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TECHNOLOGY IN OPEN ECONOMY**

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# SECTORAL FISCAL MULTIPLIERS AND TECHNOLOGY IN OPEN ECONOMY

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## Abstract

Motivated by recent evidence pointing at an increase in the TFP following higher government spending, we explore how technology affects sectoral fiscal multipliers in open economy. Our estimates for eighteen OECD countries over 1970-2015 reveal that a government spending shock increases significantly the non-traded-goods-sector share of total hours worked while the response of the value added share of non-tradables (at constant prices) is muted at all horizon. The latter finding is puzzling as government spending shocks are strongly biased toward non-tradables. Our empirical findings show that the solution to this puzzle lies in technology which responds endogenously to the government spending shock. By offsetting the effect of the biasedness of the demand shock toward non-tradables, the rise in traded relative to non-traded TFP ensures that real GDP growth is uniformly distributed across sectors (i.e., in accordance with their value added share). Because a government spending shock also leads non-traded firms to bias technological change toward labor and traded firms to bias technological change toward capital, factor-augmenting technological change rationalizes the concentration of the rise in labor in the non-traded sector. Our quantitative analysis shows that a semi-small open economy model with tradables and non-tradables can reproduce the sectoral fiscal multipliers we document empirically once we let the decision on technology improvement vary across sectors and allow firms to change the mix of labor- and capital-augmenting efficiency over time.

**Keywords:** Sector-biased government spending shocks; Endogenous technological change; Factor-augmenting efficiency; Open economy; Labor reallocation; CES production function; Labor income share.

**JEL Classification:** E25; E62; F11; F41; O33

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# 1 Introduction

In an environment of low interest rates, the limitations of monetary policy have sparked a renewed interest in the role of government spending. In their article, Delong and Summers [2012] lays out the possibility of a persistent increase in productivity following a rise in government spending. The evidence recently documented by D'Alessandro, Fella and Melosi [2019] and Jørgensen and Ravn [2019] on quarterly U.S. data reveals that an exogenous and temporary shock to government consumption leads to an increase in aggregate total factor productivity (TFP) which lends credence to Delong and Summers's hypothesis. If TFP increases, the aggregate fiscal multiplier is higher than initially thought. Because the ability of firms to increase the efficiency in the use of capital and labor may vary across sectors, we address the following questions: Are aggregate TFP gains caused by a rise in government consumption uniformly distributed across sectors? If not, how large is the discrepancy in sectoral fiscal multipliers caused by sector differences in technology improvement? We find that shocks to government consumption increase traded TFP relative to non-traded TFP significantly, thus pushing up the government spending multiplier on traded relative to non-traded value added. While efficiency gains are concentrated in the traded sector, non-traded industries bias technological change toward labor which increases the government spending multiplier on non-traded relative to traded hours worked.

Investigating the link between technology and fiscal policy at a sectoral level is important since during downturns, recent evidence suggests that non-traded firms experience the largest drop in labor, see e.g., Mian and Sufi [2014] for the U.S. (2007-2009) and De Ferra [2018] for Italy (2011-2013). Fig. 1(a) plots the cyclical components of (logged) real GDP (displayed by the red line) and the (logged) ratio of traded to non-traded hours worked (displayed by the blue line) for (the private sector of) the United States. Over 1970-2015, the two series are uncorrelated, thus suggesting that the traded and the non-traded sector are symmetrically affected during expansions and recessions. According to the evidence documented by Garìn et al. [2018] on U.S. data, the responses of sectors display more asymmetry along the business cycle in the post-1984 period, i.e., during the great moderation. When we split the whole period into two sub-samples, we find that the correlation between the cyclical components of real GDP and traded relative to non-traded hours worked moves from positive in 1970-1984 to negative (at -0.43) in the post-1984 period.<sup>1</sup> The negative correlation suggests that during recessions, non-traded industries have experienced a larger decline in hours worked than traded industries the last thirty years.

Non-traded firms are more vulnerable to downturns because non-traded labor relies heavily on local demand while traded labor relies on national and foreign demand. This

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<sup>1</sup>Using U.S. data, the correlation between the cyclical components of logged real GDP and the logged ratio of traded to non-traded hours worked stands at -0.08 over 1970-2015, 0.41 over 1970-1984 and -0.43 over 1985-2015.

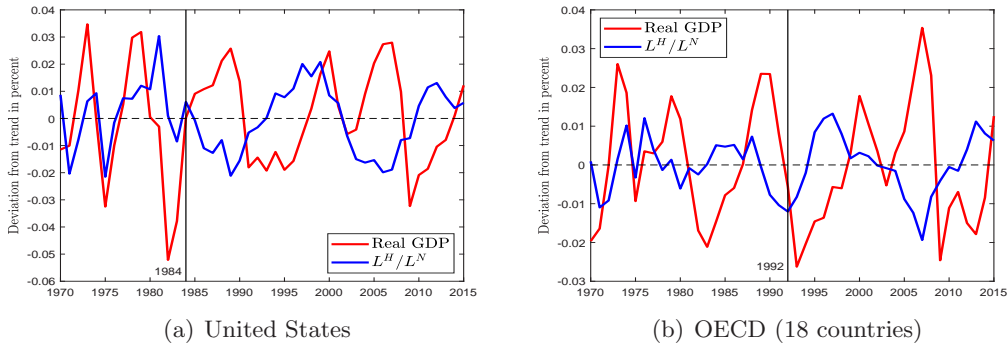


Figure 1: Real GDP and Traded relative to Non-Traded Hours Worked. *Notes:* Detrended (logged) real GDP and the detrended ratio of traded to non-traded hours worked are calculated as the difference between the actual series and the trend of time series. The trend is obtained by applying a Hodrick-Prescott filter with a smoothing parameter of  $\lambda = 100$  (as we use annual data) to the (logged) time series. Since we seek to investigate how market sectors are relatively affected by the stage of the business cycle, we abstract from the public sector and thus removed value added at constant prices of 'Community social and personal services' (which includes public services, health and education) from real GDP and non-traded hours worked. While we take the unweighted sum of time series in Fig. 1, we have alternatively used the working age population weighted sum of the eighteen OECD countries and it gives similar results. Sample: 18 OECD countries, 1970-2015, annual data.

finding is not limited to the United States. Fig. 1(b) plots the cyclical components of real GDP and the ratio of traded to non-traded hours worked for the eighteen OECD countries of our sample. When we split the whole period into two sub-periods, we find a correlation of 0.11 over 1970-1992 and a correlation of -0.44 in the post-1992 period.<sup>2</sup> Data on OECD countries thus further corroborates that sectors have not been symmetrically affected by recessions over the last thirty years as non-traded labor falls more than traded labor which raises the question of the capacity of fiscal policy to mitigate such a differential response of non-tradable versus tradable hours worked. To guide our quantitative analysis, we estimate the sectoral value added and sectoral labor effects of a shock to government consumption for eighteen OECD countries over the period running from 1970 to 2015. To conduct our empirical analysis, we adopt a two-step approach. We first estimate a VAR model in panel format. Following Blanchard and Perotti [2002], we identify exogenous shocks to government consumption by assuming that decision and implementation lags prevent government spending from responding to current output developments. In a second step, we trace out the dynamic effects of key aggregate and sectoral variables by using Jordà's [2005] projection method.

We find empirically that the aggregate fiscal multiplier is 1.2 on impact and averages 1.4 during the first six years after the shock. The rise in aggregate TFP contributes 39% of real GDP growth on average. While shocks to government consumption are strongly biased toward non-tradables, our estimates reveal that real GDP growth is uniformly distributed across sectors, i.e., in accordance with their value added share. Therefore, the value added share of non-tradables is unresponsive to the government spending shock. This finding is puzzling since according to the data taken from the World Input-Output Database, non-

<sup>2</sup>By adopting a SVAR approach, Bertinelli et al. [2019] documents evidence which reveals that the asymmetry in the sector responses along the business cycle has increased dramatically in OECD countries after 1992.

traded industries receive a share of the rise in government spending which is higher than their share in GDP. The rationale for this finding is that traded TFP rises significantly relative to non-traded TFP which neutralizes the impact of the biasedness of government spending shock on the value added share of non-tradables which thereby remains unaffected at any horizon. When we adjust TFP with capital-utilization by adapting the methodology proposed by Imbs [1999] to a sector level, our estimates confirm that technology improves in the traded sector while technology is essentially unchanged in the non-traded sector.

The allocation of labor across sectors is quite distinct from the sectoral distribution of value added. The government spending multiplier on total hours worked averages 1.15 during the first six years after the shock. A sufficient statistic to capture the distribution of the government spending multiplier on total hours worked across sectors is the change in the non-traded-goods-share of total hours worked. When the labor share of non-tradables rises, the contribution of the non-traded sector to the change in total hours worked exceeds its labor compensation share of 63%. We find empirically that the non-traded sector accounts for 88% of the rise in total hours worked at a six year-horizon. The concentration of labor growth in the non-traded sector is the result of the combined effect of the biasedness of the demand shock toward non-tradables and the biasedness of technological change toward labor in non-traded industries. More specifically, our estimates reveal that the government spending shock increases gradually non-traded relative to traded labor income share (LIS henceforth). These shifts in LISs are caused by capital-utilization-adjusted factor biased technological change (FBTC henceforth). We find empirically that non-traded firms bias (utilization adjusted-) technological change toward labor while traded firms bias (utilization adjusted-) technological change toward capital, thus increasing the government spending multiplier on non-traded labor.

We further investigate the role of technology in determining the distribution of government spending multipliers across sectors by taking advantage of the panel data dimension of our sample. We detect a negative cross-country relationship between the change in traded relative to non-traded TFP and the response of the value added share of non-tradables following a government spending shock. Cross-country data also confirms that the decision of one sector to increase the efficiency in the use of inputs is based on factor prices as technology improvements are performed in sectors/countries where the government spending shock puts upward pressure on the unit cost for producing. We also find empirically that the a government spending shock further increases the labor share of non-tradables in countries where the non-traded LIS increases relative to the traded LIS, the responses of LISs being strongly and positively correlated with utilization adjusted-FBTC within each sector.

Adapting the Sims and Zha [2006] methodology to our case allows us to answer one

key question: what would the sectoral government spending multiplier be if the technology channel were shut down? If traded relative to non-traded TFP were kept fixed, our estimates reveal that the biasedness of the government spending shock would disproportionately benefit the non-traded sector. This channel is neutralized by the adjustment of sectoral TFPs which keeps unchanged the value added share of non-tradables. When we turn to the labor share of non-tradables, we find empirically that labor reallocation almost doubles when we let the ratio of the non-traded to the traded LIS respond to the government spending shock. When we shut down capital-utilization-adjusted-FBTC, sectoral LISs remain unresponsive to the government spending shock.

To quantify the role of technology in determining the magnitude of government spending multipliers and their differences across sectors, we put forward a two-sector semi-small open economy model with tradables and non-tradables which contains specific elements detailed below. Given that government spending shocks are biased toward non-tradables and thus provide strong incentives to shift resources toward the non-traded sector, the first set of factors determining the magnitude of sectoral fiscal multipliers are barriers to factor mobility. To account for the frictions into the movement of capital and labor between the traded sector and the non-traded sector, we allow for capital adjustment costs, imperfect substitutability between sectoral hours worked and endogenous terms of trade. Likewise Kehoe and Ruhl [2009], we assume that the economy is small in world capital markets so that the world interest rate is given, but large enough in the world goods market to influence the relative price of its export good so that terms of trade are endogenous. Following a shock to government consumption biased towards non-tradables, the relative price of home-produced traded goods appreciates. By raising the marginal revenue product of inputs, the appreciation in the terms of trade stimulates the demand for labor and capital in the traded sector which in turn mitigates the reallocation of productive resources toward the non-traded sector.

The second set of factors which influence the size of sectoral fiscal multipliers is related to technology. In line with our evidence, our model features endogenous technological progress at a sectoral level. To be consistent with our measure of technological change we use in the empirical analysis, we allow for endogenous capital utilization. In the lines of Bianchi, Kung and Morales [2019], we endogenize technological change at a sectoral level by allowing for endogenous utilization of existing technologies. While we assume that the stock of knowledge is constant over time since we are interested in fiscal policy effects at business cycle frequencies and find empirically that sectoral TFPs remain unaffected in the long-run, the change in the utilization of existing technologies will move the technology frontier upward because the technology utilization rate is pro-cyclical. The extent of the rise in technology utilization depends on the cost of adjusting technology. To account for the dynamic adjustment of sectoral LISs we estimate empirically, we assume that sectoral

goods are produced from CES production functions and within each sector, the mix of labor- and capital-augmenting efficiency varies along the technology frontier, in the lines of Caselli and Coleman [2006].

