Models of Quintessential Inflation

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A unified approach to quintessence and inflation is investigated with the use of a single scalar field. It is argued that successful potentials have to approximate a combination of exponential and inverse power-law decline in the limit of large values of the scalar field. A class of such potentials is studied analytically and it is found that quintessential inflation is indeed possible. Successful models, not involving more than two natural mass scales, are obtained, which do not require fine-tuning of initial conditions and do not result in eternal acceleration.

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1   Introduction

Recent observations suggest that the Universe at present is dominated by Dark Energy [1] and undergoes accelerated expansion [2]. This can be attributed to the presence of a non-vanishing cosmological constant Λ, which, however, has to be extremely fine-tuned $\Lambda^2 \sim 10^{-120}M_P^4$, where $M_P = 1.22 \times 10^{19}$GeV is the Planck mass. Therefore, the existence of a cosmological constant is not favoured by theorists, who prefer to account for the presence of Dark Energy by different means. One novel idea is to consider a dynamical, time-varying $\Lambda = \Lambda(t)$ [3]. The most straightforward realization of this idea is to suppose that $\Lambda(t)$ is due to the potential energy of a scalar field, called quintessence [4], which comes to dominate the energy density of the Universe at present and drive the latter into a late-time accelerated expansion. However, quintessence too suffers from fine-tuning problems [5], so it is not clear that it is a better alternative to the cosmological constant.

A compelling way to minimize the fine-tunings of quintessence is to link it with inflation. This seems natural since both quintessence and inflation are based on the same idea; namely that the Universe undergoes accelerated expansion when dominated by the potential energy density of a scalar field, which slowly rolls-down its almost flat potential. Unification of inflation and quintessence is achieved by using a single scalar field ϕ to drive both. There are many merits to quintessential inflation. Firstly, one avoids the introduction of yet again another unobserved scalar field, whose nature and origin are unaccounted for. Furthermore, a single theoretical framework may be used to construct the scalar potential $V(\phi)$. Moreover, it is possible to minimize the fine-tunings of quintessence by linking them with the ones of inflation and by introducing only few mass scales and parameters in $V(\phi)$. Finally, certain fine-tunings are automatically dispensed with, such as, for example, the tuning of initial conditions for quintessence.

Apart from satisfying the requirements of inflation and quintessence, quintessential inflation needs to incorporate an additional number of features. One such requirement is that the scalar field should not be coupled to any of the standard model fields. This is so because it is necessary for ϕ to avoid decay and survive until today. The additional advantage of such a "sterile" inflaton is that one dispenses with the fine-tuning of the couplings between ϕ and the standard model fields, otherwise unavoidable in order to preserve the flatness of the inflationary potential and also because the ultra-light quintessence field would correspond to a long-range force that could violate the equivalence principle at present. Natural candidates for a sterile inflaton are hidden sector fields, moduli fields, the radion and so on. Another requirement of quintessential inflation is that the minimum of the potential (taken to be zero, i.e. there is no residual cosmological constant) should not have been reached until today, so that $V(\phi)$ may dominate the Universe at present. In order to achieve this, the minimum is typically placed at infinity. Thus, the potential features two flat regions, the inflationary plateau and the quintessential "tail".

It is not easy to construct successful quintessential inflationary models and this is why only few such models exist in the literature. Most of the existing models, however, man-

age to satisfy the requirements of inflation and quintessence by considering multi-branch potentials that change form when the field moves from the inflationary to the quintessential phase of its evolution. This is achieved either "by hand" [6] or by a phase transition [7]. In these models information is not communicated between the inflationary and the quintessential part of the field's evolution. Another option, is constructing complicated models involving a large number of parameters, usually due to the theoretical framework used, which have to be tuned correctly to achieve the desired results [8].

We adopt a different approach and attempt to formulate a single-branch potential with minimal parameter content [9]. Such a minimalistic approach avoids the large number of fine-tunings inherent in other models, and renders our quintessential inflationary models preferable compared to the cosmological constant alternative.

