AN IMPROVED MARKOV PROCESS FOR MODELING UNCERTAIN MULTI-STATE DEGRADATION OF EQUIPMENT

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Abstract

To address the challenges associated with multi-state evolution and uncertainty in equipment degradation within complex systems, this study proposes a Markov modeling approach incorporating state acceleration factors. Building on the traditional Markov framework, state acceleration factors are incorporated to dynamically modify the transition rates across degradation states. This approach preserves the simplicity of the exponential distribution while effectively capturing the non-stationary characteristics of degradation rates, thereby improving the model's accuracy in representing actual equipment degradation behavior. Furthermore, the proposed degradation model is embedded into a Dynamic Bayesian Network (DBN) to enable system-level reliability evaluation, with a case study and sensitivity analysis performed to demonstrate its effectiveness and applicability.

1 Introduction

As industrial systems become increasingly intelligent and complex, their structural complexity continues to expand [1]. Large-scale complex systems exhibit highly intricate structures, operational states, performance characteristics, and usage environments[2], making issues related to operational reliability and safety more prominent. The risks and losses induced by unreliability escalate, potentially leading to severe accidents and substantial economic damage. Currently, research on the operational behavior and failure mechanisms of these systems is deepening. However, traditional binary state recognition, which categorizes system performance simply as "normal operation" or "complete failure" is insufficient to represent the gradual changes in system performance[3]. In practical engineering scenarios, equipment performance may pass through several intermediate states, such as intact, early failure, general failure, and severe failure. The evolution, performance, and efficiency of each state differ[4]. During the initial operation phase, equipment performance indicators remain stable, allowing for efficient and stable task execution. However, over time and under environmental influences, equipment may enter the early failure phase, where performance starts to degrade, although overall functionality remains largely unaffected. In the general failure state, equipment performance indicators deviate significantly, leading to reduced operational efficiency. Severe failure, in contrast, signals critical performance degradation, with the equipment at risk of sudden shutdown, severely disrupting system operations.

For structurally complex systems, some critical components operate in unique locations and environments, making their operational states difficult to monitor. These components experience multi-state degradation during operation, which is inherently uncertain. Moreover, complex system components are often produced in small batches with high customization, resulting in limited and uncertain failure data. By analyzing fault evolution patterns and performance degradation, the multi-state performance of both equipment and systems can be identified, enabling early warnings before severe failures occur. This approach facilitates condition-based maintenance, enhances maintenance efficiency, and reduces risk.

At present, the reliability assessment theory of multi-state systems has been established, with approaches such as Monte Carlo simulation method [6], multi-state Bayesian Network (BN), Markov model [4] being widely applied. Huang et al [7] proposed a dynamic reliability assessment approach that integrates Dynamic Bayesian Networks (DBN) with evidence theory. The method quantifies component states using evidence theory and employs a Markov model to infer state transitions for evaluating system dynamic reliability. Zuo et al.[8] modeled the multi-state degradation process of nodes using dynamic fuzzy set functions, combined with BN inference, to perform dynamic reliability assessment of complex systems under concurrent degradation and uncertainty.

Current approaches for modeling equipment multi-state degradation primarily include Markov processes, semi-Markov processes, higher-order Markov chains, UGF, and physics-based modeling. In standard continuous-time Markov processes, state sojourn times follow exponential distributions [9]. Li et al.[10] developed a method for modeling the reliability of multi-state components by combining Markov processes with DBN. While semi-Markov processes allow state residence times to obey arbitrary distributions, they involve extensive computational operations when describing state transitions, leading to prohibitive computational complexity for systems with intricate structures, numerous states, and long time spans. To address this, Bo et al.[3] introduced deep neural networks to replace the cumbersome convolution calculations in traditional semi-Markov models for multi-state series-parallel systems, while employing higher-order Markov chains to account for the influence of multiple historical states on current states. Yakovyna et al.[11] used higher-order Markov processes to model complex dependencies by incorporating historical states, then applied equivalent order-reduction techniques to transform them into solvable first-order processes, thereby improving system behavior modeling accuracy. Liu et al.[12] proposed a reliability analysis framework by combining UGF with BN, where UGF handles the reliability analysis of underlying components.