To assess quantitatively the contribution of technology in determining the magnitude of sectoral fiscal multipliers, we start with a simplified version of our open economy with tradables and non-tradables model which collapses to the semi-small open model developed by Kehoe and Ruhl [2009] with capital adjustment costs and imperfect mobility of labor across sectors. In this restricted version, we shut down endogenous capital and technology utilization in both sectors and assume that sectoral goods are produced from Cobb-Douglas production functions. Under these assumptions, the restricted model considerably understates the government spending multipliers on real GDP and total hours worked we estimate empirically. By assuming fixed sectoral TFPs, the model also predicts a fall in traded value added and a disproportionate increase in non-traded value added in contradiction with our evidence. Because LISs are fixed, the model cannot generate the government spending multiplier on non-traded labor we find in the data.

Once we let capital-utilization-adjusted-technology respond endogenously to the rise in government spending and allow firms to change the mix of labor- and capital-augmenting efficiency over time, the model can account for both aggregate and sectoral effects we estimate empirically. By increasing real GDP directly and through higher wages that provide more incentives to increase labor supply, the rise in aggregate TFP allows the model to generate government spending multipliers on real GDP and total hours worked in line with our evidence. Although the government spending shock is biased toward non-tradables and technology utilization rates are pro-cyclical, traded TFP increases relative to non-traded TFP because the cost of adjusting technology is lower in the traded than in the non-traded sector. The TFP differential leads the government spending multiplier on real GDP to be symmetrically distributed across sectors. Conversely, the bulk of the rise in total hours worked is concentrated in the non-traded sector which biases technological change toward labor.

One additional key contribution of our work is to explore about the role of technology in driving international differences in sectoral government spending multipliers. When we calibrate the model to country-specific data and assume that the biasedness of the demand shock toward non-tradables is symmetric across countries, we find that the technology channel increases the government spending multiplier on real GDP by 0.64 percentage point on average. This finding masks a wide cross-country dispersion however. In two-third of OECD countries where traded relative to non-traded TFP rises, the technology channel amplifies real GDP growth by 1.5 percentage point while in the remaining economies where traded relative to non-traded TFP declines, real GDP growth is lowered by 1 percentage point.<sup>3</sup>

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<sup>3</sup>We find that in countries where technological change is concentrated in traded industries, aggregate TFP

Interestingly, technological change amplifies the magnitude of the government spending multiplier on non-traded value added by 0.18 percentage point of GDP on average and this amplification is symmetric between the two groups of countries. While on average, technological change increases the government spending multiplier on traded value added by 0.46 percentage point of GDP, international differences in the responses of sectoral TFPs generate a wide cross-country dispersion in traded value added growth.

In contrast to government spending multipliers on sectoral value added which depend on the TFP differential, the response of hours worked is driven by FBTC. We find that for half of OECD countries, technological change is biased toward labor in the non-traded sector which increases the government spending multiplier on non-traded hours worked by 0.36 percentage point of total hours worked while technological change is biased toward capital in the traded sector which lowers the government spending multiplier on traded hours worked by 0.12 percentage point. The rise in total hours worked is thus amplified by 0.24 percentage point for these OECD economies. Conversely, in the remaining nine OECD countries where technological change is biased toward capital in the non-traded sector and toward labor in the traded sector, the rise in total hours worked is reduced by 0.19 percentage point because the traded sector accounts for only one-third of labor. The combined effect of the reduction in the government spending multiplier on hours worked and technological change biased toward capital in the non-traded sector leads to a reduction in the government spending multiplier on non-traded hours worked by 0.27 percentage point of total hours worked.

The article is structured as follows. In section 2, we document a set of evidence which sheds some light on the role of technology in determining the size of sectoral government spending multipliers. In section 3, we develop a semi-small open economy model with tradables and non-tradables, endogenous technology choices and factor-biased technological change. In section 4, we compare the performance of the baseline model with endogenous technological change with the predictions of the same model shutting down the technology channel. Next, we calibrate the model to country-specific data to quantify the role of technology in driving international differences in government spending multipliers. The Online Appendix shows more empirical results, conducts robustness checks, and details the solution method.

**Related Literature.** Our paper fits into several different literature strands as we bring several distinct threads in the existing literature together.

First, we estimate the effects of government spending shocks on technological progress and our setup allows for endogenous technology choices. The literature investigating fiscal as well as monetary policy transmission has recently documented evidence pointing at the

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increases while in the remaining economies, aggregate TFP declines, thus explaining why the government spending multiplier is reduced by 1 percentage point relative to a model keeping sectoral TFPs fixed.



presence of supply side effects of stabilization policies. Jordà, Singh and Taylor [2020] find empirically that a temporary contractionary monetary policy shock leads to a decline in TFP, thus amplifying the fall in economic activity. While the authors rationalize their evidence by assuming that the endogenous response of TFP growth depends on deviations of output from its flexible-price counterpart, Baqaee, Farhi and Sangani [2021] show that the shifts in the allocation of resources across firms can generate a rise in aggregate TFP following an expansionary monetary policy. In contrast to the authors, in our paper, changes in TFP come from changes in endogenous utilization of existing technologies in the lines of Bianchi, Kung and Morales [2019]. This modelling strategy has been already introduced in a NK model with sticky prices by Jørgensen and Ravn [2019] who show that a shock to government consumption generates an increase in private consumption by lowering prices which induces the central bank to reduce the nominal interest rate. Differently, we detect empirically significant sector differences in technology and quantify the role of the technology channel in determining the size of government spending multipliers on sectoral hours worked and value added.<sup>4</sup> While the aforementioned articles assume Cobb-Douglas production functions, we find empirically a significant impact of the government spending shock on sectoral LISs and to reproduce their dynamics, we relax the assumption of Hicks-neutral technological change whilst assuming CES production functions.

Second, we contribute to the extensive literature investigating fiscal transmission both empirically and theoretically at a sectoral level. Benetrix and Lane [2010] document evidence which reveals that a government spending shock disproportionately increases non-traded value added. Cardi, Restout and Claeys [2020] find empirically that the non-traded sector is highly intensive in shocks to government consumption which cause a reallocation of labor toward this sector, and all the more so in countries where workers' costs of switching sectors are lower. The authors rationalize these findings by considering a small open economy setup with tradables and non-tradables in the lines of Fernández de Córdoba and Kehoe [2000]. In contrast to both aforementioned works, we highlight empirically the technology channel of government spending shocks and connect the TFP differential and sector differences in FBTC to the distribution of government spending multipliers across sectors.<sup>5</sup>

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<sup>4</sup>D'Alessandro, Fella and Melosi [2019] endogenize technological progress by assuming skill accumulation through past work experience which echo to learning-by-doing mechanism. In contrast to Jørgensen and Ravn [2019] and D'Alessandro, Fella and Melosi [2019], we find empirically that prices increase in both the traded and non-traded sectors. In addition, our work complements their studies as our estimates reveal that the acceleration in technological change following a rise in government spending is concentrated in traded industries. While technology improvements increase the government spending multiplier on traded value added, technological change leads non-traded firms to use labor more intensively so that the bulk of the rise in hours worked is concentrated in the non-traded sector.

<sup>5</sup>In contrast to Benetrix and Lane [2010] and Cardi, Restout and Claeys [2020], we do not find a disproportionate increase in non-traded value added, although our estimates confirm that the non-traded sector is highly intensive in the government spending shock. As shown in Online Appendix M.5, the reason is twofold. Our dataset is running from 1970-2015 instead of ending in 2005 or 2007 and includes 18 OECD countries. Importantly, we adopt a two-step estimation method where we first identify the shock by adopting the Blanchard and Perotti [2002] approach and we estimate the dynamic effects by using Jordà's [2005] projection method which does not impose the dynamic restrictions implicitly embedded in VARs and can accommodate non-linearities in the response function. Our two-step approach ensures that all variables

Third, our paper also relates to a broad literature which studies fiscal transmission by breaking down aggregate government spending into sub-categories. Like Boehm [2020], we consider a two-sector model with imperfect mobility of labor across sectors and put emphasis on the composition of government spending. In contrast to the author who estimates the fiscal multipliers by making the distinction between government consumption and government investment shocks, we restrict our attention to government consumption and disentangle the sectoral value added and sectoral labor effects into a reallocation channel caused by the biasedness of the government spending shock and a technology channel. Like Cox, Müller, Pastén, Schoenle, and Weber [2020], we find the government spending shocks are strongly biased towards a few industries and do not purely mimic consumer spending. In our model, the reallocation of productive resources toward the non-traded sector is caused by the discrepancy between the non-tradable content of government spending and the share of non-tradables in GDP. Bouakez, Rachedi and Santoro [2018] provide a decomposition of the contribution of sectors to the aggregate fiscal multiplier by evaluating the role of production networks. This research highlights the key role of both the sectoral composition of government purchases and sectoral labor intensity in determining employment effects like us but the mechanism is very different. In our paper, a government spending shock produces larger employment effects by targeting the sector that has the highest labor compensation share and biases technological change toward the labor.

Finally, recently, the literature has investigated the redistributive effects of fiscal shocks. Like Cantore and Freund [2021], we find that a shock to government consumption increases the aggregate LIS. In contrast to the authors who stress the role of household heterogeneity in a model with sticky prices, we focus on the redistributive effects at a sectoral level in a model with flexible prices. Our estimates reveal that the rise in aggregate LIS is driven by the increase in the non-traded LIS whilst the LIS in the traded sector declines. We find numerically that a semi-small open economy model with CES production functions can account for the evidence once we allow for technological change biased toward labor in the non-traded sector and biased toward capital in the traded sector. When we turn to cross-country differences, we find a strong positive cross-country relationship between the responses of sectoral LISs and FBTC, in line with our evidence.

## 2 Sectoral Fiscal Multipliers and Technology: Evidence

In this section, we document evidence about the role of technology in determining government spending multipliers on sectoral value added and labor. We first establish a set of empirical facts for a sample of eighteen OECD countries and then take advantage of the panel data dimension to investigate the role of technology in driving cross-country differences to the same identified spending shock.

ences in sectoral fiscal multipliers. To further explore the role of technological change, we adapt the Sims and Zha [2006] methodology to provide an attempt to answer the following question: what would the sectoral government spending multiplier be if the technology channel were shut down?

## 2.1 Preliminaries

Recently, the literature has uncovered the role of technology in driving fiscal policy transmission. The evidence documented by D'Alessandro, Fella and Melosi [2019] and Jørgensen and Ravn [2019] on U.S. data shows that a shock to government consumption increases aggregate TFP. Building on the work of Bianchi et al. [2019], Jørgensen and Ravn [2019] generate a rise in TFP by considering that firms can raise the utilization rate of technology to accommodate higher demand for final output. The decision to increase TFP relies on the trade-off between the rise in output generated by enhanced productivity and the cost associated with a higher utilization rate. Because such a trade-off varies across sectors, we investigate whether technology improvement is uniformly distributed across sectors and if not, we quantify the discrepancy in sectoral fiscal multipliers caused by sector differences in technology improvement. Since exporting firms are far more productive than non-exporting firms, a natural way to allow for asymmetric technological change across sectors is to make the distinction between a traded (indexed by the superscript  $H$ ) vs. non-traded sector (indexed by the superscript  $N$ ).

To discipline our empirical investigation, we decompose below the government spending multiplier and emphasize the role of technology. We consider an initial steady-state where prices are those at the base year so that real GDP,  $Y_R$ , collapses to nominal GDP,  $Y$ , initially. The percentage change in real GDP relative to its initial steady-state following a rise in government spending, or the aggregate fiscal multiplier, is denoted by  $\hat{Y}_{R,t}$ .<sup>6</sup> Using the fact that real GDP is the sum of value added at constant prices, i.e.,  $Y_{R,t} = P^H Y_t^H + P^N Y_t^N$ , and log-linearizing in the neighborhood of the initial steady-state shows that the aggregate fiscal multiplier is the sum of sectoral fiscal multipliers:

$$\hat{Y}_{R,t} = \nu^{Y,H} \hat{Y}_t^H + \nu^{Y,N} \hat{Y}_t^N, \quad (1)$$

where we denote the value added share of sector  $j$  by  $\nu^{Y,j} = \frac{P^j Y^j}{Y}$  and  $\hat{Y}_t^j = \frac{Y_t^j - Y^j}{Y^j}$  measures the percentage deviation of value added (at constant prices) relative to its initial steady-state. Note that  $\nu^{Y,H} + \nu^{Y,N} = 1$ .

