# Outage Performance Enhancement for NOMA Based Cooperative Relay Sharing Networks

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***Abstract*—This letter considers a non-orthogonal multiple access (NOMA) based cooperative relay sharing (CRS) network, where two sources communicate with their corresponding users over the same time and frequency via a shared decode-and-forward relay. A novel transmission scheme using max-min criterion based dynamic decoding order strategy is proposed to minimize the outage probability of the network at the cost of lower complexity and overhead. The closed-form expression of the overall outage probability for the proposed scheme is derived. Both analytical and simulation results show that the proposed scheme can achieve non-zero diversity order and almost the same outage performance as the dynamic power allocation based transmission scheme for NOMA based CRS networks.**

***Index Terms*—Cooperative relaying, non-orthogonal multiple access, outage probability, performance analysis.**

## I. Introduction

Non-orthogonal multiple access (NOMA) has been widely regarded as a promising candidate technology to achieve high spectral efficiency for next generation wireless networks [1]. To eliminate the effect of fading and to gain spatial degrees of freedom, NOMA has been combined with cooperative communications, which leads to a remarkable research field named cooperative NOMA.

Cooperative NOMA can be generally classified into two categories, i.e. user relaying and dedicated relaying [2]. In [3], the users with stronger channels served as relays to help the other users since they can obtain the prior information of the others when conducting successive interference cancellation (SIC). Moreover, due to the absence of direct links between the source and users, a dedicated decode-and-forward (DF) or amplify-and-forward (AF) relay was utilized to establish the communications in [4] and [5], respectively. To further improve the performance of cooperative NOMA, multiple relays are considered to be deployed in networks, and thus relay selection (RS) has drawn a lot of attention for its good character to harvest the full diversity gain while reducing the system complexity [6]–[8]. The initial work on RS for NOMA was investigated in [6], where an optimal relay was selected based on a two-stage max-min criterion, and fixed power allocation (PA) was adopted. Afterwards, the works in [7] and [8] were proposed to improve the performance of the scheme in [6], where dynamic PA and fixed decoding order strategy and dynamic PA and dynamic decoding order strategy were performed, respectively.

It is worth noting that there exists a typical cooperative NOMA transmission scenario as described in [9]–[11], where multiple sources would like to communicate with their respective users in parallel through a shared relay. This type of cooperative NOMA is also called NOMA based cooperative relay sharing (CRS). The superiority of it over orthogonal multiple access (OMA) in terms of ergodic capacity has been verified in [9], [10]. However, it is indicated that only zero diversity order is obtained for NOMA based CRS (denoted as NOMA-CRS) in the high signal-to-noise ratio (SNR) region if the conventional fixed PA based transmission scheme is employed [10]. Despite the fact that dynamic PA may break through this bottleneck under the same scenario as depicted in [11], higher complexity and system overhead will be produced as well, since the transmitters need to inform the receivers the PA coefficients if channel state changes, or the instantaneous channel state information (CSI) needs to be transferred between different sources. Therefore, a feasible transmission scheme for NOMA-CRS networks aimed at improving outage performance is desired, which motivates the study of this work.

In this letter, different from the existing fixed decoding order strategy based transmission schemes in [9]–[11], a novel NOMA-CRS scheme to minimize the overall outage probability is proposed, where the decoding order is designed to be dynamic and determined by an established max-min criterion. Despite the fact that there have been some existing works involving dynamic decoding order, they are either not optimal or not directly applicable in the considered scenario. For examples, the dynamic decoding orders used in [8] is suitable for the situations where user priorities exist; the dynamic decoding order proposed in [12] is just suboptimal and can not guarantee that non-zero diversity order can always be achieved. To characterize the performance of the proposed scheme, we derive the closed-form expression for the outage probability. The analytical result demonstrates that non-zero diversity order can always be obtained for the proposed scheme. Moreover, the provided simulation results show that the proposed scheme outperforms the existing fixed PA based NOMA-CRS scheme in the whole SNR region, achieves similar outage performance to the existing dynamic PA based NOMA-CRS scheme, and has a better outage behavior than the OMA based CRS (denoted as OMA-CRS) scheme if appropriate PA coefficients are chosen.

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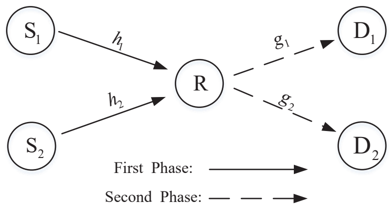


Fig. 1: A system model description of NOMA-CRS networks with two source-user pairs.

## II. System Model

Consider a similar cooperative NOMA network to the ones in [9], [11], as illustrated in Fig. 1, where two source-user pairs $\mathrm{S}_1-\mathrm{D}_1$ and $\mathrm{S}_2-\mathrm{D}_2$ are established via a shared DF relay R. All nodes are equipped with a single antenna and work in a half-duplex mode. It is assumed that the direct links from $\mathrm{S}_1$ to $\mathrm{D}_1$ and from $\mathrm{S}_2$ to $\mathrm{D}_2$ are absent due to fading and/or physical obstacles. Therefore, two successive phases are required for one information transmission. Moreover, all wireless channels are assumed to experience identically and independent Rayleigh fading, thus the channel coefficients for the links $\mathrm{S}_1\rightarrow\mathrm{R}$, $\mathrm{S}_2\rightarrow\mathrm{R}$, $\mathrm{R}\rightarrow\mathrm{D}_1$ and $\mathrm{R}\rightarrow\mathrm{D}_2$ can be expressed as $h_1 \sim CN\left(0,\Omega_1=d_{\mathrm{S_1R}}^{-\alpha}\right)$, $h_2 \sim CN\left(0,\Omega_2=d_{\mathrm{S_2R}}^{-\alpha}\right)$, $g_1 \sim CN\left(0,\Omega_3=d_{\mathrm{RD_1}}^{-\alpha}\right)$ and $g_2 \sim CN\left(0,\Omega_4=d_{\mathrm{RD_2}}^{-\alpha}\right)$, respectively, where $\alpha$ is the path loss exponent and $d$ denotes the distance measured in meters [9], [11].

