Three Essays in Macroeconomics

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Declaration

I hereby declare that the work presented in this thesis is my own work. The material has not been submitted, in whole or in part, for award of an academic degree at this, or any other university. Where I have consulted published work of others, the source has always been provided with all the major sources of help duly acknowledged and referenced appropriately.

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Abstract

This thesis explores three essays in macroeconomics with an application to Botswana. The first chapter studies the transmission path of the impact and response to a negative shock to commodity prices on resource-rich developing economies using a medium-scale dynamic stochastic general equilibrium (DSGE) model. The model incorporates a detailed fiscal block and is calibrated for Botswana. The results show that a negative shock on diamond prices has a negative impact on mining GDP, government revenues, and total GDP. The main channel of propagation is the *fiscal effect*, a fall in government spending due to the fall in resource revenues. The analysis demonstrate that without macroeconomic policy intervention, the impact of the shock is deep and prolonged. In contrast, a macroeconomic policy response that includes a modest decrease in government spending, an increase in public debt and an increase in tax rates alongside an expansionary monetary policy mitigates the impact of the shock on the economy.

The second chapter presents a large monthly macroeconomic dataset for Botswana to be used for empirical macroeconomic analysis and forecasting that require "big data". The dataset consists of 96 economic indicators that represent a broad coverage of the economy in line with earlier compilations of datasets of this type. The variance explained by static factors and dynamic factors demonstrate that the dataset allows for a factor representation. A forecasting demonstration suggests that dynamic factor models using the large macroeconomic dataset have a higher predictive ability for trimmed-mean core inflation, headline inflation and credit growth compared to a benchmark autoregressive model. This dataset will therefore be useful for macroeconomic policy and research in Botswana. Notably, the dynamic factor models could also be included in the central bank's forecasting toolkit.

Lastly, chapter three analyses the macroeconomic effects of external shocks transmitted through commodity prices using a generalised dynamic factor model based on a large quarterly macroeconomic dataset for Botswana. The study uses sign restrictions to identify three external shocks that drive the dynamics of the domestic economy. The analysis show that commodity price changes due to a global demand shock have a statistically and economically significant impact on several key macroeconomic variables: output expands, inflation rise and the trade balance improves. Commodity-specific price shocks have a moderate effect on the economy, with domestic inflation more responsive to an oil price shock while output is more sensitive to a diamond price shock in the short run. Overall, the global demand shock explain a greater share of the commodity price movements and the variation of other domestic macroeconomic variables.

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Chapter 1

Commodity Price Shocks and Macroeconomic Policy in Resource-rich Economies

1.1 Introduction

Proceeds from the extraction of natural resources form a critical source of total government revenues for most resource-rich developing countries. These revenues present opportunities for public investment and consumption necessary for employment creation and general economic development. However, the revenues are intrinsically temporary since they are from an exhaustible natural resource and are unreliable as they are subject to highly volatile commodity prices (Collier et al., 2010). This presents a challenge for the conduct of macroeconomic policy in these countries (see Céspedes and Velasco, 2012). At worst, the discovery and exploitation of natural resources has resulted in what the literature terms a *natural resource curse* (see Sachs and Warner, 2001; Agénor, 2016). The curse is characterised by high debt levels and macroeconomic instability induced by international commodity price fluctuations as well as a real exchange rate appreciation that renders non-resource sectors less competitive, commonly known as *Dutch disease effects*. It is against this background that researchers and policy makers have committed to identify fiscal policy frameworks that could reduce the impact of commodity price shocks on the economy.

Evidence shows that resource-rich developing economies lack economic and export diversification (Venables, 2016). This further exposes these economies to macroeconomic instability induced by volatile commodity prices. As a result, the debate on how such an economy responds to commodity price shocks remains a live topic for policy makers in these countries and in international development institutions. Notably, the International Monetary Fund (IMF) and World Bank have committed significant resources to help these economies manage their resource revenues productively and diversify. It is envisaged that greater economic and export diversification will act as an insurance against commodity demand and price shocks. However, this has not been an easy transition for most of the resource-rich developing economies. Consequently, for economies where the management of resource revenues is failing, and diversification remains elusive, the impact of economic shocks is severe. Despite this greater vulnerability, a major focus of the literature has been to explore policy options for new commodity discoveries and commodity price booms (see van Der Ploeg, 2019).

This paper uses a medium-scale DSGE model that characterises the Botswana economy to assess the transmission mechanism and macroeconomic policy response following a negative shock on diamond prices. The DSGE model incorporates a detailed fiscal structure that allows for the evaluation of the different fiscal policy instruments the government could use to respond to such a shock. This is important because, Bjørnland and Thorsrud (2019) show that government spending is the main link between the resource sector and the rest of the economy in countries where resource revenues constitute a significant portion of government revenues. The model also allows for the greater understanding of the interaction of fiscal and monetary policy responses to address the budget shortfall and weak economic activity following a negative shock on diamond prices.

The focus on one country, Botswana, is consistent with the observation by Céspedes and Velasco (2012) that macroeconomic responses to commodity price developments depend on the structural makeup of the economy and the unique policy frameworks in place. Notably, Botswana has escaped the resource curse and used diamond revenues to improve the economy from being one of the poorest to an upper middle-income status (Van der Ploeg, 2011). This has earned the country accolades for the prudent management of its resource revenues over the years. The achievements are a stark contrast to the economic outcomes in most resource-rich developing economies where natural resource discoveries led to bad economic performance and conflicts, more especially in Sub-Saharan Africa. Robinson et al. (2003) concluded that Botswana's success was mainly due to its good economic policies that reflected its good institutions compared to other countries in Sub-Saharan Africa.

Despite the significant economic progress and concerted efforts to diversify the economy, like most resource-rich economies, Botswana remains highly undiversified (see Sekwati, 2013). Diamond revenues constituted about 90 percent of exports and a third of government revenues in 2018. A clear signal that the economy is highly vulnerable to diamond price shocks. The sharp fall in commodity prices during the global financial crisis of 2008 revealed how vulnerable Botswana was to commodity price shocks. As consumers shifted away from luxury products in the wake of the economic downturn, diamond prices declined sharply leading to a substantial decrease in government revenues.

Notwithstanding the fall in government revenues, the government had to undertake counter-cyclical fiscal policy measures to steer the economy out of recession. This included measured public spending to foster resilience following the shock. This was supported by significant budget deficits, a rare scenario in the country's recent past then. The deficits were financed by government borrowing, which increased at a faster pace during this period. To help stimulate demand and support economic activity, the central bank also pursued an accommodative monetary policy stance. After this experience, this paper aims to engender the understanding of the dynamics of the impact on the economy and the response to a negative shock in diamond prices in a way that could aid future policy responses.

Although several studies have analysed the conduct of macroeconomic policy in commodity/resource-rich economies, most have specifically analysed monetary policy (see Ferrero and Seneca (2019) and fiscal policy (see Agénor, 2016; van Der Ploeg, 2019) in different models. Furthermore, a great part of the literature analyse the impact of the commodity price shocks on the economy with limited discussion of possible policy responses (see Rees et al. (2016) and Alberola and Benigno (2017)). On the fiscal front, studies tend to focus on new resource discoveries with the aim of identifying the best management approach for the ensuing wealth. Given the dominant position of oil in the global economy, a majority of studies in this literature analysed oil price shocks. In these papers, oil is modelled as both an export commodity and an input in the domestic production process. The diversity in the economic structure of commodity producing countries require specific setups for different countries. Hence, this paper will contribute to this literature by adding diversity in terms of the structural setup and commodity variety.

This paper takes an interesting approach for Botswana in that, first, the analysis provides a useful feedback on the appropriateness of the policy actions in response to the decline in diamond prices as experienced during the global financial crisis of 2007-08. This is a departure from the literature where a common focus is on commodity price booms, a point echoed by van Der Ploeg (2019). Second, the model combines both monetary and fiscal policy rules in line with the works of Bergholt et al. (2019) for Norway and Medina and Soto (2016) for Chile. However, fiscal policy is less detailed in these papers. The detailed fiscal policy block adopted for this paper allows for deeper analysis of the different fiscal policy instruments the government could use to respond to such a shock.

The results from the impulse response analysis show that following a negative shock on diamond prices mining GDP contract on impact, leading to a significant fall in government revenues. This indicates that the fiscal effect is the main propagation channel of the shock to the economy. The obvious response to a revenue shortfall is for the government to reduce its public spending. However, reducing the government spending will not suffice to offset the fiscal shortfall when resource revenues form a large part of government revenues. This prompts the government to issue bonds and borrow from the markets. This, in turn, increases total public debt. The debt financing moderates the decline in government spending, promoting timely recovery of the economy in a sustainable manner.

As observed in earlier works, increasing public debt levels can expose the economy to debt sustainability risks (Melina et al., 2016). Therefore, to help build government revenues, it would be necessary to increase tax rates to ensure that the budget constraint hold. Note that this is possible in resource-rich economies as they are generally characterised by low tax rates in relation to neighbouring or other comparable countries. The results also show that in response to declining output and a fall in inflation induced by weak public demand, the policy interest rate falls. This accommodative monetary policy stance leads to an increase in consumption and investment, which increases non-mining GDP. Nonetheless, the increase in non-mining GDP is not sufficient to offset the magnitude of the sharp decline in mining GDP following a negative shock in diamond prices hence a fall in total output.

Overall, the analysis shows that the ability of government to borrow is vital for a commodity dependant economy. It helps the government mitigate the impact of a downturn in commodity prices and ensures timely economic recovery. Therefore, the lesson for small resource-rich developing countries is that they should conduct policy in a way that build credit credential and promotes more resilient and diversified economies.

The rest of the paper is as follows: Section 2 reviews the related literature on DSGE models and their use in policy analysis and applications in resource-rich countries. Section 3 summarises some stylised facts of the macroeconomy of Botswana. Section 4 presents the model and its features, while Section 5 discusses the results. The last section concludes.

1.2 Related Literature

This paper is related to a vibrant and broader literature that seeks to understand the macroeconomics of resource-rich economies. The paper mainly relates to papers that use a New Keynesian DSGE model to analyse the impact of commodity price shocks on the macro-economy and the associated macroeconomic policy responses. There is a wide coverage for these studies both for developing and advanced resource-rich economies.

More work on this area is focused at finding optimal fiscal response to a newly discovered resource in the country, particularly for developing economies. Berg et al. (2013) show that a sustainable investment approach that combines public investment with a resource fund can help the economy address both the exhaustibility and volatility of resource revenues. Agénor (2016) conquers, pointing out that such a policy framework considers the capital constraints of developing economies and provides savings for use during periods of commodity downturns. In this way, policy making allows for development of the necessary infrastructure that promotes capacity building, employment creation and economic growth.

Agénor (2016) takes the analysis further by examining macroeconomic volatility in his contribution on how low-income countries with limited infrastructure should optimally respond to commodity price shocks through fiscal policy. He proposes an optimal allocation of resources that must be based on a social loss function that combines a measure of household welfare (consumption volatility) and an indicator of fiscal or macroeconomic volatility. In this way, the analysis addresses the revealed preferences of policy makers as opposed to only maximising the representative household's utility which is standard in the literature. The takeaway here is that ensuring macroeconomic stability is key for resource-rich economies. This should not only be the case when they are spending resource windfalls but also when responding to price downturns, as their impact on the economy can be severe.

Evidence shows that a commodity price boom facilitates financing by reducing the perception of risk (Alberola and Benigno, 2017). This may tempt a government to increase its borrowing and increase public investment. However, Melina et al. (2016) caution that aggressive public investment combined with increased borrowing can increase debt sustainability risks if investment efficiency declines or future resource revenues are lower than anticipated. Hence, it is imperative for governments to ensure prudent investment and restraint in borrowing. Used strategically, public debt is a key policy response tool. This paper demonstrates later how countercyclical use of public debt can help countries deal with commodity price shocks.

The analysis here and empirical evidence show that, for commodity abundant economies, fiscal policy (government spending) is the dominant path of transmission of commodity price fluctuations. This is called the *fiscal effect* in the literature. Pieschacón (2012) analyses the role of fiscal policy as the main propagation channel of oil prices in oil-exporting small open economies and finds that fiscal discipline is a key tool to regulate the impact of oil price shocks. This implies that fiscal policies that insulate the economy from exogenous oil price shocks, as it is the case in Norway, are welfare improving compared to procyclical ones. Bjørnland and Thorsrud (2019) support Pieschacón's result that Norway was able to withstand commodity price shocks. However, the authors observe that the use of the fiscal rule does not guarantee a countercyclical fiscal policy. They argue that fiscal policy remained procyclical despite the rule as the fund expanded significantly over the years. The result contrasts the view that fiscal rules, as used in Norway, could ensure fiscal countercyclicality as found in Pieschacón (2012).

The current paper will also relate to works that examine the relationship between commodity price shocks and monetary policy in resource-rich economies. Using Norway as a case study, Ferrero and Seneca (2019) observe that commodity exporting small open economies face the traditional trade-off between domestic inflation and output gap. They conclude that, although optimal monetary policy requires that interest rates should fall following an oil price drop, a central bank that has a mandate to stabilise consumer price inflation will increase interest rates to reduce the inflationary impact of an exchange rate depreciation induced by the fall in oil prices. This demonstrates the challenging nature of macroeconomic policy in resource-rich economies.

The present paper also relates to the literature on the role of commodity price shocks on business cycles. Shousha (2016) demonstrates that commodity price shocks are an important source of business cycle fluctuations for small open commodity exporters, with even larger effects on emerging economies. This is mainly attributable to the country interest rate response to the change in sovereign risk profile induced by such a shock. Positive commodity price shocks improve an economy's competitiveness and borrowing terms as higher commodity prices reduce the spread between the country's borrowing rate and world interest rate (Drechsel and Tenreyro, 2018). All these effects have a strong positive impact on the country's GDP, consumption and investment. Furthermore, the positive role of resource prices was highlighted by Rees et al. (2016) through a multisectoral DSGE model for the Australian economy. They found that increasing resource prices lead to increases in incomes and an expansion of the economy. Bergholt et al. (2019) also use a DSGE model to show that oil price movements represented an important source of macroeconomic volatility in Norway. The authors demonstrate that the fiscal regime in Norway helps shield the economy from the volatility of international oil prices induced by external shocks.

It is evident that the macroeconomic developments in resource-rich developing economies need a multi-pronged policy response. In as much as fiscal policy could be the main stabilising tool, studies show that modern monetary policy is a potent stabilisation tool (Gali and Monacelli, 2005). Thus, fiscal policy can do much more to stabilise the economy if complemented by monetary policy. The reinforcing actions should help the economy recover at a much faster pace. An analysis of the Brazilian economy shows that a fall in interest rates reduces the cost of public debt financing, increases consumption, investment and ultimately output (Cavalcanti et al., 2018). This demonstrates how effective macroeconomic policy can be when fiscal measures adjust in a way that complements the expansionary monetary policy. The analogy applies well to the policy transmission mechanism in a resource-rich developing economy. An expansionary monetary and fiscal policy following a downturn in economic activity induced by a fall in resource revenues can help reduce the impact on the economy.

This study follows the strand of literature that examines the interactions of monetary and fiscal policy allowing for a detailed fiscal policy specification in DSGE models as in Davig and Leeper (2011), Traum and Yang (2011) and Bhattarai and Trzeciakiewicz (2017). The monetary-fiscal interaction is the ideal specification to explain the impact of policy response to shocks. For example, Li and Spencer (2016) uses this approach to investigate the contributions of fiscal stimulus and monetary policy easing actions to economic recovery in the aftermath of the 2008-09 global financial crisis. Furthermore, the rich fiscal policy specification benefits policy makers in resource-rich economies as it allows for a better understanding of the implications of the use of different fiscal instruments to reduce debt. This is important because, as Leeper et al. (2010) note, the response of economic variables following a shock depends on the fiscal instrument used to finance debt.

Compared to a major focus on new resource discoveries in the literature, this study considers a well-established resource-rich economy. The fact that for Botswana the resource is diamonds, provides another opportunity to understand a different commodity other than oil, save for Medina and Soto (2016) who study copper price shocks. The nature of the commodity is important as it determines how the model accounts for direct and indirect effects. For oil there is a need to model the direct effects as an input in local production and its indirect effects as a source revenues for government while for diamond-exporting Botswana, the model accounts for the large indirect effects only. As this paper focuses on a negative shock on commodity prices, we depart from the bias towards commodity booms (positive price shocks) common in the literature. In this way, the study assesses how such a shock affects the economy and how the economy responds through monetary and fiscal policy. The detailed fiscal policy specification that allows for distortionary taxes provide some practical and informative policy choices for policy makers given a government budget that directly depends on commodity prices.

1.3 The Macroeconomy of Botswana

1.3.1 Background

Botswana remains one of the most economically stable countries in Sub-Saharan Africa. Its impressive economic performance and success has been associated with the prudent macroeconomic management of its diamond revenues. Diamond sales remain a major source of export earnings and government revenues. Notably, diamond revenues accounts for over 80 percent of the country's exports and make up over 30 percent of its government revenues.

Following modest gains in the non-mining sector, mainly led by the financial services and retail sectors, the diamond sector's contribution to total output has been declining over the years. Nonetheless, the diamond sector remains a significant player in Botswana's economic landscape. As a result, economic growth follows developments in the diamond sector. Figure 1.1 shows that when the diamond sector contracted by over 40 percent during the global financial crisis, total GDP also weakened by 7.7 percent. Similarly, the recovery in diamond prices led to an immediate positive impact on real GDP growth. The diamond price movements are represented by the rough diamond price index in Figure 1.2.

Understanding the impact of diamond price shocks is more relevant now because, like most commodity prices, diamond prices have become less stable since the turn of the 21^{st} century. This is mainly due to De Beers'[1] reduced ability to alter supply in line with global

[1] De Beers is the world largest producer and distributor of diamonds, owned 85 percent by

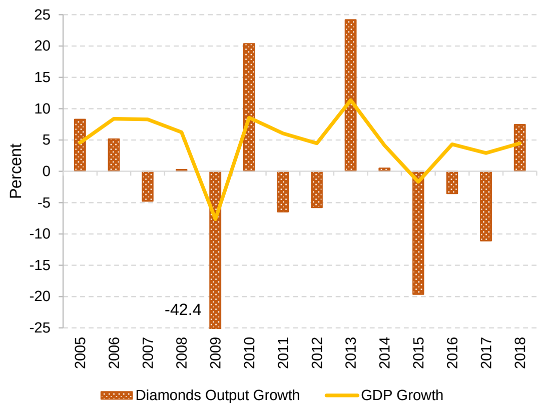


Figure 1.1: Real GDP Growth and Diamond Output Growth

Figure 1.2: Bloomberg - Rough Diamond Price index

demand patterns in a way that stabilises prices as was the case in the 20^{th} century when it had monopoly power. De Beers' monopoly has waned due to increased production in other key markets (Russia, Canada, Australia, Democratic Republic of Congo and Angola) that sell diamonds outside the De Beers framework. In addition, the advances in technology that have facilitated the production of synthetic diamonds pose a risk to natural rough diamond prices. All these market changes increase chances for diamond price shocks and, ultimately the instability in the amount of revenues that accrues to government.

Botswana's central bank, Bank of Botswana, has independence in the formulation and implementation of monetary policy. This has been the feature of macroeconomic policy since the inception of the Bank in 1976. The Bank has adopted a forward looking monetary policy framework with price stability as the main objective. The Bank defines this as a low and stable level of inflation that is within the medium-term objective range of 3 to 6 percent. This framework belongs to what the literature refers to as *inflation targeting lite* (see Carare and Stone, 2006). In this framework, the country announces a broad inflation objective range but, due to vulnerability to large economic shocks it is not able to commit to an explicit inflation target. Since international oil and food prices declined after 2009, Botswana's headline inflation has declined gradually reaching 2.2 percent in

multinational mining company Anglo American PLC and 15 percent by the Government of the Republic of Botswana.

December 2019. Inflation remained within the Bank's objective range since 2013, except for a few breaches of the lower bound. The low inflation environment provided room for the central bank to ease monetary policy. Consequently, the policy interest rate fell from 16 percent in 2009 to reach 4.75 percent in December 2019 (Figure 1.3). The policy stance complemented the government's fiscal policy measures aimed at boosting domestic demand, investment and economic activity in time to help the economy deal with the aftermath of the global financial crisis.

On the fiscal front, Botswana continues to maintain a sustainable fiscal policy stance, with an objective of government spending to GDP ratio of 30 percent and a debt-to-GDP ratio of 40 percent (distributed equally between domestic and external debt). Even during the financial crisis when government increased it's borrowing, the debt-to-GDP ratio remained way below the objective limit. Total government public debt stood at 18.7 percent of GDP at the end of 2018. Such a debt position is accompanied by good sovereign ratings[2] which makes it easier for the government to issue debt when the need arises. The government generally runs a balanced budget with either small surpluses or deficits that are less than 5 percent of GDP, with an exception of the period following the crisis (Figure 1.4). During that period the government ran deficits in order to cushion the economy with an objective that the budget will be balanced in the medium term to avoid high levels of public debt.

Apart from diamond revenues, other sources of government revenues consist of custom and excise taxes, value added tax (VAT) (12%), labour income tax ($7.5 - 25\%$) and corporate income tax (22%). Botswana has some of the lowest tax rates in the region. The government also has external reserves (fiscal savings from previous periods surpluses - stood at 37 percent of GDP in 2018) that it may tap from when the need arises. Indeed after the crisis, the government used funds from these savings to implement an economic stimulus programme.

With the weakness in the mining sector during the financial crisis, the policy options undertaken by policy makers saw the non-mining sector driving growth throughout that

[2] In 2018, S&P Global Ratings and Moodys' rated Botswana A- and A2, respectively. These upper medium grades are considered low risk.

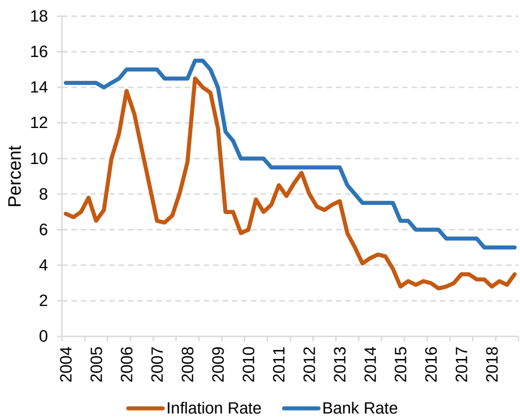


Figure 1.3: Inflation and Bank Rate

Figure 1.4: Budget balance (% of GDP)

period. This demonstrated that, with a clear policy coordination, resource rich-economies can moderate the effects of commodity price shocks on the economy. The policy actions taken by the government to increase public debt to support the budget as well as to increase VAT in order to raise additional revenues helped the economy weather the effects of the diamond price shock. The accommodative monetary policy regime also complimented these measures by reducing the cost of borrowing. This encouraged investment and consumer spending leading to an immediate recovery once the diamond prices increased. The outcomes here are consistent with the findings in the results section.

1.4 The Model

This section presents the features of the medium-scale New Keynesian DSGE model. The model consists of a continuum of households who supply labour and firms that supply goods. In this paper, firms operate in two sectors, the non-mining sector $\{n\}$ and the mining sector $\{m\}$. This setup is similar to that of Arezki et al. (2017). There is a government responsible for fiscal policy and is financed by a combination of taxes (mining sector and non-mining sector) and one period nominal government bonds. There is a central bank in charge of monetary policy and sets the policy interest rate by following a Taylor-type rule that responds to inflation and output gap. This model setup strikes a balance between being close to the real world and tractability. The following subsections outline the model, highlighting how the current model is different from earlier models that characterise a resource-rich economy.

1.4.1 Households

There is a continuum of households with identical preferences over consumption. The households provide labour to the two sectors. The representative household derives utility from consumption, C_t, and disutility from labour hours worked, L_t, and maximises the expected sum of utilities discounted by $\beta \in (0, 1)$:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ lnC_t - \frac{L_t^{1+\varphi}}{1+\varphi} \right\}, \tag{1.1}$$

where the parameter $\varphi > 0$ is the inverse of the elasticity of labour supply.

Households provide differentiated labour to the mining and non-mining sectors. To capture the imperfect substitutability of labour between the two sectors, total labour L_t is given by the following constant elasticity of substitution (CES) function in the spirit of Horvath (2000):

$$L_t = \left[(L_t^m)^{1+\gamma} + (L_t^n)^{1+\gamma} \right]^{\frac{1}{1+\gamma}}, \tag{1.2}$$

where γ captures the elasticity of labour supply between sectors. As γ approaches infinity, labour hours are perfect substitutes which implies perfect labour mobility. When $\gamma < \infty$, there is diversity of labour and relative hours respond less to wage differences across sectors. For an economy with a capital intensive resource sector, the labour mobility will also be limited by the small size of employment in the mining sector compared to the rest of the economy.

The representative household's utility maximisation is subject to a budget constraint. This presents the available resources and how they are used up as follows:

$$(1+\tau_t^c)C_t + I + b_t = (1-\tau_t^l) \sum_{i=n,m} W_t^i L_t^i + R_{t-1}\frac{b_{t-1}}{\pi_t} + (1-\tau_t^k){R^k}_t K_{t-1} + \Omega_t + TR_t. \tag{1.3}$$

The household's income consists of: (a) labour income $(1-\tau_t^l)W_t^i L_t^i$, where W_t^i represents the real wage in each sector, and τ_t^l represents labour income tax rate; (b) capital income $(1-\tau^k){R^k}_t K_{t-1}$, where R^k is the real return (rental rate) on capital, K_{t-1} is the physical stock of capital and τ^k is the tax rate on capital income; (c) transfers from the government (TR_t); (d) profits from firms (Ω_t) and (e) interest income from bonds $\left(\frac{R_{t-1}b_{t-1}}{\pi_t}\right)$, where R_{t-1} is the nominal interest on a bond (b_t). The household spends the income on consumption (C_t), investment in physical capital (I_t) and purchasing government bonds ($b_t = \frac{B_t}{P_t}$).[3] τ_t^c represents the consumption tax rate.

The build-up of capital is not instantaneous. We allow adjustment costs to account for the implicit costs associated with investments as in Christiano et al. (2005) and Berg et al. (2013). Specifically, the capital evolution process that includes investment adjustment costs is given by:

$$K_t = (1-\delta)K_{t-1} + \left[1 - \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] I_t, \tag{1.4}$$

where δ is the capital depreciation rate and $\kappa > 0$ is an adjustment cost parameter.

The household's Lagrangian function for the maximisation problem is defined as;

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left\{ \begin{array}{l} \left[lnC_t - \frac{L_t^{1+\varphi}}{1+\varphi}\right] \\ -\lambda_t[(1+\tau_t^c)C_t + I + b_t - (1-\tau_t^l) \sum\limits_{i=n,m} W_t^i L_t^i - \dfrac{R_{t-1}b_{t-1}}{\pi_t} - (1-\tau_t^k)R^k{}_t K_{t-1} \\ -\Omega_t - TR_t] - Q\left(K_t - (1-\delta)K_{t-1} - \left[1 - \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2\right] I_t\right), \end{array} \right\} \tag{1.5}$$

where λ_t is the Lagrange multiplier associated with the budget constraint. Q_t is the Lagrange multiplier for the dynamics of capital stock as in Chacon (2014). It represents the shadow price of capital commonly know as Tobin Q. It is defined as the ratio between the market value of the total installed capital and the replacement cost of the capital.

First order conditions representing the households's optimal choices of consumption, labour supply, investment and saving are given by:

$$\lambda_t(1+\tau_t^c) = \frac{1}{C_t}, \tag{1.6}$$

$$(L_t^i)^\gamma = \lambda_t(1-\tau^l)W_t^i L_t^{(\gamma-\varphi)}, \qquad \text{for} \quad i = n, m \tag{1.7}$$

[3] In Botswana households do not directly purchase bonds, commercial banks as primary dealers do on their behalf. Nonetheless, it is assumed that banks make decisions consistent with those that will be made by households.

$$\lambda_t = \beta E_t \lambda_{t+1} \frac{R_t}{\pi_{t+1}}, \tag{1.8}$$

$$Q_t = \beta E_t[Q_{t+1}(1-\delta) + \lambda_{t+1}(1-\tau_t^k)R_{t+1}^k], \tag{1.9}$$

$$\lambda_t = Q_t \left[1 - \frac{\kappa}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2 - \kappa \frac{I_t}{I_{t-1}}\left(\frac{I_t}{I_{t-1}} - 1\right)\right] + \kappa E_t \beta \left[\left(\frac{I_{t+1}}{I_t}\right)^2 \left(\frac{I_{t+1}}{I_t} - 1\right)\right]. \tag{1.10}$$

Equation (1.6) presents the marginal utility of consumption which is equal to the Lagrange multiplier. Household labour supply to the mining and non-mining sectors is given by Equation (1.7). The Euler equation for bonds is given by Equation (1.8). The investment Euler equation is given by Equation (1.10).

1.4.2 Firms

1.4.2.1 Domestic Non-Mining sector

The domestic retail firm (j) produces the composite good ($Y_{H,t}$). The firm purchases differentiated intermediate goods ($Y_{H,jt}$) from monopolistic producers and combines them using the Dixit and Stiglitz (1977) aggregation technology:

$$Y_{H,t} = \left[\int_0^1 (Y_{H,jt})^{\frac{\eta-1}{\eta}} dj\right]^{\frac{\eta}{\eta-1}}, \tag{1.11}$$

where η denotes the elasticity of substitution among the differentiated intermediate goods.

The domestic retail firm's profit maximising problem is given by:

$$max \quad \Omega_{R,t} = P_{H,t}Y_{H,t} - \int_0^1 P_{H,jt}Y_{H,jt}dj, \tag{1.12}$$

where $P_{H,t}$ is the price of a unit of output and $P_{H,jt}$ is the price of the intermediate output j. The optimisation results in the demand for the domestic intermediate good:

$$Y_{H,jt} = \left(\frac{P_{H,jt}}{P_{H,t}}\right)^{-\eta} Y_{H,t}. \tag{1.13}$$

The resulting price equation for domestically produced retail goods is:

$$P_{H,t} = \left[\int_0^1 (P_{H,jt})^{1-\eta} dj\right]^{\frac{1}{1-\eta}}. \tag{1.14}$$

1.4.2.2 Intermediate Goods Production

Each firm (j) that produces intermediate goods follows a Cobb Douglas production function featuring two factors, private capital ($K^n_{j,t-1}$) and labour ($L^n_{j,t}$):

$$Y_{H,jt}(j) = A_t(K^n_{t-1})^{\alpha}(L^n_t)^{1-\alpha}, \tag{1.15}$$

where A is technology/productivity parameter that follows an AR(1) process. In log-linearised form, it can be expressed as $\hat{a} = \rho_a \hat{a}_{t-1} + \epsilon_{a,t}$. $|\rho_a| < 1$ is an autoregressive parameter of productivity and ϵ^a_t is an independent and identically distributed (i.i.d) error term with zero mean and constant variance. $Y_{H,jt}$ is the output each firm produces, and α is the elasticity of output with respect to capital, while $(1-\alpha)$ is the elasticity of output with respect to labour.

The wholesale firm hires labour (L^n_t) and rents capital (K^n_{t-1}), for which it pays wage rate W^n_t and rental rate of capital (R^k_t), respectively. The monopolistic competition firm then solves the cost minimisation problem:

$$\underset{L_t,K_{t-1}}{min} \quad TC_{j,t} = W^n_t L^n_{j,t} + R^k_t K^n_{j,t-1}, \tag{1.16}$$

subject to the production technology in Equation (1.15). This results in the following optimisation conditions which are respectively the demand for labour and the demand for capital by intermediate good producers in the non-mining sector.

$$L^n_t = (1-\alpha) MC_{Ht} \frac{Y_{H,t}}{W^n_t}, \tag{1.17}$$

$$K^n_{t-1} = \alpha MC_{Ht} \frac{Y_{H,t}}{R^k_t}. \tag{1.18}$$

The demand functions show that there is an inverse relationship between the demand for the factors of production and their returns. A fall in the real wage increases the demand for labour. Similarly, a decrease in the rental rate of capital increases the demand for capital by firms. This adjustment continues until the marginal product of the factors of production has decreased by the same magnitude as the fall in the returns to inputs. A combination of the demand for labour and capital give the following capital-labour ratio:

$$\frac{K^n_{j,t-1}}{L^n_{j,t}} = \frac{\alpha W_t}{(1-\alpha)R^k_t}. \tag{1.19}$$

The capital-labour ratio is equal across firms therefore j can be dropped. Then the real marginal cost of firms will be:

$$MC_{Ht} = \frac{1}{A_t}\left(\frac{W_t^n}{1-\alpha}\right)^{1-\alpha}\left(\frac{R_t^k}{\alpha}\right)^{\alpha}. \tag{1.20}$$

The marginal cost relates negatively to technology and positively to the return of factor inputs, the wage rate and the rental rate of capital.

