Bimodal Transit Design with Heterogeneous Demand Elasticity under Different Fare Structures

Yi Yang [a], Xinguo Jiang [a, b], Yusong Yan [a, c], Tao Liu [a, d, *], Yu Jiang [e,f]

[a] National Engineering Laboratory of Integrated Transportation Big Data Application Technology, School of Transportation and Logistics, Southwest Jiaotong University, Chengdu 611756, China

[b] Fujian University of Technology, Fuzhou 350118, China

[c] Chengdu Technological University, Chengdu 610031, China

[d] Institute of System Science and Engineering, School of Transportation and Logistics, Southwest Jiaotong University, Chengdu 611756, China

[e] Lancaster University Management School, Lancaster University, LA1 4YX, United Kingdom

[f] Transport Division, Department of Technology, Management and Economics, Technical University of Denmark 2800, Denmark

Abstract

Bimodal transit systems (e.g., rail-bus system) have the potential to better cater to passenger demand than conventional unimodal transit systems (e.g., bus-only system). However, the optimal design of bimodal transit systems considering heterogeneous demand elasticity has received little attention in the literature. The study attempts to develop a new optimization model to design a bimodal transit system from a microeconomic view to maximize the profit of a transit agency considering different fare structures. Transit network and service characteristics, such as route and stop densities and headways, can be modified to provide a better service considering the heterogeneous demand elasticity in real-world situations. An elastic demand function is devised to include various time components and incorporate flat, distance-based, and hybrid fares. A nested iterative procedure is developed to find the optimal or near-optimal solution. Numerical experiments reveal some interesting findings. First, the increase in elasticity parameters has a knock-on effect on financial performance, consequently leading to a net profit reduction. Second, a distance-based fare scheme brings in the least actual demand but makes the most profit compared with the flat and hybrid fare schemes. Third, passengers prefer using a rail-bus system to a BRT-bus system, especially at a higher demand level. Meanwhile, bimodal systems can bring the transit agency a higher profit except in an extremely low latent demand scenario. A rail-bus system is the most profitable form when the potential demand exceeds 58,009 pass/h. Thus, bimodal transit systems are generally preferred for cities with a high demand density as opposed to unimodal transit systems.

Keywords: Public transit, bimodal transit, demand elasticity, profit maximization, transit fare

* Corresponding author, E-mail: tao.liu@swjtu.edu.cn

1. Introduction

1.1 Background

Developing an attractive urban public transit system is an efficient way to alleviate traffic congestion, reduce air pollution, and promote sustainable urban mobility. The traditional unimodal transit system (e.g., bus) is becoming inadequate to satisfy the ever-increasing and diversified travel demand. Hence, bimodal transit systems (e.g., rail-bus system) are being developed in many megacities around the world, such as Beijing, Paris, San Francisco, and Singapore (Vuchic, 2007; Gao et al., 2018; Li et al., 2020; Jiang et al., 2022). A bimodal system generally consists of two interworking transit systems, i.e., express and local transit systems. Express transit typically refers to a mode with high-speed and high-capacity, such as rail or bus rapid transit (BRT), but it is not cost-effective to be constructed densely due to its high investment. A typical local transit is the conventional bus transit, which features low investment along with low operation speed and capacity. Considering that the performance of a bimodal transit system may vary under different circumstances, this study aims to develop a new optimization model for the joint design of a bimodal transit system, particularly taking into account the effect of different fare structures and elastic demand simultaneously.

1.2 Literature review

1.2.1 Joint design of bimodal transit systems

Over the past decades, numerous research works have been conducted to find the optimal design of public transit systems. Most of them contribute to the unimodal transit system design. Some studies focused on a bus transit system design (e.g., Ceder and Wilson, 1986; Chang and Schonfeld, 1991; Guihaire and Hao, 2008; Kepaptsoglou and Karlaftis, 2009; Medina et al., 2013; Szeto and Jiang, 2014; Jiang and Szeto, 2015; Chen et al., 2017; Liu et al., 2017; Luo et al., 2020; Dou et al., 2021; Guo and Szeto. 2021), while some studies put emphasis on a rail transit system (e.g., Schobel, 2005; Repolho et al., 2013; Saidi et al., 2017; Gkiotsalitis and Cats, 2021; Wu and Schonfeld, 2022). Mathematical programming and analytic models are usually developed to obtain the optimal system characteristics (e.g., station/stop location, service frequency, timetable, and vehicle schedule) under specified assumptions.

In contrast, the joint design of bimodal transit systems needs to account for various parameters of two different transit modes as well as their interaction effects, which is very complex and cumbersome. A pioneering work was conducted by Wirasinghe (1980), where an approximate analytic model was formulated to optimize the feeder line density, feeder headway, and trunk station density under a peak-period many-to-one demand pattern. Chien and Schonfeld (1998) further incorporated the feeder stop density into the decision variables. Sivakumaran et al. (2012) optimized the service frequencies of both trunk and feeder lines in an idealized trunk-feeder system, and Sivakumaran et al. (2014) later modeled feeder lines that intersected trunk lines in a rectangular city. By maximizing the rail ridership and minimizing total passenger travel time, Sun et al. (2013) designed an integrated rail-bus transit system. Sun et al. (2017) devoted to designing the optimal length of a rail line when a single bus service was replaced by a rail-bus system under a many-to-one travel demand pattern. Fan et al. (2018) developed a bi-level continuum optimization model to design a bimodal transit system in a symmetrical grid form. With the objective of minimizing the total system cost, Li et al. (2020) jointly designed bike-sharing feeders and local-express lines in a bimodal transit system with a grid street pattern.

1.2.2 Demand elasticity

The above studies on bimodal transit design are usually conducted under the assumption of fixed travel demand. The assumption can facilitate the model formulation and solution, which is conducive to exploring the relationship between design parameters and system performances. However, in practice, passenger demand may be elastic due to the impacts of the level of service (LOS) and different fares.

Table 1 summarizes and compares the existing studies on transit design considering demand elasticity. Kocur and Hendrickson (1982) developed a linear elastic demand function to explore the effects of travel time and fare on passenger demand, which indicated an effective optimization approach to transit design with demand elasticity. Later, Chang and Schonfeld (1991) proposed multiple period models to find the closed form solutions for the optimal route spacing, headway, and fare. With an exponential elastic demand function, Yang and Kin (2000) adopted a demand-supply equilibrium model to identify how a bus route could be profitable. Deng et al. (2014) incorporated an exponential demand function into a model to address the effect of train formation length and service frequency on determining train timetables. Considering a linear elastic demand function, Chien and Tsai (2007) optimized service frequency and zone fare, while Tsai et al. (2008) optimized service frequency and station-based fare. Li et al. (2012) designed a rail transit line considering a linear elastic demand, a many-to-one uniform pattern. Kim and Schonfeld (2015) considered the elasticity demand for a bus system with fixed and flexible routes. Huang et al. (2016) formulated an optimization model to determine the bus transit fare and service frequency of an OD-based fare structure with a linear demand function. Tang et al. (2017) proposed an optimization approach with a linear elastic demand to design a bus transit system by providing a profitable fare structure and operational strategy. Sun and Szeto (2019) developed a bilevel programming model with elastic demand to jointly determine the fare and service frequency to maximize transit operators' profit. Yang et al. (2021) formulated both one-vehicle and two-vehicle models considering elastic demand to design optimal demand-responsive connector services. Qu et al. (2021) proposed a mathematical model to optimize the bus service by maximizing ridership under demand elasticity.

To sum up, when considering demand elasticity, the objective functions of optimization models are typically in the form of maximization. The objective function includes maximizing social welfare, operator profit, consumer surplus, or ridership. Most studies consider fare, mainly distance-based, station-based, flat, or zone-based fare schemes. Two commonly used demand functions are exponential and linear forms. In addition, most of them are focused on unimodal transit systems (e.g., rail or bus), while little attention was paid to the joint design of bimodal transit systems with elastic demand.

Table 1. Summary of existing and our studies on transit design considering demand elasticity.

Authors (year)	Objective	Decision variable	Demand	Fare structure	Transit mode
Kocur and Hendrickson (1982)	Max operator profit and user benefit	Route spacing, headway, and fare	Many-to-one elastic demand, linear	Flat fare	Unimodal: bus
Chang and Schonfeld	Max operator profit and	route spacing, headway, and	Many-to-one elastic demand,	Flat fare	Unimodal: bus

(1991)	social welfare	fare	linear		
Yang and Kin (2000)	Max social welfare and operator profit	Bus fleet size and fare	Elastic OD pair, exponential	Distance-based fare	Unimodal: bus
Chien and Tsai (2007)	Max operator profit	Headway and fare	Zone-to-zone elastic demand, linear	Zone fare	Unimodal: rail
Tsai et al. (2008)	Max operator profit	Headway and fare	Station-to-station elastic demand, linear	Station-based fare	Unimodal: rail
Li et al. (2012)	Max operator profit	Station location, line length, headway, and fare	Many to one and uniform elastic demand, linear	Flat and distance-based fares	Unimodal: rail
Tsai et al. (2013)	Max operator profit	Headway, fare	Station-to-station elastic demand, linear	Station-based fare	Unimodal: rail
Deng et al. (2014)	Max operator benefit and Min travel cost	Train formation length and service frequency	One-to-one elastic demand, exponential	Fixed fare	Unimodal: rail
Kim and schonfeld (2015)	Max social welfare	Headway, fleet size, fare, zones, route spacing	Time-dependent and uniform elastic demand, linear	Flat fare	Unimodal: bus
Huang et al. (2016)	Max social welfare	Frequency, effort level, fare	Elastic OD pair, linear	OD-based fare	Unimodal: bus
Tang et al. (2017)	Max social welfare	Operating strategy and fare	Elastic OD pair, linear	Flat and stop-based fares	Unimodal: bus
Sun and Szeto (2019)	Max operator profit	Route frequency, OD demand, probability of approach to be adopted, fare	Elastic OD pair, linear	Flat, sectional, and distance-based fares	Unimodal: bus
Yang et al. (2021)	Max operator profit and social welfare	service area, operating cycle, and fare	Uniform elastic demand, linear	Flat fare	Unimodal: bus
Qu et al. (2021)	Max ridership	service patterns and frequencies, the skipped stops	Elastic OD pair, linear	Flat fare	Unimodal: bus
This study	Max operator profit	Headways, station/stop density, local line density, fares	Many to one and heterogeneous elastic demand, linear	Flat, distance-based, and hybrid fares	Bimodal: rail/BRT-bus

1.2.3 Transit fare

It is generally acknowledged that transit fares directly impact passenger travel demand.

The fare is crucial to the financial situation of a transit agency. Therefore, from the perspectives of passengers and agencies, fare is a key factor in transit system design and needs to be taken into account. As discussed in previous works (e.g., Chien and Tsai, 2007; Kim and Schonfeld, 2015; Tang et al., 2017; Yang and Lim, 2017; Palma et al., 2017; Tang et al., 2019; Sun and Szeto, 2019; Tang et al., 2020; Verhoef, 2020; Yang et al., 2020), transit fares generally have different fare structures, such as flat fare, distance-based fare, and time differentiated fare. A time differentiated fare structure is related to the operation time and usually charges a higher fare for peak periods. A flat fare charges the same price for all trips. For a distance-based fare structure, the charge is related to trip length. The flat fare is normally regarded as the most convenient fare scheme, while the distance-based fare is more equitable than the flat one, and thus the distance-based fare is the most commonly adopted one in practice.

Table 1 summarizes the related transit design studies incorporating transit fare into the model formulation. Most of them only considered a single fare while losing the opportunity to compare design parameters and system performances under different fare structures. Furthermore, most studies considering fare design are mainly for a unimodal transit system, while bimodal systems are rarely studied.

1.3 Research gap, contributions, and organization

The above literature review indicates some research gaps in the existing studies. First, limited works that focus on the bimodal transit system design are generally performed to minimize system costs, including agency, user, and generalized costs. Moreover, it may be necessary to address the joint design issues from a profit point of view since the profit may affect operation strategies and policy-making. To the best of our knowledge, no such studies focused on conducting the joint design of a bimodal system considering profit maximization. Second, all the joint design studies are conducted with fixed passenger demand. This may not be true in practice because the demand may be sensitive to multi-perception factors, such as travel cost, travel time, and the number of transfers. Third, the pros and cons of flat and distance-based fares are intensively examined in the unimodal system, but the interrelations between the fare structure and bimodal system design are yet to be explored.

To bridge these research gaps, this study aims to jointly design a bimodal transit system considering heterogeneous demand elasticity. A profit maximization model is formulated from the perspective of a transit agency, considering that public transit services may be provided by profit-driven private transit agencies in some cities such as Hong Kong (Lam and Zhou, 2000; Li et al., 2012; Sun and Szeto, 2019; Guo and Szeto, 2021). Most transit services in Hong Kong are provided by private agencies, including Citybus, Long Win Bus, New World First Bus, Kowloon Motor Bus, and New Lantau Bus.

