

1     **An Integrated Modeling Approach for Analyzing the Deformation Style of**  
2                     **Active Volcanoes: Somma-Vesuvius Case Study**

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18    **Key Points:**

- 19        1. Analog and numerical modeling highlight an active spreading-sagging process at  
20        the Somma-Vesuvius volcano.  
21        2. A comparison of models with DInSAR deformation data validates the modeling  
22        procedures.  
23        3. The spreading at Vesuvius allows inference of the near-future eruption style, due  
24        to the loading stress reduction generated by the tension regime condition  
25        affecting the chemistry and explosivity index of volcanic eruptions.  
26

27 **Abstract**

28 The deformation style of active volcanoes can provide insight into the structural  
29 evolution of their edifices, volcanic activity and associated hazards. The Somma-  
30 Vesuvius volcano is considered one of the most dangerous on the planet due to its  
31 proximity to the megacity of Naples (Southern Italy). Thus, understanding its  
32 deformation style and corresponding long-term structural evolution are critical aspects  
33 for risk reduction. Although a large amount of data has already been collected about  
34 Somma-Vesuvius, the deformation style affecting its volcanic edifice is still debated.  
35 Therefore, we devised an integrated approach to clarify the current state of deformation  
36 of this volcano. In particular, we combined analog experiments and finite element (FE)  
37 modeling to constrain the current deformation style affecting Somma-Vesuvius and  
38 determine the physical parameters controlling its structural evolution. The analog  
39 models were built at a scale of 1:100,000 using sand mixtures (brittle analog) and  
40 polydimethylsiloxane (ductile analog). The FE models were implemented by  
41 considering a three-dimensional time-dependent fluid-dynamic approach performed at  
42 both the analog model scale (1:100000) and actual volcano scale (1:1). We obtained an  
43 FE model and a corresponding analog one that faithfully reproduced the observed  
44 deformation velocity patterns revealed by differential interferometric synthetic aperture  
45 radar (DInSAR) and GPS measurements at Somma-Vesuvius. Overall, our results  
46 support the hypothesis that a combined gravitational spreading-sagging process governs  
47 the deformation style of Somma-Vesuvius.

48

49 **Plain Language Summary**

50 Volcanic edifices of sufficient mass are capable of deforming substrata under their own  
51 weight; this deformation in turn can deform the volcanic edifices themselves.  
52 Identifying the deformation style characterizing a volcanic edifice is useful when  
53 considering the evolution of its volcanic activity. Vesuvius is considered one of the most  
54 dangerous volcanoes on the planet due to its proximity to the megacity of Naples  
55 (Southern Italy). Thus, understanding its deformation style and corresponding structural  
56 evolution are critical aspects for risk reduction. In order to analyze the deformation  
57 process of Vesuvius we used two different modeling techniques: analog modeling and  
58 Finite Element numerical modeling. The analog modeling approach allows us to  
59 reproduce real processes by using scaled models and media considered analog to natural  
60 materials under a physical point of view. The combination of analog and numerical  
61 modeling allowed us to constrain the current deformation style affecting Somma-  
62 Vesuvius and to determine the physical parameters controlling its structural evolution.  
63 Finally we compared our results with the observed deformation velocity patterns  
64 revealed by Differential Interferometric Synthetic Aperture Radar and GPS  
65 measurements at Vesuvius. Overall the results support the hypothesis that a combined  
66 gravitational spreading-sagging process governs the deformation style of Somma-  
67 Vesuvius.

68

69 **Keywords:** Volcano Deformation, Spreading, Sagging, Analogue Model, Finite Element Analysis,  
70 Gravitational Deformation.

71 **1 Introduction**

72 Volcanic edifices can have sufficient mass to deform their substrata, which in  
73 turn can deform the volcanic edifices themselves. Identifying the deformation style  
74 characterizing a volcanic edifice is useful because it can influence seismic and volcanic  
75 activity (Borgia et al., 2000; D’Auria et al., 2013). Borgia et al. (2005) proposed an

76 active spreading deformation hypothesis for the Somma-Vesuvius volcanic complex  
77 based on observational evidence: (i) Seismic profiles and gravimetric and magnetic  
78 surveys of the area showed that the recent strata are folded and cut by minor thrust  
79 faults (Andronico et al., 1995). (ii) Leveling surveys along a profile parallel to the  
80 coastline from Napoli to the Sorrento Peninsula (Osservatorio Vesuviano, INGV,  
81 internal reports 1990–2000; Lanari et al., 2002; Borgia et al., 2005) revealed a strong  
82 subsidence (almost 2 mm/yr) of Vesuvius and uplift (about 2 mm/yr) of the Pompeii  
83 area. (iii) DInSAR data from Somma-Vesuvius highlighted a regional scale subsidence  
84 that terminated at Pompeii, where a relative uplift was evident. Despite this evidence,  
85 the deformation style affecting Somma-Vesuvius is still debated.

## 86 1.1 Theoretical background

87 During the last decades, several studies have focused on the structural evolution and  
88 deformation processes of volcanic edifices (e.g., Merle & Borgia, 1996; van Wyk de  
89 Vries & Matela, 1998; Byrne et al., 2013; Delcamp et al., 2008; Kervyn et al., 2010).  
90 Analog and numerical modeling have highlighted the roles of rheology, volcano-  
91 tectonic features, basement geology and the sedimentary successions beneath volcanoes  
92 in defining the deformation styles of volcanic edifices.

93 Reliable modeling approaches require preliminary measurements of the natural  
94 phenomena and ground deformation analyses are fundamental in determining  
95 deformation styles. Such measurements can be used to constrain both analog and  
96 numerical models, which can generally aid each other in improving parameter  
97 estimation through several iterative simulations. At the end of modeling procedures, the  
98 models are usually compared with observed data to validate results (Kavanagh et al.,  
99 2018). The outcomes of such models are helpful for evaluating and forecasting potential  
100 volcanic hazards, such as flank collapses (e.g., van Wyk de Vries & Matela, 1998).

101 As volcanoes grow through the accumulation of erupted products, their increasing  
102 gravitational loads can exceed the mechanical strengths of volcanic substrata.  
103 Supposing that an edifice lies on a brittle layer that overlies a ductile layer (e.g.,  
104 sedimentary successions), a horizontal outward deformation of the ductile substratum  
105 can result, in turn inducing a vertical displacement of the edifice. Such gravitational  
106 deformation is mainly controlled by the rheological properties of substrata and their  
107 thicknesses (Merle & Borgia, 1996; van Wyk de Vries & Borgia, 1996). The thickness  
108 ratio between a ductile layer and a brittle layer determines the deformation style of the  
109 edifice, resulting in either flexure, sagging (also called basement extrusion), or  
110 spreading (van Wyk de Vries & Matela, 1998; Byrne et al., 2013; Fig. 1).

111 The flexure deformation style occurs when the thickness of the ductile layer beneath an  
112 edifice is significant compared with the edifice dimensions and brittle layer thickness.  
113 The ductile layer is not horizontally constrained by boundaries, and the edifice  
114 deformation is characterized by a significant vertical downward displacement. This  
115 leads to the development of normal faults at the base of the edifice, a compression of the  
116 whole edifice and a flexural bulge surrounding the edifice (Fig.1b.3).

117 In the case of sagging deformation, the deformation rate is determined by the  
118 gravitational load, edifice geometry and flexural rigidity of the basement. A pure  
119 sagging deformation is characterized by a peripheral horizontal extension of the area  
120 surrounding the volcano, associated with edifice compression (van Wyk de Vries &  
121 Matela, 1998; Byrne et al., 2013). It occurs when the ductile layer viscosity is low  
122 compared with the strength of the overlying brittle layer. In this case, there is still  
123 outward movement of the ductile material but not an associated outward displacement

124 of the volcanic slopes. As the underlying material moves away from an edifice, the  
125 edifice sags downwards and undergoes overall compression (Fig.1b.2).  
126 Finally, the spreading deformation style requires the presence of a weak ductile  
127 substratum (e.g., sedimentary successions, pyroclastic rocks, or oceanic crust affected  
128 by hydrothermal activity or partial melting) and a relatively high mass loading from an  
129 edifice. In this case, spreading is accommodated by thrust faulting around the base of  
130 the edifice and graben-style faulting at its center, in addition to a ductile flow of the  
131 weak underlying layer (Delcamp et al., 2008; Fig. 1a; 1b.1). Volcanic spreading tends to  
132 be independent of the regional tectonic setting because, locally, the volcano-tectonic  
133 stress field overcomes the regional one (Borgia et al., 2000). Generally, the spreading  
134 style can be summarized as a deformation characterized by the subsidence of the  
135 summit or upper flanks of a volcano, outward displacement at the slopes, periphery, and  
136 nearby substrate, and formation of summit horst-and-graben structures, basal thrusts,  
137 and folds (Borgia et al., 2000).  
138 The transition from the flexure style to spreading style depends on the relationship  
139 between the thickness and viscosity of the ductile layer. Spreading-style deformation  
140 can occur if there is a thin ductile layer with a high ratio of its viscosity to the volcano  
141 failure strength (van Wyk de Vries & Matela, 1998; Fig. 1b.1). Otherwise, if there is a  
142 thin ductile layer but the ratio between the viscosity of the ductile substratum and the  
143 failure strength of the edifice is low, the edifice experiences sagging deformation  
144 (Byrne et al., 2013; van Wyk de Vries & Matela, 1998; Fig. 1b.2). Low-viscosity ductile  
145 layers generally represent substrata that are decoupled from the edifice and are extruded  
146 from underneath it. In this case, the edifice sinks and is subject to an overall  
147 compressional stress field. Finally, the presence of a basal thick ductile layer allows the  
148 edifice to deform with a flexure style (van Wyk de Vries & Matela, 1998; Fig. 1b.3).  
149 A pure spreading deformation reduces the likelihood of sector collapses by reducing  
150 cone stresses by forming inward dipping normal faults and reducing slope angles (van  
151 Wyk de Vries & Borgia, 1996). Spreading-related processes range from rock creeps to  
152 large-magnitude earthquakes (Borgia et al., 2000). In contrast, basement extrusion  
153 deformation results in high compressive cone stresses, which can lead to the formation  
154 of outward dipping faults, and maintains or steepens existing slopes, thus increasing the  
155 risks of flank collapse (van Wyk de Vries & Francis, 1997).  
156

