Three Essays in Macroeconomics

MADUABUCHUKWU C. CHIKWENDU

This thesis is submitted in partial fulfilment of the requirements for the degree of

Doctor of Philosophy in Economics (PhD)

LANCASTER UNIVERSITY

DEPARTMENT OF ECONOMICS

October 2025

Declaration

I do hereby certify that this thesis presented for examination and award of a PhD degree is a product of my original research investigation. I ascertain that to the best of my knowledge, this thesis does not contain or include content that are fabricated or falsified. I certify that where other published works have been consulted, I have adequately and appropriately acknowledged and referenced the source.

I ascertain that this Thesis has been produced in accordance with the University of Lancaster's code of ethics and good practice in Research and to this effect, the copyright of this thesis rest solely with the Author, Mr Maduabuchukwu Chikwendu, and no quotation from it is permitted without full acknowledgment.

Acknowledgements

I would like to extend my profound gratitude to various Resource personnel and functionaries who have been Instrumental to the inspiring and fulfilling journey at Lancaster University over the years.

I am grateful to my Supervisors, Dr Olivier Cardi and Dr Giorgio Motta, who provided the much-needed oversight and pastoral functions while undertaking Research at the Department of Economics. Their input in a professional capacity has been very instrumental to inspiring and supporting me, while providing various resource networks and solutions which have enabled me to conclude my research and become a resourceful Economist.

I am also thankful to all the members of Staff and fellow PhD colleagues who have provided an enabling environment, helpful comments, and have inspired various research inputs and ideas in personal and professional engagements.

I am also grateful to my Family, for their unflinching support, and encouragement all through this research period.

Abstract

In the first chapter, we study the dynamic effect of real shocks in a dual-sector economy featuring asymmetric resource utilization in a medium size DSGE model under alternative pricing assumptions which underscore the ex-ante relationship between exchange rate and import price inflation. In this context, pass through is complete under Producer currency pricing (PCP) and muted under Local currency pricing (LCP). Our results show that in response to an efficient technology innovation the law of one price gap must rise, and sectoral shocks seems to be complementary. Imported inputs in production of traded goods make marginal costs and domestic inflation more sensitive than otherwise would have been in the presence of efficient technology shocks, thereby amplifying the realized real and nominal exchange rate volatility. We also find that while imported inputs in production amplify realized volatility in our theoretical model, a higher share of non-traded goods in consumption mitigates realized volatility, and in the presence of technology shocks, monetary authorities are more accommodatory thereby creating a non-inflationary economic expansion.

Next, we study the objectives, conduct and design of optimal monetary policy while accounting for the deviation from the law of one price in a fully optimizing setting, and utilizing the model framework of the previous chapter. Owing to our model characteristics, our results show that the presence of the law of one price gap and the relative price of traded goods in new Keynesian Phillips curve (NKPC) create a meaningful trade off, forcing central banks to accommodate some fluctuations in output gap and inflation in the face of efficient shocks. Our findings highlight the optimal policy response of the benevolent social planner, who adjust rates in response to the natural rate of interest, the deviation of CPI inflation from its target, and the deviations from the law of one price under discretion and commitment. Our results show that the presence of a framework that guarantees credibility significantly improves the short-run trade-off due to convexity of the flow loss function. Utilizing the welfare losses as an evaluative measure, we find that targeting Consumer prices (CITR) or domestic prices (DITR) are potentially welfare improving, optimal and outperform alternative monetary rules.

Finally, we quantify the size, magnitude, frequency and speed of adjustment of pass-through into traded goods prices and quantities. Using a dynamic and partial adjustment model alongside macro timeseries data, we find pass-through to be moderate, substantial and statistically significant, and our empirical results using an appropriately identified structured VAR model point to moderate and incomplete pass-through irrespective of time frequency and we document evidence of declining pass-through over longer periods using both dynamic and structural models.

Contents

List of Figures

List of Tables

1 Exchange rate Fluctuation and Price level determination

1.1 Introduction

In this research, we study the dynamic effect of real shocks in a dual-sector economy and exchange rate pass through into import prices and prices across the price chain. Our measure and definition of pass through is narrowed down to the changes in import prices attributable to variations in exchange rates. Our main findings show that in response to an efficient technology innovation, we find that the law of one price gap must rise, and sectoral shocks seems to be complementary. Our theoretical model show that these dynamics are driven by a complementary technology shock as in Gali (2002), and deviations from the law of one price account for asymmetric responses under LCP relative to PCP. Our results highlight the importance of using an optimizing DSGE model that account for asymmetric utilization of resource (imported inputs) in production, and accounting for the deviations from the law of one price allow for the analysis of technology and policy shocks and its impact on macroeconomic and financial outlook.

In this research, calibration and measurement are foundational to our analysis. The estimation and theoretical model must produce dynamic responses of macroeconomic variables that are accurate and plausible. To facilitate this, we model an open economy as in Gali (2005) and make significant extensions novel to our research by adding three features. In this paper, we model a framework where trade is invoiced in both producer currency (PCP) and Local currency (LCP). In doing so, we model complete pass-through under the assumption of producer currency pricing (PCP) and incomplete pass-through under the assumption of local currency pricing (LCP) in parallel, while we allow for the deviations from the law of one price under LCP and show how this endogenously generates a short run trade-off between the stabilization of inflation and output gap. This makes the design of optimal policy quite realistic and non-trivial and shift the foundations of policy doctrines that to generate meaningful policy trade-offs, we must resort to ad-hoc cost push shocks which are exogenous and acts as shifters of the Phillips curve.

Second, we account for imports both as consumption goods and production inputs. Our model allows for imported input in production of traded goods, and we discount this assumption in production of non-traded goods which enhance our analysis. These assumptions simplify our analysis and allow us to incorporate asymmetry in resource allocation across sectors and we can pin down various economic variables through solving various agents optimizing decisions. Our results show that relative to a benchmark case which is inclusive of imported input, a model without imported inputs has significant implications for labour market allocations and has significant output gap implications. We find that imported input amplify the realized volatility in our model economy, while non traded goods mitigate realized volatility.

Finally, at the heart of the misalignments caused by these deviations lies the exchange rate in its dual role of relative price in the asset and goods market, which has no place in a closed economy context. Under the sustained assumption of complete asset market, we characterize the dynamics of the model in the presence of distortions resulting from nominal rigidities and various pricing assumptions, which causes the same goods to be traded at varying prices across markets. Thus, in this framework, we show that with incomplete pass-through, the dynamics of real exchange rates is attributable to terms of trade depreciation, the deviations from the law of one price gap and the relative price of traded goods and a trade-off between policy objectives emerges in response to efficient productivity shocks and this is independent of the measure of inflation (producer price or CPI) which is featured in the loss criterion function of the central bank.

1.2 Background to the study

In a very integrated global economy, the currency in which exporters price their goods, which is referred to as currency invoicing, is a very important concept and has broad consequences in international macroeconomics. The price setting behaviors of exporters and the choice of invoicing currency underpin the role of nominal rigidities in an open economy and has broad implications for monetary policy, the international mechanism for shock transmission, and consequently, exchange rate pass-through.

Generally, the choice of currency invoicing largely redistributes the exposure to foreign exchange between producers and consumers. Of the many impulses which the exchange rate transmits into a domestic economy, we are more interested in the price-related effects of such impulses, and among many price classes, we narrow down our interest to consumer prices, import prices, and export prices.

The currency choice of exporting firms directly affect the extent and degree of exchange rate pass through, which is the degree to which fluctuations in exchange rate are reflected in import prices, and subsequently, consumer prices are important for affecting various economic outcomes. Therefore, a clear understanding of the degree and timing of pass-through of exchange rate fluctuations is important for adequately forecasting and monitoring inflation.

It is in this sense that exchange rate determination and dynamics has been a fascinating issue in international macroeconomics and as such, stakeholders in the field of economics have devoted significant amount of effort to understand, analyze and forecast exchange rate and inflation so as to craft policy aimed at achieving various stakeholders interest as variations in exchange rate have quantifiable effect on real and nominal macroeconomic variables.

Over decades, a plethora of large and persistent fluctuations have been documented across major global reserve currencies, and among country pairs of currencies, with vague and aberrational responses of prices, which has led to a focus to investigate the impact of these fluctuations and its corresponding transmission mechanism to prices across the pricing chain. These fluctuations in currency pairs are frequently evident due to structural economic transitions and changes in underlying market fundamentals such as supply and demand conditions, market expectations, differentials in foreign and domestic interest rates and relative prices of traded goods. Thus, exchange rate becomes an important macroeconomic policy target variable as changes in exchange rates have consequential impact on traded and non-traded goods of countries and regions through their impact on the relative prices of goods and services.

For emerging economies, this concept is similar and not far-fetched as exchange rate plays a major role in harmonizing different price systems across countries and presents economic agents a real-time unit to compare prices directly. Therefore, pass-through elasticities is higher for emerging economies as fluctuations in exchange rates are absorbed in import prices and prices across the price chain. This can be traced to high and persistent inflation rates emerging economies have faced, inducing widespread wage and price indexation, giving agents the motivation to lock in future wage and price inflation to anticipated exchange rate fluctuations. Also, Central banks of emerging economies are less credible and willing to accommodate temporary exchange rate shocks, which over time, become permanant and thus, affects inflation adversely via second round effects.

In this regard, rapid, large and consequential exchange rate pass-through renders the conduct and implementation of monetary policy non-trivial as it amplifies the complexities and trade-offs when conducting monetary policy in an open economy.

For small open economies, the exchange rate channel is an additional transmission channel which has an implication for the conduct of monetary policy. A variety of papers have studied the role and contribution of exchange rate in inflation dynamics, and we can highlight two unique types of such models. In the first class of models, as in Gali and Monachelli (2005) and Monachelli (2002), where the totality of imports is attributable to final consumption, but discount from imported inputs used in production.

In this class of models, exchange rate influences the economy through two channels. The first being the direct channel, where fluctuations in exchange rates directly affects import prices through the consumption of imported goods, and the second-round effects which affects consumer price inflation (CPI) through imported inflation. Hence, shocks from the external environment and exchange rate fluctuations are borne by the final consumer. In the second class of models, as in McCallum and Nelson (2000), they assume that only firms consume imports as inputs in the production process, hence discounting from households' ability to consume foreign goods. This has some implications, as external shocks or exchange rate fluctuations directly affects firms' marginal cost, and thus domestic inflation. These models highlight the second-round effects when firms, who can reset prices in the current period, can pass on the changes in exchange rates either fully or partially to consumers via the mark up channel, and finally these fluctuations in exchange rates are reflected in consumer price inflation (CPI). In these models, exchange rate pass through is dependent on the degree of nominal rigidities inherent in this models.

For both class of models, they feature rational expectations, dynamic setting optimizing agents, nominal rigidities in prices, and uniquely they both assume that all goods are traded, but these classes of models feature different implications underpinning the link between exchange rates and inflation, and different implication for monetary policy. In this research, we try to harmonize both assumptions, by allowing for imported consumption and imported inputs in production in a fully optimizing model and we also incorporate this feature that a small but sizable proportion of the economy may be producing goods and services which may be non-traded, and we investigate the impact of shocks on prices across the price chain, and quantities, and the degree to which exchange rate acts as a medium to absorb or amplify the effects of these exogenous impulses, and the challenges and complexity of monetary policy in this setting.

1.3 A Review of Literature

Prior studies have estimated the degree of exchange rate pass-through (ERPT) across the price chain and their results showed that pass-through in the short term is imperfect or incomplete while it asymptotically gets to perfect pass-through over longer periods.

From an empirical approach, Jonathan Maccarthy (1999) examined the dynamic relationship between exchange rate, import and consumer prices as well as the pass-through elasticity of exchange rates into CPI inflation across industrialized countries in a VAR model using macro time-series data from several economies. This approach largely discount from agents interaction in an optimizing environment, while investigating the dynamic relationship across macroeconomic variables and isolating the contribution of exchange rate and import prices to inflation variability. This study documents incomplete pass-through elasticities with variations across countries, and the presence of lagged effect in inflation variability and declining pass-through over time.

Campa and Goldberg (2002) empirically analyzed the pass through of exchange rate into consumer prices for 25 OECD countries and rejected the hypothesis of complete and perfect pass-through as they found imperfect pass-through for 22 out of 25 sample countries. Surprisingly, they also found incomplete pass through in the long-term for most countries in her sample observation. Coricelli et al. (2006) studied the pass-through of exchange rates into consumer prices along side Beirne et al. (2011) for central and east European countries and they found pass-through to be imperfect with estimates of about 0.6 percent (0.6%) on average.

Ghosh and wolf (2001) found a perfect pass through in the long run. Parson and Sato (2008) found similar results for pass-through results for Japan and east asian countries, however Etsuro

Shioji (2009) found results detailing the presence of imperfect pass through into import and export prices for the Japanese economy. Marzzi and Sheets (2007) studied the pass-through into import prices between 1997 and 2007 and documented a declining pass-through estimate across the sample period. Frankel, Parsley and Wei (2012) conducted research for a sample of 76 countries and observed a declining pass-through estimate across her sample observation and attributed such declining estimate to distribution cost.

Considering the empirical evidence, which significantly tilts towards the documented evidence of imperfect pass-through estimates, there is growing and prevailing evidence of imperfect pass-through owing to different factors. Dornbusch (1987) attributed this empirical discovery to the prevalent market structures which enables firms to control prices and accumulate more market power. Hellerstein (2004) explained that firms are ever more willing to absorb exchange rate shocks and are becoming reluctant to pass through these fluctuations into prices to maintain and expand their market share thereby preserving stable prices.

Burstein and Rebelo (2003) tendered alternative explanation for the prevalence of incomplete pass-through. They attribute it to the presence of non-traded goods in consumption and distribution sectors and as such, this generates a gap between border and retail prices. The wide dispersion in the direction of the results of various empirical studies is also reflected in the fact that a number of these studies capture the impact of exchange rate fluctuation on different inflation measure. They failed to harmonize their interest on a particular price metric amongst the wide variety of prices.

Recently, a variety of papers have focused on isolating import prices and its response to an exchange rate fluctuation. To this end, theoretical models which account for wage stickiness, optimizing agents, nominal stickiness, exogenous productivity, and monopolistic competition have been explored. Goldberg (2010) show that among many, the elasticity of substitution among varieties, imported inputs in production, productivity and non-traded goods all influence the elasticity of import price to exchange rate fluctuations.

Goldberg (2010) shows that as the openness index increases, the more sensitive import prices are to exchange rate changes, and the larger the elasticity of substitution amongst goods are, the smaller the response of consumer price inflation and import prices to exchange rate changes. They found that import prices are less sensitive to higher distribution margins in traded or non-traded good sector.

Campa and Goldberg (2006) document that higher exchange rate volatility leads to higher pass-through as the composition of import can alter the sensitivity of import prices to exchange rate movements. Adolfson (2001) showed that the degree of persistence of the shock under consideration can affect the sensitivity of import prices to exchange rate movements as transitory shocks have muted effects relative to more persistent or permanent shocks.

A wave of literature utilized Firm-level and industry level data to investigate the elasticity of pass-through and its link with microeconomic variables. Their results heavily leaned to the fact that pass-through is incomplete and endogenous. Thus, under this strand of literature, (Gopinath, itskhoki and Rigoboon (2010)) affirm that incomplete pass-through is influenced by the choice of invoicing currency in global trade, determinants of marginal cost, including but not limited to the use of imported inputs in production Shingal et al (2014), Amiti, Itskhoki and Konings(2014), attribute it to market structure that gives rise to pricing to market Atkeson and Burstein (2009), Itskhoki and Konings(2018), Auer and Schoenle (2016), Fitzgerald and Haller(2014), attribute incomplete pass through to Distribution costs and Burstein, Neves and Rebelo(2003) attribute it to Heterogeneous productivity (Berman, Martin and Mayer(2012); Garetto(2016)).

The aforementioned studies all discount from a theoretical general equilibrium framework and have relied on partial equilibrium and multi-equation modeling to exchange rate pass through determination and hence, motivates the need for a framework that accounts for rational agents interacting simultaneously in goods, factor and labour markets in an optimizing environment.

At the dawn of breakthrough of the RBC framework, a distinct school of thought known as the new Keynesian macroeconomics sprung up. Initially developed as a natural extension to existing RBC models which featured money neutrality in the face of non-constant wages and prices, the new Keynesian framework advanced into a pedagogical framework to highlight monetary non-neutrality. Ever since, DSGE models have inspired thought provoking research aimed at exchange rate determinations and its implications for real economic variables. This is not far-fetched, with nominal rigidities making a subset of prices across the price chain inflexible in the short term, exchange rate fluctuations have a real implication for prices and consumption of traded goods, output and trade balance.

Early studies which championed the way by using a theoretical approach to exchange rate determination in an optimizing framework while assuming producer currency pricing include Obstfeld and Rogoff (2002) and Benigno and Benigno (2003), while Bacchetta and Van Wincoop (2000), Devereux and Engel (2005), and Corsetti and Pesenti (2005) used similar framework but assumed Local currency pricing. One common draw back was that while they all analysed alternative monetary policy considerations in a two country setting, and they assumed prices were set one period in advance.

The new open economy macroeconomics (NOEM) literature initially utilized a partial equilibrium method and considers the time path of exchange rate to be exogenous. Subsequently, it reflected how markups and marginal costs are derived and how they react to exchange rate changes.

Hence, pass-through elasticities in this literature is a cumulative effect of the complete spillover effect on import prices arising from an exchange rate shock, the responsiveness of markups to ease off this spillover, and variations in marginal costs attributable to fluctuations of exchange rate which directly affects imported inputs used in production. Markups in these class of literature strictly depend on the market share of home producers relative to foreign producers. This in turn inform us on the degree of competition in the industry, and the degree and magnitude of price to market enforceable in an industry.

General predictions of these new open economy macroeconomics (NOEM) literature are that global supply is composed of a larger share of importers, which affords firms the space to utilize higher degrees of price discrimination, and hence a larger share of imported inputs used in the destination country's production process, which lead to a higher predicted pass through estimates into import prices. Therefore, exchange rate pass through may be higher if foreign exporters are larger in number relative to the number of domestic producers.

Second generation of new open economy macroeconomics (NOEM) literature made attempt to strengthen earlier models of exchange rate pass through using a general equilibrium approach, within which, they assume sticky prices in one currency, which are set in advance before the exchange rate realizations. Within this framework, exchange rate pass through is zero when prices are determined in importers currency and complete when prices are set in exporters currency. In this framework, under extreme cases of an exogenous exchange rate shock, pass through is likely to be complete or to equal unity when prices are set in producer's currency (PCP) and to be incomplete if otherwise.

Contemporary models of exchange rate pass through specifically utilize the framework of the new open economy macroeconomics (NOEM) framework, and contextualize a micro based open-economy, general equilibrium models, which produce definite predictions for exchange rate determination and behavior in relation to pass through into import and consumer prices. They postulate that pass-through elasticities depend on the underlying shock evident in the economy and the specific type of market structure which individual firms participate in.

These models feature some form of price stickiness, and make assumptions of imperfect competition as in Devereux and Engel (2001), Corsetti, Dedola, and Leduc (2004), Corsetti and Pesenti (2005) and Obstfeld and Rogoff (1995). These literature inherently still retain the price rigidity assumption of the traditional Dornbusch (1987) model, but treat exchange rate and the choice of currency invoicing by firms as endogenous. Engel (2005), Devereux, Goldberg and Storgaard (2005), show a strong correlation between optimal invoicing currency choice and optimally chosen rate of exchange rate pass through. Devereux and Engel (2001), and Devereux, Engel and Storgaad (2004), show that in steady state, countries with relatively low exchange rate volatility and relative stable monetary regimes and policies would have their currencies chosen as a currency choice and hence, exchange rate pass through will be severe for importing countries with relatively more unstable monetary policy.

The currency of invoicing exports and imports have huge implications for the evolution of consumer and producer price, terms of trade and other real variables, and subsequently, the role of monetary policy in stabilizing a variety of economic variables of interest. Within the new open economy macroeconomics (NOEM) Literature, local currency pricing and producer currency pricing has been well documented, and our work tries to investigate the role of a vehicle (dominant) currency, and its implications for real variables in the new Keynesian Framework.

Specialization, heterogeneity of technological progress, outsourcing and the prevalent use of global value chain has highlighted the unique role of imported inputs in determining pass-through elasticities and present a different channel for external shocks to directly influence marginal cost of firms, and hence renders the problem of monetary policy quite challenging.

This work will follows closely from CDGG (2017), but models the small open economy differently and in a more trivial way. We model a framework where trade is invoiced in both producer currency (PCP) and Local currency (LCP). We account for imported input in production of tradable goods, and discount this assumption in production of non-traded goods which enhance our analysis. These assumptions simplify our analysis and we can pin down various economic variables through solving various agents optimizing decisions.

In this paper, we model complete pass-through under the assumption of producer currency pricing (PCP) and incomplete pass-through under the assumption of local currency pricing (LCP) in parallel, while we allow for the deviations from the law of one price under LCP and how this endogenously generates a short run trade-off between the stabilization of inflation and of the output gap. This makes the design of optimal policy quite realistic and non-trivial, and shift the foundations of policy doctrines that in order to generate meaningful policy trade-offs, we have to resort to ad-hoc cost push shocks which are exogenous and acts as shifters of the phillips curve. At the heart of this misalignments caused by these deviations lies the exchange rate in its dual role of relative price in the asset and goods market, which has no place in a closed economy context. Under the sustained assumption of complete asset market, we characterize the dynamics of the model in the presence of distortions resulting from nominal rigidities and various pricing assumptions, which causes the same goods to be traded at varying prices across markets. Thus, in this framework, our result show that with incomplete pass-through, a trade-off between policy objectives emerges in response to efficient productivity shocks and this is independent of the measure of inflation (producer price or CPI) which is featured in the loss criterion function of the central bank.

We will also explore the optimal monetary policy for our small open economy using conventional second-order approximations to the loss in utility relative to steady state, which we can derive through the second order approximation to the social planners' loss function, which introduces a distinct term which captures the deviation from the law of one price and we find that stabilizing inflation is more costly in terms of output despite when inflation has the smallest volatility and when pass through is sufficiently low. We also explore the performance of alternative simple policy rules for our small economy under alternative pricing assumptions.

Finally we try to match our Theoretical results with our SVAR results, and investigate the transmission of shocks and how good a fit our model is. This study is structured as follows. The first section presents the small open economy DSGE model and outline the dynamics of our model. Section two explores optimal monetary policy, simple rules, trade-offs and welfare losses implied by the degree of pass-through. Section three highlight the structural Vector Auto-regressive framework where we test our model predictions under PCP and LCP using data and finally Conclusions are appended to chapter three.

1.4 Model Setup: Structure and Dynamics

We present the general framework of the dynamic stochastic general equilibrium (DSGE) model and discuss its various components, needed for a numerical simulation. We focus on the standard and applicable New Keynesian model and highlight various structural equations and necessary components needed for its simulation and implementation.

Our model economy is characterized by three major interconnected blocks: a demand block from households' optimizing behavior, a supply block which is derivative of firms' optimizing interactions, and a policy block which highlights the interaction, intervention and optimal actions of a social planner which in this case, is the central bank. These equations are derived with certain assumptions, which we describe in each section as we document agents optimizing interactions. We also make assumptions about technology endowments, expectations, and structural constraints which underpin the model equations.

We model a small open economy where the domestic economy is populated by infinitely lived households which maximize consumption in utility and minimize disutility from Labour. There are two stages of production in this economy. The intermediate good producers, who utilize a cobb-douglas type production function to produce output, and we have final good bundlers or producers who purchase intermediates from intermediate good producers and bundle them into final goods ready for domestic and foreign markets. The dynamics of this model is partly a result of the hiring, pricing and production decisions of intermediate good producers who employ factor inputs in competitive factor markets and rates, but produce and sell their products in monopolistically competitive markets. Thus, these firms make pricing decisions as they fully internalize their market power because they face a downward-sloping demand and can set prices above marginal costs.

We allow for nominal rigidities in exporting and importing sectors, and we incorporate nominal rigidities in the spirit of Calvo (1983). We account for heterogenity in sectoral resource utilization and imported input in production in a two-sector economy. The traded good sector utilizes a production technology adapted to incorporate imported inputs in production of traded goods, while the non-traded sector utilizes domestic endowments of factor inputs. We model an external sector where traded goods are invoiced in both Producer currency pricing and local currency pricing, reflecting our desire to model our economy under the two dissimilar assumptions. This is due to the fact that there has been pervasive evidence on large and persistence deviations from purchasing power parity, thus allowing us to account for deviations in law of one price.

Households trade assets in frictionless asset markets, thus connecting domestic and foreign endowments. This assumption of complete asset markets uniquely determines real exchange rate, and sets the foundation to understand the dynamics of nominal and real variables in the presence of efficient shocks. In this paper, we account for all the aforementioned components in a fully optimizing new Keynesian dynamic stochastic general equilibrium (NK-DSGE) model.

1.5 Domestic Households

The domestic economy is populated with infinitely lived households, among whom we model a representative household deriving utility by consuming Dixit-Stiglitz aggregates of traded $C_{T,t}$ and non-traded goods C_{NT} by domestic firms producing both traded and non traded goods. This representative household derives disutility from labour (N_t)and maximizes intertemporal utility subject to a budget constraint. The household maximize the following objective function:

$$E_0 \sum_{t=0}^{\infty} \beta^t U_t \tag{1.01}$$

where E_0 is the expectational operator and β is the subjective discount factor of the households. Households face isoelastic utility functions and derives utility from the composite consumption of Goods and Leisure as follows:

$$U_t = \frac{(C_t)^{1-\sigma}}{1-\sigma} - \frac{(N_t)^{1+\varphi}}{1+\varphi} \tag{1.02}$$

where C_t and N_t respectively are aggregate consumption and Labour hours or employment. The coefficients σ is the inverse of elasticity in consumption and φ is the frisch elasticity of labour supply. Consumption is a composite goods composing of traded and non traded goods which is given by the CES aggregator:

$$C_t = \left[(1-\gamma)^{\frac{1}{\eta_1}} C_{T,t}^{\frac{(\eta_1-1)}{\eta_1}} + \gamma^{\frac{1}{\eta_1}} C_{N,t}^{\frac{(\eta_1-1)}{\eta_1}} \right]^{\eta_1/(\eta_1-1)} \tag{1.03}$$

where C_{Tt}, C_{Nt} denote consumption of traded and non traded goods respectively. The coefficient η_1 measures the elasticity of intertemporal substitution of consumption between traded and non-traded goods, and as the parameter η_1 approaches infinity, goods become more substitutable. The parameter γ measures the proportion of non traded goods in aggregate consumption of households. Expenditure minimization problem yields the demand functions of traded and non-traded goods in consumption.

$$C_{T,t} = (1-\gamma)\left(\frac{P_{T,t}}{P_t}\right)^{-\eta_1} C_t \tag{1.04}$$

$$C_{N,t} = \gamma\left(\frac{P_{N,t}}{P_t}\right)^{-\eta_1} C_t \tag{1.05}$$

where P_{Tt}, P_{Nt} P_t, are the price indices of traded, non-traded, and aggregate prices respectively. It is noteworthy to mention that traded and non-traded goods are composite indices which are cumulated differentiated products produced in monopolistically competitive markets. Thus, the index of these goods are given by the Dixit-Stiglitz aggregator:

$$C_{T,t} = \left[\int_0^1 \left(C_{T,t,(j)}\right)^{\left(\frac{\varepsilon-1}{\varepsilon}\right)} dj\right]^{\varepsilon/(\varepsilon-1)} \tag{1.06}$$

$$C_{N,t} = \left[\int_0^1 \left(C_{N,t,(j)}\right)^{\left(\frac{\varepsilon-1}{\varepsilon}\right)} dj\right]^{\varepsilon/(\varepsilon-1)} \tag{1.07}$$

where j denotes each variety of traded and non traded goods and ε is the elasticity of substitution between the varieties. The Aggregate price index is given by:

$$P_t = \left[(1-\gamma)\left(P_{T,t}\right)^{1-\eta_1} + \gamma\left(P_{N,t}\right)^{1-\eta_1}\right]^{1/(1-\eta_1)} \tag{1.08}$$

The traded goods consumed domestically are either domestically produced or imported from the rest of the world. Hence, the consumption of traded is determined as a CES index composed of home produced traded and imports as follows:

$$C_{T,t} = \left[(1-\alpha)^{\frac{1}{\eta}}\left(C_{H,t}\right)^{\frac{(\eta-1)}{\eta}} + (\alpha)^{\frac{1}{\eta}}\left(C_{F,t}\right)^{\frac{(\eta-1)}{\eta}}\right]^{\eta/(\eta-1)} \tag{1.09}$$

The parameter η measures the elasticity of intertemporal substitution of consumption between domestically produced traded goods $C_{H,t}$ and imported goods $C_{F,t}$ and α denotes the share of imported goods in the total consumption of domestically consumed traded goods . As with the case of total consumption above, expenditure minimization on the traded goods yields the demand functions for domestically produced and imported traded as in the following equations.

$$C_{H,t} = (1-\alpha)\left(\frac{P_{H,t}}{P_{T,t}}\right)^{-\eta} C_{T,t} \tag{1.10}$$

$$C_{F,t} = \alpha\left(\frac{P_{F,t}}{P_{T,t}}\right)^{-\eta} C_{T,t} \tag{1.11}$$

where $P_{H,t}, P_{F,t}$ are, respectively, prices of domestically produced traded and imported goods. The traded goods price index is given by:

$$P_{T,t} = \left[(1-\alpha)\left(P_{H,t}\right)^{1-\eta} + \alpha\left(P_{F,t}\right)^{1-\eta}\right]^{\frac{1}{(1-\eta)}} \tag{1.12}$$

Total consumption expenditure by households is given by the sum of the expenditures on traded and non-traded goods they consume which yield:

$$P_t C_t = P_{T,t}C_{T,t} + P_{N,t}C_{N,t} = P_{F,t}C_{F,t} + P_{H,t}C_{H,t} + P_{N,t}C_{N,t} \tag{1.13}$$

Households enter period t holding B_{t-1} one-period bonds and also receives nominal dividends because households own monopolistic competitive firms. Households also receive W_tN_t total nominal factor payments and employment income as wages for work hours supplied in the factor markets, and make consumption decisions and fund new bond purchase at discount today, the discount factor being the inverse of the nominal riskless interest rates. We follow Woodford (2003) and assume that prices are measured in terms of a unit of account called money, but the economy is cashless. This subject households to a budget constraint as a no ponzi-game condition is imposed preventing households from excessive borrowings. This household flow budget constraint is:

$$P_tC_t + Q_tB_t \leq W_tN_t + D_t + B_{t-1} \tag{1.14}$$

This budget constraint highlights the household expenditure, as given by the left hand-side, consisting of expenditure on consumption C_t, and purchase of public bonds, B_t. The periods earned and received income, as given by the right-hand side of the budget constraint, is composed of dividends, D_t, wage income from labour services, and receipt of principal and interest income on the bond held in the previous period, B_{t-1}. The intertemporal problem faced by a representative household is given as:

$$\max_{C_t,N_t,B_t} E_0 \sum_{t=0}^{\infty} \beta^t \left[U_t = \frac{(C_t)^{1-\sigma}}{1-\sigma} - \frac{(N_t)^{1+\varphi}}{1+\varphi} - \lambda_t \left(P_t C_t + Q_t B_t - W_t N_t - D_t - B_{t-1} \right) \right] \quad (1.15)$$

The linearized first-order conditions of the optimization problem of this household are given by:

$$w_t - p_t = \sigma c_t + \varphi n_t \quad (1.16)$$

$$c_t = E_t c_{t+1} - \frac{1}{\sigma} (i_t - E_t \pi_{t+1}) \quad (1.17)$$

where π_{t+1} is next periods inflation and the above equations represent the labour supply schedule which is the marginal rate of substitution between consumption and hours and the second being the Euler's equation.

We log-linearize aggregate consumption, aggregate price, consumption of traded and non-traded goods around a symmetric steady-state to obtain the following equations:

$$c_t = (1-\gamma) c_{T,t} + \gamma c_{N,t} \quad (1.18)$$

$$p_t = (1-\gamma) p_{T,t} + \gamma p_{N,t} \quad (1.19)$$

$$c_{T,t} = (1-\alpha) c_{H,t} + \alpha c_{F,t} \quad (1.20)$$

$$p_{T,t} = (1-\alpha) p_{H,t} + \alpha p_{F,t} \quad (1.21)$$

$$c_{N,t} = -\eta_1 (p_{N,t} - p_t) + c_t \quad (1.22)$$

$$c_{H,t} = -\eta (p_{H,t} - p_{T,t}) + c_{T,t} \quad (1.23)$$

$$c_{F,t} = -\eta (p_{F,t} - p_{T,t}) + c_{T,t} \quad (1.24)$$

1.6 Domestic Firms: Production and Pricing Decisions

Monopolistic competitive intermediate good firms are a continuum, indexed by $i \in [0,1]$ in the domestic goods market. The firms are owned by consumers. Each domestic intermediate good firm produces a differentiated good. For traded good firms, they operate a constant return to scale technology (linear technology) represented by the following production function:

$$Y_{Ht}(i) = A_{ht} N_{ht}^{1-\delta}(i) M_t^{\delta} \tag{1.25}$$

$$Y_{Nt}(i) = A_{nt} N_{nt}^{1-\delta}(i) \tag{1.26}$$

Where $A_{ht} = \log A_{ht} - (\log A)$ is labor productivity and is distinct across sectors. Domestic productivity in various sectors is assumed to follow a simple stochastic autoregressive process:

$$a_{ht} = \rho_{ah} a_{ht-1} + \varsigma_{h,t}^{a} \tag{1.27}$$

$$a_{nt} = \rho_{an} a_{nt-1} + \varsigma_{n,t}^{a} \tag{1.28}$$

where $0 \leq \rho \leq 1$ is persistence parameter and $\varsigma_{h,n,t}^{a} - i.i.d$ shock. Log-linearized production function in the traded and non traded sector is given as follows:

$$y_{ht} = a_{ht} + (1-\delta) n_{ht} + \delta m_t \tag{1.29}$$

$$y_{nt} = a_{nt} + (1-\delta) n_{nt} \tag{1.30}$$

For a representative monopolistically competitive firm in both the traded and non-traded sector who utilize the production technology as expressed in equation (1.25) and (1.26), this firms employ factor inputs in perfectly competitive markets as they solve the cost minimization problem below:

$$\underset{N_{ht}, M_t}{Min} TC(j) = W_T N_{I,t}(j) + P_{f,t} M_t \tag{1.31}$$

Subject to the production technology in equation (1.25) and (1.26). Optimization and first order conditions for each sector would yield optimal factor demand for labour and imported input for the representative firm in the traded and non-traded sector as follows.

$$N_{n,t} = (1-\delta) \left[\frac{P_{N,t}}{W_t} \right] Y_{N,t} \tag{1.32}$$

$$N_{h,t} = (1-\delta)\left[\frac{P_{H,t}}{W_t}\right]Y_{H,t} \tag{1.33}$$

$$M_t = (\delta)\left[\frac{P_{H,t}}{P_{F,t}}\right]Y_{H,t} \tag{1.34}$$

$$\frac{W_t}{P_{F,t}} = \left[\frac{1-\delta}{\delta}\right]\left[\frac{M_t}{N_{H,t}}\right] \tag{1.35}$$

Equation (1.32) is the optimal demand function for labour by non-traded good firms which pins down the optimal combination of labour and wages that equilibrates the factor market. Thus, an increase in the wage rate will decrease the demand for labour by non-traded good firms until the real wage equals the marginal product of labour and an increase in the non-traded good firms desired level of output will increase the demand for labour by non-traded firms.