In the first phase, both $\mathrm{S}_1$ and $\mathrm{S}_2$ transmit their individual signals $s_1$ and $s_2$ to R with the powers $a_1P_t$ and $a_2P_t$ simultaneously, where $P_t$ denotes the total transmit power, $a_1$ and $a_2$ represent the PA coefficients satisfying $a_1+a_2=1$. Thus, the superposed signal received at R can be expressed as

$$y_{\mathrm{R}}=\sqrt{P_t}\left(\sqrt{a_1}s_1h_1+\sqrt{a_2}s_2h_2\right)+n_{\mathrm{R}}, \tag{1}$$

where $n_{\mathrm{R}}\sim CN\left(0,\sigma_{\mathrm{R}}^2\right)$ denotes the additive white Gaussian noise (AWGN) at R.

In the second phase, assume that R can decode the superposed signal correctly, then R will transmit the re-encoded signals $\hat{s}_1$ and $\hat{s}_2$ to $\mathrm{D}_1$ and $\mathrm{D}_2$, respectively, and the received superimposed signal at $\mathrm{D_i}$ $(i=1,2)$ can be given by

$$y_{\mathrm{D_i}}=\sqrt{P_r}g_i\left(\sqrt{b_1}\hat{s}_1+\sqrt{b_2}\hat{s}_2\right)+n_{\mathrm{D_i}}, \tag{2}$$

where $P_r$ denotes the transmit power, $b_1$ and $b_2$ are the PA coefficients with $b_1+b_2=1$, $n_{\mathrm{D_i}}\sim CN\left(0,\sigma_{\mathrm{D_i}}^2\right)$ denotes the AWGN at $\mathrm{D_i}$. For mathematical tractability, it is assumed that $P_t=P_r=P$ and $\sigma_{\mathrm{R}}^2=\sigma_{\mathrm{D_i}}^2=\sigma^2$ as in [9], [10].

After that, both $\mathrm{D}_1$ and $\mathrm{D}_2$ will decode their received superimposed signal according a specific decoding order strategy, which will be discussed in detail in later sections.

## III. Decoding Order Strategy

Unlike the fixed decoding order strategy used in [9], [11], where the decoding order at R and at the users are determined according to the statistical CSI, a max-min criterion based dynamic decoding order strategy using instantaneous CSI is developed and utilized in our proposed scheme. To be specific, we denote the general form of decoding order at R as $(\varphi_1,\varphi_2)\in\{(1,2),(2,1)\}$, then the available rates for R to decode $\mathrm{D}_{\varphi_1}$ and $\mathrm{D}_{\varphi_2}$'s signals can be expressed as

$$R_{\varphi_1}^u=\frac{1}{2}\log_2\left(1+\frac{a_{\varphi_1}\rho|h_{\varphi_1}|^2}{a_{\varphi_2}\rho|h_{\varphi_2}|^2+1}\right), \tag{3}$$

$$R_{\varphi_2}^u=\frac{1}{2}\log_2\left(1+a_{\varphi_2}\rho|h_{\varphi_2}|^2\right), \tag{4}$$

respectively, where $\rho=\frac{P}{\sigma^2}$ represents the SNR.

The target rates of $\mathrm{D}_{\varphi_1}$ and $\mathrm{D}_{\varphi_2}$ are denoted as $R_{\varphi_1}$ and $R_{\varphi_2}$, respectively. In order to avoid outage at R, $R_{\varphi_1}^u\geq R_{\varphi_1}$ and $R_{\varphi_2}^u\geq R_{\varphi_2}$ should always be satisfied, which can lead to $\frac{a_{\varphi_1}\rho|h_{\varphi_1}|^2}{a_{\varphi_2}\rho\varepsilon_{\varphi_1}|h_{\varphi_2}|^2+\varepsilon_{\varphi_1}}\geq 1$ and $\frac{a_{\varphi_2}\rho|h_{\varphi_2}|^2}{\varepsilon_{\varphi_2}}\geq 1$, where $\varepsilon_{\varphi_1}=2^{2R_{\varphi_1}}-1$, $\varepsilon_{\varphi_2}=2^{2R_{\varphi_2}}-1$. Based on aforementioned facts, a generic expression is derived for further comparison of different decoding order modes, which can be expressed as

$$G\left(m,n\right)=\min\left\{\frac{a_m\rho|h_m|^2}{a_n\rho\varepsilon_m|h_n|^2+\varepsilon_m},\frac{a_n\rho|h_n|^2}{\varepsilon_n}\right\}, \tag{5}$$

where $(m,n)\in\{(1,2),\ (2,1)\}$.

