Enhancing the Performance of One-Way Electric Carsharing Systems Through the Optimum Deployment of Fast Chargers.

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Abstract

One-way electric carsharing systems (OWECS) provide environmentally friendly mobility that enable users to conclude their trips at a preselected station within a region. However, the operations of OWECS are complicated mainly due to: i) unbalanced demand which causes shortages or accumulations at stations ii) excessive battery charging requirements that can reach up to 8 hours. In these systems, carsharing companies hire personnel to relocate vehicles to restore the demand-supply balance, increase the number of trips served. Fast charging technologies can reduce the charging time drastically and can help carsharing companies to cope with the inefficiencies arising from excessive battery charging times. Therefore, fast charging technologies have the potential to enhance the operational performance of OWECS.

In this study, we propose an integer programming model to find the locations of fast chargers to be implemented in OWECS while considering relocation activities and battery availability of the vehicles. We propose a time-space-battery level network model which allows vehicles to operate with sufficient battery levels. As the number of stations increases, the number of relocation variables created increases polynomially which makes the model intractable for problem instances found in real world OWECS. Therefore, we are introducing three heuristics, two of which are based on the concept of relocation reduction while the third heuristic is based on station grouping. The heuristics that are categorized as relocation reduction type heuristics, take into consideration only most likely relocations. Generating only a fraction of all possible relocation arcs significantly reduces the number of relocation variables. The third heuristic reduces the number of variables by grouping the stations and solves the problem initially at an aggregate level using groups of stations instead of individual stations, while subsequently optimizes the location of chargers withing each group of stations.

We tested different approaches combining multiple heuristics and tested them on smaller instances with exact solutions to identify the approach that is both accurate and efficient. We applied the selected heuristic approach on real-life instances taken from an OWECS based in Nice, France. The results suggest that introducing the fast chargers to OWECS can improve profitability and increase number of served trips.

Keywords: One-way electric carsharing systems, Location modelling, Fast charger, Battery electric vehicle, Integer programming

1. Introduction

Many cities around the world are seeking to improve air quality through the introduction of "Low Emission Zones". Oxford is the first to declare “Zero Emission Zone” starting from 2020 (Jones, 2017; Consultants, 2020).In Oxfordshire the county council is cooperating, within the framework of a “Zero Emission Zone” scheme, with electric car-clubs (a.k.a carsharing companies), to provide mobility services to those that do not have access to a privately owned electric vehicle (Oxford City Council, 2019).

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Carsharing is a type of car rental systems which allows users to access the vehicles and pay depending on the duration and/or driving distance of the rental. Unlike classical car rental systems, a user first registers in the system, reserves a vehicle, and drives for the reserved period. As of 2014, carsharing sharing systems have 4.8 million registered users sharing over 100,000 vehicles worldwide (Shaheen and Cohen, 2016). One of the reasons for the rapid expansion of carsharing systems is their social and environmental benefits. Firstly, most of the cars used in these systems are new and small. In Germany, 70% of the cars of carsharing companies belong to this category (Loose, 2009). Newer and smaller cars result in lower fuel consumption and reduced emissions. Secondly, registered users of carsharing systems are less likely to buy a new car or more willing to sell a car they own. Studies show that 28,000 cars have been removed from the roads in 5 cities in the U.S. because of the memberships in carsharing systems (Martin and Shaheen, 2016). Carsharing is considered as an essential key and extension of the urban public transportation system providing the convenience of driving a car while not owning it.

Carsharing systems are divided into two categories according to the allowed drop-off stations. Round-trip carsharing systems are those that require the user to return the vehicle to the station that was picked-up. One-way carsharing systems allow users to drop off at a parking spot within a specified area. This could be at stations (station-based one-way) or anywhere in a region (free-floating). Although one-way systems are more attractive since they offer more flexibility, their operation is more complex due to the imbalance between demand and supply caused by the one-way operation. Due to their flexibility, one-way carsharing systems are more attractive for the users. Therefore more and more carsharing companies are offering one-way carsharing services to their potential users. In North America, the number of vehicles used in one-way carsharing systems is reached 6,850 with 511,000 registered members as of 2015 (Martin and Shaheen, 2016).

Carsharing with electric vehicles (EV) provides more sustainable, environmentally-friendly mobility as compared to internal combustion engine vehicles. A study of Clewlow (2016) highlights that carsharing members possess fewer cars on average and when they do, the ratio of having an alternative car with low carbon footprint is higher than non-members, which shows that members of carsharing systems are highly considerate of the environment. In recent years, carsharing companies are adding more EVs to their fleet in order to lower the maintenance cost, produce less carbon dioxide, attract more customers, and prevent deterioration of air quality in urban areas. However, one of the drawbacks of EVs is the time needed to charge their battery. The time required to fully charge an EV using conventional chargers can exceed 8 hours. The excessive battery charging time reduces vehicle availability leading to lower vehicle utilization and revenues. Recent advancements in the technology of EV chargers indicate that via rapid/fast or superchargers the required time for charging a vehicle can be decreased to 30 minutes. As of May 2020, Tesla, one of the leading EV companies, provides 16,585 superchargers in the US that can charge a vehicle within 30 minutes up to 80% battery charge and 170-mile drive range (Tesla, 2020). Nissan Leaf can be charged from 20% to 80% battery level within 60 mins via ChaDeMo rapid chargers (Nissan, 2020).

There are three key elements for charging a vehicle; a connector installed at the vehicle, a cable (could be portable or tethered to the charger) and a charger. However, until the introduction of the EU Directive 2014/94/EU which introduced the Type-2 and Combo-2 plugs (as shown in Figure 1), there was no standardization (EU, 2014).

Standard Type2 connections provide alternative current (AC) which is convenient for home-charging. As the battery of an EV requires direct current (DC), Type 2 converts AC to DC which results in slow charging. For this reason, the car manufacturers in Europe are combining AC and DC chargers through the introduction of combined charging system (CCS Combo-2) (as shown in Figure 1). When in need of a DC plug for rapid chargers, the driver can open the hidden part and use as a Combo-2 plug.

One of the concerns of the DC chargers may be their effect on the battery life of the electric vehicles. Although Kia states that minimum use of DC chargers is recommended (Kia, 2019), a study conducted in Idaho National Laboratory suggests that the difference in capacity loss between using only AC and only DC chargers is insignificant compared to overall capacity loss (Shirk and Wishart, 2015). In this study; 4 identical Nissan Leaf EVs were observed for battery degradation. 2 of the EVs are charged only with AC chargers, whereas the remaining EVs are charged with DC chargers. The study indicates that after 50,000 miles of use, the battery degradation is at 22% and 25% of AC and DC charged vehicles respectively. The study also states that this may be an upper bound for capacity loss as the experiments took place in a very hot city, and vehicles were charged twice as much as it is recommended by the manufacturers. In the light of the results of this study, the mileage for battery replacement may not be affected significantly by charger type for businesses renting vehicles.

Carsharing companies with EVs are following the recent developments in rapid charging technology closely. DriveNow brought fast chargers into service for the users in Denmark (DriveNow, 2019), Autobleue, a carsharing company in Nice, France offers three rapid chargers along with regular chargers in the city. In the near

Figure 1: Type2 and CCS Charge Connectors

future, as the number of carsharing users and demand will increase, the need for rapid chargers will also grow.

The price of fast chargers is considerably higher than the price of conventional chargers and ranges between 22,000€ and 32,000€ (Çalık and Fortz, 2019). However, fast chargers can contribute significantly to the increase of vehicle utilization and the level of service offered to customers. Given the cost and operational characteristics of fast charging technologies, carsharing companies using EV's are facing decisions regarding the number and the location of fast/superchargers they should introduce in their operations.

The fast charger infrastructure investment planning problem has been studied to determine the optimal locations of different types of chargers in order to cover the charging requirements of EV users in regions (Zhang et al., 2017) and along highways (He et al., 2019). The fast charger infrastructure investment problem has been investigated from the standpoint of the charger suppliers (Guo et al., 2016).

Although the optimization of the operation of OWECS has been studied (see Boyacı et al., 2017; Gambella et al., 2018), the effect of the use of different charging technologies on the operational performance of OWECS has not been adequately addressed. Sonneberg et al. (2015) and Jiao et al. (2017) investigate the fast charger infrastructure planning problem in round-trip and OWECS respectively. However, these studies are not comprehensive enough to capture the system complications arising from the vehicle relocation and battery charging requirements of OWECSs (see Section 2 for a detailed analysis). To close this literature gap, we present a new integer programming model that optimizes the location of fast chargers that will be replacing initially installed conventional chargers. This generic charger upgrading decision model can be applied to any OWECS with a fleet of both AC and DC charger compatible vehicles.

The remainder of the paper is structured as follows: In Section 2, we provide a summary of related literature. In Section 3, a mixed-integer programming model for the charger upgrading decision is presented. Due to the computational complexity of the model, we present heuristic approaches in Section 4. Section 5 follows with the computational experiments and comparisons on both the exact model and the heuristic approaches. Finally, in Section 6, concluding remarks and future research directions are given.

2. Literature

The complexity of the operations of carsharing systems has attracted the attention of the research community over the past years. Remane et al. (2016) and Ferrero et al. (2018) provide literature reviews and taxonomies regarding typologies and use of carsharing systems. The readers are referred to Brandstatter et al. (2016) for a mathematical modeling-oriented literature review of electric vehicle carsharing systems, to Jorge and Correia (2013) for a detailed survey on carsharing demand models and one-way carsharing operations modelling, and to Laporte et al. (2018) for a survey on mathematical aspects of shared mobility systems including bike-sharing operations.

Decisions regarding the design and management of carsharing systems are categorized in three levels: strategic, tactical and operational (Boyaci et al., 2015). Strategic decisions concern with station location and capacity decisions. Tactical decisions relate to fleet sizing, while operational decisions focus on vehicle relocation, personnel scheduling, and battery charging. In this paper, we are addressing strategic decisions regarding the development of the vehicle charging infrastructure of one-way carsharing systems. In addressing this strategic decision we take into account vehicle relocation decisions, therefore, our literature review presents a brief overview of studies focused on operational OWECS decisions.

Boyaci et al. (2015) proposed a multi-objective integer-programming model that optimizes the number and location of carsharing stations, the fleet size and relocation for OWECS systems. The proposed optimization framework was applied using data from an electric carsharing company in Nice, France. Brandstatter et al. (2017) formulated the station location selection and fleet size determination in one-way carsharing systems as a two-stage stochastic integer program. The first stage determines the station locations and fleet size, while in the second stage determines the trips to be served. The model assumes that each trip is served with a full battery, and does not consider vehicle relocation operations. A heuristic approach is also presented for solving large instances. The model and the heuristic algorithm are tested for a real-life instance in Vienna for taxi trip requests. The customers are assumed to walk to origin and destination stations within 5 minutes distances. The model was solved for 201 candidate stations, and 1,060 trip requests.

Biesinger et al. (2017) treated the same location selection and fleet determination problem with a similar two-stage approach which use heuristic solution approaches in both stages. In the first stage a variable neighbourhood search heuristic is used to determine the station locations and the fleet size. In the second stage, greedy heuristic algorithms are used to determine the trip requests that should be served. The proposed model considers the battery availability in assigning trip requests to vehicles. The study was tested on data from Vienna, Austria.

Çalık and Fortz (2019) introduced a mixed-integer model and proposed a Benders decomposition algorithm to optimally locate carsharing stations. Fast chargers are assumed to be installed in all stations. However, the proposed model does not consider relocations in a one-way system and does not allow partial charging after trips, both assumptions may result to lower vehicle utilization and number of trips to be served. The proposed algorithms were tested on real data from Manhattan taxi trips.