While we estimate the size of sectoral government spending multipliers, we also aim at uncovering the factors that rationalize cross-sector differences in government spending

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<sup>6</sup>To ease the discussion in this subsection, we refer to  $\hat{Y}_{R,t}$  as the aggregate spending multiplier and  $\nu^{Y,j} \hat{Y}_t^j$  as the sectoral fiscal multiplier although it is an abuse of language as both are computed (empirically and numerically) later as the ratio of the present value of cumulative change in value added to the present value of cumulative change in government consumption over a  $t$ -year horizon.

multipliers. Subtracting  $\hat{Y}_{R,t}$  from both sides of (1) leads to the sum of the excess (measured in percentage point of GDP) of sectoral value added growth over real GDP growth, i.e.,  $0 = d\nu_t^{Y,H} + d\nu_t^{Y,N}$  where  $d\nu_t^{Y,j} = \nu^{Y,j} (\hat{Y}_t^j - \hat{Y}_{R,t})$ . By rearranging the above equality as follows  $\nu^{Y,j} \hat{Y}_t^j = \nu^{Y,j} \hat{Y}_{R,t} + d\nu_t^{Y,j}$ , it is straightforward to see that  $d\nu_t^{Y,j}$  is a sufficient statistic to measure the extent to which the aggregate government spending multiplier is distributed symmetrically across sectors. When  $d\nu_t^{Y,j} = 0$ , the rise in real GDP caused by a shock to government consumption is symmetrically distributed across sectors as each sector receives a share of real GDP growth in accordance with their value added share, i.e.,  $\nu^{Y,j} \hat{Y}_t^j = \nu^{Y,j} \hat{Y}_{R,t}$ . If  $d\nu_t^{Y,j} > 0$ , the value added (at constant prices) of sector  $j$  increases disproportionately relative to the value added of the other sector.

As shown below, the change in the sectoral value added share (i.e.,  $d\nu_t^{Y,j}$ ) can be broken down into a technology channel and a factor reallocation channel. Assuming constant returns to scale in production and perfectly competitive markets, denoting the sectoral capital-labor ratio by  $k_t^j \equiv K_t^j/L_t^j$ , log-linearizing the production function leads to  $\hat{Y}_t^j = \hat{\text{TFP}}_t^j + \hat{L}_t^j + (1 - s_L^j) \hat{k}_t^j$  where  $L^j$  is hours worked in sector  $j = H, N$ . Denoting the aggregate capital-labor ratio by  $k_t \equiv K_t/L_t$ , the percentage deviation of real GDP relative to initial steady-state reads  $\hat{Y}_{R,t} = \hat{\text{TFP}}_t + \hat{L}_t + (1 - s_L) \hat{k}_t$ . By using the fact that the percentage deviation of aggregate TFP relative to its initial steady-state is equal to the weighted sum of the percentage deviation of sectoral TFP relative to initial steady-state:

$$\hat{\text{TFP}}_t = \nu^{Y,H} \hat{\text{TFP}}_t^H + (1 - \nu^{Y,H}) \hat{\text{TFP}}_t^N, \quad (2)$$

the excess of non-traded value added growth over real GDP growth reads as follows (see Online Appendix A):

$$\begin{aligned} d\nu_t^{Y,N} &= - (1 - \nu^{Y,H}) \nu^{Y,H} \left( \hat{\text{TFP}}_t^H - \hat{\text{TFP}}_t^N \right) \\ &+ (1 - \nu^{Y,H}) \left[ \left( \hat{L}_t^N - \hat{L}_t \right) + (1 - s_L^N) \hat{k}_t^N - (1 - s_L) \hat{k}_t \right]. \end{aligned} \quad (3)$$

According to (3), keeping technological change fixed, a rise in the value added share of non-tradables (at constant prices) can be brought about by a labor and/or a capital inflow as captured by the second and the third term on the RHS of (3). Incentives for reallocating production factors toward the non-traded sector come from the biasedness of the demand shock toward non-tradables. Using data from the World Input-Output Database (WIOD) [2013], [2016], we constructed time series for sectoral government consumption and find empirically that the non-traded sector receives on average 80% of government consumption (see column 4 of Table 7). As shown by Cardi et al. [2020], when the intensity of the non-traded sector in the government spending shock, denoted by  $\omega_{GN}$ , is higher than the share of non-tradables in GDP (which averages 64%, see column 1 of Table 7), the demand shock moves productive resources toward the non-traded sector, and thus increases  $\nu_t^{Y,N}$ , keeping technology constant. If government consumption induces exporting firms to increase the

efficiency in the use of labor and capital, the rise in traded relative to non-traded TFP may neutralize the impact of the biasedness of the demand shock toward non-tradables on  $\nu_t^{Y,N}$ .

While changes in sectoral TFPs determine the distribution of real GDP growth across sectors, technology adjustment also shapes the responses of sectoral hours worked as a result of factor-biased technological change (FBTC henceforth). To shed some light on the impact of factor-biased technological adjustment on sectoral hours worked responses, we start with the sectoral decomposition of the rise in total hours worked which says that  $\hat{L}_t$  following a shock to government consumption is equal to the weighted sum of the percentage deviation of sectoral hours worked relative to initial steady-state (i.e.,  $\hat{L}_t^j$ ):

$$\hat{L}_t = \alpha_L^H \hat{L}_t^H + \alpha_L^N \hat{L}_t^N, \quad (4)$$

where  $\alpha_L^j$  is the labor compensation share in sector  $j$ ; note that  $\alpha_L^H + \alpha_L^N = 1$ . Subtracting the share of higher total hours worked received by each sector from the change in sectoral hours worked leads to  $0 = d\nu_t^{L,H} + d\nu_t^{L,N}$  where  $d\nu_t^{L,j}$  is the change in the labor share of sector  $j$ :

$$d\nu_t^{L,j} = \alpha_L^j \left( \hat{L}_t^j - \hat{L}_t \right), \quad j = H, N. \quad (5)$$

Eq. (5) can be rewritten so as to relate the government spending multiplier on non-traded hours worked,  $\alpha_L^N \hat{L}_t^N$ , to the change in the labor share of non-tradables,  $d\nu_t^{L,N}$ , i.e.,  $\alpha_L^N \hat{L}_t^N = \alpha_L^N \hat{L}_t + d\nu_t^{L,N}$ . The negative wealth effect caused by a government spending shock leads households to supply more labor and each sector  $j$  will receive a fraction  $\alpha_L^j$  of  $\hat{L}_t > 0$ . If the government spending shock is biased toward non-tradables, as evidence suggests, the non-traded sector will experience a labor inflow, i.e.,  $d\nu_t^{L,N} > 0$ . In an economy subject to frictions into the movement of capital and labor between the traded sector and the non-traded sector and where firms choose the optimal combination of labor- and capital-augmenting technological change, the ability of the non-traded sector to increase hours worked disproportionately by generating a reallocation of labor, i.e.,  $d\nu_t^{L,N} > 0$ , will depend on barriers to mobility and production technology as discussed below.

Two factors hamper the reallocation of labor toward the non-traded sector: labor mobility costs and endogenous terms of trade. Labor mobility costs amount to assuming that sectoral hours worked are imperfect substitutes. Denoting the elasticity of labor supply across sectors by  $\epsilon$ , the share of hours worked supplied to sector  $j$  is increasing in the wage differential, i.e.,  $\frac{L_t^j}{L_t} = \vartheta^j \left( \frac{W_t^j}{W_t} \right)^\epsilon$  where  $\vartheta^j$  stands for the weight attached to labor supply in sector  $j = H, N$ ,  $W_t^j$  and  $W_t$  are sectoral and aggregate wage rates, respectively. We assume perfectly competitive markets and constant returns to scale in production. Under these assumptions, labor is paid its marginal product. Denoting the labor income share by  $s_L^j$ , the marginal revenue product of labor,  $s_{L,t}^j \frac{P_t^j Y_t^j}{L_t^j}$ , must equate the wage rate  $W_t^j$ . The same logic applies at an aggregate level, i.e.,  $s_{L,t} \frac{Y_t}{L_t} = W_t$  where  $s_{L,t}$  is the aggregate LIS,  $Y_t$  is GDP at current prices. Dividing  $W_t^N$  by  $W_t$ , making use of the labor supply schedule to

eliminate the relative wage  $W_t^N/W_t$  and solving for the labor share of non-tradables leads to (see Online Appendix B):

$$\frac{L_t^N}{L_t} = (1 - \vartheta)^{\frac{1}{1+\epsilon}} \left( \frac{s_{L,t}^N}{s_{L,t}} \right)^{\frac{\epsilon}{1+\epsilon}} \left( \omega_t^{Y,N} \right)^{\frac{\epsilon}{1+\epsilon}}, \quad (6)$$

where  $\omega_t^{Y,N}$  is the value added share of non-tradables at current prices and the aggregate LIS is a weighted average of sectoral LISs, i.e.,  $s_L = \nu^{Y,H} s_L^H + (1 - \nu^{Y,H}) s_L^N$ . In a model where production functions are Cobb-Douglas, LISs remain fixed. Under this assumption, (6) says that the labor share of non-tradables,  $L_t^N/L_t$ , increases if the demand shock raises the value added share of non-tradables at current prices. For  $\omega_t^{Y,N}$  to increase, the demand shock must be biased toward non-tradables. Because the traded sector also receives a share of the rise in government spending and experiences an increase in its relative price, by mitigating the rise in  $\omega_t^{Y,N}$ , the appreciation in the terms of trade acts like a barrier to mobility. As labor mobility costs are higher (i.e.,  $\epsilon$  takes lower values),  $L_t^N/L_t$  increases less for a given change in  $\omega_t^{Y,N}$ . If sectoral goods are produced by means of CES production functions, sectoral LISs are no longer constant and their adjustment can amplify or mitigate the government spending multiplier on non-traded labor. More specifically, a rise in  $s_{L,t}^N/s_{L,t}$  tilts the demand for labor toward the non-traded sector which increases the government spending multiplier on non-traded hours worked by causing a shift of labor toward the non-traded sector.

## 2.2 VAR Model and Identification

To conduct our empirical study, we compute the responses of selected variables by using a two-step estimation procedure. We first identify shocks to government consumption by considering a baseline VAR model where government spending is ordered before the other variables. In the second step, we trace out the dynamic effects of the identified shock to government consumption by using Jordà's [2005] single-equation method.

The first step amounts to adopting the standard Cholesky decomposition pioneered by Blanchard and Perotti [2002]. More specifically, we estimate the reduced-form VAR model in panel format on annual data:

$$Z_{i,t} = \alpha_i + \alpha_t + \beta_i t + \sum_{k=1}^2 A^{-1} B_k Z_{i,t-k} + A^{-1} \epsilon_{i,t}, \quad (7)$$

where subscripts  $i$  and  $t$  denote the country and the year. The vector of endogenous variables is denoted by  $Z_{i,t}$ ,  $k$  is the number of lags; the specification includes country fixed effects,  $\alpha_i$ , time dummies,  $\alpha_t$ , and country-specific linear time trends;  $A$  is a matrix that describes the contemporaneous relation among the variables collected in vector  $Z_{i,t}$ ,  $B_k$  is a matrix of lag specific own- and cross-effects of variables on current observations, and the vector  $\epsilon_{i,t}$  contains the structural disturbances which are uncorrelated with each other. In line

with the current practice, we include two lags in the regression model and use a panel OLS regression to estimate the coefficients  $A^{-1}B_k$  and the reduced-form innovations  $A^{-1}\epsilon_{i,t}$ . Like Blanchard and Perotti [2002], we base the identification scheme on the assumption that there are some delays inherent to the legislative system which prevents government spending to respond endogenously to contemporaneous output developments. Such an assumption amounts to assuming that the matrix  $A$  is lower-triangular and thus government spending is exogenous within the year.<sup>7</sup> The VAR model we estimate in the first step includes government final consumption expenditure, real GDP, total hours worked, private investment, the real consumption wage, and aggregate total factor productivity, where all variables are logged, while all quantities are expressed in real terms and scaled by the working age population.

In the second step, we estimate the effects on selected variables detailed later by using the Jordà's [2005] single-equation method. The local projection method amounts to running a series of regressions of each variable of interest on a structural identified shock for each horizon  $h = 0, 1, 2, \dots$ :

$$x_{i,t+h} = \alpha_{i,h} + \alpha_{t,h} + \beta_{i,h}t + \psi_h(L) z_{i,t-1} + \gamma_h \epsilon_{i,t}^G + \eta_{i,t+h}, \quad (8)$$

where  $\alpha_{i,h}$  are country fixed effects,  $\alpha_{t,h}$  are time dummies, and we include country-specific linear time trends;  $x$  is the logarithm of the variable of interest,  $z$  is a vector of control variables (i.e., past values of government spending and of the variable of interest),  $\psi_h(L)$  is a polynomial (of order two) in the lag operator and  $\epsilon_{i,t}^G$  is the identified government spending shock. Country fixed effects and country-specific linear time trend control for countries' characteristics which are time-invariant and time-varying respectively, while time dummies control for macroeconomic shocks which are common across countries.

