gunawardena et al. (2008)) and nonlinear (e.g. artificial neural networks (ANN) (Kingston et al., 2000; Pape et al., 2006) and nonlinear complex empirical orthogonal functions (Ruessink et al., 2004; Rattan et al., 2005)) data-based modelling techniques have been used in the past to study beach morphology.

Transfer function (TF) models have been used over the past two decades in numerous data-based modelling applications to study linear and nonlinear stochastic dynamic systems in the fields of environmental science, ecology, engineering and social science (e.g. Young, 1993b; 2000; 2005). These models adopt a systems approach to define the input-output relationship of a studied system (Young, 1993b; 2005). The classical linear constant parameter TF modelling approach has been discussed in detail by Gunawardena et al. (2008), who used this approach to investigate and predict the linear response of the profile volume at Duck to different wave forcing variables. Although this linear model successfully predicts the long-term volume trends, it does not account for nonlinear interactions between the beach response and hydrodynamic conditions. As discussed above, examination of nonlinear interactions is important as they cause variations in the wave forcing-beach morphology relationship under different morphological conditions. Such nonlinear behaviour may be modelled using the nonlinear *state-dependent parameter* (SDP) (Young, 2000; 2005) form of the TF model. In this form, the nonlinearity of the system's input-output relationship is objectively identified by investigating its dependency on other system variable(s) (known as *state variables*). The identified nonlinearity is characterised within the model via *state-dependent parameters*, which vary with time as a function of the state variable(s). In this manner, the SDP TF model can be used to examine and predict dynamic nonlinear 'state-dependent' relationships between different system variables.

One of the most attractive features of the SDP TF modelling approach is that the nonlinearity is localised to a specific physical mechanism of the system (e.g. a state-dependent relationship between the input and any other system variable). Furthermore, dynamic properties of the system, such as time-lags between the input and output variables, are automatically identified. Thus, unlike many of the above mentioned nonlinear data-based modelling techniques, SDP TF models are not solely used for prediction but also provide useful insights into physical mechanisms of the studied system. The SDP TF modelling approach is further enhanced by powerful statistical methods used for model identification and estimation (Young, 1984; 2000). These methods enable simple, parametrically-efficient TF models that are characterised by an optimum number of parameters to be identified in a quick, objective and efficient manner. The SDP TF modelling approach has been used in several practical applications. An example of this includes, estimation and prediction of the nonlinearity in rainfall-flow processes, which results in similar rainfall rates producing different rates of river flow due a dependence on the preceding catchment conditions (e.g. wetness) (Young and Beven, 1994; Fawcett, 1999; Lees, 2000; Young, 2000; 2003).

In this study, the nonlinear SDP TF modelling approach is adopted to develop simple nonlinear data-based models for predicting the beach profile volume at Duck, North Carolina in two separate cases. Beach volume is used as a proxy for the nearshore profile morphology as it is commonly used in coastal management for characterising the overall morphological state of coastal systems (e.g. Kroon et al., 2007). In Case 1, the nonlinearity in the monthly beach volume response to monthly average wave forcing resulting from morphological feedback is investigated. Here, the model parameters associated with the wave forcing term are made to vary as a function of the beach volume itself. The results obtained are compared with those of the linear TF modelling approach adopted by Gunawardena et al. (2008) with the aim of determining the extent to which the linear and nonlinear TF models explain and forecast the beach volume response. In Case 2, the SDP TF approach is used to model the nonlinear volume response of a given beach profile in relation to those of adjacent profiles. Here, the variables within the diffusion equation by Pelnard-Considère (1956) for beach volumes are related using the SDP TF modelling approach. In this way, the ability of the latter approach to model the concept of morphological diffusion in a data-based sense is investigated, while also predicting the beach volumes.

Section 2 of this paper provides a detailed description of the nonlinear SDP TF modelling approach adopted in this study. The data employed in this study are also briefly discussed in this section. Section 3 presents the results for Case 1, where the nonlinearity between the beach volume and wave forcing is investigated. In Section 4, the results from Case 2, where beach volume is modelled in relation to the Pelnard-Considère (1956) equation, are presented. Section 5 summarises the conclusions of this paper.

2. Methodology

2.1 Nonlinear State-Dependent Parameter TF Models

Equation 1 shows the general discrete-time form of a single input-single output SDP TF model (Young, 2000; 2005).

$$y_k = \frac{B(w_k, z^{-1})}{A(v_k, z^{-1})} u_{k-\delta} + \xi_k \tag{1}$$

Here, y_k is the observed output variable and u_k is the observed input variable; where k denotes the kth discrete time sample. δ represents the pure time delay ($\delta\Delta t$ time units) associated with the input variable that accounts for a time delay between a change in the input and observation of its initial effect on the output. ξ_k is a noise term that accounts for uncertainty in the model (e.g. due to measurement noise, unmeasured inputs and uncertainty in the model). The preliminary application of the SDP TF model in this study excludes this noise term and therefore, this term is omitted from the model equations in the rest of the paper. $A(v_k, z^{-1})$ and $B(w_k, z^{-1})$ are the dynamic polynomials of the SDP TF model, which are formulated by:

$$A(v_k, z^{-1}) = 1 + a_1(v_k)z^{-1} + a_2(v_k)z^{-2} + ... + a_n(v_k)z^{-n} \tag{2}$$

$$B\left(w_k, z^{-1}\right) = b_0(w_k) + b_1(w_k)z^{-1} + b_2(w_k)z^{-2} + \ldots + b_m(w_k)z^{-m} \tag{3}$$

where: $z^{-p}, p = 1,2 \ldots n(m)$ denotes the 'backward shift' time operator (defined as $z^{-n} y_k = y_{k-n}$); v_k and w_k are *the state variables*; $a_i, i = 1,2,\ldots n$ and $b_j, j = 0,1,\ldots m$ are the time-varying TF model parameters known as *state-dependent parameters* (SDP), which vary as functions of the state variables. The numbers of '*a*' and '*b*' parameters (*n, m*) define the order of the above TF polynomials. The values of the '*a*' and '*b*' parameters as well as the values of *n*, *m* and *δ* are objectively identified from the data during the TF model calibration. The triad [*n m δ*] is generally used to describe the TF model structure. For example, a triad [1 2 5] represents a TF model comprising a 1st order $A(v_k, z^{-1})$ polynomial, 2nd order $B(w_k, z^{-1})$ polynomial and a time delay of 5 discrete-time steps. In cases where the nonlinearity is assumed to be associated only with the input variable, the parameters of only the $B(w_k, z^{-1})$ polynomial are made state-dependent while those of the $A(v_k, z^{-1})$ polynomial remain constant (i.e. $A(v_k, z^{-1})$ becomes $A(z^{-1}) = 1 + a_1 z^{-1} + \ldots + a_n z^{-n}$). The SDP TF model is then said to account only for an *input nonlinearity*. In such cases, the numerator SDP can be used to transform the input variable, such that it can be related to the output variable via a linear TF model (Young et al., 1996; Lees, 2000). Time-variation of the parameters of both numerator and denominator TF polynomials can result in model instability and poor parameter estimation. Consequently, the former 'input nonlinearity' approach has been predominantly used in the past for SDP TF modelling in natural systems (e.g. Lees (2000), Young (2000)), and is therefore adopted for this study.

The SDP TF model algorithms within the CAPTAIN toolbox (CRESS, 2004) were employed in the present study. The following outlines the steps adopted during calibration and validation of suitable SDP TF models for characterising the input nonlinearities studied in Cases 1 and 2.

1. Estimation of the state-dependent parameters (SDP) using the recursive Fixed Interval Smoothing (FIS) algorithms (Young, 1984; 1985; 2000). These time-variable parameter estimation methods allow for rapid (state-dependent) parametric changes (Young, 2000) and hence can handle nonstationary data. The estimated SDP act as a 'gain', which either enhances or suppresses the effect of the input variable, and is therefore also referred to as the *SDP gain*. The SDP gain and the state variable are plotted (e.g. Figure 3) or tabulated to provide a nonparametric estimation (i.e. in the format of a look-up table) of the state-dependency influencing the input variable (which defines the input nonlinearity).
2. Transform the input variable by multiplying it with the corresponding SDP gain to obtain the *effective* proportion of the input (i.e. *effective input*) that can be linearly related to the output variable. In this way, the input nonlinearity is incorporated via the *effective input*.
3. Identify and estimate an optimum linear TF model between the *effective input* and the output variable using the Simplified Recursive Instrumental Variable (SRIV) algorithms (Young, 1984; 1985). The selection of the most appropriate linear TF model is based on two statistical criteria, namely the coefficient of determination (R_T^2) and the Young Information Criterion (YIC). R_T^2 provides a measure of the goodness of fit of the model based on the relative measure of the variances of the model errors and the observed data (Young, 1993a). In general, the closer its value is to unity, the better is the model fit. YIC provides a measure of both the goodness of fit of the model and the efficiency of the model

parameters (Young, 1993a). It is based on minimising the sum of the total variance of all model residuals and the error variance associated with each model parameter. The more negative the YIC value, the better defined are the parameter estimates.

4. Model Validation: For purposes of validating the SDP TF model (and forecasting), the SDP gain corresponding to the validation (or forecast) period need to be estimated. For this, a shape-preserving interpolant based on the piecewise cubic Hermite interpolating polynomial in Matlab™ is fitted to the SDP gain vs state variable relationship produced during model calibration. The SDP gain associated with the state variable data corresponding to the validation period are estimated from this interpolant curve. The corresponding input data are then transformed using this SDP gain and used within the calibrated model.

The FIS and SRIV algorithms provide estimates of the standard errors (i.e. 2 × standard deviation) associated with the model parameters. In this study, the maximum range of errors associated with the SDP TF model parameters are represented by (±6 × standard errors) (Young *pers comm*).

2.2. Data

Data collected by the Field Research Facility (FRF) at Duck, North Carolina, operated by the Coastal Engineering Research Centre of the U.S. Army Engineer Waterways Experiment Station (FRF, 2003), are used in this study. The data used here cover a 20.3-year period between May 1983 and September 2003 and encompass: beach profile surveys corresponding to two pairs of profiles located north (Profiles 58 and 62) and south (Profiles 188 and 190) of the FRF pier at Duck (Figure 1); and significant wave height (H_s), peak period (T_p), wave direction (θ), water level (WL) and surge (S) measurements measured at the 8 m and 17 m depths. The beach data cover the cross-shore distance from the lower dune region (75 m) to 700 m and encloses the active portion of the profile (i.e. average position of depth of closure (4 m depth) ≈ 410 m, Larson and Kraus (1994)). Time series of volume per unit metre of shoreline for each of the above mentioned profiles were computed by integrating these beach data to the 7 m depth (illustrated in Figure 2). As these data extend beyond the average position of the depth of closure, changes in beach volume are expected to be predominantly influenced by longshore processes (as cross-shore processes would mainly influence the redistribution of sediment across the profile). Alongshore volume differences between northerly and southerly profiles are seen in Figure 2, particularly post 1991 (discussed in detail in Gunawardena (2008)). The wave data were also processed to generate regularly sampled time series of monthly average significant wave height, wave direction and peak period. Further details of the above data and the generated times series are discussed by Gunawardena et al. (2008).