Then, the adopted decoding order at R can be determined by

$$(\varphi_1,\varphi_2)=\arg\max\left\{G\left(1,2\right),G\left(2,1\right)\right\}, \tag{6}$$

which is to minimize the outage probability at R by choosing a proper decoding order mode.

On the other hand, the general form of decoding order at the users are denoted as $(\lambda_1,\lambda_2)\in\{(1,2),(2,1)\}$. According to the basic principle of NOMA, $\mathrm{D}_{\lambda_1}$ only needs to decode its own signal, while $\mathrm{D}_{\lambda_2}$ needs to decode both users' signals. Thus, the achievable rates for $\mathrm{D}_{\lambda_1}$ to decode $\hat{s}_1$ and $\mathrm{D}_{\lambda_2}$ to decode both $\hat{s}_1$ and $\hat{s}_2$ can be respectively written as

$$R_{\lambda_1}^d=\frac{1}{2}\log_2\left(1+\frac{b_{\lambda_1}\rho|g_{\lambda_1}|^2}{b_{\lambda_2}\rho|g_{\lambda_1}|^2+1}\right), \tag{7}$$

$$R_{\lambda_2\rightarrow\lambda_1}^d=\frac{1}{2}\log_2\left(1+\frac{b_{\lambda_1}\rho|g_{\lambda_2}|^2}{b_{\lambda_2}\rho|g_{\lambda_2}|^2+1}\right), \tag{8}$$

$$R_{\lambda_2}^d=\frac{1}{2}\log_2\left(1+b_{\lambda_2}\rho|g_{\lambda_2}|^2\right). \tag{9}$$

In this phase, the target rates of $\mathrm{D}_{\lambda_1}$ and $\mathrm{D}_{\lambda_2}$ can be denoted as $R_{\lambda_1}$ and $R_{\lambda_2}$, respectively. $\min\left\{R_{\lambda_1}^d,R_{\lambda_2\rightarrow\lambda_1}^d\right\}\geq R_{\lambda_1}$ and $R_{\lambda_2}^d\geq R_{\lambda_2}$ should be satisfied in order to avoid outage. As a result, there are several special cases described as follows.

(1) When $\frac{b_1}{b_2}\leq\varepsilon_1$ and $\frac{b_2}{b_1}\leq\varepsilon_2$, the outage will always occur since $\min\left\{R_{\lambda_1}^d,R_{\lambda_2\rightarrow\lambda_1}^d\right\}\geq R_{\lambda_1}$ can not be satisfied.

(2) When $\frac{b_1}{b_2}>\varepsilon_1$ and $\frac{b_2}{b_1}\leq\varepsilon_2$, only $(\lambda_1,\lambda_2)=(1,2)$ is feasible and possible to realize $\min\left\{R_{\lambda_1}^d,R_{\lambda_2\rightarrow\lambda_1}^d\right\}\geq R_{\lambda_1}$.

(3) When $\frac{b_1}{b_2}\leq\varepsilon_1$ and $\frac{b_2}{b_1}>\varepsilon_2$, only $(\lambda_1,\lambda_2)=(2,1)$ is feasible and possible to achieve $\min\left\{R_{\lambda_1}^d,R_{\lambda_2\rightarrow\lambda_1}^d\right\}\geq R_{\lambda_1}$.

(4) When $\frac{b_1}{b_2}>\varepsilon_1$ and $\frac{b_2}{b_1}>\varepsilon_2$, both $(\lambda_1,\lambda_2)=(1,2)$ and $(\lambda_1,\lambda_2)=(2,1)$ can be set to achieve possible $\min\left\{R_{\lambda_1}^d,R_{\lambda_2\rightarrow\lambda_1}^d\right\}\geq R_{\lambda_1}$. Then, by following the same analysis as that for the first phase, the equation listed below is established.

$$W\left(u,v\right)=\min\left\{\frac{z(b_u\rho-b_v\varepsilon_u\rho)}{\varepsilon_u},\frac{b_v\rho|g_v|^2}{\varepsilon_v}\right\}, \tag{10}$$

where $(u,v)\in\{(1,2),\ (2,1)\}$, $z=\min\left\{|g_1|^2,|g_2|^2\right\}$. Finally, the adopted decoding order in this case can be determined by

$$(\lambda_1,\lambda_2)=\arg\max\left\{W(1,2),W(2,1)\right\}. \tag{11}$$

## IV. Performance Analysis

In this section, the outage probability and diversity order of the proposed scheme are investigated to characterize the performance.

### A. Outage Probability for the First Phase

Based on the facts mentioned in Section III, the outage probability for the first phase can be expressed as

$$\mathrm{P}^1_{\mathrm{out}}=\Pr\left\{\max\left\{G(1,2),G(2,1)\right\}<1\right\}. \tag{12}$$