Bruglieri et al. (2014) proposed a mixed-integer linear programming model which aims to maximize the number of served trips. In this study, linear battery consumption is assumed, vehicle relocation and assignment of vehicles to trips are allowed as long as the vehicle has sufficient battery. The proposed model was tested with 30 EVs in Milan.

Boyacı et al. (2017) considered the vehicle and personnel relocation problem as a hierarchical multi-objective mixed-integer programming model. The proposed solution framework is comprised of a clustering algorithm that groups stations to reduce computational complexity, two mixed integer mathematical models for vehicle and personnel movements, and a simulation model for the charging-level feasibility control mechanism. Boyacı and Zografos (2019) proposed a framework that investigates the impacts of temporal and spatial flexibility of trip requests.

Gambella et al. (2018) proposed an integer programming model to find the route assignments of both vehicles and personnel while maximizing profit over all routes. Furthermore, a model was proposed to optimize relocation operations during the non-operating hours in order to start the next operational period with the appropriate number of vehicles. The relocation model is computationally intractable, hence a heuristic algorithm was proposed. The proposed model and solution algorithms were tested on a data set with 2 personnel, 14 stations, 20 EV and 40 time intervals.

Zhang et al. (2019) formulated vehicle assignment with relays (completing the journey by passing through an additional station and switching vehicle because of battery constraints) as a time-space–battery network flow model. Heuristic approaches based on LP relaxation were used to solve large instances. The proposed model was tested on a real-world system operating in Shanghai.

The literature of station location selection models mostly focuses on using one type of chargers. To the best of our knowledge, the only studies considered "Fast/rapid or superchargers" along with conventional chargers are Sonneberg et al. (2015) and Jiao et al. (2017). The former study is considering fast charging units for a round-trip carsharing system. The authors proposed a mathematical model to find station locations and the number of fast and regular charging units each station will have. The model assumes that each vehicle serves demand once per day. The model is integrated into a decision support system and applied to San Francisco. The latter study proposed a MIP model for a one-way electric carsharing system for locating the depots (i.e. stations) and selecting charger types. This study discretized time using independent time-intervals, hence the interaction between time intervals is not taken into account. The model was applied to a carsharing company in Beijing. Both of these studies consider the need of fast/rapid chargers based on the infrastructure cost. However, these studies cannot support decisions regarding vehicle utilization since they do not consider multiple vehicle trips and interactions between the discrete time intervals that the vehicles operate.

Table 1: List of referenced articles

Article	Decision Type	EV	Relocation operations	Allowing Partial Charging	Multiple Charger Types
Boyaci et al. (2015)	Strategic, Tactical, Operational	✓	✓	×	×
Brandstatter et al. (2017)	Strategic, Tactical	✓	×	×	×
Biesinger et al. (2017)	Strategic, Tactical	✓	×	✓	×
Çalık and Fortz (2019)	Strategic	✓	×	×	×
Bruglieri et al. (2014)	Operational	✓	✓	✓	×
Boyacı et al. (2017)	Operational	✓	✓	×	×
Gambella et al. (2018)	Operational	✓	✓	✓	×
Zhang et al. (2019)	Operational	✓	✓	✓	×
Boyacı and Zografos (2019)	Operational	✓	✓	✓	×
Jiao et al. (2017)	Strategic	✓	×	×	✓
Sonneberg et al. (2015)	Strategic	✓	×	×	✓
Proposed Study	Strategic	✓	✓	✓	✓

Table 1 summarises the existing literature on one-way electric carsharing systems. The literature review suggests that there is a gap regarding the investigation of the effect of the introduction of fast charging technologies on the profitability and performance of OWECS. Specifically, the literature review revealed that issues regarding the optimization of the number and location of fast chargers in order to serve the users of OWECS have not been adequately addressed. Although, charger acquisition is a strategic decision, this decision should be made via analysing both revenues generated from served trips and the operational costs (i.e. relocation of vehicles and personnel cost) over a period of time. This paper aims to close this literature gap by introducing models and algorithms that provide exact and near optimal solutions for making charger investment decisions considering both battery infeasibility and vehicle relocation operations. This study contributes to the current state of knowledge by introducing:

- A new ILP model that finds the optimal location of fast/rapid chargers to replace the slow/conventional chargers that are already installed. This model addresses the demand uncertainty by considering multiple scenarios with probabilities. In order to have a realistic representation of the system, the proposed model keeps track of the battery levels of the EVs and allows partial charging. We ensure that no battery infeasibility occurs while serving more trips through the consideration of partially charging, the introduction of fast/rapid chargers, and vehicle relocations.
- Heuristic algorithms to generate near-optimal solutions without compromising the quality of the ILP model. The first type of heuristic used decreases the size of the model by only considering relocations when possibly needed, whereas the second heuristic treats the problem in a three-stage approach. In the first stage, the stations are grouped using an integer programming model aiming to minimize the maximum distance within each group. Based on this grouping, the trip requests are considered between groups of stations and the ILP model is solved by considering these groups at the second stage. Finally, a new ILP model is generated to find the location of fast chargers within each group at the third stage.

3. Mathematical Model

In this paper, we consider a reservation based system where the users request trips in advance stating the origin and destination stations of their trips. The users are notified, at the end of the reservation period, whether or not their request has been accepted. Each day starts with a predetermined number of fully charged vehicles at the stations. The stations are initially equipped with conventional chargers implemented at designated parking spots. All EVs in the fleet are identical and compatible with both AC and DC chargers. After each trip/relocation, the vehicles are connected to an available charger. Please note that, if it is decided to upgrade at least one of the chargers at a station, the vehicle entering the station is notified about where to be charged. No charger or parking spot switching is allowed at the stations, i.e., if a vehicle enters a station and is connected to a charger, it occupies the allocated parking spot and the charger until it leaves the station. The battery is charged at a constant rate based on the type of the charger. The battery consumption is considered linear during the vehicle rental (trip) and its relocation. Vehicles are relocated by available personnel, and the relocation time for any origin-destination pair is fixed and not dependent on the time of the day.

The operational hours of the system have been divided into time intervals of equal length, allowing to work on discretized time space. When the system is not operating, the system is reset, i.e. vehicles are distributed to the stations and fully charged until next operational hour. The battery capacity is also divided into K equal levels. Any battery level in $\left[\frac{k-1}{K}, \frac{k}{K}\right)$ corresponds to discrete level of $k-1$ and only 100% battery charging level corresponds to level K.

Owing to its strategic nature, the proposed model does not optimize personnel relocation decisions. Personnel movement is simplified by considering only vehicle relocation movement and personnel cost is added by considering the duration of vehicle relocations.

In what follows we provide the mathematical model for location selection for fast/rapid chargers.

3.1. Sets and indices

$s \in S$	scenarios
$i \in I_s$	trip requests in scenario s
$j, l \in J$	nodes (stations)
$t, t' \in T$	time intervals
$k, k' \leq K$	battery levels
$p \in P$	charger types, $p = 0$ denotes regular charger

3.2. Parameters

Origin(i)	origin station of trip i
Dest(i)	destination station of trip i
Start(i)	start time interval of trip i
End(i)	end time interval of trip i
TR_i	revenue of trip i
P_s	occurrence probability of scenario s
DC_{jp}	depreciated cost of upgrading a regular charger to charger type p at station j per day
RC_{jl}^t	relocation cost of a vehicle from station j to l starting at time interval t
PC	personnel cost per time interval
D_{jl}	number of time intervals it takes to relocate a vehicle from node j to l
$rate_p$	number of battery levels charger type p can charge per time interval
CT_{jl}	battery consumption of relocation from station j to l
CT_i	battery consumption of trip i
IC_{jp}	current number of charger type p installed at station j
AR	average number of time intervals to relocate the vehicle, then walk to the origin of the next relocation
MaxPer	maximum number of personnel that can work at any time interval.

3.3. Variables

f_{jp}	number of charger type p decided to be installed at station j
z_{ik}^s	1 if trip i of scenario s is served by a vehicle with a battery level k, 0 otherwise
r_{jlk}^{st}	number of vehicles with battery level k relocated from station j to l starting at time interval t in scenario s
b_{jkp}^{st}	number of vehicles with battery level k start charging at charger type p at station j starting at time interval t in scenario s.
$\overline{b}_{jkp}^{st}$	number of vehicles with battery level k that left charger type p at station j at time interval t in scenario s
d_{jkp}^{st}	number of vehicles with battery level k that are being charged by a charger type p at station j during time interval t in scenario s
c_{jp}^{st}	number of charger type p at station j occupied by vehicles at time interval t in scenario s

3.4. Formulation

$$\max \overbrace{\sum_{s,k,i\in \mathrm{I}_s} \mathrm{P}_s \mathrm{TR}_i\, z^s_{ik}}^{\text{expected revenue}} - \overbrace{\underbrace{\sum_{i,p\neq 0} \mathrm{DC}_{jp}\, f_{jp}}_{\text{infrastructure upgrade cost}} - \underbrace{\sum_{j,l,t,k,s} \mathrm{P}_s \mathrm{RC}^t_{jl}\, r^{st}_{jlk}}_{\text{relocation cost}} - \underbrace{\sum_{j,l,t,k,s} \mathrm{P}_s \mathrm{D}_{jl}\, \mathrm{PC}\, r^{st}_{jlk}}_{\text{personnel cost}}}^{\text{expected cost}} \tag{1}$$

subject to

$$\sum_{\substack{(k',i\in \mathrm{I}_s):\\ k'=\mathrm{CT}_i+k\\ \mathrm{Dest}(i)=j\\ \mathrm{End}(i)=t}} z^s_{ik'} + \sum_{\substack{(l,k'):\\ k'=\mathrm{CT}_{lj}+k}} r^{s(t-\mathrm{D}_{lj})}_{ljk'} = \sum_p b^{st}_{jkp} \qquad \forall k,j,t,s \tag{2}$$

$$\sum_{\substack{(k,i\in \mathrm{I}_s):\\ \mathrm{Origin}(i)=j\\ \mathrm{Start}(i)=t\\ \mathrm{CT}_i\le k}} z^s_{ik} + \sum_{\substack{(j,k):\\ \mathrm{CT}_i\le k}} r^{st}_{jlk} = \sum_p \overline{b}^{st}_{jkp} \qquad \forall k,j,t,s \tag{3}$$

$$d^{s(t-1)}_{j(k-\mathrm{rate}_p)p} + b^{st}_{jkp} - \overline{b}^{st}_{jkp} = d^{st}_{jkp} \qquad \begin{array}{c}\forall j,t,s,\\ \mathrm{rate}_p\le k<\mathrm{K}\end{array} \tag{4}$$

$$b^{st}_{jkp} - \overline{b}^{st}_{jkp} = d^{st}_{jkp} \qquad \begin{array}{c}\forall j,t,s,p\\ 0\le k<\mathrm{rate}_p\end{array} \tag{5}$$

$$\sum_{k\ge \mathrm{K}-\mathrm{rate}_p} d^{s(t-1)}_{jkp} - \overline{b}^{st}_{j\mathrm{K}p} = d^{st}_{j\mathrm{K}p} \qquad \forall j,t,s,p \tag{6}$$

$$\sum_k d^{st}_{jkp} = c^{st}_{jp} \qquad \forall j,t,s,p \tag{7}$$

$$\mathrm{IC}_{j0} - \sum_{p\neq 0} f_{jp} \ge c^{st}_{j0} \qquad \forall j,t,s \tag{8}$$

$$\mathrm{IC}_{jp} + f_{jp} \ge c^{st}_{jp} \qquad \forall j,t,s,p\neq 0 \tag{9}$$

$$\sum_k z^s_{ik} \le 1 \qquad \forall i,s \tag{10}$$

$$\sum_{t\le t'\le t+\mathrm{AR}} \sum_{j,l,k} r^{st'}_{jlk} \le \mathrm{MaxPer} \qquad \forall t,s \tag{11}$$

$$z^s_{ik} \in \{0,1\} \qquad \forall i,k,s \tag{12}$$

$$r^{st}_{jlk}, b^{st}_{jkp}, \overline{b}^{st}_{jkp}, d^{st}_{jkp}, c^{st}_{jp}, f_{jp} \in \mathbb{N} \qquad \forall j,l,k,p,s,t \tag{13}$$

