The Queen's Gambit: Explaining the Superstar Effect Using Evidence from Chess [*]

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Abstract. Superstars exist in classrooms and workplaces. Superstars can intimidate others and create a negative performance shock, or they can encourage others by inspiring everybody to "step up their game." In this study, we examine two effects: the impact of head-to-head competition with a superstar (direct) effect and the influence of a superstar presence on players' performance (indirect) effect. We find that the direct superstar effect in theory and in the data is always negative. The indirect superstar effect is neutral in theory, but depends on the intensity of the superstar in the data: if the skill gap between the superstar and the rest is small (large), there is a positive (negative) indirect effect.

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1. Introduction

"When you play against Bobby [Fischer], it is not a question of whether you win or lose. It is a question of whether you survive."

–Boris Spassky, World Chess Champion, 1969 - 1972.

Rank-order tournaments are a prevalent aspect of numerous organizations, where performance-based bonuses and promotions are used to motivate workers to achieve high productivity. Additionally, companies continually make decisions regarding new hires, and each new addition to the workforce has the potential to alter the internal dynamics of competition within the firm.

Let's consider a scenario where an exceptionally talented individual, often referred to as a "superstar," is recruited by a Forbes top 500 firm through a lucrative hiring contract worth millions of dollars. The firm has high expectations for the superstar's output, and these expectations are taken into account during salary negotiations. However, such a hiring decision can have ripple effects in a firm that employs rank-order tournaments.

The existing employees now find themselves in competition with a highly talented superstar for bonuses, promotions, and other rewards. The overall impact of this new hire becomes uncertain. On one hand, the existing workers may rise to the challenge and exceed their expected levels of performance. On the other hand, the intensified competition resulting from the presence of the superstar could potentially lead to a decline in performance among the existing employees.

In this paper, we analyze the superstar effect using chess data. First, we present two contest models that capture the direct and indirect superstar effects in chess where exactly two players compete in every game. Unlike other sports where only one of these two effects is present, both of these effects coexist in chess. In the first model of direct competition, the superstar player has higher ability than the second player. This head-to-head competition puts pressure on the second player, who suffers from choking and intimidation. These choking and intimidation impacts depend on the ability of the superstar: the higher the superstar's ability, the higher the pressure on her competitors. Like Sanders and Walia (2012) and Benscheidt and Carpenter (2020), we introduce the choking impact into the cost function. The intimidation impact is present in the contest success function. Both choking and intimidation impacts make it more difficult (more costly) to exert effort when a player competes against the superstar and as a result the direct theoretical superstar effect is always negative.

In the second model of indirect effect, two identical-ability players compete against each other when the superstar is present in the tournament. Players can either falter in the superstar's presence or draw inspiration from it, which in turn leads to lower or higher effort in the two-player competition. Note that even though the intimidation impact of head-to-head competition with the superstar is absent in this case, the choking impact of the superstar may still be present. We find that the indirect theoretical superstar effect is neutral. This observation is intuitive: the superstar affects both identical-ability players equally, and as a result, their chances to beat each other will not change with the superstar ability.

We then empirically test the superstar effect using five different male and female chess superstars who come from different backgrounds and time periods: Magnus Carlsen, Garry Kasparov, Anatoly Karpov, Bobby Fischer, and Hou Yifan. We analyze more than 2 million move-level observations from invitation-based elite chess tournaments which took place between 1962 and 2019. Our main performance indicator is the "Average Centipawn Loss" (ACPL), which is a highly standardized performance metric unique to chess that shows the amount of error a player commits in a game by evaluating the quality of each move made. Because of the two-player nature of chess, we are able to identify direct (individual competition with a superstar) and indirect (performance in a tournament with a superstar) superstar effects from the data separately. In particular, we test

1. **Direct effect:** Do players commit more mistakes (than they are expected to) playing head-to-head against a superstar?

2. **Indirect effect:** Do players commit more mistakes (than they are expected to) in games played against each other *if* a superstar is present in the tournament as a competitor?

In chess, a player's goal is to find the optimal move(s). Failing to do so would result in mistake(s), which the ACPL metric captures. Holding all else constant, a player should be able to show similar performance in finding the best moves in two "similarly complex" chess positions. The difficulty of finding the optimal moves is related to two main factors: (1) External factors impacting a player. For instance, being under pressure can lead the player to choke and exert less effort, resulting in more mistakes, or (2) The complexity of the position that the player faces. If both players are willing to take risks, they can opt for keeping more complex positions on the board, which raises the likelihood that a player commits a mistake. To isolate the external factors, we construct a novel complexity metric using an Artificial Neural Network (ANN) algorithm that is trained on an independent sample with more than 2 million moves. This allows us to control for board complexity and compare games with similar complexity levels. Under this restriction, if a player commits more mistakes against the superstar (or in the presence of a superstar) in similarly complex games, it must be that either (i) the player is intimidated, or (ii) the player chokes under pressure, or both (i) and (ii).

Our paper is the first in the literature to use observations from chess tournaments to study the superstar effect. Chess data has a number of distinctive advantages. For example, non-player related factors are minimal to non-existent in chess since every chess board is the same for all players. Second, highly precise performance metrics can be utilized with the use of computer algorithms that can evaluate the quality of each move and estimate the complexity of each unique chess position. Third, the chess world has seen multiple superstars who existed in different time periods and come from different backgrounds. Having multiple superstars enables us to uncover patterns from varying levels of superstar strength.

We find, as the theory suggests, a strong negative **direct superstar effect**. When players compete head-to-head against the superstar, they commit more mistakes in all specifications and perform below their expected level even in similarly complex games. This result can be attributed to both the choking and the intimidation impacts.

We discover, unlike the neutral theoretical prediction, that the **indirect superstar effect** depends on the skill gap between the superstar and the competition. We find that if this gap is small, the indirect superstar effect is positive: players believe they have a chance to win the tournament and exert more effort which results in an improvement in overall performance. The data shows that the players ranked just below the superstar based on their rating points experience the largest improvement in their performance. If the skill gap is large, the indirect superstar effect is negative: players choke under the pressure of competing in the same tournament with the superstar. The top players show worse performance with more mistakes and more losses in the presence of a highly dominant superstar.

Our findings offer valuable insights for organizations: the decision to hire a superstar can result in both positive and negative spillover effects. If the superstar does not completely overshadow the rest of the group, there is a potential for overall improvement in organizational performance.

However, if the skill gap between the superstar and the rest of the group is excessively wide, negative spillover effects may occur. Managers should carefully weigh the potential benefits of hiring an exceptionally talented superstar against the potential costs of spillover effects on the entire organization.

The study of superstars originated with Rosen (1981), who made the initial contribution to understanding how skills in specific markets can become highly valuable. Subsequent works by Lazear and Rosen (1981), Green and Stokey (1983), Nalebuff and Stiglitz (1983), and Moldovanu and Sela (2001) delve into the design of optimal contracts in rank-order tournaments. Prendergast (1999) provides a comprehensive review of workplace incentives. More recently, Xiao (2020) demonstrates the potential for both positive and negative incentive effects when a superstar participates in a tournament, highlighting that these effects are influenced by the prize structure and the skill levels of the participants.

The empirical literature on superstars began with Brown (2011) and encompasses various settings, including professional track and field competitions and swimming. Yamane and Hayashi (2015) compares the performance of swimmers competing in adjacent lanes and discovers a positive superstar effect on a swimmer's performance. This effect is amplified by the visibility of the competitor's performance. Notably, in backstroke competitions with limited observability of the adjacent lane, no effect is observed, while in freestyle competitions with higher observability, the effect is present. Jane (2015) analyzes swimming competition data in Taiwan and finds that the presence of faster swimmers in a competition enhances the overall performance of all participating competitors. Jiang (2020) reveals that swimmers benefit from the presence of a teammate in a swimming contest.

Contrary to Brown's findings, Connolly and Rendleman (2014) and Babington et al. (2020) argue that the adverse superstar effect may not be as pronounced and claim that the result is not robust to alternative specifications. They suggest that the effect could even work in the opposite direction, with top competitors potentially bringing out the best in other players' performance. Additionally, Babington et al. (2020) provides further evidence from observations in men's and women's FIS World Cup Alpine Skiing competitions, showing minimal to no peer effects when skiing superstars Hermann Maier and Lindsey Vonn participate in a tournament.

Our empirical observations are consistent with a negative superstar effect reported in Brown (2011), Connolly and Rendleman (2009) and Tanaka and Ishino (2012), as well as with a positive superstar effect found in Hill (2014).

Topcoder and Kaggle are the two largest crowdsourcing platforms that facilitate online contests, where contest organizers offer prizes to contestants who excel in finding solutions to challenging technical problems presented at the start of the contest. Archak (2010) discovers that players in Topcoder competitions tend to avoid competing against superstars. Examining the impact of increased competition on participant responses, Boudreau et al. (2016) finds that lower-ability competitors tend to respond negatively to competition, while higher-ability players respond positively.

Zhang, Shunyuan and Singh, Param Vir and Ghose, Anindya (2019) suggests that competitions involving superstars may yield future benefits, as competitors can learn from the superstar's expertise. This finding resonates with the notion of positive peer effects observed in workplace and classroom settings, as explored by Mas and Moretti (2009), Duflo et al. (2011), Cornelissen et al. (2017), Moreira (2019).

Similar to the majority of superstar literature, our paper does not model and analyze the sequential nature of tournaments. Exploring this interesting topic presents an intriguing avenue for the future research where a superstar competes in sequential or elimination tournaments. This analysis will draw from both the superstar literature and existing research on sequential tournaments. See, for example, Jost and Kräkel (2005), Ryvkin (2009), and Brown and Minor (2014).

There is growing literature studying a broad range of questions using data from chess competitions. For example, Levitt et al. (2011) test whether chess masters are better at making backward induction decisions. Moul and Nye (2009) show evidence of Soviet collusion in top level chess tournaments. Gerdes and Gränsmark (2010) test for gender differences in risk-taking and report that women choose more risk-averse strategies playing against men. Dreber et al. (2013) test the relationship between attractiveness and risk-taking using chess games. Smerdon et al. (2020) and Backus et al. (2023) find that female players make more mistakes playing against male opponents with similar strength. Stafford (2018) has an opposite finding that women perform better against men with similar Elo ratings. González-Díaz and Palacios-Huerta (2016) report strong influence of psychology on performance for chess players. Künn et al. (2021) compare cognitive performance in remote against in person environments. Klingen and van Ommeren (2022) and Künn et al. (2023) report indoor air quality impacts on performance and risk-taking behavior of chess players. Smerdon (2022) finds wearing face masks have detrimental effects on cognitive performance. Bertoni et al. (2015) and Strittmatter et al. (2020) study the age dynamics and performance with chess players.

The rest of the paper is organized as follows: Section 2 presents the theory. Section 3 gives background information on chess and describes how chess data is collected and analyzed. Section 4 provides the empirical design. Section 5 presents the results, and section 6 concludes.

2. Theory

In this section we introduce two contest models that capture the direct and indirect superstar effects in chess where exactly two players compete in every game. In the first model, which focuses on the direct competition, the superstar player possesses higher ability than the second player. This head-to-head competition puts pressure on the second player, leading to choking and intimidation impacts. The extent of these impacts depends on the superstar's ability: the greater the superstar's ability, the higher the pressure exerted on their competitors. Like Sanders and Walia (2012) and Benscheidt and Carpenter (2020), we introduce the choking impact into the cost function. The intimidation impact is present in the contest success function.

In the second model, which focuses on the indirect effect, two players with identical abilities compete against each other in the presence of a superstar within the tournament. In this scenario, players have the potential to either falter or draw inspiration from the superstar's presence, resulting in lower or higher levels of effort during the two-player competition. It is important to note that while the intimidation impact of head-to-head competition with the superstar is absent in this case, the choking impact of the superstar may still be present, affecting the performance of the players. We test our theoretical predictions in Section 5.

2.1 Direct Superstar Effect: competition against the superstar

Our first model is a two-player Tullock's contest in which player 1 competes against a superstar, player 2. Player 1 maximizes his expected payoff, which is the difference between his expected prize and cost:

$$\max_{e_1} \quad \frac{e_1}{(e_1 + \theta e_2)} V_1 - \theta e_1,$$

where e_i is the effort of player $i = 1, 2$; V_1 is a monetary or rating prize which player 1 can win; and

$\theta \geq 1$ is the ability of the superstar, player 2. We normalize the ability of player 1 at one. There are intimidation and choking impacts in the model. The intimidation of player 1 is presented in his contest success function and the choking impact is in his cost function. Note that the ability of the superstar, θ, affects the intimidation and choking impacts: higher ability leads to higher impacts.

Player 2, a superstar, maximizes her expected payoff:

$$\max_{e_2} \quad \frac{\theta e_2}{(e_1 + \theta e_2)} V_2 - e_2,$$

where V_2 is the prize that player 2 can win. Note that θ is not only the ability of player 2, but also the ratio of the players' abilities. The first order conditions for players 1 and 2 are

$$\frac{\theta e_2}{(e_1 + \theta e_2)^2} V_1 - \theta = 0,$$

and

$$\frac{\theta e_1}{(e_1 + \theta e_2)^2} V_2 - 1 = 0.$$

Therefore, in an equilibrium

$$\frac{e_2}{e_1} = \theta \frac{V_2}{V_1}.$$

We can state our theoretical results now.

Proposition 1 *There exists a unique equilibrium in the two-player superstar contest model, where players exert the following effort:*

$$(e_1^D, e_2^D) = \left(\frac{\theta V_1 V_2}{(V_1 + \theta^2 V_2)^2} V_1, \frac{\theta^2 V_1 V_2}{(V_1 + \theta^2 V_2)^2} V_2 \right).$$

In the equilibrium, player $i = 1, 2$ *wins the contest with the probability* p_i^D, *where*

$$(p_1^D, p_2^D) = \left(\frac{V_1}{V_1 + \theta^2 V_2}, \frac{\theta^2 V_2}{V_1 + \theta^2 V_2} \right).$$

Note that probabilities p_1^D and p_2^D illustrate the direct superstar effect and $p_1^D(\theta)$ is decreasing with the superstar ability θ.

Since everyone expects the superstar to win the competition, her victory is neither surprising nor too rewarding, whereas the underdog's victory is special, so we can assume the prize for the underdog to be larger than the prize for the superstar in the two-player superstar contest. This prize structure is also evident in rating point calculations in chess: a lower rated player gains more rating points if he wins against a higher rated player. In this case, it follows from Proposition 1 that the underdog exerts higher effort than the superstar in the equilibrium if $V_1 > \theta V_2$, but the underdog's winning chances decrease with the superstar's ability. We have the following comparative statics results.

Proposition 2 *Suppose that $V_1 > V_2$. Then, individual superstar equilibrium effort increases with the superstar ability if $\theta^* < \sqrt{\frac{V_1}{V_2}}$ and decreases if $\theta^* > \sqrt{\frac{V_1}{V_2}}$. Individual superstar equilibrium effort is maximized if the superstar ability is $\theta^* = \sqrt{\frac{V_1}{V_2}}$.*

Proposition 3 *Suppose that $V_1 > 3V_2$. Then, individual underdog equilibrium effort increases with the superstar ability if $\theta^* < \sqrt{\frac{V_1}{3V_2}}$ and decreases if $\theta^* > \sqrt{\frac{V_1}{3V_2}}$. Individual underdog equilibrium effort is maximized if the superstar ability is $\theta^* = \sqrt{\frac{V_1}{3V_2}}$.*

Propositions 2 and 3 give the unique values for the superstar ability that maximize individual and total equilibrium efforts. Note that a large enough prize must be offered to the underdog in order to obtain the global maxima for both players. Propositions 2 and 3 suggest superstar ability and the prize ratio which maximize total effort in the case of direct competition with the superstar.