3. Case 1: Nonlinear SDP TF Modelling of the Beach Volume Response to Wave Forcing

Taking into consideration that the beach data used in this study extend well beyond the closure depth, the monthly beach volumes at Duck are, as expected, best related to the monthly average alongshore component of the wave energy flux (P) defined by Equation 4 (as in Komar (1998)).

$$P = \left(\frac{1}{8}\rho g H_b^2\right) C_g \sin\theta_b \cos\theta_b \tag{4}$$

where: ρ is density of water; g is acceleration due to gravity; H_b, θ_b, C_g are the significant wave height, wave approach angle and wave group velocity under breaking-wave conditions, which were computed using a linear wave transformation model (Gunawardena, 2008). Here, the SDP TF modelling approach was adopted to investigate the state-dependency of P on the preceding beach volume (i.e. input nonlinearity associated with P). The TF model parameters associated with P at a given time-step are made to vary with time as a function of the beach volume at the previous time-step (i.e. preceding beach volume is made the *state variable*). Equation 5 shows the SDP TF model form used in this analysis.

$$V_k = \frac{B(V_{k-1}, z^{-1})}{A(z^{-1})} P_{k-\delta} \qquad (5)$$

where: V is the monthly beach volume per unit of shoreline (m^3) (model output); P is the monthly average longshore component of the wave energy flux (normalised using $\frac{1}{8}$, ρ and g to have units of m^3/s) (model input); k and δ are as defined in Equation 1; $B(V_{k-1}, z^{-1})$ is the numerator TF polynomial comprising the SDP gain that varies with time as a function of the state variable, V_{k-1}; $A(z^{-1})$ is the denominator TF polynomial comprising constant parameters. For comparability with the linear TF model results of Gunawardena et al. (2008), the first 15 years of the monthly beach volume and P time series, starting from January 1985, were used in the calibration of a suitable SDP TF model. The remaining 2 years (2001-2003) of data were used to validate the model. In this paper, only those results corresponding to Profile 62 are presented.

3.1 Investigation of the Input Nonlinearity and Prediction of Beach Volume

Figure 3 shows the SDP gain (estimated on the basis of Equation 5) that defines the nonlinear state-dependency between the wave forcing input (P) and state variable (preceding beach volume) for Profile 62. The corresponding R_T^2 value of 0.5 indicates that this SDP relationship is well defined. In general, the strength of the identified nonlinearity is reflected by the degree of variation of the SDP gain with respect to the state variable. The variability of the SDP gain in Figure 3 over the whole range of volumes considered indicate the occurrence of this nonlinearity under all morphological conditions represented by the beach volumes. However, the variability of the SDP gain increases at volumes > 2270 m^3 (as shown by the increased negative gradient of the SDP relationship in Figure 3), which suggests that the nonlinearity becomes stronger during high profile volumes. This is also shown in Figure 4, where the largest SDP gain magnitudes correspond to 1990 - 1997 when the beach volume was > 2270 m^3. By contrast, low SDP gain values correspond to the 1985 - 1990 and 1997 - 2001 periods, which are associated with lower volumes (< 2270 m^3). Thus, these results indicate that the strength of the nonlinearity varies depending on the morphological state (high/low volume) of the profile.

From Figure 4, it is also seen that the time-variation of the SDP gain magnitude imitates that of beach volume. For example, the long-term increase in volume between 1990 - 1994 corresponds to a similar long-term increase in the SDP gain magnitude, while the more rapid decrease in volume between 1995 - 1997 is associated with a similarly rapid reduction in the SDP gain magnitude. Furthermore, increases and decreases in the SDP gain magnitude are predominantly associated with accretional and erosional events, respectively. Hence, the estimated SDP gain effectively accounts for variations in the speed of the profile's volume response as well as its direction of change (i.e. accretion or erosion). The widened standard error boundaries associated with the SDP gain for low (< 2120 m^3) and high (2470 m^3) volumes in Figure 3 reflect the higher uncertainty associated with these parameter values. This is due to the fewer data points corresponding to these low and high values of volume in the calibration data set.

The SDP gain was then used to calculate the effective proportion of the wave forcing input P (i.e. SDP gain × P) that directly influences the profile volume. Figure 5 compares the time series of beach volume (output) and P (input) with the resulting *effective* input. As observed here, the effective input significantly differs from the original P data. The effective input was then related to the beach volume using the linear TF modelling approach. Based on the R_T^2 (= 0.816) and YIC (= -6.05) values, a TF model comprising a [1 1 0] structure (i.e. 1st order numerator and denominator TF polynomials and a zero time-delay, respectively) was found to have the best-fit. This model explains a large proportion (81.6%) of the beach volume variance and has a low root-mean-square (RMS) error of 61.1 m^3. As seen in Figure 6, this model very efficiently describes the long-term trend in the data while also capturing some short-term fluctuations, particularly during periods of larger beach volume (> 2270 m^3). Equation 6 represents this identified linear TF model.

$$V_k = \frac{0.222}{1-0.897z^{-1}}\left[f\{V_{k-1}\}\cdot P_k\right] \tag{6}$$

where: V, P and k are as defined in Equation 5 ; $f\{V_{k-1}\}$ represents the time-varying SDP gain; $\lfloor f\{V_{k-1}\}\cdot P_k \rfloor$ represents the effective input, which incorporates the estimated input nonlinearity. The predominantly negative SDP gain values (Figures 3 and 4) and the positive coefficient of the numerator term in Equation 6 indicate that, in general, P forms a negative relationship with the beach volume. Equation 6 can be expanded to its difference-time form (Equation 7) by simple cross multiplication and the application of the z^{-1} operator.

$$V_k - 0.897V_{k-1} = 0.222\left\lfloor f\left\{V_{k-1}\right\} \cdot P_k \right\rfloor \qquad (7)$$

(using the same notation as Equation 6 above)

Equation 7 shows that successive beach volume differences at Duck are nonlinearly related to the longshore component of the wave energy flux via time-varying state-dependent parameters that characterise the nonlinear dependence of P on the preceding volume. It should be noted that the value of P at a given time-step represents the average wave conditions over the 1 month period prior to the corresponding profile survey and hence by default is associated with a lag of 1 month.

The SDP TF model was validated using observed P data for the 2-year period between September 2001 and September 2003. Here, volumes predicted by the model at each time-step were used for the model validation at the next time-step (i.e. simulation mode). Figure 3 shows the shape-preserving interpolant fitted to the SDP relationship for estimating the SDP gains corresponding to the latter period. As seen here, this interpolant curve reproduces the nonlinear state-dependency very well (RMS error of interpolated SDP = 0.59). Figure 6, shows the model output of beach volume produced for this validation period. The model fit is associated with a low RMS error of 72.1 m^3 and explains the longer-term patterns in the data very well. The model also does well in capturing some of the short-term events such as the peak in volume during April 2001.

3.2 Comparison of Nonlinear SDP and Linear Constant Parameter TF models

Figure 6 and Table 1 compare the results of the linear TF model previously identified for Profile 62 by Gunawardena et al. (2008) and the SDP TF model estimated above. As seen in Figure 6, in general, both linear and nonlinear TF models describe the longer-term trend in the data very well. However, the latter also captures short-term changes such as the accretional events (volume peaks) during 1990 – 1991 and 1992 - 1996. The nonlinear model typically performs better than the linear model for volume predicting during periods when the volumes are larger than 2270 m^3. The linear model, however, provides a better prediction of the longer-term volume trend than the nonlinear model for the low volume period between 1985 - 1988. The improved overall predictions provided by the nonlinear model are reflected by its higher R_T^2 value, lower YIC value and lower RMS model calibration and validation errors.

It should also be noted that there are differences in the time delays associated with the linear and nonlinear models (Table 1). That is, in the case of the linear model, the beach volume lags behind the wave forcing input P (original data) by 8 months (discussed in Gunawardena et al. (2008)). In contrast, the nonlinear model (Equations 6 and 7) is not associated with a time delay. The latter was confirmed using a cross-correlation test between the effective input of P and the beach volume of Profile 62, which showed that the maximum correlation (= 0.45) for time-lags between 0 to 12 months occurred at the zero time-lag. These differences may be explained by the different system dynamics represented by these models. That is, while the linear model considers only the linear relationship between the volume and P, the nonlinear model also accounts for the effects of morphological feedback on this relationship. This is discussed further in Section 3.4.

3.3 Forecasting Beach Volume using the Nonlinear SDP TF Model

In using the above SDP TF model for forecasting the beach volume at the k^{th} time-step, data on the preceding beach volume and wave conditions (P) at the $(k-1)^{th}$ time-step are required. Generally, wave conditions can be forecast more easily than morphological data. In this study, the Dynamic Harmonic Regression (DHR) model (Young et al., 1999) was used to forecast P corresponding to a 2-year period between September 2001 and September 2003. This was done on the basis of extrapolating the long-term trend and periodicity identified by the DHR model for the observed P data (corresponding to January 1985 - August 2001) to the latter 2-year period (Gunawardena et al., 2008). Figure 7 shows P forecast by the DHR model for this period. The forecasts follow the observed data closely and have relatively small errors (RMS error = 0.66 m^3/s). The beach volume was then computed using Equation 7, where at each k^{th} time-step the beach volume forecast for the $(k-1)^{th}$ time-step was used as the state variable in calculating the beach volume at the k^{th} time-step. Figure 8 compares the resulting beach volume forecasts with those obtained using the linear TF model adopted by Gunawardena et al. (2008) as well as the SDP TF model validation. As seen here, although both linear and nonlinear models capture the overall long-term trend in the data well, the nonlinear model performs best as it forecasts some of the shorter-term variability in the volumes (like in the case of the model validation) and consequently has smaller RMS errors (Table 1).

3.4 Discussion of Model Results

The estimated SDP relationship shows an overall nonlinearity and further suggests that the nonlinearity is strongest during high volume conditions (> 2270 m^3).

Analysis of beach profile data showed that, during periods of high volume, the profiles comprise two well-developed nearshore bars as shown in Figure 9 (also discussed in Gunawardena (2008)). Previous studies have shown that these nearshore bars at Duck interact with driving forces to generate nonlinear feedback mechanisms (Lippmann and Holman, 1990; Lippmann et al., 1993; Plant et al. 2001). The stronger nonlinearity identified in this study for periods of increased beach volume is thus inferred to be associated with these nonlinear bar mechanisms.

The SDP TF modelling approach describes the beach volume response to P under all conditions of the SDP relationship (irrespective of the occurrence of a strong or mild nonlinearity). Thus, a key advantage of this technique is that it enables prediction of the beach response when there is no clear separation between conditions and timescales for approximately linear and/or nonlinear behaviour. Comparison of the model output and forecasts of the nonlinear SDP TF model with those of the linear TF model shows that both these models very efficiently predict the long-term trends in the data. However, the nonlinear model also predicts the shorter-term volume fluctuations about these trends during periods of increased beach volume (Figures 6 and 8). Thus, the results suggest that, in general:

- the dominant longer-term volume trend, which is successfully predicted by both linear and nonlinear models, is a direct consequence of the long-term changes in P (i.e. wave height and direction).
- the shorter-term volume fluctuations occurring about this long-term trend may be associated with the nonlinear mechanisms of morphological feedback considered here (particularly during periods of increased volume).