2 Requirements of inflation and quintessence

Recent CMB observations [10] suggest that we live in a spatially-flat FRW Universe. We model the Universe content as a collection of perfect fluids, the background fluid with density ρ_B comprised by matter (including baryons and CDM) and radiation (including relativistic particles) and the scalar field ϕ with density $\rho_\phi \equiv \rho_{\text{kin}} + V$ and pressure $p_\phi \equiv \rho_{\text{kin}} - V$, where $\rho_{\text{kin}} \equiv \frac{1}{2}\dot{\phi}^2$. The evolution of ϕ is determined by the equation,

$$\ddot{\phi} + 3H\dot{\phi} + V' = 0 \tag{1}$$

where $H \equiv \dot{a}/a$ is the Hubble parameter with $a(t)$ being the scale factor of the Universe and the prime {dot} denotes derivative with respect to ϕ {the cosmic time t}.

2.1 Inflationary requirements

During inflation $V(\phi)$ dominates the Universe and the above equation becomes,

$$3H\dot{\phi} \simeq -V'(\phi) \tag{2}$$

Inflation occurs when $|\varepsilon|, |\eta| < 1$, where, $\varepsilon \equiv \frac{1}{\sqrt{6}} m_P(V'/V)$ and $\eta \equiv \frac{1}{3} m_P^2(V''/V)$.

2.1.1 Horizon and flatness problems

The horizon and flatness problems are solved if the scale that corresponds to the observable Universe at present did exit the horizon during inflation. The number of e-foldings before the end of inflation when this happened is estimated as $N_H \simeq \ln(t_0 T_{\text{CMB}}) - N_{\text{reh}}$, where t_0 is the age of the Universe according to the Hot Big Bang, T_{CMB} is the temperature of the CMB at present and $N_{\text{reh}} \equiv \ln(T_{\text{reh}}/H_{\text{end}})$ with T_{reh} being the reheating temperature and H_{end} the Hubble parameter at the time t_{end} when inflation ends.

To solve the horizon and flatness problems the total number of inflationary e-foldings has to be larger than N_H, which translates into a constraint on the initial value of ϕ.

2.1.2 COBE normalization

This constraint corresponds to the amplitude of the density perturbations as deduced by the CMB temperature anisotropies observed by COBE. The constraint reeds,

$$\left.\frac{\Delta T}{T}\right|_{\mathrm{dec}} \simeq \frac{\delta\rho}{\rho}(N_{\mathrm{CMB}}) \simeq 2 \times 10^{-5} \tag{3}$$

where

$$\frac{\delta\rho}{\rho} \simeq \left.\frac{H^2}{\pi\dot{\phi}}\right|_{\mathrm{exit}} \simeq -\frac{1}{\sqrt{3}\pi}\frac{V^{3/2}}{m_P^3 V'} \tag{4}$$

and $N_{\mathrm{CMB}} \simeq \ln(t_{\mathrm{dec}} T_{\mathrm{dec}}) - N_{\mathrm{reh}}$, with $T_{\mathrm{dec}} \equiv T(t_{\mathrm{dec}})$ being the temperature of the last scattering surface and t_{dec} being the time of decoupling between matter and radiation. Typically, the COBE normalization constraint determines the inflationary energy scale.

2.1.3 Spectral index

It can be shown that the spectral index n_s of the density perturbations is given by,

$$n_s - 1 \simeq 6(\eta - 3\varepsilon^2) \tag{5}$$

Large scale structure and CMB observations provide the following constraint for n_s,

$$|n(N_{\mathrm{CMB}}) - 1| \leq 0.1 \tag{6}$$

which constraints the slope and curvature of the potential $|V'|$ and $|V''|$ respectively.

2.2 Quintessential requirements

2.2.1 Coincidence

The scalar field should account for the required Dark Energy at present. Thus, ρ_ϕ needs to be comparable with ρ_B today, but subdominant during the Hot Big Bang. The constraint reeds, $V(\phi_0) = \Omega_\phi \rho_0$, where $\phi_0 \equiv \phi(t_0)$, ρ_0 is the overall energy density of the Universe at present and $\Omega_\phi \approx 2/3$ is the currently observed abundance of Dark Energy [1].