Equation (1.33) and (1.34) are the optimal demand for labour and imported inputs for firms in the traded sector which show that the demand for labour and imported input is inversely related to the wage rate and the price of imported input respectively, and positively related to the output levels in the traded sector while equation (1.35), is the optimal factor combination that produces firms desired output levels, given factor prices. Thus we can derive the nominal marginal cost of the traded and non-traded sector as follows:

$$MC_{H,t} = \frac{1}{A_{h,t}}\left[\frac{W_t}{1-\delta}\right]^{1-\delta}\left[\frac{P_{F,t}}{\delta}\right]^{\delta} \tag{1.36}$$

$$MC_{N,t} = \frac{1}{A_{n,t}}\left[\frac{W_t}{1-\delta}\right]N_{n,t}^{\delta} \tag{1.37}$$

When we fully loglinearize the expression of nominal marginal cost, and we obtain the expression for nominal marginal cost as follows:

$$mc_{h,t}^{n} = (1-\delta)w_t - a_{h,t} + \delta p_{f,t} \tag{1.38}$$

$$mc_{n,t}^{n} = w_t - a_{n,t} + \delta n_{n,t} \tag{1.39}$$

The above expressions highlights that nominal[1] marginal costs are increasing in factor prices (wage rates and prices of imported input) and decreasing in the level of technology and productivity.

[1]in appendix A, we show the transformation from nominal to real marginal cost

1.7 Incomplete Pass-through, the terms of trade and the real exchange rate

To account for incomplete pass-through and its link to the deviation from the law of one price gap, and also the relationship between the deviations from law of one price and the real exchange rate, we utilize the aggregate price level and terms of trade. First, we can define terms of trade as price of import in terms of export expressed in domestic currency.

$$S_t = \frac{P_{F,t}}{P_{H,t}} \tag{1.40}$$

$$\Delta s_t = \pi_{f,t} - \pi_{h,t} \tag{1.41}$$

log-linearization of the CPI expression around a steady-state and using the terms of trade above yields

$$\pi_t = p_t - p_{t-1} = (1-\gamma)p_{T,t} + \gamma p_{T,t} \tag{1.42}$$

$$\pi_t = (1-\gamma)\pi_{H,t} + \alpha(1-\gamma)\Delta s_t + \gamma\pi_{nt} \tag{1.43}$$

Also, we define the internal terms of trade as the price of traded goods expressed in terms of non-traded goods.

$$V_t = \frac{P_{T,t}}{P_{N,t}} \tag{1.44}$$

$$v_t = \alpha s_t + p_{ht} - p_{nt} \tag{1.45}$$

We model two opposing assumptions of trade invoicing. One where traded goods are invoiced in the currency of the producer, generally referred to as producer currency pricing (PCP) and where traded goods are invoiced in the currency of the destination country, referred to as Local currency pricing (LCP). Under PCP, the law of one price hold and pass-through is perfect and under LCP, pass-through is assumed to be incomplete in the model and the LOP does not hold. Under this pricing paradigm, the price of any imported goods in the market of small open economy is not equal to the price of an identical good in the world market in terms of domestic currency. In other words, the economy is identified by deviation of the world price from the domestic currency price of imports as follows:

$$PCP : P_{F,t} = \varepsilon_t P_t^* \tag{1.46}$$

$$LCP : P_{F,t} \neq \varepsilon_t P_t^* \tag{1.47}$$

where ε_t is the nominal exchange rate, which is the price of foreign currency in the terms of its domestic currency. P_t^* is the price of domestic imports expressed in foreign currency. In log-linearized form:

$$\psi_{F,t} = (e_t + p_t^*) - p_{F,t} \tag{1.48}$$

where $\psi_{F,t}$ measures the law-of-one-price gap (l.o.p gap). In order to show the effect of l.o.p gap, in what follows, it is useful to define the relationship between the real exchange rate and l.o.p gap. Real exchange rate in structural and loglinearized form is given by:

$$Q_T = \frac{E_t P_t^*}{P_t} \tag{1.49}$$

$$q_t = e_t + p_t^* - p_t \tag{1.50}$$

Combining the expressions of terms of trade and l.o.p gap, the real exchange rate in log-linearized form can be rewritten:

$$PCP : q_t = (1 - \alpha)s_t + \gamma v_t \tag{1.51}$$

$$LCP : q_t = \psi_{F,t} + (1 - \alpha)s_t + \gamma v_t \tag{1.52}$$

Some important observations are worth pointing out here from the equations. First, from the deviations of law of one price equation, we see that the law of one price gap increases when there is a deviation of the domestic currency price of imports from the world price of import.

Also from equation (1.52), there are three sources of fluctuations that determine the real exchange rate. Under the assumption of LCP, where the law of one price does not hold and there exist a deviation in the law of one price gap, these deviations contribute to the volatility of real exchange rate under incomplete pass through. Also, the incongruity of the consumption basket between the domestic economy and the rest of the world pins down the real exchange rate via the external terms of trade, alongside the internal terms of trade. These three factors drive the fluctuation of real exchange rate in the model under incomplete pass-through while the law of one gap channel is muted under perfect pass-through and under PCP.

1.8 Supply Block: Price Setting decisions

Under this framework, domestic firms to reset their prices in a staggered fashion, as in Calvo-Yun rule (1983). Each intermediate good firm sets new prices with probability $1-\theta_H^k$ in any given period and the price set at time t will still hold at time $t+k$. Hence, a measure of $1-\theta_H$ of producers reset their prices, while a fraction θ_H keep their prices unchanged. A firm reoptimizing in period t need to set the optimal new price, $P_{H,t}^{new}$ that maximizes the current market value of the profits generated while that price remains effective. Firm i 's production is constrained by the sum of dometic and foreign demand. The export price of the domestic good, $P_H^*(i)$, is assumed to be flexible and determined by the law of one price (LOOP). The optimal price setting problem is defined as follows:

$$\max_{P_{H,t}^{new}} \sum_{k=0}^{\infty} \left(\beta \theta^k\right) E_t \left\{ Q_{t,t+k} \left[Y_{t+k|t}(i) \left(P_{H,t}^{new} - MC_{t+k}^n \right) \right] \right\} \tag{1.53}$$

$$s.t Y_{t+k}(i) = C_{H,t+k}(i) + C_{H,t+k}^*(i) \tag{1.54}$$

After taking the first-order condition by solving the optimal pricing problem, we get the optimal pricing equation

$$p_{H,t}^{new}(j) = \left(\frac{\varepsilon}{1-\varepsilon}\right) \sum_{k=0}^{\infty} \left(\theta_H \beta\right)^k E_t \left\{ mc_{t+k}^r \right\} \tag{1.55}$$

Where $p_{H,t}^{new}(j)$ is the log of newly set prices by firms who have the opportunity to reset prices at time t and the above relation reveal that firms prices are a constant markup over marginal costs and for all intermediate good firms in both sectors who receive this signal and can set prices in time t will do so in a homogenous pattern thereby setting the same prices as a markup over marginal cost, therefore $p_{H,t}^{new}(j)$ will be the same across the $(1-\theta_h)$ firms and across sectors that adjust prices at time t. When we update the domestic traded good sector pricing rule, and using the assumption that all firms who do not receive the price signal and therefore cannot change their prices will keep the prices prevailing in the previous period, we get the pricing rule as follows:

$$P_{H,t} = \left[\theta_H P_{H,t}^{1-\varepsilon} + (1-\theta_H) \left(P_{H,t}^{new} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}} \tag{1.56}$$

When we log linearize equation (1.55) around a zero inflation steady state and combine with equation (1.56), we arrive at the following equation below:

$$p_{H,t}^{new} = (1-\theta_i\beta)\sum_{k=0}^{\infty}(\theta_H\beta)^k E_t\left\{mc_{t+k}^r\right\} \tag{1.57}$$

The above equation is indicative of the fact that firms setting prices today will choose a price that corresponds to their desired markups over the weighted value of current and expected future marginal costs. After some algebra, we arrive at the new Keynesian Phillips curve for both sectors as follows:[2]

$$\pi_{H,t} = \beta E_t\left\{\pi_{H,t+1}\right\} + \lambda\widehat{mc}_{ht}^r \tag{1.58}$$

$$\pi_{N,t} = \beta E_t\left\{\pi_{N,t+1}\right\} + \lambda\widehat{mc}_{nt}^r \tag{1.59}$$

Where λ measures the elasticity of current inflation to current levels of real marginal costs and the degree of price rigidity. This expression gives us a full forward looking Phillips curve for both traded and non traded sectors indicating that inflation will rise when optimizing intermediate good firms expect markups to be below steady state level, and therefore, inflation is a direct result of firms purposeful price setting decision by intermediate good firms in all sectors.

1.9 Import Pricing

Consider a local retailer who import good j at a cost $\varepsilon_t P_{F,t}^*(j)$. This cost is the price paid in the world market. ε_t is the level of the nominal exchange rate. The retailers face a downward sloping demand for imported goods like the local producers.

The retailers of imported goods are assumed to reset their prices in a staggered fashion like domestic intermediate good firms. Each retailer firm sets new prices with probability $1-\theta_F^k$ in any given period and the price set at time t will still hold at time $t+k$. Hence, a measure of $1-\theta_F$ of retailers reset their prices, while a fraction θ_F keep their prices unchanged. A retailer firm, who wants to change their price in period t, need to choose new price, $P_{F,t}^{new}(j)$ expressed in domestic currency to maximizes the current market value of the profit. Retailer firm j 's imported goods are constrained by the demand of imported goods j in the domestic market.

[2] similar steps is used to derive the optimal pricing rule and NKPC for the non-traded good sector which is reported in equation 1.59. Also, θ_H is homogeneous across sectors

The optimal price setting problem is defined as follows:

$$\max_{P_{F,t}^{new}} \quad E_t \left\{ \sum_{k=0}^{\infty} \beta^k \Lambda_{t,t+k} \theta_F^k \left(P_{F,t}^{\text{new}}(j) - \varepsilon_{t+k} P_{F,t+k}^{*}(j) \right) C_{F,t+k}(j) \right\} \tag{1.60}$$

$$s.t C_{F,t+k}(j) = \left(\frac{P_{F,t}^{\text{new}}(j)}{P_{F,t+k}} \right)^{-\varepsilon} C_{F,t+k} \tag{1.61}$$

where $P_{F,t}^{*}(j)$ denotes the foreign-currency price of the imported good, θ_F^k is the probability that the price $P_{F,t}^{new}(j)$ set for good j at time t still holds k periods ahead, and $\beta^k \Lambda_{t,t+k}$ is a relevant stochastic discount factor. The degree of domestic price stickness θ_H is allowed to be different from the degree of the import price stickness θ_F. As a result of the FOC, the newly set import price is given by:

$$P_{F,t}^{\text{new}}(j) = \frac{\varepsilon}{\varepsilon - 1} \cdot \frac{E_t \left\{ \sum_{k=0}^{\infty} \beta^k \Lambda_{t,t+k} \theta_F^k \left(\varepsilon_{t+k} P_{F,t+k}^{*}(j) C_{F,t+k}(j) \right) \right\}}{E_t \left\{ \sum_{k=0}^{\infty} \beta^k \Lambda_{t,t+k} \theta_F^k C_{F,t+k}(j) \right\}} \tag{1.62}$$

After taking logarithms of the first order condition and using a first order Taylor approximation around steady state which yields the inflation equation and the NKPC for the retail importer below:[3]

$$\pi_{F,t} = \beta E_t \left\{ \pi_{F,t+1} \right\} + \lambda_F \psi_{F,t} \tag{1.63}$$

1.10 Complete asset Markets, Exchange rate determination and Uncovered interest parity

The set of choices facing a household is dependent on the opportunity for trading financial assets in the financial markets. The existence of a full set of state contingent nominal traded assets has implication for consumption risk sharing and can uniquely pin down real exchange rate in a perfect foresight steady state. Exchange rate determination depends critically on the nature of asset markets. We discount from financial autarky and the case where limited assets classes are traded across borders, and subscribe to the case where each household can engage in trade and acquire a full set of state contingent claims or in this case, nominal bonds, and as

[3]The LOOP does not hold yielding eq (1.63), however, once we collapse this assumption, eq (1.63) collapses to eq (1.46)

shown in Gali. Et al. (2002), the following condition is obtained:

$$c_t = c_t^* + \frac{1}{\sigma} q_t \tag{1.64}$$

The above condition reflects complete markets, where price equalization in state-contingent claims which are invoiced in home currency implied by the equilibrium risk-sharing condition. From the above equation, consumption will vary across countries to the extent that exchange rate varies. And for a given policy, this equation fully determines both real and nominal exchange rate. As evident from the log-linearized relation, for a given level of preference, an increase in per capita home consumption relative to world consumption would invoke a depreciation of real exchange rate. Also, as we combine efficiency conditions from an optimal bond portfolio by home and foreign households, and after log-linearizing, we obtain the following equation:

$$i_t = i_t^* + E_t \Delta e_{t+1} \tag{1.65}$$

The above equation highlights the uncovered interest parity condition, where i_t is domestic nominal interest rate equals the world interest rate, i_t^* plus expected nominal exchange rate depreciation.

1.11 Labour Market Clearing

Equilibrium in the labour market requires that the aggregate household supply of labour is equal to the cumulative demand of labour by traded and non traded good firms. In a loglinearized equilibrium, we have:

$$n_t = n_{h,t} + n_{n,t} \tag{1.66}$$

Thus, in the absence of structural impediments that constrain the flow of human capital across sectors, we assume that labour is perfectly mobile across sectors, yielding a symmetric wage rate across sectors that clear the labour market.

A deeper delve into the dynamics between traded goods and non-traded sector output highlights some interesting dynamics. First, when we combine the aggregate labour market clearing condition with the non-traded good sector production function, and utilize the traded good production function, we derive a unique relation between traded good output and traded good technology, non-traded good technology, aggregate employment and non-traded good output as stated below:

$$Y_{h,t} = a_{h,t} + a_{n,t} + (1-\delta)n_t + \delta m_t - y_{n,t} \tag{1.67}$$

From the above relation, we see that traded sector output moves one for one with both traded and non-traded sector productivity, and negatively with non-traded sector output. Also, traded sector output is positively influenced by aggregate employment and to the extent that either traded and non-traded good sector employment improves aggregate employment, it also affects traded sector output positively.

1.12 Sectoral Output Dynamics and Composition of Real Marginal Cost

Following intermediate good firm's first order condition, we derive firm's marginal cost function, and after log linearizing and combining it with firms' production function, we arrive at the marginal cost function for both traded and non-traded firms as follows:[4]

$$mc_{h,t} = \left[\frac{\varphi(1-\delta)}{1-\gamma}\right] y_t - (1+\varphi)a_{h,t} - \varphi a_{n,t} + \theta_{h,y*}y* + \theta_{h,\psi}\psi_t + \theta_{h,v}V_t \tag{1.68}$$

$$mc_{n,t} = \left[\frac{\varphi}{1-\gamma}\right] y_t - \left[\frac{1+\varphi}{1-\delta}\right] a_{n,t} - \left[\frac{\varphi}{1-\delta}\right] a_{ht} + \theta_{n,y*}y* + \theta_{n\psi}\psi_t + \theta_{n,v}V_t \tag{1.69}$$

The above relation pins down the marginal cost of the traded sector and the marginal cost for the non-traded sector and shows that the real marginal cost is increasing in aggregate domestic output via its effect on employment vis a vis the real wage, and via the relative share of traded goods in aggregate output. Real marginal cost is also decreasing in domestic traded and non-traded technology improvement, through its amplifying impact on direct labour productivity. Also, the external environment through open economy factors affects real marginal cost. World output, through its effect on labour supply via risk sharing, and a relative price effects captured by terms of trade, the deviations from law of one price gap, and the relative price of non-traded goods all affect marginal cost.

When prices are flexible, real marginal cost is a constant, and the corresponding output level under flexible prices is called natural output. If we denote natural output by y^n, then equilibrium

[4]the parameters $\theta_{h,y*}, \theta_{s,ht}, \theta_{h,\psi}, \theta_{h,v}$ across eq (1.68) & (1.69) are composite parameters thus represented for analytical convenience and we document its derivation in appendix A

marginal cost under flexible prices is given by:[5]

$$mc_h = \left[\frac{\varphi(1-\delta)}{1-\gamma}\right] y_t^n - (1+\varphi)a_{h,t} - \varphi a_{n,t} + \theta_{h,y*}y* \tag{1.70}$$

From the above equation, we can derive the flexi-price level of output to be

$$y_t^n = \left[\frac{1+\varphi}{\theta_{y,t}}\right] a_{ht} + \left[\frac{\varphi}{\theta_{y,t}}\right] a_{nt} + \left[\frac{\theta_{y,*}}{\theta_{y,t}}\right] y_y^* \tag{1.71}$$

From the above equation, natural output is an increasing function of exogenous forces comprising technological innovations and changes in world output. Utilizing equation (1.68) and (1.70), we can derive the real marginal cost as a function of output gap, technology, foreign incomes and open economy variables as follows:

$$\hat{mc}_{h,t} = \left[\frac{\varphi(1-\delta)}{1-\gamma}\right] \tilde{y}_t + \theta_{h,\psi}\psi_t + \theta_{h,v}V_t \tag{1.72}$$

we substitute the above expression into the new Keynesian phillips curve for the traded sector in equation (1.58) and we have:

$$PCP : \pi_{H,t} = \beta E_t \left\{\pi_{H,t+1}\right\} + k_{yh}\tilde{y}_t + k_{vh}v_t \tag{1.73}$$

$$LCP : \pi_{H,t} = \beta E_t \left\{\pi_{H,t+1}\right\} + k_{yh}\tilde{y}_t + k_{\psi h}\psi_{F,t} + k_{vh}v_t \tag{1.74}$$

For the non-traded sector we derive the Phillips curve of the non-traded sector as:

$$PCP : \pi_{Nt} = \beta E_t \left\{\pi_{nt+1}\right\} + k_{yn}\tilde{y}_{t_t} + k_{vn}v_t \tag{1.75}$$

$$LCP : \pi_{Nt} = \beta E_t \left\{\pi_{nt+1}\right\} + k_{yn}\tilde{y}_{t_t} + k_{\psi n}\psi_{F,t} + k_{vn}v_t \tag{1.76}$$

where $\pi_{H,t}$ and $\pi_{N,t}$ are traded and non-traded good inflation respectively. Here, we model the case where the law of one price does not hold, under LCP. We have expressed our sectoral Philips curve in terms of domestic traded and non-traded inflation. Similarly, we can utilize price and inflation identities and express the new Keynesian Phillips curve in terms of CPI

[5] in eq (1.68) & (1.70) $mc_{h,t} \neq mc_h$. See appendix A

inflation as follows:

$$PCP: \pi_t = \beta E_t \{\pi_{t+1}\} + k_{ya}\tilde{y}_t + k_{va}v_t \tag{1.77}$$

$$LCP: \pi_t = \beta E_t \{\pi_{t+1}\} + k_{ya}\tilde{y}_t + k_{\psi a}\psi_{F,t} + k_{va}v_t \tag{1.78}$$

1.13 World Equilibrum

Here, we characterize the equilibrium in the rest of the world by assuming that in the rest of the world, policy makers aim at achieving the flexi-price equilibrium. We derive the evolution of flexi-price output from the domestic traded good sector marginal cost expressions by imposing autarky. This gives the marginal cost, and hence, natural output equation under flexible prices, to be:

$$p_t^* = p_{ft}^* : \pi_t^* = \pi_{ft}^*$$

$$\tilde{y_t}^* = \left(\frac{1+\varphi}{\sigma(1-\delta)+\varphi(2-\delta)}\right) a_{h,t}^* + \left(\frac{\varphi}{\sigma(1-\delta)+\varphi(2-\delta)}\right) a_{n,t}^* \tag{1.79}$$

Thus, under our assumption, there is an equivalence between domestic and CPI inflation which holds for the world economy as we invoke the divine coincidence for the rest of the world, where monetary authorities simultaneously stabilize both inflation and output gap, and these variables are exogenous to domestic allocations.

1.14 Equilibrium Aggregate Output

Aggregate equilibrium in the goods market is an expression of three components which reflects the domestic production and consumption of non-traded goods, domestic and foreign consumption of domestically produced traded goods. The latter which pins down exports gives us an understanding of the dynamics of goods market equilibrium. In this, export demand yield:

$$C_{h,t}^* = -\eta(P_{h,t} - P_{T,t}^*) - \eta_1(P_{T,t}^* - P_t^*) + C_t^*$$

This simplifies out under complete (PCP) and incomplete pass-through (LCP) respectively to be:

$$LCP: c_{h,t}^* = [\eta_1 + \alpha(\eta - \eta_1)]s_t + \eta_1\psi_t + c_t^* \tag{1.80}$$

$$PCP: c_{h,t}^* = [\eta_1 + \alpha(\eta - \eta_1)]s_t + c_t^* \tag{1.81}$$

Evidently, the above equations represent foreign demand for domestically produced traded goods. From the above equation, under LCP, export demand for domestic produced traded goods rises (falls) when the terms of trade depreciates (appreciates), and when the world price of export rise relative to the domestic currency price (a rise in the deviation of law of one price), and when foreign income increases. In the light of this, after substitution and aggregation, we obtain an expression for the aggregate output under both perfect and incomplete pass-through respectively to be:

$$Y_t = (1-\gamma)Y_{H,T} + (\gamma)Y_{N,T} \tag{1.82}$$

$$Y_t = (1-\gamma)\left[C_{H,T} + C^*_{H,T}\right] + (\gamma)C_{N,T} \tag{1.83}$$

$$y_t = (2-\gamma)y^*_t + \frac{\omega_\psi}{\sigma}\psi_t + \frac{\omega_s}{\sigma}s_t + \frac{\gamma}{\sigma}V_t \tag{1.84}$$

Where the following parameters ω_s and ω_ψ are the elasticities of output with respect to foreign incomes and relative prices consisting of terms of trade, deviation from law of one price gap, and the internal terms of trade. The aggregate output expression highlights some interesting dynamics. A real depreciation, which can come in either a fall in the domestic currency price of traded goods relative to foreign goods, reflecting a rise in terms of trade or a nominal depreciation of exchange rates which increases the deviation of the law of one price gap is necessary in equilibrium to sustain a rise in domestic output relative to foreign output.

1.15 Demand Block: Dynamic IS Relation

To consolidate the non-policy block of the model, we derive our aggregate demand relation which is expressed by the dynamic I-S equation which shows the interest rate and output combination that equilibrates the goods market. We utilize the aggregate output relation alongside with the complete market relation and the Euler relation from households first order conditions, and we arrive at the aggregate demand equation expressed as a function of domestic inflation below:

$$\tilde{y}_t = \tilde{y}_{t+1} - \frac{\omega_s}{\sigma}(i_t - E_t\pi_{H,t+1} - r^n) + \left[\frac{\omega_s - \omega_\psi}{\sigma}\right]\Delta\psi_{t+1} + \frac{\gamma}{\sigma}\Delta v_{t+1} \tag{1.85}$$

From our aggregate demand equation, we can extract the natural rate of interest which is dependent on a combination of traded and non-traded sector technology shock and foreign income shock.[6] From the above equation, current output gap is negatively related to the nominal interest rate, positively related to expected changes in output gap and expected changes in the law of one price gap. Also, expected changes in the law of one price gap are positively related to the expected changes in output gap. Natural or flexi-price allocations are indifferent to policy variables but totally dependent on exogenous shocks.

1.16 Policy Block: Interest rate specification.

In this model, we follow the conventional Taylor-type policy rule as specified in Gali and Monachelli (2005), we slightly adapt it to incorporate depreciation as a policy target which the monetary authorities respond to.

$$i_t = \rho + \phi_\pi \pi_t + \phi_{\tilde{y}} \tilde{y}_t + \phi_e \Delta e_t + u_t \tag{1.86}$$

Here, the monetary authorities adjust the nominal interest rate in response to deviation of inflation, output gap and depreciation from their target which happens to be the steady state value, where ϕ_π,$\phi_{\tilde{y}}$ and ϕ_e are weights attached to inflation deviations, output gap deviations and depreciation deviations from targets respectively. The monetary policy and foreign income shocks are captured by the following equations:

$$u_t = \rho_u u_{t-1} + \varsigma^u_{u,t} \tag{1.87}$$

$$y^*_t = \rho_{y^*} y^*_{t-1} + \varsigma^{y*}_{y,t} \tag{1.88}$$

where ρ_u and ρ_{y^*} are the auto-regressive persistence parameters in the shock specifications.

1.17 Solution and approximation Methods

Using the above DSGE model for theoretical and empirical investigations require finding a solution to the dynamic system. Hence a simple applicable approximation needs to be derived as most large and complex dynamic models lack a closed form solution. To minimize any complexity that may arise during computation, a variety of studies involving simulation and estimation use linear approximations of the true model, for which there exist a variety of methods

[6] we derive the expression for natural rate of interest in appendix A. Also, we can express our IS curve as a function of CPI inflation

which can be used to solve large linear dynamic difference models under rational expectations.

To describe in detail, our solution method which we implement in simulation to solve our model, we embarked on log-linearization of our dynamic non-linear system into a linear block of equations in terms of log-deviations of our variables from their steady state values. To Log-linearized our model, we employ the use of a first-order Taylor approximation of the model around its steady states.

A detailed survey of historical developments on a variety of solution methods is well documented by Canova (2007), stemming from the seminal work of Blanchard and Khan (1980), Unlig (1999) and Sims (2002), and despite the varying framework and procedures in methodologies, one harmonizing conclusion reflects the ability for each of the aforementioned solution technique to express the solution in a state space form, enabling the use of the Kalman-filter to perform likelihoods- based analysis and solution of DSGE models.

We utilize two methods which are the Blanchard and Khan (1980) method of undetermined coefficients, and the Klien's (2000) solution, which can be obtained by solving for the linear difference equations under rational expectation. Both frameworks are quite similar and we document the method of Klien's solution to a dynamic systems of equations which can be represented in the form below:

$$FE_t x_{t+1} = Gx_t + H\mu_t$$

$$\mu_t = J\mu_{t-1} + \varepsilon_t$$

Where F,G,and H, are matrix of coefficients, and J retains the auto-regressive parameter of the exogenous processes, and μ_t is the exogenous driving force of the model, while ε_t are the series of serially and mutually uncorrelated innovations. Just as in Blanchard and Khan, we can partition x_t into:

$$x_t = \begin{bmatrix} x_{1,t} & x_{2,t} \end{bmatrix}'$$

Where G is a vector of predetermined variables, and F represents a vector of non-predetermined variables, thus satisfying:

$$x_{t+1} = \begin{bmatrix} x_{1,t+1} & E_t x_{2,t+1} \end{bmatrix}'$$

To find a solution, this framework decouples the dynamic system into stable and unstable portions, using a complex process of generalized Shur decompositions, and recursively solving for the various components. When the number of unstable generalized eigenvalues of the loading matrix is equal to the number of non-predetermined variables, then the dynamic system is on a stable saddle-path and a unique local solution exists. This method, produces the following solution below:

$$x_{t+1} = K_0 x_t + K_1 \mu_t$$

$$h_t = K_2 x_t$$

where x_t is a vector of the model's state variables, including endogenous predetermined and exogenous variables. also, the matrices K_1 and K_2 are all a function of the models parameters. The Blanchard and Khan method of undetermined coefficients we also employ depends on a Jordan transformation or decomposition, which requires the invertibility of the anchor/full information matrix.

1.18 Simulation, Calibration and Interpretation of Results

Calibration of our model parameters helps us validate the extension, modification and prototype of our theoretical model, therefore we embark on the calibration of our model parameters to compute the model dynamics and to investigate if our calibrated model underscores stylized fact. The convention approach to calibration is to lean of pre-established parameter estimates from the literature, or to perform a parsimonious estimation using actual data for a specific country, or utilize a mixture of both.

In this research, we utilize the former and lean upon existing parameter estimates, with slight modifications to suit our model peculiarities. Our parameter choice for the discount factor follows convention, as well as the degree of price stickiness in the traded and non-traded good sector. The parameter governing the degree of pass-through θ_f is in line with the specification in Monachelli (2002), but will vary when we carry out sensitivity analysis in the proceeding chapters.[7]

1.19 Effect of Technology shock (Traded Good Sector)

For a technology shock in the traded sector, an improvement in technology has an expansionary effect on real activities evident in the response of consumption, non traded sector employment, traded sector output and aggregate output, which all rise on impact. This traded sector shock is complementary to the non-traded sector, as non-traded sector output experience a positive spillover effect, thereby rising on impact. Net export is not excluded from the positive first round effect which increases significantly.

Due to a traded sector positive technology innovation, this effectively goes to reduce firms' cost of production, as reflected in equation (1.68), through its effect on direct labour productivity. As firms can increase output for a given cost, this leads to an expansion of economic activity, as firms utilize more of all inputs to take advantage of this positive exogenous innovation, and employment and imported inputs increase.

Evidently, a positive innovation in technology increases aggregate output but by less than its natural counterpart, which causes a decline in output gap. Marginal cost is decreasing in the face of improvement in technology through its direct effect on labour productivity, which amplifies the effect of negative output gap via the Philips curve, leading to a decline in traded and non traded good inflation.

We expect the law of one price gap to rise in response to efficient productivity shocks, and thus, the response of output gap is magnified under LCP relative to PCP because expected changes in output gap are tied to expected changes in the relative price of traded goods which is amplified under LCP relative to PCP and expected changes in output gap are tied to expected changes in the deviations from law of one price gap via the aggregate demand equation or the dynamic IS equation. The latter channel is muted if the law of one price holds under PCP.

The response of nominal and real interest rate is declining upon impact, suggestive of the

[7]tabular display of our model parameters and calibrated values are in Appendix A

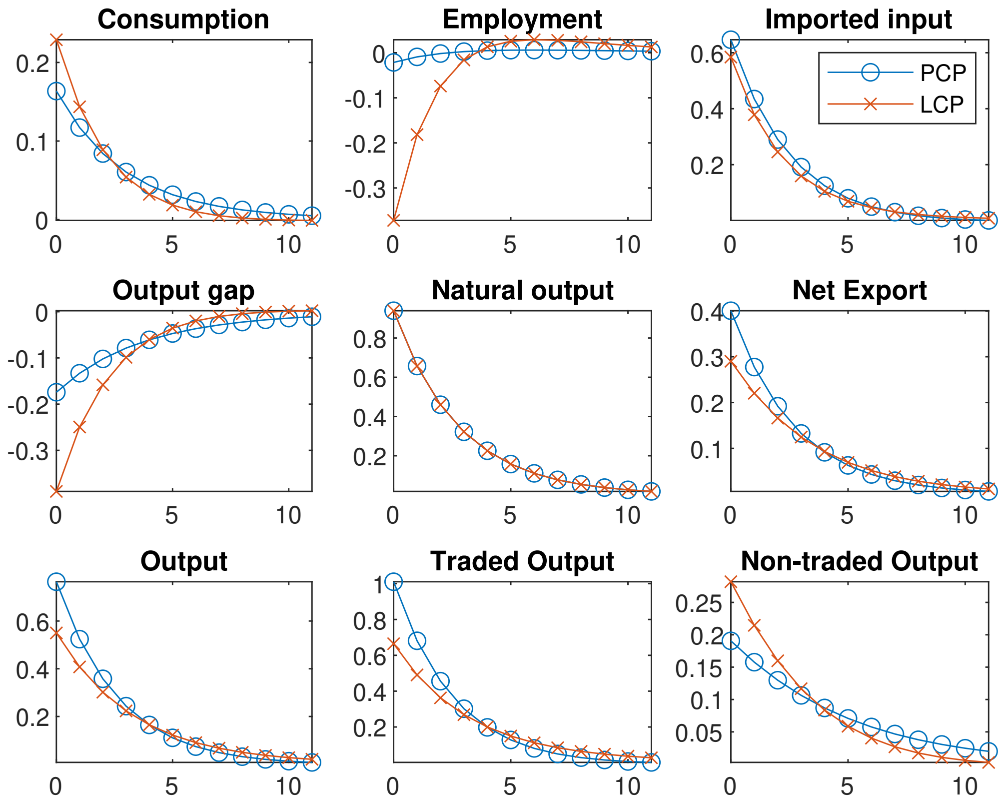

Figure 1: Response to a Traded sector Technology shock (Real quantities)

fact that monetary authorities are accommodating the positive innovations in technology. This eventually stimulate aggregate activity, causing a rise in non-traded good sector employment, consumption, sectoral and aggregate output, but this is not sufficient to close the negative output gap. However, it is evident that monetary authorities are less accommodating under PCP, as interest rates adjust to a less severe negative output gap under PCP relative to LCP.

In response to a technology shock, we see a depreciation of exchange rates and terms of trade. This triggers expenditure switching effects as exports become relatively cheaper and imports become relatively expensive thereby switching demand and improving net export which is rising. This open economy channel further amplifies aggregate demand, but this effect is dampened by the expected appreciation of exchange rates from the uncovered interest parity relation, as exchange rates overshoots steady state one period after the shock and slowly reverts to equilibrium.