## 157 1.2 Geological setting

158 The Somma-Vesuvius volcano complex of the Neapolitan volcanic district comprises  
159 explosive and effusive products (Borgia et al., 2005). It is characterized by the  
160 asymmetric shapes and truncated cone of Mt. Somma, remnants of various caldera-  
161 forming eruptions, and the smaller cone of Vesuvius Gran Cono, which grew in the last  
162 two millennia and is offset from the axis of Mt. Somma (Bonasia et al., 1985; Fig. 2).  
163 The oldest evidence of Mt. Somma-Vesuvius activity dates back to  $0.369 \pm 0.028$  Ma  
164 ( $^{40}\text{Ar}/^{39}\text{Ar}$ ) (Jashemsky, 2002), and its last eruption occurred in 1944. Somma-  
165 Vesuvius grew on a substratum consisting of Mesozoic carbonates displaced by SW-  
166 and NW-dipping normal fault systems and, secondarily, by NE-SW and E-W faults  
167 (Fusi et al., 1991; Brocchini et al., 2001; D’Auria et al., 2014a; see also the detailed  
168 geological map in Sbrana et al. 2020). NW-SE, NE-SW, and ENE-WSW faulting also  
169 affect the volcanic units cropping out in the Somma caldera (Santacroce, 1987; Borgia  
170 et al., 2003; D’Auria et al., 2014a; Fig. 2). Somma-Vesuvius is currently quiescent,  
171 showing only fumarolic activity, low-energy seismicity (Ventura & Vilardo, 1999), and  
172 slow ground deformation. The latter is characterized by the subsidence of the edifice



173 and uplift in the surrounding area (Lanari et al., 2002; Borgia et al., 2005; Marturano et  
174 al., 2013).

175 The hypothesis of the active spreading deformation at Somma-Vesuvius was first  
176 proposed by Borgia et al., (2005). Seismicity at Somma-Vesuvius can be separated  
177 into two seismogenic volumes, located at different depths and dominated by different  
178 stress patterns (Bianco et al., 1998; D’Auria et al., 2014). The seismicity in the lower  
179 volume is confined between about 1 and 5 km b.s.l. and is related to the background  
180 regional stress field. In contrast, seismicity in the upper volume, located above sea level,  
181 can be related to the gravitational deformation of the edifice (D’Auria et al., 2014).

182 In this work, to understand the deformation style at Somma-Vesuvius better, we built an  
183 original set of analog and finite element (FE) scale models to reproduce the deformation  
184 processes currently active at Mt. Somma-Vesuvius. DInSAR measurements derived  
185 from ERS-1/2 and ENVISAT SAR data during 1993–2010 were used as constraints in  
186 the modeling procedure. The analyses of DInSAR mean velocity maps and the  
187 corresponding time series (dataset from Tizzani et al., 2020) allowed us to obtain a  
188 reliable picture of the active deformation processes at Somma-Vesuvius and their  
189 temporal evolution. Figure 3 shows sensor line of sight (LOS) mean velocity maps  
190 along ascending and descending orbits. The processed data are relevant to the complete  
191 SAR image catalogs of ERS-1/2 and ENVISAT from 1993 to 2010, and they highlight  
192 three main aspects of the ground deformation patterns of Somma-Vesuvius: (i)  
193 subsidence on the summit area, (ii) generalized subsidence of the S-SW flank of the  
194 edifice and a semi-circular area surrounding the volcano spanning from NW to SE, and  
195 (iii) uplift of the area located at W-NW of the edifice and the area of Pompeii, located  
196 SE, a few kilometers away from the volcanic edifice.

## 197 **2 Methods**

### 198 **2.1 Scaling approach and dimensionless analysis**

199 Analog models need to be scaled to faithfully represent volcano deformation processes  
200 (e.g., Hubbert, 1937; Ramberg, 1981). Our experiments were arranged with a main  
201 length scale ratio ( $L^* = L$  at the model scale/ $L$  at the volcano scale) set at 1:100,000, a  
202 density ratio ( $\rho^* = \rho$  at the model scale/ $\rho$  at the volcano scale) set at 0.6, and a gravity  
203 ratio ( $g^* = g$  at the model scale/ $g$  at the volcano scale) of 1, resulting in a stress ratio ( $\sigma^*$   
204 =  $\sigma$  at the model scale/ $\sigma$  at the volcano scale) of:

$$205 \quad \sigma^* = \rho^* \times g^* \times L^* \approx 6 \times 10^{-6} \quad (1)$$

206 Since cohesion has the same stress dimension ([Pa]),  $\sigma^*$  also represents the scaling ratio  
207 for cohesion ( $c^* = c$  at the model scale/ $c$  at the volcano scale). This allows setting the  
208 sand mixture cohesion used for modeling (~65 Pa) to correspond to an unfractured rock  
209 with cohesion of ~10 MPa (Jaeger & Cook, 1971; Handin, 1996).

210 In the context of scaling analyses, volcano deformation styles can be characterized  
211 using dimensionless analyses (Merle & Borgia, 1996; van Wyk de Vries & Borgia,  
212 1996; Barenblatt, 2003; Tizzani et al., 2010; Gibbings, 2011). Such analyses entail  
213 relating the geometrical and physical parameters of a volcano through a set of  
214 dimensionless numbers derived based on the Buckingham  $\Pi$  theorem (Buckingham,  
215 1914, 1915). According to it, we selected eleven variables (volcano height, volcano base  
216 diameter, ductile layer thickness, brittle layer thickness, internal friction angle, ductile  
217 layer density, ductile layer viscosity, volcano bulk density, gravity acceleration,  
218 deformation velocity, and brittle material cohesion) and three dimensions (length, time,  
219 and mass) defining eight dimensionless numbers (Table 1).

220 The dimensionless numbers ( $\Pi_1$  to  $\Pi_5$ ) consider the geometrical characteristics, while  
221  $\Pi_7$  and  $\Pi_8$  represent the force ratios. The numbers from  $\Pi_5$  to  $\Pi_8$  have a minor role in  
222 interpreting experimental results because they consider the brittle layer behavior ( $\Pi_5$ -  
223  $\Pi_7$ ) or the ductile layer ( $\Pi_8$ ) alone. The numbers from  $\Pi_5$  to  $\Pi_8$  were only used for  
224 scaling.

## 225 2.2 Analog modeling

226 We realized twelve analog models. The first series of models ( $60 \times 50$  cm planar  
227 dimensions) were constrained by fixed walls and comprised two layers: a brittle upper  
228 layer (0.2–0.9-cm thick) made of a mixture of dry quartz-sand and K-feldspar powder  
229 (30% in weight) as a bulking agent, and a lower ductile layer of polydimethylsiloxane  
230 (PDMS) with a thickness of 0.4–0.7 cm (Fig. 4a; Table 2). The volcanic edifice was  
231 represented by sand emplaced on the brittle layer. We constructed models that  
232 reproduced the asymmetric shape of the Somma-Vesuvius volcano, with a truncated  
233 cone analog of Mt. Somma topped by a smaller cone analog of Vesuvius Gran Cono,  
234 which was set off the axis of the central cone (Fig. 4b and c). We used a higher density  
235 mix (quartz-sand and rutile powder; 50% in weight) for a few models, which helped  
236 widen the parameter space explored by the experiments (Table 2). To provide  
237 confidence in any asymmetry in results interpreted as due to edifice asymmetry, we  
238 also carried out control models with a symmetric edifice to provide a benchmark  
239 reference.

240 To estimate the vertical and horizontal deformations, we monitored the experiments  
241 using four digital cameras. Image sets were processed into sequences of 3-D surface  
242 models using structure-from-motion photogrammetry, allowing vertical deformation  
243 analyses (AgiSoft PhotoScan Professional, version 1.4.3; 21 July 2018. ©2018 Agisoft  
244 LLC). Horizontal deformation was assessed by tracking feature displacements within  
245 the vertical camera image sequence using the Pointcatcher software  
246 ([www.lancaster.ac.uk/staff/jamesm/software/pointcatcher.htm](http://www.lancaster.ac.uk/staff/jamesm/software/pointcatcher.htm); Delcamp et al., 2008;  
247 James et al., 2015).

248 Our twelve models comprised variations in brittle and ductile layers thickness and  
249 bulk density (Table 3). We also varied the symmetry of the volcanic edifice to provide  
250 a benchmark reference to the asymmetric ones.

251

## 252 2.3 Finite element modeling

253 We not only performed FE simulations to reproduce and validate the analog models but  
254 also to analyze the deformation affecting the volcanic edifice while using the same  
255 geometrical and physical characteristics of the analog models but with the dimensions  
256 of the Somma-Vesuvius volcano. Indeed, the numerical method is the only approach  
257 allowing a 1:1 scale simulation. The correspondence between the results of the  
258 numerical simulations reproducing the analog models and the numerical simulations  
259 made with the real dimensions of the volcano was a key result confirming the validity of  
260 our analog model. This correspondence could allow us to compare the results of our  
261 scaled analog model with the DInSAR data, thus demonstrating the actual deformations  
262 of the Somma-Vesuvius volcano.