The time-lag associated with the linear model and the nonlinear dependence on the preceding morphology incorporated by the nonlinear model both suggest that the longer-term beach morphology at Duck evolves at a slower rate compared to the wave forcing. This was also observed by Plant et al. (1999) for bar migration at Duck. Furthermore, the negative state-dependency between P and the beach volume identified in this study, and the negative linear TF relation found between these variables in Gunawardena et al. (2008), suggest that P is negatively related (either linearly and/or nonlinearly) to the beach volume. Thus, in general, low magnitude negative values of P (corresponding to reduced wave heights approaching from directions south of the FRF pier) generate volume increases. Conversely, large positive values of P (corresponding to increased wave heights approaching from directions north of the FRF pier) are associated with decreases in volume. These results show that profile volume changes at Duck are associated with longshore wave processes. Similar observations have been made in the past by Miller et al. (1983) and more recently by Miller and Dean (2007b) via an analysis of longshore shoreline variability at Duck.

The short-term fluctuations in volume that remain unpredicted by both linear and nonlinear TF models may be attributed to: other system inputs (e.g. storm waves and/or surge effects); more intricate nonlinear feedback mechanisms (e.g. "destabilising" feedback effects that result in the decay of nearshore bars under continuous non-breaking conditions (Plant et al., 2001)); and limitations imposed by the differences in the sampling frequencies of the beach profile and wave data. It should also be recalled that the state-dependent nonlinearity investigated here is associated only with P (i.e. only the numerator parameters of the SDP TF model vary in time). Further research is required to determine if time-variation of the denominator TF model parameters instead would alter the results of this analysis. This is beyond the scope of the present study and will be considered in the future.

4. Case 2: Nonlinear SDP TF Modelling of the Alongshore Interactions of the Beach Profile Volume

In this analysis, the nonlinear SDP transfer function modelling approach is used to investigate the dynamic evolution of the beach profile in relation to the alongshore diffusion equation of Pelnard-Considère (1956) for beach volumes. The latter author introduced the idea of modelling the planform evolution of beaches using the following diffusion equation:

$$\frac{\partial V}{\partial t} = G\frac{\partial^2 V}{\partial x^2} \tag{8}$$

where: V is the volume of sand (*m^3*) at an alongshore position x (*m*) at a given time t (*unit of time*), and G is the time varying *sediment longshore diffusivity* (*m^2/unit of time*) that represents the wave conditions and is given by:

$$G = \frac{KH_b^{5/2}\sqrt{g/\kappa}}{8(s-1)(1-p)(h_* + B)} \tag{9}$$

where: K = sediment transport coefficient, H_b = breaking wave height, g = acceleration due to gravity, κ = ratio of breaking wave height to local water depth, s = sediment specific gravity, p = sediment porosity, $(h_* + B)$ = height of the active beach profile (closure depth, h_*, and berm height, B). Equation 8 assumes that no significant cross-shore exchanges of sediment occur beyond the active portion of the profile enclosed by the berm and closure depth. This equation can be solved in space and time, using analytical and/or different numerical schemes, to predict the volume V at any given alongshore beach position at a given time. However, the latter requires a regular spatial grid. The data-based approach adopted here provides a very simple and statistically efficient alternative.

Using the simplest finite difference approximations to the differential terms in Equation 8, shown in Equation 10, the above diffusion equation can be expanded to give its approximate difference-time form shown in Equation 11.

$$\frac{V_j^k - V_j^{k-1}}{\Delta t} = G(t).\left[\frac{V_{j+1}^k - 2V_j^k + V_{j-1}^k}{\Delta x^2}\right] \tag{10}$$

$$\frac{\left(2\Delta t.G(t) + \Delta x^2\right)V_j^k - \Delta x^2 V_j^{k-1}}{\Delta x^2 \Delta t} = G(t).\left[\frac{V_{j+1}^k + V_{j-1}^k}{\Delta x^2}\right] \tag{11}$$

where: V_j represents the beach volume at the alongshore positions j, V_{j+1} and V_{j-1} represent the beach volume at the alongshore positions $j+1$ and $j-1$, k is the k[th] discrete-time sample, Δt is the sampling time interval and Δx is the sampling interval between adjacent profile lines.

By introducing the backward shift operator z^{-1} (where, for e.g. $z^{-n}V_k = V_{k-n}$) in Equation 11 to obtain Equation 12, the approximate discrete-time transfer function form of the diffusion equation can be presented by Equation 13.

$$\frac{\left(2\Delta t.G(t) + \Delta x^2\right)V_j^k - \Delta x^2 z^{-1} V_j^k}{\Delta x^2 \Delta t} = G(t).\left[\frac{V_{j+1}^k + V_{j-1}^k}{\Delta x^2}\right] \tag{12}$$

$$V_j^k = \left[\frac{\Delta t.G(t)}{\left((2\Delta t.G(t) + \Delta x^2) - \Delta x^2 z^{-1}\right)}\right]\left(V_{j+1}^k + V_{j-1}^k\right) \tag{13}$$

Equation 13 has the form of a 1st order discrete-time transfer function model given by:

$$V_j^k = \frac{b_0\{G(t)\}}{a_0\{G(t)\} - a_1 z^{-1}} V_{j+1}^k + \frac{b_0\{G(t)\}}{a_0\{G(t)\} - a_1 z^{-1}} V_{j-1}^k \tag{14}$$

where: $b_0 = \Delta t.G(t)$; $a_0 = (2\Delta t.G(t) + \Delta x^2)$ and $a_1 = \Delta x^2$ are the TF model parameters, V_j is the output, and V_{j+1} and V_{j-1} are the inputs. The above TF model is nonlinear as the parameters b_o and a_o are time-varying and are dependent on the longshore sediment diffusivity term, G.

In this analysis, the SDP TF approach is used to model the input nonlinearities associated with the state-dependencies of the V_{j+1} and V_{j-1} terms in Equation (14) on G. The objectives of this work are to: (a) investigate the ability to model the concept of alongshore sediment diffusion using a data-based TF approach and monthly beach volume data; (b) identify and estimate a statistically efficient TF model from the observed data that best explains this behaviour; (c) predict the beach volume of a given profile line. Here, the beach volume of a given profile line (say Profile 62) at the kth time-step is used as the TF model output, while the beach volume at adjacent profile lines (in this case Profiles 58 and 188) at the (k-1)th time-step are used as the model inputs. The longshore sediment diffusivity in deep water at the (k-1)th time-step is used as the state variable on which the inputs, V_{j+1} and V_{j-1}, are made to depend. The input nonlinearity associated with each of these variables is investigated individually so that the nonlinear contributions of adjacent profile lines on either sides of the investigated profile could be studied independently. The input and state variable data used within this model are lagged by 1 month relative to the output data. This was done in order to investigate the ability of predicting the volume of a given profile (at a given time) on the basis of the preceding volume of the adjacent profiles and the preceding wave conditions. Thus, the general SDP TF relation investigated here can be represented by:

$$V_k^{62} = \frac{B^1(G_{k-1}, z^{-1})}{A(z^{-1})} V_{k-1}^{58} + \frac{B^2(G_{k-1}, z^{-1})}{A(z^{-1})} V_{k-1}^{188} \tag{15}$$

where: V^{62} (*output*), V^{58} and V^{188} (*inputs*) are the beach volume time series of Profiles 62, 58 and 188; $A(z^{-1})$ is the denominator TF model polynomial, while $B^1(G_{k-1}, z^{-1})$ and $B^2(G_{k-1}, z^{-1})$ are the dynamic numerator TF model polynomials associated with the inputs V^{58} and V^{188}, respectively. The numerator polynomials would comprise *state-dependent parameters* (SDP) that vary in time, depending on G (*state variable*). Here, G is the longshore sediment diffusivity in deep water, which was calculated using Equation 16 (Dean, 2001). The latter was used instead of the shallow water estimate of G (defined in Equation 9) so that it could be used with different beach profiles (i.e. because it is generalised and not associated with a specific bathymetry).

$$G = \frac{H_o^{2.4} C_{Go}^{1.2} g^{0.4} \cos^{1.2}(\beta_o - \alpha_o) \cos 2(\beta_o - \alpha_o)}{8(s-1)(1-p) C_* \kappa^{0.4} (h_* + B) \cos(\beta_o - \alpha_*)} \tag{16}$$

The subscript 'o' in the above equation denotes the deep water conditions of the variables defined in Equation 9, while C_{Go} is the deep water wave group velocity, C_* and α_* are the wave celerity and wave approach angle, respectively, in the water depth, h_* (= 8 m depth), β_o and α_o are the azimuth of an outward normal drawn to the shoreline and the wave approach angle relative to the true north, respectively. The above deep water wave properties were computed by transforming the wave data measured at the 8 m and 17 m depths to deep water using linear wave theory. Values of s = 2.65, p = 0.42 (Komar, 1998) and κ = 0.4 (Sallenger and Holman, 1985) were used in the computation of G.

Thus, using the above approach, the variables in the diffusion equation were related to determine a statistically efficient data-based SDP TF model that best describes the observed data. This approach ensures that the SDP TF model structure is determined directly from the observed data itself, rather than fixing it a priori on the basis of the 1st order TF approximation given in Equation 14. The results obtained by relating the volume of Profile 62 to the volume of Profiles 58 and 188 via the dependence on the deep water longshore sediment diffusivity (as seen in Equation 15) are presented here. The first 14 years of data (starting May 1983) (Figure 10) were used to calibrate the nonlinear SDP TF model between these variables. The remaining 5 years of data were used to validate the model.

4.1 SDP TF Model Calibration and Prediction of Beach Volume

Figures 11 and 12 show the state-dependent parameter gains corresponding to the two input variables (i.e. volume of Profiles 58 and 188, respectively) estimated on the basis of Equation 15. These SDP gains are associated with an R_T^2 value of 0.70, which suggests that the identified nonlinearities are well defined. As seen in Figure 11, at values of $G < 0.2\text{x}10^6$ *m^2/month*, the SDP gain associated with Profile 58 initially decreases with increasing G and thereafter progressively increases until $G \approx 2\text{x}10^6$ *m^2/month*, after which it remains approximately constant. This indicates that for $0.2\text{x}10^6 < G < 2\text{x}10^6$ *m^2/month*, the SDP gain associated with the volume of Profile 58 increases with G. This in turn suggests that under these conditions, the effective contribution of the volume of Profile 58 in characterising the volume of Profile 62 increases with the longshore sediment diffusivity. The converse is seen in Figure 12, where the SDP gain associated with Profile 188 initially increases with G (up to G =

0.2×10^6 *m^2/month*) and thereafter decreases to exhibit negative values until $G = 2 \times 10^6$ *m^2/month,* after which it remains constant. This suggests that the effective contribution of the volumes of Profile 188 in characterising those of Profile 62 increases negatively with increasing longshore sediment diffusivity up to $G < 2 \times 10^6$ *m^2/month*, beyond which no significant state-dependent nonlinearity is estimated. From Figures 11 and 12, it is seen that the standard errors associated with the estimated SDP gains increase for $G > 2 \times 10^6$ *m^2/month*. This can be explained by the availability of fewer data points associated with these values of G. This also explains the relative constancy of the SDP vs G relationships (i.e. decay of the state-dependent nonlinearity) at these larger values of G.