Although existing methods each have theoretical advantages, traditional Markov processes generally assume constant state transition rates for equipment[9]. In reality, however, failure probability increases as equipment experiences wear, fatigue, and aging, with internal micro-damage accumulating over time[13]. During the early failure stage, degradation progresses slowly, but in the mid-to-late stages, crack propagation and severe fatigue lead to significantly accelerated degradation. Observational data from actual equipment often show an increasing failure rate curve, corresponding to the later stage of the bathtub curve[14]. Therefore, using a single fixed degradation rate to describe transitions among different states fails to accurately reflect the actual degradation behavior. While semi-Markov processes using Weibull or log-normal distributions are commonly employed to model accelerated degradation or aging, these approaches significantly increase modeling complexity and require more experimental data to fit the sojourn time distributions, in addition to more intensive analytical and computational work[11].

Thus, this study proposes introducing state acceleration factors within the Markov framework. Specifically, by incorporating corresponding acceleration factors into each equipment state, the base failure rate is adjusted. By maintaining the exponential distribution assumption for state sojourn times, different degradation rates are assigned to each state, enhancing degradation modeling accuracy while preserving model practicality. The improved method is further embedded into a DBN to enable reliability evaluation of complex systems. Compared with conventional Markov models that assume constant transition rates across all degradation stages, the proposed method incorporates state acceleration factors to dynamically regulate transition intensities according to the current degradation state. This enhances the model's ability to capture real-world nonlinear degradation behaviors, such as accelerated damage accumulation in later stages. Unlike semi-Markov models, which require complex distribution fitting and more data to describe sojourn times, our method retains the computational simplicity of the exponential distribution while offering a more flexible and realistic representation of state-dependent degradation. Therefore, the proposed approach achieves a practical balance between modeling accuracy and implementation efficiency, making it suitable for systems with limited failure data or real-time evaluation requirements.

2 Methodology for multi-state degradation modeling and system reliability evaluation

2.1 Analysis of method applicability and limitations

To more accurately characterize the acceleration of equipment degradation over time, this study introduces state acceleration factors within a continuous-time Markov process framework to adjust the transition rates between different equipment states. This approach preserves the simplicity of exponential distribution modeling while offering a more realistic representation of failure characteristics across various degradation stages. However, the application of state acceleration factors is contingent on certain prerequisites.

State transitions still adhere to the memoryless assumption, with degradation acceleration reflected solely through the adjustment of failure intensity rates across states. As such, this method is applicable to systems whose overall degradation process can be approximated by piecewise exponential. This includes mechanical, electrical, and structural components subject to physical degradation mechanisms such as fatigue, wear, corrosion, and aging, where failure rates tend to progressively increase at different stages. Typical scenarios where this approach applies include pumps and valves, generator components, and avionics system modules.

The method is unsuitable for equipment with significant phase transitions or multi-modal degradation characteristics. For example, systems subjected to sudden shock loads or extreme random environmental disturbances often feature complex failure mechanisms that cannot be uniformly adjusted via simple rate modifiers. Moreover, for equipment lacking baseline failure rate estimates or with ambiguous state definitions, it becomes difficult to reasonably assign acceleration factors, potentially leading to model distortion.

2.2 Modeling of multi-state degenerative process

The equipment degradation process is divided into four states according to the performance levels: "0", "1", "2", and "3". Among these, states "1", "2", and "3" represent early failure, general failure, and severe failure states, respectively, while state "0" indicates perfect operating condition. Each equipment starts in "0", and over time, it may transition to "1", "2", or "3". The state transition diagram of each piece of equipment is shown in Fig. 1.

The following assumptions are made based on the Markov process:

State transitions are assumed to follow a first-order Markov process and state residence times follow the exponential distribution.

(2) Equipment and systems may randomly fail in any operating state.

(3) This paper focuses on proposing a reliability modeling framework that introduces state acceleration factors and verifies the feasibility and applicability of this method in system reliability modeling and inference. Considering the differences in degradation characteristics and acceleration mechanisms of various types of equipment, the state acceleration factors can be obtained by fitting failure models under different equipment states and converting them into state-level acceleration coefficients, or by analyzing historical maintenance data and failure records to estimate the relative degradation rates between states. This study does not delve into the detailed fitting process at this stage.