Under PCP, we see the full expenditure switching effects as exports quantities rise and import

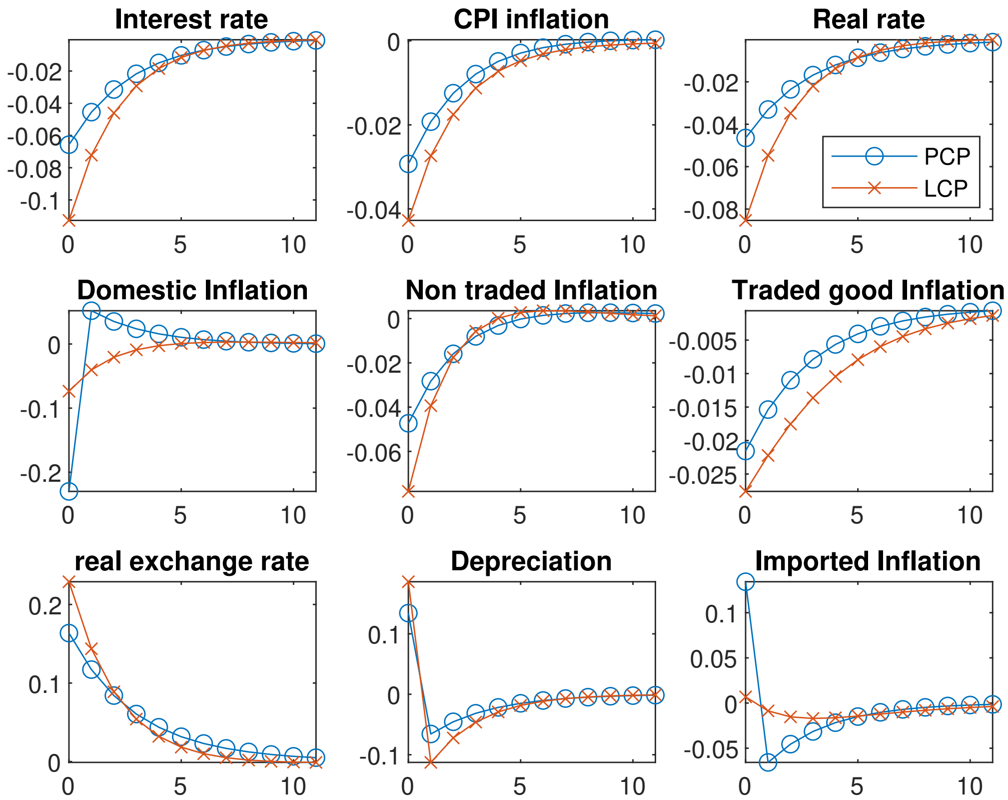

Figure 2: Response to a Traded Sector Technology shock (Rates and prices)

quantities fall, causing net exports to rise. Under LCP, net exports rise on impact as export quantities are rising faster than import quantities, however, we document a non-trivial dynamics as pass through is muted under LCP, and import prices are not fully reflective of the changes in exchange rates, thereby making import quantities to be less sensitive to conventional exchange rate impulses, and accounting for the magnified response of net exports under PCP relative to LCP.

Under efficient technology shocks, the distinction between PCP and LCP are evermore clear, as for a given level of depreciation of nominal exchange rate highlights a perfect pass-through under PCP and muted pass-through under LCP. From the results, a 1.2 percent depreciation is fully absorbed by import prices under PCP, as imported inflation rises by 1.2 percent while under LCP, the impact on import inflation is muted, i.e zero on impact. This accounts for the heterogeneous response of CPI inflation under PCP relative to LCP, as CPI and aggregate traded good inflation absorbs the impact of the depreciation in exchange rate through imported inflation.

Worthy of mention is the presence of excess volatility evident under LCP relative to PCP under short horizons. On impact, we see a more pronounced depreciation of real and nominal rates, and one period after, we see a more pronounced expected appreciation under LCP. The response of real variables is not proportional to this excess volatility indicating the presence of exchange rate disconnect. Finally, terms of trade[8] are more muted under LCP relative to PCP because importers pass-through the realized exchange rate changes into the prices of imports only slowly on the short term, and pass-through are complete over time.

1.20 Effect of Technology shock (Non-Traded Sector)

For the non-traded sector, a positive technology innovation has similar responses to that of the traded sector, with subtle differences which I would highlight. A positive technology shock in the non-traded sector reduces non-traded firms' cost of production, as reflected in the marginal cost in equation (1.69), through its effect on direct labour productivity. As firms can increase output for a given cost, this shock is contagious and spills over to the traded sector, causing traded, non-traded, and aggregate output to increase. As firms scramble to expand economic activity, they employ more factor inputs, thus tangible imported inputs see a considerable increase and we document the muted utilization of human capital.

As the non-traded sector accounts for about 30 percent of total output, this positive shock has less dire effects on non-traded good inflation as well as traded goods inflation relative to the response of the nominal variables to a traded sector technology shock. For the non-traded technology shock, this produces moderate deflationary pressures, causing the monetary authorities to respond mildly to these pressures relative to the response to a traded sector technology shock, thereby accommodating this shock.

This moderate response triggers a moderate depreciation of real and nominal exchange rates and terms of trade relative to that of traded tech shock, and triggers the expenditure switching mechanism, thereby increasing net export and amplifying the effect of an expansionary monetary policy coupled with positive technological improvement, causing aggregate demand and

[8] the response of terms of trade is in figure 12 situated in the appendix

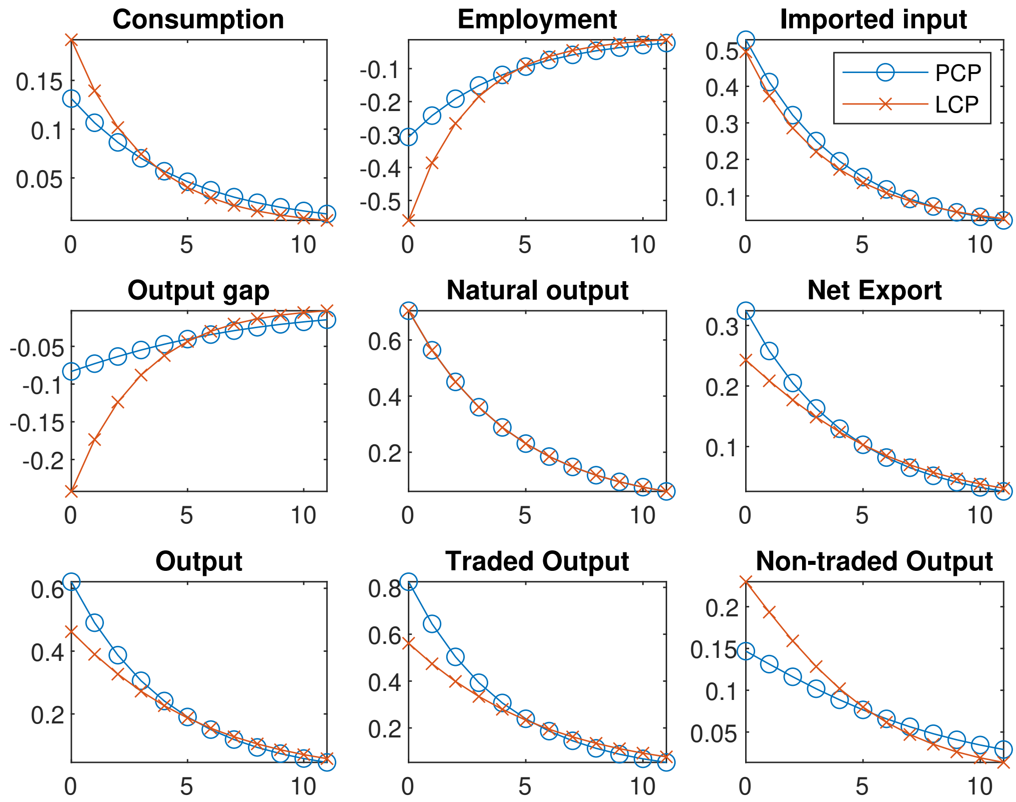

Figure 3: Response to a Non-Traded Technology shock (Real quantities)

output to rise.

Therefore, under efficient technology shocks in the non-traded goods sector, we document a distinction between PCP and LCP, as the depreciation of nominal exchange rate highlights a perfect pass-through into import prices under PCP and a muted response on impact under LCP. This in turn, induces a more muted response of terms of trade under LCP, relative to PCP.

Under PCP, we see the full expenditure switching effects as exports quantities rise and import quantities fall, causing net exports to rise. Under LCP, net exports rise on impact as export quantities are rising faster than import quantities, however, we document a non-trivial dynamics as pass through is muted under LCP, and import prices are not fully reflective of the changes in exchange rates, thereby making import quantities to be less sensitive to conventional exchange rate impulses, and accounting for the magnified response of net exports under PCP relative to LCP in response to positive technology innovations in the non-traded good sector.

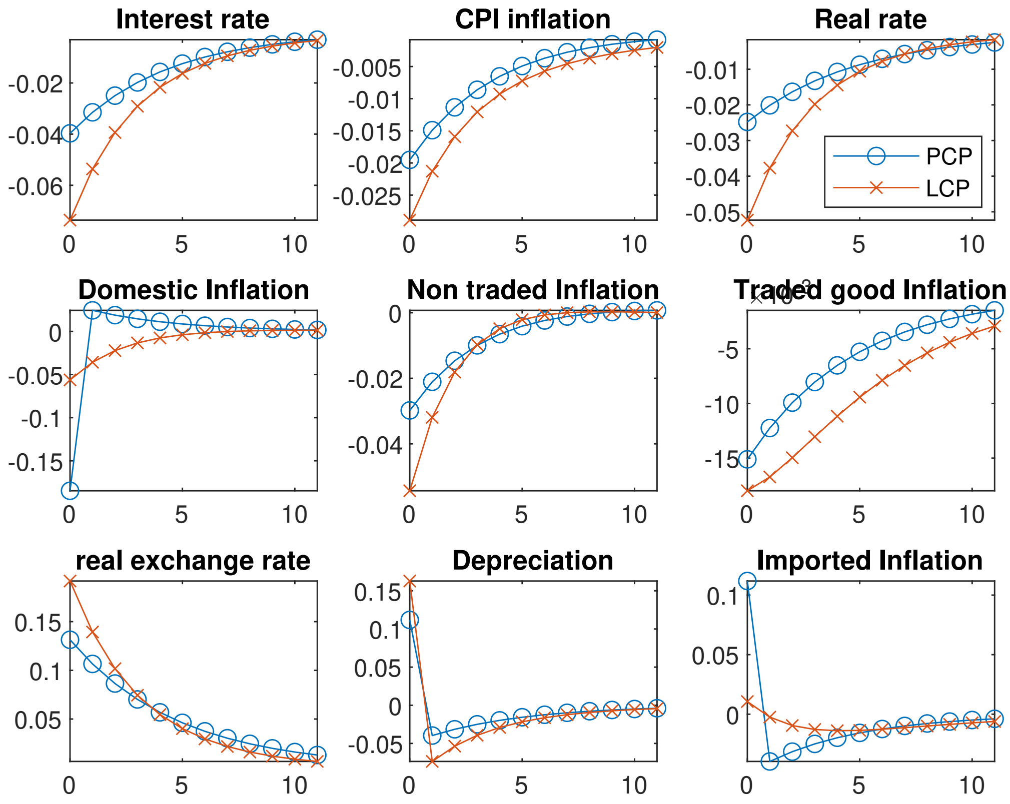

Figure 4: Response to a Non-Traded Technology shock (Rates and prices)

1.21 Effect of Monetary Policy shock

In response to an exogenous monetary policy shock which raises the nominal interest rate exogenously, the immediate impact is an increase in firm's cost of production, thus firms making input and output decisions are internalizing this shock, and as a result, downsizing on the use of imported input, and employment in both sectors which drops significantly. This results in a decline in real quantities as reflected in a fall in traded sector and non-traded sector output, and cumulatively, aggregate output declines.

Also, in a way consistent with our analytical results, we see from the graph that in response to an exogenous monetary policy shock, there is a decline in real rate, domestic, imported and consumer price inflation, terms of trade and real exchange rate, and output on impact.

The response of output gap is synonymous to output, as natural output is invariant to a monetary policy shock and thus the response of output gap mirrors the decline in output one for one.

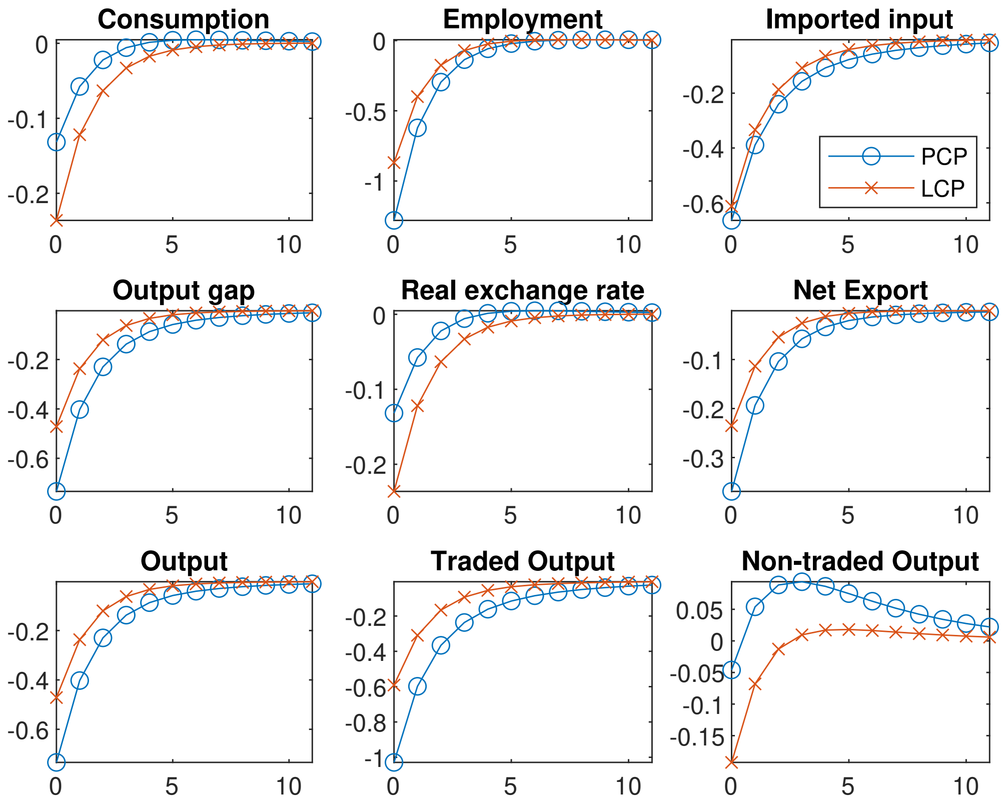

Figure 5: Response to a monetary policy shock(Real Quantities)

Note that under out calibration, the nominal interest rate goes up but by less that the exogenous component, due to the joint decline of both output gap and domestic price inflation, thus leading to a contractionary response as the real rate goes up.

This increase in real rate has a contractionary impact on real variables as consumption, aggregate and sectoral output and employment all fall, causing a fall in aggregate demand. This is further amplified by the exchange rate channel, as the fall in domestic price and CPI cause a real appreciation of terms of trade and real exchange rates, as evident in Fig1.2. Expenditure switching effects kick in as exports become relatively more expensive and imports become relatively cheaper thereby causing net export to fall, amplifying the decline in aggregate demand and real activity.

From the Dynamic IS equation, output gap is very sensitive to changes in nominal and real interest rates, and in response to an increase in interest rates , the uncovered interest parity relation kicks in and we see a sharp appreciation of exchange rate, which will be partly offset by

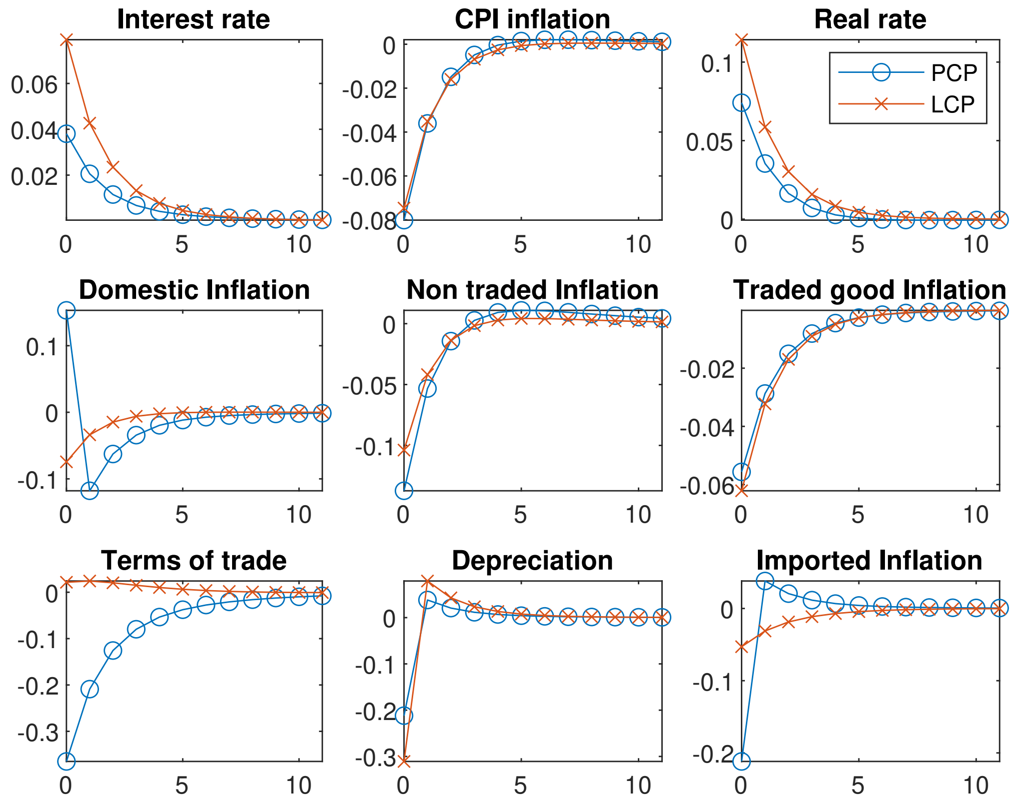

Figure 6: Response to a monetary policy shock (Rates and Prices)

an increase in CPI inflation relative to domestic inflation, induced by an expected real depreciation, which would dampen the change in consumption-based real rate, there by engineering a soft landing of real quantities and variables.

From the graph, a key observation arises in the LCP model where pass through is incomplete relative to PCP. Under PCP, in response to the exogenous policy shock, we see a 0.2 percent appreciation of nominal exchange rate on impact and for PCP, import price absorbed all the shock response, falling by 0.2 percent, and pass-through is conceivably 100 percent. Under LCP, the story is not far fetched, for a 0.3 percent appreciation of nominal exchange rates, import price falls about 0.05 percent, absorbing about 20 percent of the exchange rate change and evidently, pass-through is incomplete. But for export price, we do not expect such effect as domestic inflation/export prices are influenced by the determinants of marginal costs and law of one price hold for export prices.

This pass-through variation is better understood as law of one price holds at the dock, and importers purchase imports where we assume law of one price hold at the dock but in setting the domestic prices for these imports, these retailers solve a dynamic markup problem which causes the domestic prices they set to deviate from the world price slowly in the short term, but which converge only asymptotically, in the long run.

One period after the shock as we expect a nominal depreciation of exchange rate, we document the evidence of an overshooting of the exchange rate, but one thing is clear, in the LCP model, the law of one price gap contributes to the volatility of nominal exchange rate highlighting the fact that the lower the pass through, the larger the nominal exchange rate variation required to achieve a given equilibrium.

Under PCP, we see that the appreciation of both nominal and real exchange rates produces conventional expenditure switching effects, but under LCP, the LOOP holds for exports prices and the real and nominal appreciation produces similar effects when it comes to export volumes as we see export volumes/quantities[9] fall because exports are now relatively more expensive. On the import side, the effect is seemingly conventional as imports become relatively cheaper and import quantities rise.

Thus, due to the deviation from Law of one price, a nominal and real depreciation would decrease the world price of imports, but the totality of such variation is not be reflected in the domestic price of import due to the wedge, or the Law of one price gap, and thus, import quantities are unresponsive to this conventional relative price signals caused by the real appreciation, and thus both exports and imports are falling, but exports are declining faster than imports. The effect of this on net export is that on impact, net exports fall considerably but we see a amplified response of net export under LCP relative to PCP, and this magnified response is attributed to the import volumes being unconventionally responsive to the relative price changes caused by a nominal and real appreciation.

[9]the response of trade quantities are documented in figure 11 situated in the appendix

1.22 The role of Imported Input

In the presence of a positive innovation in traded good technology, we see conventional responses across both models. In our benchmark model with imported input, traded good technology reduces firms' cost per unit of production, and we see a decline in domestic inflation, and this technological innovation is complementary to both sectors as we see a decline in non-traded good inflation, and as output and as consumption increases, the risk sharing condition necessitates a real and nominal depreciation of exchange rates and terms of trade, and this impulse is fully absorbed by import prices as our model underscores perfect pass-through.

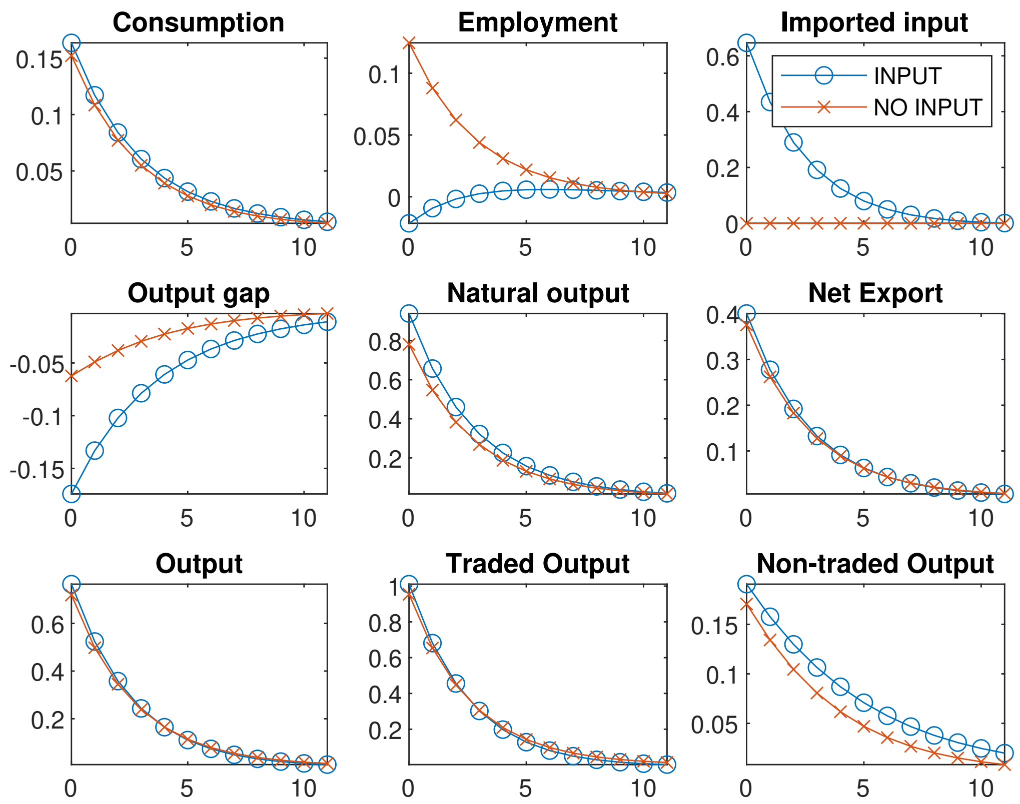

Figure 7: Isolating the role of Imported input: Traded Technology shock

Monetary authorities accommodate this positive technology shock as evident by the expansionary decline in nominal interest rates and coupled with declining prices, real interest rates decline, further stimulating production, consumption and aggregate demand.

The risk sharing condition which necessitates a depreciation, coupled with a declining interest rate ensure that agents reallocate portfolio away from the domestic assets and this portfolio rebalancing triggers a real and nominal depreciation which further amplifies aggregate demand as exports become relatively cheaper and imports relatively expensive, leading to the conventional expenditure switching effects thereby strengthening and supporting the expansion of economic activities across both models.

We highlight the role of imported input in our model by highlighting the difference between our benchmark model and the model without imported input. The impact of imported input on the dynamics of our model can be traced to two sources, owing to firms' decision making, and due to the structural framework of our model. In the model without imported input, firms utilize only one factor input, namely labour, in production of goods and services and an increase in technology which reduces firms' marginal cost of production enable firms to produce a given level of output with less labour inputs as technology is labour augmenting.

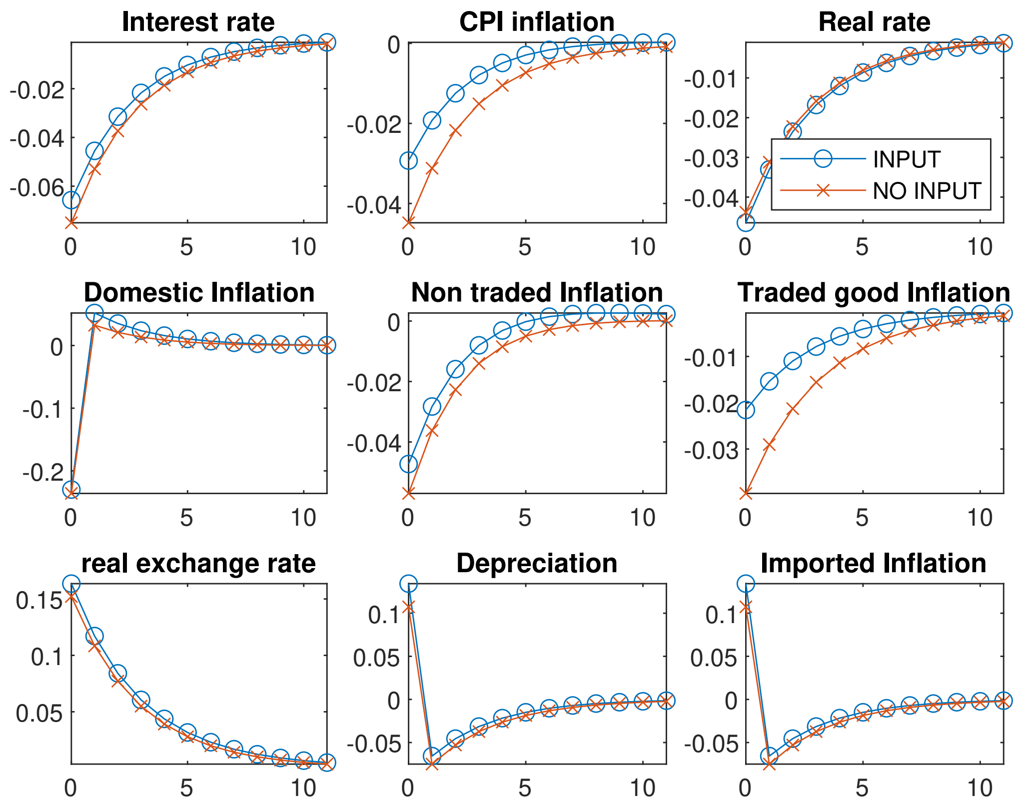

Figure 8: Isolating the role of Imported input: Traded Technology shock

Therefore, we see a mild decline in traded sector employment and this labour resources are absorbed by the non-traded sector, who can utilize more resources to boost production, thereby reaping the increase in marginal productivity of labour. As aggregate demand for labour increase, firms bid up the wage rate and we see a sharp increase in real wages and aggregate employment in the model without imported input. We also document an increase in consumption and aggregate output and in both traded and non-traded sectors.

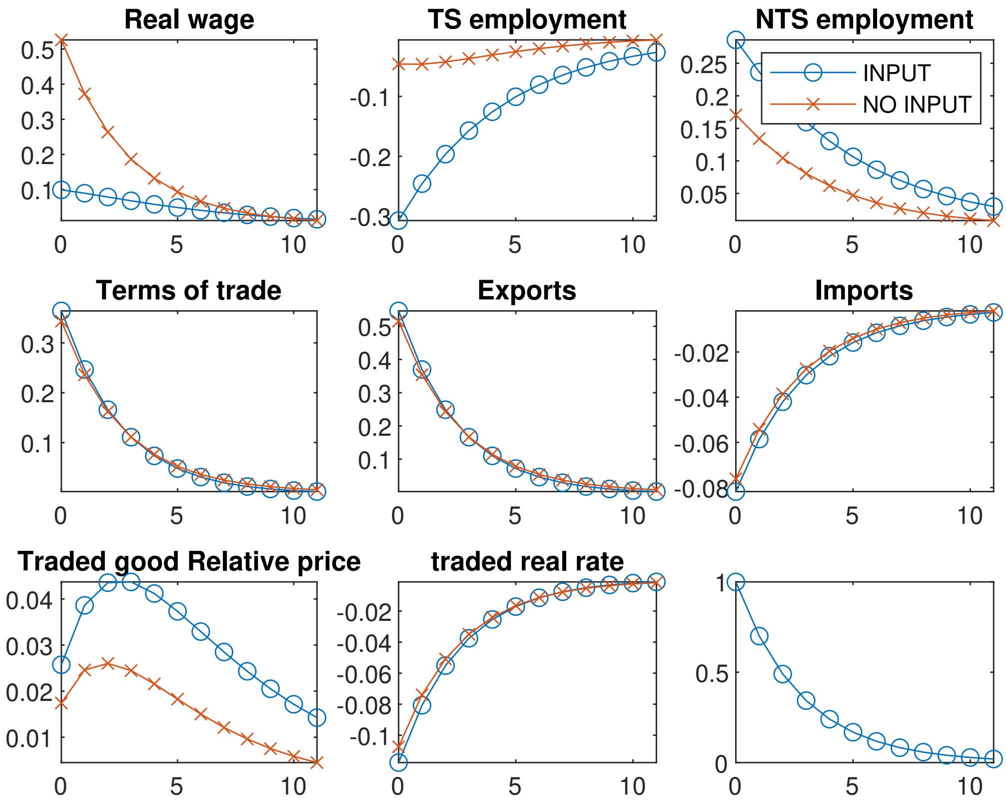

Figure 9: Isolating the role of Imported input: Traded Technology shock)

From the structural side, in the model without imported input, natural output as calibrated with the possibility of labour endowments as the only productive resource for this economy, is lower than with the model with imported inputs and we witness an increase in natural output in response to a traded good technology shock, which grows faster than aggregate output inducing a decline in output gap. This decline in output gap is relatively amplified for the benchmark model as the rise in aggregate output is homogeneous across models, yielding an amplified decline in output gap in the benchmark model. This in turn is responsible for generating a decline in inflation via the new Keynesian Phillips curve (NKPC).

Under the benchmark model with imported input, we document a slightly higher relative consumption, causing a real and nominal depreciation which is amplified under the benchmark model. This amplified response feeds into import prices causing an amplified increase in imported inflation under the benchmark model. As this amplified response feeds into aggregate traded good prices and consumer price inflation, we have a more muted response under the benchmark model.

In conclusion, the presence of imported input in production amplifies the volatility of real and nominal exchange rates and softens the demand for labour in the traded sector leading to a mild decline in traded sector labour demand, but a significant increase labour demand by the non-traded sector, causing an increase in aggregate employment and bidding up real wages and this effect is amplified in the model without imported input.

1.23 Effect of Foreign Income shock

In the presence of a positive foreign income shock, our results are non-trivial and conventional. Our linearized model provides an insight into the dynamics we see in response to a positive foreign income shock. This positive innovation in foreign incomes is synonymous to a huge tidal wave that raises all the small ships on the sea.

For our small open economy, an increase in foreign incomes is complementary to domestic allocations as sectoral and aggregate output, incomes and consumption all rise on impact. Monetary authorities are accommodating of this non-inflationary positive innovation in foreign incomes as evidenced by an expansionary monetary policy as nominal interest rates decline on impact.

Prices across the price chain from non-traded to traded prices, import prices and CPI inflation all experience a decline on impact creating deflationary pressures, however, monetary authorities are quite accommodative as the size of the rate cuts are more pronounced relative to the fall in prices, causing real interest rates to fall, thereby stimulating consumption, production of traded and non-traded goods and aggregate output.

As expected, natural output increases as reflected in equation (1.71), but less than aggregate output as the joint impact of rising foreign incomes on aggregate output via risk sharing and

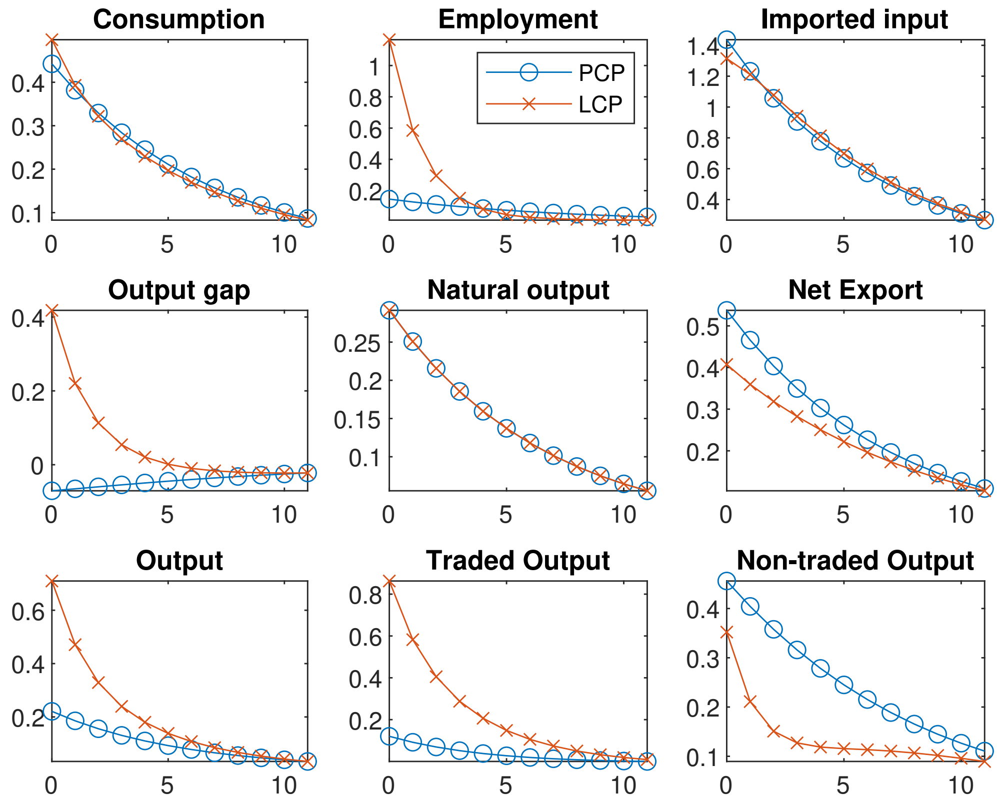

Figure 10: Response to a Foreign Income shock(Real Quantities)

market clearing relations and the expansionary policy posture of monetary authorities supersede the rising impact on natural output, causing output gap to rise. This rising output gap is however not sufficient to moderate the deflationary pressures via the Phillips curve, thus sustaining a non-inflationary output growth and an expansion of economic activities.