Objective function (1) maximizes expected daily profit considering all demand scenarios. It is equal to the expected revenue generated by the served trips minus the infrastructure upgrade cost, expected total salaries of the personnel and vehicle relocation cost. Since the fast charger implementation decision is a strategic decision, it is not scenario dependent. Expected revenue, vehicle relocation and personnel costs are scenario dependent and therefore a coefficient expressing the scenario probability is used. Scenario revenue is calculated as the sum of the revenues of all trips corresponding to the scenario under consideration. Relocation cost represents the expected driving cost associated with the relocation activities. The parameter RC^t_{jl} in the relocation cost component, can be calculated by the energy consumption cost of driving the route connecting station j to station l at time interval t. Similarly, relocation personnel cost per scenario can be calculated by multiplying personnel cost per time interval by the total number of time intervals required to relocate all vehicles of that scenario. We do not consider maintenance and parking related cost as done in Boyaci et al. (2015), since the number of vehicles and parking areas are considered fixed. The additional cost associated with the reduction of battery and vehicle life due to high utilization of vehicles is not taken into consideration.

Constraints (2) and (3) require that the total number of vehicles arriving to and departing from station j with charging level k is equal to the number of vehicles arriving to and departing from station j for relocations or trips with charging level j at time interval t in scenario s respectively.

Constraints (4), (5), and (6) ensure that in scenario s at charger type p of station j, the number of vehicles being charged at time interval t is equal to the number of vehicles being charged at time interval $t-1$ plus the number of vehicles arrived at time interval t minus the number of vehicles departed from station j at time interval t. Note that these constraints are written separately for charging levels $\mathrm{rate}_p \le k < \mathrm{K}$, $0 \le k < \mathrm{rate}_p$ and

K. Constraints (5) are needed to define the number of vehicles being charged at charger type p when the initial battery level is less than what could be charged at one time interval (rate_p). This indicates that there would be no vehicle flow coming from the previous time interval to charger type p. Constraints (6) are used to keep track of the fully charged vehicles. Since there is an upper limit on the battery capacity (battery levels of K), the number of vehicles with battery level $k \geq \text{K} - \text{rate}_p$ at time interval $t-1$ will be fully charged and be at the charger type p at the station unless they do not serve a trip or relocation at time t.

Constraints (7)-(9) are to ensure enough number of chargers of each type are present at each station. Constraints (7) keep track of the occupied capacity of each charger type at each station. Constraints (8) ensure that the total number of purchased chargers for station j cannot exceed the initial regular (slow) charger capacity of station j. We assume that the operator is allowed to replace only regular/slow chargers with non-regular chargers. Constraints (9) ensure that the number of vehicles that are being charged or parked at a charger type p at station j will not exceed the number of charger type p at station j.

Constraints (10) ensure that every demand request is served not more than once. Lastly, Constraints (11) restrict the number of personnel and the total number of relocations performed within consecutive AR time intervals. Here, AR is the average relocation time in the system plus the average movement time between one relocation's destination and the other's origin station. The rationale behind creating Constraints (11) is to prevent generating solutions that have relocations accumulated at certain time intervals. For instance, consider a solution with (w) number of relocations where $w = \sum_{j,l,k} \text{R}_{jlk}^{s^* t^*}$ for a time interval t^* at scenario s^*. This means that the company needs to hire at least w number of personnel to relocate the vehicles at time t^*. This might not be practically feasible when w is high. In order to prevent such cases to occur, we limit the model to have at most MaxPer number of relocations within AR consecutive time intervals with Constraints (11).

4. Heuristic Approaches

The model provided in Section 3 may not be computationally tractable due to the large number of variables and constraints. In this section, we provide two heuristic approaches, namely selective relocation and grouping approaches, that reduce the size of the problem.

4.1. Heuristic 1: Selective Relocation

Number of relocation variables created in mathematical model (1)-(13) is in the order of $|\text{J}|^2|\text{T}||\text{K}||\text{S}|$. With a sufficient battery level, a relocation may happen between any origin-destination pair at any time. Given 64 time intervals and 32 battery levels in a 60 station system like the system in Nice, France, the total number of relocation variables created is equal to 7,063,414 even for a single scenario. Yet, the total number of realized relocations accounts for the % 0.017 of the created relocation variables for small-sized instances analysed in Section 5.

As most of the created relocation variables take value of zero, we decrease the number of relocation variables. Reducing the density of the relocation variables (arcs) has been proposed in studies (Carlier et al., 2015) and (Gambella et al., 2018). The former study employs the relocation arcs at specific time steps whereas the latter considers inclusion of relocation arcs gradually. Both of these reduction techniques generate good quality, near-optimal solutions.

In this study, we propose two heuristics by creating relocations akin to trip requests. There are two main reasons to perform relocations at a system; namely, to ensure the availability of vehicles and/or parking spaces. In the first heuristic, relocation arcs are created to serve at least one of the reasons. To serve the first reason, relocation arcs are created such that the arrival time of relocation to its destination (also the origin of the trip) must be prior to the starting time of the trip. The time between the end of the relocation and start of the trip is limited to a predetermined time window. The same predetermined time window is applied to the relocation arcs which provides a parking spot for an end of a trip. By this method, the created number of relocation arcs is in the order of $|\text{I}_s||\text{J}||\text{S}||\text{K}|$.

To illustrate this heuristic method, consider a simple OWECS having four stations with a capacity of one charger and one parking spot at each station as shown in Figure 2A. Trip requests and relocation arcs are demonstrated with green and red arrows respectively. A trip is requested from Station 1 to Station 2 at time interval 4 with a duration of five time intervals. In order to provide a vehicle to Station 1, possible relocation arcs are created from Station 2, 3 and 4. Similarly, in order to provide a parking spot at Station 2, a relocation arc from Station 2 to Station 1, 3 and 4 at time interval 9 are created.

In the second relocation selection approach, three types the relocation arcs are inserted to the network. The first type of arcs are the ones when relocation may be required both at the origin and destination stations.

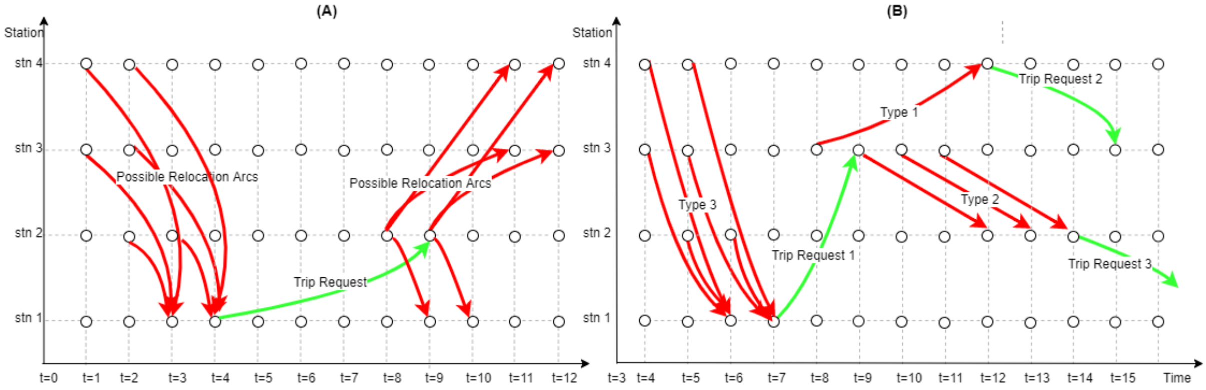

Figure 2: Examples of Selective Relocation 1 (A) and Selective Relocation 2 (B)

Consider again the same system with three trip requests as shown in Figure 2B. A relocation arc starting at Station 3 at time 8 and ending at Station 4 at time 12 is created to provide a parking spot for arrival of a trip and provide a vehicle to a start of another trip. The start time of the relocation ($t = 8$) is earlier than the end time of trip1 ($t = 9$) and it is limited by a time window. It should be stressed that relocation arc is added if the end time of the relocation is prior to the start of Trip2 and it falls into the time window. The second type of relocation arcs start after a trip has been concluded, and it provides a vehicle to another trip as shown "Type 2" in Figure 2B. The last type of relocation arcs (Type 3) are added to serve trips that start at the beginning of the day.

We observed 2,778,912 and 851,800 number of relocation variables on the average for relocation selection 1 and 2 respectively for 10 different instances with the system of 60 stations and 600 demand requests. (The time window was taken as 2 in both approaches). The reduction rate of relocation variables were 60.7% in the first and 87.9% in the second approach.

4.2. Heuristic 2: Station Grouping - Three Step Approach

Even though the relocation variables accounts for the highest proportion of variables, reducing their number may not be suffice to solve large scale problems. Variables $b^{st}_{jkp}, \overline{b}^{st}_{jkp}, d^{st}_{jkp}$ are in the order of |J||K||S||T||P| and having this many variables could prevent us to have a tractable model that is applicable to real-life problems. To decrease the number of variables even further, we propose a second type of heuristic approach based on grouping the stations. This method reduces |J| hence reduce the sizes of all of the variable set.

This approach has three steps: (i) station grouping ((14)-(21)), (ii) solving the mathematical model ((1)-(13)) using the identified groups, and (iii) solving the within-group models ((22)-(30)) (see Figure 3). We start with solving the station grouping model provided in Section 4.2.1 by employing walking distance matrix. Each station group is formed in such a way that any pair of stations within the group is accessible by the relocation personnel through walking from the stations in the group. The main reason for aggregating the stations is to run the model provided in Section 3 for the groups considering them as "large stations". The Station Grouping model aims to minimize the maximum walking distance within the groups and is solved for different |G| values where G denotes the group set ($1 \leq |G| \leq |J|$). After observing the objective function values for different |G| values, the final |G| value is determined by selecting the number of groups corresponding to the elbow of the curve (point of diminishing returns) describing the relationship between the objective function value and the number of the corresponding groups. It should be noted that if |G| = |J|, this heuristic method becomes the exact model we have proposed in Section 3. On the other hand, the fewer the groups are, the higher the deviation from the optimal solution is. It is a trade-off between accuracy and efficiency.

After determining the number of groups and stations at each group, the exact model is solved for the groups decided in the first step. The groups are treated as large stations with the total capacities of the stations in them. For the second step, Model 3.4 is solved for the variables $r^{st}_{guk}, b^{st}_{gkp}, \overline{b}^{st}_{gkp}, d^{st}_{gkp}, c^{st}_{gkp}, f_{gp}$ while g and u denote groups instead of stations. In this implementation of the model, the only variable that takes stations into account is z^{s}_{ik}. The distance parameter D_{gu} is considered as the maximum distance of any stations of group g to group u.

The solution of the model with groups (Step 2) determines the number of fast chargers to be installed at each group,and the number of relocations between groups and the accepted trips. After solving Step 2, a further optimization is required to find where to locate the chargers within groups.