Figure 13 illustrates the time variation of these SDP gains. The positive SDP gain associated with Profile 58 indicates that the volumes of this profile are positively related to those of Profile 62 (i.e. volume increases on Profile 62 are linked with volume increases on Profile 58, and vice versa). By contrast, the SDP gain associated with Profile 188 is positive or negative at varied times, the latter which indicates that the volumes of this profile are negatively related with those of Profile 62 (i.e. volume increases on Profile 62 are linked with volume decreases on Profile 188 and vice versa) at particular periods of time. These contrasting relationships between the volumes of Profile 62 and those of Profiles 58 and 188 are illustrated in Figure 10, where it is seen that increased volumes on Profiles 58 and 62 during certain periods coincide with significantly reduced volumes on Profile 188, and vice versa (e.g. Dec 1984 – Sep 1986, Dec 1986 – Feb 1987, 1989 – 1990, 1992 – 1997). This explains the opposing patterns observed in the state-dependent relationships in Figures 11 and 12. Furthermore, the SDP gain associated with Profile 58 has larger magnitudes than those estimated for

Profile 188, suggesting that the former has a more dominant influence in characterising the volume of Profile 62. This can be expected as Profile 62 is located at a closer proximity to Profile 58 (~ 100 m apart) than Profile 188 (~ 1000 m apart). It should be noted that the SDP gains associated with the latter two profiles are normalised during their estimation such that their summation at each k^{th} time-step is 1 (as seen clearly from Figure 13) while still representing weights for relating the volumes of these two profiles to those of Profile 62.

Figure 14 shows the *effective* volume contributions of Profiles 58 and 188 computed using the estimated state-dependent parameters. As seen in this figure, the effective input associated with Profile 188 comprises increased short-term fluctuations and shows a significant transformation from the original volume data. In contrast, the effective input associated with Profile 58 is similar to the original volume time series, except that it comprises slightly smaller magnitudes. These effective beach volume inputs were then related to the volume of Profile 62 using the linear TF modelling approach. Equation 17 represents the SDP TF model that was identified to best fit the observed beach volume of Profile 62, while also having statistically reliable parameters (YIC = -5.45). This 1st order model has a R_T^2 value of 0.912, indicating that it explains a very large proportion (91.2%) of the variance in the volume data of Profile 62. The standard errors (in this case, = 2 × standard deviation) associated with the model parameters are enclosed within the brackets. Equation 18 represents the difference-time form of this model.

$$V_k^{62} = \frac{0.56(0.06)}{1-0.45(0.05)z^{-1}}\left[f\{G_{k-1}\}^{58}\cdot V_{k-1}^{58}\right] + \frac{0.05(0.008)}{1-0.45(0.05)z^{-1}}\left[f\{G_{k-1}\}^{188}\cdot V_{k-1}^{188}\right] + \xi_k \quad (17)$$

$$V_k^{62} - 0.45V_{k-1}^{62} = 0.56\left(f\{G_{k-1}\}^{58}\cdot V_{k-1}^{58}\right) + 0.05\left(f\{G_{k-1}\}^{188}\cdot V_{k-1}^{188}\right) + \xi_k \quad (18)$$

where: V_k^{62}, V_k^{58} and V_k^{188} are the beach volumes of Profiles 62, 58 and 188; $f\{G_{k-1}\}^{58}$ and $f\{G_{k-1}\}^{188}$ represents the time-varying SDP gains associated with Profiles 58 and 188, respectively; $\left(f\{G_{k-1}\}^{58} \cdot V_{k-1}^{58}\right)$ and $\left(f\{G_{k-1}\}^{188} \cdot V_{k-1}^{188}\right)$ represent the effective inputs associated with Profiles 58 and 188, respectively. Equation 18 shows that the above SDP TF model defines the successive monthly volume differences of Profile 62 as a linear combination of time-varying weighted proportions of the volumes of Profiles 58 and 188. These weights represent the nonlinear state-dependency of the latter variables on the longshore sediment diffusivity.

Figure 15 compares the calibrated model output of beach volume with the observed data for Profile 62. As seen here, the model very effectively captures the long-term and short-term patterns in the data with small errors (RMS error = 38 m^3). The identified model was also validated using data for the 5 year period between September 1998 and September 2003. The SDP gains associated with the input variables for this period were estimated from the interpolant curves fitted to the SDP relationships in Figures 11 and 12. As seen in the latter figures, these interpolant curves reproduce the identified state-dependent nonlinearities well (in both cases, RMS errors of interpolated SDP gain = 0.1). Figure 15 shows the resulting beach volume computed for this validation period. As seen here, the model very successfully predicts the beach volume (RMS error = 73 m^3) for the latter period. This shows that the TF model in Equation 18, which was calibrated using a portion of the data set, can efficiently describe the beach volume over long validation periods without the re-estimation of its constant parameters.

4.2 Discussion of the SDP TF Model Results

The above results show that the variables within the well-known diffusion equation by Pelnard-Considère (1956) can be successfully related using the SDP TF modelling approach to predict the monthly beach profile volume at Duck. Prediction of a profile's volume on the basis of adjacent profile volumes and the deep water diffusivity term indicates that volumetric changes of adjacent profiles at Duck are related (either positively or negatively, as indicated by the estimated SDP gains). The latter is consistent with Miller and Dean (2007a) who also found that during certain periods, accretion of northerly profiles coincide with erosion of southerly profiles and vice versa. The results of Case 2 further suggest that the beach volumes at this site are influenced by alongshore sediment exchanges between profiles. As these volume data characterise the combined net product of both short-term (1-12 months) and long-term (> 12 months) sediment transport processes, both shorter and longer-term volume changes of a given profile can be predicted using this approach.

A comparison of the data-based SDP TF model in Equation 17 and the TF approximation of the diffusion equation (Equation 14) shows that, even though both these models comprise 1st order transfer functions, they have differences. That is, while the latter is fully nonlinear (i.e. both numerator and denominator TF parameters vary as a function of G), the former only incorporates the input nonlinearities associated with the state-dependency of each input variable on G. In addition, the latter applies to data comprising a regular spatial (alongshore) sampling interval, Δx. In contrast, the alongshore profiles considered in the SDP TF model in Equation 17 are not equally spaced (i.e. Δx is not constant). The results of this analysis show that, despite considering only the input nonlinearities, the SDP TF model explains the beach volume extremely well, which demonstrates that the concept of alongshore sediment diffusion can be successfully modelled using this alternative approach and monthly beach and wave data. Furthermore, the SDP TF approach can be adopted to approximately represent the diffusion equation without the need of data sampled at equal spatial intervals. This is a key advantage of the latter approach because in practice field measurements are often taken at irregular intervals.

It should be noted that the TF models presented in both Cases 1 and 2 adopt the same assumption that profile volume changes at Duck are associated with longshore processes; although different variables and relationships are modelled in each case giving different prediction results. The nonlinear SDP TF model identified in this section is clearly far superior in fitting the beach volume data to the linear and nonlinear TF models previously discussed in Section 3. However, as the former operates on the basis of adjacent profile volumes and wave data at the previous time-step (month), the maximum lead-time for volume forecasts is only 1 month. Hence, unlike the linear and nonlinear TF models considered in Section 3, the above nonlinear SDP TF model cannot be used to produce longer-term predictions of beach volume, but instead provides one step-ahead (i.e. 1 month-ahead) predictions. Nevertheless, the simplicity, inexpensive nature and good performance of this model make it attractive for practical applications requiring the prediction of the volume of different cross-shore sections of a beach. This model could also be used for data-processing to fill gaps in volume data of a given profile by considering those of adjacent profiles.

5. Conclusions

In this paper, a simple data-based nonlinear SDP TF modelling approach is introduced for identifying and estimating dynamic nonlinearities in the context of beach morphology. Here, input nonlinearities associated with the dependency of system inputs on other system variables are explored. A key advantage of this approach is that the nonlinearities investigated are confined to particular system variables and are exposed, enabling their interpretation in terms of physical mechanisms influencing the system.

The application of the SDP TF modelling approach is demonstrated via two case examples, both involving the prediction of the monthly beach profile volume at Duck, North Carolina. In the first of these cases, the input nonlinearity arising from the dependence of the longshore component of the wave energy flux on the antecedent beach volume is investigated. The strength of this nonlinearity is generally found to vary with time depending on the volume of the beach profile, and is strongest during periods of high volume (> 2270 m^3). The resulting SDP TF model identified efficiently predicts the long-term volume trend (similar to a previously identified linear TF model) but also predicts the shorter-term volume fluctuations particularly during periods of high beach volume. In the second case example, the variables within the well-known diffusion equation (Pelnard-Considère) for beach volumes were successfully related via the SDP TF modelling approach to predict both the short and long-term volume changes. The TF models considered in these two cases can be used for both long-term (up to 2 years) and short-term (1 month ahead) forecasting applications, respectively. Furthermore, the results of this study show that monthly volumetric changes of the beach profiles at Duck that extend beyond the average position of the closure depth are predominantly related to longshore processes.

The SDP TF models presented in this paper may be refined further by modelling the associated noise term to account for uncertainty in the data and other system inputs. However, the fact that the prediction and forecasting performances of these models are good, despite these limitations, is a testament to the robustness of this method. Further research is required to validate the applicability of this modelling approach for predicting the beach morphology at other coastal sites exhibiting varied environmental conditions (e.g. stronger tidal influences).

Acknowledgements

TEAMS Consultants of Sri Lanka are gratefully acknowledged for providing the funding to support this study. We also acknowledge the skill and dedication of the staff at the Field Research Facility at Duck, North Carolina, USA, in collecting and compiling the field data. We also thank Prof. Peter Young and Dr. Howard Southgate for their useful comments and discussions.