### 2.3 Data Construction

Before presenting evidence on fiscal transmission across sectors, we briefly discuss the dataset we use. Our sample contains annual observations and consists of a panel of 18 OECD countries. The baseline period is running from 1970 to 2015. All quantities are logged, expressed in real terms and scaled by the working age population. Government final consumption expenditure ( $G_{i,t}$ ) in volume is taken from OECD Economic outlook.

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<sup>7</sup>While using annual data makes the assumption of government spending being unresponsive to current output developments due to decision and implementation lags in the legislative process less relevant, the test performed by Born and Müller [2012] reveals that the assumption that government spending is predetermined within the year cannot be rejected. In Online Appendix M.1, based on the presumption that this industry is government-dominated, we perform a robustness analysis by excluding the industry "Community Social and Personal Services" from non-tradable industries'. In doing this, we purge for the potential and automatic link between non-traded value added and public spending, see e.g., Beetsma and Giuliodori [2008]. While the labor effects are mitigated quantitatively when we remove this industry from the non-traded sector, all of our conclusions hold. In Online Appendix M.2, we also conduct an investigation of the potential presence of anticipation effects by using a dataset constructed by Born, Juessen and Müller [2013] which contains one year-ahead OECD forecasts for government spending. Our estimates show that our main results are not affected by the inclusion of forecasts for government spending growth.



We describe below how we construct time series at a sectoral level. For more details, see Online Appendix C.

Since our primary objective is to quantify the role of the technology channel in determining the sectoral effects of a government spending shock, we describe below how we construct time series at a sectoral level. Our sample covers eleven 1-digit ISIC-rev.3 industries which are split into traded and non-traded sectors by adopting the classification by De Gregorio et al. [1994]. Agriculture, hunting, forestry and fishing; Mining and quarrying; Total manufacturing; Transport, storage and communication are classified as traded industries. Following Jensen and Kletzer [2006], we updated the classification by De Gregorio et al. [1994] by treating Financial intermediation as a traded industry. Electricity, gas and water supply; Construction; Wholesale and retail trade; Hotels and restaurants; Real estate, renting and business services; Community, social and personal services are classified as non-traded industries.<sup>8</sup>

Once industries have been classified as traded or non-traded, series for sectoral value added in current (constant) prices are constructed by adding value added in current (constant) prices for all sub-industries  $k$  in sector  $j = H, N$ , i.e.,  $P_{it}^j Y_{it}^j = \sum_k P_{k,it}^j Y_{k,it}^j$  ( $\bar{P}_{it}^j Y_{it}^j = \sum_k \bar{P}_{k,it}^j Y_{k,it}^j$  where the bar indicates that prices  $P^j$  are those of the base year), from which we construct price indices (or sectoral value added deflators),  $P_{it}^j$ . Normalizing base year price indices  $\bar{P}^j$  to 1, the relative price of non-tradables,  $P_{it}$ , is defined as the ratio of the non-traded value added deflator to the traded value added deflator (i.e.,  $P_{it} = P_{it}^N / P_{it}^H$ ). The relative price of home-produced traded goods (or the TOT, denoted by  $P_{it}^H$ ) is constructed as the ratio of the traded value added deflator ( $P_{it}^H$ ) to the price deflator of imported goods and services ( $P_{it}^F$ ). The same logic applies to constructing series for hours worked ( $L^j = \sum_k L_{k,it}^j$ ) and labor compensation in the traded and the non-traded sectors which allow us to construct sectoral wages,  $W_{it}^j$ . We also construct the share of hours worked and value added (at constant prices) of sector  $j$  in total hours worked and GDP, respectively, denoted by  $\nu_{it}^{L,j}$  and  $\nu_{it}^{Y,j}$ . To estimate the redistributive effects and infer FBTC, we calculate the LIS for each sector  $j$ , denoted by  $s_{L,it}^j$ , as the ratio of labor compensation to valued added at current prices in sector  $j$ .

We construct time series for capital-utilization-adjusted sectoral TFPs,  $Z^j$ , to approximate technical change. Sectoral TFPs are constructed as Solow residuals from constant-price (domestic currency) series of value added,  $Y_{it}^j$ , capital stock,  $K_{it}^j$ , and hours worked,  $L_{it}^j$ :

$$\widehat{\text{TFP}}_{it}^j = \hat{Y}_{it}^j - s_{L,i}^j \hat{L}_{it}^j - \left(1 - s_{L,i}^j\right) \hat{K}_{it}^j, \quad (9)$$

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<sup>8</sup>Because "Financial Intermediation" and "Real Estate, Renting and Business Services" are made up of sub-sectors which display a high heterogeneity in terms of tradability and "Hotels and Restaurants" has experienced a large increase in tradability over the last fifty years, we perform a sensitivity analysis with respect to the classification for the three aforementioned sectors in Online Appendix M.1. Treating "Financial Intermediation" as non-tradables or classifying "Hotels and Restaurants" or "Real Estate, Renting and Business Services" as tradables does not affect our main results.



where  $s_{L,i}^j$  is the LIS in sector  $j$  averaged over the period 1970-2015. To obtain series for the capital stock in sector  $j$ , we first compute the overall capital stock by adopting the perpetual inventory approach, using constant-price investment series taken from the OECD's Annual National Accounts. Following Garofalo and Yamarik [2002], we split the gross capital stock into traded and non-traded industries by using sectoral valued added shares. Once we have constructed the Solow residual for the traded and the non-traded sector, we construct a measure for technological change by adjusting the Solow residual with the capital utilization rate denote by  $u_{it}^{K,j}$ :

$$\hat{Z}_{it}^j = \hat{\text{TFP}}_{it}^j - \left(1 - s_{L,i}^j\right) \hat{u}_{it}^{K,j}, \quad (10)$$

where we follow Imbs [1999] in constructing time series for  $u_{it}^{K,j}$ , see Appendix D.

## 2.4 Sectoral Effects of Government Spending Shocks: VAR Evidence

We generated impulse response functions by means of local projections. The dynamic adjustment of variables to an exogenous increase in government spending by 1% of GDP is displayed by the solid blue line in Fig. 2. The shaded areas indicate 90% confidence bounds. The horizontal axis of each panel measures the time after the shock in years and the vertical axis measures deviations from trend. Responses of sectoral value added and sectoral hours worked are re-scaled by the sample average of sectoral value added to GDP and sectoral labor compensation share, respectively. As such, on impact the responses of sectoral value added at constant prices and sectoral hours worked can be interpreted as government spending multipliers on value added and labor as they are expressed in percentage point of GDP and total hours worked, respectively. We also compute the government spending multipliers over a six-year horizon by computing the ratio of the present value of the cumulative change in value added/labor to the present value of the cumulative change in government consumption, setting the world interest rate to 3% in line with our estimates summarized in Table 6.

**Aggregate effects.** The first row of Fig. 2 displays the aggregate effects of a shock to government consumption. As shown in Fig. 2(a), government consumption follows a hump-shaped response and displays a high level of persistence since it takes more than eight years before government consumption is restored back toward its initial level. Fig. 2(b) and Fig. 2(c) reveals that a rise in government consumption has a strong expansionary effect on total hours worked and real GDP. Total hours worked increase by 0.9% on impact while real GDP increases by 1.2%. The government spending multiplier on real GDP and total hours worked average 1.4 and 1.15, respectively, the first six years, both responses being statistically significant over this period. One key factor that generates a multiplier on real GDP larger than one is technology since 39% of real GDP growth is driven by aggregate

TFP growth (displayed by Fig. 2(d)) over a six-year horizon.<sup>9</sup> We find some strong support of this finding by adapting the methodology proposed by Sims and Zha [2006], Bachman and Sims [2012]. We find that the fiscal multiplier is reduced by 42% when the response of TFP to a shock to government consumption is shut down.<sup>10</sup>

**Government spending multiplier on sectoral labor.** The second row of Fig. 2 displays the dynamic adjustment of sectoral hours worked. Fig. 2(e) and 2(f) reveals that a shock to government consumption by 1% of GDP increase both traded and non-traded hours worked but only the latter is statistically significant. More specifically, the government spending multiplier on non-traded hours worked averages 1.02 ppt of total hours worked while the government spending multiplier on traded hours worked averages 0.13 ppt of total hours worked. Therefore, the rise in non-traded hours worked contribute 88% to the increase in total hours worked. Fig. 2(g) shows the response of the labor share of non-tradables, i.e.,  $L^N/L$ , which measures the change in non-traded hours worked driven by labor reallocation only. On average, over the first six years, the non-traded goods-sector share of total hours worked increases by 0.3 ppt of total hours worked. The shift of labor toward the non-traded sector contributes 29% to the rise in  $L^N$ . As mentioned previously, the reallocation of labor toward the non-traded sector is driven by the biasedness of the demand shock toward non-tradables.<sup>11</sup> The second factor that encourages labor to move toward the non-traded sector is the non-traded LIS which builds up relative to the traded LIS, as displayed by Fig. 2(h). As detailed below, the rise in  $s_L^N/s_L^H$  is brought about by technological change biased toward labor which amplifies the reallocation of labor toward the non-traded sector and thus rationalizes a government spending multiplier on non-traded hours worked of about one.<sup>12</sup>

**Government spending multiplier on sectoral value added and technology.** The third row of Fig. 2 shows that a rise in government consumption increases both traded and non-traded value added at constant prices. Both responses are statistically significant. Over the first six years, the government spending multiplier on traded value added averages 0.52 ppt of GDP while the government spending multiplier on non-traded value added averages 0.89 ppt. In contrast to labor, the non-traded sector contributes 64% only to real GDP growth, a value which collapses to the share of non-tradables in GDP. In accordance with

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<sup>9</sup>There is a slight discrepancy between the response of real GDP which is constructed as the sum of traded and non-traded value added and the sum of the responses of traded and non-traded value added. To ensure consistency, we calculate the government spending multiplier and the contribution of TFP growth to real GDP growth by calculating the response of real GDP as the sum of the responses of  $Y^H$  and  $Y^N$ .

<sup>10</sup>When we estimate a VAR model which includes government consumption, aggregate TFP and real GDP, the government spending multiplier in real GDP over a six-year horizon averages 1.25 and thus is slightly smaller than that obtained when we adopt a two-step approach. When we shut down technological change, the government spending multiplier averages 0.73 the first six years.

<sup>11</sup>See Fig. 9 relegated to the Online Appendix E which shows that a shock to government consumption by 1% of GDP is associated with a rise in  $G^N$  by 0.8% of GDP on impact.

<sup>12</sup>We compute the LIS like Gollin [2002], i.e., labor compensation is defined as the sum of compensation of employees plus compensation of self-employed. We find that our results are robust to alternative constructions of the LIS, see Online Appendix M.3.

this observation, Fig. 2(k) reveals that the value added share of non-tradables (at constant prices) remains unresponsive to the shock, thus confirming that the government spending multiplier on real GDP is symmetrically distributed across sectors, i.e., in accordance with their value added share. This result is puzzling because the government spending shock is strongly biased toward non-tradables and triggers a reallocation of productive resources toward the non-traded sector. As shown in Fig. 2(l), traded TFP increases significantly relative to non-traded TFP. On average, over the first six years, the TFP differential between tradables and non-tradables amounts to 1.5% on average (per year). The technology gap is large enough to offset the impact of the reallocation of productive resources toward the non-traded sector and leaves unchanged  $\nu_t^{Y,N}$ .<sup>13</sup>

**Fiscal policy and utilization-adjusted TFP.** The last row of Fig. 2 displays the dynamic adjustment of TFP and FBTC for tradables and non-tradables, which are both adjusted with capital utilization to reflect the true variations of technological change, see Basu, Fernald and Kimball [2006]. Fig. 2(m) and 2(n) show the responses of traded and non-traded TFP once we control for varying utilization of capital at a sectoral level. See Online Appendix D where we detail the adaptation of the approach proposed by Imbs [1999] to measure the capital utilization rate in the traded and non-traded sector by considering CES production functions. After correcting for capital utilization, Fig. 2(m) and Fig. 2(n) confirm that technology improves in the traded sector and is essentially unchanged in the non-traded sector. Because the capital utilization rate increases in the traded relative to the non-traded sector, these findings indicate that the rise in the relative TFP of tradables shown in Fig. 2(l) is driven by both a higher utilization of capital and a technology improvement in the traded sector.