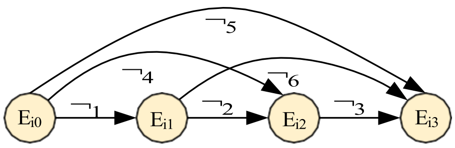


Figure 1. Equipment state transition diagram

According to the Markov process, let the current time be t, the interval between successive time be $\mathrm{M}t$, and the device failure rate be A, the node transfer probabilities within two consecutive slices are presented in Table 1[15].

Table 1. Transfer relationships among non-repairable discrete nodes

t		0	1	2	3
	0	$e^{-(A_1+A_4+A_5)\mathrm{M}t}$	0	0	0
	1	$\frac{A_4}{A_1+A_4+A_5}(1-e^{-(A_1+A_4+A_5)\mathrm{M}t})$	$e^{-(A_2+A_6)\mathrm{M}t}$	0	0
$t+\mathrm{M}t$	2	$\frac{A_5}{A_1+A_4+A_5}(1-e^{-(A_1+A_4+A_5)\mathrm{M}t})$	$\frac{A_6}{A_2+A_6}(1-e^{-(A_2+A_6)\mathrm{M}t})$	$e^{-A_3\mathrm{M}t}$	0
	3	$\frac{A_1}{A_1+A_4+A_5}(1-e^{-(A_1+A_4+A_5)\mathrm{M}t})$	$\frac{A_2}{A_2+A_6}(1-e^{-(A_2+A_6)\mathrm{M}t})$	$1-e^{-A_3\mathrm{M}t}$	1

$$\begin{cases} A_1 + A_4 + A_5 = A \\ A_2 + A_6 = A_P \\ A_3 = A_F \end{cases} \quad (1)$$

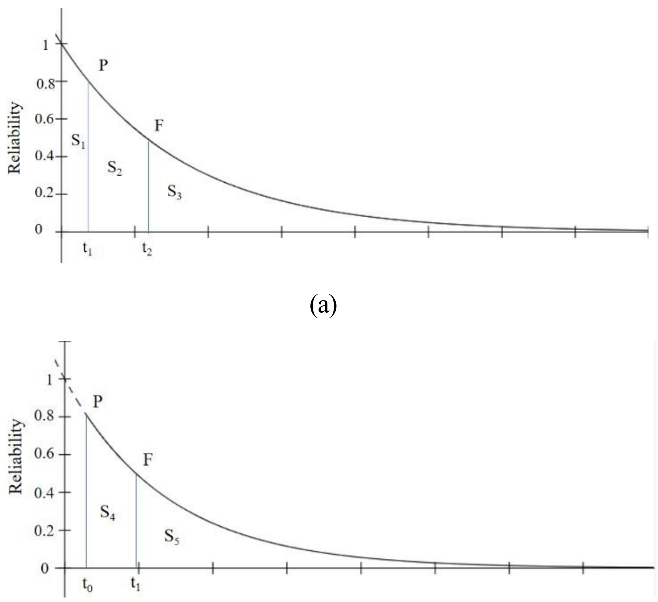
It is assumed that potential failure points (P) and functional failure points (F) exist during equipment degradation to demarcate different stages of state transition. The potential failure point P indicates the transition from state 1 to state 2, while the functional failure point F represents the boundary between the state 2 and state 3.

Eqs. (1) describe the relationships among the failure rates of the various states. During equipment degradation, factors like material fatigue, component wear, and stress concentration gradually accumulate, causing failure rates to increase. Consequently, equipment and systems become more susceptible to functional loss or critical performance degradation. To capture the accelerating failure behavior during degradation, a state acceleration factor α_i is introduced to adjust the transition rates between different states. The state acceleration factor is a proportional coefficient that adjusts the base failure rate based on degradation severity, reflecting the dynamic increase in failure risk across performance states. The failure rate for state i is calculated as shown in Equation (2), where $\alpha_i > 1$ and increases progressively with worsening state conditions.