It is found that the above equation consists of six specific cases, which can be written as

$$\begin{aligned}
&\mathrm{P}^1_{\mathrm{o}}=\Pr\left\{|h_1|^2\geq\frac{a_2\rho\varepsilon_1|h_2|^2+\varepsilon_1}{a_1\rho},|h_2|^2<\frac{\varepsilon_2}{a_2\rho}\right\},\\
&\mathrm{P}^2_{\mathrm{o}}=\Pr\left\{|h_1|^2<\frac{\varepsilon_1}{a_1\rho},\frac{\varepsilon_2}{a_2\rho}\leq|h_2|^2<\frac{a_1\rho\varepsilon_2|h_1|^2+\varepsilon_2}{a_2\rho}\right\},\\
&\mathrm{P}^3_{\mathrm{o}}=\Pr\left\{|h_1|^2<\frac{\varepsilon_1}{a_1\rho},|h_2|^2\geq\frac{a_1\rho\varepsilon_2|h_1|^2+\varepsilon_2}{a_2\rho}\right\},\\
&\mathrm{P}^4_{\mathrm{o}}=\Pr\left\{\frac{\varepsilon_1}{a_1\rho}\leq|h_1|^2<\frac{a_2\rho\varepsilon_1|h_2|^2+\varepsilon_1}{a_1\rho},|h_2|^2<\frac{\varepsilon_2}{a_2\rho}\right\},\\
&\mathrm{P}^5_{\mathrm{o}}=\Pr\left\{\frac{\varepsilon_1}{a_1\rho}\leq|h_1|^2<\frac{a_2\rho\varepsilon_1|h_2|^2+\varepsilon_1}{a_1\rho},\frac{\varepsilon_2}{a_2\rho}\leq|h_2|^2<\frac{a_1\rho\varepsilon_2|h_1|^2+\varepsilon_2}{a_2\rho}\right\},\\
&\mathrm{P}^6_{\mathrm{o}}=\Pr\left\{|h_1|^2<\frac{\varepsilon_1}{a_1\rho},|h_2|^2<\frac{\varepsilon_2}{a_2\rho}\right\}.
\end{aligned} \tag{13}$$

For $\mathrm{P}^1_{\mathrm{o}}$, from $|h_1|^2\geq\frac{a_2\rho\varepsilon_1|h_2|^2+\varepsilon_1}{a_1\rho}$, we can derive $|h_2|^2\leq\frac{a_1\rho|h_1|^2-\varepsilon_1}{a_2\rho\varepsilon_1}$, then $\frac{a_1\rho|h_1|^2-\varepsilon_1}{a_2\rho\varepsilon_1}$ and $\frac{\varepsilon_2}{a_2\rho}$ should be compared.

When $\frac{a_1\rho|h_1|^2-\varepsilon_1}{a_2\rho\varepsilon_1}\geq\frac{\varepsilon_2}{a_2\rho}$, we have $|h_1|^2\geq\frac{\eta_1}{a_1\rho}$, where $\eta_1=\varepsilon_1+\varepsilon_1\varepsilon_2$. As a result, $\mathrm{P}^1_{\mathrm{o}}$ in this case (denoted as $\mathrm{P}^{1,1}_{\mathrm{o}}$) can be derived as

$$\begin{aligned}
\mathrm{P}^{1,1}_{\mathrm{o}}&=\Pr\left\{|h_1|^2\geq\frac{\eta_1}{a_1\rho},|h_2|^2<\frac{\varepsilon_2}{a_2\rho}\right\}\\
&=\left(1-\mathrm{F}_{|h_1|^2}\left(\frac{\eta_1}{a_1\rho}\right)\right)\mathrm{F}_{|h_2|^2}\left(\frac{\varepsilon_2}{a_2\rho}\right)\\
&=e^{-\frac{\eta_1}{\Omega_1a_1\rho}}-e^{-\frac{\eta_1}{\Omega_1a_1\rho}}e^{-\frac{\varepsilon_2}{\Omega_2a_2\rho}}.
\end{aligned} \tag{14}$$

When $\frac{a_1\rho|h_1|^2-\varepsilon_1}{a_2\rho\varepsilon_1}<\frac{\varepsilon_2}{a_2\rho}$, we have $|h_1|^2<\frac{\eta_1}{a_1\rho}$. Also, $\mathrm{P}^1_{\mathrm{o}}$ in this case (denoted as $\mathrm{P}^{1,2}_{\mathrm{o}}$) can be derived as

$$\begin{aligned}
\mathrm{P}^{1,2}_{\mathrm{o}}&=\Pr\left\{\frac{\varepsilon_1}{a_1\rho}\leq|h_1|^2<\frac{\eta_1}{a_1\rho},|h_2|^2<\frac{\varepsilon_2}{a_2\rho}\right\}\\
&=\left(\mathrm{F}_{|h_1|^2}\left(\frac{\eta_1}{a_1\rho}\right)-\mathrm{F}_{|h_1|^2}\left(\frac{\varepsilon_1}{a_1\rho}\right)\right)\mathrm{F}_{|h_2|^2}\left(\frac{\varepsilon_2}{a_2\rho}\right)\\
&=e^{-\frac{\varepsilon_1}{\Omega_1a_1\rho}}-e^{-\frac{\eta_1}{\Omega_1a_1\rho}}+\frac{\Omega_2a_2\varepsilon_1}{\Omega_1a_1+\Omega_2a_2\varepsilon_1}e^{\frac{1}{\Omega_2a_2\rho}}\\
&\times\left(e^{-\frac{\Omega_1a_1\eta_1+\Omega_2a_2\eta_1}{\Omega_1\Omega_2a_1a_2\varepsilon_1\rho}}-e^{-\frac{\Omega_1a_1+\Omega_2a_2}{\Omega_1\Omega_2a_1a_2\rho}}\right).
\end{aligned} \tag{15}$$

Based on the above analysis, the final expression of $\mathrm{P}^1_{\mathrm{o}}$ can be obtained by combing $\mathrm{P}^{1,1}_{\mathrm{o}}$ and $\mathrm{P}^{1,2}_{\mathrm{o}}$, i.e. $\mathrm{P}^1_{\mathrm{o}}=\mathrm{P}^{1,1}_{\mathrm{o}}+\mathrm{P}^{1,2}_{\mathrm{o}}$.

