PH.D. THESIS

Mathematical Models and Algorithms for Managing Carsharing Systems

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Declaration

I declare that this thesis entitled ”Mathematical Models and Algorithms for Managing Carsharing Systems” is carried out by me under the supervision of Dr Burak Boyacı and Prof. Konstantinos G. Zografos. It has not been submitted for a degree at this or any other university. It is an original research written by me except where explicitly stated otherwise in the text.

The main body of this thesis consists of three papers that can be read as separate entities. Chapter 2 has been published. Chapter 3 and Chapter 4 are in preparation for submission.

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Abstract

Carsharing systems provide sustainable, environmentally friendly short-distance inner urban transportation. One-way carsharing systems, a type of carsharing system with a significant portion of registered members worldwide, allow users to leave their vehicles at any station or anywhere in the region. However, this leads to vehicle stock imbalances since the demand is not symmetrical. One-way carsharing systems require demand-balancing relocation activities with the involvement of personnel bringing vehicles where and when needed. Therefore, one-way carsharing systems are complex regarding strategic and operational decisions. The main goal of this thesis is to contribute to the one-way carsharing literature by providing i) new mathematical models and heuristic algorithms for the strategic decisions in infrastructure investments regarding the new technological developments in electric vehicle chargers ii) an in-depth literature review for user-based relocation and pricing studies and iii) new mathematical models and heuristic algorithms for operational decisions where users are offered counteroffers that increase the operational profitability.

The first work conducted in this thesis exploits new technological developments corresponding to electric vehicle chargers. A new mathematical model is proposed to determine the number and location of fast/rapid chargers to be implemented in one-way carsharing systems with an electric vehicle fleet. The proposed model takes into account vehicle relocation, battery availability and

partial charging. As the model becomes intractable for large-sized instances, we introduce heuristic algorithms that reduce the size of the variables and constraints created. The results suggest that the proposed algorithms increase profitability by providing charger infrastructure upgrade decisions at the stations.

The second work of this thesis presents a literature review on user-based relocations and pricing studies. Other than operator-based relocations, user-based relocations (providing alternative trips to users) and pricing are powerful tools to balance the vehicle supply and trip demand. This part of the thesis provides a discussion on the main lines of the research pertinent to user-based relocation and pricing methods applied to carsharing systems by categorizing the literature based on the aim and methods of the studies. Furthermore, we identified current research gaps and shed some light on directions for future research.

Finally, the last work presented here provides an operational-level decision support system in one-way carsharing systems. The users are offered counteroffers (user-based relocations) to increase profitability by mainly decreasing operator-based relocation costs. A mathematical model is introduced to find the best possible offer to the user at each trip request. The model considers the users' acceptance and rejection rates of the offers. Additionally, heuristic algorithms that work efficiently are presented. The results suggest that incorporating user-based relocations in operational level decisions increase profitability.

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Chapter 1

Introduction

1.1 Background and motivation

Carsharing is a transportation mode similar to car-rental systems with a short term access to a shared vehicle fleet. Carsharing systems provide sustainable transportation without sacrificing private vehicle comfort. Users registered in such systems benefit from the access of private vehicles without considering the costs that are associated with owning one.

The earliest carsharing system can be traced back to a carsharing initiative in Switzerland called "Sefage" in 1948. Yet, carsharing has become an essential actor in the transportation sector after 2000, with new technological advancements and traditional car-rental companies offering carsharing systems (Mindur et al., 2018). Especially, the growing smartphone penetration has had an undeniable effect on the growth of carsharing systems. As a result, the number of countries with carsharing systems increased to 47, with almost 32m registered users and 200k vehicles worldwide as of 2018 (Shaheen and Cohen, 2020).

The impacts of carsharing can be classified as environmental, land use, social and transportation. Reduced vehicle ownership, vehicle km travelled and mobility emissions are the positive environmental effect of carsharing. Studies reveal that each vehicle in a carsharing system decreases the need for at least four vehicles on the road. Additionally, at least 15.6% of the users have sold their private vehicles after joining the system (Shaheen and Cohen, 2012). According to Shaheen et al. (1999), private vehicle owners tend to drive more than required since the variable cost of driving is relatively low compared to fixed costs. This does not apply to carsharing systems where the users pay for actual use, resulting in reduced vehicle km travelled. This is also one of the reasons for reduced mobility emissions. Carsharing systems' impact on land use can be explained by reducing the demand for parking spaces and freeing up public areas by up to 22% (Tchervenkov et al., 2019). Next, the affordable private car access to all, including the disadvantaged groups, defines the social effects. Finally, the decrease in vehicle ownership, decrease in vehicle purchases, and change in travel modes by encouraging active lifestyles such as cycling and walking are the effects of carsharing systems on transportation in general (Shaheen and Cohen, 2012).

The effects of carsharing systems with an electric vehicle fleet are enriched compared to an internal combustion engine fleet (Liao et al., 2020). Carsharing operators benefit from the decreased operational cost and enjoy a positive environmental image when deploying EVs in their fleets. EV fleets in carsharing systems also affect consumer behaviour in the users, providing easy access to EVs and promoting EV use. On the other hand, providing EVs in the fleet imposes additional burdens on the operational complexity of the carsharing companies. First, the carsharing companies should equip their stations with EV chargers. Second, the state of charge (SoC) level must either be above a predefined threshold or meet the trip's battery requirements before each rental. This reduces vehicle availability as the time required to charge an EV fully can exceed eight hours using conventional chargers.

In terms of vehicle drop-off locations, carsharing can be divided into two categories. The first category, round-trip, requires users to return the vehicle to the origin station. The second category, one-way systems, does not force users to return to the origin station (or location). In one-way systems, users may leave the vehicle at any station (station-based one-way systems) or anywhere permitted in the region (free-floating systems). Although the second category can attract more users as these systems are more flexible regarding the destination locations, it comes with a cost of operational complexity. The fact that the trip's origin and destination are not forced to be identical causes demand asymmetry in vehicle stocks at stations (or subregions for free-floating systems). Providing a sufficient number of vehicles at stations is vital to maintain a high level of customer service. Therefore, one-way carsharing companies often interfere with the vehicle configuration at the stations to serve more customers. One way to alter the vehicle stock is to relocate vehicles by personnel from over-saturated to under-saturated stations (operator-based relocation). Operator-based relocations are very complex since they involve both vehicle and personnel movements. The personnel drives the vehicle to the station where the vehicle stock is low and then proceeds to the next task on foot, by bicycle or by public transport. These movements result in high personnel costs. Another way to rebalance vehicle distribution is to incentivise users to change their original travel requests so that vehicle flow is maintained from a low-demand station to a high-demand one (user-based relocation). Finally, the last method in balancing the uneven distribution of vehicles is through pricing. By setting lower prices on the routes from low-demand to high-demand stations, and vice versa, it is aimed to change customers' travel behaviour in favour of the system. Note that user-based relocation and pricing strategies depend on the stochastic nature of human travel decisions.

The operational complexity of carsharing systems has attracted the attention of the Operational Research Community, especially over the last decade. However, despite the vast literature on carsharing systems, there are still several aspects in research on carsharing decisions regarding advancements in EV charging technology and innovative approaches that increase profitability.

The current doctoral thesis aims to:

- Develop mathematical models and heuristic algorithms to solve strategic-level decision-making problems pertinent to EV charging technologies.

- Provide a comprehensive and in-depth literature review on operational-level decisions focusing on user-based relocations and pricing.
- Provide solution algorithms to spatiotemporal user-based relocation problems that increase profitability of the system.

1.2 Related literature

This section provides an introductory literature review on carsharing systems and a brief overview of relevant studies that are pertinent to the presented work in this thesis.

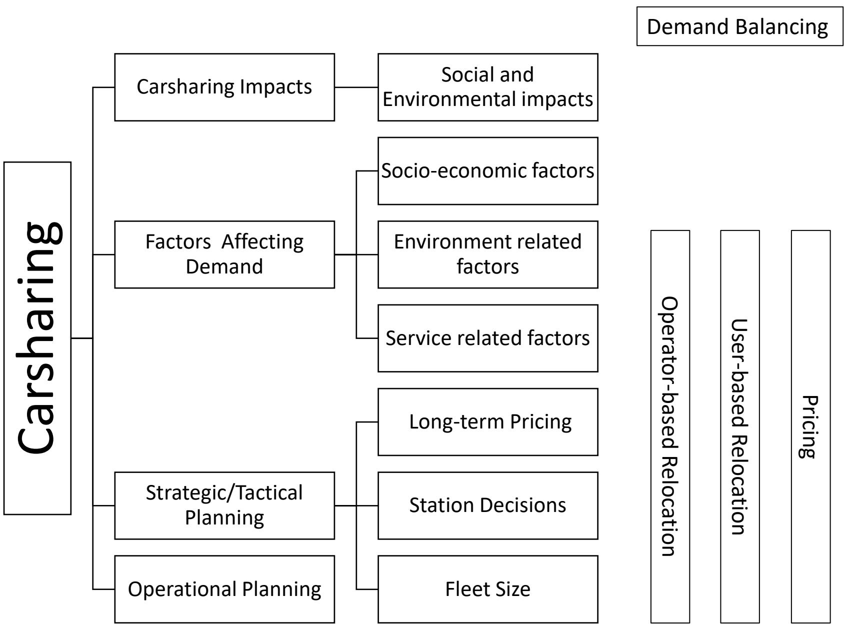

Figure 1.1: Classification of carsharing literature

Figure 1.1 provides an overview of the carsharing literature. Vehicle sharing can be divided into four categories: vehicle sharing effects, factors affecting demand, strategic/tactical and operational decisions. The first two categories can be considered descriptive studies, while strategic, tactical and operational planning decision studies are the prescriptive studies which constitute the main focus of the Operational Research Community regarding carsharing systems. We considered strategic and tactical decision studies in the same category because these decisions were mostly considered together.

Another dimension of the carsharing literature is the demand balancing methods. Vehicle supply and trip demand can be balanced through operator relocations, user relocations or pricing. Service-related factors (under the factors affecting demand category) and the prescriptive studies are mostly concerned with one of the vehicle balancing methods. Note that Chapter 3 is the review study of user-based and pricing studies regarding carsharing systems. Therefore, we do not present a review of demand rebalancing methods in this section. Instead, we refer readers to Chapter 3 for more information.

1.2.1 Carsharing impacts

We review carsharing impacts based on the aims of the studies. First, one major impact of carsharing is on vehicle ownership. Martin et al. (2010) conducted a survey to analyze the effects of carsharing household vehicle ownership in North America. The before-and-after analysis showed that the carsharing membership greatly influences the car ownership of the members, reducing the average number of vehicles owned from 0.47 to 0.24. Similarly, Vine and Polak (2019) analysed the early-stage impact of carsharing on vehicle ownership. Surveys performed three months after the free-floating system was introduced in London showed that 37% of the respondents said the presence of the free-floating system had affected their vehicle ownership. Furthermore, 4% of the respondents have already disposed of their vehicles, and 2% were willing to sell a car within the following months.

One other impact of carsharing is the reduction of greenhouse gas (GHG) emissions. By surveying the members of carsharing companies, Martin and Shaheen (2011) showed that the decrease in vehicle kilometres travelled (VKT) and vehicle ownership due to carsharing membership led to a significant reduction in greenhouse gas emissions (-0.84 t GHG/year per household). Another study by the same authors, Martin and Shaheen (2016), focused on 5 US cities to explore the effects of Car2Go, one of the leading carsharing companies. The study showed that the net annual emission change ranged between -10,027 and -2,270 t/year and greenhouse gas emission was decreased between -18% and -4% per household for the five cities due to Car2Go membership. Similarly, VKT was also reduced between -16% and -6% per household due to the vehicles sold and Car2Go use.

Other than the impacts of reduction in vehicle ownership, greenhouse gas emissions, and VKT, carsharing has social impacts on society by providing "pay-as-you-go" mobility to those who cannot afford to own a vehicle (Shaheen et al., 2012)

1.2.2 Factors affecting demand

Demand is affected by the socio-economical factors of the users, environmental factors, and service related factors. We first investigate the socio-economical factors (i.e. age, gender, occupation, and so forth). Kim et al. (2015) surveyed electric vehicle sharing program participants to analyse the factors that affect the participation of the carsharing systems. The analysis showed that age, household income and marital status influence the participation in the program significantly. The study showed that older participants tend to continue to use the carsharing program despite the common belief that young people use carsharing more. Furthermore, single participants and participants with higher incomes were likely to continue using carsharing.

Secondly, environmental factors such as weather, temperature, geographical characteristics of the area, and seasons affect the demand. Schmöller et al. (2015) showed that the weather conditions affect short-term demand of free-floating carsharing systems in Berlin and Munich. Similarly, Yoon et al. (2017) examined the environmental factors that may affect the demand for carsharing systems. It was concluded that precipitation and air quality do not significantly impact demand. It was also stated that warmer weather conditions are more favourable than colder weather conditions. On the contrary, through a stated preference experiment in Beijing, Wang et al. (2020) showed that daily weather condition's effect on the daily demand is negligible.

Finally, service-related factors, such as vehicle age and pricing, affect the demand. By using regression models, de Lorimier and El-Geneidy (2013) showed that the age of the vehicle and having a child seat have a significant impact on vehicle usage. Note that how pricing affects the demand is explored in Chapter 3.

1.2.3 Strategic and tactical level decisions

Here, we provide optimization-based studies that are on strategic and tactical level decisions that are pertinent to Chapter 2 of this thesis. Strategic decisions in carsharing systems involve determination of the number, location and parking capacity of the stations and infrastructure investment decisions. Furthermore, the tactical decisions of fleet sizing decisions are mostly adopted in strategical decision-making models. Therefore, we examine station planning and/or fleet sizing decision studies together.

The most distinctive categorization in such studies is the fleet type. Depending on the fleet type (combustion engine or electric), station planning is concerned with providing parking spaces with or without chargers. Note that the location and size of the stations are found by aiming to serve as many trips as possible (or maximize profit) in combustion engine fleet, while in the electric vehicle fleet, it is also aimed to serve trips with sufficient battery (state of charge, aka SoC) level.

Correia and Antunes (2012) and Huang et al. (2018) studied station location and capacity problem in carsharing systems considering combustion engine fleet. In Correia and Antunes (2012), it was aimed to maximize the profit of one-way carsharing systems while deciding on the location and capacity of the locations. It is assumed that the relocation activities are performed at the end of the day. The problem was modelled as an integer programming model and applied to a real-world system in Lisbon.

Huang et al. (2018) considered station location, station capacity and fleet sizing problems with simultaneous operator-based relocations. A mixed integer nonlinear programming (MINLP) model that considers nonlinear demand that is shaped by the competition between private vehicles and carsharing systems was proposed. Note that the model did not take into account the personnel movements. A customized gradient algorithm was proposed to solve the MINLP model.

Unlike Correia and Antunes (2012) and Huang et al. (2018), station location problems of carsharing systems with electric vehicle fleets should consider i) the time required to partially/fully charge the vehicle, ii) battery SoC level to cover the trip.

Boyaci et al. (2015) considered the problems of station location, fleet sizing and the number of personnel to be employed together at a one-way electric carsharing system. It was assumed that each vehicle needed to be fully charged to be assigned to any relocation activity or trip. A multi-objective integer programming model was proposed to maximize the company's profit and users' net benefits. In order to reduce the size of the variables and constraints created, an imaginary hub through which the relocations are performed was considered. The model was tested at a real-world system in Nice, France.

Hua et al. (2019) proposed a multistage nonlinear stochastic integer program to determine the station location and fleet size while considering time-varying uncertain demand. The demand was assumed to be known at the beginning of each stage, and real-time fleet operations (relocation and charging decisions) were optimized at each stage. Note that partial charging was allowed, and vehicles with sufficient SoC levels (but may not be fully charged) were considered for either relocations or trips.

Çalık and Fortz (2019) modelled the station location problem of one-way electric carsharing systems as a mixed integer stochastic program. A Benders decomposition algorithm was proposed to optimally locate the stations.

Huang et al. (2020a) proposed a MINLP model to optimize the fleet size and station capacity problem at one-way electric carsharing systems. It was assumed that the SoC of the vehicles that are parked at the stations follow a continuous distribution. A golden section line search method and a shadow price algorithm were built to solve the proposed MINLP.

Xu and Meng (2019) addressed the fleet sizing problem in one-way electric carsharing systems considering nonlinear charging functions by developing a set partitioning model. To solve the problem, an exact method based on the branch and price algorithm was proposed.

Xu et al. (2021) also studied fleet sizing problem with consideration of battery degradation. Battery wear cost was assumed to be nonlinear when charging and discharging the vehicles. A MINLP model was proposed and linearized by using a piecewise linear approximation approach and the outer-approximation method.

Even though many studies in the literature have addressed station location and sizing problems, infrastructure planning involving different charging technologies has not been taken into account. Especially, infrastructure upgrade decisions for the systems with stations equipped with slow chargers have room to increase profitability by optimally locating fast/rapid chargers. This is why in Chapter 2, we have solved the problem of determining the number and location of fast chargers to be implemented in one-way electric carsharing systems while considering vehicle relocation and battery availability.

1.2.4 Operational level decisions

Operational level decision models determine daily vehicle and personnel schedules, decide on pricing by assuming the strategical decisions are already made, station location and capacities are defined beforehand.

Optimization-based operational planning studies aim to maximize profit while deciding on the trip selection. Note that higher profit values are achievable by applying demand balancing methods (operator-based / user-based relocation, pricing). Therefore, operational studies mainly incorporate at least one of the rebalancing methods. In this section, we provide a brief introduction to optimization-based operational planning studies that include operator-based relocations. In Chapter 3, we present a comprehensive literature review on user-based relocations and pricing.

Nourinejad et al. (2015) considered joint optimization of vehicle and personnel relocations at one-way carsharing systems where the demand is assumed to be known. By considering all the trip requests to be served, a mixed integer linear programming (MILP) model was developed to minimize the cost of the operations. In order to solve the proposed MILP efficiently, a heuristic model that divides the MILP into two problems was proposed. Vehicle relocation and staff relocation decisions are made at separate problems in the proposed heuristic.

Boyacı et al. (2017) addressed the same problem considering electric vehicle fleets. To solve the problem, a decision framework which consists of optimization and simulation modules was proposed. The optimization module consisted of three optimization models, namely, a station clustering model, a vehicle operations optimization model, and a personnel flow model. The station clustering model reduced the size of the problem. The vehicle and personnel schedules were found by solving the last two models by disregarding the battery requirements. Then, through the simulation module, the feasibility was checked. If an infeasible schedule was found, an additional constraint was added to the model, and the framework was solved iteratively until a feasible solution was found.

Gambella et al. (2018) modelled the same operational level problem by tracking the battery level of each vehicle. Each arc in the system (either charging or driving) was associated with a battery value, and by adding constraints, it was made sure that the battery level of each vehicle was changed according to the arc's battery value. A secondary model was also proposed to solve the overnight relocation problem to prepare the system for the next operational day.

Zhang et al. (2019) considered relays (completing the trip with multiple vehicles) due to the battery requirements of the electric vehicle fleet in operational level problems. A space–time–battery network flow model was proposed to solve the operational level problem without fully considering the staff scheduling problem. A diving heuristic approach was developed to solve the model efficiently.

We have provided the literature of the other demand balancing methods, user-based relocation and pricing, in Chapter 3 and presented research gaps and possible future directions.

In user-based relocations, the users receive a counteroffer from the system. The counteroffer is made due to the reasons: i) the original trip request is not feasible, but the counteroffer is ii) the counteroffer generates more profit. As we explored the literature of user-based relocations, we have concluded that one possible future direction is solving the dynamic user-based relocation problem where the users might reject the counteroffer. In Chapter 4 we have modelled this problem and proposed solution algorithms.

1.3 Contributions

Despite the increasing number of studies in the field of one-way carsharing systems, especially in recent years, there are still open strategic and operational-level decision problems that need to be addressed. For strategical level decisions, the incorporation of technological developments related to advanced charging technologies has not been explored. Likewise, there is still more to study for operational level decisions on user-based relocation decisions that represent the real operating conditions of the one-way systems and user behaviour. This thesis addresses some of these gaps by providing mathematical model-based solution frameworks and algorithms for both strategical and operational-level decision problems in one-way carsharing systems.

We summarize the contributions of the thesis as follows :

- To close the gap in strategical decisions regarding charger technologies, this thesis introduces a novel model and solution algorithms to solve the fast charger location problem in one-way electric carsharing systems. The proposed mathematical model finds the optimal location of fast chargers that replace already installed conventional chargers. The model represents real operating conditions by allowing partial charging and keeping track of the battery levels of the vehicles.

- Due to the computational intractability of the model to solve the fast charger location problem, this thesis presents matheuristic algorithms to provide near-optimal solutions. The first type of heuristic algorithm reduces the size of the model by generating relocation variables only where and when relocation activities might be needed. The second heuristic algorithm decreases the size of the model by grouping the stations.
- To identify under-explored areas in user-based relocation and pricing studies in carsharing systems, this thesis provides an in-depth literature review of the current state of research focusing on vehicle stock balancing methods pertinent to user-based relocations and pricing. Furthermore, this thesis proposes future directions for interested researchers and practitioners focusing on user-based relocations and pricing.
- To close the gap in operational-level decisions regarding user-based relocations, this thesis introduces a mathematical model which provides users with a discounted counteroffer that generates the maximum expected profit considering user acceptance/rejection rates. Furthermore, to represent real-world systems better, the proposed model addresses the trips and relocations in different time-space networks since, in real-world systems, the relocation activities may start and end anytime.
- To solve the problem efficiently, heuristic algorithms that reduce the size of the model by i) working on a graph-spanner network structure and ii) decreasing the number of the relocation variables, as in the fast charger location problem, have been proposed.

1.4 Structure of the thesis

Chapter 2, Chapter 3, and Chapter 4 of this thesis are either published or in preparation for submission to journals. A brief description of each chapter is presented below.

- Chapter 2 *Enhancing the performance of one-way electric carsharing systems through the optimum deployment of fast chargers.* This chapter introduces a novel MILP model to introduce fast/rapid chargers at one-way electric vehicle sharing systems. This chapter also proposes heuristics based on i) clustering/grouping the stations and ii) removing the relocation arcs that are likely to get a value of zero.
- Chapter 3 *Balancing carsharing systems through user-based relocation and pricing: A literature review.* This chapter presents a comprehensive and in-depth literature review on operational-level decisions focusing on user-based relocations and pricing. In addition, this chapter identifies research gaps and possible future directions in carsharing demand balancing studies considering user-based or pricing methods.
- Chapter 4 *User-based relocations in one-way carsharing systems with consideration of flexible demands with user acceptance probabilities.* This chapter presents a MILP model that optimally provides a counteroffer to the user considering the acceptance and rejection rates of the offers. Like the arc-removing heuristic presented in Chapter 2, an arc-removing algorithm for driving and moving relocations is provided. Additionally, a graph-spanner based heuristic is introduced to solve the problem efficiently.
- Chapter 5 is the concluding remarks chapter of the thesis.

1.5 Status of publications

Chapter 2 is published as: Seyma Bekli, Burak Boyacı, Konstantinos G. Zografos, Enhancing the performance of one-way electric carsharing systems through the optimum deployment of fast chargers, Transportation Research Part B: Methodological, Volume 152, 2021, Pages 118-139, https://doi.org/10.1016/j.trb.2021.08.001. This work was presented at ”The OR Society’s 63rd Annual Conference (OR60)” on September 11, 2018 in Lancaster, UK, at ”2nd IMA and OR Society Conference on Mathematics of Operational Research” on April 26, 2019 in Birmingham, UK and at ”31st European Conference on Operational Research (EURO 2021)” on July 14, 2021 in Athens, Greece.

Chapter 4 was presented at ”The 32nd European Conference on Operational Research (EURO2022)” on July 4, 2022 in Espoo, Finland and at ”Conference on Advanced Systems in Public Transport and TransitData 2022 (CASPT2022)” on November 10, 2022 in Tel Aviv, Israel.

Chapter 2

Enhancing the performance of one-way electric carsharing systems through the optimum deployment of fast chargers

Abstract

One-way electric carsharing systems (OWECS) provide environmentally friendly mobility that enables users to commence and terminate their trips at a preselected station within a region. However, the operations of OWECS are complicated mainly due to: i) unbalanced spatial and temporal distribution of demand which causes shortage or surplus of vehicles at stations, and ii) excessive battery charging requirements that can reach up to 8 hours. To ensure vehicle availability to their customers, carsharing companies hire personnel to relocate vehicles to restore the demand-supply balance, and increase the number of trips served. On the other hand, fast charging technologies can reduce the charging time drastically and can help carsharing companies to cope with the inefficiencies arising from excessive battery charging times. Therefore, fast charging technologies have the potential to enhance the operational performance of OWECS.

In this study, we propose an integer programming model to determine the number and location of fast chargers to be implemented in OWECS while considering vehicle relocation and battery availability. We propose a time-space-battery level network model which allows to track battery levels of each vehicle. As the number of stations increases, the number of relocation variables created increases polynomially which makes the model intractable for problem instances found in real world OWECS. Therefore, we are introducing three heuristics, two of which are based on the concept of reduction of vehicle relocation variables, while the third heuristic is based on station grouping. The heuristics that are categorized as relocation reduction type heuristics generate only a fraction of all possible relocation arcs significantly reducing the number of relocation variables. The third heuristic reduces the number of variables by grouping the stations and solves the problem initially at an aggregate level using groups of stations instead of individual stations, while subsequently optimizes the location of chargers within each group of stations.

We tested different approaches combining heuristics and tested them on smaller instances with exact solutions to identify the approach that is both accurate and efficient. We applied the selected heuristic approach on real-life instances taken from an OWECS based in Nice, France. The results suggest that the use of the proposed modelling and solution framework leads to a configuration of the network of fast and conventional chargers that improves the profitability and the number of trips served by the OWECS system.

keywords : One-way electric carsharing systems; Location modelling; Fast charger; Battery electric vehicle; Integer programming

2.1 Introduction

Many cities around the world are seeking to improve air quality through the introduction of "Low Emission Zones". Oxford is the first city to declare "Zero Emission Zone" starting from 2020 (Jones, 2017; Consultants, 2020). In Oxfordshire the county council is cooperating, within the framework of a "Zero Emission Zone" scheme, with electric car-clubs (a.k.a. carsharing companies), to provide mobility services to those that do not have access to a privately owned electric vehicle (Council, 2019).

Carsharing is a type of short term car rental system which allows users to access the vehicles and pay depending on the duration and/or driving distance of the rental. Unlike classical car rental systems, a user first registers in the system, reserves a vehicle, and drives for the reserved period. As of 2014, carsharing systems have 4.8 million registered users sharing over 100,000 vehicles worldwide (Shaheen and Cohen, 2016). One of the reasons for the rapid expansion of carsharing systems is their social and environmental benefits. Firstly, most of the cars used in these systems are new and small. In Germany, 70% of the cars of carsharing companies belong to this category (Loose, 2010). Newer and smaller cars result in lower fuel consumption and reduced emissions. Secondly, registered users of carsharing systems are less likely to buy a new car or more willing to sell a car they own. Studies show that 28,000 cars have been removed from the roads in 5 cities in the U.S. because of the memberships in carsharing systems (Martin and Shaheen, 2016). Carsharing may be considered as an essential complement of the urban public transportation system providing the convenience of driving a car without owning it.

Carsharing systems are divided into two categories according to the locations that the cars can be dropped-off. Round-trip carsharing systems are those that require the user to return the vehicle to the station where it was picked-up. One-way carsharing systems allow users to drop off at a parking spot within a specified area. This could be at designated stations (station-based one-way) or anywhere in a designated region (free-floating). Although the operations of one-way systems are more complex due to the spatial and temporal demand-supply imbalance, they are more attractive since they offer more flexibility. Therefore, more and more carsharing companies are offering one-way carsharing services to their potential users. In North America, the number of vehicles used in one-way carsharing systems has reached 6,850 with 511,000 registered members as of 2015 (Martin and Shaheen, 2016).

Carsharing with electric vehicles (EV) provides more sustainable, environmentally-friendly mobility as compared to that with internal combustion engine vehicles. A study of Clewlow (2016) highlights that carsharing members possess fewer cars on average and when they do, the ratio of having an alternative car with low carbon footprint is higher than non-members, which shows that members of carsharing systems are highly considerate of the environment. In recent years, carsharing companies are adding more EVs to their fleets in order to improve maintenance needs[1] and produce less carbon dioxide. However, one of the drawbacks of EVs is the time needed to charge their battery. The time required to fully charge

[1] U.S. DOE (United States Department of Energy) (2020) suggests that electric vehicles require less maintenance compared to internal combustion engine vehicles.

an EV using conventional chargers can exceed 8 hours. The excessive battery charging time reduces vehicle availability leading to lower vehicle utilization and revenues. Recent advancements in the technology of EV chargers indicate that via fast chargers[2] the required time for charging a vehicle can be decreased to 30 minutes. As of May 2020, Tesla, one of the leading EV companies, provides 16,585 fast chargers in the US that can charge a vehicle within 30 minutes up to 80% battery charge and 170-mile drive range (Tesla, 2020). Nissan Leaf can be charged from 20% to 80% battery level within 60 mins via ChaDeMo fast chargers (Nissan, 2020).

There are three key elements for charging a vehicle; a connector installed at the vehicle, a cable (could be portable or tethered to the charger) and a charger. However, until the introduction of the EU Directive 2014/94/EU which introduced the Type-2 and Combo-2 plugs (as shown in Figure 2.1), there was no standardization (EU, 2014).

Standard Type2 connections provide alternative current (AC) which is convenient for home-charging. As the battery of an EV requires direct current (DC), Type 2 converts AC to DC which results in slow charging. For this reason, the car manufacturers in Europe are combining AC and DC chargers through the introduction of combined charging system (CCS Combo-2) (as shown in Figure 2.1). When in need of a DC plug for fast chargers, the driver can open the hidden part and use as a Combo-2 plug.

One of the concerns of the DC chargers may be their effect on the battery life of the electric vehicles. Although Kia states that minimum use of DC chargers is recommended (Kia, 2019), a study conducted in Idaho National Laboratory suggests that the difference in capacity loss between using only AC and only DC chargers is insignificant compared to overall capacity loss (Shirk and Wishart, 2015). Four identical Nissan Leaf EVs were observed for battery degradation in

[2]In some sources this term is called rapid or super charger depending on the manufacturer. The term fast charger is used thoughout this study to address superior charging units. Moreoever, we use the term conventional chargers when referring both regular and slow chargers.

Figure 2.1: Type2 and CCS charge connectors (Trucks, 2020; InsideEVs, 2018)

Shirk and Wishart (2015). Two of the EVs are charged only with AC chargers, whereas the remaining EVs are charged with DC chargers. The study indicates that after 50,000 miles of use, the battery degradation is at 22% and 25% of AC and DC charged vehicles, respectively. The study also states that this may be an upper bound for capacity loss as the experiments took place in a very hot city and vehicles were charged twice as much as it is recommended by the manufacturers. In the light of the results of this study, the mileage for battery replacement may not be affected significantly by charger type for businesses renting vehicles.

Carsharing companies with EVs are following the recent developments in fast charging technology closely. DriveNow brought fast chargers into the service for the users in Denmark (DriveNow, 2019), Autobleue, a carsharing company in Nice, France offered three fast chargers along with conventional chargers in the city. In the near future, as the number of carsharing users and demand will increase, the need for fast chargers will also grow.

The price of fast chargers is considerably higher than the price of conventional chargers and ranges between 22,000€ and 32,000€ (Çalık and Fortz, 2019). However, fast chargers can contribute significantly to the increase of vehicle utilization and the level of service offered to customers. Given the cost and operational characteristics of fast charging technologies, carsharing companies using EV's are facing decisions regarding the number and the locations of fast chargers they should introduce in their operations.

The fast charger infrastructure investment planning problem has been studied to determine the optimal locations of different types of chargers in order to cover the charging requirements of EV users in regions (Zhang et al., 2017) and along highways (He et al., 2019). The fast charger infrastructure investment problem has been investigated from the standpoint of the charger suppliers (Guo et al., 2016).

Although the optimization of the operations of OWECS has been studied (see Boyacı et al., 2017; Gambella et al., 2018), the effect of the use of different charging technologies on the operational performance of OWECS has not been adequately addressed. Sonneberg et al. (2015) and Jiao et al. (2017) investigate the fast charger infrastructure planning problem in round-trip and OWECS, respectively. However, these studies are not comprehensive enough to capture the system complications arising from the vehicle relocation and battery charging requirements of OWECSs (see Section 2.2 for a detailed analysis). To close this literature gap, we present a new integer programming model that optimizes the location of fast chargers that will be replacing initially installed conventional chargers. This generic charger upgrading decision model can be applied to any OWECS with a fleet of both AC and DC charger compatible vehicles.

The remainder of the paper is structured as follows: In Section 2.2, we provide a summary of related literature. In Section 2.3, an integer linear programming (ILP) model for the charger upgrading decision is presented. Due to the computational complexity of the model, we present heuristic approaches in Section 2.4. Section 2.5 follows with the computational experiments and comparisons on both the exact model and the heuristic approaches. Finally, in Section 2.6, concluding remarks and future research directions are given.

2.2 Related literature

The complexity of the operations of carsharing systems has attracted the attention of the research community over the past years. Remane et al. (2016) and Ferrero et al. (2018) provide literature reviews and taxonomies regarding typologies and use of carsharing systems. The readers are referred to Brandstatter et al. (2016) for a mathematical modelling-oriented literature review of electric vehicle carsharing systems, to Jorge and Correia (2013) for a detailed survey on carsharing demand models and one-way carsharing operations modelling, and to Laporte et al. (2018) for a survey on mathematical aspects of shared mobility systems including bike-sharing operations.

Decisions regarding the design and management of carsharing systems are categorized into three levels: strategic, tactical and operational (Boyaci et al., 2015). Strategic decisions concern with station location and capacity decisions. Tactical decisions are related to fleet sizing, while operational decisions focus on vehicle relocation, personnel scheduling, and battery charging. In this paper, we are addressing strategic decisions regarding the improvement of the vehicle charging infrastructure of one-way carsharing systems. Please note that the focus of our research is to make the charger upgrading decisions for OWECS systems that are already in operation and therefore the strategic decisions regarding the size, the number, and the location of the stations have already been made and constitute inputs to our problem. In addressing charger upgrading decisions, we take into account vehicle relocation decisions which are usually made at an operational level. However, the consideration of vehicle relocations is necessary when making strategic decisions as vehicle relocation operations affect the cost of the operation and the spatial and temporal availability of the vehicles of OWECS systems. Therefore, in our literature review we also provide a brief overview of studies focused on strategic and/or operational OWECS decisions.

Strategic decisions regarding the location of stations have been investigated by

Boyaci et al. (2015); Brandstatter et al. (2017, 2020); Biesinger et al. (2017); Çalık and Fortz (2019); Hua et al. (2019) and Huang et al. (2020a).

The first study that considers the location of the charging stations in one-way systems is Boyaci et al. (2015). The study proposed a multi-objective ILP model that determines the number and location of carsharing stations, the fleet size, the number of relocation personnel and relocation for OWECS systems while maximizing company's profit and users' net benefits. In order to decrease the size of the problem, it was assumed that each relocation operation is performed through an imaginary hub. It was also assumed that vehicles after serving a trip, are waiting at the stations to be fully charged before they are assigned to the next trip. This paper does not consider different types of chargers and does not allow partially charged vehicles to be allocated to trip requests. Therefore, this model and the associated solution framework can not address the charger selection decisions. The proposed optimization framework was applied using data from an electric carsharing company in Nice, France.

Hua et al. (2019) solved station planning and fleet size determination problem under time-varying uncertain demand. The study proposed a multistage nonlinear stochastic integer program seeking to minimize total operational and infrastructure cost, unmet demand, and cost of waiting customers. It was assumed that the demand was known at the beginning of each stage. Via the multistage approach, a real-time fleet management was aimed. A Lagrangian relaxation and a stochastic dual dynamic programming methods were used to solve the problem. The study was tested on data from New York City.

Huang et al. (2020a) considered mixed integer nonlinear program to solve the location and fleet size problem by maximizing the profit of the system. The study assumes that the charging levels (state of charge) follow a continuous distribution. Due to computational complexity, a heuristic algorithm based on dividing the original model into two submodels and solving iteratively to tackle strategic and operational level problems is proposed. The submodel for strategic level problem is solved via shadow price and golden section line search algorithms. The operational level submodel is solved via a rolling horizon method. The proposed approach was tested in Suzhou, China.

Boyaci et al. (2015), Hua et al. (2019) and Huang et al. (2020a) consider relocation operations in modelling strategic decisions while studies Brandstatter et al. (2020, 2017), Çalık and Fortz (2019) and Biesinger et al. (2017) disregard the relocation operations, arguing that this might not be necessary for strategic decision making.

Brandstatter et al. (2020) proposed integer programming formulations and constructive heuristics for solving the station location and fleet size determination problem with explicit consideration of the battery levels. The proposed approach was tested on data from Vienna, Austria.

Biesinger et al. (2017) considered the same problem with a two-stage approach which uses heuristic solution approaches in both stages. In the first stage, a variable neighbourhood search heuristic is used to determine the station locations and the fleet size. In the second stage, greedy heuristic algorithms are used to determine the trip requests that should be served. The proposed model considers the battery availability in assigning trip requests to vehicles. The study was compared with the models proposed by Brandstatter et al. (2020) for the Vienna instance. Although the proposed heuristic was not able to find the optimal solutions which are found by Brandstatter et al. (2020) for the small size problems, it was able to find better feasible solutions for the large size problems.

Brandstatter et al. (2017) formulated the station location selection and fleet size determination in one-way carsharing systems as a two-stage stochastic integer program. The first stage determines the station locations and fleet size, while the second stage determines the trips to be served. The model assumes that each trip is served with a full battery, and does not consider vehicle relocation operations. A heuristic approach is also presented for solving large instances. The model and the heuristic algorithm are tested for a real-life instance in Vienna for taxi trip requests.

The customers are assumed to walk to origin and destination stations within 5 minutes distances.

Çalık and Fortz (2019) introduced a mixed integer linear programming (MILP) model and proposed a Benders decomposition algorithm to optimally locate carsharing stations. Fast chargers are assumed to be installed in all stations. The proposed algorithms were tested on real data from Manhattan taxi trips.

It is worth noting that the models proposed by Brandstatter et al. (2017) and Çalık and Fortz (2019) do not allow partial charging of vehicles following their return from a trip. This assumption coupled with the lack of consideration of vehicle relocation may result to lower vehicle utilization and number of served trips.

In what follows, we provide a literature review on operational decision level problems that incorporate vehicle relocation operations.

Boyacı et al. (2017) considered the vehicle and personnel relocation problem in a hierarchical multi-objective MILP model aiming to maximize the number of trip requests served, and minimize vehicle relocation cost. The proposed solution framework incorporates both vehicle and personnel relocation operations and allows partial charging of the vehicles following their return from a trip. A simulation module is included in the proposed framework for checking vehicle charging requirements. To reduce the computational complexity resulting from the incorporation of vehicle and personnel movements, the proposed model introduced the concept of station clustering. Boyacı and Zografos (2019) proposed a framework that investigates the impacts of temporal and spatial flexibility of trip requests.