$$A_i = A * \alpha_i \quad (2)$$

State transition probabilities are determined based on the integral areas under the reliability curve within adjacent state intervals. Specifically, the transition probability from a given state to a future state is determined by the area under the reliability function for the relevant interval. As shown in Figure 2, subfigures (a) and (b) illustrate reliability curves during degradation from states 0 and 1, respectively. These curves represent the area enclosed by the degradation curve and the corresponding state interval. The relationships among failure rates are governed by Equation (3). Based on these equations and the selected time step, the state transition matrix for each equipment unit is determined.

$$\begin{cases} A_1 : A_4 : A_5 = S_1 : S_2 : S_3 \\ A_2 : A_6 = S_4 : S_5 \end{cases} \quad (3)$$



(a)

(b)

Figure 2. The degradation curve of a device

2.3 Dynamic reliability modeling of multi-state complex systems

System reliability modeling is performed to demonstrate the applicability of the proposed degradation method at the system level. DBNs, extended by incorporating temporal evolution, provide a unique probabilistic framework for representing uncertainty. They describe the likelihood of equipment being in different failure states over time, aligning well with the variable and uncertain nature of complex system behaviors. Thus, system reliability is analyzed using a DBN-based model.

The DBN model is constructed based on the structural logic of the complex system. As shown in Figure 3, equipment states are defined as root nodes, while the system state is represented as the output node. The task execution time is divided into equal time slices, and state transition probabilities between adjacent time slices are determined based on the equipment degradation process.

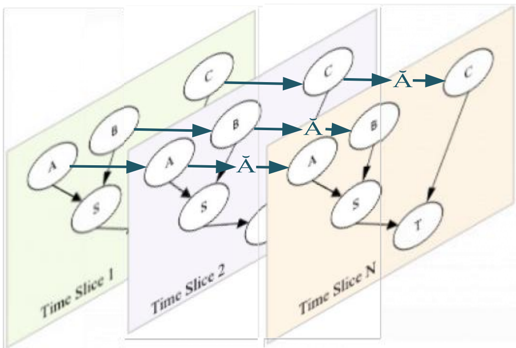


Figure 3. DBN model

In multi-state complex systems, failure of a single-component does not directly cause overall system failure, and the relationship between component states and system state is inherently uncertain. To characterize this uncertainty, a Conditional Probability Table (CPT) is constructed via a multi-source information fusion approach combining expert judgment and historical system failure data using Analytic Hierarchy Process (AHP) and Triangular Fuzzy Numbers (TFN)[15]. First, AHP is applied to assign weights to expert assessments and traditional system failure data. Experts then evaluate the impact of component failures using linguistic terms, which are mapped to fuzzy sets represented by triangular fuzzy numbers. Similarly, historical failure probabilities from traditional data sources are also mapped to fuzzy sets. These two data sources are then combined through weighted aggregation. Aggregated fuzzy values are then defuzzified and normalized to derive child node conditional probabilities given to parent nodes. This process is repeated to compute the complete CPT for the model. Fig. 4 shows the overall method flow of the proposed multi-state equipment degradation modeling and system reliability assessment.

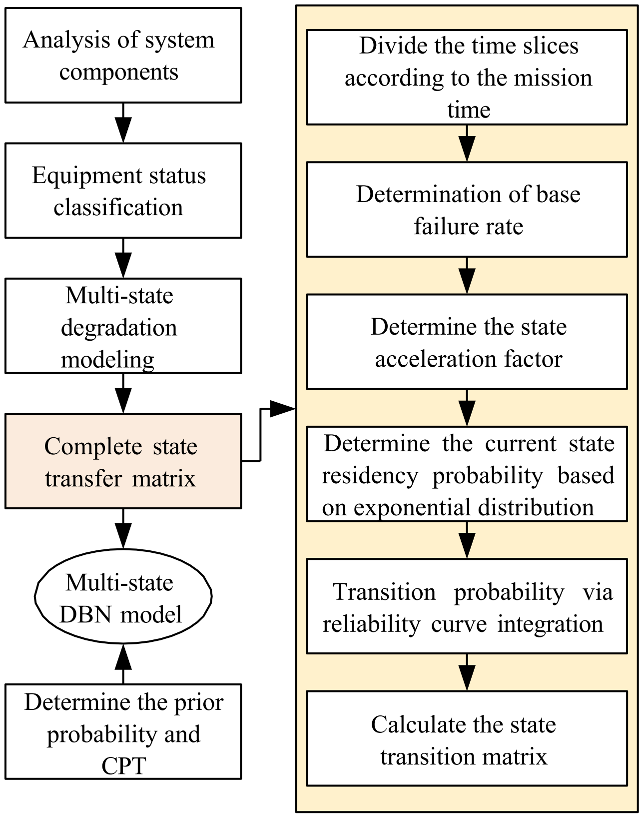


Figure 4. Flowchart of the methodology for multi-state equipment degradation and system reliability assessment