The main contributions of the study are threefold. First, to the best of our knowledge, the study appears to be the first one devoted exclusively to jointly designing a bimodal transit system to maximize profit, considering heterogeneous demand elasticity under different fare structures. Second, a heterogeneous environment is the premise of the study, which may effectively improve the applicability of the model and results. That is, the network characteristics (e.g., route and station densities) can vary to better satisfy the passenger demand. Third, the study evaluates the effects of key factors (e.g., demand density and elasticity parameters) on the optimal system design.

The remainder of the paper is organized as follows. Section 2 presents the details of the proposed model, mainly including the demand, fare, investment, revenue, and net profit. Section 3 describes the solution procedure. Section 4 conducts two numerical scenarios to examine the effectiveness of the model. Section 5 performs sensitivity analyses to further demonstrate the performances of the model and solution method. Section 6 concludes the work and enlightens some promising future research directions.

2. Model formulation

2.1. Problem statement

Inspired by previous studies (e.g., Sivakumaran et al., 2012; Jara-Díaz and Muñoz-Paulsen, 2021; Luo and Kang, 2022), we consider a bimodal transit system with one CBD area (Fig. 1). It consists of two parts: an express line, denoted by a thick line with length L, which runs in x-direction to CBD at location $(L, 0)$, and local lines, represented by the thin lines with length $l_i(x)$, which connects the express transit stations. i = 1 or 2 represents the upper or lower side of the express line, respectively.

The transit service considered in this study is heterogeneous; that is, the transit system characteristics such as route/station spacing are distributed arbitrarily and may not uniformly appear over the space, which may considerably reflect a real-world situation. Therefore, stations and lines may be theoretically placed at any location. Meanwhile, local transit vehicles may operate vertically and then turn parallel upon reaching the express line to access the station. Let $\phi(x), \psi_i(x)$, and $\phi_i(x,y)$ represent the densities of express stations, local lines, and local stops, respectively.

To facilitate the presentation of the essential ideas, in line with the previous studies (e.g., Lam and Huang, 1995; Chien and Yang, 2000; Sivakumaran et al., 2012; Li et al., 2012; Dijoseph and Chien, 2013; Sun et al., 2017; Guo et al., 2018; Tian et al., 2021), the following assumptions are made in this study.

A1: The passenger demand is assumed to follow a many-to-one demand pattern with all passengers bound for the CBD, which typically indicates the morning peak hour or gathering events. Thus, the study focuses on a one-hour peak-period design.

A2: The passenger demand is assumed to be elastic, which is sensitive to the level of service and different fares.

A3: Passengers are assumed to walk to the nearest local route in terms of access time and take local buses to transfer to the express line to accomplish their travel.

A4: The average passenger waiting time is assumed to be half the headway when vehicles are operated regularly (i.e., the variance of vehicle time headways is zero), passengers randomly arrive at stops, and the arrival process is independent with the vehicle arrival process (Chien and Tsai, 2007; Daganzo, 2010; Li et al., 2012; Sivakumaran et al., 2014; Kim and Schonfeld, 2015; Gu et al., 2016; Mei et al., 2021).

A5: The operations of express and local transit vehicles are assumed to be regular without random disturbance. Potential travel demand is heterogeneously distributed over the service region with density $D_i^0(x,y)$. With reference to the existing literature (e.g., Lam and Huang, 1995; Chien and Yang, 2000; Li et al., 2012; Dijoseph and Chien, 2013; Guo et al., 2018; Tian et al., 2021), passenger trips are distributed in a many-to-one pattern, with all passengers taking local vehicles to transfer to the express line and ultimately travel to the CBD. A case like this might appear during peak hours, such as the morning rush. It is assumed that passengers always go straight to the nearest local line upon accessing local services. Table 2 lists the main notations used in the model formulation.

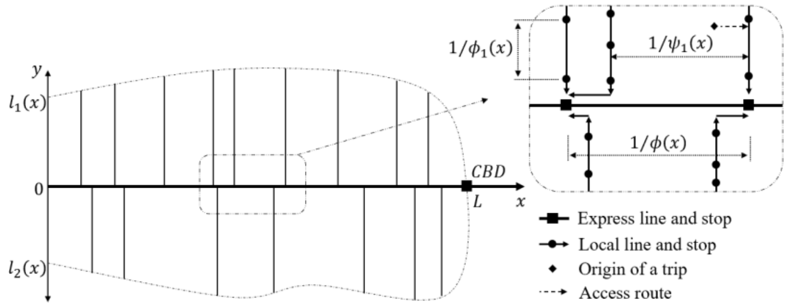


Fig. 1. Illustration of a hypothetical bimodal transit system in a linear city.

Table 2. Key notations with definitions and units.

Notation	Definition	Unit
Decision variables		
$\phi(x)$	Express stop density	station/km
$\psi_i(x)$	Local line density (side i)	line/km
$\phi_i(x,y)$	Local stop density	stop/km^2
H	Express service headway	hour
$h_i(x)$	Local service headway	hour
$\bar{F}_i$	Flat fare	$/pass
$\tilde{F}_i$	Fixed component of a distance-based fare	$/pass
λ_i	Variable component of a distance-based fare	$/km
Auxiliary variables and parameters		
$D_i^0(x,y)$	Potential demand density	pass/km^2-h
$D_i(x,y)$	Actual demand density	pass/km^2-h
$e_i^U, e_i^W, e_i^T, e_i^F$	Elasticity parameters	1/h
$U_i(x,y)$	Passengers' access time	h
$W_i(x,y)$	Passengers' wait time	h
$T_i(x,y)$	Passengers' in-vehicle travel time	h
$F_i(x,y)$	Fare for a trip	$
P	Transit agency's net profit	$
R	Transit agency's operating revenue	$
I	Transit agency's infrastructure expenditure	$
O	Transit agency's operating expenditure	$
π_s^{ex}, π_s^{lo}	Unit costs per express and local stops	$/stop
π_l^{ex}, π_l^{lo}	Unit costs per km of line infrastructure for express and local systems	$/km
$\pi_{tb}^{ex}, \pi_{tb}^{lo}$	Unit costs per vehicle-hour traveled	$/veh-h
$\pi_{db}^{ex}, \pi_{db}^{lo}$	Unit costs per vehicle-km traveled	$/veh-km
$d_i(x,y)$	Distance of a trip	km
ε	Value of time	$/h
v_U	Average walking speed	km/h
v_{ex}, v_{lo}	Cruising speeds of express and local transit vehicles, respectively	km/h
C^{ex}, C^{lo}	Vehicle capacities of express and local services, respectively	pass/veh

2.2. Demand characteristics

Under demand elasticity, the demand density is generally sensitive to the level of service (LOS) and fare structure. LOS is the service quality perceived by passengers, which can be reflected by some service factors, such as travel time. Multi-form linear demand functions are applied in bus transit system design (Kim and Schonfeld, 2015; Sun and Szeto, 2019) and rail transit system design (Chien and Tsai, 2007; Li et al., 2012). A typical form is:

$$D_i(x,y) = \delta D_i^0(x,y) \tag{1}$$

where $D_i^0(x,y)$ represents the potential demand density at location (x,y) in the service region, while the actual demand is denoted by $D_i(x,y)$. δ is an elasticity coefficient measuring how demand is sensitive to the travel impedance.

To better reflect the impact of LOS on demand, various time components are considered in the bimodal system, and a linear demand function is formulated as:

$$D_i(x,y) = \left(1 - e_i^U U_i(x,y) - e_i^W W_i(x,y) - e_i^T T_i(x,y) - e_i^F F_i(x,y)\right) D_i^0(x,y) \tag{2}$$

where the content in the bracket denotes the elasticity coefficient δ in Eq. (1). $F_i(x,y)$ is the fare that a passenger pays to complete a trip. $U_i(x,y)$ represents the passenger access time from location (x,y) to the nearest transit; $W_i(x)$ is the wait time at location (x,y); $T_i(x,y)$ is the in-vehicle time experienced by an on-board passenger. Coefficients e_i^U, e_i^W, e_i^T, and e_i^F, are the elasticity parameters for the corresponding time components. The time components can be specified as:

$$U_i(x,y) = \frac{d_i^U(x,y)}{v_U} \tag{3}$$

$$W_i(x,y) = \mu\left(H + h_i(x,y)\right) \tag{4}$$

$$T_i(x,y) = \frac{L-x}{v_{ex}} + \tau^{ex} N^{ex} + \frac{1}{4\phi(x) v_{lo}} + \frac{y}{v_{lo}} + \tau^{lo} N^{lo} \tag{5}$$

where $U_i(x,y)$ is determined by the walking distance $d_i^U(x,y)$ between location (x,y) and the nearest stop with walking speed v_U. Following previous studies (e.g., Chien and Tsai, 2007; Daganzo, 2010; Sivakumaran et al., 2014; Gu et al., 2016; Mei et al., 2021), $d_i^U(x,y)$ can be estimated as $d_i^U(x,y) = 1/4\psi_i(x) + 1/4\phi_i(x,y)$. In Eq. (4), H and $h_i(x)$ represent the headways of express and local services, respectively. μ is a calibration parameter specified as 0.5 when vehicles operated regularly according to previous studies (Chien and Tsai, 2007; Daganzo, 2010; Li et al., 2012; Sivakumaran et al., 2014; Kim and Schonfeld, 2015; Gu et al., 2016; Mei et al., 2021). In Eq. (5), the in-vehicle time includes the travel time between stops and the delay at stops, where the travel time is determined by the distance and average vehicle speed, and the delay is related to the number of stops, which is adapted from the previous works (e.g., Daganzo, 2010; Sivakumaran et al., 2014; Gu et al., 2016; Mei et al., 2021). v_{ex} and v_{lo} are the cruising speeds of express and local transit vehicles, respectively. N^{ex} and N^{lo} represent the number of express and feeder stops to ride, which are given by $\int_x^L \phi(x)dx$ and $\int_0^y \phi_i(x,y)dy$, respectively. The third item in Eq. (5) is the in-vehicle time spent on the local line in x direction. $1/4\phi(x)$ denotes the average length of the local line in the x direction. τ^{ex} and τ^{lo} are the average delays per stop along express and local routes, respectively.

2.3. Financial characteristics

The net profit P is the result of the total operating revenue minus the total investment, which can be formulated as:

$$P = R - A \tag{6}$$

where R and A are the operating revenue and investment of a transit agency, respectively. In the following context, we present the formulation of the operating revenue for the transit agency.

As a key factor in the financial elements of the transit system, revenue R is the sum of the passenger demand multiplied by the related fare.

$$R = \sum_{i=1}^{2} \int_0^L \int_0^{l_i(x)} D_i(x,y) F_i(x,y) dxdy \tag{7}$$

where the actual passenger demand $D_i(x,y)$ is derived from the demand function Eq. (2). Transit fare, $F_i(x,y)$, is a crucial factor in urban transit planning, which affects passenger demand and acts on the financial situation of the operator. Generally, it consists of flat and differentiated fares. A flat fare is a single ticket price for all trips, which is the simplest and most convenient fare scheme for passengers and operators but ignores equity. Let $\bar{F}_i^m$ denote the flat fare (FF) for transit mode m, including both express and local transit modes in this study. There are different forms of differentiated fares. The distance-based fare is one of the most common ones, which is expressed by:

$$F_i(x,y) = \sum_m \tilde{F}_i^m + \lambda_i^m d_i^m(x,y) \tag{8}$$

where $\tilde{F}_i^m$, λ_i^m, $d_i^m(x,y)$ are the fixed component, variable component, and travel distance, respectively. Obviously, $F_i(x,y)$ will be a FF when $\lambda_i^m = 0$. This study further considers a hybrid fare scheme for a bimodal transit system; that is, a FF is used for a local transit and a DBF is used for an express transit.

As for the cost of a transit agency A, it mainly includes infrastructure and operating expenditures and can be calculated by:

$$A = I + O \tag{9}$$

where the infrastructure expenditure I specifically refers to the station/stop and line infrastructure costs. It can be determined by:

$$I = I_s + I_l \tag{10}$$

$$I_s = \pi_s^{ex} \int_{x=0}^{L} \phi(x)dx + \pi_s^{lo} \sum_{i=1}^{2} \int_{x=0}^{L} \int_{y=0}^{l_i(x)} \psi_i(x)\phi_i(x,y)dxdy \tag{11}$$

$$I_l = \pi_l^{ex} L + \pi_l^{lo} \sum_{i=1}^{2} \int_{x=0}^{L} \psi_i(x)/2\phi(x)\, dx + \pi_l^{lo} 2 \sum_{i=1}^{2} \int_{x=0}^{L} \psi_i(x) l_i(x) dx \tag{12}$$

where I_s and I_l represent the station/stop cost and line cost, respectively. π_s^{ex} and π_s^{lo} are the hourly infrastructure costs per express and local stops, respectively. π_l^{ex} and π_l^{lo} are the hourly infrastructure costs per kilometer of express and local lines, respectively. In Eq. (12), the second and the last items are the local line construction costs in x and y directions, respectively. $1/2\phi(x)$ is the average length of the back-and-forth local line in x direction.