Gambella et al. (2018) proposed a MILP model to find the route assignments for both vehicles and personnel while maximizing profit over all routes. Furthermore, a model was proposed to optimize relocation operations during the non-operating hours in order to start the next operational period with the appropriate number of vehicles. The relocation model was computationally intractable; hence two heuristic algorithms based on performing relocations at certain time steps and adding relocation arcs gradually (rolling horizon approach) were proposed. The proposed model and solution algorithms were tested on a data set from Singapore.

Zhang et al. (2019) formulated vehicle assignment with relays (completing the journey by passing through an additional station and switching the vehicles because of battery constraints) as a time-space–battery network flow model. Heuristic approaches based on LP relaxation were used to solve large instances. The proposed model was tested on a real-world system operating in Shanghai.

The literature on station location selection models mostly focuses on using one type of chargers. To the best of our knowledge, the only studies that considered ”fast chargers” along with conventional chargers are Sonneberg et al. (2015) and Jiao et al. (2017). The former study is considering fast charging units for a round-trip carsharing system and proposed a mathematical model to find station locations and the number of fast and conventional charging units to be allocated at each station. The model assumes that each vehicle serves demand once per day. The model is integrated into a decision support system and applied to San Francisco. The latter study proposed a MILP model for a one-way electric carsharing system for locating the depots (i.e. stations) and selecting charger types. Jiao et al. (2017) discretized time using independent time-intervals, hence the interaction between time intervals is not taken into account. The model was applied to a carsharing company in Beijing. Both of these studies consider the need of fast chargers based on the infrastructure cost. However, these studies do not consider multiple vehicle trips and interactions between the discrete time intervals that the vehicles operate.

Table 2.1: List of referenced articles

		Strategic Decision			Tactical Decision	Operational Decision					
Decision Type	Article	Station Location	Station Size	Infrastructure Upgrade	Fleet Size	Number of Staff	Vehicle Movements	Staff Movements	Relocation Operations	Allowing Partial Charging	Multiple Charger Types
Strategic	Boyaci et al. (2015)	✓	✓		✓	✓	✓	✓	✓		
	Hua et al. (2019)	✓			✓		✓		✓	✓	
	Huang et al. (2020a)	✓	✓		✓		✓		✓	✓	
	Brandstatter et al. (2020)	✓	✓		✓		✓			✓	
	Biesinger et al. (2017)	✓	✓		✓		✓			✓	
	Brandstatter et al. (2017)	✓	✓		✓		✓				
	Çalık and Fortz (2019)	✓			✓		✓				
	Jiao et al. (2017)	✓		✓			✓		✓		✓
	Sonneberg et al. (2015) (round trip)	✓	✓	✓	✓		✓				✓
Operational	Boyacı et al. (2017)					✓	✓	✓	✓	✓	
	Gambella et al. (2018)						✓	✓	✓	✓	
	Zhang et al. (2019)						✓		✓	✓	
	Boyacı and Zografos (2019)					✓	✓	✓	✓	✓	
Strategic	Proposed Study			✓			✓		✓	✓	✓

The focus of the research reported in this paper is on modeling and solving the problem regarding the optimization of strategic decisions relevant to the determination of the number, location, and type of chargers (fast and conventional), for OWECS systems. The realistic representation of this problem requires the: i) consideration of different charger types, ii) recognition that vehicles of OWECS systems can be partially charged when allocated to users, and iii) incorporation of relocation operations in modeling the spatial and temporal availability of vehicles. As discussed in the preceding paragraphs of the literature review section and presented in Table 2.1, none of the currently available strategic and/or operational models address simultaneously all these three important attributes. This paper aims to close this literature gap by introducing models and algorithms that provide exact and near-optimal solutions for making charger investment decisions for existing OWECS systems considering both battery charging level and vehicle relocation operations. This study contributes to the current state of knowledge by introducing:

- A new ILP model that finds the optimal location of fast chargers to replace already installed conventional chargers. This model addresses the demand uncertainty by considering multiple scenarios, each associated with an occurrence probability. In order to have a realistic representation of the system, the proposed model keeps track of the battery levels of the EVs and allows partial charging. We ensure that no battery infeasibility, i.e., vehicle movements without sufficient battery charge, occurs while serving multiple trips. We incorporate partial charging, the introduction of fast chargers, and vehicle relocations in our model.
- Heuristic algorithms to generate near-optimal solutions without compromising the quality of the ILP model. The first type of heuristic used decreases the size of the model by considering only the relocations needed. The second heuristic treats the problem in a three-stage approach. In the first stage, the stations are grouped using an integer programming model aiming

to minimize the maximum distance within each group of stations. At the second stage, the ILP problem is solved using trip requests between stations. At the third stage, a new ILP model is used to locate fast chargers within each group of stations.

2.3 Mathematical model

In this paper, we consider a reservation-based system where the users request trips in advance, stating the origin and destination stations of their trips. The users are notified, at the end of the reservation period, whether or not their requests have been accepted. Each day starts with a predetermined number of fully charged vehicles at the stations. The stations are initially equipped with conventional chargers implemented at designated parking spots. All EVs in the fleet are identical and compatible with both AC and DC chargers. After each vehicle trip/relocation, the vehicles are connected to an available charger. Please note that if it is decided to upgrade at least one of the chargers at a station, the vehicle entering the station is notified where it will be charged. No charger or parking spot switching is allowed at the stations, i.e., if a vehicle enters a station and is connected to a charger, it occupies the allocated parking spot and the charger until it leaves the station. The battery is charged at a constant rate that depends on the type of the charger. When the vehicle battery is fully charged, it stops charging, but remains connected to the charger until the vehicle will be assigned to a trip request. The battery consumption is considered linear during the vehicle rental (trip) and its relocation. Vehicles are relocated by available personnel, and the relocation time for any origin-destination pair is fixed and does not depend on the time of the day that the vehicle trip/relocation takes place.

The operational hours of the system are discretized in time intervals of equal length. When the system is not operating, the system is reset, i.e. vehicles are distributed to the stations and fully charged until the first time interval of the next operational day starts. The battery capacity is also divided into $\mathcal{K}+1$ equal levels. Any battery level in $\left[\frac{k-1}{\mathcal{K}}, \frac{k}{\mathcal{K}}\right)$ corresponds to discrete level of $k-1$ and only 100% battery charging level corresponds to level $\mathcal{K}$.

Owing to its strategic nature, the proposed model does not optimize personnel relocation decisions. Personnel movement is simplified by considering only vehicle relocation movement and personnel cost is added by considering the duration of vehicle relocations.

In what follows, we provide the mathematical model that optimizes the location selection for fast chargers.

2.3.1 Sets and indices

$s \in S$	scenarios
$i \in I_s$	trip requests in scenario s
$j, l \in J$	nodes (stations)
$t, t' \in T = \{0, ..., \mathcal{T}\}$	time intervals
$k, k' \in K = \{0, ..., \mathcal{K}\}$	battery levels
$p \in P$	charger types, $p = 0$ denotes the conventional charger

2.3.2 Parameters

Origin(i)	origin station of trip i
Dest(i)	destination station of trip i
Start(i)	start time interval of trip i
End(i)	end time interval of trip i
TR_i	revenue of trip i
P_s	occurrence probability of scenario s
DC_{jp}	depreciated cost of upgrading a conventional charger to charger type p at station j per day
RC_{jl}	relocation cost of a vehicle from station j to l
PC	personnel cost per time interval
D_{jl}	number of time intervals it takes to relocate a vehicle from node j to l
rate_p	number of battery levels charger type p can charge per time interval
CT_{jl}	number of battery levels utilized for relocation from station j to l

CT_i number of battery levels utilized for trip i

IC_{jp} current number of charger type p installed at station j

AR average number of time intervals to relocate a vehicle, then walk to the origin of the next relocation

MaxPer maximum number of personnel that can work at any time interval

V_j number of vehicles that are present at station j at the beginning of the time horizon

2.3.3 Variables

f_{jp} number of charger type p $(p \neq 0)$ decided to be installed for upgrading the charging capacity of station j

z^s_{ik} 1, if trip i of scenario s is served by a vehicle with a battery level k; 0, otherwise

r^{st}_{jlk} number of vehicles with battery level k relocated from station j to l starting at time interval t in scenario s

b^{st}_{jkp} number of vehicles with battery level k that start charging at charger type p at station j starting at time interval t in scenario s

e^{st}_{jkp} number of vehicles with battery level k that leave charger type p at station j at time interval t in scenario s

d^{st}_{jkp} number of vehicles with battery level k that are being charged by a charger type p at station j during time interval t in scenario s

c^{st}_{jp} number of charger type p at station j occupied by vehicles at time interval t in scenario s

2.3.4 Formulation

$$\max \overbrace{\sum_{s\in S,k\in K,i\in I_s} \mathrm{P}_s\,\mathrm{TR}_i\, z^s_{ik}}^{\text{expected revenue}} - \overbrace{\Big(\underbrace{\sum_{j\in J,p\in P-\{0\}} \mathrm{DC}_{jp}\, f_{jp}}_{\text{infrastructure upgrade cost}} + \underbrace{\sum_{j,l\in J,t\in T,k\in K,s\in S} \mathrm{P}_s\,\mathrm{RC}_{jl}\, r^{st}_{jlk}}_{\text{relocation cost}} + \underbrace{\sum_{j,l\in J,t\in T,k\in K,s\in S} \mathrm{P}_s\,\mathrm{D}_{jl}\,\mathrm{PC}\, r^{st}_{jlk}}_{\text{personnel cost}} \Big)}^{\text{expected cost}} \tag{2.1}$$

subject to

$$\sum_{\substack{(k'\in K,i\in I_s):\\ k'=CT_i+k\\ \mathrm{Dest}(i)=j\\ \mathrm{End}(i)=t}} z^s_{ik'} + \sum_{\substack{(l\in J,k'\in K):\\ k'=CT_{lj}+k}} r^{s(t-D_{lj})}_{ljk'} = \sum_{p\in P} b^{st}_{jkp} \qquad \forall j\in J, s\in S, k\in K, t\in T \tag{2.2}$$

$$\sum_{\substack{(k\in K,i\in I_s):\\ \mathrm{Origin}(i)=j\\ \mathrm{Start}(i)=t\\ CT_i\le k}} z^s_{ik} + \sum_{\substack{(l\in J,k\in K):\\ CT_{jl}\le k}} r^{st}_{jlk} = \sum_{p\in P} e^{st}_{jkp} \qquad \forall j\in J, s\in S, k\in K, t\in T \tag{2.3}$$

$$d^{s(t-1)}_{j(k-\mathrm{rate}_p)p} + b^{st}_{jkp} - e^{st}_{jkp} = d^{st}_{jkp} \qquad \begin{array}{c}\forall j\in J, t\in T, s\in S, p\in P\\ \mathrm{rate}_p \le k < \mathcal{K}\end{array} \tag{2.4}$$

$$b^{st}_{jkp} - e^{st}_{jkp} = d^{st}_{jkp} \qquad \begin{array}{c}\forall j\in J, t\in T, s\in S, p\in P\\ 0\le k < \mathrm{rate}_p\end{array} \tag{2.5}$$

$$\sum_{\substack{(k\in K):\\ k\ge \mathcal{K}-\mathrm{rate}_p}} d^{s(t-1)}_{jkp} - e^{st}_{j\mathcal{K}p} = d^{st}_{j\mathcal{K}p} \qquad \forall j\in J, t\in T, s\in S, p\in P \tag{2.6}$$

$$\sum_{k\in K} d^{st}_{jkp} = c^{st}_{jp} \qquad \forall j\in J, t\in T, s\in S, p\in P \tag{2.7}$$

$$IC_{j0} - \sum_{p\in P-\{0\}} f_{jp} \ge c^{st}_{j0} \qquad \forall j\in J, t\in T, s\in S \tag{2.8}$$

$$IC_{jp} + f_{jp} \ge c^{st}_{jp} \qquad \forall j\in J, t\in T, s\in S, p\in P-\{0\} \tag{2.9}$$

$$\sum_{k\in K} z^s_{ik} \le 1 \qquad \forall s\in S, i\in I_s \tag{2.10}$$

$$\sum_{\substack{(t'\in T):\\ t\le t'\le t+\mathrm{AR}}} \sum_{j,l\in J,k\in K} r^{st'}_{jlk} \le \mathrm{MaxPer} \qquad \forall t\in T, s\in S \tag{2.11}$$

$$\sum_{p\in P} d^{s0}_{j\mathcal{K}p} = V_j \qquad \forall j\in J, s\in S \tag{2.12}$$

$$\sum_{p\in P} \sum_{\substack{(k\in K):\\ 0\le k\le \mathcal{K}-1}} d^{s0}_{jkp} = 0 \qquad \forall j\in J, s\in S \tag{2.13}$$

$$\sum_{k\in K,p\in P} d^{s\mathcal{T}}_{jkp} = V_j \qquad \forall j\in J, s\in S \tag{2.14}$$

$$z^s_{ik} \in \{0,1\} \qquad \forall s\in S, i\in I_s, k\in K \tag{2.15}$$

$$r^{st}_{jlk}, b^{st}_{jkp}, e^{st}_{jkp}, d^{st}_{jkp}, c^{st}_{jp}, f_{jp} \in \{0\}\cup \mathbb{Z}^+ \qquad \forall j,l\in J, k\in K, p\in P, s\in S, t\in T \tag{2.16}$$

Objective function (2.1) maximizes expected daily profit considering all demand scenarios. The daily profit is calculated as the difference between the expected daily revenues generated by all served trips and the expected daily cost consisted of the infrastructure upgrade cost, expected total relocation personnel cost and vehicle relocation cost. Since the fast charger implementation decision is a strategic decision, it is not scenario dependent. Expected revenue, vehicle relocation and personnel costs are scenario dependent and therefore, a coefficient expressing the scenario probability is used. Scenario revenue is calculated as the sum of the revenues of all trips corresponding to the scenario under consideration. Relocation cost represents the expected driving cost associated with the relocation activities. The parameter RC_{jl} in the relocation cost component is the unit energy consumption cost of driving the route connecting station j to station l. Similarly, relocation personnel cost per scenario can be calculated by multiplying personnel cost per time interval by the total number of time intervals required to relocate all vehicles of that scenario. We do not consider maintenance and parking related cost as done in Boyaci et al. (2015), since the number of vehicles and parking areas are considered fixed. The additional cost associated with the reduction of battery and vehicle life due to high utilization of vehicles is not taken into consideration.

Constraints (2.2) are used to keep track of the number of vehicles that arrive at station j with charging level k at time interval t in scenario s by trips or relocation activities. Similarly, Constraints (2.3) are introduced to keep track of vehicles that leave station j. Please note that the variables z_{ik}^{s} and r_{jlk}^{st} are generated only if battery levels are sufficient (i.e. $k \geq \mathrm{CT}_i$ and $k \geq \mathrm{CT}_{jl}$ for z_{ik}^{s} and r_{jlk}^{st}, respectively).

Constraints (2.4), (2.5), and (2.6) ensure that in scenario s at charger type p of station j, the number of vehicles being charged at time interval t is equal to the number of vehicles being charged at time interval $t-1$ plus the number of vehicles arrived at time interval t minus the number of vehicles departed from station j at time interval t. Note that these constraints are written separately for charging levels $\mathrm{rate}_p \leq k < \mathcal{K}$, $0 \leq k < \mathrm{rate}_p$ and $\mathcal{K}$. Constraints (2.5) are needed to define the number of vehicles being charged at charger type p when the initial battery level is less than what could be charged at one time interval (rate_p).

This indicates that there would be no vehicle flow coming from the previous time interval to charger type p. Constraints (2.6) are used to keep track of the fully charged vehicles. Since there is an upper limit on the battery capacity (battery levels of $\mathcal{K}$), the number of vehicles with battery level $k \geq \mathcal{K} - \text{rate}_p$ at time interval $t-1$ will be fully charged and be at the charger type p at the station unless they serve a trip or relocation at time t.

Constraints (2.7)-(2.9) are to ensure enough number of chargers of each type are present at each station. Constraints (2.7) keep track of the occupied capacity of each charger type at each station. Constraints (2.8) ensure that the number of vehicles parked at the conventional chargers of station j cannot exceed the station's conventional charger capacity. Here, it is also ensured that the total number of purchased chargers for station j cannot exceed the initial conventional charger capacity of station j. We assume that the operator is allowed to replace only conventional chargers with non-conventional chargers. Constraints (2.9) ensure that the number of vehicles that are being charged or parked at a charger type p at station j will not exceed the number of charger type p at station j.

Constraints (2.10) ensure that every demand request is served at most once. Constraints (2.11) restrict the number of personnel and the total number of relocations performed within consecutive AR time intervals. Here, AR is the average relocation time in the system plus the average movement time between one relocation's destination and the other's origin station. The rationale behind creating Constraints (2.11) is to prevent generating solutions that have relocations accumulated at certain time intervals. For instance, consider a solution with (w) number of relocations where $w = \sum_{j,l,k} R_{jlk}^{s^*t^*}$ for a time interval t^* at scenario s^*. This means that the company needs to hire at least w number of personnel to relocate the vehicles at time t^*. This might not be practically feasible when w is high. In order to prevent such cases to occur, we limit the model to have at most MaxPer number of relocations within AR consecutive time intervals with Constraints (2.11).

Constraints (2.12) and (2.13) ensure that a predetermined number of fully charged vehicles are present at the beginning of the time horizon at each station in each scenario. Contraints (2.14) ensure that the same predetermined number of vehicles are available at each station in each scenario at the end of the day. Constraints (2.15) and (2.16) define the domains for the binary and integer variables.

2.4 Heuristic approaches

The model provided in Section 2.3 may not be computationally tractable due to the large number of variables and constraints. In this section, we provide two heuristic approaches, namely selective relocation and grouping approaches, that reduce the size of the problem.

2.4.1 Heuristic 1: Selective Relocation

Number of relocation variables created in mathematical model (2.1)-(2.16) is in the order of $|J|^2|T||K||S|$. With a sufficient battery level, a relocation may happen between any origin-destination pair at any time. Given 64 time intervals and 33 battery levels in a 60 station system like the system in Nice, France, the total number of relocation variables created is equal to 7,063,414 even for a single scenario. Yet, we observed that the total number of realized relocations account for the 0.017% of the created relocation variables for small-sized instances analyzed in Section 2.5.

As most of the created relocation variables take value of zero, we decrease the number of relocation variables. Reducing the density of the relocation variables (arcs) has been proposed in studies (Carlier et al., 2015) and (Gambella et al., 2018). The former study employs the relocation arcs at specific time steps, whereas the latter considers the inclusion of relocation arcs gradually. Both of these reduction techniques generate good quality, near-optimal solutions.

In this study, we propose two heuristics by creating relocations akin to trip requests. There are two main reasons to perform relocations at a system; namely, to ensure the availability of vehicles and/or parking spaces. In the first heuristic, relocation arcs are created to serve at least one of the reasons. To serve the first reason, relocation arcs are created such that the arrival time of a relocation to its destination (also the origin of the trip) must be prior to the starting time of the trip. The time between the end of the relocation and start of the trip is limited to a predetermined time window. The same predetermined time window is applied to the relocation arcs which provide a parking spot for an end of a trip. By this method, the created number of relocation arcs become in the order of $|I_s||J||S||K|$.

To illustrate this heuristic method, consider a simple OWECS having four stations with a capacity of one charger and one parking spot at each station as shown in Figure 2.2A. Trip requests and relocation arcs are demonstrated with green and red arrows, respectively. A trip is requested from Station 1 to Station 2 at time interval 4 with a duration of five time intervals. In order to provide a vehicle to Station 1, possible relocation arcs are created from Station 2, 3 and 4. Similarly, in order to provide a parking spot at Station 2, a relocation arc from Station 2 to Station 1, 3 and 4 at time interval 9 are created.

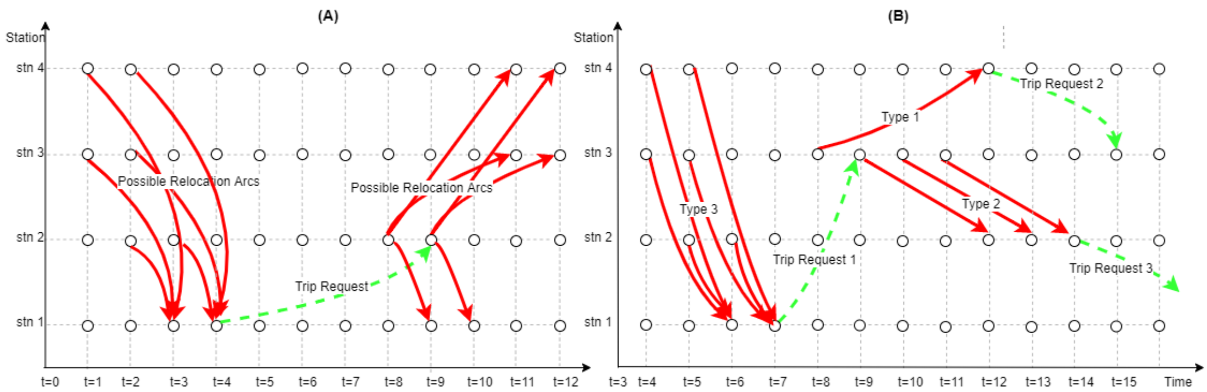

Figure 2.2: Examples of Relocation Restriction 1 (A) and Relocation Restriction 2 (B)

In the second relocation selection approach, three types of the relocation arcs are inserted to the network. The first type of arcs are the ones when relocation may be required both at the origin and destination stations. Consider the same system again with three trip requests, as shown in Figure 2.2B. A relocation arc (Type 1) starting at Station 3 at time 8 and ending at Station 4 at time 12 is created to provide a parking spot for arrival of a trip and provide a vehicle to a start of another trip. The start time of the relocation ($t = 8$) is earlier than the end time of "Trip Request 1" ($t = 9$) and it is limited by a time window. It should be stressed that a relocation arc is added if the end time of the relocation is prior to the start of "Trip Request 2" and it falls into the time window. The second type of relocation arcs start after a trip has been concluded, and it provides a vehicle to another trip as shown "Type 2" in Figure 2.2B. The last type of relocation arcs (Type 3) is added to serve trips that start at the beginning of the day.

We observed 2,778,912 and 851,800 relocation variables are needed on average for relocation selection 1 and 2, respectively, for our 10 instances each contains 60 stations and 600 demand requests. (The time window was taken as 2 in both approaches). The reduction rate of relocation variables were 60.7% in the first and 87.9% in the second approach.

2.4.2 Heuristic 2: Station Grouping - Three Step Approach

Even though the relocation variables account for the highest proportion of variables, reducing their number may not suffice to solve large scale problems. Variables $b^{st}_{jkp}, e^{st}_{jkp}, d^{st}_{jkp}$ are in the order of $|J||K||S||T||P|$ and having these many variables could prevent us from having a tractable model that is applicable to real-life problems. To decrease the number of variables even further, we propose a second type of heuristic approach based on grouping the stations. This method reduces $|J|$; hence, reduces the sizes of all of the variable sets.

This approach has three steps: (i) station grouping ((2.17)-(2.24)), (ii) solving the mathematical model ((2.1)-(2.16)) using the identified groups, and (iii) solving the within-group models ((2.25)-(2.33)) (see Figure 2.3). We start by solving the station grouping model provided in Section 2.4.2.1 by employing a walking distance matrix. Each station group is formed in such a way that any pair of stations within the group is accessible by the relocation personnel through walking from the stations in the group. The main motivation for aggregating the stations is to run the model provided in Section 2.3 for the groups considering them as "large stations". The Station Grouping model aims to minimize the maximum walking distance within the groups and is solved for different $|G|$ values where G denotes the group set ($1 \leq |G| \leq |J|$). After observing the objective function values for different $|G|$ values, the final $|G|$ value is determined by selecting the number of groups corresponding to the elbow of the curve (point of diminishing returns) describing the relationship between the maximum walking distance within the groups and the number of the corresponding groups. It should be noted that if we set $|G| = |J|$, the grouping heuristic method becomes the exact model we have proposed in Section 2.3. On the other hand, the fewer the groups are, the higher the deviation from the optimal solution is. It is a trade-off between accuracy and efficiency.

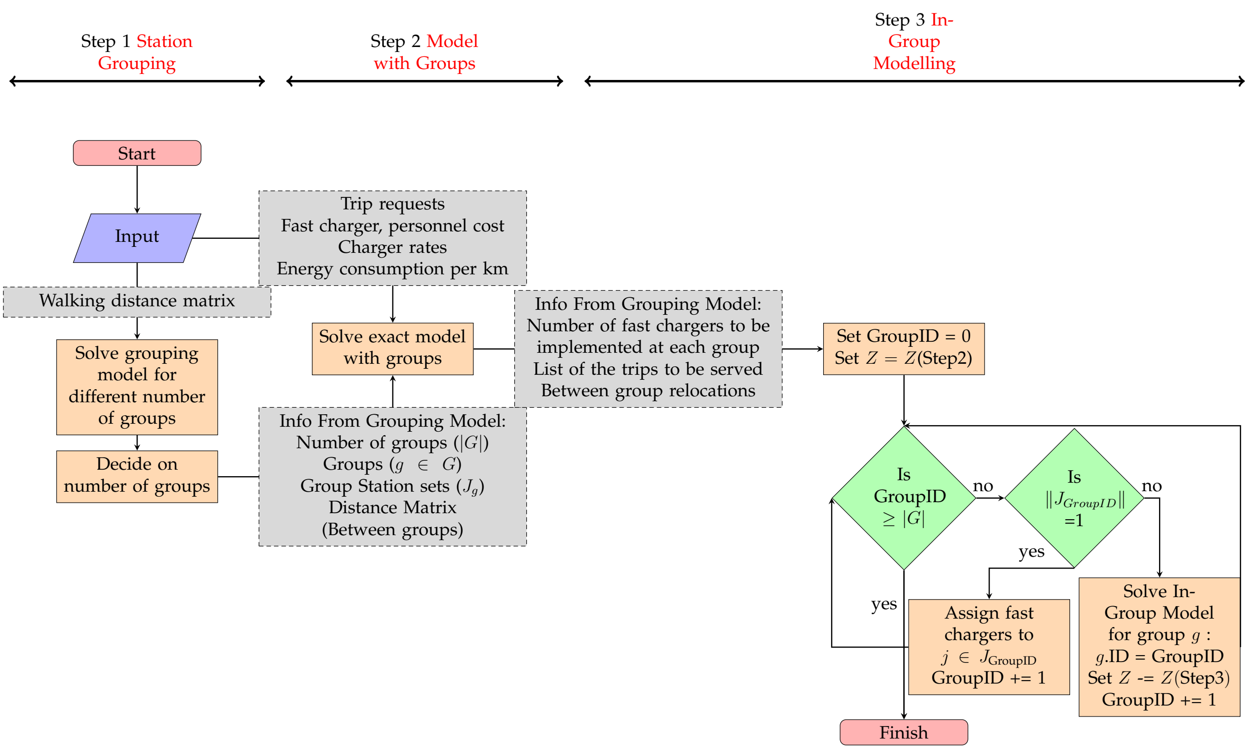

Figure 2.3: Flowchart of Grouping Heuristic Method

After determining the number of groups and stations at each group, the mathematical model is solved for the groups decided in the first step. The groups are treated as large stations with the total capacities of the stations in them. For the second step, Model 2.3.4 is solved for the variables $\widetilde{r}^{st}_{guk}, \widetilde{b}^{st}_{gkp}, \widetilde{e}^{st}_{gkp}, \widetilde{d}^{st}_{gkp}, \widetilde{c}^{st}_{gkp}, \widetilde{f}_{gp}$ which differ from the variables of the exact model only by handling groups instead of stations. The indices g and u denote groups instead of stations. In this implementation of the model, the only variable that takes stations into account is z^{s}_{ik}. The distance parameter $\widetilde{D}_{gu}$ is considered as the maximum distance of any stations of group g to group u. The rest of the parameters are also considered as taking the indices of groups (g, u) instead of stations (j, l). Please note that $\widetilde{V}_g = \sum_{j \in J_g} V_j$ where J_g denotes the station set of group g.

The solution of the model with groups (Step 2) determines the number of fast chargers to be installed at each group, the number of relocations between groups and the accepted trips. After solving Step 2, a further optimization is required to find where to locate the chargers within groups.

Figure 2.4 demonstrates an instance of 6-station system where the Step 2 Model is applied. In this example, the first group is formed by combining Station 1, 2, and 3. The second group is comprised of Station 4 and 5, while the last group includes only Station 6. As illustrated in Figure 2.4, after solving the Between-Group Model (Step 2), the accepted trips are assigned from one station to another. However, the relocations are considered to be performed between groups. The fast charger decision is also made for groups. In Figure 2.4, two fast chargers decided to be implemented to Group 1. Then, the corresponding stations for relocation activities along with the within relocations for Group 1 must be decided. This is why, a new Within-Group model (Step 3) is created (see Section 2.4.2.2). The outputs of the main model (Step 2) are used as the parameters of the Within-Group Model. This model must be iteratively solved for each group, unless the number of stations within the group is one as shown in flowchart Figure 2.3. Clearly, if the number of stations in a group is 1, then all the fast charger decisions of the group must be implemented at this station, the relocations involving the group must be operated to/from that station, and there will be no within-group relocation. The aim of the Within-Group Model is to minimize the within-group relocation cost while serving all the accepted trips, performing relocations, and implementing fast chargers, all of which are decided by the main model.

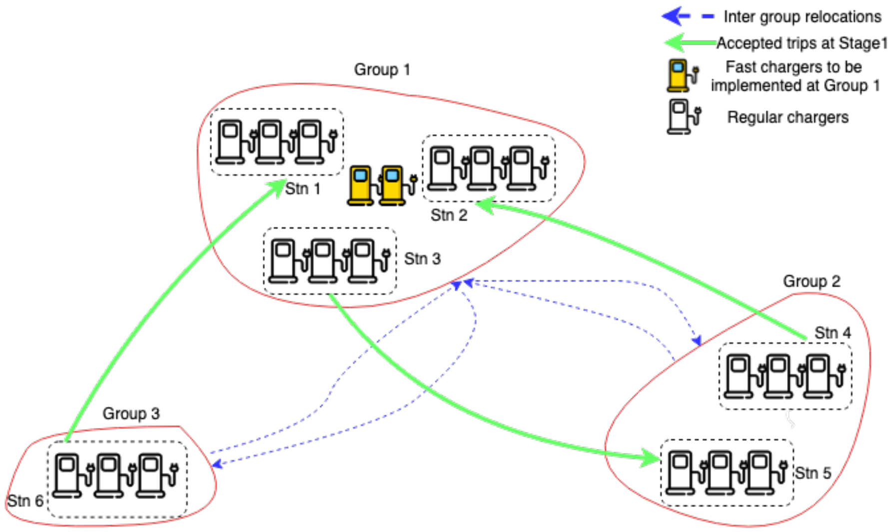

Figure 2.4: Representation of groups at time interval t, scenario s after solving the exact model with groups

2.4.2.1 Step 1: Station Grouping Model

This model aims to minimize the maximum distance within station groups. It allocates the stations to a given $|G|$ groups and determines the maximum distance between any stations within any group.

Sets and indices

$j, l \in J$ nodes (stations)

$g \in G$ groups

Parameters

W_{jl} walking distance between station j and l

Variables

a_{jg} 1, if station j is assigned to group g; 0, otherwise

h_{jlg} 1, if both stations j and l are assigned to group g; 0 otherwise

v maximum distance within groups

Grouping Mathematical Model

$$\min \quad v \tag{2.17}$$

subject to

$$\sum_{g \in G} a_{jg} = 1 \qquad \forall j \in J \tag{2.18}$$

$$h_{jlg} \leq a_{jg} \qquad \forall j, l \in J, g \in G \tag{2.19}$$

$$h_{jlg} \leq a_{lg} \qquad \forall j, l \in J, g \in G \tag{2.20}$$

$$a_{jg} + a_{lg} - 1 \leq h_{jlg} \qquad \forall j, l \in J, g \in G \tag{2.21}$$

$$\sum_{g \in G} W_{jl} h_{jlg} \leq v \qquad \forall j, l \in J \tag{2.22}$$

$$\sum_{g \in G} W_{lj} h_{jlg} \leq v \qquad \forall j, l \in J \tag{2.23}$$

$$v \in \mathbb{Q}^+, a_{jl}, h_{jlg} \in \{0, 1\} \quad \forall j, l \in J, g \in G \tag{2.24}$$

A similar clustering model has been proposed by (Rao, 1971). Constraints (2.18) ensure that each station is assigned to a cluster (group). Constraints (2.19), (2.20) and (2.21) are the linearization of $h_{jlg} = a_{jg}a_{lg}$. Constraints (2.22) and (2.23) define the maximum distance within groups.

2.4.2.2 Step 3: Within-Group Model

After determining the number of station groups (Step 1) and solving the exact model for groups (Step 2), the model below is solved for all of the groups separately. For group g, Within-Group model finds the locations of the fast chargers inside g if it is decided at Step 2 that g has fast charger(s) to be located. Furthermore, this model also determines the corresponding stations inside g for the relocations that are coming in or going out of g while minimizing the relocation costs within g. The summation of objective function values (in group relocation costs) of all of the groups is subtracted from the solution found in Step 2, resulting in the total revenue of the heuristic method.

Sets and indices

$s \in S$	scenarios
$i \in I_{As}$	accepted trip request in scenario s at Stage 2
$j, l \in J_g$	nodes(stations) in group g
$t \in T = \{0, ..., \mathcal{T}\}$	time intervals
$k, k' \in K = \{0, ..., \mathcal{K}\}$	battery levels
$p \in P$	charger types, $p = 0$ denotes conventional chargers

Additional Parameters

R_k^{st}	number of vehicles arriving to group g by relocation at time interval t in scenario s with battery level k decided at Stage 2
$\overline{R}_k^{st}$	number of vehicles arriving to group g by relocation at time interval t in scenario s with battery level k decided at Stage 2
Z_{ik}^s	trip requests accepted at Stage 2 and will be served with battery level k
F_p	decided number of charger type p to be installed at group g

Additional Variables

q_{jk}^{st}	number of vehicles arriving to group g from outside of the group by relocation, at time interval t in scenario s with battery level k
$\overline{q}_{jk}^{st}$	number of vehicles leaving from group g by relocation, at time interval t in scenario s with battery level k

Within-Group Mathematical Model

$$\min \sum_{j,l \in J_g, k \in K, s \in S, t \in T} \mathrm{P}_s \; (\mathrm{RC}_{jl} + \mathrm{D}_{jl}\mathrm{PC}) \; r_{ljk}^{st} \tag{2.25}$$

subject to

$$\sum_{j \in J_g} q_{jk}^{st} = R_k^{st} \qquad \forall k \in K, t \in T, s \in S \tag{2.26}$$

$$\sum_{j \in J_g} \overline{q}_{jk}^{st} = \overline{R}_k^{st} \qquad \forall k \in K, t \in T, s \in S \tag{2.27}$$

$$q_{jk}^{st} + \sum_{\substack{(k' \in K, i \in I_{A_s}): \\ \mathrm{Dest}(i)=j \\ \mathrm{End}(i)=t \\ k'=CT_i+k}} Z_{ik'}^{s} + \sum_{l \in J_g} r_{ljk}^{st} = \sum_{p \in P} b_{jkp}^{st} \qquad \forall k \in K, j \in J_g, t \in T, s \in S \tag{2.28}$$

$$\sum_{\substack{(i \in I_{A_s}): \\ \mathrm{Origin}(i)=j \\ \mathrm{Start}(i)=t}} Z_{ik}^{s} + \sum_{l \in J_g} r_{jlk}^{st} + \overline{q}_{jk}^{st} = \sum_{p \in P} e_{jkp}^{st} \qquad \forall k \in K, j \in J_g, t \in T, s \in S \tag{2.29}$$

$$\sum_{j \in J_g} f_{jp} = F_p \qquad \forall p \in P \tag{2.30}$$

(2.4), (2.5), (2.6), (2.7), (2.8), (2.9) — Same with Model in Section 2.3 only $\forall j \in J$ is replaced with $\forall j \in J_g$ (2.31)

(2.16) — $\forall (j,l) \in J_g, k \in K, p \in P, s \in S, t \in T$ (2.32)

$$q_{jk}^{st}, \overline{q}_{jk}^{st} \in \{0\} \cup \mathbb{Z}^{+} \qquad \forall j \in J_g, k \in K, s \in S, t \in T \tag{2.33}$$

This model aims to provide a feasible solution for the results generated in the second stage. Constraints (2.26) and (2.27) guarantee that arriving (R_k^{st}) and departing ($\overline{R}_k^{st}$) relocations (as shown in Figure 2.4) are assigned to stations of group g. Constraints (2.28) and (2.29) are flow conservation equations similar to (2.2) and (2.3) respectively. Please note that the trip requests in the first model are now provided as parameters (accepted trips). The within-group relocation arcs (r_{jlk}^{st}) are assumed to be only within the station dimension, i.e., no battery or time consumption is considered. These variables are only ensuring feasibility. Constraints (2.30) are for the charger implementation. The remaining of the model is the same with the original model, only applied for the stations within-group g.

Since the goal is to find a feasible solution, we minimize the relocation cost within group g (Objective Function (2.25)) in order to have a more realistic representation of the system.

To demonstrate the steps of the grouping heuristic, a small instance with 10 stations is provided in Figure 2.5. In this example, first, we divided 10 stations to 5 groups: J_{g_0}={Station 0, Station 4, Station 7}, J_{g_1}={Station 1}, J_{g_2}={Station 3}, J_{g_3}={Station 5} and J_{g_4}={Station 2, Station 6, Station 8, Station 9} with the help of Station Grouping Model. Then, we found the best station groups for the fast chargers by solving "Model with Groups" (see Figure 2.5A). Finally, we solved Within-Group Mathematical Model for each group formed in Step 1 to identify stations that should have fast chargers installed (see Figure 2.5B). In this last step, we used between-group relocations and accepted trips provided in Step 2. In Step 3, we identified the exact origin-destination stations of between-group relocations and generated new within-group relocations while considering carried-over accepted trips and between-group relocations. In Figure 2.5, the colour of the cells is associated with the battery levels. As shown in the figure key, the darkest shades of black and red colours represent full (32) and empty battery (0) levels, respectively. In this example, we allow vehicles to be rented only if their charging level is over 40%. To indicate this, the charging levels from 0% to 40% and from 40% to 100% are shown with red and black colours with increasing darker shades respectively. Also, a cell is added for every vehicle leaving a station that shows if the vehicle is leaving for a trip (green), or relocation within (yellow) or between (blue) groups. Each number in the colored cells refers to a certain vehicle. Additionally, the color of each line is associated with a particular vehicle, and represents the movement of the corresponding vehicle. Based on the representation of the state of the usage of each vehicle described above, one can follow each vehicle's path for the day. For instance, dark blue illustrates the movements of vehicle 0 which starts the day at g_0 in Figure 2.5A and at Station 0 in Figure 2.5B. Solid, dashed and dash-dotted lines represent trips, relocations between groups and relocations within groups, respectively. Please note that Figure 2.5A indicates trips originating/terminating at grouping levels, while in

Figure 2.5B, trips are illustrated from/to stations.