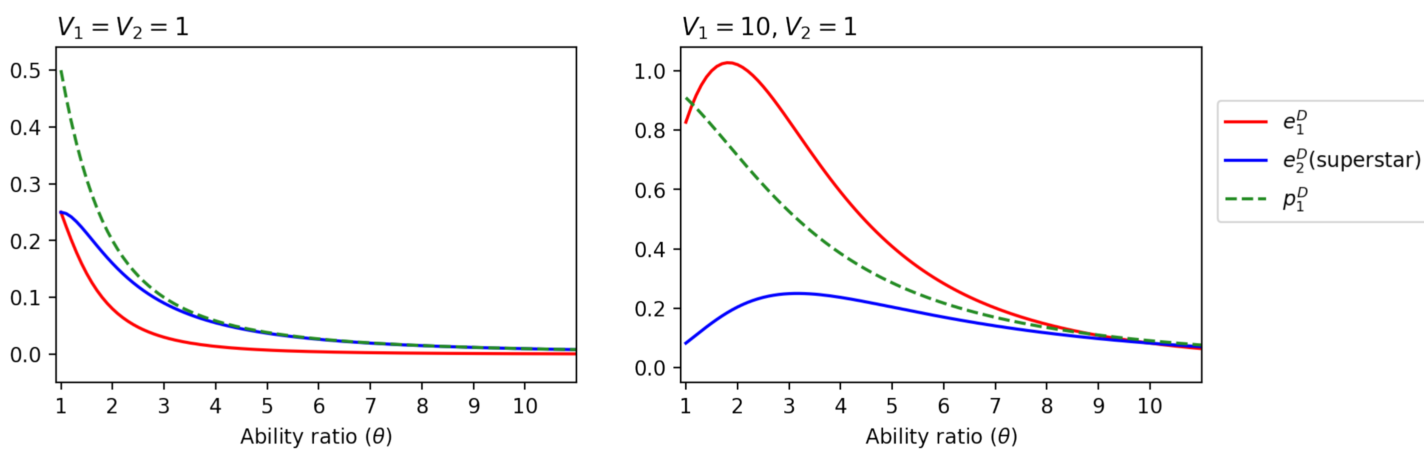


Figure 1: Contest against the superstar. Equilibrium efforts, e_1^D, e_2^D, and winning probability, p_1^D, when $V_1 = V_2 = 1$ (left figure) and $V_1 = 10, V_2 = 1$ (right figure). Player 2 is the superstar. p_2^D is omitted in the graphs since $p_2^D = 1 - p_1^D$. V_1 and V_2 are prizes for the underdog and the superstar, respectively.

Figure 1 illustrates Propositions 2 and 3 and shows how equilibrium efforts (red curve for the underdog, blue curve for the superstar) and winning probabilities (green curve for the underdog) change for different levels of superstar abilities, first if $V_1 = V_2 = 1$ (left graph), then $V_1 = 10$ and $V_2 = 1$ (right graph). When players compete for the same prize, introducing any level of heterogeneity between the players lowers individual and total efforts.

When the contest designer provides a large enough prize to the underdog, the effort levels for both players increase as long as the ability ratio is small. As the ability ratio increases, both players start to decrease their efforts. A very extreme level of heterogeneity in ability results in both players exerting very low efforts, but the probability of the underdog winning diminishes with heterogeneity. From our illustrative model, we observe that higher superstar ability, or higher ability heterogeneity, leads to more pressure on the underdog, who is intimidated and chokes under this pressure, thereby exerting less effort. This, in turn, results in lower underdog performance.

It is important to emphasize that the condition of Proposition 3, $\theta < \sqrt{\frac{V_1}{3V_2}}$, does not hold if the superstar ability is high or if the prize difference is not large. In this case, the individual effort is decreasing with the superstar ability θ. At the same time, the direct superstar effect on the player is always negative, or the probability $p_1^D(\theta)$ is decreasing with the superstar ability θ.

2.2 Indirect Competition: competition against another player when the superstar is in the tournament

In this section, we consider a two-player contest in which player 1 competes against player 2 in a tournament where a superstar is present. Then, player i maximizes his expected payoff:

$$\max_{e_i} \quad \frac{e_i}{(e_1 + e_2)} V_i - A(\theta) e_i,$$

where e_i is the effort of player $i = 1, 2$; $A(\theta) > 0$ for any $\theta \geq 1$; V_i is a monetary or rating prize which player i can win. We assume that player i is "influenced" by the presence of the superstar in the tournament. This influence is modelled by the positive function $A(\theta)$. If player i is choking under the presence of the superstar, then $A'(\theta) > 0$. However, if player i is motivated or encouraged by the presence of the superstar, then $A'(\theta) < 0$.

The first order condition for player $i = 1, 2$ is

$$\frac{e_j}{(e_1 + e_2)^2} V_i - A(\theta) = 0,$$

where $j \neq i$.

Therefore, in an equilibrium

$$\frac{e_2}{e_1} = \frac{V_2}{V_1}.$$

We can state our main results now.

Proposition 4 *There exists a unique equilibrium in the two-player contest model with a superstar, where players exert the following effort:*

$$(e_1^I, e_2^I) = \left(\frac{V_1 V_2}{(V_1 + V_2)^2} \frac{V_1}{A(\theta)}, \frac{V_1 V_2}{(V_1 + V_2)^2} \frac{V_2}{A(\theta)} \right).$$

In the equilibrium, player $i = 1, 2$ wins the contest with the probability p_i^I, where

$$(p_1^I, p_2^I) = \left(\frac{V_1}{V_1 + V_2}, \frac{V_2}{V_1 + V_2} \right).$$

From Proposition 4, we have the following comparative statics results.

Proposition 5 *The contest winning probabilities p_1^I and p_2^I are independent from the superstar ability, θ.*

If players are choking under the presence of the superstar or $A'(\theta) > 0$, then players' efforts in the equilibrium are decreasing with the superstar ability, θ.

However, if players are encouraged by the presence of the superstar or $A'(\theta) < 0$, then players' efforts in the equilibrium are increasing with the superstar ability, θ.

Note that probabilities p_1^I and p_2^I illustrate the indirect superstar effect, which is neutral here. This observation is intuitive: the superstar will affect both identical-ability players equally, and as a result, their chances to beat each other will not change with the superstar ability θ.

Figure 2 shows the equilibrium effort for two identical-ability players competing against each other when a superstar is present in the same tournament and $A(\theta) = \theta$. The graph on the left-hand side has identical prizes; the graph on the right has one of the players competing for a larger prize. There are two main predictions: First, competing in the presence of a superstar results in more pressure on both participants, who choke under this pressure since $A'(\theta) > 0$, thereby exerting less effort. Both graphs show that efforts decline monotonically. Second, if one of the players is competing for a larger prize (the graph on the right), then this player exerts more effort than the other player. Lastly, the green line shows a constant win probability with θ. This suggests that the players' performances are not impacted by the superstar's presence.

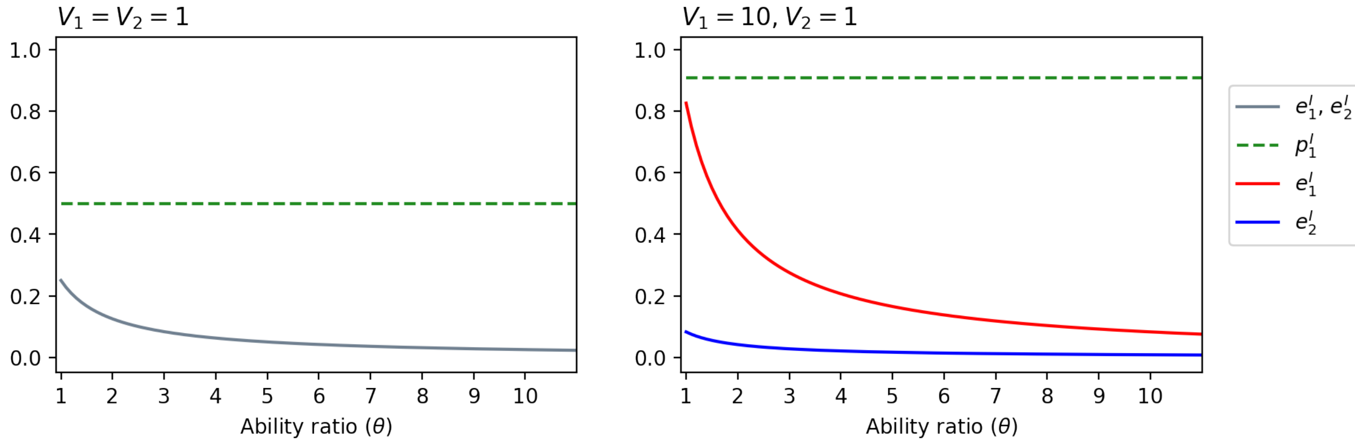


Figure 2: Identical-ability players compete against each other when the superstar is present in the same tournament and $A(\theta) = \theta$. Equilibrium efforts, e_1^I, e_2^I, when $V_1 = V_2 = 1$ (left figure) and $V_1 = 10, V_2 = 1$ (right figure). Players have identical abilities. V_1 and V_2 are prizes for each player. Analytically, $p_1^I = p_2^I = 0.5$ for $V_1 = V_2 = 1$ and $p_1^I = 0.91,\ p_2^I = 0.09$ for $V_1 = 10,\ V_2 = 1$.

3. Data

3.1 Chess: Background

"It is an entire world of just 64 squares."

−Beth Harmon, The Queen's Gambit,
Netflix Mini-Series (2020)

Chess is a two-player game with origins dating back to 6th century AD. Chess is played over a 8x8 board with 16 pieces for each side (8 pawns, 2 knights, 2 bishops, 2 rooks, 1 queen, and 1 king). Figure 3 shows a chess board. Players make moves in turns, and the player with the white pieces moves first. The ultimate goal of the game is to capture the enemy king. A game can end in three ways: white player wins, black player wins, or the game ends in a draw.

The possible combinations of moves in a chess game is estimated to be more than the number of atoms in the universe. However, some moves are better than others. With years of vigorous training, professional chess players learn how to find the best moves by employing backward-induction and calculating consequences of moves to a certain complexity level. Failing to find the

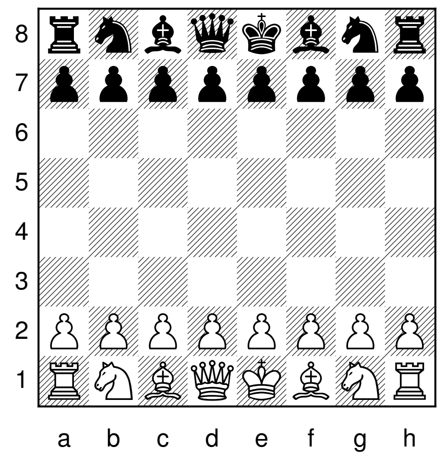


Figure 3: A chess board

best move(s) in a position would result in a "blunder" or a "mistake" which typically leads to the player losing their game at the top level if a player commits multiple blunders or mistakes. The player who performs better overall is the player who manages to find the correct moves more often.

The standard measure of player strength in chess is the Elo rating system first adopted by FIDE (The International Chess Federation) in 1970. This system was created by the Hungarian physicist Arpad Elo (Elo 1978). Elo considers the performance of a player in a given game as a random variable normally distributed around her unobservable true ability. Each player gets a starting Elo rating which is updated according to the outcome of each game via

$$ELO_{R,t+1} = ELO_{R,t} + K\left[S_i - E_t\left(S_i \mid R_i, R_j\right)\right], \tag{1}$$

where S_i is the outcome of a game such that $S_i = 1$ if player i wins the game, $S_i = 0$ if player i loses the game, and $S_i = 1/2$ if the game ended in a draw. $E_t\left(S_i \mid R_i, R_j\right)$ is the expected probability of player i winning the game given the Elo ratings of the two players R_i and R_j which equals $E\left(S_i \mid R_i, R_j\right) = \Phi\left(\frac{R_i - R_j}{400}\right)$ where $\Phi(.)$ is the c.d.f. of the normal distribution. K is a parameter for rate of adjustment.

This rating system allows comparisons of players' strengths. For instance, every month, FIDE publishes Elo ratings of all chess players. The Top 10 players are considered the most elite players in the world who earn significant amounts of prizes and sponsorships. Moreover, chess titles have specific Elo rating requirements. For instance, the highest title in chess, Grandmaster, requires the player to have an Elo rating 2500 or higher. Note that our sample consists of the very elite chess players, often called "Super GMs", with Elo ratings higher than 2700 in most cases.

Over the past decades, computer scientists have developed algorithms, or "chess engines" that exploit the game-tree structure of chess. These engines analyze each possible tree branch to come up with the best moves. The early chess engines were inferior to humans. After a few decades, however, one chess engine developed by IBM in the 1990s, Deep Blue, famously defeated the world chess champion at the time, Garry Kasparov, in 1997. This was the first time a world chess champion lost to a chess engine under tournament conditions. Since then, chess engines have passed well beyond the human skills. Currently, Stockfish 15 is the strongest chess engine with an Elo rating of 3531. In comparison, the current world chess champion, Magnus Carlsen, has an Elo rating of 2862. We use Stockfish in our analyses.

In addition to finding the best moves in a given position, a chess engine can be used to analyze the games played between human players. The quality of a move can be measured numerically by evaluating the move chosen by a player and comparing it to the list of moves suggested by the chess engine. If the move played by a player is considered a bad move by the engine, then that move is assigned a negative value with its magnitude depending on the engine's evaluation. We explain in detail in Section 3.4.

3.2 Chess Superstars

The first official world chess champion is Wilhelm Steinitz who won the title in 1886. Since Steinitz, there have been sixteen world chess champions in total. Among these sixteen players, four have shown an extraordinary dominance over their peers: Magnus Carlsen, Garry Kasparov, Anatoly Karpov, and Bobby Fischer. In his classic series, "My Great Predecessors", Kasparov (2003) gives in-depth explanations about his predecessors, outlining qualities of each world champion before him. In this paper, we consider the "greatest of the greatest" world champions as "superstars" in their eras. We present evidence why these players were so dominant and considered "superstars" in their eras. Specifically, we define a superstar as a player who satisfies the following conditions: (i) be the world chess champion; (ii) win at least 50% of all tournaments participated in (Note that, for comparison, Tiger Woods won 24.2 percent of his PGA Tour events.); (iii) maintain an Elo rating at least 50 points above the average Elo rating of the world's top 10 players (this condition must hold for the post-1970 era when Elo rating was introduced); (iv) have a substantially high Elo rating such that winning an elite tournament is not sufficient to gain Elo rating points. We define an elite tournament, a tournament which has (1) at least two players from the world's Top 10 and (2) the average Elo rating in the tournaments is within 50 points of the average Elo rating in tournaments with a superstar.

Magnus Carlsen is the current world chess champion, who first became champion in 2013 at age 22. He reached the highest Elo rating ever achieved in history. Garry Kasparov was the world champion from 1985-2000 and was the number one ranked chess player for 255 months, setting a record for maintaining the number one position for the longest duration of time. (For comparison, Tiger Woods was the number one ranked player in the world for a total of 683 weeks, the longest ever in golf history.) Anatoly Karpov was the world champion before Kasparov in the years 1975-1985. He won over 160 tournaments, which is a record for the highest number of tournaments won by a chess player.

Bobby Fischer was the world champion before Karpov between 1972 - 1975, winning all U.S. championships he played in from 1957 (at age 14) to 1966. Fischer won the 1963 U.S. chess championship with a perfect 11 out of 11 score, a feat no other player has ever achieved.