263 Also, using the COMSOL Multiphysics<sup>®</sup> software package, we performed 3D time-  
264 dependent fluid-dynamic models in an FE environment. FE simulations of the ground  
265 deformation velocities were performed using a Newtonian viscous flow approximation  
266 for the media behavior, which allowed us to evaluate how the thickness and viscosity

267 contrast between the ductile and brittle domains modulated the observed gravity-driven  
268 deformation. We first reproduced the analog experiments in the FE environment and  
269 then reproduced the best model of the Somma-Vesuvius at natural scale (1:1), scaling  
270 all parameters with the same scale factor used to build the analog models (see Section  
271 1.2 for details on scaling approach). To model the sand mixture, we approximated the  
272 rheology using a viscous approximation by applying “apparent viscosity” (i.e., ratio  
273 between the applied shear stress and the shear rate). The use of apparent viscosity can  
274 be considered a valid approximation when the material has a high cohesion value as in  
275 our analog experiments due to the addition of the bulking agent (see Section 2.2).  
276 The FE models reproduced the analog experiments through domain dimensions of  $60 \times$   
277  $60 \times 10$  cm. We defined two regions of appropriate thickness to represent (1) the edifice  
278 topography and upper sand mixture and (2) the lower PDMS layer (e.g., Fig. 5a), which  
279 has a density of  $\rho_d = 965 \text{ kg m}^{-3}$  and a viscosity of  $\eta = 2 \times 10^4 \text{ Pa}\cdot\text{s}$  (Weijermars, 1986).  
280 The sand mixture was represented by bulk density of  $1550 \text{ kg m}^{-3}$  (Montanari et al.,  
281 2017) and an apparent viscosity value of  $0.8 \times 10^8 \text{ Pa s}$ . This apparent viscosity value  
282 was determined as the optimum from a series of FE tests. The cohesion of the brittle  
283 layer, thickness of the ductile layer, and DEM of the model were changed for each  
284 simulation based on the corresponding analog models.  
285 We set free boundary conditions at the surface, roller conditions (movement only  
286 parallel to the boundary) at the four lateral sides, and fixed constraints at the bottom of  
287 the computational domain. The computational domain was discretized into tetrahedral  
288 elements (Fig. 5b), which enabled a fine meshing adapted to the complex topography.  
289 The domain was discretized into 164,800 tetrahedral elements ranging in size from  
290 0.002 m to 0.01 m, with the coarser elements located along with the boundaries of the  
291 domain. To validate the simulations of the analog models, we realized FE simulations at  
292 a real scale (i.e., 1:1), including the actual topography of Somma-Vesuvius. The digital  
293 topography was defined using the SRTM DEM of the volcano (Farr et al., 2007).

294

### 295 **3 Experimental results**

#### 296 **3.1 Analog modeling results**

297 The benchmark reference symmetric models (i.e., models 02, 03, 05, 07, and 10)  
298 generated almost symmetric deformation patterns and gave confidence that asymmetry  
299 observed in other results reflected asymmetry in the edifice. Consequently, we focus our  
300 discussion on the asymmetric models (01, 04, 06, 08, 09, 11, and 12; Table 3) with  
301 results most closely resembling the deformation of Somma-Vesuvius (for further  
302 information on all results, see Appendix A).

303 In model 01 (Fig. 6a), the whole model edifice generally subsided quite symmetrically  
304 by 0.25–0.5 cm but with peak values of almost 1.0 cm. The area surrounding the  
305 edifice showed a little diffuse uplift with values of up to 0.25 cm. The highest  
306 values, which ranged from 0.5 to 0.75 cm, were located in a small circular area  
307 close to the base of the edifice.

308 In model 04 (Fig. 6b), the model edifice was characterized by a significant subsidence  
309 with values ranging from 0.75 to 1.0 cm, showing a little asymmetric behavior, which  
310 was more prominent on the side of the Somma caldera rim. The whole area surrounding  
311 the model edifice had a gentle uplift with values of  $\sim 0.25$  cm. In comparison, the area  
312 close to the base of the edifice showed high values, with an uplift of  $\sim 1.0$  cm arranged  
313 in concentric circular sectors. The surrounding area on the side close to the Somma  
314 caldera rim showed a relevant uplift (flexural bulge in section 1.2).

315 In model 06 (Fig. 6c), the whole edifice had a prominent subsidence with peak values of  
316 ~1.0 cm, and the area close to its base was characterized by a diffuse uplift reaching  
317 values of ~1.0 cm.

318 Model 08 (Fig. 6d) was characterized by an overall deformation comparable with that  
319 of model 01. Both models showed a subsidence of the whole edifice and an uplift of the  
320 area close to its base, while the surrounding area was subjected to a slight uplift.  
321 However, in contrast to model 01, the structures forming at the top and base of the  
322 edifice in model 08 clearly showed flexural bulges around the volcano and radial faults  
323 cutting the edifice from the center to the slopes.

324 Model 09 (Fig. 6e) is characterized by the general subsidence of the edifice along with  
325 the formation of almost radial faults, which were more developed on the side with the  
326 Somma caldera rim. As for the previous models, a dominant shear fracture separated the  
327 Somma caldera rim from the side of the edifice on which Vesuvius Gran Cono rises. An  
328 uplift was highlighted all around the edifice base but was greatest along the area close  
329 to the Somma caldera rim.

330 Model 11 (Fig. A1 in Appendix A), with a large edifice and a thin ductile layer (0.4  
331 cm), highlights the asymmetric gravitational deformation of the volcano due to its  
332 asymmetric shape. It shows a strong subsidence of the Vesuvius Gran Cono and Somma  
333 caldera rim as well as a really strong uplift of the surface close to the edifice on the side  
334 of the Somma caldera rim. This model is characterized by a flexural bulge, particularly  
335 on the Somma rim side, and by the formation of a few main radial faults.

336 Finally, the results of model 12 (Fig. 6f) are generally comparable with those of models  
337 05 and 08. Still, the deformation affecting model 12 appears to be less prominent: the  
338 subsidence affected the whole edifice, and the uplift affected almost only the base of the  
339 volcano slopes. The edifice was not cut by a fault as in model 08. Conversely, faults  
340 were only formed at the edge of the subsiding area. Both the subsidence and uplift were  
341 less prominent than in the other models.

342 We focus our discussion on the asymmetric model results, which closely match the  
343 geometry of the Somma-Vesuvius asymmetric volcanic edifice (Table 3). Comparing  
344 the maximum positive velocities and the maximum negative velocities (Fig. 7d)  
345 confirmed that the edifice mass had a significant influence on determining the extent of  
346 the deformation pattern. An overall analysis (Fig. 7a–c) showed that Somma-Vesuvius  
347 (black star, whose value was derived from the analysis of the DInSAR data (sec. 1.2))  
348 falls in the proximity of the point corresponding to model 08, confirming a similar  
349 behavior (Fig. 7a–c).

350 The relationship between the area and velocity ratios (Fig. 7d) suggest that model 08 is  
351 the best Somma-Vesuvius volcano analog. Considering the velocity and diameter ratios  
352 suggests that models 09 and 06, along with the asymmetrical and heavier model 08, also  
353 approximate the behavior of Somma-Vesuvius quite well (Fig. 7e). If the diameter ratio  
354 and the ratio of the areas are also considered, it can be clearly seen that model 08 is the  
355 best Somma-Vesuvius analog. This last evidence supports the use of the denser medium  
356 (50% sand–50% bulking agent) to represent the volcanic edifice (Fig. 7f).  
357

### 358 3.2 Finite element modeling results

359 The FE simulations of the analog models were run for a simulation time of 3 h  
360 (reflecting the run time of the analog models), and the results were output at intervals of  
361 0.5 h. FE simulations of the analog models reproduced the general subsidence of the  
362 whole edifice and the uplift of a circular ring around the edifice base observed in the  
363 models. The simulations also reproduced the diffuse uplift affecting a larger area for the

364 models with smaller scale lengths (models 01 and 06 in Fig. 8). This correspondence  
365 was visible when comparing the vertical deformation profiles (Fig. 8). As for the analog  
366 models, we now focus our discussion on the simulation that showed results most  
367 representative of Somma-Vesuvius.

368 Figure 9 shows the results obtained for the FE simulation corresponding to the analog  
369 model 08 (model from Castaldo & De Matteo, 2020), as it was considered as the analog  
370 model that best reproduces the deformation affecting Somma-Vesuvius. In this case,  
371 similar to the results of the analog model, the FE simulation results had a slightly high  
372 uplift value at the volcanic edifice base. In the sector close to the Somma rim, we  
373 postulated that this occurred due to the heavier load generated by the presence of the  
374 high density and homogeneous structure of the Somma rim on that side. The area  
375 characterized by the most horizontal movement is located at the base of the volcano  
376 slopes and in the close surrounding area (Fig. 9). The similarity between the natural-  
377 scale (1:1) FE simulation results, reduced-scale simulations, and surface velocities  
378 measured by DInSAR confirm that the parameters adopted for the reduced-scale models  
379 efficiently reproduced the deformation style affecting Somma-Vesuvius.  
380

#### 381 **4 Discussion**

382 The proposed combined approach represents a reliable way to overcome the intrinsic  
383 limitations resulting from using a single modeling approach. Specifically, the  
384 combination of analog experiments and numerical modeling made it possible to analyze  
385 both the kinematic aspects and those relating to stress distribution, significantly  
386 clarifying our understanding of the current deformation style of Somma-Vesuvius.  
387 Thus, we compared the modeling results with both ground-based measurements and  
388 remote sensing data.

389 The analog models could reproduce the overall Somma-Vesuvius ground deformation  
390 pattern but did not include local effects, such as the development of individual fractures  
391 or other details. Therefore, analog modeling provides a “low pass filtered”  
392 representation of the active deformation processes of long-term deformation patterns.

393 To compute both the stress distribution and deformation affecting Somma-Vesuvius in  
394 FE simulations, we used the parameters derived from the analog modeling, which are a  
395 simplification of the actual case, as they do not consider the existence of structural  
396 discontinuities and/or lateral mechanical heterogeneity. Thus, in the FE modeling, we  
397 chose to consider the mean velocity field rather than displacements.

398 By analyzing the long-term deformation processes affecting Somma-Vesuvius, we  
399 followed the approach proposed by van Wyk de Vries and Matela (1998). We calculated  
400 the dimensionless numbers ( $\Pi_3$  and  $\Pi_5$ ) for the real volcanic edifice shape and those for  
401 each of our models. We plotted our results along with the results of different  
402 deformation styles based on the distributions of  $\Pi_3$  versus those of  $\Pi_5$  (van Wyk de  
403 Vries & Matela, 1998; Fig. 10). The points related to the real Somma-Vesuvius edifice  
404 fell in the black ellipse plotted in Figure 10. Our dimensionless analysis placed Somma-  
405 Vesuvius in a field with high  $\Pi_3$  and  $\Pi_7$  values, defined as “volcano and basement  
406 spreading” (Fig. 10).