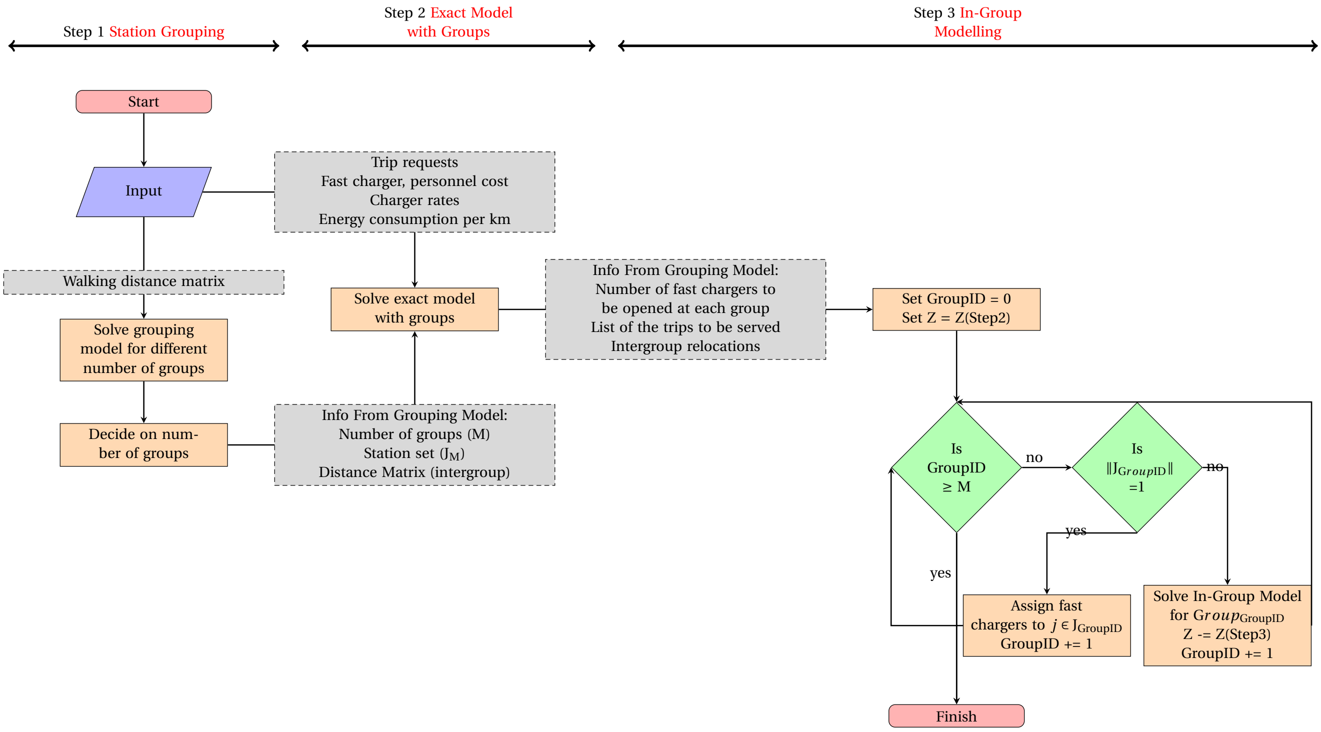

Figure 3: Flowchart of Grouping Heuristic Method

Figure 4 demonstrates an instance of 6-station system where the Step 2 Model is applied. In this example, the first group is formed by combining Station 1, 2, and 3. The second group is comprised of Station 4 and 5, while the last group includes only Station 6. As illustrated in Figure 4 , after solving the Inter-Group Model (Step 2), the accepted trips are assigned from one station to another. However, the relocations are considered to be performed between groups. The fast charger decision is also made for groups. In Figure 4, two fast chargers decided to be implemented to Group 1, and the corresponding stations for relocation activities along with the within relocations for Group 1 must be decided. This is why, a new Within-Group model (Step 3) is created (see Section 4.2.2). The outputs of the main model (Step 2) are used as parameters at the Within-Group Model. This model must be iteratively solved for each group, unless the number of stations within the group is one as shown in flowchart Figure 3. Clearly, if the number of stations in a group is 1, then all the fast charger decisions of the group must be implemented at this station, the relocations involving the group must be operated to/from that station, and there will be no within-group relocation. The aim of the Within-Group Model is to minimize the within-group relocation cost while serving all the accepted trips, and relocations from the main model and implementing the fast chargers.

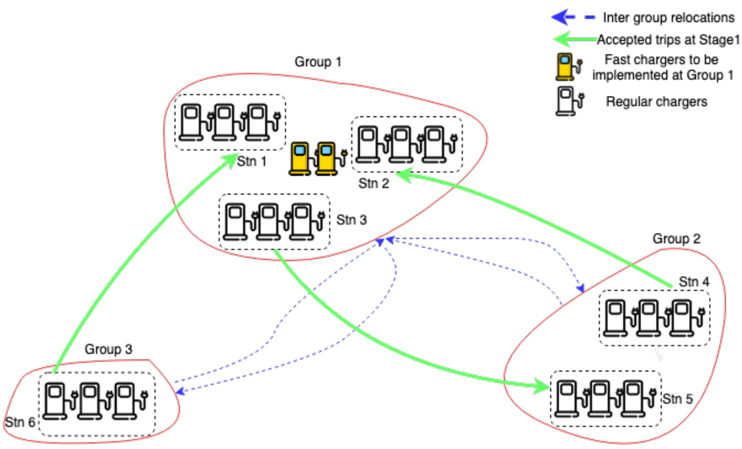

Figure 4: Representation of groups at time interval t, scenario s after solving the exact model with groups

4.2.1. *Step 1: Station Grouping Model*

This model aims to minimize the maximum distance within station groups. It allocates the stations to a given number of |G| groups and determines the maximum distance between any stations within any group.

4.2.1.1. Sets and indices.

$j, l \in \mathrm{J}$ nodes (stations)
$g \in \mathrm{G}$ groups

4.2.1.2. Parameters.

D_{jl} walking distance between station j and l

4.2.1.3. Variables.

a_{jg} binary variable, takes 1 if station j is assigned to group g
b_{jlg} binary variable, takes 1 if both stations j and l are assigned to group g
v maximum distance within groups.

4.2.1.4. Grouping Mathematical Model.

$$\min \quad v \tag{14}$$

subject to

$$\sum a_{jg} = 1 \qquad \forall j \tag{15}$$

$$b_{jlg} \leq a_{jg} \qquad \forall j, l, g \tag{16}$$

$$b_{jlg} \leq a_{lg} \qquad \forall j, l, g \tag{17}$$

$$a_{jg} + a_{lg} - 1 \leq b_{jlg} \qquad \forall j, l, g \tag{18}$$

$$\sum_g \mathrm{D}_{jl} b_{jlg} \leq v \qquad \forall j, l \tag{19}$$

$$\sum_g \mathrm{D}_{lj} b_{jlg} \leq v \qquad \forall j, l \tag{20}$$

$$v \in \mathrm{Q}^+, a_{jl}, b_{jlg} \in \{0, 1\} \qquad \forall j, l, g \tag{21}$$

A similar clustering model has been proposed by (Rao, 1971). Constraints (15) ensure that each station is assigned to a cluster (group). Constraints (16), (17) and (18) are the linearization of $b_{jlg} = a_{jg}a_{lg}$. Constraints (19) and (20) define the maximum distance within groups.

4.2.2. *Step 3: Within-Group Model*

After determining number of station groups (Step 1) and solving the exact model for groups (Step 2), the model below is solved for all of the groups seperately. For a group g, Within-Group model finds the locations of the fast chargers inside g if it is decided at Step 2 that g has fast charger(s) to be located. Furthermore, this model also determines the corresponding stations inside g for the relocations that are coming in or going out of g while minimizing the relocation costs within g. The summation of objective function values (in group relocation costs) of all of the groups is subtracted from the solution found in Step 2, resulting in the total revenue of the heuristic method.

4.2.2.1. Sets and indices.

$s \in \mathrm{S}$ scenarios
$i \in \mathrm{I}_{\mathrm{A}s}$ accepted trip request in scenario s at Stage 2
$j, l \in \mathrm{J}_g$ nodes(stations) in group g
$t \in \mathrm{T}$ time intervals
$k, k' \in \mathrm{K}$ battery levels
$p \in \mathrm{P}$ charger types, $p = 0$ denotes regular chargers

4.2.2.2. Additional Parameters.

- R_k^{st} decided number of vehicles arriving to group g by relocation at time interval t in scenario s with battery level k decided at Stage 2
- $\overline{\mathrm{R}}_k^{st}$ number of vehicles arriving to group g by relocation at time interval t in scenario s with battery level k decided at Stage 2
- Z_{ik}^{s} trip requests accepted at Stage 2 and will be served with battery level k.
- F_p decided number of charger type p to be installed at group g

4.2.2.3. Additional Variables.

- q_{jk}^{st} number of vehicles arriving to group g from outside of the group by relocation, at time interval t in scenario s with battery level k
- $\overline{q}_{jk}^{st}$ number of vehicles leaving from group g by relocation, at time interval t in scenario s with battery level k

4.2.2.4. Within-Group Mathematical Model.

$$\min \sum_{\substack{(j,l,k,s,t):\\(j,l)\in \mathrm{J}_g}} \left(\mathrm{RC}_{jl}^{t} + \mathrm{PC}\right) \mathrm{P}_s\, r_{jlk}^{st} \tag{22}$$

subject to

$$\sum_{j\in \mathrm{J}_g} q_{jk}^{st} = \mathrm{R}_k^{st} \qquad \forall k,t,s \tag{23}$$

$$\sum_{j\in \mathrm{J}_g} \overline{q}_{jk}^{st} = \overline{\mathrm{R}}_k^{st} \qquad \forall k,t,s \tag{24}$$

$$q_{jk}^{st} + \sum_{\substack{(k,i\in \mathrm{I}_{\mathrm{A}s}):\\ \mathrm{Dest}(i)=j\\ \mathrm{End}(i)=t\\ k'=\mathrm{CT}_i+k}} \mathrm{Z}_{ik'}^{s} + \sum_{l\in \mathrm{J}_g} r_{ljk}^{st} = \sum_p b_{jkp}^{st} \qquad \forall k, j\in \mathrm{J}_g, t, s \tag{25}$$

$$\sum_{\substack{(k,i\in \mathrm{I}_{\mathrm{A}s}):\\ \mathrm{Origin}(i)=j\\ \mathrm{Start}(i)=t\\ \mathrm{CT}_i\le k}} \mathrm{Z}_{ik}^{s} + \sum_{l\in \mathrm{J}_g} r_{jlk}^{st} + \overline{q}_{jk}^{st} = \sum_p \overline{b}_{jkp}^{st} \qquad \forall k, j\in \mathrm{J}_g, t, s \tag{26}$$

$$\sum_{j\in \mathrm{J}_g} f_{jp} = \mathrm{F}_p \qquad \forall p \tag{27}$$

$$(4),(5),(6),(7),(8),(9) \qquad Same\ \ with\ \ \forall j\in \mathrm{J}_g \tag{28}$$

$$(13) \qquad \forall (j,l)\in \mathrm{J}_g, k, p, s, t \tag{29}$$

$$q_{jk}^{st}, \overline{q}_{jk}^{st} \in \mathbb{N} \qquad \forall (j,l)\in \mathrm{J}_g, k, s, t \tag{30}$$

This model aims to provide a feasible solution for the results generated in the second stage. Constraints (23) and (24) guarantee that arriving (R_k^{st}) and departing ($\overline{\mathrm{R}}_k^{st}$) relocations (as shown in Figure 4) are assigned to stations of group g. Constraints (25) and (26) are flow conservation equations similar to (2) and (3) respectively. Please note that, the trip requests in the first model are now provided as parameters (accepted trips). The relocation arcs (r_{jlk}^{st}) are assumed to move only within the station dimension (i.e. no battery or time consumption is considered). These variables are only ensuring feasibility. Constraints (27) are for the charger implementation. The remaining of the model is the same with the original model, only applied for the stations within group g.