**Fiscal policy and utilization-adjusted FBTC.** While the rise in traded relative to non-traded TFP leads real GDP growth to be uniformly distributed across sectors, the last two panels of the last row of Fig. 2 show that the differential in FBTC between non-tradables and tradables can rationalize the concentration of labor growth in the non-traded sector. To measure capital-utilization-adjusted-FBTC in the traded and non-traded sector, we draw on Caselli and Coleman [2006] and Caselli [2016]. Denoting the elasticity of substitution between capital and labor by  $\sigma^j$ , capital- and labor-augmenting efficiency by  $B_t^j$  and  $A_t^j$ , respectively, our measure of capital-utilization-adjusted-FBTC, denoted by

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<sup>13</sup>We are aware that the traded and non-traded sectors are made-up of several industries and variations in TFP in broad sectors could be the result of changes in the value added share of sub-sectors (between-effect) rather than a technology improvement within the industry (within-effect). Our dataset covers eleven industries and in Online Appendix M.4, we conduct the same empirical analysis as in the main text but at a disaggregate industry level. First, we find that the behavior of industries classified as tradables experience an increase in traded TFP while the responses of TFP in non-traded industries are more heterogenous and clustered around the horizontal axis.

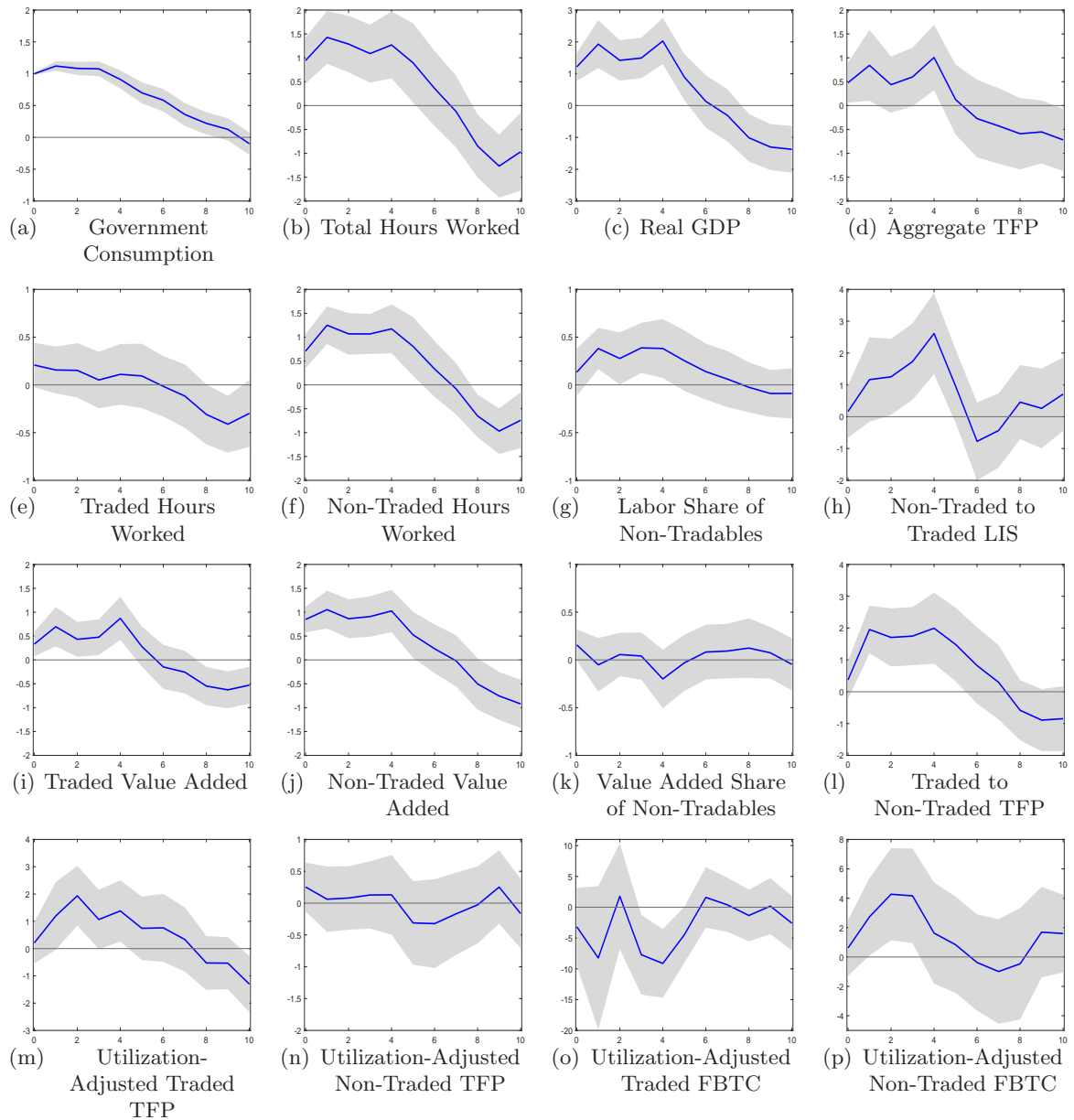


Figure 2: Sectoral Effects of a Shock to Government Consumption. *Notes:* The solid blue line shows the response of aggregate and sectoral variables to an exogenous increase in government final consumption expenditure by 1% of GDP. Shaded areas indicate the 90 percent confidence bounds. To estimate the dynamic responses to a shock to government consumption, we adopt a two-step method. In the first step, the government spending shock is identified by estimating a VAR model that includes real government final consumption expenditure, real GDP, total hours worked, the real consumption wage, and aggregate TFP. In the second step, we estimate the effects by using Jordà's [2005] single-equation method. Horizontal axes indicate years. Vertical axes measure percentage deviation from trend in GDP units (sectoral value added, sectoral value added share, labor income share), percentage deviation from trend in total hours worked units (sectoral hours worked, labor share), percentage deviation from trend (sectoral TFPs, sectoral FBCT). Sample: 18 OECD countries, 1970-2015, annual data.

FTBC $_{t,adjK}^j$ , reads (see Online Appendix F):

$$\text{FTBC}_{t,adjK}^j = \left( \frac{B_t^j / \bar{B}^j}{A_t^j / \bar{A}^j} \right)^{\frac{1-\sigma^j}{\sigma^j}} = \frac{S_t^j}{\bar{S}^j} \left( \frac{k_t^j}{\bar{k}^j} \right)^{-\frac{1-\sigma^j}{\sigma^j}} \left( \frac{u_t^{K,j}}{\bar{u}^{K,j}} \right)^{-\frac{1-\sigma^j}{\sigma^j}}, \quad (11)$$

where a bar refers to averaged values of the corresponding variable over 1970-2015. To construct time series for FTBC $_{t,adjK}^j$ , we plug estimates for the elasticity of substitution between capital and labor,  $\sigma^j$ , and time series for the ratio of labor to capital income share,  $S_t^j = \frac{s_{L,t}^j}{1-s_{L,t}^j}$ , the capital-labor ratio,  $k_t^j$ , and the capital utilization rate,  $u_t^{K,j}$ , in sector  $j = H, N$ . An increase in our measure FTBC $_{t,adjK}^j$  described by (11) means that technological change is biased toward labor. Since this measure crucially depends on  $\sigma^j$ , we have estimated this parameter for both sectors, see Online Appendix J.3. We find empirically that  $\sigma^H = 0.64$  for the traded sector and  $\sigma^N = 0.80$  for the non-traded sector for the whole sample, as summarized in columns 18 and 19 of Table 7. Evidence displayed by Fig. 2(o) and 2(p) suggests that technological change is biased toward capital in the traded sector while technological change is biased toward labor in the non-traded sector. These findings are consistent with the rise in non-traded LIS relative to the traded LIS shown in Fig. 2(h). Because capital and labor are gross complements in production, our evidence indicates that traded firms tend to lower  $B^H/A^H$  and non-traded firms to increase  $B^N/A^N$ . In Online Appendix K.2, we document evidence which rationalizes the decision to bias technological change toward one specific factor. Because the non-traded sector must pay higher wages to encourage workers to shift, non-traded firms increase labor-augmenting productivity to mitigate the rise in the labor cost. Since labor- and capital-augmenting productivity are strong complements along the technology frontier, capital productivity disproportionately increases, thus generating a rise in  $B^N/A^N$ . The other way around is true in the traded sector. For both sectors, point estimates of FTBC $_{t,adjK}^j$  are associated with wide confidence bounds which may suggest that the direction of FBTC varies substantially across countries. We explore this assumption in the next subsection.

## 2.5 Fiscal Transmission and Technology: Cross-Country Differences

Our evidence above reveals that a shock to government consumption leads traded firms to improve their technology and non-traded firms to increase capital- relative to labor-augmenting efficiency. In this subsection, we further explore the role of the technology channel following a shock to government consumption by considering cross-country differences. To conduct this analysis, we use a two-step estimation procedure as in section 2.2, except that we consider one country at a time and plot responses of sectoral variables against measures of technology in Fig. 3. In Fig. 3(a), we plot the change in the value added share of non-tradables against the TFP differential between tradables and non-tradables, while in Fig. 3(d) we plot the variation in the non-traded-goods-share of total hours worked

against the capital-utilization-adjusted FBTC differential between non-tradables and tradables. Because  $d\nu_t^{Y,N}$  and  $d\nu_t^{L,N}$  determine the size of the government spending multiplier on non-traded value added and hours worked, respectively, in the first column we show the present value of the cumulative change of the corresponding variable divided by the present value of the cumulative change in government consumption, both computed over a six-year horizon.<sup>14</sup> In the second and third column Fig. 3, we focus on impact responses.

**Value added share of non-tradables and relative TFP.** Evidence in Fig. 3(a) reveals that  $\text{TFP}_t^H/\text{TFP}_t^N$  increases in two-third of the countries of our sample while traded TFP declines significantly relative to non-traded TFP in six countries, including Canada, France, Korea, Netherlands, Spain, Sweden.<sup>15</sup> The downward sloping trend line shows that the value added share of non-tradables increases less in countries where traded relative to non-traded TFP increases more since the TFP differential offsets the impact of the reallocation of labor triggered by the biasedness of the demand shock toward non-tradables (see eq. (3)).

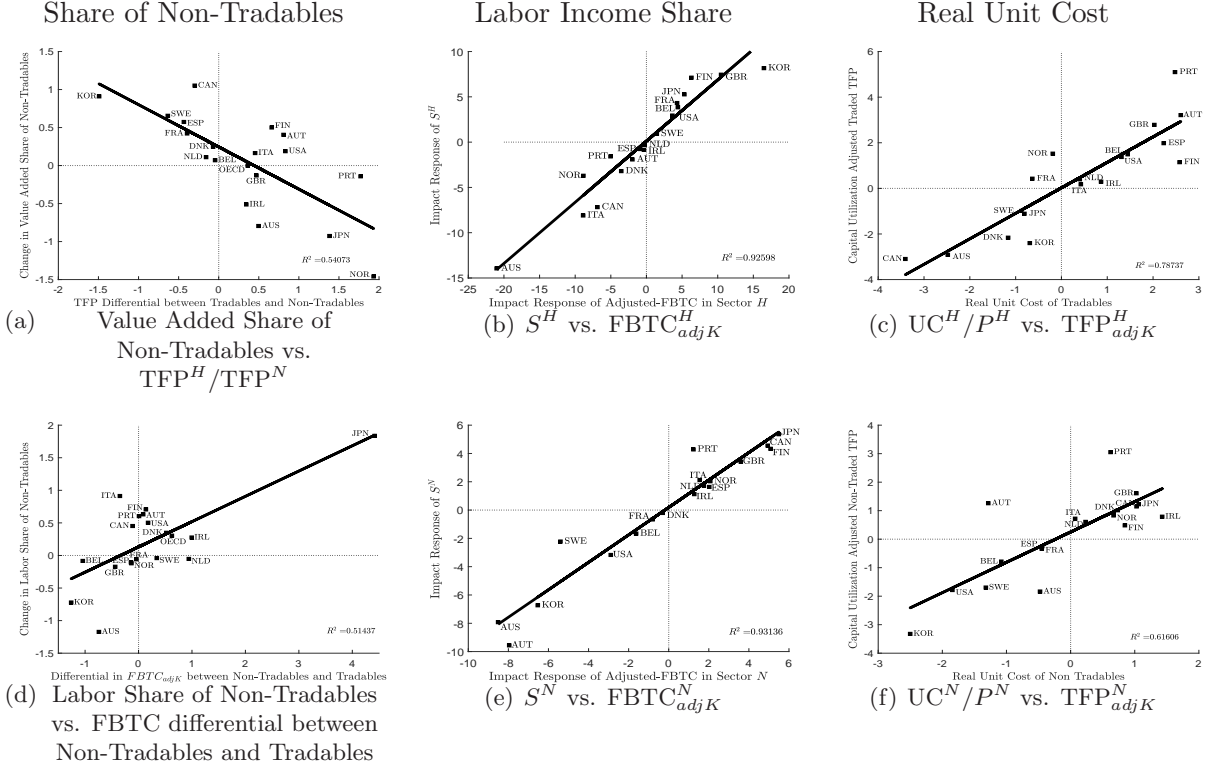
**Unit cost and technology adjustment.** The wide cross-country dispersion in the adjustment of the relative TFP of tradables shown in Fig. 3(a) suggests that technology decisions vary substantially between OECD economies. In the top panel of the third column of Fig. 3, we shed some light on the factor which leads firms to increase the efficiency in the use of inputs. In a model with flexible prices, firms equate their prices to the unit cost for producing divided by the capital-utilization-adjusted-TFP. In face of a higher unit cost for producing, sectors can increase prices or improve technology or both. Firms will decide to further increase the efficiency in the use of inputs as the cost of improving technology is lower. As detailed in Online Appendix G, we construct a measure of the unit cost for producing and we divide this measure by the value added deflator of the corresponding sector to control for price adjustments. In Fig. 3(c) and Fig. 3(f), we plot the response of the capital-utilization-adjusted-TFP on the vertical axis for tradables and non-tradables, respectively, against the change in the real unit cost on the horizontal axis. The trend line reveals a strong positive cross-country relationship between the decision to adjust technology and the real cost for producing which suggests that technology improvements are driven by a cost-minimization strategy.