In addition to the four male superstars, our sample includes a female chess superstar: Hou Yifan, a four time women's world chess champion between the years 2010-2017. She played three women's world chess championship matches in this period and did not lose a single game against her opponents, dominating the tournaments from 2014 until she decided to stop playing in 2017.

Figures A.2–A.6 show how the four world chess champions: Carlsen, Kasparov, Karpov and Hou Yifan performed compared to their peers across years. The Elo rating difference between each superstar and the average of world's top 10 players in each corresponding era is about 100 points. This rating gap is very significant, especially at top-level competitive chess. For instance, the expected win probabilities between two players with a gap of 100 Elo rating points are approximately 64%-36%.

Figures A.12–A.16 show individual tournament performances across years for each superstar with the vertical axis showing whether the superstar gained or lost rating points at the end of a tournament. For instance in 2001, Kasparov played in four tournaments and won all of them. In one of these tournaments, he even lost rating points despite winning. For the world's strongest player, winning a tournament is not sufficient to maintain or gain rating points. Figures A.12–

A.16 show cases where the superstar won a tournament, but nevertheless lost rating points. The superstar must typically win a tournament by a large margin to maintain their #1 rating level.

Table 1 presents statistics showing the superstars' dominance. Panels A-E include the World's Top 10 chess players for the corresponding era and a summary of their tournament performances. For example, Magnus Carlsen participated in 35 tournaments with classical time controls between 2013 and 2019, winning 21 of them. This 60% tournament win rate is two times higher than World's #2 chess player, Fabiano Caruana, who has a tournament win rate of 30%. A more extreme case is Anatoly Karpov, who won 26 out of 32 tournaments, which converts to an 81% tournament win rate while the runner up Jan Timman had a tournament win rate of 20%.

Table 1: World's Top 10 chess players and their tournament performances

years: 2013-2019 *PANEL A*

Name	# of tournament wins	# of tournaments played	% tournament wins	$\overline{ELO}$	proportion of games won	proportion of draws	proportion of games lost	$\overline{ACPL}$	complexity	# of moves	# of games
Carlsen, Magnus	21	35	60%	2855	0.352	0.576	0.072	14.413	26.802	16,104	303
Caruana, Fabiano	15	49	30%	2802	0.283	0.592	0.126	16.758	28.280	22,205	447
So, Wesley	6	27	22%	2777	0.226	0.666	0.108	14.928	25.717	11,622	263
Aronian, Levon	5	33	15%	2788	0.196	0.662	0.142	16.059	25.654	13,455	294
Giri, Anish	3	31	9%	2770	0.149	0.719	0.131	14.873	26.202	14,224	304
Karjakin, Sergey	3	28	10%	2768	0.168	0.689	0.143	15.947	26.938	12,764	281
Mamedyarov, Shakhriyar	3	22	13%	2777	0.172	0.674	0.154	15.050	26.339	9,405	216
Nakamura, Hikaru	3	35	8%	2779	0.218	0.622	0.160	15.823	27.398	15,349	327
Vachier Lagrave, Maxime	3	26	11%	2777	0.163	0.703	0.134	14.539	26.842	10,227	232
Grischuk, Alexander	0	15	0%	2777	0.183	0.633	0.184	18.081	27.539	6,852	146

years: 1995-2001 *PANEL B*

Name	# of tournament wins	# of tournaments played	% tournament wins	$\overline{ELO}$	proportion of games won	proportion of draws	proportion of games lost	$\overline{ACPL}$	complexity	# of moves	# of games
Kasparov, Garry	17	22	77%	2816	0.439	0.510	0.051	17.523	28.509	8,344	225
Kramnik, Vladimir	12	29	41%	2759	0.323	0.620	0.057	16.468	25.966	10,183	276
Anand, Viswanathan	7	24	29%	2761	0.304	0.589	0.108	19.142	28.477	8,740	239
Topalov, Veselin	6	25	24%	2708	0.270	0.512	0.218	21.868	28.752	10,262	240
Ivanchuk, Vassily	4	17	23%	2727	0.254	0.583	0.163	19.763	27.044	6,051	162
Adams, Michael	3	20	15%	2693	0.249	0.569	0.182	19.784	27.360	7,679	187
Short, Nigel D	3	18	16%	2673	0.272	0.475	0.253	22.717	29.080	6,359	164
Svidler, Peter	3	11	27%	2685	0.261	0.590	0.149	19.621	27.844	4,044	108
Karpov, Anatoly	2	11	18%	2739	0.203	0.696	0.101	18.099	26.256	4,138	94
Shirov, Alexei	1	25	4%	2707	0.282	0.465	0.253	21.979	29.097	10,098	244

Table 1 (cont): World's Top 10 chess players and their tournament performances

years: 1976-1983 *PANEL C*

Name	# of tournament wins	# of tournaments played	% tournament wins	$\overline{ELO}$	proportion of games won	proportion of draws	proportion of games lost	$\overline{ACPL}$	complexity	# of moves	# of games
Karpov, Anatoly	26	32	81%	2707	0.432	0.524	0.044	17.377	25.244	13,794	367
Timman, Jan H	8	39	20%	2607	0.331	0.519	0.150	20.396	26.292	20,152	502
Larsen, Bent	5	28	17%	2606	0.381	0.338	0.281	22.666	27.291	17,462	371
Portisch, Lajos	4	21	19%	2634	0.330	0.518	0.152	19.937	25.689	10,794	281
Kasparov, Garry	3	5	60%	2652	0.429	0.487	0.084	19.217	25.634	2,207	64
Kortschnoj, V L	3	7	42%	2667	0.448	0.388	0.164	20.807	26.962	3,683	88
Tal, Mihail	3	13	23%	2629	0.280	0.624	0.096	19.073	25.064	6,313	191
Petrosian, Tigran V	2	12	16%	2608	0.244	0.652	0.103	19.936	23.089	5,257	167
Spassky, Boris Vasilievich	2	16	12%	2624	0.196	0.697	0.107	19.119	24.771	6,115	192
Beliavsky, Alexander G	1	5	20%	2596	0.320	0.457	0.223	22.644	28.210	2,758	69

years: 1962-1970 *PANEL D*

Name	# of tournament wins	# of tournaments played	% tournament wins	$\overline{ELO}$	proportion of games won	proportion of draws	proportion of games lost	$\overline{ACPL}$	complexity	# of moves	# of games
Fischer, Robert James	13	16	81%	0	0.647	0.280	0.073	18.487	28.634	10,598	250
Kortschnoj, V L	7	12	58%	0	0.469	0.459	0.072	19.964	26.378	7,679	196
Keres, Paul	5	9	55%	0	0.427	0.544	0.029	19.324	25.043	4,830	139
Spassky, Boris Vasilievich	4	9	44%	0	0.410	0.570	0.020	18.240	24.341	4,365	138
Tal, Mihail	4	8	50%	0	0.474	0.388	0.138	21.847	26.624	4,615	123
Botvinnik, Mikhail	3	5	60%	0	0.529	0.414	0.056	18.769	26.065	2,251	63
Geller, Efim P	2	12	16%	0	0.422	0.508	0.069	18.542	25.033	8,365	219
Petrosian, Tigran V	1	13	7%	0	0.334	0.621	0.045	20.311	24.675	7,622	227
Reshevsky, Samuel H	1	11	9%	0	0.258	0.505	0.237	24.871	25.505	5,253	140
Bronstein, David Ionovich	0	6	0%	0	0.283	0.628	0.089	21.604	25.199	3,043	94

Table 1 (cont): World's Top 10 chess players and their tournament performances

years: 2014-2019 *PANEL E*

Name	# of tournament wins	# of tournaments played	% tournament wins	$\overline{ELO}$	proportion of games won	proportion of draws	proportion of games lost	$\overline{ACPL}$	complexity	# of moves	# of games
Hou, Yifan	4	4	100%	2644	0.614	0.364	0.023	16.222	26.820	2,014	44
Ju, Wenjun	3	6	50%	2563	0.400	0.523	0.077	16.565	26.079	2,913	65
Koneru, Humpy	2	6	33%	2584	0.379	0.424	0.197	20.033	27.516	2,916	66
Dzagnidze, Nana	0	6	0%	2540	0.359	0.347	0.295	24.728	28.335	3,040	64
Goryachkina, Aleksandra	0	1	0%	2564	0.364	0.636	0.000	14.322	27.547	558	11
Kosteniuk, Alexandra	0	7	0%	2532	0.297	0.508	0.195	22.841	29.092	3,439	75
Lagno, Kateryna	0	2	0%	2544	0.227	0.682	0.091	16.283	26.304	833	22
Muzychuk, Anna	0	5	0%	2554	0.218	0.582	0.200	18.416	26.901	2,407	55
Muzychuk, Mariya	0	3	0%	2544	0.242	0.576	0.182	18.302	27.178	1,528	33
Zhao, Xue	0	6	0%	2519	0.288	0.424	0.288	24.058	27.521	2,995	66

Notes: Panels A-E show the tournament performance for the World's Top 10 chess players for the corresponding time period. Elo rating system was first adopted by the International Chess Federation (FIDE) in 1970, hence this information is absent in Panel D for Fischer's sample. Variables and their definitions are presented in table Table A.1.

3.3 ChessBase Mega Database

Our data comes from the 2020 ChessBase Mega Database containing over 8 million chess games dating back to the 1400s. Every chess game is contained in a PGN file which includes information about player names, player sides (White or Black), Elo ratings, date and location of the game, tournament name, round, and the moves played. An example of a PGN file and a tournament table is provided in the appendix. See Table A.2 and Figure A.1.

Table A.1 in the appendix provides a summary of variables used and their definitions. Table 2 presents the summary statistics for each era with tournaments grouped according to the superstar presence. In total, our study analyzes over 2 million moves from approximately 25,000 games played in over 300 tournaments between 1962 and 2019. A list of the tournaments is provided in the online appendix.

Table 2: Summary statistics for all samples

	years: 2013-2019				*years: 1995-2001*			
	with Carlsen		*without Carlsen*		*with Kasparov*		*without Kasparov*	
variable	mean	sd	mean	sd	mean	sd	mean	sd
ACPL	16.580	10.755	17.818	11.783	21.101	12.436	21.592	13.235
Complexity	26.784	5.309	27.040	5.419	27.189	5.656	26.850	5.811
TotalBlunder	.179	.508	.229	.576	.279	.635	.317	.684
TotalMistake	1.432	1.797	1.682	1.943	1.879	1.903	1.922	2.028
win	.173	.378	.204	.403	.228	.419	.234	.423
draw	.654	.476	.592	.491	.545	.498	.533	.499
loss	.173	.378	.204	.403	.228	.419	.234	.423
ELO	2759	47.13	2714	80	2615	57.53	2590	55.97
Moves	43.031	15.682	45.198	17.682	39.537	17.294	39.635	16.810
#of tournaments	=35		=37		=22		=43	
#of games	=1,336		=1,774		=805		=1,793	
#of moves	=114,898		=160,362		=61,936		=139,469	

Notes: Superstar player observations are exluded in each sample. Data comes from Chessbase Mega Database 2020.

Table 2: Summary statistics for all samples (cont.)

	years: 1987-1994				*years: 1976-1983*			
	with Kasparov & Karpov		*without Kasparov & Karpov*		*with Karpov*		*without Karpov*	
ACPL	20.762	12.068	21.592	13.235	21.777	13.501	23.099	14.091
Complexity	27.139	5.555	26.850	5.811	25.461	5.873	26.007	5.787
TotalBlunder	.264	.603	.317	.684	.271	.639	.327	.727
TotalMistake	1.821	1.902	1.922	2.028	1.850	2.040	2.061	2.089
win	.221	.415	.234	.423	.223	.416	.251	.433
draw	.561	.496	.533	.499	.553	.497	.499	.500
loss	.218	.413	.234	.423	.224	.417	.250	.433
ELO	2615	60.68	2590	55.97	2558	68.05	2531	76.24
Moves	39.537	17.294	39.635	16.810	36.699	17.542	37.964	17.118
#of tournaments	=11		=37		=32		=39	
#of games	=635		=1,989		=1,882		=3,593	
#of moves	=50,212		=157,668		=138,223		=273,351	

	years: 1962-1970				*years: 2014-2019*			
	with Fischer		*without Fischer*		*with Hou Yifan*		*without Hou Yifan*	
ACPL	24.017	15.634	25.284	15.670	22.466	14.500	21.128	12.301
Complexity	25.835	5.599	26.077	5.747	27.193	5.225	27.162	5.045
TotalBlunder	.342	.768	.349	.746	.405	.736	.380	.755
TotalMistake	2.173	2.128	2.311	2.254	2.341	2.344	2.267	2.209
win	.254	.435	.250	.433	.270	.444	.241	.428
draw	.492	.500	.500	.500	.459	.499	.519	.500
loss	.254	.435	.250	.433	.270	.444	.241	.428
ELO*	.	.	.	.	2493	72.50	2499	43.43
Moves	38.126	16.458	36.112	15.628	45.823	19.041	46.179	17.511
#of tournaments	=15		=82		=4		=6	
#of games	=1,657		=7,826		=220		=374	
#of moves	=126,288		=565,078		=20,162		=34,542	

Notes: Superstar player observations are exluded in each sample. Data comes from Chessbase Mega Database 2020.
$*$: Elo rating system was first adopted by FIDE beginning 1970.

3.4 Measuring Performance

3.4.1 Average Centipawn Loss

We follow Guid and Bratko (2006) and Regan et al. (2011) and obtain computer evaluations for mistakes committed by each player in a given game. A chess game g consists of moves $m \in \{1, \ldots, M\}$ where player i makes an individual move m_{ig}. A chess engine can evaluate a given position by calculating layers with depth n at each decision node and make suggestions about the best moves to play. Given a best move is played, the engine provides the relative (dis)advantage in a given position $C_{igm}^{computer}$. This evaluation is then compared to the actual evaluation score C_{igm}^{player} once a player makes his or her move. The difference in scores reached via the engine's top suggested move(s) and the actual move a player makes can be captured by

$$error_{igm} = \left| C_{igm}^{computer} - C_{igm}^{player} \right|. \tag{2}$$

If the player makes a top suggested move, the player has committed zero error, i.e., $C_{igm}^{computer} = C_{igm}^{player}$. We can think of chess as a game of attrition where the player who makes less mistakes eventually wins the game. While staying constant if top moves are played, the evaluation shows an advantage for the opponent if a player commits a mistake by playing a bad move.

We then take the average of all the mistakes committed by player i in game g via

$$\overline{error_{ig}} = \frac{\sum_{m=1}^{M} \left| C_{igm}^{computer} - C_{igm}^{player} \right|}{M}, \tag{3}$$

which is a widely accepted metric named Average Centipawn Loss (ACPL). ACPL is the average of all the penalties a player is assigned by the chess engine for the mistakes they committed in a game. If the player plays the best moves in a game, his ACPL score will be small where a smaller number implies the player performed better. On the other hand, if the player makes moves that are considered bad by the engine, the player's ACPL score would be higher.

We used Stockfish 11 in our analyses with depth $n = 19$ moves. For each move, the engine was given half a second to analyze the position and assess $|C_{igm}^{computer} - C_{igm}^{player}|$. Figure A.7 shows an example of how a game was analyzed. For instance, at move 30, the computer evaluation is +3.2, which means that the white player has the advantage by a score of 3.2: roughly the equivalent of being one piece (knight or bishop) up compared to his opponent. If the white player comes up with the best moves throughout the rest of the game, the evaluation can also stay 3.2 (if the black player also makes perfect moves) or only go up leading to a possible win toward the end of the game. In the actual game, the player with the white pieces lost his advantage by making bad moves and eventually lost the game. The engine analyzes all 162 moves played in the game and evaluates the quality of each move. Dividing the sum of mistakes committed by player i to the total number of moves played by player i gives the player-specific ACPL score.