1.4.2.3 Price Setting

The other problem for the monopolistically competitive firm is to define the price of the intermediate goods. We assume that domestic firms set prices in a staggered manner *à* la Calvo (1983). Following the Calvo price setting mechanism, it is assumed that a share of the firms (θ_H) are not able to adjust the price therefore uses the previous period price $P_{H,t-1}$. Another share $(1-\theta_H)$ are able to adjust prices to maximise profits and set an optimal price $\tilde{P}_{H,t}$. The average time which a price is fixed is $\frac{1}{1-\theta}$ (Clarida et al., 1999).

A firm that optimises in period t therefore chooses a price $\tilde{P}_{H,t}$ that maximises present value of the profits generated when that price remains effective (see Walsh, 2010; Gali, 2015). Thus, the profit maximisation problem of a firm j that is able to set a new price in period t for its good is given by:

$$max_{\tilde{P}^*_{H,jt}} E_t \sum_{t=0}^{\infty} (\beta\theta_H)^k \left[(\tilde{P}_{H,it} - MC_{H,jt+i})Y_{j,t+i}\right]. \tag{1.21}$$

The optimal price set $\tilde{P}^*_{H,jt}$ is given by:

$$\tilde{P}_{H,t} = \left(\frac{\eta}{\eta-1}\right)\sum_{t=0}^{\infty} (\beta\theta)^k E_t\{MC^n_{H,t+k}\}. \tag{1.22}$$

Firms that can adjust their prices set the same price because they face the same production technology. Therefore we can drop j. Using the pricing rule in Equation (1.14), the fact that groups of firms define their prices and firms subject to price stickiness use similar prices, the aggregate price level is given by:

$$P_{H,t} = \left[\theta_H P^{1-\eta}_{H,t-1} + (1-\theta_H)\tilde{P}^{1-\eta}_{H,t}\right]^{\frac{1}{1-\eta}}. \tag{1.23}$$

Equations (1.22) and (1.23) can be expressed in log-linearised form respectively as:

$$\hat{\tilde{p}}_{H,t} = (1-\beta\theta_H)\sum_{t=0}^{\infty}(\beta\theta_H)^k E_t\{\widehat{mc}_{H,t+k}\} \tag{1.24}$$

and

$$\hat{p}_{H,t} = \theta_H \hat{p}_{H,t-1} + (1-\theta_H)\hat{\tilde{p}}_{H,t}. \tag{1.25}$$

After some algebra, combining Equations (1.24) and (1.25) yields the New-Keynesian Phillips Curve (NKPC) equation (inflation equation):

$$\pi_{H,t} = \beta E_t \pi_{H,t+1} + \tilde{\kappa}\widehat{mc}_{H,t}, \tag{1.26}$$

where $\pi_{H,t} = p_{H,t} - p_{H,t-1}$ is inflation in the domestic sector. The parameter $\tilde{\kappa} = \frac{(1-\theta_H)(1-\beta\theta_H)}{\theta_H}$ measures the degree of price rigidity. Higher price stickiness (higher θ_H) leads to lower values of $\tilde{\kappa}$, implying that deviations in marginal cost will only lead to low levels of inflation in the short run. The NKPC is forward looking, meaning firms take into account future inflation as they may not be able to adjust their prices in the next period.

1.4.2.4 Imports Sector

There is a continuum of imported good retailers indexed by $i \in [0,1]$, who purchase homogeneous intermediate products from abroad at the foreign price (expressed in foreign currency) $P_{F,t}$. By way of branding technology each firm transforms the imported products into different varieties $Y_{F,it}$. Monopolistically competitive importers assemble these differentiated import varieties into a composite import good using the following CES production technology:

$$Y_{F,t} = \left[\int_0^1 (Y_{F,it})^{\frac{\eta_f-1}{\eta_f}} di\right]^{\frac{\eta_f}{\eta_f-1}}, \tag{1.27}$$

where η_f governs the degree of substitution between firms' imported goods varieties. The demand function for each variety is given by:

$$Y_{F,it} = \left(\frac{P_{F,it}}{P_{F,t}}\right)^{-\eta_f} Y_{F,t}, \tag{1.28}$$

where $P_{F,it}$ is the price for firm i's variety and $P_{F,t}$ is the composite imported good price.

Importing firms operate in a market of monopolistic competition and set prices according to a Calvo (1983) price setting mechanism to optimise profits. To maximise profits, they set prices similar to domestic producing firms with the following optimisation problem:

$$max_{P_{F,it}} E_t \sum_{t=0}^{\infty} (\beta\theta_F)^k \, [\tilde{P}_{F,it} - MC_{F,t+i})Y_{F,t+i}], \tag{1.29}$$

where the parameter θ_F is the Calvo parameter for the share of firms not adjusting their price. It is assumed that importers buy foreign products at their marginal cost (expressed in domestic currency) as $MC_{f,t} = e_t P_{F,t}$, where e_t is the nominal exchange rate. The real marginal cost will be equal to the real exchange rate. The aggregate import price index evolves according to the following expression:

$$P_{F,t} = \left[\theta_F P_{F,t-1}^{1-\eta_f} + (1-\theta_F)\tilde{P}_{F,t}^{1-\eta_f}\right]^{\frac{1}{1-\eta_f}}, \tag{1.30}$$

where $\tilde{P}_{F,t}$ is the optimal price for firms that are able to adjust the price. In line with the domestic good setup, the New Keynesian Phillips curve for imported good retailers will be $\pi_{F,t} = \beta E_t \pi_{F,t+1} + \tilde{\kappa}\widehat{mc}_{F,t}$, where $\pi_{F,t} = p_{F,t} - p_{F,t-1}$ is price inflation in the import sector. The parameter $\tilde{\kappa}_f = \frac{(1-\theta_F)(1-\beta\theta_F)}{\theta_F}$ measures the degree of price rigidity in the imports sector and θ_F is the degree of price stickiness in the imports sector.

1.4.2.5 Final Goods Sector

Following Rees et al. (2016), the aggregate final good Y_t^N is a composite index of domestic goods $Y_{H,t}$ and imported goods $Y_{F,t}$:

$$Y_t^N = \left[(1-\mu)^{\frac{1}{\psi}} Y_{H,t}^{\frac{\psi-1}{\psi}} + \mu^{\frac{1}{\psi}} Y_{F,t}^{\frac{\psi-1}{\psi}}\right]^{\frac{\psi}{\psi-1}}, \tag{1.31}$$

where Y_t^N is the is the domestic final good which is the domestic final demand. Parameter $\mu \in (0,1)$ captures the degree of openness as measured by the fraction of imports in total final goods. The parameter ψ is the elasticity of substitution between domestic and foreign goods. Given the price of the final good P_t, the price of domestic output P_H and the price of foreign goods $P_{F,t}$ the final goods producer chooses Y_H and Y_F to maximise profits:

$$\max \quad \Pi = P_t Y_t^n - P_{H,t} Y_{H,t} - P_{F,t} t Y_{F,t}^n, \tag{1.32}$$

subject to the CES aggregation in Equation (1.31). Profit maximisation leads to the following downward sloping demand functions for the domestic good and foreign good, respectively:

$$Y_{H,t} = (1-\mu)\left(\frac{P_{H,t}}{P_t}\right)^{-\psi} Y_t^N \qquad \text{and} \qquad Y_{F,t} = \mu\left(\frac{P_{F,t}}{P_t}\right)^{-\psi} Y_t^N, \tag{1.33}$$

The final good price level, which is the consumer price index (CPI), is linked to the domestic good price and foreign good price by the following equation:

$$P_t = \left[(1-\mu)P_{H,t}^{1-\psi} + \mu P_{F,t}^{1-\psi}\right]^{\frac{\psi}{1-\psi}}. \tag{1.34}$$

Given the formula CPI and using the domestic and foreign NKPC derived earlier, the log-linear form of overall inflation is defined as:

$$\pi_t = (1-\mu)\pi_{H,t} + \mu\pi_{F,t}. \tag{1.35}$$

This concludes the inflation dynamics of a small open economy.

1.4.2.6 Resource/Mining Sector

For developing countries, the extraction of natural resources is mainly financed by foreign direct investment. Hence, most papers modelling this sector tend to assume both resource production and prices as exogenous processes (see Dagher et al., 2012; Berg et al., 2013; Agénor, 2016). This is relevant for new discoveries. However, for an established natural resource producer (like Botswana) this may not necessarily be the case. Following the work of Alberola and Benigno (2017), the resource sector is modelled to produce under a decreasing return to scale technology with labour the only variable input [4] in the mining of the resource such that:

$$Y_t^m = A_t^m \left(L_t^m\right)^{\alpha_m}, \tag{1.36}$$

where Y_t^m is mining output and A_t^m is total factor productivity specific to the mining sector and follows an AR(1) process. In log-linearised form it can be expressed as

[4] This assumption is valid to reflect the long-term nature of capital changes in the mining sector that will be difficult to incorporate for a model that seeks to help with short-run policy analysis.

$\hat{a}^m = \rho_{a_m}\hat{a}^m_{t-1} + \epsilon^{a^m}_t$. L^m_t is the amount of labour employed in the mining sector. The parameter α_m is the elasticity of output with respect to labour.

Commodity prices are determined in the international markets and the economy take such as given. Therefore, it is assumed that diamond prices p^{d*}_t follow an exogenous process. This is standard in papers that model resource rich economies.

$$p^{d*}_t = \rho_{p_d} p^{d*}_{t-1} + \epsilon^{p_{d*}}_t, \tag{1.37}$$

where $\rho_{p_{d*}} \in (0, 1)$ is the autoregressive parameter and $\epsilon^{p_{d*}}_t$ is an i.i.d normal error with mean zero and constant variance. The real domestic diamond price can be expressed as $p^d_t = s_t p^{d*}_t$, where s_t is the real exchange rate.

The mining sector seeks to maximise profits Π^m by choosing the right amount of labour

$$max \quad \Pi^m = P^d_t Y^m_t - W^m_t L^m_t, \tag{1.38}$$

subject to the mining production function in Equation 1.36. The resulting first order conditions implies that the demand for labour in the mining sector is:

$$L^m_t = \alpha_m \frac{P^d_t Y^m_t}{W^m_t}. \tag{1.39}$$

This expression shows that a fall in the price of diamonds leads to a fall in the demand for labour in the mining sector.

The government is entitled to dividends as a shareholder in the mining company. It also levies mineral taxes and royalties. All these are combined to determine the revenue government obtains from the mining sector (T^m_t) levied on the level of production and are given by:

$$T^m_t = \tau^m P^d_t Y^m_t, \tag{1.40}$$

where τ^m represent the constant share of government in mining output. Following Ferrero and Seneca (2019), this setup does not specify how the rest of the world demand the domestic diamond output as it is not necessary for the characterisation of the small open economy.

1.4.3 The Terms of Trade and Real Exchange Rate

The terms of trade, $\mathcal{T}$, between the home country and the foreign country is defined as the price on imported goods in terms of the price of domestic goods ($\mathcal{T}_t = P_{F,t}/P_{H,t}$). Log-linearisation of the CPI formula from Equation (1.34) around the steady state yields:

$$\begin{aligned} p_t &\equiv (1-\mu)p_{H,t} + \mu p_{F,t} \\ &= p_{H,t} + \mu\tau_t, \end{aligned} \tag{1.41}$$

where $\tau_t = p_{F,t} - p_{H,t}$ is the log effective terms of trade as in Gali and Monacelli (2005). It is assumed that the law of one price holds ($P_{F,t} = e_t P_t^i$). This can be expressed in a log-linearised form as $p_{F,t} = e_t + p_t^*$, where p_t^* is the log world price index. Combining the definition of law of one price with the terms of trade yields:

$$\tau_t = e_t + p_t^* - p_{H,t}. \tag{1.42}$$

The bilateral real exchange rate with country i is given by:

$$S_t = \frac{\varepsilon_{it} P_t^i}{P_t}. \tag{1.43}$$

Then consistent with Gali and Monacelli (2005), let s_t be the log effective real exchange rate.

$$\begin{aligned} s_t &= e_t + p_t^* - p_t \\ &= \tau_t + p_{H,t} - p_t \\ &= (1-\mu)\tau_t. \end{aligned} \tag{1.44}$$

The expression yields a relation between the real exchange rate and the terms of trade.

1.4.4 Government

1.4.4.1 Fiscal Policy

Government's income is made up of tax collection (consumption tax, labour income tax, capital income tax and resource tax) and new debt (it is assumed that fiscal authorities borrow from both the domestic and external market). Expenditure includes government spending, debt service and transfers to households. The fiscal authority's budget constraint is given by:

$$\tau_t^c C_t + \tau_t^l W_t L_t + \tau_t^k R_t^k K_{t-1} + b_t + s_t b_t^* + T_t^m = R_{t-1}\frac{b_{t-1}}{\pi_t} + R_{t-1}^s \frac{s_t b_{t-1}^*}{\pi_t^*} + TR_t + G_t, \quad (1.45)$$

where b_t^* is the amount borrowed from abroad, π_t^* is foreign inflation and R_t^s is the specific cost of foreign borrowing.

There is no consensus among researchers on the specification of fiscal rules. In the spirit of Leeper et al. (2010) and Zubairy (2014), the expenditures are designed to allow for response to the state of the economy (automatic stabilisers) and debt to avoid higher debt-to-GDP ratios. Meanwhile, given that in developing economies taxes are hardly used for economic stabilisation given the revenue constraints, we assume that tax rates only respond to debt. Botswana's tax rates are low compared to its peers in the region, hence, upward movements could be implemented if the need arises. With the aim to protect vulnerable sections of society in times of downturns it is considered reasonable to have government transfers to households as exogenous.

Similar to Leeper et al. (2010), and consistent with Frankel et al. (2013)'s observation on Botswana's graduation from fiscal procyclicality, government spending responds counter-cyclically to lagged values of GDP and debt deviations to steady state. Taxes respond procyclically to debt as in Sims and Wolff (2018b). In this way, fiscal instruments ensure fiscal solvency. For example, a decline in resource revenues that accrue to the government will reduce the overall government revenue. To finance the deficit in the budget, the government borrows and the level of debt increases. This triggers the response mechanism which ensures that government spending reduces and taxes increase to avoid explosive debt-to-GDP ratio. The adjustment mechanism ensure that the government budget constraint is maintained following a shock. The fiscal rules are expressed as:

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + (1-\rho_g)(-\phi_{gy}\hat{y}_{t-1} - \phi_{gb}\hat{b}_{t-1}) + \epsilon_{g,t}, \quad (1.46)$$

$$\widehat{tr}_t = \rho_{tr}\widehat{tr}_{t-1} + \epsilon_{tr,t}, \quad (1.47)$$

$$\hat{\tau}_t^c = \rho_c \hat{\tau}_{t-1}^c + (1-\rho_c)(\phi_c \hat{b}_{t-1}) + \epsilon_{\tau^c,t}, \quad (1.48)$$

$$\hat{\tau}_t^l = \rho_l \hat{\tau}_{t-1}^l + (1-\rho_l)(\phi_l \hat{b}_{t-1}) + \epsilon_{\tau^l,t}, \quad (1.49)$$

$$\hat{\tau}_t^k = \rho_k \hat{\tau}_{t-1}^k + (1-\rho_k)(\phi_k \hat{b}_{t-1}) + \epsilon_{\tau^k,t}, \tag{1.50}$$

where $\phi_i \geq 0$ for $i = \{gy, gb, c, l, k\}$ capture the fiscal measures response to the cyclical position of the economy and to the government debt level. ϵ_t^i for $i = \{g, tr, l, k, c, d\}$ are i.i.d error terms that capture the unexpected changes in policy. Parameters ρ_i for $i = \{g, tr, c, l, k\}$ are the persistence parameters of fiscal measures (government spending, transfers, consumption, labour and capital income tax rates).

1.4.4.2 Monetary Policy

Monetary policy is implemented independently by the Bank of Botswana with the ultimate objective of price stability. The Bank defines this as a low and stable level of inflation within the objective range of $3-6$ percent. It is assumed that the Bank follows a log-linearised Taylor rule, with the set interest rates responding to previous interest rate and deviations in inflation from the steady state and growth in output as follows:

$$\hat{r}_t = \rho_r \hat{r}_{t-1} + (1-\rho_r)(\phi_\pi \hat{\pi}_t + \phi_y(\hat{y}_t - \hat{y}_{t-1})) + \epsilon_t^r, \tag{1.51}$$

where ρ_r is the interest rate smoothing parameter. The parameter $\phi_\pi > 1$ measures the adjustment of nominal interest rate to deviations in the inflation rate from the steady state, while the parameter $\phi_y > 0$ measure how the nominal interest rates adjust to output growth. The parameter is non-negative to have a stabilising effect. $\epsilon_{r,t}$ is an i.i.d normal error term with zero mean and constant variance.

1.4.5 Market Clearing and the Current Account

Goods market clearing requires that final good is equal to consumption by households, government spending and investment so that:

$$Y_t^n = C_t + I_t + G_t. \tag{1.52}$$

In equilibrium, the total labour supply by households should be equal the demand for labour by firms in each sector:

$$L_t = L_t^m + L_t^n. \tag{1.53}$$

Total public debt (B_t^P) is made by domestic (B_t) and foreign (B_t^*) borrowing:

$$B_t^P = B_t + B_t^*. \tag{1.54}$$

Total output (GDP) is defined as the sum of output in the non-mining sector and mining/resource sector:

$$Y_t = Y_t^n + P_t^d Y_t^m. \tag{1.55}$$

The net exports (NX) or trade balance is the value of resource (commodity) exports less the value of imports. Recall that given a significant proportion (over 80%) of the commodity exports in total exports it is assumed that the country does not export other goods:

$$NX_t = P^d Y^m - s_t P_t^* Y_{F,t}. \tag{1.56}$$

The external debt accumulation equation for a small open economy evolve according to the following equation:

$$s_t B_t^* = s_t R_{t-1}^s B_{t-1}^* + NX_t, \tag{1.57}$$

where R_t^s is the specific cost of foreign borrowing that depends on the world risk free rate (R^w) and the country specific risk premium, RP_t, as in Agénor (2016). R^w follows an AR(1) process as it is assumed to be exogenous to the small open economy. The risk premium is positively related to the country's foreign debt-to-GDP ratio:

$$RP_t = \left(\frac{s_t B_t^p}{Y_t} \right)^{\rho_{rp}}, \tag{1.58}$$

where $\rho_{rp} > 0$ governs how quickly debt returns to its steady state level. The inclusion of a risk premium implies that even though government has access to external funds, increased borrowing will lead to a higher risk premium. This increases the cost of borrowing and ensures that public debt is not explosive (Alberola and Benigno, 2017).

1.4.6 Calibration

The model is calibrated to reflect Botswana's economic conditions. Botswana is a relevant candidate, with a diamond mining sector that is a key contributor to economic activity and government revenues. The economy represents well the framework in that the mining sector is well established and has been in existence for decades. Therefore, it is ideal to model the resource sector in a way that give us the opportunity to better understand the transmission of the diamond price shocks to the rest of the economy.

As observed by Dagher et al. (2012), the calibration of developing countries can be a challenging task. However, as it is common in the DSGE literature, the use of calibration measures from prior studies and using the available data to match the characteristics of the economy is adopted. The discount factor $\beta = 0.975$ which imply a steady state risk free nominal interest rate of 5 percent. This reflects the higher interest rates in developing economies. The depreciation rate of capital, δ, is set equal to 0.025, which implies an annual depreciation rate of 10 percent as in Smets and Wouters (2003). This is a very common specification in the literature. The parameter for adjustment cost is set at 5.2. The inverse elasticity of labour supply $\varphi = 8$ so that the Frisch elasticity of labour supply is 0.125 as in Agénor (2016). This is reasonable to capture the somewhat inelastic labour supply in developing countries.

The share of capital in the non-mining sector production is set at 0.35 (i.e. $\alpha = 0.35$) which leaves a share of 0.65 for labour. This is consistent with earlier papers like Berg et al. (2013). For the mining sector, the share of labour $\alpha_m = 0.3$, which reflects the capital intensive nature of mining. There is no empirical guide for this parameter, but the figure is fairly reasonable to represent the diamond mining industry of Botswana. The persistence parameter for productivity for both sectors is set at 0.7 to ensure consistency of the model.

As is common in the literature, setting the index of price stickiness $\theta_H = \theta_F = 0.75$ is consistent with an average duration of one year between price changes. The elasticity of substitution between differentiated goods $\eta = 11$, which implies a steady-state price markup of 10% as in Sims and Wolff (2018a). For interest rate rule, the smoothing parameter is set at 0.75. The parameter $\phi_\pi = 1.5$ ensures that the central bank responds more than proportionate to inflation developments. In periods of shocks, the central bank responds aggressively to the output developments and therefore $\phi_y = 0.825$. This Taylor type policy parameters are applicable in developing economies when price stability is the primary goal of monetary policy and interest rates are the main policy instruments.

Table 1.1 Baseline Calibration

Parameter	**Description**	**Value**
β	Discount factor	0.975
α	Capital share: non-mining	0.35
α^m	Labour share:mining	0.3
δ	Capital depreciation rate	0.025
φ	Inverse of Frisch labour elasticity	8
γ	Sectoral labour elasticity	1
θ_h, θ_f	Calvo parameters: domestic (import) prices	0.75
ϵ	Goods elasticity of substitution	6
μ	Degree of openness	0.5
τ^c, τ^l, τ^k	Consumption, Labour and capital income tax rates	0.12, 0.15, 0.22
τ^d	Share of diamond revenues to government	0.6
ρ_α, ρ_α^m	productivity persistence:non-mining/mining	0.7
ρ_d	Diamond price persistence	0.7
ϕ_{gb}, ϕ_{gy}	Government spending response to debt (output)	0.1 (0.025)
$\phi_{cb}, \phi_{hb}, \phi_{kb}$	Tax rates response to debt	0.1
ρ_i	Fiscal parameters persistence	0.7
ρ_r,ρ_{r^*}	interest rate smoothing parameter	0.75
ϕ_π	interest rate response to inflation	1.2
ϕ_y	Interest rate response to output growth	0.825
ρ_w	World interest rate persistence	0.95
ρ_{rp}	Debt cost response to risk premium	0.025
C/GDP	Private consumption to GDP	0.5
I/GDP	Private investment to GDP	0.1
G/GDP	Government spending to GDP	0.25
P^dY^m/GDP	Mining output to GDP	0.2
B/GDP	Public debt to GDP	0.20
TR/GDP	Public transfers to GDP	0.04

Steady state measures calibrated to match the Botswana economy data are presented in Table 1.1. The government spending to GDP ratio of 0.25 is set to ensure that the steady state values are consistent with data averages for the period 2004 - 2018. The calibration yields 0.1 for investment. Consumption is calibrated at 50 percent of GDP, consistent with the data. The response of fiscal measures to debt innovations and output developments is adjustable consistent with the policy experiments. Where the fiscal measures are not responsive to output and debt developments the parameters are set at 0. While, where they are allowed to respond, they are set at 0.1 as in Lim and McNelis (2013). The persistence parameter for fiscal measures is set at 0.7. Meanwhile the mining sector is calibrated to reflect the 20 percent share of total output. The persistence of diamond price shocks ρ_d is set at 0.7 to ensure that there is recovery within about two years. The public debt-to-GDP ratio is set at 0.2 to reflect the averages for the sample period. In the model this is made up of domestic and foreign borrowing in equal proportions.

1.5 Results

This section explores the dynamic effects of shocks to the model using impulse response functions (IRFs). The IRFs help understand how the effects of the exogenous shocks propagate through the economy. To showcase the consistency of the model to the common shocks, productivity and monetary policy shocks are discussed prior to the diamond price shock. This set of shocks allows for a greater understanding of the dynamics of the economy and facilitates for the model to be compared with other models in the literature. Results from these shocks are consistent with the literature (see, among others, Smets and Wouters (2007), Clancy and Merola (2016)).

1.5.1 Productivity Shock

The effect of an exogenous shock to productivity in the non-mining sector is analysed in Figure 1.5. The shock is simulated as a 1 percent increase in total factor productivity of the non-mining good production function. An increase in the productivity of the factor inputs has a positive impact on output. An increased efficiency in the use of factor inputs leads to a fall in marginal cost which implies lower prices. The lower inflation creates room for an accommodative monetary policy stance (fall in interest rate). In addition, the fall in inflation increases real wages which leads to an increase in household incomes. Due to higher household incomes, lower prices and low interest rates, household consumption increases.

The increased factor efficiency reduces the demand for labour shifting employment (hours) downwards. This result is in line with previous studies that show negative labour (hours) response to positive technology shocks (for example, Gali (1999) and Smets and Wouters (2007) among others). The increased marginal product of capital following a productivity shock increases investment, contributing to an increase in total output. Government spending acts as an automatic stabiliser as it declines due to output expansion. These results show the potential benefits of the measures put in place to promote productivity in Botswana. These measures include, labour force training, increased adoption of latest technologies and providing an enabling business environment that should help promote sustainable output growth.

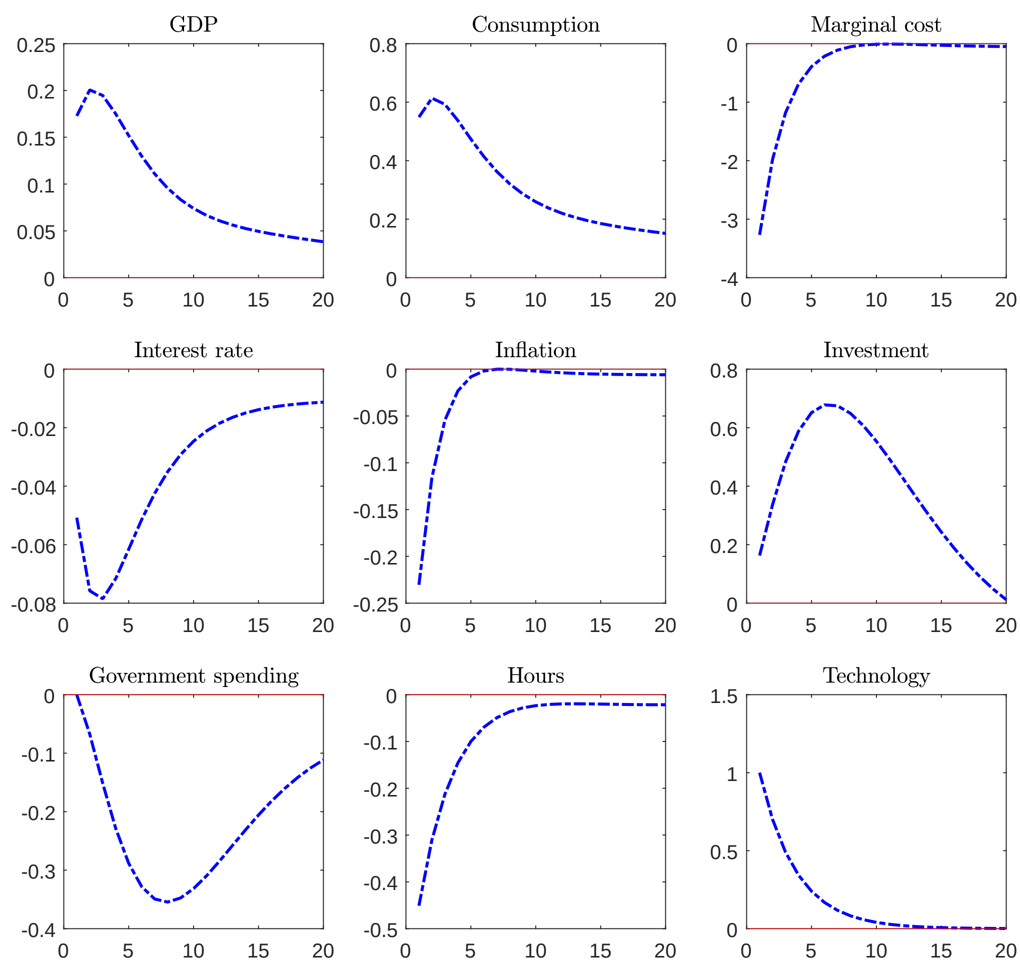


Figure 1.5: Impulse Responses to a Positive Productivity Shock

1.5.2 Monetary Policy Shock

The contractionary monetary policy shock is illustrated in Figure 1.6. The shock is simulated by a one standard deviation positive shock which increases the interest rate. The impulse responses show that variables respond in a way consistent with economic literature. An increase in interest rates lead to a contraction in output. In a similar manner, consumption and investment also contract in response to higher interest rates. Higher interest rates lead to a fall in aggregate demand resulting in a decline in inflation. Government spending also contract due to lower revenues on account of lower tax collection and high borrowing costs following an increase in interest rates. The reduced domestic demand following a tighter monetary policy stance leads to a fall in demand for factor inputs. As a result, employment (hours worked) and the returns to labour (wages) decrease.

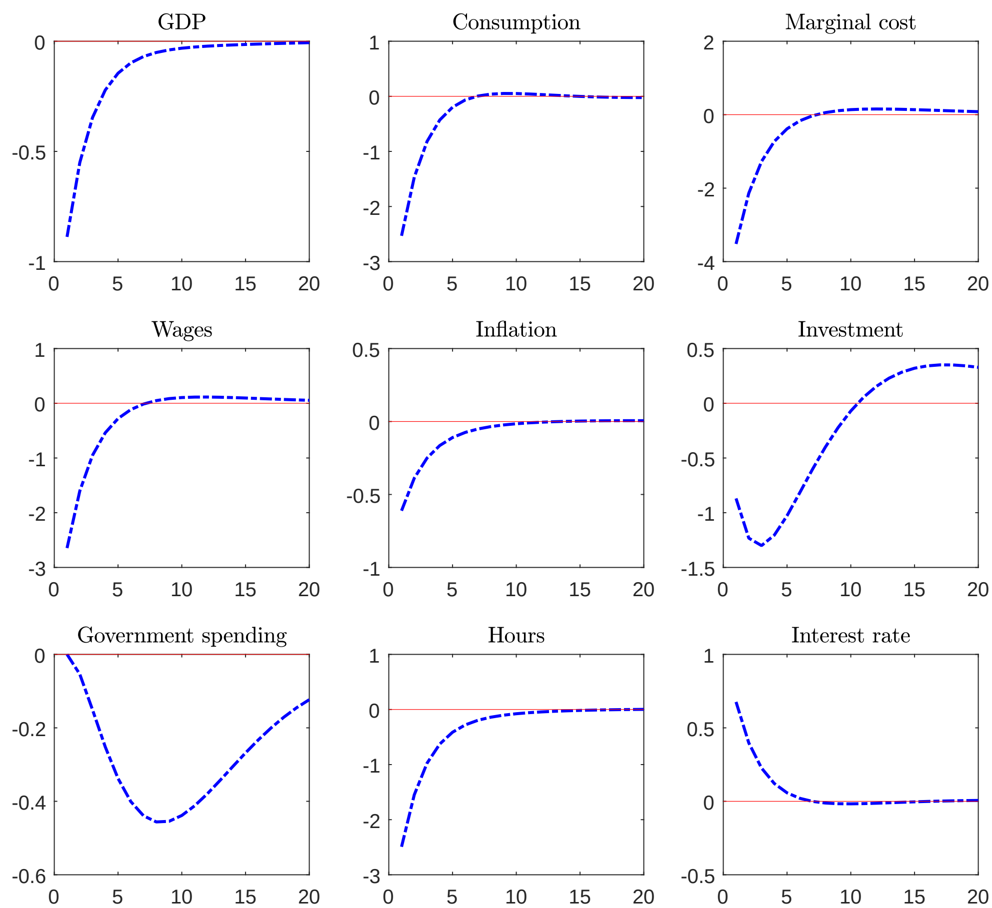


Figure 1.6: Impulse Responses to a Contractionary Monetary Policy Shock

1.5.3 Negative Diamond Prices Shock

Figure 1.7 presents the IRFs for a negative shock to diamond prices in the model. The analysis considers in the main two cases where for the benchmark case, government spending is the only fiscal instrument adjusting following the shock with the fiscal shortfall financed through an increase in public debt. In the alternative setups, we present scenarios where fiscal policy responds through a combination of government spending and an increase in taxation. This includes an increase in consumption tax, labour income tax and capital income tax, respectively.

The idea of increasing taxes when the economy's output is contracting may be seen as against the well established macroeconomic wisdom of providing stimulus through tax cuts. However, Correia et al. (2013), by adopting an unconventional fiscal policy framework show that distortionary taxes could be used to replicate the effect of negative interest rates and avoid the conditions of the zero lower bound. This involves generating an increasing path for consumption taxes alongside a decreasing path for labour income taxes. This suggests that increasing tax rates in periods of recession is an alternative policy response for countries experiencing zero lower bound. Although used to reflect the effects of a negative nominal interest rate in their framework, the application shows that in order for policy to address some macroeconomic objectives it may have to be against the conventional wisdom. For instance, in this paper's setup, the increase in tax rates is about raising government revenues to ensure fiscal solvency. As observed by Correia et al. (2013), such a framework require flexibility in tax rates.