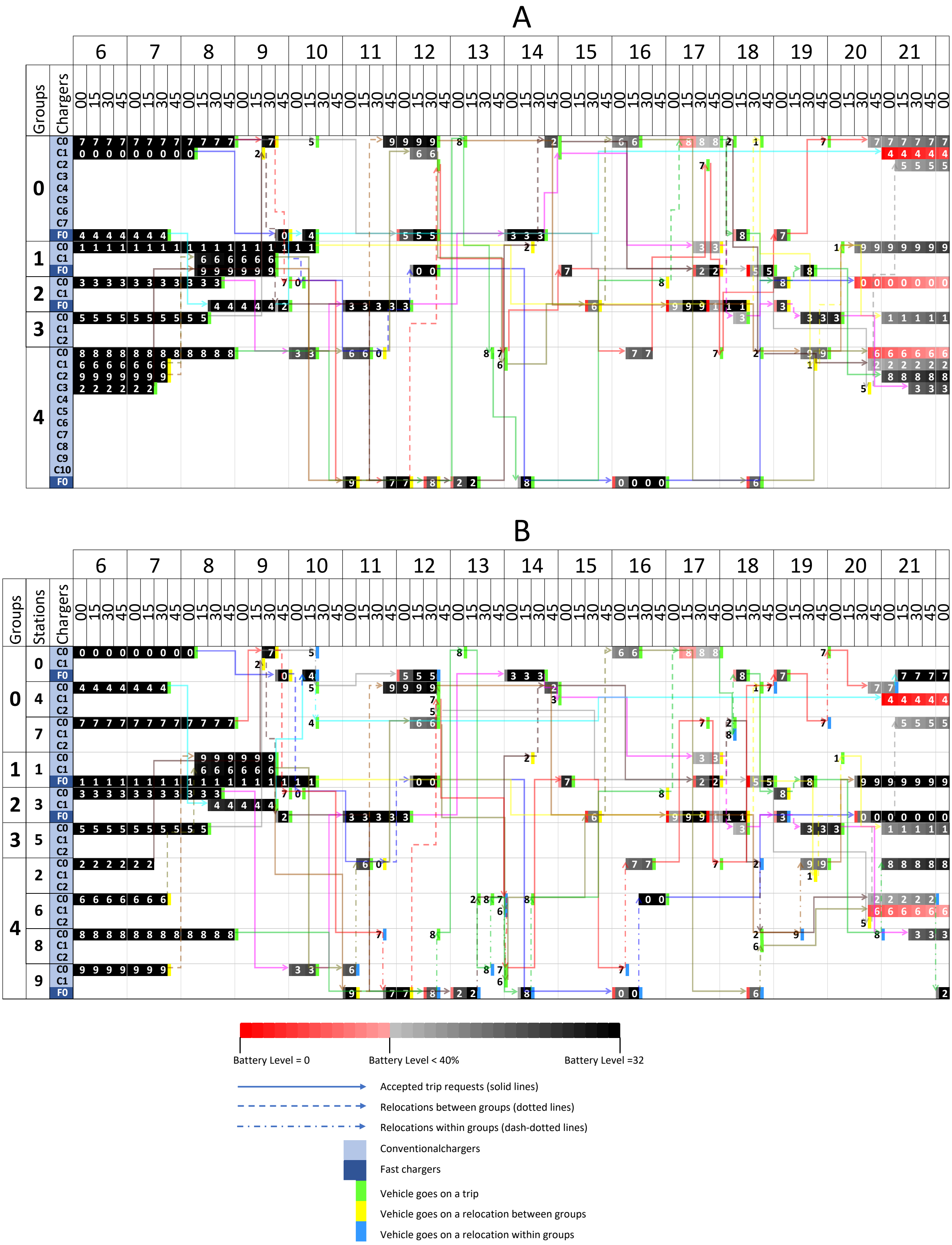

Figure 2.5: Vehicle flows in Step 2 (A) and Step 3 (B) when the Grouping Heuristic is applied to an instance with 10 stations.

2.5 Computational experiments

The model and heuristic approaches presented in Sections 2.3 and 2.4 are applied to historical data of Auto Bleue, which used to be a one-way electric carsharing system operating in Nice, France. The system has 60 stations. Each station has 3 parking spots along with conventional chargers provided at each parking spot. Each day is assumed to start at 6:00 and end at 22:00. As discretization in time is assumed, the 16-hour time horizon is divided into 64 15-minute time intervals.

The proposed models and heuristics assume discrete battery levels. The installed conventional chargers need 8 hours (32 time intervals) to fully charge an empty battery. In the experiments, one type of fast charger that can charge an empty battery in 30 mins is assumed to be implemented. The battery is discretized into 33 (from 0 to 32) charging levels. Therefore, the charging rates of conventional and fast charger are taken as 1 and 16 battery levels per time interval, respectively (i.e. $\text{rate}_0 = 1, \text{rate}_1 = 16$). Although the charging speed is not linear and slows down after a certain charging level, we assume linear charging rates. It should be noted that the models are still compatible with nonlinear (with a fixed charging speed in each time interval, i.e., piecewise linear) charging rates because of the discretization of the battery levels.

The experiments are conducted in C# in .NET environment, on a workstation with Intel Xeon E5-2640 v3 processor. The integer programming models are solved by IBM ILOG Cplex 12.10.

The parameters used in the model and heuristic algorithms except for the personnel cost are taken from Boyaci et al. (2015) and provided in Table 2.2. In this study, the personnel cost per hour is taken as the minimum hourly wage in France in 2018 (OECD, 2018). We employ the personnel based on the need of relocation, and the labour cost is calculated by the summation of the relocation duration multiplied by the personnel cost per time interval as shown as personnel cost in Objective Function (2.1). Revenue of trip i (TR_i) is calculated by multiplying the trip duration (End(i)-Start(i) in hours) by the hourly revenue given in Table 2.2. Relocation cost (RC_{jl}) is based on the driving distance between station j and l, and vehicle relocation cost per km. Serradilla et al. (2017) considers the investment horizon of fast charger implementation as 10 years, and the salvage value at the end of the useful life as 5% of the purchase value. Çalık and Fortz (2019) indicates fast charger prices in the range (€22,000 - €32,000). In our calculations, we have €30,000 for the cost of the fast chargers. The resulting daily depreciated upgrade cost of a fast charger is €11.15.

Table 2.2: Parameters used in computations

Parameter	Value
Revenue per hour	€13
Vehicle relocation cost per km	€0.01
Relocation speed (km/hour)	30
Personnel cost per hour	€10
MaxPer	3, 4, and 6
	for 10, 20 and 60 stations systems, respectively
AR	3 time intervals

The number of vehicles is assumed to be equal to the number of stations. The trips and relocations are only allowed when the battery level is higher than 40% and sufficient to drive the origin-destination distance of trip requests. In the proposed model, a vehicle is present with a full battery at one of the parking spots of each station before the start of the day (i.e. 6 AM).

2.5.1 Heuristic selection

Since the model (Section 2.3) is intractable for the real life system with 60 stations, we run initial experiments on systems that are composed of 10 and 20 busiest stations to compare the performance of the proposed heuristic methods with the exact method. Scenarios are created by randomly selecting 100 and 200 trips from the historical data set for systems with 10 and 20 stations, respectively. Problem instances are solved using the solution methods proposed in Sections 2.3 and 2.4.

As the grouping heuristic approach assumes "instant within-group relocation"

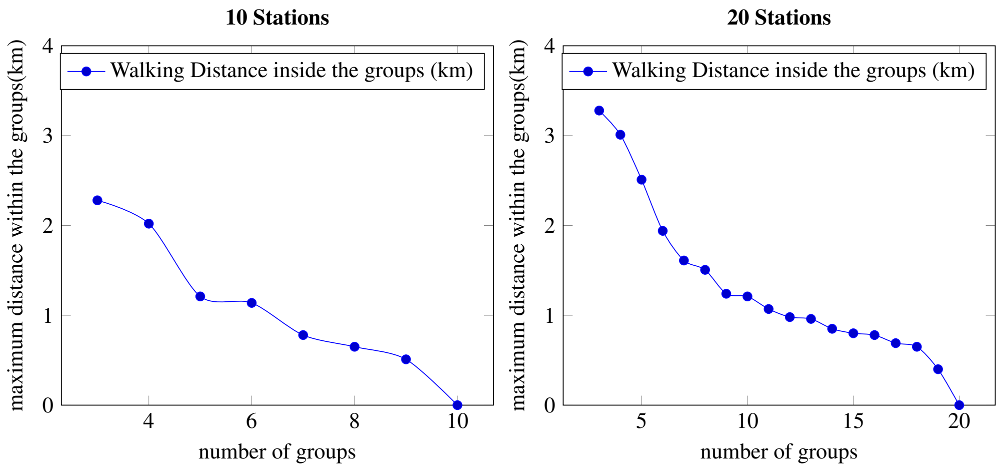

Figure 2.6: Maximum walking distance within groups for 10 and 20 stations

(no battery consumption or travel time is encountered due to the very close proximity of the stations comprising a group), the groups are selected such that the maximum walking distance within the group stations is minimized. It should be noted that the number of groups to be used affects the resulting computational complexity. If the number of groups is equal to the number of stations, i.e. no grouping is considered, then the resulting model is the exact model proposed in Section 2.3. If it can be solved, it provides the exact solution but could be intractable for real-life instances. If the number of groups is too small, the model would be inaccurate. The battery consumption and travel time assumptions may not be realistic since the size of the groups would be too big and station distances within groups would be too high.

The numbers of groups selected for the experiments with 10 and 20 station instances are 5 and 9, respectively. This choice was made because these number of stations correspond to the elbow of the curves of the graphs illustrating the relationship between the number of stations and the maximum distance within the stations belonging to the identified clusters (See Figure 2.6).

Table 2.3 summarizes the values of the objective function obtained by the (i) exact model, (ii) exact model without relocation variables (No Reloc.), (iii) exact model without charger upgrade variables (No Fast Charger) and, (iv) relocation restriction 1 (RelRes1), (v) relocation restriction 2 (RelRes2), (vi) grouping, (vii) grouping with relocation restriction 1 (GroRelRes1) and (viii) grouping with relocation restriction 2 (GroRelRes2) heuristics. It should be pointed out that except for the computational time values, we do not present the criteria values of the heuristics directly as the objective functions are treating different distance values for grouping heuristics. Instead, the expected profit, number of served trips, and vehicle utilization values of the heuristic methods are calculated in 2 steps; (i) solving the heuristic method and finding the locations of the fast chargers (f_{jp}), and (ii) solving the exact method by fixing the (f_{jp}) values found in (i). This way we ensure that the resulting profits are computed on the basis of the same objective function. Each cell in the table represents the average value of 5 runs for different instances. 1% MIP gap and 3 hours (10800 sec) are set as termination parameters for the mathematical models used in all of the algorithms. The runs were stopped when one of the termination parameters is reached.

Table 2.3: Expected profit, number of served trips, time and vehicle utilization values of the solutions

Problem Type	Criteria	Exact	No Reloc.	No Fast Charger	RelRes1	RelRes2	Grouping	GroRelRes1	GroRelRes2
10 Nodes-1 Scenario	Expected Profit (obj)(€)	1343.1	1246.8	1105.5	1340.8	1338.7	1287.7	1287.3	1286.1
	Time (sec)	376.5	2.2	60.5	166.3	25.9	71.4	44.2	25.9
	# of served trips	53.2	48.4	35.4	53.2	53.4	52.2	50.0	50.0
	Avg. Vehicle Utilization	62%	57%	45%	62%	63%	60%	59%	59%
10 Nodes-2 Scenarios	Expected Profit (obj)(€)	1331.1	1282.1	1094.7	1330.2	1324.7	1285.5	1293.9	1297.1
	Time (sec)	949.6	4.3	130.8	351.1	45.8	139.3	97.4	49.7
	# of served trips	50.2	50.2	34.2	50.3	51.2	50.4	50.3	50.6
	Avg. Vehicle Utilization	62%	60%	45%	62%	63%	60%	60%	60%
10 Nodes-3 Scenarios	Expected Profit (obj)(€)	1324.0	1278.3	1096.0	1323.7	1320.4	1284.1	1284.2	1285.8
	Time (sec)	1618.8	6.5	277.6	584.3	83.4	323.9	197.9	84.8
	# of served trips	52.4	48.7	33.8	51.9	52.7	50.5	50.6	50.2
	Avg. Vehicle Utilization	62%	59%	45%	62%	62%	60%	60%	60%
10 Nodes-4 Scenarios	Expected Profit (obj)(€)	1305.0	1276.1	1092.5	1305.0	1302.7	1275.7	1278.6	1278.2
	Time (sec)	3233.0	8.5	453.8	1122.6	147.1	481.9	296.4	116.8
	# of served trips	53.1	52.2	34.7	53.1	53.4	51.7	51.3	51.3
	Avg. Vehicle Utilization	62%	60%	45%	62%	63%	60%	60%	60%
20 Nodes-1 Scenario	Expected Profit (obj)(€)	2653.6	2548.7	2175.2	2648.5	2648.1	2575.9	2589.0	2582.8
	Time (sec)	4917.4	4.1	408.6	1950.8	237.5	242.7	182.3	145.4
	# of served trips	105.0	103.2	69.4	110.4	110.4	104.4	106.4	105.0
	Avg. Vehicle Utilization	64%	60%	45%	63%	63%	61%	61%	61%

In Table 2.3, the comparison of the exact and no fast charger (fixing $f_{jp} = 0$ for $p \neq 0$) algorithms suggests that fast charger purchases will increase the expected profit (17.3%), number of served trips (33.8%) and average vehicle utilization (27.3%) dramatically.

When the heuristic methods are compared with the exact model, it is clear that RelRes1 and RelRes2 methods are performing better than the rest of the heuristic methods in terms of expected profit (objective) values. The error of a heuristic algorithm's expected profit value is calculated by $\frac{z-z^{heur}}{z} * 100$, where z and z^{heur}represent the expected profit values of the exact model, and the exact model with fixed (f_{jp}) values found by the heuristic algorithm, respectively. The average errors compared to the exact method are 0.09% and 0.29% for RelRes1 and RelRes2 methods, respectively. However, especially for 20-node instances, the time required to solve RelRes1 reaches the 3-hour limitation even for two-scenario instances. On the other hand, the average computation time of RelRes2 is 237.5 sec which is less than 1/8 of the time required for RelRes1 for 20-Node, one-scenario instances.

The average error to the optimal value of the heuristics No Relocation (setting $r_{jlk}^{st} = 0 \ \forall j, l, k, s, t$), Grouping, GroRelRes1 and GroRelRes2 are 4.04%, 3.05%, 2.81%, and 2.80%, respectively, for 25 different instances. Figure 2.7 illustrates how the errors of the heuristic methods are distributed. The boxplot shows that the grouping heuristics behave similarly. Solving the exact method without the relocations (No Relocation method) spreads wider than the grouping heuristics. In fact, the standard deviation of No Relocation method is 4% whereas it is 2.21% for Grouping and 2.20% for GroRelRes1 and GroRelRes2. As the numbers of nodes and groups increase, the time required to solve Grouping and GroRelRes1 shows a similar pattern with Exact and RelRes1 algorithms. This is why, among No Relocation, Grouping, GroRelRes1 and GroRelRes2, we decided to use GroRelRes2 heuristic.

At this point of the analysis, we compare RelRes2 and GroRelRes2 computation times on the system with 60 stations.

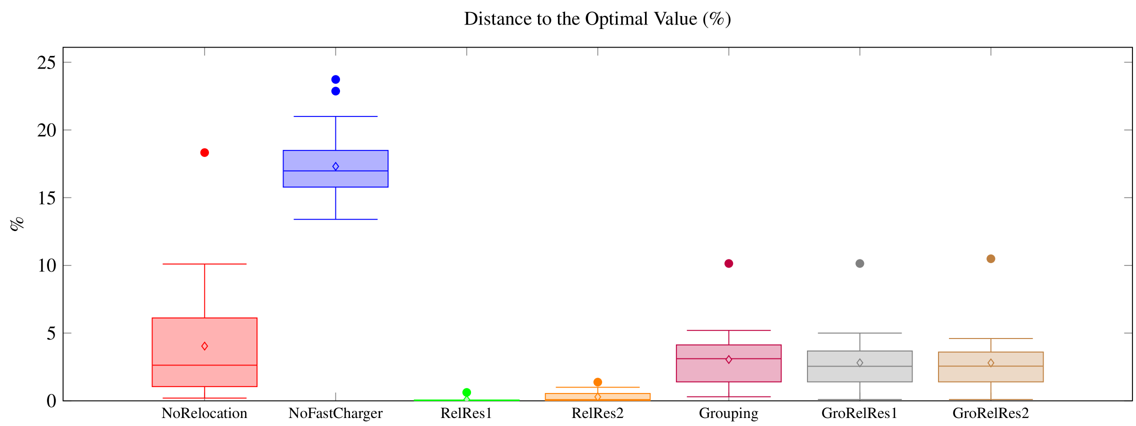

Figure 2.7: Heuristic methods' expected profit value distances to the exact model's expected profit (%)

In order to compare the computation times of the heuristics when solving instances with 60 stations, the number of groups should be determined first. Figure 2.8 shows the maximum within distances for different number of groups. On the graph, we observe a sharp decrease when the number of groups is changing from 10 to 15, and a milder change when the number of stations becomes larger than 15 (elbow of the curve). At 15, the maximum distance within the groups is 1.5km, which denotes negligible battery consumption between stations inside the groups. This is why the number of groups is selected as 15 for the system with 60 stations.

Table 2.4 shows the computation time of heuristics RelRes2 and GroRelRes2 when the number of groups is set to 15. Each cell represents the average of four different runs. RelRes2 heuristic is able to solve instances only with 60 stations and 1 scenario within 3 hours (10800sec) time limitation. On the other hand, GroRelRes2 heuristic method is able to solve instances with more scenarios within the time limit.

Table 2.4: Duration of RelRest 2 and Grouping RelRest 2 heuristics for 60 Node systems

Problem Type	RelRes2	GroRelRes2 ($\|G\|$=15)
60Node-1 Scenario	5476 (sec)	1066 (sec)
60Node-2 Scenarios	NA	3023 (sec)
60Node-3 Scenarios	NA	6405 (sec)

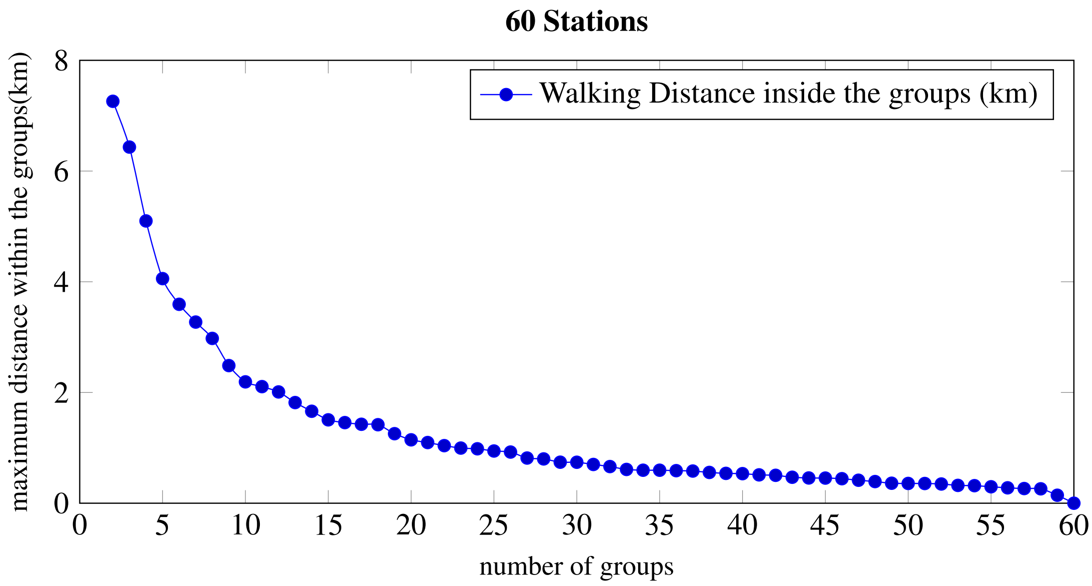

Figure 2.8: Maximum distance within groups for different number of groups

It is concluded RelRes2 is able to find near-optimal solutions for smaller size problems whereas GroRelRes2 performs well for both small and large size problems.

We performed sensitivity analysis in order to investigate the effect of the length of the time interval on the calculation of the optimal profit value. In our sensitivity analysis we used 5 and 15-minute intervals and we observed that the average difference on the objective function value was negligible (1.03% on the average). Thereby, the rest of the analysis is carried out via GroRelRes2 heuristic algorithm using 15-minute time intervals.

Figure 2.9 illustrates the sample solution for the 60-node system in Nice. Optimal locations of fast chargers are obtained by solving the GroRelRes2 heuristic algorithm that considers 15 scenarios. Before solving the model, we identify the groups first. Stations 14, 36, 37, 43 and 44 form groups on their own. In addition sets of {17, 29}, {21, 35}, {15, 54, 55}, {10, 11, 34, 40}, {0, 27, 50, 56, 58}, {2, 3, 7, 8, 45, 46}, {4, 5, 22, 24, 32, 57}, {1, 18, 20, 25, 26, 41, 52}, {9, 19, 23, 31, 33, 38, 49, 53, 59}, {6, 12, 13, 16, 28, 30, 39, 42, 47, 48, 51} are suggested by the grouping algorithm. The optimal solution suggests that 17 conventional chargers should be upgraded to fast chargers in various groups, e.g. one in group composed of stations 15, 54, 55; two in group composed of 2, 3, 7, 8, 45, 46. The rest of the assignments can be seen in Figure 2.9. In the second stage of the algorithm, it is identified that one of the conventional chargers located in stations 0, 1, 5, 6, 7, 8, 9, 11, 14, 15, 16, 17, 22, 31, 35, 36 and 41 should be upgraded to maximize the profit.

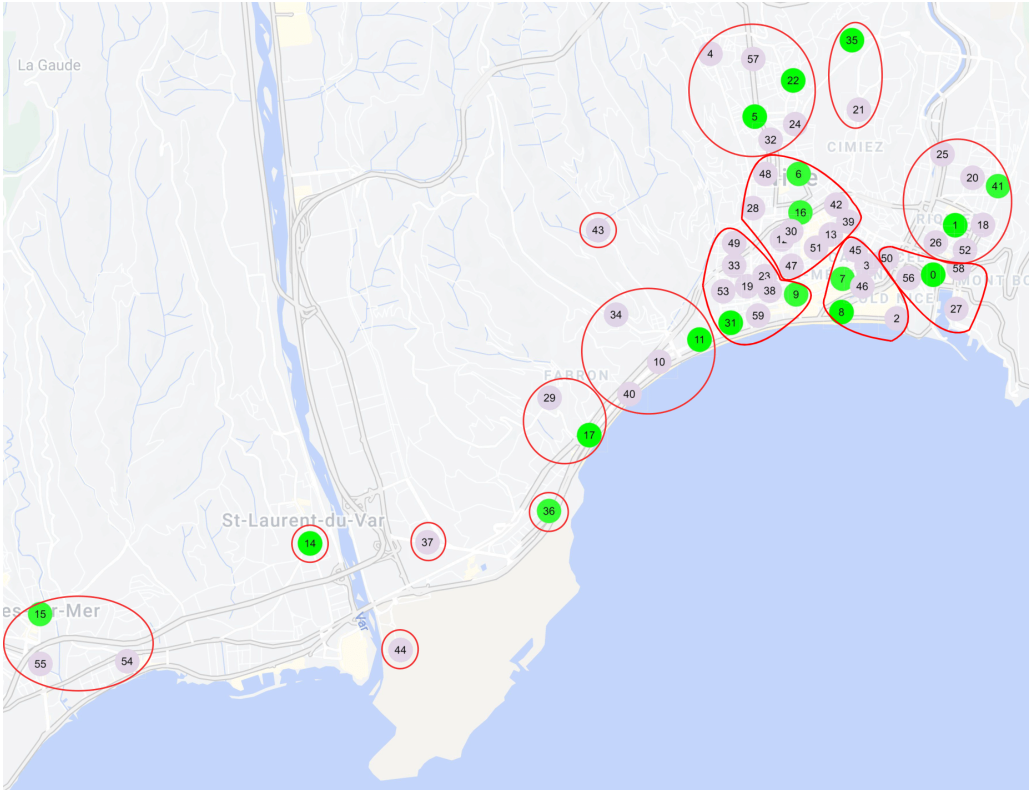

Figure 2.9: Sample fast charger location solution for the system in Nice

2.5.2 Sensitivity analysis on the number of chargers

In this part, we evaluate the effects of different number of fast chargers on the total profit for a 60-Station OWECS. Figure 2.10 is obtained via fixing the number of chargers at GroRelRes2 heuristic algorithm on a 60 Node-1 Scenario instance. Please note that we also solve the same instance without considering charging levels. In other words, we assumed a system with internal combustion engine (ICE) vehicles. There are many bottlenecks that could prevent a carsharing system from serving all requested demand, such as station capacity, demand distribution, vehicle count. Running the system with ICE vehicles allows us to see the bottleneck created by factors other than the battery restrictions. We consider this value as the upper bound for the profit in our analysis. As the Grouping heuristics maximize the profit of the model of Step 2 (solving the exact model by using the station groups), it can be observed that the second step profit values behave as a discrete concave function, and the optimal solution is obtained when the number of fast chargers is fixed at 17 (see Step 2 profit values in Figure 2.10). On the other hand, the total profit fluctuates especially when the number of fast chargers is greater than 4. The total profit has its maximum value when the fast chargers are fixed to 20. We observe that the within-group relocation cost when locating 17 fast chargers is higher than that of 20. Yet the difference in total profit when locating 17 and 20 fast chargers is less than 0.2%. Figure 2.10 also illustrates the marginal increase in total profit at each additional fast charger implementation. Implementing 17 fast chargers will result in 25% increase in the total profit. The largest increase (7.5%) for an additional fast charger is observed when the first fast charger is implemented. The increase in the total profit is 12.5%, 16.1%, 18.6% and 20.1% for 2, 3, 4, and 5 fast charger implementations, respectively. As expected, the impact on the second step profit is decreasing as the number of fast chargers increases, and after reaching the optimal solution (17 fast chargers), the second step profit is decreasing because of the fast charger costs. With some fluctuations, the total profit also behaves similarly. Using 17 fast chargers increases the number of served trips from 207 to 351 by 69.6% compared to not implementing any fast chargers. We observe a steep rise in the number of served trips when 4 fast chargers are implemented, in fact, 61.4% of 69.6% increase is due to the purchase of the first 4 chargers. After the implementation of 20 chargers, we see almost no change in the number of served trips with additional charger implementations. There is 7.6% difference between the total profit of optimally locating fast chargers and the upper bound (ICE total profit value). This result is based on the requirement of having compulsory battery level (40%) before trips/relocations, charging times and battery capacity. Please notice that, the optimal solution of the proposed algorithm does not require to upgrade all conventional chargers to fast chargers. For instance, the daily profit in the optimal solution of a 60-Node 2-scenario instance is €9,188 whereas it is €7,558 when all chargers are upgraded to fast chargers. The profit difference is mainly due to the upgrade cost of the chargers that are not upgraded in the optimal solution.

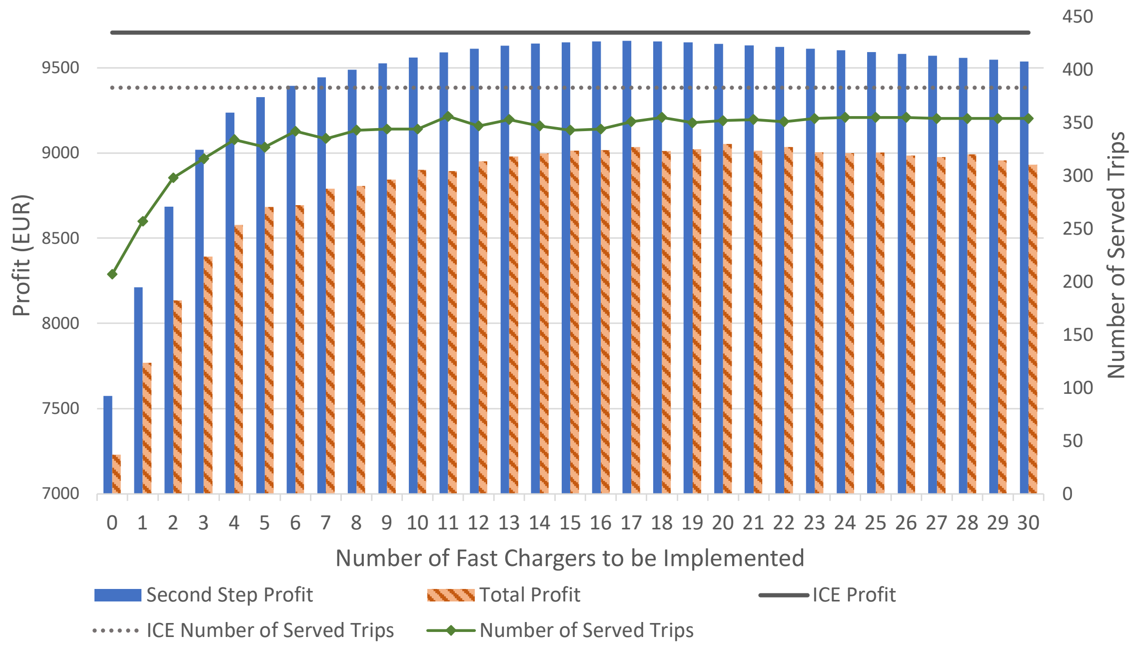

Figure 2.10: Second stage profit, total profit and number of served trips when different numbers of fast chargers are implemented

Figure 2.11 illustrates how the number of relocations (both between and within-group) and relocation cost vary with the number of fast chargers implemented to the system. The number of within-group relocations per served trips shows a downward trend as the number of fast chargers increases. This could be explained by the availability of the fast chargers at the stations. If a trip has already been accepted at the grouping stage of the algorithm, a vehicle with a sufficient battery level must be available at the origin station of the trip. In certain cases, this may be only possible by relocating a vehicle to a station that has an available fast charger within the group. If there is a fast charger available at the origin of the trip, within-group relocation is usually not needed to serve the trip.

Clearly, the proposed algorithm generates more relocation activities than the exact algorithm. In comparing the number of required relocations between the exact and heuristic solutions for problem instances with 10 and 20 stations, we found that the exact solution generates relocations for 57% of the accepted trips while the heuristic generates relocations for 82% of the accepted trips. However, as seen in Figure 2.11, the majority of the generated relocations of the GroRelRes2 (76% on the average) is resulting from the within-group relocations. The average cost of a within-group relocation is significantly less than the average cost of a between-group relocation. Although we observe more relocations in our heuristic approach, i.e. GroRelRes2, as it can be seen in the analysis conducted in Section 2.5.1, the difference between the optimal profit and profit resulting from the heuristic decision is not more than 5% in 95% of the cases (see Figure 2.7). In other words, relatively high relocation numbers in the heuristic approach do not affect the quality of the heuristic solutions significantly and give close to optimal results in most of the cases.

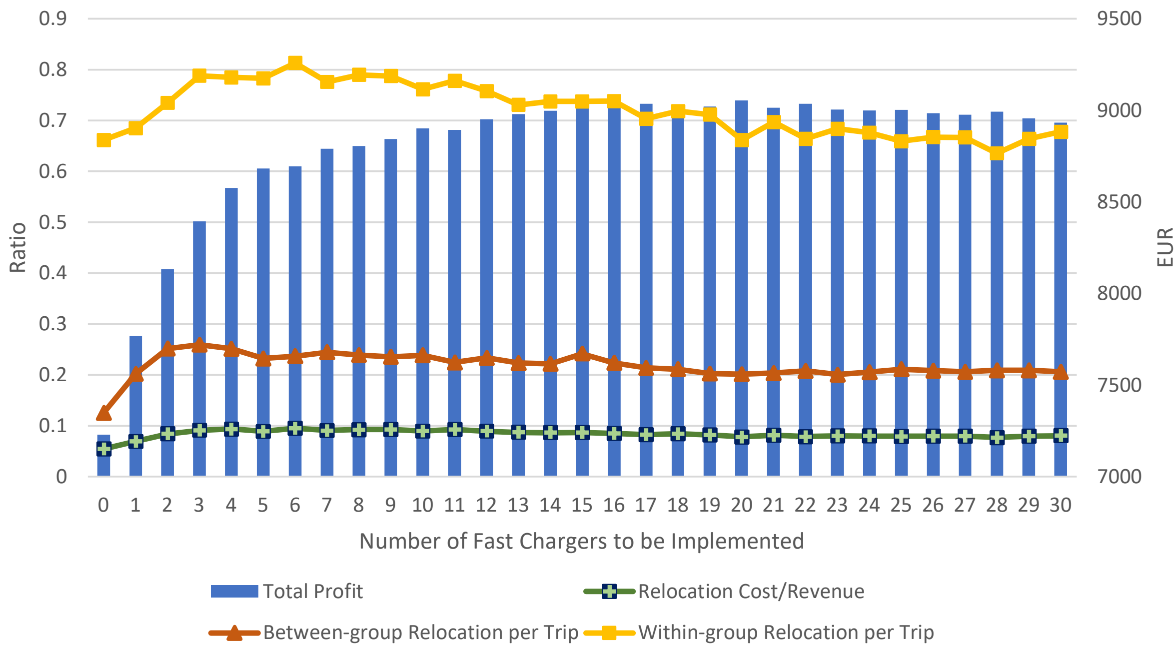

Figure 2.11: Second stage profit, total profit and number of served trips when different number of fast chargers implemented

2.5.3 Sensitivity analysis on charger prices

To understand the effects of the cost of fast chargers on the optimal solution, we vary the cost of fast chargers. We solve the problem by considering five different costs per day (€11.15, €8.36,€5.57, €2.79,€0) per fast charger installation. We also consider the option without fast charger installation (indicated with ∞ sign in Table 2.5) and report the percent improvements of other fast charger installation price levels compared to this option. Table 2.5 provides information on the total profit, number of served trips, number of fast chargers to be implemented and vehicle utilization. Figure 2.12 illustrates how these values are compared to "without fast charger option" when the daily depreciated cost of fast charger is varied.

From Figure 2.12, it can be seen that introducing fast chargers increases the expected daily profit, the number of served trips and vehicle utilization remarkably by 25.3-29.1%, 63.1-65.2%, and 37.3-37.9%, respectively. However, it is observed that the system performance is not sensitive to the cost of the fast chargers. When 25% gradual discounts are analyzed for (€11.15 to €8.36), (€8.36 to €5.57),(€5.57 to €2.79),and (€2.79 to €0), the average increase in the total profit value, number of served trips and vehicle utilization are 0.8%, 0.4% and 0.1%, respectively.

We observe that when the daily depreciation of a single charger drops from €11.15 to €0, the model advises to open 28 chargers (not 180, i.e. total number of charging points, even though they are free to install) instead of 17. In addition, the profit increases by 3.08%, 2.06% of which comes from additional profit of serving more trips.

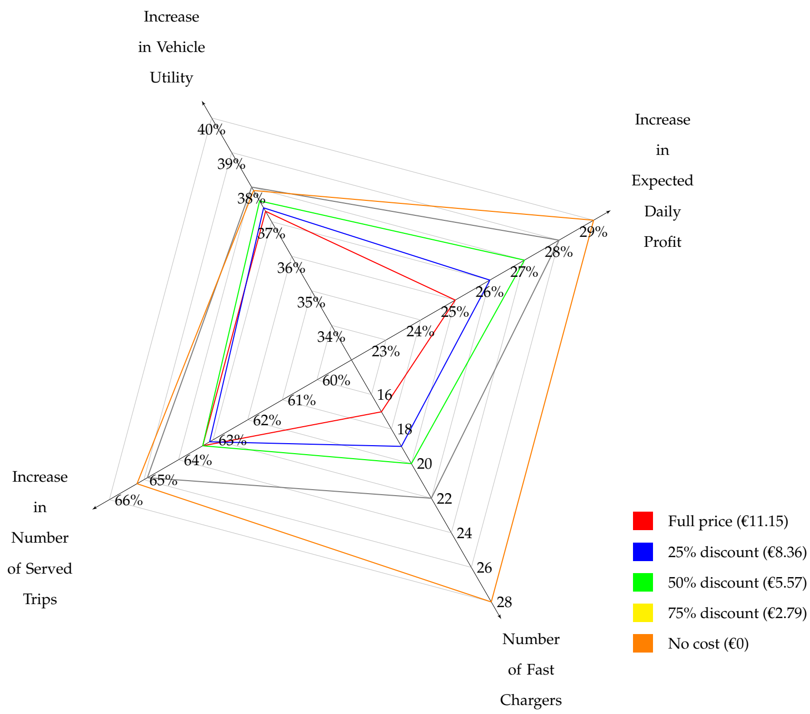

Figure 2.12: Price sensitivity analysis

Table 2.5: Total profit, number of served trips, fast chargers and vehicle utilization under different price scenarios for fast chargers

Depreciated Cost	Total Profit	Av. Number of Served Trips	Number of Fast Chargers	Vehicle Utilization
∞	€7335.0	189.5	0	49%
€11.15	€9188.2	309.5	17	67%
€8.36	€9232.7	309.0	19	67%
€5.57	€9294.6	309.5	20	67%
€2.79	€9353.5	312.5	22	68%
€0	€9470.0	313.0	28	68%

2.5.4 Sensitivity analysis on demand levels

In this section, we evaluate the effects of demand levels on the fast charger acquisition decision. Alternative demand scenarios were generated by decreasing the initial demand of 600 requests to 250, 300, 350, 400, 450, 500 and 550, and increasing the demand for trip requests to 650, 700, 750, 800, 850, 900, 950 and 1000 requests. Figure 2.13 depicts how the total profit and number of fast chargers change when demand size varies from 250 to 1000 for EVs. We also added the same analysis, obviously without the consideration of chargers, for an ICE fleet for comparison purposes. When the demand increases between 250 and 500, there is a significant growth in the number of fast chargers with 3 fast charger increase per additional 50 requests on the average. When the demand exceeds 500, the number of fast chargers fluctuates between 16 and 18. There may be many reasons for the fluctuations. First, we are using both duration and distance information for the trips in our experiments. Since the model is selecting the trips to serve, each additional 50 trips may alter the trips selected to be served. Please note that in our model we are not trying to serve all demand but select the most profitable ones as much as the capacity allows. We have many constraints in our system, including station capacities. We cannot have more than 3 rentals from any station at any time interval. If the model can find more profitable trips that reduce the relocation activities or trips with longer duration but shorter driving distance, it can use its limited capacity for these trips. As the selected trips and generated relocations change, the fast charger location and number may vary. Clearly, the profit of the system increases as the demand size increases. However, an additional 50 requests for smaller demand sizes (sizes between 250 to 500) yield more profit than larger demand sizes. For example, additional 50 requests provide 22% and 0.7% increase in the total profit for demand sizes 300 and 900, respectively. For the smaller demand sets, the station and personnel relocation capacity is still sufficient to serve more trips, whereas for the larger demand sets, there is not enough capacity for accepting additional trips, and extra profit is generated by accepting longer trips or trips that require fewer relocation operations. It should be noted that as the size of demand increases, the difference between optimal solutions of EV and ICE fleet increases. The difference increased from 3.07% when demand is equal to 250 to 9.3% when demand increases to 1000. The difference between the performance of ICE and EV fleets is the outcome of restrictions imposed by the batteries. As the system gets more congested, the profit difference between the system with ICE vehicles and EVs increases. When the vehicles are utilized more, EVs require more frequent charging. Although the system with EVs can install more fast chargers, since it is not profitable, it is not advised by our model.