Abbreviations

ANN - Artificial Neural Networks

FRF - Field Research Facility

FIS - Fixed Interval Smoothing

RMS - Root-Mean-Square

SRIV - Simplified Recursive Instrumental Variable

SDP - State Dependent Parameter

TF - Transfer Function

YIC - Young Information Criterion

Notation

$a_i, i = 1,2,..n$	denominator state-dependent parameters
$A(v_k, z^{-1})$	denominator polynomial of the SDP TF model
$b_j, j = 0,1,..m$	numerator state-dependent parameters
$B(w_k, z^{-1})$	numerator polynomial of the SDP TF model
B	berm height
C_{Go}	deep water group wave velocity
C_*	wave celerity in water depth,
g	acceleration due to gravity (m/s^2)
G	sediment longshore diffusivity (m^2/unit of time)
h_*	closure depth
H_o	deep water wave height (m)
H_b	breaking wave height (m)
H_s	significant wave height (m)
k	discrete-time sample (months)
K	sediment transport coefficient
m	order of numerator TF polynomial
n	order of denominator TF polynomial
p	porosity
R_T^2	coefficient of determination
s	specific gravity
S	surge (m)

t	time
Δt	sampling time interval
T_p	peak period (s)
u	TF input variable
U	cross-shore and alongshore contributions of the normalised wave energy
V	beach profile volume per unit of shoreline (m^3)
$v_{i,k}, i = 1,2,...n$	state variables
$w_{j,k}, j = 0,1,...m$	state variables
WL	water level (m)
x	alongshore distance (m)
Δx	sampling interval between adjacent profile lines
y	TF output variable
z^{-1}	backward shift operator
α_o	wave approach angle relative to the true north
β_o	azimuth of an outward normal drawn to the shoreline
θ	wave direction (degrees relative to the FRF pier)
κ	ratio of breaking wave height to local water depth
δ	pure time delay
ξ	noise term

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Table Captions

Table 1. Comparison of the linear and nonlinear TF models estimated between beach volume and P for Profile 62.

Figure Captions

Figure 1. Bathymetry of the Field Research Facility at Duck showing the locations of Profile lines 58, 62, 188 and 190.

Figure 2 Time series of monthly beach volume at Profiles 58, 62, 188 and 190.

Figure 3. State-dependent parameters (or SDP gain) associated with the wave forcing term P. This SDP vs beach volume (Profile 62) relationship (red circles) characterises the nonparametric estimation of the input nonlinearity resulting from the dependence of P on the antecedent beach volume. The standard error boundaries associated with the SDP gain are shown by the dotted black lines. The shape-preserving cubic interpolant fitted to these data, for purposes of estimating the SDP gain during model validation and forecasting, is also shown (black line). The corresponding SDP gain determined from this interpolant curve (i.e. "interpolated parameters") are marked by the blue diamonds.

Figure 4. Time-variation of the state-dependent parameters (SDP gain), associated with P, in relation to beach volume and P data for the period between 1985 and 2001. The SDP gain magnitude increases during periods when the beach volume is larger than 2270 m^3 indicating stronger nonlinearity under these conditions.

Figure 5. Comparison of time series of observed beach volume of Profile 62, wave forcing P and the effective nonlinear input of P. The latter represents the effective proportion of P that directly influences the beach volume of Profile 62.

Figure 6. Comparison of the nonlinear SDP TF (Case 1) and linear TF (Gunawardena et al., 2008) model performance in fitting the beach volume of Profile 62. Here, the beach volume predicted by these models during model calibration and validation are compared with observed data. The corresponding standard error boundaries estimated are represented by the respectively coloured dotted lines.

Figure 7: DHR model output and forecasts for P. The vertical black line represents the start of the 2 year forecasting horizon. The errors corresponding to the calibrated model output and the forecasts (dotted red line) are offset by -6 m^3/s.

Figure 8. Comparison of the beach volumes forecast by the nonlinear SDP TF model (Case 1) and linear TF model (Gunawardena et al., 2008), and the SDP TF model validation results (Case 1) for Profile 62 over the 2 year period between September 2001 and September 2003. The standard errors associated with the volume forecasts and model validation are presented by the respectively coloured and marked dotted lines.

Figure 9. Comparison of beach profiles during periods of high and low beach volume. It can be seen that the beach profile comprises two prominent nearshore bars during periods of high beach volume (> 2270 m^3) (e.g. June 1995, September 1995, April 1996 and February 1997). By contrast, periods of low volume (< 2270 m^3) are associated with a flatter beach profile and/or less well-developed sand bars (e.g. July 1986 and September 1987).

Figure 10. Time series of beach volume for Profiles 62, 58 and 188 and the longshore sediment diffusivity in deep water between 1983 and 1997. These data were used in the SDP TF model calibration in Case 2).

Figure 11. State-dependent parameters (or SDP gain) associated with the volume of Profile 58 (red circles), fitted cubic interpolant (black line), and interpolated SDP

gain (used for model validation) (blue diamonds). The standard error boundaries associated with the SDP gain are shown by the black dotted lines. This SDP vs the longshore sediment diffusivity (G) relation characterises the nonparametric estimation of the input nonlinearity resulting from the dependence of the beach volume of Profile 58 on G .

Figure 12. State-dependent parameters (or SDP gain) associated with the volume of Profile 188 (red circles), fitted cubic interpolant (black line) and interpolated SDP gain (used for model validation) (blue diamonds). The standard error boundaries associated with the SDP gain are shown by the black dotted lines. This SDP vs the longshore sediment diffusivity (G) relation characterises the nonparametric estimation of the input nonlinearity resulting from the dependence of the beach volume of Profile 188 on G .

Figure 13. Time variation of the state-dependent parameters associated with Profiles 58 and 188.

Figure 14. Comparison of the observed beach volume of Profiles 188 and 58 (original inputs) with the transformed *effective* inputs estimated via the input nonlinearities. Here, the observed and effective beach volume data are scaled to have a maximum value of 1 for comparison. It should be noted that, the effective input associated with Profile 188 comprises negative values at certain times (corresponding to the negative SDP estimated).

Figure 15. Beach volume (Profile 62) predicted using the nonlinear SDP TF model (Case 2). Here, the beach volumes estimated by this model during the calibration (blue) and validation (red) stages are compared with the observed data. The corresponding standard errors are represented by the respectively coloured dotted lines.

Figure

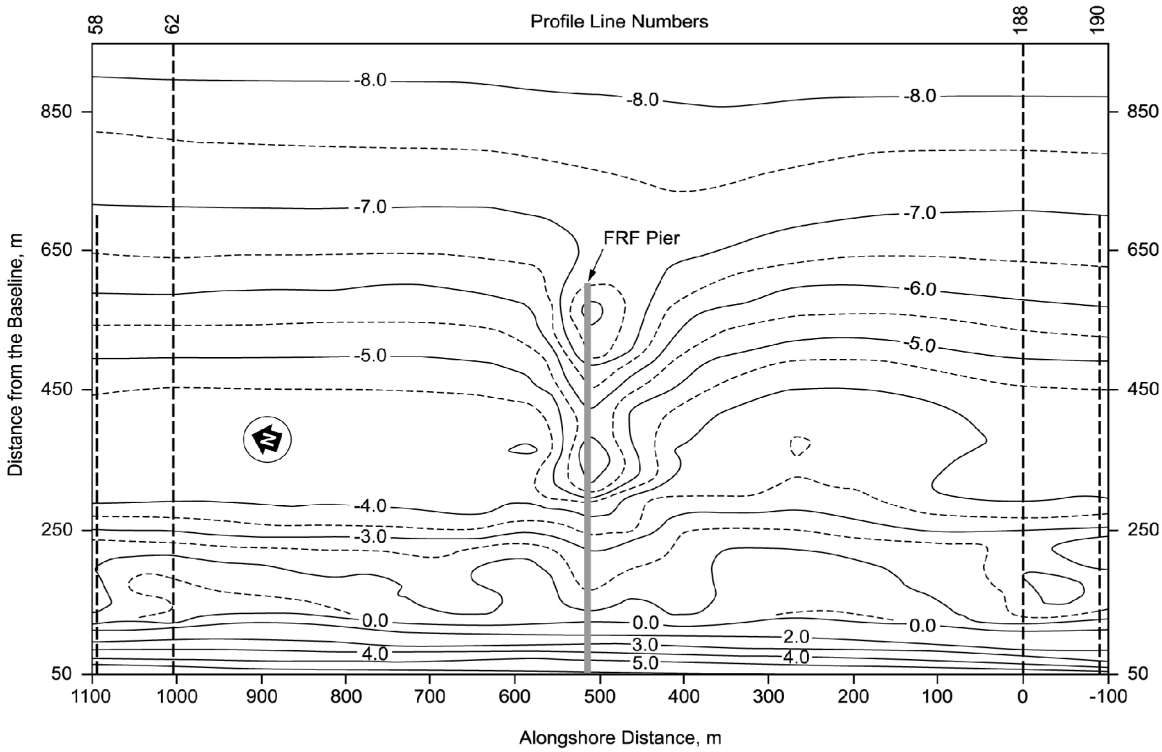


Figure 1. Bathymetry of the Field Research Facility at Duck showing the locations of Profile lines 58, 62, 188 and 190.

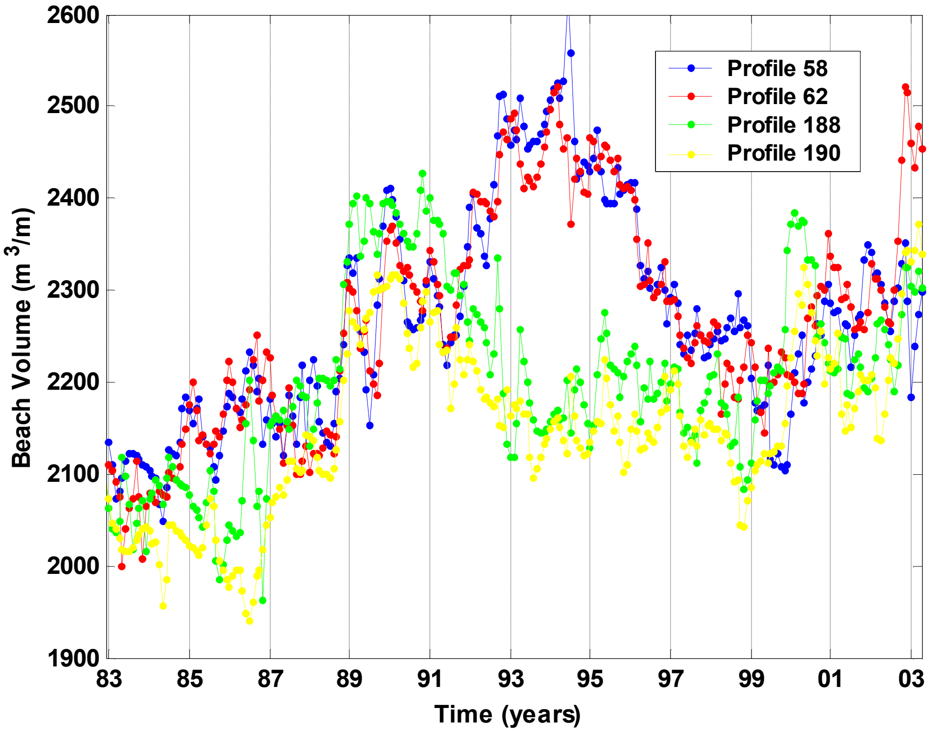


Figure 2 Time series of monthly beach volume at Profiles 58, 62, 188 and 190.