407 The deformation velocity, area and diameter values of model 08 (Figs. 7a–c), which are  
408 based on geometrical characteristics, showed good conformity with the corresponding  
409 values from Somma-Vesuvius, suggesting that this model can be regarded as a good  
410 approximation of the actual volcano. Therefore, from the deformation pattern of model  
411 08, it can be said that Somma-Vesuvius shows typical features of a spreading process,  
412 such as (i) onset of normal faults in the summit region of the edifice (Merle & Borgia,

413 1996 (Fig. 1a and 11) and (ii) a pronounced symmetrical uplift at the base of the edifice  
414 (Fig. 11). Features typical of a sagging deformation should also develop, such as (i) an  
415 encircling trough at the base of the edifice (Byrne et al., 2013; Fig. 11), (ii) a peripheral  
416 flexural bulge (Kervyn et al., 2010; Fig. 11), and (iii) an uprising of the ductile material  
417 along with the fracture opening at the base of the brittle layer as a consequence of the  
418 extension generated at the brittle-ductile interface due to the subsidence of the edifice  
419 (Fig. 11).

420 In conclusion, we argue that the Somma-Vesuvius edifice is affected by a hybrid  
421 sagging/spreading deformation style, with a dominance of spreading over sagging. This  
422 spreading dominance is demonstrated by the development of near-radial shear fractures  
423 on the edifice slopes rather than the formation of semi-circular, tensile, and shear  
424 fractures, which are typical features of a sagging-dominated deformation process (see  
425 Fig. 11).

426 Based on the comparison of the vertical deformations retrieved from our analog (Fig.  
427 12a) and FE simulations (Fig. 12c) with the DInSAR LOS mean velocity (Fig. 12b), it  
428 can be observed that the proposed modeling procedure very well reproduces the current  
429 ground deformation pattern of Somma-Vesuvius. Also, these results show a good fit  
430 between our FE model and the DInSAR measurements. In detail, both the analog and  
431 FE models, as well as and the DInSAR measurements showed an overall subsidence of  
432 the whole edifice along with an uplift in a circular ring at the edifice base. Additionally,  
433 structures compatible with the compression of the SW-sector (i.e., the opposite side to  
434 the Somma caldera rim) were also developed (Fig. 12).

435

## 436 **5 Conclusions**

437 Our approach supports the integration of different modeling techniques as a  
438 successful method to reliably estimate parameters related to the ground deformation  
439 patterns in volcanoes. This integrated approach effectively increases the possibility of  
440 better understanding the variables affecting volcanic systems and could also be applied  
441 to other geodynamic settings.

442 Our models reproduced the real mean velocities of the Somma-Vesuvius volcano as  
443 determined from DInSAR LOS, and demonstrated that the volcano is affected by a  
444 hybrid sagging-spreading deformation style, characterized by a predominance of  
445 spreading. The strong subsidence in the region of the Somma caldera rim and the most  
446 significant uplift along the adjacent base of the edifice illustrate that the edifice  
447 asymmetry influences the deformation pattern because it affects the geometry of the  
448 structures forming and evolving on the volcano.

449 The recognition of the active spreading processes at Somma-Vesuvius has substantial  
450 consequences for inferring the evolution of volcanic activity as the decreasing of the  
451 volcanic explosive index (VEI), due to the establishment of a tension regime condition  
452 that significantly reduces the loading stress on magmatic reservoir systems. This vertical  
453 load reduction could have favored the ascent of less evolved magmatic bodies with a  
454 consequent impact on the chemistry of erupted products and consequently on the  
455 explosivity index of volcanic eruptions (Borgia et al., 2005).

456 Thus, the knowledge of the deformation process affecting the Somma-Vesuvius volcano  
457 during its quiescent phase is valuable for understanding the future changes in its  
458 deformation pattern due to volcanic reactivation processes. Since a renewed activity will  
459 interact with the present deformation field, changing and increasing the deformation of  
460 the area, the knowledge of the current deformation process affecting Somma-Vesuvius  
461 is definitely a key point for a reliable volcanic surveillance system.



463 **Appendix A**

464 All the models (Fig. A1) showed the general subsidence of the volcanic edifice  
465 and the general uplift of the surrounding area, but were characterized by different  
466 deformation rates. The majority of the models, especially the larger ones, showed a  
467 flexural bulge around the volcano and radial faults cutting the edifice from the center to  
468 the slopes. The symmetric models (i.e., models 02, 03, 05, 07, and 10) showed an  
469 almost symmetric deformation pattern, and some of those were characterized by the  
470 formation of “flower faults” (e.g., models 05 and 07), which is typical of a spreading  
471 deformation process (Merle & Borgia, 1996).

472 In model 02, the whole volcanic edifice was characterized by a significant subsidence  
473 with values ranging from 0.75 to 1.0 cm. Few radial faults cut the edifice and an uplift  
474 ring surrounds it, with values ranging from 0.75 to 1.0 cm.

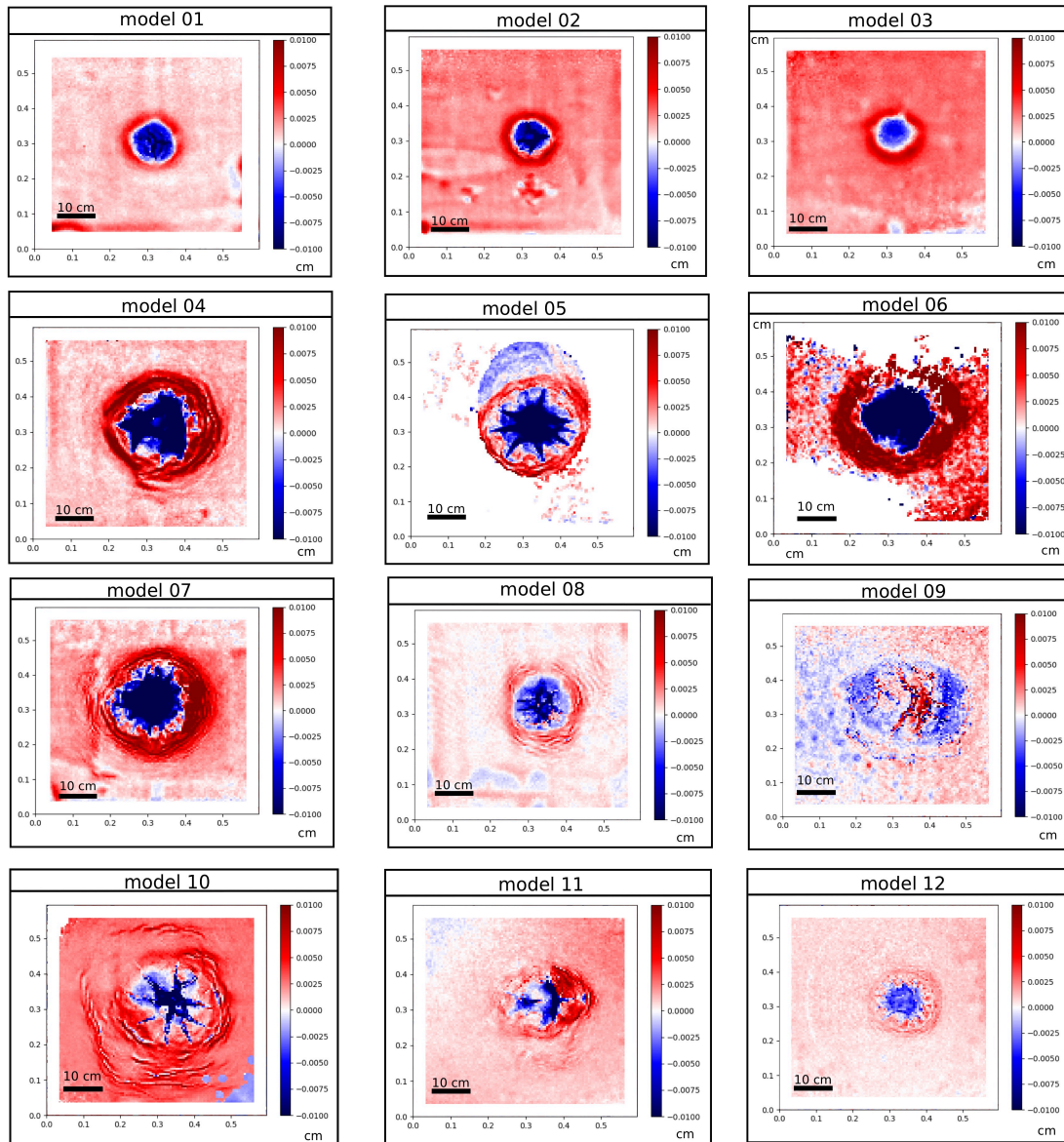
475 Model 03 was characterized by a subsidence of the volcanic edifice and an uplift of the  
476 surrounding area as it occurred in model 02 but, in this case, the subsidence and uplift  
477 were less prominent due to the thicker brittle layer.

478 In model 05, the volcanic edifice deformed with a typical spreading deformation style:  
479 subsidence of the volcano, forming a moderate flexural bulge of the surrounding area  
480 and radial “leaf” faults.

481 Model 07, built with a thinner brittle layer than model 05, was characterized by a  
482 significant subsidence, by a wide area affected by uplift (with the formation of several  
483 bulging surrounding the edifice base) and by the formation of a lot of radial faults  
484 cutting the lower sides of the slopes.

485 Model 10 showed a subsidence focused on the center of the volcano, while some faults  
486 cut the edifice from the center to the slopes, and a lot of flexural bulge in the area  
487 surrounding the volcanic edifice.





**Figure A1. Vertical deformation of the performed twelve analog models. For the model parameters, see Table 3.**

**Code and data availability:** the code and data necessary to run our FE model named model08 are stored in a Zenodo data repository (Castaldo & De Matteo, 2020). The DInSAR mean velocity measurements of the Somma-Vesuvius volcano from 1992 to 2010 are available in the Zenodo data repository (Tizzani et al., 2020). The dense points clouds dataset of the “best” analog model (model 08) are collected in the Zenodo data repository (De Matteo & Massa, 2021).