The results show that a negative one standard deviation shock to diamond prices leads to a negative impact on mining GDP. The decline in mining output leads to a decline in total output. Furthermore, the contraction in mining GDP has a negative fiscal effect as it reduces resource revenues due to the government. As a result, there would be a financing shortfall that needs to be addressed for the government budget constraint to hold. It is assumed that the government finances the fiscal gap by issuing bonds. This increased public borrowing triggers an adjustment in other fiscal instruments to ensure fiscal solvency. For the benchmark case, this includes a marginal decline in government spending (given that government also engages in a counter-cyclical policy to address the fall in output). For the alternative cases, a decrease in government spending is complemented by an increase in tax rates.[5] Their combined effect leads to a gradual fall in the debt-to-GDP ratio.

The decline in government spending has a demand effect - it reduces the demand for goods and services which results in lower prices and consequently a fall in inflation. The deflationary pressures and weak economic activity triggers a response by the central bank to undertake an expansionary monetary policy by reducing the policy interest rate, consistent with the Taylor rule. Meanwhile, the resultant low prices and interest rates lead to an increase in private consumption on impact through the demand effect and inter-temporal effect.

The increase in consumption needs further clarification. Conventional wisdom postulates that a decrease in commodity prices should lead to a decline in purchasing power within an economy, resulting in a decline in consumption (see Shousha, 2016; Drechsel and Tenreyro, 2018). However, for an economy that can leverage on debt for some time, the pass through of commodity prices could be limited. Instead the expansionary conditions created by falling public demand and lower interest rates may increase aggregate private demand resulting in an increase in consumption. This postulation is also consistent (albeit on the opposite side) with Bergholt et al. (2019)'s observation that public demand crowds out private consumption following increases in commodity prices. Put another way, falling public demand may crowd in private consumption in periods of falling commodity prices.

For the labour markets, the fall in diamond prices results in reduced demand for labour in the mining sector. The weak demand for labour and resultant low wages in the sector leads to a decline in labour income. Thus, the negative diamond price shock will have a negative wealth effect on households that work in the mining sector. On the other hand, the non-mining sector will experience a positive labour supply shock induced by an increase in labour that moves from the mining sector. This, in turn, leads to a decline in wages in the non-mining sector. A decline in wages reduces the marginal cost which reinforces the decline in inflation.

It is worthy to note that low interest rates exert a downward pressure on return to

[5] This case represent the policy options implemented by Botswana in the aftermath of the financial crisis of 2008-09 where public debt increased and consumption tax (VAT) also increased from 10% to 12%.

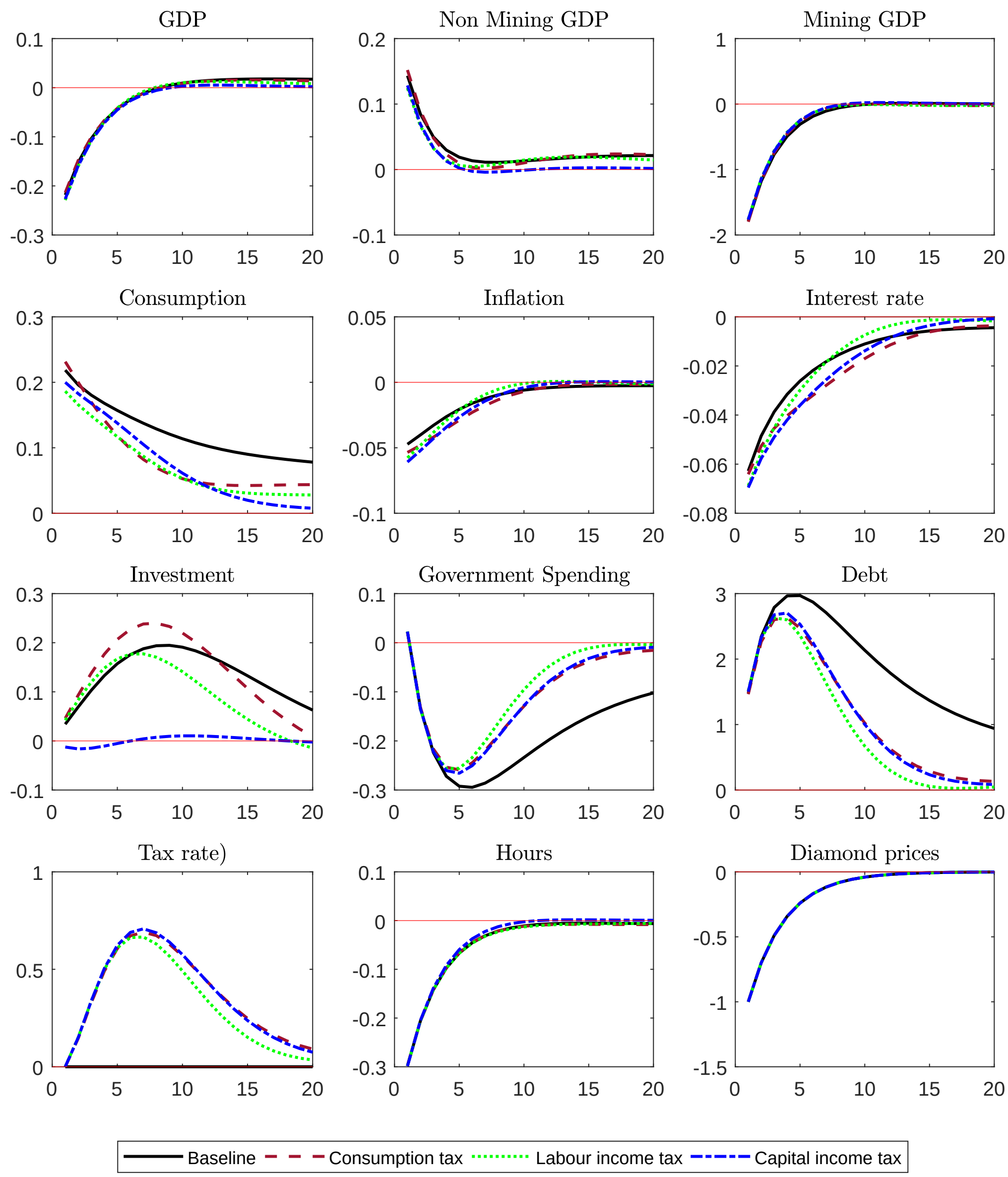


Figure 1.7: Impulse Responses to a Negative Diamond Price Shock

capital which promotes investment in the non-mining sector and leads to a build up of private capital. This increases the demand for labour in the non-mining sector. A good match for the labour supply shock which ensures that wages do not take long to return to the steady state. A combination of an increase in the capital stock and labour in the non-mining sector leads to an expansion in non-mining GDP. The outcome here reflects Botswana's experiences of 2011, where despite the decline in diamond prices (exports) and a decline in government spending, non-mining GDP posted a positive growth (see International Monetary Fund, 2012).

Overall, a negative shock on diamond prices leads to a sharp decline in mining GDP on impact. This contributes to a fall in total GDP as the slight increase in non-mining GDP is not enough to offset the fall in mining GDP. GDP recovers gradually as investment and production in the non-mining sector increase and the impact of the shock wanes. The differences of the benchmark and alternative cases are more pronounced for government spending and the debt-to-GDP ratio. Similar to Sims and Wolff (2018b), we considered the different tax rates in financing the fiscal gap. The alternatives allow a quicker recovery of the government spending as well as a faster decline in public debt. In general, the impact of the tax adjustments in the economy are similar with the exception of the increase in capital income tax which makes saving less attractive and households switch their spending towards consumption. This reduces investment and capital accumulation. The changes have a limited impact on total output since investment is a small proportion of GDP. Similarly, the response of consumption and labour income tax rates have offsetting impacts over consumption and government spending. As a result, their responses have no material impact on total output.

1.5.4 Public Debt

A negative diamond price shock has a direct impact on government revenues. The decline in revenues in turn has implications for the government's ability to spend. It is evident from the results that one of the key ways to finance the resulting government budget shortfall is through public debt. In many developing countries it may not be easy to access credit for a number of reasons. These could include poor credit ratings and generally weaker fiscal positions to afford debt. Therefore, despite the significance of public debt in helping the economy recover it may be an option that is out of reach. Next, we assess the implications of the government issuing and not issuing debt while in the alternative the government uses public debt to help smooth the impact of the shock to the economy.

1.5.4.1 No Debt Financing

The first scenario, a situation of no public debt financing underscores the vital role of public debt for commodity producers in the wake of commodity price downturns. With no access to debt, the inevitable macroeconomic policy response to the negative shock on commodity prices is a significant fall in government spending due to a fall in government revenues from the mining sector. The major contraction of the economy and a decline in inflation given reduced public demand prompts an aggressive expansionary monetary policy. The fall in public demand weighs down investment despite lower interest rates. Meanwhile, the low prices and expansionary monetary policy leads to an increase in private consumption by households. However, given the significant fall in government spending and investment, non mining GDP contracts.

It is evident from Figure 1.8 that in the absence of public debt to smooth a negative commodity price shock the combined fall of mining GDP and non mining GDP will lead to a substantial contraction of the economy. Thus, in the absence of debt to offset the financing shortfall, the fall in economic activity will be deep and prolonged with the recovery setting in only when the impact of the shock dissipates. This implies that it is critical for a commodity producer to be in a position that it can increase its borrowing in the event of downturns.

1.5.4.2 Debt Financing

Where the government uses public debt to minimise the impact of the fall in resource revenues, investment in the non-mining sector increases and the decrease in government spending is modest. This allows for an increase in non-mining GDP although it is not enough to offset the contraction in mining GDP which leads to a slighter decline in total GDP.

The literature on resource-rich economies has shown that although debt could be used to manage commodity price shocks, in practice this has been a challenge. According to Frankel (2010), ideally a country should borrow when it experiences downturns to sustain consumption and investment and should repay the debt or accumulate foreign assets when commodity prices recover. However, for developing countries, debt has been used more

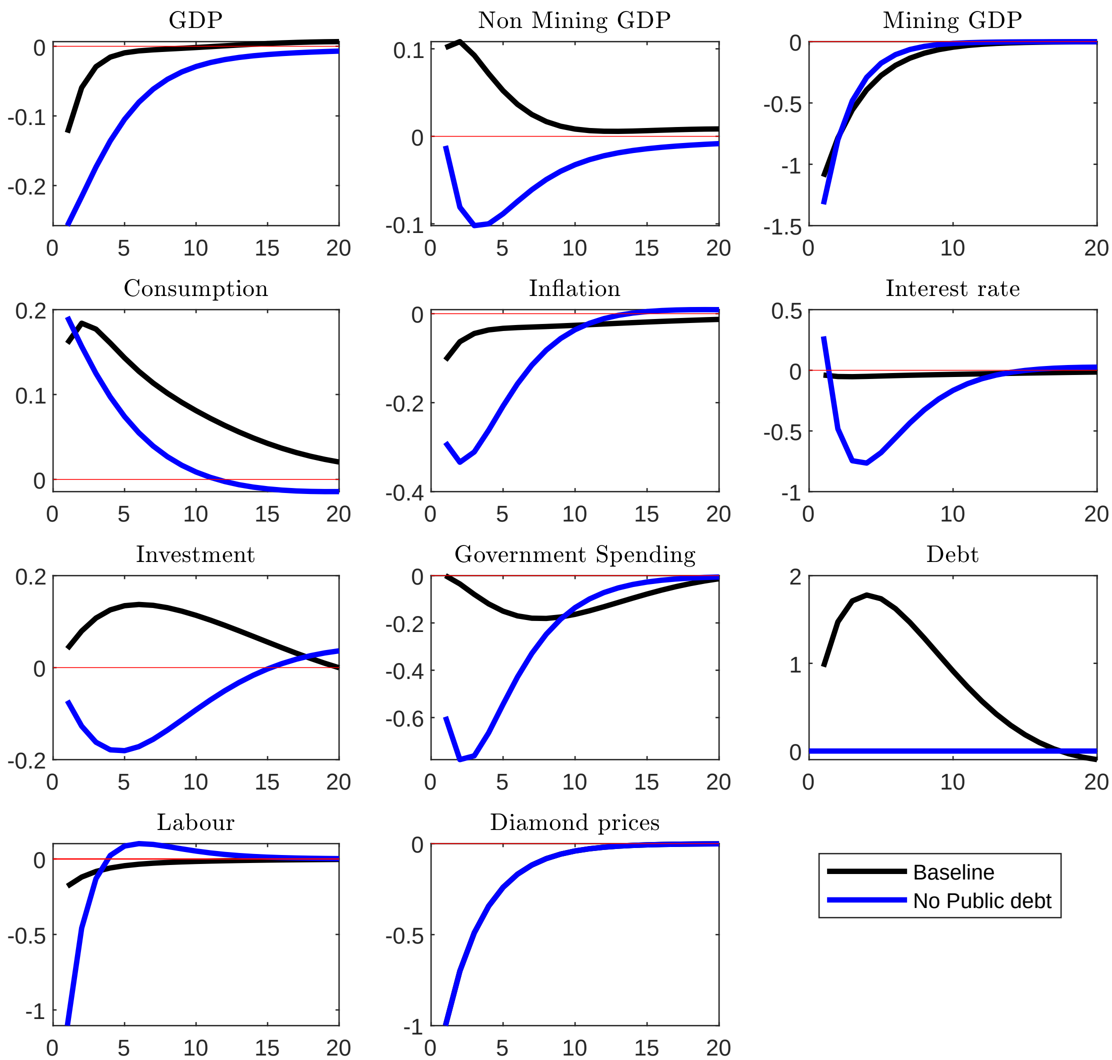


Figure 1.8: Impulse Responses to a Negative Diamond Price Shock: With and Without Public Debt Smoothing

procyclically than countercyclically. In periods of a boom, governments found it easy to access credit with lenders convinced that they can recover their money. As a result, when there are downturns they find it difficult to access credit when they need it the most. This is where Céspedes and Velasco (2012)'s conclusion that macro responses reflect the country's unique structural characteristics and policy frameworks come into play. A country with very low debt levels, high levels of external reserves and a high credit rating can be able to use debt countercyclically. In this way, fiscal policy is able to mitigate the impact of the commodity price shocks on the economy. Such is the lesson that comes from Botswana. After the 2008 crisis and resulting sharp fall in diamond revenues the government increased its borrowing (still remaining below the statutory limit of 40 percent of GDP) and repaid the debt when prices increased. Hence, for countries that debt is used in a more procyclical way the results here will not be applicable.

1.5.5 Robustness Analysis

Since the parameters of the model were not directly estimated it is important to consider the robustness of our results for a set of plausible parameter values. We specifically test for the different values of the share of labour in the mining sector, Frisch labour elasticity and elasticity of labour between sectors. For share of labour in mining production, we test for values from 0.2 to 0.35 and there is negligible change in the macroeconomic responses. Similarly, a change in Frisch labour elasticity has no material change in the results of the model. Increasing the labour mobility between sectors leads to output overshooting the steady state after the shock. In this model it is ideal to maintain low levels of mobility as the mining sector is specialised and there are rigidities associated with the size of employment and the unique skill sets required in the mining sector. The results are also robust to a range of values for the investment adjustment cost parameter and the substitutability of goods.

1.6 Conclusion

Resource revenues are a lifeline to developing economies as they provide the much required funding for economic development. However, these resource revenues are unstable due to volatile commodity prices. As a result, the conduct of policy can be challenging for these economies. The challenge of increasing commodity prices has been discussed extensively in the literature with the proposition that developing countries should engage in sustainable investment. This allows countries to invest part of the funds domestically and save the rest in a sovereign wealth fund. In this way, they both prepare for downturns and provide capital necessary for domestic public investment. Falling commodity prices can be more challenging for commodity exporters. If not well managed, the consequences can be severe. Nonetheless, the impact of falling commodity prices and the policy responses have received limited attention in the literature.

In this paper, we study the transmission path of the commodity prices and identify the macroeconomic policy responses to counter the impact of a fall in commodity prices in the short to medium term. Using a medium scale DSGE model calibrated for Botswana, we present the macroeconomic effects of a negative diamond price shock on the economy. The results point to a sharp fall in mining GDP and contraction of total output following a negative diamond price shock. The shock is mainly transmitted to the economy through the fiscal channel of a cut in public spending following a fall in government revenues. The economy can respond through a modest fall in government spending financed by borrowing and an increase in tax rates as well as an expansionary monetary policy. All these responses lead to a modest growth in non-mining sector which moderates the decline of total output and helps the economy recover as the impact of the shock wanes. The reinforcing effect of the combined monetary and fiscal policy responses to commodity price shocks prove to be superior to a single policy response.

The analysis shows that the use of public debt in the policy intervention is critical. Where there is no access to credit, the impact of the shock is deep and prolonged. This highlights the need to build the credit worthiness of a country as well as the need to use public debt in a countercyclical manner. Lastly, it has to be appreciated that all these policy measures can only address a temporary shock. Hence, commodity producers need to diversify their economies to build economies more resilient to a protracted fall in commodity prices.

1.7 Appendix

1.7.1 Steady State

In the steady state $A = 1$, $A^m = 0.2$, $\pi = 1$, $L^M = L^N = 1/3$, $P^D = 1$, $G = g_y Y$ where $g_y = \frac{G}{GDP} = 0.25$ and $B = b_y Y$ where $b_y = \frac{B}{GDP} = 0.2$

$$R = \frac{1}{\beta} \tag{1.59}$$

$$R^k = \frac{\frac{1}{\beta} - 1 + \delta}{1 - \tau^k} \tag{1.60}$$

$$MC = \frac{\eta - 1}{\eta} \tag{1.61}$$

$$I = \delta K \tag{1.62}$$

$$C = Y - G - I - P^D Y^m \tag{1.63}$$

$$K = L^N \left(\frac{R^k}{\alpha MC} \right)^{\frac{1}{\alpha - 1}} \tag{1.64}$$

$$Y^N = K^\alpha L^{N(1-\alpha)} \tag{1.65}$$

$$Y = Y^N + P^D Y^M \tag{1.66}$$

$$T^M = \tau^d P^D Y^M \tag{1.67}$$

$$W^N = \frac{MC(1-\alpha) Y^N}{L^N} \tag{1.68}$$

$$W^M = \frac{\alpha^M P^D Y^M}{L^M} \tag{1.69}$$

$$TR = T^M + \tau^c C + \tau^k K + \tau^l W^N L^N + \tau^l W^M L^M - G + (1 - R_s) B \tag{1.70}$$

$$L = \left(L^{M(1+\gamma)} + L^{N(1+\gamma)} \right)^{\frac{1}{1+\gamma}} \tag{1.71}$$

1.7.2 The Log-linearised Model

Equilibrium condition for household consumption

$$\hat{c}_t = \hat{c}_{t+1} - \left(\hat{r}_t - \hat{\pi}_{t+1} + \frac{\tau^c}{1+\tau^c} (\hat{\tau}^c_t - \hat{\tau}^c_{t+1}) \right) \tag{1.72}$$

Household labour supply in the non-mining sector

$$\hat{l}_t^n = \frac{1}{\gamma}(\hat{w}_t^n - \hat{c}_t - \frac{\tau^c}{1+\tau^c}\hat{\tau}_t^c - \frac{\tau^l}{1-\tau^l}\hat{\tau}_t^l) + (\gamma - \varphi)\hat{l}_t \tag{1.73}$$

Household labour supply in the mining sector

$$\hat{l}_t^m = \gamma(\hat{w}_t^m - \hat{c}_t - \frac{\tau^c}{1+\tau^c}\hat{\tau}_t^c - \frac{\tau^l}{1-\tau^l}\hat{\tau}_t^l) + (\gamma - \varphi)\hat{l}_t \tag{1.74}$$

Capital accumulation process

$$\hat{k}_t = (1-\delta)\hat{k}_{t-1} + \delta\hat{i}_t \tag{1.75}$$

Tobin's Q

$$\hat{q}_t = \hat{\pi}_{t+1} - \hat{r}_t + \frac{1}{(1-\delta+(1-\tau^k)R^k)}[(1-\tau^k)R^k\hat{r}_{t+1}^k - R^k\tau^k\hat{\tau}_t^k + (1-\delta)\hat{q}_{t+1}] \tag{1.76}$$

Household choice of investment

$$\hat{i}_t = \frac{1}{1+\beta}\hat{i}_{t-1} + \frac{\beta}{1+\beta}\hat{i}_{t+1} + \frac{1}{\kappa(1+\beta)}\hat{q}_t \tag{1.77}$$

Domestic production of the non-mining goods

$$\widehat{yn}_t^h = a_t + \alpha\hat{k}_{t-1} + (1-\alpha)\hat{l}_t \tag{1.78}$$

Total factor productivity in the non-mining sector

$$\hat{a} = \rho_a\hat{a}_{t-1} + \epsilon_a \tag{1.79}$$

Cost minimisation - optimal wages

$$\hat{w}_t^n = \widehat{mc}_t^h + \widehat{yn}_t^h - \hat{w}_t^n \tag{1.80}$$

Cost minimisation - optimal return to capital

$$\hat{r}_t^k = \widehat{mc}_t^h + \widehat{yn}_t^h - \hat{k}_{t-1} \tag{1.81}$$

Real marginal cost in domestic non-mining sector

$$\widehat{mc}_t^h = \alpha\hat{r}_t^k + (1-\alpha)\hat{w}_t - \hat{p}_t^h - \hat{a}_t \tag{1.82}$$

Phillips curve for domestic non-mining goods

$$\hat{\pi}_t^h = \beta \hat{\pi}_{t+1}^h + \left[\frac{(1-\theta^h)(1-\beta\theta^h)}{\theta^h} \right] \widehat{mc}_t^h \tag{1.83}$$

Phillips curve for the imported good retailer

$$\hat{\pi}_t^h = \beta \hat{\pi}_{t+1}^h + \left[\frac{(1-\theta^f)(1-\beta\theta^f)}{\theta^f} \right] \widehat{mc}_t^f \tag{1.84}$$

Real marginal cost for the imported good retailer

$$\widehat{mc}_t^h = \hat{s}_t - \hat{p}_t^f \tag{1.85}$$

Household demand for domestically produced goods

$$yd^n = -\psi p_t^h + y_t^n \tag{1.86}$$

Household demand for foreign produced goods

$$y_t^f = -\psi p_t^f + y_t^n \tag{1.87}$$

Mining sector production

$$\widehat{y}_t^m = \hat{a}_t^m + \hat{l}_t^m \tag{1.88}$$

Total factor productivity in the mining sector

$$\hat{a}_m = \rho_{a_m} \hat{a}_{m,t-1} + \epsilon_{a_m} \tag{1.89}$$

Diamond prices in foreign currency

$$p_t^{d*} = \rho_{p_d*} p_{t-1}^{d*} - \epsilon_t^{p_d*} \tag{1.90}$$

Diamond price in domestic currency

$$p_t^d = s_t + p_t^{d*} \tag{1.91}$$

Demand for labour in the mining sector

$$\hat{l}_t^d = \hat{p}_t^d - \hat{y}_t^m - \hat{w}_t^d \tag{1.92}$$

Total diamond revenues

$$\hat{t}^d = \hat{p}^d + \hat{y}^m \tag{1.93}$$

Government budget

$$TM\widehat{tm}_t + \tau^c C(\hat{\tau}^c_t + \hat{c}_t) + \tau^l W^N L^N(\hat{\tau}^l_t + \hat{w}^n_t + \hat{l}^n_t) + \tau^l W^M L^M(\hat{\tau}^l_t + \hat{w}^m_t + \hat{l}^m_t)$$
$$+ \tau^k R^k K(\hat{\tau}^k_t + \hat{r}^k_t + \hat{k}_{t-1}) + B\hat{b}_t = R^s B(\hat{r}^s_{t-1} - \widehat{\pi}_t + \hat{b}_{t-1}) + G\hat{g}_t + TR\hat{tr}_t \quad (1.94)$$

Government spending process

$$\hat{g}_t = \rho_g \hat{g}_{t-1} + (1-\rho_g)(-\phi_{gy}\hat{y}_t - \phi_{gb}\hat{b}_{t-1}) + \epsilon_{g,t} \quad (1.95)$$

Consumption taxes

$$\hat{\tau}^c_t = \rho_{\tau^c}\hat{\tau}^c_{t-1} + (1-\rho_{\tau^c})(\phi_c \hat{b}_{t-1}) + \epsilon_{\tau^c,t} \quad (1.96)$$

Labour income taxes

$$\hat{\tau}^l_t = \rho_{\tau^l}\hat{\tau}^l_{t-1} + (1-\rho_{\tau^l})(\phi_l \hat{b}_{t-1}) + \epsilon_{\tau^l,t} \quad (1.97)$$

Capital income taxes

$$\hat{\tau}^k_t = \rho_{\tau^k}\hat{\tau}^k_{t-1} + (1-\rho_{\tau^k})(\phi_k \hat{b}_{t-1}) + \epsilon_{\tau^k,t} \quad (1.98)$$

Transfers to households

$$\hat{tr}_t = \rho_t r\hat{t}_{t-1} + \epsilon_{tr,t} \quad (1.99)$$

Monetary policy

$$\hat{r}_t = \rho_r \hat{r}_{t-1} + (1-\rho_r)(\phi_\pi \hat{\pi}_t + \phi_y \hat{y}_t) + \epsilon_{r,t} \quad (1.100)$$

Debt accumulation equation

$$B(\hat{s}_t + \hat{b}_t) = R^s B(\hat{s}_t + \hat{r}^s_t + \hat{b}_{t-1}) + P^D Y^M(\widehat{p}^d_t + \widehat{y}^m_t) - (\hat{p}^f_t + \hat{y}^f_t) \quad (1.101)$$

Uncovered interest rate parity condition

$$\hat{s}_t - \hat{s}_t = \hat{\pi}^*_t - \hat{\pi}^*_t + \hat{r}_t - \hat{r}^*_t - rp_t \quad (1.102)$$

Real exchage rate

$$\hat{s}_t = (1-\mu)\hat{\tau}_t \quad (1.103)$$

Terms of trade

$$\hat{\tau}_t = \hat{p}^f_t - \hat{p}^h_t \quad (1.104)$$

Country specific debt elastic interest rate

$$\hat{r}^s_t = \hat{r}^w_t + \rho_{wr}(\hat{s}_t + \hat{b}_t - \hat{y}_t) \tag{1.105}$$

World interest rate

$$\hat{r}^w_t = \rho_w \hat{r}^w_{t-1} + \epsilon^{r^w}_t \tag{1.106}$$

Foreign inflation

$$\hat{\pi}^*_t = \rho_{\pi^*} \pi^*_{t-1} + \epsilon^{\pi^*}_t \tag{1.107}$$

Debt aggregator

$$B^P = B\hat{b}_t + B\hat{b}^*_t \tag{1.108}$$

Labour aggregator

$$\hat{l}_t = \left[\frac{L^N}{L}\right]^{1+\gamma} \hat{l}^n_t + \left[\frac{L^M}{L}\right]^{1+\gamma} \hat{l}^m_t \tag{1.109}$$

Non mining final good uses

$$\hat{y}^n_t = \frac{C}{Y^n}\hat{c}_t + \frac{I}{Y^n}\hat{i}_t + \frac{G}{Y^n}\hat{g}_t \tag{1.110}$$

Total output

$$Y\hat{y}_t = Y^n\hat{y}^n_t + P^d Y^m(\hat{p}^d_t + \hat{y}^m_t) \tag{1.111}$$

Chapter 2

A Large Dataset for Macroeconomic Forecasting in Botswana

2.1 Introduction

The use of large macroeconomic datasets for macroeconomic research is growing. This is largely inspired by the need to extract as much information from the several macroeconomic indicators available to macroeconomists about the state of the economy and the behaviour of economic agents. When processed, this information could be of benefit to central bankers for the conduct of monetary policy, fiscal authorities analysing fiscal policy options or businesses and investors making investment decisions. Although in the past it would have been almost impossible to analyse such large amounts of data, advances in research through factor models and other dimensionality reduction techniques allow for greater applications of large macroeconomic datasets. In light of these data requirements, assembling an informative database that helps explain economic variables of interest has become an integral part of macroeconomic research (McCracken and Ng, 2020).

In the past two decades, researchers have made significant advances in compilation of macroeconomic datasets that allow for the extraction of useful information from a large panel of economic predictors. These databases among others, are compiled in Stock and Watson (2002b) and McCracken and Ng (2016, 2020) for the United States of America, Forni et al. (2001) for the Euro Area and Fortin-Gagnon et al. (2018) for Canada. The large macroeconomic databases are then used for forecasting macroeconomic series (Stock and Watson, 2002a; Marcellino et al., 2003; D'Agostino and Giannone, 2012), structural macroeconomic analysis (Forni and Gambetti, 2010; Boivin et al., 2020), business cycle analysis (Aruoba et al., 2009) and measuring macroeconomic uncertainty (Jurado et al., 2015).

In this paper, we compile a macroeconomic database for Botswana. This work is inspired by US databases in McCracken and Ng (2016, 2020). Compiling a database of this nature provides capacity for macroeconomic analysis, forecasting and monitoring in a data-rich environment. This is important for developing countries given the success of factor models and large macroeconomic datasets in macroeconomic forecasting and monitoring reviewed in Stock and Watson (2016). The key advantage of such a compilation is that it creates a standardised and readily available database that provides for timely macroeconomic analysis when the need arise. As such, it is a valuable resource for researchers in academia, public policy institutions and the private sector.

Due to data availability constraints and differences in economic structures, the coverage of variables is different from that of McCraken and Ng. Similar to these authors, the choice of variables for building this dataset is guided by the key considerations of the pioneering work of Stock and Watson (1996) on data compilations. These include the need to have (i) main monthly economic aggregates and coincident indicators (ii) leading economic indicators and (iii) a broad class of variables and lastly identifying series with consistent historical definitions. The main categories of variables that are similar to those in McCracken and Ng are money and credit, consumer prices, interest rates, exchange rates and stock market indicators.

Like most developing countries, some macroeconomic indicators available in US studies are not available in Botswana. Given the limited industrial activity, there are no industrial production indicators. The more relevant mining production index is only available on a quarterly basis. Economic surveys of consumer sentiments and business conditions are considered relevant in these datasets. However, no such measures are available at a high frequency in Botswana. Bank of Botswana's business expectations survey was carried out biannually with a transition to a quarterly publication in 2019. Furthermore, these macroeconomic databases usually include a broad range of labour and employment series. Unfortunately, labour statistics are not available at a high frequency, with unemployment data only available intermittently.

On a positive note, for Botswana, government spending figures are available on a monthly basis. In addition, trade values (both aggregated and disaggregated exports and imports) are available at a monthly frequency. This ensures that the monthly dataset has key macroeconomic variables. For a small open economy like Botswana, it is important that the database include important indicators of economic conditions in trading partner countries. Overall, the dataset consist of 96 variables over the period 2005:01 - 2019:12. The variables are categorised into eight groups: (1) commodity prices, (2) fiscal, (3) consumer prices (4) trade, (5) money and credit, (6) financial, (7) regional and (8) global.

Our analysis shows that the extracted factor estimates explain a reasonable variation of the Botswana data. This implies that the macroeconomic dataset accommodates a factor structure. This is a necessary condition for the suitability of these dataset to be analysed using factor models. A forecasting exercise demonstrates that some of the dynamic factor models lead to improvements in forecasting inflation indicators and credit growth (one of the measures used to represent economic activity in the absence of industrial production) in comparison to a benchmark autoregressive model. These results indicate that the dataset will be useful for macroeconomic forecasting and policy analysis.

The rest of the paper is organised as follows. The next section presents Botswana's macroeconomic dataset. Section 3 presents the static factor model and generalised dynamic factor models used to study the factor structure of the macroeconomic dataset. Section 4 discusses the factor estimates and demonstrates that the macroeconomic dataset for Botswana accommodates a factor structure. Section 5 presents a forecasting exercise that shows how some dynamic factor models improve on an autoregressive benchmark in forecasting Botswana's inflation and economic activity indicators. Section 6 concludes and emphasises the need to collect timely data on other key macroeconomic indicators.

2.2 Macroeconomic Dataset

This section presents a large macroeconomic dataset for Botswana in the spirit of McCracken and Ng (2016) for the United States of America. Most of the data is sourced from Bank of Botswana's Botswana Financial Statistics, a monthly statistical publication of the Bank. The selection of monthly macroeconomic indicators into Botswana's macroeconomic dataset is guided by their relevance, consistency and timely availability. In choosing variables, researchers also consider variables that Bloomberg market participants and central banks monitor as it was the case for Luciani et al. (2018) and Caruso (2018) for Indonesia and Mexico, respectively. In this paper, most of the selected variables feature prominently in Bank of Botswana's Monetary Policy Reports. This shows that the central bank monitors these variables to form expectations about the state of the economy. For a small open economy like Botswana, it is also important that the dataset includes key indicators of economic activity, price developments and business conditions in trading partner countries (see Aastveit et al., 2014; Caruso, 2018). These foreign indicators are sourced from the respective central banks, statistical agencies and international institutions.