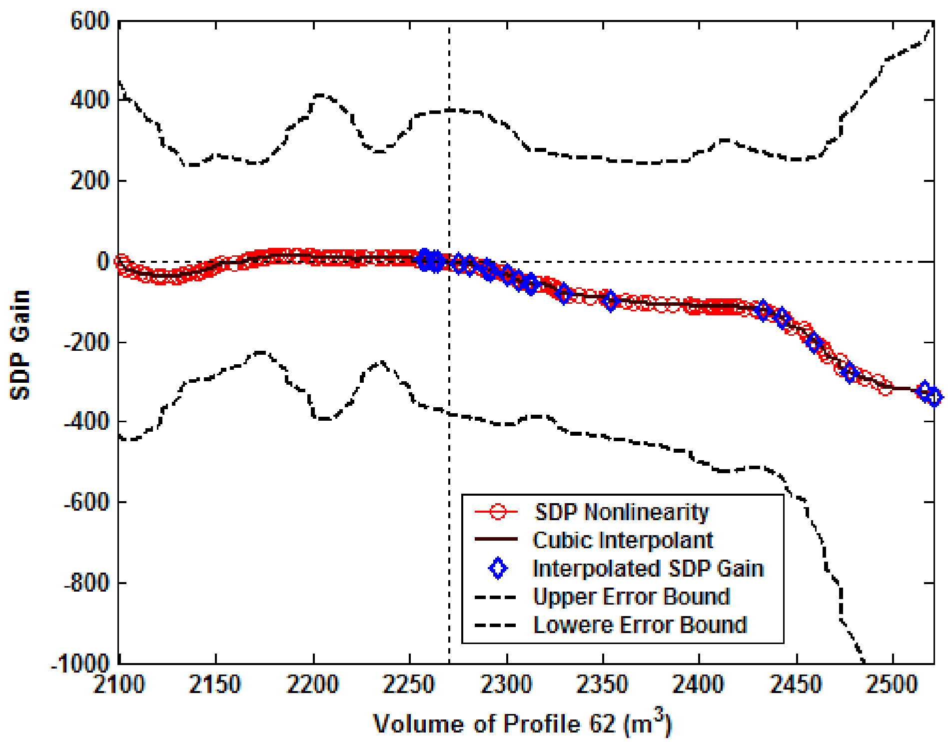


Figure 3. State-dependent parameters (or SDP gain) associated with the wave forcing term P. This SDP vs beach volume (Profile 62) relationship (red circles) characterises the nonparametric estimation of the input nonlinearity resulting from the dependence of P on the antecedent beach volume. The standard error boundaries associated with the SDP gain are shown by the dotted black lines. The shape-preserving cubic interpolant fitted to these data, for purposes of estimating the SDP gain during model validation and forecasting, is also shown (black line). The corresponding SDP gain determined from this interpolant curve (i.e. "interpolated parameters") are marked by the blue diamonds.

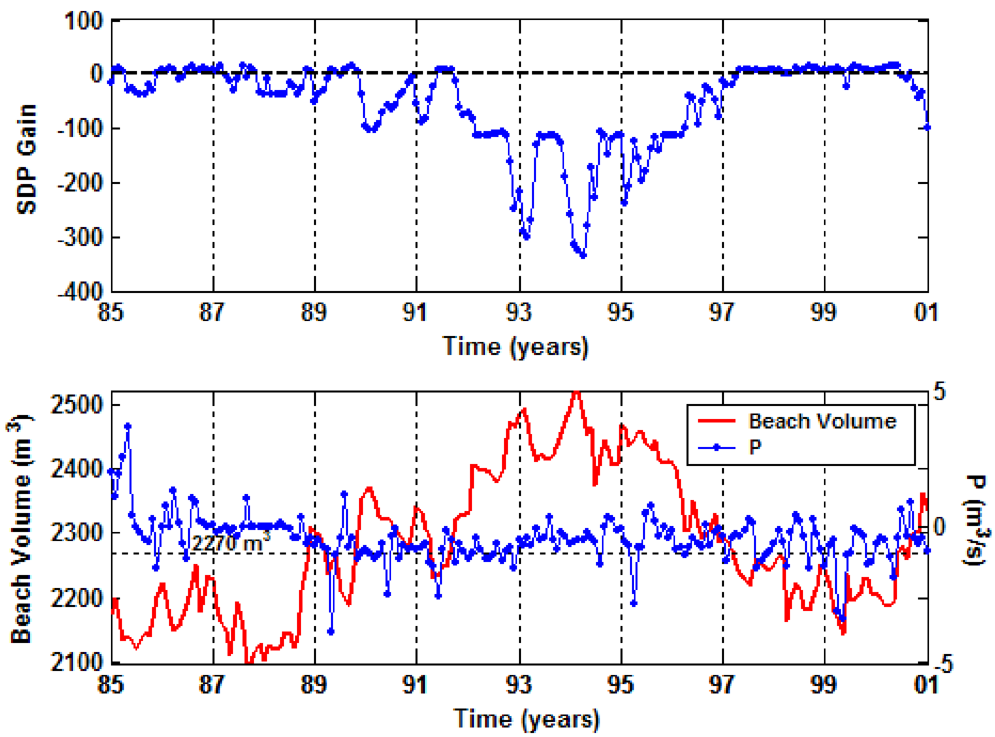


Figure 4. Time-variation of the state-dependent parameters (SDP gain), associated with P, in relation to beach volume and P data for the period between 1985 and 2001. The SDP gain magnitude increases during periods when the beach volume is larger than 2270 m^3 indicating stronger nonlinearity under these conditions.

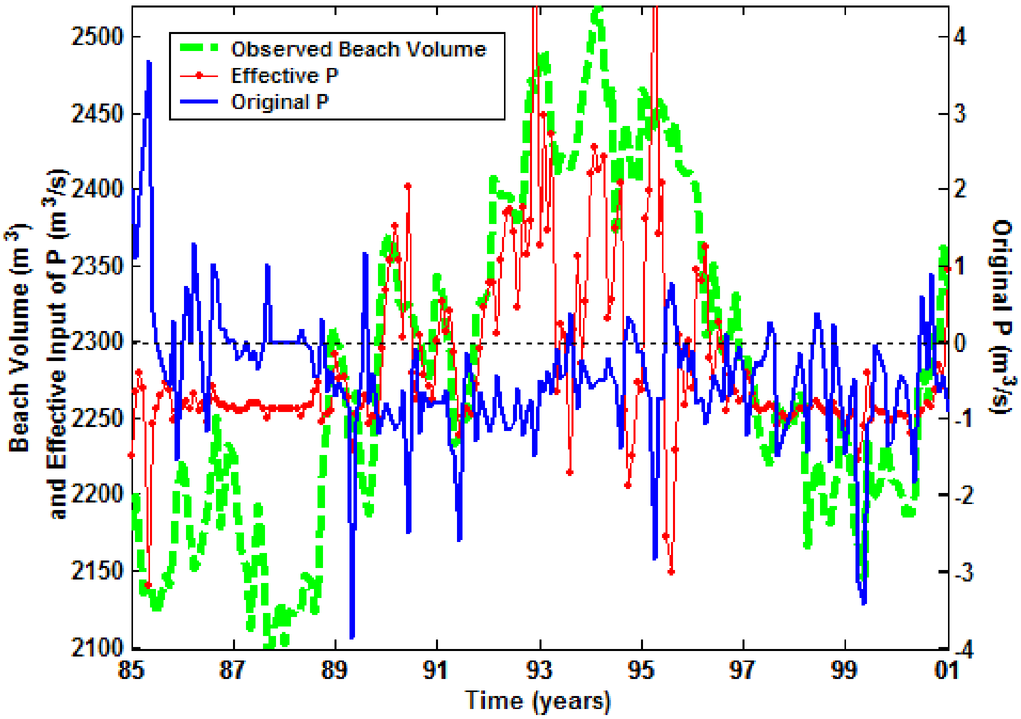


Figure 5. Comparison of time series of observed beach volume of Profile 62, wave forcing P and the effective nonlinear input of P. The latter represents the effective proportion of P that directly influences the beach volume of Profile 62.

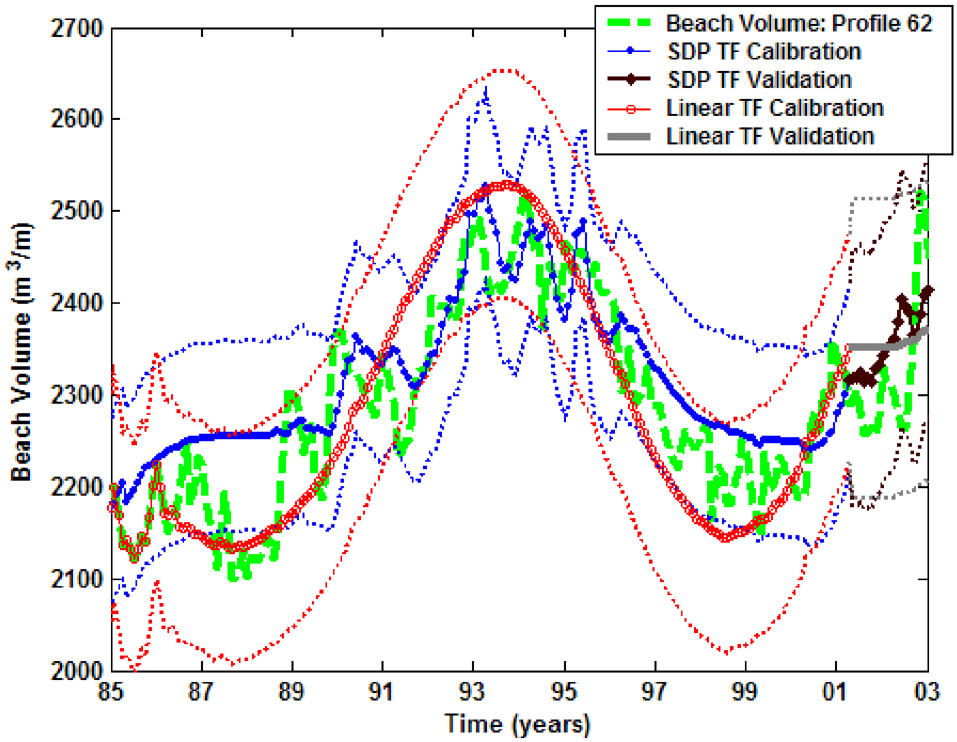
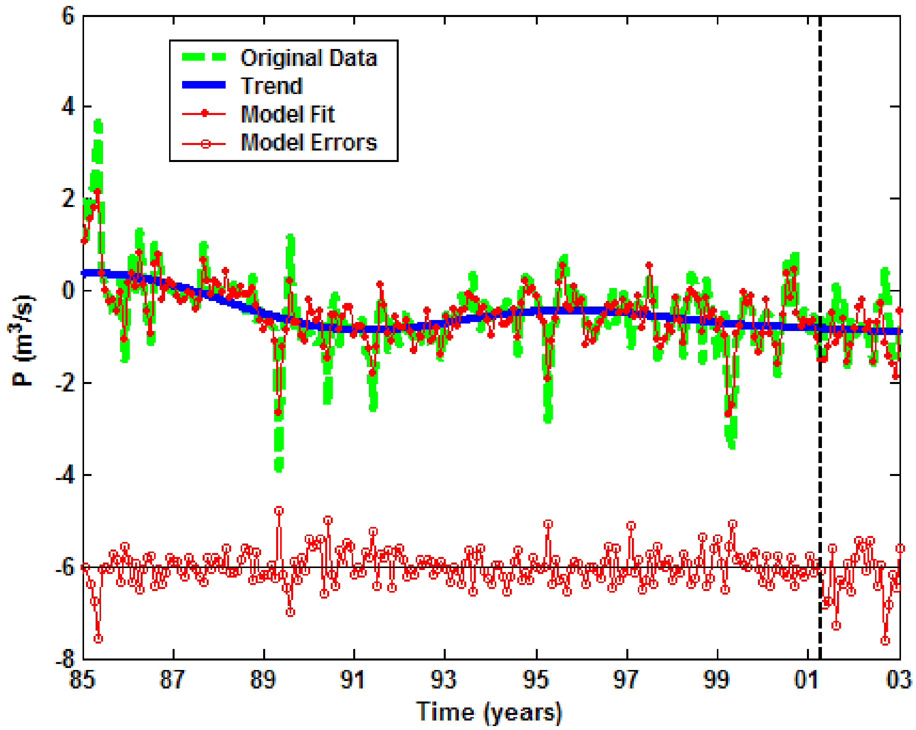


Figure 6. Comparison of the nonlinear SDP TF (Case 1) and linear TF (Gunawardena et al., 2008) model performance in fitting the beach volume of Profile 62. Here, the beach volume predicted by these models during model calibration and validation are compared with observed data. The corresponding standard error boundaries estimated are represented by the respectively coloured dotted lines.