Since the goal is to find a feasible solution, we minimize the relocation cost within group g (Objective Function 22) in order to have a more realistic representation of the system.

To demonstrate the steps of the grouping heuristic, a small instance with 10 stations are provided in Figure 5. In this example, first, we divided 10 stations to 5 groups: g_0={Station 0, Station 4, Station 7}, g_1={Station 1}, g_2={Station 3}, g_3={Station 5} and g_4={Station 2, Station 6, Station 8, Station 9} with the help of Station Grouping Model. Then, we found the best station groups for the fast chargers by solving the Grouping Mathematical Model (see Figure 5A). Finally, we solved Within-Group Mathematical Model for each group formed in Step 1 to identify stations that should have fast chargers installed (see Figure 5B). In this last step, we use between group relocations and accepted trips provided in Step 2. In Step 3, we identify the exact origin destination stations of between group relocations and generated new within group relocations while considering carried-over accepted trips and between group relocations. Please note that, although we are serving the same trips, since stations are aggregated in Step 2, trips are originating from (destined to) the groups of the origin (destination) station.

5. Computational Experiments

The model and heuristic approaches presented in Sections 3 and 4 are applied to historic data of Auto Bleue, which used to be a one-way electric carsharing system operating in Nice, France. The system has 60 stations. Each station has 3 parking spots along with regular chargers provided at each parking spot. Each day is assumed to start at 6:00 and end at 22:00. As discretization in time is assumed, the 16-hour time horizon is divided into 64 15-minute time intervals.

The proposed models and heuristics assume discrete battery levels. The installed conventional chargers need 8 hours (32 time intervals) to fully charge an empty battery. In the experiments, one type of rapid charger that can charge an empty battery in 30 mins is assumed to be implemented. The battery is discretized into 32 charging levels. Therefore, the charging rates of conventional and fast charger are taken as 1 and 16 battery levels per time interval ($\text{rate}_0 = 1, \text{rate}_1 = 16$). Although the charging speed is not linear and slows down after a certain charging level, we assume linear charging rates. It should be noted that the models are still compatible with nonlinear (with a fixed charging speed in each time interval) charging rates because of the discretization of the battery levels.

The experiments are conducted in C# in .NET environment, on a workstation with Intel Xeon E5-2640 v3 processor. The integer programming models are solved by IBM ILOG Cplex 12.10.

The parameters used in the model and heuristic algorithms except for the personnel cost are taken from Boyaci et al. (2015) and provided in Table 2. In this study, the personnel cost per hour is taken as the minimal hourly wage in France in 2018 (OECD, 2018). We employ the personnel based on the need of relocation, and the labour cost is calculated by the summation of relocation duration multiplied by the personnel cost per time interval as shown as personnel cost in Objective Function (1).

Table 2: Parameters used in computations

Parameter	Value
Revenue per hour	13€
Vehicle relocation cost per km	0.01€
Relocation speed (km/hour)	30
Personnel cost per hour	$11.5
MaxPer	3, 4, and 6 for 10, 20 and 60 stations systems respectively
AR	3 time intervals

The number of vehicles is assumed to be equal to the number of stations. The trips and relocations are only allowed when the battery level is higher than 40% and sufficient to drive the origin-destination distance of trip requests. In the proposed model, a vehicle is present with a full battery at one of the parking spots of each station before the start of the day (i.e. 6AM). This is done by adding the following constraints (31 and 32) to the model provided in Section 3. Please note that in order to add Constraints (31 and 32) a dummy time interval (time zero) is created which represents one time interval before the actual system starts. Hence, the scope of the constraints is altered from $\forall t$ to $\forall (t \neq 0)$ in the models.

$$\sum_{p} d^{s0}_{j\mathrm{K}p} = 1 \qquad \forall j, s \quad (31)$$

$$\sum_{p} \sum_{0 \le k \le \mathrm{K}-1} d^{s0}_{jkp} = 0 \qquad \forall j, s \quad (32)$$

For the second step of grouping heuristic approach where the groups are treated as large stations, Constraints (33) are added instead of Constraints (31) where g and $|\mathrm{J}_g|$ denotes groups and number of stations at group g respectively. Constraints (31) is also applied in Within-Group models.

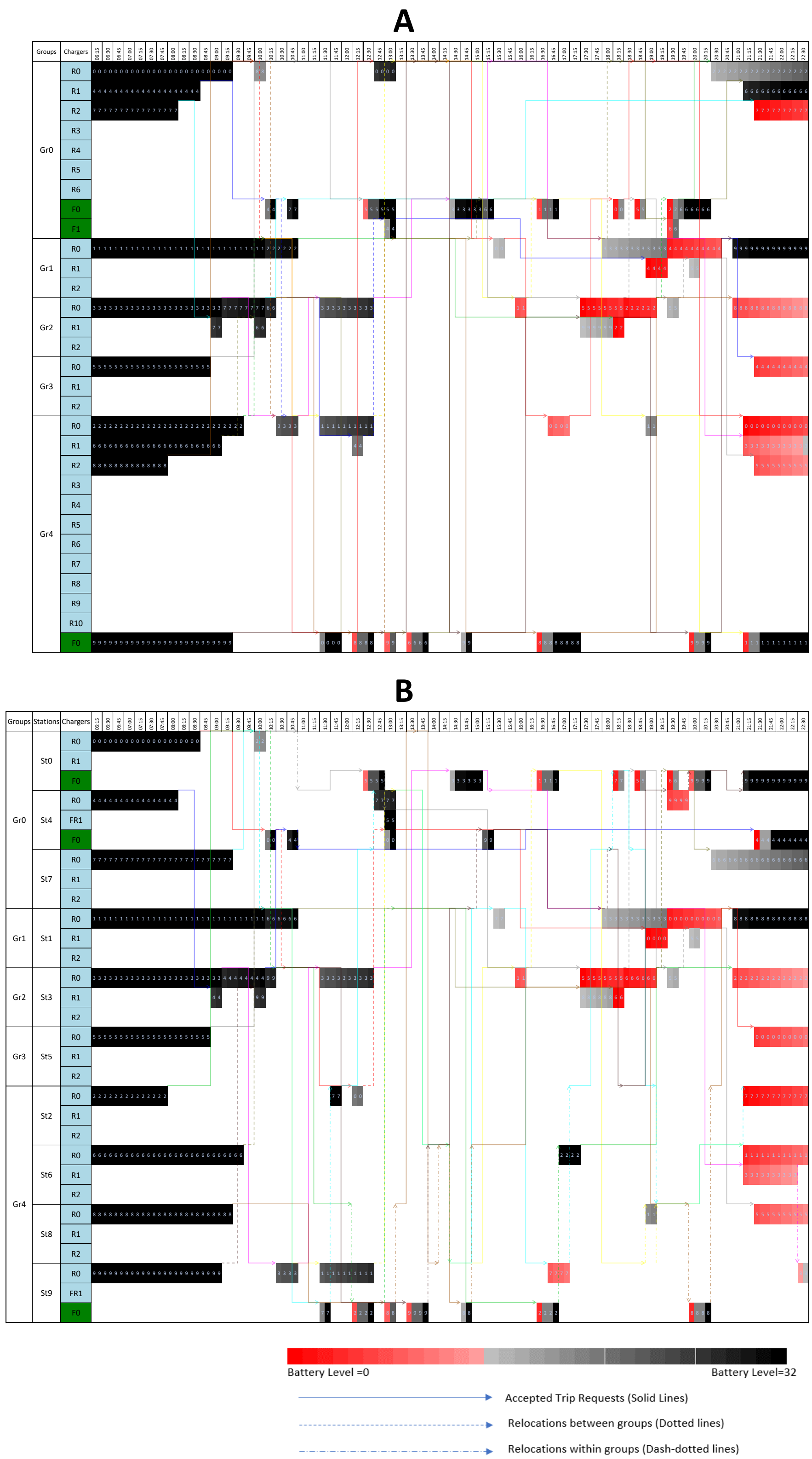

Figure 5: Vehicle flows in Step 2 (A) and Step 3 (B) when the Grouping Heuristic is applied to an instance with 10 stations.

$$\sum_{p} d^{s0}_{g\mathrm{K}p} = |\mathrm{J}_g| \qquad \forall g,s \quad (33)$$

To ensure that each station starts the day with a predefined number of fully charged vehicles, the following constraints ((34) & (35)) are added to the model in Section 3. Please note that TIL denotes the time interval length in minutes.

$$\sum_{k,p} d^{s\mathrm{T}}_{jkp} = 1 \qquad \forall j,s \quad (34)$$

$$\sum_{k,p} (d^{s\mathrm{T}}_{jkp}(\mathrm{K}-k)) \leq \left(\frac{1440}{\mathrm{TIL}} - |\mathrm{T}|\right) \mathrm{rate}_p \qquad \forall j,s \quad (35)$$

Constraints (34) ensure that at the end of the time horizon, each station has one vehicle parked. From the end of a day until the beginning of the next day, it is guaranteed that the parked vehicles are fully charged by Constraint (35). The left hand side of the inequality of Constraint (35) gives the battery need ($\mathrm{K}-k$ levels) of the vehicle that is parked at a station with k levels of battery , while the right hand side gives the number of battery levels that the charger which the vehicle is parked at can charge between the end of the day and the start of the next day. Although this constraint is redundant in our experiments since the duration of the non-operational hours is 8 hours which is sufficient to fully charge a vehicle even at a regular charger, this could be useful for the systems when the length of the non-operational hours is less than the time required to fully charge a vehicle of any charger in the system.

The same rationale is applied to Step 2 in Grouping Heuristic Method. The Constraints (36) confirm that each group g has $|\mathrm{J}_g|$ vehicles parked at the end of the operational hours each day. Constraints (34) is also added to Within-Group Model (Step 3) for the same reasons.

$$\sum_{k,p} d^{s\mathrm{T}}_{gkp} = |\mathrm{J}_g| \qquad \forall g,s \quad (36)$$

5.1. Heuristic Selection

Since the model (Section 3) is intractable for the real life system with 60 stations, we have selected 10 and 20 of the busiest stations in order to test the performance of the proposed heuristic methods. Problem instances with 100 and 200 trips are created for these station sets, and solved using the solution methods proposed in Sections 3 and 4.