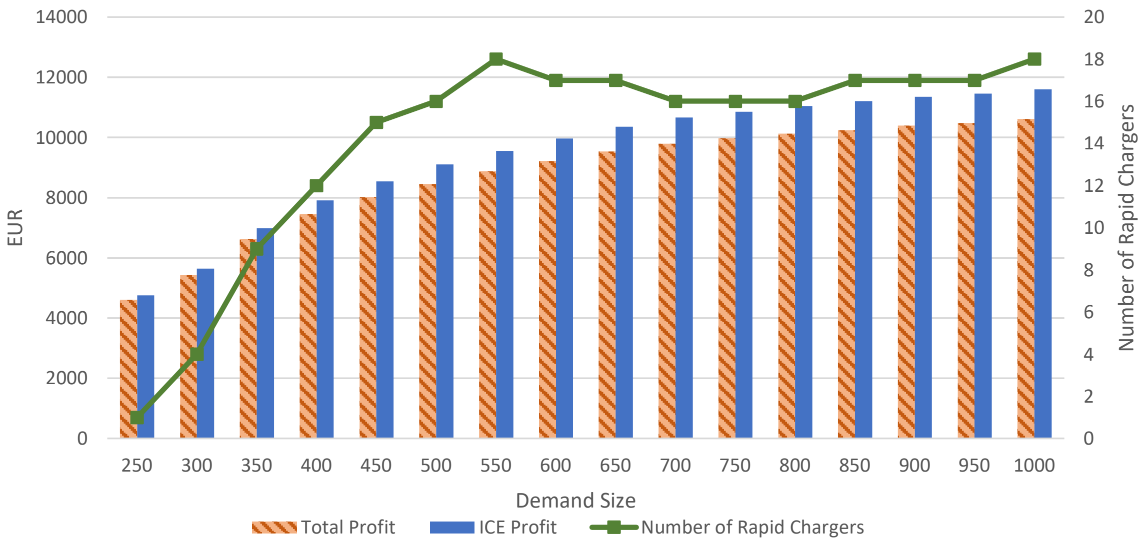

Figure 2.13: Changes in the total profit and number of fast chargers when demand size differs

2.5.5 Sensitivity analysis on charger locations

In this analysis, we explore the effect of the location of the fast charger to the profitability of the system. We want to observe how much more we can gain by choosing the right fast charger location. We consider a system without a fast charger as a start and compare the total profit of the system when we install a single fast charger to one of the stations. We consider 60 different instances. We install a fast charger to one of the stations at every instance. Figure 2.14 shows the total profit and the number of served trips when only one fast charger is implemented at different stations. The green dotted line located on the figure shows the total profit level if we do not consider restrictions on the total number of fast chargers. Gray line, on the other hand, shows the total profit level if no fast chargers are installed to the system. Station 16 and 42 are the most and least profitable stations to install a fast charger, respectively. In this system, installing the fast charger to a wrong station could lead up to 7.2% loss in profits. However, the distribution of the profitability of the stations is left-skewed. The average profit of installing a fast charger to any station is just 1.6% worse than the maximum profit of installing a fast charger.

We have also analyzed the vehicle utilization when only one fast charger is implemented at different stations at each run. The utilization of vehicles varies between 49% and 55%, which shows that if only one fast charger purchase is possible, the utilization of vehicles might increase by 12.25% if the right location is selected.

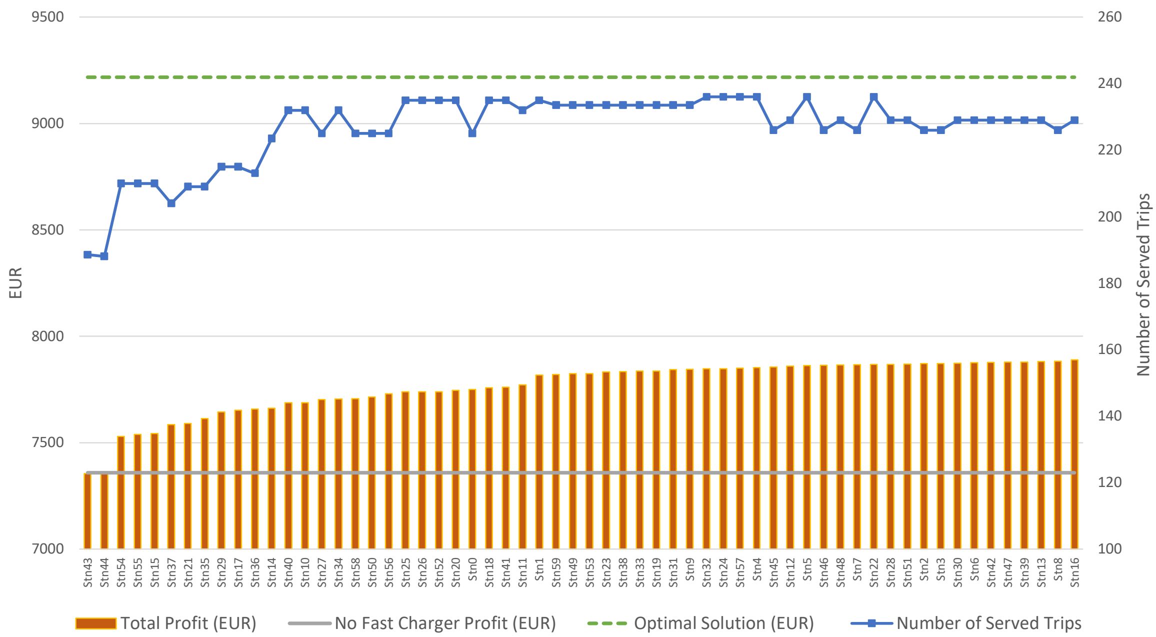

Figure 2.14: Changes in total profit and number of served trips when only one fast charger is implemented to different locations

2.5.6 Sensitivity analysis on the compulsory battery level

Carsharing companies often follow a policy of minimum battery level requirements. In this section, we investigate the effect of the minimum battery level of a vehicle required to start a trip. We name it as the compulsory battery level for brevity. Table 2.6 illustrates profit values and number of fast chargers to be implemented when the compulsory battery level varies between 0% to 100%. When the compulsory battery level is increased from 0% to 100%, the total profit is decreased by 4.5% (€420.1 /day). This is due to the fact that higher battery availability thresholds render a higher number of vehicles unavailable and require additional charging time. The number of fast chargers implemented shows almost a non-decreasing trend. As the compulsory battery level increases, even though the optimal number of fast chargers in almost all of the cases increases, the total profit of the system decreases. Fast charger implementation is becoming increasingly important as the compulsory battery level increases.

Table 2.6: Profit values and number of fast chargers for different compulsory battery levels

Compulsory Battery Level(%)	Second Stage Profit(€)	Total Profit (€)	Number of Fast Chargers
0	9820.05	9249.33	16
10	9813.73	9226.82	18
20	9804.12	9220.82	17
30	9790.55	9183.46	18
40	9772.61	9188.15	17
50	9745.20	9161.96	18
60	9671.99	9105.02	19
70	9620.46	9058.52	19
80	9565.50	9029.89	21
90	9474.24	8939.92	22
100	9334.61	8829.26	25

2.6 Conclusions and future research directions

In this study, we have introduced a time-space-battery level integer programming model to solve the fast charger location problem in one-way electric carsharing systems. Besides the exact solution, we propose two types of heuristic methods based on relocation reduction and station grouping. The first type of heuristic method creates relocation arcs based on the trip request data set rather than creating relocation arcs between all station pairs for all time intervals and battery levels. The second type of heuristic method decreases the size of the network by grouping the stations and treating the groups as large stations. This heuristic method involves three stages: (i) determine the number of groups, (ii) solve the integer programming model for groups, (iii) model the problem of locating fast chargers within the groups. Along with these two types of heuristic methods, a heuristic combining relocation reduction and grouping methods was also presented.

The proposed model and heuristic methods are tested on 10 and 20 station subset settings of a real world one-way electric carsharing system from Nice, France. It is observed that the mixed heuristic gives good quality solutions in a reasonable computational time. Our analysis on the real world instance with 60 stations suggests that the introduction of the fast chargers increases the profitability of the system and the level of service offered to its users as more trip requests can be served with lower relocation cost without increasing the fleet size. Furthermore, we found that the performance of the system is not very sensitive to the price of fast chargers. The analysis on the number of demand requests suggests that the number of fast chargers increases as the number of demand requests increases until the system reaches the capacity of either relocation or stations, then it remains fairly constant. For the location analysis, only one fast charger is allowed in the system and applied to different locations. In this analysis, we observe that the distribution of the profitability of the system is left-skewed. Although there are a few stations that installing a fast charger to one of these stations increases profit as much as installing the fast charger to the best station, installing fast charger to the wrong station could lead to huge losses in profit. The analysis on different compulsory battery levels shows that as the minimum battery level that is required to assign a vehicle to start a trip increases, the profitability of the system decreases and the system requires more fast chargers. Please note that the presented results reflect the characteristics of the system under consideration. Managerial insights derived from the study are only applicable to the data set used.

Although the exact model can only be applied to very small scale problems, the proposed combined heuristic approach is scalable to address problem instances found in real world OWECS and can be used by decision makers to optimize fast charger location decisions.

In our study, we consider fixed station capacities in terms of the available parking spots. In reality, one of the major costs of the one-way electric carsharing companies is the parking spot cost since most of these companies are established in urban areas. With minor adjustments, the proposed models can be used to determine the optimum number of fast chargers and the optimum number of parking spots per station. In addition to the proposed single objective model considering profit maximization, multi-objective models can be introduced by considering maximization of the number of served trips, vehicle utilization, and other performance measures. In our study we have used randomly selected scenarios for analyzing the OWECS system instance under consideration. A shortcoming of the random selection of the scenarios to be analyzed is their representativeness. Therefore, a fertile ground for future research is the development of a systematic procedure for scenario reduction leading to the identification of a representative scenario set to be used as the basis for analyzing the OWECS system.

Chapter 3

Balancing carsharing systems through user-based relocation and pricing : A literature review

Abstract

Carsharing systems (CSS) provide sustainable and affordable access to private vehicles. In recent years, CSS gained more importance in urban mobility as an attractive alternative to privately owned vehicles and a crucial component in public transport. However, the imbalanced demand pattern in CSS makes these systems operationally challenging. To rebalance the vehicle and demand distribution, carsharing companies often hire personnel to relocate the vehicles over the stations. In addition, to make the system more profitable, these companies promote certain trips, enabling users to relocate vehicles, and thereby reducing staff dependency. In this paper, we review studies on carsharing literature that involve user-based relocation and pricing strategies to obtain more balanced systems in terms of demand and supply. First, we investigate the descriptive studies that explore the willingness of users to participate in user-based relocations. Second, user-based related optimization problems and their solution methods are surveyed. Third, we provide an in-depth analysis of pricing studies that affect the demand to achieve a more balanced system. Finally, by highlighting the gaps in the literature, we propose future research directions and areas for improvement for carsharing studies that address pricing mechanisms or user-based relocation activities.

keywords : literature review; carsharing; user-based relocations; pricing; imbalanced demand

3.1 Introduction

Privately owned vehicles have reached a mounting number of 34.5 million in the UK alone (Bates and Leibling, 2012). Increasing population and urbanization have escalated the need for transportation and, therefore, the number of private vehicles. Vehicles are in the parking position 96.5% of the time, which results in fewer parking areas in the city centres (Bates and Leibling, 2012). While the increase in the number of private vehicles causes traffic congestion, idly sitting vehicles cause a decrease in urban public spaces. Therefore, the importance of sustainable sharing economy models in transportation, such as carsharing, has emerged.

The introduction of carsharing dates back to 1948, with a pioneering start-up called Sefage providing short-term vehicles in Zurich, Switzerland. Although it has been more than seventy years since the first launch of carsharing, it has made its transformation as an important player in the transportation sector over the past decade with the era of digitalization and mobile technologies. From 2006 to 2018, the number of vehicles in carsharing globally increased by more than 16-fold and reached almost two hundred thousand. A more significant increase (more than 90-fold) for the same period was observed in the number of registered users, which has reached almost 32 million worldwide in 2018 (Shaheen et al., 2020).

Carsharing can be categorized into three types based on the pick-up and drop-off points. The first category, round trip systems, requires users to return the vehicle to the pick-up station. In the second category, one-way systems, drop-off is possible at any station, while in the third category, free-floating systems, users may leave their vehicle at any parking point in the region. Although one-way and free-floating systems provide flexibility and convenience to the users, they are operationally challenging due to the imbalanced vehicle-demand distributions. As a result, these systems may face over or undersupply of vehicles and may require corrective actions to reconfigure the vehicle distribution. To tackle this problem, carsharing companies employ three schemes (Brendel et al., 2022):

- Operator-based relocation: Reactive approach which requires personnel to relocate the vehicles to meet future demands, balance the vehicle distribution or regain idle vehicles in low-demand areas.
- User-based relocation: Proactive approach where users are convinced to alter their original trips through incentive schemes or rewards.
- Pricing: Preventive approach of carsharing companies by analyzing the overall demand and promoting the prices of routes from excess supply to excess demand.

Particularly, complex relocation operations have attracted the attention of the Operations Research Community in terms of strategic, tactical, and operational decisions in carsharing systems. As a result, several literature review papers have been published to analyse studies focusing on demand estimation, optimization models, business practices and models.

One of the literature review papers, Jorge and Correia (2013), provides an overview of the studies conducted on demand estimation of the carsharing market, mathematical models, and vehicle demand-supply balancing. A recent study, Yao et al. (2022), investigates similar aspects of carsharing considering electric vehicle fleets. Laporte et al. (2018) investigates mostly studied questions, namely station location, fleet size, station inventory, incentives, and vehicle repositioning for vehicle sharing systems (including bike sharing) from an operational research perspective. Wu and Xu (2022) also provides literature focusing on optimization problems and solution methods for carsharing systems. Illgen and Höck (2019) provides an in-depth analysis of decision problems related to operator-based relocations in one-way carsharing systems. Ferrero et al. (2018) provides a taxonomy along with an overview of the papers included in the taxonomy. Another taxonomy study Remane et al. (2016) provides a general introduction to the carsharing operator profiles with a focus on business models. Nansubuga and Kowalkowski (2021) provides a broader literature review with consideration of business models among carsharing operators, drivers and barriers to the use of carsharing, customer behaviour, and the studies in vehicle rebalancing. Finally, Golalikhani et al. (2021) provides a literature review on descriptive research on user behaviour and prescriptive research on modelling problems. Table 3.1 summarizes the literature review papers.

Table 3.1: Summary of literature review studies on carsharing systems

		Descriptive and Predictive Studies		Supply Demand Balancing							
	Taxonomy	Factors in adoption of CSS	Demand analysis	Operator-based relocation	User-based relocation	Pricing	Rebalancing Incentives	Decision Based Analysis	Solution Methodology	EV Charging	Business Models
Study											
(Ferrero et al., 2018)	X	X	X					X	X		
(Jorge and Correia, 2013)				X	X						
(Remane et al., 2016)	X										X
(Wu and Xu, 2022)				X	X	X		X			
(Yao et al., 2022)			X	X	X			X		X	
(Illgen and Höck, 2019)			X	X		X			X	X	
(Laporte et al., 2018)				X			X	X	X		
(Nansubuga and Kowalkowski, 2021)		X	X	X	X						X
(Golalikhani et al., 2021)		X	X	X	X	X		X			X

The supply-demand balance in carsharing has been examined in the mentioned literature review studies with a focus on operator-based relocation. Even though most of the literature review papers mention user-based relocation strategies and pricing, an in-depth analysis has not been provided. The literature lacks review papers that present an extensive analysis and categorization of the user-based and pricing approaches to balance the demand.

Influencing demand through incentives or pricing has been studied in different sectors, including electricity markets, hotel and airline management. A summary of incentive-based and pricing-based demand response in the electricity market is given in Albadi and El-Saadany (2008). A literature review of pricing optimization techniques in hotel management is presented in Vives et al. (2018). In transportation, acting on demand by incentive-based and pricing-based methods has been intensely studied for ridesourcing, bike sharing and carsharing systems. Wang and Yang (2019) has provided a literature review on ridesourcing with a focus on demand pricing and supply incentives. The studies Reiss and Bogenberger (2017), Pfrommer et al. (2014), Fricker and Gast (2016) and Singla et al. (2015) aim to rebalance bike sharing systems through user-based relocation with given incentives to users, whereas Haider et al. (2018) uses pricing tools to achieve a bike-stock and demand balance. Similarly, the same approaches to achieving balanced systems in carsharing systems are gaining importance.

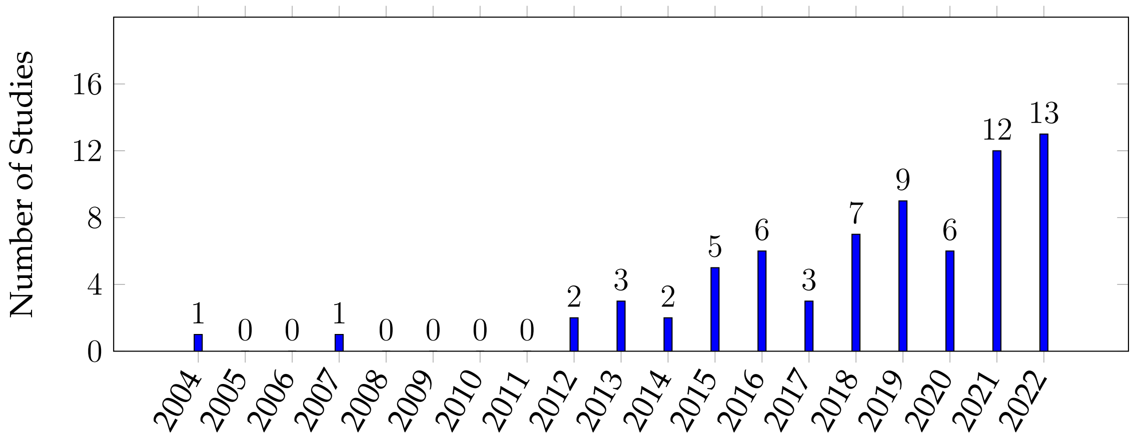

Figure 3.1: Number of user-based relocation and/or pricing studies in carsharing over years

Figure 3.1 points out pricing and user-based relocation strategies' importance over the years in the operational research community. Particularly over the last six years, an increasing number of studies have addressed user-based relocation and pricing in carsharing literature. In this paper, our purpose is to explore the papers that try to maintain a supply-demand balance through user-based relocations and pricing. The contribution of this review is twofold. First, we provide a comprehensive overview of the current state of research on descriptive and prescriptive analysis focusing on vehicle stock balancing methods, excluding studies involving only operator-based relocations. Second, we identify under-explored areas and propose future directions for interested researchers and practitioners.

The literature review is conducted through a keyword search ("user-based relocation"), ("pricing"+"carsharing") and ("incentive"+"carsharing") of the databases

Scopus and Web of Science. For the relevant papers, we have also conducted a backward and forward search and, finally, achieved 70 papers that include the user-based and pricing aspects in carsharing. Please note that throughout this study, we use the term ”user-based relocations” to refer to the studies that consider changing the routes of certain trips by offering incentives (monetary rewards/free driving minutes) and lessening the need for operator-based relocations. On the other hand, we use ”pricing” for the studies that use different minute-based prices to affect the overall demand on the routes and achieve a more balanced vehicle supply demand among the stations. These terms might be used differently by the related studies but are categorized according to the definitions we assumed in this literature review.

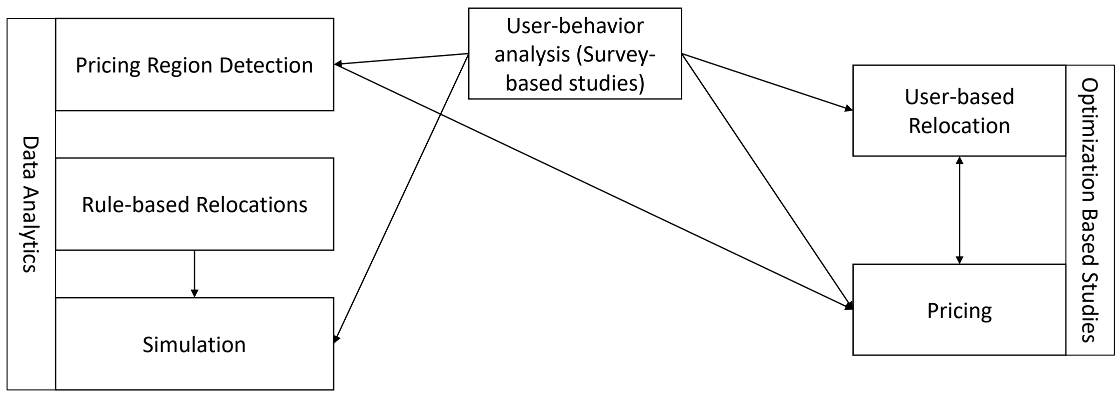

Figure 3.2: Classification of pricing and user-based relocation review

The remainder of this paper is structured as follows. Section 3.2 provides a review of survey-based studies on pricing and user-based relocations, followed by Section 3.3, a summary of studies using data analytics on pricing region detection, rule-based relocation policies and simulation approaches. Section 3.4 focuses on the mathematical modelling aspects in the related literature. Finally, in Section 3.5, this paper addresses the gaps in the literature to shed light on future research. Figure 3.2 illustrates the relations between sections. Note that the sections provided in this review are not disjoint sections, and certain studies are discussed under more than one section.

3.2 Descriptive analysis

In this section, we examine the exploratory studies investigating the incentive mechanisms and users' willingness to accept flexibility when considering incentives. These studies involve both supply and demand-side factors that affect carsharing participation ratios. User-based relocations require the users to walk to stations with available vehicles or drive to stations with parking availabilities other than their original pick-up or drop-off stations or wait more than expected (supply-side factors). The factors affecting acceptance of the spatiotemporal flexibilities are not only limited to additional walking/driving distance and time required but also weather conditions, temperature, time of the day, day of the week, public holidays, seasons, the purpose of the trip, the number of people travelling together, location of stations, road conditions, the incentives on the flexibility, as well as the demographic indicators including gender, education, income, vehicle ownership (demand-side factors).

Shaheen et al. (2018) designed two types of incentives that reduce the need for operators in one-way electric carsharing systems. The first type requires the user to leave the vehicles at stations close to the central charging station, while the second type provides 10-minute driving credit for moving the vehicle from low-demand to high-demand zones. The analysis shows that, for the charging-focused incentives, 85% of the respondents reported that they could participate and earn credit before the incentives were implemented. After the incentives were implemented, 22% of the respondents received credit, and the majority of the non-credit takers responded that incentivized stations were not in their destination areas. For the rebalancing incentives, 7% of the respondents indicated that they would move the vehicle to the requested station for 10-minute driving credit. When driving credit is 30-minutes, the participation rate rises to 65%.

Zoepf and Keith (2016) conducted a discrete choice survey by providing different alternatives (price, distance, availability and vehicle type) to the users.

The analysis shows that one-mile access distance is almost equal in value to one hour shift in the scheduled hour. Ströhle et al. (2019) provided the additional cost of the flexibility of the users to a mixed integer programming model (see Section 3.4) by carrying out a discrete choice survey as done in (Zoepf and Keith, 2016). It is assessed how much the user would pay to reduce the flexibility (i.e. decrease the walking distance or time shift). In general, the participants would pay more for a 250m drop from 1km to 750m than for a drop from 500m to 250m. The same is valid for the time shifts.

Herrmann et al. (2014) designed a survey to understand the users' willingness to participate in an incentivized scheme in free-floating systems. The survey indicates that 20% of the respondents accept walking more than 500m, and 95% of the respondents do not agree to wait more than 30 mins. When asked if the user accepts a more distant vehicle, 85% of the customers accept, whereas 13% of the respondents expect more discount than 10 cents/km in order to accept the distant vehicles. With the insights gained from the survey, user-relocation strategies are developed and tested on car2go data via simulation.

Wu et al. (2020) conducted a survey on willingness to pay and accept incentivization mechanisms on free-floating carsharing users in Washington D.C. and Vancouver. Among the relocation mechanisms, namely, delivery, pick-up, drop-off changes, and paid relocation, drop-off with incentives gets the highest ratio of willingness to accept (57%). The study suggests that user experience factors influence the willingness to pay/accept mechanisms more than socio-demographic factors. Curtale et al. (2021) examined the user preferences based on flexibility in origin-destination stations and vehicle sharing through a survey administered on one-way carsharing users in the Netherlands. Users offered 3€, 1€, and 3€ for alternative pick up, drop off and sharing the vehicle, respectively. Unlike Wu et al. (2020), the analysis indicates that users are more willing to participate in the incentive program for pick-up alternatives.

Wu et al. (2019) considered spatiotemporal vehicle balancing through two mechanisms, virtual queueing (VQ) and guaranteed advanced reservation (GAR), and examined the attractiveness of these mechanisms to customers. By implementing GAR and VQ, demand-vehicle stock balance can be achieved while not losing the customer. The survey for the analysis of trip behaviour based on social activity purpose revealed that users were more willing to use GAR over VQ, preferring to pay more for guaranteed trips rather than risking long waiting times.

Herrenkind et al. (2019) conducted a survey to feed a self-learning algorithm on incentive computation. Unlike the other survey approaches mentioned earlier, factors affecting the willingness to walk, such as weather and temperature, are included to assess the incentive acceptance rates. The survey data is used to train a machine learning algorithm to provide the best incentive value to the user. The algorithm is further trained by providing the accepted and rejected offers with lower and higher incentives to induce a learning process.

Table 3.2 provides a brief introduction to survey-based studies regarding user-based relocations and pricing. In summary, both demand and supply-side factors that affect carsharing membership are extensively studied. Survey analysis studies can reveal general user travel behaviour regarding user-based relocation. Surveys that can analyse the effects of the geo-location characteristics of the region/stations and incentives on origin-destination pairs can provide companies with more realistic insights. Moreover, the surrounding points of interest of the stations might be additional influencing factors on the willingness to accept the flexibilities. More research is needed to analyse user behaviour under uncertainty in vehicle availability, dynamic pricing, and vehicle location (Wu et al., 2020). Additionally, the effects of the existence of travel modes that complement carsharing systems on carsharing prices can be further analysed. Survey studies on pricing in carsharing systems can be used to reveal the relationship between user behaviour and distance to systems such as public transportation and bike sharing. Finally, further analysis can be made on how incentive schemes shape demand by seeking to answer the following questions: i) does the volume of demand increase through incentives, or ii) is the demand just shifted to the incentivized vehicles within the system? (Lippoldt et al., 2018)

Table 3.2: Summary of the survey studies on user flexibility behaviour in carsharing systems

Study	Number of respondents	Flexibility	Incentive	Key findings of the analysis
Shaheen et al. (2018)	1081+448+473	Dropping station	Discount or points in the system	Minutes are more favourable than discount, Almost half of the respondents who did not take incentive state that the destination is not in the incentive area
Herrmann et al. (2014)	87	Walking to the stations, and waiting	10cent/km discount, driving minutes	20% of the respondents accepts to walk more than 500m Majority of the respondents do not accept to wait more than 30mins A few of the respondents expect more discount than 10cents/km to the far stations One third of the respondents are interested in free minutes.
Curtale et al. (2021)	739	Walking to stations/ sharing vehicle	3 Eur for pickup, 1 eur for drop off	Alternative pick up is more favourable than drop off or sharing vehicle
Wu et al. (2020)	311	Walking to stations	delivery fee $7, station change with a discount $0.06/min, paid relocation 10%discount	Willingness to change the drop off station is highest (57%) among other relocation mechanisms
Wu et al. (2019)	232	Walking to stations waiting at the queue	£0.54 pay for journey	Positive willing to pay for GAR
Herrenkind et al. (2019)	274	Walking to the stations (between 800m to 1500m)	0.50€ to 6€ with 0.50€ increments	The responds are used in a self-learning algorithm to compute the incentives
Ströhle et al. (2019)	1529	Spatio-temporal flexbility	Choice task with different walking time, vehicle availability, and price	Half of the customers can be persuaded with very low incentives.
Zoepf and Keith (2016)	3958	Spatio-temporal flexbility	Choice task with different walking time, vehicle availability, and price	Traveling additional one mile is equivalent to shifting the trip one hour

3.3 Data analytics

In this section, we explore how data analytics applications can improve the service quality of carsharing systems in terms of vehicle availability. Carsharing companies generate data at each rental and booking search. The generated data include helpful information on vehicle position, battery/fuel consumption, km driven, personnel movements, customer behaviour and various others. To address the spatiotemporal demand asymmetricities, data analytics are used to distinguish pricing areas, predict demand, compute incentives and assess the effectiveness of already implemented policies.

3.3.1 Pricing region detection

This section presents studies that investigate data and provide spatiotemporal decision support on how the coverage area is divided into pricing areas. Specifically, we provide a summary of the studies that divide the business areas into either same-sized grids or areas with homogeneous characteristics. Especially in free-floating systems with no clear definitions of subregions, it is necessary to provide differentiated pricing regions to keep the balance on the vehicle stock.

Wagner et al. (2015) developed a location-based support system for free-floating carsharing systems, providing a user-based relocation scheme to reduce vehicle idle time. In the proposed support system, first, the business area of the system is divided into a grid of individual tiles. Then, a user is offered free minutes for altering the destination to an area (i.e. tile) with a lower idle time within a 500m radius. It is assumed that the probability of acceptance depends on the distance between the original destination and the proposed station. Average vehicle idle time was reduced by 16% with a simulation analysis performed on a free-floating system in Vancouver. Brandt and Dlugosch (2021) proposed a similar approach on user-based relocation incentivization or nudging (i.e. offering alternative drop-off stations by highlighting the parking availabilities of low idle-time areas) scheme where the aim is to decrease the expected idle times of the vehicles. Simulations on Vancouver and Berlin data show that a significant impact on the utilization of vehicles can be achieved.

Lippoldt et al. (2018) evaluated the incentive programs implemented by a free-floating carsharing system operating in Milan. The business district in Milan was divided into two categories, central and peripheral areas, with high and low demands, respectively. The users were offered discounted rates to pick up a vehicle from the peripheral area and free driving minutes to drop off the vehicle at the central area. Rental data were analysed, and it was stated that district categorization might not fully reflect attractiveness levels. However, improved distribution of vehicles is achieved through incentivization schemes. The same authors conducted a similar study for assessment of already placed incentive schemes in Cologne (Lippoldt et al., 2019). It is shown that free-driving minute incentives decrease the average idle time of vehicles in low-demand areas. Both in Lippoldt et al. (2018), and Lippoldt et al. (2019),the incentives are applied in the system through manual interference of the personnel, which demonstrate room for improvement.

Weikl and Bogenberger (2013) conceptualize a two-stage approach for demand prediction and relocation decisions. Yet, the focus of the study is on the first stage, where the data is partitioned into clusters based on the principal component analysis taking the number of bookings, vehicle idle times, and unsatisfied bookings for the time of the days as indicators. After predicting demand for a time interval of a day for each section (even though section identification is not provided), a mixed integer programming model (MIP) is provided.

Brendel et al. (2020) presented a decision support system to distinguish pricing areas in free-floating systems and applied to a real-world system in Göttingen. Using the historical rental data, a heat map is generated, and the k-means clustering algorithm is applied. Finally, a grid representation of pricing areas is formed for the carsharing provider. Increased vehicle supply in high-demand areas is achieved by offering bonus minutes. Please note that unlike Wagner et al. (2015) and Brandt and Dlugosch (2021), the user acceptance rates are not considered in the study.

Willing et al. (2017) proposed two types of spatial decision support systems (SDSS) for balancing supply-demand in free-floating systems. The first SDSS is for existing systems, whereas the second is for the systems that aim to expand their operation areas. By incorporating points of interest (POI, i.e. the aim of the trips, such as arriving at restaurants and banks), heat-maps that show high, medium and low-demand areas for different time intervals are created. The proposed SDSS is applied to data collected from a carsharing operator in Amsterdam and tested on data from Berlin to predict demand. Application of SDSS indicates that the demand can be explained by POI to a certain extent which may help with pricing decisions that lower the risk of vehicle imbalance in the system.

Table 3.3: Summary of the studies that detect pricing regions

Study	Carsharing Type	Flexibility	Aim	Methods Used	Application
Wagner et al. (2015)	Free-Floating	Destination station	Decrease the vehicle idle time through user-based relocation	Business area segregation into tiles Computation of expected idle times of each tile. User-based relocation policy to offer lower idle-vehicle-time stations that are close to destination station.	Vancouver
Brandt and Dlugosch (2021)	Free-Floating	Destination station	Identification relocation policy and implementation	Map-based visualization of the rentals User-based relocations based on idle vehicle times at stations	Vancouver and Berlin
Lippoldt et al. (2018)	Free-Floating	Origin and destination stations	Evaluation of user-based relocation schemes	Partitioning operating area into zones. Idle time and vehicle distribution assessment	Milan
Lippoldt et al. (2019)	Free-Floating	Origin and destination stations	Evaluation of user-based relocation schemes	Idle time and vehicle distribution assessment	Cologne
Weikl and Bogenberger (2013)	Free-Floating	spatio-temporal	Demand clustering	Principle component analysis	Munich
Brendel et al. (2020)	Free-Floating	Destination station	Pricing areas	Heat map on the rental data	Göttingen
Willing et al. (2017)	Free-Floating	Destination station	Pricing areas	POI-based heat maps to show the demand areas	Berlin

By displaying the regions that may offer price incentives to users on the map via mobile applications (Wagner et al., 2015), companies can raise awareness about user-based relocation and increase the participation rate. One challenge in providing such guidelines for incentives is the dynamic nature of the system. It requires regular updates to the map, making management more complex than static incentive schemes. Furthermore, with the integration of historical data for different time-intervals of the day-month-year into dynamic mapping, the current and future state of the system can be analysed and combined with the pricing area detection process. Table 3.3 provides summary information of the studies on pricing area detection.

3.3.2 Rule-based relocation policies

This section presents a summary of studies focusing on rule-based relocation strategies in carsharing systems. As mentioned earlier, user-based relocation is one of the tools that balance vehicle availability throughout operational hours. However, vehicle flow from a station with an excess of vehicles to a station with a shortage of vehicles does not necessarily guarantee a balanced distribution of vehicles, as it might affect future vehicle movements within the system. Therefore, determining the status of stations in terms of vehicle flow requirement and availability is a challenging task (Laarabi et al., 2019).

Brendel et al. (2018) and Brendel et al. (2016) provided decision support frameworks for user-based relocation assignments to vehicles with or without considering EVs, respectively. The framework proposed in Brendel et al. (2016) first predicts over and under-supplied stations by taking the historical data into account. Second, the trips that are moving towards a vicinity of an under-supplied station are prioritized and offered incentives for relocation. The same approach is used in Brendel et al. (2018) with additional consideration of the battery charging process and vehicle state of charge (SoC).

Wang et al. (2019) proposed a ranking method that offers users incentives to reduce stations' vehicle idle times and zero vehicle times (i.e. the time that stations are without vehicles). The ranking method prioritizes the user-based relocations based on a linear weighted sum of factor values that affect the supply-demand ratio. These factor values include vehicle idle time, pickup and drop-off demands for neighbouring stations and the reserved vehicle. The ranking method is applied to a real-world system in Shanghai for a brief period of time and compared to an incentive plan based on the experience of company personnel. The results show that the ranking method outperforms the empirical method and yields better profit.

Barth et al. (2004) considers two types of user-based relocation strategies, where users join or split trips. The former strategy is applied when two users request vehicles for the same route with an under-supplied origin station and share one vehicle. The latter strategy is offered to requests with multiple passengers wishing to move to an under-supplied station. With this strategy, passengers are asked to move separately. Simulations of the strategies show that with a 100% participation rate, the number of relocations is decreased by 42%. Uesugi et al. (2007) also adopted trip splitting and merging strategies based on minimizing the square error sum for the number of available vehicles and the favourable number of vehicles at stations.

Laarabi et al. (2019) considered user-based relocation policies in one-way carsharing systems with lightweight stackable vehicles (i.e. ESPRIT cars) where users are assumed to be able to drive a maximum of two vehicles per trip. Relocation strategies based on vehicle and parking spot availabilities show that leveraging historical data and forecast results in better alignment of vehicles over the operational period.

Iacobucci et al. (2022) proposed a multi-stage DSS to solve the relocation problem in one-way systems with stackable vehicles. First, the inventory imbalances at each station, considering future demands, are calculated. Then, solution algorithms are proposed based on the relocation scheme (operator-based, user-based and autonomous). For the operator-based scheme, two distinct integer programming (IP) models are formulated to find the vehicle flow quantity and the personnel assignments. The user-based scheme is designed according to the state of the station (feeder or receiver). Finally, the autonomous scheme considers the first IP model's solution to vehicle flow and assigns vehicles to relocations, taking into account the maximum time to complete the relocation instead of immediately dispatching the vehicles.

Cepolina et al. (2015) proposed a user-based relocation scheme along with two autonomous sharing systems for Personal Intelligent City Accessible Vehicles (PICAVs). PICAVs are urban electric vehicles which are suitable mode of travel for historical centres. Cepolina et al. (2015) offers a user-based relocation strategy for users travelling outside of the urban area after completing the trip with a PICAV. A set of PICAV stations is offered to the user, guaranteeing that public transport to the user's final destination is available.

Table 3.4 summarizes the rule-based user relocation studies. The aforementioned rule-based studies lack incorporating user behaviour and pricing decisions. Users accepting any alternative offer may result in higher profit values, hence, mislead the operators. Integrating the user acceptance probabilities into user-based relocation triggering mechanisms will yield a more realistic view of the system. The next section presents such mechanisms through agent-based simulations.

3.3.3 Simulation based studies on user-based relocation and pricing

This section introduces the studies that use simulation-based decision support systems in the pricing and incentive decisions to balance the demand distribution. In particular, we investigate myopic approaches with decisions regarding the current state of the system. Although some of the studies mentioned in this section use rule-based mechanisms with threshold values, they are mentioned in this section due to the primary focus on simulation.