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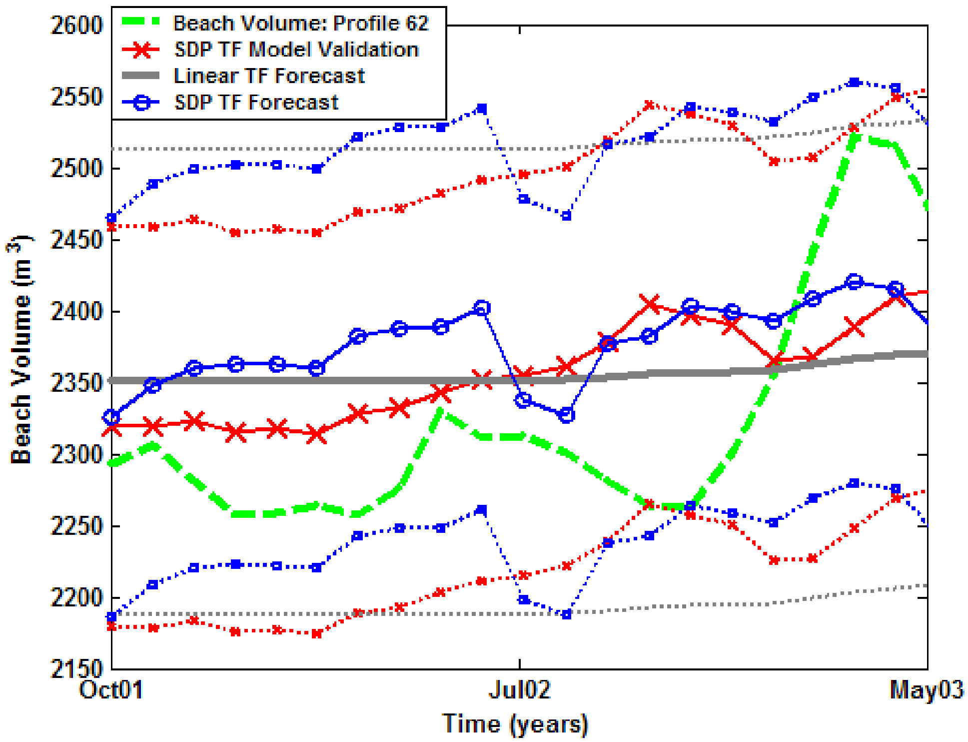

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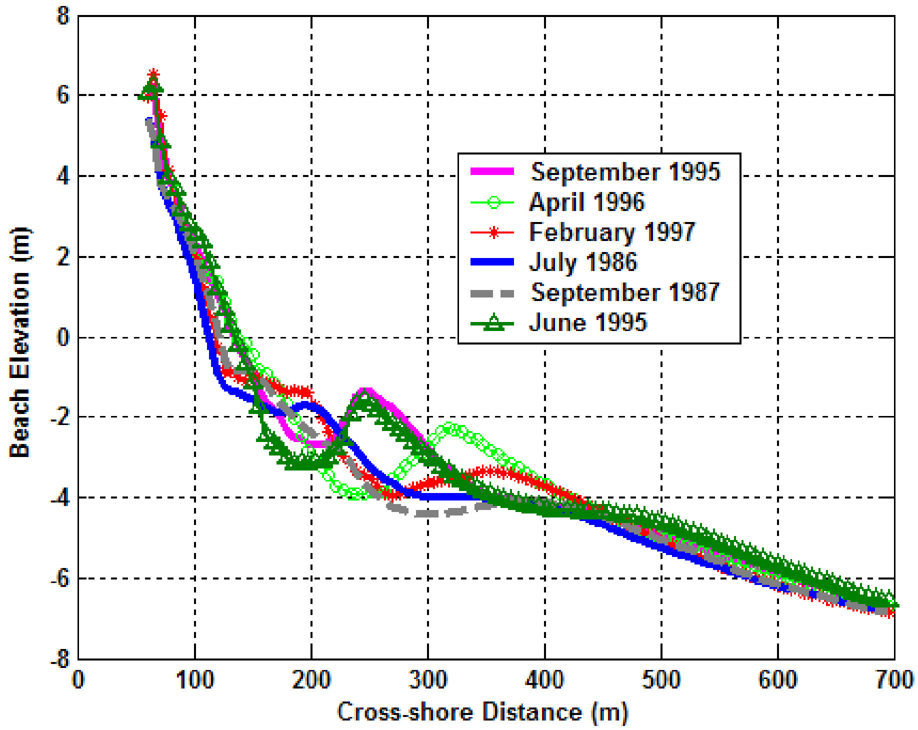

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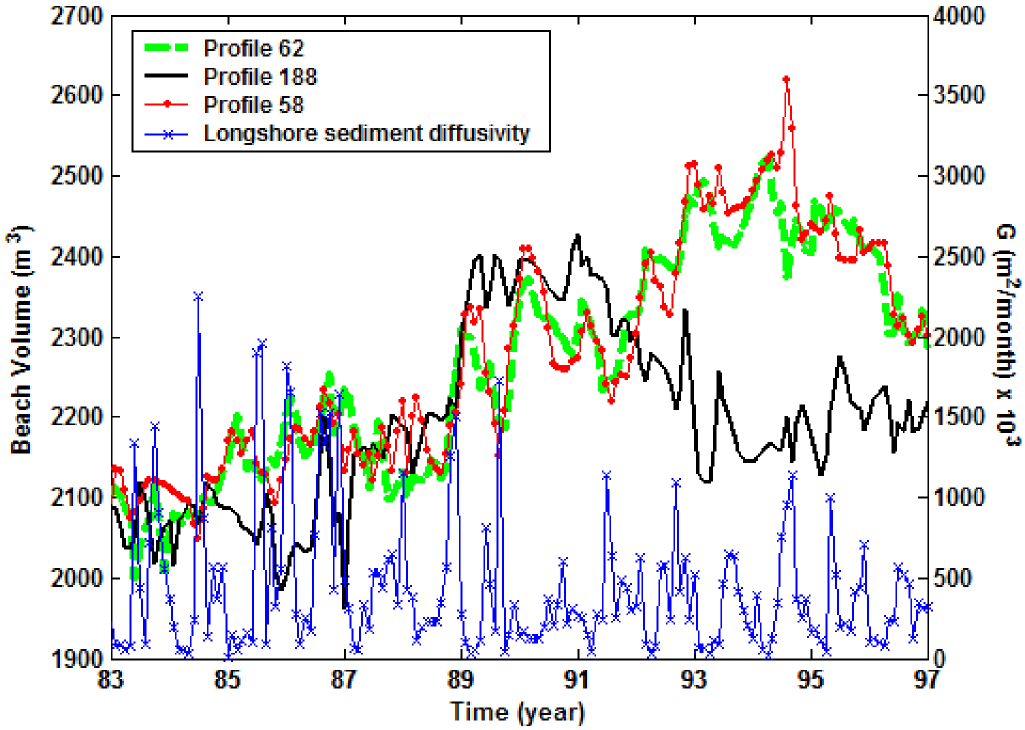

Figure 10. Time series of beach volume for Profiles 62, 58 and 188 and the longshore sediment diffusivity in deep water between 1983 and 1997. These data were used in the SDP TF model calibration in Case 2).

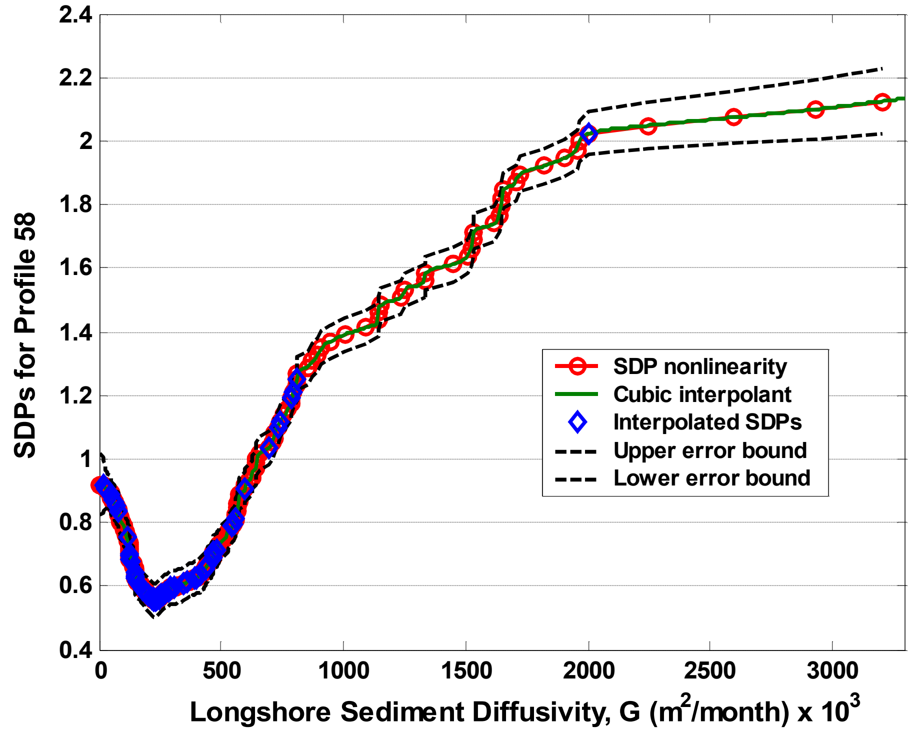

Figure 11. State-dependent parameters (or SDP gain) associated with the volume of Profile 58 (red circles), fitted cubic interpolant (black line), and interpolated SDP gain (used for model validation) (blue diamonds). The standard error boundaries associated with the SDP gain are shown by the black dotted lines. This SDP vs the longshore sediment diffusivity (G) relation characterises the nonparametric estimation of the input nonlinearity resulting from the dependence of the beach volume of Profile 58 on G.