The dataset covers the period from January 2005 to December 2019, $T = 180$. The series consists of $n = 96$ economic variables that can be categorised into eight blocks as presented in Table 2.1. The commodity prices block captures the key global commodity price indicators (Brent crude oil price, Food and Agriculture Organisation (FAO) food price indices and the International Monetary Fund (IMF)'s aggregate global commodity price index) and the rough diamond price index relevant for Botswana. The fiscal block consists of the monthly government outlays and captures government spending on both recurrent and development budget items. This block represents the direct effect of government spending on economic activity in Botswana. Unlike more private sector led economies, most private companies rely on government projects for their business in Botswana. Therefore, the government expenditure patterns highly signals the business conditions in the country.

The consumer prices block is made up of the overall consumer price index (CPI) and its sub-components. It would have been ideal to also have a producer price index (PPI) included in the data, but Statistics Botswana only started producing PPI indicators

(Mining and Electricity & Water Supply industries) in 2017. The money and credit block captures developments in the banking sector. Credit developments are a key determinant of economic activity and are closely monitored by the central bank to inform the conduct of monetary policy. Total credit and the constituent loans to households and businesses form part of the dataset. Monetary aggregates also reflect the level of business activity in the economy. As a result, M1, M2 and monetary base are included.

The financial category consists of interest rates, exchange rates and stock market indicators. Interest rates determine the cost of borrowing and influences credit developments in the economy. In this way, interest rates affect the spending on goods and services and ultimately their prices. To capture the role of interest rates, the dataset consists of policy rates, lending rates, saving rates, the government's long term government bond yield and interest rate spread between the prime lending rate and the savings rate. Botswana maintains a crawling peg exchange rate framework which allows for modest movements in bilateral exchange rates which has implications for trade, price developments and the overall economic developments. To capture the role of exchange rates we include the currencies that make up the Botswana Pula currency basket, the South African Rand (ZAR) and the Special Drawing Rights (SDR) of the IMF. In addition, the bilateral exchange rate with leading countries (United States of America, United Kingdom, Japan and Euro area) are included to reflect their role in international trade and finance.

Botswana's exports and imports are consolidated under the trade category. For exports, only diamond exports are added given their dominant share (80 - 90%) in total exports. All the imports subcategories are added to the database. For a regional trading partner, selected variables that reflect economic developments in South Africa are included in the dataset. The variables cover South Africa's economic activity (manufacturing index and mining index), business cycle (coincident indicator), monetary policy (interest rate and CPI) and the stock market (JSE Allshare index). Finally, a set of economic indicators for the United States of America, China and the Euro area are considered to account for the role of global economic developments. They cover business cycle indicators, monetary policy interest rates, stock market indices and industrial production indicators in these key markets.

Some data transformations are necessary to prepare the data for analysis. First, where necessary, monthly series are seasonally adjusted using the US census x13 method to remove distortions associated with seasonal factors (e.g. violation of constant parameter assumption). Second, series are transformed to achieve stationarity and harmonise the different variable scales. Lastly, the data are demeaned and standardised to facilitate the estimation using a factor model. A detailed table on the data transformation for each variable is available in the Appendix.

Table 2.1 Number of Variables per Category

Category	Number
Commodity prices	6
Fiscal and other	4
Prices	11
Trade	13
Money and Credit	7
Financial	16
Regional	14
Global	25

2.3 The Factor Model

In this section, we describe the factor models used to demonstrate the existence of a factor structure using Botswana's macroeconomic dataset. The section borrows extensively from the prominent works in this area (see Forni et al., 2000; Stock and Watson, 2002b; Forni et al., 2005, 2015, 2017).

Factor models aim to describe the co-movement between a large number of macroeconomic variables by a few latent factors. This allows factor models to be parsimonious and avoids the estimation of a large number of parameters given a very small degrees of freedom (the *curse of dimensionality*) associated with standard estimation methods like vector autoregressive (VAR) models. With a VAR model, the number of parameters increases with the square of n and estimation becomes unfeasible as the number of variables increase.

On the contrary, for factor models, the number of parameters does not increase with n. The fact that factors will be easily recovered with a large cross-sectional dataset implies that factor models have the *blessing of dimensionality*.

Let the panel, $\mathrm{x}_t = (x_{1t}, \ldots, x_{nt})'$, be an n dimensional vector of covariance stationary time series $(x_{it} \mid t = 1, \ldots, T\})$ with zero mean and unit variance. Let $\Gamma_k = E(\mathrm{x}_t \mathrm{x}'_{t-k})$ be the covariance matrix of x_t and $\Sigma(\theta)$ its spectral density matrix at frequency $\theta \in [-\pi, \pi]$. Further, define $\{v_j, z_j\}_{j=1}^{n}$ and $\{\lambda_j(\theta), p_j(\theta)\}_{j=1}^{n}$ as the eigenvalues (in decreasing order) and the corresponding eigenvectors of Γ_k and $\Sigma(\theta)$, respectively.

A factor representation implies that each variable in the dataset can be expressed as the sum of two orthogonal components. The common component driven by few common shocks and the idiosyncratic component that captures the variable specific shocks and measurement errors. The factor model is expressed as:

$$x_{it} = \chi_{it} + \xi_{it}, \tag{2.1}$$

where χ_{it} is the *common* component and ξ_{it} is the *idiosyncratic* component.

The following subsections presents the three versions of large-dimensional factor models. The models differ in how they account for time series dependence of the common components, estimation strategy and the forecasting equation (Giovannelli et al., 2021).

2.3.1 Static Method, Static Representation: SW

Stock and Watson (2002a) (SW) use principal component analysis (PCA) to extract static factors. In the static factor model, the common component is a linear combination of a few $(r \leq n)$ static factors $f_{1t}, f_{2t}, \ldots, f_{rt}$ such that:

$$\chi_{it} = \lambda_{i1} f_{1t} + \lambda_{i2} f_{2t}, \ldots, +\lambda_{ir} f_{rt}, \tag{2.2}$$

for $i = 1, \ldots, n$ and $t = 1, \ldots, T$. Loadings (λ_{ij}) for $i = 1, \ldots, n$ and $j = 1, \ldots, r$ represent the contributions of variable i to the common factor. The static factor model can be expressed in vector form as:

$$\mathrm{x}_t = \mathbf{\Lambda} \mathbf{F}_t + \boldsymbol{\xi}_t, \tag{2.3}$$

where $\mathbf{\Lambda} = (\lambda_1, \ldots, \lambda_n)'$ is $n \times r$ matrix of factor loadings, $\mathbf{F}_t = (f_{1t}, \ldots, f_{rt})'$ is an $r \times 1$ vector of *static factors* and have only a contemporaneous effect on x_t. $\boldsymbol{\xi}_t = (\xi_{1t}, \ldots, \xi_{nt})'$ is an $n \times 1$ vector of idiosyncratic components uncorrelated with the common component at all leads and lags. The PCA simultaneously estimate the factors and the factor loadings. The r principal components are estimated by a linear combination of the data that maximises the variance of the estimated factors $\hat{z}\hat{\Gamma}_0\hat{z}'$, where $\hat{\Gamma}_0$ is the sample variance-covariance matrix of x_t. The maximisation problem can be transformed into solving the eigenvalue problem

$$z_j\hat{\Gamma}_0 = v_j z_j, \qquad j = 1, \ldots, r.$$

where v_j is the j^{th} eigenvalue (sorted in decreasing order) and z_j is the associated eigenvector of dimension $n \times 1$. The estimated factors $\mathbf{F}_t^{\mathrm{SW}} = \hat{z}\mathrm{x}$, where $\hat{z}$ is a $n \times r$ matrix of stacked eigenvectors $\hat{z} = (\hat{z}_1, \ldots, \hat{z}_r)'$.

2.3.2 Dynamic Method, Static Representation: FHLR

The dynamic factor model proposed by Forni et al. (2005) (FHLR henceforth) is a type of *generalised dynamic factor model* (GDFM) first proposed by Forni et al. (2000). The model uses a dynamic principal component approach to extract the factors. The common component is driven by a q-dimensional vector of common factors $f_t = (f_{1t}, f_{2t}, \ldots, f_{qt})'$:

$$\chi_{it} = b_{i1}(L)f_{1t} + b_{i2}(L)f_{2t}, \ldots, +b_{iq}(L)f_{qt}. \tag{2.4}$$

By defining $B(L)$ as the $n \times q$ matrix whose (i, j) entry is $b_{ij}(L)$, the dynamic factor model is:

$$\mathrm{x}_t = B(L)f_t + \xi_t. \tag{2.5}$$

The FHLR methodology follows two steps to estimate the factor space and the associated covariance matrices.

Step 1: The estimation of common components is based on the dynamic principal components method that relies on spectral densities. The $n \times n$ spectral density of x_t for frequency $\theta \in [-\pi, \pi]$ is defined as:

$$\Sigma^{\mathrm{x}}(\theta) = \frac{1}{2\pi} \sum_{k=-\infty}^{\infty} e^{ik\theta}\Gamma_{\mathrm{x}}(k), \tag{2.6}$$

where $\Gamma_{\mathrm{x}}(k) = E[\mathrm{x}, \mathrm{x}'_{t-k}]$.

Let $\hat{p}_j(\theta)$ and $\hat{\lambda}_j(\theta)$ be the eigenvectors and eigenvalues of $\hat{\Sigma}^{\mathrm{x}}(\theta)$, the spectral density matrices of the common and idiosyncratic components are defined respectively as:

$$\widehat{\Sigma}^{\chi}(\theta) = \sum_{j=1}^{q} \hat{\lambda}_j(\theta)\hat{p}'_j(\theta)\hat{p}_j(\theta) \tag{2.7}$$

$$\widehat{\Sigma}^{\xi}(\theta) = \sum_{j=q+1}^{n} \hat{\lambda}_j(\theta)\hat{p}_j(\theta)\hat{p}_j(\theta), \tag{2.8}$$

where orthogonality conditions imply that $\Sigma^{\mathrm{x}}(\theta) = \widehat{\Sigma}^{\chi}(\theta) + \widehat{\Sigma}^{\xi}(\theta)$. The corresponding common and idiosyncratic covariances using the Fourier transforms are estimated as:

$$\hat{\Gamma}^{\chi}_{k} = \frac{2\pi}{2M+1} \sum_{k=-M}^{M} e^{ik\theta}\widehat{\Sigma}^{\chi}(\theta_j) \tag{2.9}$$

$$\hat{\Gamma}^{\xi}_{k} = \frac{2\pi}{2M+1} \sum_{k=-M}^{M} e^{ik\theta}\widehat{\Sigma}^{\xi}(\theta_j), \tag{2.10}$$

where $M = [T^{1/2}]$ and Fourier frequencies $\theta_j = \frac{2\pi}{2M+1}$

Step 2: The factor space can be estimated by linear combinations $a\mathrm{x} = a\chi + a\xi$ of the x's as n approaches infinity. The aim is to compute r independent linear combination $\hat{F}_{jt} = \hat{Z}_j \mathrm{x}_t$, where the weights $\hat{Z}_j$ maximise the variance of $\hat{Z}_j\mathrm{x}$ estimated in step 1. The weights are defined recursively as

$$\hat{Z}_j = \arg\max_{a\in\mathbb{R}^n} a\hat{\Gamma}^{\chi}_0 a' \quad \text{s.t.} \quad a\hat{\Gamma}^{\xi}_0 a' = 1 \quad \text{and} \quad a\hat{\Gamma}^{\chi}_0 \hat{Z}'_m = 0 \tag{2.11}$$

for $j = 1, \dots, r$ and $1 \leq m \leq j-1$ (for $j = 1$, only the first constraint applies). The solutions Z_j are the *generalised eigenvectors* associated with the *generalised eigenvalues*, $\hat{v}_j$, of the matrices $\hat{\Gamma}^{\chi}_0$ and $\hat{\Gamma}^{\xi}_0$.

$$\hat{\mathrm{Z}}_j\hat{\Gamma}^{\chi}_0 = \hat{v}_j\hat{\mathrm{Z}}_j\hat{\Gamma}^{\xi}_0, \qquad j = 1, \dots, n, \tag{2.12}$$

with the normalisation constraints $\widehat{\mathrm{Z}}_j\widehat{\mathbf{\Gamma}}^{\xi}_0\hat{\mathrm{Z}}'_j = 1$ and $\hat{\mathrm{Z}}_i\hat{\Gamma}^{\xi}_0\hat{\mathrm{Z}}'_j = 0$ for $i \neq j$.

Define $\hat{\mathrm{Z}} = (\hat{\mathrm{Z}}_1, \dots, \hat{\mathrm{Z}}_r)'$ as a vector of the first r *generalised principal components* of x. The estimated factor $\mathbf{F}^{\mathbf{FHLR}}_{\mathbf{t}} = \hat{\mathrm{Z}}\mathrm{x}_t$.

The linear combinations of the x's extracted from the information in the covariance matrices estimated in step 1 are more efficient than standard principal components.

2.3.3 Fully Dynamic Method: FHLZ

This model developed in Forni et al. (2015, 2017) (FHLZ) is also a type of *generalised dynamic factor model*. The FHLZ model relaxes the assumption that the common component spans a finite-dimensional space. In this way, this approach addresses the restrictiveness of finite-dimension assumption that rules out simple cases like

$$x_{it} = \frac{c_i}{1-\alpha_i L}u_t + \xi_{it} = c_i(u_t + \alpha_i u_{t-1} + \alpha_i^2 u_{t-2} + \ldots) + \xi_{it}. \tag{2.13}$$

Starting with a representation of GDFM model of the form

$$x_{it} = \chi_{it} + \xi_{it} = \sum_{k=1}^{q} b_{ik}(L)u_{kt} + \xi_{it}, \qquad i = 1, \ldots, n \tag{2.14}$$

where $\mathbf{u}_t = (u_{1t} \ldots u_{qt})'$ is q-dimensional orthonormal white process ($q \leq n$). L is the lag operator and the filters $b_{ik}(L)$ are one-sided and square summable for any $i = 1, \ldots, n$ and any $k = 1, \ldots, q$. The *common component* χ_{it} is driven by common shocks u_{kt} (dynamic factors). The *idiosyncratic component* ξ_{it} is weakly cross correlated, that is, the eigenvalues of its spectral density matrix are uniformly bounded as $N \to \infty$. The common component and the idiosyncratic component are mutually orthogonal at all leads and lags.

From equation (2.14), the common component can be expressed in vector form as

$$\boldsymbol{\chi}_t = \mathbf{B}(L)\mathbf{u}_t, \tag{2.15}$$

with $\boldsymbol{\chi}_t = (\chi_{1t} \ldots \chi_{nt} \ldots)'$ and $\mathbf{B}(L) = b_{i1}(L) \ldots b_{iq}(L)$. Forni and Lippi (2011) and Forni et al. (2015, 2017) show that if the common component has a rational spectral density it admits a unique autoregressive representation with block structure of the form:

$$\underbrace{\begin{pmatrix} \mathbf{A}^{(\mathbf{1})}(L) & \mathbf{0} & \ldots & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^{(\mathbf{2})}(L) & \ldots & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \ldots & \mathbf{A}^{(K)}(L) \end{pmatrix}}_{\mathbf{A(L)}} \underbrace{\begin{pmatrix} \boldsymbol{\chi_t}^{(1)} \\ \boldsymbol{\chi_t}^{(2)} \\ \vdots \\ \boldsymbol{\chi_t}^{(K)} \end{pmatrix}}_{\boldsymbol{\chi}_t} = \underbrace{\begin{pmatrix} \mathbf{R}^{(1)} \\ \mathbf{R}^{(2)} \\ \vdots \\ \mathbf{R}^{(K)} \end{pmatrix}}_{\mathbf{R}} \mathbf{u}_t, \tag{2.16}$$

where $\mathbf{R}$ is a $(q+1) \times q$ matrix and each diagonal block $\mathbf{A}^{(i)}(L)$, for any $i = 1, 2, \ldots K$, is a $(q+1) \times (q+1)$ polynomial matrix. Furthermore, $\det(\mathbf{A}^{(i)}(L)) \neq 0$ for $|z| \leq 1$, the filters

$\mathbf{A}^{(i)}(L)$ are one sided, and each polynomial has finite order. According to Barigozzi et al. (2019), the K autoregressive operators $\mathbf{A}^{(i)}(L)$ invert into fundamental moving average filters which are unique for a subset of $q+1$ dimensional subset of common components $(\chi_{(i-1)(q+1)+1}, \ldots, \chi_{i(q+1)})'$. That is, the polynomial matrix have no roots inside the unit circle.

Then, by inverting the polynomial matrix $\mathbf{A}(L)$ in Equation (2.16), it follows that the common component admits a moving average representation

$$\boldsymbol{\chi}_t = [\mathbf{A}(L)]^{-1}\mathbf{R}\mathbf{u}_t = \mathbf{B}(L)\mathbf{u}_t. \tag{2.17}$$

Finally, using $\boldsymbol{\chi}_t = \mathbf{x}_t - \boldsymbol{\xi}_t$ the GDFM can be expressed as:

$$\mathbf{A}(L)\mathbf{x}_t = \mathbf{R}\mathbf{u}_t + \mathbf{A}(L)\boldsymbol{\xi}_t. \tag{2.18}$$

Forni and Lippi (2011) show that the last term on the right hand side remain idiosyncratic. Estimation of Equation (2.18) is detailed in the following steps, consistent with Forni et al. (2015, 2017):

i. Estimate the spectral density matrix $\hat{\boldsymbol{\Sigma}}^{\mathbf{x}}(\theta)$ of $\mathbf{x}_t$: The $n \times n$ spectral density of $\mathbf{x}_t$ for frequency $\theta \in [-\pi, \pi]$ is defined as in Forni et al. (2000) as:

$$\hat{\boldsymbol{\Sigma}}^{\mathrm{x}}(\theta) = \frac{1}{2\pi} \sum_{k=-M}^{M} w_k e^{ik\theta} \Gamma_{\mathrm{x}}(k), \tag{2.19}$$

where $\Gamma_{\mathrm{x}}(k) = E[\mathrm{x}, \mathrm{x}'_{t-k}]$ and $w_k = 1 - \frac{|k|}{M+1}$ are the weights corresponding to the Bartlett lag with window size M. Empirical applications use $M = \sqrt{T}$ (see Forni et al., 2005; D'Agostino and Giannone, 2012).

ii. Estimate the spectral density matrix $\hat{\boldsymbol{\Sigma}}^{\chi}(\theta)$ of $\boldsymbol{\chi_t}$: This is obtained by means of dynamic principal component analysis that relies on spectral densities with the selection of q principal components associated with the largest eigenvalues. Let $\hat{p}_j(\theta)$ and $\hat{\lambda}_j(\theta)$ be the eigenvectors and eigenvalues of $\hat{\boldsymbol{\Sigma}}^{\mathrm{x}}(\theta)$, the spectral density matrix of the common component is defined as

$$\hat{\boldsymbol{\Sigma}}^{\chi}(\theta) = \sum_{j=1}^{q} \hat{\lambda}_j(\theta)\hat{p}'_j(\theta)\hat{p}_j(\theta). \tag{2.20}$$

iii. Estimate the autocovariance matrix $\hat{\mathbf{\Gamma}}_k^{\chi}$ of $\boldsymbol{\chi_t}$: By means of inverse Fourier transform we have

$$\hat{\mathbf{\Gamma}}_k^{\chi} = \frac{2\pi}{2M+1} \sum_{m=-M}^{M} e^{ik\theta_m} \widehat{\Sigma}^{\chi}(\theta_m), \tag{2.21}$$

where Fourier frequencies $\theta_m = \frac{2\pi m}{2M+1}$, $m = -M, \ldots, M$.

iv. Compute the VAR matrices $\mathbf{A}^k(L)$: These are obtained from common covariances by means of Yule-Walker equations.

v. Compute the matrices $\mathbf{R}$ and the shock $\mathbf{u}_t$: These are obtained via standard principal component analysis (Bai and Ng, 2002; Stock and Watson, 2002b) of $\tilde{\mathbf{x}}_t = \mathbf{R}\mathbf{u}_t + \tilde{\boldsymbol{\xi}}_t$, where $\tilde{\mathbf{x}}_t = \mathbf{A}(L)\mathbf{x}_t$ and $\tilde{\boldsymbol{\xi}}_t = \mathbf{A}(L)\boldsymbol{\xi}_t$. Given all the definitions the estimated impulse response functions of the common component to the q common shocks are defined as $\hat{\mathbf{B}}(L) = [\widehat{\mathbf{A}(L)}]^{-1}\hat{\mathbf{R}}$. It follows then that the estimated common component is given by $\hat{\chi}_{\mathbf{t}} = \hat{\mathbf{B}}(L)\hat{\mathbf{u}}_t$.

2.3.4 Factor Estimates

The use of factor models helps extract information contained in large macroeconomic datasets. This factor transformation facilitates the use of such data for forecasting of macroeconomic aggregates, structural macroeconomic analysis and estimation of factor based regressions. The next task after transforming the data is to identify the number of common factors that summarises the macroeconomic data in a parsimonious way. [1]

An information criterion proposed by Bai and Ng (2002) is used to determine the number of static factors, r. With the maximum number of factors set at 15, the selection procedure PC_{p2} gives an estimated number of static factors $\hat{r} = 10$. The PC_{p2} is considered to have good finite sample properties (Bai and Ng, 2002) and more stable in empirical work (Bai and Ng, 2008). The Hallin and Liška (2007) information criterion is used to determine the number of dynamic factors q. The criterion IC_1, penalty p_1 and a $q_{\max} = 10$ suggest that the estimated number of dynamic factors $\hat{q} = 3$.

Table 2.2 presents the number of factors and variance explained by the first ten (10)

[1] The estimations of the models are implemented in MATLAB using some of the codes shared by Professor Matteo Barigozzi through his website www.barigozzi.eu.

common (static/dynamic) components. The variance explained is the share of total variation in the dataset that can be explained by the common component. The total variance explained is constructed as a cumulative sum of the estimated eigenvalues. Note that the eigenvalues are basically the variance explained by each particular factor. Therefore, the ten static eigenvalues account for 48.59% of the variation in the dataset. It should be noted that the static and dynamic factors are alternative approaches to represent the common component of the dataset. Hence, the results in the table are consistent with earlier works (see Amengual and Watson, 2007; Barigozzi et al., 2014) as they show that the variance explained by the static factors is approximately equal to the variance explained by the selected three dynamic factors (48.63%).

Table 2.2 Total Variance Explained by First r/q Principal Components

Number of factors	1	2	3	4	5	6	7	8	9	10
Static factors (r)	11.79	19.29	24.43	29.08	32.91	36.50	39.96	42.97	45.86	**48.59**
Dynamic factors (q)	24.86	37.88	**48.63**	56.63	63.25	68.56	73.07	77.01	80.35	83.27

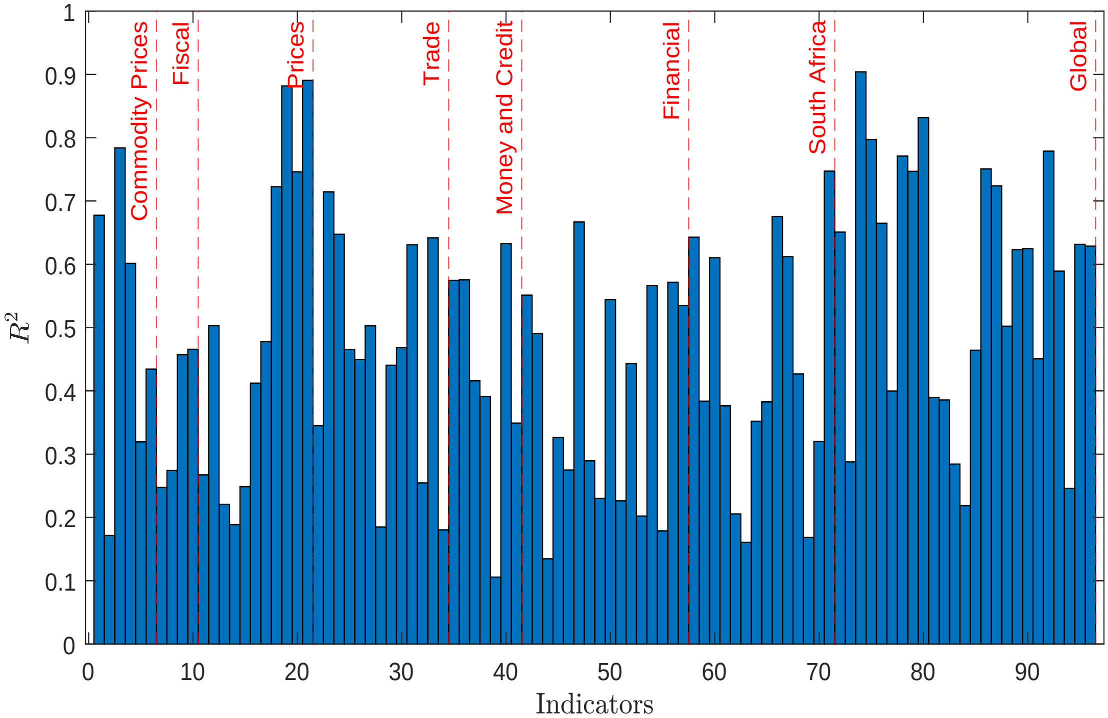
The first principal component (static/dynamic) explains 11.79% and 24.86% of the variation in the dataset. For both cases this factor is dominated by global predictors. This include, global industrial production, global stock market indices (MSCI World Index, MSCI Emerging Market index, S&P 500, FTSE100 and EUROSTOXX50) and global commodity prices. Hence, the first factor can be interpreted as the global factor.

2.3.5 Factor Importance

This part presents the share of each variable's variance that can be explained by the selected factors from both the static and dynamic factor models. High values indicate that variables are well represented by the common factor space, while variables with low values are not well represented as they experience considerable idiosyncratic variation. Therefore, the results here show the extent to which the macroeconomic dataset accommodates a factor structure.

Figure 2.1 presents the explanatory power of the first ten static factors for each series i

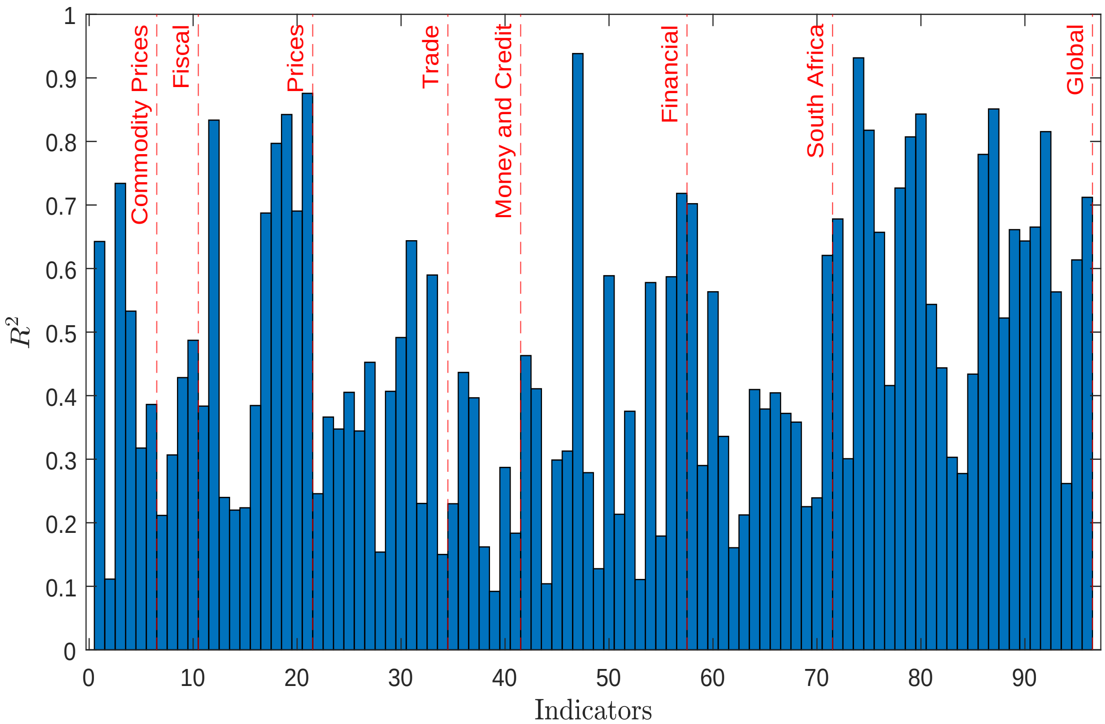
Figure 2.1: Importance of Static Factors



Note: Explanatory power of the first ten static factors on the macroeconomic indicators in the dataset. The bars are ordered according to the labelled categories. The horizontal axis are the series represented by the variable identity numbers as presented in the appendix. The vertical axis represents the fraction of variation in each series explained by the ten common factors. The relative importance of the ten common factors varies across the economic indicators. The fractions for some variables under prices, trade, financial and global indicators are relatively high which implies that they are subject to common variation. Some financial variables have a low R^2 which implies that the variables are subject to idiosyncratic variation.

The are three dynamic factors estimated from the GDFM presented in Equation (2.5). As with the static factors, Figure 2.2 presents the explanatory power of the three dynamic factors for the different groups. Similar to the results of the static factors, the relative importance of the dynamic factors varies across the series. For most variables, the fraction of the variation in each series explained by the three dynamic factors is slightly lower compared to the static factors.

Figure 2.2: Importance of Dynamic Factors



Explanatory power of the first three dynamic factors on the macroeconomic indicators

Figures 2.1 and 2.2 show that there is a high level of commonality among global variables. Table 2.3 presents the share of variance explained by the estimated common components for selected Botswana variables to highlight their commonality in the dataset. Common variance explained by static factors for the consumer price index, exports, bank rate and domestic company index is 89%, 71%, 55% and 57%, respectively. Similarly, the dynamic factors explain a significant proportion of the domestic variables. These outcomes show that Botswana data accommodates a factor structure. This is a validation that paves way for the dataset to be used in macroeconomic research that require a data-rich environment.

Table 2.3 Variance Explained for Selected Botswana Variables

Variable	S_VE	D_VE	Variable	S_VE	D_VE
Government spending	45.71	42.83	Credit	57.54	43.66
Consumer price index	89.06	87.57	Money supply (M2)	63.27	28.71
Exports	71.44	36.64	Bank rate	55.15	46.30
Imports	46.56	40.53	DCI index	57.16	58.71

Note: S_VE and D_VE represents the variance explained by static and dynamic factors, respectively.

2.4 Forecasting Inflation and Economic Activity Indicators

This section explores the potential of the large macroeconomic dataset in forecasting inflation indicators (headline inflation and core inflation - trimmed mean) and economic activity indicators (electricity consumption growth and credit growth). [2] To achieve this, we study the predictive ability of dynamic factor models compared to a benchmark autoregressive (AR) model of order p - AR(p) model of the target variable, defined as

$$\hat{y}_{t+h} = \hat{c} + \sum_{j=1}^{p} \hat{\varphi}_j y_{t-j+1}, \tag{2.22}$$

where y_t is the target variable, c is a constant, h is the forecast horizon and p is the number of lags. Since a monthly dataset is used, the maximum lag length is set at twelve with the optimal p chosen using a Bayesian Information Criterion (BIC) for every out-of-sample forecast period.

The next step is to evaluate the usefulness of additional information in predicting inflation indicators and the other selected economic indicators by comparing the benchmark AR(p) against a set of dynamic factor models. This is achieved by assessing the ratio of the Mean Square Forecast Error (MSFE) of each factor model to the MSFE of the AR(p).

[2] In Botswana, industrial production index which is normally used as a timely indicator of economic activity in other markets is not available while the more relevant mining production index is only available at a quarterly frequency.

A ratio less than one indicates that the dynamic factor model outperforms the benchmark AR(p) on average. To validate these results, we study the global predictive ability of the dynamic factor models over the autoregressive benchmark using the Diebold and Mariano (1995) (DM) test. The DM test tests the null hypothesis that the two models have equal performance. The DM test statistics are assumed to follow a normal distribution which determines their significance or otherwise at a given level of significance.

Following Giovannelli et al. (2021), we also use the Giacomini and Rossi (2010) fluctuation test to study the local predictive ability of the dynamic factor models over the benchmark. This procedure tests the null hypothesis that two models have equal out-of-sample performance at each point in time. In this way, the fluctuation test provides useful information on the evolution of model performance over the entire forecast path with respect to the benchmark. The fluctuation test results are presented in Figures 2.3 to 2.6 in the Appendix. The blue lines are the test statistics, estimated as the normalised and smoothed difference between the square forecast errors of the dynamic factor model (FHLZ, FHLR and SW) and the benchmark. Along the zero line there is equal performance while the red dashed lines indicate the 5% critical values. When the blue solid line is below the lower dashed red line, the selected factor model outperforms the benchmark locally in time at 5% level of significance (see Forni et al., 2018)

2.4.1 Forecasting Using Factor Models

There are a number of approaches to handle large macroeconomic datasets for forecasting purposes. In this paper, we focus on the dynamic factor models used to establish the ability of the dataset to accommodate a factor representation in section 3. The opportunity to compare the predictive ability of the three models will help establish the value of using a large macroeconomic dataset for Botswana in forecasting the variables of interest. This approach has been embraced in related work (see Forni et al., 2018; Giovannelli et al., 2021). The three forecasting equations of the factor models are as follows.