3.4.2 Board Complexity

Our second measure that reinforces our ACPL metric is "board complexity" which we obtain via an Artificial Neural Network (ANN) approach. The recent developments with AlphaGo and AlphaZero demonstrated the strength of using heuristic-based algorithms that perform at least as good as the traditional approaches, if not better. Instead of learning from self-play, our neural-network algorithm "learns" from human players. Sabatelli et al. (2018) and McIlroy-Young et al. (2020) are two recent implementations of such architecture. To train the network, we use an independent sample published as part of a Kaggle contest consisting of 25,000 games and more than 2 million moves, with Stockfish evaluation included for each move. The average player in this sample has an Elo rating of 2280, which corresponds to the "National Master" level according to the United States Chess Federation (USCF).

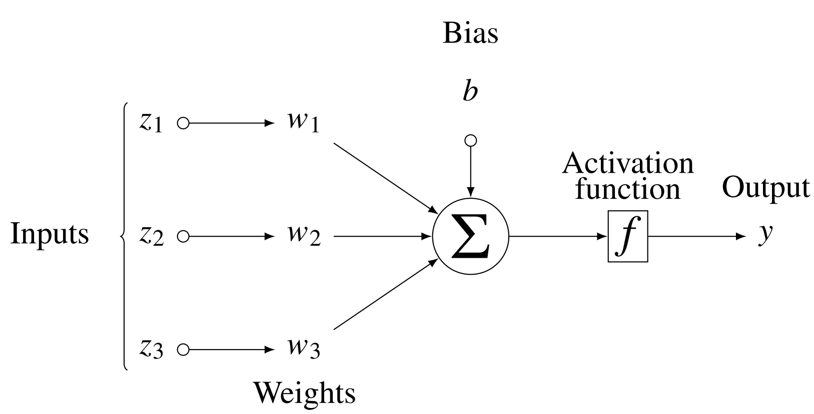


Figure 4: Example of a simple perceptron, with 3 input units (each with its unique weight) and 1 output unit.

The goal of the network is to predict the probability of a player making a mistake with its magnitude. This task would be trivial to solve for positions that were previously played. How-

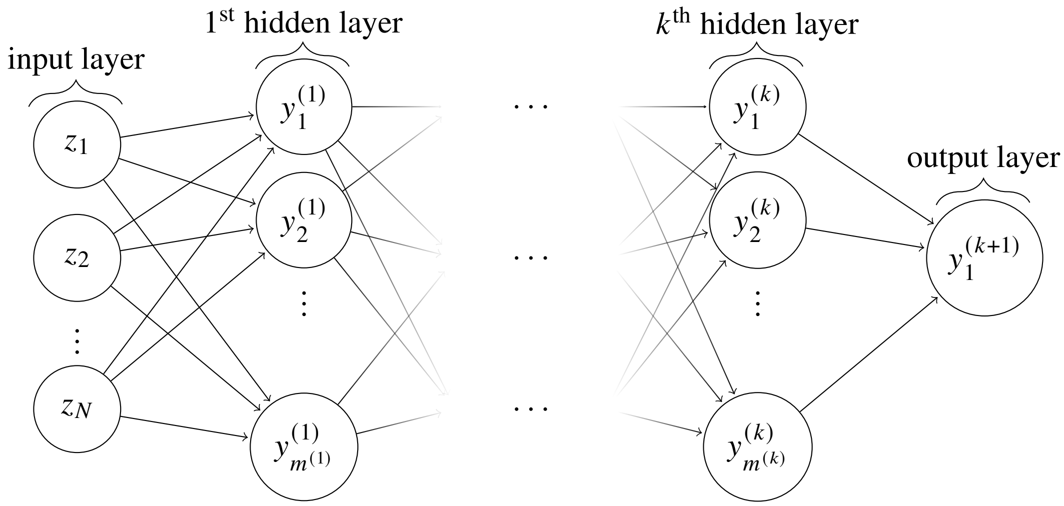


Figure 5: Network graph of a multilayer neural network with $(k + 1)$ layers, N input units, and 1 output unit. Each neuron collects unique weights from each previous unit. The k^{th} hidden layer contains $m^{(k)}$ neurons.

ever, each chess game reaches a unique position after the opening stage which requires accurate extrapolation of human play in order to predict the errors. This approach is vastly different than traditional analysis with an engine such as Stockfish. Engines are very strong and can find the best moves. However, they cannot give any information about how a human would play in a given situation because they are designed to find the best moves without any human characteristics. Our neural-network algorithm is specifically designed to learn how and when humans make mistakes in given positions from analyzing mistakes committed by humans from a sample of 2 million moves. We represent a chess position through the use of its 12 binary features, corresponding to the 12 unique pieces on the board (6 for White, 6 for Black). A chess board has $8 \times 8 = 64$ squares. We split the board into 12 separate 8×8 boards (one for each piece) where a square gets "1" if the piece is present on that particular square and gets "0" otherwise. In total, we represent a given position using $12 \times 8 \times 8 = 768$ inputs. We add one additional feature to represent the players' turn (white to move, or black to move) and thus have $768 + 1 = 769$ inputs in total per position. We use a network architecture with three layers. The layers have 1048, 500, and 50 neurons, each with its unique weight. In order to prevent over-fitting, a 20% dropout regularization on each layer is used. Each hidden layer is connected with the Rectified Linear Unit (ReLU) activation function. The Adam optimizer was used with a learning rate of 0.001. Figures 4–5 illustrate.

The neural network "learns" from 25,000 games by observing each of the approximately two million positions and estimates the optimal weights by minimizing the error rate that results from each possible set of weights with the Gradient Descent algorithm. A set of 1,356,612 optimal weights uniquely characterizes our network. We use two networks to make a prediction on two statistics for a given position: (i) probability that a player commits an error and (ii) the amount of error measured in centipawns. For a full game, the two statistics multiplied (and averaged out across moves) gives us an estimate for the ACPL that each player is expected to get as the result of the complexity of the game

$$E(\overline{error_{ig}}) = \frac{\sum_{m=1}^{M} P\left(\left|C_{igm}^{computer} - C_{igm}^{network}\right| > 0\right) \left|C_{igm}^{computer} - C_{igm}^{network}\right|}{M}, \tag{4}$$

where $\left|C_{igm}^{computer} - C_{igm}^{network}\right|$ is the expected centipawn loss in a given position predicted by the neural network. We test our network's performance on our main "superstars" sample. The mean ACPL for the whole sample with 35,000 games is 25.87, and our board complexity measure, which is the expected ACPL that we obtained through our network, is 26.56. The reason why our network −which was trained with games played at on average 2280 Elo level− makes a close estimate for the ACPL in the main sample is that the estimates come from not a single player with Elo rating 2280, but rather from a "committee" of players with Elo rating 2280 on average. Hence, the network is slightly "stronger" compared to an actual 2280 player. Figure A.10 shows a scatterplot of ACPL and the expected ACPL. The slope coefficient is 1.14, which implies that a point increase in our complexity measure results in a 1.14 point increase in the actual ACPL score. The highest ACPL prediction of the network is 50.2 while about 8% of the sample has an actual ACPL > 50.2. These extreme ACPL cases are under-predicted by the network due to the network's behavior as a "committee" rather than a single player, where the idiosyncratic shocks are averaged out. Figure A.11 shows the distributions of ACPL and the expected ACPL.

The board complexity measure addresses the main drawback of using only ACPL scores. The ACPL score of a player is a function of his or her opponent's strength and their strategic choices. For instance, if both players find it optimal to not take any risks, they can have a simple game where players make little to no mistakes, resulting in low ACPL scores. Yet, this would not imply that players showed a great performance compared to their other −potentially more complex− games. Being able to control for complexity of a game enables us to compare mistakes committed in similarly-complex games.

3.4.2 Game outcomes

The third measure we use is game-level outcomes. Every chess game ends in a win, a loss, or a draw. The player who wins a tournament is the one who accumulates more wins and fewer losses, as the winner of a game receives a full point toward his or her tournament score. A draw brings half a point, while a loss brings no points in a tournament. In other words, a player who has more wins in a tournament shows a higher performance. In terms of losses, the opposite is true. If a player has many losses in a tournament, their chances to win the tournament are slim. Of course, a draw is considered better than a loss and worse than a win.

4. Empirical Design

We test the direct (head-to-head) superstar effect using the following specification:

$$Performance_{ij} = \alpha_0 + \alpha_1 AgainstSuperstar_{ij} + \mathbf{\Phi X_{ij}} + \eta_i + \epsilon_{ij}, \tag{5}$$

where $Performance_{ij}$ is the performance of player i in their game j measured by the methods discussed in section 3.4. $AgainstSuperstar_{ij}$ equals one if player i in their game j plays against a superstar. $\mathbf{\Theta X_{ij}}$ contains player and game level controls. η_i are player fixed effects. ϵ_{ij} is an idiosyncratic shock. In this specification, α_1 captures the effect of head-to-head competition against a superstar.

Our second specification tests the indirect effect by comparing a player's performance in a tournament where a superstar is present with their performance in a tournament without a superstar. This can be captured by the following specification:

$$Performance_{ijk} = \beta_0 + \beta_1 SuperstarPresent_k + \mathbf{\Theta X_{ijk}} + \eta_i + \epsilon_{ijk}, \tag{6}$$

where $Superstar_k$ is an indicator for the superstar being present in tournament k. $\mathbf{\Theta X_{ijk}}$ contains player, game, tournament level controls. ϵ_{ijk} is an idiosyncratic shock. Having a negative sign for β_1 would indicate that the superstar presence is associated with an adverse effect. We break down (6) by the tournament participants' rating quartiles at the time of their tournament to study the heterogeneous effects of the superstar's presence with the following specification:

$$
\begin{aligned}
Performance_{ijk} = \phi_0 + \phi_1 Superstar_k \times HighELO_{ik} + \phi_2 Superstar_k \times MidELO_{ik} \\
+ \phi_3 Superstar_k \times LowELO_{ik} + \phi_4 HighELO_{ik} + \phi_5 MidELO_{ik} \\
+ \boldsymbol{\Theta X_{ijk}} + \eta_i + \epsilon_{ijk}, \quad (7)
\end{aligned}
$$

where $HighELO_{ik}$ equals one if player i has an Elo rating within the top quartile in the Elo rating distribution of the tournament participants at the time of tournament k. $MidELO_{ik}$ captures the second and third quartiles, and $LowELO_{ik}$ captures the bottom quartile.

The main variation in the superstar's participation in elite tournaments comes from the superstar's inability to participate in all available elite tournaments in a given year. There are multiple sources that create variation for the superstar's presence: (1) Candidate tournaments: some elite tournaments are run to determine the challenger against the world champion for the world championship title, which by definition implies that the world champion cannot participate. (2) Nationality: some elite tournaments are run to determine a national champion. If the world champion has a different nationality, he or she is not allowed to attend. (3) World Championship match years. If a World Championship match is planned to take place, the superstar (who is the champion) decreases participation in elite tournaments. (4) Political: For Soviet-era champions such as Karpov and Kasparov, traveling to an elite tournament outside the U.S.S.R. requires getting travel approval from the government, which limits their ability to participate in more than what they receive permission for.

It is possible that players may face a different prize or have different motivations depending on whether the event is for national championship, candidacy, or an event where the superstar is participating. However, for all the tournaments mentioned, players need to seriously prepare and exert as much effort as possible both before and during the tournament due to the potential of losing rating points in *any* game during the event. A minuscule drop in effort or concentration can lead to a loss which will result in losing valuable rating points. These points are an important determinant of a player's elite status and future invitations.

Linnemer and Visser (2016) document self-selection in chess tournaments with stronger players being more likely to play in tournaments with higher prizes. A central difference between their sample and ours is the level of tournaments, with their data coming from the World Open tournament, which is an open tournament with non-master participants with Elo ratings between 1400-2200. Meanwhile, our sample consists of players from a much more restricted sample with only the most elite Grandmasters having Elo ratings often above 2700. Moreover, each high-level tournament in our sample is invitation based; i.e., tournament organizers send invitations to a select group of strong players (10-12 players in each tournament). These restrictions work against selection problems.

Moreover, another potential threat to our identification is if non-superstar elite players decline invitations strategically to play in elite tournaments in a way that avoids playing (and losing) against the superstar. Avoiding the superstar is not an optimal strategy for several reasons: (1) Declining an invitation to play in an elite tournament (where the superstar is participating) means declining a guaranteed prize. Elite tournaments pay out higher prizes relative to other chess events, often times including an "appearance fee". (2) Playing in the same tournament with the superstar promotes the player in the media which can create opportunities for the player to sign more sponsorship and endorsement deals. (3) Declining an invitation to play may reduce the chance of being invited again to a future event by the same organizer. Typically, an emergency or a conflict in schedule is a reason one might decline an elite tournament invitation. In all other cases, an elite player whose livelihood depends on tournament prizes and endorsements has very strong incentives to accept the invitation and participate.

It is important to emphasize that we do not have a controlled experiment where a designer randomly assigns a superstar to tournaments. Such a study would be impossible to conduct as a field experiment, and the results from an experimental study could have little external validity. With this word of caution, we believe that our results still provide valuable information on how contest participants respond to superstars through a collection of correlative evidence from multiple superstars with varying dominance.

5. Results

Table 3 shows the performance of non-superstar players playing against a superstar for each sample. There is a distinct pattern that is true for all superstars: playing against them is associated with a higher ACPL score, more blunders, more mistakes, lower chances to win, and higher chances to lose. What is more, games played against superstars are more complex. This higher complexity could be due to the superstar's willingness to reach more complex positions in order to make the ability-gap more salient. It could also be linked to a non-superstar player taking more risk. Taken as a whole, players commit more blunders and mistakes, holding board complexity constant. For instance, a player who plays against Fischer shows an ACPL that is 4.4 points higher compared to his games against other players with a similar complexity level. The probability estimates are 10 percentage points less for a win, 18 percentage points less for a draw, and 28 percentage points higher for a loss compared to his typical games.

This finding verifies our theoretical observations that the direct effect is always negative: the superstars in our sample show dominance over their peers. Moreover, Hou Yifan demonstrates the strongest domination, with Fischer closely following behind. The magnitudes for ACPL, win, and loss probabilities are larger for these players compared to other superstars such as Carlsen, Kasparov, and Karpov.

To illustrate, Kasparov (2003) shares an observation of Karpov's direct effect on other players during a game in Moscow in 1974: *"Tal, who arrived in the auditorium at this moment, gives an interesting account: "The first thing that struck me (I had not yet seen the position) was this: with measured steps Karpov was calmly walking from one end of the stage to the other. His opponent was sitting with his head in his hands, and simply physically it was felt that he was in trouble. 'Everything would appear to be clear,' I thought to myself, 'things are difficult for Polugayevsky.' But the demonstration board showed just the opposite! White was a clear exchange to the good – about such positions it is customary to say that the rest is a matter of technique. Who knows, perhaps Karpov's confidence, his habit of retaining composure in the most desperate situations, was transmitted to his opponent and made Polugayevsky excessively nervous."*

Hou Yifan's level of dominance is not seen in any of our other samples consisting of male superstars. A potential explanation for why the most dominant superstar in our sample is a female chess player could be related to the Central Limit Theorem. There are much fewer female chess players than male chess players. A smaller sample has higher variance, making it more likely to produce outliers. See Bilalić et al. (2009) for a discussion of this phenomenon.

We now turn to the indirect superstar effect. Table 4 shows the impact of superstar presence for all samples aggregated. We focus on each superstar in Table 5 (and Figure 6) where we break down each superstar's sample and regress Equation 6 separately for each superstar. We present

Table 3: Performance against a superstar.