The operation expenditure O consists of the time-based vehicle operating cost (e.g., staff wage) and the distance-based vehicle operating cost (e.g., fuel cost). They can be calculated by:

$$O = O_{tb} + O_{db} \tag{13}$$

$$O_{tb} = \pi_{tb}^{ex} \int_{x=0}^{L} \left(2/v_{ex} + \tau^{ex}\phi(x)\right) dx/H + O_{tb}^{lo} \tag{14}$$

$$O_{db} = \pi_{db}^{ex}\, 2L/H + O_{db}^{lo} \tag{15}$$

where O_{tb} and O_{db} denote the time-based and distance-based vehicle operating costs, respectively. π_{tb}^{ex} and π_{db}^{ex} are the unit costs related to the vehicle hours traveled and vehicle kilometers traveled for the express transit, respectively. In Eq. (14), $1/v_{ex}$ is the time taken by the express vehicle to travel per kilometer. It is doubled in the expression to calculate the cycle time. Parameter τ^{ex} is the estimated average delay per express transit stop. O_{tb}^{lo} and O_{db}^{lo} indicate the time-based operating cost and distance-based operating cost for local transit, respectively, which are derived by:

$$O_{tb}^{lo} = O_{tb}^{lox} + O_{tb}^{loy} \tag{16}$$

$$O_{tb}^{lox} = \pi_{tb}^{lo} \sum_{i=1}^{2} \int_{x=0}^{L} \frac{\psi_i(x)}{2\phi(x) v_{lo}} \frac{1}{h_i(x)} dx \tag{17}$$

$$O_{tb}^{loy} = \pi_{tb}^{lo} \sum_{i=1}^{2} \int_{x=0}^{L} \int_{y=0}^{l_i(x)} \frac{\psi_i(x)}{h_i(x)} \left(2/v_{lo} + \tau^{lo} \phi_i(x,y)\right) dx dy \tag{18}$$

$$O_{db}^{lo} = O_{db}^{lox} + O_{db}^{loy} \tag{19}$$

$$O_{db}^{lox} = \pi_{db}^{lo} \sum_{i=1}^{2} \int_{x=0}^{L} \frac{\psi_i(x)}{2\phi(x) h_i(x)} dx \tag{20}$$

$$O_{db}^{loy} = \pi_{db}^{lo}\, 2 \sum_{i=1}^{2} \int_{x=0}^{L} \frac{l_i(x)\psi_i(x)}{h_i(x)} dx \tag{21}$$

where O_{tb}^{lox} and O_{tb}^{loy} are the time-based operating costs incurred by local vehicles in x and y directions, respectively. Parameter π_{tb}^{lo} is the operating cost per vehicle-hour of the local service. O_{db}^{lox} and O_{db}^{loy} are the distance-based operating costs of local vehicles in x and y directions, respectively. π_{db}^{lo} is the operating cost per vehicle-kilometer of the local service. τ^{lo} is the estimated delay per stop for the local transit.

2.4. Optimization model

The profit maximization optimization problem can be formulated as a multivariate nonlinear mathematical programming model.

$$\max_{H, h_i(x), \phi(x), \psi_i(x), \phi_i(x,y), \bar{F}_i^m, \tilde{F}_i^m, \lambda_i^m} P = R - A \tag{22}$$

subject to：

$$H \max_{0\le x\le L} \sum_{i=1}^{2} \int_{0}^{x} \int_{y=0}^{l_i(x)} D_i(x,y)dxdy \le C^{ex} \tag{23}$$

$$h_i(x) \int_{y=0}^{l_i(x)} D_i(x,y)dy/\psi_i(x) \le C^{lo} \tag{24}$$

$$0 \le e_i^U U_i(x,y) + e_i^W W_i(x,y) + e_i^T T_i(x,y) + e_i^F F_i(x,y) \le 1 \tag{25}$$

$$H, h_i(x), \phi(x), \psi_i(x), \phi_i(x,y), \bar{F}_i^m, \tilde{F}_i^m, \lambda_i^m > 0 \tag{26}$$

where the decision variables include express service headway H, local service headway $h_i(x)$, express transit stop density $\phi(x)$, local transit line density $\psi_i(x)$, local transit stop density $\phi_i(x,y)$, flat fare $\bar{F}_i^m$, and distance-based fare components $\tilde{F}_i^m$ and λ_i^m. Eqs. (23) and (24) ensure that the on-board passengers do not exceed the vehicle capacities of express and local transit systems, C^{ex} and C^{lo}, respectively. The expression $\sum_{i=1}^{2} \int_0^x \int_{y=0}^{l_i(x)} D_i(x,y)dxdy$ is used to calculate the hourly maximum on-board passenger flow for the express transit, while the hourly maximum on-board passenger flow for a local line at location x is determined by $\int_{y=0}^{l_i(x)} D_i(x,y)dy/\psi_i(x)$. A financial budget constraint can be measured by, for example, the required fleet size. Constraint Eq. (25) is derived from the demand function Eq. (1), which is assigned to ensure the actual passenger demand is non-negative and less than the potential demand. Eq. (26) guarantees the non-negativity of the decision variables. The solution space and decision variables of the model are continuous. The solution space S is a two-dimensional Euclidean space $\mathbb{R}^2$. The normal on S is a function from the two-dimensional Euclidean space to a non-negative real number: $S \rightarrow \mathbb{R}^2$, and captured by the formula $\|\boldsymbol{x}\|_2 := \sqrt{x^2+y^2}$, which indicates the ordinary distance from the coordinate origin to the point (x, y).

3. Solution method

Motivated by previous studies (e.g., Chien and Tsai, 2007; Li et al., 2012; Medina et al., 2013; Ouyang et al., 2014; Sivakumaran et al., 2014; Gu et al., 2016; Chen et al., 2018; Sun and Szeto, 2019; Li et al., 2020; Wu et al., 2020; Guo and Szeto, 2021; Fan and Ran, 2021; Luo et al., 2021; Mei et al., 2021), this study develops a multi-step nested iterative procedure to solve the multi-variables nonlinear math-programming model.

We derive the analytical expressions of the optimal decision variables with different transit fare structures. It is easy to derive that the objective function (Eq. (22)) is concave with respect to one decision variable while keeping the others fixed. The detailed proof is provided in Appendix A, which follows the proof method used in previous studies (e.g., Ouyang et al., 2014; Gu et al., 2016; Luo et al., 2021; Fan and Ran, 2021; Mei et al., 2021). We can update one variable with its analytical expression by using other given variables in each iteration. The analytical expression for the optimal variable is derived by setting the first-order partial derivative of the net profit function to zero. By doing so, the values of the decision variables can be updated iteratively. The derived results for all decision variables are shown in Table 3. Taking into account the model constraints, the general solution procedure is furnished with the DBF as an example and outlined as follows:

Step 1: Randomly generate a set of non-negative feasible initial values for variables $H, h_i(x), \phi(x), \psi_i(x), \phi_i(x,y), \tilde{F}_i^{ex}, \lambda_i^{ex}, \tilde{F}_i^{lo}, \lambda_i^{lo}$ considering constraints Eqs. (23) - (26).

Step 2: Calculate the net profit P by Eq. (22) using the variables $H, h_i(x), \phi(x), \psi_i(x), \phi_i(x,y), \tilde{F}_i^{ex}, \lambda_i^{ex}, \tilde{F}_i^{lo}, \lambda_i^{lo}$ and go to Step 3.

Step 3: Sequentially update the decision variables for the profit maximization model $H, h_i(x), \phi(x), \psi_i(x), \phi_i(x,y), \tilde{F}_i^{ex}, \lambda_i^{ex}, \tilde{F}_i^{lo}, \lambda_i^{lo}$ based on the first-order derivatives that are presented in Table 3.

Step 3.1: Update the express transit headway H' by its corresponding first-order derivative shown in Table 3 using $\phi(x), \tilde{F}_i^{ex}, \lambda_i^{ex}, \tilde{F}_i^{lo}, \lambda_i^{lo}$. Check the constraints Eqs. (23) - (26). Following previous studies (e.g., Li et al., 2012; Sun and Szeto, 2019; Guo and Szeto, 2021; Fan and Ran, 2021; Luo et al., 2021; Mei et al., 2021), if some constraints are not satisfied, the bounds of these constraints are employed to update the values of the decision variables and then go to Step 3.2.

Step 3.2: Update the local transit headway $h_i(x)'$ by substituting $\phi(x), \psi_i(x), \phi_i(x,y), \tilde{F}_i^{ex}, \lambda_i^{ex}, \tilde{F}_i^{lo}, \lambda_i^{lo}$ into the first-order derivative shown in Table 3. Check the constraints Eqs. (23) - (26). If some constraints are not satisfied, the bounds of these constraints are employed to update the values of the decision variables and then go to Step 3.3.

Step 3.3: Update the express station density $\phi(x)'$ by the corresponding first-order derivative in Table 3 using the updated H' and $h_i(x)'$ (steps 3.1 and 3.2) and $\psi_i(x), \tilde{F}_i^{ex}, \lambda_i^{ex}, \tilde{F}_i^{lo}, \lambda_i^{lo}$. Check the constraints Eqs. (23) - (26). If some constraints are not satisfied, the bounds of these constraints are employed to update the values of the decision variables and then go to Step 3.4.

Step 3.4: Use the same principle as the above steps to update other variables $\psi_i(x)', \phi_i(x,y)', \tilde{F}_i^{ex\prime}, \lambda_i^{ex\prime}, \tilde{F}_i^{lo\prime}, \lambda_i^{lo\prime}$ in sequence. By doing so, we can update all the decision variables and go to Step 4.

Step 4: Update the net profit P' by objective function Eq. (22) with the new values of variables $H', h_i(x)', \phi(x)', \psi_i(x)', \phi_i(x,y)', \tilde{F}_i^{ex\prime}, \lambda_i^{ex\prime}, \tilde{F}_i^{lo\prime}, \lambda_i^{lo\prime}$ derived from Step 3.

Step 5: Check the convergence. If the objective function values of two successive iterations are very close, i.e., $|P' - P| < \xi$, where ξ is a pre-specified threshold, then terminate the iteration and output the objective function value P' and the solutions $H', h_i(x)', \phi(x)', \psi_i(x)', \phi_i(x,y)', \tilde{F}_i^{ex\prime}, \lambda_i^{ex\prime}, \tilde{F}_i^{lo\prime}, \lambda_i^{lo\prime}$. Otherwise, update the decision variables $\phi(x) = \phi(x)', \psi_i(x) = \psi_i(x)', \phi_i(x,y) = \phi_i(x,y)', \tilde{F}_i^{ex} = \tilde{F}_i^{ex\prime}, \lambda_i^{ex} = \lambda_i^{ex\prime}, \tilde{F}_i^{lo} = \tilde{F}_i^{lo\prime}, \lambda_i^{lo} = \lambda_i^{lo\prime}$, and go to Step 3. Ultimately, the solution can be obtained when the convergence of the nested iterative procedure is satisfied. Following previous studies (e.g., Gu et al., 2016; Fan et al., 2018; Li et al., 2020; Wu et al., 2020; Luo et al., 2021), the solution procedure is repeated several times with different initial solutions. If the same final solution can be obtained, it is regarded as the final near-optimal solution.

Table 3. Analytical expressions of the decision variables under different fare schemes.

FF scheme	
Transit service headways:	$H^* = \sqrt{\dfrac{\pi_{tb}^{ex}\int_0^L\left(2/v_{ex} + \tau^{ex}\phi(x)\right)dx + \pi_{db}^{ex}2L}{\sum_{i=1}^{2}\int_0^L\int_0^{l_i(x)} D_i^0(x,y)e_i^W \sum_m \bar{F}_i^m\, dxdy/2}}$
	$h_i(x)^* = \sqrt{\dfrac{\pi_H^f \sum_{i=1}^{2}\psi_i(x)\left(\frac{1}{2\phi(x)v_{lo}} + \int_{y=0}^{l_i(x)}\left(\frac{2}{v_{lo}} + \tau^{lo}\phi_i(x,y)\right)dy\right) + \pi_{db}^{lo}\sum_{i=1}^{2}\psi_i(x)\left(\frac{1}{2\phi(x)} + 2l_i(x)\right)}{\sum_{i=1}^{2}\int_0^{l_i(x)} D_i^0(x,y)e_i^W \sum_m \bar{F}_i^m\, dy/2}}$