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$\Pi 1$	$\frac{\text{volcano height}}{\text{volcano diameter}}$	$\frac{H_v}{D_v}$
$\Pi 2$	$\frac{\text{volcano height}}{\text{ductile layer thickness}}$	$\frac{H_v}{H_d}$
$\Pi 3$	$\frac{\text{volcano diameter}}{\text{ductile layer thickness}}$	$\frac{D_v}{H_d}$
$\Pi 4$	$\frac{\text{brittle layer thickness}}{\text{ductile layer thickness}}$	$\frac{H_b}{H_d}$
$\Pi 5$	Coefficient of internal friction	$\tan \Phi$
$\Pi 6$	$\frac{\text{volcano bulk density}}{\text{ductile layer density}}$	$\frac{Bd_v}{\rho_d}$
$\Pi 7$	$\frac{\text{viscous force}}{\text{Mohr Coulomb failure resistanc}}$	$\frac{\eta \cdot v}{[\tau_0(1 + 2 \tan \Phi \sqrt{H_b}) + g Bd_v H_v \tan \Phi (1 + H_b)] \cdot H_v}$
$\Pi 8$	$\frac{\text{inertial force}}{\text{viscous force}}$	$\frac{v \cdot \rho_d \cdot H_d}{\eta}$

Table 1. Description of the  $\Pi$  numbers and their formulas. For a description of the symbols, see Table 2.  $g$  and  $v$  represent the gravity acceleration and velocity, respectively.

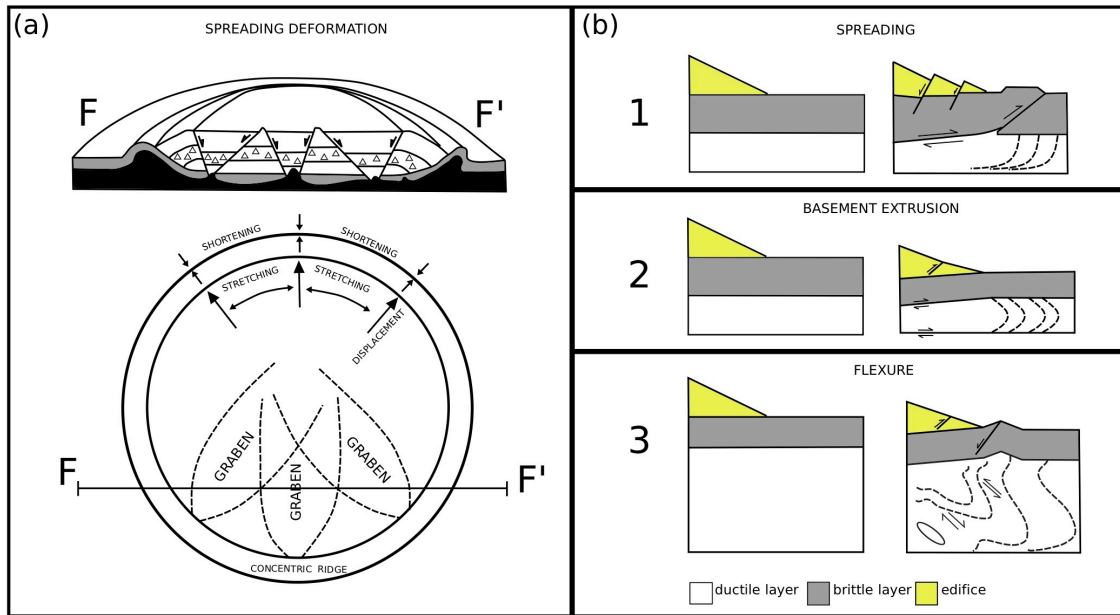
Parameters	Descriptions	Values		Units
		Models	Natural equivalent	
$H_d$	Ductile layer thickness	0.004–0.007	400–700	m
$H_b$	Brittle layer thickness	0.002–0.005	200–500	m
$H_v$	Volcano height	0.012	1200	m
$D_v$	Volcano diameter	0.122	12200	m
$\tau_0$	Cohesion	65	107	Pa
$\Phi$	Angle of internal friction	39	30	°
$\eta$	Ductile layer viscosity	$2 \times 10^4$	$1 \times 10^{19}$	Pa s
$Bd_b$	Brittle layer bulk density	1550	2580	$\text{kg m}^{-3}$
$Bd_v$	Volcano bulk density	1550–1950	2580–3245	$\text{kg m}^{-3}$
$\rho_d$	Ductile layer density	1100	1830	$\text{kg m}^{-3}$

Table 2. Parameter values adopted for the modeling and corresponding natural equivalent.

Model	Hd (cm)	Hb (cm)	Hv (cm)	Dv (cm)	Symmetry	Edifice bulk density (kg m <sup>-3</sup> )
01	0.7	0.2	1.1	12.2	no	1550 (sand:K-feldspar = 70:30)
02	0.7	0.2	1.1	12.2	yes	1550 (sand:K-feldspar = 70:30)
03	0.7	0.5	1.1	12.2	yes	1550 (sand:K-feldspar = 70:30)
04	0.7	0.5	2.2	24.4	no	1550 (sand:K-feldspar = 70:30)
05	0.7	0.5	2.2	24.4	yes	1550 (sand:K-feldspar = 70:30)
06	0.7	0.2	2.2	24.4	no	1550 (sand:K-feldspar = 70:30)
07	0.7	0.2	2.2	24.4	yes	1550 (sand:K-feldspar = 70:30)
08	0.7	0.2	1.1	12.2	no	1950 (sand:K-feldspar = 50:50)
09	0.7	0.2	2.2	24.4	no	1950 (sand:K-feldspar = 50:50)
10	0.7	0.2	2.2	24.4	yes	1950 (sand:K-feldspar = 50:50)
11	0.4	0.2	2.2	24.2	no	1950 (sand:K-feldspar = 50:50)
12	0.4	0.2	1.1	12.2	no	1950 (sand:K-feldspar = 50:50)

Table 3. Parameter values (Table 2) for the analog models.

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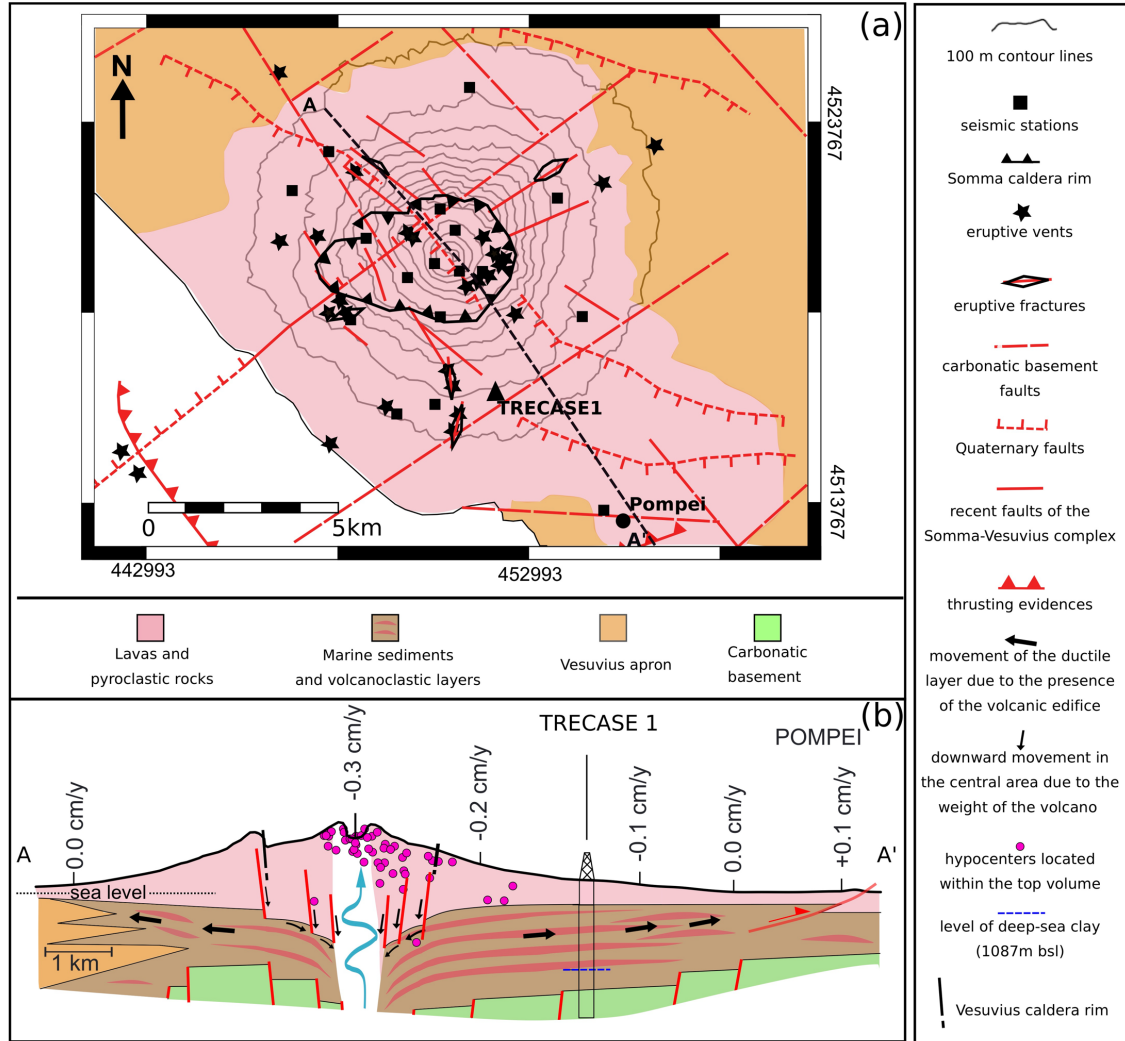


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**Figure 1.** (a) Scheme of the volcanic spreading process (after Merle and Borgia, 1996). On the top, a cross-section of the initial stage shows the flow of the weak layer (in black) and the horst-and-graben structures developing within the volcanic edifice. On the bottom, a surface projection shows the relationship between the volcano-tectonic structures and the strain pattern associated with the spreading process: radial displacement, concentric stretching in the volcano, and radial shortening in the substratum surrounding the volcano (from Merle and Borgia, 1996, modified). (b) Schematic diagrams of the principal deformation styles. The dotted lines show the movement of the ductile material. The small arrows indicate the movement directions. 1: Spreading deformation. 2: Basement extrusion (or sagging) deformation. 3: Flexure deformation (from van Wyk de Vries and Mattela, 1998, modified).