**Sectoral LIS and FBTC.** In the second column of Fig. 3, we plot the responses of the ratio of labor to capital income share,  $S_t^j = \frac{s_{L,t}^j}{1-s_{L,t}^j}$ , against the adjustment of our measure of capital-utilization-adjusted-FBTC (see eq. (11)).<sup>16</sup> As can be seen in Fig. 3(b)

<sup>14</sup>In doing this, we control for cross-country differences in the adjustment of  $G_t$ . When we compute the present value of the cumulative change of a variable at a country level, we take the (country-specific) interest rate from the first column of Table 6.

<sup>15</sup>While traded TFP also declines relative to non-traded TFP in Belgium and Denmark, this rise is not confirmed when we reconstruct the change in the sectoral TFP as follows  $\text{TFP}_t^j = \hat{Y}_t^j - \hat{L}_t^j - (1 - s_L^j) \hat{k}_t^j$  due to the uncertainty surrounding estimates at a country level.

<sup>16</sup>Since  $\hat{S}_{it}^j = \frac{s_{L,it}^j}{1-s_{L,it}^j}$  and thus the percentage deviation of the ratio of labor to capital income share relative



**Figure 3: Effects of Government Spending Shocks and Cross-Country Differences in Technology Adjustment.** *Notes:* Fig. 3 plots responses to an exogenous increase in government consumption by 1% of GDP against measures of technological change. Fig. 3(a) plots the present value of the cumulative change in the value added share of non-tradables,  $\nu_t^{Y,N}$  (vertical axis) against the present value of the cumulative change in the ratio of traded TFP to non-traded TFP (horizontal axis), both computed over a six-year horizon and divided by the present value of the cumulative change in government consumption. In accordance with eq. (3), the TFP differential is scaled by multiplying by  $(1 - \nu^{Y,H}) \nu^{Y,H}$ . Fig. 3(d) plots the present value of the cumulative change in the share of non-traded hours worked in total hours worked (vertical axis) against the present value of the cumulative change in the differential in capital-utilization-adjusted FBTC between non-tradables and tradables (horizontal axis), both computed over a six-year horizon and divided by the present value of the cumulative change in government consumption. To construct time series for  $FBTC_{t,adjK}^j$  for each country, we use eq. (11) and take estimates of the elasticity of substitution between capital and labor  $\sigma^j$  from columns 18 and 19 of Table 7. The response of  $FBTC_{t,adjK}^j$  in sector  $j$  is adjusted with  $1 - s_L^j$  and the differential is scaled by  $\alpha_L^H \alpha_L^N$ , see the last of eq. (178) in Online Appendix K.2. The second column of Fig. 3 plots impact responses of sectoral LISs (vertical axis) against the adjustment of capital-utilization adjusted FBTC (the horizontal axis). The third column of Fig. 3 plots impact responses of the ratio of the unit cost for producing to the value added deflator (vertical axis) against the adjustment of capital-utilization adjusted TFP (horizontal axis). The construction of the unit cost for producing is detailed in Online Appendix G. Sample: 18 OECD countries, 1970–2015, annual data.



for tradables and Fig. 3(e) for non-tradables, a rise in government spending increases the share of the value added paid to workers in half of the countries while the LIS falls in the rest of the sample. The trend line reveals that there exists a strong positive cross-country relationship between the variations of the LIS and capital-utilization-adjusted-FBTC. For example, countries which lie in the north-east of the scatter-plot experience an increase in the LIS driven by technological change biased toward labor. By contrast, for countries which lie in the south-west of the scatter-plot, technological change biased toward capital lowers the LIS.

**Labor share of non-tradables and relative non-traded LIS.** While the non-traded-goods-share of total hours worked increases as the result of the biasedness of the government spending shock toward non-tradables, the reallocation of labor toward the non-traded sector will be amplified if technological change makes non-traded production more labor intensive. This channel is captured a rise in  $s_{L,t}^N/s_{L,t}$  in eq. (6). According to the evidence displayed by the second column of Fig. 3, the non-traded LIS increases relative to the aggregate LIS as long as technological change is biased toward labor in the non-traded sector and is biased toward capital in the traded sector. Fig. 3(d) shows that the change in the labor share of non-tradables is positively correlated with the differential in capital-utilization-adjusted FBTC between non-tradables and tradables. More specifically, in countries positioned in the north-east, technological change is more biased toward labor in the non-traded relative to the traded sector which amplifies the rise in  $L_t^N/L_t$ . Conversely, as can be seen in the south-west of the scatter-plot, when technological change is more biased toward labor (or less biased toward capital) in the traded sector, the labor share of non-tradables can decline such as in Australia, Korea, Norway, Spain, the UK.

## 2.6 Isolating the Pure Technology Effect in Driving Fiscal Transmission

In this subsection, we move a step further and isolate the pure technology transmission mechanism following a shock to government consumption. More specifically, the responses of sectoral value added (and labor) to an exogenous increase in government spending can be broken down into two effects. First, a government spending shock has a direct impact on sectoral value added and sectoral hours worked by increasing the demand for the sectoral good. In addition to this standard mechanism, there is a second effect passing through technological change. Because a rise in government consumption puts upward pressure on the unit cost for producing, firms may decide to increase the efficiency in the use of inputs and to modify the mix of capital- and labor-augmenting efficiency. By increasing its sectoral TFP, technology improvements performed by one sector increase its output multiplier relative to the other sector. If firms increase capital- relative to labor-augmenting

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to its initial steady-state is proportional to the percentage change in the LIS,  $\hat{s}_{L,t}^j$ , we refer interchangeably to the LIS or the ratio of factor income share as long as it does not cause confusion.



productivity in one sector, the technology of production becomes more intensive in labor if capital and labor are gross complements which increases the relative magnitude of the fiscal multiplier on labor of this sector. To study how important the response of technology is in the transmission of a government spending shock, we generate the dynamic adjustment of sectoral variables if technological factors were unresponsive to the fiscal shock and compare it to the actual response of sectoral variables.

The decomposition implemented below is based on the methodology proposed by Sims and Zha [2006], and Bachmann and Sims [2012]. This method amounts to constructing the hypothetical sequence of the other shocks in the system so that the response of technological factors to a government spending shock is zero at all horizons. Online Appendix I details our empirical strategy. We consider three VAR models where all variables are logged and quantities are divided by population. Within each VAR specification, government consumption is ordered first, technology is ordered second and sectoral variables are ordered third. Fig. 4 shows the dynamic effects of an exogenous increase in government consumption by 1% of GDP. The blue line displays the actual response of variables while the red line shows the hypothetical response of the same variable when we restrict government consumption not to move technology at any horizon. Whilst the first row shows the response of government consumption, the second and the third row shows the dynamic adjustment of technology and sectoral variables. To start with, as can be seen in the first row, the endogenous response of government consumption remains fairly unchanged whether technology is shut down or not.

**Value added share of non-tradables.** In the first column of Fig. 4, we plot the response of the ratio of traded to non-traded TFP and the dynamic adjustment of the value added share of non-tradables to a shock to government consumption, in the middle and lower panel, respectively.<sup>17</sup> In Fig. 4(d), the blue line shows that traded TFP increases relative to non-traded TFP by 1% on average the first six years. As the productivity differential builds up, Fig. 4(g) reveals that the value added share of non-tradables does not increase significantly. In the red line in Fig. 4(d), we shut down the response of the relative productivity of tradables and as can be seen in Fig. 4(g), the hypothetical adjustment of the value added share of non-tradables is distinct from its actual response. Quantitatively, when we divide the present value of the cumulative change in  $\nu_t^{Y,N}$  by the present value of the cumulative change in  $G_t$  over a six-year horizon, we find a rise in the value added share of non-tradables by 0.26 ppt of GDP as the result of the shift of labor and capital toward the non-traded sector when holding the response of  $TFP_t^H/TFP_t^N$  constant. The rise in  $\nu_t^{Y,N}$  is almost divided by a factor of three, as it amounts to 0.09 ppt of GDP, when we allow sectoral TFPs to react to the demand shock. In the latter case, the response

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<sup>17</sup>We estimate a VAR model which includes government consumption, the ratio of traded to non-traded TFP and the value added share of non-tradables.

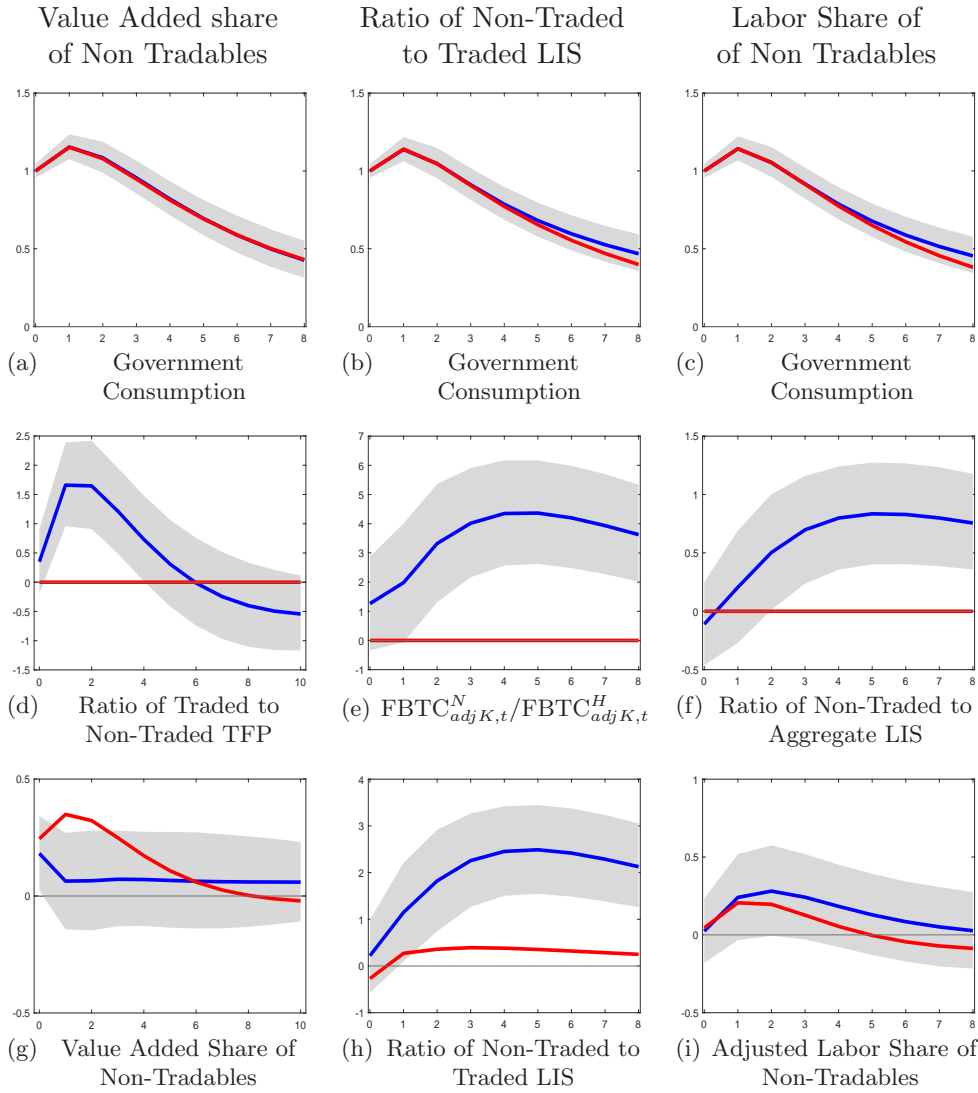


Figure 4: Dynamic Adjustment to Government Spending Shocks: Isolating the Technology Channel. *Notes:* Fig. 4 plots the dynamic adjustment of sectoral variables to an exogenous increase in government consumption by 1% of GDP by isolating the pure technology effect. We plot the actual dynamic adjustment of sectoral variables to a government spending shock in the blue line. The red line shows the hypothetical dynamic adjustment of sectoral variables if technology were unresponsive to the spending shock at all horizons. The first row displays the endogenous response of government consumption to the exogenous fiscal shock. The second row shows the actual dynamic adjustment of technology to a government spending shock in the blue line while the red line keeps the dynamic response of technology unchanged. In the third row, we plot the actual dynamic adjustment of sectoral variables to a government spending shock in the blue line while the red line shows the responses if technology were shut down. The first and the third column displays the dynamic response of the value added share and the labor share of non-tradables, respectively. The second column plots the dynamic response of the ratio of the non-traded to the traded LIS. Sample: 18 OECD countries, 1970-2015, annual data.

of  $\nu_t^{Y,N}$  is not statistically significant however.