Route and stop densities:

$$\phi(x)^{*}=\sqrt{\frac{\sum_{i=1}^{2}\int_{0}^{l_i(x)}\frac{D_i^0(x,y)e_i^T\sum_m\bar{F}_i^m}{4v_{lo}}dy+\sum_{i=1}^{2}\left(\frac{\pi_l^{lo}\psi_i(x)}{2}+\frac{\pi_{tb}^{lo}\psi_i(x)}{2h_i(x)v_{lo}}+\frac{\pi_{db}^{lo}\psi_i(x)}{2h_i(x)}\right)}{\sum_{i=1}^{2}\int_{0}^{l_i(x)}\int_{0}^{x}D_i^0(x,y)e_i^T\tau^{ex}\sum_m\bar{F}_i^m\,dxdy+\pi_s^{ex}+\pi_{tb}^{ex}\tau^{ex}/H}}$$

$$\phi_i(x,y)^{*}=\sqrt{\frac{\sum_{i=1}^{2}D_i^0(x,y)e_i^U\sum_m\bar{F}_i^m/4v_U}{\sum_{i=1}^{2}\int_{y}^{l_i(x)}D_i^0(x,y)e_i^T\tau^{lo}\sum_m\bar{F}_i^m\,dy+\pi_s^{lo}\sum_{i=1}^{2}\psi_i(x)+\pi_{tb}^{lo}\sum_{i=1}^{2}\psi_i(x)\,\tau^{lo}/h_i(x)}}$$

$$\psi_i(x)^{*}=\sqrt{\frac{\sum_{i=1}^{2}\int_{0}^{l_i(x)}D_i^0(x,y)e_i^U\sum_m\bar{F}_i^m\,dy/4v_U}{\begin{array}{c}\pi_s^{lo}\int_0^{l_i(x)}\phi_i(x,y)dy+\pi_l^{lo}\left(\frac{1}{2\phi(x)}+2l_i(x)\right)+\pi_K^{lo}\left(\frac{1}{2\phi(x)h_i(x)}+\frac{2l_i(x)}{h_i(x)}\right)\\+\pi_{tb}^{lo}\sum_{i=1}^{2}\left(\frac{1}{2\phi(x)h_i(x)v_{lo}}+\int_{y=0}^{l_i(x)}\frac{1}{h_i(x)}\left(\frac{2}{v_{lo}}+\tau^{lo}\phi_i(x,y)\right)dy\right)\end{array}}}$$

Fare components:

$$\bar{F}_i^{ex(lo)^{*}}=\sqrt{\frac{\sum_{i=1}^{2}\int_0^L\int_0^{l_i(x)}D_i^0(x,y)\left(1-e_i^UU_i(x,y)-e_i^WW_i(x,y)-e_i^TT_i(x,y)-e_i^F2\bar{F}_i^{lo(ex)}\right)dxdy}{\sum_{i=1}^{2}\int_0^L\int_0^{l_i(x)}2D_i^0(x,y)e_i^F\,dxdy}}$$

DBF scheme

Transit service headways:

$$H^{*}=\sqrt{\frac{\pi_{tb}^{ex}\int_0^L\left(2/v_{ex}+\tau^{ex}\phi(x)\right)dx+\pi_{db}^{ex}2L}{\sum_{i=1}^{2}\int_0^L\int_0^{l_i(x)}D_i^0(x,y)e_i^W\left(\sum_m\tilde{F}_i^m+\lambda_i^md_i^m(x,y)\right)dxdy/2}}$$

$$h_i(x)^{*}=\sqrt{\frac{\pi_H^f\sum_{i=1}^{2}\psi_i(x)\left(\frac{1}{2\phi(x)v_{lo}}+\int_{y=0}^{l_i(x)}\left(\frac{2}{v_{lo}}+\tau^{lo}\phi_i(x,y)\right)dy\right)+\pi_{db}^{lo}\sum_{i=1}^{2}\psi_i(x)\left(\frac{1}{2\phi(x)}+2l_i(x)\right)}{\sum_{i=1}^{2}\int_0^{l_i(x)}D_i^0(x,y)e_i^W\left(\sum_m\tilde{F}_i^m+\lambda_i^md_i^m(x,y)\right)dy/2}}$$

Route and stop densities:

$$\phi(x)^{*}=\sqrt{\frac{\sum_{i=1}^{2}\int_{0}^{l_i(x)}\frac{D_i^0(x,y)e_i^T\left(\sum_m\tilde{F}_i^m+\lambda_i^md_i^m(x,y)\right)}{4v_{lo}}dy+\sum_{i=1}^{2}\left(\frac{\pi_l^{lo}\psi_i(x)}{2}+\frac{\pi_{tb}^{lo}\psi_i(x)}{2h_i(x)v_{lo}}+\frac{\pi_{db}^{lo}\psi_i(x)}{2h_i(x)}\right)}{\sum_{i=1}^{2}\int_{0}^{l_i(x)}\int_{0}^{x}D_i^0(x,y)e_i^T\tau^{ex}\left(\sum_m\tilde{F}_i^m+\lambda_i^md_i^m(x,y)\right)dxdy+\pi_s^{ex}+\pi_{tb}^{ex}\tau^{ex}/H}}$$

$$\phi_i(x,y)^{*}=\sqrt{\frac{\sum_{i=1}^{2}D_i^0(x,y)e_i^U\left(\sum_m\tilde{F}_i^m+\lambda_i^md_i^m(x,y)\right)/4v_U}{\sum_{i=1}^{2}\int_{y}^{l_i(x)}D_i^0(x,y)e_i^T\tau^{lo}\left(\sum_m\tilde{F}_i^m+\lambda_i^md_i^m(x,y)\right)dy+\pi_s^{lo}\sum_{i=1}^{2}\psi_i(x)+\pi_{tb}^{lo}\sum_{i=1}^{2}\psi_i(x)\,\tau^{lo}/h_i(x)}}$$

$$\psi_i(x)^{*}=\sqrt{\frac{\sum_{i=1}^{2}\int_{0}^{l_i(x)}D_i^0(x,y)e_i^U\left(\sum_m\tilde{F}_i^m+\lambda_i^md_i^m(x,y)\right)dy/4v_U}{\begin{array}{c}\pi_s^{lo}\int_0^{l_i(x)}\phi_i(x,y)dy+\pi_l^{lo}\left(\frac{1}{2\phi(x)}+2l_i(x)\right)+\pi_K^{lo}\left(\frac{1}{2\phi(x)h_i(x)}+\frac{2l_i(x)}{h_i(x)}\right)\\+\pi_{tb}^{lo}\sum_{i=1}^{2}\left(\frac{1}{2\phi(x)h_i(x)v_{lo}}+\int_{y=0}^{l_i(x)}\frac{1}{h_i(x)}\left(\frac{2}{v_{lo}}+\tau^{lo}\phi_i(x,y)\right)dy\right)\end{array}}}$$

Fare components:

$$\tilde{F}_i^{ex(lo)^{*}}=\sqrt{\frac{\sum_{i=1}^{2}\begin{array}{l}\int_0^L\int_0^{l_i(x)}D_i^0(x,y)\left(1-e_i^UU_i(x,y)-e_i^WW_i(x,y)-e_i^TT_i(x,y)\right)dxdy\\-\int_0^L\int_0^{l_i(x)}D_i^0(x,y)e_i^F2\left(\lambda_i^{ex(lo)}d_i^{ex(lo)}(x,y)+F_i^{lo(ex)}(x,y)\right)dxdy\end{array}}{\sum_{i=1}^{2}\int_0^L\int_0^{l_i(x)}2D_i^0(x,y)e_i^F\,dxdy}}$$

$$\lambda_i^{ex(lo)^{*}}=\sqrt{\frac{\sum_{i=1}^{2}\begin{array}{l}\int_0^L\int_0^{l_i(x)}D_i^0(x,y)\left(1-e_i^UU_i(x,y)-e_i^WW_i(x,y)-e_i^TT_i(x,y)\right)d_i^{ex(lo)}(x,y)dxdy\\-\int_0^L\int_0^{l_i(x)}D_i^0(x,y)e_i^F2\left(\tilde{F}_i^{ex(lo)}+\tilde{F}_i^{lo(ex)}+\lambda_i^{lo(ex)}d_i^{lo(ex)}(x,y)\right)d_i^{ex(lo)}(x,y)dxdy\end{array}}{\sum_{i=1}^{2}\int_0^L\int_0^{l_i(x)}d_i^{ex(lo)}(x,y)^2D_i^0(x,y)e_i^F2dxdy}}$$

4. Numerical studies

Two test scenarios are schemed to validate the performances of the optimization model and solution algorithm. The first scenario is a special case with a homogeneous potential demand density, i.e., $D_i^0(x,y)=D_i^0$. The second scenario is a general case with a heterogeneous potential demand density; that is, the service requests in the service region follow a non-uniform distribution. Based on the actual demand density $D_i(x,y)$, the total number of passengers in the whole service region is denoted by Z, which can be calculated by $Z=\int_0^L\int_0^{l_i(x)}D_i(x,y)dxdy$. With reference to the previous studies (e.g., Sivakumaran et al., 2012; Sivakumaran et al., 2014; Kim and Schonfeld, 2015; Yang et al., 2021; Luo and Kang, 2022), we consider a rectangular service area with L= 20km and $l_i(x)$ = 5km, $\forall x \in [0, L]$. For the sake of brevity, the study investigates the upper side of the service area, i.e., i=1, while the same principle can be applied to the lower-side area.

Typical transit modes, including rail, bus rapid transit (BRT), and ordinary bus, are considered. According to the features of bimodal transit systems, rail and BRT are regarded as high-speed and low-speed express transit modes, respectively. The ordinary bus is considered as a local transit mode. Three representative fare schemes are considered in the study, i.e., FF, DBF, and HF. Please note that in this study, under a HF scheme, a FF is used for local transit and a DBF is used for express transit in the numerical studies.

4.1. Parameter settings

The estimated average walking speed v_U is set as 2km/h with respect to the natural delays at street junctions (Daganzo, 2010; Li et al., 2020). Following the previous works (e.g., Vuchic, 2007; Daganzo, 2010; Sivakumaran et al., 2014; Gu et al., 2016; Fan et al., 2018; Li et al., 2020), the value of time ε is set to be 20 ($/h). Table 4 lists the values of the operating and cost parameters used for the three transit modes (i.e., rail, BRT, and bus transit) considered in the study. These parameter settings are referred from previous studies (e.g., Vuchic, 2007; Daganzo, 2010; Sivakumaran et al., 2014; Gu et al., 2016; Fan et al., 2018; Li et al., 2020; Wu et al., 2020; Fan and Ran, 2021; Luo et al., 2021; Mei et al., 2021). For practical implementation, these parameter values can be calibrated by conducting field surveys on the economic and technical characteristics of the local public transit systems.

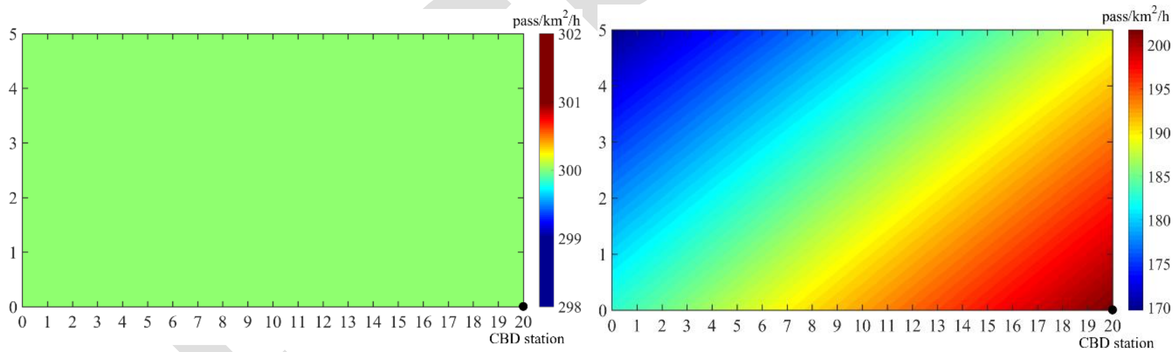
Table 4. Operating and cost parameters of three transit modes.

Modes	Operating parameters			Cost parameters			
Express	v_{ex} (km/h)	τ^{ex} (s/station)	C^{ex} (pass/veh)	π_s^{ex} ($/station)	π_l^{ex} ($/km)	π_{tb}^{ex} ($/veh-h)	π_{db}^{ex} ($/veh-km)
Rail	60	45	2400	294+9.8ε	594+19.8ε	101+5ε	2.20
BRT	40	30	160	4.2+0.14ε	162+5.4ε	3.81+4ε	0.66
	v_{lo} (km/h)	τ^{lo} (s/station)	C^{lo} (pass/veh)	π_s^{lo} ($/station)	π_l^{lo} ($/km)	π_{tb}^{lo} ($/veh-h)	π_{db}^{lo} ($/veh-km)
Local Bus	25	30	80	0.42+0.014ε	6+0.2ε	2.66+3ε	0.59

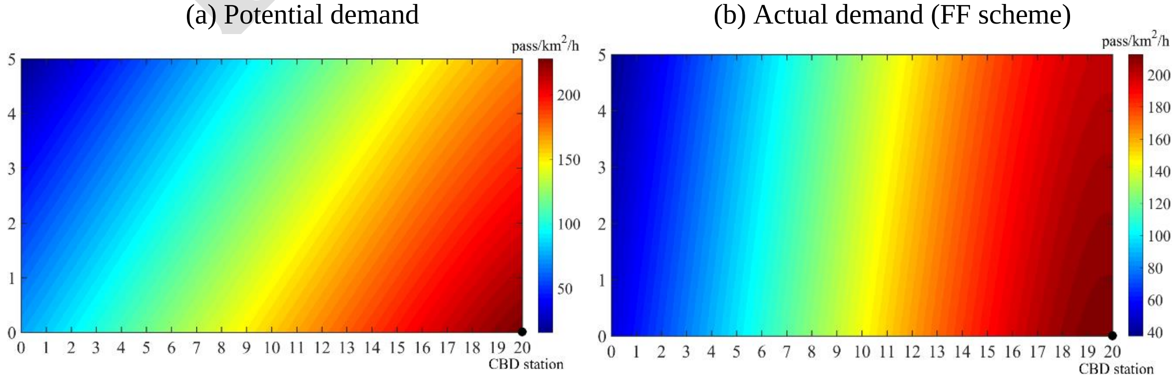
4.2. Scenario 1

For Scenario 1, a rail-bus bimodal system is considered to test the validity of the model and solution algorithm. Adapted from the previous works (e.g., Sivakumaran et al., 2012; Li et al., 2012; Kim and Schonfeld, 2015; Huang et al., 2016; Li et al., 2020; Yang et al., 2021), a homogeneous potential demand density with $D_i^0(x,y)$=300 (pass/km^2-h) is considered. Thus, the total demand in the study area is 30,000 (pass/h). Fig. 2(a) illustrates the potential demand distribution. Since different colors reflect different levels of demand, the service requests are consistent over the whole service region.