	(1) ACPL	(2) TotalBlunder	(3) TotalMistake	(4) win	(5) draw	(6) loss	(7) Complexity	*# of games*	*# of moves*
Against Carlsen	1.868***	0.080***	0.174**	-0.069***	-0.100***	0.169***	0.172	3,316	294,876
2013-2019	(0.449)	(0.026)	(0.085)	(0.015)	(0.023)	(0.025)	(0.252)		
Against Kasparov	2.667***	0.116***	0.272***	-0.105***	-0.068*	0.173***	1.575***	2,689	212,347
1995-2001	(0.578)	(0.041)	(0.101)	(0.012)	(0.035)	(0.034)	(0.364)		
Against Kasparov/Karpov	2.685***	0.151***	0.205**	-0.108***	-0.077***	0.185***	-0.423	2,764	219,567
1987-1994	(0.659)	(0.030)	(0.096)	(0.015)	(0.028)	(0.032)	(0.314)		
Against Karpov	3.011***	0.175***	0.109	-0.101***	-0.094***	0.196***	0.623**	5,198	427,503
1976-1983	(0.558)	(0.034)	(0.082)	(0.012)	(0.023)	(0.024)	(0.256)		
Against Fischer	4.410***	0.148***	0.232	-0.106***	-0.179***	0.285***	2.635***	9,430	702,544
1962-1970	(0.873)	(0.041)	(0.142)	(0.020)	(0.031)	(0.039)	(0.364)		
Against Hou Yifan	4.490**	0.211***	0.539	-0.113***	-0.212***	0.325***	0.748*	616	56,718
2014-2019	(1.591)	(0.046)	(0.451)	(0.032)	(0.036)	(0.031)	(0.352)		
Against Superstar	2.887***	0.125***	0.207***	-0.099***	-0.084***	0.183***	0.666***	23,614	1,825,871
1962-2019	(0.263)	(0.013)	(0.041)	(0.007)	(0.012)	(0.014)	(0.155)		

Notes: All regressions include player and year fixed effects, round fixed effects, event site fixed effects, board complexity measured by our neural-network algorithm (except in column (7) where it is the outcome variable), opponent ACPL, player's side (white or black), and number of moves played. Clustered standard errors (clustered by tournament) are shown in parentheses.
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

the results together with our estimates of the effect on the top quartile of players from Equation 7. Going back to Table 3 and the direct effect, the least dominant superstar in our sample was Carlsen, and the most dominant superstar was Hou Yifan. Even though the theory suggests that the indirect effect has to be neutral, the data shows that this effect depends on the intensity of the superstar: Moving from Carlsen to Hou Yifan, we observe increases in the committed mistakes; in blunders; in the loss rates; and decreases in win rates. When the gap between the superstar and the rest of the participants is not too wide, all players perform better, but as the gap widens, performance drops. The indirect effect is amplified for the top players. For example, Hou Yifan's presence is associated with an ACPL score that is 5 points higher; 13 percentage points less chances of winning; 18 percentage points higher chances of losing. The top players are those who are arguably impacted the most by the superstar's presence, as they have the highest ex-ante probability to win the event in the absence of the superstar.

Another situation with intense competition is when two superstars, Kasparov and Karpov, both participate in a tournament. This means that for a given player, he or she will have to face both Kasparov and Karpov and perform better than both of them in order to win the tournament. This tough competition appears to be not associated with higher ACPL or more blunders, or mistakes. The top quartile of players experience more losses with marginal significance. The results have suggestive evidence of a spillover effect: the win rate for all players is positive and significant. Potentially, the rest of the group could attain slightly more wins and fewer losses as a result of a worsened performance by the top quartile players.

Table 4: Performance in tournaments with and without a Superstar (overall effect).

	(1)	(2)	(3)	(4)	(5)	(6)	(7)
	ACPL	TotalBlunder	TotalMistake	win	draw	loss	Complexity
Superstar effect for							
Top 25% players	0.133	0.012	0.022	-0.008	0.008	0.000	0.302**
	(0.265)	(0.015)	(0.047)	(0.010)	(0.014)	(0.011)	(0.136)
Mid 50% players	-0.067	0.003	-0.051*	-0.005	0.012	-0.007	0.168*
	(0.186)	(0.011)	(0.028)	(0.006)	(0.009)	(0.007)	(0.092)
Bottom 25% players	-0.279	-0.026*	0.058	0.001	0.013	-0.014	-0.009
	(0.323)	(0.015)	(0.046)	(0.008)	(0.012)	(0.012)	(0.149)
Number of moves	1,045,441	1,045,441	1,045,441	1,045,441	1,045,441	1,045,441	1,045,441
Number of games	13,067	13,067	13,067	13,067	13,067	13,067	13,067

Notes: Superstars' games are excluded. Top 25% is defined as having an Elo rating in the top 25% among the competitors at the time of the tournament. Bottom 25% is defined as having an Elo in the bottom quartile. All regressions include player and year fixed effects, round fixed effects, average Elo rating in the tournament, player's Elo rating, board complexity measured by our neural-network algorithm, opponent ACPL, player's side (white or black), and number of moves played. Fischer's sample is excluded due to Elo rating system not being in use during his period, preventing identification of rating quartiles of players. Clustered standard errors (clustered by tournament) are shown in parentheses.
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Players perform better if they face only Kasparov or Karpov in the tournament compared to facing both superstars at once. With one superstar, either Kasparov or Karpov in the tournament, players play more accurately and manage to get more wins with substantial gains in the ACPL score and less mistakes committed. The improvement is the strongest for the top quartile of players. For example, the top players show approximately 1.7 lower ACPL scores in tournaments with either Kasparov or Karpov. These two superstars' samples are where the indirect effects are the most positive for the players. The number of superstars in the tournament seems to matter to players: facing one superstar instead of two at once lowers the barrier for victory, and the players respond positively.

Lastly, Carlsen's presence is correlated with a slight positive indirect effect on his competitors' performance. This indirect effect is similar, albeit slighly weaker than facing Kasparov or Karpov alone. Players play more accurately and make fewer mistakes under higher challenges with more complex positions. The positive indirect effect is present on all tournament participants.

Table 5: Performance in tournaments with and without a superstar for all and top players.

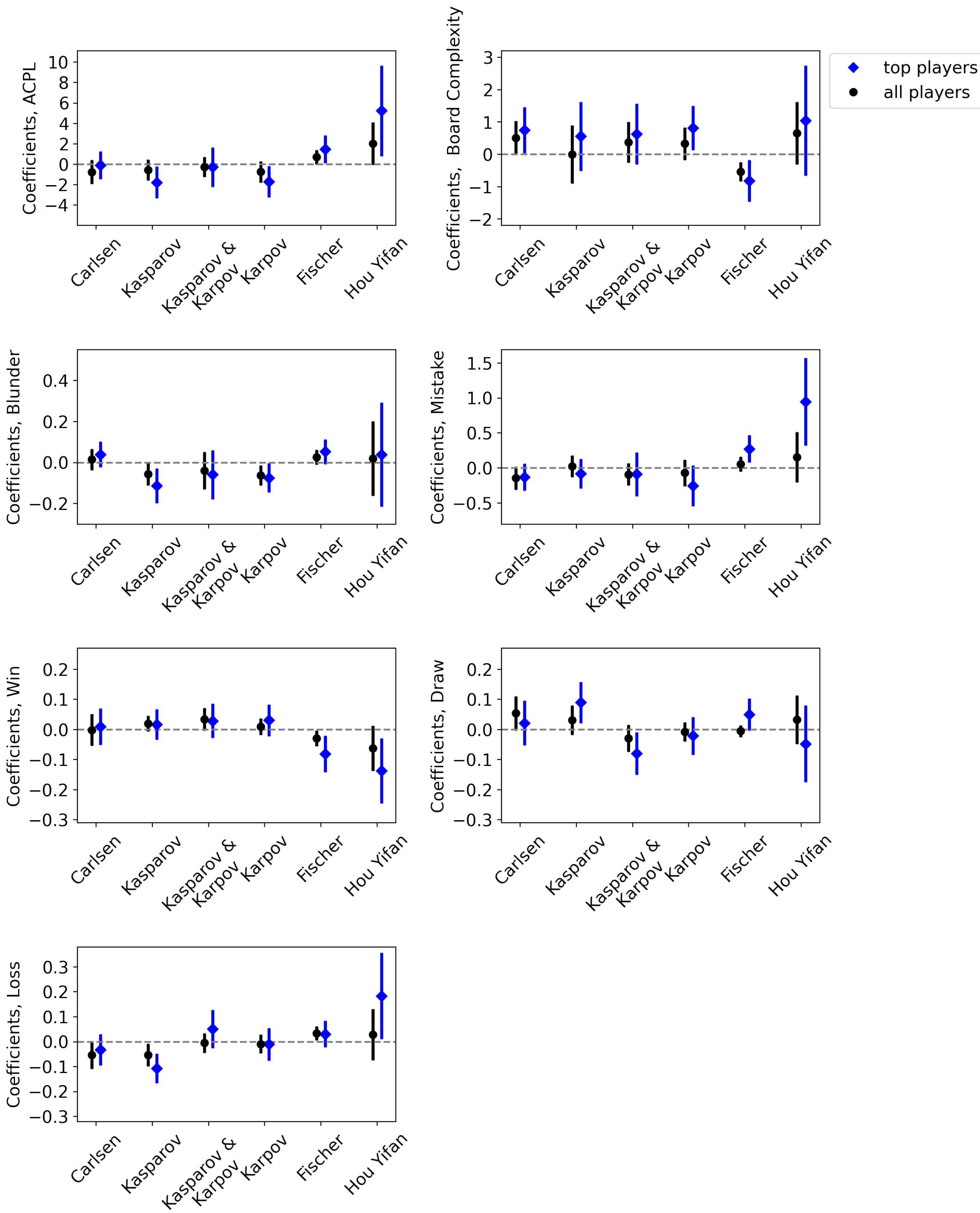
	(1)		(2)		(3)		(4)		(5)		(6)		(7)			
	ACPL		TotalBlunder		TotalMistake		win		draw		loss		Complexity		*# of games*	*# of moves*
	All	Top	All	Top	All	Top	All	Top	All	Top	All	Top	All	Top		
Carlsen present	-0.751	-0.080	0.015	0.040	-0.143*	-0.128	-0.001	0.010	0.054*	0.022	-0.053*	-0.032	0.506*	0.756**	3,110	275,260
2013-2019	(0.592)	(0.698)	(0.026)	(0.032)	(0.084)	(0.097)	(0.027)	(0.031)	(0.029)	(0.038)	(0.028)	(0.032)	(0.266)	(0.362)		
Kasparov present	-0.563	-1.770**	-0.055*	-0.113**	0.027	-0.078	0.020	0.017	0.032	0.090**	-0.053**	-0.107***	0.002	0.563	2,588	200,560
1995-2001	(0.526)	(0.785)	(0.028)	(0.043)	(0.079)	(0.108)	(0.013)	(0.026)	(0.025)	(0.035)	(0.023)	(0.030)	(0.458)	(0.544)		
Kasparov&Karpov present	-0.265	-0.277	-0.039	-0.059	-0.088	-0.087	0.034*	0.029	-0.029	-0.080**	-0.005	0.051	0.377	0.632	2,619	207,482
1987-1994	(0.501)	(0.988)	(0.046)	(0.061)	(0.080)	(0.159)	(0.019)	(0.029)	(0.023)	(0.036)	(0.020)	(0.039)	(0.323)	(0.476)		
Karpov present	-0.748	-1.721**	-0.062**	-0.074**	-0.069	-0.252*	0.018	0.031	-0.007	-0.021	-0.009	-0.010	0.331	0.816**	5,028	377,902
1976-1983	(0.522)	(0.782)	(0.025)	(0.036)	(0.096)	(0.148)	(0.014)	(0.027)	(0.016)	(0.032)	(0.019)	(0.033)	(0.257)	(0.349)		
Fischer present+	0.700**	1.479**	0.026	0.053*	0.055	0.275***	-0.029**	-0.081**	-0.005	0.050*	0.034**	0.031	-0.536***	-0.816**	9,299	677,962
1962-1970	(0.352)	(0.694)	(0.018)	(0.031)	(0.054)	(0.100)	(0.013)	(0.031)	(0.010)	(0.027)	(0.014)	(0.027)	(0.148)	(0.329)		
Hou Yifan present	2.036*	5.242**	0.020	0.039	0.157	0.946***	-0.062	-0.137**	0.033	-0.047	0.029	0.184**	0.659	1.052	594	54,704
2014-2019	(1.061)	(2.262)	(0.093)	(0.129)	(0.184)	(0.319)	(0.038)	(0.055)	(0.041)	(0.065)	(0.052)	(0.088)	(0.495)	(0.869)		
Aggregate effect	-0.201	0.133	-0.021*	0.012	-0.004	0.022	0.006	-0.008	-0.012	0.008	0.007	0.000	-0.118	0.302**	23,238	1,793,870
	(0.211)	(0.265)	(0.011)	(0.015)	(0.032)	(0.047)	(0.006)	(0.010)	(0.008)	(0.014)	(0.008)	(0.011)	(0.105)	(0.136)		

Notes: Superstars' games are excluded. The rows report the coefficients for each superstar's presence effect on all and top players in their eras separately. A top player is defined as having an Elo rating in the top 25% among the competitors at the time of the tournament. All regressions include player and year fixed effects, round fixed effects, event site fixed effects, average Elo rating in the tournament (except for pre-1970 games, Elo rating was adopted in 1970 by FIDE), player's Elo rating (except pre-1970 games), board complexity measured by our neural-network algorithm, opponent ACPL, player's side (white or black), and number of moves played. Clustered standard errors (clustered by tournament) are shown in parentheses.
+: Since no Elo rating information was available in Fischer's era, we define the top players as the top chess players in the world from 1962-1970 other than Fischer. These players are Tigran Petrosian, Viktor Korchnoi, Boris Spassky, Vasily Smyslov, Mikhail Tal, Mikhail Botvinnik, Paul Keres, Efim Geller, David Bronstein, and Samuel Reshevsky. Kasparov (2003) provides a detailed overview on each of these players.
* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Why do players commit more mistakes playing against other non-superstar players when a superstar is participating in the tournament? A superstar creates a mental pressure on players. The literature documents detrimental effects of stress and pressure on performance. See Heaton and Sigall (1991), Ariely et al. (2009), Yu (2015), González-Díaz and Palacios-Huerta (2016) among others. Neuropsychological mechanisms vary from choking due to loss of focus to over-arousal in high stake situations. Playing in the tournament with a superstar creates such a high pressure situation.

We find that when the superstar is present, the top players play more complex games against other non-superstar players. However, they end up committing more mistakes, resulting in more losses.

Figure 6: Coefficients for superstar presence for top players over different superstars.



Note: The figure presents the coefficients in Table 5 with 95% confidence intervals for each superstar in the sample. The superstars are sorted by their intensity levels on the x-axis using Table 3 as reference. The order is ascending from left to right.

6. Conclusion

The empirical superstar literature finds evidence for both positive and negative superstar effects. In golf, players perform poorly in tournaments where a highly talented competitor is present. In the 100-meter running and swimming contests, performance improves if a superstar is present. In this paper, we analyze elite chess tournaments going back to 1960s. We show theoretically and empirically that the direct superstar effect is always negative. We find that even though the indirect effect is supposed to be neutral in theory, the chess data demonstrate that this effect depends on the intensity of the superstar. If the skill gap between the superstar and other players is small (large), the indirect superstar effect is positive (negative).