**The relative LIS of non-tradables.** In the second column of Fig. 4, we plot the response of the differential in capital-utilization-adjusted-FBTC between non-tradables and tradables in the upper panel and the dynamic adjustment of the ratio of the non-traded to the traded LIS,  $s_{L,it}^N/s_{L,it}^H$ , in the lower panel.<sup>18</sup> As shown in the blue line in Fig. 4(h),  $s_{L,it}^N/s_{L,it}^H$  increases significantly after two years and the rise in the relative LIS of non-tradables averages 1.73% the first six years. The blue line in Fig. 4(e) reveals that the rise in the non-traded relative to the traded LIS is driven by the differential in FBTC.

<sup>18</sup>In accordance with the decomposition of the labor share of non-tradables detailed in Online Appendix K.2, see. eq. (178), we scale sectoral FBTC by the capital income share, i.e., the blue line in Fig. 4(e) shows  $(1 - s_{L,i}^N) \hat{\text{FBTC}}_{adjK,it}^N - (1 - s_{L,i}^H) \hat{\text{FBTC}}_{adjK,it}^H$

In Online Appendix I, see Fig. 11, we find empirically that technological change is biased toward labor in the non-traded sector and biased toward capital in the traded sector. When we shut down FBTC in the red line, the annual increase in the non-traded relative to the traded LIS averages 0.25% only, the rise in the non-traded LIS being driven by the capital inflow which pushes  $k^N$  up.

**The adjusted labor share of non-tradables.** In the third column of Fig. 4, we quantify the reallocation of labor toward the non-traded sector driven by the response of the non-traded relative to aggregate LIS to a government spending shock. To isolate the pure effect of the movement in the LIS on the labor share of non-tradables, in accordance with (6), we adjust the share of non-traded hours worked in total hours worked with the value added share of non-tradables at current prices augmented with the elasticity of labor supply across sectors, i.e.,  $\frac{L_{it}^N}{L_{it}} \left( \omega_{it}^{Y,N} \right)^{-\frac{\epsilon_i}{1+\epsilon_i}}$ .<sup>19</sup> The blue line in Fig. 4(i) shows the actual rise in the (adjusted) labor share of non-tradables while the red line displays its adjustment if the LIS were unresponsive to the government spending shock. As displayed by the blue line in Fig. 4(f), the shock to government consumption increases the non-traded LIS relative to the aggregate LIS which reflects the fact that the technology of production becomes more labor intensive. When we divide the present value of the cumulative change in  $\nu_t^{L,N}$  by the present value of the cumulative change in  $G_t$  over a six-year horizon, we find a rise in the (adjusted) labor share of non-tradables by 0.11 ppt of total hours worked when we shut down the response of the non-traded relative to the aggregate LIS, and by 0.2 ppt of total hours worked when we allow  $s_{L,it}^N/s_{L,it}$  to respond to the government spending shock. Since the bulk of the variation in LIS is driven by FBTC, our empirical findings reveal that technological change biased toward labor in the non-traded sector and biased toward capital in the traded sector almost doubles the reallocation of labor toward the non-traded sector.

### 3 A Semi-Small Open Economy Model with Tradables and Non-Tradables

We consider a semi-small open economy that is populated by a constant number of identical households and firms that have perfect foresight and live forever. The country is assumed to be semi-small in the sense that it is price-taker in international capital markets, and thus faces a given world interest rate,  $r^*$ , but is large enough on world good markets to influence the price of its export goods. The open economy produces a traded good which can be exported, consumed or invested and imports consumption and investment goods.

<sup>19</sup>Our sample includes eighteen OECD countries which differ in terms of labor mobility costs and intensity of the non-traded sector in the government spending shock. To control for cross-country differences in labor mobility and biasedness of the government spending shock toward non-tradables, we adjust the ratio of non-traded hours worked to total hours worked with  $\left( \omega_{it}^{Y,N} \right)^{-\frac{\epsilon_i}{1+\epsilon_i}}$ . In doing this, we ensure that we capture the rise in the labor share of non-tradables driven by an increase in  $s_L^N/s_L$ .

Besides the home-produced traded good, denoted by the superscript  $H$ , a non-traded sector produces a good, denoted by the superscript  $N$ , for domestic absorption only. The foreign good is chosen as the numeraire. Time is continuous and indexed by  $t$ .

### 3.1 Households

At each instant the representative household consumes traded and non-traded goods denoted by  $C^T(t)$  and  $C^N(t)$ , respectively, which are aggregated by means of a CES function:

$$C(t) = \left[ \varphi^{\frac{1}{\phi}} (C^T(t))^{\frac{\phi-1}{\phi}} + (1-\varphi)^{\frac{1}{\phi}} (C^N(t))^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}}, \quad (12)$$

where  $0 < \varphi < 1$  is the weight of the traded good in the overall consumption bundle and  $\phi$  corresponds to the elasticity of substitution between traded goods and non-traded goods. The traded consumption index  $C^T(t)$  is defined as a CES aggregator of home-produced traded goods,  $C^H(t)$ , and foreign-produced traded goods,  $C^F(t)$ :

$$C^T(t) = \left[ (\varphi^H)^{\frac{1}{\rho}} (C^H(t))^{\frac{\rho-1}{\rho}} + (1-\varphi^H)^{\frac{1}{\rho}} (C^F(t))^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}, \quad (13)$$

where  $0 < \varphi^H < 1$  is the weight of the home-produced traded good and  $\rho$  corresponds to the elasticity of substitution between home- and foreign-produced traded goods. The consumption-based price index  $P_C(t)$  is a function of traded and non-traded prices, denoted by  $P^T(t)$  and  $P^N(t)$ , respectively:

$$P_C(t) = \left[ \varphi (P^T(t))^{1-\phi} + (1-\varphi) (P^N(t))^{1-\phi} \right]^{\frac{1}{1-\phi}}, \quad (14)$$

where the price index for traded goods is a function of the terms of trade denoted by  $P^H(t)$ :

$$P^T(t) = \left[ \varphi^H (P^H(t))^{1-\rho} + (1-\varphi^H) \right]^{\frac{1}{1-\rho}}. \quad (15)$$

As shall be useful later in the quantitative analysis, we denote the relative price of non-tradables by  $P(t) = P^N(t)/P^H(t)$ .

The representative household supplies labor to the traded and non-traded sectors, denoted by  $L^H(t)$  and  $L^N(t)$ , respectively. To put frictions into the movement of labor between the traded sector and the non-traded sector, we assume that sectoral hours worked are imperfect substitutes in the lines of Horvath [2000]:

$$L(t) = \left[ \vartheta^{-1/\epsilon} (L^H(t))^{\frac{\epsilon+1}{\epsilon}} + (1-\vartheta)^{-1/\epsilon} (L^N(t))^{\frac{\epsilon+1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon+1}}, \quad (16)$$

where  $0 < \vartheta < 1$  parametrizes the weight attached to the supply of hours worked in the traded sector and  $\epsilon$  is the elasticity of substitution between sectoral hours worked. The aggregate wage index  $W(\cdot)$  associated with the above defined labor index (16) is:

$$W(t) = \left[ \vartheta (W^H(t))^{\epsilon+1} + (1-\vartheta) (W^N(t))^{\epsilon+1} \right]^{\frac{1}{\epsilon+1}}, \quad (17)$$

where  $W^j(t)$  is the wage rate paid in sector  $j = H, N$ .

The representative agent is endowed with one unit of time, supplies a fraction  $L(t)$  as labor, and consumes the remainder  $1 - L(t)$  as leisure. At any instant of time, households derive utility from their consumption and experience disutility from working. Assuming that the felicity function is additively separable in consumption and labor, the representative household maximizes the following objective function:

$$U = \int_0^{\infty} \left\{ \frac{1}{1 - \frac{1}{\sigma_C}} C(t)^{1 - \frac{1}{\sigma_C}} - \frac{1}{1 + \frac{1}{\sigma_L}} L(t)^{1 + \frac{1}{\sigma_L}} \right\} e^{-\beta t} dt, \quad (18)$$

where  $\beta > 0$  is the discount rate,  $\sigma_C > 0$  the intertemporal elasticity of substitution for consumption, and  $\sigma_L > 0$  the Frisch elasticity of (aggregate) labor supply.

Households supply labor  $L(t)$  and capital services  $K(t)$  and in exchange receive a wage rate  $W(t)$  and a capital rental rate  $R(t)$ . We assume that households choose the level of capital utilization  $u^{K,j}(t)$  in sector  $j$ . They also own the stock of intangible capital  $\bar{Z}^j$  and decide about the level of utilization  $u^{Z,j}(t)$  of existing technology in sector  $j$ .<sup>20</sup> In the sequel, we normalize the stock of knowledge,  $\bar{Z}^j$ , to one as we abstract from endogenous choices on the stock of knowledge.<sup>21</sup> Because households may decide to use more intensively the stock of knowledge in sector  $j$  which increases the efficiency in the use of inputs and thus value added, the counterpart is a rise in factor prices in accordance with the Euler Theorem, i.e.,  $P^j(t)u^{Z,j}(t)Y^j(t) = u^{Z,j}(t)W^j(t)L^j(t) + u^{Z,j}(t)R(t)u^{K,j}(t)K^j(t)$  where  $P^j$  is the value added deflator and  $Y^j$  stands for technology-utilization-adjusted value added. Both the capital  $u^{K,j}(t)$  and technology utilization rate  $u^{Z,j}(t)$  collapse to one at the steady-state. We let the function  $C^{K,j}(t)$  and  $C^{Z,j}(t)$  denote the adjustment costs associated with the choice of capital and technology utilization rates which are increasing and convex functions of utilization rates  $u^{K,j}(t)$  and  $u^{Z,j}(t)$ :

$$C^{K,j}(t) = \xi_1^j (u^{K,j}(t) - 1) + \frac{\xi_2^j}{2} (u^{K,j}(t) - 1)^2, \quad (19a)$$

$$C^{Z,j}(t) = \chi_1^j (u^{Z,j}(t) - 1) + \frac{\chi_2^j}{2} (u^{Z,j}(t) - 1)^2, \quad (19b)$$

where  $\xi_2^j > 0$ ,  $\chi_2^j > 0$  are free parameters; as  $\xi_2 \rightarrow \infty$ ,  $\chi_2 \rightarrow \infty$ , utilization is fixed at unity.

Households can accumulate internationally traded bonds (expressed in foreign good units),  $N(t)$ , that yield net interest rate earnings of  $r^*N(t)$ . Denoting lump-sum taxes by  $T(t)$ , household's flow budget constraint states that real disposable income (on the RHS of the equation below) can be saved by accumulating traded bonds, consumed,  $P_C(t)C(t)$ , or

<sup>20</sup>We assume that the technology utilization rate is Hicks-neutral. We could alternatively assume that households choose the utilization rate of capital- and labor-augmenting efficiency. We have considered this possibility both theoretically and numerically. Since the decision to use more intensively the existing factor-augmenting technology is pro-cyclical, we have to assume an implausible high utilization cost of labor-augmenting productivity in the non-traded sector to produce a rise in capital-augmenting relative to labor-augmenting productivity. The increase in relative capital efficiency cannot generate the magnitude of the rise in the non-traded LIS we estimate empirically.