The rest three demand distributions depicted in Fig. 2(b), Fig. 2(c), and Fig. 2(d) show the actual demand status of the optimized bimodal transit system under the FF, DBF, and HF schemes, respectively. For the first two actual demand scenarios, shown in Fig. 2(b) and Figure 2(c), the FF and DBF schemes are adopted in the whole service region. The actual demand distribution with the FF scheme is more concentrated than those with the DBF and HF schemes, while the distance-based fare scheme demonstrates the most dispersed demand distribution. This phenomenon is incurred by the fare structure, i.e., the further from the CBD, the higher the ticket price under the distance-based fare scheme, which directly reduces passengers' willingness to use transit service.



(a) Potential demand (b) Actual demand (FF scheme)



(c) Actual demand (DBF scheme) (d) Actual demand (HF scheme)

Fig. 2. Demand distributions under different fare schemes (Scenario 1).

Table 5 shows the primary model results under three different fare schemes. The total number of passengers in the whole service region under the DBF, HF, and FF schemes are 12,666, 13,501, and 18,592 (pass), respectively. It can be seen that the FF scheme attracts the maximum number of passengers, but it has the worst financial performance with a minimum net profit, while the best financial performance is achieved by the DBF scheme. Although the operating cost for DBF is slightly higher than those of other fare schemes, it brings more revenue to the agency. Thus, the DBF scheme is preferable for the transit agency. This finding is consistent with the previous related studies (e.g., Chien and Tsai, 2007; Huang et al., 2016; Sun and Szeto, 2019).

Table 5. Model results under different transit fare schemes (Scenario 1)

Results	Fare schemes		
	Case 1: DBF	Case 2: HF	Case 3: FF
Z (pass)	12666	13501	18592
P (\$/h)	91640.89	86108.58	72453.35
R (\$/h)	134603.30	128514.07	110640.44
A (\$/h)	42962.41	42405.49	38187.09
I_s (\$/h)	2792.74	2721.15	2435.29
I_l (\$/h)	22346.58	22158.40	21383.77
O_{tb} (\$/h)	13727.98	13521.23	11160.04
O_{db} (\$/h)	4095.11	4004.72	3207.99
$F(x,y)$ (\$)	0.89+0.74$d(x,y)^{ex}$; 0.89+0.88$d(x,y)^{lo}$	1.52+0.82$d(x,y)^{ex}$; 1.52	2.98; 2.98
δ	0.42	0.45	0.62
H (min)	2.40	2.50	3.42
h_{mean} (min)	4.79	4.45	3.38
$\phi(x)$ (station/km)	0.26	0.26	0.24
$\psi(x)$ (line/km)	1.07	0.99	0.65
$\phi(x,y)$ (stop/km^2)	2.85	2.80	2.53

4.3. Scenario 2

In this scenario, following previous studies (e.g., Li et al., 2012; Dijoseph and Chien, 2013; Saidi et al., 2017; Luo and Nie, 2019; Qu et al., 2021; Xu et al., 2022), a heterogeneous potential demand density is considered. It is specified as an exponential function, $D_i^0(x,y)=\alpha e^{\beta\sqrt{x^2+y^2}}$(pass/km^2/h), where $\sqrt{x^2+y^2}$ denotes the distance from location (x,y) to the left-most point (origin) of the express line. The CBD station is the right-most point on the express line. To match the total number of passengers in Scenario 1, the coefficients in the function are set as α=215 and β=0.03. By doing so, the demand increases as the distance increases and the highest demand occurs at the CBD station.

Fig. 3 depicts the demand distributions of the heterogeneous demand scenario. Fig. 3(a) shows the potential demand distribution, where the color deepens with the decreasing distance from CBD, meaning that demand increases when approaching CBD. The actual demands under the FF, DBF, and HF schemes are shown in Fig. 3(b), Fig. 3(c), and Fig. 3(d), respectively. It can be seen that the actual demand distribution under the FF scheme is closer to the potential demand compared to the other two schemes. The reason is that the ticket price directly affects the willingness to travel with respect to the demand elasticity.

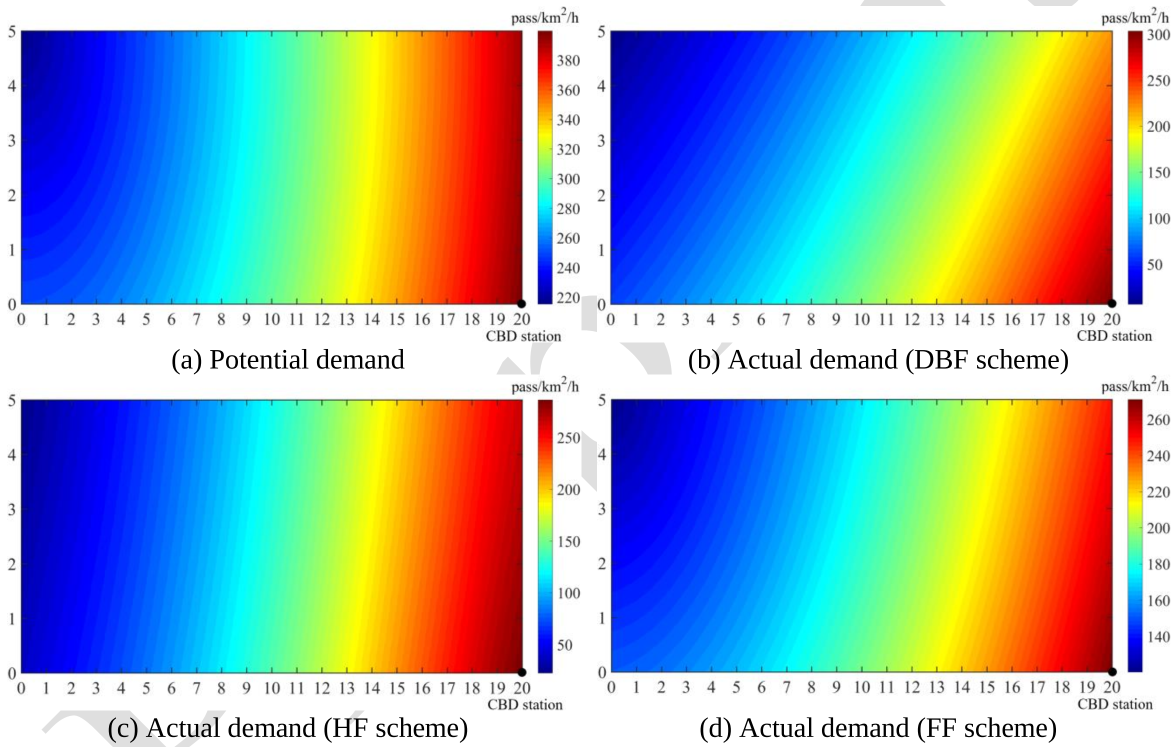


Fig. 3. Demand distributions under different fare schemes (Scenario 2).

Table 6 summarizes the primary model results under three different fare schemes. Similar to the results of Scenario 1, the FF scheme brings the largest actual demand. The net profit of the DBF scheme is the greatest among all the fare schemes. Thus, similar to Scenario 1, a DBF is advisable for the transit agency. The HF scheme, where a DBF is applied to the rail system and an FF is adopted by ordinary buses, shows balanced results in terms of financial performances and the values of design variables.

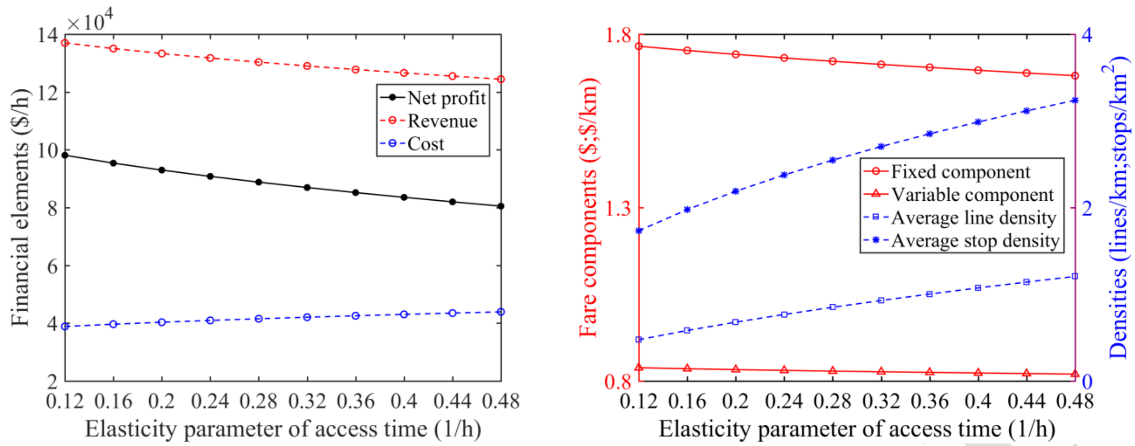
Table 6. Model results with different transit fare schemes (Scenario 2)

Results	Fare schemes		
	Case 1: DBF	Case 2: HF	Case 3: FF
Z (pass)	12988	13972	18674
P ($/h)	92646.83	85635.12	73146.70
R ($/h)	135680.77	128089.00	111371.93
A ($/h)	43033.94	42453.88	38225.23
I_s ($/h)	2832.34	2755.82	2441.97
I_l ($/h)	22344.32	22143.17	21382.03
O_{tb} ($/h)	13769.12	13565.64	11187.72
O_{db} ($/h)	4088.16	3989.25	3213.51
$F(x,y)$ ($)	1.14+0.73$d(x,y)^{ex}$	1.71+0.83$d(x,y)^{ex}$	2.98
	1.14+0.92$d(x,y)^{lo}$	1.71	2.98
δ	0.41	0.44	0.62
H (min)	2.42	2.54	3.41
h_{mean} (min)	4.79	4.48	3.49
$\phi(x)$ (station/km)	0.27	0.26	0.24
$\psi(x)$ (line/km)	1.07	0.98	0.65
$\phi(x,y)$ (stop/km^2)	2.86	2.81	2.53

5. Sensitivity analysis

To understand the robustness of the proposed model and make further in-depth explorations on the system, sensitivity analyses on some key parameters and demand levels are conducted. To simplify the analyses, only the HF scheme is investigated.

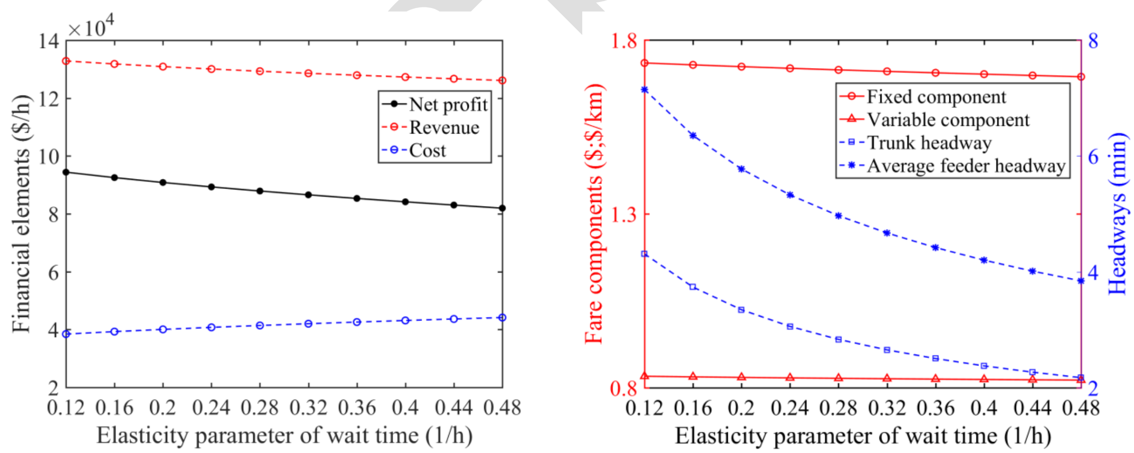
By varying the values of the related elasticity parameters, we can obtain different results (Figs. 4-7). Fig. 4 shows the sensitivity analysis results on the elasticity parameter of the access time. Fig. 4(a) shows the results of financial performances, including revenue, cost, and net profit. It can be seen that with the increase of the elasticity parameter of the access time, the agency cost slightly increases while the revenue significantly decreases, which leads to a reduction of the net profit. This is attributed to the reduction of the fixed and variable components of the fare; that is, with the increase of the elasticity parameter of the access time, according to the related derivatives shown in Table 3, both the fixed and variable components of the fare decrease, as shown in Fig. 4(b). The reduction of the fare leads to reduced revenue. Moreover, as we can see from Fig. 4(b) that with the increase of the elasticity parameter of the access time, both the average line density and stop density increase, which increases the operating cost.