The response of open economy variables (exchange rates, terms of trade, and law of one price gap) are reflective of the intuitive predictions as reflected in the market clearing and risk sharing relations. A rise in foreign incomes relative to the domestic economy would in equilibrium, require a real appreciation of exchange rates. Our results document this, as on impact, we witness a real appreciation and a critical look at the determinants of real exchange rates would imply that this can only be achieved via two channels. First, a fall in the domestic currency price of imported goods relative to domestic goods (a fall in s_t, or an appreciation of terms of trade) and a nominal appreciation which would trigger a deviation from the law of one price and a fall in the law of one price gap, ψ_t, all of which are evident from our results.

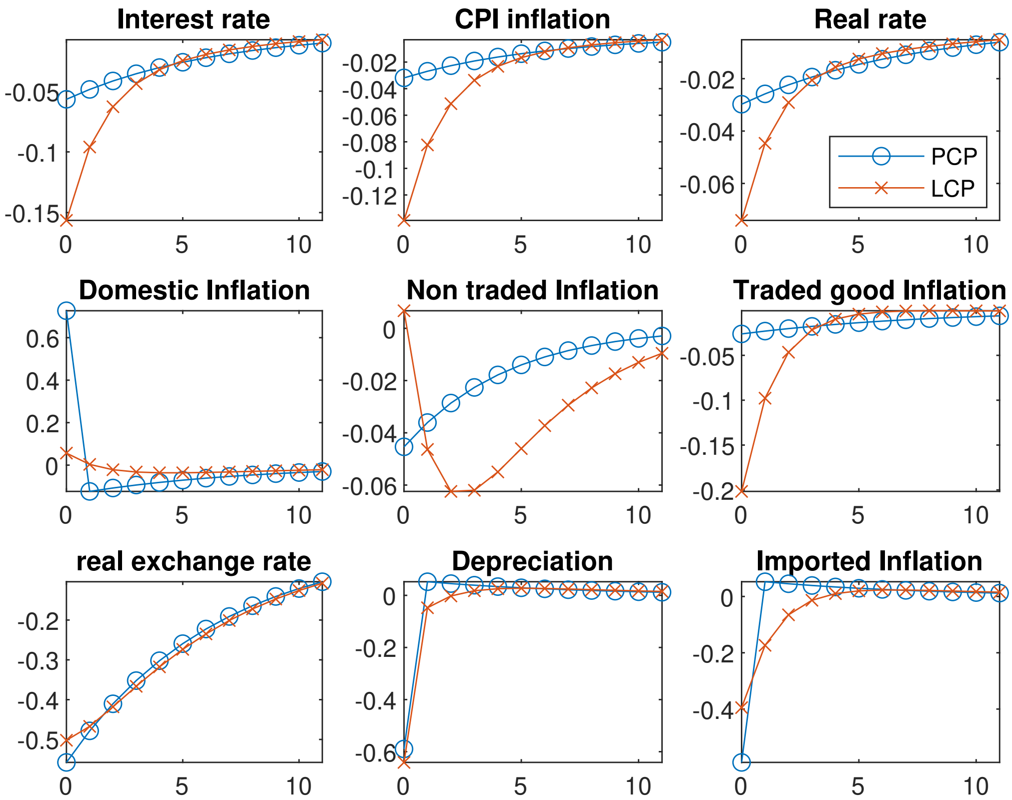

Figure 11: Response to a Foreign Income shock(Rates and Prices)

The response of net exports is non-trivial. From the export relation, a positive shock to foreign incomes should increase exports and hence, net exports and this is quite evident from our results. However, contrary headwinds from a real appreciation of exchange rates due to risk sharing now works to make exports relatively more expensive as evident from our results. However, this contrary impulse does not neutralize the positive effect from a rising foreign income, as foreign incomes rise faster than export prices,[10] sustaining the rise in export quantities on impact under both PCP and LCP. Under LCP, as imports are relatively cheaper as evidenced by falling import prices, import quantities rise on impact in response to a real appreciation. As exports rise faster than imports, this makes net exports rise on impact, which faster accelerates aggregate demand, and aggregate output.

In summary, for a technology shock in either sectors, this shock seemed to be positively contagious to the other sectors, and monetary authorities accommodate these shocks which expand

[10]here income effects dominates the substitution effects causing quantities to respond positively despite rising prices

economic activity. Our model also highlighted the fact that in the presence of an efficient productivity shock, law of one price gap must rise, and a greater exchange rate variation which is needed on the path to equilibrium because firms utilize imported inputs in production, thus transmitting these price shocks into their domestic pricing decisions. For a monetary policy shock, our model displayed conventional results, clearly highlighting perfect and incomplete pass through under PCP and LCP respectively. A foreign income shock as witnessed, is expansionary with non inflationary effects as domestic inflation, traded good and non traded good inflation, and CPI inflation are falling, and with monetary authorities accommodating its effect, as central banks take an expansionary stance thereby further stimulating and expanding sectoral and aggregate output, consumption and general economic activity. We also find that while imported inputs in production amplify realized volatility in our theoretical model, a higher share of non-traded goods in consumption mitigates realized volatility, and in the presence of technology shocks, monetary authorities are more accommodatory thereby creating a non-inflationary economic expansion.

1.24 Appendix

1.25 A1: LogLinearized Model

Labour supply

$$w_t - p_t = \sigma c_t + \varphi n_t \tag{1.89}$$

Interest rate rule (Monetary policy)

$$i_t = \rho + \phi_\pi \pi_{h,t} + \phi_{\tilde{y}} \tilde{y}_t + \phi_e \Delta e_t + u_t \tag{1.90}$$

Dynamic IS equation

$$\tilde{y}_t = \tilde{y_{t+1}} - \frac{\omega_s}{\sigma}(i_t - E_t \pi_{H,t+1} - r^n) + \left(\frac{\omega_s - \omega_\psi}{\sigma}\right) \Delta \psi_{t+1} + \frac{\gamma}{\sigma} \Delta v_{t+1} \tag{1.91}$$

Traded sector Phillips curve

$$LCP: \pi_{H,t} = \beta E_t \left\{ \pi_{H,t+1} \right\} + k_{y,h} \tilde{y}_t + k_{\psi,h} \psi_{F,t} + k_{vh} v_t \tag{1.92}$$

Non traded Phillips curve

$$LCP: \pi_{Nt} = \beta E_t \{ \pi_{nt+1} \} + k_{yn} \tilde{y}_{t\,t} + k_{\psi n} \psi_{F,t} + k_{vn} v_t \tag{1.93}$$

Retail importer Phillips curve

$$\pi_{F,t} = \beta E_t \left\{ \pi_{F,t+1} \right\} + \lambda_F \psi_{F,t} \tag{1.94}$$

output gap

$$\tilde{y}_t = y_t - y_t^n \tag{1.95}$$

Natural output

$$y_t^n = \frac{1+\varphi}{\theta_{y,t}} a_{ht} + \frac{\varphi}{\theta_{y,t}} a_{nt} + \frac{\theta_{y,*}}{\theta_{y,t}} y_t^* \tag{1.96}$$

aggregate labour supply

$$n_t = n_{h,t} + n_{n,t} \tag{1.97}$$

goods market clearing

$$y_t = (2-\gamma)y_t^* + \frac{\omega_\psi}{\sigma}\psi_t + \frac{\omega_s}{\sigma}s_t + \frac{\gamma}{\sigma}V_t \tag{1.98}$$

Risk sharing

$$c_t = c_t * + \frac{1}{\sigma}q_t \tag{1.99}$$

law of one price gap

$$\psi_{F,t} = (e_t + p_t^*) - p_{F,t} \tag{1.100}$$

traded production function

$$y_{ht} = a_{ht} + (1-\delta)n_{ht} + \delta m_t \tag{1.101}$$

non traded production funciton

$$y_{nt} = a_{nt} + (1-\delta)n_{nt} \tag{1.102}$$

non traded output

$$y_n = \eta_1(1-\gamma)V_t + c_t \tag{1.103}$$

imported input supply

$$m_t = y_h + s_t \tag{1.104}$$

Aggregate output

$$y = (1-\gamma)y_{ht} + \gamma y_{nt} \tag{1.105}$$

exports

$$LCP : c_{h,t} = [\eta_1 + \alpha(\eta - \eta_1)]s_t + \eta_1\psi_t + c_t^* \tag{1.106}$$

imports

$$c_{f,t} = c - \eta(1-\alpha)s_t - \eta_1\gamma V_t \tag{1.107}$$

net exports

$$nx_t = y_t - c_t - \alpha s_t - \gamma V_t \tag{1.108}$$

Aggregate traded goods inflation

$$\pi_{T,t} = (1-\alpha)\pi_{ht} + \alpha\pi_{ft} \tag{1.109}$$

Consumer price inflation

$$\pi_t = \pi_{nt} + (1-\gamma)[V_t - V_{t-1}] \tag{1.110}$$

real exchange rate

$$q_t = \psi_{F,t} + (1-\alpha)s_t + \gamma V_t \tag{1.111}$$

traded goods technology process

$$a_{ht} = \rho_{ah} a_{ht-1} + \varsigma^{a}_{h,t} \tag{1.112}$$

nontraded goods technology process

$$a_{nt} = \rho_{an} a_{nt-1} + \varsigma^{a}_{n,t} \tag{1.113}$$

Monetary policy shock process

$$u_t = \rho_u u_{t-1} + \varsigma^{u}_{u,t} \tag{1.114}$$

foreign income shock process

$$y^*_t = \rho_{y^*} y^*_{t-1} + \varsigma^{y*}_{y,t} \tag{1.115}$$

Natural interest rate

$$r^n = \left[\frac{\sigma}{\omega_s}\right] \Delta y^n_{t+1} + \left[\frac{\omega_s - (2-\gamma)}{\omega_s}\right] \Delta y^*_{t+1} \tag{1.116a}$$

Real interest rate

$$r_t = i_t - E_t \pi_{t+1} \tag{1.116b}$$

1.26 A2: Derivation of traded and non traded sector NKPC

For the traded sector, we show the derivation of real marginal cost as expressed in equation (1.68). We start by adding and subtracting $(1-\delta)p_t$ to the RHS of equ. 1.38 to have

$$mc^n_{h,t} = (1-\delta)w_t - a_{h,t} + \delta p_{f,t} + (1-\delta)p_t - (1-\delta)p_t \tag{1.117a}$$

$$mc_{h,t} = (1-\delta)\left[w_t - p_t\right] - a_{h,t} - \delta[p_t - p_{f,t}] \tag{1.117b}$$

For the non-traded sector, we add and subtract p_t to the RHS of equ 1.39

$$mc^n_{n,t} = w_t - a_{n,t} + \delta n_{n,t} + p_t - p_t \tag{1.118a}$$

$$mc_{n,t} = [w_t - p_t] - a_{n,t} + \delta n_{n,t} \tag{1.118b}$$

Equation (1.117b) and equ (1.118b) are the expressions for real marginal costs for the traded and non-traded sector respectively.

We substitute equ (1.89),(1.97),(1.98),(1.99),(1.101),(1.103),(1.1074),(1.111) and (1.117) to arrive at equation (1.68) and (1.69) in the text respectively. We derive the flexiprice level of equ (1.68) which is (1.70) by imposing on (1.68) flexible prices i.e $p_{T,t} = p_{N,t} = 1, \psi_t = 0$, as our natural output characterize the flexi price level of output under PCP, i.e when the law of one price hold.[11] When we subtract equ (1.70) from (1.68) we have equ (1.72) recognizing that $\hat{mc}_{h,t} = mc_{h,t} - mc_h$ and $\bar{y}_t = y_t - y^n_t$

To derive our Dynamic IS relation in equ (1.85), we use the complete market relation and real exchange rate relation in (1.64) and (1.84) to eliminate the terms of trade s_t in our market clearing relation in (1.84) to have:

$$c_t = \left[\frac{1-\alpha}{\omega_s}\right] y_t + \left[\frac{\omega_s - (2-\gamma)(1-\alpha)}{\omega_s}\right] y^*_t + \left[\frac{\omega_s - \omega_\psi(1-\alpha)}{\sigma\omega_s}\right] \psi_t + \gamma \left[\frac{\omega_s - (1-\alpha)}{\sigma\omega_s}\right] V_t \tag{1.119}$$

We substitute this into our euler relation in equ (1.52) to have our plain vanilla version of equ (1.85) of which we impose flexiprice and when we subtract it from the plain vanilla version, we arrive at our dynamic IS relation in equ (1.85)

[11] see monachelli 2002

1.27 A3: Parameter description for Calibrated Model

Description	Parameter	Value
Discount factor	β	0.99
Risk aversion	σ	1
Inverse frisch elasticity	φ	3
Openness	α	0.572
Share on Nontraded good	γ	0.3
Home/foreign elasticity of substitution	η	1.5
Traded/non traded elasticity of substitution	η_1	1.5
Calvo probability for domestic firms	θ_H	0.75
Calvo probability for importing firms	θ_F	0.75
Dixit-stiglitz demand elasticity of substitution	ε	6
Labour share of traded goods	δ	0.33
Traded tech autocorr process	ρ_{ah}	0.8
Non-traded tech autocorr process	ρ_{an}	0.9
M-P autocorr process	ρ_u	0.5
Foreign Income autocorr process	ρ_{y^*}	0.86
Inflation feedback	ϕ_π	1.5
Output gap feedback	ϕ_y	0.125
Depreciation feedback	ϕ_e	1.8

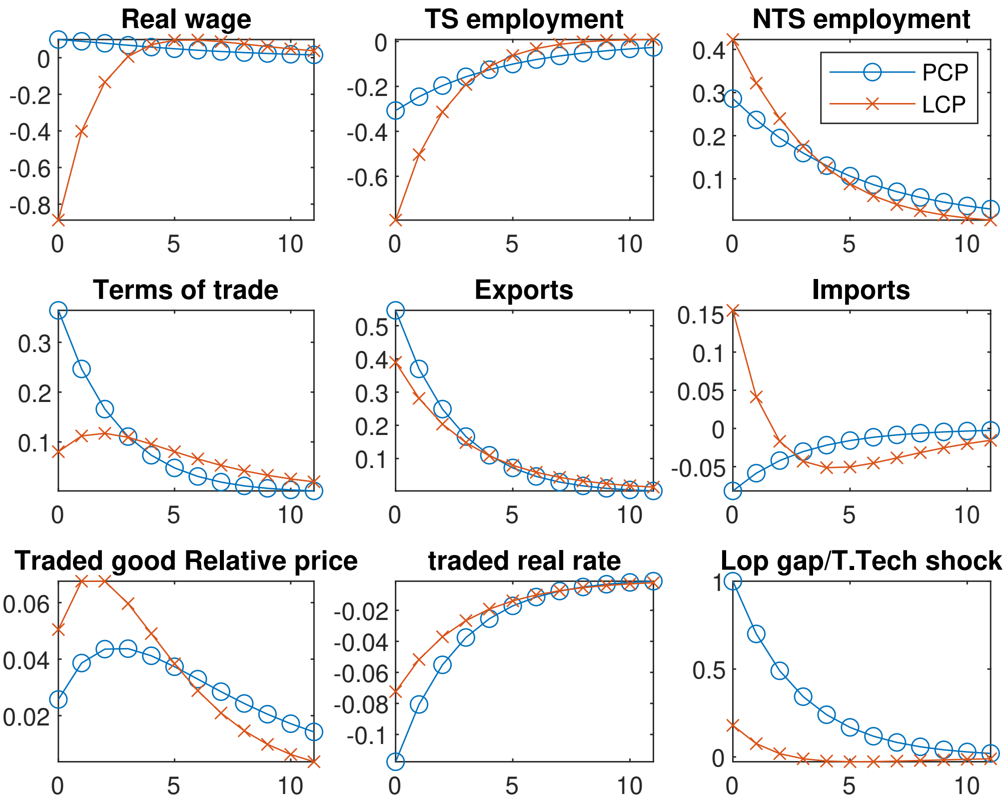

Figure 12: Response to a Traded Technology shock (Supplementary Variables)

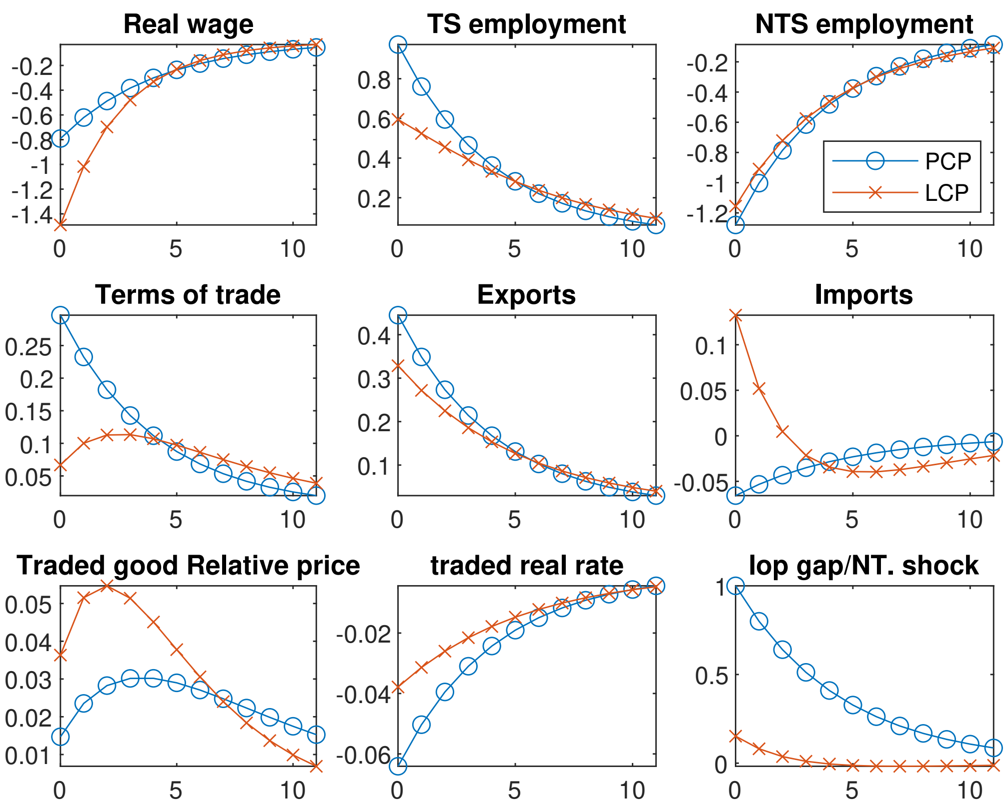

Figure 13: Response to a Non-Traded Technology shock (Supplementary Variables)

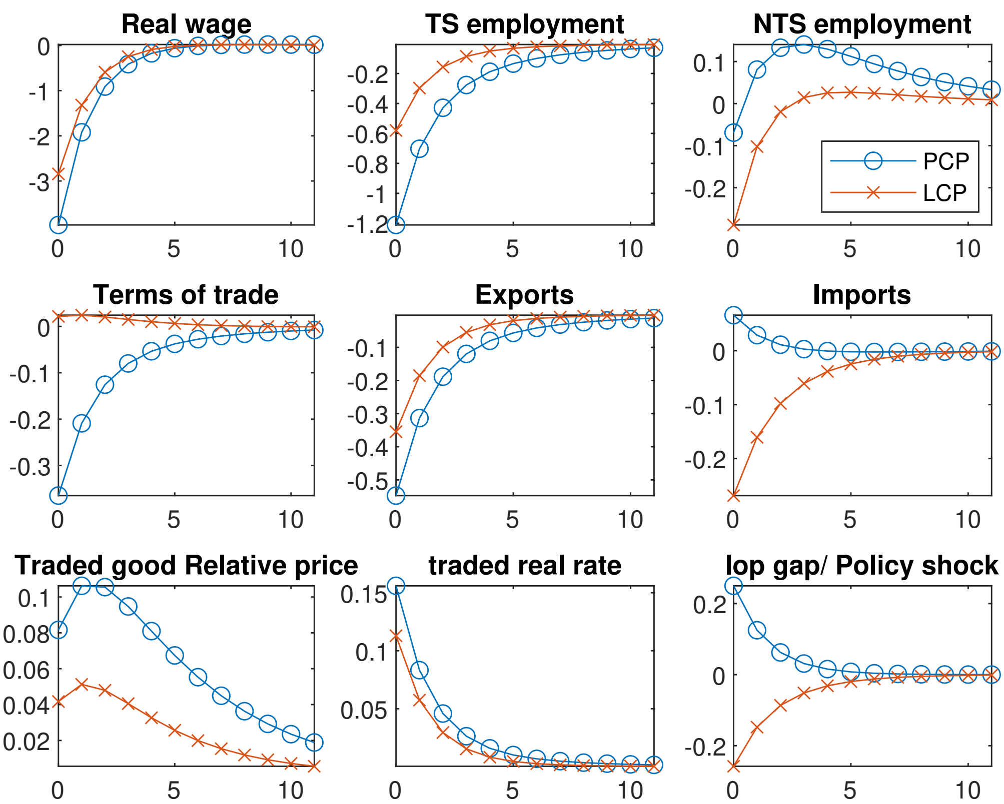

Figure 14: Response to a Monetary policy shock (Supplementary Variables)

2 OPTIMAL MONETARY POLICY

2.1 Introduction

In this chapter, we study the design of monetary policy aimed at improving and stabilizing the economy modelled in the previous chapter. We have earlier presented a detailed new Keynesian DSGE model, where structural equations are a derivative of the optimizing decision making of various agents, households and firms alike. The goal here is to design monetary policy to achieve the objectives of a benevolent social planner while taking as constraints, the various economic assumptions, resource endowments and economic environment that underscores agents' interactions.

The conventional view stemming from established literature show that under simple models' featuring nominal rigidity and imperfect competition the social planner would stabilize inflation, which is synonymous to stabilizing the output gap, which is welfare optimal, highlighting the divine coincidence (see Gali (2002) and Woodford (2003) for a survey). Accordingly, models that introduce a trade-off do so either by arbitrarily introducing a cost push shock or through sticky wages, and in this research, we make a novel contribution to the analysis of optimal policy owing to the unique features of our model economy under consideration. This novel contribution lends itself to the breakdown of the divine coincidence via two sources. One, by accounting for the deviations from the law of one price and second stemming from the relative sectoral price distortion.

Our main results validate the presence of a stabilization bias as central banks can manage expectations effectively under commitment, therefore, improving the per period trade-offs and yield welfare optimal outcomes. We also find the optimal interest rate function that recover the static and dynamic equilibrium under time inconsistency and under commitment, which is a function of the deviations of the law of one price gap and the relative price of traded goods.

As we evaluate welfare-based policies under alternative regimes, we find that as the degree of pass-through increases, optimal policy constrains the policy maker to be outward looking, targeting CPI inflation which yield the best welfare outcome relative to other regimes, and as the share of non-traded goods in consumption increases, the policy maker is constrained to be more inward looking in targeting Domestic Inflation.

The literature on RBC models have been the foundation for understanding economic dynamics in general equilibrium and optimizing models and a natural extension of RBC models is the limited role of monetary policy as the predictions of such framework highlight the neutrality of monetary policy in the short term. But as documented in the previous section, the detailed presentation of the new Keynesian model highlights the potency of monetary policy in the short term in influencing aggregate activity, and thus the motivation for the design of optimal monetary policy.

It is well documented in traditional small open economy models that in the presence of real and nominal rigidities, an economy's natural allocation becomes inefficient, and by means of an employment subsidy that adequately offsets the distortion caused by the presence of real rigidities, which features through market power of firms operating in monopolistically competitive environments, if policy makers can stabilize markups at their frictionless level, the nominal rigidity and resulting consequence of price stickiness becomes non-binding, and the natural allocation becomes efficient.

Under such environment, we have a divine coincidence, as the benevolent social planner can fully stabilize both inflation and output gap at no cost,[12] due to the lack of trade-offs, and any deviation from this optimal policy could become costly and suboptimal and yield welfare losses.

In the open economy, in the presence of a mechanism that offset the distortions caused by real rigidities, there exist a strong disincentive from sticking to the optimal policy through the terms of trade. This is due to the imperfect substitutability between home and foreign goods, thus, nominal rigidity creates a perfect environment that highlights the non-neutrality of monetary policy as a policy maker will embark on competitive devaluations, and manipulate the terms of trade which would divert demand from foreign competitors to domestic industries, thereby improving output and employment domestically.

It is well documented that in a special case, featuring unique parameter configurations, an

[12]see Gali (2002) for a summary of literature and braoda ideas

employment subsidy that exactly offset the joint distortion from nominal rigidity and terms of trade which rules out the existence of inflation or deflation bias can be derived, thereby ensuring the natural allocation becomes efficient.

The natural consequence of this is that the optimal policy becomes inward looking as central banks stabilize domestic inflation. In these models, strict inflation targeting[13] becomes the optimal candidate for policy makers as a policy rule that responds to output gap becomes suboptimal due to the cost of such policy when measured in terms of welfare losses.

In the model presented in this work, we highlight the assumptions which characterize the price setting decisions firms engage in the face of nominal rigidity, and this assumption yields two basic inferences. Most obvious is that it presents a gap for monetary policy to influence the level of economic activity in the short term, and it also emphasize agent's outlook in decision making reflecting the expectational channels, as agents incorporates faith and belief about future policies and outcomes into their decision making today. This outcome presents a social planner, the opportunity to affect agents' decisions in current time by pledging to take a certain action in a future date based on mutually agreed realizations or state of the world at that time, and this concept highlights the role of commitment to a future path of policy action, and credibility becomes relevant.

In this context, we also must answer an important question in view of the earlier documented framework in the previous chapter, whether there exist gains from enhancing credibility, either by commitment to a policy rule, or by some institutional framework that naturally incentivizes agents' decision to arrive at the same end. One pivotal issue we will wrestle with is regarding the instrument of monetary policy, and this has sparked a sharp debate over decades, and we settle for the interest rate as the instrument the social planner uses to achieve its policy targets and objectives. In this chapter, we also justify the reasons for the choice of this policy rule relative to monetary aggregates, and we derive the optimal policy rule which unveils itself to be inflation targeting, as it prescribes the gradual adjustment to the inflation rate.

We do this by incorporating both nominal and real rigidities in the model and characterize broad

[13] Gali shows that a central bank's function that respond to deviations of inflation from target and the deviation of output from target yield suboptimal welfare outcomes relative to a rule that strictly reacts to inflation

principles that underpin the formulation of optimal policy, while assessing how our theoretical results sit relative to policy making. In doing so, we accommodate some practical limitation, by realizing that a binding commitment to a policy rule is not always feasible, as we sustain a limited knowledge of the underlying structure of the economy coupled with exogenous innovations that cast itself over the economy in real time.

We then seek to identify an implement some optimal simple rules which is well suited for practice, which is a function of observables in the economy, and which can replicate or approximate the numerical and statistical features of the optimal policy, understanding the existence of trade-offs or not, and to what extent and degree, does the trade-off affect the objectives of monetary policy, and the gains of commitment relative to discretion, of optimal policy in presence of these trade-offs.

2.2 The design problem

With the design of optimal monetary policy, when facing limited or imperfect information stemming from agents expectations, the exact structure of the economy that underpins the natural allocation, data problems or lags in effect of policy, the type and statistical characteristics of the shocks, whether or not they are transitory, persistent or permanent, the optimal policy is expressed in relation to forecasts of target variables, and in doing so, monetary institutions may decide for a rule involving a monetary aggregate relative to interest rate which can be practical, with a lot of negative consequences not restricted to but includes that pervasive evidence documents that these target instruments are generally too unstable to be the one used in practice.

In an optimizing model which was detailed in the previous chapter, we discount from a utility function having real money balances in its argument, but we can account for real money balances at no cost and doing so leads to the equilibrium in the money market linking both instruments, namely, the interest rate and real balances, which in equilibrium, equates the supply of powered money. If this equation is subjects to shocks which are not observable, a monetary aggregate as an instrument would allow output and interest rate variations clear the market, thus inducing an unpleasant volatile path of interest rates, in the presence of interest rate inelasticity in the short term. In the case of interest rate as the instruments, central banks allow money stock to adjust to money demand shocks, thereby muting the impact on output and inflation as the central bank accommodates them. Thus, strengthening the incentives for policy makers to adopt interest rates as the preferred instruments. We allow for this practical reality in this work, and interest rate is adopted as the instrument for policy making.

In the presence of nominal rigidity, as highlighted previously, monetary policy can influence economic activity in the short term, and if the only friction a social planer is faced with is attributable to nominal rigidity, then inflation targeting becomes the optimal policy, as a policy objective to achieve zero inflation also stabilizes the output gap, thus, highlighting the classic divine coincidence. However, in the presence or real imperfections, policy makers now face a trade-off in the short term and a policy that seeks to close the output gap or influence output to vary one for one with the efficient level becomes suboptimal. Under PCP, in the presence of real imperfection, there is a meaningful trade-off, as the natural level becomes inefficient, and generates a variation between output and its efficient counterpart. We dispense with this approach and utilize a framework that highlights the implications of our model setup and structure which features two relative price variables which are ever present as an argument in our sectoral new Keynesian Phillips curve (NKPC). These variables are the relative price of tradables and the law of one price gap. We show that under LCP and PCP, the existence of one or both variables creates a meaningful trade-off between stabilizing inflation and output gap, thus breaking the divine coincidence. We now characterize the policy regime under Discretion and commitment.

Worthy of mention is that which ever price metrics and index the policy maker chooses to target, either domestic or CPI inflation, in the case of CPI inflation, central bank inherently targets a mixture of both nominal rigidities, which are stickiness in domestic prices and import prices. And to the extent that imported inputs affect domestic marginal cost, targeting domestic inflation accumulates both rigidities in both import and domestic prices, at lease to some degree.

The rational expectation equilibrium can be computed as a set of process that solves the system of equations for a given set of exogenous variables and in doing so, we assume that conditional upon the choice of an appropriate rule, the world economy pursues strict price stability.

2.3 Optimal Policy: The Small Open Economy under Local currency pricing

We characterize our optimal policy by analyzing the welfare maximizing policy by taking both a time consistent and a timeless perspective under both discretion and commitment. To underscore tractability and analytical convenience, we characterize the case where a subsidy that exactly offsets all distortions caused by real and nominal rigidities are in place. This approach ensures that we have an efficient steady state, which enables us in the spirit of Gali (2005) to derive a flow loss function using second order approximations.

Here, we derive the optimal labour allocation under a competitive equilibrium, which enables us under some specific parameter restrictions, to retrieve the appropriate subsidy which can offset any distortion. We proceed to derive the welfare loss function, as well as equilibrium conditions and reaction functions and targeting rules.

From a social planner perspective, he faces a limited set of possibilities constrained by both resource, technological and risk sharing constraints. The period optimization problem is:

$$\max_{c_t, n_t} = E_0 \sum_{t=0}^{\infty} u(C_t, N_t) \tag{3.01}$$

subject to:

$$c_t = \frac{1-\alpha}{\omega_s} y_t + \frac{\omega_s - (2-\gamma(1-\alpha))}{\omega_s} y_t^* + \frac{\omega_s - \omega_\psi(1-\alpha)}{\sigma\omega_s} \psi_t + \frac{\gamma[\omega_s - (1-\alpha)]}{\sigma\omega_s} v_t \tag{3.02}$$

optimization result in an equilibrium which after utilizing labor market clearing, must satisfy and yields the following constant supply of labor:

$$V'(N_t)N_t = \frac{(1-\alpha)}{\omega_s} U'(C_t)C_t \tag{3.03}$$

The above condition yield a constant unemployment[14] of $N = \frac{(1-\alpha)}{\omega_s}^{\frac{1}{1+\varphi}}$. We can show that

[14]The detailed derivation of this decentralized allocation is shown in the Appendix

using labour market clearing $N_t = \int N_{nt} N_{ht}$ and after substitutions, we have:

$$n_t = f\left(A_{nt}, A_{ht}, y_{ht}, y_{nt}, d_t\right) \tag{3.04}$$

$$d_t = \left[\left(\frac{P_{ht}(i)}{P_{ht}} \right)^{-\varepsilon} \left(\frac{P_{ft}(i)}{P_{ft}} \right)^{-\varepsilon} \left(\frac{P_{nt}(i)}{P_{nt}} \right)^{-\varepsilon} \delta i \right] \tag{3.05}$$

Let us partition d_t and let h_t be equal to the first component of our aggregate price dispersion term, then it follows that:

$$h_t = log \left[\int_0^1 \left(\frac{p_{h,t}(i)}{p_{h,t}} \right) \delta i \right] \tag{3.06}$$

LEMMA 1. $h_t = \frac{1-\varepsilon}{2} Var_i P_{h,t}(i) + o||a||^3$

From the labour market clearing condition, aggregate labour is a function of traded and non-traded technology, output, alongside relative price terms which after a second-order approximation of those price indices, yield the output in lemma. This price dispersion terms can now be conveniently expressed in terms of its variance.