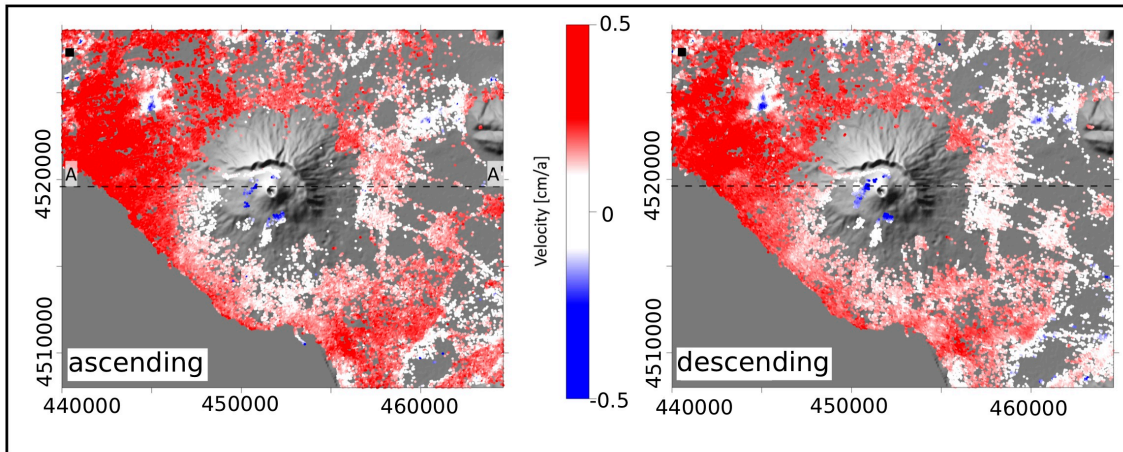


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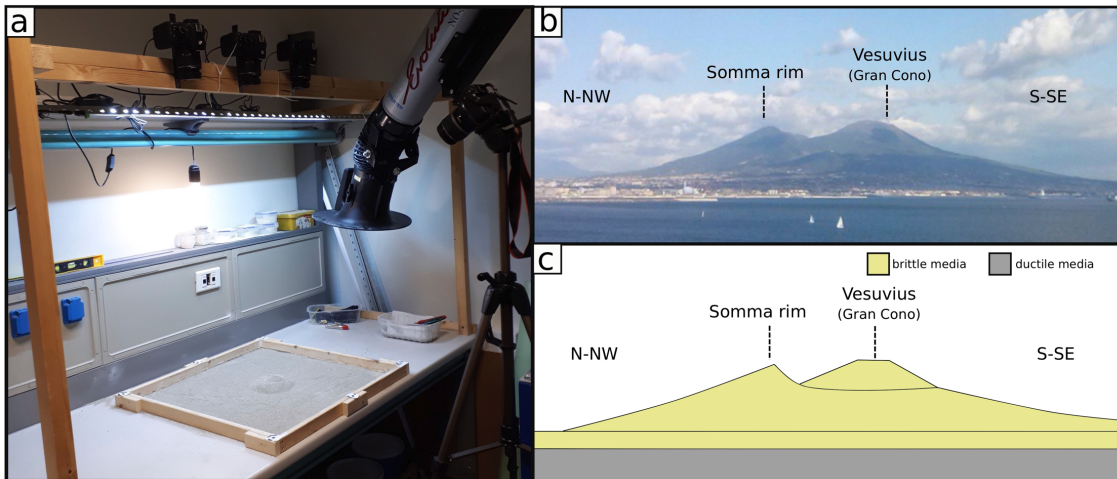


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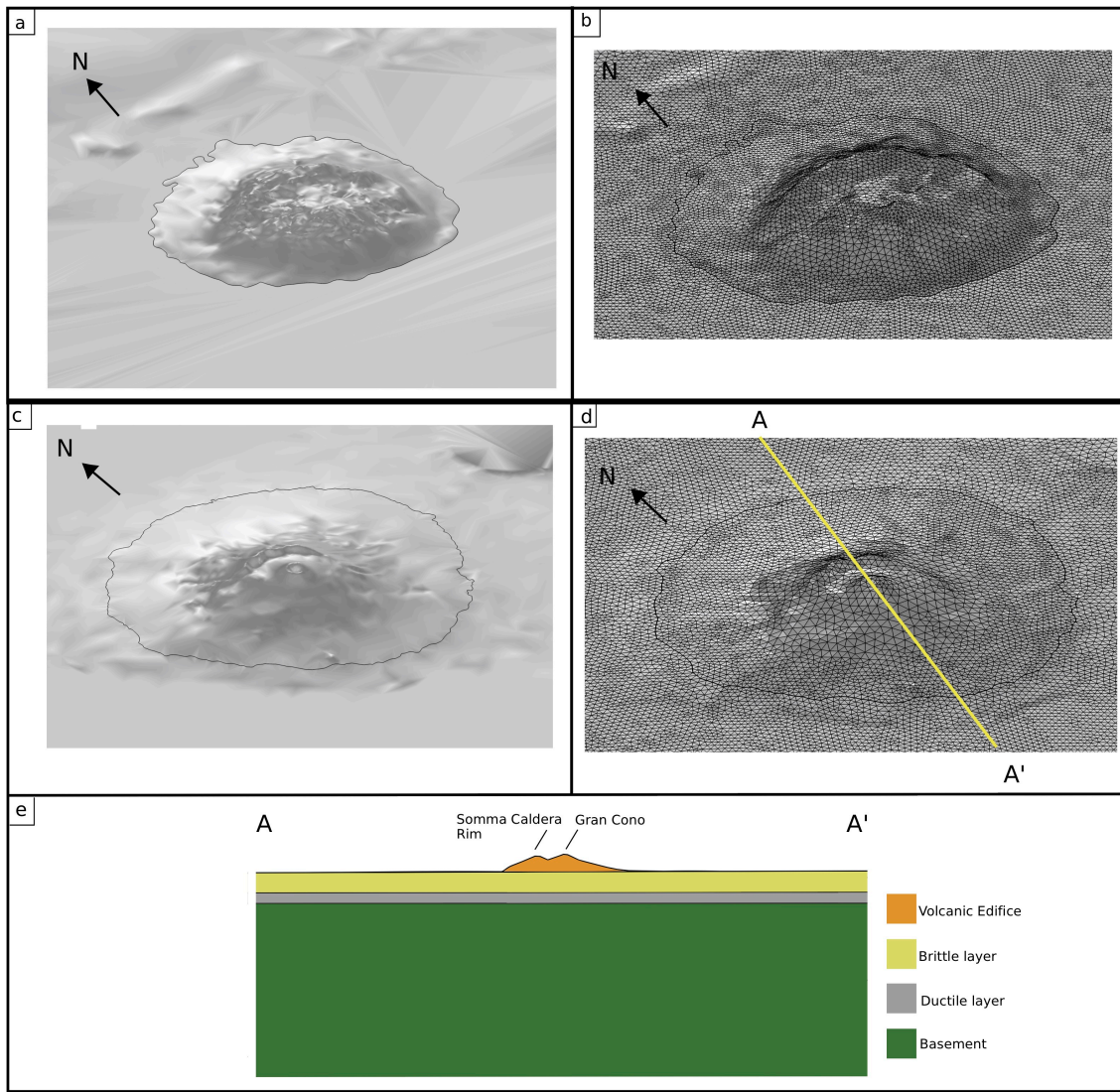
**Figure 2.** (a) Map of the Vesuvius area with an outline of the major tectonic features. The coordinates are in UTM WGS84. Data from Ippolito et al. (1973), Bianco et al. (1998), Bruno and Rapolla (1999), Ventura and Vilardo (1999), Orsi et al. (2003), Borgia et al. (2005), and Milia et al. (2012). (b) Schematic cross-section along with A–A'. The numbers along the profile indicate the vertical ground deformation rate. Along the Trecase well, we schematically reported the stratigraphic succession. The top layer consists of lavas interbedded with pyroclastic rocks, the intermediate layer mostly consists of sandy and clayey marine deposits interbedded with volcanoclastic layers, and the bottom layer consists of carbonatic rocks of the basement.