As for $\mathrm{P}^2_{\mathrm{o}}$, $\mathrm{P}^3_{\mathrm{o}}$, $\mathrm{P}^4_{\mathrm{o}}$, $\mathrm{P}^5_{\mathrm{o}}$ and $\mathrm{P}^6_{\mathrm{o}}$, they can be calculated according to the similar process of deriving $\mathrm{P}^1_{\mathrm{o}}$, thus the details of the calculation are omitted herein due to the limited space.

Finally, by combing the results of six cases, the final expression of $\mathrm{P}^1_{\mathrm{out}}$ can be expressed as

$$\mathrm{P}^1_{\mathrm{out}}=\mathrm{P}^1_{\mathrm{o}}+\mathrm{P}^2_{\mathrm{o}}+\mathrm{P}^3_{\mathrm{o}}+\mathrm{P}^4_{\mathrm{o}}+\mathrm{P}^5_{\mathrm{o}}+\mathrm{P}^6_{\mathrm{o}}, \tag{16}$$

and the complete form of $\mathrm{P}^1_{\mathrm{out}}$ is shown in (17), at the top of the next page.

### B. Outage Probability for the Second Phase

It has been concluded that there are four cases regarding different settings of fixed PA coefficients in Section III, thus we will discuss the outage probability of each case orderly in the following.

(1) When $\frac{b_1}{b_2}\leq\varepsilon_1$ and $\frac{b_2}{b_1}\leq\varepsilon_2$, the outage probability for the second phase is $\mathrm{P}^2_{\mathrm{out}}=1$.

(2) When $\frac{b_1}{b_2}>\varepsilon_1$ and $\frac{b_2}{b_1}\leq\varepsilon_2$, $\mathrm{P}^2_{\mathrm{out}}$ can be rewritten as $\mathrm{P}^2_{\mathrm{out}}=\Pr\left\{W(1,2)<1\right\}$. This time we will directly calculate the coverage probability instead of calculating the outage probability for simplicity, thus $\mathrm{P}^2_{\mathrm{cov}}=\Pr\left\{W(1,2)\geq1\right\}$ is required, which can be derived as follows.

When $|g_1|^2\leq|g_2|^2$, we have

$$\begin{aligned}
\mathrm{P}^{2,1}_{\mathrm{cov}}&=\Pr\left\{|g_1|^2\leq|g_2|^2,|g_1|^2\geq\frac{\varepsilon_1}{b_1\rho-b_2\varepsilon_1\rho},|g_2|^2\geq\frac{\varepsilon_2}{b_2\rho}\right\}\\
&=\int_{\nu_1}^{+\infty}\frac{1}{\Omega_4}e^{-\frac{x}{\Omega_4}}dx\int_{\frac{\varepsilon_1}{b_1\rho-b_2\varepsilon_1\rho}}^{x}\frac{1}{\Omega_3}e^{-\frac{y}{\Omega_3}}dy\\
&=e^{-\frac{\varepsilon_1}{\Omega_3(b_1\rho-b_2\varepsilon_1\rho)}}e^{-\frac{\nu_1}{\Omega_4}}-\frac{\Omega_3}{\Omega_3+\Omega_4}e^{-\frac{(\Omega_3+\Omega_4)\nu_1}{\Omega_3\Omega_4}},
\end{aligned} \tag{18}$$

where $\nu_1=\max\left(\frac{\varepsilon_1}{b_1\rho-b_2\varepsilon_1\rho},\frac{\varepsilon_2}{b_2\rho}\right)$.

When $|g_1|^2>|g_2|^2$, we have

$$\begin{aligned}
\mathrm{P}^{2,2}_{\mathrm{cov}}&=\Pr\left\{|g_1|^2>|g_2|^2,|g_2|^2\geq\frac{\varepsilon_1}{b_1\rho-b_2\varepsilon_1\rho},|g_2|^2\geq\frac{\varepsilon_2}{b_2\rho}\right\}\\
&=\int_{\nu_1}^{+\infty}\frac{1}{\Omega_4}e^{-\frac{x}{\Omega_3}}e^{-\frac{x}{\Omega_4}}dx=\frac{\Omega_3}{\Omega_3+\Omega_4}e^{-\frac{(\Omega_3+\Omega_4)\nu_1}{\Omega_3\Omega_4}}.
\end{aligned} \tag{19}$$

Then, by combing $\mathrm{P}^{2,1}_{\mathrm{cov}}$ and $\mathrm{P}^{2,2}_{\mathrm{cov}}$ from (18) and (19), we can have

$$\begin{aligned}
\mathrm{P}^2_{\mathrm{out}}&=1-\mathrm{P}^2_{\mathrm{cov}}=1-\left(\mathrm{P}^{2,1}_{\mathrm{cov}}+\mathrm{P}^{2,2}_{\mathrm{cov}}\right)\\
&=1-e^{-\frac{\varepsilon_1}{\Omega_3(b_1\rho-b_2\varepsilon_1\rho)}}e^{-\frac{\nu_1}{\Omega_4}}.
\end{aligned} \tag{20}$$