2.2.2 Acceleration

The SN Ia observations suggest that the Universe at present is undergoing accelerated expansion [2]. In order to achieve this we need $\rho_\phi(t_0) > \rho_B(t_0)$ and also $-1 \leq w_\phi(t_0) < -\frac{1}{3}$, where $w_\phi \equiv p_\phi/\rho_\phi$. String theory considerations disfavor eternal acceleration because it results in the existence of a future event horizons [11]. If we take this constraint into account we can allow only for a brief acceleration period occurring today.

3   Evolution

The particular characteristics of quintessential inflationary potentials result in a particular scenario for the Universe evolution, which imposes a few additional constraints.

3.1   After the end of inflation

Reheating, in quintessential inflation, is due to the gravitational production of non-conformally invariant, effectively massless fields, facilitated by the change of the vacuum at the end of inflation [12]. The reheating temperature is, $T_{\mathrm{reh}} = \alpha(H_{\mathrm{end}}/2\pi)$, where $\alpha \sim 0.01$ is an (in)efficiency factor [13]. Therefore, $N_{\mathrm{reh}} = \ln(\alpha/2\pi) \simeq -6$ regardless of the inflationary energy scale, so that, $N_H \simeq 74$ and $N_{\mathrm{CMB}} \simeq 69$.

3.1.1   Kination

Because, at the end of inflation, $\rho_B(t_{\mathrm{end}}) \sim T_{\mathrm{reh}}^4 \ll V_{\mathrm{end}}$ the Universe remains ϕ–dominated. However, after the end of slow roll we have kination with $\rho_{\mathrm{kin}} \gg V$ [13] and (1) becomes,

$$\ddot{\phi} + (\dot{\phi}/t) \simeq 0 \tag{7}$$

Thus, the evolution of ϕ is independent of $V(\phi)$. Because $\rho_{\mathrm{kin}} \propto a^{-6}$ soon ρ_B comes to dominate and the Hot Big Bang begins. The temperature at the end of kination is,

$$T_* = \pi \sqrt{\frac{g_*}{30}} \frac{T_{\mathrm{reh}}^3}{V_{\mathrm{end}}^{1/2}} = \frac{\alpha^3}{72\pi^2} \sqrt{\frac{g_*}{30}} \frac{V_{\mathrm{end}}}{m_P^3} \tag{8}$$

where $V_{\mathrm{end}} \equiv V(t_{\mathrm{end}})$ and g_* is the number of relativistic degrees of freedom, which, for the standard model in the early Universe, is $g_* = 106.75$. Kination has to be over before Big Bang Nucleosynthesis (BBN). Therefore, $T_{\mathrm{BBN}} < T_*$, where $T_{\mathrm{BBN}} \simeq 0.5$ MeV is the temperature at the onset of BBN. The BBN constraint provides a lower bound on V_{end}.

3.1.2   Hot Big Bang

After the end of kination $\rho_{\mathrm{kin}} \propto t^{-3}$ so that $\dot{\phi} \to 0$ rapidly. Consequently, the scalar field freezes at some value ϕ_F, estimated as,

$$\phi_F \simeq \phi_{\mathrm{end}} + 2\sqrt{\frac{2}{3}} \left[1 + \frac{3}{2} \ln\left(\frac{12\pi}{\alpha^2} \sqrt{\frac{30}{g_*}} \right) + 3 \ln\left(\frac{m_P}{V_{\mathrm{end}}^{1/4}} \right) \right] m_P \gg m_P \tag{9}$$

where $m_P \equiv M_P/8\pi$ and $\phi_{\mathrm{end}} \equiv \phi(t_{\mathrm{end}})$.

3.2   Attractors and trackers

During the quintessential part of its evolution the scalar field is again dominated by its potential energy density so that (2) is still applicable. However, during the Hot Big Bang, $H \propto t^{-1}$ and the roll of the field is not as much restrained as in inflation.