3 Case study

3.1 Analysis of the system architecture

Taking the autonomous ship navigation and positioning system as a case study, this research investigates its reliability over a 30-day maritime operation period. The system comprises several key components, as shown in Table 2. The failure parameters for these devices are determined by referencing the NPRD, OREDA, electronic equipment reliability prediction models, data handbooks, and relevant literature. The number of equipment units and baseline failure rates are provided in the corresponding Table 2.

Among these, the high-precision attitude sensor serves as a representative example of equipment primarily governed by internal physical degradation mechanisms. Over time, its internal gyroscope and accelerometer components degrade due to microstructural fatigue, electronic drift, etc., exhibiting a degradation process that progresses from slow to accelerated stages. This process is categorized into four states. Since the degradation behavior within each state is relatively stable, it is reasonable to approximate the state sojourn time with an exponential distribution.

Table 2. Equipment composition and numbering of navigation and positioning system

System	Equipment	Number	Failure Rate/h−1
Navigation and positioning system(S)	High-precision attitude sensor	E01	$9.4718*10^{-5}$
	Navigation satellite system	E02	$6.6667*10^{-4}$
	Electronic chart system	E03	$1.6672*10^{-5}$
	Automatic identification system	E04	$2.2614*10^{-5}$
	Satellite communication receiver	E05	$1.3635*10^{-5}$

3.2 *Equipment degradation process modeling*

According to the proposed approach, a Markov state transition model is established based on the baseline failure rate. For the high-precision attitude sensor, state acceleration factors are introduced. Specifically, the acceleration factor from state 1 to state 2 is set to 1.2, and the acceleration factor from state 2 to state 3 is set to 1.5. The sojourn times for other devices follow an exponential distribution. Using a 24-hour time slice, the autonomous ship's 30-day operation comprises 30 time slices. Based on the equipment failure characteristics, performance thresholds of 80% and 50% are defined at points P and F, respectively. The state transition matrix for the high-precision attitude sensor is calculated as shown in Equation (4).

$$A_o = \begin{bmatrix} 0.997729 & 0.000454 & 0.000681 & 0.001136 \\ 0 & 0.997276 & 0.001022 & 0.001702 \\ 0 & 0 & 0.996596 & 0.003404 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

3.3 *System reliability analysis*

This study employs GeNIe software to compute the reliability of an autonomous ship navigation and positioning system during autonomous sailing missions. Each root node starts degradation from state 0. Prior probabilities and state transition matrices for each root node, along with conditional probabilities for intermediate and top nodes, are input. This allows for the calculation of the probabilities of the navigation system during the phase, as presented in Table 6. The corresponding DBN model is illustrated in Fig. 5. The performance state of "state 3" is considered an unacceptable state for task completion. The reliability of the task during the autonomous sailing phase is calculated to be 0.849060.

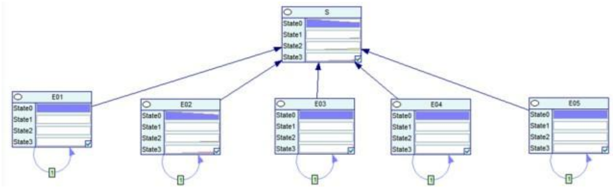


Figure 5. Fig. 5 DBN of navigation and positioning system

Table 3. Table 3 System State Probability

State	Probability
State 0	0.583802
State 1	0.138222
State 2	0.127036
State 3	0.150940

3.4 *Sensitivity analysis*

To verify the necessity and practical effect of introducing state acceleration factors, two analyses are conducted: one on baseline failure rate variation and the other on acceleration factor magnitude variation. Baseline failure rates are set at different levels to analyze how acceleration factors influence system reliability across varying equipment reliability conditions. Additionally, for a given baseline failure rate, different acceleration factor magnitudes are applied to evaluate system reliability sensitivity to acceleration factor value changes.