(3) When $\frac{b_1}{b_2}\leq\varepsilon_1$ and $\frac{b_2}{b_1}>\varepsilon_2$, $\mathrm{P}^2_{\mathrm{out}}$ can be rewritten as $\mathrm{P}^2_{\mathrm{out}}=\Pr\left\{W(2,1)<1\right\}$. Then, by following the similar procedure of calculating $\mathrm{P}^2_{\mathrm{out}}$ in case (2), the final expression of $\mathrm{P}^2_{\mathrm{out}}$ in this case can be given by

$$\mathrm{P}^2_{\mathrm{out}}=1-e^{-\frac{\varepsilon_2}{\Omega_4(b_2\rho-b_1\varepsilon_2\rho)}}e^{-\frac{\nu_2}{\Omega_3}}, \tag{21}$$

where $\nu_2=\max\left(\frac{\varepsilon_2}{b_2\rho-b_1\varepsilon_2\rho},\frac{\varepsilon_1}{b_1\rho}\right)$.

(4) When $\frac{b_1}{b_2}>\varepsilon_1$ and $\frac{b_2}{b_1}>\varepsilon_2$, $\mathrm{P}^2_{\mathrm{out}}$ can be rewritten as $\mathrm{P}^2_{\mathrm{out}}=\Pr\left\{\max\left\{W(1,2),W(2,1)\right\}<1\right\}$, which can be further expressed as

$$\mathrm{P}^2_{\mathrm{out}}=1-\left(\mathrm{P}^2_{\mathrm{cov}}+\hat{\mathrm{P}}^2_{\mathrm{cov}}-\tilde{\mathrm{P}}^2_{\mathrm{cov}}\right), \tag{22}$$

$$\mathrm{P}_{\mathrm{out}}^1=\frac{\Omega_2a_2}{\Omega_1a_1\varepsilon_2+\Omega_2a_2}e^{-\frac{\varepsilon_1}{\Omega_2a_2\rho}}\left(e^{-\frac{\Omega_1a_1\varepsilon_2\eta_1+\Omega_2a_2\eta_1}{\Omega_1\Omega_2a_1a_2(1-\varepsilon_1\varepsilon_2)\rho}}-e^{-\frac{\Omega_1a_1\varepsilon_2\eta_1+\Omega_2a_2\eta_1}{\Omega_1\Omega_2a_1a_2\rho}}\right)-e^{-\frac{\varepsilon_2}{\Omega_2a_2\rho}}e^{-\frac{\eta_1}{\Omega_1a_1\rho}}+1+\frac{\Omega_2a_2\varepsilon_1}{\Omega_1a_1+\Omega_2a_2\varepsilon_1}e^{\frac{1}{\Omega_2a_2\rho}}$$
$$\times\left(e^{-\frac{\Omega_1a_1\eta_1+\Omega_2a_2\varepsilon_1\eta_1}{\Omega_1\Omega_2a_1a_2\varepsilon_1\rho}}-e^{-\frac{\Omega_1a_1\eta_1+\Omega_2a_2\varepsilon_1\eta_1}{\Omega_1\Omega_2a_1a_2\varepsilon_1(1-\varepsilon_1\varepsilon_2)\rho}}\right)+\frac{\Omega_2a_2}{\Omega_1a_1\varepsilon_2+\Omega_2a_2}e^{-\frac{\varepsilon_2}{\Omega_2a_2\rho}}\left(e^{-\frac{\Omega_1a_1\varepsilon_2\eta_1+\Omega_2a_2\eta_1}{\Omega_1\Omega_2a_1a_2\rho}}-e^{-\frac{\Omega_1a_1\varepsilon_1\varepsilon_2+\Omega_2a_2\varepsilon_1}{\Omega_1\Omega_2a_1a_2\rho}}\right). \quad (17)$$

where $\mathrm{P}_{\mathrm{cov}}^2$ has been presented in case (2), $\hat{\mathrm{P}}_{\mathrm{cov}}^2$ denotes the coverage probability of case (3) and $\tilde{\mathrm{P}}_{\mathrm{cov}}^2$ can be expressed as

$$\tilde{\mathrm{P}}_{\mathrm{cov}}^2=\Pr\left\{W(1,2)\geq 1, W(2,1)\geq 1\right\}. \quad (23)$$

Also, $|g_1|^2$ and $|g_2|^2$ should be compared. Therefore, $\tilde{\mathrm{P}}_{\mathrm{cov}}^2$ is classified into the cases as follows.

When $|g_1|^2\leq|g_2|^2$, we have

$$\tilde{\mathrm{P}}_{\mathrm{cov}}^{2,1}=\Pr\left\{|g_1|^2\leq|g_2|^2,|g_1|^2\geq\nu_3,|g_2|^2\geq\frac{\varepsilon_2}{b_2\rho}\right\}$$
$$=\int_{\nu_3}^{+\infty}\frac{1}{\Omega_4}e^{-\frac{x}{\Omega_4}}dx\int_{\nu_3}^{x}\frac{1}{\Omega_3}e^{-\frac{y}{\Omega_3}}dy=\frac{\Omega_4}{\Omega_3+\Omega_4}e^{-\frac{(\Omega_3+\Omega_4)\nu_3}{\Omega_3\Omega_4}}, \quad (24)$$

where $\nu_3=\max\left\{\frac{\varepsilon_1}{b_1\rho-b_2\varepsilon_1\rho},\frac{\varepsilon_2}{b_2\rho-b_1\varepsilon_2\rho}\right\}$.