LEMMA 2. $\sum_{t=0}^{\infty} \beta^t var_i p_{h,t}(i) = \frac{1}{\lambda} \sum_{t=0}^{\infty} \pi_{h,t}^2$

Subject to Lemma 1 and a sequence of constraints, by utilizing statistical transformations and mathematical forward iterations, we can transform the previously expressed price dispersion term which was transformed into the variance of such a price metrics to now a term expressed in inflation of the underlying variable. Utilizing the above expressions, we can proceed with expressing the second order approximations of the consumer's utility function in this small economy as below:

$$L = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \delta_x \tilde{y_t}^2 + \left[\frac{\omega_s - \omega_\psi (1-\alpha)}{\sigma \omega_s} \right] \psi_t + \left[\frac{\gamma[\omega_s - (1-\alpha)]}{\sigma \omega_s} \right] v_t \right) \tag{3.07}$$

From the above welfare loss function, we have a combination of both squared and non-squared terms inside the flow loss function, and if we take expectation, recognizing that the law of one price gap and the relative price of tradable are independent and mean zero, and thus, they drop out in variances and we have:

$$L = \frac{(1-\alpha)}{2\omega_s} \left[\frac{\varepsilon}{\lambda} Var \pi_t^2 + (1+\varphi) Var \tilde{y_t}^2 \right] \tag{3.08}$$

This is the social planners flow loss function expressed in terms of dispersion of inflation and output from their respective target levels.

2.4 Optimal Discretionary Policy

Under Discretion, the policy maker does not internalize the effect of its behavior on expectations. In this light, we consider the case where central banks treats the problem as one of sequential optimization in the absence of tools for commitment. In this setting, expectations of central bank behavior are exogenously given and thus, the social planner re-optimizes at each point in time when it realizes the shocks, without committing to any future policy. If we define δ_x as the weight attached to output variability in the loss criterion as originally developed and highlighted by Woodford (2000), hence monetary authorities minimize a one period problem given as:

$$\max_{\pi_t, i_t, \tilde{y}_t} \frac{(1-\alpha)}{2\omega_s}\left[\pi_t^2 + \delta_x \tilde{y}_t^2\right] \tag{3.09}$$

subject to the following constraints:

$$\tilde{y}_t = \tilde{y}_{t+1} + \frac{\omega_s}{\sigma}(i_t - E_t\pi_{H,t+1} - r^n) + \left[\frac{\omega_s - \omega_\psi(1-\alpha)}{\sigma(1-\alpha)}\right]\Delta\psi_{t+1} + \left[\frac{\gamma[\omega_s - (1-\alpha)]}{\sigma(1-\alpha)}\right]\Delta v_{t+1} \tag{3.10}$$

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + k_{yh}\tilde{y}_t + k_{\psi h}\psi_{F,t} + k_{vh}v_t \tag{3.11}$$

$$\pi_{N,t} = \beta E_t \pi_{N,t+1} + k_{yn}\tilde{y}_t + k_{\psi n}\psi_{F,t} + k_{vn}v_t \tag{3.12}$$

$$\pi_{F,t} = \beta E_t \pi_{F,t+1} + \lambda_F \psi_{F,t} \tag{3.13}$$

Here, we take as given, agents' expectation as the benevolent social planner cannot affect or influence agent's future expectations. In the spirit of Woodford (2006), an analytical solution to the model is obtained using the first order condition after optimization which yield a unique relationship between output gap and inflation below.

$$\tilde{y}_t = -\delta_e \pi_t \tag{3.14}$$

where $\delta_e = \frac{k_{ya}}{\delta_x}$. This condition highlights the equilibrium relationship between output gap and inflation, that in response to a cost push shock, inflation would increase above target causing the central bank to contract output below the efficient level until the above condition is reached. The parameter δ_e dictates the sensitivity of output gap to inflation and is increasing in the degree of pass-through θ_f thus under discretion, CPI inflation must rise in response to current and future expected deviations from the law of one price.

Substituting the above equilibrium condition into the CPI new Keynesian phillips curve, we have:[15]

$$\pi_t = \left(\frac{\beta \delta_x}{\delta_x + k_{ya}^2}\right) E_t \pi_{t+1} + \left(\frac{k_{\psi a}\delta_x}{\delta_x + k_{ya}^2}\right) \psi_t + \left(\frac{k_{va}\delta_x}{\delta_x + k_{ya}^2}\right) v_t \tag{3.15}$$

Iterating forward reveals that current inflation is a function of expected deviations from the law of one price gaps and the relative price differentials of traded goods.

For tractability and computational convenience, we assume ψ_t is an ar(1) cost push process, and that the price of tradable and non-tradable move one for one, then we recover the closed form solution for both inflation and output gap as:

$$\pi_t = \left[\frac{\delta_x k_{\psi a}}{k_{ya}^2 + \delta_x(1-\beta\rho_\psi)}\right] \psi_t \tag{3.16}$$

$$\tilde{y}_t = -\left[\frac{k_{ya} k_{\psi a}}{k_{ya}^2 + \delta_x(1-\beta\rho_\psi)}\right] \psi_t \tag{3.17}$$

From the above, it is evident that in the presence of a cost push shock, inflation increases and in response to this, as central banks tighten policy, they drive output below its efficient level, causing output gap to fall. This continues until the equilibrium condition in equation (3.14) which the central bank upholds at all times as derived above is satisfied. if we allow

$$\Omega_\psi = \left[\frac{k_{\psi a}}{k_{ya}^2 + \delta_x(1-\beta\rho_\psi)}\right] \tag{3.18}$$

Using the condition derived above, we can recover the interest rate rule that attains the aforementioned equilibrium process as:

$$i_t = r^n + \phi_\pi \pi_t + (1-\rho_\psi)\left[\frac{\sigma\Omega_\psi(1-\alpha)k_{ya} - \Omega_\psi\delta_x\omega_s - (\omega_s - \omega_\psi(1-\alpha))}{\omega_s}\right]\psi_t + \left[\frac{\gamma(\omega_s - (1-\alpha))}{\sigma(1-\alpha)}\right]\Delta v_{t+1} \tag{3.19}$$

[15]we show a detailed derivation in the appendix

2.5 Optimal Policy under commitment

Here we consider the action of the central bank that has tools at it disposal with which it can credible commit to a contingent policy path in the current period, as it realizes the shock it faces in the future, and in doing so, the expectations of agents' are well anchored and affect decision making today. If we define δ_x as the weight attached to output variability in the loss criterion as originally developed and highlighted by Woodford (2000), hence monetary authorities minimize an infinite problem given as:

$$\max_{\pi_t, i_t, \tilde{y}_t} \frac{(1-\alpha)}{2\omega_s} \left[\pi_{t+k}^2 + \delta_x \widetilde{y}_{t+k}^2\right] \tag{3.20}$$

subject to the following constraints:

$$\tilde{y}_t = \tilde{y}_{t+1} + \frac{\omega_s}{\sigma}(i_t - E_t \pi_{H,t+1} - r^n) + \left[\frac{\omega_s - \omega_\psi(1-\alpha)}{\sigma(1-\alpha)}\right] \Delta\psi_{t+1} + \left[\frac{\gamma[\omega_s - (1-\alpha)]}{\sigma(1-\alpha)}\right] \Delta v_{t+1} \tag{3.21}$$

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + k_{yh}\tilde{y}_t + k_{\psi h}\psi_{F,t} + k_{vh}v_t \tag{3.22}$$

$$\pi_{N,t} = \beta E_t \pi_{N,t+1} + k_{yn}\tilde{y}_t + k_{\psi n}\psi_{F,t} + k_{vn}v_t \tag{3.23}$$

$$\pi_{F,t} = \beta E_t \pi_{F,t+1} + \lambda_F \psi_{F,t} \tag{3.24}$$

This dynamic optimization result in first order conditions, when combined, yield the following relationship between output gap and inflation.

$$\widetilde{y}_o = -\delta_e \pi_o \tag{3.25}$$

$$\widetilde{y}_t - \widetilde{y}_{t-1} = -\delta_e \pi_t \tag{3.25}$$

where $\delta_e = \frac{k_{ya}}{\delta_x}$. And after due transformations, $\widetilde{y}_t = -\delta_e \tilde{p}_t$ for each period, where $\widetilde{p}_t$ is the deviation between price level and its implicit target given by the prevailing price level one period before the choice of this optimal plan. The core difference here is that the central bank can choose the sign and size of the output gap in proportion to deviations from the previous periods price level which is the implicit target.

Substituting the above equilibrium condition into the CPI new Keynesian phillips curve, we have the following second order difference equation:

$$\hat{p}_t - \frac{\hat{p_{t-1}}}{a\beta} + \frac{\hat{p_{t-2}}}{\beta} + \frac{k_{za}}{\beta}V_{t-1} = -\frac{k_{\psi a}}{\beta}\psi_{t-1} \tag{3.27}$$

where $a = \left(\frac{\delta_x}{\delta_x(1+\beta)+k_{ya}^2}\right)$ and assuming ψ is a cost push ar(1) process without persistence, and for analytical convenience, traded goods and non traded goods prices move one for one, then the stationary solution to the above second order difference equation is:

$$\hat{p}_t = \delta \hat{p_{t-1}} + \delta k_{\psi a}\psi_t \tag{3.28}$$

Substituting the above expression into our equilibrium targeting rule under commitment becomes:

$$\tilde{y}_t = -\frac{k_{ya}}{\delta_x}[\delta \hat{p_{t-1}} + \delta k_{\psi a}\psi_t] \tag{3.29}$$

The above equations highlight a similar relationship between inflation and output gap in response to an exogenous cost push shock. In the presence of a cost push shock, prices rise, and central banks respond by contracting economic activity below its efficient allocation, there by, contracting output gap until the equilibrium relationship is satisfied.

Using the condition derived above, we can recover the interest rate rule that supports the aforementioned equilibrium process as[16]:

$$i_t = r^n - \left[\phi_p + (1-\delta)k_{\psi a}\left(1 - \frac{\sigma(1-\alpha)k_{ya}}{\delta_x\omega_s}\right)\right]\sum_{k=0}^{t}\delta^{t+1}\psi_{t-k} - \beta_0\psi_t + \beta_1\Delta v_{t+1} \tag{3.30}$$

The results above indicate the response of output gap, and domestic inflation to a transitory and persistent cost push shock under local currency pricing (LCP). The first row highlights the response to a transitory shock and the latter to a persistent shock. To the extent that we have efficiently incorporated an endogenous cost push shock, this suffices for generating the trade-off under this economic environment for policy makers.

The case of the transitory shock highlights one clear difference, both variables contained in the argument of the loss function of the central bank, output gap and inflation revert back to their steady state one period after the shock has dissipated, but from the results, the central bank is

[16]The broad representation of β_0 and β_1 are described in the appendix

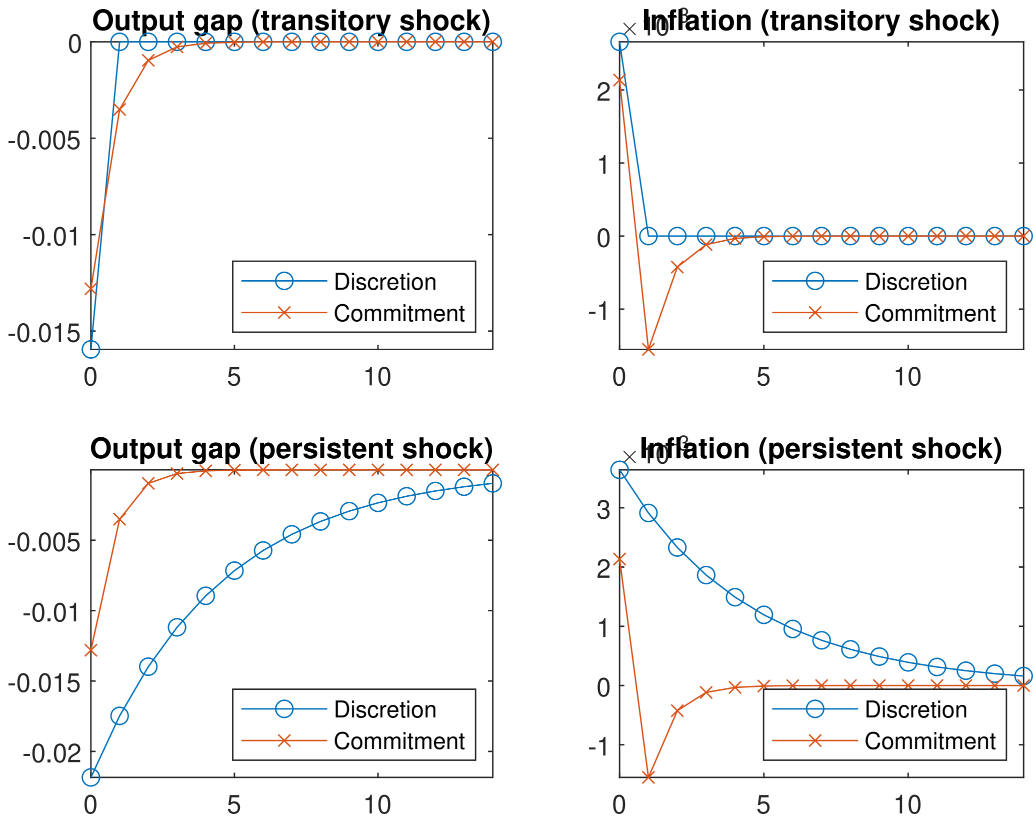

Figure 15: Response to a cost-push shock

happy to accommodate and allow deviations in output gap and inflation to persist well beyond the life of the shock. This is motivated by the fact that by committing to such a response, the social planner improves the inflation/output gap trade-off in the period the shock occurs. The forward looking nature of inflation enables the social planner to influence the decisions of agents, such that it lowers the initial impact of the shock relative to discretion, affording the central banks the ability to achieve any given level of inflation, with smaller declines in output gaps.

Evidently from the results, the central bank is able to improve the inflation/output gap trade-off by a technology of committing to a contingent policy path. Also, the central bank can achieve an improvement in welfare via commitment relative to discretion and this is made possible by the convexity of the loss function in output and inflation.

However, if we discount from the assumption that this shock is not reflective of a cost-push shock, it allows us to uncover the interesting dynamics and evolution of real and nominal variables in the face of efficient shocks which we document below under various monetary frameworks and regimes.

2.6 Analytical Evaluation of Simple Rules Under Various Monetary Regimes

An inference from the previous section, shows that there are definite gains from commitment, and any of such deviations from strict inflation targeting is very costly in terms of welfare losses. Also, due to the impracticability of responding to output gap as it require exact knowledge of the economy's true model and structure, all exogenous disturbances hitting the economy in real time, which presents a huge constraint for policy makers, we evaluate the evolution of variables under this macroeconomic environment, as we discount the option of interest rate rules responding to output gap as strict inflation targeting is the first best policy.

2.7 Domestic Inflation Targeting Rule (DITR)

In characterizing the evolution of real and nominal variables for a small open economy in the presence of efficient traded goods shock, we assume that the monetary authorities adopt a domestic inflation targeting approach, reflecting $\pi_{h,t} = 0$. Which implies from equation (3.22), that:

$$\tilde{y}_t = -\alpha \left(\frac{\kappa_{zh}}{\kappa_{yh}} \right) s_t - \left(\frac{\kappa_{\psi h}}{\kappa_{yh}} \right) \psi_t + \left(\frac{\kappa_{zh}}{\kappa_{yh}} \right) p_{nt} \tag{3.31}$$

Under this monetary framework, we jointly assume through out that the monetary authorities in the world economy achieves CPI price stability, in what follows, both world prices and domestic inflation are zero, and we can express the terms of trade under LCP and PCP as:

$$LCP : p_{f,t} = s_t = e_t - \psi_t \tag{3.32}$$

$$PCP : p_{f,t} = s_t = e_t \tag{3.33}$$

Thus, under the LCP model, the existence of a law of one price gap breaks the parallel between nominal exchange rate and the terms of trade, while under PCP, this homogeneity holds, as terms of trade moves one for one with the terms of trade and inherits all the statistical properties of the exchange rate and will exhibit mean reversion. Thus under both PCP and LCP, terms of trade and exchange rates depreciate in response to a relative rise in traded goods productivity. Under careful observation of our natural output relation, under our specific calibration in domestic inflation targeting, output rises in the face of a positive traded good technological improvement though by less than its natural counterpart and thus, output gap would always decline in response to a positive innovation, which requires a nominal and real depreciation and an expenditure switching towards domestic output. We can derive the equilibrium response of exchange rate, terms of trade, and the real exchange rate after combining domestic and world efficient output allocation to have:

$$\bar{e}_t = \Gamma_{ah}\left(a_{h,t} - a^*_{h,t}\right) + \Gamma_{an}\left(a_{n,t} - a^*_{n,t}\right) + \left[\frac{\alpha\gamma + \omega_s - \omega_\psi}{\alpha\gamma + \omega_s}\right]\psi_t + \left[\frac{\gamma}{\alpha\gamma + \omega_s}\right]p_{nt} \tag{3.34}$$

$$\bar{s}_t = \Gamma_{ah}\left(a_{h,t} - a^*_{h,t}\right) + \Gamma_{an}\left(a_{n,t} - a^*_{n,t}\right) - \left[\frac{\omega_\psi}{\alpha\gamma + \omega_s}\right]\psi_t + \left[\frac{\gamma}{\alpha\gamma + \omega_s}\right]p_{nt} \tag{3.35}$$

$$q_t = [1 - \alpha(1-\gamma)]s_t + \psi - \gamma p_{n,t} \tag{3.36}$$

It has been well documented in the previous chapter that the law of one price gap must rise in response to an increase in relative productivity shocks, and thus from the above equation, in the face of a technology shock, for given levels of relative prices, terms of trade depreciates and the law of one price gap increases. This dual impact on both terms of trade and law of one price gap, would induce a decline in output gap via the dynamic IS equation. Thus, the driving factors of variations in output gap is solely due to the terms of trade, non-traded good prices and law of one price gap, and as such, to minimize the variation in output gap in the presence of domestic inflation targeting, a policy that aims to minimize the variations in the former would also minimize the variation in output gap.

2.8 Consumer price Inflation Targeting Rule (CITR)

When monetary authorities decides to pursue a policy that targets the consumer price inflation, the evolution of variables is considerably similar to that where the central banks target domestic price inflation with very unique differences.

Under this regime, $\pi_t = 0$ for all t. Stacking this outcome upon the assumption that world monetary authorities pursue an optimal policy synonymous to that facing a small economy with financial autarky, this means: $p^*_t = p^*_{f,t}$ and as a result $\pi^*_t = \pi^*_{f,t}$, highlighting an equivalence between domestic prices and consumer prices for the world economy. Using the definition for consumer prices, it is evident that:

$$p_{h,t} = -\alpha s_t - \left[\frac{\gamma}{1-\gamma}\right] p_{n,t} \tag{3.37}$$

we can derive the allocations of economy wide variables under consumer price inflation targeting. First, output gap becomes:

$$\tilde{y_t} = \left[\frac{\kappa_{z,a} - \gamma\kappa_{\psi a}}{\gamma\kappa_{y,a}}\right] \psi_t - \left[\frac{\kappa_{z,a}}{\gamma\kappa_{y,a}}\right] q_t + \left[\frac{\kappa_{z,a}(1-\alpha)}{\gamma\kappa_{y,a}}\right] s_t \tag{3.38}$$

As output is a function of relative prices, once we pin down all the various metrics of relative prices, we can identify the response of output to a relative rise in domestic traded goods shock. Using aggregate market clearing, and substituting that into our traded goods marginal cost function, we have:

$$mc_t = -\left[\frac{\Gamma_y \omega_s}{\sigma\alpha}\right] p_{h,t} - (1+\varphi)\left(a_{h,t} - a^*_{h,t}\right) - \varphi\left(a_{n,t} - a^*_{n,t}\right) + \Gamma_\psi \psi_t + \Gamma_n p_{n,t} \tag{3.39}$$

using the above equation and substituting into the domestic traded good new Keynesian phillips curve, we obtain the second order difference equation below:

$$\varsigma p_{h,t} = p_{ht-1} + \beta p_{ht} - \lambda_h(1+\varphi)\left(a_{h,t} - a^*_{h,t}\right) - \lambda_h\varphi\left(a_{n,t} - a^*_{n,t}\right) + \lambda_h\Gamma_\psi\psi_t + \lambda_h\Gamma_n p_{n,t} \tag{3.40}$$

where $\varsigma = \frac{\lambda_h[\sigma\alpha(1+\beta)+\Gamma_y\omega_s]}{\sigma\alpha}$, and under plausible and simplifying assumptions, we get a unique stationary solution to the above second order difference equation. This enables us to derive the unique solution for both real and nominal exchange rates which are homogeneous under this monetary regime as expressed below:

$$q_t = \Gamma_{ah}\left(a_{h,t} - a^*_{h,t}\right) + \Gamma_{an}\left(a_{n,t} - a^*_{n,t}\right) + \left[\frac{\omega_s - \omega_\psi(1-\alpha)}{\omega_s}\right]\psi_t + \left[\frac{\gamma(\omega_s - 1 + \alpha)}{(1-\gamma)\omega_s}\right] p_{n,t} \tag{3.41}$$

Evidently, the response of domestic prices to a relative increase in domestic traded good shock is negative. This can be traced to the fact that first, in response to a traded technology shock, from the above equation, there is a nominal and real depreciation of exchange rates with endogenous persistence which are independent from the productivity differential. This impulse from exchange rates should be fed into the consumer prices, but in the monetary framework under consideration, consumer prices remain invariant as monetary authorities stabilize consumer prices. This in turn, can only mean that domestic prices and traded good inflation would decline significantly, thereby absorbing the impulses from the exchange rates to achieve equilibrium. This decline in domestic prices also reflect an depreciation of terms of trade and reflected in a negative output gap.

2.9 MONETARY INTEGRATION VIA A CURRENCY PEG

We highlight the analytical allocations for the small open economy under the arrangement that the central bank embarks on a permanent and credible exchange rate peg vis a vis the world economy. This framework is synonymous to the small economy under consideration, adopting the currency of the world economy, thereby dispensing from the independence and autonomy of its domestic monetary authorities. When combined with the assumption that the world monetary authorities embark on a Ramsey policy and seeks to replicate the flexi price allocation of output, which is homogeneous to domestic and CPI targeting in the world economy.

Under this framework, in the absence of capital control, $r_t = r_t^*$, and with an invariant nominal exchange rates and world prices, relative price allocations become $s_t = -p_{h,t}$ and

$$q_t = -(1-\alpha)(1-\gamma)p_{h,t} - \gamma p_{n,t} \tag{3.42}$$

Using the market clearing equation alongside the traded goods inflation equation, we derive a second-order difference equation for domestic price level as:

$$\varsigma_e p_{h,t} = p_{t-1} + \beta p_t - \Gamma_{eah}[a_{h,t} - a^*_{h,t}] - \Gamma_{ean}[a_{n,t} - a^*_{n,t}] + \frac{\lambda_h(\sigma\Gamma_z - \gamma\Gamma_y)}{\sigma} p_{n,t} \tag{3.43}$$

where $\varsigma_e = \frac{\sigma(1+\beta)+\lambda_h[\Gamma_z\sigma(1-\alpha)+\Gamma_y(\gamma(1-\alpha)-\omega_s]}{\sigma)}$ and under some simplifying assumptions, we can derive a unique stationary solution to the above system.

The statistical properties of the above system is reflective of the stationarity of the terms of trade under this monetary framework. Domestic prices respond in identical ways as in comparable monetary regimes, as this framework also reveal similar statistical properties of the variables under consideration displaying endogenous persistence uncorrelated to the relative productivity differential. Worthy of note is that under this regime, the variation of output gap and domestic prices is more prevalent.

Under a monetary union, there is a complete stabilization of the law of one price gap, and under this framework, the equilibrium allocations of variables under LCP are harmonized with that of PCP, while generating a higher volatility in consumer price inflation and output gap respectively.

2.10 Evolution of Variables in response to a Traded good Technology shock

We begin by analyzing the evolution of variables under the various monetary framework of domestic inflation targeting rule (DITR), consumer inflation targeting rule (CITR), exchange rate peg, which is characterized by a monetary integration or a currency area, and we also look at two unique frameworks, one where the monetary authorities embark on non-traded good inflation targeting rule (PINR) and traded good inflation targeting rule (PITR), and we characterize the response of the small open economy under LCP assuming the law of one price gap do not hold.

First, under DITR, where monetary authorities aim at stabilizing the domestic inflation where $\pi_{h,t} = 0$ for all t, we witness a decline, on impact in both output gap and domestic inflation as evidenced in the graph and after wards, a reversal back towards steady state. This impact is relatively less pervasive and prevalent when compared with CITR and PEG.

While the response of nominal interest rates are similar under DITR and CITR, on impact, there is a significant decline in nominal interest rates with the response under DITR being slightly more responsive to a traded technology shock and this decline is followed by a persistent increase and return to steady state. Under the PEG framework, the interest rate is completely nonresponsive as the domestic monetary authority has dispensed with its monetary autonomy, has adopted the foreign currency as its domestic currency and hence its interest rate is pegged at the level of the world economy, which is exogenous to the allocation of the small economy under consideration. This decline in domestic nominal interest rate under DITR is sufficient to anchor and stimulate the transitory expansion of consumption and output, which increases less than its natural counterpart, to generate a decline in output gap.

Under DIT, world interest rates is exogenous to domestic allocations which means that they are effectively anchored at a stationary point, and with uncovered interest parity, coupled with declining interest rates and falling prices, we expect, and we witness an initial sharp depreciation and then an expected future appreciation and overshooting of nominal exchange rate. This response is more prevalent under DITR relative to CITR, causing the evolution of prices one period after the shock to be more moderate and smoother under CITR relative to DITR. Under DIT, the invariability of domestic price would support a larger real depreciation of terms of trade and real exchange rate, which further amplifies the expansion of output, consumption, and net exports via the expenditure switching channels, thereby, leading to a more moderate decline in output gap under DITR, relative to both CITR and PEG.

Thus, under DITR, as we see a larger real and nominal depreciation of exchange rates and terms of trade, and this pervasive impulse affects the law of one price gap and imported inflation is obvious ways. First, the law of one price gap rises in response to efficient productivity shocks, but under DITR, its response is more prevalent relative to CITR and PEG. Also, with incomplete pass-through, under DITR, import price inflation is a proportion of equilibrium exchange rates variations and more volatile while being moderate under CITR. Import price inflation and the law of one price gap is stabilized under a peg.

How does traded good inflation fare under the various regimes? First, as traded good inflation accumulates the impulses to both domestic and import price inflation which move in opposite directions, under DITR, the effect of domestic inflation dominate the effect on import price inflation and traded goods inflation decline on impact and reverses course back to equilibrium. Under CITR and PEG, the pattern hold true as the decline in domestic inflation outweighs the positive innovation in imported inflation owing from a large depreciation. However, under DITR, the decline in traded good aggregate inflation is sustained and it persist up to a period after the shock before reverting to its stationary level, and this response is reflected in other traded good prices.

A domestic traded good technology improvement spills over and affects the allocations in the non-traded good sector. Specifically, the prices of non-traded goods decline, leading to a fall in non-traded good inflation on impact. This effect is less muted under DITR and relatively moderate under CITR and PEG respectively.

As consumer price inflation accumulates impulses from both traded goods and non-traded goods inflation, we see a measured response of CPI inflation which declines on impact under DITR, and a less dominant effect under the PEG, with CITR exhibiting comparable and moderate responses. For relative price of tradable, its responses track traded goods inflation closely, as the impulse of traded good inflation reveals considerable similarities across the 3 monetary framework which is not far fetched. The responses are identical under the 3 monetary frameworks with the impulses exhibiting large and pervasive variations under DITR, and minimal impulses recorded under the PEG, and moderate response under CITR.

It is evident that under the PEG, all relative prices, namely, real exchange rates, terms of trade, relative price of tradables and law of one price gap are more stable under the PEG, and most volatile under DIT, and moderate under CITR. However, this excess of smoothness of relative prices, and nominal exchange rates under the PEG is associated with very volatile responses of output gap and inflation across the board. Also, as we highlight higher volatility of relative prices under DITR, this volatility of relative prices and exchange rates are associated with more stable, smoother, and softer effects of output gap and inflation measures, with the response being moderate under CITR.

This fact, however suggests that the stabilization of import price inflation and the law of one price gap generated significant volatility in output gap and domestic and consumer price inflation under PEG relative to DITR, and CITR and as such would not be a close candidate for the optimal policy. Also, as output gap volatility is lower for domestic, traded and consumer price inflation under both DITR and CITR, then it follows that DITR and CITR can be ranked above alternative monetary regimes.

The core difference from the allocations under LCP relative to PCP is the absence of the law of one price gap which, first, moderates the responses of variables to efficient traded good technology shock, and moderates the volatility of various relative and traded goods prices more than the volatility consistent with the flexi-price allocations. Thus, under PCP, we document smoother and softer response of variables than in the former, thereby amplifying the likelihood of such models generating more welfare gains.

2.11 Optimal policy Vs Simple rules

Having characterized the optimal policy by utilizing the second order approximation to derive the planners flow loss function, optimal allocations and price systems, we then characterize the dynamics of economy wide variables under this framework. We also investigate to what extent simple rules which are expressed as Taylor type rules can recover or approximate the dynamics of the social planner. The Ramsey policy is the set of process in i_t associated with a competitive efficient equilibrium that yields the highest level of utility or minimizes the loss measured as a fraction of steady state consumption to the rational expectation agents for the small economy.

In the spirit of Woodford (2002), we build on the foundation of a timeless perspective and assume policy makers can sufficiently commit with credibility to a preannounced policy at a fixed point in time. Figure 13 and 14, displays the dynamic responses of macroeconomic variables to an economy wide shock for different monetary frameworks and regime. In the face of a domestic traded shock, its impact is severe on domestic inflation in the benchmark model relative to the optimal policy which stabilizes the impact significantly. In the face of this shock, it is evident that policy makers accommodate this shock, by an expansionary policy as reflected in the decline in nominal and real interest rates.

A careful observation of the various inflation metrics especially under the optimal policy highlights clearly that the efficient traded good technology shock is neither purely inflationary or deflationary, as they are all dependent on the dynamics of other important macroeconomic variables. In response to this shock and policy response of the policy makers, output expand, though by less that its efficient allocation, thereby leading to a decline in output gap, which is relatively less volatile and stabilized under the optimal policy compared to the rest of the targeting regimes. These economic trends are just sufficient to sustain an increase consumption, which is welfare improving. In this economy, the duplicity of inflation metrics gives the policy maker a plethora of targets to aim for. Particularly of interest, is the possibility to both craft policy that respond to traded and non-traded good inflation.

When the policy rule is responding to non-traded good inflation (PINR), domestic allocations

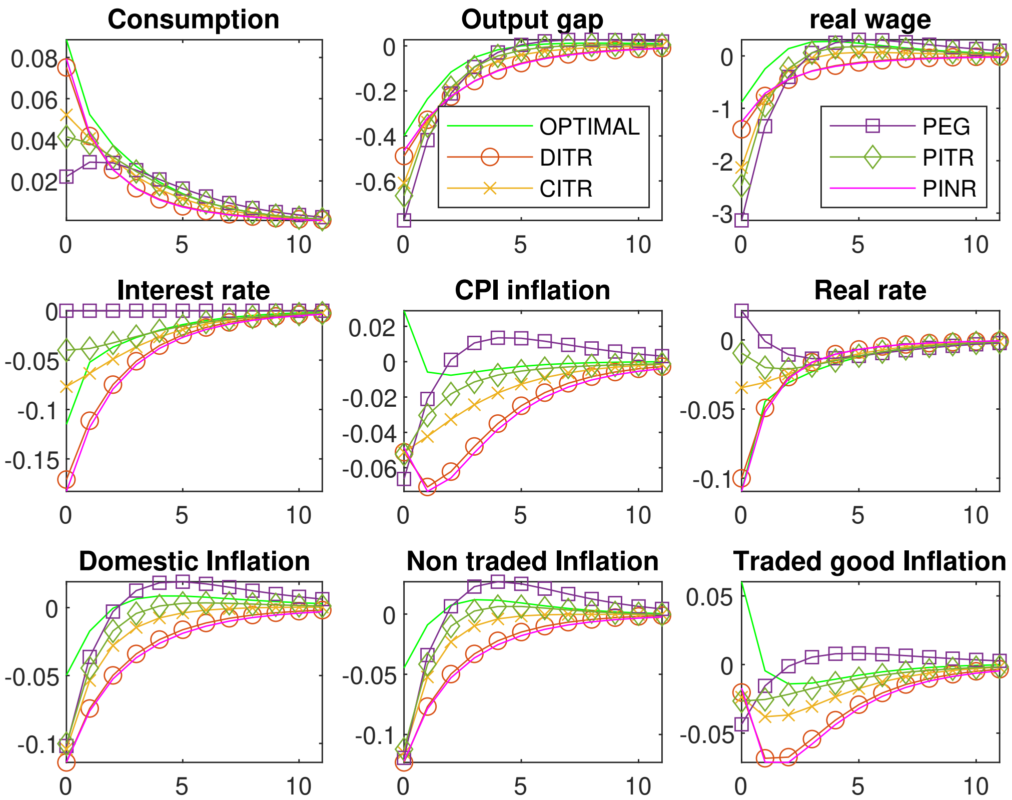

Figure 16: Response to a Traded good tech shock Under Alternative Monetary policy regimes

appear to be very volatile. The response of Domestic inflation, traded good inflation, non-traded good inflation, CPI inflation, real rate, and the nominal interest rate all seem to underscore high macroeconomic volatility under PINR, but it tends to amplify the real and nominal exchange rate volatility, though by less than the response of the optimal policy, and law of one price gap, and appears to fare mildly on terms of trade and imported inflation.

However, the response of CPI inflation and output gap under PINR is without convenience, explosive and volatile. This response is not enough to approximate closely to the optimal policy or simple rules that approximate the optimal policy rule.