The takeaway for firms seeking to hire a superstar employee is that such hiring decision may introduce a positive or negative effect on workplace performance depending on the skill gap. If the gap is too large, there may be a negative spillover effect from hiring a superstar employee. In these cases, a highly skilled team member hurts competition and creates an adverse effect on the rest of the team members. If a team member is forced to compete head-to-head against the superstar, the manager can similarly expect under-performance due to direct superstar effect. Such adverse effects can occur not just in workplaces, but in many other environments. For example, in a classroom, a superstar student may discourage other students from learning under high peer competition. If rank order tournaments were to be employed in an organization, it is critical for the designer to ensure that the skill heterogeneity among their members is not too large.

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Appendix (For Online Publication)

Table A.1: Variables list

Variable Name	*Variable Meaning*
Superstar Present	=1 if a superstar is present in a tournament.
Against Superstar	=1 if a game is played against a superstar.
ELO	Elo rating of a player.
ACPL	Average Centipawn Loss of a player in a game.
TotalBlunder	Total number of blunders committed by a player in a game. A move is considered a blunder if the change in centipawn score is more than 300 centipawns.
TotalMistake	Total number of mistakes committed by a player in a game. A move is considered a mistake if the change in centipawn score is between 100-300 centipawns.
Complexity	The board complexity metric estimated via an Artificial Neural Network algorithm.
win	=1 if a player wins his or her game.
draw	=1 if a games ends in a draw.
loss	=1 if a player loses his or her game.

Table A.1: Variables list (cont.)

Variable Name	*Variable Meaning*
white	=1 if a player's side is white.
moves	Total number of moves played by a player in a game.
Round-robin tournament	An invitation based tournament system with a limited number of participants. Each participant plays against participants once or twice, depending on the tournament length. The participant who accumulates the highest number of points wins the tournament.
Swiss tournament	A tournament system that is typically used in open tournaments with a large pool of participants. Following the results of the first round, winners are paired with other winners. Towards the end of the tournament, strongest players with the highest number of scores get paired. The participant who accumulates the highest number of points wins the tournament.

Table A.2: An example pgn file.

[Event "GRENKE Chess Classic"]
[Site "Karlsruhe GER"]
[Date "2019.04.20"]
[EventDate "2019.04.20"]
[Round "1"]
[Result "0-1"]
[White "Vincent Keymer"]
[Black "Magnus Carlsen"]
[ECO "A56"]
[WhiteElo "2516"]
[BlackElo "2845"]
[PlyCount "162"]

1 d4 ♘f6 2 c4 c5 3 d5 g6 4 ♘c3 d6 5 e4 ♗g7 6 ♘f3 O-O 7 ♗e2 e5 8 O-O ♘e8 9 ♘e1 f5 10 e×f5 g×f5 11 f4 ♘d7 12 ♘d3 e4 13 ♘f2 ♗×c3 14 b×c3 ♘df6 15 ♗e3 ♘g7 16 ♕e1 ♗d7 17 ♘d1 ♗a4 18 h3 ♗×d1 19 ♕×d1 ♕e8 20 ♔f2 ♕g6 21 ♖g1 ♔h8 22 a4 ♖g8 23 ♕f1 ♘fh5 24 g3 ♖af8 25 ♕g2 ♕f6 26 ♖ac1 ♕d8 27 ♕h2 ♘f6 28 g4 ♘d7 29 g5 ♕a5 30 g6 h6 31 ♖b1 ♖b8 32 ♕g3 ♕d8 33 ♔e1 ♘e8 34 ♔d2 ♘f8 35 ♗f2 ♕e7 36 ♔e3 ♕f6 37 ♔d2 ♘×g6 38 h4 ♘e7 39 ♕h3 ♖×g1 40 ♖×g1 ♕f7 41 h5 ♘f6 42 ♗h4 b6 43 ♖b1 ♕f8 44 ♖g1 ♕f7 45 ♖b1 ♕g7 46 ♖g1 ♕f8 47 ♔c2 ♘fg8 48 ♔d2 ♕f7 49 ♔c2 ♖f8 50 ♔d2 ♕e8 51 ♖a1 ♖f7 52 a5 b×a5 53 ♖×a5 ♘c8 54 ♖a1 ♕f8 55 ♖b1 ♘b6 56 ♖g1 ♖g7 57 ♖×g7 ♔×g7 58 ♕g3+ ♔h8 59 ♕g6 a5 60 ♗f1 a4 61 ♔c2 a3 62 ♔b3 ♘a4 63 ♗h3 ♕g7 64 ♕×g7+ ♔×g7 65 ♗×f5 ♘f6 66 ♔×a3 ♘×c3 67 ♗f2 ♘e2 68 ♔a4 ♘×h5 69 ♔a5 ♘f6 70 ♔b6 ♔f7 71 ♔c7 ♔e7 72 ♗e3 ♘d4 73 ♗g6 h5 74 ♗f2 ♘f3 75 ♗f5 ♘d2 76 ♗h4 e3 77 ♗d3 ♘f3 78 ♗×f6+ ♔×f6 79 ♔×d6 h4 80 ♔c7 ♘d4 81 ♔c8 e2 0-1

Figure A.1: An example of an invitation-based round robin tournament table from the sample.

Grenke Chess Classic 6th 2019

				1	2	3	4	5	6	7	8	9	10		TB	Perf.	+/-
1		Carlsen,Magnus	2845	*	½	½	1	½	1	1	1	1	1	7.5 / 9		2990	+14
2		Caruana,Fabiano	2819	½	*	1	½	½	½	½	½	1	1	6.0 / 9		2833	+3
3		Naiditsch,Arkadij	2695	½	0	*	½	1	½	0	½	1	1	5.0 / 9	19.00	2766	+9
4		Vachier Lagrave,Maxime	2773	0	½	½	*	½	½	½	½	1	1	5.0 / 9	18.25	2758	-1
5		Anand,Viswanathan	2774	½	½	0	½	*	½	½	1	0	1	4.5 / 9	19.75	2719	-6
6		Aronian,Levon	2763	0	½	½	½	½	*	1	½	½	½	4.5 / 9	18.75	2720	-5
7		Svidler,Peter	2735	0	½	1	½	½	0	*	½	1	½	4.5 / 9	17.75	2723	-1
8		Vallejo Pons,Francisco	2693	0	½	½	½	0	½	½	*	½	1	4.0 / 9		2689	-1
9		Meier,Georg	2628	0	0	0	0	1	½	0	½	*	0	2.0 / 9	8.75	2518	-12
10		Keymer,Vincent	2516	0	0	0	0	0	½	½	0	1	*	2.0 / 9	6.50	2529	+1

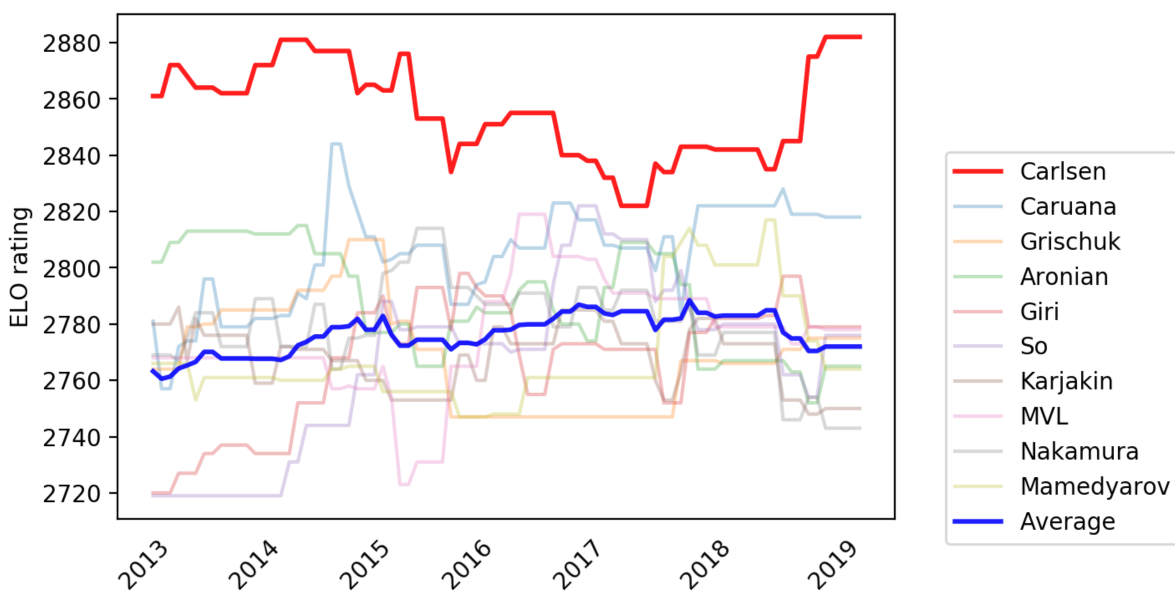
Average Elo: 2724 < = > Cat: 19

gm = 3.24 m = 1.44

(45 Games)

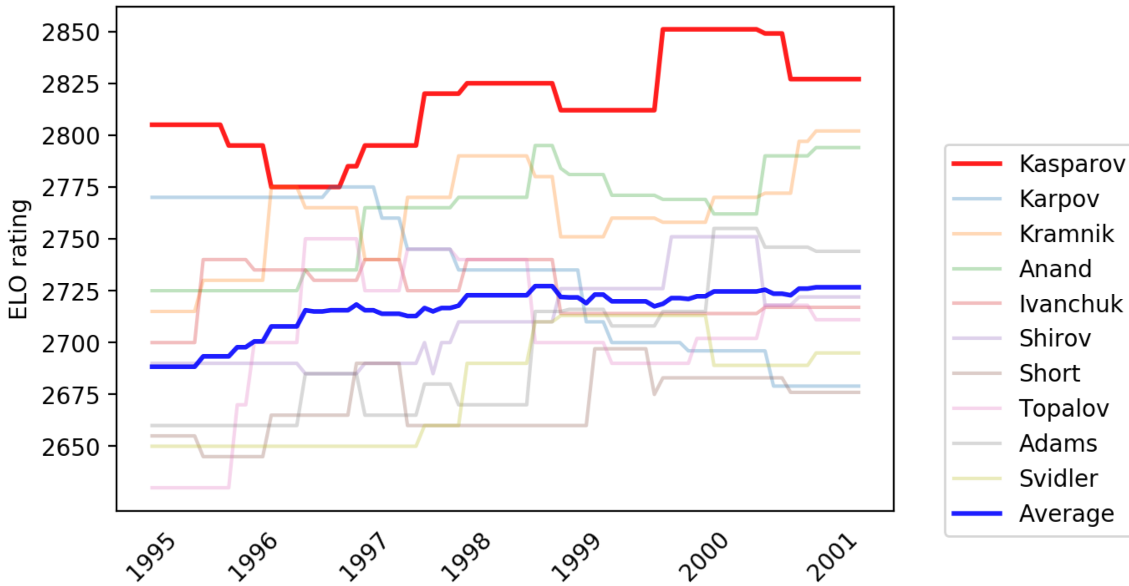
Note: The tournament table is obtained from Chessbase Mega Database 2020.

Figure A.2: Elo ratings of top chess players between 2013-2019.



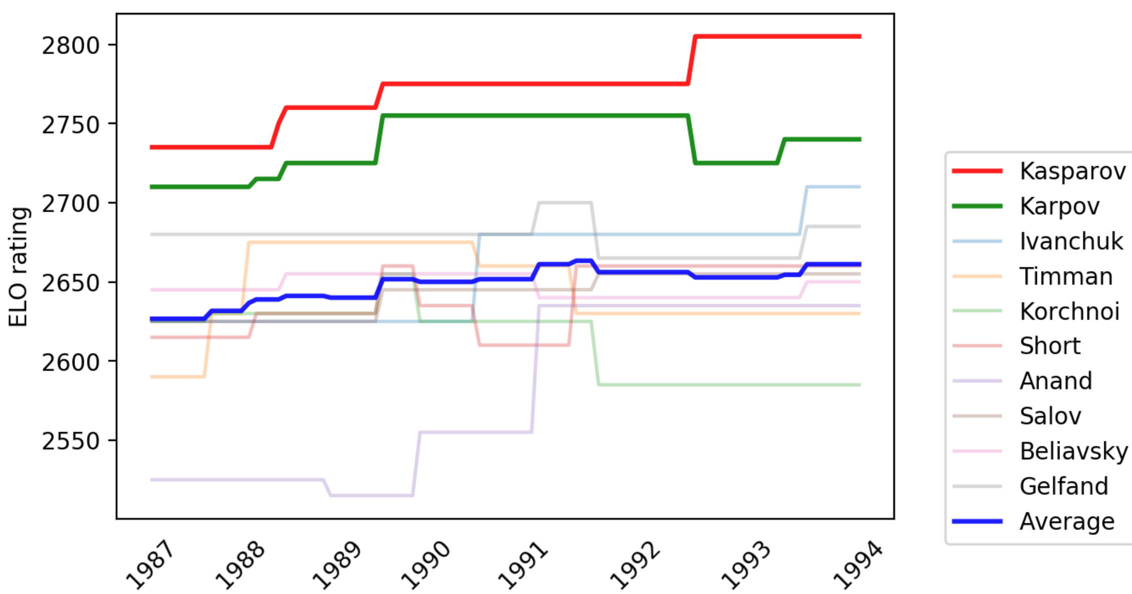
Note: The blue line shows the average Elo rating of top chess players other than Carlsen (World ranking 2–10). Elo rating data is obtained from Chessbase Mega Database 2020.

Figure A.3: Elo ratings of top chess players between 1995-2001.



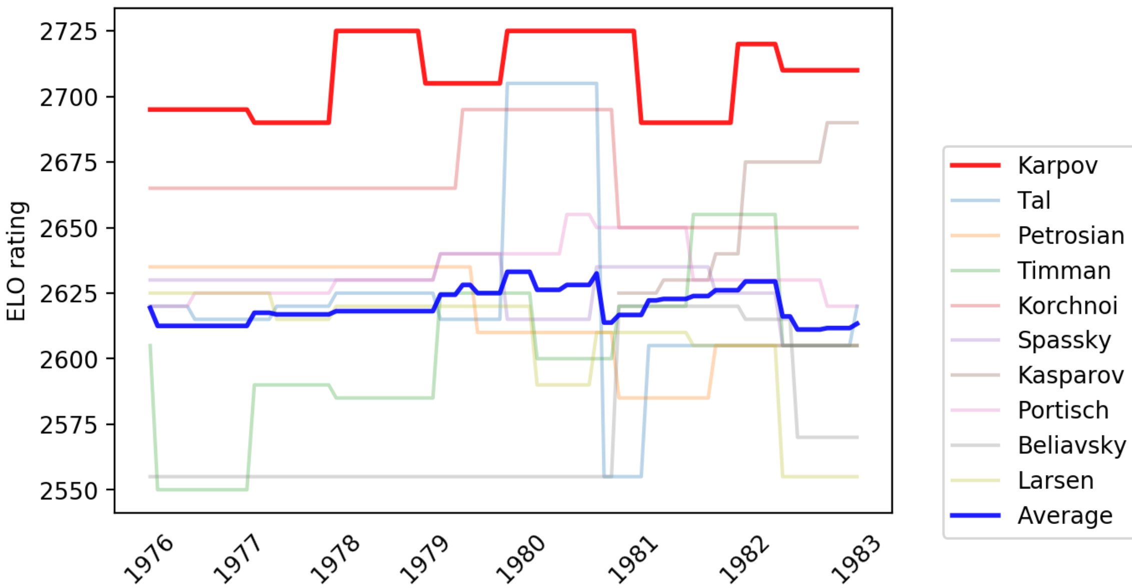
Note: Elo rating data is obtained from Chessbase Mega Database 2020.