When $|g_1|^2>|g_2|^2$, we have

$$\tilde{\mathrm{P}}_{\mathrm{cov}}^{2,2}=\Pr\left\{|g_1|^2>|g_2|^2,|g_2|^2\geq\nu_3,|g_1|^2\geq\frac{\varepsilon_1}{b_1\rho}\right\}$$
$$=\int_{\nu_3}^{+\infty}\frac{1}{\Omega_3}e^{-\frac{x}{\Omega_3}}dx\int_{\nu_3}^{x}\frac{1}{\Omega_4}e^{-\frac{y}{\Omega_4}}dy=\frac{\Omega_3}{\Omega_3+\Omega_4}e^{-\frac{(\Omega_3+\Omega_4)\nu_3}{\Omega_3\Omega_4}}. \quad (25)$$

As a consequence, we obtain $\tilde{\mathrm{P}}_{\mathrm{cov}}^2=\tilde{\mathrm{P}}_{\mathrm{cov}}^{2,1}+\tilde{\mathrm{P}}_{\mathrm{cov}}^{2,2}$, and the final expression of $\mathrm{P}_{\mathrm{out}}^2$ can be expressed as

$$\mathrm{P}_{\mathrm{out}}^2=1-\left(e^{-\kappa_1}e^{-\frac{\nu_1}{\Omega_4}}+e^{-\kappa_2}e^{-\frac{\nu_2}{\Omega_3}}-e^{-\frac{(\Omega_3+\Omega_4)\nu_3}{\Omega_3\Omega_4}}\right), \quad (26)$$

where $\kappa_1=\frac{\varepsilon_1}{\Omega_3(b_1\rho-b_2\varepsilon_1\rho)}$ and $\kappa_2=\frac{\varepsilon_2}{\Omega_4(b_2\rho-b_1\varepsilon_2\rho)}$.

Finally, by combing (17) and (26), the overall outage probability for the proposed NOMA-CRS scheme can be expressed as

$$\mathrm{P}_{\mathrm{out}}^{\mathrm{o}}\overset{a}{=}1-\left(1-\mathrm{P}_{\mathrm{out}}^1\right)\left(1-\mathrm{P}_{\mathrm{out}}^2\right)$$
$$=\mathrm{P}_{\mathrm{out}}^1+\mathrm{P}_{\mathrm{out}}^2-\mathrm{P}_{\mathrm{out}}^1\mathrm{P}_{\mathrm{out}}^2, \quad (27)$$

where step a is obtained by using the independence of the outage behavior between two transmission phases.

### C. Diversity Order

To gain more insights, the diversity order of the proposed NOMA-CRS scheme is also derived by assuming $\rho\to\infty$. Specifically, by using the fact that $e^{-x}\approx 1-x$ when $x\to 0$, the approximations listed below can be calculated:

$$\mathrm{P}_{\mathrm{out}}^1\approx\frac{\varepsilon_2}{\Omega_2a_2\rho}+\frac{\varepsilon_1}{\Omega_1a_1\rho}, \quad (28)$$

$$\mathrm{P}_{\mathrm{out}}^1\approx\frac{\kappa_1+\upsilon_2}{\Omega_3}+\frac{\kappa_2+\upsilon_1}{\Omega_4}-\frac{(\Omega_3+\Omega_4)\nu_3}{\Omega_3\Omega_4}. \quad (29)$$

Then, by substituting (28) and (29) into (27), $\mathrm{P}_{\mathrm{out}}^{\mathrm{o}}\propto\frac{1}{\rho}$ can be easily obtained, thus the diversity order of the proposed scheme can be derived as

$$d=-\lim_{\rho\to\infty}\frac{\log\left(\mathrm{P}_{\mathrm{out}}^{\mathrm{o}}\right)}{\log(\rho)}=1. \quad (30)$$

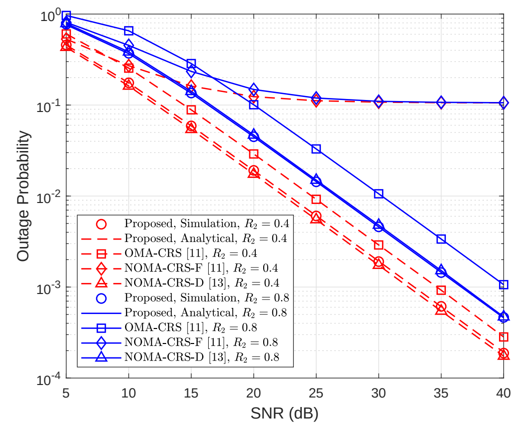


Fig. 2: Outage probability versus SNR $\rho$ with $R_1=0.1$ bit per channel use (BPCU).

It is observed that the full diversity order can always be obtained, which demonstrates the superiority of the proposed decoding order strategy over those designed in [12].