Clemente et al. (2018) proposed a DSS that gives users incentives in parking areas triggered by a threshold value. The system is described as a discrete event system (DES) model, and optimal threshold values are found by maximizing the level of service considering the system state knowledge. A particle swarm

Table 3.4: Studies that investigate rule-based user relocations

Study	Carsharing Type	Flexibility	Aim	Methods Used	Application
Brendel et al. (2016)	One-way	Destination station	Designing a user-based relocation system	Predicting number of vehicles needed at each station Based on the treshold values, customers are asked to change destinations.	Generated one-way data from a round-trip system's data
Brendel et al. (2018)	One-way Mixed Fleet	Destination station	Maximize battery electric vehicle utilization	Based on treshold value, vehicles are charged. Battery electric vehicle are prioritized over internal combustion engine User-based policy of Brendel et al. (2016) is applied	Simulation on a German system
Wang et al. (2019)	One-way	Destination stations	Incentive-based approach to influence the demand patterns	Predict demand for each time period Make a ranking list of vehicles and stations	China
Barth et al. (2004)	One-way EV	Trip splitting and joining	Introduction of user-based mechanisms	Relocations are offered when the conditions on number of passenger, trip status and treshold on balancing operations are met	Real-world university campus
Uesugi et al. (2007)	One-way	Trip splitting and joining	Assigning optimum number of vehicles to users	Minimizing the residual error of optimum number of vehicles and available number of vehicles	Simulation
Laarabi et al. (2019)	One-way ESPRIT fleet	vehicle train	user-based relocations in ESPRIT fleet systems	Rule based relocation policies based on number of vehicles both at origin and destination stations	Lyon
Iacobucci et al. (2022)	One-way stackable fleet	vehicle train	operator-based, user-based and autonomous relocations in stackable fleet systems	Inventory balance prediction, integer programming for operator-based and autonomous relocations, Rule based relocation policies based the vehicle stock for user-based policy	New York Taxi data
Cepolina et al. (2015)	one-way one-way autonomous free-floating autonomous	Destination station	Comparing user-based relocations with autonomous systems	Rule-based assignment of a station near the destination station of the user	Genoa

optimization algorithm is proposed to solve the optimization problem. Schulte and Voß (2015) designed a DES with four types of user-based relocation strategies, namely incentives for remote vehicles, remote drop-off locations, paid relocations (routes), and demand pooling (trip joining). Acceptance probabilities are generated based on the survey results of Herrmann et al. (2014). All the strategies are applied together in the simulation model. The results suggest that the application of user-based relocation strategies yields a more economical and environmental-friendly system.

Bianchessi et al. (2013) considered a dynamic pricing simulation scheme in which the prices are updated according to the vehicle availability at each station. If the number of vehicles at a station is below the average number of available vehicles in the system, the price increases depending on the difference. Unlike the rest of the literature, this is the only study that assumes that users' decisions follow a Normal distribution when accepting a station in the vicinity.

Ciari et al. (2015) designed a multi-agent modelling framework to explore how demand is affected by pricing schemes on free-floating systems in a system where free-floating and round-trip systems coexist. Full price, half price, and half price at some time intervals during the day are taken as pricing schemes on the free-floating system. The analysis shows that pricing policies on free-floating systems also change the demand on the round-trip. In addition, the spatiotemporal profile of carsharing demand has been shown to be sensitive to the pricing policies adopted.

Chen and Kockelman (2016) considered autonomous vehicle-sharing systems in an agent-based simulation framework. A multinomial logit model is presented for the agents to select a transportation mode among private vehicles, city bus service or shared autonomous electric vehicles (SAEV). The utility of each transport mode is calculated by functions based on the service cost and the value of travel time (VOTT) of the agents. VOTT is assigned to each agent in the simulation, considering US census data of metropolitan residents. User-based pricing schemes (origin-based, destination-based and combined) are offered to the agents and compared to distance-based pricing schemes. Note that the pricing schemes consider the current system state and the next time interval's anticipated demand. The results show that user-based pricing schemes can significantly reduce agents' average wait times.

Another agent-based pricing study was conducted by Giorgione et al. (2019) to investigate the effects of availability-based pricing on round-trip systems. Agents' travel disutility in choosing the transport mode is defined by a function that depends on VOTT, the number of available vehicles, the cost of the trip, and the time required to complete the trip. Analysis of the study indicates that the number of reservations decreases slightly when the dynamic pricing scheme is applied. Additionally, users with low VOTT tend to switch to other transport modes.

Clemente et al. (2013) considered one-way electric carsharing systems and modelled three incentive schemes (regular pricing, 20% off for the vehicles returned back to the origin station, 20% off for the vehicles reducing the imbalance of the vehicle stock across stations) in a Timed Petri Net environment. The simulation results suggest that incentives with balance considerations improve the LOS and profitability, whereas the incentives requiring immediate return of the vehicles do not.

Table 3.5 provides a summary of the simulation-based studies regarding pricing and user-based relocations in carsharing systems. The utility users get by preferring carsharing systems or changing their trip choices not only depends on price and time differences but also the region's geographic characteristics. Incorporating more information into the system, such as traffic evolution and geographic characteristics, provides a more realistic view of the system. Such studies form a groundwork for future studies (Bianchessi et al., 2013). Furthermore, inclusion of the operator-based relocations might alter the systems' dynamics which provide a perspective for future directions.

Table 3.5: Simulation focused user-based relocation or pricing studies

Study	Carsharing Type	Aim	Methods Used	Simulation
Clemente et al. (2018)	One-way	Designing a decison support system with incentives	Incentive threshold values are found by Particle Swarm Optimization Algorithm	Discrete Event Simulation
Schulte and Voß (2015)	Free-floating	Cost and emission efficient system	Different user-based relocation schemes proposed and applied at a simulation model. The relocation mechanisms are steadily triggered	Discrete Event Simulation
Bianchessi et al. (2013)	One-way	Dynamic fleet balancing with pricing	Dynamic pricing based on the error between the number of vehicles at a station and average number of vehicles	System Dynamics
Ciari et al. (2015)	Free-floating	Understanding how pricing affects demand	5 scenarios with different pricing schemes are compared	Multi-agent Modelling
Chen and Kockelman (2016)	Autonomous	Understanding affects of pricing when choosing travel mode including shared autonomous EV systems	Value of travel time based utility calculation for each travel mode	Agent-based simulation
Giorgione et al. (2019)	Round trip	Understanding affects of availability-based dynamic pricing on balancing the system	Travel disutility functions	Agent-based simulation
Clemente et al. (2013)	One-way EV	Understanding affects of economic incentives	Providing incentives based on the availabilities at the stations.	Timed Petri Net

3.3.4 Dynamic anticipative approaches

Waserhole et al. (2013) considered dynamic pricing of the routes. Since the number of vehicles in the system remains constant all the time, a closed queuing model on a Markovian formulation is introduced. The proposed model decides on the pricing from a discrete pricing set at each trip request. Various heuristic approaches are applied to a toy city example. Similarly, Waserhole and Jost (2016) presents a study on a closed queuing network with infinite buffer capacity and Markovian demands in one-way systems. In order to solve the problem of maximizing the number of trips sold, a Maximum Circulation problem and a greedy algorithm are proposed. Another study on queuing theory has been conducted by Boldrini et al. (2018). Instead of influencing demand by differentiated pricing as done in Waserhole et al. (2013) and Waserhole and Jost (2016), a user-based approach has been studied at one-way systems with stackable vehicle fleets. The proposed model is validated against real data.

A predictive, incentive-based, Markovian one-way carsharing system study was conducted by Stokkink and Geroliminis (2021). The study aims to determine user-based relocations by offering incentives on alternative trips considering the system's current state and expected future demand. The discount value of an incentive is found by maximizing the welfare of the system, taking into account the omitted demand loss (recovered expected future loss), which the Markovian model determines. Furthermore, a learning algorithm for the customers' acceptance rate function of the alternative trips (binomial logit function) is proposed to approximate the probabilities accurately.

3.4 Mathematical modelling based studies on user-based relocation and pricing

In this section, we provide the studies concerned with mathematical modelling approaches to solve the operational level - user-based relocation or pricing problems. First, we describe a general mathematical model formulation of the operational level problem where personnel and vehicle schedules are to be generated. This model is adapted from the studies Gambella et al. (2018) and Boyacı et al. (2017) without battery charging requirements of the electric vehicle fleet. Then, we present a series of papers addressing user-based relocation and pricing. Finally, we specify how an operational-level model is adapted to solve such problems.

3.4.1 Operational level mathematical model for one-way carsharing systems

The operational level problem in a station-based one-way carsharing system with reservation seeks to provide schedules for both vehicles and personnel while maximizing profit or number of served trip requests (or, in some cases minimizing cost). The system is composed of a set of stations ($j, l \in J$) with finite capacity (C_j), a fleet of identical vehicles positioned at the stations, and a number of personnel waiting to be assigned for relocations at the beginning of the day. The time horizon (i.e. operational day) is divided into equal-length time intervals ($t \in T$). The problem is represented by a time-space network with arc set A. Trips, vehicle relocations, personnel walking relocations, and personnel driving relocations are allowed to flow on subsets of A. Decision variables x_i are the binary variables, denoting the vehicle flow on arc $(j_t, l_{t'}) \in A_i \subset A$ taking value of 1 if the trip request i is accepted; 0 otherwise. Variables y_r stand for the number of relocated vehicle flow on arc $r = (j_t, l_{t+DD_{jl}}) \in A_r \subset A$ where DD_{jl} is the driving distance between j and l in time intervals. Similarly, variables u_r stand for the number of personnel flow that travel by a vehicle on the arc $r = (j_t, l_{t+DD_{jl}}) \in A_r \subset A$ and variables $\bar{u}_m$ are for the personnel that walks on the arc $m = (j_t, l_{t+WD_{jl}}) \in A_m \subset A$ where WD is the walking distance from j to l in time intervals. Note that arcs $(j_t, j_{t+1}) \in A_r$ are used to express the waiting vehicle/personnel flow at each station j.

The objective function for the operational level problems is mostly the maximization of profit (revenue - operational cost including relocation cost, daily personnel cost, battery charging cost) or the number of accepted trip requests. Note that the objective function may only minimize the cost if the system does not decide on trip requests.

$$\max \text{Total Profit}$$

or

$$\max \sum_i x_i$$

subject to

$$\sum_{r \in FS(j_t) \cap A_r} y_r + \sum_{i \in FS(j_t) \cap A_i} x_i = \sum_{r \in RS(j_t) \cap A_r} y_r + \sum_{i \in RS(j_t) \cap A_i} x_i \qquad \forall j, t \tag{3.1}$$

$$\sum_{r \in FS(j_t) \cap A_r} u_r + \sum_{m \in FS(j_t) \cap A_m} \bar{u}_m = \sum_{r \in RS(j_t) \cap A_r} u_r + \sum_{m \in RS(j_t) \cap A_m} \bar{u}_m \qquad \forall j, t \tag{3.2}$$

$$y_r \leq u_r \leq CapV \; y_r \qquad \forall r \in A_r \tag{3.3}$$

$$y_r = IV_j \qquad \forall j, \forall (r | r = (j_{t_0}, j_{t_1})) \tag{3.4}$$

$$\bar{u}_m = IP_j \qquad \forall j, \forall (m | m = (j_{t_0}, j_{t_1})) \tag{3.5}$$

$$y_r \leq C_j \qquad \forall j, \forall (r | r = (j_t, j_{t+1})) \tag{3.6}$$

$$x_i \in \{0, 1\}, y_r, u_r, \bar{u}_m \in \mathbb{Z}^+ \cup \{0\} \qquad \forall i \in A_i, r \in A_r, m \in A_m \tag{3.7}$$

Constraints (3.1) and (3.2) are the flow constraints for vehicles and personnel, respectively. Forward star (FS) and reverse star (RS) in these constraints denote the outgoing and incoming flows. Constraints (3.3) ensure that vehicles are driven by personnel, and the driving personnel is bounded by the vehicle seater capacity on each arc in A_r. Constraints (3.4) and (3.5) are for the initial number of vehicles and personnel at each station at the beginning of the day. Constraints (3.6) limit the number of cars that are waiting at the stations. Finally, constraints (3.7) define the decision variables.

Optimization-focused user-based relocation studies mainly modify decision variables x_i to binary variables $x_{ii'}$ where $i' \in N(i)$, $N(i)$ denotes the neighbouring trips to i. On the other hand, pricing studies shape the demand at the routes. Therefore, instead of a variable type x_i focusing on a single trip demand, a variable type D^t_{jl} is defined to express the trip flow from station j to l at time t under certain pricing functions.

3.4.2 User-based relocation

This section includes studies that integrate mathematical modelling approaches into the decision-making process. The studies were classified according to the prominent features of the problem setting.

The only study that considers demand-flexibility in round-trip systems is Ströhle et al. (2019). Spatial flexibility in round-trip systems requires that both the pick-up and drop-off stations be assigned to the same station so that the business structure does not change. To understand the potential effects of demand-side flexibility on the round-trip systems, Ströhle et al. (2019) considered systems with complete (offline) and instant (online) information. A mixed integer linear programming (MILP) model is proposed to minimize the fleet size by spatiotemporal flexibility without considering the users' rejection of the proposed trips. The tests on a real-world data set from a city in Germany for the offline case showed that temporal flexibility does not affect the fleet size as much as spatial flexibility, with a reduction of 4% compared to 12%. The online case also demonstrates that the fleet size reduction found in the offline case remains viable. Furthermore, a survey is conducted to find the cost of offering flexibility (see Section 3.2) and implemented into the model. The new cost minimization model resulted in a 25% reduction in cost.

Correia et al. (2014) considered spatial flexibility as the closest three stations (first as closest to the original request, second as closest to the first, and third as closest to the second station) in one-way systems. A MILP model is proposed to solve the profit maximization problem without consideration of operator-based relocation. The proposed MILP model can take different user types, namely, inflexible, flexible without information, and flexible with information, as input to see the effect of flexibility and knowledge on the vehicle distribution of the system. To set flexibility on x_i used in the model in Section 3.4.1, the binary variables $X^i_{j_t, l_{t+DD_{jl}}}$, taking the value of 1, if trip request i is served through arc $(j_t, l_{t+DD_{jl}})$; 0, otherwise, are introduced. By adding constraints $\sum_{j,l,t} X^i_{j_t, l_{t+DD_{jl}}} \leq 1 \quad \forall i$, at most one offer is made to the user. Note that the model also finds the daily operational station locations and employed fleet size. The proposed model is tested in Lisbon, and results show that there is a significant profit increase (from 428Eur to 1732Eur) when the users become flexible. It is also stated that accessing the system's vehicle stock in real-time via the mobile app can further increase profits (up to 2380Eur).

Schiffer et al. (2021) studied revenue maximization problem with flexible users in free-floating system. Rather than a commodity flow consideration used in the model in Section 3.4.1, an integer programming model that treats each vehicle and their trip chain separately is introduced. Relocation activities are assumed to be managed only by flexible users regardless of user acceptance possibilities. Through a graph representation, the model is reformulated as a k-disjoint shortest path problem, and an exact algorithm that can be solved in polynomial time is proposed.

Monteiro et al. (2021) considered an agent-based simulation at round-trip and one-way systems where users are assumed to be flexible in walking to neighbouring stations. A multiobjective MILP model is proposed to maximize the number of accepted trips and minimize the maximum number of vehicles allocated to a station. Unlike other models in the literature, the proposed MILP gives priority to the users who enter the system earlier by forcing the model to serve the customer if there is any vehicle available at the origin station. The model is tested using data from Sao Paulo. Results show that round-trip services provide better improvement than one-way, considering flexibility.

Incorporating Acceptance Rates

Zhang et al. (2022) developed a personalized price discounting scheme for one-way systems, where users are grouped according to certain characteristics. It is assumed that the whole demand set is known in advance. Through a mixed integer nonlinear programming (MINLP) model that considers origin station and start time flexibility, profit maximization is aimed. Each user class's acceptance rate is assumed to be dependent on the related population's ratio (and probability density function) of high/low requirements for the level of service. Similar to Correia et al. (2014), a binary decision variable $x_i^{t,j}$ is created to denote if a trip i is accepted originating at station j at time t as an alternative route. An additional decision variable p_m is included in the model to find the corresponding discount on a user class m per km. Please note that, in the study, the original request's walking distance and time shift to the proposed trip do not affect acceptance probabilities.

Febbraro et al. (2012) evaluated a user-based relocation methodology through a discrete event simulator (DES) that employs an integer programming model to minimize the rejection ratio of reservations. The model offers alternative drop-off stations or time intervals to the users. The acceptance rate of the proposed alternative (p_{accept}) is calculated by a binomial logit model that is dependent on the discount s and distance between the original and alternative trip's destination stations d. β_0, $\beta_d d$ and $\beta_s s$ values are the estimated coefficients of the logit model.

$$p_{accept} = 1 \bigg/ \left(1 + e^{\beta_0 - (\beta_d d + \beta_s s)}\right) \quad \forall d, s \tag{3.8}$$

The proposed simulator is applied to an operating system in Turin, Italy. The same authors applied a similar acceptance rate calculation method in Febbraro et al. (2019). However, this time, a two-stage optimization model is proposed. The first stage determines the optimal drop-off locations, considering the future demand. The acceptance rate is used to determine the optimal discounts to be offered to the users in the second stage of the model.

Wu et al. (2022) incorporated user behaviour modelling approach at a free-floating system in which the users can make on-demand or advanced reservations. The system receives a request from each user and responds with a set of options from which the user can select. A MINLP model is proposed to maximize the expected revenue, assuming that each user will prefer the option with the highest utility value they will receive.

Liu et al. (2021) proposed a decision framework for real-time operations in one-way carsharing systems. The study considers users to modify their destination stations suggested by the operator. The vehicle schedules and relocation decisions are determined through a dynamic programming approach. The acceptance rate of the flexibility is assumed to depend on the customer's extra walking time and incentives. Furthermore, the acceptance ratio function is calibrated through the Bayesian learning approach at each revealed customer preference data.

Incorporating Charging Decisions

Zou et al. (2020) proposed a mixed integer programming model addressing strategic planning decisions regarding station location for one-way EV carsharing systems. Rewarding and punishment mechanisms are introduced when picking up and leaving the vehicle, respectively, in the social parking lot. Battery levels of the vehicles are not monitored, and it is assumed that each vehicle parked is unavailable for at least two time intervals (i.e. 30 minutes) for charging purposes.

Boyacı and Zografos (2019) proposed a framework that considers operational-level activities in one-way electric carsharing systems. Vehicle and personnel schedules are generated while promoting flexibility through price incentives. The proposed framework consists of pre-processing for clustering the stations, optimization models to find the vehicle and personnel schedules without battery feasibility consideration, and a simulation module that checks and takes actions against battery infeasibility. The purpose of the station clustering approach is to reduce the size of the vehicle and personnel relocation variables, and therefore, the model provides solutions in a reasonable time. Furthermore, the vehicle and personnel flows are considered through station to cluster, cluster to cluster and cluster to station rather than creating direct arcs between two stations. Instead of direct variables y_r that directly find the vehicle flow between two stations in the model in Section (3.4.1), three complementary variables r^t_{sj}, $\bar{r}^t_{sj}$ and $\tilde{r}^{tu}_{sbd}$ are created to find the vehicle relocation flows to station j, from station j and between clusters b and d at time interval t / shift s, respectively. The proposed framework is tested at a real-life instance from Nice, France. The results suggest that flexibility can increase the profit by 10% and reduce the number of relocation personnel by 80%.

Lai et al. (2022) considered a rolling horizon approach optimization model in which the alternative routes to the newcomer users are provided at each time interval. In the proposed model, each vehicle is tracked separately. A time-space network model similar to the one given in Section (3.4.1) is proposed with battery feasibility consideration of each vehicle. Each arc on the system is associated with a b_e value representing the change in battery level (if the vehicle is being charged, $b_e \geq 0$; if the vehicle is in use, $b_e < 0$). Battery consumption/charging is controlled by constraints (3.9) limiting the state-of-charge (SoC) level of vehicle h at time t. Notice that the continuous decision variables SOC_{ht} indicate the state-of-charge of vehicle h at time t and binary decision variables z^h_e show whether the vehicle h passes through arc e. Since the model is computationally expensive, an iterated local search algorithm is proposed for large-size problems.

$$SoC_{h,t} \leq SoC_{h,t-1} + \sum_{e} b_e \, z^h_e \quad \forall h, t \tag{3.9}$$

Wang et al. (2022) considered a one-way carsharing and bike sharing joint system where the users are asked to reach adjacent stations by bike when there are no available EVs at stations. The first step of the proposed framework is to establish the over-supplied or under-supplied stations. Then, by a greedy heuristic, user-based relocations from over to under-supplied stations are assigned regardless of the SOC levels. Next, SOC levels of the assigned vehicles are monitored. In case of battery infeasibility, the system is updated, and new assignments are made. Please note that the system may still face a supply-demand imbalance; therefore, there may be rejected users. Through a MILP, the user-based relocation cost and customer dissatisfaction are minimized at this stage. Finally, the number of user-based relocations is provided to a separate MILP to balance the bike supply-demand at the stations.

Fanti et al. (2022) proposed a system where the users are notified through an IT application, stating the set of vehicles to be relocated, which results in a free ride or rewards. Two integer programming models are developed to minimize the relocation cost in the system. The first one considers only the relocations done by operators, whereas the second allows the company to crowdsource relocations. Although the proposed models focus on relocation activities at the end of the operations (i.e. electric vehicle relocation problem, aka E-VREP), the personnel schedule is not considered. Battery feasibility is controlled by simple constraints stating that vehicle k cannot traverse arc (j, l) if the initial SoC level of k cannot cover the battery level requirement of arc (j, l). A randomized metaheuristic algorithm is proposed where limitations on the number of vehicles at stations are relaxed.

Table 3.6: Summary of optimization focused user-based studies

Study	Carsharing Type	Relocation Type	Flexibility Definition	Acceptance Probability	Aim of the Study	Methods Used
Ströhle et al. (2019)	Round-trip	User-based	temporal postponement and spatial flexibility (origin=destination)	No	Minimize the number of vehicles in a round trip system by providing user-flexibility Minimize the cost associated with flexibility.	MILP, Survey
Zou et al. (2020)	One-way EV	Operator-based and user-based	Using social parking lots	No	Station location	MILP
Zhang et al. (2022)	One-way	User-based	Pick-up station , start time	Yes	Operational level decision on the trips and discounts.	MINLP
Fanti et al. (2022)	One-way EV	Crowdsourcing	Crowdsourcing	NA	E-VREP problem, minimization of relocation cost	ILP and methaheuristic algorithm
Schiffer et al. (2021)	Free-floating	User-based	Spatial and temporal flexibility	No	Profit maximization - operational level decisions	ILP and an exact algorithm based on k-disjoint shortest path.
Boyacı and Zografos (2019)	One-way EV	Operator-based and user-based	Spatial flexibility and pick up station	No	Profit maximization - operational level decisions	MILP + Clustering Model + Feasibility control simulator
Correia et al. (2014)	One-way	User-based	3 closest stations to origin and destination	No	Maximization of daily profit	MILP
Febbraro et al. (2012)	One-way	User-based	Destination station	Yes	Minimize the rejection ratio of reservations	Discrete Event Simulation and ILP
Febbraro et al. (2019)	One-way	User-based	Destination station	Yes	Optimal drop-off locations and discount amount	2-stage optimization, MILP and MINLP with simulator
Monteiro et al. (2021)	One-way and round-trip	User-based	Spatial flexibility	No	maximize the number of accepted trips and minimize the highest number of vehicles to be allocated to a station	Agent-based simulation + Multiobjective MILP
Liu et al. (2021)	One-way	Operator-based and user-based	Destination station	Yes	Maximize the profit for operational level decisions	Approximate Dynamic Programming
Wang et al. (2022)	One-way EV	User-based	Spatial flexibility	Yes (Not a function, but a general ratio is used)	Joint relocation model for vehicle and bicycle fleet distributions	
Wu et al. (2022)	Free-floating	User-based	Spatial and temporal flexibility	Yes	Revenue maximization	MINLP
Lai et al. (2022)	One-way EV	User-based	Spatial flexibility	No	Profit maximization - operational level (including charging) decisions	MILP + Iterated local search heuristic algorithm
Wang et al. (2022)	One-way EV and bike	User-based	Spatial flexibility and bikeshare use	No	User-based relocation and customer dissatisfaction cost minimization	Greedy Heuristic + MILP + Genetic Algorithm

Table 3.6 summarizes the optimization-based studies that aim to find station location, fleet size or the vehicle/personnel routes in carsharing systems with flexible users. The studies mentioned in this section mostly prefer to avoid adopting operator-based relocations along with user-based relocations due to the increased model size. Although user-based relocation is a powerful tool to balance the supply-demand distribution of vehicles, without operator-based relocations, isolated stations may still suffer from over or under-supplied vehicles. This is why combining user-based and operator-based approaches will bring a more balanced vehicle distribution. As this would be computationally expensive, effective algorithms for such systems could be a future direction. Crowdsourcing can also be considered in a combined operator- and user-based system by adding a stochastic component of user reliability. In cases where operator-based or user-based displacements cannot be performed, a more robust and balanced system can be achieved by considering crowdsourcing. Next, another possible future direction is the inclusion of historical/predicted trip data, especially in dynamic systems where decisions are made at each user request. Finally, reinforcement learning might be a useful tool for incorporating user-behavioural modelling.

3.4.3 Pricing

This section presents studies that use prices to manage vehicle stock at stations. Trip pricing differs from user-based relocation in the sense that user-based relocation treats each trip request individually and provides incentives for certain trip requests. In contrast, in trip pricing, certain routes are promoted with differentiated prices, which influence the travel decisions and overall demand in the system.

Jorge et al. (2015) proposed a mixed integer nonlinear programming model to find prices of routes that maximizes the daily profit. The demand for a given reference price is assumed to be known. A price-demand relation is established by taking a fixed value of price elasticity. The proposed model does not consider relocation activities. Instead, vehicle supply-demand balance is achieved by varying the demand on these routes through the route prices. An iterative local search algorithm is proposed to solve the nonlinear model. The algorithm has been tested on a dataset from Lisbon. The results show that trip pricing is a powerful tool for achieving a profitable system. Ren et al. (2019) integrated the pricing problem with the decisions on selling the energy back to the grid (i.e. vehicle-to-grid electricity selling). They assumed a similar elastic behaviour function proposed in Jorge et al. (2015).

Unlike Jorge et al. (2015), Xu et al. (2018) considered a nonlinear logit-based elastic demand function to better represent the existing demand elasticity in real systems. The elastic demand function f for price p is given by the Equation (3.10)

$$f(p_{ij}^{tg}) = \begin{cases} \left\lfloor D_{jl}^{tg} e^{-\theta_{jl}^{tg} p_{jl}^{tg}} \right\rfloor & , 0 \leq p_{jl}^{tg} \leq \frac{\ln D_{jl}^{tg}}{\theta_{jl}^{tg}} \\ 0 & , \text{otherwise} \end{cases} \quad \forall j, l, t, g \tag{3.10}$$

where D_{jl}^{tg} denotes the maximum number of group g users that travel from station j to l at time t with similar battery and time requirements. θ is the degree of demand variation. The authors proposed a mixed integer nonlinear programming (MINLP) model to maximize the profit while deciding on the pricing, vehicle fleet and personnel movements. A mixed integer convex programming is developed as a solution algorithm.

A similar demand function is discussed in Huang et al. (2020b), which compares operator-based and user-based (trip-based and station-based pricing) methods in a one-way electric carsharing system. All three schemes mentioned are modelled as MINLP models with varying demand elasticity functions. The price values in the elasticity functions are taken as fixed, similar pricing variables to p_{jl}^{tg} in 3.10 and a pricing function based on the rates for trips leaving/arriving at the stations for operator-based, trip-based pricing, and station-based pricing schemes, respectively. ϵ-optimal and iterated local search algorithms are applied to the rolling horizon approach as solution methods. Results show that all three schemes perform well in balancing the demand. The trip-based pricing scheme is more flexible than the station-based pricing scheme.

Huang et al. (2021) proposed a mixed integer nonlinear programming model to maximize the expected daily profit considering demand variations over the days and aimed to find the vehicle fleet size and prices on the routes. They used a similar elastic demand function to Xu et al. (2018) with a demand uncertainty associated with the daily demand variations. One distinct difference between Huang et al. (2021) and Xu et al. (2018) is that the former considers customers' changing stations when no vehicle is present at the origin station. Note that in Huang et al. (2021), even though the customer is diverted to a station that is not initially the trip's origin, the prices are not changed accordingly.

Jiao et al. (2020) studied pricing problem with EV fleet. Instead of using price-elasticity functions of the demand, they have employed a similar approach, service adoption rate. The service adoption rate is a probability function based on the customers' willingness to pay for the service. If the customers' expected price for the service is higher than what is offered, they accept the service. The study proposes a MINLP to solve the profit maximization problem considering service adoption rates and fully charged vehicles.

Pantuso (2022) proposed a mixed integer two-stage stochastic program in which relocation and pricing decisions are made. It is assumed that along with a minute-based base pricing , each route between any station pair (j, l) is associated with a drop-off fee that is determined before the beginning of each time period (i.e. 1 hour, morning-afternoon-evening periods). The company's pricing decisions are made under the assumption that a customer can have a set of options for travelling the target route, including public transportation. Each element in the option set yields a utility to the customer, among which he chooses the item with the highest utility. The model's stochasticity stems from the utility function's unknown components. The model is solved by the L-Shaped method and applied to the data set from Milan.

Similar to Pantuso (2022), Lu et al. (2021) studied pricing problem under different transport modes. However, instead of having multiple options to choose over carsharing, in Lu et al. (2021), users only have a private vehicle option assuming each user in the system owns one. In the study, a bilevel optimization model is proposed to find i) the fleet size, prices and relocations at the upper level and ii) users' private vehicle or carsharing travel modes. The bi-level model is transformed into a single-level model using the Karush-Kuhn-Tucker conditions and solved using a genetic algorithm.

Most pricing studies consider that users either agree on the price or quit the carsharing transport mode. Wang et al. (2021a), Wang et al. (2021b), and Wang and Ma (2019) considered that neighbouring stations' demand is correlated, and therefore, there is a need to consider demand shift between stations rather than assuming that demand would disappear.

Wang et al. (2021a) proposed a pricing study on incentives and surcharges applied to stations. The proposed two-level approach is concerned with carsharing transport mode selection probability in the first level, and departure/arrival station selection probability in the second level, given the origin and destination locations. A two-level nested model and a simulated annealing-based heuristic approach are presented.

Wang et al. (2021b) has integrated user-based relocations into pricing decision study, considering users' waiting, cancelling or shifting the demand. By conducting a survey, the price elasticity function is obtained. The proposed multi-objective optimization model aims to maximize the daily profit and minimize the failure rate (trip cancellation or alterations).

Wang and Ma (2019) also investigated the effects of station-based incentivization (or penalties) on demand. The authors developed a quadratic program which aims to keep the vehicle stock at the stations within a certain range and to find the pricing schemes for the stations.

Soppert et al. (2022) proposed a model that differentiates pricing based on the origin station and start time of the trip request. The assumption of origin-based pricing is consistent with practice, as most real-world systems do not require destination station to be specified. Similar to Monteiro et al. (2021), additional constraints are added to the model to ensure that trip requests are accepted when vehicles are available at the stations. Note that this is not considered in the rest of the relevant literature. It is generally assumed that the model can reject the trip request for more profitable future requests. The proposed model is proven to be NP-hard and solved through an approximate dynamic programming method.

Marecek et al. (2016) was considered a dynamic pricing scheme where prices are partially differentiated based on the distance to the closest parked vehicles in a free-floating system. It is aimed to have a system of evenly spread vehicles by charging more when there are vehicles in the vicinity. By defining an ″inconvenience″ function based on the distances to the surrounding vehicles, the cost of not capturing demand elsewhere is calculated.

Wu et al. (2021) provided a dynamic programming model for pricing and relocation problem in one-way EV systems facing random demand and electricity cost. An Input Convex Neural Network method has been proposed to solve the problem.

Lastly, Yang et al. (2022) considered pricing in the carsharing market where multiple carsharing companies operate. A multi-leader-follower model is proposed to maximize companies' (leaders) profit values and minimize the users' (followers) disutility (i.e. combination of price, total time, discomfort and possibility of rejection). Nash equilibrium and Stackelberg equilibrium are considered at the upper level among the companies and between the two levels at the bi-level model, respectively.

Table 3.7 provides a list of the studies that focus on optimization-based pricing in carsharing systems. Unlike user-based relocations, pricing studies are suitable for creating decision support systems for long-term strategic decisions such as location selection. However, there is not a sufficient number of studies that focus

Table 3.7: Summary of optimization focused pricing studies

Study	Carsharing Type	Personnel Movements	Demand Function	Aim of the Study	Methods Used
Jorge et al. (2015)	One-way	No	Fixed price elasticity value	Maximizing total daily profit	MINLP, Iterated Local Search
Ren et al. (2019)	One-way EV	No	Simple elastic behavior, similar to Jorge et al. (2015)	Profit maximization including profit generated from vehicle-grid integration services	MILP
Xu et al. (2018)	One-way EV	Yes	Logit-based elastic demand function	Profit maximization by finding EV fleet size, trip price, personnel schedule	MINLP, outer approximation algorithm
Huang et al. (2020b)	One-way EV	Comparison of with and without + pricing	Logit-based elastic demand function	Comparison of systems with operator-based relocation and pricing-based balancing	MINLP, rolling horizon, ϵ-optimal and iterated local search algorithm
Huang et al. (2021)	One-way	Yes	Logit-based elastic demand function	Long term pricing strategy	MILNP, two-stage stochastic programming, Gradient search algorithm
Jiao et al. (2020)	One-way EV	Yes	Service adoption rate	Profit maximization	MINLP, outer approximation algorithm
Pantuso (2022)	One-way	Yes	Demand based on utility value	Joint decisions on pricing and relocation activities	Two-stage stochastic IP, L-Shaped Method
Lu et al. (2021)	One-way	Yes	Logit-based elastic demand function	Joint decisions on pricing and relocation activities in carsharing considering private car option	Bilevel nonlinear mathematical model, genetic algorithm
Wang et al. (2021a)	One-way	No	Generalized nested logit	Profit maximization through differentiated pricing on stations	A two-level nested logit model
Wang et al. (2021b)	One-way EV	No	Survey-based demand function	Profit maximization and failure rate minimization	Back propagation neural network prediction, a multi objective MINLP
Wang and Ma (2019)	One-way	No	Similar to Waserhole and Jost (2016), S curve, reverse logistic function	Pricing approach to keep the vehicle stock at certain level	Quadratic programming
Soppert et al. (2022)	One-way	No	Based demand times origin-dest-time specific sensitivity factor	Profit maximization considering origin-based differentiated pricing	MILP, approximate dynamic programming
Marecek et al. (2016)	Free-floating	No	NA	To achieve a geographically well distributed system	
Yang et al. (2022)	One-way	Yes	Logit-based elastic demand function	Pricing and relocation decisions of competitor carsharing companies	Multi-leader-follower model
Wu et al. (2021)	One-way EV	Yes	Linear demand curve	Pricing and relocation decisions under demand quantity and electricity cost uncertainty	Dynamic programming model, Input Convex Neural Network method

on station location considering long-term pricing mechanisms. A very fruitful future direction would be aiming for station locations while considering the customer flows that stem from the cancelled station locations to the other stations. Clearly, this type of problem can be enhanced with charger location decisions. Next, logit-based behavioural modelling can be implemented in vehicle-to-grid studies, with further consideration of operator-based relocations.

Another future study direction is the pricing decisions of companies that involve more than one transportation system (such as carsharing and bike-sharing systems together). The route between any station pair can be covered by i) carsharing directly, ii) carsharing + alternative transportation system. Consumer behaviour can be analysed to understand the flow that chooses each pattern and modelled into the pricing decisions. One of the most important problems of optimization-based approaches is providing effective algorithms in terms of computational time and objective function. This is why effective heuristic algorithm studies will remain as future studies.

3.5 Concluding remarks

As carsharing systems are gaining importance as a sustainable urban transport mode and reaching more people, the operations in these systems are tempting researchers, especially from the operational research community. Clearly, rebalancing the vehicle stock across the stations is one aspect that makes these systems challenging to design. Therefore, the studies involving carsharing systems with imbalanced demand (in other words, one-way and free-floating) constitute 66% of the related literature Remane et al. (2016).

There are three ways to achieve a balanced system in carsharing, namely operator-based relocation, user-based relocation and pricing. This study focuses on the latter two, providing a categorization and summary. The previous sections have summarized the user-based and pricing studies regarding carsharing operations. Specifically, we have analysed the studies that have conducted surveys to understand customer travel behaviour when incentives and differentiated pricing are applied. Next, we provided a comprehensive review of data analytics application areas related to both pricing and user-based relocation. To explain in more detail, in the data analytics part of this study, we examined the effect of rule-based studies on the system's profitability, simulation-based studies, and region-determination studies in pricing. Finally, we provide an in-depth review of optimization-based studies. Even though this part could be reviewed under the data analytics umbrella term, it is also suitable to examine in a separate section since 52% of the related literature falls into this category.

Based on the review we conducted, we have highlighted several research gaps to shed light on the possible future research directions in each section. Here we provide more general research directions that could be implemented in many of the sections reviewed in this study.

(i) Use of pricing and user-based relocation decisions together: Although certain studies used pricing and user-based decisions through optimization-focused approaches, agent-based simulation studies could implement data arrival processes by pricing strategies and still divert the necessary flow by rule-based user-based relocations.

(ii) Mixed fleet use: Users have different attitudes towards different vehicle types. Carsharing companies often offer different sizes/brands of vehicles to their users. Each vehicle is mainly associated with a minute-based price dependent on vehicle purchasing and gasoline costs (or electricity costs). This price calculation can be enhanced by user-behavioural modelling and demand balancing purposes.

(iii) Consideration of different transport modes: The availability of different transport modes in the vicinity of the stations affects the demand distribution of the carsharing systems. Survey-based user behaviour analysis can show possible positive/negative effects of available transport modes. Alternatively, pricing or user-based incentive calculation studies may consider incorporating collaborations with public transport services, such as providing a free or discounted ticket with coupon to public transport to the vehicles parked close to railway/subway stations.

(iv) Incentive calculations in different fleet types: The majority of the literature in the pricing/user-based relocation studies focuses on non-autonomous vehicles. One possible future direction is pricing and incentive calculations considering autonomous vehicles. Such vehicles do not require a driver and position themselves automatically. By providing safety separators inside the vehicle, more than one customer may be served at a time, which results in new research areas to explore. In stackable fleet systems, more than one vehicle can be driven to a desired station for balancing purposes. Although rule-based studies are already conducted considering stackable fleets, there is no optimization-based pricing study that deals with 1) operations of the vehicles, 2) charging decisions of the vehicle train.