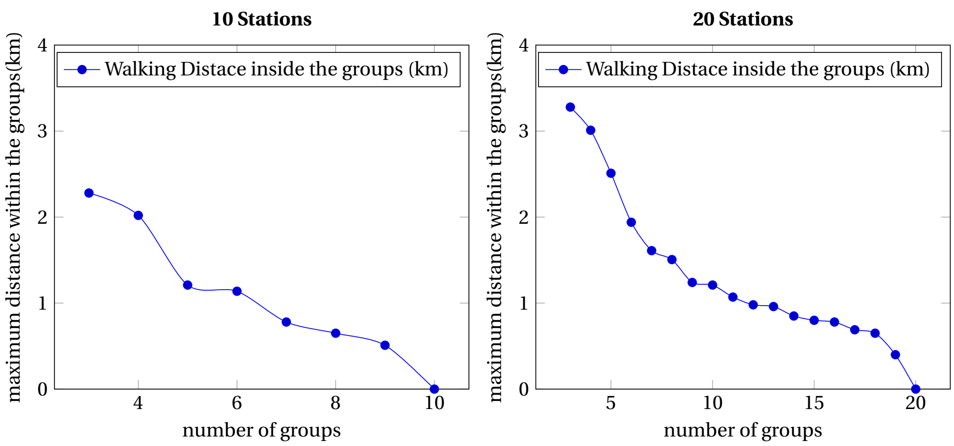

Figure 6: Maximum walking distance within groups for 10 and 20 stations

As the grouping heuristic approach assumes "instant within group relocation" (no battery consumption or travel time is encountered due to the very close proximity of the stations comprising a group of stations), the groups are selected such that the maximum walking distance within the group stations is minimized. It should be noted that the number of groups to be used affects the resulting computational complexity. If the number of groups will be selected to be equal to the number of stations, i.e no grouping is considered, then the resulting model is the exact model proposed in Section 3. If it can be solved, it gives the exact solution but could be intractable for real-life instances. If the number of groups is too small, the model is less accurate. The battery consumption and travel time assumptions may not be realistic since the size of the groups are too big and station distances within groups are too high.

The number of groups are selected for the experiments with the 10 and 20 station instances are 5 and 9 respectively. This choice was made because these number of stations correspond to the elbow of the curves of the graphs illustrating the relationship between the number of stations and the maximum distance within the stations belonging to the identified clusters (See Figure 6).

The values of the objective functions corresponding to the different heuristics used to solve the test instances are presented in Table 3. It should be pointed out that we do not present the criteria values of the heuristics directly as the objective functions are treating different distance values for grouping heuristics. Instead, the expected profit, number of served trips, and vehicle utilization values of the heuristic methods are calculated in 2 steps; (i) solving the heuristic method and finding the locations of the fast chargers (f_{jp}) and (ii) solving the exact method by fixing the (f_{jp}) values found in (i). This way we ensure that the resulting profits are computed on the basis of the same objective function. Each cell in the table represents the average value of 5 runs for different instances. 1% MIP gap and 3 hours (10800 sec) are set as termination parameters. The runs were stopped when one of the termination parameters is reached.

Table 3: Expected Profit, Number of Served Trips, Time and Vehicle Utilization values of the solutions

Problem Type	Criteria	Exact	No Relocation	No Fast Charger	RelRes1	RelRes2	Grouping	GroRelRes1	GroRelRes2
10 Stations- 1 Scenario	Expected Profit (obj)	1531.1	1421.3	1260.3	1528.5	1526.1	1468.0	1467.5	1466.2
	Time (sec)	376.5	2.2	60.5	166.3	25.9	71.4	44.2	25.9
	# of served trips	53.2	48.4	35.4	53.2	53.4	52.2	50.0	50.0
	Average Vehicle Utilization	62%	57%	45%	62%	63%	60%	59%	59%
10 Stations- 2 Scenarios	Expected Profit (obj)	1517.5	1461.6	1248.0	1516.4	1510.2	1465.5	1475.0	1478.7
	Time (sec)	949.6	4.3	130.8	351.1	45.8	139.3	97.4	49.7
	# of served trips	50.2	50.2	34.2	50.3	51.2	50.4	50.3	50.6
	Average Vehicle Utilization	62%	60%	45%	62%	63%	60%	60%	60%
10 Stations- 3 scenarios	Expected Profit (obj)	1509.4	1457.3	1249.4	1509.0	1505.2	1463.9	1464.0	1465.8
	Time (sec)	1618.8	6.5	277.6	584.3	83.4	323.9	197.9	84.8
	# of served trips	52.4	48.7	33.8	51.9	52.7	50.5	50.6	50.2
	Average Vehicle Utilization	62%	59%	45%	62%	62%	60%	60%	60%
10 Stations- 4 scenarios	Expected Profit (obj)	1487.7	1454.8	1245.5	1487.7	1485.1	1454.3	1457.6	1457.1
	Time (sec)	3233.0	8.5	453.8	1122.6	147.1	481.9	296.4	116.8
	# of served trips	53.1	52.2	34.7	53.1	53.4	51.7	51.3	51.3
	Average Vehicle Utilization	62%	60%	45%	62%	63%	60%	60%	60%
20 Stations- 1 scenarios	Expected Profit (obj)	3025.1	2905.5	2479.7	3019.3	3018.8	2936.5	2951.5	2944.4
	Time (sec)	4917.4	4.1	408.6	1950.8	237.5	242.7	182.3	145.4
	# of served trips	105.0	103.2	69.4	110.4	110.4	104.4	106.4	105.0
	Average Vehicle Utilization	64%	60%	45%	63%	63%	61%	61%	61%

In Table 3, the comparison of the exact and no fast charger (fixing $f_{jp} = 0$ for $p \neq 0$) algorithms suggests that fast charger purchases will increase the expected profit (17.3%), number of served trips (33.8%) and average vehicle utilization (27.3%) dramatically.

When the heuristic methods are compared with the exact model, it is clear that Relocation Restriction 1 (RelRes1) and Relocation Restriction 2 (RelRes2) methods are performing better than the rest of the heuristic methods in terms of expected profit (objective) values. The average errors compared to the exact method are 0.09% and 0.29% for RelRes1 and RelRes2 methods respectively. However, especially for 20-node instances, the time required to solve RelRes1 reaches the 3-hour limitation even for two-scenario instances. On the other hand, the average computation time of RelRes2 is 237.5 sec which is less than 1/8 of the time required for RelRes1 for 20-Node, one-scenario instances.

The average error to the optimal value of the heuristics No Relocation (setting $r^{st}_{jlk} = 0 \ \forall j, l, k, s, t$), Grouping, Grouping with relocation restriction 1 (GroRelRes1) and Grouping with relocation restriction 2 (GroRelRes2) are 4.04%, 3.05%, 2.81%, and 2.80% respectively for 25 different instances. Figure 7 illustrates how the errors of the heuristic methods are distributed. The boxplot shows that the grouping heuristics behave similarly. Solving the exact method without the relocations (No Relocation method) spreads wider than the grouping heuristics. In fact, the standard deviation of No Relocation method is 4% whereas it is 2.21% for Grouping and 2.20% for GroRelRes1 and GroRelRes2. As the number of nodes and groups increase, the time required to solve Grouping and GroRelRes1 will show a similar pattern with Exact and RelRes1 algorithms. This is why, among

No Relocation, Grouping, GroRelRes1 and GroRelRes2, we decided to use GroRelRes2 heuristic.

At this point of the analysis, we will compare RelRes2 and GroRelRes2 computational times on the system with 60 stations.

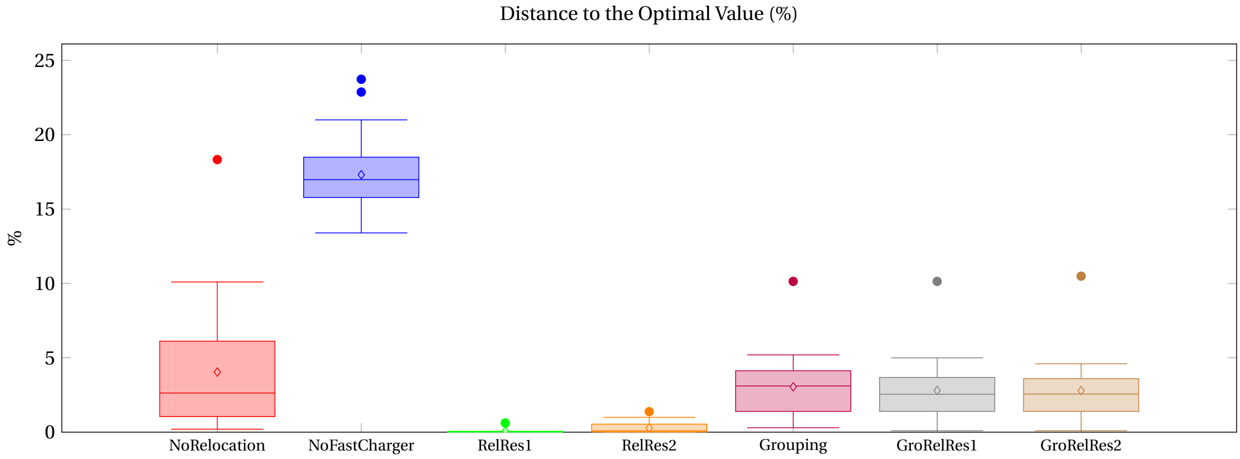

Figure 7: Heuristic methods' expected profit value distances to the exact model's expected profit (%)

In order to compare the computational times of the heuristics when solving instances with 60 stations, the number of groups should be determined first. Figure 8 shows the maximum within distances for different number of groups. On the graph, we observe a sharp decrease when the number of groups is changing from 10 to 15, and a milder change when the number of stations becomes larger than 15 (elbow of the curve). At 15, the maximum distance within the groups is 1.5km, which denotes negligible battery consumption between stations inside the groups. This is also one of the other reasons, the number of groups is selected as 15 for the system with 60 stations.

Table 4 shows the computational time of heuristics RelRes2 and GroRelRes2 when the group numbers is set to 15. Each cell represents the average of four different runs. RelRes2 heuristic is able to solve instances only with 60 stations and 1 scenario within 3 hours (10800sec) time limitation. On the other hand, GroRelRes2 heuristic method is able to solve instances with more scenarios within the time limit.

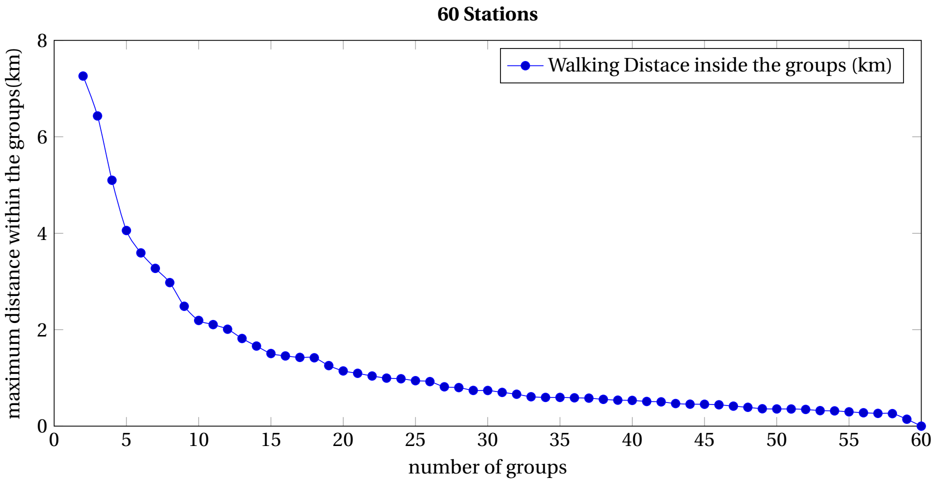

Figure 8: Maximum distance within groups for different number of groups

It is concluded RelRes2 is able to find near-optimal solutions for smaller size problems whereas GroRelRes2 performs well for both small and large size problems. The rest of the analysis is carried out via GroRelRes2 heuristic algorithm.