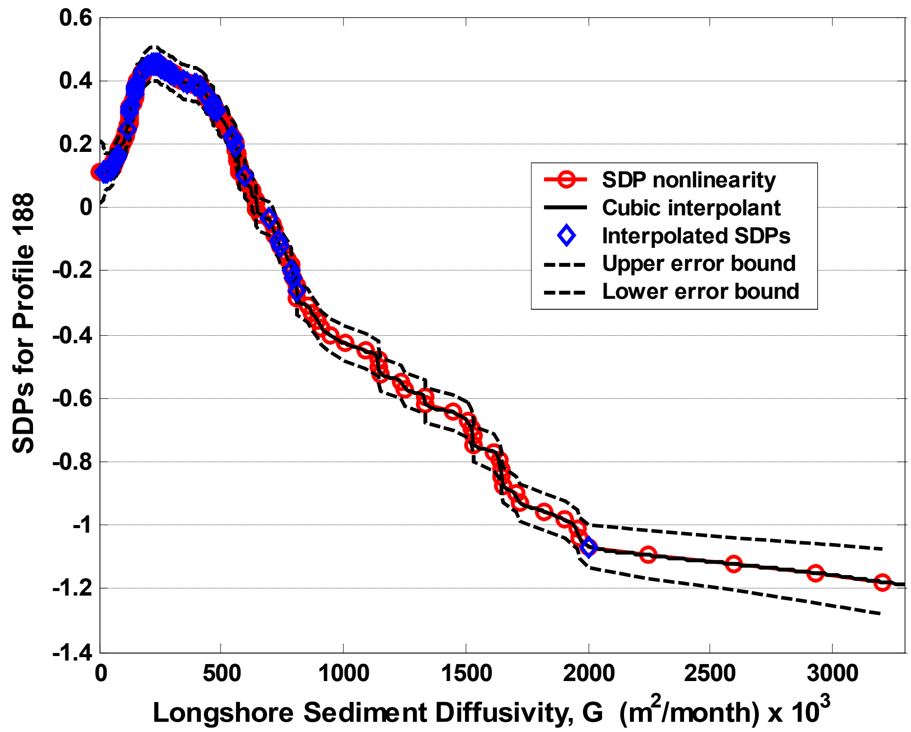

Figure 12. State-dependent parameters (or SDP gain) associated with the volume of Profile 188 (red circles), fitted cubic interpolant (black line) and interpolated SDP gain (used for model validation) (blue diamonds). The standard error boundaries associated with the SDP gain are shown by the black dotted lines. This SDP vs the longshore sediment diffusivity (G) relation characterises the nonparametric estimation of the input nonlinearity resulting from the dependence of the beach volume of Profile 188 on G.

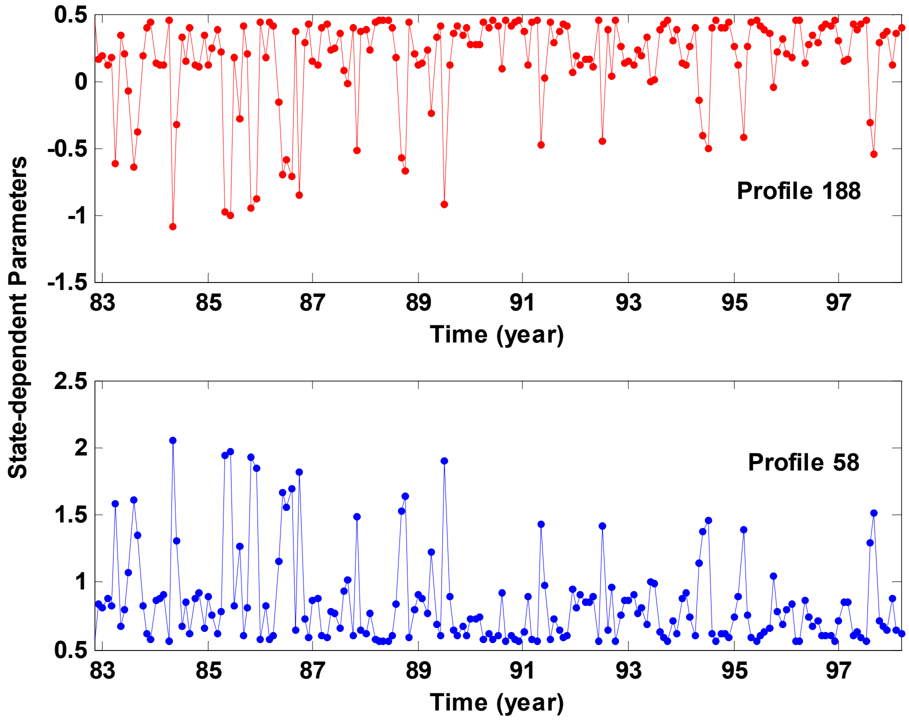

Figure 13. Time variation of the state-dependent parameters associated with Profiles 58 and 188.

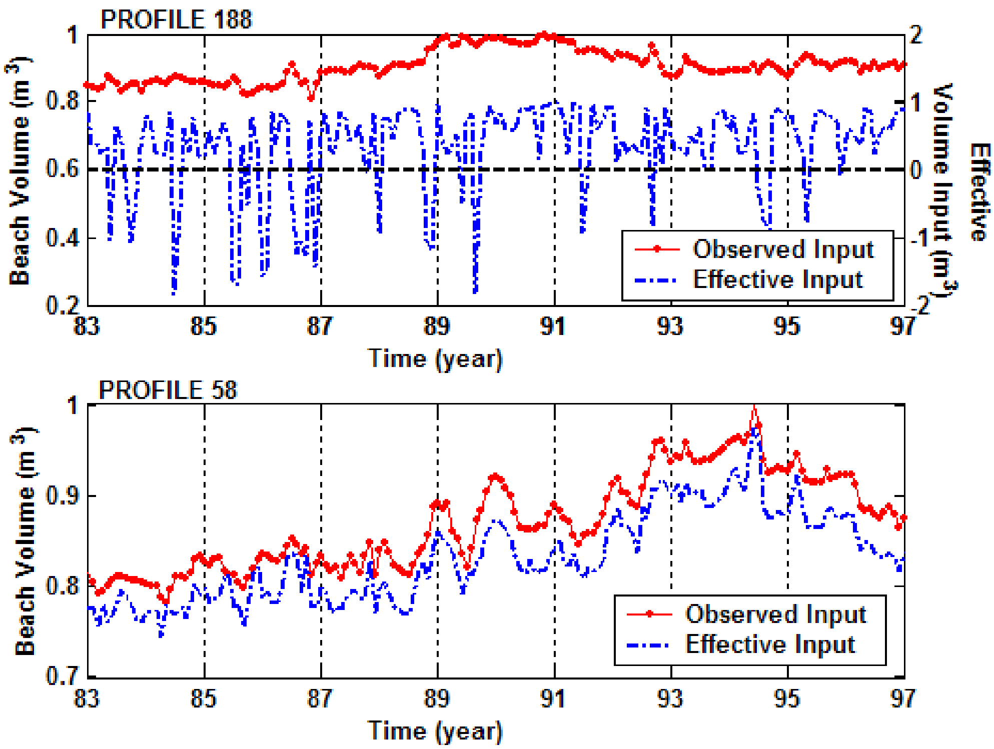

Figure 14. Comparison of the observed beach volume of Profiles 188 and 58 (original inputs) with the transformed *effective* inputs estimated via the input nonlinearities. Here, the observed and effective beach volume data are scaled to have a maximum value of 1 for comparison. It should be noted that, the effective input associated with Profile 188 comprises negative values at certain times (corresponding to the negative SDP estimated).

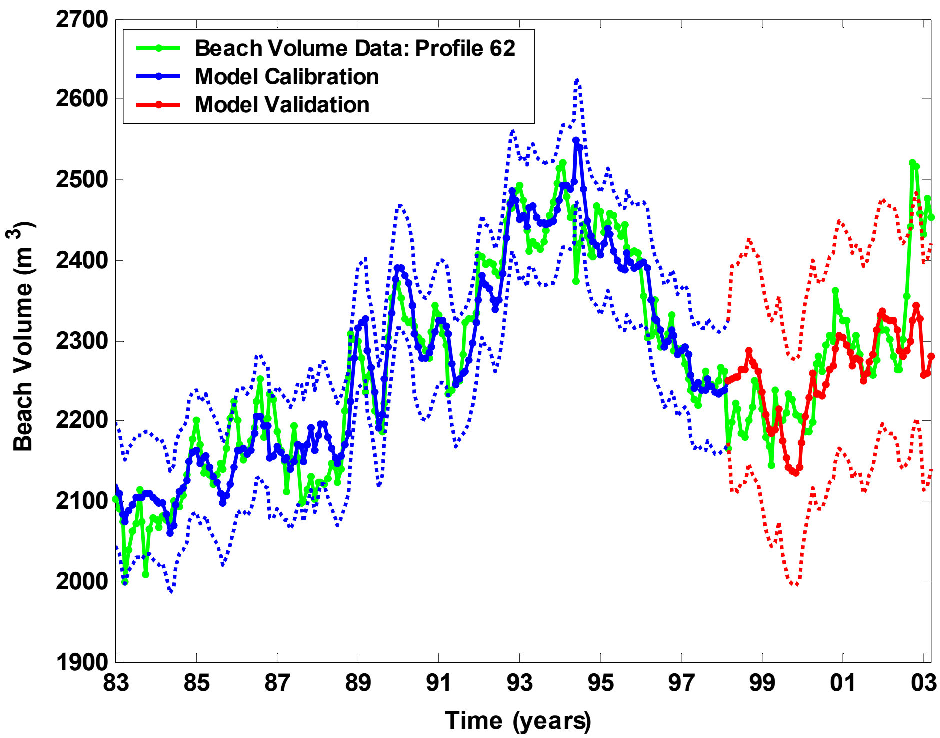

Figure 15. Beach volume (Profile 62) predicted using the nonlinear SDP TF model (Case 2). Here, the beach volumes estimated by this model during the calibration (blue) and validation (red) stages are compared with the observed data. The corresponding standard errors are represented by the respectively coloured dotted lines.

Table

Table 1. Comparison of the linear and nonlinear TF models estimated between beach volume and P for Profile 62.

Model Type	**Model Structure**	**Calibration**			**Validation (2 yrs): RMS error m^3/m**	**Forecasting (2 yrs): RMS error m^3/m**
		R_T^2	**YIC**	**RMS error m^3/m**		
Linear	[4 3 8]	0.67	-1.39	70.3	81.2	90.3
Nonlinear	[1 1 0]	0.82	-6.05	61.1	72.0	75.9