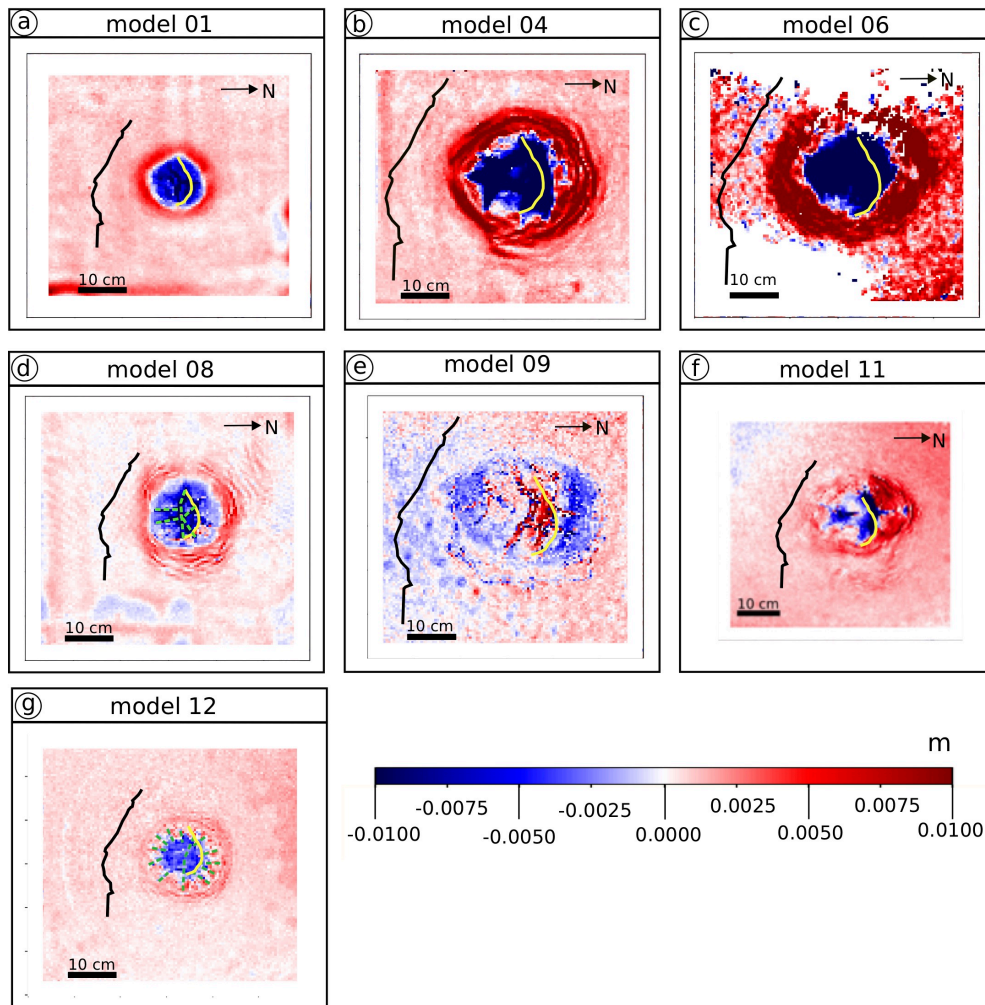
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 525 **Figure 3.** LOS projection components (ascending and descending) of the mean  
 526 deformation velocity observed at Somma-Vesuvius from 1993 to 2010 (achieved by  
 527 DInSAR-SBAS processing).  
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 531 **Figure 4.** (a) Experimental setup of the analog model, (b) profile of the Somma-  
 532 Vesuvius edifice (for scale, the summit is 1281 m above sea level), and (c) a sketch of  
 533 the double truncated cone geometry used to represent the edifice in the experiments.



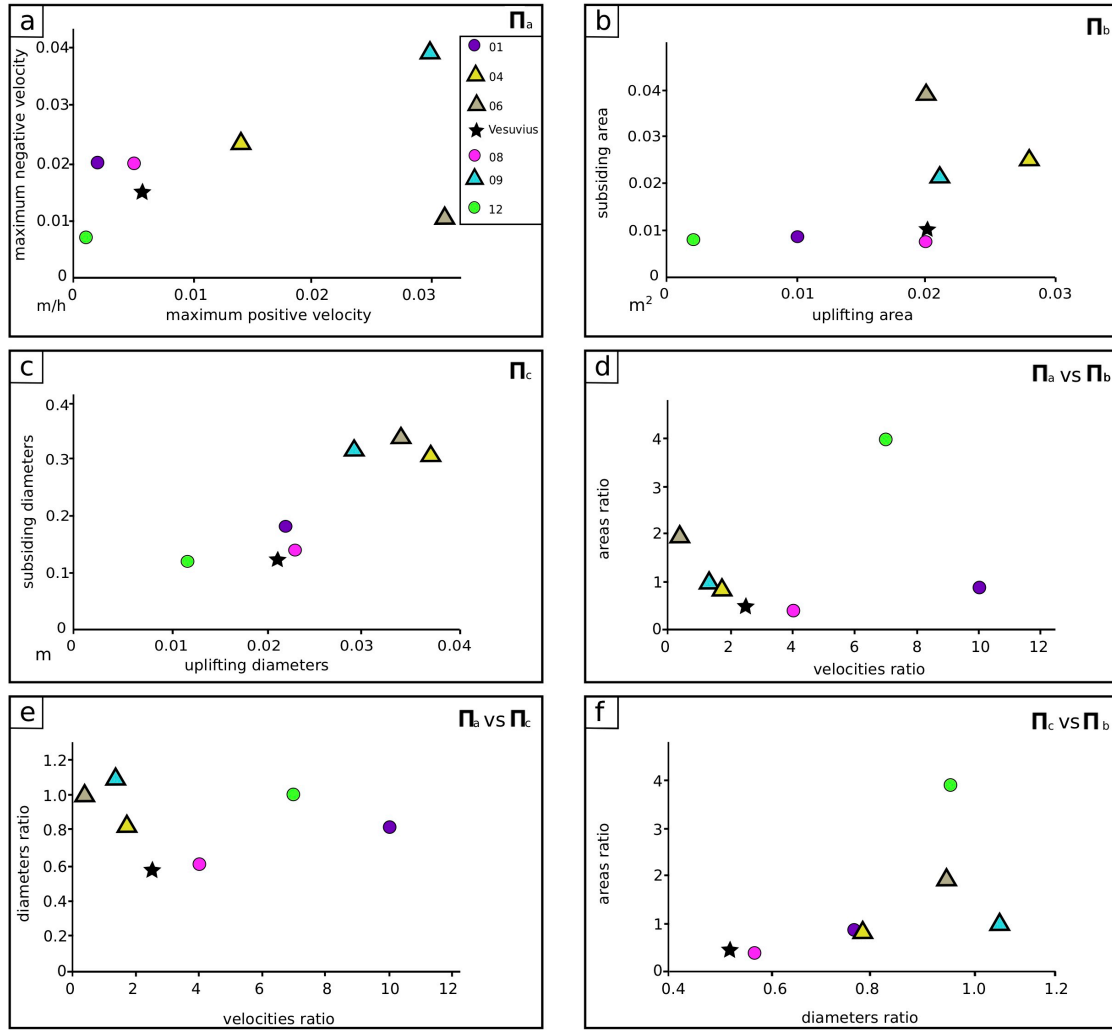
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 536 **Figure 5.** (a) 3D geometry and (b) mesh of the tetrahedral elements used for the FE  
 537 modeling related to, as an example, the analog model 08. (c) 3D geometry and (d)  
 538 mesh of the tetrahedral elements used for the finite element model related to the real Somma-  
 539 Vesuvius. (e) Sketch of the geometry used to represent the real Somma-Vesuvius in the  
 540 FE simulations, taken along with AA' (shown in panel d).



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**Figure 6.** Vertical deformations of the selected analog models. The black and yellow lines represent the coastline and the Somma caldera rim, respectively.

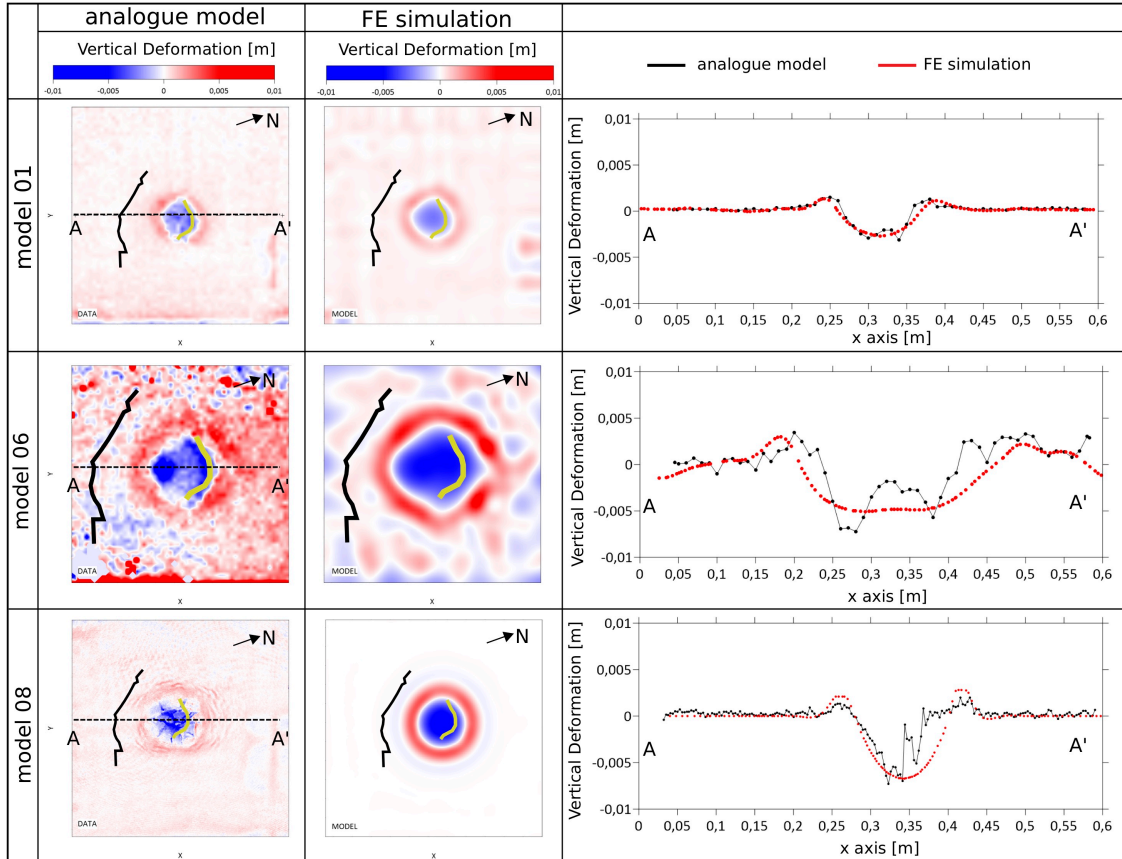




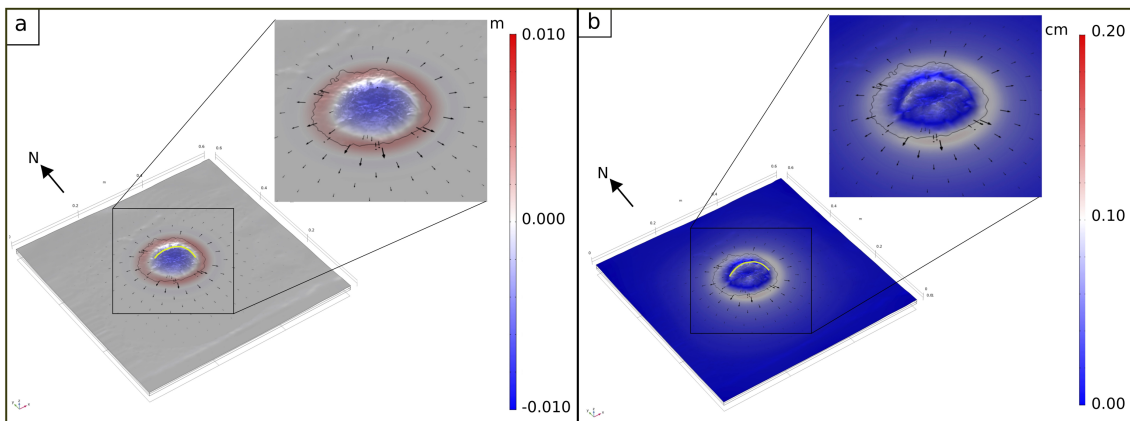
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**Figure 7.** Relationships between the main parameters of the analog models, FE models, and scaled monitoring data of real Somma-Vesuvius volcano (see the main text paragraph 2.1 and Table 2 for details). (a), (b), and (c) show the relationship between the parameters defining  $\Pi_a$ ,  $\Pi_b$ , and  $\Pi_c$ , respectively.