## V. Simulation Results

This section provides the simulation results to evaluate and compare the outage performance of the proposed scheme, the fixed PA based NOMA-CRS (NOMA-CRS-F) scheme and OMA-CRS scheme in [9], along with the dynamic PA based NOMA-CRS (NOMA-CRS-D) scheme in [11]. Recall that the statistical CSI based PA is adopted by the proposed scheme to further improve the outage performance, where the PA strategies developed in [13] and [14] will be employed in the first phase and the second phase of the proposed transmission scheme, respectively. Unless otherwise stated, it is assumed that $d_{\mathrm{S_1R}}=0.5$ m, $d_{\mathrm{S_2R}}=1$ m, $d_{\mathrm{RD_1}}=1$ m, $d_{\mathrm{RD_2}}=0.5$ m, $\alpha=2$, and $a_1=0.5$ and $b_1=0.6$ will be adopted for the NOMA-CRS-F scheme.

Fig. 2 depicts the outage probability versus $\rho$ with different setting of $R_1$ and $R_2$. Firstly, it is found that the analytical curves of the proposed scheme match precisely well with the simulation curves, which corroborates the accuracy of our analysis. Then, it is shown that the non-zero diversity order can be indeed obtained by the proposed scheme, while there exists an error floor causing zero diversity order for the NOMA-CRS-F scheme. This indicates the superiority of the proposed decoding order strategy since it utilizes the instantaneous CSI to actively avoid outage. Moreover, it is observed that NOMA-CRS-D scheme with higher complexity and overhead can achieve optimal outage performance, but the gap between it and the proposed scheme is narrow. Finally, it can be seen that a larger $R_2$ will lead to a higher outage probability, but the

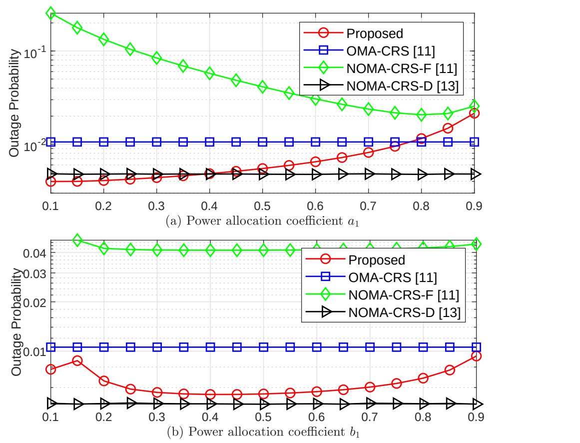


Fig. 3: Outage probability versus the PA coefficient $a_1$ and $b_1$ with $R_1 = 0.1$ BPCU, $R_2 = 0.8$ BPCU and SNR $\rho = 30$ dB.

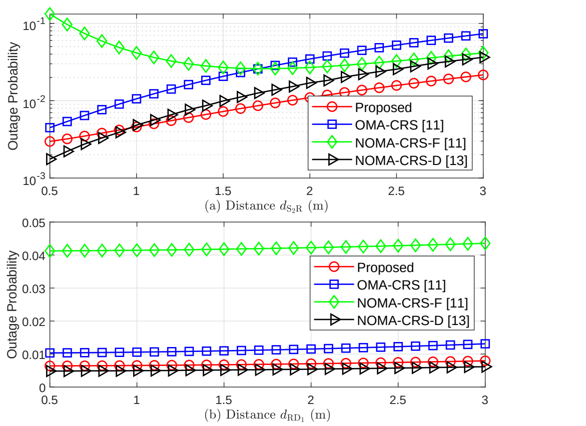


Fig. 4: Outage probability versus the distance $d_{S_2R}$ and $d_{RD_1}$ with $R_1 = 0.1$ BPCU, $R_2 = 0.8$ BPCU and SNR $\rho = 30$ dB.

trends of the curves for the same scheme under different $R_2$ are similar.

Fig. 3 illustrates the outage probability versus the PA coefficient $a_1$ and $b_1$, respectively. Since the concept of PA coefficient is not involved in the OMA-CRS scheme and the PA coefficients in the NOMA-CRS-D scheme are dynamically adjusted according to the instantaneous CSI, it is found that there is no change in their curves. Then, it is observed that the proposed scheme and OMA-CRS scheme always outperform the NOMA-CRS-F scheme. Furthermore, the proposed scheme can achieve a better outage performance than the OMA-CRS scheme if appropriate PA coefficients are set, which is effective for both adjusting $a_1$ and $b_1$. This is due to the reason that NOMA allows different users to transmit their individual signals using the same time and frequency, which is a more effective way to utilize the transmission resource than OMA, and the advantage will appear when decoding order and PA strategies are well designed.

Fig. 4 depicts the outage probability versus the distance $d_{S_2R}$ and $d_{RD_1}$, respectively. It is found that the proposed scheme achieves almost the same/even better outage performance as/than the NOMA-CRS-D scheme. Furthermore, a similar trend to OMA-CRS scheme can be obtained by these two schemes, which attributes to the reason that the decoding order or PA coefficients they adopt can be dynamically adjusted to minimize the outage probability. While the mutual effect from the mixture of $d_{S_1R}$ and $d_{S_2R}$ on decoding first user's signal at R is inevitable for the NOMA-CRS-F scheme, which leads to a convex curve as shown in Fig. 4 (a). This is due to the fact that its decoding order and PA coefficients are both fixed.

## VI. Conclusions

In this letter, we have proposed a new max-min criterion based dynamic decoding order strategy to minimize the outage probability of NOMA-CRS networks. A closed-form outage probability expression has been derived. Simulation results have demonstrated that the proposed scheme can achieve the suboptimal outage performance among all the considered transmission schemes for NOMA-CRS networks.

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