Chapter 4

User-based relocations in one-way carsharing systems with consideration of flexible demands with user acceptance probabilities

Abstract

One-way carsharing systems allow users to leave vehicles at stations that are not necessarily the same as the pick-up stations. Due to the demand asymmetricities, one-way carsharing systems experience vehicle supply and trip demand imbalances at stations during their operations. Carsharing companies often hire personnel to relocate vehicles to restore the vehicle supply-demand among the stations. These relocation operations constitute a significant portion of the operational costs. One other way to reduce vehicle imbalance in the system is by interfering with the vehicle configuration at the stations by providing alternative routes to the users.

In this study, we present a reservation-decision framework to determine the alternative routes and incentive amounts to be offered to the users. At each trip request, the system either offers a route (including the original request) to the user or rejects the user considering the probability of the user's acceptance and rejection rates. We propose a mixed integer linear programming model to determine the offer that maximize the expected profit while considering the acceptance probabilities of the offers. Since the proposed MILP is computationally intractable, we present two heuristic algorithms that reduce the number of created relocation variables. The first one creates all relocation variables based on the need for the vehicles or parking spots. The second heuristic creates relocation variables based on a graph-spanner-based network. We have tested both of the algorithms using real-world data from Nice, France. The results show that both of the proposed heuristics work well compared to the MILP model. Our analysis indicates that systems using both of the operator and user-based relocation types at the same time will yield a noteworthy increase in profit.

keywords : One-way carsharing, user-based relocation, imbalanced demand, spatial and temporal flexibility, consumer behaviour, incentives

4.1 Introduction

The sharing economy is gaining significant importance in people's daily lives, from transport to video streaming. It is expected to reach a value of $335 billion by 2025 (Bothun et al., 2015). Two key drivers of the rapid expansion of the sharing economy are changing consumer behaviour and mobile/electronic devices (Puschmann and Alt, 2016). These key drivers are also the motives behind the growth in carsharing, a sector in the sharing economy.

Carsharing is a car rental system that provides temporary access to a fleet of vehicles. Just as the sharing economy focuses on underutilized assets to increase sustainability (Hossain, 2020), carsharing systems help to build sustainable urban mobility by increasing vehicle usage time and providing effective use of vehicles. Carsharing systems reduce car ownership's adverse effects, such as pollution, traffic congestion, and excessive energy consumption. Furthermore, these systems provide the comfort of private vehicle use to its users without owning one. They provide an economically viable transportation mode compared to owning a vehicle that is driven less than 6,000 miles/year on the average (Litman, 2000). As of 2018, the number of vehicles and registered users have reached to 198.4k and 31.8 million, respectively (Shaheen and Cohen, 2020).

Carsharing systems can be categorized into two systems depending on the drop-off stations. The first category, round trip systems, requires the users to complete their trip at the origin station. The second category, one-way systems, allows users to park at any station (station-based one-way systems) or anywhere specified by the company (free-floating systems). Since one-way systems can attract more people compared to round trip systems, the market share of one-way systems is increasing. The number of registered users in one-way systems has boosted by 238% in only two years (from 2016 to 2018) and accounted for almost 50% of the overall membership in the industry. The fleets deployed in one-way systems have also increased by 103% in order to capture the rise in the demand (Shaheen and Cohen, 2020).

Managing the operations of one-way systems is cumbersome due to the imbalance between vehicle supply and demand at the stations. When a vehicle is picked up from a station, it automatically affects the vehicle and parking availabilities at that station and destination station, respectively (Banerjee et al., 2022). Limited vehicle supply and parking spots affect the service level. Therefore, one-way carsharing companies often hire personnel to relocate vehicles to ensure that vehicles are available at the right time at the right place to meet customers' demand requests. In general, two-types of relocation activities can increase the system's level of service. The first is providing a vehicle to under-supplied stations with high demand, and the second is emptying a parking spot of an over-supplied station where the demand is low. To perform a relocation activity, personnel must arrive at the under/oversupplied station on foot (or by using public transport, foldable bikes, e.t.c.), and continue on their way by driving the relevant vehicle. Due to the relocation operations performed by the operators, the system faces higher operation costs and lessened vehicle utilization rates (Boyacı and Zografos, 2019). Lack of relocation can lead to larger fleets and, therefore, higher investment costs. Another way to rebalance vehicle supply and demand heterogeneity is through user involvement in the relocation operations. By incentivizing the users to make changes in their original trip requests, the relocation cost can be reduced, and the system's profitability can be increased. The spatial and temporal flexibility of the demand allows the system to rearrange the relocation needs and realign the vehicles across time and space.

In this paper, we present a decision-framework for the operational level decisions at a one-way carsharing system, where spatio-temporal flexible demands are taken into consideration.

The remainder of the paper is as follows: In Section 4.2, related literature on user-based relocation studies is presented. Then, in Section 4.3, we introduce the reservation-decision framework and provide a MILP to solve the decision problem where the user-based relocation and discount values are decided. Since we work on operational level decisions which need to be solved in a timely manner, we introduce time-efficient heuristic algorithms in Section 4.3.2. Next, we apply the proposed methods to a real-world system and present the results in Section 4.4. Finally, we provide concluding remarks and future directions in Section 4.5.

4.2 Related literature

Strategic, tactical and operational decisions in carsharing systems have captured the attention of the operational research community over the past two decades. Readers are referred to studies Jorge and Correia (2013), Laporte et al. (2018), Yao et al. (2022), Nansubuga and Kowalkowski (2021) for more information on the literature related to carsharing systems. In accordance with the content of this paper, we have limited this literature review section to a brief discussion on operational-level decisions and a more detailed analysis on user-based relocations where flexible demand is considered. While operational-level decisions are concerned with personnel and vehicle schedules, studies involving user-based relocation and flexibility work on how users' requests are shaped and how the associated price is calculated.

Nourinejad et al. (2015) proposed two integrated multi-travelling salesman formulations to solve vehicle and staff relocations simultaneously. Since all demand is assumed to be satisfied, cost minimization is applied in the model. Boyacı et al. (2017) studied vehicle and personnel relocations at electric one-way carsharing systems while maximizing the number of trips served and minimizing the relocation cost. In addition to a multi-objective MILP model, a simulator is proposed for feasibility concerns of vehicle battery charge levels. Gambella et al. (2018) addressed the operational level problem in one-way electric carsharing systems and proposed a MILP that finds both personnel and vehicle schedules while keeping track of the battery level of each vehicle. Bruglieri and Colorni (2014) considers personnel schedule at a one-way electric carsharing system and formulates it as a paired pick-up and delivery problem with time windows. Please note that, Bruglieri and Colorni (2014) differs from Nourinejad et al. (2015), Boyacı et al. (2017) and Gambella et al. (2018) in the sense that in Bruglieri and Colorni (2014), it is assumed that vehicle routes are already known and scheduled.

User-based relocation is first introduced by Barth et al. (2004) for a carsharing system where the vehicles can be connected via towbars (similar to stackable vehicles). Instead of spatio-temporal flexibility in demand, trip-joining and trip-splitting flexibility are considered. Trip-joining is offered to users who have similar trip requests and want to move from a high-demand station to a low-demand station. Trip-splitting is offered to multi-passenger user groups who want to travel from low-demand stations to high-demand stations. Simulations show that with a high user-participation rate, the number of relocations could be reduced by 42%. Still, it would be challenging to implement such policies to large systems, as it requires ”trust” in-between customers (Boyaci et al., 2015).

Correia et al. (2014) aimed to assess the effects of user-flexibility and information provided to the users on the profitability in carsharing systems. They have considered three types of scenarios where users are called inflexible (only walk to the closest station, and pick up a vehicle if there is any), flexible (if do not find any available vehicle at the closest, may check the next 2 closest stations) and flexible with information (have the vehicle stock information on the closest 3 stations). By applying the proposed system to data from Lisbon, it is suggested that flexible users with information could increase the profit significantly.

Boyacı and Zografos (2019) investigated the effects of spatial and temporal flexibility in one-way electric carsharing systems. It is assumed that the users are flexible and would accept any offer provided by the system. In the proposed framework, first, the stations are clustered through a MILP. Then, personnel and vehicle flows are determined by an optimization model that takes the relocation flows from(to) clusters to(from) stations. Please note that creating relocation arcs through clusters reduces the size of the generated variables. Finally, a simulator module is used to detect any infeasible pattern in terms of vehicle battery level and to feed the optimization module with additional constraints. The proposed framework is applied to a system in Nice, France, and the results indicate that spatial flexibility has a more substantial effect than temporal flexibility.

A recent study, Lai et al. (2022), considers a rolling horizon decision framework for one-way electric carsharing systems where users are considered flexible and accept alternative offers provided by the system. At each time interval (e.g. 15 minutes), the decisions are optimized with the new demand requests received. The model tracks the battery level of each vehicle, similar to what has been proposed in Gambella et al. (2018). In order to reduce the size of the relocation arcs, a clustering approach is employed. For large-sized problems, a greedy heuristic algorithm is introduced.

Another recent study, Zhang et al. (2022) considers flexibility as start time and origin station changes for demand requests. Unlike Boyacı and Zografos (2019), Correia et al. (2014) and Lai et al. (2022), it is considered that users might reject the offer. Users are divided into classes with different acceptance rates of the offers based on their demographic characteristics. In the proposed framework, the demand is treated as a whole for the operational period (i.e. daily). Using the acceptance/rejection probabilities of each class, a MILP model is proposed to maximize the profit. Additionally, equity constraints are added to the model to achieve fair offerings on any OD pairs in the system. Without fair offering constraints, the system may serve more demand through certain stations, which causes imbalances.

Acceptance rates are also considered in the studies Febbraro et al. (2012) and Febbraro et al. (2019). Acceptance rates are calculated using a binomial logit model that depends on the discount value and the distance between the destination stations of the original trip request and the offer. In Febbraro et al. (2012), a discrete event simulator is designed for one-way carsharing systems. The relocation event is optimized through a MILP, providing users with destination offers. Although a binomial logit model is proposed to calculate the acceptance rates of the offers, integrating the acceptance rates and corresponding discount values to the simulator is considered in Febbraro et al. (2019). Febbraro et al. (2019) used the same simulator, and proposed a two-stage optimization model where the first stage is an extended model that is provided in Febbraro et al. (2012). In the second stage, the discount value of the offer is found through a mathematical programming model.

Different from the studies on user-based relocation mentioned earlier, Ströhle et al. (2019) considered user-flexibility in round-trip systems. Spatial flexibility is provided by the change so that the origin and destination stations remain identical. An integer programming model is proposed to reduce the fleet size. The proposed model is applied to real-world data from Germany and concluded that spatial flexibility has a higher potential to generate additional profit compared to temporal flexibility.

This paper addresses the operational level decision problem in one-way car-sharing systems where users are offered alternative routes that yield a more profitable system. The proposed framework optimizes decisions at each trip request dynamically. At each request, there are many decisions, such as staff schedules, vehicle schedules and trip offers, that should be made by the operator. We aim to provide an efficient algorithm and an algorithm that offers a benchmark for small and large-sized problems, respectively.

The contribution of this paper is two-fold;

- A new MILP model that aims to find the offer (both spatio-temporal changes in the original demand and the price discount) with the highest expected profit to be presented to the user. Unlike Boyacı and Zografos (2019) and Lai et al. (2022), we assume that users do not always accept the offer. User acceptance rate of the offer is dependent on how much the offer spatio-temporally differs from the original request. In order to increase the level of service, we considered operator-based and user-based relocations simultaneously. Even though operator-based relocations result in additional complexity and decisions in the system, the generated profit and the number of served customers are higher compared to systems without operator-based relocations. Please note that in real-life systems, the operator may ask the users to specify the time of the request as distinctive time points (e.g. 15-minute time intervals, 14:00, 14:15, ..) in the time horizon. However, the related literature also treats operator-based relocations as taking actions at these time points. In reality, relocations can start/end anytime, without restriction. The proposed MILP manages the trips and relocations in different time-space networks to present a more realistic representation.

- Math-heuristic algorithms that provide near-optimal solutions. We present a novel math-heuristic approach that aims to reduce the size of the problem without compromising solution quality. The proposed heuristic is comprised of i) a multiplicative graph spanner and ii) shortest path edges that connect any station pair. Instead of a direct arc, we create paths through additional stations with a total distance always guaranteed to be below a multiplicative constant. Please note that in reality, when an operator's schedule is changed, they are directed to the new arrival station without completing the previous task. Thanks to our new heuristic, we can capture and evaluate this feature of real-life systems by having additional stops on personnels' movement paths
.

4.3 Methodology

In this paper, we consider the operational level decision problem at a station-based one-way carsharing system where the users reserve a vehicle during the time interval the system operates (i.e. a day). Each station is assumed to have a finite capacity of parking spots. Only stations are allowed to be used for parking purposes. Operational time is discretized into $|T|$ equal-length time intervals. $T_{trip} \in T$ denotes the time interval set where users are allowed to pick up or drop off a vehicle. This assumption is consistent with the real-world systems where users are required to select the start/end time of their trips from a drop-down menu. For any trip request $i \in I$, users need to identify origin-destination stations, start and end times of the trip. Each trip $i \in I$ is represented by a tuple (origin(i), destination(i), start(i), end(i), booking(i)), where start(i), end(i) $\in T_{trip}$. Booking time of trip i (booking(i)) is recorded by the system and coupled with i. Note that, booking(i) is not necessarily an element of the set T_{trip}. At the beginning of each day, it is assumed that a fixed number of vehicles are positioned at each station. Each trip has an associated price which is dependent on the trip distance. Except for the trips, the vehicles can only be moved for relocation operations by dedicated personnel. Each vehicle can be only in one state at each time interval; waiting at a station, under relocation or serving a trip request. Personnel can also be in one of the three states: waiting at a station, moving from a station to another station to reach a vehicle, driving a vehicle for relocation.

The flowchart of the decision process in the reservation system is illustrated in Figure 4.1. First, the users enter the system individually. Upon entering, each user specifies the desired trip request. For each trip request i^*, a set of neighboring offers (N_i^*) is formed. In the framework, the optimization module finds the offer i and discount value that generates the highest expected profit considering acceptance and rejection rates. The initial trip request i^* is an element of N_i^*, and optimization module includes i^* with a discount value of zero. Note that with

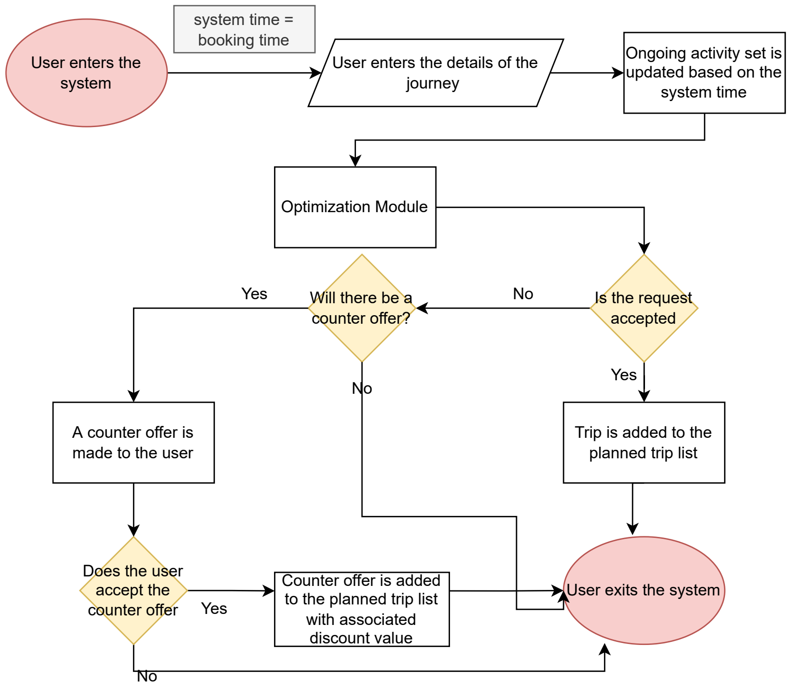

Figure 4.1: Flowchart of proposed reservation system

each trip request, the system optimizes future events such as where and when to relocate vehicles, where and when to make personnel move and drive while at the same time ensuring that the accepted trips are realized in the future.

4.3.1 Mathematical model

In the optimization module, a MILP is employed to find the offer and its associated discount value that maximizes the expected profit. In the next section, we present the parameters and variables used in the model and the model formulation. At each iteration, the model takes the output of the previous iteration and uses it as a parameter for the upcoming iteration.

We provide the mathematical model of a single iteration, where the user makes a request of trip i^* .

4.3.1.1 Sets and indices

$i \in I$	trips/offers
N_{i^*}	Neighbouring offer set for request i^*

ATL accepted trip list

$j, l \in J$ nodes (stations)

$t \in T$ time intervals

T_{iter} time intervals for the upcoming iteration, $T_{iter} = \{t \in T | t > booking(i^*)\}$

T_{trip} time intervals that trips can start/end, $T_{Trip} \subset T_{iter}$

$q \in Q$ discount value for the counter offer

ORL ongoing relocation list

OWL ongoing personnels' movement relocation list

ODL ongoing personnels' driving relocation list

4.3.1.2 Parameters

origin(i)/destination(i) origin/destination station of trip/offer i

start(i)/end(i) start/end time interval of trip/offer i, start(i), end(i) $\in T_{trip}$

booking(i) booking time interval of trip i, booking(i) $\in T$

Pr_{iq} acceptance probability of offer i with discount value q

TP_i price of trip i, $i \in ATL \cup \{i^*\}$

VC_{jl} vehicle relocation cost from station j to station l

MC_{jl} personnel moving cost from station j to station l

DC_{jl} personnel driving cost from station j to station l

DD_{jl} driving distance between stations j to l in time intervals

WD_{jl} walking distance between stations j to l in time intervals

IV_j number of vehicles available at station j at the beginning of time interval booking(i^*)

IP_j number of personnel available at station j at the beginning of time interval booking(i^*)

LastPr profit value found from the last iteration

RC$_{i^*}$ realized vehicle relocation, personnel driving and movement cost at $booking(i^*)$

CapV vehicle capacity

A_i ongoing and accepted trip flows

B^t_{jl} ongoing vehicle relocation flows from station j to l at the beginning of time interval $booking(i^*)$

C^t_{jl} ongoing personnel movement relocation flows from station j to l at the beginning of time interval $booking(i^*)$

D^t_{jl} ongoing personnel driving relocation flows from station j to l at the beginning of time interval $booking(i^*)$

4.3.1.3 Variables

x_{iq} binary variable, takes value 1, if neighbouring trip i with discount value q is offered to the user; 0, otherwise

r^t_{jl} number of vehicles relocated from station j to l starting at time interval t

d^t_j number of vehicles that are waiting at station j starting at time interval t

w^t_j number of personnel that are waiting at station j starting at time interval t

u^t_{jl} number of personnel that are moving from station j to l starting at time interval t

$\bar{u}^t_{jl}$ number of personnel that are driving (or being driven in) a vehicle from station j to l starting at time interval t

4.3.1.4 Mathematical model that provides the most profitable offer at each iteration

$$\max \sum_{i \in N_{i^*}, q} \Bigg[x_{iq} \Big[\mathrm{Pr}_{iq} \Big((\mathrm{TP}_{i^*} - q) - \sum_{j,l,t} \big(\mathrm{VC}_{jl}\, r^t_{jl} + \mathrm{MC}_{jl}\, u^t_{jl} + \mathrm{DC}_{jl}\, \bar{u}^t_{jl} \big) + \sum_{i \in ATL} TP_i - \mathrm{RC}_{i^*} \Big) + (1 - \mathrm{Pr}_{iq}) \mathrm{LastPr} \Big] \Bigg] \tag{4.1}$$

subject to

$$\sum_{\substack{(i \in ATL): \\ \mathrm{Dest}(i)=j \\ \mathrm{End}(i)=t}} A_i + \sum_{((l,j,t-DD_{lj}) \in ORL)} B^{t-DD_{lj}}_{lj} + \sum_l r^{t-DD_{lj}}_{lj} + \sum_{\substack{(q,i \in N_{i*}): \\ \mathrm{Dest}(i)=j \\ \mathrm{End}(i)=t}} x_{iq} + d^{t-1}_j = \sum_{\substack{(i \in ATL): \\ \mathrm{Origin}(i)=j \\ \mathrm{Start}(i)=t}} A_i + \sum_l r^t_{jl} + \sum_{\substack{(q,i \in N_{i*}): \\ \mathrm{Origin}(i)=j \\ \mathrm{Start}(i)=t}} x_{iq} + d^t_j \quad \forall j, t \in T_{trip} \tag{4.2}$$

$$\sum_{l} r_{lj}^{t-DD_{lj}} + d_j^{t-1} + \sum_{((l,j,t-DD_{lj})\in ORL)} B_{lj}^{t-DD_{lj}} = \sum_{l} r_{jl}^{t} + d_j^{t} \qquad \forall j, t \in T_{iter} - T_{trip} \quad (4.3)$$

$$d_j^t \leq CapS_j \qquad \forall j, t \in T_{iter} \quad (4.4)$$

$$\sum_{(q,i\in N_{i*})} x_{iq} \leq 1 \qquad (4.5)$$

$$\sum_{l} u_{lj}^{(t-WD_{lj})} + \sum_{l} \bar{u}_{lj}^{(t-DD_{lj})} + \sum_{((l,j,t-WD_{lj})\in OWL)} C_{lj}^{t-WD_{lj}} + \sum_{((l,j,t-DD_{lj})\in ODL)} D_{lj}^{t-DD_{lj}} + w_j^{t-1} = \sum_{l} u_{jl}^{t} + \sum_{l} \bar{u}_{jl}^{t} + w_j^{t} \qquad \forall j, t \in T_{iter} \quad (4.6)$$

$$r_{jl}^t \leq \bar{u}_{jl}^t \leq CapV\, r_{jl}^t \qquad \forall j, l, t \in T_{iter} \quad (4.7)$$

$$d_j^{\text{booking}(i^*)} = IV_j \qquad \forall j \quad (4.8)$$

$$w_j^{\text{booking}(i^*)} = IP_j \qquad \forall j \quad (4.9)$$

$$x_{iq} \in \{0,1\} \qquad \forall q \in P, i \in N_{i*} \quad (4.10)$$

$$r_{jl}^t, d_j^t, w_j^t, u_{jl}^t, \bar{u}_{jl}^t \in \mathbb{Z}^+ \cup \{0\} \qquad \forall j, l, t \in T_{iter} \quad (4.11)$$

Objective function (4.1) maximizes the expected profit value over the neighbouring offers for the trip request i^*. The objective function gives the expected profit value of the system when the selected offer is made to the user. The component that calculates the profit when the user accepts the selected offer is the summation of the revenue of the offer and realized profit (revenue coming from accepted trips minus realized cost) minus the future costs found in this iteration (vehicle relocation and personnel cost). If the user does not accept the offer, the last iteration's profit remains the same. Note that realized profit and last iteration's profit are not the same since the last iteration's profit includes future costs found at the previous iteration.

Constraints (4.2) and (4.3) are the flow balance constraints for the vehicles for the time intervals in which trips are allowed and not allowed, respectively.

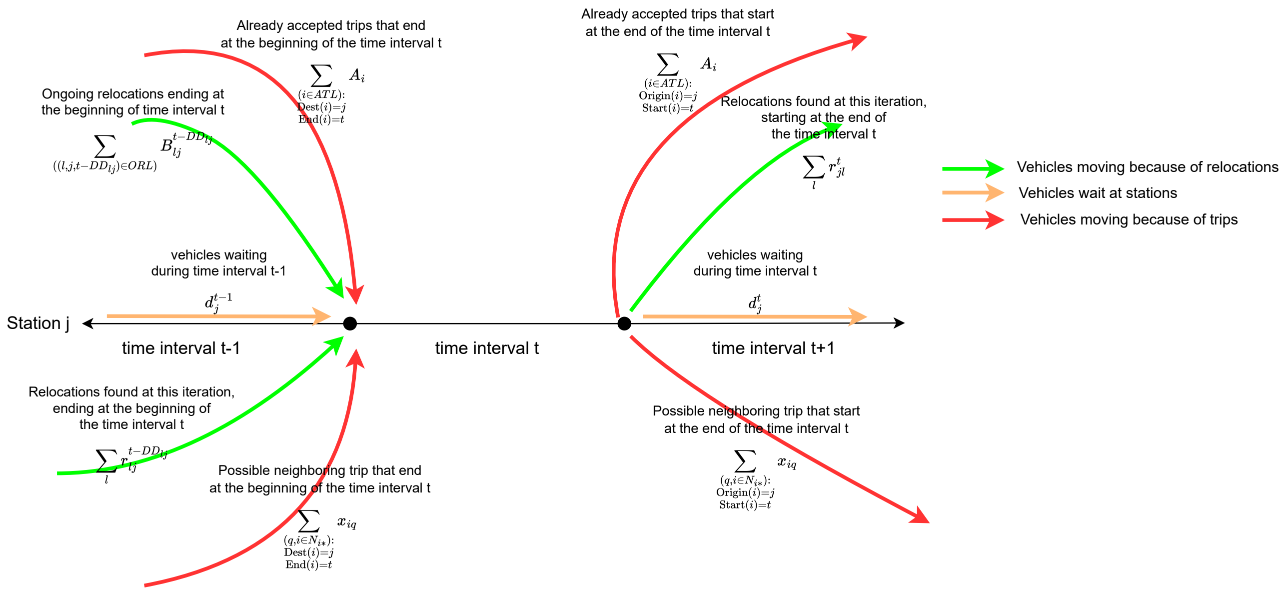

Figure 4.2: Flow diagram of vehicles at station j at time interval t

The flow diagram of the vehicles at station j at time interval t is illustrated in Figure 4.2. The figure shows the vehicle flow relationship of Constraints (4.2) where $t \in T_{trip}$. It can also provide the vehicle flows pointed out in Constraints (4.3) for the time intervals $t \in T_{iter}$ by removing the trip arcs, i.e. red arcs given in the figure. Constraints (4.4) ensure that the vehicles parked at a station cannot exceed the station's capacity. Constraint (4.5) is to offer at most one trip to the user. Constraints (4.6) are similar to Constraints (4.2) and (4.3), ensuring the personnel flow balance in the network. For constraints (4.2), (4.3) and (4.6), the vehicles/personnel that appear on the network due to the ongoing events at time booking(i^*) are included in the model. Constraints (4.7) guarantee that each relocation is done by a personnel, and the number of personnel on a relocation route is bounded by the vehicle seater capacity of the vehicles on that arc. Constraints (4.8) and (4.9) state the number of vehicles and personnel waiting at station j at the beginning of the iteration, respectively.

Multiplication of variables in the objective function (4.1) makes the model nonlinear. In order to linearize the objective function, we introduce variables:

y_{iq} ; denoting the expected profit if trip $i \in N_i^*$ is offered to the customer with price q, 0 otherwise.

Then the model is replaced by ;

$$\max \sum_{i \in N_{i^*}, q} y_{iq} \tag{4.12}$$

subject to

$$y_{iq} \leq \text{Pr}_{iq}\Big((\text{TP}_{i^*} - q)x_{iq} - \textstyle\sum_{j,l,t}\left(\text{VC}_{jl}\, r_{jl}^t + \text{MC}_{jl}\, u_{jl}^t + \text{DC}_{jl}\, \bar{u}_{jl}^t\right) + \sum_{i \in ATL} TR_i - \text{RC}_{i^*}\Big)$$
$$+ (1 - \text{Pr}_{iq})\text{LastPr} \qquad \forall q \in P, i \in N_{i*} \tag{4.13}$$

$$y_{iq} \leq M\, x_{iq} \qquad \forall q \in P, i \in N_{i*} \tag{4.14}$$

(4.2), (4.3), (4.4), (4.5), (4.6), (4.7), (4.8), (4.9), (4.10), (4.11)

$$y_{iq} \in \mathbb{R} \qquad \forall q \in P, i \in N_{i*} \tag{4.15}$$

Objective function (4.12) is equivalent of (4.1) when Constraints (4.13) and (4.14) are included in the model. Note that constraints (4.13) define the variable y_{iq}, and constraints (4.14) limit y_{iq} to have at most one value greater than zero.

4.3.1.5 Probability function

In this study, each user individually decides to accept or reject the offer provided by the system. For a user n, selecting from the choice set $C_n = \{a, b\}$ where a and b denotes accepting and rejecting the offer, respectively, the probability of choosing to accept the offer is $P_n(a) = 1 - P_n(b)$. There are many factors that influence the user's decision to accept the offer, such as distance to the original travel request, discount amount, weather, road conditions, demographic profile of the user. The relation between binary responses (a, b) and explanatory variables (affecting factors) can be defined by binomial logit functions (Trueck and Svetlozar, 2009). In binomial logit functions, probability $P_n(a)$ is equal to the value of the logistic function of linear combination of the explanatory variables;

$$P_n(a) = 1 \Big/ \left(1 + e^{-(\beta_0+\beta_1\alpha_1+\beta_2\alpha_2+\ldots+\beta_k\alpha_k)}\right) \tag{4.16}$$

where β_i and α_i $\forall 0 \leq i \leq k$ denotes the regression coefficients and explanatory variables, respectively.

In our study, we consider the decision affecting factors as the discount value and the spatio-temporal distance to the original travel demand. We propose the probability of user n with an initial trip request i^* accepting the provided offer i with a discount value q is given by Eq.(4.16) where k=5, $\alpha_1 = WD_{\text{origin}(i^*),\text{origin}(i)}$, $\alpha_2 = WD_{\text{dest}(i^*),\text{dest}(i)}$, $\alpha_3 = |\text{start}(i^*) - \text{start}(i)|$, $\alpha_4 = |\text{end}(i^*) - \text{end}(i)|$, and $\alpha_5 = q$. Please note that the model provided in Section 4.3.1.4 takes P_{iq} values as parameters provided (calculated) before the relevant iteration. When the system is put into use sufficiently long enough, data gathered through customer preferences can be used to determine β_i values.

4.3.2 Heuristic Approach 1: Relocation Restriction

The mathematical model provided in Section 4.3.1.4 is computationally intractable due to the number of movement, driving relocation variables of personnel and relocation variables of vehicles created at each iteration. Providing a time-efficient algorithm with a high-quality solution is crucial in operational-level problems. In order to reduce the size of the relocation variables created, we adopt the "Relocation Restriction" heuristic proposed in Bekli et al. (2021) to our study.

Instead of creating all the relocation arcs for any station pair and time interval, the relocation restriction heuristic focuses on the relocation needs in the system, and creates relocation arcs accordingly. These created arcs either bring a vehicle to a trip's origin station at the start time of the trip (or within a certain time gap) or empty a spot of the destination station of the trip at the end-time (or within a certain time gap). The heuristic algorithm does not create movement relocation arcs of personnel since the study Bekli et al. (2021) does not consider personnel schedules. This is why, in our study, we adapt the mentioned heuristic algorithm by creating arcs for movement relocation variables.

The pseudo code for the relocation restriction algorithm is presented in Algorithm (1) where VR, PD and PW sets denote vehicle relocation, personnel driving, and personnel movement arcs with tuple (node, node, time).

Algorithm 1 Relocation Restriction Heuristic

```
Result: VR, PD, PW
VR, PD, PW ← ∅
foreach i ∈ N_{i*} do
    for m ← 0 to timegap do
        foreach j ≠ origin(i) do
            Create arc (j, origin(i), start(i)-timegap)
            VR ← VR ∪ {arc}                          ▷ vehicle relocation arcs
            PD ← PD ∪ {arc}                          ▷ personnel driving arcs
            foreach l ≠ j do
                Create arc2 (l, j, start(i)-timegap - WD_{lj})
                PW ← PW ∪ {arc2}                     ▷ personnel movement arcs
            end
        end
    end
end
```

VR, PD, PW inserted to the model proposed in Section 4.3.1.4 such that r^t_{jl}, u^t_{jl}, $\bar{u}^t_{jl}$ are defined for $(j, l, t) \in VR, PD, PW$, respectively.

4.3.3 Heuristic Approach 2: Graph spanner based heuristic algorithm

In the carsharing literature, it is assumed that once activities begin, they will necessarily end. This assumption does not match reality, considering the long duration of personnel's movement paths. Each new trip request might alter the schedule of the vehicles/personnel. By taking into account the live location of personnel (and/or vehicle), the previous schedule might be cancelled, and a new route is generated. The carsharing literature disregards this cost-reduction action and assumes that any personnel (and/or vehicle) would follow the given schedule until the ongoing action is completed. In this section, we provide an algorithm to investigate the effects of interrupting necessary movement paths to have a more profitable system. To be able to do that, we propose combining the Relocation Restriction algorithm with generating movement arcs by employing graph spanners.

A graph spanner is defined as a connected subgraph in a graph G, where the shortest path between any given nodes (a, b), sp_{ab}, is within a certain error range (additively, or multiplicatively) (Ahmed et al., 2020). For given t and β values, and direct$_{ab}$ denoting the edge distance of nodes a and b, the inequalities (17) or (18) must hold based on the types of the graph spanner.

$$sp_{ab} \leq t \times direct_{ab} \qquad \forall a, b \text{ (for multiplicative, i.e. t-spanner)} \tag{4.17}$$

$$sp_{ab} \leq direct_{ab} + \beta \qquad \forall a, b \text{ (for additive)} \tag{4.18}$$

The additive method might produce shortest paths with a high per cent error rate for node pairs with short direct distances. Therefore, the Relocation Restriction algorithm is combined with a multiplicative graph spanner.

In the proposed algorithm, we assume that personnel at node j is reachable from any node l, but the path between j and l does not need to be the direct edge (j, l), it could span multiple edges (i.e. connected graph). This algorithm aims to provide edges to the graph that generates a connected graph while keeping the distortion in the path length within a certain range. By this method, we create personnel's movement relocation paths that are geographically close to the direct paths for any node pair and still have stops along the way to provide flexibility in adopting upcoming schedules. This is why, we propose to create the connected graph by graph spanners.

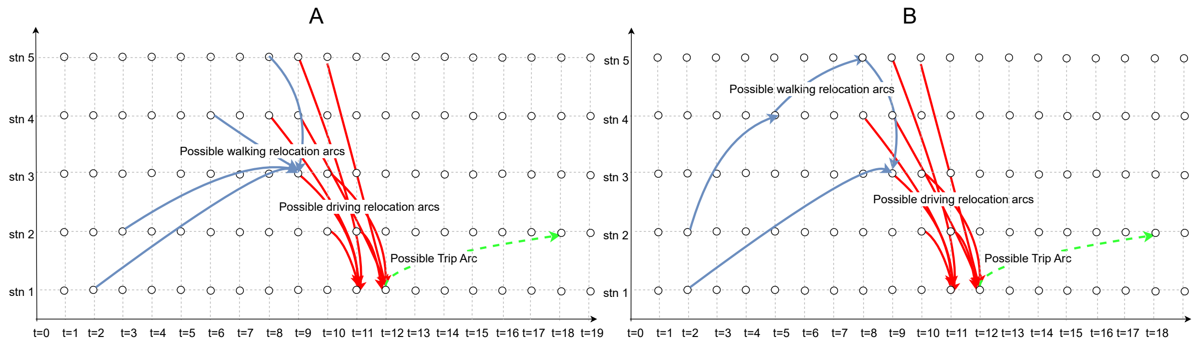

Figure 4.3: Illustration of Relocation Restriction (A) and Graph Spanner Based heuristics (B)

Figure 4.3 illustrates both of the heuristic approaches presented in this study. In Figure 4.3, a trip is requested by a customer. Out of many neighbouring trips in the neighbouring trip request set, only one possible trip arc from Station 1 to Station 2, starting at t=12 and ending at the beginning of t=18 has been shown. In order to provide a vehicle to station 1 either at t=11 or t=12, possible relocation arcs are created serving from Station 2, 3, 4 and 5. To demonstrate the possible movement arcs created, we only consider the relocation arc from Station 3 to Station 1 starting at t=9. The relocation restriction algorithm creates all the direct arcs to Station 3 at time 9 from all the other stations. However, the graph spanner-based heuristic algorithm considers personnel movement through a set of stations to arrive at a station with a vehicle. For instance, instead of moving directly from Station 2 to

Station 3, the operator will start from Station 2, then move through Stations 4 and 5, and arrive at Station 3. The graph spanner-based heuristic algorithm ensures that the journey from Station 2 to Station 3 is at most t times higher than the direct edge between Station 2 and 3.

We adopted the greedy algorithm proposed in Althöfer et al. (1993) to form the graph spanner. During our tests, providing a warm start of Voronoi graphs yielded graphs spanners with fewer edges. To form a t-spanner graph G with vertice set V and edge set E, $G = (V, E)$, first, a connected voronoi graph $G' = (V, E_v)$ is provided. Let E_{vl} denote the subset of E_v with shortest paths that have error rate greater than t.

$$sp_{ab} > t \times direct_{ab} \qquad \forall (a, b) \in E_{vl} \tag{4.19}$$

The pseudo code for the greedy algorithm with a warm start of Voronoi graph is demonstrated in Algorithm (2).

Algorithm 2 Graph Spanner - Greedy Heuristic with a warm start

Result: $G = (V, E)$

$E \leftarrow E_v$

foreach $(a, b) \in E_{vl}$ *in non-decreasing order* **do**

  if $sp_{ab} > t \times direct_{ab}$ **then**

    $E \leftarrow E \cup (a, b)$

    Update E_{vl} by applying dijkstra(a, b) $\forall a, b \in E$

  end

end

Note that the function dijkstra(a, b) finds the shortest path between vertices a and b using Dijkstra's algorithm. The graph $G = (V, E)$ found in Algorithm (2) is used for creating personnel's movement relocation paths in the mathematical model. Driving relocation and vehicle relocation paths are created by applying Algorithm (1).

4.4 Computational results

We applied the reservation system (both using the mathematical model and the heuristic algorithms) to a real-world system, Auto Bleue, which formerly operated in Nice, France. Auto Bleue was a station-based one-way carsharing company with 60 stations scattered in Nice. Figure 4.4 illustrates the map of the city and the stations. We used realized rental data with information on origin/destination stations, start and end times. Booking time is not available for the rental information. This is why we have generated booking times for any trip request i^*, booking(i^*), by assigning a random time within the interval of (start of the day, start of the trip - 30 minutes). Note that the reservation system provided in this study considers advanced reservations with at least a 30-minute gap between the booking time and the start time of the trip request. It differs from the systems where demand data is known a priori (generally a day before the operations). In our proposed reservation system, the data is revealed gradually. Therefore, it is necessary to perform the corresponding action for the system repeatedly. Last-minute reservations (walk-in customers) are not a part of this study.