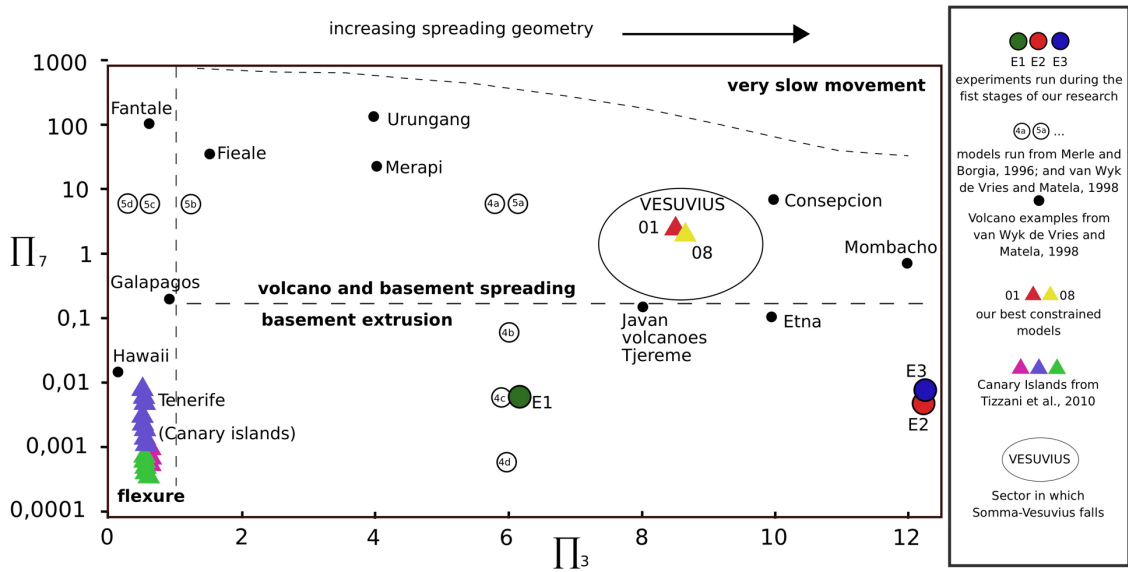
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551 **Figure 8.** Left column: retrieved vertical deformation of the selected analog models.  
 552 Central column: Equivalent FE simulations. The black and yellow lines represent the  
 553 coastline and Somma caldera rim, respectively. Right column: Profiles showing the  
 554 comparison between the results of the analog and FE models, taken along the dashed  
 555 line AA' (shown in left panels).  
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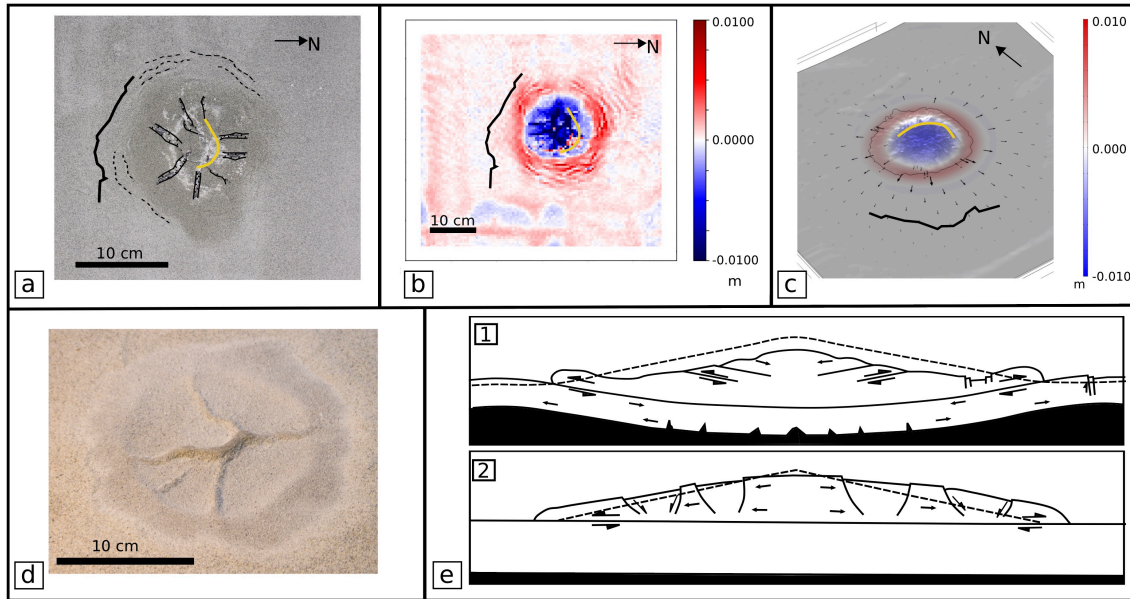


558 **Figure 9.** (a) Vertical (scale in  $10^{-3}$ ) and (b) horizontal (scale in  $10^{-4}$ ) deformation of  
 559 the FE model 08. The black arrows represent the horizontal movement direction; their  
 560 size is proportional to the movement amount.  
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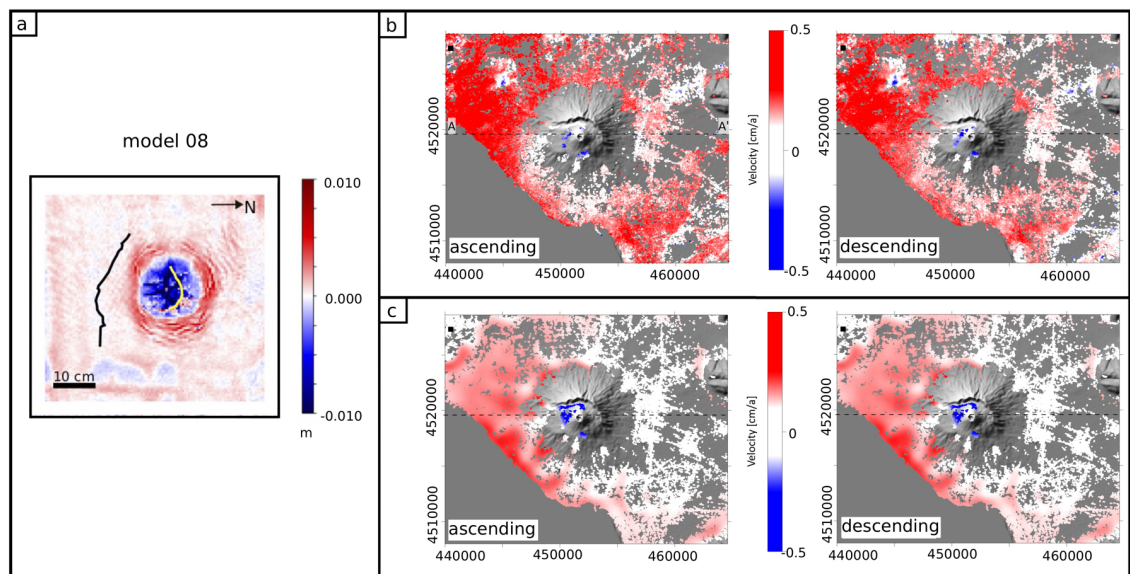


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**Figure 10.** Plot of  $\Pi_3$  against  $\Pi_7$  illustrating the fields occupied by flexure, spreading, and extrusion deformations. Somma-Vesuvius (Vesuvius) is represented by an ellipse reflecting the uncertainty of a few geometrical parameter values, such as the thickness of the ductile layer. The models above discussed and not plotted here fall in sectors of the diagram too far from the areas of interest (modified from Wyk de Vries and Matela, 1998).



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 585 **Figure 11.** (a) Top view of model 08 at the end of the experiment, peripheral flexural  
 586 bulge (dashed lines), radial faults, and graben are highlighted. The black and yellow  
 587 lines represent the coastline and Somma caldera rim, respectively. (b) Vertical  
 588 deformation maps of model 08 (dense points cloud in De Matteo & Massa, 2021). (c)  
 589 Vertical deformation of the FE model 08; the black and yellow lines represent the  
 590 coastline and Somma caldera rim, respectively, while the black arrows represent the  
 591 horizontal movement direction. (d) Top view of the ductile layer of model 08 at the end  
 592 of the experiment: the uprising of the ductile material along with the fracture opening at  
 593 the base of the brittle layer are evident. (e) Schematic block diagrams of the hybrid  
 594 sagging-spreading architecture (1) and endmember spreading architecture (2) (modified  
 595 from Byrne et al., 2013).  
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 599 **Figure 12.** (a) Vertical deformation of the analog model 08. (b) LOS projection  
 600 components (ascending and descending ones) of the mean deformation velocity  
 601 observed at Somma-Vesuvius from 1993 to 2010 by DInSAR-SBAS processing. (c)



602 LOS projection components of the mean deformation velocity modeled with the FE  
603 method at a natural scale (1:1).  
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