(*a*) Financial performances (*b*) Fare components, line and stop densities

Fig. 4. Sensitivity analyses on the elasticity parameter of the access time

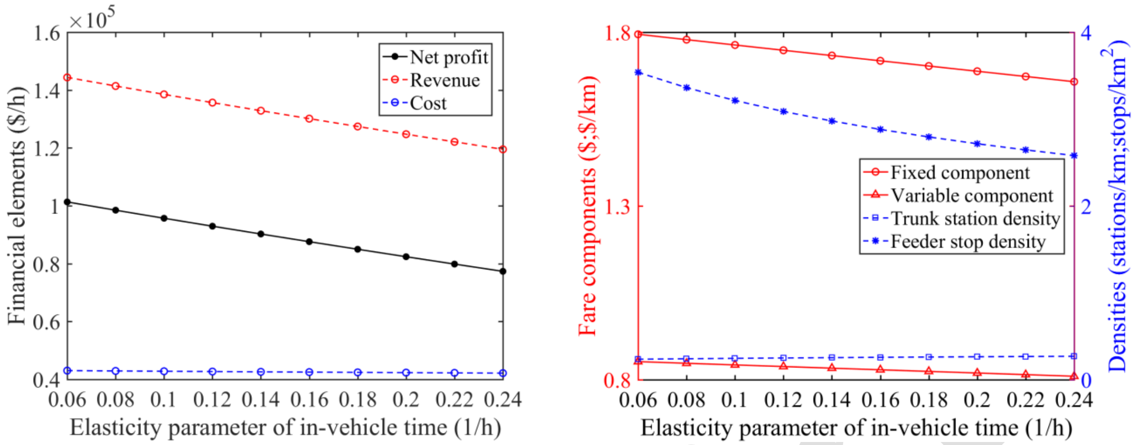
The sensitivity analyses results on the elasticity parameter of the wait time are shown in Fig. 5. It shows that with the increase of the elasticity parameter of the wait time, both the fixed and variable components of the fare decrease, which leads to a reduction of the net profit, as shown in Fig. 5(a). In addition, the increase of the elasticity parameter of the wait time leads to significant reductions in both the trunk and feeder headways. A shorter headway will increase the total fleet size, increasing the total operating cost, as shown in Eqs. (14)-(21).



(*a*) Financial performances (*b*) Fare components and headway

Fig. 5. Sensitivity analyses on the elasticity parameter of the wait time

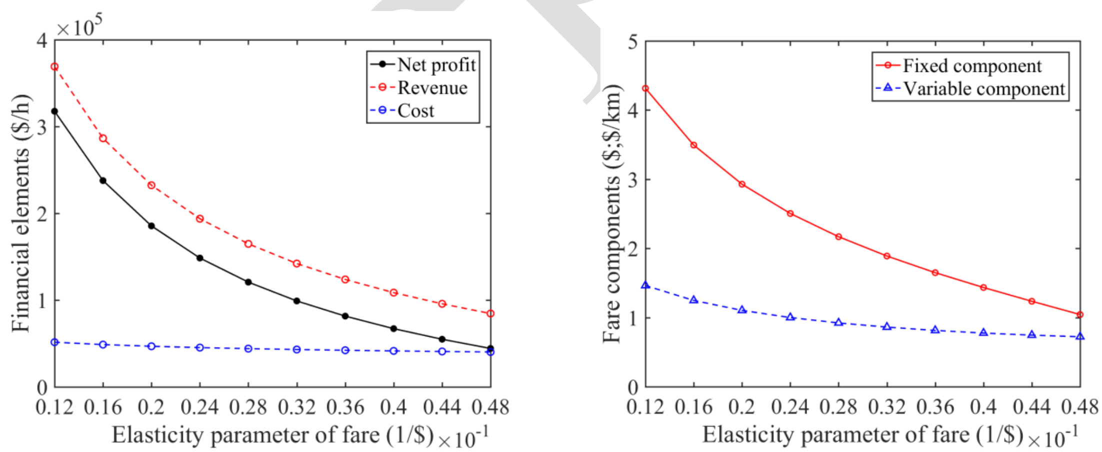
Fig. 6 shows the sensitivity analyses results on the elasticity parameter of the in-vehicle time. It shows that with the increase of the elasticity parameter of the in-vehicle time, both the fixed and variable components of the fare decrease, which reduces the net profit. The local transit stop density demonstrates a remarkable decrease, while the express transit station density shows a slight increase. The total operating cost slightly increases.



(*a*) Financial performances (*b*) Fare components and station/stop density

Fig. 6. Sensitivity analyses on the elasticity parameter of the in-vehicle time

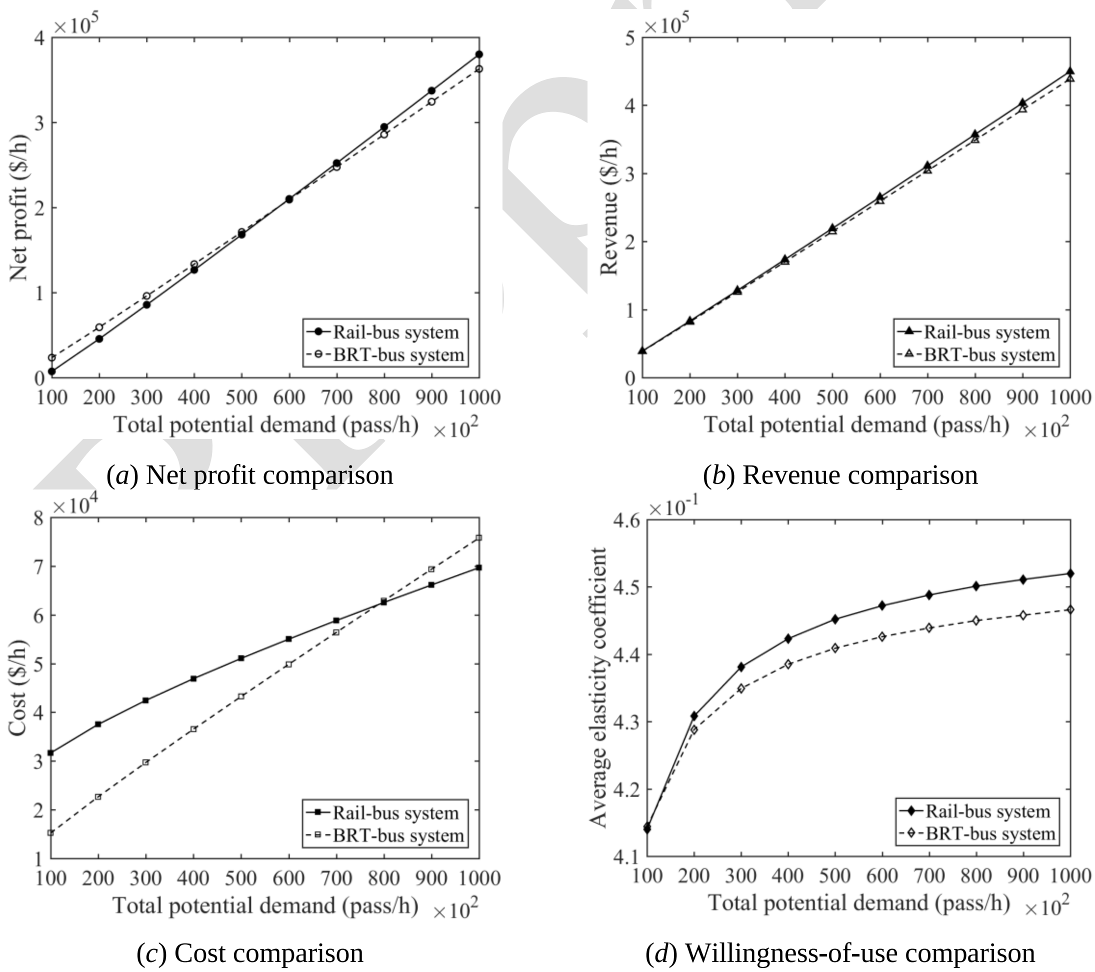
The sensitivity analyses results on the elasticity parameter of the fare are shown in Fig. 7. It shows that with the increase of this parameter, both the fixed and variable components of the fare significantly decrease, which reduces the revenue and net profit. The total operating cost does hardly change. In general, the elasticity parameter of the fare has a knock-on effect on the net profit of the transit agency.



(*a*) Financial performances (*b*) Fare components

Fig. 7. Sensitivity analyses on the elasticity parameter values of the fare

In practice, rail and BRT are two common express transit modes considered by transit agencies. Rail-bus and BRT-bus systems are two common forms of bimodal transit systems where the bus serves as the local transit. We hereby further explore the performances of the two different bimodal transit systems under different demand levels. Following previous studies (e.g., Sivakumaran et al., 2012; Li et al., 2012; Yang et al., 2021), a set of potential demands is generated by adjusting the coefficient α in scenario 2. Fig. 8 shows the comparisons of the two bimodal transit systems in terms of net profit, revenue, operating cost, and passengers' willingness-of-use under different potential demand levels. It can be seen from Fig. 8(a) that both the net profits of the two systems increase with the increase of the potential demand. When the demand reaches 58,009 pass/h, the two systems has the same net profit of 201,661 \$/h. With the further increase of the demand, the rail-bus bimodal system has a higher net profit than that of the BRT-bus bimodal system. Thus, a rail-bus system is preferable to a BRT-bus system for a demand exceeding 58,009 pass/h.



(*a*) Net profit comparison (*b*) Revenue comparison

(*c*) Cost comparison (*d*) Willingness-of-use comparison

Fig. 8. Comparisons of two bimodal transit systems (rail-bus and BRT-bus) under different demand

levels.

Fig. 8(b) shows the revenue comparisons of the two bimodal transit systems. It can be found that with the increase in passenger demand, the rail-bus system can result in slightly more revenue for a transit agency than a BRT-bus system. Fig. 8(c) unfolds that a rail-bus system ultimately costs less than a BRT-bus system with an increased demand level. According to Eq. (1), the elasticity coefficient δ indicates the ratio of the actual demand to the latent demand, which can reflect the willingness-of-use of passengers. Fig. 8(d) indicates that passengers prefer using a rail-bus system, especially at a higher demand level. This may be attributed to a lower travel time of the rail-bus system compared to that of the BRT-bus system.

6. Conclusions

This study addresses the bimodal transit design problem considering heterogeneous demand elasticity under different fare structures. The study considers the heterogeneous environment, where station/stop spacing, line spacing and length, and service frequencies may vary in space or time to better cater to the heterogeneous demand. The demand elasticity is incorporated to capture the passengers' expectations and perceptions of a certain quality-of-service level, where the impact of fare structures (e.g., DBF, FF, and HF) and times (e.g., access time, wait time, and in-vehicle time) are explicitly considered. Numerical studies and sensitivity analyses are conducted to explore the cause-and-effect relationship between design parameters and optimal design schemes and assess the performances of the proposed model and solution method.

Numerical results show that: (i) the proposed model is suitable for designing bimodal transit systems considering heterogeneous demand elasticity; (ii) among flat fare, hybrid fare, and distance-based fare, the distance-based fare scheme has a better financial performance and, thus, is preferable for the transit agency; (iii) the increase in elasticity parameters has a knock-on effect on financial performances, consequently leading to a net profit reduction; (iv) except in an extremely low potential demand case, bimodal transit systems are preferable to unimodal systems in terms of agency's profit, where a rail-bus system is recommended for a service area of 100(km^2) with a potential demand exceeding 58,009(pass/h); otherwise, the BRT-bus system should be used instead; and (v) passengers prefer using a rail-bus system to a BRT-bus system, especially at a higher demand level.

This study has some limitations. First, the proposed model considers a many-to-one demand pattern with one CBD as the destination in the service region, which represents the commute demand in peak hours, while there are diversified and differentiated demands that need to be considered. Second, the study focuses on bimodal transit design under the FF, DBF, and HF schemes, while there are multi-form fare structures that deserve further exploration. Third, the research emphasises rail, BRT, and ordinary bus transit modes, while there are other different ones. In the future study, some potential extensions could be explored, including (i) inclusion of personalized and customized transit mode in the joint design (Nuzzolo and Lam, 2017; Daganzo and Ouyang, 2019; Guo et al., 2019; Jiang and Ceder, 2021; Dou et al., 2021); (ii) investigation of the interrelations between bimodal transit design and other fare structures, e.g., time differentiated fares, discount fares, and special fares for particular passenger groups (Tang et al., 2020; Fu et al., 2020; Guo and Szeto, 2020); (iii) consideration of a polycentric network topology (Park et al., 2020; Cats and Birch, 2021; Huai et al., 2021); (iv) consideration of other forms of nonlinear demand functions, and the inclusion of transfer penalty in the demand function; (v) inclusion of diversified travel modes, e.g., shared bikes, on-demand transit, and automated transit, to conduct a multi-modal transit system design (Wu et al., 2020; Li et al., 2020; Liu et al., 2021; Luo et al., 2021; Tian et al., 2021; Liu and Ouyang, 2022; Li et al., 2022); and (vi) consideration of other forms of objective functions, such as social welfare.