(a) SW Approach

Following from how the static factors are recovered for the SW model presented in subsection 3.1, the $h-$step ahead forecasting equation is recovered by a projection of $x_{i,t+h}$

on the space spanned by $(\hat{\mathbf{F}}_t, \hat{\mathbf{F}}_{t-1}, \dots x_{it}, x_{i,t-1}, \dots)$, i.e regressing $x_{i,t+h}$ on the factors $\hat{\mathbf{F}}_t$ and x_{it} with an option to add the lags of both (Stock and Watson, 2002b). Thus, the forecasting equation is defined as

$$\hat{x}^{SW}_{i,t+h} = \hat{\alpha}_{ih}(L)\hat{\mathbf{F}}_t + \hat{\beta}_{ih}(L)x_{it}, \tag{2.23}$$

where $\hat{\alpha}_{ih}(L)$ is a $1 \times r$ matrix polynomial and $\beta(L)$ is a scalar polynomial.

(b) FHLR Approach

Compared to the SW model that uses standard principal components, the FHLR method uses generalised principal components. However, the prediction equation has a form similar to the SW forecasting model presented above in Equation (2.23). The FHLR forecasting equation is given as

$$\hat{x}^{\mathrm{FHLR}}_{i,t+h} = \hat{\alpha}^G_{ih}(L)\hat{\mathbf{F}}^G_t + \hat{\beta}^G_{ih}(L)x_{it}, \tag{2.24}$$

with $\hat{F}^G_{jt}, j = 1, 2, \dots, r$ the j^{th} generalised principal components recovered by frequency-domain techniques (Forni et al., 2018).

(c) FHLZ Approach

Following Forni et al. (2018), the common component under the FHLZ method in section 3 is defined as

$$\hat{\boldsymbol{\chi}}_{\boldsymbol{t}} = \hat{\mathbf{B}}(L)\hat{\mathbf{u}}_t = \hat{\mathbf{B}}_0\hat{\mathbf{u}}_t + \hat{\mathbf{B}}_1\hat{\mathbf{u}}_{t-1} + \cdots . \tag{2.25}$$

Hence, the prediction equation for the common components at horizon h is modelled as

$$\hat{\boldsymbol{\chi}}^{\mathrm{FHLZ}}_{t+h|t} = \hat{\mathbf{B}}_h\hat{\mathbf{u}}_t + \hat{\mathbf{B}}_{h+1}\hat{\mathbf{u}}_{t-1} + \cdots . \tag{2.26}$$

Then using $\hat{\boldsymbol{\xi}}_t = \mathrm{X}_t - \hat{\boldsymbol{\chi}}_{\boldsymbol{t}}$ each variable x_{it} can be predicted by

$$\hat{x}^{\mathrm{FHLZ}}_{i,t+h|t} = \hat{\chi}^{\mathrm{FHLZ}}_{i,t+h|t} + \hat{\xi}^{\mathrm{FHLZ}}_{i,t+h|t}. \tag{2.27}$$

It is important to recall that the estimation of factor models is implemented on standardised data. Therefore, for all the factor models, the forecast of the target variables is recovered by reincorporating the estimated standard deviation (σ_i) and mean (μ_i) as in D'Agostino and Giannone (2012) so that

$$\hat{y}^{\mathrm{FM}}_{i,t+h} = \hat{\sigma}_i\hat{x}^{\mathrm{FM}}_{i,t+h} + \hat{\mu}_i, \tag{2.28}$$

where $\mathrm{FM} \in (\mathrm{SW}, \mathrm{FHLR}, \mathrm{FHLZ})$ represents the respective factor model.

2.4.2 Inflation Indicators

The monthly dataset spanning the period 2005m1 to 2019m12 is used to forecast inflation indicators (headline inflation and core inflation). The out-of-sample forecast period is 2012m1 to December 2019m12 with forecasting horizons for 1,3, 6 and 12 months using a rolling window of the recent five years (i.e., 60 monthly observations). The length of this rolling window scheme is consistent with applications in the macroeconomic forecasting literature (see Giovannelli et al., 2021). In order to use the best benchmark, the commonly used univariate autoregressive model was compared with $AR(p)$ with $p = 2$ selected based on the BIC criterion. The AR(1) model outperformed the $AR(p)$ in forecasting inflation indicators and was therefore treated as the benchmark for the inflation indicators.

Table 2.4 presents the Relative Mean Square Forecast Error (RMSFE) of the factor models to the benchmark $AR(1)$ model for the forecast period. A RMSFE below one implies that the FHLZ outperforms the benchmark at horizon 1 in forecasting core inflation. Using the Diebold and Mariano (1995) test to compare the models show that the null hypothesis of equal predictive performance is rejected at 10% level of significance. For the rest of the horizons the FHLZ underperforms the benchmark. The FHLR outperforms the benchmark at horizons 1, 3 and 6. However, it is only at horizon 6 where the null hypothesis of equal performance is rejected at 5% level of significance. SW outperforms the benchmark at horizons 3, 6 and 12 and the null hypothesis is rejected for horizons 6 and 12 at 10% and 5% respectively. The results show that the SW improves on the benchmark by about 26.1% on 12 months ahead forecasts. The Harvey et al. (1998) test, an adjustment of the Diebold and Mariano test for short datasets, confirms the forecast accuracy at the same significance level.

Going a step further by using the fluctuation test developed by Giacomini and Rossi (2010) we show that the FHLR and SW models perform better than the benchmark in multiple points around 2018. This is because the null hypothesis of equal performance will be rejected around this period as the test statistic is below the lower bound 5% critical value. Meanwhile, the fluctuation test also demonstrate that the FHLZ model is outperformed by the benchmark in forecasting core inflation over the forecast period at horizons 3, 6 and 12. The null hypothesis of equal performance is not rejected as the test statistic is not below the bottom 5% significance level red dashed line.

Table 2.4 Mean Square Forecast Error Relative to AR - Inflation Indicators

	CORE INFLATION			**HEADLINE INFLATION**		
	FHLZ	FHLR	SW	FHLZ	FHLR	SW
$h = 1$	**0.988***	0.989	1.101	0.939	**0.865****	0.964
$h = 3$	1.048	0.900	0.955	1.012	**0.832****	**0.926****
$h = 6$	1.106	**0.891****	**0.941***	1.125	**0.835****	**0.853***
$h = 12$	1.312	1.097	**0.739****	1.274	1.005	**0.903***

Notes: Relative mean square forecast error (RMSFE) for core inflation and headline inflation out-of-sample predictions from 2012m1 to 2019m12. Significance of the forecast accuracy assessed through the Diebold and Mariano (1995) test. Asterisks indicate 1% ($***$), 5% ($**$) and 10% ($*$) level of significance.

For headline inflation, the FHLZ outperforms the benchmark at horizon 1. However, we can not reject the null hypothesis of equal performance between FHLZ and benchmark at horizon 1. Meanwhile, the FHLR outperforms the benchmark at horizons 1, 3 and 6. The DM test resutlts show that the predictive superiority of the FHLR over the benchmark is statistically significant at 5% level of significance. The improvements of the FHLR over the AR ranges from 14.5% at horizon 1 to 16.8% at horizon 3. The SW model outperforms the benchmark at all horizons, although not statistically significant at horizon 1.

These results are confirmed by the fluctuation test in Figure 2.4. The test show that FHLZ performs better than the benchmark in 2014 in 1-step ahead headline inflation forecasts. The fluctuation test results also show that the FHLR and SW outperform the benchmark at several points over the entire forecast horizon. In other words, the null hypothesis of equal local performance is rejected at 5% level of significance as the test statistic breaches the lower bound 5% critical value at multiple points.

Overall, these results show that using a large dataset to forecast Botswana's inflation indicators improves on the AR benchmark. The results are highly significant for headline inflation and are consistent with earlier works in this area (see Forni et al., 2018; Della Marra, 2017). Thus, the dynamic factor models can be added as an alternative in Bank of Botswana's inflation forecasting toolkit.

2.4.3 Economic Activity Indicators

Table 2.5 presents the relative mean square error of the factor models and the benchmark $AR(p = 1)$ model for selected economic activity indicators. Similar to the inflation indicators discussed before, the sample period is 2005m1 to 2019m12 and the forecast period 2012m1 to 2019m12 with 1, 3, 6 and 12 months ahead forecasts. Although statistically insignificant, the results show that the FHLZ outperforms the benchmark AR model in forecasting growth in electricity consumption. The FHLR and SW perform more or less the same as the benchmark AR or worse. The fact that the factor models offer no improvement over the benchmark is demonstrated in the fluctuation test result presented in Figure 2.5. The poor performance of the factor models forecasts over the benchmark for electricity use growth are largely a result of an outlier in October 2013 where electricity consumption dropped significantly.

Table 2.5 Mean Square Forecast Error Relative to AR - Economic Activity Indicators

	ELECTRICITY USE GROWTH			**CREDIT GROWTH**		
	FHLZ	FHLR	SW	FHLZ	FHLR	SW
$h = 1$	0.968	1.090	1.233	0.989	**0.941*****	1.079
$h = 3$	0.945	1.008	1.029	1.151	**0.793*****	**0.853*****
$h = 6$	0.957	1.075	1.027	1.214	**0.770****	**0.747*****
$h = 12$	0.875	1.036	1.105	1.339	0.913	**0.803***

Notes: Relative mean square forecast error (RMSFE) for electricity use growth and credit growth out-of-sample predictions from 2012m1 to 2019m12. Significance of the forecast accuracy assessed through the Diebold and Mariano (1995) test. Asterisks indicate 1% $(***)$, 5% $(**)$ and 10% $(*)$ level of significance.

For credit growth forecasts, both the FHLR (5.9% - 23%) and SW (14.7% - 25.3%) show improvements over the benchmark AR. The gains of the FHLR are statistically significant at 1% level of significance for horizon 1 and 3 and at 5% level of significance for 6-step ahead forecasts. The SW (except for horizon $h = 1$) outperforms the AR and this is statistically significant (1% level of significance for horizon 3 and 6 and 5% level of significance for 12 months step ahead). Evaluation of local performance through the fluctuation test show that the null hypothesis that the FHLR/SW and AR models have an equal out-of-sample performance for credit growth is rejected at 5% level of significance. The gains are statistically significant at 5% level of significance (Figure 2.6). Therefore, results of the fluctuation test validates those of the relative MSFE that the FHLR and SW outperform the benchmark model in forecasting credit growth.

2.4.4 Discussion of Limitations of Small Sample

Compared to data from advanced economies, ours is a small sample. The out-of-sample forecast is of size $S = 84$. Therefore, it is worthy to think about the performance of the inferential approaches used in finite samples. First, for small samples, the estimation window may be too small so that the normal approximation of the test for global predictive accuracy like that of Diebold and Mariano (1995) may not be correct. In their Monte Carlo analysis, the authors show that the Diebold-Mariano test has a higher performance in terms of size and power for samples of size $S > 64$. Our results are reasonable because the forecast sample falls within this range. Additionally, the Diebold-Mariano test was found to be oversized for small samples, with the problem increasing as the forecast horizon, h, increases. To address this, we use the modified Diebold-Mariano test of Harvey et al. (1998) that has a better finite-sample performance in the sense that it is normal-sized at all horizons. The results leads to similar conclusions as the original test. This is important because it demonstrates that our predictive accuracy results are robust to the potential finite sample bias.

Second, it is possible that predictive models are subject to structural breaks of possibly unknown form (i.e temporal variation in the factor loadings and/or factor number). In such an unstable environment, the relative forecasting performance of the models may be changing over time. As explained in Section 2.4, the fluctuation test of Giacomini and Rossi (2010) is specifically designed for possibly unstable environments so that we can test the null of equal predictive ability at any point in time of our sample. All in all, our fluctuation test results lead us to conclude that our main results on the good performance of the factor models considered are robust to structural breaks. Of course, also the fluctuation test is based on an asymptotic approximation in finite samples. However, Giacomini and Rossi (2010) find that the fluctuation test is normal-sized and powerful for a sample size as little as $S = 50$, (i.e. way smaller than $S = 84$ as in this work). Hence, our conclusions are robust to potential breaks and finite sample bias.

2.5 Conclusion

This article presents a set of 96 monthly macroeconomic indicators for Botswana. The dataset covers the period from January 2005 to December 2019. Factor estimates show that the data panel accommodates a factor structure as both the static and dynamic factors explain a considerable variation of the macroeconomic series. The analysis show that there is commonality even with Botswana specific variables consumer price index, exports, bank rate and the domestic company index. The ability to handle a factor structure facilitates for the data to be used for structural macroeconomic analysis (e.g application implemented in the next chapter) and macroeconomic forecasting using factor models.

The forecasting exercise has demonstrated that the large macroeconomic dataset have predictive power for core inflation and headline inflation compared to the benchmark autoregressive model. Notably, the FHLR model is more dominant compared to the SW model and FHLZ model in forecasting the inflation measures. Similarly, for the selected economic activity indicators the FHLR and SW show gains over the benchmark AR in forecasting credit growth. The results show that these gains are statistically significant in terms of their global and local predictive ability. Thus, a GDFM approach using this dataset could be added to the suite of models that Bank of Botswana uses for macroeconomic forecasting.

Going forward, there is a need for Statistics Botswana and other statistics generating institutions to transition to high frequency data collection. This data should also be available timely to promote its use in macroeconomic analysis. For variables where data is not formally compiled, authorities should put in place measures that will ensure that data is available in the near future. This could include leveraging on technology to make online surveys that could give guidance on employment for example.

2.6 Appendix

2.6.1 The Number of Static and Dynamic factors

The use of factor models helps extract information contained in large macroeconomic datasets. The factor transformation facilitates the use of such data for forecasting of macroeconomic aggregates and estimation of factor based regressions. The next task then is to identify the number of common factors that summarises the macroeconomic data in a parsimonious way.

Static factors

There are many procedures developed to identify the number of factors for large datasets. Bai and Ng (2002) propose a criterion that chooses r static factors that best capture the variance, $V(k, F^k)$ in x_t. Let the variance be the sum of squared residuals (divided by NT)

$$V(k, F^k) = \min_{\Lambda} \frac{1}{NT} \sum_{i=1}^{T} (\mathrm{x}_t - \hat{\Lambda}\hat{F}_t^k)'(\mathrm{x}_t - \hat{\Lambda}\hat{F}_t^k), \tag{2.29}$$

where F^k is a matrix of k factors and Λ is the matrix of estimated factor loadings. The objective is to choose the number of factors that minimise the sum of squared residuals in a way that keeps the model parsimonious. Bai and Ng (2002) proposed a number of penalised objective functions to achieve this trade-off. For this paper, I use the penalty $\frac{N+T}{NT}\log(\min(N,T))$, so that the optimised criterion is

$$PC_{p2} = V(k, F^k) + k\frac{N+T}{NT}\log(\min(N,T)). \tag{2.30}$$

The number of r factors obtained by the minimisation of the criterion for $k = 0, \ldots, r_{max}$ where r_{max} is the maximum number of static factors. The selection procedure is called PC_{p2}. It is considered to have good finite sample properties (Bai and Ng, 2002) and more stable in empirical work (Bai and Ng, 2008).

Dynamic factors

To determine the number of q dynamic factors, Hallin and Liška (2007) proposes an information criterion based on the spectral density of the matrix.

$$IC_{1,n}^{T} = \frac{1}{n}\sum_{i=k+1}^{n}\frac{1}{2M_T+1}\sum_{h=-M_T}^{M_T}\lambda_{ni}^{T}(\theta_h) + ckp(n,T), \qquad 0 \leq k \leq q_{\max}, \tag{2.31}$$

where $\theta_h = \pi h/(M_T + 1/2)$ for $h = -M_T, \ldots, M_T$, $M_T > 0$ is a truncation parameter and c is a positive constant . $q_{\max}$ is a predetermined upper bound and $\lambda_{ni}^{T}(\theta)$ are the eigevalues of $\Sigma^{\mathrm{x}}(\theta)$. $p(n,T)$ is a penalty function such that

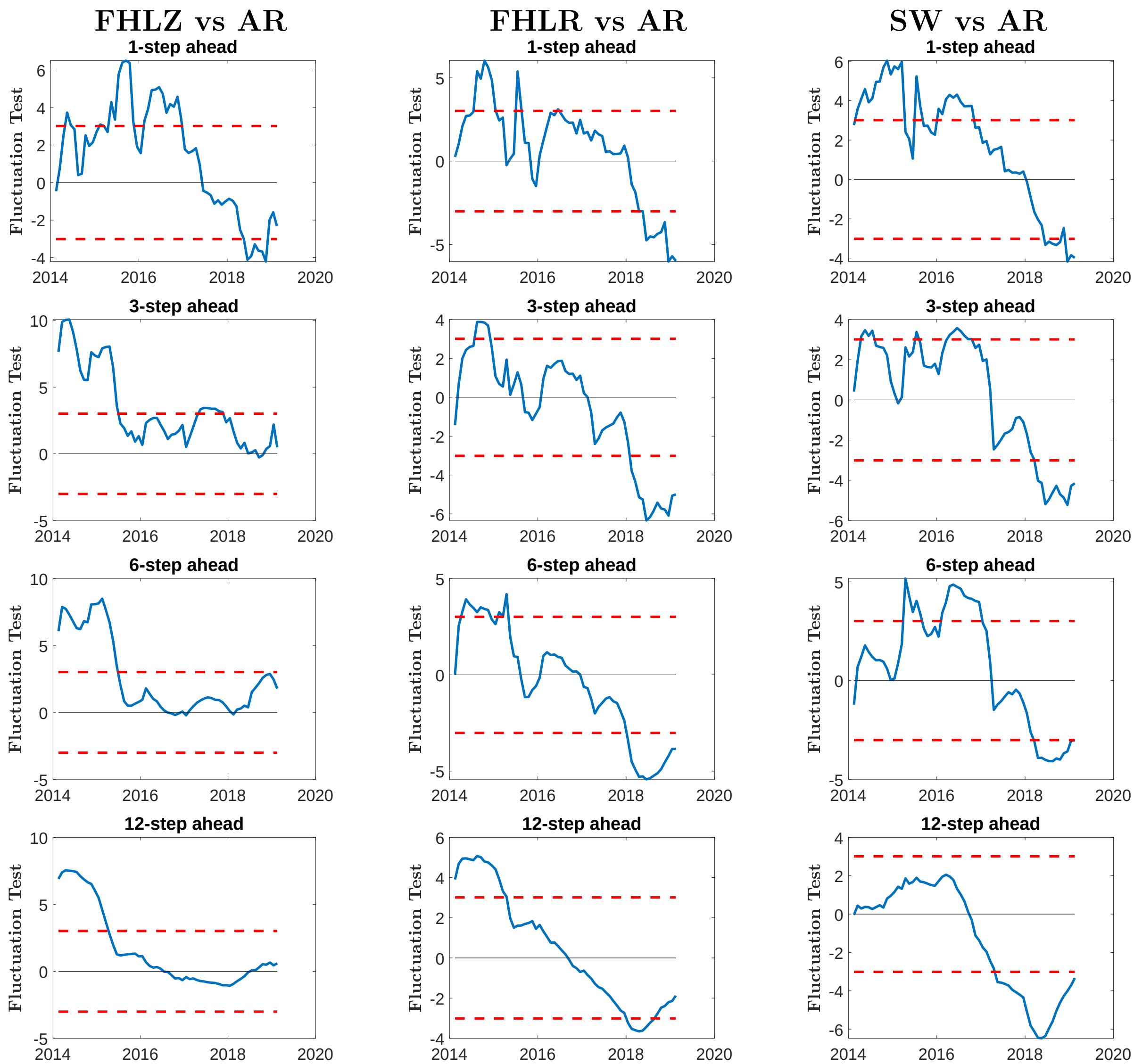
$$p(n,T) = (M_T^{-2} + M_T^{1/2}T^{-1/2} + n^{-1}) \times \log(\min[n, M_T^2, M_T^{-1/2}T^{1/2}]). \tag{2.32}$$

This penalty function is called p_1 in Hallin and Liška (2007). The estimated number of factors is given by

$$\hat{q} = \underset{0\leq k\leq q_{\max}}{\arg\max}\, IC_{1,n}^{T}(k). \tag{2.33}$$

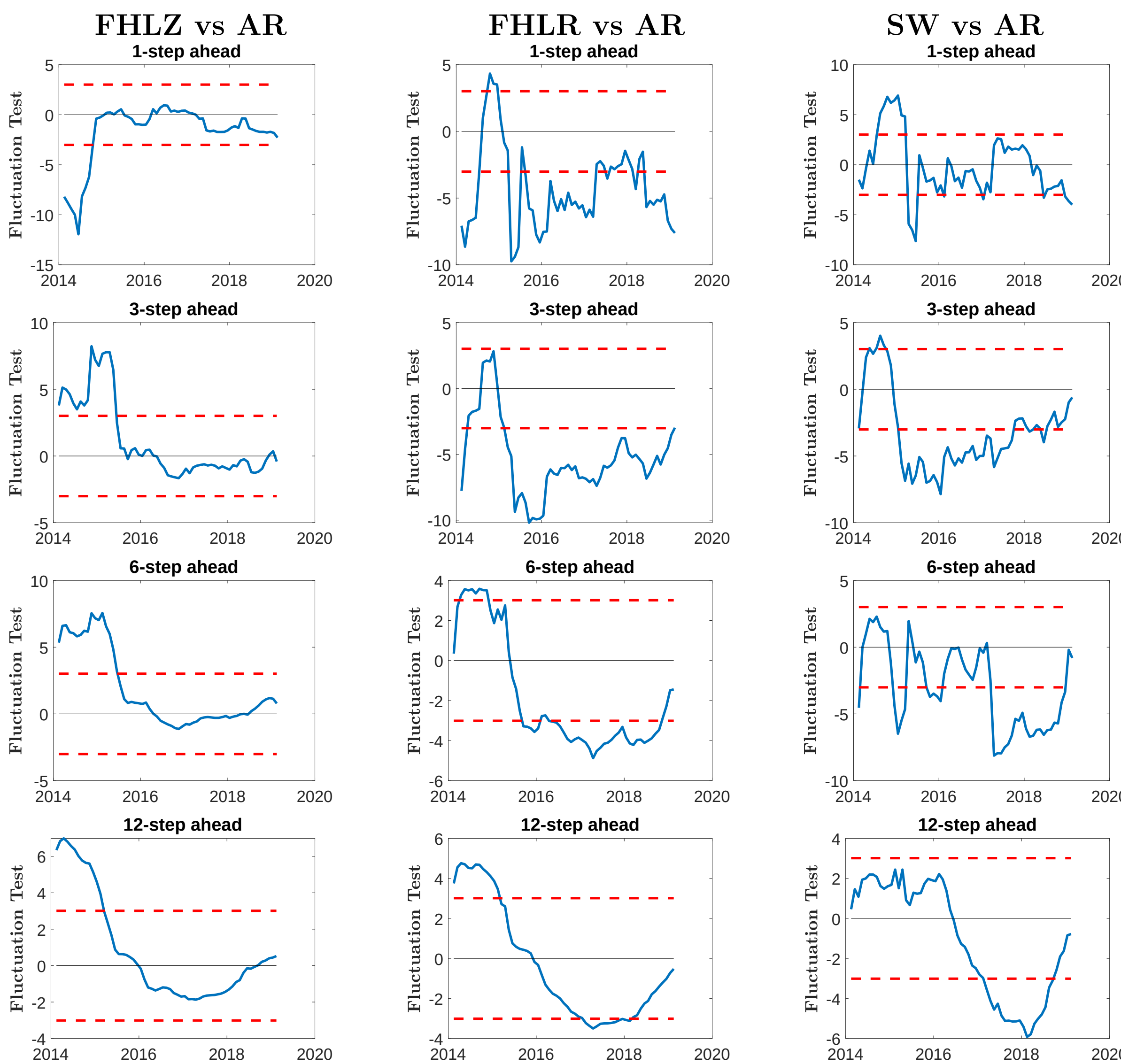
2.6.2 Fluctuation Tests

Figure 2.3: Fluctuation Test: Core CPI



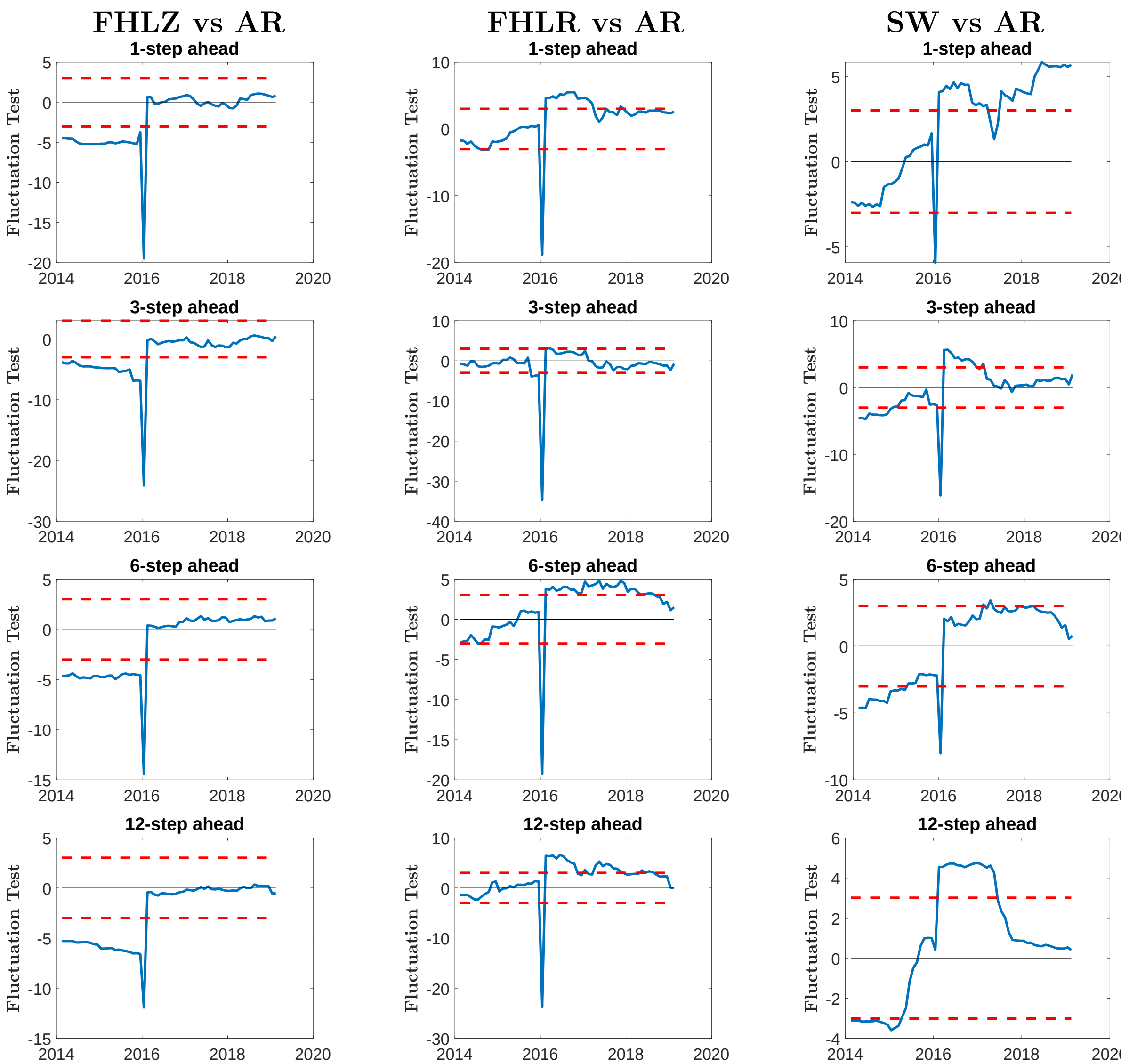
Notes: Column 1, 2, 3 are respectively the FHLZ vs AR, FHLR vs AR and SW vs AR with each row the horizons $h = 1, 3, 6, 12$. The blue line is the Giacomini and Rossi test statistics while the red dashed lines are the 5% confidence bands. The smoothing adopted is 20 data points.

Figure 2.4: Fluctuation Test: CPI



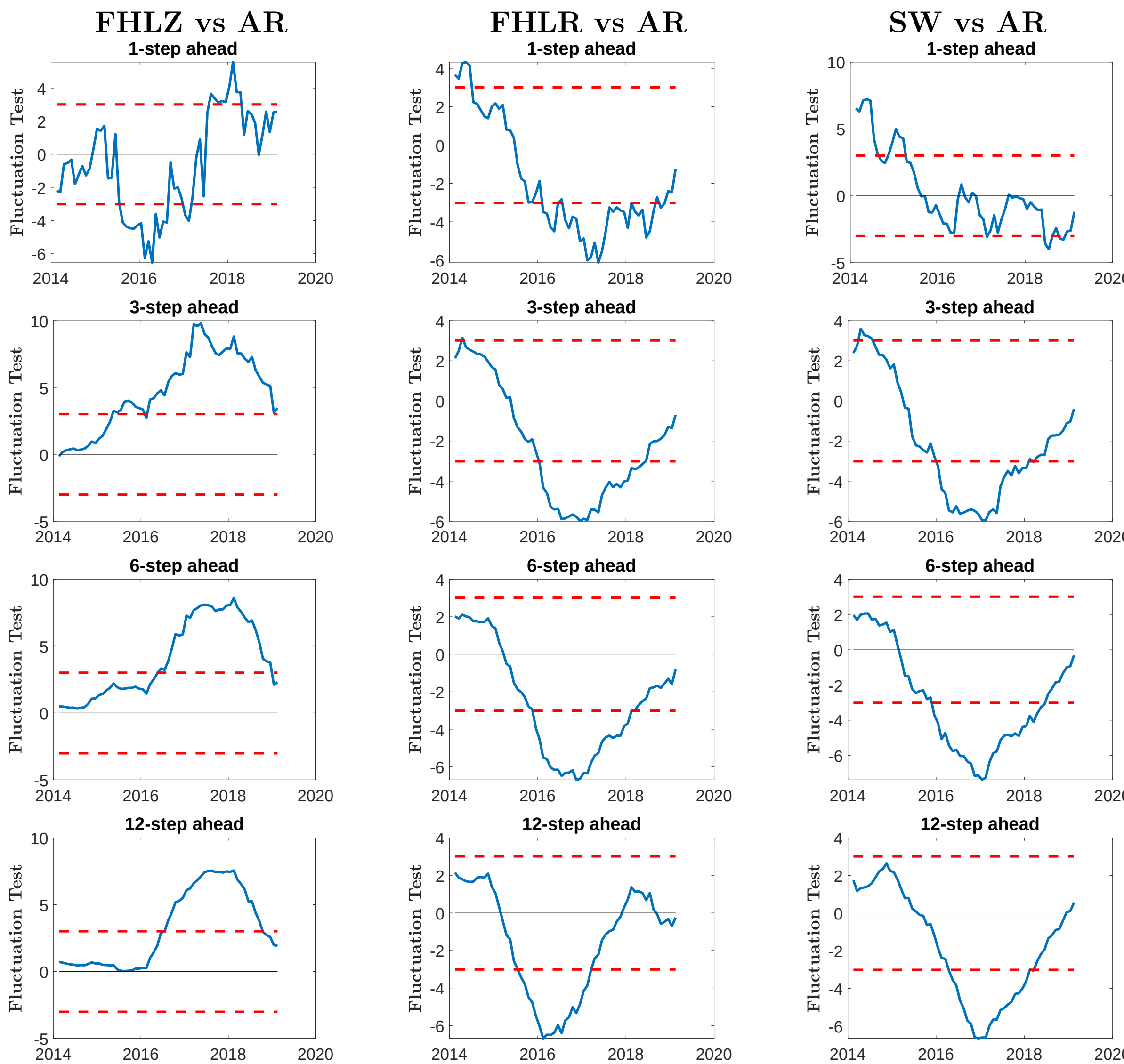
Notes: Column 1, 2, 3 are respectively the FHLZ vs AR, FHLR vs AR and SW vs AR with each row the horizons $h = 1, 3, 6, 12$. The blue line is the Giacomini and Rossi test statistics while the red dashed lines are the 5% confidence bands. The smoothing adopted is 20 data points.

Figure 2.5: Fluctuation Test: Electricity Consumption



Notes: Column 1, 2, 3 are respectively the FHLZ vs AR, FHLR vs AR and SW vs AR with each row the horizons $h = 1, 3, 6, 12$. The blue line is the Giacomini and Rossi test statistics while the red dashed lines are the 5% confidence bands. The smoothing adopted is 20 data points.

Figure 2.6: Fluctuation Test: Credit



Notes: Column 1, 2, 3 are respectively the FHLZ vs AR, FHLR vs AR and SW vs AR with each row the horizons $h = 1, 3, 6, 12$. The blue line is the Giacomini and Rossi test statistics while the red dashed lines are the 5% confidence bands. The smoothing adopted is 20 data points.