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3.2.1 Attractor solution

The solution of (2) is of the form, $f(\phi(t)) - f(\phi_F) = F(t)$, where $F(t) \equiv \frac{1}{4}(1 + w_B)t^2$ and $f(\phi) \equiv -\int' d\phi/V'$ and the prime on the integral means that one should not consider constants of integration. The above suggests

$$\phi(t) \simeq \begin{cases} \phi_F & \text{when} \quad f(\phi_F) \gg F(t) \quad \text{Frozen Quintessence} \\ \\ \phi_{\text{atr}}(t) & \text{when} \quad f(\phi_F) \ll F(t) \quad \text{Attractor Quintessence} \end{cases} \tag{10}$$

where $\phi_{\text{atr}}(t)$ is the solution of

$$f(\phi) \simeq F(t) \Leftrightarrow \frac{1}{4}(1 + w_B)t^2 = -\int' \frac{d\phi}{V'(\phi)} \tag{11}$$

referred to as attractor solution [14]. Because $F(t)$ is a growing function of time, even though initially the field remains frozen at ϕ_F, later on it unfreezes and starts following the attractor. This occurs when $V_{\text{atr}} \equiv V(\phi_{\text{atr}}) \simeq V_F \equiv V(\phi_F)$ so that, at all times, $V(\phi) = \min\{V_F, V_{\text{atr}}\}$. Note that the attractor solution is independent of initial conditions.

3.2.2 The choice of the quintessential tail

Depending on the slope of the quintessential tail V_{atr} may fall more {less} rapidly than ρ_B. We will call such an attractor steep {mild}. If the attractor is steep, only frozen-quintessence may achieve coincidence. However, steep attractors begin soon after the end of inflation. Hence, potentials with steep quintessential tails are ruled out. Mild attractors are different. Such attractors assist ϕ to eventually dominate ρ_B and, hence, they are called "trackers" [14]. Mild quintessential tails and all cases of frozen-quintessence result in eternal acceleration, disfavored by string theory [11], and are, therefore, also ruled out.*

Therefore, the only acceptable case is to consider attractors according to which V_{atr} falls as rapidly as ρ_B (see Fig. 1), so that $V_{\text{atr}}/\rho_B = \text{constant}$. This corresponds to exponential quintessential tails [16], for which the attractor (11) in the matter era is,

$$\frac{\rho_B}{V_{\text{atr}}} = 4\varepsilon^2 = \text{constant} \tag{12}$$

Although it may explain the missing Dark Energy, the exponential attractor is seemingly unable to result in accelerated expansion, because it mimics ρ_B and, hence, $w_\phi(t_0) \simeq 0$. However, a brief period of acceleration is indeed possible to achieve if the field unfreezes at present [17]. This is because, when unfreezing, the field oscillates briefly around the attractor. Thus, at first crossing of V_F and V_{atr} the field remains "super-frozen" and dominates the Universe, causing accelerated expansion. Soon, however, it settles onto the attractor and acceleration is terminated (Fig. 2). Thus, coincidence requires $\phi_{\text{atr}}(t_0) \simeq \phi_F$. Moreover, "super-freezing" results in $w_\phi(t_0) \simeq -1$ which guarantees acceleration.

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*Quintessential tails, which change slope from mild to steep [15], may be constructed only at the expense of additional mass-scales and parameters, which contrasts our minimalistic approach.
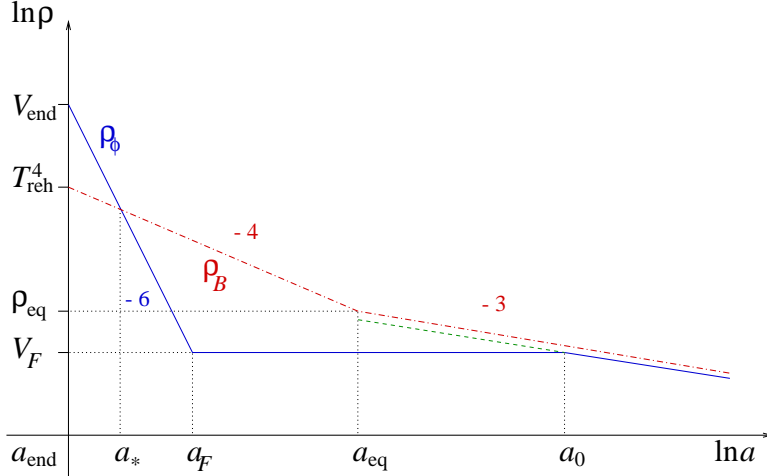
Figure 1: Exponential attractor (dashed line). After the end of kination, ρ_ϕ (solid line) remains frozen until today, when it begins to mimic the evolution of ρ_B (dash-dot line).