Under traded good inflation targeting (PITR), the policy approach is akin to stabilizing a weighted average of traded good inflation and imported inflation. The evolution of macroeconomic variables under this regime are not farfetched. But in comparison, the optimal policy having successfully stabilized both output gap, CPI inflation, real wages, interest rates and domestic, traded and non-traded inflation, is a candidate policy that ensures maximum volatility

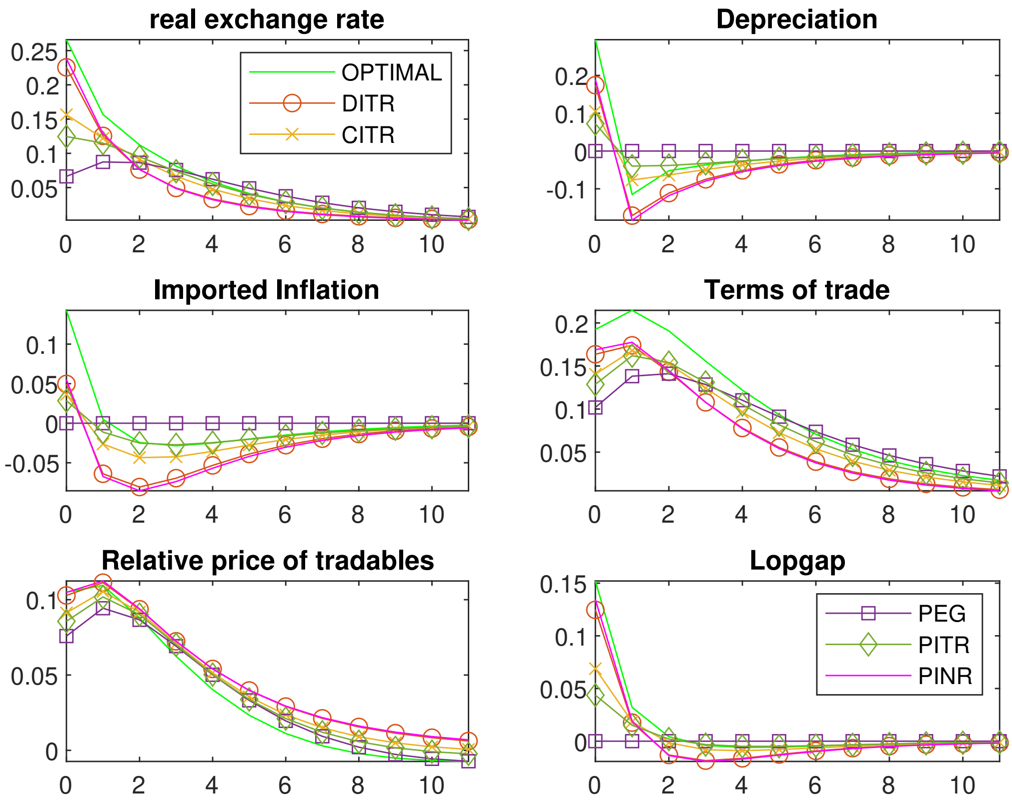

Figure 17: Response to a Traded good tech shock Under Alternative Monetary policy regimes

of real and nominal exchange rates. Terms of trade, law of one price gap, imported inflation and the relative price of tradable are seeing wide variations and acute volatility, but this is an efficient outcome and an appropriate response to efficient technology shocks.

In summary, the Ramsey planner has a distaste for sectoral inflation but is willing to accommodate some price innovations in CPI inflation recognizing that the social planner responds to this inflation metric. Having documented the various responses of macroeconomic variables under alternative monetary regimes, we can rank them in order of minimal contributions to welfare losses, but to do so accurately, we rely on the cyclical properties of various monetary regimes and the welfare losses, as captured by the expected variance of both inflation and output gap. Also, in response to inflation, the Ramsey planner chooses a nominal interest rate level that corresponds to the most sensitive response of real interest rates, thereby, moderately stabilizing output and enhancing consumption which is welfare improving.

While the Ramsey planner stabilizes the average inflation rate which is a weighted average

Table 1: Cyclical properties of Alternative policy regimes

	DITR	CITR	PITR	PINR	PEG
Output	0.81	0.67	0.63	0.79	0.53
Output gap	0.40	0.60	0.68	0.43	0.82
Domestic inflation	0.08	0.08	0.08	0.09	0.10
CPI inflation	0.07	0.03	0.03	0.07	0.07
Traded good inflation	0.09	0.04	0.02	0.09	0.12
NonTraded good	0.08	0.08	0.09	0.08	0.12
Nominal interest rate	0.21	0.09	0.05	0.21	0.00
Real interest rate	0.14	0.06	0.04	0.14	0.03
Real exchange rate	0.33	0.24	0.21	0.32	0.17
Terms of trade	0.36	0.33	0.32	0.35	0.29
Nominal depreciation	0.35	0.16	0.10	0.34	0.00
LOP GAP	0.17	0.08	0.05	0.17	0.00
Internal terms of trade	0.19	0.17	0.17	0.19	0.16

Note: Standard deviations are expressed in percent

of all the measures of inflation rates, he manages to improve welfare, but at a cost of higher volatility in the real and nominal exchange rates, terms of trade, imported inflation. Law of one price gap and the relative price of tradable goods. In this small economy, targeting singular or sectoral inflation or exchange rates are sub-optimal and a policy that seeks to stabilize the average inflation rate seems to be welfare improving, optimal and would outperform alternative monetary policy rules.

2.12 Second Moments of variables under alternative monetary regimes

To strengthen our understanding of the dynamics of our model under alternative monetary regimes, table 1, documents the standard deviation of key variables and our results are quite complementary to the results from our impulse response.

From the table, worthy of note is the distinguishing response of key variables highlighting the excess smoothness of nominal and real exchange rates, and law of one price gap. These open economy variables under the PEG cannot absorb impulses and therefore lack the capacity to insulate but rather transmits these impulses to domestic allocations. As evident from the results, the volatility of output gap and inflation across different classes (domestic, traded good, non-traded good and CPI inflation) is amplified under the PEG relative to alternative simple rules. Our model properties under domestic inflation targeting (DITR) and consumer price inflation targeting (CITR) are a close mirror of each other, yielding very stable and desirable properties.

From the table, the volatility of output gap is optimized under DITR relative to CITR, while consumer price inflation is smoother under CITR relative to DITR, and other policy regimes and somewhere in between. This evidential trade-off which is identifiable under CITR and DITR from visual inspection of the model properties, relative to alternative regimes is sufficient to rank the best policy regimes as CITR and DITR perform the best. To stress test our ranking of policy regimes, we would utilize the welfare evaluation of various policy regimes under varying assumptions to test and validate our ranking of the first best policy.

2.13 MACROECONOMIC ANALYSIS AND SENSITIVITY OF VARIABLES TO VARYING ELASTICITIES

We have previously ranked the various monetary frameworks based on its performance relative to the Ramsey allocation, and in this section, we examine how sensitive the allocations are and thus our ranking, to the calibration of two central parameters, which are not only important for our small open economy model with non -traded goods, but also affect the various policy regimes, and our Ramsey policy. In this exercise, as we vary the parameter governing the degree of pass-through, this can have a huge implication for the efficient allocation of our models under various regimes and thus heighten the complexity of the problem for policy makers. I also vary the share of non-traded goods to see if any, the effect or not of varying the share of non-traded goods and how it amplifies or dampens the transmission of exogenous impulses in our small open economy.

2.14 Higher degree of pass-through

In the baseline model, the Ramsey planner in the face of a traded good technology shock accommodates this shock but as the degree of pass-through increases, the Ramsey planner is more introspective, allowing this impulse to be absorbed by other nominal open economy variables, as he acts less aggressively in accommodating this shock thereby stabilizing the volatility in output moderately while remaining committed to fighting inflation aggressively. As pass-through increases, output still grows but even more slowly than in the baseline model, thereby softening consumption as the decline in real interest rates necessary for supporting a robust consumption increase is more muted. Overall, in the wake of an increase in the degree of pass-through, real quantities appear to be more insulated, as Ramsey planner stand ready

Table 2: Contribution to Welfare Losses

	DITR	**CITR**	**PINR**	**PITR**	**PEG**
Benchmark $\theta_F = 0.48, \gamma = 0.3$					
Var(CPI inflation)	0.0174	0.0038	0.0172	0.0039	0.0148
Var(output gap)	0.0075	0.0166	0.0084	0.0211	0.0311
Total	0.0249	0.0204	0.0257	0.0250	0.0459
Higher degree of Pass through $\theta_F = 0.40, \gamma = 0.3$					
Var(CPI inflation)	0.0174	0.0038	0.0172	0.0039	0.0148
Var(output gap)	0.0075	0.0166	0.0084	0.0211	0.0311
Total	0.0249	0.0204	0.0257	0.0250	0.0459
High share of non traded goods $\theta_F = 0.48, \gamma = 0.36$					
Var(CPI inflation)	0.0108	0.0038	0.0184	0.0046	0.0160
Var(output gap)	0.0101	0.0181	0.0095	0.0244	0.0335
Total	0.0209	0.0219	0.0279	0.0290	0.0495
High Pass-through and share of non-traded good $\theta_F = 0.4, \gamma = 0.36$					
Var(CPI inflation)	0.0108	0.0038	0.0184	0.0046	0.0160
Var(output gap)	0.0101	0.0181	0.0095	0.0244	0.0335
Total	0.0209	0.0219	0.0279	0.0290	0.0495

Note: Entries are percentage units of steady consumption.

stabilize the weighted average of various inflation metric in CPI inflation.

However, when we look through the various monetary regimes, the policy maker appears to be indifferent to an increase in the degree of pass through, as they adjust the interest rates to their preferred inflation metric under (DITR, CITR, PINR, PITR, PEG). While it is worth noting that varying the degree of pass-through has consequential and explosive effects on the volatility of exchange rates, consumer price inflation and output gap variability, under CITR, the welfare losses are minimized as policy makers adjust nominal interest rates to effectively all price impulses via the consumer price Inflation (CPI), and the model properties show that under CITR, welfare is significantly optimized relative to alternative pricing regimes in the midst of a higher degree of pass through and therefore ranks as the first best policy, assuming a structural shift which increases the degree of pass through.

Generally, the welfare measure of the economy's allocation under the various regimes continues to be invariant to a twenty percent (20%) increase in the degree of pass through, underscoring the resilience of our small open economy in its ability to be adaptable to adverse structural changes in both domestic and international environments.

2.15 Higher share of non-traded goods.

From table 2, in the presence of an increase in the share of non-traded good by twenty percent (20%), the Ramsey planner is seen to accommodate this variation through a decline in nominal interest rate, which increase output and consumption. This increase in consumption depreciates the real exchange rate and the terms of trade as the optimal policy is seen to generate more volatility. However, this increase is relatively muted compared to the benchmark model. As we focus on the various monetary regimes, this increase in the share of non-traded goods is generally not welfare improving under monetary integration (PEG), traded and non-traded inflation targeting (PINR, PITR) and consumer price inflation targeting (CITR), as it appears to amplify the volatility of inflation as well as output gap, thereby, being very costly in welfare terms.

From table 2, this increase in the share of non-traded goods under domestic inflation targeting (DITR), appears to stabilize the volatility of inflation creating a channel for potential welfare improving outcome. However, this stability in inflation comes at the cost of more variability of output gap, but generally, DITR proves to be welfare improving and the first best amongst the simple rules in the face of an increasing share of non-traded good consumption and superior to CITR (2.19%) which ranks better than PINR(2.79%) and better than the PITR(2.90%) and finally PEG with a (4.95%) loss relative to steady state consumption.

2.16 Conclusion

We have constructed a small open economy which accounted for incomplete pass-through within the assumption of local currency pricing which served as a foundation to investigate the optimal monetary policy in a fully forward-looking two-sector model. In this model, the law of one price gap in conjunction with the relative price of non traded goods renders the design and implementation of the optimal policy non-trivial, thereby generating a trade-off implicit in the Ramsey planner's objective. A micro founded approach using second order approximation of the household's utility function presents the benevolent social planner a flow function which inherently underpins the trade-off that exists between stabilizing inflation and output gap independent of the measure of inflation.

A key result stands clear that in the presence of a cost-push shock, the discretionary social planner faces similar targeting rules and objectives to the planner in possession of a framework that guarantees credible commitment, and we show that under commitment, the social planner improves the inflation and output gap trade offs in the period of the shock relative to the discretionary case without any corresponding welfare losses. Also, we derived the interest rate rule adequate to replicate and guarantee a unique equilibrium outcome under commitment.

We document evidence that strict consumer price inflation targeting (CITR) and domestic inflation targeting rule (DITR) stands out as the first best policy capable of approximating the optimal policy. Both the optimal policy, CITR, DITR and other targeting rules all subscribe to trading off more stability of inflation rates and output gaps while accommodating more volatility in real and nominal exchange rates. However, as the degree of pass through and the share of non-traded goods are increased by twenty percent (20%), this prevailing shift in the underlying structure of the economy further underscores Consumer Inflation Targeting rule (CITR) and Domestic inflation targeting rule (DITR) as the first best policy capable of approximating the optimal policy as it can neutralize potential avenues that tend to be welfare suboptimal and improves the model allocations under these scenarios.

Due to imported input in production, we can rationalize why CITR and DITR comes close and is ranked the first best policy that approximates the model properties of the optimal policy as to the extent that imported input is an argument of the domestic traded good firm's marginal cost, imported input potentially can transmit impulses from the external environment and thus stabilizing domestic inflation is also a partial stabilization of exchange rate impulses and the law of one price gap.

In conclusion, We have embarked on the design of optimal monetary policy while accounting for the deviation from the law of one price in a fully optimizing setting, and utilizing the model framework of the previous chapter. Owing to our model characteristics, our results show that the presence of the law of one price gap and the relative price of traded goods in new Keynesian Phillips curve (NKPC) create a meaningful trade off, forcing central banks to accommodate some fluctuations in output gap and inflation in the face of efficient shocks. Our findings highlight the optimal policy response of the benevolent social planner, who adjust rates in response to the natural rate of interest, the deviation of CPI inflation from its target, and the deviations from the law of one price under discretion and commitment. Our results show that the presence of a framework that guarantees credibility significantly improves the short-run trade-off due to convexity of the flow loss function. Utilizing the welfare losses as an evaluative measure, we find that targeting Consumer prices (CITR) or domestic prices (DITR) are potentially welfare improving, optimal and outperform alternative monetary rules.

2.17 Appendix B

2.18 B1: Derivation of optimal constant Employment

From risk sharing equ (1.99) and (1.111), when we subtitute into market clearing, we get:

$$c_t = \left[\frac{1-\alpha}{\omega_s}\right] y_t + \left[\frac{\omega_s-(2-\gamma)(1-\alpha)}{\omega_s}\right] y_t^* + \left[\frac{\omega_s-\omega_\psi(1-\alpha)}{\sigma\omega_s}\right] \psi_t + \gamma\left[\frac{\omega_s-(1-\alpha)}{\sigma\omega_s}\right] V_t \tag{1.119}$$

our Aggregate good market clearing can be written in exponential expressions as:

$$Y_t = Y_{h,t}^{1-\gamma} Y_{n,t}^{\gamma} \tag{3.44}$$

Substitute equ (1.25), (1.26) and (1.97) into equ (3.44), we get

$$Y_t = \left[A_{ht} M_t^{\delta}\right]^{1-\gamma} A_{nt}^{\gamma} N_t \tag{3.45}$$

The period optimization problem of the benevolent social planner becomes a problem in N_t to maximize

$$\max\left[E_0 \sum_{t=0}^{\infty} U[Y_t^{\frac{1-\alpha}{\omega_s}} Y_t^{\frac{\omega_s-(2-\gamma)(1-\alpha)}{\omega_s}} \psi_t^{\frac{\omega_s-\omega_\psi(1-\alpha)}{\sigma\omega_s}} V_t^{\frac{\omega_s-(1-\alpha)}{\sigma\omega_s}}, N_t]\right] \tag{3.46}$$

FOC: N_t

$$U_{ct}\frac{1-\alpha}{\omega_s}C_t + U_{nt} = 0 \tag{3.47}$$

$$\frac{N_t^{1+\varphi}}{1/C_t} = \frac{1-\alpha}{\omega_s}C_t \tag{3.48}$$

$$N_t = \left[\frac{1-\alpha}{\omega_s}\right]^{\frac{1}{1+\varphi}} \tag{3.49}$$

The above equation is the decentralized allocation of constant employment as featured in the text.

2.19 B2: Preamble & Derivation of the price dispersion term (Lemma 1)

To derive the social planners flow loss function, we write the aggregate labour market clearing condition in exponential form as:

$$N_t = \int_0^1 N_{ht}^{1-\gamma} N_{n,t}^{\gamma} \delta i \tag{3.50}$$

Substituting the traded and non-traded firm production technology, we have:

$$N_t = \int_0^1 \left[\left(\frac{Y_{ht}}{A_{ht} M_t^{\delta}} \right)^{\frac{1}{1-\delta}} \right]^{1-\gamma} \left[\left(\frac{Y_{nt}}{A_{nt}} \right)^{\frac{1}{1-\delta}} \right]^{\gamma} \delta i \tag{3.51}$$

When we utilize equ (1.82), (1.83) and our demand functions in equ (1.04),(1.05),(1.10) and (1.11), and substitute it into equ (3.51) above, we get:

$$N_t = [A_{ht} A_{nt}]^{-\frac{1}{1-\delta}} \left[C_{ht} C_{ft} C_{nt} \right]^{\frac{1}{1-\delta}} \left[\left(\frac{P_{ht}(i)}{P_{ht}} \right)^{-\varepsilon} \left(\frac{P_{ft}(i)}{P_{ft}} \right)^{-\varepsilon} \left(\frac{P_{nt}(i)}{P_{nt}} \right)^{-\varepsilon} \right] \tag{3.52}$$

taking Logs of the above expression, and after substitution, we have:

$$n_t = \left(\frac{1}{1-\delta} \right) [y_t - a_{ht} - a_{nt}] + ln \left[\int \left(\frac{P_{ht}(i)}{P_{ht}} \right)^{-\varepsilon} \left(\frac{P_{ft}(i)}{P_{ft}} \right)^{-\varepsilon} \left(\frac{P_{nt}(i)}{P_{nt}} \right)^{-\varepsilon} \delta i \right] \tag{3.53}$$

let $d_t = ln \left[\int \left(\frac{P_{ht}(i)}{P_{ht}} \right)^{-\varepsilon} \left(\frac{P_{ft}(i)}{P_{ft}} \right)^{-\varepsilon} \left(\frac{P_{nt}(i)}{P_{nt}} \right)^{-\varepsilon} \delta i \right]$ represent the aggregate dispersion term, then we can write:

$$n_t = \left(\frac{1}{1-\delta} \right) [y_t - a_{ht} - a_{nt}] + d_t \tag{3.54}$$

when we decouple the aggregate price dispersion term d_t, and let $h_t = log \int_0^1 \frac{p_{ht}(i)}{p_{ht}} \delta i$ We do a second order taylor approximation on h_t which necessitates us to first, do a first order taylor approximation of the above expression, as documented in Gali (2002), up to a second order,

$$e^{(1-\varepsilon)(p_{ht}(i)-p_{ht})\delta i} = \frac{(1-\varepsilon)^2}{2} E_i \hat{p_{ht}}^2(i) = -\frac{(1-\varepsilon)}{2} Var_i\, P_{ht}(i) \tag{3.55}$$

with subsequent algebraic transformations, we arrive at Lemma 1. Here, we show the transformation for domestic traded good price dispersion. Similar procedure is followed for import price and non-traded good price dispersion, which when accumulated gives us our flow loss function as a function of CPI inflation.

2.20 B3: Derivation of the Loss function

To derive the Loss function, we take a second order approximation of the social planner's utility function as follows:

$$u_t - u \approx u_c C \frac{C_t - C}{C} + u_N N \frac{N_t - N}{N} + \frac{1}{2} u_{CC} C^2 \left(\frac{C_t - C}{C} \right)^2 + \frac{1}{2} u_{NN} N^2 \left(\frac{N_t - N}{N} \right)^2 \tag{3.56}$$

$$\approx u_c C \left(\hat{c}_t + \frac{1}{2} \hat{c}_t^2 \right) + u_N N \left(\hat{n}_t + \frac{1}{2} \hat{n}_t^2 \right) + \frac{1}{2} u_{CC} C^2 \hat{c}_t^2 + \frac{1}{2} u_{NN} N^2 \hat{n}_t^2 \tag{3.57}$$

$$= u_c C \left(\hat{c}_t + \frac{1}{2} \hat{c}_t^2 \right) + u_N N \left(\hat{n}_t + \frac{1}{2} \hat{n}_t^2 \right) \tag{3.58}$$

$$\frac{u_t - u}{u_c C} \approx c_t + \frac{1 + \frac{u_c}{u_c} C}{2} c_t^2 + \frac{u_N}{u_c} \frac{N}{C} \left(\hat{n}_t + \frac{1 + \frac{u_N}{u_N}}{2} \hat{n}_t^2 \right) \tag{3.59}$$

after substituting equ (1.119)(3.54)(3.55) into the above expression, and with further algebraic transformations, we have our flow loss function in the text in equ (3.08).

$$L = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\pi_t^2 + \delta_x \tilde{y}_t{}^2 + \frac{\omega_s - \omega_\psi (1 - \alpha)}{\sigma \omega_s} \psi_t + \frac{\gamma [\omega_s - (1 - \alpha)]}{\sigma \omega_s} z_t \right) \tag{3.60}$$

2.21 B4: Derivation of Interest rate rule under discretion

To derive the interest rate rule under discretion, we first we combine our traded, non traded and retail importers NKPC to have:

$$\pi_t = \beta E_t \{\pi_{t+1}\} + k_{ya} \tilde{y}_t + k_{\psi a} \psi_{F,t} + k_{va} v_t \tag{3.61}$$

Our objective function is:

$$L = \mathbb{E}_0 \sum_{t=0}^{\infty} \pi_t^2 + \delta_x \tilde{y}_t{}^2 + \Theta_{1t} \left(\tilde{y}_t - \tilde{y}_{t+1} + \frac{\omega_s}{\sigma} (i_t - E_t \pi_{H,t+1} - r^n) - \left[\frac{\omega_s - \omega_\psi}{\sigma} \right] \Delta \psi_{t+1} - \frac{\gamma}{\sigma} \Delta v_{t+1} \right) \tag{3.61a}$$

$$-\Theta_{2t} (\pi_t - \beta E_t \{\pi_{t+1}\} - k_{ya} \tilde{y}_t - k_{\psi a} \psi_{F,t} - k_{va} v_t) \tag{3.61b}$$

FOC WRT $\pi_t, \hat{y}_t, i_t$ is

$$\frac{\delta L}{\delta \pi_t} = -\pi_t - \Theta_{2t} = 0 \tag{3.62}$$

$$\frac{\delta L}{\delta \hat{y}_t} = -\delta_x \hat{y}_t - \Theta_{1t} + \Theta_{2t} k_{ya} = 0 \tag{3.63}$$

$$\frac{\delta L}{\delta i_t} = \frac{\omega_s}{1-\alpha} \Theta_{1t} = 0 \tag{3.64}$$

$$-b_w \hat{y}_t - k_{ya} \pi_t = 0 \tag{3.65}$$

$$\hat{y}_t = -\frac{k_{ya}}{\delta_x} \hat{y}_t \tag{3.66}$$

Substituting the above expression into the CPI NKPC, and iterating forward, we have equation (3.15). Also, from equ (3.18), we can rewrite equ (3.16) and (3.17) as:

$$\pi_t = \delta_x \Omega_\psi \psi_t \tag{3.67}$$

$$\hat{y}_t = -k_{ya} \Omega_\psi \psi_t \tag{3.68}$$

when we substitute the above expression into the Dynamic IS relation in equ (3.21), we have:

$$i_t = r^n + \phi_\pi \pi_t + (1-\rho_\psi) \left[\frac{\sigma \Omega_\psi (1-\alpha) k_{ya} - \Omega_\psi \delta_x \omega_s - (\omega_s - \omega_\psi (1-\alpha))}{\omega_s} \right] \psi_t + \left[\frac{\gamma(\omega_s - (1-\alpha))}{\sigma(1-\alpha)} \right] \Delta v_{t+1} \quad ()$$

Note, This is equation (3.19) in the text which is the optimal interest rate that guarantees a unique equilibrium under discretion. We follow similar steps, but we utilize dynamic optimization as opposed to static optimization to retrieve the optimal interest rate that guarantees equilibrium under commitment to arrive at equ (3.30).

From page 67, section 2.5, we define our composite parameters as follows:

$$\beta_0 = \left[\frac{(\omega_s - \omega_\psi (1-\alpha))}{\sigma(1-\alpha)} \right] \tag{1}$$

$$\beta_1 = \left[\frac{\gamma(\omega_s - (1-\alpha))}{\sigma(1-\alpha)} \right] \tag{2}$$

2.22 B5: Economys' allocations under CITR

Here, central banks set $\pi_t = 0$ for all $t$and the world economy replicates the same allocation, hence, $\pi_t^* = \pi_{ft}^* = 0$, and with the definition of relative price of traded goods, we have:

$$V_t = -\frac{1}{1-\gamma} p_{nt} \tag{3.69}$$

utilizing eq (3.69), and the expression for CPI, while constraining $\pi_t = 0$, we have:

$$s_t = -\frac{1}{\alpha} p_{ht} - \frac{\gamma}{\alpha(1-\gamma)} p_{nt} \tag{3.70}$$

Substituting equ (3.70) into the market clearing in equ (1.98), to eliminate s_t, we have:

$$y_t = -\frac{\omega_s}{\sigma\alpha} p_{ht} + (2-\gamma) y_t^* + \frac{\omega_\psi}{\sigma} \psi_t + \frac{\gamma(\omega_s - \alpha)}{\sigma\alpha(1-\gamma)} p_{nt} \tag{3.71}$$

substitute the above expression in the traded goods marginal cost from equ (1.68) and the world allocation of world output in equ (1.79) and we have equ (3.39) in the text. To derive the allocation for real exchange rate, we recognize that

$$q_t = e_t + p_t^* - p_t \tag{3.72}$$

it is clear that in the presence of CITR, where $\pi_t = 0$ and $\pi_t^* = \pi_{ft}^* = 0$ thus $q_t = e_t$. utilizing the risk sharing condition and real exchange rate expression in eq (1.111)and eq (1.99) to have

$$q_t = e_t = \psi_t - \frac{1-\alpha}{\alpha} p_{ht} - \frac{\gamma(1-2\alpha)}{\alpha(1-\gamma)} p_{nt} \tag{3.73}$$

Next we subtitute equ (3.73) and world market clearing in equ (1.98) to have equ (3.41) in the text.

3 Empirical Estimation of Exchange rate pass through in Dynamic and Structural models

3.1 Introduction

The price setting behaviors of firms underpin the role of nominal rigidities in an open economy and has broad implications for monetary policy, the international mechanism for shock transmission, and consequently, exchange rate pass-through.

Empirical evidence document large and persistent fluctuations in exchange rates over the past five decades and this variations in exchange rate have quantifiable effect on real and nominal macroeconomic variables. These fluctuations in currency pairs are frequently evident due to structural economic transitions and changes in underlying market fundamentals such as supply and demand conditions, market expectations, differentials in foreign and domestic interest rates and relative prices of traded goods. Thus, exchange rate becomes an important macroeconomic signal variable as changes in exchange rates have consequential impact on traded and non-traded goods of countries and regions through their impact on the relative prices of goods and services.

An understanding of the co-movement between exchange rate fluctuations and prices across the pricing chain complement our understanding of how firms respond to cost shocks, both in terms of frequency and magnitude and of the many impulses which the exchange rate transmits into a domestic economy, we are more interested in the price-related effects of such impulses, and across the price chain (import prices, export prices, wages, producer prices, and consumer prices), we narrow down our interest to import prices and export prices.

A vast number of empirical research has been devoted to study the relationship between exchange rate fluctuations and prices. Considering the empirical evidence, which significantly tilts towards the documented evidence of imperfect pass-through estimates, there is growing and prevailing evidence of imperfect pass-through owing to different factors.

Dornbusch (1987) attributed this empirical discovery to the prevalent market structures which enables firms to control prices and accumulate more market power. Hellerstein (2004) explained that firms are ever more willing to absorb exchange rate shocks and are becoming reluctant to pass through these fluctuations into prices to maintain and expand their market share thereby preserving stable prices.

Burstein and Rebelo (2003) tendered alternative explanation for the prevalence of incomplete pass-through. They attribute it to the presence of non-traded goods in consumption and distribution sectors and as such, this generates a gap between border and retail prices. The wide dispersion in the direction of the results of various empirical studies is also reflected in the fact that a number of these studies capture the impact of exchange rate fluctuation on different inflation measure. They failed to harmonize their interest on a particular price metric amongst the wide variety of prices.

Recently, a variety of papers have focused on isolating import prices and its response to an exchange rate fluctuation. To this end, theoretical models which account for wage stickiness, optimizing agents, nominal stickiness, exogenous productivity, and monopolistic competition have been explored. Goldberg (2010) show that among many, the elasticity of substitution among varieties, imported inputs in production, productivity and non-traded goods all influence the elasticity of import price to exchange rate fluctuations.

Goldberg (2010) shows that as the openness index increases, the more sensitive import prices are to exchange rate changes, and the larger the elasticity of substitution amongst goods are, the smaller the response of consumer price inflation and import prices to exchange rate changes. Also, they find that import prices are less sensitive to higher distribution margins in traded or non-traded good sector.

Campa and Goldberg (2006) document that higher exchange rate volatility leads to higher pass-through therefore currency pairs that exhibits more stability or less volatility would stabilize not just trade prices, but trade quantities as the composition of import can alter the sensitivity of import prices to exchange rate movements.

Campa and Goldberg (2002) estimated the pass-through elasticities across twenty-three countries using extensive cross-country and time series data and found considerable differences in pass through elasticities across countries. This can be attributed to a term called pricing to

Market, which has been well documented in industrial organization literature.

Bett and Devereux (2001) allowed for Pricing to market in a general equilibrium framework while Krugman (1987) and Dornbusch (1987) introduced pricing to market by assuming that firms set prices in the currency of the producer, PCP, while other firms invoice their prices in the currency of the Local or destination market, LCP. This framework which had PPP at its foundation attributed the deviations from PPP to a combination of sticky prices and PTM and a key conclusion is that pass-through is complete in the long run.

Using disaggregated industry data, Corsetti and Dedola (2003) attributed the heterogeneity of pass-through across countries and industries to micro factors which include elasticities of substitution, productivity and markups. Low demand elasticities imply higher markups and higher exchange rate pass-through. They further document that cross-country differences in the use of imported input in production generates strong heterogeneity in pass-through elasticities.

3.2 Micro foundations: Exchange rate and prices

The micro foundations that enable the understanding of the relationship between exchange rate and inflation is the theory of purchasing power parity, (PPP). The central idea highlights the fact that prices and the changes thereof, should be equalized across location when measured in a common denominator or currency. This equalization, which presupposes perfect pass-through hinges on the assumption that traded goods are priced in the currency of the producer, an assumption known as producer currency pricing, (PCP). Thus, the import price of a country imported goods is a linear combination of the export prices of that country's trading partners denominated in the foreign Countrys' currency, and the exchange rate, E_t. We can represent import prices as follows:

$$P_t^{m,j} = E_t P_t^{x,j} \tag{3.01}$$

The equation above highlights the law of one price, where $P_t^{x,j}$ is the price of world exports denominated in foreign currency. A further decomposition of exporters prices, $P_t^{x,j}$ dictates that prices are a markup over exporters marginal cost as expressed below.

$$P_t^{x,j} = U_{it} MC_t^x \tag{3.02}$$

where U_{it} is exporters markup and MC_t^x is exporters marginal cost expressed in exporters currency. The equation above highlights the fact that markup could vary, reflecting the changes in competitive conditions facing exporters in a destination market. Furthermore, the elasticity of markups will be sensitive to the market share of the home firms relative to foreign counterparts, market concentration and density which underscores industry competition and the extent of discrimination possible within an industry.

To gain a deeper insight into the economic and structural relationship between exchange rate and prices, we follow conventional industry organization literature as in Dornbusch (1987) partial equilibrium approach to cost decomposition, which focuses on the determinants of marginal costs, while modeling the exchange rate as exogenous. In the spirit of Campa and Goldberg (2002), we model markups to be a function of exchange rate at destination markets as follows:

$$U_{I,t} = \alpha_0 + \alpha_1 e_t \tag{3.03}$$

Where α_0 is the autonomous components and α_1 is the elasticity of markups to markup determinants across destination countries. Following the theoretical framework in earlier chapters, we allow for imported input in production, as firms face a downward sloping demand leading to marginal costs determined by input prices, output levels and exchange rate as follows:

$$MC_t^{x,j} = \alpha_2 e_t + \alpha_3 w_t + \alpha_4 y_t \tag{3.04}$$

where w_t are wages representing exporters labour input costs, y_t is aggregate output or level of production and E_t is trade weighted nominal exchange rate. After substitution and aggregating the above equations, we have the following equation that tells us that import prices evolves according to exchange rates, and marginal cost determinants as follows:

$$P_t^{m,j} = \alpha_0 + (1 + \alpha_1 + \alpha_2)e_t + \alpha_3 w_t + \alpha_4 y_t \tag{3.05}$$

From the above equation, the elasticity of import price $P_t^{m,j}$ with respect to exchange rate e_t is defined as the exchange rate pass through. Therefore, it follows that

$$\frac{\delta P_t^{m,j}}{\delta e_t} = 1 + \alpha_1 + \alpha_2 \tag{3.06}$$

From the above, we are presented with a structural equation that will guide our empirical estimation to model exchange rate pass through which highlights the elasticity of import and export prices to exchange rates changes. This approach helps our empirical estimation to be predicated upon economic theory as opposed to a regression estimates of level exchange rate on prices which yield $cov(P_t^{I,j}, e_t)/var(e_t)$ which gives a statistical correlation between two variables and void of economic significance.

3.3 MODEL SPECIFICATION AND DATA DESCRIPTION

We estimate the pass through of exchange rate into import and export prices using a dynamic lagged model which allows for partial adjustment of prices to exchange rates and marginal cost components overtime as specified below:

$$\Delta P_t^{m,j} = \alpha + \sum_{1=0}^{n} \beta_i^j \Delta e_{t-i}^j + \gamma Z_t + \varepsilon_t \tag{3.07}$$

Where $\Delta P_t^{m,j}$ is the quarterly log change of import and export prices expressed in domestic currency and Δe_t is the log change in the nominal effective exchange rate of the Norwegian Krone. In our specification, we allow for contemporaneous as well as dynamic effects. Z_t is a vector of control variables which captures changes in foreign exporters marginal costs, competitor's markets prices, the role of monetary policy and relative output.