2.6.3 Dataset

Transformation codes (Tcode) denotes the respective data transformation for each series x:

1 - No transformation x_t;

2 - $\triangle x_t$;

3 - $\triangle^2 x_t$

4 - $\log(x_t)$

5 - $\triangle \log(x_t)$

6 - $(1-L)(1-L^{12})\log(x_t)$

Sources : Bank of Botswana **(BoB)**, Statistics Botswana **(SB)**, South African Reserve Bank **(SARB)**, Statistics South Africa **(StatsSA)**, Johannesburg Stock Exchange **(JSE)**, Bureau for Economic Research **(BER)** - South Africa, Federal Reserve Economic Data **(FRED)**, international Monetary Fund **(IMF)**, Food and Agriculture Organization **(FAO)**, Organisation for Economic Co-operation and Development **(OECD)**, Baumeister and Hamilton (2019) **(B-H)** and Yahoo Finance **(Yahoo)**.

Table 2.6 List of Variables

Id.	Series code	Variable	Tcode	Source
		COMMODITY PRICES		
1	BRENT_OILPRICE	Brent crude oil price	5	IMF
2	DIAMOND_PIND	Diamond Price Index	5	BoB
3	GLOBAL_COMM	Global Price Index of All Commodities	5	IMF
4	FOOD_PIND	Food price index	5	FAO
5	CEREAL_PIND	Cereal price index	5	FAO
6	METAL_PIND	Metal Price Index	5	IMF
		FISCAL AND OTHER		
7	GOVSPEND_DEV	Government spending: Development	5	BoB
8	GOVSPEND_REC	Government spending: Recurrent	5	BoB
9	GOVSPEND_T	Government spending: Total	5	BoB
10	ELEC_CONS	Electricity consumption	4	SB
		PRICES		
11	CPI_FOOD	CPI: Food	6	BoB
12	CPI_ALCTOB	CPI: Alcohol and Tobacco	6	BoB
13	CPI_CLO	CPI: Clothing	6	BoB
14	CPI_HOUSING	CPI: Housing	6	BoB
15	CPI_EDUCATION	CPI: Education	6	BoB
16	CPI_NT	CPI: Non Tradeables	6	BoB
17	CPI_DT	CPI: Domestic Tradeables	6	BoB
18	CPI_IT	CPI: Imported Tradeables	6	BoB
19	CPI_AT	CPI: All Tradeables	6	BoB
20	CPI_CORE	CPI: Core Trimmed Mean	6	BoB
21	CPI_ALL	CPI: All Items	6	BoB
		TRADE		
22	RESERVES	Foreign reserves	5	BoB
23	EXPORTS	Exports: Total	5	BoB
24	EXP_DIAMONDS	Exports: Diamonds	5	BoB
25	IMPORTS	Imports: Total	5	SB
26	IMP_CRP	Imports: Chemical and rubber products	5	SB
27	IMP_FOOD	Imports: Food	5	SB
28	IMP_FUEL	Imports: Fuel	5	SB
29	IMP_MEE	Imports: Machinery and electrical Equipment	5	SB
30	IMP_METAL	Imports: Metal and metal products	5	SB
31	IMP_TEXT	Imports: Textiles	5	SB
32	IMP_VTE	Imports: Vehicle and Transport Equipment	5	SB
33	IMP_WPP	Imports: Wood and Paper products	5	SB
34	IMP_OTHER	Imports: Other	5	SB
		MONEY AND CREDIT		
35	DEPOSITS	Deposits	5	BoB
36	CREDIT	Credit: Total	5	BoB
37	CREDIT_H	Credit: Household	5	BoB
38	CREDIT_B	Credit: Business	5	BoB
39	M1	Money supply: M1	5	BoB
40	M2	Money supply: M2	5	BoB
41	MONEYBASE	Monetary base	5	BoB

		FINANCIAL		
42	BANK_RATE	Bank rate	2	BoB
43	PRIME_RATE	Prime lending rate	2	BoB
44	GOVBOND_LTY	Long term Government bond yield	2	BoB
45	MORTG_RATE	Mortgage rate	2	BoB
46	SAVING_RATE	Saving rate	2	BoB
47	INT_SPREAD	Interest rate spread (Prime - saving)	1	BoB
48	USD_BWP	Exchange rate: USD/BWP	5	BoB
49	GBP_BWP	Exchange rate: GBP/BWP	5	BoB
50	JPY_BWP	Exchange rate: JPY/BWP	5	BoB
51	EUR_BWP	Exchange rate: EUR/BWP	5	BoB
52	ZAR_BWP	Exchange rate: ZAR/BWP	5	BoB
53	SDR_BWP	Exchange rate: SDR/BWP	5	BoB
54	MKT_CAP	Market capitalisation	5	BSE
55	DIV_YIELD	Dividend yield	2	BSE
56	DCI	Domestic Company Index (DCI)	5	BSE
57	FCI	Foreign Company Index (FCI)	5	BSE
		REGIONAL - SOUTH AFRICA		
58	JSEINDEX	JSE Allshare index	5	JSE
59	RSA_CPI	RSA Consumer Price Index	6	StatsSA
60	RSA_COIN	RSA Coincident Indicator	5	SARB
61	RSA_MANUF	RSA Manufacturing production index	5	SARB
62	RSA_MININGIND	RSA Mining production index	5	SARB
63	RSA_CCI	RSA Consumer Confidence Index	5	OECD
64	RSA_BCI	RSA Business Confidence Index	5	OECD
65	SARB_REPO	SARB repo rate	2	SARB
66	RSA_BACT	Business activity	5	BER
67	RSA_NSO	New Sales Orders	5	BER
68	RSA_INVENTO	Inventories	5	BER
69	RSA_SUP_PER	Suppliers Performance	5	BER
70	RSA_EMP	Employment	5	BER
71	RSA_PMI	Purchasing Managers' Index	5	BER
		GLOBAL		
72	GIP	Global Industrial Production Index	5	B-H
73	GLOBAL_UNCERT	Global Economic Policy Uncertainty Index	5	Fred
74	MSCIWI	MSCI World Index	5	MSCI
75	MSCIEM	MSCI Emerging Market Index	5	MSCI
76	NIKKEI225	Nikkei 225	5	Yahoo
77	SHANGHAI_SI	Shanghai Stock Index	5	Yahoo
78	EUROSTOXX50	Euro Stoxx 50	5	Yahoo
79	FTSE100	FTSE 100	5	Fred
80	S&P500	S&P 500	5	Fred
81	CHINA_BCI	China Business Confidence index	5	OECD
82	CHINA_IND	China Industrial Production Index	5	OECD
83	CHINA_CPI	China Consumer Price Index	5	OECD
84	CHINA_CCI	Consumer Confidence Index	5	OECD
85	EURO_INDPROD	Euro (28) Industrial production Index	5	Eurostat
86	EU_CCI	EU Consumer Confidence Index	5	OECD
87	EU_BCI	EU Business Conditions Index	5	OECD
88	EU_HICP	Harmonised Index of Consumer Prices	6	Eurostat
89	ECB_INT	ECB interest rate	5	ECB
90	US_INDPROD	US Industrial Production Index	5	Fred
91	FED_RATE	Fedfunds rate	2	Fred
92	ADS_index	Aruoba-Diebold-Scott Business conditions	2	Fred
93	VIX	CBOE Volatility Index	5	Fred
94	UMCSENT	University of Michigan Consumer Sentiment	5	Fred
95	US_UNRATE	US Unemployment rate	5	Fred
96	US_CPI	US Consumer Price Index	6	Fred

Chapter 3

The Macroeconomic Effects of External Shocks on Developing Commodity-Exporting Economies: A Generalised Dynamic Factor Model (GDFM) Approach

3.1 Introduction

The recent experience of large commodity price fluctuations has led to a renewed focus on the role of commodity prices as key drivers of business cycle fluctuations in small emerging market economies (Fernández et al., 2018). Indeed, empirical evidence show that commodity price shocks have a significant impact on investment, output, consumption, inflation, exchange rates and trade balances of producer countries (see Céspedes and Velasco, 2012; McGregor, 2017; Fernández et al., 2018; Drechsel and Tenreyro, 2018). For developing countries, particularly those reliant on commodities for export and government earnings, there is heightened vulnerability to external shocks transmitted through commodity prices. Consequently, policy makers and researchers closely monitor the price of the exported commodity as an important determinant of economic developments in the small open economy. However, commodity exporters are also vulnerable to external shocks transmitted through imported key commodities like oil. The analysis by Peersman and Van Robays (2009) on the macroeconomic effects of oil shocks in the Euro area show that imported commodity price developments have a significant effect on the domestic business cycles as well.

Fernández et al. (2017) demonstrate that the transmission of export and import commodity prices is different across the sectors of the economy as a given commodity price change may transmit the external shock to a few (even one) macroeconomic indicators. For instance, in the case of a net oil importer, export price shocks may have a significant impact on domestic production (output) while the oil price shocks may result in material changes in consumption and domestic prices. These findings affirm the need for a detailed analysis of the external shocks and their propagation to the economy in a way that will be more informative for policymakers in their decision making process. Although most studies have analysed the dynamic effects of commodity export prices on small commodity exporting countries (see Pedersen, 2019; Bjørnland and Thorsrud, 2016; Fernández et al., 2018), there has been limited attention to the joint dynamic effects of commodity export and import prices.

Our goal is to study the macroeconomic impact of global shocks transmitted through commodity prices on commodity exporting developing economies. Consistent with developments in the macroeconomic literature, we identify global demand shocks and commodity specific shocks in a way that recognises the source of the shock. This approach acknowledges not only that both commodity export price and commodity import price may affect the economy in different ways (Fernández et al., 2017) but also that the source of the shock as envisaged in Kilian (2009) is important in understanding the ultimate impact on response variables. Furthermore, this approach also takes into account the fact that not all terms of trade shocks are the same as demonstrated by Di Pace et al. (2020) and that disaggregated commodity prices are more helpful in mapping the link between external shocks and the macroeconomy (Fernández et al., 2017). This promotes a detailed understanding of the role of external shocks in the domestic economy compared to the commonly used single price specifications like the conventional measure of terms of trade (export to import price index ratio) that was found to underestimate the importance of global shocks on domestic business cycles (see Schmitt-Grohé and Uribe, 2018; Fernández et al., 2017).

Although extensive research is ongoing to explore the importance and macroeconomic effects of global shocks in cross-country studies, not much is covered on the role of such shocks on individual small developing commodity exporting economies. This limits the ability to assess the full impact of global shocks on developing economies in a way that recognises their uniqueness. This is important because there is diversity in the macroeconomic impact of commodity price shocks due to structural characteristics and policy frameworks in the economy (see Céspedes and Velasco, 2012; Peersman and Van Robays, 2009).

This paper contributes to the literature by exploring the role of global shocks transmitted through commodity prices on a developing economy, Botswana. Botswana is an ideal candidate because it is a net commodity exporter (diamond exports make up 80-90% of total exports) but also relies on commodity imports that are key to economic activity in a developing economy. The country's commodity export price captured by the diamond price index and the commodity import price represented by the Brent crude oil price are considered fundamental for the transmission of external shocks to the Botswana economy. Therefore, we analyse the macroeconomic effects of three types of external shocks and their propagation. These are a positive global demand shock that increases global economic activity and commodity prices, a positive oil-specific price shock and a positive diamond-specific price shock that capture increases in commodity prices (oil and diamonds, respectively) due to changes in the respective markets and not associated with the global business cycle.

In this paper, we study the macroeconomic impact of the identified external shocks to the economy using a generalised dynamic factor model (GDFM) proposed by Forni et al. (2015, 2017) and a large quarterly macroeconomic dataset for Botswana. This builds on the existing works on structural macroeconomic analysis of global shocks using factor models in data-rich environments (see Aastveit, 2014; Charnavoki and Dolado, 2014;

Juvenal and Petrella, 2015; Bjørnland and Thorsrud, 2016). Aastveit (2014) notes that the use of a large dataset and a factor model is useful in two important ways. First, it allows for the incorporation of large information closely monitored by policy makers. Second, it accommodates the analysis of impulse responses of a wide range of macroeconomic variables. These advantages provide for a broader understanding of the propagation mechanism of the identified shocks on selected macroeconomic variables. The external shocks are identified by imposing restrictions on the responses of a few selected macroeconomic indicators.

The results show that a positive global demand shock is associated with a rise in global economic activity and an increase in commodity prices which is consistent with the results of Juvenal and Petrella (2015) and Jääskelä and Smith (2013). This shock leads to a pick-up in domestic economic activity, a rise in domestic inflation and an improvement in the trade balance. Our findings also show that a positive oil-specific price shock tend to have a contractionary effect on global and domestic economic activity. The shock is inflationary, which prompts the central bank to increase the monetary policy interest rate to ensure price stability. Lastly, a positive diamond-specific price shock that increases diamond prices is short-lived, as the shock dissipates within a quarter. This shock has a positive effect on mining GDP, exports, total output and the trade balance.

Our framework allows us to orthogonalise the changes in commodity prices against the global and commodity specific conditions in a way that gauges the importance of the source of shock to the changes in commodity prices. As a result, the forecast error variance decomposition estimates show that global demand shock contributes more to the variation in commodity prices and other domestic macroeconomic variables. This highlights the need to account for the underlying sources of external shocks in structural macroeconomic analysis.

The rest of the paper is organised as follows. Section 2 reviews the literature on external shocks on small open economies and the role of commodity prices as a conduit for transmission of external shocks to these economies. Section 3 presents the generalised dynamic factor model and the identification strategy. Section 4 reports the empirical results and analysis. Section 5 concludes the paper.

3.2 Related Literature

This paper contributes to a well-established literature on the impact of global shocks on emerging and developing economies. This section presents a brief review of the literature on the role of external shocks on macroeconomic variables and how commodity price changes can be an alternative to capture the transmission of global shocks to commodity-exporting countries.

External shocks to small open economies: There has always been an interest to understand the role of global shocks on developing and emerging countries given their welfare implications (see Bidarkota and Crucini, 2000). Researchers have presented different approaches to account for the transmission of global shocks to small open economies. Using real business cycle models, Mendoza (1995) uses a terms of trade measure as the transmission channel and concluded that terms of trade shocks explained about half the variation in output. Kose (2002) uses a similar model to show that world price shocks explained about 88% of output variation. These studies set the bar on the role of external shocks as major sources of business cycle fluctuations in developed and developing economies.

Notwithstanding the previous evidence, recent empirical research by Schmitt-Grohé and Uribe (2018) show that the terms of trade shocks explain about 10% of the variation in output of emerging and developing economies. This led to the notion of a disconnect between the theoretical models and empirical models on the importance of terms of trade shocks on business cycles. Although not emphasised at the time, it could be argued that this disconnect was observed in Broda (2004). Through an SVAR model, he estimated that terms of trade shocks explained 30% of the variance in output of developing countries with a fixed exchange rate regime and 10% for those with a floating exchange rate regime. These figures were significantly lower than the results of the theoretical models of Mendoza (1995) and Kose (2002).

The conclusion derived from the latest evidence is that, single price specifications like the conventional measure of terms of trade (export-to-import price index ratio) underestimate the importance of world shocks on domestic business cycles (Fernández et al., 2018;

Schmitt-Grohé and Uribe, 2018). This observation is consistent with the view that an aggregate price index like the terms-of-trade measure fails to fully reflect sharp peaks and deep troughs observed in major export items of developing countries (see Kose and Riezman, 2001). Therefore, for a proper design and conduct of macroeconomic policy in developing economies, it is crucial to identify and use price indicators that account for the different sources of world shocks and their transmission to the respective economies.

For developing and emerging countries that are commodity dependent, a specific commodity terms of trade is considered an alternative measure to capture global shocks compared to the standard terms of trade. The commodity terms of trade is derived from the relative prices of a country's commodity exports and imports weighted by the country specific GDP shares (Aizenman et al., 2012). In response to Schmitt-Grohé and Uribe (2018), Roch (2019) show that commodity terms of trade shocks are an important driver of business cycles, explaining around 30% of variations in output for a selected group of developing and advanced commodity exporting economies. In another study, Zeev et al. (2017) conclude that accounting for anticipated shocks (i.e. news component) in the terms of trade doubles the contribution of commodity terms of trade as a source of business cycle fluctuations. For five selected commodity exporting Latin American countries, the authors show that the news-augmented commodity terms of trade explain about half of the variation in output.

Recent work by Di Pace et al. (2020) conclude that the terms of trade disconnect between theory and the empirical analysis is due to the fact that "not all terms of trade are the same". This is consistent with the assessment in Jääskelä and Smith (2013). Their view is that a single price indicator like the terms of trade (export-import price ratio) implicitly assumes that the economy responds symmetrically to an increase in export prices and a decline in import prices. However, their evidence show that this is not the case. The impact of a positive export price shock will not directly match the effects of a negative import price shock because of the different weights of the sectors on the economy and differences in the transmission channels of the shocks. By disaggregating the terms of trade components, Di Pace et al. (2020) show that the export price shock has a larger and more persistent effect when the commodity export share is greater than the commodity import share.

The other reason advanced by Di Pace et al. (2020) for the limited importance of terms of trade on business cycles is that global demand shocks that reflect changes in global economic activity may have a minimal or no impact on the terms of trade. This is due to simultaneous increase in commodity export and import price indices following an increase in global demand. Despite the offsetting price movements on the terms of trade measure, it would be wrong to conclude that the change in global economic activity will have no impact on the local economy. This observation also implies that the use of a commodity terms of trade measure as in Roch (2019) may fail to fully explain the impact of global shocks even though explaining a higher (30%) share of the variation in output compared to the standard terms of trade. This highlights the need to identify approaches that can allow for a greater detail in accounting for the impact of world shocks on a small open economy.

Since single relative price indicators are considered to be less informative, there is a shift towards identifying explicitly the underlying sources of shocks to the terms of trade. In the case of the Australian economy, Jääskelä and Smith (2013) demonstrated that macroeconomic effects of the terms of trade shocks depends on the characteristics of the underlying shock and policy response. To this end, they identify three terms of trade shocks, a world demand shock, a commodity-markets-specific shock and a globalisation shock. They conclude that the three positive terms of trade shocks propagate through the economy in different ways with the contribution of each shock to output, inflation, interest rates and exchange rates changing over time.

For Botswana, Mangadi and Sheen (2017) used a sign-restricted structural vector autoregression to evaluate the macroeconomic responses to the same terms of trade shocks in Jääskelä and Smith (2013) as well as a global supply shock. Their results show that a positive global demand and globalisation shock had expansionary effects while commodity market specific shock had a contractionary effect indicating possible Dutch disease response. This paper is different from Mangadi and Sheen (2017) in that we consider specifically two commodity prices and a large dataset that allows for the assessment of the impact of the shock on a number of variables.

Commodity prices as a medium for transmitting global shocks: There is a well-documented literature on the role of commodity prices as a conduit of the transmission of global shocks to commodity-dependent economies. Bidarkota and Crucini (2000) concluded that commodity price fluctuations have an important role on the variation of the terms of trade of a developing country. The changes in commodity prices also have a major effect on government revenues and therefore they are main drivers of fiscal outcomes in these countries (Céspedes and Velasco, 2014). Furthermore, Drechsel and Tenreyro (2018) show that commodity price changes affect economies through their impact on the competitiveness of the economy and its borrowing terms. Through the different transmission channels commodity price fluctuations ultimately affect the economy (Collier and Goderis, 2012; Céspedes and Velasco, 2012; Fernández et al., 2018).

Using a panel of poor, emerging and rich countries, Fernández et al. (2017) show that world disturbances transmitted via commodity prices explain more than a third of the variation in output, consumption, investment and trade balance. Here the authors are not concerned with the source of the global shocks but rather the amount of variation of business cycle fluctuations explained by world shocks transmitted through the commodity prices (agricultural, fuel and metal).

Charnavoki and Dolado (2014) use Kilian (2009)'s approach to identify global shocks and assess their impact on the Canadian economy. The authors use a principal component analysis to extract a factor to represent real commodity price movements and found that a rise in commodity prices generate a positive effect on external balances while a positive global demand shock stimulates real output and expenditure across industries. In a recent study, Fernández et al. (2020) uses a commodity super cycle (defined as the principal component of the commodity prices) to show that world shocks that affect commodity prices explain more than half of the variation in output growth in developed and emerging economies.

The surveyed literature shows that a common approach to examine the impact of exogenous international shocks to the economy is by using standard export-to-import price ratio and the commodity export-to-import price ratio. In some instances, aggregate commodity prices are used (e.g commodity factor and commodity super cycle). The use of aggregate commodity price indicators may be helpful to explain the role of global shocks in cross-country studies. However, the aggregate price may fail to account for the peaks and troughs of commodity prices relevant for a specific country. Further, the aggregated indicators suffer the same fate of single price indicators of an implicit assumption of a symmetrical impact of export price increases and import price decreases.

This study contributes to this literature by a way of an extension of the work by Di Pace et al. (2020). The paper adopts their approach and extends by using a data-rich environment to map the macroeconomic effects of global shocks transmitted by explicit country-specific commodity (export and import) prices. Their approach has two main benefits to explain the impact of global shocks on emerging market economies. First, it recognises that "not all terms of trade are the same". The idea is that single price indicators like the export-import price ratio that implicitly assumes the economy responds symmetrically to an increase in export prices and a decline in import prices may underestimate the impact of global shocks on the economy. Therefore, examining how export and import price shocks propagate to macroeconomic variables through different transmission channels and capturing the asymmetry in the economic responses to these shocks is informative. Secondly, the approach accounts for the role of global demand shocks that increases both export and import prices. This impact is not well captured by a single price indicator as potentially offsetting commodity price movements due to a global demand shock has a minimal impact on the terms of trade measure.

3.3 Econometric Strategy

3.3.1 Data

The analysis in this paper uses 85 quarterly time series for the sample period 1994Q1 - 2019Q4. This study period is informed by data availability. Nonetheless, this time-frame is considered to be sufficient as it covers periods of relative commodity price stability prior to the year 2000, the boom in commodity prices from the early 2000s through to the 2008 global financial crises as well as the increased volatility in prices thereafter. The macroeconomic dataset is made up of Botswana variables and key indicators of regional economic developments (South Africa) and global economic developments (China, United Kingdom and the United States of America). The domestic variables are sourced from Bank of Botswana's monthly Botswana Financial Statistics (BFS) report and Statistics Botswana website. For the United States variables, data are from Federal Reserve Economic Data (FRED) available at the Federal Reserve Bank of St Louis website. The rest of the data are sourced from the respective central banks, national statistics organisations and international institutions.

The macroeconomic variables in the dataset are selected to capture the main drivers of economic developments in Botswana. The variables are categorised into 8 groups, being, commodity prices, GDP expenditures, economic activities, prices, financial (interest rates, stock market and exchange rates), trade, regional and global. Similar to Bjørnland and Thorsrud (2016), most variables are transformed to four-quarter logarithmic changes to capture the dynamics in the variables of interest. The variables are demeaned and standardised before estimation. A detailed list of the variables, their source and corresponding transformation is presented in the appendix.

3.3.2 Generalised Dynamic Factor Model

A generalised dynamic factor model (GDFM) proposed by Forni et al. (2017) (FHLZ model) is used to analyse the macroeconomic effect of global shocks transmitted via commodity prices to the Botswana economy. The FHLZ model is a special case of the generalised dynamic factor models proposed in Forni et al. (2000). To help describe the model, let the macroeconomic dataset be a panel, $\mathrm{x}_t = (x_{1t}, \dots, x_{nt})'$. The panel is an n dimensional vector of covariance stationary time series $(x_{it} \mid t = 1, \dots, T\})$ with zero mean and unit variance. Let $\Gamma_k = E(\mathrm{x}_t \mathrm{x}'_{t-k})$ be the covariance matrix of x_t and $\Sigma(\theta)$ its spectral density matrix at frequency $\theta \in [-\pi, \pi]$. Further, define $\{v_j, z_j\}_{j=1}^{n}$ and $\{\lambda_j(\theta), p_j(\theta)\}_{j=1}^{n}$ the eigenvalues (in decreasing order) and the corresponding eigenvectors of Γ_k and $\Sigma(\theta)$, respectively.

The *generalised dynamic factor model* representation of x is;

$$x_{it} = \chi_{it} + \xi_{it} = \sum_{k=1}^{q} b_{ik}(L) u_{kt} + \xi_{it}, \qquad i = 1, \dots, n \tag{3.1}$$

where $\mathbf{u}_t = (u_{1t} \dots u_{qt})'$ is q-dimensional orthonormal white process ($q \leq n$). L is the lag operator and the filters $b_{ik}(L)$ are one-sided and square summable for any $i = 1, \dots, n$ and any $k = 1, \dots, q$. The *common component* χ_{it} is driven by common shocks u_{kt} (dynamic factors). The *idiosyncratic component* ξ_{it} is weakly cross correlated, that is, the eigenvalues of its spectral density matrix are uniformly bounded as $n \to \infty$. The common component and the idiosyncratic component are mutually orthogonal at all leads and lags.

From Equation (3.1), the common component can be expressed in vector form as:

$$\boldsymbol{\chi}_t = \mathbf{B}(L)\mathbf{u}_t, \tag{3.2}$$

with $\boldsymbol{\chi}_t = (\chi_{1t} \dots \chi_{nt} \dots)'$ and $\mathbf{B}(L) = b_{i1}(L) \dots b_{iq}(L)$. Forni and Lippi (2011) and Forni et al. (2015, 2017) show that if the common component has a rational spectral density it admits a unique autoregressive representation with block structure of the form:

$$\underbrace{\begin{pmatrix} \mathbf{A}^{(\mathbf{1})}(L) & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{A}^{(\mathbf{2})}(L) & \dots & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{A}^{(K)}(L) \end{pmatrix}}_{\mathbf{A(L)}} \underbrace{\begin{pmatrix} \boldsymbol{\chi_t}^{(1)} \\ \boldsymbol{\chi_t}^{(2)} \\ \vdots \\ \boldsymbol{\chi_t}^{(K)} \end{pmatrix}}_{\boldsymbol{\chi}_t} = \underbrace{\begin{pmatrix} \mathbf{R}^{(1)} \\ \mathbf{R}^{(2)} \\ \vdots \\ \mathbf{R}^{(K)} \end{pmatrix}}_{\mathbf{R}} \mathbf{u}_t, \tag{3.3}$$

where $\mathbf{R}$ is a $(q+1) \times q$ matrix and each diagonal block $\mathbf{A}^{(i)}(L)$, for any $i = 1, 2, \dots K$, is a $(q+1) \times (q+1)$ polynomial matrix. Furthermore, $\det(\mathbf{A}^{(i)}(L)) \neq 0$ for $|z| \leq 1$, the filters $\mathbf{A}^{(i)}(L)$ are one sided, and each polynomial has finite order. According to Barigozzi et al. (2019), the K autoregressive operators $\mathbf{A}^{(i)}(L)$ invert into fundamental moving average filters which are unique for a subset of $q+1$ dimensional subset of common components $(\chi_{(i-1)(q+1)+1}, \dots, \chi_{i(q+1)})'$. That is, the polynomial matrix have no roots inside the unit circle.

Then, by inverting the polynomial matrix $\mathbf{A}(L)$ in Equation (2.16), it follows that the common component admits a moving average representation

$$\boldsymbol{\chi}_t = [\mathbf{A}(L)]^{-1}\mathbf{R}\mathbf{u}_t = \mathbf{B}(L)\mathbf{u}_t. \tag{3.4}$$

This implies that the shocks $\mathbf{u}_t$ are *fundamental*[1] for the common component $\boldsymbol{\chi}$. Using

[1] According to Lippi and Reichlin (1994), the moving average representation in Equation (3.4) where

$\boldsymbol{\chi}_t = \mathbf{x}_t - \boldsymbol{\xi}_t$, the GDFM can be written as:

$$\mathbf{A}(L)\mathbf{x}_t = \mathbf{R}\mathbf{u}_t + \mathbf{A}(L)\boldsymbol{\xi}_t. \tag{3.5}$$

Forni and Lippi (2011) show that the last term on the right hand side remain idiosyncratic.

Estimation of Equation (3.5) is detailed in the following steps, consistent with Forni et al. (2015, 2017):

i. Estimate the spectral density matrix $\hat{\boldsymbol{\Sigma}}^{\mathbf{x}}(\theta)$ of $\mathbf{x}_t$: The $n \times n$ spectral density of $\mathbf{x}_t$ for frequency $\theta \in [-\pi, \pi]$ is defined as in Forni et al. (2000) as:

$$\hat{\boldsymbol{\Sigma}}^{\mathrm{x}}(\theta) = \frac{1}{2\pi} \sum_{k=-M}^{M} w_k e^{ik\theta} \Gamma_{\mathrm{x}}(k), \tag{3.6}$$

where $\Gamma_{\mathrm{x}}(k) = E[\mathrm{x}, \mathrm{x}'_{t-k}]$ and $w_k = 1 - \frac{|k|}{M+1}$ are the weights corresponding to the Bartlett lag with window size M. Empirical applications use $M = \sqrt{T}$ (see Forni et al., 2005; D'Agostino and Giannone, 2012).

ii. Estimate the spectral density matrix $\hat{\boldsymbol{\Sigma}}^{\chi}(\theta)$ of $\boldsymbol{\chi_t}$: This is obtained by means of dynamic principal component analysis that relies on spectral densities with the selection of q principal components associated with the largest eigenvalues. Let $\hat{p}_j(\theta)$ and $\hat{\lambda}_j(\theta)$ be the eigenvectors and eigenvalues of $\hat{\boldsymbol{\Sigma}}^{\mathrm{x}}(\theta)$, the spectral density matrix of the common component is defined as:

$$\hat{\boldsymbol{\Sigma}}^{\chi}(\theta) = \sum_{j=1}^{q} \hat{\lambda}_j(\theta) \hat{p}'_j(\theta) \hat{p}_j(\theta). \tag{3.7}$$

iii. Estimate the autocovariance matrix $\hat{\boldsymbol{\Gamma}}_k^{\chi}$ of $\boldsymbol{\chi_t}$: By means of inverse Fourier transform we have

$$\hat{\boldsymbol{\Gamma}}_k^{\chi} = \frac{2\pi}{2M+1} \sum_{m=-M}^{M} e^{ik\theta_m} \widehat{\Sigma}^{\chi}(\theta_m), \tag{3.8}$$

where Fourier frequencies $\theta_m = \frac{2\pi m}{2M+1}$, $m = -M, \ldots, M$.

iv. Compute the VAR matrices $\mathbf{A}^{(i)}(L)$: These are obtained from common covariances by means of Yule-Walker equations.

v. Compute the matrices $\mathbf{R}$ and the shock $\mathbf{u}_t$: These are obtained via standard principal component analysis (Bai and Ng, 2002; Stock and Watson, 2002b) of $\tilde{\mathbf{x}}_t = \mathbf{R}\mathbf{u}_t + \tilde{\boldsymbol{\xi}}_t$,

(i) $\mathbf{u}_t$ is a weak white noise vector, (ii) $\mathbf{B}(L)$ is a matrix of rational functions L with no poles of modulus smaller or equal to unity is *fundamental* if (iii) det $\mathbf{B}(L)$ has no roots in the unit circle.

where $\tilde{\mathbf{x}}_t = \mathbf{A}(L)\mathbf{x}_t$ and $\tilde{\boldsymbol{\xi}}_t = \mathbf{A}(L)\boldsymbol{\xi}_t$. Given all the definitions the estimated impulse response functions of the common component to the q common shocks are defined as $\widehat{\mathbf{B}}(L) = [\widehat{\mathbf{A}(L)}]^{-1}\widehat{\mathbf{R}}$. It follows then that the estimated common component is expressed as $\widehat{\chi}_{\mathbf{t}} = \widehat{\mathbf{B}}(L)\widehat{\mathbf{u}}_t$.

3.3.3 Identification of Structural Shocks

Impulse responses and the common shocks are identified up to an orthogonal transformation (multiplication by a $q \times q$ rotation matrix $\mathbf{H}$). From the representation of generalised dynamic factor model in equation 3.1, the common component can then be defined as:

$$\boldsymbol{\chi}_t = \mathbf{B}(L)\mathbf{H}'\mathbf{H}\mathbf{u}_t. \tag{3.9}$$

Identification is then attained by the choice of $\mathbf{H}$ and imposing restrictions guided by economic intuition as in Forni and Gambetti (2010) and Barigozzi et al. (2014). Specifically, shocks of interest are identified by placing sign restrictions on the impulse response functions. It is sufficient to impose orthogonality and restrict $\mathbf{H} \equiv \mathbf{H}(\theta)$ to an orthogonal matrix that depends on an iterative draw vector of $q(q-1)/2$ angles θ from a uniform distribution on $[0, 2\pi]$. The orthogonal matrices $\mathbf{H}$ are constructed by means of Givens rotation matrices. For our case, the Hallin and Liška (2007) criterion suggests that the number of dynamic factors $q = 3$. This implies that:

$$H(\theta) = \begin{bmatrix} cos(\theta_1) & -sin(\theta_1) & 0 \\ sin(\theta_1) & cos(\theta_1) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos(\theta_2) & 0 & -sin(\theta_2) \\ 0 & 1 & 0 \\ sin(\theta_2) & 0 & cos(\theta_2) \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos(\theta_3) & -sin(\theta_3) \\ 0 & sin(\theta_3) & cos(\theta_3) \end{bmatrix}.$$

Next, we estimate the associated impulse responses and only those that satisfy the set of sign restrictions are accepted while the rest are discarded. Accordingly, $\widehat{\mathbf{D}}(L) = \widehat{\mathbf{B}}(L)\widehat{\mathbf{H}}'(\theta^*)$ are the identified impulse response functions to the structural shocks $\widehat{\mathbf{w}}_t = \widehat{\mathbf{H}}(\theta^*)\widehat{\mathbf{u}}_t$.[2] Where θ^* is the set of angles that ensures $\widehat{\mathbf{H}}$ fulfil the identification criteria.