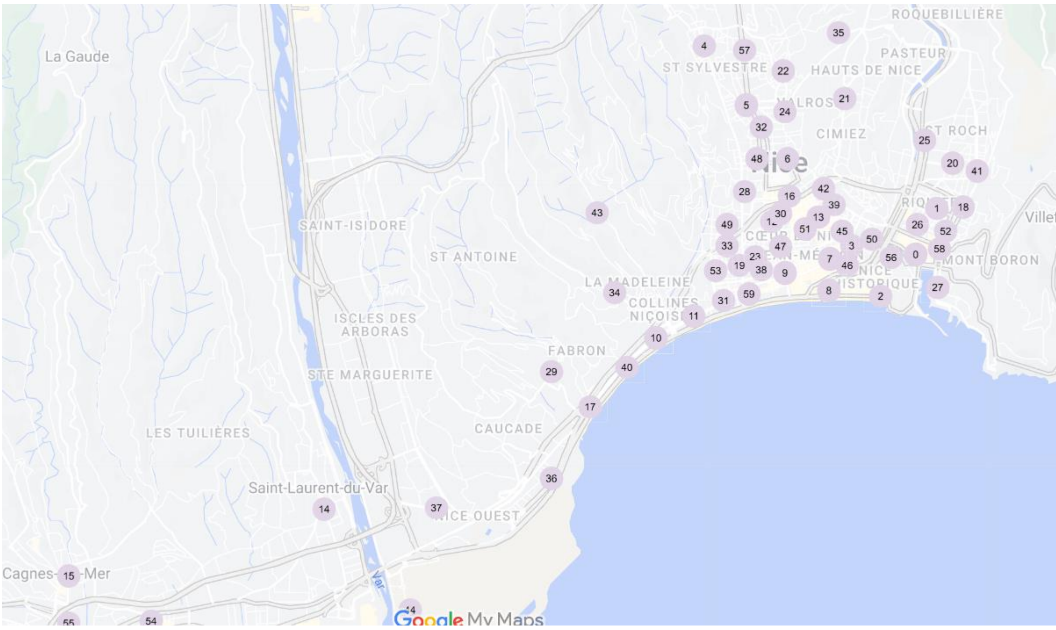

Figure 4.4: Auto Bleue System in Nice

The system starts at 6:00 AM and ends its operations at 10 PM. The time horizon is discretized and divided into equal time lengths. Each activity for vehicles (wait, serve trip, relocate) and personnel (wait, walk, drive) is assumed to start at the beginning of a time interval, and each activity lasts an integer number of time slots. To capture real-life operations more accurately, we allowed relocation operations to be done more frequently than trip travels. To do that, we set the time length to 5 minutes for relocations and 15 minutes for trips.

The computational experiments are conducted in C# in .NET environment using a workstation with Intel Xeon E5-2640 v3 processor. IBM ILOG Cplex 12.10. is used to solve the integer programming models.

The parameters used in this study are taken from Boyaci et al. (2015). Revenue per hour is €13. The fee for any trip i is calculated by $13 * a/4$ where a is the number of 15-minute time intervals to complete the journey. Personnel moving from one station to another to arrive at a station for a vehicle is only made by walking. The walking and driving distances are generated using Google Maps API. The driving speed is taken as 30km/hour. Walking distance in terms of time data is retrieved from Google Maps API. Note that we did not consider finding the optimal number of personnel to be employed for the operational day. This is because the daily cost of a single operator is too high to be decided at any iteration. Hence, we took the personnel cost as €10 per hour as in Bekli et al. (2021). The walking (MC_{jl}) and driving personnel cost (DC_{jl}) parameters are calculated using the number of time intervals it takes to walk and drive, respectively. At the beginning of each day, it is assumed that a vehicle is present at each station (i.e. $IV_j = 1\ \forall j$ at time $t = 0$). Vehicle relocation cost is taken as €0.01 per km.

M value in Constraints (4.14) is iteratively updated as

$$M = \mathrm{TP}_{i^*} + \sum_{i \in ATL} TR_i - \mathrm{RC}_{i^*} \tag{4.20}$$

for each trip request i^*. Intuitively, an iteration's profit cannot be higher than the revenue from accepted trips, plus revenue from the request itself minus realized costs. Below, we provide proof that the selected M always holds (4.14).

Proof. We start with Constraints (4.13). We continue to the proof by providing LastPr = $(\sum_{i\in ATL} TR_i - \mathrm{RC}_{i^*} - \mathrm{UC}_{i^*})$, where UC_{i^*} denotes the unrealized cost at bookingtime(i^*) which is calculated at the previous iteration's bookingtime.

$$y_{iq} \le \mathrm{Pr}_{iq}\Big((\mathrm{TP}_{i^*} - q)x_{iq} - \textstyle\sum_{j,l,t} (\mathrm{VC}_{jl}\, r_{jl}^t + \mathrm{MC}_{jl}\, u_{jl}^t + \mathrm{DC}_{jl}\, \bar{u}_{jl}^t) + \sum_{i\in ATL} TR_i - \mathrm{RC}_{i^*}\Big) + (1-\mathrm{Pr}_{iq})\mathrm{LastPr} \qquad \forall q \in P, i \in N_{i*} \tag{4.21}$$

$$\le \mathrm{Pr}_{iq}\Big((\mathrm{TP}_{i^*}) + \textstyle\sum_{i\in ATL} TR_i - \mathrm{RC}_{i^*}\Big) + (1-\mathrm{Pr}_{iq})\mathrm{LastPr} \qquad \forall q \in P, i \in N_{i*} \tag{4.22}$$

$$\le \mathrm{Pr}_{iq}\Big((\mathrm{TP}_{i^*}) + \textstyle\sum_{i\in ATL} TR_i - \mathrm{RC}_{i^*}\Big) + (1-\mathrm{Pr}_{iq})(\sum_{i\in ATL} TR_i - \mathrm{RC}_{i^*} - \mathrm{UC}_{i^*}) \qquad \forall q \in P, i \in N_{i*} \tag{4.23}$$

$$\le \mathrm{Pr}_{iq}\Big((\mathrm{TP}_{i^*}) + \textstyle\sum_{i\in ATL} TR_i - \mathrm{RC}_{i^*}\Big) + (1-\mathrm{Pr}_{iq})(\sum_{i\in ATL} TR_i - \mathrm{RC}_{i^*}) \qquad \forall q \in P, i \in N_{i*} \tag{4.24}$$

$$\le \mathrm{TP}_{i^*} + \sum_{i\in ATL} TR_i - \mathrm{RC}_{i^*} \tag{4.25}$$

□

4.4.1 Algorithm comparison

In order to see the spatiotemporal impacts of the proposed reservation system and the mathematical model provided in Section 4.3, we first tested the algorithms on a 10-station system taken from the busiest stations in the 60-station system. One of the main factors that affect the overall profit at the end of the operational day is the acceptance rate of the offered trip to the users. The probability function for the offered trip i with a discount value of q is assumed to be calculated by $P_n(a) = 1\Big/\Big(1 + e^{-(\beta_0+\beta_1 WD_{\mathrm{origin}(i^*),\mathrm{origin}(i)}+\beta_2 WD_{\mathrm{dest}(i^*),\mathrm{dest}(i)}+\beta_3|\mathrm{start}(i^*)-\mathrm{start}(i)|+\beta_4|\mathrm{end}(i^*)-\mathrm{end}(i)|+\beta_5 q)}\Big)$. The initial trip requests of the users are assumed to be accepted by the users with a probability of 1. Therefore, we have selected β_0 as -5, making $Pr_{i^*q} = 1/(1+e^{-5}) = 0.993307$.

We considered two scenarios where users are more likely and relatively less likely to accept offered trips. One may think that offering neighbouring trips (or having higher acceptance rates) will lead to a higher profit. However, this is not true for all cases, as changing the paths at an iteration may or may not lead to a higher profit at the end of the day.

The coefficients of the first and the second probability functions are set as $\{\beta_0 = -5, \beta_1 = 5, \beta_2 = 5, \beta_3 = 5, \beta_4 = 5,\ \beta_5 = 0.5\}$ and $\{\beta_0 = -5, \beta_1 = 3, \beta_2 = 3, \beta_3 = 3, \beta_4 = 3, \beta_5 = 1\}$, respectively. The probability of accepting an offer i with $WD_{\text{origin}(i^*),\text{origin}(i)} = 1$ (and every other feature remains the same with the original request i^*) is 0.5 and 0.88 for the first and second functions. The probability values are the same, i.e. 0.5 and 0.88, when the time shift is 15 minutes for either the start or the end time interval. We call the first function as the lower acceptance rate and the second function as the higher acceptance rate.

Table 4.1 provides information on the solutions for the small instances. Note that the instance names are created by "<Number of Stations> _ <Number of Iterations> _ <Instance number>". No User-Based Relocation columns refer to the solutions where the users are not provided with any neighbouring options and are treated with only the original trip requests. Probability Function 1 and 2 columns refer to the solutions using the reservation system provided in this study with the lower and higher probability functions. For graph-spanner based heuristic algorithm, the multiplicative constant value is chosen as 1.2. We have also provided solutions for the systems with or without operator-based relocations. Note that we refer to the No User-Based and No Operator-Based case as the base case. At each row, the results of the rest of the cases are compared with the base case.

Table 4.1: Algorithm comparison for small instances

Instance	No User-based Relocation					Probability Function 1										Probability Function 2									
	Without Operator-based (Base)		With Operator-based			Without Operator-based		With Operator-based								Without Operator-based		With Operator-based							
								Exact				Relocation Restriction Based Heuristic		Graph Spanner Based Heuristic				Exact				Relocation Restriction Based Heuristic		Graph Spanner Based Heuristic	
	Profit	# of Refusal by the Res. System	Profit	# of Refusal by the Res. System	% Improvement	Exact Profit	% Improvement Compared to Base	Profit	% Improvement Compared to Base	# of Refusal by the Res. System	# of Offer Not Accepted by User	Profit	% Improvement Compared to Base	Profit	% Improvement Compared to Base	Exact Profit	% Improvement Compared to Base	Profit	% Improvement Compared to Base	# of Refusal by the Res. System	# of Offer Not Accepted	Profit	% Improvement Compared to Base	Profit	% Improvement Compared to Base
10_100_1	519.4	66	1344.9	17	159%	644.8	24%	1413.5	172%	10	21	1372.7	164%	1389.6	168%	656.9	26%	1508.0	190%	9	9	1416.5	173%	1378.3	165%
10_100_2	807.1	65	1503.4	22	86%	810.0	0%	1576.5	95%	17	20	1532.7	90%	1469.0	82%	717.2	-11%	1525.9	89%	17	3	1552.0	92%	1488.8	84%
10_100_3	524.1	71	1345.4	13	157%	574.9	10%	1283.9	145%	9	21	1286.4	145%	1275.0	143%	594.9	14%	1413.9	170%	9	5	1387.9	165%	1383.0	164%
10_100_4	857.2	64	1466.6	22	71%	1080.7	26%	1536.3	79%	15	17	1498.3	75%	1535.5	79%	1180.8	38%	1511.8	76%	19	1	1536.4	79%	1514.3	77%
10_100_5	732.5	66	1324.1	24	81%	720.3	-2%	1384.0	89%	17	15	1354.3	85%	1335.3	82%	746.9	2%	1393.3	90%	16	5	1414.5	93%	1367.7	87%
10_100_6	611.3	72	1321.8	17	116%	725.1	19%	1300.0	113%	8	21	1321.9	116%	1356.1	122%	702.1	15%	1432.4	134%	11	4	1394.0	128%	1389.2	127%
10_100_7	583.5	74	1637.2	31	181%	975.8	67%	1772.2	204%	21	15	1725.2	196%	1659.6	184%	858.3	47%	1712.0	193%	24	2	1639.9	181%	1631.2	180%
10_100_8	527.1	70	1313.0	24	149%	683.5	30%	1297.9	146%	22	14	1263.2	140%	1182.3	124%	630.5	20%	1364.3	159%	21	3	1300.9	147%	1168.4	122%
10_100_9	753.1	69	1363.6	22	81%	843.2	12%	1353.1	80%	14	18	1374.6	83%	1341.3	78%	818.6	9%	1390.2	85%	17	3	1404.8	87%	1383.7	84%
10_100_10	601.9	63	1294.3	20	115%	694.6	15%	1285.0	114%	14	17	1272.8	111%	1264.7	110%	741.4	23%	1340.1	123%	18	3	1307.0	117%	1326.4	120%
Average		68.0		21.2	120%		20%		124%	14.7	17.9		120%		117%		18%		131%	16.1	3.8		126%		121%
10_150_1	478.6	121	1238.3	59	159%	806.2	68%	1522.2	218%	29	34	1450.0	203%	1365.3	185%	723.41	51%	1446.3	202%	35	10	1448.8	203%	1499.7	213%
10_150_2	787.3	125	1812.5	78	130%	878.9	12%	1832.9	133%	58	23	1807.6	130%	1887.8	140%	896.77	14%	1871.1	138%	67	7	1844.2	134%	1794.2	128%
10_150_3	965.2	113	1366.0	66	42%	971.4	1%	1480.9	53%	36	30	1493.3	55%	1376.1	43%	1016.86	5%	1585.5	64%	44	7	1452.2	50%	1432.0	48%
10_150_4	770.1	110	1429.6	55	86%	832.3	8%	1593.7	107%	35	38	1683.8	119%	1676.7	118%	810.10	5%	1497.3	94%	40	6	1560.8	103%	1485.7	93%
10_150_5	714.3	109	1517.0	63	112%	921.0	29%	1602.3	124%	36	36	1595.2	123%	1569.7	120%	846.14	18%	1537.2	115%	49	7	1512.8	112%	1533.3	115%
Average		115.6		64.2	106%		24%		127%	38.8	32.2		126%		121%		19%		123%	47.0	7.4		120%		119%

First, we consider the number of iterations in which the users do not receive any positive reply from the system (i.e., the system refuses the trip). In the base case where the system does not consider any type of relocations, the users are rejected by the system in 100 and 150 demand set scenarios for 68% and 77% of the iterations, respectively. When only operator-based relocations are considered, these values are decreased to 21% and 43%. With the inclusion of user-based relocations, the no-offer iterations are further decreased. When the lower probability function (Probability Function 1) is considered, the percentage of the iterations where users do not receive any offer is 15% and 26% for 100 and 150 demand instances, respectively. When the higher probability function (Probability Function 2) is considered, the values are 16% and 31% for 100 and 150 demand instances, respectively. In terms of increasing customer service rates, operator-based relocations are highly effective. Applying both operator and user-based relocations in the system results in the highest customer service rates. Note that by service rate, we only consider users receiving any offer (including the original trip request itself).

Second, we compare the total number of iterations that the users either do not receive any offer (number of refusals by the reservation system) or have rejected the provided offer for Probability Function 1 and 2. Although the average number of iterations in which the system refuses to make any offer for 100 demand set instances are very close, surprisingly, the system with the lower probability function rejects fewer iterations in 150 demand set instances. However, the total number of users who do not participate (either by the system or by choice) has decreased significantly when the higher probability function is applied. 37.5% and 31.4% of the iterations, the users did not travel by the carsharing system (due to refusal by the system or not accepting the offer) when Probability Function 1 and 2 were applied, respectively.

We compare the profit values at the end of the operational day for each instance. The columns with the title ”% Improvement” provide information on how the profit values are changed compared to the base case. Table 4.1 shows that without the operator-based relocations, the system increases its profitability by 20% on average. With operator-based relocation, a significant increase, 115% on the average considering 100 and 150 demand set instances together, in the profit values are achieved. These values were further improved by the inclusion of user-based relocations. The exact solutions of the instances where both operator and user-based relocations are applied with Probability Function 1 and 2 exhibit an average of 125% and 128% increase in the profit values, respectively, compared to the base case.

Next, we compare the heuristic algorithms by the profit values generated at the end of the operational day. In most of the cases, Relocation Restriction algorithm provides higher profits compared to the Graph-Spanner based heuristic algorithm. For 150-demand cases, the higher acceptance rate values of Graph-Spanner based algorithm almost caught the improvement observed in the Relocation Restriction algorithm. Still, both of the heuristic approaches provide good quality solutions and improvements in the profit compared to the base case.

4.4.2 Spatial and temporal flexibility comparison

In this part, we analyse and compare the effects of spatial and temporal flexibility. We have solved nine 20-node instances as follows: i) no balancing method is applied, i.e. no operator and no user-based relocations are applied (base), ii) only operator-based relocation is applied, iii) only user-based relocation is applied, iv) for relocation restriction heuristic, 1) both of the relocation types are applied, 2) operator-based and temporal user-based relocation is applied, 3) operator-based relocation and spatial user-based relocation is applied v) for graph-spanner based heuristic, 1) both of the relocation types are applied, 2) operator-based and temporal user-based relocation is applied, 3) operator-based relocation and spatial user-based relocation is applied. Table 4.2 presents the comparison values of the approaches summarized above. First, any type of vehicle stock balancing method (either user- or operator-based) is more effective than the base approach. Applying only operator-based relocations generate more profitable schedules when compared to applying only user-based relocations. Note that considering only user-based relocation for relocation restriction and graph-spanner based heuristics generates the same values. These heuristic approaches reduce the number of relocation variables (both vehicle and personnel movements). When no relocation variable is created, the rest of the calculations remain the same.

Table 4.2: Spatial and temporal flexibility comparison for 20-station systems

	No User-Based, No Operator-Based Relocation (Base Case) Profit	Only User Based	Relocation Restriction Based Heuristic				Graph Spanner Based Heuristic			
			Only Operator Based	Temporal and Spatial Together	Only Temporal (15 mins)	Only Spatial (1km)	Only Operator Based	Temporal and Spatial Together	Only Temporal (15 mins)	Only Spatial (1km)
		Profit increase compared to Base								
20_200_1	1418.8	21%	90%	113%	88%	97%	86%	105%	95%	95%
20_200_2	1239.1	34%	105%	129%	114%	129%	81%	142%	117%	128%
20_200_3	1124.5	37%	144%	172%	152%	174%	140%	167%	155%	182%
20_200_4	794.4	85%	203%	238%	217%	236%	194%	218%	215%	226%
20_200_5	1507.7	18%	53%	77%	47%	80%	47%	77%	55%	79%
20_200_6	1435.0	21%	94%	105%	100%	123%	86%	122%	85%	113%
20_200_7	828.2	64%	211%	249%	208%	236%	210%	236%	212%	198%
20_200_8	1167.9	37%	139%	168%	136%	172%	119%	153%	127%	159%
20_200_9	1085.7	47%	174%	214%	173%	187%	130%	191%	172%	156%
Average		41%	135%	163%	137%	159%	121%	157%	137%	149%

Considering operator and user-based relocations together generates high profit values for both relocation restriction and graph-spanner based algorithms. Both of the heuristic types produce similar profit values that are significantly higher than the cases where only one relocation type is applied.

The average improvement values observed for the heuristic algorithms when both spatial and temporal flexibility is considered are similar as the cases when temporal flexibility is not considered. Yet, temporal flexibility makes a significant impact on profitability when defining flexibility alone. However, when considered together with spatial flexibility, no significant profit improvement is generated (4% in Relocation Restriction and 8% in Graph-Spanner based heuristic) due to the temporal flexibility. It is consistent with Boyacı and Zografos (2019) that spatial flexibility has stronger effects on the system than temporal flexibility.

4.4.3 Sensitivity analysis on the number of personnel

As shown in 4.4.2, the inclusion of the operator-based relocation in the proposed system significantly increases the profitability. Therefore, we analyzed the effects of the number of personnel on profitability and the number of accepted/rejected offers and counteroffers.

Table 4.3 provides information on the 20-node 200 demand instances considering the number of personnel ranging between 0 and 4 for both of the heuristic algorithms. The average profit increases when Relocation Restriction algorithm is applied are 116%, 163%, 165% and 169% for 1,2,3 and 4 personnel, respectively, compared to base case where no relocation is considered. The average profit increases are 112%, 149%, 163% and 167% for 1,2,3 and 4 personnel, respectively, for the Graph-spanner based heuristic algorithm. The first two personnel have a noteworthy impact on the profit with the introduction of operator-based relocations in the system. The inclusion of the third personnel also results in a considerable profit increase, whereas the fourth personnel do not have a significant impact compared to the case where three personnel are employed. Note that as the number of personnel increases, the average number of iterations where no offers are presented to the user decreases. When only user-based relocation is considered, the average number of refusals by the system is 111 out of 200 trip requests. This number is decreased to 20 and 23 when four personnel are hired to relocate the vehicles in Relocation Restriction and Graph-spanner based algorithms, respectively.

Table 4.3: Comparison of profitability when different number of personnel are employed

	No user based, no operator based relocation (Base)	No Operator-Based Relocation				Relocation Restriction Based Heuristic															
						1 Personnel				2 Personnel				3 Personnel				4 Personnel			
		Profit Improvement	Number of Counter Offers	Number of Rejected Offers by the User	Number of Refusal by the Res. System	Profit Improvement	Number of Counter Offers	Number of Rejected Offers by the User	Number of Refusal by the Res. System	Profit Improvement	Number of Counter Offers	Number of Rejected Offers by the User	Number of Refusal by the Res. System	Profit Improvement	Number of Counter Offers	Number of Rejected Offers by the User	Number of Refusal by the Res. System	Profit Improvement	Number of Counter Offers	Number of Rejected Offers by the User	Number of Refusal by the Res. System
20_200_1	1418.8	21%	34	1	94	87%	91	10	45	113%	100	7	21	109%	108	8	10	116%	111	9	11
20_200_2	1239.1	34%	42	1	109	91%	70	3	79	129%	105	4	44	141%	102	3	34	150%	116	4	25
20_200_3	1124.5	37%	36	1	114	147%	81	3	67	172%	93	5	53	188%	111	8	32	193%	103	6	30
20_200_4	794.4	85%	34	0	118	198%	84	3	67	238%	98	5	47	261%	114	6	23	261%	114	6	22
20_200_5	1507.7	18%	37	0	109	49%	72	3	71	77%	87	1	61	89%	108	4	35	93%	124	7	20
20_200_6	1435.0	21%	39	1	114	79%	76	4	78	105%	105	5	49	124%	131	9	22	124%	127	9	19
20_200_7	828.2	64%	41	3	114	183%	88	5	59	249%	103	6	27	258%	112	7	15	261%	118	8	11
20_200_8	1167.9	37%	41	1	109	108%	63	2	83	168%	98	4	41	174%	104	6	31	177%	117	7	14
20_200_9	1085.7	47%	37	2	112	153%	68	4	70	214%	99	8	41	219%	107	9	34	222%	113	10	19
Average		38%	37	1.1	111	116%	77	4	69	163%	99	5	43	165%	110	7	28	169%	116	8	20

	Graph-Spanner Heuristic Based															
	1 Personnel				2 Personnel				3 Personnel				4 Personnel			
	Profit Improvement	Number of Counter Offers	Number of Rejected Offers by the User	Number of Refusal by the Res. System	Profit Improvement	Number of Counter Offers	Number of Rejected Offers by the User	Number of Refusal by the Res. System	Profit Improvement	Number of Counter Offers	Number of Rejected Offers by the User	Number of Refusal by the Res. System	Profit Improvement	Number of Counter Offers	Number of Rejected Offers by the User	Number of Refusal by the Res. System
20_200_1	77%	74	6	47	105%	89	7	30	116%	111	8	11	107%	92	6	16
20_200_2	103%	82	4	65	142%	103	3	41	143%	94	4	38	148%	108	3	25
20_200_3	136%	74	4	69	167%	88	4	53	186%	112	9	30	187%	105	7	36
20_200_4	198%	91	5	57	218%	103	6	44	252%	104	6	30	260%	110	6	25
20_200_5	37%	69	3	78	77%	98	6	46	90%	107	4	34	91%	116	6	23
20_200_6	74%	80	2	76	122%	111	8	34	125%	127	7	22	131%	121	9	21
20_200_7	176%	88	5	59	236%	97	7	34	258%	101	7	20	262%	115	9	11
20_200_8	123%	72	5	69	153%	95	4	46	166%	100	5	35	168%	109	4	25
20_200_9	144%	68	3	68	191%	92	1	47	209%	102	5	34	222%	118	9	20
Average	112%	75	4	69	149%	96	5	43	163%	107	6	28	167%	110	7	23

The number of personnel has a positive impact on the number of counteroffers. Clearly, the employment of a higher number of personnel results in personnel being scattered around the stations and able to provide a vehicle to the vicinity of the trip request. The analysis shows that the average number of counter offers has boosted from 37 to 114 in Relocation Restriction algorithm, and from 37 to 106.2 in Graph-spanner based algorithm, when the number of personnel increases from 0 to 4. Note that the number of rejected offers and counteroffers have a positive correlation. As the number of counteroffers increases, the number of rejected offers also increases.

4.4.4 Comparison of computational time for heuristic approaches

The performances of the proposed heuristic algorithms are further investigated through the comparison of required computational time to solve each iteration. Figure 4.5 presents the required CPU time (in seconds) for Relocation Restriction and Graph-Spanner based heuristic algorithms. As expected, fluctuations in the CPU times are not observed for the first few iterations. As the system becomes busier, the system is exposed to more fluctuations in required computational time. Eventually, the system becomes emptier, and the fluctuations in CPU time decreases.

The proposed system seeks to provide a solution to the operational level user-based relocation problem, and therefore, providing an algorithm with low computational time is crucial. First, we have checked how many times the required CPU time exceeds 1-minute duration for both of the heuristic algorithms. On the average, 9 and 3 iterations (out of 200) exceed 1-minute time duration for Relocation Restriction and Graph-Spanner based heuristic algorithms, respectively. When we compare the total outlier points in Figure 4.5, we observed 9 of 11 iterations that exceed 4-minute time duration belong to Relocation Restriction algorithm. Please note that the average CPU times are 23.4 and 17.3 seconds for Relocation Restriction and Graph-Spanner based heuristic algorithms, respectively.

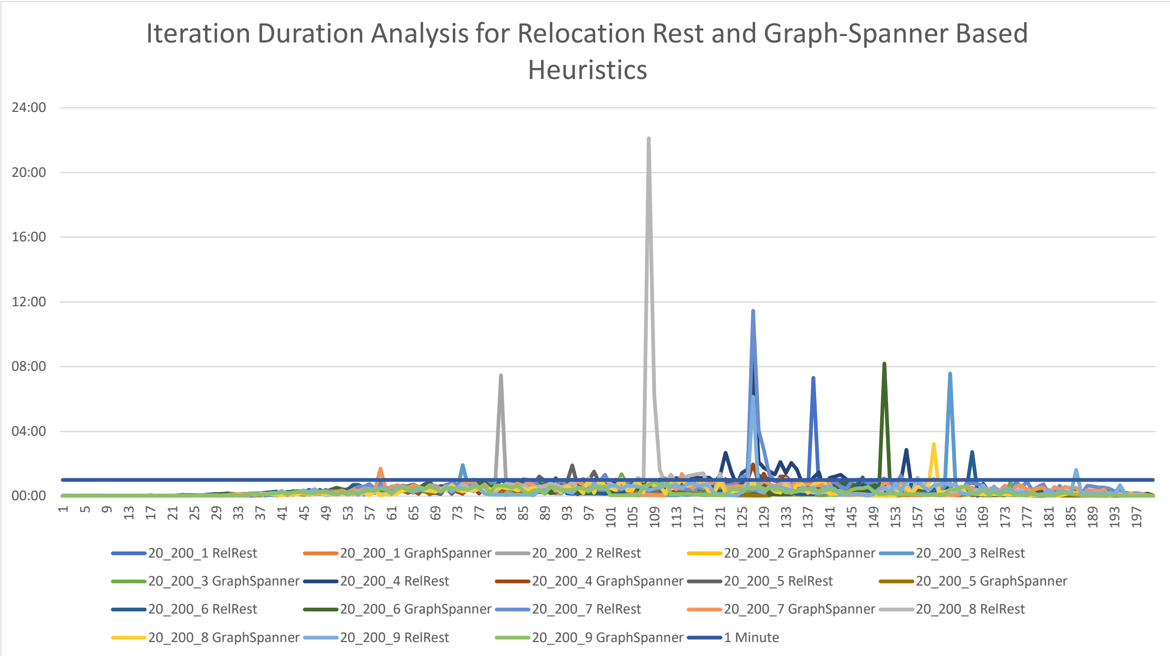

Figure 4.5: Comparison of required CPU times of each iteration for heuristic approaches

Next, we have compared the average required time to solve each iteration for the instances. Average times for Relocation Restriction algorithm is greater than Graph-Spanner based heuristic algorithm for 8 out of 9 instances. One example of the comparison of the same instance (instance 20_200_1) is given in Figure 4.6. Blue and orange lines in Figure 4.6 denote CPU times for Relocation Restriction and Graph-Spanner based heuristic algorithms, respectively. As can be seen from the figure, the blue line dominates the orange line in most of the iterations. We compared the average CPU times of the heuristic algorithms using Welch's t-test on all of the nine instances. For the instance used in Figure 4.6 (instance 20_200_1), it is concluded that the time required to compute Relocation Restriction algorithm is statistically significantly greater than that of Graph-Spanner based algorithm with a test statistic value of 0.0016. 5 out of 9 instances have statistically significantly higher average CPU times for Relocation Restriction algorithm than Graph-Spanner based algorithm in the significance level of 0.05.

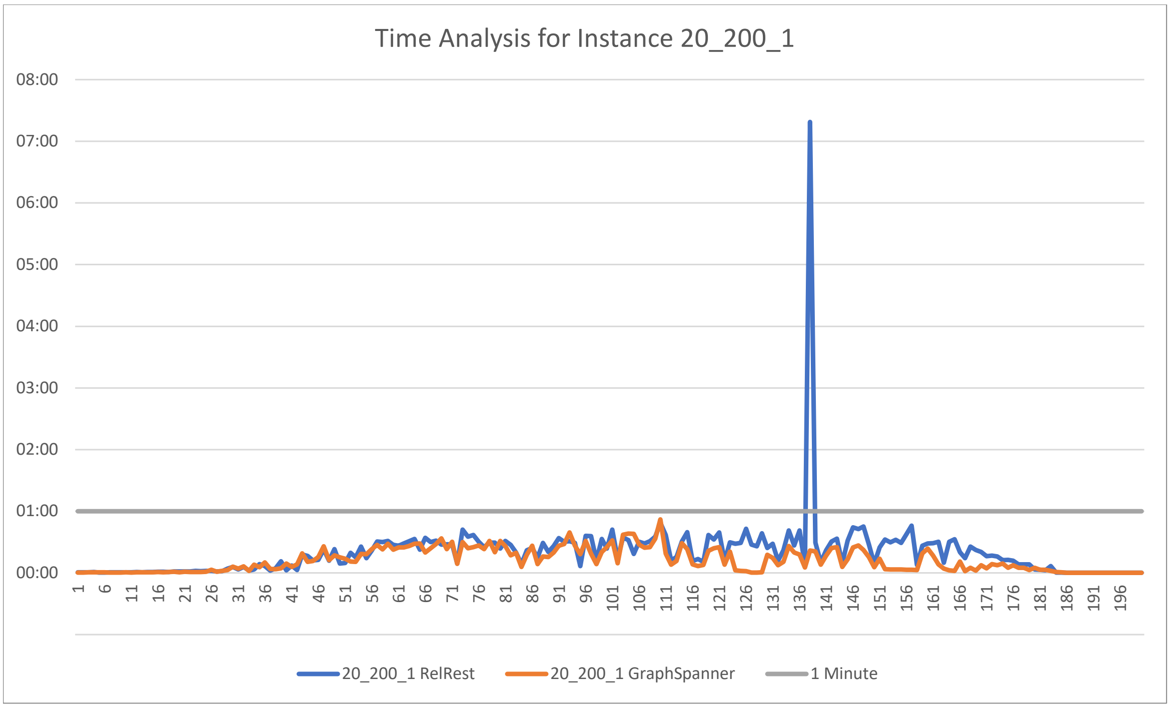

Figure 4.6: Comparison of required CPU times of each iteration for instance 20_200_1

4.5 Concluding remarks and future directions

In this research, we have introduced a reservation framework in which advanced reservations with at least a 30-minute gap between the booking time and the start time of the trip requests are processed. Each user is either served with their original trip request, offered a counteroffer that generates higher profits than the original request, or rejected as no neighbouring trip is feasible in the system. It is assumed that the users may accept or reject the offered trips. To choose the most profitable neighbouring trip, a mathematical model which considers the users' acceptance/rejection rates is proposed. This model also provides vehicle and personnel schedules, ensuring that the accepted trips from the previous iterations are going to be realized. As the system gets larger, the provided mathematical model becomes intractable. Hence, we proposed two matheuristic algorithms to reduce the size of the model. The first heuristic algorithm creates relocation arcs based on the neighbouring trip set. The second heuristic algorithm creates the walking-relocation variables on a graph-spanner network.

We have tested both of the heuristic algorithms and compared them with the proposed mathematical model on 10-node instances. The analysis shows that, in general, both relocation restriction and graph-spanner based heuristic algorithms provide better solutions than the case where no relocation is applied.

We conducted a sensitivity analysis on spatial and temporal flexibility using both of the heuristic algorithms. Consistent with the literature, spatial flexibility provides higher profits than temporal flexibility. We have shown that considering operator- and user-based relocations together significantly increases profit values. Secondly, we have analysed the effects of the number of personnel on the system for 20-node instances. As the number of personnel increases, the profit values increase significantly until a certain number of personnel; after that, the improvement in the profit is negligible. We have compared the CPU times for the heuristic algorithms and concluded that the average required CPU time for Relocation Restriction algorithm is significantly greater than that of Graph-Spanner based algorithm for most of the instances.

The proposed real-time operational-level problem is realistic but difficult to solve. Operational-level problems require efficient solution algorithms that work fast and provide near-optimal solutions. The main limitation of the proposed algorithms is that there is still a need to have efficient heuristic algorithms that can work on large-scale systems. There are several research opportunities pertinent to the proposed system and solution algorithms. In the proposed model, we have not considered electric vehicle fleets. Incorporating battery limitations, charging and discharging procedures may be a good extension to this research. Furthermore, another possible future direction is the inclusion of last-minute reservations in the reservation system. Moreover, exploiting the historical data to prepare the system for future reservations and using predicted states of the system in the solution algorithms is another future direction.

Chapter 5

Conclusion

5.1 Thesis summary

Carsharing, a part of urban mobility, is an alternative to vehicle ownership by providing short-term vehicle access. In carsharing systems, users leave the vehicle at the station where they start their journey (round-trip) or at any station/anywhere in the region (one-way) after the rental period. Due to flexibility in destination stations, one-way systems attract more users compared to round-trip systems. However, one-way systems face a vehicle supply-demand imbalance among the stations, which requires corrective actions. Carsharing companies often employ imbalance mitigating measures which make the system complex to operate. For this reason, it is very difficult to make strategic, tactical and operational decisions in one-way car sharing systems.

With the developments in information technology, the number of one-way carsharing systems, and therefore the number of studies carried out, have increased significantly in the last ten years. Yet, there exist several research questions regarding strategic and operational-level decisions in one-way carsharing systems that remain unaddressed. This thesis aims to provide solution algorithms to i) the strategic decision of charger infrastructure renewal decisions and ii) the operational level decision of providing the most profitable offer to the user considering user acceptance/rejection rates. Furthermore, this thesis provides an in-depth literature review on user-based relocation and pricing in carsharing systems. Each goal of this thesis has been the subject of separate research studies, summarized below.

The first research study, Chapter 2 of this thesis, focuses on the strategic decisions of charger infrastructure renewals in one-way electric carsharing systems. A time-space-battery level network model has been introduced to solve the fast charger location problem. The proposed model can mimic real-world battery charging conditions of different types of chargers. The model optimizes the location and number of fast chargers (or different types of chargers) to be implemented over a set of scenarios. Additionally, the vehicle schedules, including relocation activities as well as trip selection, are found by the model. Note that the considered system is a reservation-based system where the users request trips in advance and wait for approval. The proposed model is challenging to solve, as the variables and constraints created increase greatly when the system gets larger. Ergo, we propose matheuristic algorithms that reduce the size of the model.

The first type of heuristic algorithm focuses on reducing the number of relocation variables created. It should be noted that, according to the proposed model, a relocation activity may happen anytime between any station pair if the battery level allows. Hence, the created relocation variables in the exact model constitute a significant portion of the variables. Yet, our analysis shows that the majority of the relocation variables get the value of zero. Therefore, we propose heuristics focusing on where and when the relocation activities are needed. The logic behind the heuristic algorithms is to create a relocation variable if it serves a trip request. This could be done by bringing a vehicle to the trip request's origin station before the start time of the trip request and/or emptying a parking spot at the trip request's destination station before the end time of the trip request.

The second type of heuristic algorithm reduces the number of variables by grouping the stations and solves the problem initially at an aggregate level using groups of stations instead of individual stations while optimizing the chargers' location within each station group. By combining both of the heuristic algorithms, we proposed a heuristic approach that works efficiently with minimal compromise in the solution quality. The results of the proposed heuristic algorithm reveal that introducing fast chargers increases the profitability of the system (by 25%) and the level of service offered to its users.

The second research study of the thesis, which is presented in Chapter 3, provides an in-depth literature review on user-based relocations and pricing in carsharing systems. Apart from operator-based relocations, there exist vehicle supply-demand balancing methods, namely user-based relocations and pricing. The aim of this study is to provide a comprehensive overview of the current understanding of user-based relocations and pricing in carsharing systems. There are several literature review studies focusing on many aspects of the carsharing literature, including operator-based relocations. To the best of our knowledge, this paper is the first structured classification of user-based relocations and pricing studies. The studies reviewed in Chapter 3 are categorized into three groups, namely (i) user behaviour analysis, (ii) data analytics, and (iii) optimization-based approaches. These categories are then subcategorized and examined in detail. Besides providing a summary of the studies mentioned, we presented a mathematical model for operational-level decisions. Studies with similar approaches are examined to reveal how user-based relocations and pricing are adapted to the mathematical model for operational-level decisions. Finally, each subcategory in the study is enhanced by providing future research directions.

The last research study of this thesis, presented in Chapter 4, focuses on the real-time operational-level decisions of the system. This study exploits user-based relocations to increase profitability. The proposed study mimics real-world conditions by taking requests from the beginning of the day. We propose a reservation system in which each trip request receives a response in the order of the request's booking time. At each request, the system generates alternative offers based on the origin/destination stations and start/end time of the original request. Out of the alternative offer set (which also includes the original trip request), the system offers the offer with the highest expected profit to the user considering the acceptance/rejection rates of the user. In order to do that, we propose a mathematical model that finds the best counteroffer to provide to the user while ensuring that the offers accepted before the current offer will be realized according to the vehicle/personnel schedules generated. In real-world reservation systems, trips can start and end at specific time intervals. However, there is no restriction on the relocation activities, as the operator may start and end the relocations anytime the system allows. Therefore, in the model, we treated trip movements and relocation activities in different time interval sets.

The proposed mathematical model is intractable for large-sized systems. We propose two matheuristics to reduce the number of arcs created in the model. The first type is reducing the number of relocation arcs similar to what we have proposed in Chapter 2. The second heuristic algorithm creates relocation arcs based on the multiplicative graph-spanner network. The analysis shows that proposed algorithms can generate higher profits by employing user-based relocations compared to both no-relocation cases, and only operator-based relocation cases. The profit values for 20-node cases are improved by 142% and 146% on average for the graph-spanner based and relocation restriction algorithms, respectively. The profit increase for the same instance set when only operator-based relocation is considered reached 109%. Additionally, we have seen that the number of users that are directly rejected from the system are lowered by the proposed system.