To complement and test the model predictions from the invoicing assumptions of our previous chapters, we take a bird's eye view using macro quarterly time series data which spans from 1981-2024 all of which are sourced from the international monetary fund's international financial statistics data repository. Monetary aggregates for Norway are sourced from historical Norwegian Central banks data repository. For import and export prices, we utilize the quarterly series of import and export price deflators, and we use the United States PPI index to control for changes in foreign exporters marginal costs and Norwegian PPI index to account for foreign competitors market price in the home market (Norway).

In previous chapters, we have highlighted two pricing assumptions which had clear cut implication for exchange rate pass through and yielded testable implications. Thus, under producer currency pricing, PCP, pass through into import prices are complete as the law of one price holds and under local currency pricing, LCP, pass through into import prices are muted, reflecting a complete misalignment from the law of one price.

Thus, in this chapter, our objective is to model and quantify the relationship between exchange rate and trade prices as we estimate the pass-through elasticity to test the validity and predictions of various pricing assumptions using Norwegian data. In our theoretical model, we calibrated our model parameters for the Norwegian economy through the Lens of Gali and Monachelli (2005) which calibrated the autoregressive properties of the efficient shocks for the Canadian economy, as a reasonable framework for model comparison. As we motivate the choice of Norway, we recognize that Norway, which is synonymous to Canada, fits the description of a small open economy which is geographically located next to a very large open economy and Norway has such similarities, as it is proximal to the European Union, which is a large economic unit and a huge trading bloc. Also, Canada retains its unique currency and is a resource abundant and a net exporter of its energy resources like Norway which is a net exporter of its abundant energy resource and retains its unique currency and uniquely for us, is the prevalence of the use of the currency of its large economic neighbor which serves as a reserve currency, for Canada (the US dollar) and in our context, the Euro. Hence, the significance of the choice of Norway, both for our calibrated model and for empirical estimation.

Thus, under PCP, if pass through is perfect, then we expect our β coefficient of the contemporaneous effect of changes in exchange rate to be equal to unity and terms of trade to vary one for one with fluctuations in trade weighted nominal exchange rates. Under LCP, the prediction of pass through is zero, and we expect terms of trade to be invariant to changes in trade weighted nominal exchange rates.

We test this hypothesis and see if our results validates the pricing assumption of producer currency pricing, PCP or local currency pricing, LCP or if we are to be open to a growing invoicing assumption which has gained traction over half of the last decade, dominant currency pricing, DCP. In this regard, our choice of country is significant beyond testing our model predictions, as it helps us to uniquely validate the possibility of dominant currency pricing in the context of the dual conventional pricing assumptions of PCP and LCP.

Under DCP, firms invoice traded goods in a currency other than the producer's currency or the destination currency, however, both PCP and LCP nests special cases of DCP, and Norway as a country allows us to hypothesize and test this assumption, at least partially. In a two-country setting, say assume Norway, the home country and the United States, when Norwegian firms invoice traded goods in the US Dollar, in this case, the Dollar doubles as both a dominant currency and the destination currency, so a dollar pricing by firms nests both DCP and LCP. In this similar setting, if we take the United States as the home country, then pricing in the US dollar doubles as one of DCP and PCP.

Trade flow statistics show that the European union is Norway's biggest trading partner accounting for about 62% of total trade in goods in 2024 and the United States accounted for about 7% of total trade, jointly accounting for more than two-thirds or about 70% of the Norwegian total trade. We also characterize the Euro as a dominant currency due to the growing prevalence of its use by nations and corporations to diversify their wealth and portfolio holdings accounting for 20% of global trade and Invoicing. Considering this macroeconomic evidence, Norwegian firms pricing in the Euro allows us to capture and understand both LCP and DCP concurrently, as we account for this under our testable hypothesis to see if our results could validate the pricing assumption and predictions of DCP.

In addition to estimating the pass-through elasticities and validating the testable predictions from our theoretical model, we estimate a system of dynamic equations and give it some structure to enable us trace out the response of price variables to structural exchange rate shocks.

3.4 Results

3.5 Import price pass through

Table 3 report estimates from a quarterly dynamic regression of the log change in trade weighted exchange rates on log change in import price index while accounting for export firms' marginal cost components as controls. The short run or contemporaneous effect of the log changes in trade weighted exchange rates is given by the lag-0 β coefficient, while the long-run estimates is given by the cumulative sum of the dynamic coefficient lag 1-4 of exchange rates on the log change of import price.

Table 3: ERPT into the Log change of Norwegian Import Prices

	(1)	(2)	(3)	(4)	(5)
	Δp^m	Δp^m	Δp^m	Δp^m	Δp^m
Δe_{neer}	-0.324***	-0.415***	-0.441***	-0.438***	-0.4385***
	(0.07)	(0.075)	(0.076)	(0.078)	(0.078)
ΔPPI_{us}		0.338***	0.263**	0.266**	0.266**
		(0.115)	(0.111)	(0.108)	(0.101)
ΔPPI_{nor}			0.0986***	0.0977***	0.097***
			(0.0306)	(0.0311)	(0.0326)
$\Delta Y_{nor/us}$				-0.036	-0.037
				(0.163)	(0.166)
$\Delta Intr_{nor}$					0.0004**
					(0.008)
const	.0064	0.004	0.003	0.003	0.003
Obs	180	180	180	180	180
R-squared	0.08	0.116	0.139	0.139	0.139
Sample	Q	Q	Q	Q	Q

Note: All regressions include dynamic lags of our trade weighted exchange rate series not reported here. Standard errors are all robust. The sample are quarterly time series data of various macroeconomic variables and ***, **, and * indicate significance at 1, 5, and 10 percent level respectively.

As reported in table 3 in column (5), we find that estimates of exchange rate pass-through are largely moderate and significant. In the model, we allow for partial adjustment of import prices to log changes in exchange rates overtime, the nominal effective exchange rate is our primary regressor, is defined as the foreign currency price of a unit of domestic currency, signifying that an increase in exchange rate is an appreciation, and therefore, we expect a negative relationship with import prices. Therefore, a 1% depreciation will increase import prices by 0.44% on impact in the first quarter and this result is statistically significant. We find that the contemporaneous estimate of pass through of exchange rate into import prices is 0.44 as seen in column 5 in the short run, which coincides with a quarter as our primary regressor enters the model with up to four lags, and we find that across various specification, pass through is realized after the first two lags and these results underscores the empirical evidence of pass through into import prices for both OECD and European countries which is about 0.40 – 0.55 in the short run.

To capture the impact of foreign exporter marginal cost, we utilize the producer price index of the United States PPI_{us}, and this price index which are correlated with real and nominal exchange rates improves our model specification and the results in table 3, column 5, show that an increase in exporters marginal cost by 1% increases import prices by 0.27% and in motivating pass through using micro foundation, we show that marginal costs vary with firms desired output level, and we use the united states real gross domestic product (GDP) to capture foreign exporters output level, but we do so relative to the output level of the domestic economy, Norway, hence the relative Norwegian-US output level has a negative relationship with import prices indicative from table 3 column 5, and we show that a 1% increase in foreign exporters desired output level, here, US gross domestic product, relative to domestic output level, will increase exporters marginal cost and import prices by 0.04 %. This is economically intuitive despite the negative sign, as an increase in US output level reduces the relative output metric, thereby, increasing import prices by 0.04%.

When we control for domestic firms marginal cost components, From micro theory of exchange rate pass through, domestic firms and foreign exporters utilize imported inputs in production. And for foreign exporters who rely on domestic inputs, changes in domestic marginal costs, as captured by Norwegian producer price index and interest rate which is reflective of domestic borrowing costs adequately captures the elasticity of foreign exporters marginal costs and trade prices to changes domestic marginal costs.

Evidently, a 1% increase in domestic firms PPI index would increase foreign exporters costs due to dependence on imported input with cumulative effects increasing import prices by 0.097% and a 1% increase in interest rates would increase foreign exporters marginal cost with cumulative effects increasing import prices by 0.0004% as exporting firms adjust prices to reflect output, cost and demand conditions.

The Norwegian PPI index can also be used to capture competition and market dynamics in the domestic market as firms internalizing an exogenous exchange rate shocks, vis a vis competing firms increasing prices in face of this perceived exchange rate impulse, would also increase their prices, without loosing market share, as the impulse is one that is forcing firms in the industry to respond symmetrically under monopolistic competition and unchanging demand conditions. Our model results reflect parameter stability as the inclusion and exclusion of variables is inconsequential to our results, as evident across columns 2 - 5 in table 3.

Our econometric model is sensitive to the data properties. If the log of import prices, exchange rates and exporters marginal costs are cointegrated, our dynamic lagged model becomes mis-specified, and a vector error correction model would be an appropriate solution.

To that effect, we test for non-stationarity, and we were unable to reject the null hypothesis that our series are non-stationary at level form and only became stationary after taking a first difference using a Dickey fuller test with time trend at 5 percent significance. To capture the long-run relationship, a linear combination of our series should result in a stationary process to be cointegrated, but in this model, we rejected the cointegrating hypothesis, and we do not estimate a vector error correction model. Therefore, our dynamic lag model yields unbiased, consistent and efficient estimates.

After accounting for exporters marginal cost, and relative output of trading partners which are strongly correlated with the exchange rates, results in our full model show that the long run pass through elasticity is 0.63. This result is reflective of the cumulative effect of log changes of exchange rates on the lags of import prices, $\sum_{k=0}^{4} \beta_j^t$. This finding is consistent with Corsetti and Dedola (2002), which estimate pass-through to be incomplete in the long run, but at odds with the empirical results of Ghosh and wold (2001) and Parson and Sato (2008) both studies of which, found pass-through to be perfect in the long run.

3.6 Export price pass through

We estimate the exchange rate pass-through into export prices using parallel model specification and estimation technique. Our model specification has similar micro-foundations as that of pass-through into import prices. Here, home firms export prices are a markup over domestic marginal costs. In turn, firms' markups are a function of domestic and international market factors and determinants, while firms' marginal costs are reflective of input costs, output levels, and exchange rates, to the extent that domestic firms utilize imported inputs in production.

Under this framework, the short run pass through into export price is the contemporaneous effect of a log change in the trade weighted exchange rate on changes in export prices. Our results show that for Norway, export price pass through is about 0.5 in the short run and near unity in the long term, and our model specification yield consistent and efficient estimates for similar reasons articulated earlier. Therefore, in the face of exchange rate changes, domestic

Table 4: ERPT into Norwegian Export Prices and Trade Quantities

	(1)	(2)	(3)	(4)	(5)
	Δp^{ex}	Δq^{m}	Δq^{m}	Δq^{ex}	Δq^{ex}
Δe_{neer}	-0.503**	-0.684**	-0.699**	-0.669*	-0.6742*
	(0.227)	(0.323)	(0.324)	(0.369)	(0.280)
ΔPPI_{us}	0.548*	0.055	0.013	-0.332**	-0.348
	(0.308)	(0.364)	(0.371)	(0.296)	(0.299)
ΔPPI_{nor}	1.060***	0.212	0.19	0.237	0.229
	(0.15)	(0.138)	(0.147)	(0.153)	(0.154)
$\Delta Y_{nor/us}$	0.048	1.976***	1.826***	-0.739	-0.794
	(0.458)	(0.533)	(0.528)	(0.699)	(0.688)
$\Delta Intr_{nor}$	-0.034		0.034		0.0124
	(0.025)		(0.026)		(0.023)
const	$-.007$	-0.005	-0.005	-0.004	0.004
Obs	180	180	180	180	180
R-squared	0.646	0.224	0.235	0.245	0.247
Sample	Q	Q	Q	Q	Q

Note: All regressions include dynamic lags of our trade weighted exchange rate series not reported here. Standard errors are all robust. The sample are quarterly time series data of various macroeconomic variables and ***, **, and * indicate significance at 1, 5, and 10 percent level respectively.

exporting firms pass through about half of this impulses into trade prices.

An increase in domestic firms desired output level has a positive relationship with export prices as domestic firms' marginal cost positively varies with an increase in firms desired output level. We capture this by the relative output of Norway to United state gross output and our results show that an increase in domestic firms output by 1% relative to US GDP will increase domestic firms' marginal cost and cumulatively, export prices will increase by 0.05%.

The domestic interest rate variable tells an important story when it comes to its elasticity with respect to export prices because on the one hand, we could use this to account for borrowing and marginal costs, but from table 4, in column 1, we document a negative relationship between interest rate and export prices. Here an increase in domestic interest rate by 1% point will induce economic agents to rebalance their portfolios towards domestic financial assets, causing an appreciation of exchange rates, which would make export prices relatively expensive, explaining the negative response of export prices to interest rates by about 0.034%.

PPI_{us} and PPI_{nor} which are the marginal cost component have symmetrical influence on export price pass through as an increase in the domestic exporting firms marginal cost by 1% increases export prices by 1.06% units, i.e, more than one for one, making export prices very elastic and sensitive to changes in exporter's costs, while an increase by 1% in the marginal cost of foreign exporting firms would increase the cost of imported inputs and subsequently export prices, hence, the symmetrical effect of raising marginal cost across trading partners country as evident from column 1 on table 4.

3.7 Import and Export quantity pass through

We estimate pass through into import and export quantities and our results are in parallel with the predictions from our calibrated theoretical models. In response to log changes in trade weighted exchange rate, a 1% appreciation of exchange rates induces a 0.68% and 0.67% decline in both import and export volumes, respectively. A 1% appreciation which is an increase in nominal effective exchange rates, would make exports relatively more expensive, leading to an erosion of export demand by 0.67% as documented in our result in table 4, column 4 & 5. Here, the response of export volume is quite conventional.

However, for import volumes, our results are non-trivial. We expect an increase or an appreciation of exchange rates to make imports pries relatively cheaper and stimulate import demand, however, we see a 0.68% decline in import volumes which is clearly attributable to changing preferences amongst the domestic residents which is diverting demand away from foreign goods to domestically produced goods. In general, export quantities hold stable and solid in response to changes in relative prices of traded goods. However, the elasticity of import and export volumes from our empirical model is twice the response from our calibrated theoretical DSGE model as in Rigobon and Gopinath (2010).

These results recasts the exchange rate puzzles as asymmetric shocks which generate changes in exchange rates are absorbed by the exchange rate than can be accounted for by changes in relative prices and real quantities, thus highlighting exchange rate disconnect which is attributable to the presence of Local currency pricing or Dominant currency pricing and a structure of international pricing and product distribution such that the effects of exchange rate changes are minimized as documented by Devereux and Engel(2002).

3.8 Rethinking Conventional Pricing Assumptions

Our model specification allow us to carry out extensive test to enable us to validate our pricing assumption in earlier chapters. Under producer currency pricing (PCP), assumption, conventional theoretical predictions indicate that pass-through elasticity into import prices is complete, i.e. 1 and using quarterly series, results from the test of our pass-through coefficient show that we can reject the model prediction of the PCP paradigm as we rejected the hypothesis that the contemporaneous parameter estimate is equal to unity in the short term.

Concurrently, under local currency pricing conventional theoretical predictions are that, pass through into import prices is muted (i.e zero) and using quarterly series, result of the test of our contemporaneous pass-through estimates show that we can reject the model prediction of the LCP paradigm and conclude that the contemporaneous parameter is not equal to zero in the short term.

Our model is sufficient to validate the possibility of the dominant currency paradigm, DCP. Under the conventional pricing assumptions of producer and local currency pricing, pass-through or variations of import prices and terms of trade should be sensitive to the trade weighted exchange rate and invariant to the bilateral exchange rate of the home currency relative to the dominant currency. However, our results show that in the pass through regression of import prices as evident in table 5, column 3 and 4, the inclusion of the bilateral exchange rate of the Norwegian krone relative to the Euro and US dollar in column 4, reduces the short run pass-through elasticity by 10 percentage points from 0.44 to 0.34, which is congruent to the theoretical prediction of Dominant currency pricing assumption which yield more stable trade prices, as evident from the decline in the price impact in column 4.

Complementary regression of the bilateral exchange rate on the terms of trade in column 5 reveal itself to be statistically significant and non-zero, all of which strongly supports and validates the possibility of Dominant currency pricing at work.

All this finding and tests unveils that the two conventional pricing assumptions can be rejected and pass through is neither complete nor muted. However, our findings reveal that across price classes, pass through in the short term is incomplete and bounded as contemporary empirical

Table 5: ERPT into the Log change of Norwegian Import Prices

	(1)	(2)	(3)	(4)	(5)
	Δp^m	Δp^m	Δp^m	Δp^m	ΔTOT_{nor}
Δe_{neer}	-0.693***	-0.356***	-0.4385***	-0.346***	-2.726***
	(0.244)	(0.063)	(0.078)	(0.092)	(0.744)
ΔPPI_{us}	0.191	0.209***	0.266**	0.269**	
	(0.748)	(0.067)	(0.101)	(0.11)	
ΔPPI_{nor}	0.6085	0.0736***	0.097***	0.102***	
	(0.551)	(0.027)	(0.0326)	(0.032)	
$\Delta Y_{nor/us}$	0.075	-0.224^*	-0.037	-0.06	
	(0.326)	(0.134)	(0.166)	(0.169)	
$\Delta Intr_{nor}$	-0.042	0.0095	0.0004	0.003	
	(0.029)	(0.008)	(0.008)	(0.009)	
$\Delta e_{kr/e}$				0.122*	-1.819***
				(0.073)	(0.574)
$\Delta e_{kr/s}$				-0.006	-0.447^{**}
				(0.044)	0.177
const	0.002	0.005	0.003	0.004	17.166
Obs	76	105	180	180	180
R-squared	0.161	0.361	0.139	0.139	0.136
Sample	Q	Q	Q	Q	Q

Note: All regressions include dynamic lags of our trade weighted exchange rate series not reported here except column 5. Standard errors are all robust. The sample are quarterly time series data of various macroeconomic variables and ***, **, and * indicate significance at 1, 5, and 10 percent level respectively.

evidence points to the prevalence of dominant currency pricing as a practical pricing assumption.

The local currency pricing literature makes an important prediction about pass through elasticity over the long term, stating that asymptotically, pass through will be complete and equal to unity in the long term. We test this hypothesis using our import price dynamic model and our results strongly reject this prediction of a cumulative pass through estimate of unity in the long term, and we find that time frequency is inconsequential and pass through is incomplete both over short horizons and over longer periods.

3.9 Decline in Aggregate pass through

Our results reveal that long run pass through estimates into import prices is incomplete in contrast to a priori expectation of convergence to unity and this concept has been partially documented in the empirical literature. In our model, we test for a decline in pass through by exogenously imposing a structural break in 1999 which coincides with the formation of the Euro area and using the Chow test, we test for equality of means across sample proportions.

We reject the hypothesis of equal mean estimates across sample proportions as seen from column 1 and 2 of table 5, we document a significant decline in pass through elasticity, as pass through is halved from 0.7 to 0.36 across samples. This decline could be attributed to changes in macroeconomic policy variables and protocols as well as structural changes overtime which affect market and competition conditions.

Devereux and Engel (2002) document that import prices in domestic currency are more stable if firms price their traded goods in the currency of countries with relatively stable monetary policy. Thus we expect a higher pass through estimate for countries with more arbitrary and volatile monetary policy, and our result pick up on this on the validation of dominant currency pricing (DCP), as the inclusion of the bilateral exchange rate relative to the US dollar and the Euro in column 4 of table 5, trimmed the pass through elasticity by 10 percentage points, reflecting a relatively more stable import price fluctuations.

Consequently, as countries transition to inflation targeting which anchors expectations and with a complementary forward guidance framework and approach to crafting, design and implementation monetary policy, this will results in more stable prices generally and we account for this by controlling for monetary policy via the inclusion of domestic interest rates or broad monetary aggregates which moderated import price variations.

Over short and longer period as firms absorb more of the perceived changes in exchange rate due to strategic interaction and market segmentation, sticky prices which define markups, guarantee incomplete pass through across all time periods and as the share of imported inputs and distribution cost make up a larger component of final goods, variations in cost significantly influence the elasticity of demand leading to incomplete pass through over longer periods.

3.10 Structured VAR: Shock dependent Exchange rate pass through

3.11 Dynamics of exchange rate pass through

In the previous section, our objective was to estimate the size and magnitude of exchange rate pass through and investigate the stability of the degree of pass through over time. We accounted for a variety of micro-dependent factors responsible for the heterogeneity of pass through across time to validate our major conclusion that exchange rate pass-through is incomplete in the short run.

In this section, we would take a microscopic approach in investigating the degree, frequency, persistence and speed of adjustment of pass through across time, with an emphasis on the sources of asymmetric shocks which affect the varying degree of pass through over longer horizons while utilizing a method that efficiently estimates a system of equations and produces unbiased, consistent and efficient estimates using a VAR model.

Previously we have utilized ordinary least squares to estimate the size and the impact of Pass-through with polynomial distributed lags to capture the dynamic response of traded goods prices, while controlling for other relevant economic and model drivers, and in explaining our results, we assumed there were no movements in other regressors as they were assumed to be purely exogenous.

This section complements our previous approach by allowing and accounting for endogeneity of the variables by introducing symmetry in the way we account for variables while allowing us to trace out the dynamic response of traded good prices to purely exogenous identified structural exchange rate shocks. Thus, in addition to the size and magnitude of the impact of exchange rates shocks, we investigate and understand the speed and frequency of the impact of exchange rate on the dynamical response of prices in the model.

Therefore, the main empirical contribution we make in this section is to ascertain and explain the shock dependent nature of exchange rate pass through which is the complementary cause of time-varying exchange rate pass through or the decline in pass-through over time. To fulfill this objective, we utilize a vector auto regression, (VAR) framework to investigate the shock dependent nature of exchange rate pass through to complement the single equation approach which helped us test the predictions of our theoretical assumptions relative to the DCP assumption.

Vector auto regression has been used as a credible alternative to single equation modelling and its use to investigate the relationship between exchange rate and traded good prices is not novel. In this section, we utilize the VAR model to estimate and investigate the persistence, degree and speed of adjustment of the pass through relative to the source of the identified exogenous shock which have desirable and policy relevant applications in contemporary times.

Consider a multivariate generalization of a VAR model of the form:[17]

$$BX_t = \Omega X_{t-1} + DV_t, V_t \sim (0, \Theta) \tag{3.08}$$

Premultiplying equation (3.08) by B^{-1} yields

$$X_t = B^{-1}\Omega X_{t-1} + B^{-1}DV_t \tag{3.08a}$$

$$B_0 = B^{-1}\Omega, B_1 = B^{-1}D \tag{3.09}$$

$$X_t = B_0 X_{t-1} + U_t, U_t \sim (0, \sum) \tag{3.10}$$

[17] here, Θ is the variance vector of the structural model and $\sum$ is the variance covariance matrix of the reduced form model

equivalently, we can have

$$X_t = A(L)X_{t-1} + F(L)Z_t + U_t \tag{3.11}$$

Where matrix B is the contemporaneous effects of the vector of endogenous variables within the system and we assume X_t is stationary, and V_t are uncorrelated white noise processes of the structural system with constant mean and variances, and U_t are a vector of residuals which are individually and independently correlated and having constant mean and variances and $A(L)$ and $F(L)$ are matrix polynomials in the lag operator and Z_t are a vector of constant, trend, dummies and exogenous variables.

For our reduced form VAR, an arbitrary estimation is void of economic content, however, with innovative accounting using a Cholesky ordering and decompositions, the results inherit an economic meaning which is quite arbitrary as the Cholesky decomposition makes very strong assumption about the underlying structural shocks, and in the case where the correlation coefficient of the V_t sequence is low, the ordering of variables is likely unimportant.

3.12 Identification

Sims and Bernanke (1980) pioneered modelling the structural innovation using economic analysis in a bit to estimate the relationship among the structural shocks using an appropriate economic model in such a way that the structural model can be identified from the reduced form VAR model.

We follow this approach to identify the structural model necessitating us to impose a minimum of $(n^2 - n)/2$ restrictions on our model and to this effect, we restrict our system so as to recover the structural shocks while preserving the error structure concerning the independence of the various structural shocks through a credible and well founded identification strategy.

We utilize a vector or quarterly time series macroeconomic variables, where:[18]

$$x_t = [\Delta y_t, \Delta int, \Delta ppi^n, \Delta PPI^*, \Delta neer, \Delta p^m] \tag{3}$$

Our VAR specification utilizes the following variables which includes the log of united states

[18]All variables are for the domestic (Norwegian) economy under consideration except for the US PPI index

producer price index PPI^*, as a proxy for foreign exporters marginal cost, and the log of domestic (Norwegian) producer price index PPI_t^n, as a proxy for domestic marginal costs and foreign competitors price in domestic markets, and the log of domestic (Norwegian) interest rates int_t, the log of Norwegian nominal effective exchange rates $neer_t$ and the log of Norwegian import price index, p_t^m and y_t is the log of real gross domestic product.

The quarterly data for each time series are seasonally adjusted from the source and the exchange rate is expressed as a the foreign currency price of a unit of domestic currency, and as a result, an increase in exchange rate is an appreciation, and our exchange rate pass through estimates are expected to have a negative sign and complete ERPT corresponds to a value of -1.

The econometric technique we adopt to identify exchange rate shock is an extension of the Souki and Enders (2000) approach, where we utilize a six variable structural VAR to obtain global supply shock, exchange rate shock amongst others. We classify shocks not by their source, but by their impact, and for this, we follow the small open economy (SOE) theoretical assumption that foreign or world allocations are exogenous to the domestic economy, as we allow the global price shock to have symmetric effects on the system.

Exchange rate shock is identified using the uncovered interest parity (UIP) condition. The application and use of interest rate is twofold, one as a proxy for domestic marginal costs and the other as a tool of monetary policy. We identify monetary policy shock via its use as a policy tool, and like in conventional SOE literature, central banks adjust rates in reaction to deviations of inflation from target. The domestic PPIs and import price equation are identified using the SOE assumption where producer prices are a function of marginal costs. World price equation follows our SOE assumption that foreign variables are exogenous to domestic allocations and foreign impulses have a symmetric impact on the rest of the world.

This identification strategy guides our methodological approach and enables us to restrict the system and identify structural shocks in a way that is consistent with our theoretical model in previous chapter, and broad NOEM literature.

3.13 Results

We begin with pre-estimation test of stationarity and all of our series are integrated at the first order, thus, our variables enter the model in differenced form as growth rates relative to level form estimation. Subsequently, we estimate our unrestricted VAR with 8 lags to inform our lag selection, and we settle for 3 lags as prescribed by the Akaike information criteria (AIC) and (FPE & LR) recommendations.[19]

We re-estimate our VAR with the appropriate chosen lag, and we run some diagnostic checks, and we confirm that our residuals are serially correlated as expected. The stability properties of the dynamic system are examined, and it is quite evident that the inverse roots of the ARMA process for our model lie strictly in the unit circle, which indicates that our VAR system is dynamically stable.

This model checks are important as we add a structure to our VAR model, to enable the forecast errors, which are composites of our structural shocks to yield a stable system and improve the quality of our structural analysis, estimation and forecast.

From the results, a one percent transitory increase in exchange rate (1% increase in V_{ner}) has an immediate impact on exchange rate, triggering a sharp appreciation, which is persistent after a quarter, and becomes statistically zero between quarter 4 and 5 and at the end of quarter 4, the effect of the shock on exchange rate is muted all through the horizon and vanishes out after the 7th quarter. The declining positive impact up until quarter 4 witnessed initially after impact displays the persistence of exchange rate, due to the response of the central bank tightening interest rates as the UIP equation predicts an eventual stabilization as can been seen from the results.

The response of import prices is of great interest, and it spotlights the objectives of our research. From figure 18, The response of import prices is quite significant as on impact, the effect of a transitory shock to exchange rate is negatively propagated across the time path of import prices for the first 4 quarters, becomes positive, and then displays a muted response after the quarter 6. We see a 0.009% decline in import prices which is expected and this impact declines

[19]see appendix for all lag length recommendations using diverse information criteria

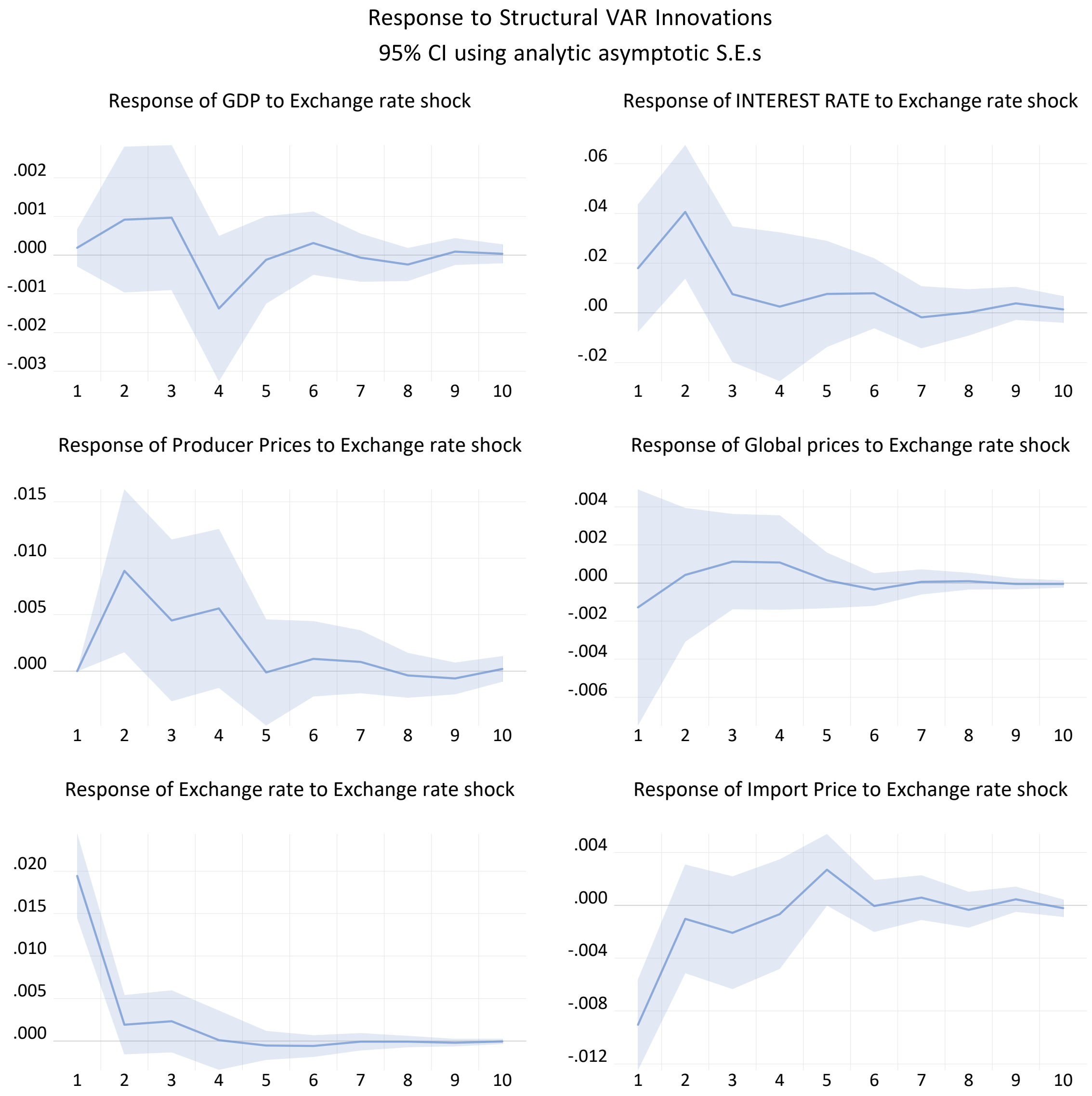

Figure 18: Response to an exchange rate shock

monotonically afterwards. We can extract an economic interpretation as the structural exchange rate shock being propagated is an appreciation of exchange rate, and to understand the degree of pass through, we take the absolute value of the effects on import prices and it becomes apparent that there is a sharp and sudden decline in pass through from the second quarter as we still document incomplete pass through both at short and longer horizons of the time path of import prices.

The response of domestic producer prices is economically appealing and intuitive. First, in response to an appreciation which dampened import prices, imported components and inputs become relatively affordable and firms witness a significant decline in their marginal cost per unit, but this effect is neutralized by the sharp rise in interest rates, thereby increasing the cost of production leading to a sharp increase in prices as the effect of the shock is positively propagated across the time path of traded goods prices until the fifth quarter when prices witnessed a moderation followed by a muted response after.

From the response of domestic interest rates, we see an increase in interest rates as the effect of a transitory increase in exchange rates which triggered an appreciation necessitates an ex ante monetary tightening to sustain the realized appreciation. As such, monetary authorities are forced to tighten policy to mitigate the effect of the inflationary pressures by embarking on a contractionary policy.

For the response of foreign prices, our structural relationship underscores the fact that either of the endogenous vector have no contemporaneous impact on US PPI but only through the lag structure of the VAR, accounting for the muted contemporaneous response on impact as the shock declines monotonically.

The response of real output to an exchange rate shock, shows a slight and moderate increase which last up to the first two quarters, and this increase in output is attributable to the fall in import prices which enables firms to utilize more imported inputs thereby boosting production. Following this initial response of output which last for about two quarters, we witness a steep decline in output after quarter three. This decline is in response to the contractionary policy stance of the central banks evidenced by an increase in interest rates, triggering a decline in economic activity after the third quarter.

From fig 19, A critical examination of a temporary exogenous shock to foreign exporters marginal cost would increase US PPI index on impact by 0.015%, as the shock has positive effects for the first three quarters, peaking on impact, and being negative for the following two quarters and then vanishing asymptotically. The implication of this global price shock is felt through the system as traded goods prices increase on impact as evident from the response of import prices, p_t^m and producer prices PPI_t^{nor}.

The effect of this transitory increase in foreign exporters marginal cost is positively propagated

Figure 19: Response to a global price shock

over the time path of our prices across the price chain, as central banks respond with a tighter monetary policy to stem the inflationary impact of this global supply shock exchange rates appreciate in response to this domestic tightening of policy and strong enough to produce an overshooting of exchange rates.

The response of the vector of regressors to a sudden transitory increase in exchange rates by 1% reveal that across time, there is a significant decline in pass through and this results presents an empirical foundation for testing the model predictions of our theoretical pricing assumptions of PCP and LCP.