Therefore, we are led to try a quintessential tail of the form, $V(\phi \gg \phi_{\text{end}}) \simeq V_{\text{end}} \, e^{-\lambda\phi/m_P}$ where $\lambda > 0$ is a parameter for which $\varepsilon = -\lambda/\sqrt{6}$. In order to avoid frozen quintessence we require $V_{\text{atr}} \lesssim \rho_B$, which means that λ should be close but not smaller than $\sqrt{3/2}$. However, it turns out that the corresponding value for V_{end} cannot satisfy the BBN constraint. Fortunately, one can overcome this difficulty by modifying the the quintessential tail in a way that preserves the exponential attractor. This is achieved by introducing an Inverse Power-Law (IPL) factor so that, $V(\phi \gg \phi_{\text{end}}) \simeq V_{\text{end}} \, e^{-\lambda\phi/m_P} (m/\phi)^k$, where $k \geq 1$ is an integer and $m \leq m_P$. The attractor for this quasi-exponential tail is identical with the pure exponential case because $\phi_F \gg m_P$ suggests,

$$V'(\phi_F) = -\frac{V}{m_P}\Big[\lambda + k\Big(\frac{m_P}{\phi_F}\Big)\Big] \simeq -\frac{\lambda V}{m_P} \tag{13}$$

The quasi-exponential behaviour cannot carry over to the inflationary era because of the steepness it results to. Thus, both the exponential and the IPL features have to be modified in inflation. This modification is not trivial because of the huge difference $V_{\text{end}} \gg V_F \sim \rho_0$ as required by BBN and coincidence. A steep inflationary plateau results either in too brief inflation or in strongly super-Planckian inflationary energy scale. A flat inflationary plateau, however, because it has to "prepare" for the deep dive towards V_F, typically features a large value of $|V''|$, which results in too large n_s [c.f. (5)].

4   A concrete example

Designing a quintessential inflationary potential is not easy, let alone using few mass scales and parameters. Nevertheless, it is indeed possible. For example, consider the potential,
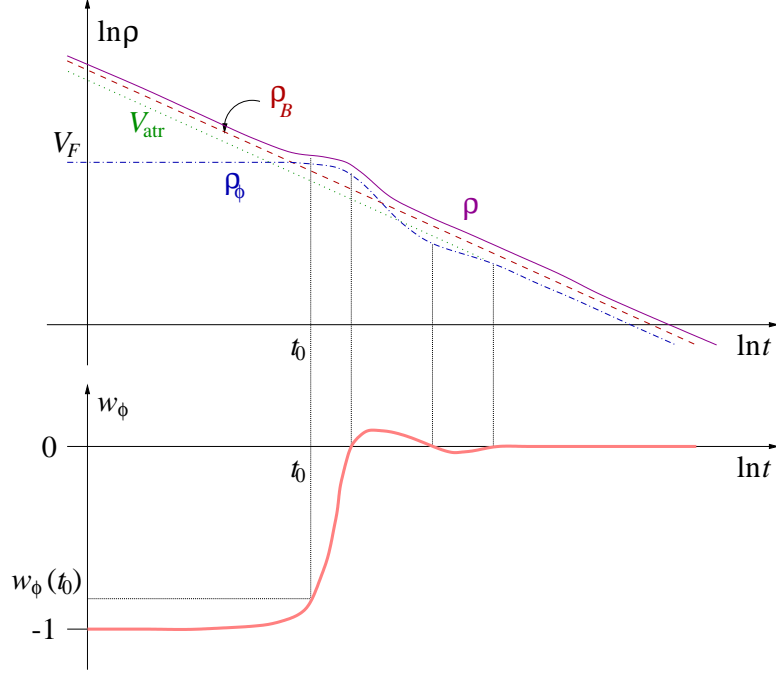
Figure 2: At unfreezing, ρ_ϕ (dash-dot line) briefly oscillates around the attractor (dotted line), which mimics ρ_B (dashed line). Consequently, there is a "bump" on the overall density ρ (solid line). During the "bump", $w_\phi \simeq -1$, which results into brief acceleration.