Acknowledgements

This work was partially supported by the National Natural Science Foundation of China (No. 72271206, No. 72271207), Chunhui Cooperative Research Project of the Ministry of Education of the People’s Republic of China (No. CH2019lt), and the Science and Technology Department of Sichuan Province (No. 2021YJ0536). The authors would like to thank the editorial team and the anonymous referees for their constructive comments throughout the review process, which helped improve the quality of the manuscript significantly.

Appendix A. Proof of the concavity of the objective function with respect to each individual variable

The proof follows a similar approach used in previous studies (e.g., Ouyang et al., 2014; Gu et al., 2016; Fan and Ran, 2021; Luo et al., 2021; Mei et al., 2021). By reorganizing the items that include the local transit stop density $\phi_i(x,y)$, the objective function, Eq. (22), can be reformulated as:

$$P = -\int_0^L \int_0^{l_i(x)} (\Upsilon_1 \phi_i(x,y) + \Upsilon_2 \phi_i^{-1}(x,y))dxdy + \Upsilon_3 \quad \text{(A1)}$$

where Υ_1, Υ_2, and Υ_3 are the terms that are irrelevant to $\phi_i(x,y)$, and $\Upsilon_1, \Upsilon_2 > 0$, where $\Upsilon_1 = \pi_s^{lo}\psi_i(x) + \pi_{tb}^{lo}\tau^{lo}\frac{\psi_i(x)}{h_i(x)}$ and $\Upsilon_2 = D_i^0(x,y)F_i(x,y)\left(\frac{e_i^U}{4v_U} + \int_0^y e_i^T \tau^{lo} dy\right)$. Thus, P is a concave function of $\phi_i(x,y)$ when $\phi_i(x,y) > 0$.

Similarly, in terms of the local transit headway $h_i(x)$, we can reformulate the objective function as:

$$P = -\int_0^L \int_0^{l_i(x)} \Omega_1 h_i(x)dxdy - \int_0^L \Omega_2 h_i^{-1}(x)dx + \Omega_3 \quad \text{(A2)}$$

where Ω_1, Ω_2, and Ω_3 are independent with $h_i(x)$, and $\Omega_1, \Omega_2 > 0$, where $\Omega_1 = e_i^W \mu D_i^0(x,y)F_i(x,y)$ and $\Omega_2 = \frac{\pi_{tb}^{lo}\psi_i(x)}{2\phi(x)v_{lo}} + \int_{y=0}^{l_i(x)} \pi_{tb}^{lo}\psi_i(x)\left(2/v_{lo} + \tau^{lo}\phi_i(x,y)\right)dy + \pi_{db}^{lo}2l_i(x)\psi_i(x)$. Hence, the objective function P is concave with respect to $h_i(x)$ when $h_i(x) > 0$. For other decision variables, the concavity of the objective function can be proved in the same way. By doing so, we can prove that the objective function is concave with respect to each decision variable. This completes the proof.□

References

Allen, Jaime, Juan Carlos Muñoz, and Jordi Rosell. 2019. "Effect of a Major Network Reform on Bus Transit Satisfaction." *Transportation Research Part A: Policy and Practice* 124 (April). Elsevier: 310–333. doi:10.1016/j.tra.2019.04.002.

Broome, Kieran, Linda Worrall, Jennifer Fleming, and Duncan Boldy. 2012. "Evaluation of Flexible Route Bus Transport for Older People." *Transport Policy* 21. Elsevier: 85–91. doi:10.1016/j.tranpol.2012.02.005.

Cats, Oded, and Nigel Birch. 2021. “Multi-Modal Network Evolution in Polycentric Regions.” *Journal of Transport Geography* 96 (July). Elsevier Ltd: 103159. doi:10.1016/j.jtrangeo.2021.103159.

Ceder, Avishai, and Nigel H.M. Wilson. 1986. “Bus Network Design.” *Transportation Research Part B* 20 (4): 331–344. doi:10.1016/0191-2615(86)90047-0.

Chen, Ya Juan, Zhi Chun Li, and William H.K. Lam. 2018. “Cordon Toll Pricing in a Multi-Modal Linear Monocentric City with Stochastic Auto Travel Time.” *Transportmetrica A: Transport Science* 14 (1–2): 22–49. doi:10.1080/23249935.2017.1286411.

Chen, Jingxu, Shuaian Wang, Zhiyuan Liu, and Wei Wang. 2017. “Design of Suburban Bus Route for Airport Access.” *Transportmetrica A: Transport Science* 13 (6). Taylor & Francis: 568–589. doi:10.1080/23249935.2017.1306896.

Chien, Steven, and Chuck F.M. Tsai. 2007. “Optimization of Fare Structure and Service Frequency for Maximum Profitability of Transit Systems.” *Transportation Planning and Technology* 30 (5): 477–500. doi:10.1080/03081060701599961.

Chien, Steven, and Paul Schonfeld. 1998. “Joint Optimization of a Rail Transit Line and Its Feeder Bus System.” *Journal of Advanced Transportation* 32 (3): 253–284. doi:10.1002/atr.5670320302.

Chien, Steven, and Zhaowei Yang. 2000. “Optimal Feeder Bus Routes on Irregular Street Networks.” *Journal of Advanced Transportation* 34 (2): 213–248. doi:10.1002/atr.5670340204.

Daganzo, Carlos F. 2010. “Structure of Competitive Transit Networks.” *Transportation Research Part B: Methodological* 44 (4): 434–446. doi:10.1016/j.trb.2009.11.001.

Daganzo, Carlos F., and Yanfeng Ouyang. 2019. “A General Model of Demand-Responsive Transportation Services: From Taxi to Ridesharing to Dial-a-Ride.” *Transportation Research Part B: Methodological* 126 (August): 213–224. doi:10.1016/j.trb.2019.06.001.

Deng, Lianbo, Qiang Zeng, Wenliang Zhou, and Feng Shi. 2014. “The Effect of Train Formation Length and Service Frequency on the Determination of Train Schedules.” *Proceedings of the Institution of Mechanical Engineers, Part F: Journal of Rail and Rapid Transit* 228 (4): 378–388. doi:10.1177/0954409713478589.

de Palma, André, Robin Lindsey, and Guillaume Monchambert. 2017. “The Economics of Crowding in Rail Transit.” *Journal of Urban Economics* 101 (September): 106–122. doi:10.1016/j.jue.2017.06.003.

Dijoseph, Patricia, and Steven I.Jy Chien. 2013. “Optimizing Sustainable Feeder Bus Operation Considering Realistic Networks and Heterogeneous Demand.” *Journal of Advanced Transportation* 47 (5): 483–497. doi:10.1002/atr.174.

Dou, Xueping, Qiang Meng, and Kai Liu. 2021. “Customized Bus Service Design for Uncertain Commuting Travel Demand.” *Transportmetrica A: Transport Science* 17 (4). Taylor & Francis: 1405–1430. doi:10.1080/23249935.2020.1864509.

Fan, Wenbo, Yu Mei, and Weihua Gu. 2018. “Optimal Design of Intersecting Bimodal Transit Networks in a Grid City.” *Transportation Research Part B: Methodological* 111. Elsevier

Ltd: 203–226. doi:10.1016/j.trb.2018.03.007.

Fan, Wenbo, and Yu Ran. 2021. "Planning Skip-Stop Services with Schedule Coordination." *Transportation Research Part E: Logistics and Transportation Review* 145 (December 2020). Elsevier Ltd: 102119. doi:10.1016/j.tre.2020.102119.

Fu, Xiao, William H.K. Lam, Bi Yu Chen, and Zhiyuan Liu. 2020. "Maximizing Space-Time Accessibility in Multi-Modal Transit Networks: An Activity-Based Approach." *Transportmetrica A: Transport Science* 18 (2). Taylor & Francis: 192–220. doi:10.1080/23249935.2020.1806372.

Gao, Yuan, Lixing Yang, and Ziyou Gao. 2018. "Energy Consumption and Travel Time Analysis for Metro Lines with Express/Local Mode." *Transportation Research Part D: Transport and Environment* 60: 7–27. doi:10.1016/j.trd.2016.10.009.

Gkiotsalitis, Konstantinos, and Oded Cats. 2021. "Optimal Frequency Setting of Metro Services in the Age of COVID-19 Distancing Measures." *Transportmetrica A: Transport Science*. doi:10.1080/23249935.2021.1896593.

Gu, Weihua, Zahra Amini, and Michael J. Cassidy. 2016. "Exploring Alternative Service Schemes for Busy Transit Corridors." *Transportation Research Part B: Methodological* 93. Elsevier Ltd: 126–145. doi:10.1016/j.trb.2016.07.010.

Guihaire, Valérie, and Jin Kao Hao. 2008. "Transit Network Design and Scheduling: A Global Review." *Transportation Research Part A: Policy and Practice* 42 (10). Elsevier Ltd: 1251–1273. doi:10.1016/j.tra.2008.03.011.

Guo, Qian Wen, Joseph Y.J. Chow, and Paul Schonfeld. 2018. "Stochastic Dynamic Switching in Fixed and Flexible Transit Services as Market Entry-Exit Real Options." *Transportation Research Part C: Emerging Technologies* 94. Elsevier Ltd: 288–306. doi:10.1016/j.trc.2017.08.008.

Guo, Rongge, Wei Guan, Wenyi Zhang, Fanting Meng, and Zixian Zhang. 2019. "Customized Bus Routing Problem with Time Window Restrictions: Model and Case Study." *Transportmetrica A: Transport Science* 15 (2): 1804–1824. doi:10.1080/23249935.2019.1644566.

Guo, Ren Yong, and W. Y. Szeto. 2021. "Profit Optimization of Public Transit Operators: Examining Both Interior and Boundary Solutions." *Transportmetrica A: Transport Science* 17 (4). Taylor & Francis: 824–855. doi:10.1080/23249935.2020.1817170.

Huai, Yue, Hong K. Lo, and Ka Fai Ng. 2021. "Monocentric versus Polycentric Urban Structure: Case Study in Hong Kong." *Transportation Research Part A: Policy and Practice* 151 (July). Elsevier Ltd: 99–118. doi:10.1016/j.tra.2021.05.004.

Huang, Di, Zhiyuan Liu, Pan Liu, and Jun Chen. 2016. "Optimal Transit Fare and Service Frequency of a Nonlinear Origin-Destination Based Fare Structure." *Transportation Research Part E: Logistics and Transportation Review* 96. Elsevier Ltd: 1–19. doi:10.1016/j.tre.2016.10.004.

Jara-Díaz, Sergio R., and Esteban Muñoz-Paulsen. 2021. "Lessons from the Strategic Design of a Bimodal Public Transport System on a Linear City." *Research in Transportation Economics*, 94. Elsevier Ltd: 101167. doi:10.1016/j.retrec.2021.101167.

Jiang, Y., and W. Y. Szeto. 2015. “Time-Dependent Transportation Network Design That Considers Health Cost.” *Transportmetrica A: Transport Science* 11 (1): 74–101. doi:10.1080/23249935.2014.927938.

Jiang, Yu, and Avishai (Avi) Ceder. 2021. “Incorporating Personalization and Bounded Rationality into Stochastic Transit Assignment Model.” *Transportation Research Part C: Emerging Technologies* 127 (April). Elsevier Ltd: 103127. doi:10.1016/j.trc.2021.103127.

Jiang, Yu, Thomas Kjaer Rasmussen, and Otto Anker Nielsen. 2022. “Integrated Optimization of Transit Networks with Schedule-and Frequency-Based Services Subject to the Bounded Stochastic User Equilibrium.” *Transportation Science*, Published Online: doi.org/10.1287/trsc.2022.1148.

Kepaptsoglou, Konstantinos, and Matthew Karlaftis. 2009. “Transit Route Network Design Problem: Review.” *Journal of Transportation Engineering* 135 (8): 491–505. doi:10.1061/(ASCE)0733-947X(2009)135:8(491).

Kim, Myungseob Edward, and Paul Schonfeld. 2015. “Maximizing Net Benefits for Conventional and Flexible Bus Services.” *Transportation Research Part A: Policy and Practice* 80. Elsevier Ltd: 116–133. doi:10.1016/j.tra.2015.07.016.

Kocur, George, and Chris Hendrickson. 1982. “Design of Local Bus Service With Demand Equilibration.” *Transportation Science* 16 (2): 149–170. doi:10.1287/trsc.16.2.149.

Lam, William H.K., and Hai Jun Huang. 1995. “Dynamic User Optimal Traffic Assignment Model for Many to One Travel Demand.” *Transportation Research Part B* 29 (4): 243–259. doi:10.1016/0191-2615(95)00001-T.

Lam, W. H.K., and J. Zhou. 2000. “Optimal Fare Structure for Transit Networks with Elastic Demand.” *Transportation Research Record*, no. 1733: 8–14. doi:10.3141/1733-02.