We consider exchange rate pass through as complete at a particular horizon if the corresponding confidence interval and bounds around the impulse response point estimates contain -1 and excludes zero, 0. It is evident that across the time path of the response of import price, the confidence bounds does not include one,1 nor zero, 0 , even after 10 quarters, and as such we conclude that the overwhelming empirical evidence does not support the predictions of PCP or LCP, thus giving credence to other pricing assumptions, as pass-through is overwhelmingly incomplete at short and longer horizons.

We try to empirically validate our structural model by conducting robustness checks to see if our results are uniquely sensitive to our identification method and strategy. To do this, we utilize the Cholesky orthogonalization by ordering $\Delta PPI_t^* \rightarrow \Delta y_t$ as fixed points in the ordering space after which the vector of regressors are fitted in line leading to a maximum of 24 unique orderings, and we show that our results are invariant to the Cholesky orthogonalization at various horizons which underscores our unique identification for producing efficient estimates and a rich model dynamics as the true model.

3.14 Forecast Error variance decomposition

We have identified an appropriate economic structure for our vector autoregressive model and have documented the orthogonalized impulse response of our variables to the identified structural shocks, and here, we aim to provide a deeper insight into the dynamic interaction amongst our system of variables to help us understand which shock influences variability at different horizons. In practice, variance decomposition utilizes quite similar information retractable from impulse response, it is highly likely for a variable to explain almost all of its forecast error variance at short horizons and smaller proportions at longer horizons.

For our structured VAR representation in equation 3.11, we document how the system responds to a unit structural shock at time t, and we express it in a vector moving average (VMA) as follows:

$$x_t = \sum_{k=0}^{\infty} \Phi_k \varepsilon_{t-k} \tag{3.12}$$

To identify the amount of information or how much of the forecast error variance of each variable can be explained by exogenous shocks to other variables, we have

$$\theta_{ij}(h) = \frac{\sum_{k=0}^{h-1}\left[e_i\Phi_k\Sigma\Phi_k e_j\right]^2}{\sum_{k=0}^{h-1}[e_i\Phi_k\Sigma\Phi_k e_i]} \tag{3.13}$$

Here, e_i is a selection vector and the denominator is the total forecast error variance of variable i up to horizon h. Therefore, $\theta_{ij}(h)$ is the forecast error variance decomposition which measures the relative quantitative importance of each structural shock, by evaluating its relative contribution to the variance of the forecast error of variable i at horizon h.

From figure 24, a critical look at our results reveal that exchange rate shocks contribute mildly to the realized variations in import prices while foreign exporters marginal costs, after accounting for macroeconomic factors, show moderate contribution to import prices and jointly, both exchange rate and global price shocks make up about 10% of the realized variation over both short and longer horizons.

However, import price shocks contribute significantly and accounting for about 90% of variations in import prices and these contributions are not short-lived, but they persist and are sustained over longer periods. Our results also reveal that import price shocks contribute significantly and accounting for about 90% of variations in nominal exchange rate. In our results, a clear thread becomes visible, as the structure of our economic model accounts for a complex structure of global supply in a trivial way, via the use of imported inputs in production. This is amplified by the empirical fact that most exports are importers, who utilize imported inputs at various and multiple stages of production, and the elasticity of trade volumes and aggregate trade values to marginal cost shocks eventually affect the demand and supply signals in the foreign exchange markets, and subsequently the nominal exchange rate, therefore accounting for this two empirical results in this model.

Finally, we document little evidence for, and a limited role of monetary policy shocks which is negligible across the entire horizon in accounting for variations in import prices, as they are only relevant in explaining variations in domestic interest rates at shorter and longer horizons.

In conclusion, we document moderate and significant exchange rate pass through into import and export price of about 0.44 and 0.5 respectively, using a dynamic lagged model, and we document a decline in pass-through over time and we show that pass-through is fully realize fully after four lags and this is evident both from our dynamic lagged and fully structured model. Our dynamic lagged model show that our results are consistent and significant and we reject the two primary pricing assumptions of the New open macroeconomic model (NOEM) literature of local currency pricing (LCP) and producer currency pricing (PCP), along with their predictions and we conclude that exchange rate pass-through is neither perfect(PCP) nor muted(LCP) over short periods or longer periods and document the evidence that time frequency is inconsequential and pass-through is incomplete and over a short horizon and over longer periods.

3.15 Appendix

Table 6: List of Variables

Id.	Label	Description	Source
1	P^m	Import Price (Unit value)	IMF
2	P^{ex}	Export Price (Unit value)	IMF
3	INT	Domestic interest rate	IMF
4	PPI_{us}	US/Global Producer Price Index	IMF
5	PPI_{nor}	Norwegian Producer price index	IMF
6	$GDP_{nor/us}$	Relative output	IMF
7	NEER	Nominal effective exchange rate	IMF
8	TOT	Terms of trade	IMF
9	$EXR_{kr/usd}$	Bilateral Krone/USD exchanger rate	IMF
10	$EXR_{kr/eur}$	Bilateral Krone/EURO exchanger rate	IMF
11	IMPQTY	Import Volume Index	IMF
12	EXPQTY	Export Volume Index	IMF

Table 7: Stationary Properties of Time series

Variables	ADF Statistics	5% Critical Value	Order of Integration
Import Price	-4.0713	-2.8784	I(1)
NEER	-7.9315	-2.8779	I(1)
PPI_{us}	-10.0533	-2.8777	I(1)
PPI_{nor}	-6.6653	-2.8780	I(1)
INTR	-6.9165	-2.8780	I(1)
$GDP_{nor/us}$	-17.720	-2.8776	I(1)

VAR Lag Order Selection Criteria
Endogenous variables: GDP INTERESTRATE PPI USPPI DNEER IMPORTPRICE
Exogenous variables: C
Date: 10/16/25 Time: 17:04
Sample: 1980Q1 2024Q4
Included observations: 171

Lag	LogL	LR	FPE	AIC	SC	HQ
0	2144.964	NA	5.50e-19	-25.01712	-24.90689*	-24.97240
1	2227.963	159.2019	3.18e-19	-25.56681	-24.79518	-25.25372*
2	2263.615	65.88378	3.19e-19	-25.56275	-24.12971	-24.98128
3	2300.655	65.84811	3.17e-19*	-25.57491*	-23.48046	-24.72507
4	2329.495	49.24792	3.47e-19	-25.49117	-22.73532	-24.37296
5	2365.908	59.62359	3.49e-19	-25.49600	-22.07875	-24.10942
6	2389.018	36.22013	4.13e-19	-25.34525	-21.26659	-23.69030
7	2424.302	52.82283*	4.27e-19	-25.33687	-20.59682	-23.41356
8	2451.608	38.96161	4.88e-19	-25.23518	-19.83372	-23.04349

* indicates lag order selected by the criterion
LR: sequential modified LR test statistic (each test at 5% level)
FPE: Final prediction error
AIC: Akaike information criterion
SC: Schwarz information criterion
HQ: Hannan-Quinn information criterion

Figure 20: Lag selection criteria

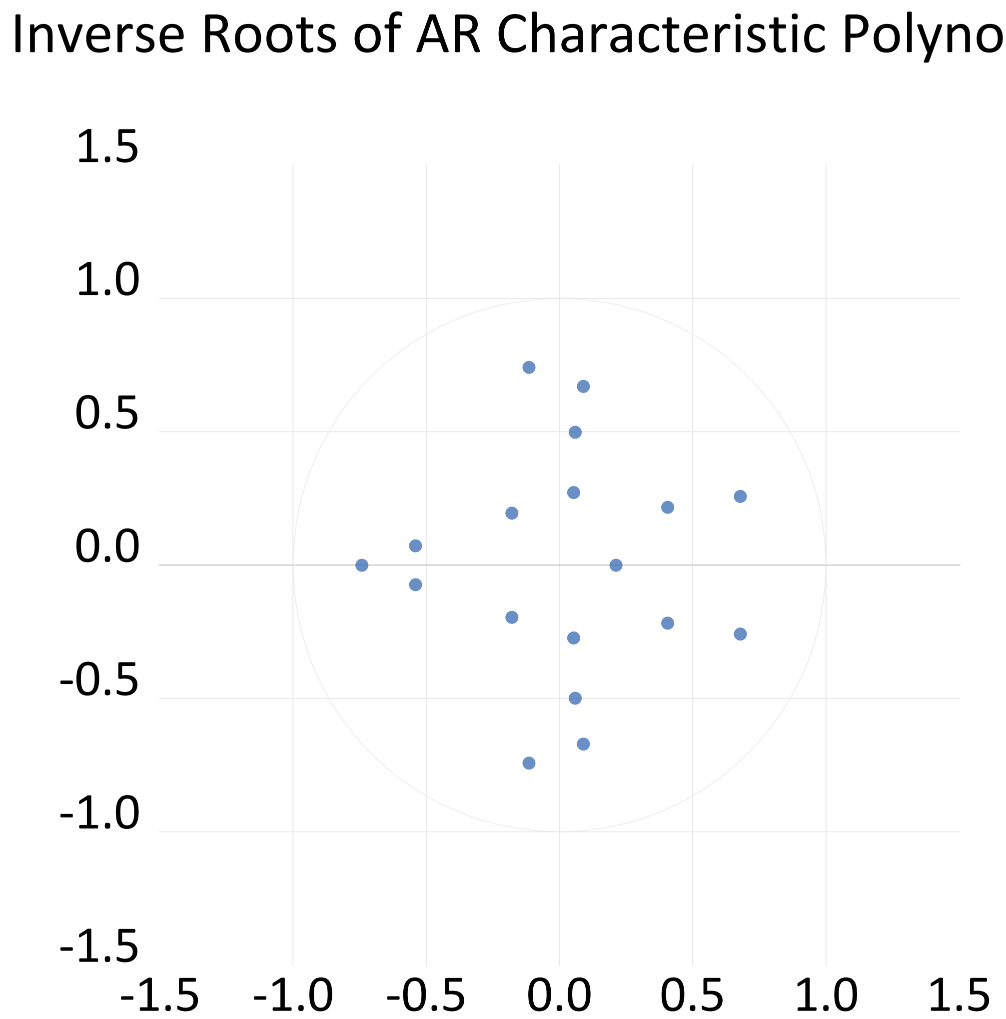

Figure 21: Dynamic Stability of our AR(3) model

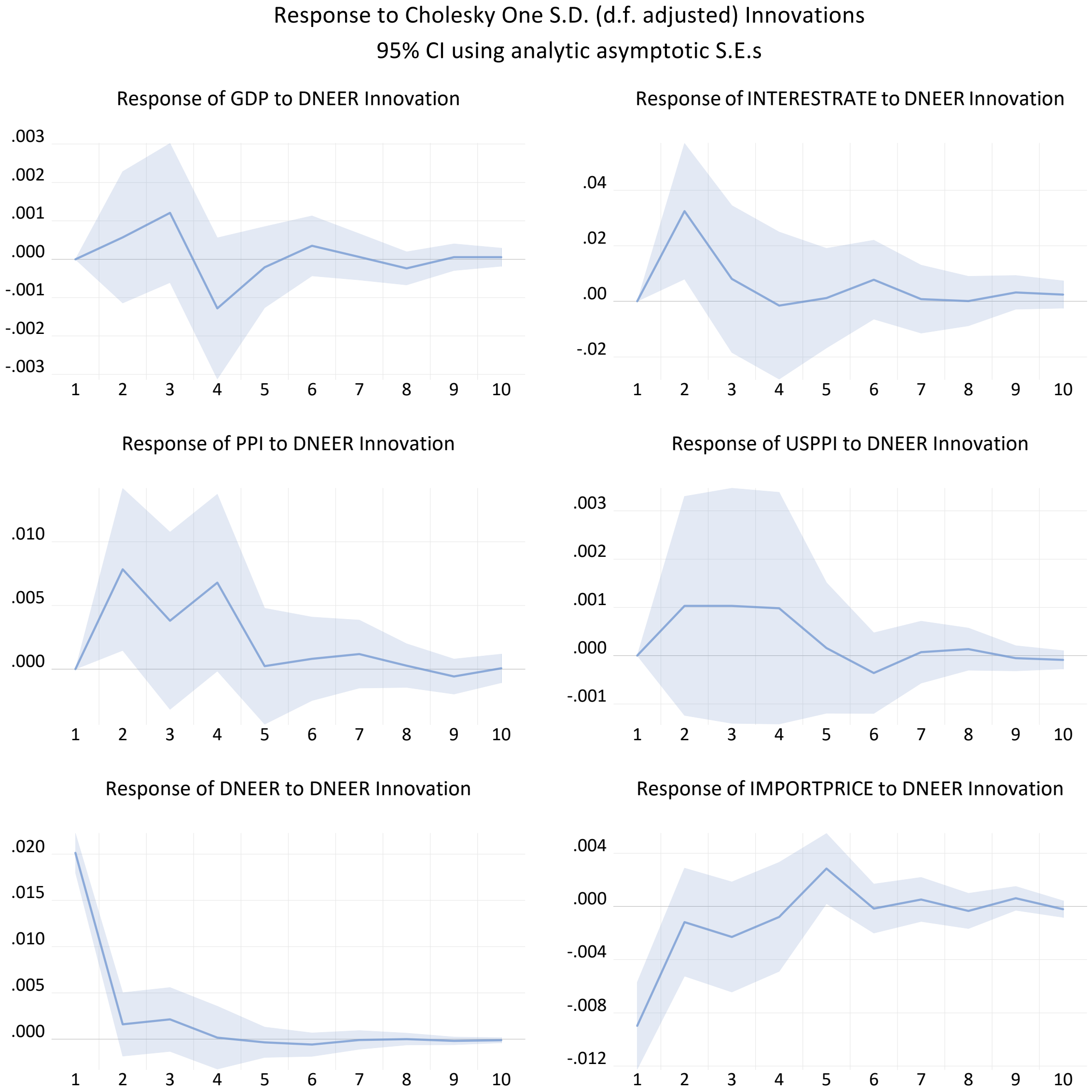

Figure 22: Order 1 $x_t = [\Delta y, \Delta int, \Delta ppi^n, \Delta PPI^*, \Delta neer, \Delta p^m]$: Response to a exchange rate price shock

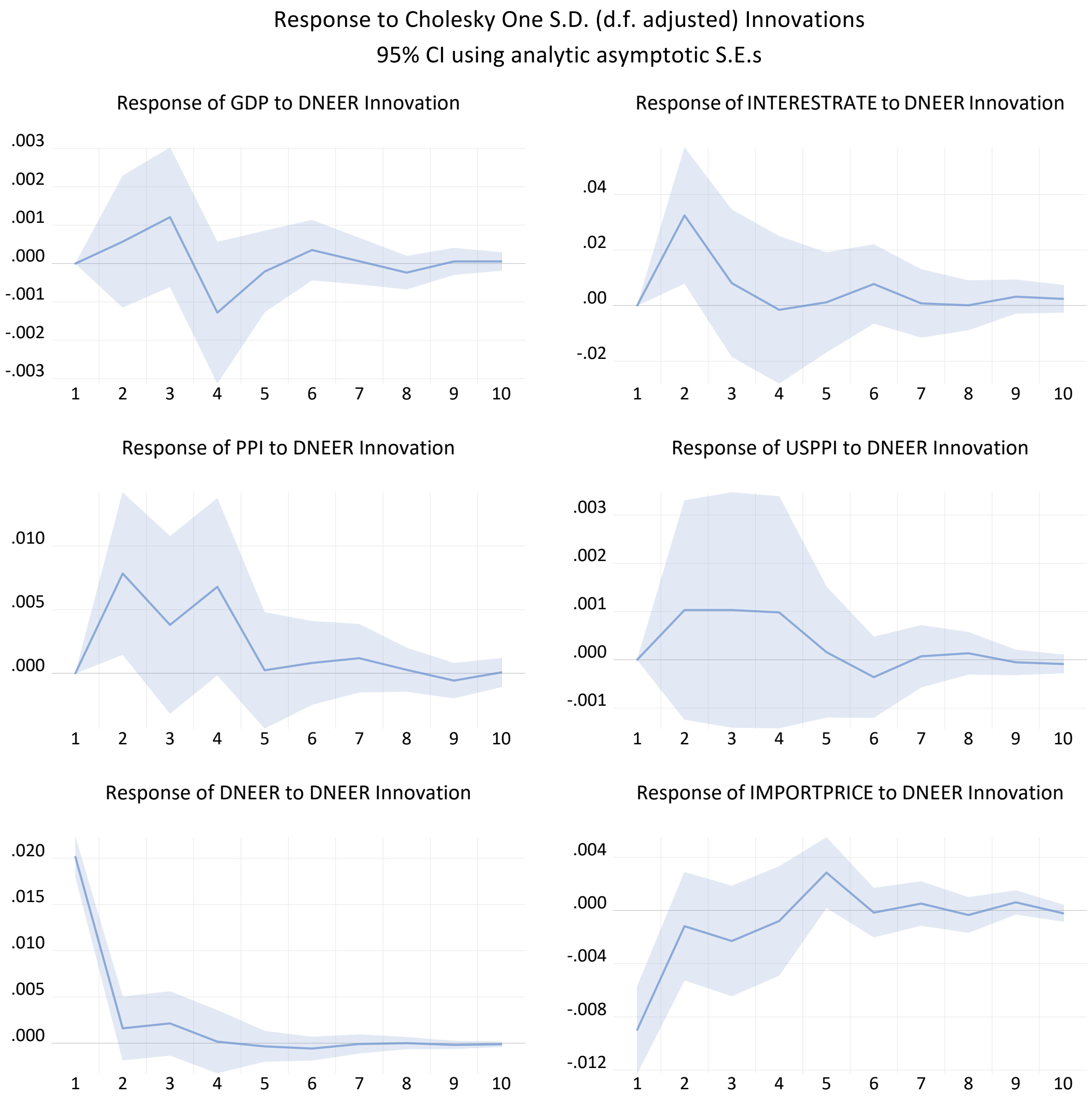

Figure 23: Order 2 $x_t = [\Delta PPI^*, \Delta y, \Delta int, \Delta ppi^n, \Delta neer, \Delta p^m]$: Response to a exchange rate price shock

Variance Decomposition using Structural VAR Factors

Variance Decomposition of GDP:

Period	S.E.	Shock1	Shock2	Shock3	Shock4	Shock5	Shock6
1	0.011029	97.06536	0.406215	1.793655	0.705389	0.029379	0.000000
2	0.011903	85.92048	8.932221	2.532226	1.148547	0.620687	0.845841
3	0.012381	79.80563	11.34572	5.808456	1.074467	1.184008	0.781725
4	0.012550	78.04550	11.51559	5.653926	1.086603	2.362275	1.336105
5	0.012611	77.78055	11.40718	5.603773	1.370166	2.348789	1.489546
6	0.012668	77.43511	11.30942	5.806845	1.494995	2.387056	1.566576
7	0.012702	77.02840	11.55699	5.875134	1.579045	2.377003	1.583432
8	0.012709	76.95435	11.54567	5.879597	1.578061	2.411579	1.630738
9	0.012717	76.85919	11.55474	5.929567	1.580051	2.413497	1.662954
10	0.012720	76.83615	11.55124	5.932245	1.585025	2.413147	1.682192

Variance Decomposition of INTERESTRATE:

Period	S.E.	Shock1	Shock2	Shock3	Shock4	Shock5	Shock6
1	0.156182	0.000000	98.28245	0.036851	0.348707	1.331990	0.000000
2	0.174641	0.040540	92.22666	0.058939	0.293608	6.472719	0.907537
3	0.178909	0.039488	88.47175	2.988171	0.516425	6.344777	1.639389
4	0.186856	1.636811	81.15976	6.296969	3.105887	5.834460	1.966111
5	0.191340	1.563123	80.10300	6.160064	4.489584	5.722220	1.962007
6	0.191959	1.620783	79.58739	6.523442	4.461363	5.853828	1.953198
7	0.193373	1.605048	79.15263	7.085897	4.454908	5.776772	1.924747
8	0.193584	1.630121	79.01094	7.138457	4.483230	5.764253	1.972995
9	0.193713	1.652473	78.92393	7.149859	4.495004	5.795876	1.982852
10	0.193854	1.679601	78.87918	7.140746	4.521649	5.792985	1.985837

Variance Decomposition of PPI:

Period	S.E.	Shock1	Shock2	Shock3	Shock4	Shock5	Shock6
1	0.040847	0.000000	0.000000	100.0000	0.000000	0.000000	0.000000
2	0.045233	0.708131	7.035996	87.00709	1.173568	3.842642	0.232571
3	0.045987	0.910482	6.945938	85.27533	1.832305	4.674488	0.361462
4	0.047715	1.178031	10.37967	80.38734	1.816570	5.694836	0.543550
5	0.048199	1.158427	10.86571	79.97848	1.859107	5.581354	0.556926
6	0.048282	1.202507	10.92094	79.77857	1.887107	5.613916	0.596958
7	0.048416	1.378242	10.98447	79.44468	1.961732	5.612260	0.618623
8	0.048594	1.381341	11.46880	78.86627	2.091449	5.576984	0.615158
9	0.048608	1.384080	11.47052	78.84961	2.090238	5.590743	0.614814
10	0.048649	1.384373	11.49114	78.83852	2.087319	5.583048	0.615602

Variance Decomposition of USPPI:

Period	S.E.	Shock1	Shock2	Shock3	Shock4	Shock5	Shock6
1	0.014580	0.000000	1.665581	9.324304	88.23326	0.776854	0.000000
2	0.015949	0.014845	1.433806	9.834142	87.82619	0.719879	0.171142
3	0.016153	0.016462	1.462388	10.31635	86.03360	1.182755	0.988444
4	0.016246	0.023462	1.522453	10.24111	85.62595	1.606713	0.980310
5	0.016271	0.043775	1.517830	10.33803	85.37494	1.609112	1.116318
6	0.016281	0.044130	1.552046	10.32519	85.27349	1.652382	1.152765
7	0.016290	0.044953	1.552361	10.38367	85.18294	1.651856	1.184227
8	0.016295	0.044937	1.592898	10.37890	85.13892	1.654578	1.189767
9	0.016295	0.045823	1.592788	10.38050	85.13356	1.655346	1.191976
10	0.016297	0.045813	1.610085	10.38017	85.11168	1.656056	1.196187

Variance Decomposition of DNEER:

Period	S.E.	Shock1	Shock2	Shock3	Shock4	Shock5	Shock6
1	0.022101	0.000000	0.381789	2.137345	20.22509	77.25577	0.000000
2	0.022576	0.286932	0.702924	4.298622	19.69541	74.76134	0.254771
3	0.022900	0.358959	0.813028	4.635471	20.22843	73.68340	0.280708
4	0.023088	0.588334	1.259764	5.165048	20.21545	72.48812	0.283275

Figure 24: Structural Model: Variance decomposition

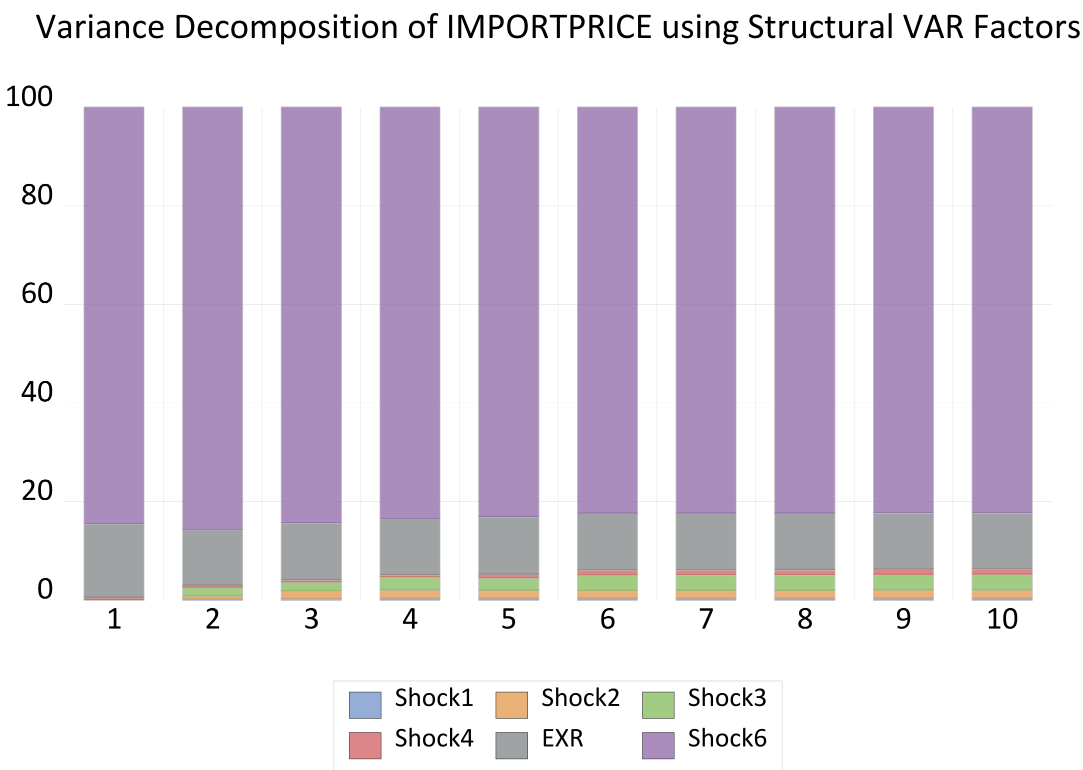

Figure 25: Structural Model: Variance decomposition of Import Price

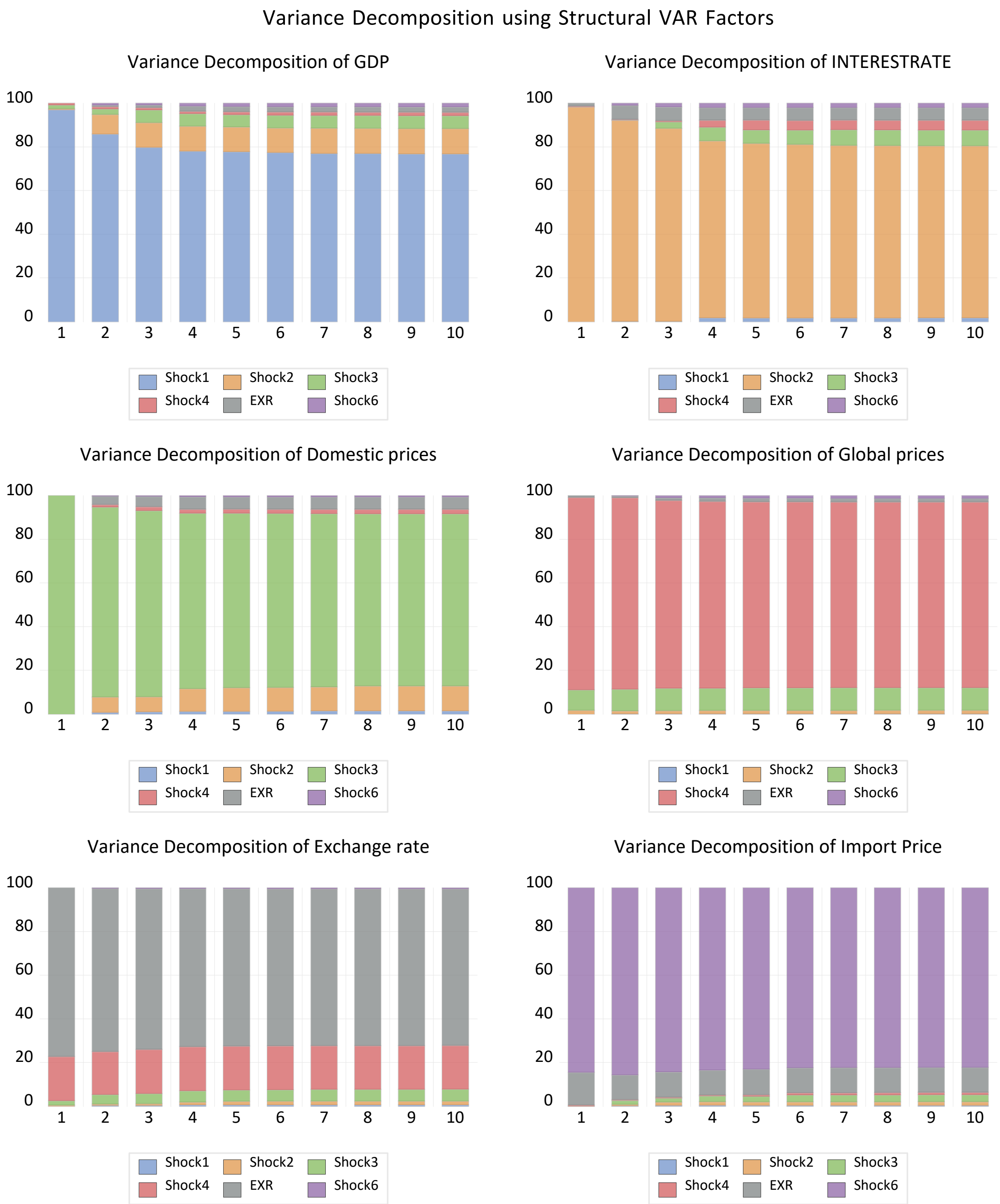

Figure 26: Variance decomposition

4 REFERENCES

Amiti, M., Itskhoki, O., and Konings, J. (2014). Importers, exporters, and exchange rate disconnect. *American Economic Review*, 104(7):1942-78.

Atkeson, A. and Burstein, A. (2008). Trade costs, pricing-to-market, and international relative prices. *American Economic Review*, 98(5):1998-2031.

Bacchetta, Philippe, and Eric Van Wincoop, "A Theory of the Currency Denomination of International Trade," *Journal of International Economics* 67(2): 295-319 (2005).

Benigno, Gianluca, and Pierpaolo Benigno, "Price Stability in Open Economics," *Review of Economic Studies* 70: 743-764 (2003).

Burstein, Ariel, Martin S. Eichenbaum, and Sergio Rebelo, "Modeling Exchange Rate Pass Through After Large Devaluations," *Journal of Monetary Economics* 54(2): 346-368 (2007).

Burstein, A. and Gopinath, G. (2014). International prices and exchange rates. In Gopinath, G., Helpman, E., and Rogoff, K., editors, *Handbook of International Economics*, volume 4, pages 391 - 451. Elsevier.

Camila, C. Diez, F.J. Gita, G. and Gourinchas, P.O 2018. Dominant Currency Paradigm: A new model for small open economies. IMF working paper, WP/17/264

Campa, J. and Goldberg, L. (2003). Exchange Rate Pass-Through into Import Prices: A Macro or Micro Phenomenon. SSRN Electronic Journal.

Campa, Jose Manuel, and Linda Goldberg, "Exchange Rate Pass Through into Import Prices", *Review of Economics and Statistics* 87(4): 679-690 (2005).

Canetti, E. and Greene, J. (1991). Monetary Growth and Exchange Rate Depreciation as Causes of Inflation in African Countries: An Empirical Analysis. IMF Working Papers, 91(67), p.1.

Corsetti, Giancarlo, and Paolo Pesenti, "International Dimensions of Optimal Monetary Policy," *Journal of Monetary Economics* 52: 281-305 (2005).

Corsetti, G., Dedola, L. and Leduc, S. (2008). High exchange-rate volatility and low pass-through. Journal of Monetary Economics, 55(6), pp.1113-1128.

Devereux, M. and Engel, C. (2002). Exchange rate pass-through, exchange rate volatility, and exchange rate disconnect. Journal of Monetary Economics, 49(5), pp.913-940.

Devereux, M., Engel, C. and Storgaard, P. (2004). Endogenous exchange rate pass-through when nominal prices are set in advance. Journal of International Economics, 63(2), pp.263-291.

Devereux, Michael B., Charles Engel and Peter E. Storgaard, "Endogenous Exchange Rate Pass-through when Nominal Prices are Set in Advance," *Journal of International Economics* 63: 263-291 (2004).

Devereux, Michael B., Kang Shi, and Juanyi Xu, "Global Monetary Policy under a Dollar Standard," *Journal of International Economics* 71(1): 113-132 (2005). Dornbusch, R. (1987). Exchange Rate Economics: 1986. The Economic Journal, 97(385), p.1.

Fitzgerald, D. and Haller, S. (2012). Exchange rates and producer prices: Evidence from micro data. Working Paper, Stanford University.

Gali J and Monacelli T. (2005). Monetary policy and Exchange rate volatility in a small open economy. Review of Economic Studies 72, 707-734

Goldberg, L. and Tille, C. (2009). Macroeconomic interdependence and the international role of the dollar. *Journal of Monetary Economics*, 56(7):990-1003.

Gopinath, G. and Itskhoki, O. (2010). In search of real rigidities. In Acemoglu, D. and Woodford, M., editors, *NBER Macroeconomics Annual*, volume 25. University of Chicago Press.

Jonathan Maccarthy (1999). Pass-through of exchange rate and import prices into domestic inflation in some industrialized countries *Bank of international settlement working paper* BIS-79, 1999.

Levin, J. (1987). The Dornbusch Model, Trade Flow Lags, and Exchange Rate Overshooting. Journal of Economic Integration, 2(1), pp.71-12.

MacDonald, R. and Taylor, M. (1991). The monetary approach to the exchange rate. Economics Letters, 37(2), pp.179-185.

McCarthy, J. (1999). Pass-Through of Exchange Rates and Import Prices to Domestic Inflation in Some Industrialised Economies. SSRN Electronic Journal.

Monacelli, Tommaso, "Monetary Policy in a Low Pass-Through Environment," *Journal of Money, Credit and Banking* 37(6): 1047-1066 (2005).

McCallum and Nelson (2002) "Recent developments in Monetary policy analysis : The role of theory and evidence" *Federal reserve bank of Richmond Economic quarterly volume 88/1* 2002

Obstfeld, M. and Rogoff, K. (1995). Exchange Rate Dynamics Redux. Journal of Political Economy, 103(3), pp.624-660.

Obstfeld, Maurice, and Kenneth Rogoff, "Global Implications of Self-Oriented National Monetary Rules," *Quarterly Journal of Economics* 117: 503-36 (2002).

Salyer, k. (1989). exchange rate volatility: the role of real shocks and the velocity of money. Economic Inquiry, 27(3), pp.387-409.

Sekine, T. (2006). Time-Varying Exchange Rate Pass-Through: Experiences of Some Industrial Countries. SSRN Electronic Journal.

Woodford, Michael, *Interest and Prices: Foundation of a Theory of Monetary Policy*, Princeton University Press, 2003.