$$V(\phi) = M^4[1 - \tanh(\phi/m_P)]\Big[1 - \sin\Big(\frac{\pi\phi/2}{\sqrt{\phi^2 + m^2}}\Big)\Big]^k \tag{14}$$

where $-\infty < \phi < +\infty$, M, $m < m_P$ and $k > 0$ is an integer (Fig. 3). The asymptotic forms of the above far from the origin are,

$$V(\phi \ll 0) \simeq 2^{k+1} M^4 \left[1 - e^{2\phi/m_P} - \frac{k\pi^2}{64}(\frac{m}{\phi})^4\right] \simeq 2^{k+1} M^4 \tag{15}$$

and

$$V(\phi \gg 0) \simeq 2^{1-k}(\frac{\pi}{4})^{2k} e^{-2\phi/m_P} M^4 \Big(\frac{m}{\phi}\Big)^{4k} \propto e^{-2\phi/m_P}(m/\phi)^{4k} \tag{16}$$

Thus, for negative values of ϕ the potential approaches a constant, non-zero, false vacuum energy density responsible for inflation, whereas for positive values of ϕ the potential attains the desired quasi-exponential form with $\lambda = 2$. Enforcing the constraint (3) gives,

$$\frac{2^{\frac{k+1}{2}}}{\sqrt{3}\,\pi}\Big(\frac{M}{m_P}\Big)^2(N_{\mathrm{CMB}} + 1/3) = 10^{-5} \Rightarrow M \sim 10^{15}\mathrm{GeV} \tag{17}$$
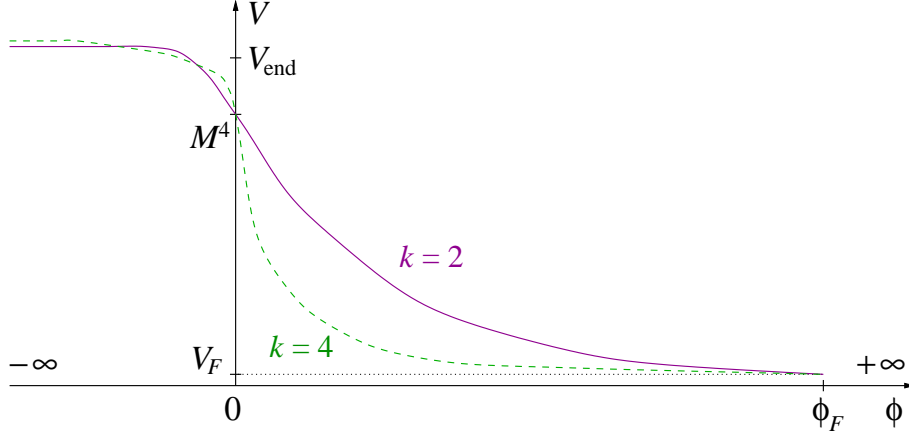
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Figure 3: Artist's view of model (14) for $k = 2, 4$ (solid, dashed line respectively).