5.2. Sensitivity Analysis on the Number of Chargers

In this part, we evaluate the effects of different number of fast chargers on the total profit for a 60-Station OWECS. Figure 9 is obtained via fixing the number of chargers at GroRelRes2 heuristic algorithm on a 60 Node-

Table 4: Duration of RelRest 2 and Grouping RelRest 2 heuristics for 60 Node systems

Problem Type	RelRes2	GroRelRes2 (m=15)
60Node-1 Scenario	5476 (sec)	1066 (sec)
60Node-2 Scenarios	NA	3023 (sec)
60Node-3 Scenarios	NA	6405 (sec)

1 Scenario instance. Please note that, we also solve the same instance without considering charging levels. In other words, we assumed a system with internal combustion engine (ICE) vehicles. There are many bottlenecks that could prevent a carsharing system to serve all requested demand, such as station capacity, demand distribution, vehicle count. Running the system with ICE vehicles allow us to see the bottleneck created by factors other than battery restrictions. We consider this value as the upper bound for the profit in our analysis. As the Grouping heuristics maximize the profit of the model of Step 2 (solving the exact model by using the station groups), it can be observed that the second step profit values behave as discrete concave function, and the optimal solution is obtained when the number of fast chargers is fixed at 17 (see Step 2 profit values on Figure 9). On the other hand, the total profit fluctuates especially when the number of fast chargers is greater than 4. The total profit has its maximum value when the fast chargers are fixed to 20. We observe that the within-group relocation cost when locating 17 fast chargers is higher than that of 20. Yet the difference in total profit of when locating 17 and 20 fast chargers is less than 0.2%. Figure 9 also illustrates the marginal increase in total profit at each additional fast charger implementation. Implementing 17 fast chargers will result in 25% increase in the total profit. The largest increase (7.5%) for an additional fast charger observed when the first fast charger is implemented. The increase in the total profit will be 12.5%, 16.1%, 18.6% and 20.1% for 2, 3, 4, and 5 fast charger implementations. As expected, the impact on the second stage profit is decreasing as the number of fast chargers increases, and after reaching the optimal solution (17 fast chargers), the second stage profit is decreasing because of the fast charger costs. With some fluctuations, the total profit also behaves similarly. Using 17 fast chargers increased the number of served trips from 207 to 351 by 69.6% compared to not buying any. We observe a steep rise in the number of served trips when 4 fast chargers are implemented, in fact, 61.4% of 69.6% increase is due to the purchase of the first 4 chargers. After 20 fast chargers implementation, we see almost no change in the number of served trips. There is 7.6% difference between the total profit of optimally locating fast chargers and the upper bound (ICE total profit value). This result is based on the requirement of having compulsory battery level (40%) before trips/relocations, charging times and battery capacity.

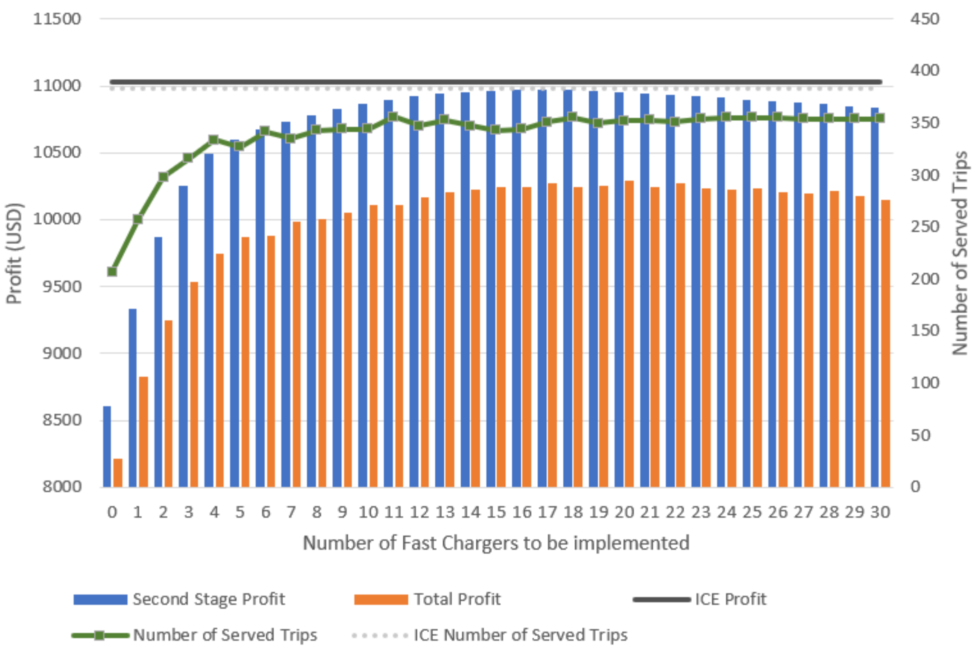

Figure 9: Second Stage Profit, Total Profit and Number of Served Trips when different number of fast chargers implemented

Figure 10 illustrates how the number of relocations (both inter and within group) and relocation cost vary with the number of fast chargers implemented to the system. The number of within group relocations per served trips shows a downward trend as the number of fast chargers increases. This could be explained by the availability of the fast chargers at the stations. If a trip has already been accepted at the grouping stage of the algorithm, a vehicle with a sufficient battery level must be available at the origin station of the trip. In certain cases, this may be only possible by relocating a vehicle to a station that has an available fast charger within the group. If there is a fast charger available at the origin of the trip, within group relocation is usually not needed to serve the trip.

Clearly the proposed algoirthm generates more relocation activities than the exact algorithm. In comparing the number of required relocations between the exact and heuristic solutions for problem instances with 10 and 20 stations, we found that the exact solution generates relocations for 57% of the accepted trips while the heuristic generates relocations for 82% of the accepted trips. However, as seen in Figure 10, the majority of the generated relocations of the GroRelRes2 (76% on the average) is resulting from the within group relocations. The average cost of a within group relocation is less than the average cost of an inter group relocation. We observe that the relocation cost is \$0.053 and \$0.105 per \$1 trip revenue on the average with exact and heuristic algorithms applied to the problem instances in Table 3. We investigate the effects of additional fast chargers on (relocation cost/revenue) ratio. (Relocation cost/revenue) ratio decreases slightly compared to within group relocations per trip values as the number of fast chargers increase.

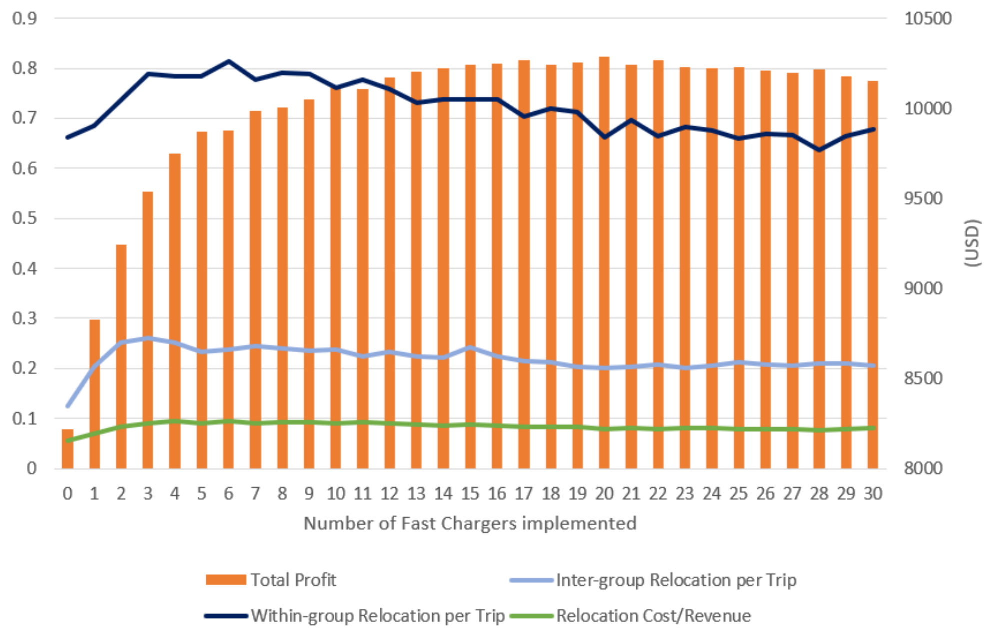

Figure 10: Second Stage Profit, Total Profit and Number of Served Trips when different number of fast chargers implemented

5.3. Sensitivity Analysis on Charger Prices

To understand the effects of the cost of fast chargers on the optimal solution, we vary the cost of fast chargers. We solve the problem by considering five different costs per day (\$12.7,\$9.525, \$6.35, \$3.175,\$0) per fast charger installation. We also consider the option without fast charger installation (indicated with ∞ sign in Table 5) and report the percent improvements of other fast charger installation price levels compared to this option. Table 5 provides information on the total profit, number of served trips, number of fast chargers to be implemented and vehicle utilization. Figure 11 illustrates how these values are compared to "without fast charger option" when daily depreciated cost of fast charger is varied.

From Figure 11, it can be seen that introducing fast charger increases the expected daily profit, the number of served trips and vehicle utilization remarkably by 25.3-29.1%, 63.1-65.2%, and 37.3-37.9% respectively. However, it is observed that the system performance is not sensitive to the cost of the fast chargers. When 25% gradual discounts are analysed for (\$12.7 to \$9.525), (\$9.525 to \$6.35),(\$6.35 to \$3.175),and (\$3.175 to \$0), the average increase in the total profit value, number of served trips and vehicle utilization are 0.8%, 0.4% and 0.1% respectively.

We observe that, when the daily depreciation of a single charger drops from \$12.7 to \$0, the model advises to open 28 chargers (not 180, i.e. number of charging points, even though they are free to install) instead of 17. In addition, the profit increases by 3.08%, 2.06% of which comes from additional profit of serving more trips.

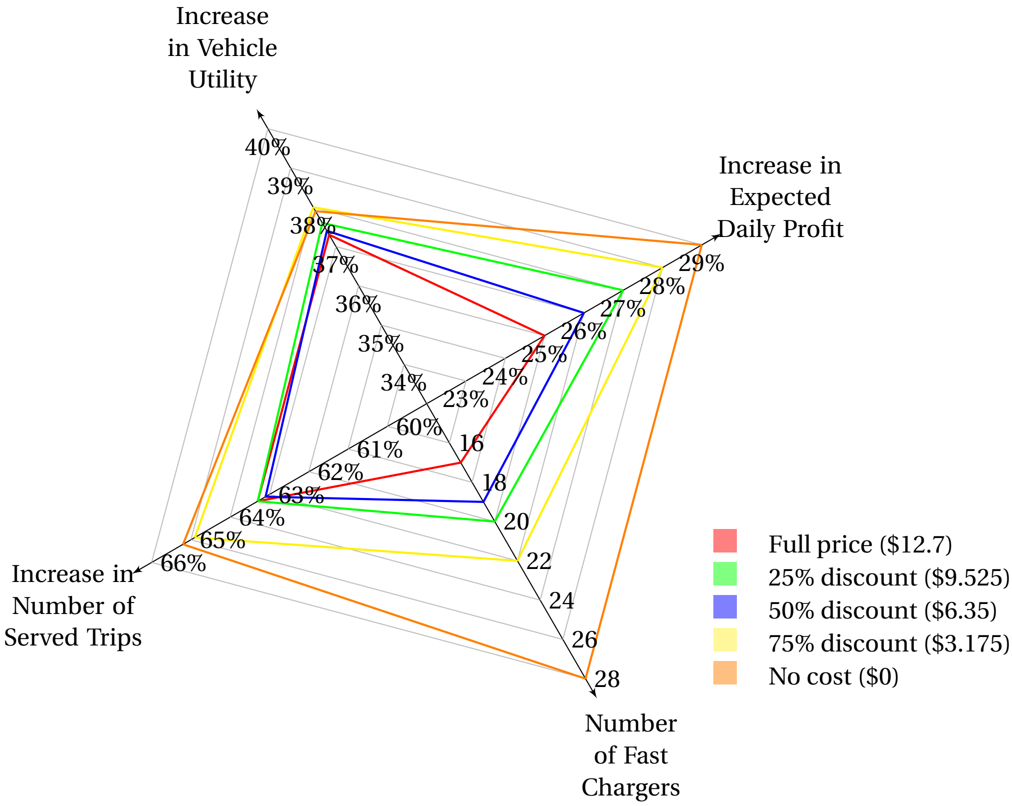

Figure 11: Price sensitivity analysis

Table 5: Total Profit, Number of Served Trips, Fast Chargers and Vehicle Utility under different price scenarios for fast chargers