5.2 Future directions

A large number of research studies on carsharing systems regarding strategic and operational level decisions have appeared in the last decade. Yet, there still exist several open research problems pertinent to the studies provided in this thesis.

Future studies regarding strategic decisions of charger infrastructure renewal may extend the proposed study by incorporating the station sizing decisions. In our approach, a fixed capacity for each station is considered. With the introduction of fast chargers, the required time to charge the vehicles at the stations is lowered, and vehicle utilization values are increased. This may decrease the need for parking spots, decreasing the station rental cost. With minor alterations, the proposed models can be used to determine the optimum number of fast chargers and the optimum number of parking spots per station. Additionally, multi-objective models can be introduced by considering different objectives, such as maximization of the number of served trips, vehicle utilization, and other performance measures.

There are several future directions of research related to the vehicle supply-demand balancing mechanisms other than operator-based relocations. Regarding descriptive studies, surveys that can analyze the effects of the geo-location characteristics on the incentives or pricing decisions can be further investigated. The effect of synergies between other transport modes and carsharing on the pricing strategies can also be investigated. Moreover, user purchasing behaviour to the pricing strategies of carsharing systems and mobility service providers that share the same Mobility as a Service platforms can be further analyzed. In addition, regarding prescriptive studies, incorporating the stochastic nature of external factors, such as traffic evolution and geographic characteristics, will enhance the existing studies. There is still future work needed to be done to combine the operator-based, user-based relocations and pricing. One other direction that needs attention in the future is the various fleet types, such as autonomous or stackable fleets. The operations of these fleets differ from the regular combustion engine or electric vehicle fleets. Therefore, studies regarding pricing or user-based relocations would require different solution approaches. Furthermore, providing a user-based relocation and/or pricing-based literature review on other vehicle-sharing systems can also be a future direction.

Promising areas in the real-time operational level decision problems in carsharing can be summarized as providing efficient algorithms, inclusion of different fleet types, last-minute reservations and historical data. First of all, operational-level problems require efficient solution algorithms which work fast and provide near-optimal solutions. Therefore, we need to have efficient heuristic algorithms that can work on large-scale systems. Furthermore, the proposed reservation system can be adapted to systems with an electric vehicle fleet. One other possible future direction is the inclusion of last-minute reservations to the system. Finally, keeping the system in a state that is more profitable using the historical data and using the predicted states of the stations within the algorithms would improve the proposed reservation system.

References

Ahmed, R., Bodwin, G., Sahneh, F. D., Hamm, K., Jebelli, M. J. L., Kobourov, S., Spence, R., 2020. Graph spanners: A tutorial review. Computer Science Review 37. http://dx.doi.org/10.1016/j.cosrev.2020.100253.

Albadi, M. H., El-Saadany, E. F., 2008. A summary of demand response in electricity markets. Electric Power Systems Research 78 (11), 1989–1996. http://dx.doi.org/10.1016/j.epsr.2008.04.002.

Althöfer, I., Das, G., Dobkin, D., Joseph, D., Soares, J., 1993. On sparse spanners of weighted graphs. Discrete Computational Geometry 9 (1), 81–100. http://dx.doi.org/10.1007/BF02189308.

Banerjee, S., Freund, D., Lykouris, T., 2022. Pricing and optimization in shared vehicle systems: An approximation framework. Operations Research 70 (3), 1783–1805. http://dx.doi.org/10.1287/OPRE.2021.2165.

Barth, M., Todd, M., Xue, L., 2004. User-based vehicle relocation techniques for multiple-station shared-use vehicle systems. Transportation Research Board 83rd Annual Meeting.

Bates, J., Leibling, D., 2012. Spaced out perspectives on parking policy. RAC Foundation (July), 132, http://www.racfoundation.org/assets/rac_foundation/content/downloadables/spaced_out-bates_leibling-jul12.pdf.

Bekli, S., Boyacı, B., Zografos, K. G., 2021. Enhancing the performance of one-way electric carsharing systems through the optimum deployment of fast chargers. Transportation Research Part B: Methodological 152, 118–139. http://dx.doi.org/10.1016/J.TRB.2021.08.001.

Bianchessi, A. G., Formentin, S., Savaresi, S. M., 2013. Active fleet balancing in vehicle sharing systems via feedback dynamic pricing. IEEE Conference on Intelligent Transportation Systems, Proceedings, ITSC (Itsc), 1619–1624. http://dx.doi.org/10.1109/ITSC.2013.6728461.

Biesinger, B., Hu, B., Stubenschrott, M., Ritzinger, U., Prandtstetter, M., 2017. Optimizing charging station locations for electric car-sharing systems. Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics) 10197 LNCS (April 2018), 157–172. http://dx.doi.org/10.1007/978-3-319-55453-2$_1$1.

Boldrini, C., Incaini, R., Bruno, R., 2018. Relocation in car sharing systems with shared stackable vehicles: Modelling challenges and outlook. IEEE Conference on Intelligent Transportation Systems, Proceedings, ITSC 2018-March, 1–8. http://dx.doi.org/10.1109/ITSC.2017.8317752.

Bothun, D., Lieberman, M., Egol, M., Clarke, D., Atkinson, J., Blumenthal, J., Decker, B., Hobbs, M., Shirsekar, S., 2015. The sharing economy.

Boyaci, B., Zografos, K. G., Geroliminis, N., 2015. An optimization framework for the development of efficient one-way car-sharing systems. European Journal of Operational Research 240 (3), 718–733. http://dx.doi.org/10.1016/j.ejor.2014.07.020.

Boyacı, B., Zografos, K. G., 2019. Investigating the effect of temporal and spatial flexibility on the performance of one-way electric carsharing systems. Transportation Research Part B: Methodological 129, 244–272. http://dx.doi.org/10.1016/j.trb.2019.09.003.

Boyacı, B., Zografos, K. G., Geroliminis, N., 2017. An integrated optimization-simulation framework for vehicle and personnel relocations of electric carsharing systems with reservations. Transportation Research Part B: Methodological 95, 214–237. http://dx.doi.org/10.1016/j.trb.2016.10.007.

Brandstatter, G., Gambella, C., Leitner, M., Malaguti, E., Masini, F., Puchinger, J., Ruthmair, M., Vigo, D., 2016. Overview of optimization problems in electric car-sharing system design and management.

Brandstatter, G., Kahr, M., Leitner, M., 2017. Determining optimal locations for charging stations of electric car-sharing systems under stochastic demand. Transportation Research Part B: Methodological 104, 17–35. http://dx.doi.org/10.1016/j.trb.2017.06.009.

Brandstatter, G., Leitner, M., Ljubi, I., 2020. Location of charging stations in electric car sharing systems. Transportation Science 54 (5), 1408–1438. http://dx.doi.org/10.1287/TRSC.2019.0931.

Brandt, T., Dlugosch, O., 2021. Exploratory data science for discovery and ex-ante assessment of operational policies: Insights from vehicle sharing. Journal of Operations Management 67 (3), 307–328. http://dx.doi.org/10.1002/joom.1125.

Brendel, A. B., Brauer, B., Hildebrandt, B., 2016. Toward user-based relocation information systems in station-based one-way car sharing. pp. 1–10.

Brendel, A. B., Brennecke, J. T., Hillmann, B. M., Kolbe, L. M., 2020. The design of a decision support system for computation of carsharing pricing areas and its influence on vehicle distribution. IEEE Transactions on Engineering Management, 1–15http://dx.doi.org/10.1109/TEM.2020.3028997.

Brendel, A. B., Lichtenberg, S., Brauer, B., Nastjuk, I., Kolbe, L. M., 2018. Improving electric vehicle utilization in carsharing: A framework and simulation of an e-carsharing vehicle utilization management system.

Transportation Research Part D: Transport and Environment 64 (March), 230–245. http://dx.doi.org/10.1016/j.trd.2018.01.024.

Brendel, A. B., Lichtenberg, S., Morana, S., Prinz, C., Hillmann, B. M., 2022. Designing a crowd-based relocation system — the case of car-sharing. Sustainability 14, 1–28. http://dx.doi.org/10.3390/su14127090.

Bruglieri, M., Colorni, A., 2014. The relocation problem for the one-way electric vehicle sharing. Networks 64 (4), 292–305. http://dx.doi.org/10.1002/net.21585.

Çalık, H., Fortz, B., 2019. A Benders decomposition method for locating stations in a one-way electric car sharing system under demand uncertainty. Transportation Research Part B: Methodological 125, 121–150. http://dx.doi.org/10.1016/j.trb.2019.05.004.

Carlier, A., Munier-Kordon, A., Klaudel, W., 2015. Mathematical model for the study of relocation strategies in one-way carsharing systems. Vol. 10. Elsevier, pp. 374–383.

Cepolina, E. M., Farina, A., Holloway, C., Tyler, N., 2015. Innovative strategies for urban car-sharing systems and a simulator to assess their performance. Transportation Planning and Technology 38 (4), 375–391. http://dx.doi.org/10.1080/03081060.2015.1026098.

Chen, T. D., Kockelman, K. M., 2016. Management of a shared autonomous electric vehicle fleet: Implications of pricing schemes. Transportation Research Record 2572 (2572), 37–46. http://dx.doi.org/10.3141/2572-05.

Ciari, F., Balac, M., Balmer, M., 2015. Modelling the effect of different pricing schemes on free-floating carsharing travel demand: A test case for Zurich, Switzerland. Transportation 42 (3), 413–433. http://dx.doi.org/10.1007/s11116-015-9608-z.

Clemente, M., Fanti, M. P., Iacobellis, G., Nolich, M., Ukovich, W., 2018. A decision support system for user-based vehicle relocation in car sharing systems. IEEE Transactions on Systems, Man, and Cybernetics: Systems 48 (8), 1283–1296. http://dx.doi.org/10.1109/TSMC.2017.2665881.

Clemente, M., Fanti, M. P., Mangini, A. M., Ukovich, W., 2013. The vehicle relocation problem in car sharing systems: Modeling and simulation in a Petri net framework. Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics) 7927 LNCS, 250–269. http://dx.doi.org/10.1007/978-3-642-38697-8$_1$4.

Clewlow, R. R., 2016. Carsharing and sustainable travel behavior: Results from the San Francisco Bay Area. Transport Policy 51, 158–164. http://dx.doi.org/10.1016/j.tranpol.2016.01.013.

Consultants, S., 2020. Urban access regulations in europe. https://www.urbanaccessregulations.eu/

Correia, G. H. A., Antunes, A. P., 2012. Optimization approach to depot location and trip selection in one-way carsharing systems. Transportation Research Part E: Logistics and Transportation Review 48 (1), 233–247. http://dx.doi.org/10.1016/j.tre.2011.06.003.

Correia, G. H. D. A., Jorge, D. R., Antunes, D. M., 2014. The added value of accounting for users' flexibility and information on the potential of a station-based one-way car-sharing system: An application in Lisbon, Portugal. Journal of Intelligent Transportation Systems 18 (3), 299–308. http://dx.doi.org/10.1080/15472450.2013.836928.

Council, O. C., 2019. Oxford city zero emission zone proposal frequently asked questions. https://consultations.oxfordshire.gov.uk/gf2.ti/f/862530/31286213.1/PDF/-/Zero_Emission_Zone__Frequently_Asked_Questions.pdf

Curtale, R., Liao, F., van der Waerden, P., 2021. Understanding travel preferences for user-based relocation strategies of one-way electric car-sharing services. Transportation Research Part C: Emerging Technologies 127 (November 2020), 103135. http://dx.doi.org/10.1016/j.trc.2021.103135.

de Lorimier, A., El-Geneidy, A., 2013. Understanding the factors affecting vehicle usage and availability in carsharing networks: A case study of Communauto carsharing system from Montréal, Canada. International Journal of Sustainable Transportation, 35–51.

DriveNow, 2019. Quick Charge — Fast Charge the BMW i3 — DriveNow Car Sharing. `https://www.drive-now.com/dk/en/tips/quickcharge`

EU, 2014. Directive 2014/94/EU of the European Parliament and of the council of 22 October 2014 on the deployment of alternative fuels infrastructure . `https://eur-lex.europa.eu/legal-content/EN/TXT/PDF/?uri=CELEX:32014L0094`

Fanti, M. P., Mangini, A. M., Roccotelli, M., Silvestri, B., 2022. Innovative approaches for electric vehicles relocation in sharing systems. IEEE Transactions on Automation Science and Engineering 19 (1), 21–36. http://dx.doi.org/10.1109/TASE.2021.3103808.

Febbraro, A. D., Sacco, N., Saeednia, M., 2012. One-way carsharing solving the relocation problem. Transportation Research Record (2319), 113–120. http://dx.doi.org/10.3141/2319-13.

Febbraro, A. D., Sacco, N., Saeednia, M., 2019. One-way car-sharing profit maximization by means of user-based vehicle relocation. IEEE Transactions on Intelligent Transportation Systems 20 (2), 628–641. http://dx.doi.org/10.1109/TITS.2018.2824119.

Ferrero, F., Perboli, G., Rosano, M., Vesco, A., 2018. Car-sharing services: An annotated review. Sustainable Cities and Society 37 (October 2017), 501–518. http://dx.doi.org/10.1016/j.scs.2017.09.020.

Fricker, C., Gast, N., 2016. Incentives and redistribution in homogeneous bike-sharing systems with stations of finite capacity. EURO Journal on Transportation and Logistics 5 (3), 261–291. http://dx.doi.org/10.1007/s13676-014-0053-5.

Gambella, C., Malaguti, E., Masini, F., Vigo, D., 2018. Optimizing relocation operations in electric car-sharing. Omega (United Kingdom) 81, 234–245. http://dx.doi.org/10.1016/j.omega.2017.11.007.

Giorgione, G., Ciari, F., Viti, F., 2019. Availability-based dynamic pricing on a round-trip carsharing service: An explorative analysis using agent-based simulation. Procedia Computer Science 151 (2018), 248–255. http://dx.doi.org/10.1016/j.procs.2019.04.036.

Golalikhani, M., Oliveira, B. B., Carravilla, M. A., Oliveira, J. F., Antunes, A. P., 2021. Carsharing: A review of academic literature and business practices toward an integrated decision-support framework. Transportation Research Part E: Logistics and Transportation Review 149 (March), 102280. http://dx.doi.org/10.1016/j.tre.2021.102280.

Guo, Z., Deride, J., Fan, Y., 2016. Infrastructure planning for fast charging stations in a competitive market. Transportation Research Part C: Emerging Technologies 68, 215–227. http://dx.doi.org/10.1016/j.trc.2016.04.010.

Haider, Z., Nikolaev, A., Kang, J. E., Kwon, C., 2018. Inventory rebalancing through pricing in public bike sharing systems. European Journal of Operational Research 270 (1), 103–117. http://dx.doi.org/10.1016/j.ejor.2018.02.053.

He, Y., Kockelman, K. M., Perrine, K. A., 2019. Optimal locations of u.s. fast charging stations for long-distance trip completion by battery electric vehicles. Journal of Cleaner Production 214, 452–461. http://dx.doi.org/10.1016/j.jclepro.2018.12.188.

Herrenkind, B., Brendel, A. B., Lichtenberg, S., Kolbe, L., 2019. Computing incentives for user-based relocation in carsharing. Wirtschaftsinformatik, 1388–1402.

Herrmann, S., Schulte, F., Voß, S., 2014. Increasing acceptance of free-floating car sharing systems using smart relocation strategies: A survey based study of car2go Hamburg. Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics) 8760, 151–162. http://dx.doi.org/10.1007/978-3-319-11421-7$_1$0.

Hossain, M., 2020. Sharing economy: A comprehensive literature review. International Journal of Hospitality Management 87 (February). http://dx.doi.org/10.1016/j.ijhm.2020.102470.

Hua, Y., Zhao, D., Wang, X., Li, X., 2019. Joint infrastructure planning and fleet management for one-way electric car sharing under time-varying uncertain demand. Transportation Research Part B: Methodological 128, 185–206. http://dx.doi.org/10.1016/j.trb.2019.07.005.

Huang, K., An, K., de Almeida Correia, G. H., 2020a. Planning station capacity and fleet size of one-way electric carsharing systems with continuous state of charge functions. European Journal of Operational Research 287 (3), 1075–1091. http://dx.doi.org/10.1016/j.ejor.2020.05.001.

Huang, K., An, K., de Almeida Correia, G. H., Rich, J., Ma, W., 2021. An innovative approach to solve the carsharing demand-supply imbalance problem under demand uncertainty. Transportation Research Part C: Emerging Technologies 132 (September), 103369. http://dx.doi.org/10.1016/j.trc.2021.103369.

Huang, K., An, K., Rich, J., Ma, W., 2020b. Vehicle relocation in one-way station-based electric carsharing systems: A comparative study of operator-based and user-based methods. Transportation Research Part

E: Logistics and Transportation Review 142 (November 2018), 102081. http://dx.doi.org/10.1016/j.tre.2020.102081.

Huang, K., de Almeida Correia, G. H., An, K., 2018. Solving the station-based one-way carsharing network planning problem with relocations and non-linear demand. Transportation Research Part C: Emerging Technologies 90, 1–17. http://dx.doi.org/10.1016/j.trc.2018.02.020.

Iacobucci, R., Bruno, R., Boldrini, C., 2022. A multi-stage optimisation approach to design relocation strategies in one-way car-sharing systems with stackable cars. IEEE Transactions on Intelligent Transportation Systems 23 (10), 17048–17061. http://dx.doi.org/10.1109/TITS.2022.3164989.

Illgen, S., Höck, M., 2019. Literature review of the vehicle relocation problem in one-way car sharing networks. Transportation Research Part B: Methodological 120 (xxxx), 193–204. http://dx.doi.org/10.1016/j.trb.2018.12.006.

InsideEVs, 2018. European ccs (type 2 / combo 2) conquers world - ccs combo 1 exclusive to north america, digital image. https://insideevs.com/news/333637/european-ccs-type-2-combo-2-conquers-world-ccs-combo-1-exclusive-tc

Jiao, Z., Ran, L., Chen, J., Meng, H., Li, C., 2017. Data-driven approach to operation and location considering range anxiety of one-way electric vehicles sharing system.

Jiao, Z., Ran, L., Liu, X., Zhang, Y., Qiu, R. G., 2020. Integrating price-incentive and trip-selection policies to rebalance shared electric vehicles. Service Science 12 (4), 148–173. http://dx.doi.org/10.1287/serv.2020.0266.

Jones, H., 2017. Oxford aims for world's first zero emissions zone with petrol car ban - environment - the guardian. https://www.theguardian.com/uk-news/2017/oct/11/oxford-aims-to-cut-air-pollution-car-ban-zero-emissions-zone

Jorge, D., Correia, G., 2013. Carsharing systems demand estimation and defined operations: A literature review. European Journal of Transport and Infrastructure Research 13 (3), 201–220. http://dx.doi.org/10.18757/ejtir.2013.13.3.2999.

Jorge, D., Molnar, G., de Almeida Correia, G. H., 2015. Trip pricing of one-way station-based carsharing networks with zone and time of day price variations. Transportation Research Part B: Methodological 81, 461–482. http://dx.doi.org/10.1016/j.trb.2015.06.003.

Kia, 2019. 2019 Kia Niro EV - Crossover SUV Pricing Features . https://www.kia.com/us/en/niro-ev

Kim, D., Ko, J., Park, Y., 2015. Factors affecting electric vehicle sharing program participants' attitudes about car ownership and program participation. Transportation Research Part D: Transport and Environment 36, 96–106. http://dx.doi.org/10.1016/j.trd.2015.02.009.

Laarabi, H., Boldrini, C., Bruno, R., Porter, H., Davidson, P., 2019. User-based relocation of stackable car sharing. Vol. 921. Springer International Publishing. http://dx.doi.org/10.1007/978-3-030-02907-4$_1$3.

Lai, M., Hu, Q., Liu, Y., Lang, Z., 2022. A rolling-horizon decision framework for integrating relocation and user flexibility in one-way electric carsharing systems. Transportation Research Part C: Emerging Technologies 144 (September), 103867. http://dx.doi.org/10.1016/j.trc.2022.103867.

Laporte, G., Meunier, F., Calvo, R. W., 2018. Shared mobility systems: An updated survey. Annals of Operations Research 271 (1), 105–126. http://dx.doi.org/10.1007/s10479-018-3076-8.

Liao, F., Molin, E., Timmermans, H., van Wee, B., 2020. Carsharing: the impact of system characteristics on its potential to replace private car trips and reduce car ownership. Vol. 47. Springer US. http://dx.doi.org/10.1007/s11116-018-9929-9.

Lippoldt, K., Niels, T., Bogenberger, K., 2018. Effectiveness of different incentive models in free-floating carsharing systems: A case study in Milan. IEEE Conference on Intelligent Transportation Systems, Proceedings, ITSC 2018-Novem, 1179–1185. http://dx.doi.org/10.1109/ITSC.2018.8569242.

Lippoldt, K., Niels, T., Bogenberger, K., 2019. Analyzing the potential of user-based relocations on a free-floating carsharing system in Cologne. Transportation Research Procedia 37 (September 2018), 147–154. http://dx.doi.org/10.1016/j.trpro.2018.12.177.

Litman, T., 2000. Evaluating carsharing benefits. Transportation Research Record (1702), 31–35. http://dx.doi.org/10.3141/1702-04.

Liu, Y., Xie, J., Chen, N., 2021. Offline-online approximate dynamic programming for stochastic carsharing systems with relocation incentives. SSRN Electronic Journal (September). http://dx.doi.org/10.2139/ssrn.3882532.

Loose, W., 2010. The State of European Car-Sharing: Final Report D 2.4 Work Package 2. http://www.carsharing.de/images/stories/pdf_dateien/wp2_report__englisch_final_2.pdf

Lu, R., de Almeida Correia, G. H., Zhao, X., Liang, X., Lv, Y., 2021. Performance of one-way carsharing systems under combined strategy of pricing and relocations. Transportmetrica B: Transport Dynamics 9 (1), 134–152. http://dx.doi.org/10.1080/21680566.2020.1819912.

Marecek, J., Shorten, R., Yu, J. Y., 2016. Pricing vehicle sharing with proximity information. 2016 3rd MEC International Conference on Big Data and Smart City, ICBDSC 2016, 261–267http://dx.doi.org/10.1109/ICBDSC.2016.7460378.

Martin, E., Shaheen, S., 2016. Impacts of car2go on vehicle ownership, modal shift, vehicle miles traveled, and greenhouse gas emissions: An analysis of five North American cities. http://innovativemobility.org/wp-content/uploads/2016/07/Impactsofcar2go_FiveCities_2016.pdf

Martin, E., Shaheen, S., Lidicker, J., 2010. Impact of carsharing on household vehicle holdings. Transportation Research Record (2143), 150–158. http://dx.doi.org/10.3141/2143-19.

Martin, E. W., Shaheen, S. A., 2011. Greenhouse gas emission impacts of carsharing in North America. IEEE Transactions on Intelligent Transportation Systems 12 (4), 1074–1086. http://dx.doi.org/10.1109/TITS.2011.2158539.

Mindur, L., Turoń, K., Sierpiński, G., 2018. Car-sharing development – current state and perspective. Logistics and Transport 39 (3), 5–14. http://dx.doi.org/10.26411/83-1734-2015-3-39-1-18.

Monteiro, C. M., Machado, C. A., de Oliveira Lage, M., Berssaneti, F. T., Davis, C. A., Quintanilha, J. A., 2021. Optimization of carsharing fleet size to maximize the number of clients served. Computers, Environment and Urban Systems 87, 101623. http://dx.doi.org/10.1016/j.compenvurbsys.2021.101623.

Nansubuga, B., Kowalkowski, C., 2021. Carsharing: A systematic literature review and research agenda. Journal of Service Management 32 (6), 55–91. http://dx.doi.org/10.1108/JOSM-10-2020-0344.

Nissan, 2020. Range charging — 2019 nissan leaf electric vehicle — nissan. https://www.nissan.co.uk/vehicles/new-vehicles/leaf/range-charging.html

Nourinejad, M., Zhu, S., Bahrami, S., Roorda, M. J., 2015. Vehicle relocation and staff rebalancing in one-way carsharing systems. Transportation Research Part E: Logistics and Transportation Review 81, 98–113. http://dx.doi.org/10.1016/j.tre.2015.06.012.

OECD, 2018. Real minimum wages - hourly pay period. https://stats.oecd.org/Index.aspx?DataSetCode=RMW

Pantuso, G., 2022. Exact solutions to a carsharing pricing and relocation problem under uncertainty. Computers Operations Research 144, 105802. http://dx.doi.org/https://doi.org/10.1016/j.cor.2022.105802.

Pfrommer, J., Warrington, J., Schildbach, G., Morari, M., 2014. Dynamic vehicle redistribution and online price incentives in shared mobility systems. IEEE Transactions on Intelligent Transportation Systems 15 (4), 1567–1578. http://dx.doi.org/10.1109/TITS.2014.2303986.

Puschmann, T., Alt, R., 2016. Sharing economy. Business and Information Systems Engineering 58 (1), 93–99. http://dx.doi.org/10.1007/s12599-015-0420-2.

Rao, M. R., 1971. Cluster analysis and mathematical programming. Journal of the American Statistical Association 66 (335), 622. http://dx.doi.org/10.2307/2283542.

Reiss, S., Bogenberger, K., 2017. A relocation strategy for Munich's Bike Sharing System: Combining an operator-based and a user-based scheme. Transportation Research Procedia 22, 105–114. http://dx.doi.org/10.1016/j.trpro.2017.03.016.

Remane, G., Nickerson, R. C., Hanelt, A., Tesch, J. F., Kolbe, L. M., 2016. A taxonomy of carsharing business models. 2016 International Conference on Information Systems, ICIS 2016 (December).

Ren, S., Luo, F., Lin, L., Hsu, S. C., LI, X. I., 2019. A novel dynamic pricing scheme for a large-scale electric vehicle sharing network considering vehicle relocation and vehicle-grid-integration. International Journal of Production Economics 218 (July), 339–351. http://dx.doi.org/10.1016/j.ijpe.2019.06.020.

Schiffer, M., Hiermann, G., Rüdel, F., Walther, G., 2021. A polynomial-time algorithm for user-based relocation in free-floating car sharing systems. Transportation Research Part B: Methodological 143, 65–85. http://dx.doi.org/10.1016/j.trb.2020.11.001.

Schmöller, S., Weikl, S., Müller, J., Bogenberger, K., 2015. Empirical analysis of free-floating carsharing usage: The Munich and Berlin case. Transportation Research Part C: Emerging Technologies 56, 34–51. http://dx.doi.org/10.1016/j.trc.2015.03.008.

Schulte, F., Voß, S., 2015. Decision support for environmental-friendly vehicle relocations in freefloating car sharing systems: The case of car2go. Procedia CIRP 30, 275–280. http://dx.doi.org/10.1016/j.procir.2015.02.090.

Serradilla, J., Wardle, J., Blythe, P., Gibbon, J., 2017. An evidence-based approach for investment in rapid-charging infrastructure. Energy Policy 106, 514–524. http://dx.doi.org/10.1016/j.enpol.2017.04.007.

Shaheen, S., Cohen, A., 2016. Innovative mobility carsharing outlook - carsharing market overview, analysis and trends winter 2016. http://innovativemobility.org/wp-content/uploads/2016/02/Innovative-Mobility-Industry-Outlook_World-2016-Final.pdf

Shaheen, S., Cohen, A., 2020. Innovative mobility: Carsharing outlook; carsharing market overview, analysis, and trends. UC Berkeley: Transportation Sustainability Research Centerhttp://dx.doi.org/10.7922/G2125QWJ.

Shaheen, S., Cohen, A., Jaffee, M., 2020. Innovative mobility: Carsharing outlook - worldwide carsharing growth. Transportation Sustainability Research Center - University Of California, Berkeley, 6http://dx.doi.org/10.7922/G2125QWJ.

Shaheen, S., Martin, E., Bansal, A., 2018. One-way electric vehicle carsharing in San Diego: An exploration of the behavioral impacts of pricing incentives on operational efficiency. https://escholarship.org/uc/item/25x091bh

Shaheen, S. A., Cohen, A. P., 2012. Carsharing and personal vehicle services: Worldwide market developments and emerging trends. International Journal of Sustainable Transportation 7 (1), 5–34. http://dx.doi.org/10.1080/15568318.2012.660103.

Shaheen, S. A., Mallery, M. A., Kingsley, K. J., 2012. Personal vehicle sharing services in North America. Research in Transportation Business Management 3, 71–81. http://dx.doi.org/https://doi.org/10.1016/j.rtbm.2012.04.005.

Shaheen, S. A., Sperling, D., Wagner, C., 1999. A short history of carsharing in the 90's. The Journal of World Transport Policy Practice 5 (3), 18 – 40, https://escholarship.org/uc/item/6p3305b0.

Shirk, M., Wishart, J., 2015. Effects of electric vehicle fast charging on battery life and vehicle performance. http://avt.inl.gov

Singla, A., Santoni, M., Bartók, G., Mukerji, P., Meenen, M., Krause, A., 2015. Incentivizing users for balancing bike sharing systems. Proceedings of the National Conference on Artificial Intelligence 1 (June 2014), 723–729. http://dx.doi.org/10.1609/aaai.v29i1.9251.

Sonneberg, M. O., Kühne, K., Breitner, M. H., 2015. A decision support system for the optimization of electric car sharing stations. 2015 International Conference on Information Systems: Exploring the Information Frontier, ICIS 2015, 1–19.

Soppert, M., Steinhardt, C., Müller, C., Gönsch, J., 2022. Differentiated pricing of shared mobility systems considering network effects. Transportation Science 56 (5), 1279–1303. http://dx.doi.org/10.1287/trsc.2022.1131.

Stokkink, P., Geroliminis, N., 2021. Predictive user-based relocation through incentives in one-way car-sharing systems. Transportation Research Part B: Methodological 149, 230–249. http://dx.doi.org/10.1016/j.trb.2021.05.008.

Ströhle, P., Flath, C. M., Gärttner, J., 2019. Leveraging customer flexibility for car-sharing fleet optimization. Transportation Science 53 (1), 42–61. http://dx.doi.org/10.1287/trsc.2017.0813.

Tchervenkov, C., Balać, M., Hörl, S., Becker, H., Axhausen, K. W., 2019. How much parking space can carsharing save? https://www.research-collection.ethz.ch/handle/20.500.11850/293883

Tesla, 2020. Supercharger — tesla. https://www.tesla.com/supercharger?redirect=no

Trucks, R., 2020. Leading the charge, digital image. http://www.renault-trucks.co.uk/leading-the-charge

Trueck, S., Svetlozar, R. R., 2009. Rating Based Modeling of Credit Risk. Elsevier Inc. http://dx.doi.org/https://doi.org/10.1016/B978-0-12-373683-3.X0001-2.

Uesugi, K., Mukai, N., Watanabe, T., 2007. Optimization of vehicle assignment for car sharing system. Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics) 4693 LNAI (PART 2), 1105–1111. http://dx.doi.org/10.1007/978-3-540-74827-4$_1$38.

U.S. DOE (United States Department of Energy), 2020. Fuel economy guide model year 2020. https://www.fueleconomy.gov/feg/pdfs/guides/FEG2020.pdf

Vine, S. L., Polak, J., 2019. The impact of free-floating carsharing on car ownership: Early-stage findings from London. Transport Policy 75, 119–127. http://dx.doi.org/10.1016/j.tranpol.2017.02.004.

Vives, A., Jacob, M., Payeras, M., 2018. Revenue management and price optimization techniques in the hotel sector: A critical literature review. Tourism Economics 24 (6), 720–752. http://dx.doi.org/10.1177/1354816618777590.

Wagner, S., Willing, C., Brandt, T., Neumann, D., 2015. Data analytics for location-based services: Enabling user-based relocation of carsharing vehicles. 2015 International Conference on Information Systems: Exploring the Information Frontier, ICIS 2015 (December). http://dx.doi.org/10.13140/RG.2.1.1595.6966.

Wang, H., Yang, H., 2019. Ridesourcing systems: A framework and review. Transportation Research Part B: Methodological 129, 122–155. http://dx.doi.org/10.1016/j.trb.2019.07.009.

Wang, L., Jin, Y., Wang, L., Ma, W., Li, T., 2019. Incentive-based approach to control demands for operation of one-way carsharing system. Transportation Research Record 2673 (4), 427–438. http://dx.doi.org/10.1177/0361198119837497.

Wang, L., Ma, W., 2019. Pricing approach to balance demands for one-way carsharing systems. 2019 IEEE Intelligent Transportation Systems Conference, ITSC 2019, 1697–1702http://dx.doi.org/10.1109/ITSC.2019.8917414.

Wang, L., Ma, W., Wang, M., Qu, X., 2021a. Demand control model with combinatorial incentives and surcharges for one-way carsharing operation. Transportation Research Part C: Emerging Technologies 125 (November 2019), 102999. http://dx.doi.org/10.1016/j.trc.2021.102999.

Wang, N., Guo, J., Liu, X., Fang, T., 2020. A service demand forecasting model for one-way electric car-sharing systems combining long short-term memory networks with Granger causality test. Journal of Cleaner Production 244. http://dx.doi.org/10.1016/j.jclepro.2019.118812.

Wang, N., Jia, S., Liu, Q., 2021b. A user-based relocation model for one-way electric carsharing system based on micro demand prediction and multi-objective optimization. Journal of Cleaner Production 296. http://dx.doi.org/10.1016/j.jclepro.2021.126485.

Wang, N., Liu, Q., Guo, J., Fang, T., 2022. A user-based adaptive joint relocation model combining electric car-sharing and bicycle-sharing. Transportmetrica B: Transport Dynamics 10 (1), 1046–1069. http://dx.doi.org/10.1080/21680566.2021.2007174.

Waserhole, A., Jost, V., 2016. Pricing in vehicle sharing systems: Optimization in queuing networks with product forms. EURO Journal on Transportation and Logistics 5 (3), 293–320. http://dx.doi.org/10.1007/s13676-014-0054-4.

Waserhole, A., Jost, V., Brauner, N., 2013. Pricing techniques for self regulation in vehicle sharing systems. Electronic Notes in Discrete Mathematics 41, 149–156. http://dx.doi.org/10.1016/j.endm.2013.05.087.

Weikl, S., Bogenberger, K., 2013. Relocation strategies and algorithms for free-floating car sharing systems. IEEE Intelligent Transportation Systems Magazine 5 (4), 100–111. http://dx.doi.org/10.1109/MITS.2013.2267810.

Willing, C., Klemmer, K., Brandt, T., Neumann, D., 2017. Moving in time and space – Location intelligence for carsharing decision support. Decision Support Systems 99, 75–85. http://dx.doi.org/10.1016/j.dss.2017.05.005.

Wu, C., Vine, S. L., Philips, S., Tang, W., Polak, J., 2020. Free-floating carsharing users' willingness-to-pay/accept for logistics management mechanisms. Travel Behaviour and Society 21 (June), 154–166. http://dx.doi.org/10.1016/j.tbs.2020.06.008.

Wu, C., Vine, S. L., Sivakumar, A., Polak, J., 2019. Traveller preferences for free-floating carsharing vehicle allocation mechanisms. Transportation Research Part C: Emerging Technologies 102 (February), 1–19. http://dx.doi.org/10.1016/j.trc.2019.02.019.

Wu, C., Vine, S. L., Sivakumar, A., Polak, J., 2022. Dynamic pricing of free-floating carsharing networks with sensitivity to travellers' attitudes towards risk. Transportation 49 (2), 679–702. http://dx.doi.org/10.1007/s11116-021-10190-8.

Wu, T., Xu, M., 2022. Modeling and optimization for carsharing services: A literature review. Multimodal Transportation 1 (3), 100028. http://dx.doi.org/10.1016/j.multra.2022.100028.

Wu, Y., Chen, M., Wang, X., 2021. Incentivized self-rebalancing fleet in electric vehicle sharing. IISE Transactions 54 (2), 173–185. http://dx.doi.org/10.1080/24725854.2021.1928340.

Xu, M., Meng, Q., 2019. Fleet sizing for one-way electric carsharing services considering dynamic vehicle relocation and nonlinear charging profile. Transportation Research Part B: Methodological 128, 23–49. http://dx.doi.org/10.1016/j.trb.2019.07.016.

Xu, M., Meng, Q., Liu, Z., 2018. Electric vehicle fleet size and trip pricing for one-way carsharing services considering vehicle relocation and personnel assignment. Transportation Research Part B: Methodological 111, 60–82. http://dx.doi.org/10.1016/j.trb.2018.03.001.

Xu, M., Wu, T., Tan, Z., 2021. Electric vehicle fleet size for carsharing services considering on-demand charging strategy and battery degradation. Transportation Research Part C: Emerging Technologies 127. http://dx.doi.org/10.1016/j.trc.2021.103146.

Yang, S., Wu, J., Sun, H., Qu, Y., Wang, D. Z., 2022. Integrated optimization of pricing and relocation in the competitive carsharing market: A multi-leader-follower game model. Transportation Research Part C: Emerging Technologies 138 (February), 103613. http://dx.doi.org/10.1016/j.trc.2022.103613.

Yao, Z., Gendreau, M., Li, M., Ran, L., Wang, Z., 2022. Service operations of electric vehicle carsharing systems from the perspectives of supply and demand: A literature review. Transportation Research Part C: Emerging Technologies 140 (January 2021), 103702. http://dx.doi.org/10.1016/j.trc.2022.103702.

Yoon, T., Cherry, C. R., Jones, L. R., 2017. One-way and round-trip carsharing: A stated preference experiment in Beijing. Transportation Research Part D: Transport and Environment 53, 102–114. http://dx.doi.org/10.1016/j.trd.2017.04.009.

Zhang, A., Kang, J. E., Kwon, C., 2017. Incorporating demand dynamics in multi-period capacitated fast-charging location planning for electric vehicles. Transportation Research Part B: Methodological 103, 5–29. http://dx.doi.org/10.1016/j.trb.2017.04.016.

Zhang, D., Liu, Y., He, S., 2019. Vehicle assignment and relays for one-way electric car-sharing systems. Transportation Research Part B: Methodological 120, 125–146. http://dx.doi.org/10.1016/j.trb.2018.12.004.

Zhang, S., Sun, H., Wang, X., Lv, Y., Wu, J., 2022. Optimization of personalized price discounting scheme for one-way station-based carsharing systems. European Journal of Operational Research 303 (1), 220–238. http://dx.doi.org/10.1016/j.ejor.2022.02.026.

Zoepf, S. M., Keith, D. R., 2016. User decision-making and technology choices in the U.S. carsharing market. Transport Policy 51, 150–157. http://dx.doi.org/10.1016/j.tranpol.2016.01.010.

Zou, Y., Yang, C., Yang, J., 2020. EV-sharing station location and vehicle relocation with parking rewards and punishments mechanism. International Journal of Sustainable Transportationhttp://dx.doi.org/10.1080/15568318.2020.1750076.