Thus, we see that M is of the scale of grand unification. Now, employing (5) we obtain,

$$n_s(N) - 1 \simeq -\frac{6}{3N + 1}\left[1 + \frac{9}{8(3N + 1)}\right] \Rightarrow n_s(N_{\mathrm{CMB}}) \simeq 0.97$$

which is in excellent agreement with the observations [10]. Also, $V_{\mathrm{end}} \simeq 2^{k-1} M^4$ gives,

$$T_{\mathrm{reh}} \simeq \frac{\alpha}{4} \frac{10^{-5} m_P}{\sqrt{N_{\mathrm{CMB}} + 1/3}} \sim 10^9 \,\mathrm{GeV} \tag{18}$$

which saturates but does not violate the gravitino constraint. Using the above, (8) gives,

$$T_* \simeq \frac{\alpha^3 \pi^2}{96} \sqrt{\frac{g_*}{30}} \frac{10^{-10} m_P}{(N_{\mathrm{CMB}} + 1/3)^2} \sim 10 \,\mathrm{MeV} > T_{\mathrm{BBN}} \tag{19}$$

Thus, the BBN constraint is satisfied. Finally, solving the horizon and flatness problems requires the initial condition for the inflaton, $\phi_i \leq \phi(N_H) \simeq -3\, m_P$, which does not require fine-tuning since we expect $\phi_i \sim |M_P|$. Note that, $n_s, T_{\mathrm{reh}}, T_*$ are all k-independent.

Moving on to quintessential requirements, from (11), the attractor for the matter era is, $V_{\mathrm{atr}}/\rho_B = 3/8$. Super–freezing is expected to boost this fraction up to $\Omega_\phi \approx 2/3$ and also to give $w_\phi \approx -1$. The freezing value of ϕ is found using (9),

$$\phi_F = \sqrt{6}\left\{\frac{2}{3} - \frac{1}{\sqrt{6}}\ln(2/\sqrt{3}) + \ln\left[\frac{24}{\alpha^2}\sqrt{\frac{10}{g_*}}\left(N_{\mathrm{CMB}} + \frac{1}{3}\right) \times 10^5\right]\right\} m_P \simeq 67 m_P \tag{20}$$

Using the coincidence constraint $\phi_0 \simeq \phi_F$ and after some algebra one finds,

$$m \sim 4 \times 10^2 (4 \times 10^{70})^{1/4k} 10^{-30/k}\, m_P \tag{21}$$

which gives,

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k	1	2	3	4
m	10^8GeV	10^{15}GeV	10^{17}GeV	10^{18}GeV

Thus, we can identify m with M, m_P for $k = 2, 4$ respectively, and obtain the models,

- **Model 1:**

$$V(\phi) = M^4[1 - \tanh(\phi/m_P)]\Big[1 - \sin\Big(\frac{\pi\phi/2}{\sqrt{\phi^2 + M^2}}\Big)\Big]^2 \qquad (22)$$

- **Model 2:**

$$V(\phi) = M^4(\phi)\,[1 - \tanh(\phi/m_P)] \quad \text{and} \quad M(\phi) \equiv M\Big[1 - \sin\Big(\frac{\pi\phi/2}{\sqrt{\phi^2 + m_P^2}}\Big)\Big] \qquad (23)$$

5 Conclusions

We have shown that it is indeed possible to unify inflation and quintessence using a single scalar field without incorporating too many mass scales and parameters. The best approach suggests a quasi-exponential tail, which manages to meet BBN requirements, while retaining the pure-exponential attractor solution. The quasi-exponential tail achieves a brief acceleration period if the scalar field is about to unfreeze today and lies, at present, in a super-frozen state. Both the exponential and inverse-power law features of the tail have to be suppressed at the inflationary plateau.

Two successful models are presented, which satisfy all requirements of inflation and quintessence with the use of only two natural mass-scales: the Planck and the grand unification scale, and no other parameters. Moreover, no fine tuning of initial conditions is required. In these models the inflationary scale is that of grand unification, which is low enough to ensure safety from radiative corrections. Also, $T_{\text{reh}} \sim 10^9$GeV, which saturates the gravitino constraint and $n_s \simeq 0.97$, in excellent agreement with observations. Finally, we expect $w_\phi \approx -1$ because of super-freezing. Due to the plethora of constraints and requirements we believe that any successful model should not differ much from the toy-models presented.

In summary, successful quintessential inflationary models without additional fine-tuning for quintessence are possible to construct. Such models outshine the cosmological constant alternative.

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