Table 4. System reliability for various acceleration factor settings

Acceleration Factor	$9.4718*10^{-4}$	$9.4718*10^{-5}$	$9.4718*10^{-6}$
(1.0'1.0)	0.71065710	0.84919921	0.86517300
(1.2'1.5)	0.70324018	0.84906009	0.86517151
(1.4'1.8)	0.69877868	0.84896678	0.86517051
(1.6'2.2)	0.69393600	0.84885279	0.86516926

For the high-precision attitude sensor, a failure rate of 9.4718×10^{-5} is used as the baseline value. To conduct a comparative analysis of system reliability under varying failure rate levels, the baseline failure rates are also set to 9.4718×10^{-4} and 9.4718×10^{-6}, respectively. At each failure rate level, different combinations of acceleration factors are tested. Based on the current setting (1.2 from state 1 to 2, and 1.5 from state 2 to 3), three scenarios are considered: no acceleration factors (1, 1), a moderate-to-high setting (1.4, 1.8), and a high setting (1.6, 2.2). The variation in system reliability under each combination is analyzed. The calculated system reliability values under different baseline failure rates and acceleration factor settings are summarized in Table 4.

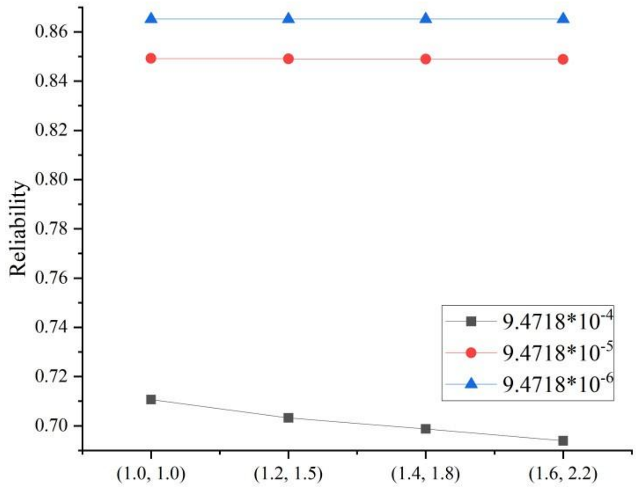


Figure 6. Fig. 6 Reliability variation chart

Sensitivity analysis in Fig. 6 shows that under high baseline failure rates, system reliability differs significantly across acceleration factor settings. Neglecting degradation acceleration clearly overestimates system reliability, challenging modeling accuracy in high-risk scenarios. At moderate failure rates, introducing acceleration factors more accurately represents the uncertain degradation process, substantially correcting system reliability estimates. Although the impact is limited under low failure rates due to the system's inherent robustness, precise modeling of the degradation process remains essential for high-reliability and long-duration mission systems. State acceleration factors not only enhance the Markov degradation model's ability to capture actual equipment aging, but also deliver meaningful modeling accuracy improvements across failure rate levels. This approach is particularly well-suited to system modeling for high-safety and high-reliability mission requirements.

4 Conclusion

This study addresses the multi-state and uncertain characteristics of equipment degradation in complex systems by proposing a multi-state degradation model based on an enhanced Markov process. The introduction of state acceleration factors allows the model to capture the acceleration of equipment degradation under aging effects across various states, facilitating dynamic modeling of complex system degradation patterns. This degradation model is integrated into a DBN framework to support system-level task reliability assessments. The proposed method preserves the simplicity of exponential distributions while improving the representation of realistic degradation behavior through state-specific adjustments to transition rates.

Case studies and sensitivity analyses reveal that, under high baseline failure rates, neglecting acceleration factors results in a significant overestimation of system reliability. Even in scenarios with moderate failure rates, noticeable differences in reliability are evident. In cases with low failure rates, incorporating acceleration factors remains crucial for modeling high-reliability missions. These findings confirm the method's capacity to enhance modeling accuracy and adaptability while maintaining computational simplicity.

This research presents a novel approach to modeling uncertain equipment degradation and analyzing system reliability. Future studies will focus on exploring methods to estimate state acceleration factors for specific types of equipment.

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