Figure A.4: Elo ratings of top chess players between 1987-1994.



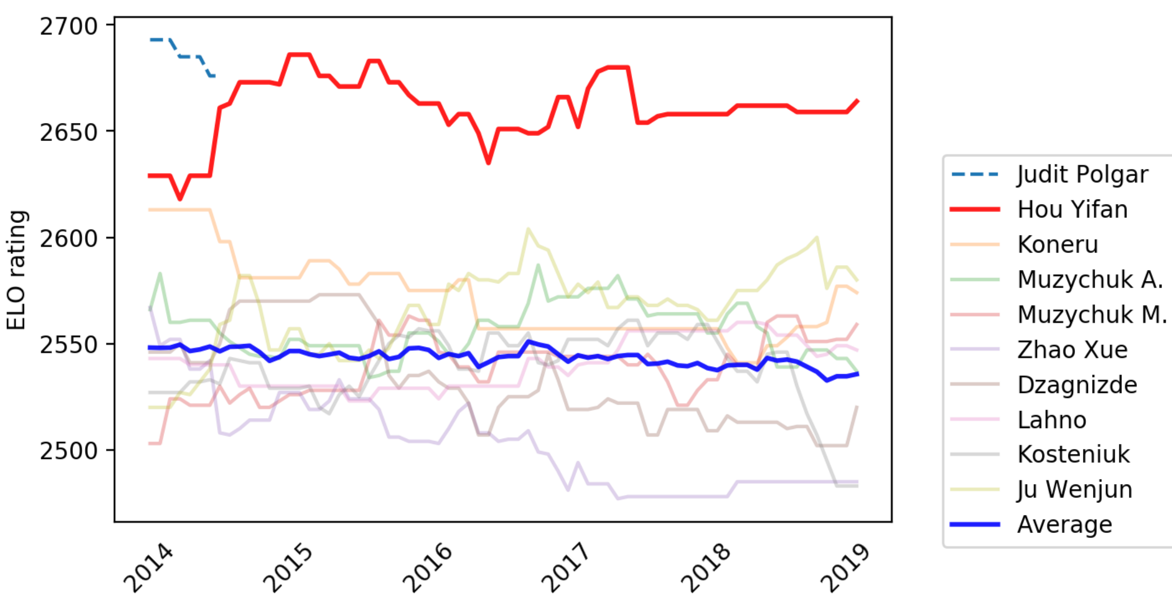
Note: Elo rating data is obtained from Chessbase Mega Database 2020.

Figure A.5: Elo ratings of top chess players between 1976-1983.



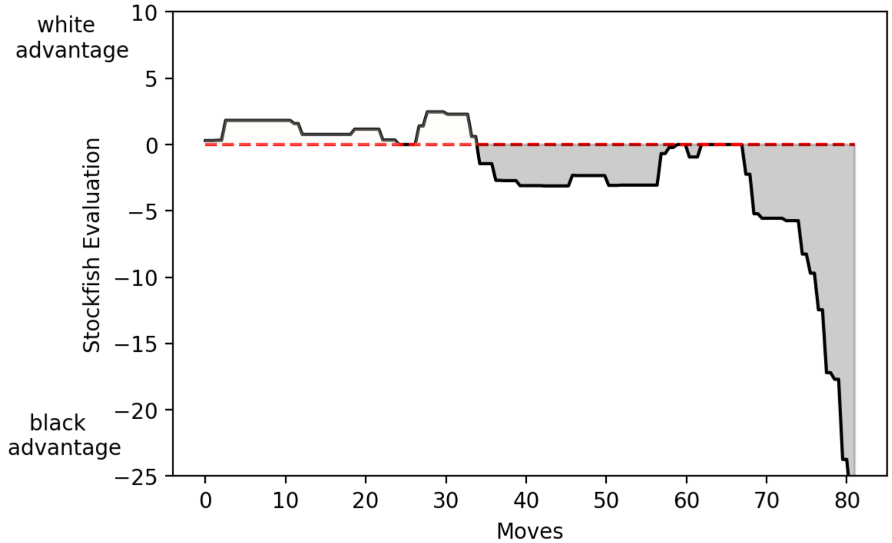
Note: Elo rating data is obtained from Chessbase Mega Database 2020.

Figure A.6: Elo ratings of top female chess players between 2014-2019.



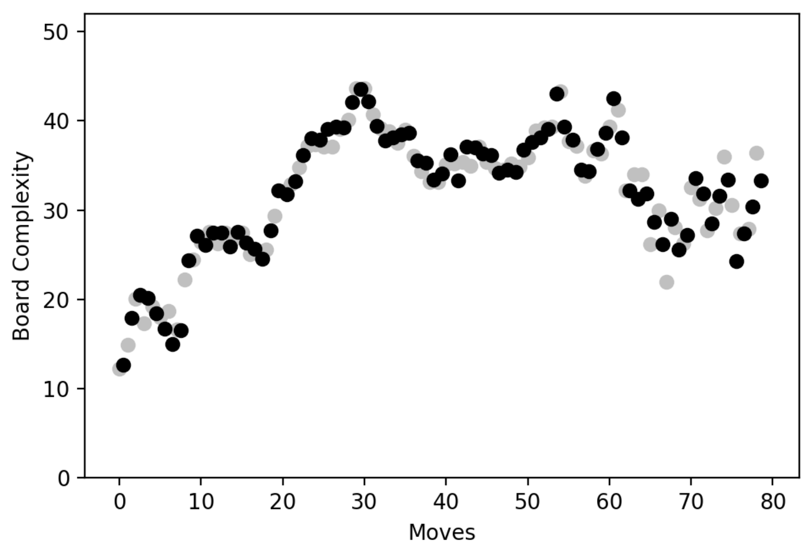
Note: Judit Polgar is considered the strongest female chess player of all time, however she stopped competing in female tournaments in 1990 when she was 14 years old. Hou Yifan stopped competing in female tournaments after 2017. Elo rating data is obtained from FIDE available online at https://ratings.fide.com

Figure A.7: Computer evaluation of a game played by Carlsen in 2019.



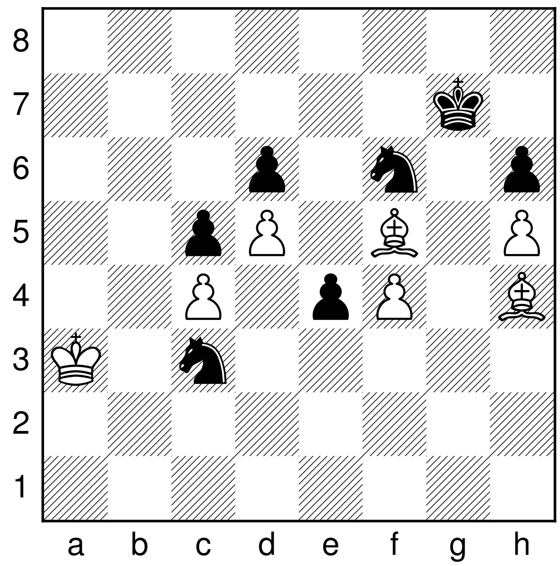
Note: The game was played between Vincent Keymer (White) and Magnus Carlsen (Black) on April 20, 2019 during the first round of Grenke Chess Classic 2019. Keymer's Average Centipawn Loss (ACPL) was 35.22 and Carlsen's 26.17, calculated by using Equation 3. A higher ACPL means the player made more mistakes according to the chess engine. The chess engine used for evaluations is Stockfish 11 with a depth of 19 moves.

Figure A.8: Complexity evaluation of a game played by Carlsen in 2019 using an Artificial Neural Network (ANN) algorithm.



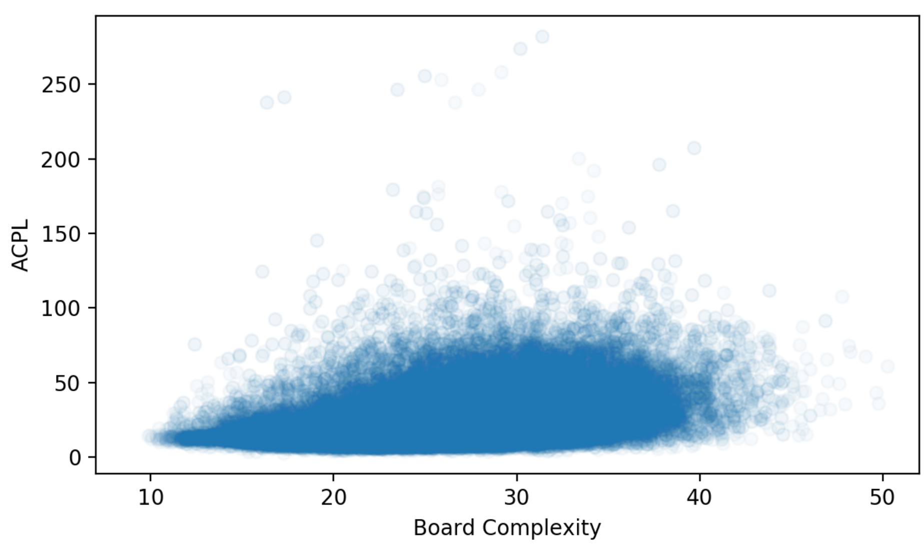
Note: The game was played between Vincent Keymer (White) and Magnus Carlsen (Black) on April 20, 2019 during the first round of Grenke Chess Classic 2019. Keymer's Average Centipawn Loss (ACPL) was 35.22 and Carlsen's 26.17 using our algorithm. Our neural-network board complexity estimate assigns an expected ACPL score of 34.87. This score is substantially higher than the sample average, 26.56. The game is within the top 10% of the sample in terms of complexity.

Figure A.9: A position from Keymer vs. Carlsen (2019).



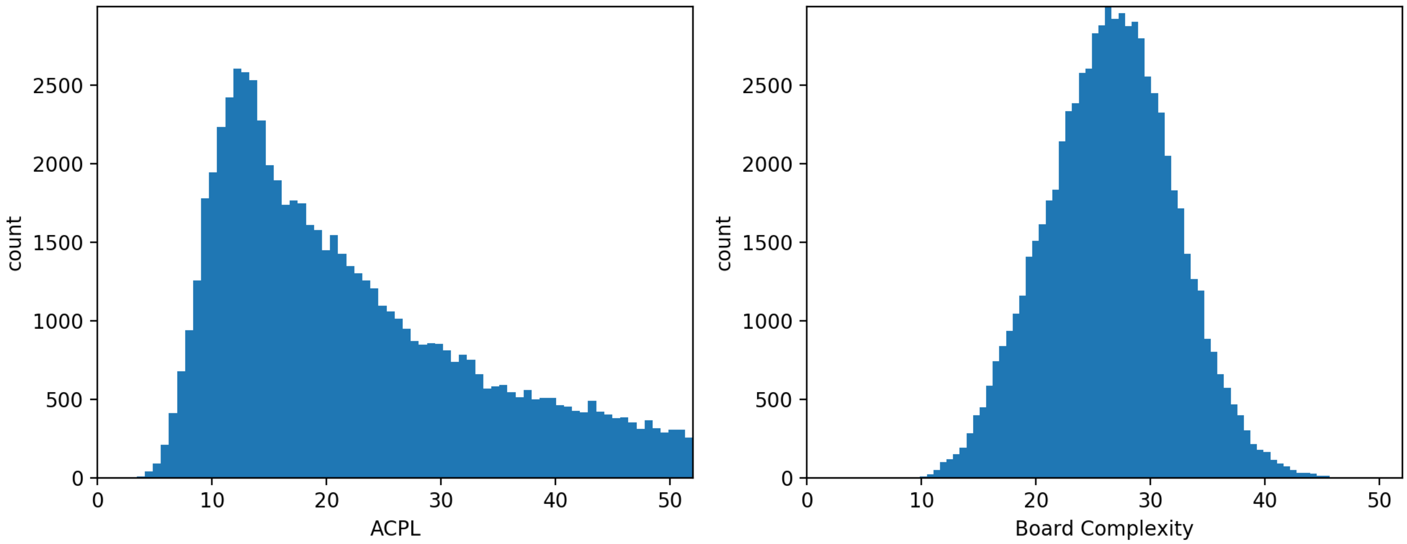
Note: This position is from Vincent Keymer (White) vs. Magnus Carlsen (Black), Grenke Chess Classic 2019 (white to play). Our neural-network algorithm calculates the probability of making an error as 0.52 (about twice as high as the sample average) in an amount of 65 centipawns. In the game, white blundered (by playing Bf2) in an amount of 180 centipawns, according to Stockfish. Before this blunder, the position was a forced draw.

Figure A.10: Scatterplot of board complexity and ACPL scores.



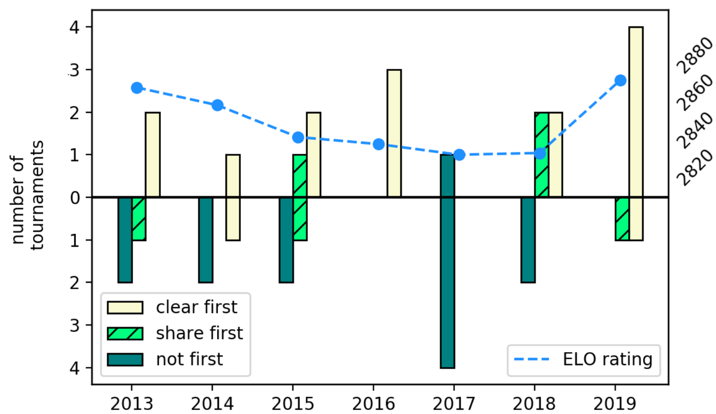
Note: The board complexity measure is obtained via a neural-network algorithm. It is the "expected ACPL score" according to the AI, depending on the complexity of a game. The estimated slope is 1.14 for the overall sample of 32,000 games and 2.1 million moves.

Figure A.11: Distribution of ACPL and board complexity scores.



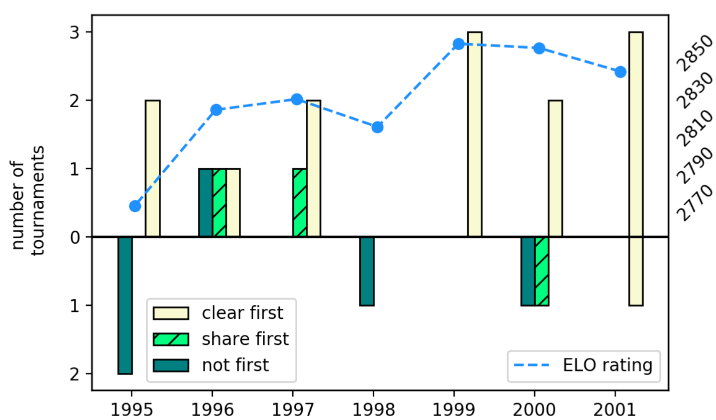
Note: The board complexity measure is obtained via a neural-network algorithm. It is the "expected ACPL score" according to the AI which depends on the complexity of a game. The average ACPL score in the sample is 25.49 and the board complexity score is 26.57 for the overall sample with 32,000 games and 2.1 million moves. The neural-network was trained with an independent sample consisting of 25,000 games and 2 million moves with games played between players with "National Master" ranking on average.