To address estimation uncertainty, we follow Forni and Gambetti (2010) to construct

[2] The dynamic effect of the shock w_j on x_i is calculated as $\frac{\partial x_{it+h}}{\partial w_{jt}} = D_{ij,h}$ for $h = 0, 1, 2, \ldots$. All these collects into $\widehat{\mathbf{D}}(L)$.

confidence bands for the selected impulse responses using a standard block bootstrap technique. Recall that the dataset $\mathrm{X} = [x_{it}]$ is a $T \times n$ matrix. We choose the length of the blocks (L) that evenly divide the length of the series (T) to partition the dataset into blocks, $l = 1, ..., L$, of dimension $k \times n$. $k = T/L$ is an integer. Draw with replacement an integer m_l between 1 and L and concatenate the resulting blocks to get a bootstrap sample $\mathrm{X}^* = [\mathrm{X}'_{m1} \quad \mathrm{X}'_{m2}...\mathrm{X}'_{ml}]'$ of dimension $kl \times n$. A distribution of impulse response functions is obtained from repeated drawing and estimation of the generated artificial X^*. These are used to map the confidence bands.

The three external shocks, global demand shock, oil-specific price shock and a diamond-specific price shock are identified by means of sign restrictions. Therefore, sign restrictions are imposed on the estimated response levels of global economic activity (GEA) represented by global industrial production, oil price (OIL) and diamond price (DIA). For the domestic variables, restrictions are placed on the responses of gross domestic product (GDP) to help identify the shocks, particularly the diamond-specific price shock. No restrictions are required for the rest of the domestic variables. The imposed sign restrictions are summarised in Table 3.1 followed by a brief description of each of the identified shocks.

Table 3.1 Sign Restrictions

	GEA	OIL	DIA	GDP
Global demand shock	+	+	+	+
Oil-specific price shock	−	+		−
Diamond-specific price shock	•		+	+

Note: A positive (+) / negative (−) sign show that the impulse response of a variable is restrained to be positive/negative. A blank space represents a case where no restrictions are imposed. The bullet (•) captures the role of the minimisation criterion that ensures that the impact of the diamond price shock on global economic activity is negligible for the entire horizon.

A *global demand shock* captures unanticipated fluctuations in the global business cycle that increases global economic activity. In our case, global economic activity is represented by the aggregated industrial production index of Organisation for Economic Co-operation and Development (OECD) countries plus six major emerging economies (Brazil, China, India, Indonesia, Russia and South Africa) created by Baumeister and Hamilton (2019)). Increased global activity stimulates global demand for commodities, leading to an increase in both commodity import (oil) and export (diamond) prices. Rising commodity prices due to the rapid economic expansion and industrialisation of China, India and other emerging markets is a representative case of this shock (Kilian and Hicks, 2013).

A positive global demand shock that enhances the demand for commodities promotes productivity and investment in new productive capacity and leads to an increase in output of commodity-exporting countries that results in a rise in overall global economic activity (see Stuermer, 2018). Therefore, we impose positive restrictions on the response of commodity prices, domestic output and global economic activity in response to a positive global demand shock. This is consistent with earlier identifications (see Peersman and Van Robays, 2009; Kilian and Park, 2009; Charnavoki and Dolado, 2014).

An *oil-specific price shock* captures the changes in oil (Brent crude) prices not driven by changes in global economic activity. This shock captures the effect of changes in oil prices due to developments specific to the oil market and not due to global demand changes explained above. This include among others, changes in oil prices due to speculation in the oil market, precautionary oil demand and shifts in oil supply induced by temporary disruptions in oil-exporting countries. In other studies, these drivers of oil price fluctuations have been identified separately (see Kilian, 2009; Juvenal and Petrella, 2015).

The increase in international oil prices due to any of the above-mentioned oil market developments is likely to lead to a fall in global economic activity. Notably, an increase in oil prices due to a precautionary or speculative demand has a contractionary effect on overall consumer and business demand, leading to weaker economic activity (Juvenal and Petrella, 2015). Equally, an increase in oil prices due to a temporary fall in oil production in an oil-exporting country will have a negative impact on global economic activity (Peersman and Van Robays, 2009). Therefore, to identify the positive oil-specific price shock we impose a positive restriction on Brent crude oil price and a negative restriction on the response of global economic activity and domestic real GDP.

A *diamond-specific price shock* accounts for fluctuations in diamond prices that are not associated with any changes in global economic activity. The shock captures changes in the diamond pipeline fundamentals indicated in the global diamond industry report (Bain & Company, 2019). This includes, the production performance of existing mines, the introduction of new mines or new deposit discoveries and midstream financial performance and efficiency. Therefore, a diamond price decrease induced by funding challenges experienced in the diamond market of India, the world's largest diamond polishing hub in 2019 (Dempsey and Parkin, 2019) is a clear example of this shock. The positive diamond-specific shock is identified by a positive response of the diamond price and Botswana GDP.

In relation to the global market, the diamond market is a very small component. According to the Kimberly Process Certification Scheme, rough diamond market was about $37 billion in 2019. The diamond-specific price shock is therefore not expected to have any material effect on global economic activity. It is common in the literature on commodities that are of global importance like oil (Peersman and Van Robays, 2009) and copper (Pedersen, 2019) to identify such a shock as one that also lead to a contraction in global economic activity. In our case, it is also important to disentangle the diamond price shock from the global demand shock by ensuring that the diamond price increase is not associated with an increase in global economic activity. To achieve that, we extract impulse responses from the diamond price shock that is close (in the mean square sense) to a no impact on global economic activity for all the rotations (**gr**) that fulfil the identification. Unlike the sign restrictions where the sign is for on impact, this restriction ensures that the response is minimised for the entire horizon. As in Barigozzi et al. (2019), that is attained by choosing the angles θ^* such that

$$\theta^* = \underset{\theta \in [0,2\pi]}{\arg\min} \sum_{i \in \mathbf{gr}} [\mathbf{B}(L)\mathbf{H}'(\theta)]^2_{i,1} \tag{3.10}$$

This criterion allows us to recover impulse responses from a unique model out of the many that would have fulfilled the sign restrictions draws. This will be the model for the point estimates.

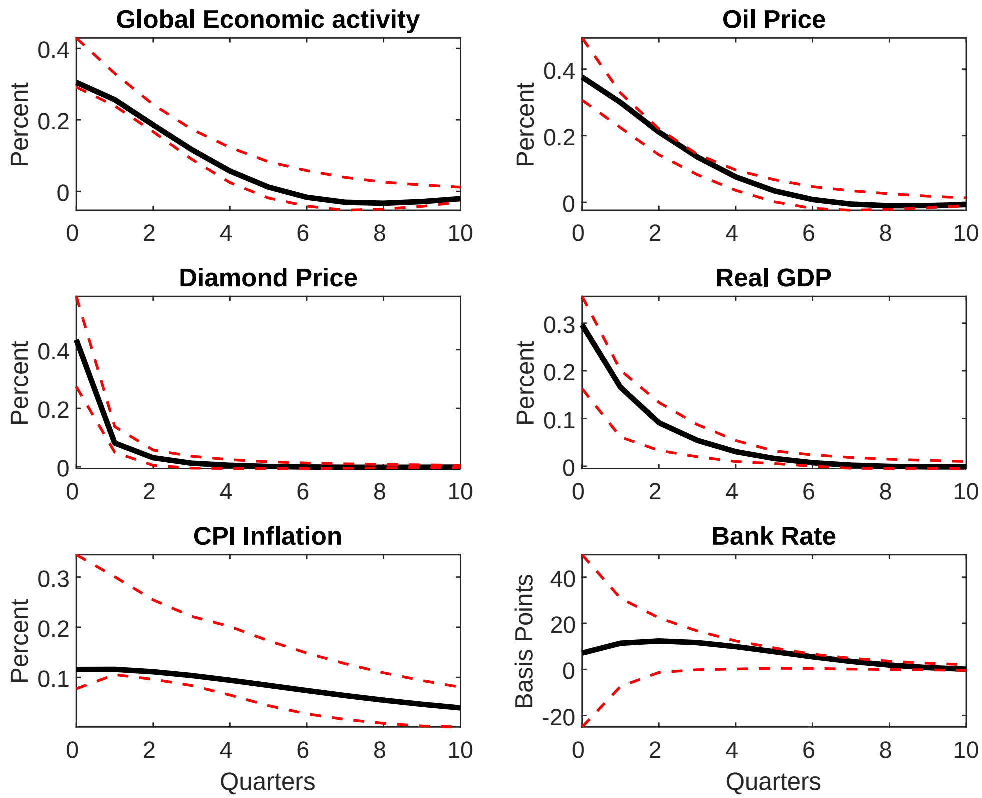
3.4 Results

This section presents the impulse response functions (IRFs) used to map the dynamic effects of the identified external shocks on Botswana's macroeconomic variables of interest. One of the main advantages of the GDFM approach is that it allows for the recovery of impulse responses of all the variables in the dataset. This allows for a detailed analysis of impact of the shocks on the Botswana economy. The sign restrictions approach / baseline impulse responses are those selected based on the criterion discussed in Equation (3.10) mentioned earlier. The black line presents the point estimate of the IRFs of the selected model while the red dashed lines reports the 80% confidence bands based on 1000 block bootstrap replications of the dataset discussed in Section 3.3.1. The variance decomposition (VD) is also presented to further understand the relationship between the structural shocks and the economy. Lastly, the macroeconomic effects of the shocks are evaluated using an alternative identification method that combines zero restriction and sign restrictions with the unique IRFs selected based on Fry and Pagan (2011)'s Median Target approach.

3.4.1 Global Demand Shock

Figure 3.1 plots the impulse responses of the selected key variables to a global demand shock. An unexpected increase in global demand increases global economic activity (global industrial production) on impact. The positive response in global economic activity declines gradually reaching the steady state in about two years. The positive demand shock and the resultant increased economic activity leads to an increase in the demand for commodities which increases oil and diamond prices. Recall that oil is an imported commodity while diamonds are an export commodity. The increase in oil price is persistent and reaches the steady state as the impact of the shock dissipates. Although significant on impact, the positive response of diamond prices lasts for a year. The rise in commodity prices due to demand associated with increased global economic activity is consistent with the results of Kilian and Murphy (2014), Juvenal and Petrella (2015) and Stuermer (2018) that show that commodity price fluctuations are mainly driven by global demand shocks.

Figure 3.1: Impulse Responses to a Global Demand Shock



Notes: Impulse responses to a positive global demand shock. The black line is the point estimate while the red dash lines represent the 80% confidence bands from 1000 block bootstrap replications.

Domestic output increases in response to the positive global demand shock. The increase in domestic real GDP dissipates in about seven quarters. The increase in real GDP reflects the impact of the increase in diamond prices. The price increase boosts the value of exports leading to a favourable trade balance and an increase in mining activity that leads to a higher level of output. This result is consistent with the results of Pedersen (2019) and Jääskelä and Smith (2013) who found that global demand shocks have a positive impact on Chilean GDP and Australian GDP, respectively.

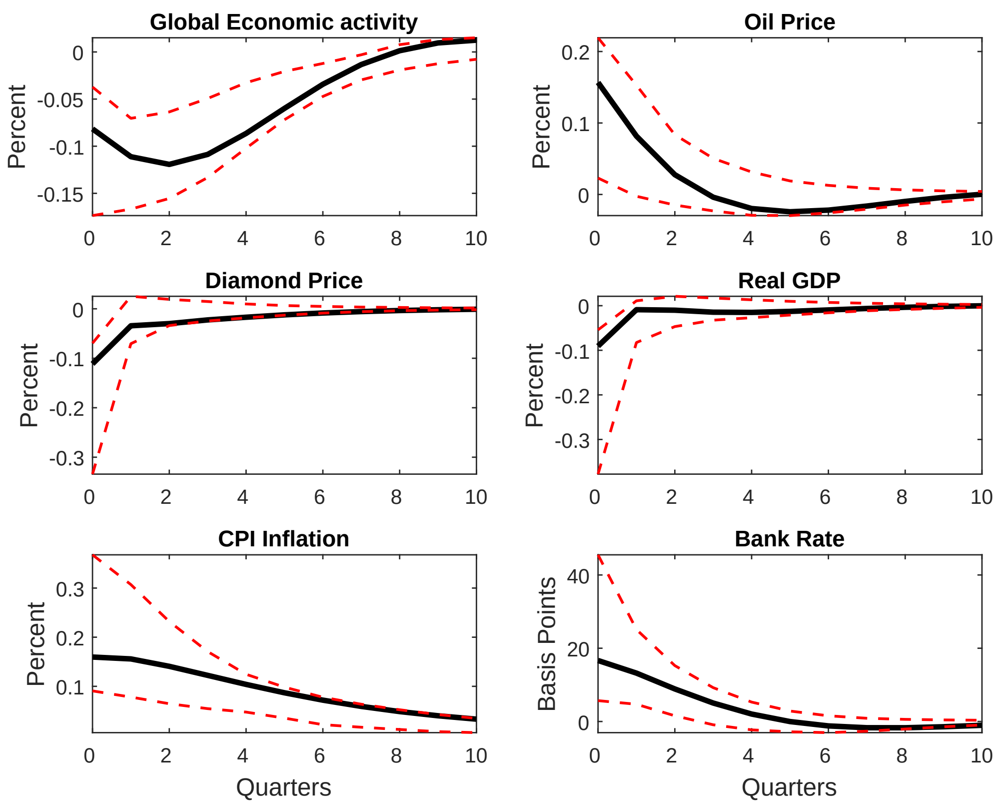
The increased global demand also leads to a rise in domestic CPI inflation. The rise in inflation reflects the impact of increased demand for general goods and services in the global market that increases domestic prices. Notably, the rise in inflation is reflective of the impact of an increase in oil prices given the pervasive nature of oil prices in the consumer basket. As noted by Peersman and Van Robays (2009), oil prices have both a direct and indirect impact on domestic inflation through rising energy prices and rising production costs that are transferred to consumer prices. The response of the domestic interest rate to a global demand shock is statistically insignificant.

3.4.2 Oil-Specific Price Shock

The IRFs in Figure 3.2 show that a positive oil specific-price shock leads to a statistically significant increase in oil price on impact. The shock is also associated with a contraction in global economic activity. This is consistent with literature on the macroeconomic effects of oil prices that show that increases in oil prices not associated with an increase in global demand lead to a contraction in economic activity as noted in Peersman and Van Robays (2009) and Aastveit et al. (2015). The results are also consistent with the findings of Kilian (2009) that a positive precautionary oil demand shock lowers real GDP and increases consumer prices. It is important to highlight that although oil price changes have a direct impact on economic activity, Segal (2011) notes that the most important transmission channel through which oil prices affect output is through monetary policy. That is, the contractionary effect of the increase in interest rate necessary to mitigate inflationary pressures outweighs the direct effect of oil prices on the global economy.

Given the negative impact on global economic activity, the oil price shock leads to a short-lived negative effect on the diamond prices. Diamonds are luxury goods, therefore more responsive to income changes (income elastic). A fall in global economic activity which also implies a fall in incomes will lead to a weaker demand and a decline in diamond prices. As diamond mining contributes a significant share to the Botswana economy, the fall in diamond prices leads to a contraction in domestic output. The oil price shock has a positive effect in domestic inflation that leads to a rise in domestic bank rate (Bank of Botswana's monetary policy interest rate) to counter the inflationary pressures. It is evident from the results that the increase in CPI inflation is persistent and statistically significant. These results demonstrate that it is very important for the central bank to

Figure 3.2: Impulse Responses to an Oil-Specific Price Shock



Notes: Impulse responses to a positive oil-specific price shock. The black line is the point estimate while the red dash lines represent the 80% confidence bands.

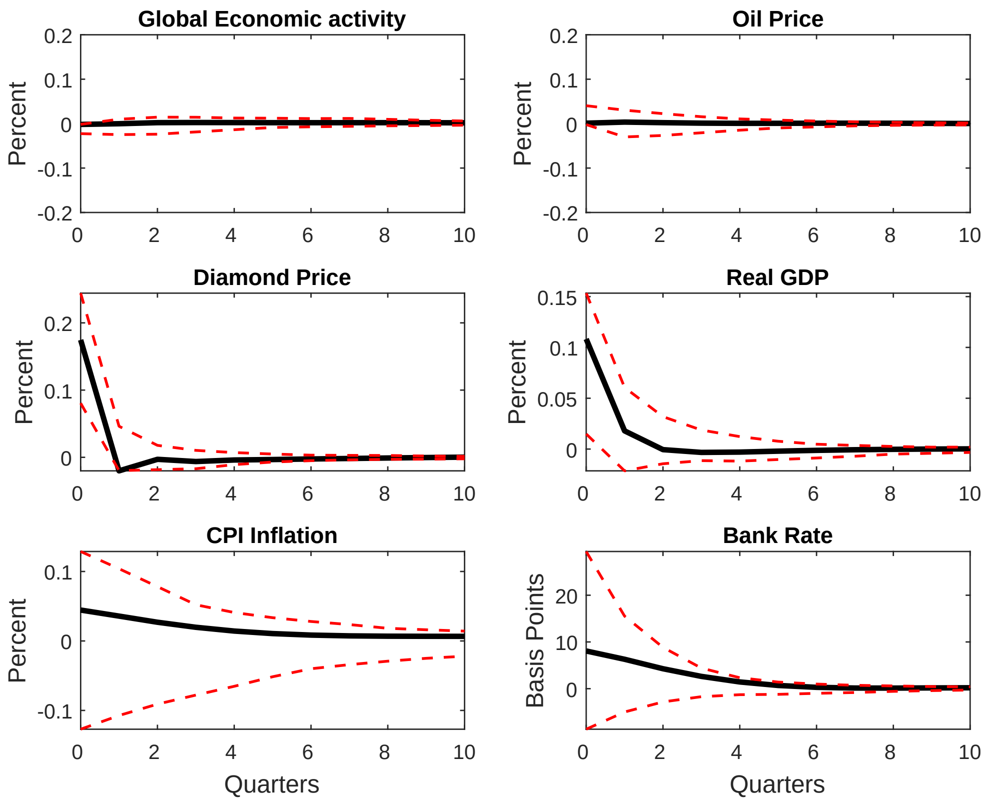
closely monitor oil price developments in the conduct of monetary policy.

3.4.3 Diamond-Specific Price Shock

Figure 3.3 presents the impulse responses to a positive diamond specific-price shock that leads to an increase in diamond prices. The rise in the diamond prices due to the diamond specific shock is short-lived, lasting only one quarter. This is consistent with McGregor (2017) who concluded that the effect of shocks on the price of commodities produced by low-income resource rich countries tend to last less than a year. As it is evident that the diamond market specific shocks are temporary, it may be appropriate for policymakers to take note of these shocks but not respond as they would if the shock were permanent.

Consistent with the criterion in equation 3.10, the results show that positive diamond price shock has no material impact on oil prices and global economic activity. This is an important feature that differentiate the diamond price shock from the global demand shock.

Figure 3.3: Impulse Responses to a Diamond-Specific Price Shock



Notes: Impulse response to a positive diamond price shock. The black line is the point estimate while the red dash lines represent the 80% confidence bands.

For the domestic variables, IRFs show that diamond price shock has a temporary positive impact on real domestic output. The expansion in domestic output captures the significant role of the diamond sector on the domestic economy. The responses of domestic prices and interest rates to a diamond-specific price shock are statistically insignificant. This result implies that the temporary nature of this shock will be of limited implication for domestic price and monetary policy developments.

3.4.4 Disaggreagated Macroeconomic Effects

A great benefit of the use of a GDFM is that we can assess the dynamic effects of the identified structural shocks on a number of macroeconomic variables included in the dataset. This subsection presents the impulse response functions of a set of variables that represent Botswana's economic activity, aggregate expenditures, trade variables and financial variables. The results here further demonstrate the importance of the global demand shocks in explaining the dynamics of domestic macroeconomic variables. More responses to this shock are in line with economic theory and statistically significant compared to responses to commodity-specific price shocks.

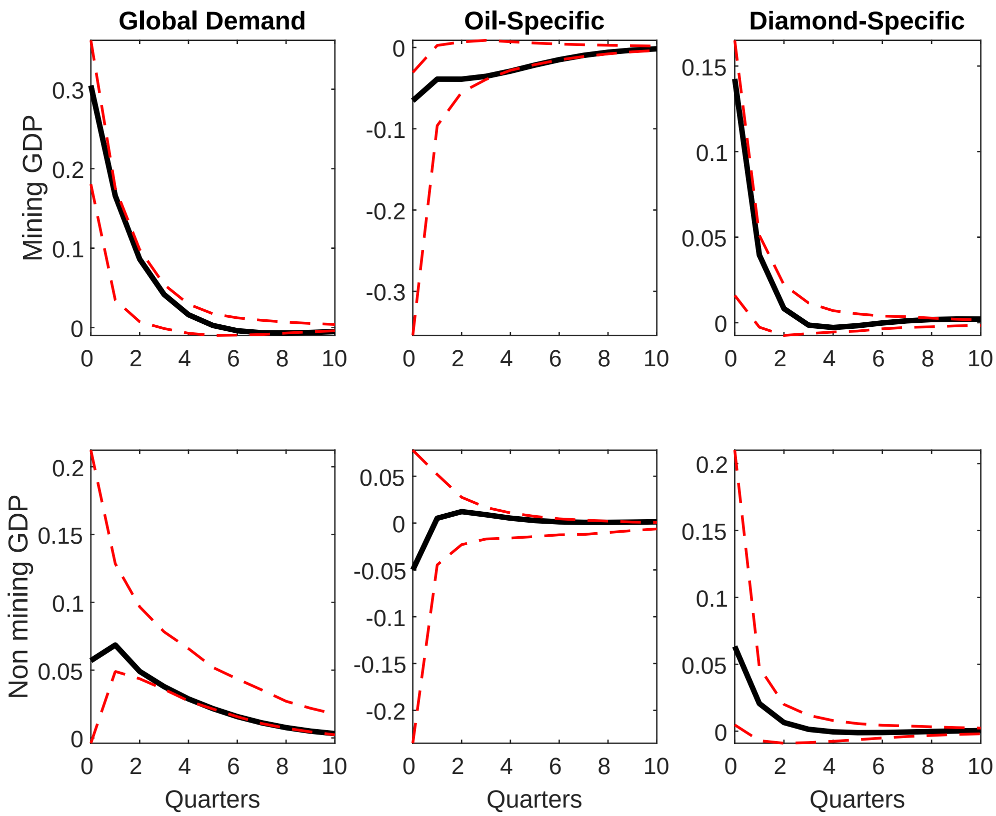
3.4.4.1 Economic Activity

The impulse responses in Figure 3.4 show that a global demand shock and a diamond-specific price shock lead to a significant increase in mining GDP on impact. These results highlight the importance of accounting for the underlying drivers of changes in diamond prices. Notably, the positive response of mining GDP to a global demand shock is double that due to a diamond-specific price shock. This could be associated with the fact that a more persistent global boom increases both production and diamond prices while short-lived diamond-specific price shocks are more likely to be associated with an increase in diamond prices and a limited to no effect on output.

A global demand shock and a diamond-specific price shock have a quantitatively similar positive effect on non-mining GDP on impact. Both responses are statistically significant with the response to a global demand shock permanent while it is transitory to a diamond price shock (lasting only 3 quarters). The higher positive response of mining GDP compared to non-mining GDP show that these external shocks are mainly transferred to the Botswana economy through the mining sector. This is consistent with the result of Pedersen (2019) on the macroeconomic effects of copper prices in Chile.

Meanwhile, an unexpected increase in oil prices due to changes in the oil market has a statistically significant negative effect on mining GDP. Its impact on non-mining GDP is not statistically significant. The negative effect of the oil price shock to mining activity largely reflects its impact on global economic activity and the resulting negative impact

Figure 3.4: Impulse Responses of Mining and Non-mining GDP



Notes: Impulse responses of mining and non-mining GDP to the three external shocks. The black line is the point estimate while the red dash lines represent the 80% confidence bands.

on diamond prices. A fall in diamond prices and increased costs of production leads to weaker activity in the mining sector. Overall, the effect of an oil-specific shock to economic activity in Botswana is limited compared to the role of the other two external shocks considered.

3.4.4.2 Domestic Expenditures

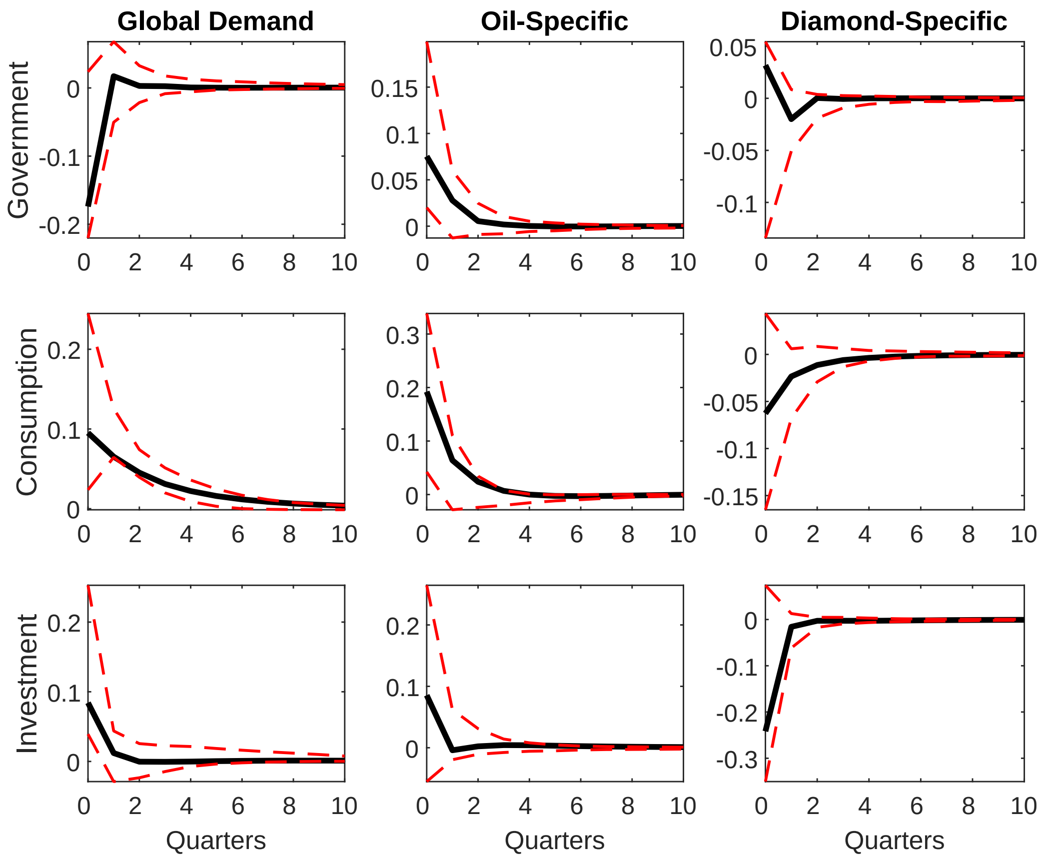
The results of the spending effects in the first column of Figure 3.5 show that government spending response to a global demand shock is statistically insignificant. As expected and in line with theory, a global demand shock leads to a statistically significant increase in household consumption and investment. The positive effect on consumption is reflective of the increase in incomes following a boom in global economic activity while the increase in investment reflects the need to satisfy the general increase in demand.

A positive oil specific shock leads to a positive effect on government spending. This implies that government spending responds countercyclically since this shock has a negative effect on domestic economic activity. This fiscal policy response ensures that the full strength of the impact of the shock is mitigated and the economy recovers from the shock in a timely manner. The response of household consumption to the oil-specific price shock is rather challenging. A positive oil price shock leads to an increase in household consumption. This is not consistent with economic wisdom that an increase in oil prices increases consumer prices and erodes the purchasing power of domestic customers. This should in turn lead to a fall in demand for goods and services and ultimately consumption.

The positive response of consumption to a positive oil price shock is not a first. In the case of the Euro area, Peersman and Van Robays (2009) found that the rise in private consumption following a positive oil price shock was due to an increase in real consumer wages that offset the negative income effect. We doubt this will be the case for this study and we don't have the data to assess this view in support of the results here. However, we can postulate that this increase could likely be explained by uncertainty effects also highlighted by Peersman and Van Robays (2009). The idea is that consumers may increase their immediate consumption when uncertain about the evolution of oil prices going forward so as to avoid higher costs in the future. In relation to investment, the effect of an oil-specific price shock is statistically insignificant.

Although a positive diamond-specific price shock has a positive and statistically significant impact on overall GDP, it has a statistically insignificant effect on the aggregate spending components (government spending, consumption and investment). This could be attributable to the temporary nature of the shock and the observation by Charnavoki and Dolado (2014) that the overall effect of the shock depends on how the income from commodity exports is spent. If a larger share of these windfall revenues are saved abroad, there will be limited effect on the domestic economy.

Figure 3.5: Impulse Responses of Domestic Expenditures



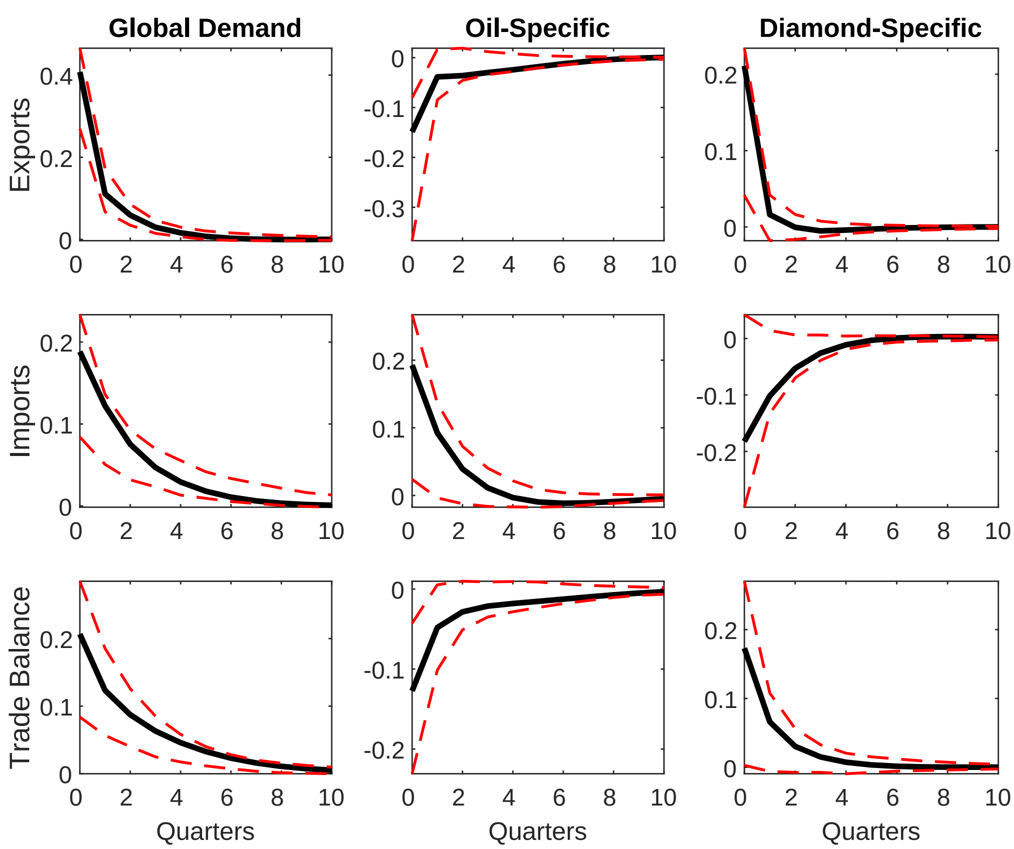
Notes: Impulse responses of domestic expenditure variables to the three external shocks. The black line is the point estimate while the red dash lines represent the 80% confidence bands.

3.4.4.3 Trade Variables

Impulse responses in Figure 3.6 show that external shocks are largely transmitted to the domestic economy through the trade variables. Notably, a positive global demand shock stimulates global trade, leading to a statistically significant increase in trade variables. Increasing global demand and economic activity raises demand for the domestic goods leading to a significant growth in total exports. The rising exports imply a transfer of incomes from the rest of the world to the domestic economy which, in turn, increases the demand for goods and services from the rest of the world. The rise in exports dominates the increase in imports leading to a positive trade balance (as a share of GDP).