Depreciated Cost (USD)	Total Profit	Number of Served Trips	Number of Fast Chargers	Vehicle Utilization
∞	8361.9	189.5	0	49%
12.7	10474.5	309.5	17	67%
9.525	10525.3	309	19	67%
6.35	10595.8	309.5	20	67%
3.175	10663.0	312.5	22	68%
0	10795.8	313	28	68%

5.4. Demand Based Sensitivity Analysis

In this section, we evaluate the effects of demand on fast charger acquisition decision. Alternative demand scenarios were generated by decreasing the initial demand of 600 requests to 250, 300, 350, 400, 450, 500 and 550, and increasing the demand for trip requests to 650, 700, 750, 800, 850, 900, 950 and 1000 requests. Figure 12 depicts how the total profit and number of fast chargers change when demand size varies from 250 to 1000 for EVs. We also added the same analysis, obviously without the consideration of chargers, for an ICE fleet for comparison purposes. When the demand increases between 250 and 500, there is a significant growth in the number of fast chargers with 3 fast charger increases per additional 50 requests on the average. When the demand exceeds 500, the number of fast chargers fluctuates between 16 and 18. Clearly, the profit of the system increases as the demand size increases. However, an additional 50 requests for smaller demand sizes (sizes between 250 to 500) yield more profit than larger demand sizes. For example, additional 50 requests provide 22% and 0.7% increase in total profit for demand sizes 300 and 900, respectively. For the smaller demand sets, the station and personnel relocation capacity is still sufficient to serve more trips,whereas for the larger demand sets, there is not enough capacity for accepting additional trips, and extra profit is generated by accepting longer trips or trips such that fewer relocation operations are required. It should be noted that as the size of demand increases, the difference between optimal solutions of EV and ICE fleet increases. The difference increased from 3.07% when demand is equal to 250 to 9.3% when demand increases to 1000. The difference between the performance of ICE and EV fleets is the outcome of restrictions imposed by batteries. As the system gets more congested, the profit difference between the system with ICE vehicles and EVs increases. When the vehicles are utilised more, EVs require more frequent charging. Although the system with EVs can install more fast chargers, since it is not profitable, it is not advised by our model.

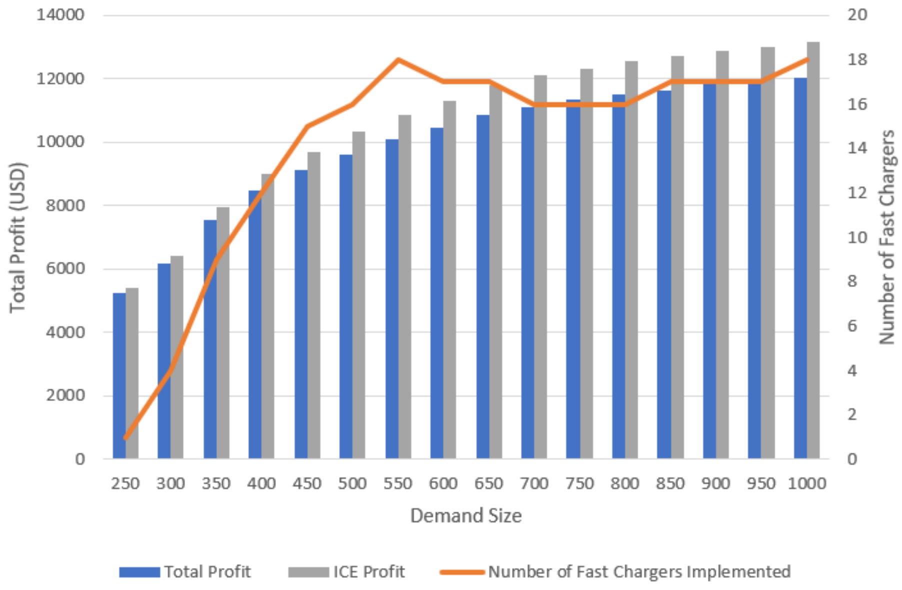

Figure 12: Changes in Total Profit and Number of Fast Chargers When Demand Size Differs

5.5. Location Based Sensitivity Analysis

In this analysis, we explore the effect of the location of the fast charger to the profitability of the system. We want to observe how much more we can gain by choosing the right fast charger location. We consider a system without a fast charger as a start and compare the total profit of the system when we install a fast charger to one of the stations. We consider 60 different instances. We install a fast charger to one of the stations at every instance. Figure 13 shows total profit and number of served trips when only one fast charger is implemented at different stations. The blue line located on the figure shows the total profit level if we do not consider restrictions on the total number of fast chargers. Green line on the other hand shows the total profit level if no fast chargers is installed to the system. Station 16 and 42 are the most and least profitable station to install a fast charger respectively. In this system, installing fast charger to a wrong station could lead up to 7.2% loss in profits. However, the distribution of the profitability of the stations is left-skewed. The average profit of installing a fast charger to any station is just 1.6% worse than the maximum profit of installing a fast charger.

5.6. Sensitivity Analysis on the Compulsory Battery Level

Carsharing companies often follow a policy of minimum battery level requirements. In this section, we investigate the effect of minimum battery level of a vehicle required to start a trip. We name it as compulsory battery level for brevity. Table 6 illustrates profit values and number of fast chargers to be implemented when the compulsory battery level varies between 20% to 100%. When the compulsory battery level is increased from 20% to 100%, the total profit is decreased by 4.4% (446$ per day). This is due to the fact that higher battery availability thresholds render a higher number of vehicles unavailable and require additional charging time. The number of fast chargers implemented shows almost a non-decreasing trend. As the compulsory battery level increases, even though the optimal number of fast chargers in almost all of the cases increases, the total profit of the system decreases. Fast charger implementation is becoming incresingly important as the battery level requirement increases.

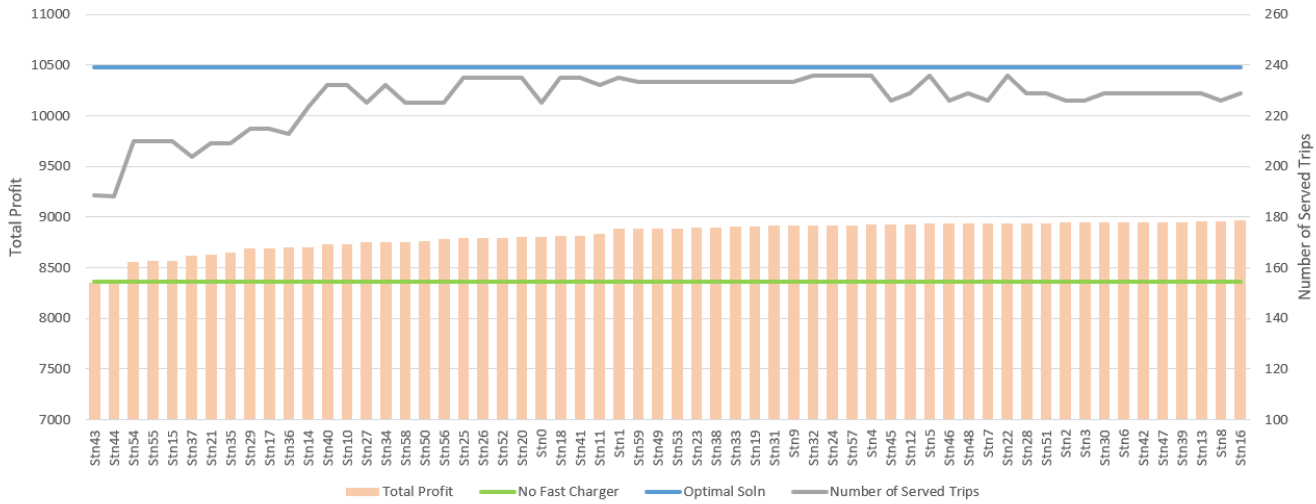

Figure 13: Changes in Total Profit and Number of Fast Chargers When Demand Size Differs

Table 6: Profit values and number of fast chargers for different compulsory battery levels

Compulsory Battery Level (%)	Second Stage Profit ($)	Total Profit ($)	Number of Fast Chargers
20	11176.70	10511.73	17
30	11161.23	10469.14	18
40	11140.77	10474.49	17
50	11109.53	10444.64	18
60	11026.07	10379.72	19
70	10967.32	10326.71	19
80	10904.67	10294.07	21
90	10800.63	10191.51	22
100	10641.45	10065.36	25

6. Conclusions and Future Directions

In this study, we have introduced a time-space-battery level integer programming model to solve the rapid/fast charger location problem in one-way electric carsharing systems. Besides the exact solution, we have proposed two types of heuristic methods based on relocation reduction and station grouping. The first type of heuristic method creates relocation arcs based on the trip request data set rather than creating relocation arcs between all station pairs for all time intervals and battery levels. The second type of heuristic method decreases the size of the network by grouping the stations and treating the groups as large stations. This heuristic method involves three stages: i) determine the number of groups, ii) solve the integer programming model for groups, iii) model the problem of locating fast/superior chargers within the groups. Along with these two types of heuristic methods, a heuristic combining relocation reduction and grouping methods was also presented.

The proposed model and heuristic methods are tested on 10 and 20 station subset settings of a real world one-way electric carsharing system from Nice, France. It is observed that the mixed heuristic gives good quality solutions in a reasonable computational time. Our analysis suggests that the introduction of the fast chargers increases the profitability of the system and the level of service offered to its users as more trip requests can be served with lower relocation cost without increasing the fleet size. Furthermore, we found that the performance of the system is not very sensitive to the price of fast chargers. The analysis on the number of demand requests suggests that the number of fast chargers increases as the number of demand requests increases until the system reaches the capacity of either relocation or stations, then it remains fairly constant. For the location analysis, only one fast charger is allowed in the system and applied to different locations. In this analysis, we observe that the distribution of the profitability of the system is left-skewed. Although there are a few stations that installing a fast charger to one of these stations increases profit as much as installing the fast charger to the best station, installing fast charger to the wrong station could lead to huge losses in profit. The analysis on different compulsory battery levels show that as the minimum battery level that is required to assign a vehicle to start a trip increases, the profitability of the system decreases and the system requires more fast chargers.

Although the exact model can only be applied to very small scale problems, the proposed combined heuristic approach is scalable to address problem instances found in real world OWECS and can be used by decision makers to optimize fast charger location decisions.

In our study, we consider fixed station capacities in terms of the available parking spots. In reality, one of the major costs of the one-way electric carsharing companies is the parking spot cost since most of these companies are established in urban areas. With minor adjustments, the proposed models can be used to determine the optimum number of fast chargers and the optimum number of the parking spots per station. In addition to the proposed single objective model considering profit maximization, multi-objective models can be introduced by considering maximization of the number of served trips, vehicle utilization, and other performance measures.

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