Figure A.12: Carlsen's tournament performance (classical)



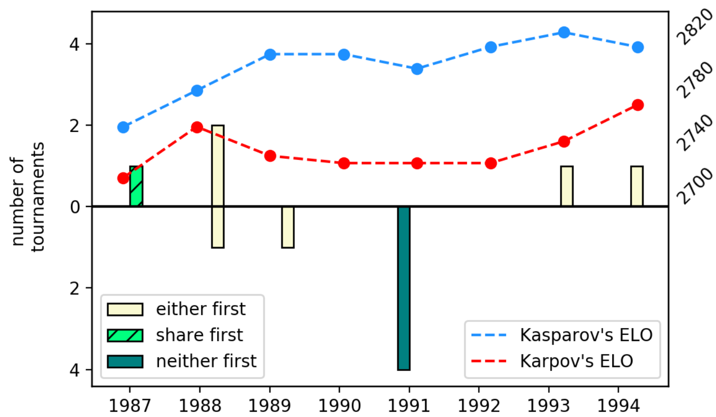
Note: Carlsen's Elo rating data is obtained from FIDE. Bars above (below) count the number of tournaments in which Carlsen gained (lost) Elo rating at the end of the tournament.

Figure A.13: Kasparov's tournament performance (classical)



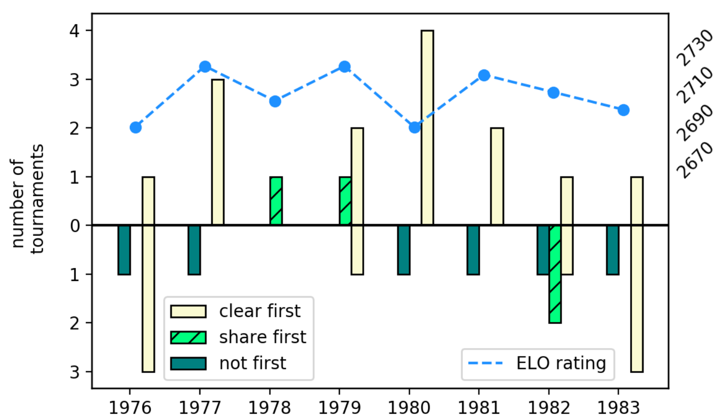
Note: Kasparov's Elo rating data is obtained from Chessbase Mega Database 2020. Bars above (below) count the number of tournaments in which Kasparov gained (lost) Elo rating at the end of the tournament.

Figure A.14: Kasparov and Karpov's tournament performance (classical)



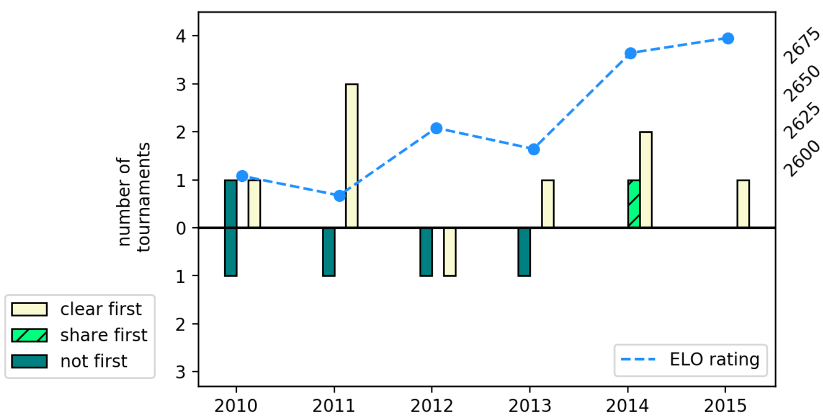
Note: Kasparov's and Karpov's Elo rating data are obtained from Chessbase Mega Database 2020. Bars above (below) count the number of tournaments in which both Kasparov and Karpov gained (lost) Elo rating at the end of the tournament.

Figure A.15: Karpov's tournament performance (classical)



Note: Karpov's Elo rating data is obtained from Chessbase Mega Database 2020. Bars above (below) count the number of tournaments in which Karpov gained (lost) Elo rating at the end of the tournament.

Figure A.16: Hou Yifan's tournament performance (classical)



Note: Hou Yifan's Elo rating data is obtained from Chessbase Mega Database 2020. Bars above (below) count the number of tournaments in which Hou Yifan gained (lost) Elo rating at the end of the tournament.

Table A.17: List of tournaments (classical)

Year	*Tournament Name*
	Panel A. Carlsen Present
2019	GCT Croatia 2019, Grenke Chess Classic 2019, Gashimov Memorial 2019, Norway Chess 2019, Sinquefield 2019, Tata Steel 2019
2018	Gashimov Memorial 2018, Sinquefield 2018, Biel 2018, Norway Chess 2018, Grenke Chess Classic 2018, Tata Steel 2018
2017	London Classic 2017, Norway Chess 2017, Sinquefield 2017, Grenke Chess Classic 2017, Tata Steel 2017
2016	Norway Chess 2016, Tata Steel 2016, Bilbao Masters 2016
2015	London Classic 2015, Sinquefield 2015, Norway Chess 2015, Gashimov Memorial 2015, Grenke Chess Classic 2015, Tata Steel 2015
2014	Norway Chess 2014, Zuerich Chess Challange 2014, Sinquefield 2014, Gashimov Memorial 2014
2013	Moscow Tal Memorial 2013, Norway Chess 2013, Candidates Tournament 2013, Tata Steel 2013, Sinquefield 2013
	Panel B. Carlsen Not Present
2019	U.S. Championship 2019, Dortmund 2019
2018	Candidates Tournament 2018, U.S. Championship 2018, Dortmund 2018
2017	U.S. Championship 2017, Dortmund 2017, Gashimov Memorial 2017
2016	London Classic 2016, Sinquefield 2016, Gashimov Memorial 2016, Candidates Tournament 2016, Moscow Tal Memorial 2016, U.S. Championship 2016, Dortmund 2016
2015	Dortmund 2015, Zuerich Chess Challenge 2015, Tbilisi FIDE GP 2015, Khanty-Mansiysk FIDE GP 2015, Capablanca Memorial 2015, U.S. Championship 2015
2014	Beijing Sportaccord Basque 2014, London Classic 2014, Tashkent FIDE GP 2014, Dortmund 2014, Tata Steel 2014, U.S. Championship 2014, Candidates Tournament 2014, Baku FIDE GP 2014, Capablanca Memorial 2014, Bergomo ACP Golden Classic 2014
2013	Paris FIDE GP 2013, Dortmund 2013, Thessaloniki FIDE GP 2013, Zug FIDE GP 2013, Beijing FIDE GP 2013, Zuerich Chess Challenge 2013, Grenke Chess Classic 2013, Capablanca Memorial 2013, U.S. Championship 2013

Table A.18: List of tournaments (classical)

Year	*Tournament Name*
	Panel A. Kasparov Present
2001	Astana 2001, Zuerich 2001, Linares 2001, Corus Wijk aan Zee 2001
2000	Fujitsu Siemens Giants 2000, Sarajevo Bosnia 2000, Linares 2000, Corus Wijk aan Zee 2000
1999	Sarajevo Bosnia 1999, Linares 1999, Hoogovens Wijk aan Zee 1999
1998	Linares 1998
1997	Tilburg 1997, Novgorod 1997, Linares 1997
1996	Las Palmas 1996, Dos Hermanas 1996, Amsterdam Euwe Memorial 1996
1995	Horgen 1995, Amsterdam Euwe Memorial 1995, Novgorod 1995
	Riga Tal Memorial 1995
	Panel B. Kasparov Not Present
2001	Sigeman & Co 2001, Biel 2001, Dortmund 2001, Pamplona 2001, Dos Hermanas 2001
2000	Japfa Classic 2000, Dortmund 2000, Sigeman & Co 2000, Biel 2000
1999	Pamplona 1999, Lost Boys Amsterdam 1999, Dortmund 1999, Sigeman & Co 1999
	Dos Hermanas 1999, Biel 1999
1998	Hoogovens Wijk aan Zee 1998, Tilburg 1998, Dortmund 1998, Madrid 1998, Pamplona 1998
1997	Hoogovens Merrillville 1997, Hoogovens Wijk aan Zee 1997, Sigeman & Co 1997, Ubeda 1997,
	Dos Hermanas 1997, Lost Boys 1997, Dortmund 1997, Madrid 1997, Belgrade Investbank 1997
1996	Koop Tjuchem 1996, Donner Memorial 1996, Hoogovens Wijk aan Zee 1996,
	Tilburg 1996, Dortmund 1996, Dos Hermanas 1996, Madrid 1996
1995	Belgrade Investbank 1995, Donner Memorial 1995, Biel 1995, Madrid 1995,
	Dos Hermanas 1995, Groningen 1995, Dortmund 1995

Table A.19: List of tournaments (classical)

Year	*Tournament Name*
	Panel A. Kasparov & Karpov Both Present
1994	Linares 1994
1993	Linares 1993
1992	
1991	Reggio Emilia 1991, Tilburg 1991, Amsterdam Euwe Memorial 1991, Linares 1991
1990	
1989	World Cup Skelleftea 1989
1988	USSR Championship 1988, World Cup Belfort 1988, Optiebeurs Amsterdam 1988
1987	Brussels 1987
	Panel B. Kasparov & Karpov Neither Present
1994	Donner Memorial 1994, Dortmund 1994, Hoogovens Wijk aan Zee 1994, Groningen 1994, Munich 1994
1993	Antwerp 1993, Amsterdam VSB 1993, Madrid 1993, Las Palmas 1993, Munich 1993
1992	Alekhine Memorial 1992, Amsterdam Euwe Memorial 1992, Hoogovens Wijk aan Zee 1992, Groningen 1992, Munich 1992
1991	World Cup Reykjavik 1991, Hoogovens Wijk aan Zee 1991, Groningen 1991, Munich 1991
1990	Tilburg 1990, Hoogovens Wijk aan Zee 1990, Prague 1990, Groningen 1990, Munich 1990
1989	Hoogovens Wijk aan Zee 1989, Groningen 1989, Munich 1989, Amsterdam Euwe Memorial 1989
1988	Amsterdam Euwe Memorial 1988, OHRA Amsterdam 1988, Linares 1988, Hastings 1988
1987	Belgrade Investbanka 1987, Hoogovens Wijk aan Zee 1987, Interpolis 1987, OHRA Amsterdam 1987, Reykjavik 1987

Table A.20: List of tournaments (classical)

Year	*Tournament Name*
	Panel A. Karpov Present
1983	Interpolis 1983, International DSB Mephisto GM 1983, USSR Final 1983, Bath 1983, Linares 1983
1982	Interpolis 1982, Turin 1982, Hamburg 1982, London Phillips 1982, Mar del Plata Clarin Masters 1982
1981	IBM Herinnerungs Toernooi 1981, Moscow 1981, Linares 1981
1980	Buenos Aires 1980, Interpolis 1980, IBM Kroongroep 1980, Bugojno 1980, Bad Kissingen 1980
1979	Interpolis 1979, Waddinxveen KATS 1979, Montreal International 1979, GER International 1979
1978	Bugojno 1978
1977	Interpolis 1977, October Revolution 1977, Las Palmas 1977, GER International 1977
1976	USSR Final 1976, Montilla 1976, Manila Marlboro 1976, Amsterdam 1976, Skopje Solidarnost 1976
	Panel B. Karpov Not Present
1983	Jakarta International 1983, Hoogovens Wijk aan Zee 1983
1982	Bugojno 1982, Moscow Interzonal 1982, Las Palmas Interzonal 1982, Toluca Interzonal 1982, Niksic International 1982, Hoogovens Wijk aan Zee 1982
1981	Las Palmas 1981, Interpolis 1981, Hoogovens Wijk aan Zee 1981
1980	Buenos Aires 1980, London Phillips 1980, Hoogovens Wijk aan Zee 1980, Las Palmas 1980, Reykjavik International 1980
1979	Buenos Aires Clarin 1979, Riga Interzonal 1979, Buenos Aires Interzonal 1979, Vidmar Memorial 1979, IBM 1979, Hoogovens Wijk aan Zee 1979, Buenos Aires Konex 1979
1978	Interpolis 1978, Reykjavik International 1978, Hoogovens Wijk aan Zee 1978, Las Palmas 1978 IBM 1978, Clarin 1978
1977	Geneve 1977, Vidmar Memorial 1977, Hoogovens Wijk aan Zee 1977, IBM 1977
1976	Interzonal 1976, Las Palmas 1976, Reykjavik International 1976, Hoogovens Wijk aan Zee 1976, IBM 1976

Table A.21: List of tournaments (classical)

Year	*Tournament Name*
	Panel A. Fischer Present
1970	Interzonal 1970, Buenos Aires 1970, Rovinj Zagreb 1970
1969	
1968	Vinkovci 1968, Nathanya 1968,
1967	Skopje 1967, Monaco Grand Prix 1967
1966	Piatigorsky Cup 1966, U.S. Championship 1966
1965	U.S. Championship 1965, Capablanca Memorial 1965
1964	
1963	U.S. Championship 1963
1962	U.S. Championship 1962, Candidates Tournament 1962, Interzonal 1962
	Panel B. Fischer Not Present
1970	Vinkovci 1970, IBM Amsterdam 1970, Budapest 1970, Sarajevo 1970, Caracas 1970, Hoogovens Wijk an Zee 1970, Costa del Sol 1970, Skopje 1970, Rubinstein Memorial 1970, Christmas Congress 1970
1969	Monaco Grand Prix 1969, Hoogovens Wijk an Zee 1969, Venice 1969 U.S. Championship 1969, Palma de Mallorca 1969, IBM Amsterdam 1969, Sarajevo 1969, Christmas Congress 1969, Rubinstein Memorial 1969, Capablanca Memorial 1969
1968	Rubinstein Memorial 1968, Christmas Congress 1968, Palma de Mallorca 1968, U.S. Championship 1968, Bamberg 1968, IBM Amsterdam 1968, Sarajevo 1968 Hoogovens Wijk an Zee 1968, Monaco Grand Prix 1968, Skopje 1968
1967	Winnipeg 1967, October Revolution Leningrad 1967, October Revolution Moscow 1967, Capablanca Memorial 1967, Palma de Mallorca 1967, Sarajevo 1967, Hoogovens Beverwijk 1967, Christmas Congress 1967, Rubinstein Memorial 1967, Venice 1967, IBM Amsterdam 1967
1966	IBM Amsterdam 1966, Sarajevo 1966, Palma de Mallorca 1966 Hoogovens Beverwijk 1966, Venice 1966, Rubinstein Memorial 1966, Christmas Congress 1966
1965	ZSK International 1965, Zagreb 1965, Mer del Plata 1965, IBM Amsterdam 1965, Sarajevo 1965, Hoogovens Beverwijk 1965, Christmas Congress 1965, Rubinstein Memorial 1965
1964	Buenos Aires 1964, Capablanca Memorial 1964, Rubinstein Memorial 1964, Interzonal 1964, IBM Amsterdam 1964, Sarajevo 1964, Hoogovens Beverwijk 1964, Christmas Congress 1964, ZSK International 1964
1963	Piatigorsky Cup 1963, Alekhine Memorial 1963, IBM Amsterdam 1963, Sarajevo 1963, Hoogovens Beverwijk 1963, Rubinstein Memorial 1963, Christmas Congress 1963, Capablanca Memorial 1963
1962	Mer del Plata 1962, Sarajevo 1962, Hoogovens Beverwijk 1962, Rubinstein Memorial 1962, Christmas Congress 1962, Capablanca Memorial 1962

Table A.22: List of tournaments (classical)

Year	*Tournament Name*
	Panel A. Hou Yifan Present
2015	Monte Carlo FIDE GP 2015
2014	Lopota FIDE GP 2014, Khanty-Mansiysk FIDE GP 2014, Sharjah FIDE GP 2014
	Panel B. Hou Yifan Not Present
2019	Skolkovo FIDE GP 2019, Saint Louis Cairns Cup 2019
2016	Khanty-Mansiysk FIDE GP 2016, Chengdu FIDE GP 2016, Batumi FIDE GP 2016, Tehran FIDE GP 2016