Meanwhile, the IRFs show that a positive oil-specific price shock has a negative effect on domestic exports. This, may in part, reflect the negative effect of a fall in global economic activity on diamond prices following an oil price shock. As expected for an oil importing country, an increase in oil prices will lead to a rise in the cost of total imports to the domestic economy resulting in an overall trade deficit. The increase in total imports reflect both the direct impact of a rise in the value of fuel imports and the indirect impact of an increase in other non-fuel imports given the increased global production and distribution costs. Further demonstrating that these external structural shocks are transmitted through the trade variables, all the responses to the oil price shock are sizeable and statistically significant.

Figure 3.6: Impulse Responses of Trade Variables



Notes: Impulse response functions of trade variables to the three external shocks. The black line is the point estimate while the red dash lines represent the 80% confidence bands.

As anticipated, a diamond-specific shock has a positive but short-lived impact on total exports. The response of imports to the diamond-specific shock is statistically insignificant. This lack of significance could be attributable to the change in the composition of total imports following the relocation of Diamond Trading Company from London to Gaborone, Botswana between 2012 and 2013. For the period (2003 to 2011) diamonds contributed on average 5% to total imports and this proportion increased significantly to an average of about 30% for the period 2012 to 2019. Overall, the diamond-specific price shock has a positive and statistically significant impact on the trade balance.

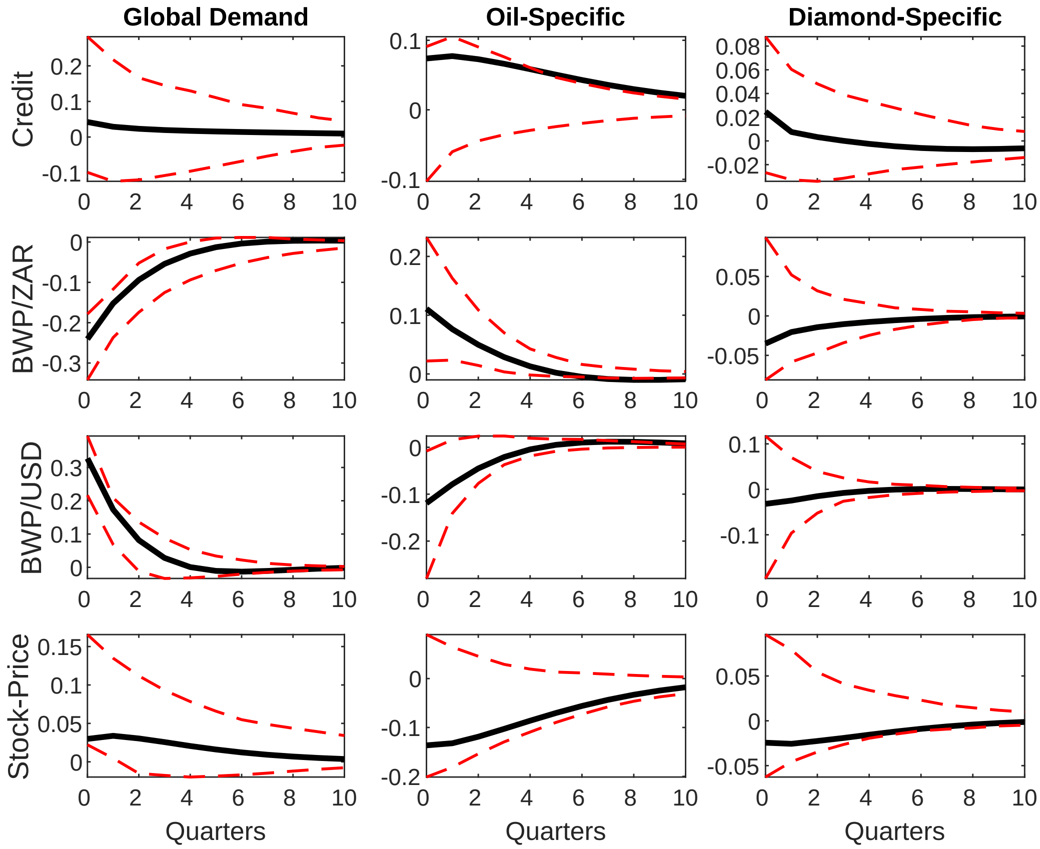
3.4.4.4 Financial Variables

A positive global demand shock has a negligible statistically insignificant impact on total credit. This could imply that variations in credit developments are explained by domestic shocks. The positive shock to global demand leads to a depreciation of the domestic currency against the South African Rand while it appreciates against the US dollar. These exchange rate movements largely reflects the positive impact of a global demand shock on commodity exporting countries as well documented in the literature (see Bjørnland and Thorsrud, 2016). Note that the opposing movements of the Pula against the South African Rand and US Dollar is also mainly reflective of Botswana's exchange rate system [3]. The positive global demand shock also leads to a small increase in stock prices represented by the Domestic Company Index (DCI).

From Figure 3.7 the impulse responses show that a positive oil-specific price shock is associated with an appreciation of the domestic currency against the South African Rand and a depreciation against the US Dollar. An increase in oil prices leads to a deterioration of the trade balance and a slowdown in economic activity which explains the depreciation of the Pula against the US dollar. A positive oil price shock has no statistically significant impact on domestic credit and stock market developments. The diamond-specific price shock has no statistically significant impact on these financial variables. This could be due to the temporary nature of the shock.

[3] The Botswana Pula is pegged to a weighted basket of currencies consisting 45% South African Rand and 55% Special Drawing Rights of the International Monetary Fund

Figure 3.7: Impulse Responses of Financial Variables



Notes: Impulse response of financial variables to the three external shocks. The black line is the point estimate while the red dash lines represent the 80% confidence bands.

3.4.5 Variance Decomposition

The impulse response results presented earlier show that the identified structural external shocks have an impact on the Botswana economy. This subsection presents the structural forecast error variance decomposition ($FEVD$) that allow us to evaluate the importance of the source of shocks on Botswana's macroeconomic fluctuations. The $FEVD_{h,ij}$ measures the quantitative importance of each j^{th} structural shock by evaluating the relative contribution to the variance of the forecast error of a variable x_i due to a particular structural shock $\mathbf{w}_j$, $j = 0, 1, \ldots, q$, at a given horizon $h = 0, 1, 2, \ldots H$. Following Stock and Watson (2016), the fraction of the variance of the forecast error in predicting the variable $\mathrm{x}_{i,t+h}$ due to the j^{th} structural shock at horizon h is given by

$$FEVD_{h,ij} = \frac{\sum_{k=0}^{h} \mathbf{D}_{k,ij}^2}{\sum_{j=1}^{n} \sum_{k=0}^{h} \mathbf{D}_{k,ij}^2} \tag{3.11}$$

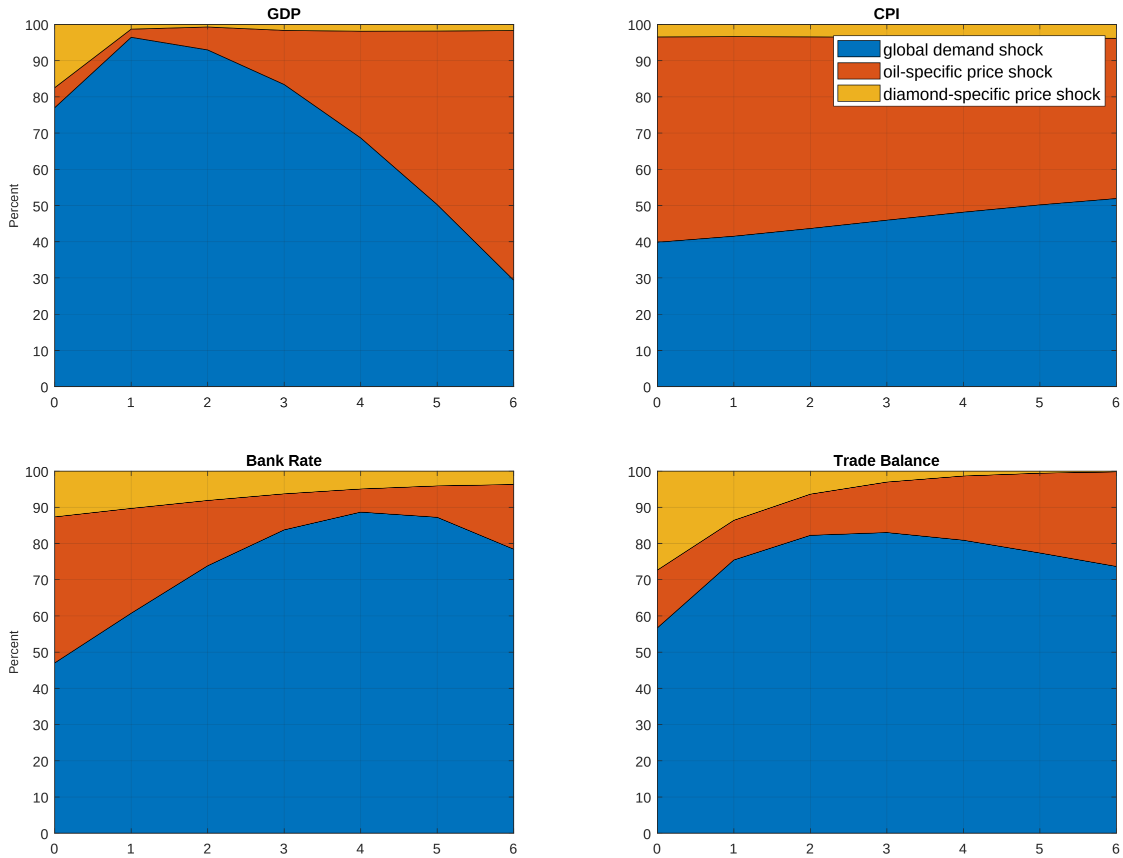
where $\mathbf{D}(\mathbf{L}) = \mathbf{B}(\mathbf{L})\mathbf{H}'$, the identified impulse response functions.

Figure 3.8 presents the FEVD for the four key macroeconomic variables, GDP, CPI, Bank rate and trade balance while Table 3.2 report the FEVD for all the variables analysed earlier. The results show that global demand shocks contribute significantly to the variation of commodity prices (oil and diamond). These results corroborate the well-established evidence of the importance of global demand shocks as the main drivers of commodity price developments (see Kilian, 2009; Juvenal and Petrella, 2015; Stuermer, 2018).

As with the commodity prices, the variance decomposition results show that the global demand shock account for a larger proportion of the variation of Botswana's macroeconomic variables. The global demand shock explain a higher share of variability in real GDP. The oil-specific price shock has very limited influence on real GDP within the first year of the shock but explain half of the variation in real GDP after five quarters (Figure 3.8). Meanwhile, reflecting the short-lived nature of the diamond-specific price shock, it only explains about 18% percent of the variation in real GDP on impact and its effect is almost negligible after the first quarter.

For inflation, the variation is explained almost equally by the global demand shock and the oil-specific price shock while the role of the diamond price shock is limited for the entire horizon. This demonstrates that the effect of oil prices changes generated by developments specific to the oil market are as important as those due to changes in global demand. Similar to inflation, the bulk of the variations in the bank rate are explained by the global demand shock and the oil-specific price shock. The analysis show that the central bank is responsive to inflation and economic developments induced by the two shocks on impact but more responsive to global demand shocks from the first quarter to just under two years. Meanwhile, the diamond-specific price shock contribute only 13% of the variation on impact and its influence on monetary policy decreases gradually over time. The results indicate that the diamond-specific price shock explain a very small part of the variations in the aggregate variables. This is reflective of the temporary nature

Figure 3.8: Forecast Error Variance Decomposition



Note: Forecast error variance decomposition for GDP, CPI, Bank Rate and Trade Balance

and speedy market correction to developments in the midstream of the diamond pipeline. For example, historical events like financing or liquidity issues in the diamond polishing industry that affected diamond prices did not last beyond one quarter.

The dominance of the global demand shock in influencing the rest of the domestic macroeconomic variables is evident. Government spending, household consumption, mining and non-mining GDP are more responsive to global demand shocks. Similarly, the variance decomposition show that a higher share of the variation in trade variables and the exchange rates is explained by the global demand shock. This is in line with the result of Aastveit et al. (2016) on the significant influence of world activity shocks on the variation in trade variables in small open economies. The diamond-specific price shock explains 14% and 22% of the variation in mining GDP and non-mining GDP on impact, respectively.

Table 3.2 Variance Decomposition

	Shocks								
	Global demand			Oil price			Diamond price		
Variable/Horizon	$h = 0$	$h = 2$	$h = 6$	$h = 0$	$h = 2$	$h = 6$	$h = 0$	$h = 2$	$h = 6$
Key-External									
Global IP	0.79	0.59	0.41	0.21	0.41	0.58	0.00	0.00	0.00
Oil price	0.89	0.96	0.51	0.10	0.04	0.48	0.01	0.01	0.01
Diamond price	0.76	0.36	0.03	0.06	0.60	0.96	0.18	0.04	0.02
Key-Domestic									
Real GDP	0.77	0.93	0.50	0.06	0.06	0.48	0.17	0.01	0.02
Inflation	0.40	0.44	0.50	0.57	0.53	0.46	0.03	0.03	0.04
Bank Rate	0.47	0.74	0.87	0.40	0.18	0.09	0.13	0.08	0.04
Expenditures									
Government	0.79	0.33	0.77	0.17	0.66	0.22	0.04	0.01	0.01
Consumption	0.27	0.83	0.94	0.65	0.13	0.05	0.07	0.04	0.02
Investment	0.10	0.06	0.14	0.10	0.61	0.59	0.80	0.33	0.28
GDP aggregates									
Mining GDP	0.80	0.68	0.17	0.06	0.31	0.81	0.14	0.01	0.02
Non-mining GDP	0.65	0.95	0.98	0.13	0.05	0.01	0.22	0.01	0.00
Trade									
Exports	0.72	0.58	0.10	0.11	0.41	0.10	0.18	0.01	0.02
Imports	0.39	0.61	0.47	0.31	0.14	0.48	0.31	0.24	0.05
Trade balance	0.57	0.82	0.77	0.16	0.11	0.22	0.27	0.06	0.01
Financial									
Credit	0.13	0.09	0.11	0.85	0.89	0.86	0.02	0.01	0.03
BWP/ZAR	0.80	0.75	0.33	0.19	0.23	0.59	0.02	0.02	0.08
BWP/USD	0.85	0.70	0.46	0.14	0.27	0.54	0.01	0.03	0.00
Stock price (DCI)	0.05	0.05	0.05	0.92	0.92	0.93	0.03	0.03	0.02

Note: Each row-column intersection reports the variance decomposition for the selected IRFs based on criterion in equation 3.10 for the horizons $h = 0$, 2 and 6 respectively.

Furthermore, about a third of the variation of trade variables on impact are explained by the diamond-specific price shock. Lastly, the diamond-specific price shock has a limited role in explaining the variations of financial variables.

Overall, the results show that global demand shocks explain a significant part of the dynamics of commodity prices and Botswana's macroeconomy. These findings highlight the need to determine the source of external shocks when evaluating their impact on an economy. The fact that the commodity specific shocks play a limited role in driving the domestic macroeconomic fluctuations does not suggest that the commodity prices are not important. The important conclusion is that, commodity price changes have a significant impact on the economy if the changes are driven by developments in global demand.

3.4.6 Alternative Identification

The baseline results are based on a sign restrictions approach and a minimisation criterion. Here we test the robustness of these results by using a specification that combines zero and sign restrictions to identify the three external shocks. This is guided by the assumption that diamond prices will not have any influence or material impact on global economic activity and oil prices. Therefore, a combination of sign and zero restrictions are used as an alternative to recover the structural shocks.

Table 3.3 Sign and Zero Restrictions

	GEA	OIL	DIA	GDP
Global demand shock	+	+	+	+
Oil-specific price shock	−	+		−
Diamond-specific price shock	0	0	+	+

Note: A positive (+)/ negative (−) sign show that the impulse response of a variable is restrained to be positive/negative. Zeros (0) indicate that the diamond specific price shock has no contemporaneous impact on global economic activity and international oil prices. A blank space represents a case where no restrictions are imposed.

The alternative identification is made up of two steps, first is the Cholesky factorisation that identifies the diamond-specific price shock while the second step appends a 2×2 sign restrictions that identifies the global demand shock and the oil-specific price shock.

Step1: Identification of the diamond-specific price shock

We assume that a diamond price shock has no contemporaneous effect on global economic activity and the oil price. Ordering of the key identification variables in the dataset is important. The approach requires listing of global economic activity first, followed by oil price and lastly the diamond price. This ensures that when all shocks are recovered we have the global demand shock, oil price shock and diamond price shock in that order. Recalling that the common component is defined as $\boldsymbol{\chi}_t = \mathbf{B}(L)\mathbf{u}_t$, then identification of the diamond-specific price shock is attained by zero contemporaneous restrictions implemented by the Cholesky decomposition of the variance-covariance matrix $\Sigma_u = \mathbb{E}(u_t u_t')$. Starting with a lower-triangular Cholesky factor $\mathbf{P} = Chol(\Sigma_u) \implies \mathbf{P}'\mathbf{P} = \Sigma_u$, partial identification implies that

$$\begin{aligned} \boldsymbol{\chi}_t &= \mathbf{B}(L)\mathbf{P}'\mathbf{P}\mathbf{u}_t \\ &= \tilde{\mathbf{C}}(L)\tilde{\mathbf{w}}_t, \end{aligned} \tag{3.12}$$

with

$$\tilde{\mathbf{C}}(L) = \begin{pmatrix} \tilde{C}_{11}(L) & \tilde{C}_{12}(L) & C_{13}(L) \\ \tilde{C}_{21}(L) & \tilde{C}_{22}(L) & C_{23}(L) \\ \vdots & \vdots & \vdots \\ \tilde{C}_{m1}(L) & \tilde{C}_{m2}(L) & C_{m3}(L) \end{pmatrix} \text{ and } \tilde{\mathbf{w}}_t = \begin{pmatrix} \tilde{w}_{1t} \\ \tilde{w}_{2t} \\ w_{3t} \end{pmatrix}$$

where $\tilde{w}_{1t}$ and $\tilde{w}_{2t}$ are unidentified and w_{3t} is the diamond-specific price shock and the only identified shock. Thus, the impulse responses $\tilde{\mathbf{C}}(L)$ are semi structural as only column three captures responses to the identified diamond-specific price shock. The responses of the first two columns will be structural when the other two shocks are identified next by means of sign restrictions.

Step2: Identification of the global demand shock and oil-specific price shock

We need a 2×2 orthogonal matrix $\mathbf{H}(\theta)$ where $\mathbf{H}'\mathbf{H} = \mathbf{I}_2$ for the identification of the global demand shock and the oil-specific price shock. This entails building an orthonormal matrix Ψ that combines $\mathbf{H}(\theta)$ with a vector that ensures that we recover the identified column three of $\tilde{\mathbf{C}}(L)$. Define $\mathbf{H}(\theta) = \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix}$ and $\Psi = \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$. It follows then that

$$\begin{aligned} \boldsymbol{\chi}_t &= \tilde{\mathbf{C}}(L)\Psi'\Psi\tilde{\mathbf{w}}_t \\ &= \mathbf{D}(L)\mathbf{v}_t, \end{aligned} \tag{3.13}$$

where $\mathbf{D}(L) = \tilde{\mathbf{C}}(L)\Psi'$ are the structural impulse responses and $\mathbf{v}_t = \Psi\tilde{\mathbf{w}}_t$ are the identified structural shocks of the alternative criteria. Similar to the sign restrictions in the benchmark case, the restrictions require a positive response of global economic activity and oil prices to a positive global demand shock as well as a positive response of oil prices and a negative response of global economic activity to an oil-specific price shock.

As the identification scheme has a sign restrictions component to identify the global demand shock and the oil-specific price shock, $\mathbf{D}(L)$ is made up by drawing θ from a uniform distribution on $[0, 2\pi]$ and keep only impulse responses that satisfies the sign restrictions. While in the baseline (sign restriction approach in Table 3.1) a criterion was used to recover a unique model, in this alternative scheme (zero-sign restictions in Table 3.3) we follow the analysis of Fry and Pagan (2011) to find a unique model through the Median Target (MT) method. This approach recovers the model that produce impulse responses that are as close to the median responses as possible. The results from this exercise are presented in the appendix Figure A.2-1. The results are similar with the main result except for a few statistically insignificant differences. This implies that our analysis is robust to the alternative identification strategy.

To address the potential concern that the order of the variables, global economic activity (GEA) and brent crude oil price (OIL) may affect the results, an alternative ordering is also considered. Figure A.2-2 presents the result when oil price is ordered first followed by global economic activity in the dataset. The results are qualitatively similar to those presented in Figure A.2-1.

3.5 Conclusion

This paper uses a generalised dynamic factor model to estimate the macroeconomic effects of external shocks transmitted through commodity prices on a small commodity-exporting economy. To achieve this objective, we evaluate the impact of a global demand shock, an oil-specific price shock and a diamond-specific price shock on a representative country, Botswana. The two latter shocks cover the influence of an imported commodity and an exported commodity, respectively. The key advantage of this approach is that it recognises that the source of the shock is very important in modern empirical structural macroeconomic analysis. The other benefit is that the approach acknowledges that commodity exporting countries are not only vulnerable to the commodity export prices but to commodity import prices as well. The final benefit of this approach is that the econometric framework that facilitates analysis of a large dataset allows for a broader assessment of the macroeconomic effects of external shocks on a large number of macroeconomic variables. This enhances our understanding of the impact of external shocks.

Results show that the macroeconomic effects of external shocks are mainly determined by the source of the shock. Notably, the adjustment in commodity prices due to a positive global demand shock play a crucial role in driving macroeconomic fluctuations in Botswana. The shock is pervasive and has a significant impact on key macroeconomic variables. A positive global demand shock's positive impact on output reflects its positive effect on diamond prices (export commodity) while the increased inflation is largely due to rising oil prices (import commodity) in response to the shock. A positive oil-specific price shock had a significant impact on CPI inflation while a positive diamond-specific price shock had a short-lived significant impact on real GDP.

Evidence from the forecast error variance decomposition demonstrate that global demand shocks explain a significant share of business cycle fluctuations of small commodity exporting economies. The oil-specific shock explain about half of the variations in inflation while the short-lived diamond-specific price shock explain about a fifth of the variations in output in the first quarter. In general, the diamond price shock has a limited role in explaining the variation of macroeconomic variables compared to the global demand shock and an oil-specific price shock. This has implications for any structural analysis that assesses the role of diamond prices on the Botswana economy. Not taking into account the source of the shock may lead to misleading results. In terms of policy response, more focus should be geared towards the dominant global demand shock and oil price shock. Notably, for developing commodity exporting countries the need for economic diversification to ensure resilience against shocks transmitted through the exported commodity prices cannot be overemphasised. In the same spirit, a timely transition from oil related energy to alternative renewable energies will help countries mitigate external shocks transmitted via international oil prices.

3.6 Appendix

3.6.1 Dataset

Transformation codes (Tcode) denotes the respective data transformation for each series x:

1 - No transformation x_t;

2 - $\triangle x_t$;

3 - $\triangle^2 x_t$

4 - $\log(x_t)$

5 - $\triangle_4 \log(x_t)$

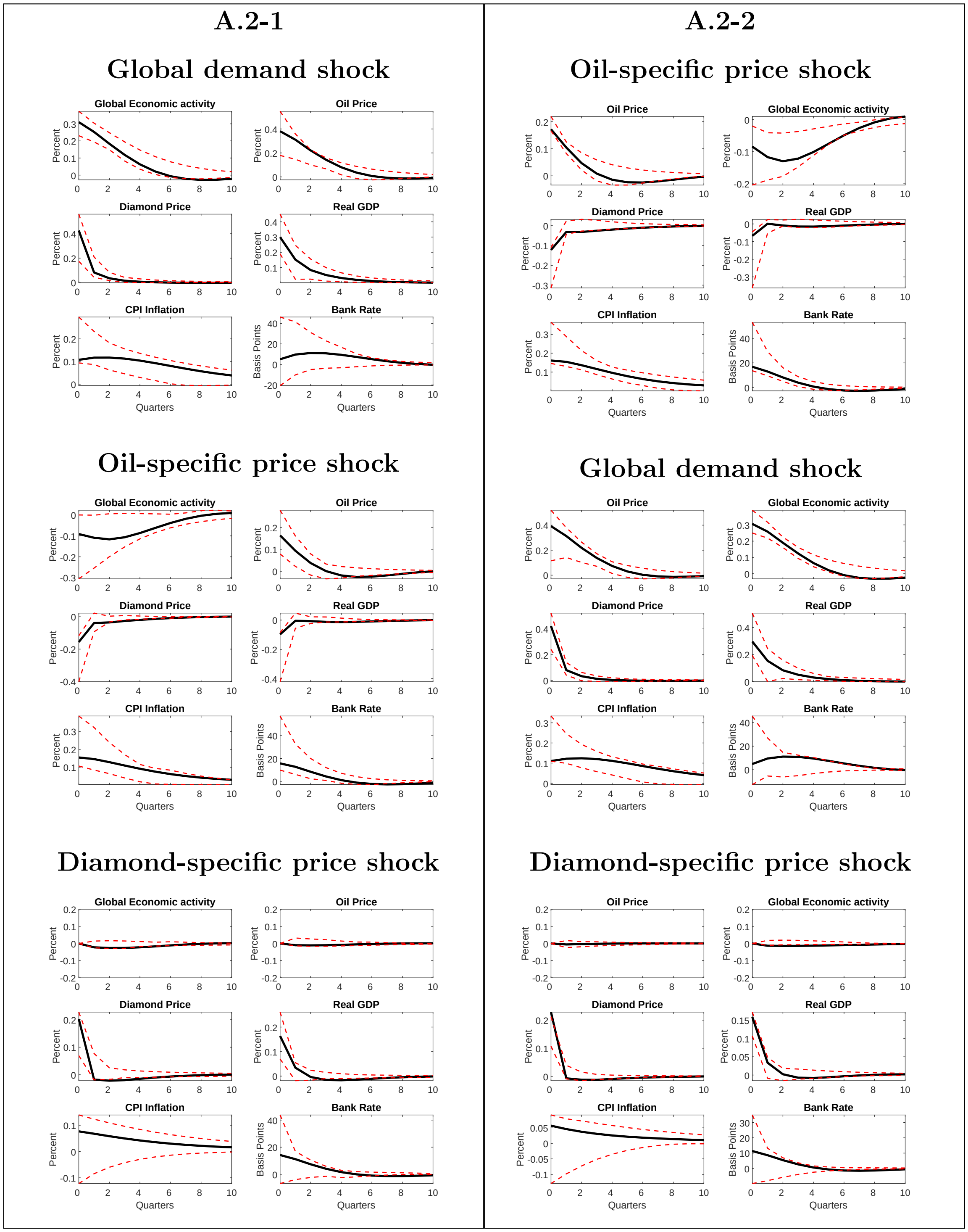
Sources : Bank of Botswana **(BoB)**, Statistics Botswana **(SB)**, South African Reserve Bank **(SARB)**, Statistics South Africa **(StatsSA)**, Johannesburg Stock Exchange **(JSE)**, Bureau for Economic Research **(BER)** - South Africa, Federal Reserve Economic Data **(FRED)**, international Monetary Fund **(IMF)**, Food and Agriculture Organization **(FAO)**, Organisation for Economic Co-operation and Development **(OECD)**, Baumeister and Hamilton (2019) **(B-H)**, US Energy Information Administration **(EIA)** and Yahoo Finance **(Yahoo)**.

Table 3.4 List of Variables

Id.	Series code	Variable	Tcode	Source
		COMMODITY PRICES		
1	GIP	Global Industrial Production	5	B-H
2	BRENT_OILPRICE	Brent crude oil price	5	EIA
3	DIAMOND_PIND	Diamond Price Index	5	BoB
4	GLOBAL_COMM	Global Price Index of All Commodities	5	IMF
5	FOOD_PIND	Food price index	5	FAO
6	CEREAL_PIND	Cereal price index	5	FAO
7	METAL_PIND	Metal Price Index	5	IMF
		EXPENDITURE		
8	GDP	Gross Domestic Product	5	BoB
9	GFC_CG	GFC: Central Government	5	BoB
10	GFC_LG	GFC: Local Government	5	BoB
11	GFC	Government Final Consumption	5	SB
12	HFC_NPI	HFC: Non Profit Services	5	SB
13	HFC_HCM	HFC: Consumption Market	5	SB
14	HFC_HCNM	HFC: Non-Consumption Market	5	SB
15	HFC	Household Final Consumption	5	SB
16	GFCF_CONST	GFCF: Construction	5	SB
17	GFCF_ME	GFCF: Machinery and Equipment	5	SB
18	GFCF_TE	GFCF: Transport and Equipment	5	SB
19	GFCF_MP	GFCF: Mineral Prospecting	5	SB
20	GFCF	Gross Fixed Capital Formation	5	SB
		ECONOMIC ACTIVITY		
21	AGRIC	Agriculture	5	SB
22	MINING	Mining	5	SB
23	MANUF	Manufacturing	5	SB
24	CONSTR	Construction	5	SB
25	TRADEHOT	Trade, Hotels and Restaurants	5	SB
26	TRANS_COMM	Transport and Communication	5	SB
27	FIN_BUS	Finance and Business Services	5	SB
28	GENGOVT	CPI: General government	5	SB
29	SOCIAL_PERSERV	Social and Personal Services	5	SB
30	N_MGDP	Non mining GDP	5	SB
		TRADE		
31	EXP_G	Exports: Goods	5	BoB
32	EXP_S	Exports: Services	5	BoB
33	EXP	Exports: Total	5	BoB
34	IMP_G	Imports: Goods	5	SB
35	IMP_S	Imports: Services	5	SB
36	IMP	Imports: Total	5	SB
37	TRADEBALS	Trade Balance	5	SB
38	RESERVES	Foreign reserves	5	BoB
		PRICES		
39	CPI_ALL	CPI: All Items	5	BoB
40	CPI_NT	CPI: Non Tradeables	5	BoB
41	CPI_DT	CPI: Domestic Tradeables	5	BoB
42	CPI_IT	CPI: Imported Tradeables	5	BoB
43	CPI_AT	CPI: All Tradeables	5	BoB

		FINANCIAL		
44	CREDIT	Credit: Total	5	BoB
45	CREDIT_H	Credit: Household	5	BoB
46	CREDIT_B	Credit: Business	5	BoB
47	BANK_RATE	Bank rate	2	BoB
48	3MONTH_BOBC	3 Month BOBC rate	2	BoB
49	PRIME_RATE	Prime lending rate	2	BoB
50	SAVING_RATE	Saving rate	2	BoB
51	INT_SPREAD	Interest rate spread (Prime - saving)	1	BoB
52	USD_BWP	Exchange rate: BWP/USD	5	BoB
53	EUR_BWP	Exchange rate: BWP/EUR	5	BoB
54	GBP_BWP	Exchange rate: BWP/GBP	5	BoB
55	JPY_BWP	Exchange rate: BWP/JPY	5	BoB
56	ZAR_BWP	Exchange rate: BWP/ZAR	5	BoB
57	SDR_BWP	Exchange rate: BWP/SDR	5	BoB
58	MKT_CAP	Market capitalisation	5	BSE
59	DCI	Domestic Company Index (DCI)	5	BSE
		REGIONAL - SOUTH AFRICA		
60	JSEINDEX	JSE Allshare index	5	JSE
61	RSA_GDP	RSA Gross Domestic Product	5	StatsSA
62	RSA_CPI	RSA Consumer Price Index	5	StatsSA
63	RSA_COIN	RSA Coincident Indicator	5	SARB
64	RSA_MANUF	RSA Manufacturing production index	5	SARB
65	RSA_MININGIND	RSA Mining production index	5	SARB
66	RSA_CCI	RSA Consumer Confidence Index	5	OECD
67	RSA_BCI	RSA Business Confidence Index	5	OECD
68	SARB_REPO	SARB repo rate	2	SARB
		GLOBAL		
69	NIKKEI225	Nikkei 225	5	Yahoo
70	SHANGHAI_SI	Shanghai Stock Index	5	Yahoo
71	FTSE100	FTSE 100	5	Fred
72	S&P500	S&P 500	5	Fred
73	CHINA_IND	China Industrial Production Index	5	OECD
74	CHINA_CPI	China Consumer Price Index	5	OECD
75	CHINA_CCI	Consumer Confidence Index	5	OECD
76	UK_CPI	UK Consumer Price Index	5	ONS
77	UK_BCI	UK Business Conditions Index	5	ONS
78	UK_CCI	UK Consumer Confidence Index	5	ONS
79	UK_INT	UK interest rate	5	BoE
80	US_INDPROD	US Industrial Production Index	5	Fred
81	FED_RATE	Fedfunds rate	2	Fred
82	ADS_index	Aruoba-Diebold-Scott Business conditions	2	Fred
83	UMCSENT	University of Michigan Consumer Sentiment	5	Fred
84	US_UNRATE	US Unemployment rate	5	Fred
85	US_CPI	US Consumer Price Index	5	Fred

3.6.2 Alternative Identification IRFs



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