Li, Zhi Chun, William H.K. Lam, S. C. Wong, and A. Sumalee. 2012. “Design of a Rail Transit Line for Profit Maximization in a Linear Transportation Corridor.” *Transportation Research Part E: Logistics and Transportation Review* 48 (1). Elsevier Ltd: 50–70. doi:10.1016/j.tre.2011.05.003.

Li, Xin, Yue Luo, Tianqi Wang, Peng Jia, and Haibo Kuang. 2020. “An Integrated Approach for Optimizing Bi-Modal Transit Networks Fed by Shared Bikes.” *Transportation Research Part E: Logistics and Transportation Review* 141 (May). Elsevier: 102016. doi:10.1016/j.tre.2020.102016.

Li, Xin, Yue Luo, Yanhao Li, Huaiyue Li, and Wenbo Fan. 2022. “A Joint Optimisation Model for Designing Demand Responsive Connectors Fed by Shared Bikes.” *Transportmetrica A: Transport Science*. doi:10.1080/23249935.2022.2025949.

Liu, Tao, Oded Cats, and Konstantinos Gkiotsalitis. 2021. “A Review of Public Transport Transfer Coordination at the Tactical Planning Phase.” *Transportation Research Part C: Emerging Technologies* 133. Elsevier Ltd: 103450. doi:10.1016/j.trc.2021.103450.

Liu, Tao, Avishai (Avi) Ceder, and Subeh Chowdhury. 2017. “Integrated Public Transport Timetable Synchronization with Vehicle Scheduling.” *Transportmetrica A: Transport Science* 13 (10). Taylor & Francis: 932–954. doi:10.1080/23249935.2017.1353555.

Liu, Yining, and Yanfeng Ouyang. 2022. “Planning Ride-Sharing Services with Detour

Restrictions for Spatially Heterogeneous Demand: A Multi-Zone Queuing Network Approach," 1–34. http://arxiv.org/abs/2208.02219.

Luo, Sida, and Liujiang Kang. 2022. "A Bimodal Transit System for Large Cities: Cost Efficiency and Environment Friendliness." *Transportmetrica A: Transport Science*, 18(3), 1154-1177. May. doi:10.1080/23249935.2021.1931549.

Luo, Sida, and Yu (Marco) Nie. 2019. "Paired-Line Hybrid Transit System in a Monocentric City with Exponential Demand Pattern." *Transportation Research Record*. 2673(9), 614-626, doi:10.1177/0361198119839985.

Luo, Xiaoling, Wenbo Fan, Yangsheng Jiang, and Jun Zhang. 2020. "Optimal Design of Bus Stop Locations Integrating Continuum Approximation and Discrete Models." *Journal of Advanced Transportation* 2020. doi:10.1155/2020/8872748.

Luo, Xiaoling, Weihua Gu, and Wenbo Fan. 2021. "Joint Design of Shared-Bike and Transit Services in Corridors." *Transportation Research Part C: Emerging Technologies* 132 (November): 103366. doi:10.1016/j.trc.2021.103366.

Medina, Marcos, Ricardo Giesen, and Juan Muñoz. 2013. "Model for the Optimal Location of Bus Stops and Its Application to a Public Transport Corridor in Santiago, Chile." *Transportation Research Record*, no. 2352: 84–93. doi:10.3141/2352-10.

Mei, Yu, Weihua Gu, Michael Cassidy, and Wenbo Fan. 2021. "Planning Skip-Stop Transit Service under Heterogeneous Demands." *Transportation Research Part B: Methodological* 150 (December 2020). Elsevier Ltd: 503–523. doi:10.1016/j.trb.2021.06.008.

Nuzzolo, Agostino, and William H.K. Lam. 2021. *Modelling Intelligent Multi-Modal Transit Systems*. 1st Edition, CRC Press, doi:10.1201/9781315368986.

Ouyang, Yanfeng, Seyed Mohammad Nourbakhsh, and Michael J. Cassidy. 2014. "Continuum Approximation Approach to Bus Network Design under Spatially Heterogeneous Demand." *Transportation Research Part B: Methodological* 68. Elsevier Ltd: 333–344. doi:10.1016/j.trb.2014.05.018.

Park, Keunhyun, Reid Ewing, Sadegh Sabouri, Dong ah Choi, Shima Hamidi, and Guang Tian. 2020. "Guidelines for a Polycentric Region to Reduce Vehicle Use and Increase Walking and Transit Use." *Journal of the American Planning Association* 86 (2). Routledge: 236–249. doi:10.1080/01944363.2019.1692690.

Qu, Hezhou, Ruijie Li, and Steven Chien. 2021. "Maximizing Ridership through Integrated Bus Service Considering Travel Demand Elasticity with Genetic Algorithm." *Journal of Transportation Engineering, Part A: Systems* 147 (4): 04021010. doi:10.1061/jtepbs.0000511.

Repolho, Hugo M., António P. Antunes, and Richard L. Church. 2013. "Optimal Location of Railway Stations: The Lisbon-Porto High-Speed Rail Line." *Transportation Science* 47 (3): 330–343. doi:10.1287/trsc.1120.0425.

Saidi, Saeid, S. C. Wirasinghe, Lina Kattan, and Seyedshahab Esmaeilnejad. 2017. "A Generalized Framework for Complex Urban Rail Transit Network Analysis." *Transportmetrica A: Transport Science* 13 (10): 874–892. doi:10.1080/23249935.2017.1348401.

Schöbel, Anita. 2005. "Locating Stops along Bus or Railway Lines-a Bicriteria Problem." *Annals of Operations Research* 136 (1): 211–227. doi:10.1007/s10479-005-2046-0.

Shen, Zuo Jun Max, Bo Feng, Chao Mao, and Lun Ran. 2019. "Optimization Models for Electric Vehicle Service Operations: A Literature Review." *Transportation Research Part B: Methodological* 128 (2019). Elsevier Ltd: 462–477. doi:10.1016/j.trb.2019.08.006.

Shyue Koong Chang, and Paul M. Schonfeld. 1991. "Multiple Period Optimization of Bus Transit Systems." *Transportation Research Part B: Methodological* 25 (6): 453–478. doi:10.1016/0191-2615(91)90038-K.

Sivakumaran, Karthikgeyan, Yuwei Li, Michael J. Cassidy, and Samer Madanat. 2012. "Cost-Saving Properties of Schedule Coordination in a Simple Trunk-and-Feeder Transit System." *Transportation Research Part A: Policy and Practice* 46 (1): 131–139. doi:10.1016/j.tra.2011.09.013.

Sivakumaran, Karthik, Yuwei Li, Michael Cassidy, and Samer Madanat. 2014. "Access and the Choice of Transit Technology." *Transportation Research Part A: Policy and Practice* 59. Elsevier Ltd: 204–221. doi:10.1016/j.tra.2013.09.006.

Sun, S., and W. Y. Szeto. 2019. "Optimal Sectional Fare and Frequency Settings for Transit Networks with Elastic Demand." *Transportation Research Part B: Methodological* 127. Elsevier Ltd: 147–177. doi:10.1016/j.trb.2019.06.011.

Sun, Yang, Xiaonian Sun, Baoqing Li, and Dehui Gao. 2013. "Joint Optimization of a Rail Transit Route and Bus Routes in a Transit Corridor." *Procedia - Social and Behavioral Sciences* 96 (Cictp). Elsevier B.V.: 1218–1226. doi:10.1016/j.sbspro.2013.08.139.

Sun, Yanshuo, Qianwen Guo, Paul Schonfeld, and Zhongfei Li. 2017. "Evolution of Public Transit Modes in a Commuter Corridor." *Transportation Research Part C: Emerging Technologies* 75. Elsevier Ltd: 84–102. doi:10.1016/j.trc.2016.12.007.

Szeto, W. Y., and Y. Jiang. 2014. "Transit Route and Frequency Design: Bi-Level Modeling and Hybrid Artificial Bee Colony Algorithm Approach." *Transportation Research Part B: Methodological* 67. Elsevier Ltd: 235–263. doi:10.1016/j.trb.2014.05.008.

Tang, Chunyan, Avishai Avi Ceder, and Ying En Ge. 2017. "Integrated Optimization of Bus Line Fare and Operational Strategies Using Elastic Demand." *Journal of Advanced Transportation* 2017. doi:10.1155/2017/7058789.

Tang, Yili, Hai Yang, Bo Wang, Jianling Huang, and Yunyun Bai. 2019. "A Pareto-Improving and Revenue-Neutral Scheme to Manage Mass Transit Congestion with Heterogeneous Commuters." *Transportation Research Procedia* 38: 586–605.

Tang, Yili, Yu Jiang, Hai Yang, and Otto Anker Nielsen. 2020. "Modeling and Optimizing a Fare Incentive Strategy to Manage Queuing and Crowding in Mass Transit Systems." *Transportation Research Part B: Methodological* 138 (August): 247–267. doi:10.1016/j.trb.2020.05.006.

Tian, Qingyun, Yun Hui Lin, and David Z. W. Wang. 2021. "Autonomous and Conventional Bus Fleet Optimization for Fixed-Route Operations Considering Demand Uncertainty." *Transportation* 48 (5): 2735–2763. doi:10.1007/s11116-020-10146-4.

Tian, Qiong, Peng Liu, Ghim Ping Ong, and Hai Jun Huang. 2021. "Morning Commuting Pattern and Crowding Pricing in a Many-to-One Public Transit System with Heterogeneous Users." *Transportation Research Part E: Logistics and Transportation Review* 145 (October 2020). Elsevier Ltd: 102182. doi:10.1016/j.tre.2020.102182.

Tsai, Feng Ming, Steven I.Jy Chien, and Lazar N. Spasovic. 2008. "Optimizing Distance-Based Fares and Headway of an Intercity Transportation System with Elastic Demand and Trip Length Differentiation." *Transportation Research Record*, no. 2089: 101–109. doi:10.3141/2089-13.

Tsai, Feng Ming, Steven Chien, and Chien Hung Wei. 2013. "Joint Optimization of Temporal Headway and Differential Fare for Transit Systems Considering Heterogeneous Demand Elasticity." *Journal of Transportation Engineering* 139 (1): 30–39. doi:10.1061/(ASCE)TE.1943-5436.0000468.

Verhoef, Erik T. 2020. "Optimal Congestion Pricing with Diverging Long-Run and Short-Run Scheduling Preferences." *Transportation Research Part B: Methodological* 134 (April): 191–209. doi:10.1016/j.trb.2020.02.009.

Vuchic, V.R., 2007. Urban Transit System and Technology. *John Wiley & Sons*.

Wirasinghe, S. C., and P. N. Seneviratne. 1986. "Rail Line Length in an Urban Transportation Corridor." *Transportation Science* 20 (4): 237–245. doi:10.1287/trsc.20.4.237.

Wirasinghe, S.C. 1980. "Nearly Optimal Parameters for a Rail/Feeder-Bus System on a Rectangular Grid." *Transportation Research Part A: General* 14 (1): 33–40. doi:10.1016/0191-2607(80)90092-8.

Wu, Liyu, Weihua Gu, Wenbo Fan, and Michael J. Cassidy. 2020. "Optimal Design of Transit Networks Fed by Shared Bikes." *Transportation Research Part B: Methodological* 131. Elsevier Ltd: 63–83. doi:10.1016/j.trb.2019.11.003.

Wu, Fei, and Paul Schonfeld. 2022. "Optimized Two-Directional Phased Development of a Rail Transit Line." *Transportation Research Part B: Methodological* 155 (December 2021). Elsevier Ltd: 424–447. doi:10.1016/j.trb.2021.12.007.

Xu, Guangming, Linhuan Zhong, Xinlei Hu, and Wei Liu. 2022. "Optimal Pricing and Seat Allocation Schemes in Passenger Railway Systems." *Transportation Research Part E: Logistics and Transportation Review* 157 (April 2021). Elsevier Ltd: 102580. doi:10.1016/j.tre.2021.102580.

Yang, Hai, and Woo Ka Kin. 2000. "Modeling Bus Service under Competition and Regulation." *Journal of Transportation Engineering* 126 (5): 419–425. doi:10.1061/(ASCE)0733-947X(2000)126:5(419).

Yang, Nan, and Yong Long Lim. 2018. "Temporary Incentives Change Daily Routines: Evidence from a Field Experiment on Singapore's Subways." *Management Science* 64 (7): 3365–3379. doi:10.1287/mnsc.2017.2731.

Yang, Hongtai, Zhaolin Zhang, Wenbo Fan, and Feng Xiao. 2021. "Optimal Design for Demand Responsive Connector Service Considering Elastic Demand." *IEEE Transactions on Intelligent Transportation Systems* 22 (4): 2476–2486. doi:10.1109/TITS.2021.3054678.

Yang, Yi, Lianbo Deng, Qing Wang, and Wenliang Zhou. 2020. "Zone Fare System Design in a Rail Transit Line." *Journal of Advanced Transportation* 2020. doi:10.1155/2020/2470579.

Zhou, Feng, Changfeng Li, Zhiyuan Huang, Ruihua Xu, and Wei David Fan. 2020. “Fare Incentive Strategies for Managing Peak-Hour Congestion in Urban Rail Transit Networks.” *Transportmetrica A: Transport Science* 18 (1). Taylor & Francis: 166–187. doi:10.1080/23249935.2020.1793028.