<sup>21</sup>Bianchi et al. [2019] assume that firms can choose both the technology utilization rate and the stock of knowledge. Because we consider a temporary demand shock and find empirically that utilization-adjusted-sectoral TFP, is restored back toward its initial steady-state level, see Fig. 2(m) and Fig. 2(n), we abstract from endogenous choices on the stock of knowledge.

invested,  $P_J(t)J(t)$ :

$$\begin{aligned}
& \dot{N}(t) + P_C(t)C(t) + P_J(t)J(t) + P^H(t)C^{K,H}(t)\alpha_K(t)K(t) + P^N(t)C^{K,N}(t)(1 - \alpha_K(t))K(t) \\
& + P^H(t)C^{Z,H}(t) + P^N(t)C^{Z,N}(t) = [\alpha_L(t)u^{Z,H}(t) + (1 - \alpha_L(t))u^{Z,N}(t)]W(t)L(t) \\
& + [\alpha_K(t)u^{K,H}(t)u^{Z,H}(t) + (1 - \alpha_K(t))u^{K,N}(t)u^{Z,N}(t)]R(t)K(t) + r^*N(t) - T(t), \tag{20}
\end{aligned}$$

where we denote the share of traded capital in the aggregate capital stock by  $\alpha_K(t) = K^H(t)/K(t)$  and the labor compensation share of tradables by  $\alpha_L(t) = \frac{W^H(t)L^H(t)}{W(t)L(t)}$ .

The investment good is (costlessly) produced using inputs of the traded good and the non-traded good by means of a CES technology:

$$J(t) = \left[ \varphi_J^{\frac{1}{\phi_J}} (J^T(t))^{\frac{\phi_J-1}{\phi_J}} + (1 - \varphi_J)^{\frac{1}{\phi_J}} (J^N(t))^{\frac{\phi_J-1}{\phi_J}} \right]^{\frac{\phi_J}{\phi_J-1}}, \tag{21}$$

where  $0 < \varphi_J < 1$  is the weight of the investment traded input and  $\phi_J$  corresponds to the elasticity of substitution between investment traded goods and investment non-traded goods. The index  $J^T(t)$  is defined as a CES aggregator of home-produced traded inputs,  $J^H(t)$ , and foreign-produced traded inputs,  $J^F(t)$ :

$$J^T(t) = \left[ (\iota^H)^{\frac{1}{\rho_J}} (J^H(t))^{\frac{\rho_J-1}{\rho_J}} + (1 - \iota^H)^{\frac{1}{\rho_J}} (J^F(t))^{\frac{\rho_J-1}{\rho_J}} \right]^{\frac{\rho_J}{\rho_J-1}}, \tag{22}$$

where  $0 < \iota^H < 1$  is the weight of the home-produced traded input and  $\rho_J$  corresponds to the elasticity of substitution between home- and foreign-produced traded inputs. The investment-based price index  $P_J(t)$  is a function of traded and non-traded prices:

$$P_J(t) = \left[ \iota (P_J^T(t))^{1-\phi_J} + (1 - \iota) (P^N(t))^{1-\phi_J} \right]^{\frac{1}{1-\phi_J}}, \tag{23}$$

where the price index for traded investment goods reads:

$$P_J^T(t) = \left[ \iota^H (P^H(t))^{1-\rho_J} + (1 - \iota^H) \right]^{\frac{1}{1-\rho_J}}. \tag{24}$$

Installation of new investment goods involves convex costs, assumed quadratic. Thus, total investment  $J(t)$  differs from effectively installed new capital:

$$J(t) = I(t) + \frac{\kappa}{2} \left( \frac{I(t)}{K(t)} - \delta_K \right)^2 K(t), \tag{25}$$

where the parameter  $\kappa > 0$  governs the magnitude of adjustment costs to capital accumulation. Denoting the fixed capital depreciation rate by  $0 \leq \delta_K < 1$ , aggregate investment,  $I(t)$ , gives rise to capital accumulation according to the dynamic equation:

$$\dot{K}(t) = I(t) - \delta_K K(t). \tag{26}$$

Households choose consumption, worked hours and investment in physical capital by maximizing lifetime utility (18) subject to (20) and (26) together with (25). Denoting by

$\lambda$  and  $Q'$  the co-state variables associated with (20) and (26), the first-order conditions characterizing the representative household's optimal plans are:

$$(C(t))^{-\frac{1}{\sigma_C}} = P_C(t)\lambda(t), \quad (27a)$$

$$\gamma(L(t))^{\frac{1}{\sigma_L}} = \lambda(t)\tilde{W}(t), \quad (27b)$$

$$Q(t) = P_J(t) \left[ 1 + \kappa \left( \frac{I(t)}{K(t)} - \delta_K \right) \right], \quad (27c)$$

$$\dot{\lambda}(t) = \lambda(\beta - r^*), \quad (27d)$$

$$\begin{aligned} \dot{Q}(t) = (r^* + \delta_K) Q(t) - \left\{ [\alpha_K(t)u^{K,H}(t)u^{Z,H}(t) + (1 - \alpha_K(t))u^{K,N}(t)u^{Z,N}(t)] \right. \\ \left. - P^H(t)C^{K,H}(t)\alpha_K(t) - P^N(t)C^{K,N}(t)(1 - \alpha_K(t)) + P_J(t)\frac{\kappa}{2} \left( \frac{I(t)}{K(t)} - \delta_K \right) \left( \frac{I(t)}{K(t)} + \delta_K \right) \right\}, \end{aligned} \quad (27e)$$

$$R(t)u^{Z,H}(t) = P^H(t) [\xi_1^H + \xi_2^H (u^{K,H}(t) - 1)], \quad (27f)$$

$$R(t)u^{Z,N}(t) = P^N(t) [\xi_1^N + \xi_2^N (u^{K,N}(t) - 1)], \quad (27g)$$

$$R(t)u^{K,H}(t)K^H(t) + W^H(t)L^H(t) = P^H(t) [\chi_1^H + \chi_2^H (u^{Z,H}(t) - 1)], \quad (27h)$$

$$R(t)u^{K,N}(t)K^N(t) + W^N(t)L^N(t) = P^N(t) [\chi_1^N + \chi_2^N (u^{Z,N}(t) - 1)], \quad (27i)$$

and the transversality conditions  $\lim_{t \rightarrow \infty} \bar{\lambda}N(t)e^{-\beta t} = 0$  and  $\lim_{t \rightarrow \infty} Q(t)K(t)e^{-\beta t} = 0$ ; to derive the labor supply decision (27b), we use the fact that  $[\alpha_L(t)u^{Z,H}(t) + (1 - \alpha_L(t))u^{Z,N}(t)]W(t) = \tilde{W}^H(t)L^H(t) + \tilde{W}^N(t)L^N(t)$  where we add a tilde when factor prices are inclusive of technology utilization. Plugging  $\tilde{W}^j(t) = u^{Z,j}(t)W^j(t)$  into (17) leads to  $\tilde{W}(t)$ . To derive (27c) and (27e), we used the fact that  $Q(t) = Q'(t)/\lambda(t)$ . In an open economy model with a representative agent having perfect foresight, a constant rate of time preference and perfect access to world capital markets, we impose  $\beta = r^*$  in order to generate an interior solution. Setting  $\beta = r^*$  into (27d) implies that the shadow value of wealth is constant over time, i.e.,  $\lambda(t) = \lambda$ . When new information about the fiscal shock arrives,  $\lambda$  jumps (to fulfill the intertemporal solvency condition determined later) and remains constant afterwards.

Solving (27c) for investment, i.e.,  $\frac{I(t)}{K(t)} = \frac{1}{\kappa} \left( \frac{Q(t)}{P_J(t)} - 1 \right) + \delta_K$ , leads to a positive relationship between investment and Tobin's  $q$  which is defined as the shadow value to the firm of installed capital,  $Q(t)$ , divided by its replacement cost,  $P_J(t)$ . For the sake of clarity, we drop the time argument below provided this causes no confusion.

Applying Shephard's lemma (or the envelope theorem) yields the following demand for the home- and the foreign-produced traded good for consumption and investment:

$$C^H = \varphi \left( \frac{P^T}{P_C} \right)^{-\phi} \varphi^H \left( \frac{P^H}{P^T} \right)^{-\rho} C, \quad C^F = \varphi \left( \frac{P^T}{P_C} \right)^{-\phi} (1 - \varphi^H) \left( \frac{1}{P^T} \right)^{-\rho} C, \quad (28a)$$

$$J^H = \iota \left( \frac{P_J^T}{P_J} \right)^{-\phi_J} \iota^H \left( \frac{P^H}{P_J^T} \right)^{-\rho_J} J, \quad J^F = \iota \left( \frac{P_J^T}{P_J} \right)^{-\phi_J} (1 - \iota^H) \left( \frac{1}{P_J^T} \right)^{-\rho_J} J, \quad (28b)$$

and the demand for non-traded consumption and investment goods, respectively:

$$C^N = (1 - \varphi) (P^N/P_C)^{-\phi} C, \quad J^N = (1 - \iota) (P^N/P_J)^{-\phi_J} J. \quad (29)$$



Given the aggregate wage index (17) and  $\tilde{W}^j(t) = u^{Z,j}(t)W^j(t)$ , the allocation of aggregate labor supply to the traded and the non-traded sector reads:

$$L^H = \vartheta \left( \frac{\tilde{W}^H}{\tilde{W}} \right)^\epsilon L, \quad L^N = (1 - \vartheta) \left( \frac{\tilde{W}^N}{\tilde{W}} \right)^\epsilon L, \quad (30)$$

where  $\epsilon$  determines the percentage change in the share of hours worked in sector  $j$ ,  $L^j/L$ , following a rise in the relative wage,  $\tilde{W}^j/\tilde{W}$ , by 1%. As the elasticity of labor supply across sectors,  $\epsilon$ , takes higher values, workers experience lower mobility costs and thus more labor shifts from one sector to another.

### 3.2 Firms

We denote the value added in sector  $j = H, N$  by  $Y^j$ . When we add a tilde, it means that value added is inclusive of the technology utilization rate, i.e.,  $\tilde{Y}^j(t) = u^Z(t)Y^j(t)$ . Both the traded and non-traded sectors use physical capital (inclusive of capital utilization), denoted by  $\tilde{K}^j(t) = u^{K,j}(t)K^j(t)$ , and labor,  $L^j$ , according to a constant returns to scale technology described by a CES production function:

$$\tilde{Y}^j(t) = \left[ \gamma^j \left( \tilde{A}^j(t)L^j(t) \right)^{\frac{\sigma^j-1}{\sigma^j}} + (1 - \gamma^j) \left( \tilde{B}^j(t)\tilde{K}^j(t) \right)^{\frac{\sigma^j-1}{\sigma^j}} \right]^{\frac{\sigma^j}{\sigma^j-1}}, \quad (31)$$

where  $0 < \gamma^j < 1$  and  $0 < 1 - \gamma^j < 1$  are the weight of labor and capital in the production technology, respectively,  $\sigma^j$  is the elasticity of substitution between capital and labor in sector  $j = H, N$ . We allow for labor- and capital-augmenting efficiency denoted by  $\tilde{A}^j(t)$  and  $\tilde{B}^j(t)$ . We assume that factor-augmenting productivity has a symmetric time-varying component which collapses to  $u^{Z,j}(t)$  such that  $\tilde{A}^j(t) = u^{Z,j}(t)A^j(t)$  and  $\tilde{B}^j(t) = u^{Z,j}(t)B^j(t)$ .

Firms lease the capital from households and hire workers. They face two cost components: a capital rental cost equal to  $R(t)$ , and a labor cost equal to the wage rate  $W^j(t)$ . Both sectors are assumed to be perfectly competitive and thus choose capital services and labor by taking prices as given. While capital can move freely between the two sectors, costly labor mobility implies a wage differential across sectors:<sup>22</sup>

$$P^j(t)\gamma^j \left( A^j(t) \right)^{\frac{\sigma^j-1}{\sigma^j}} \left( y^j(t) \right)^{\frac{1}{\sigma^j}} = W^j(t), \quad (32a)$$

$$P^j(t) (1 - \gamma^j) \left( B^j(t) \right)^{\frac{\sigma^j-1}{\sigma^j}} \left( u^{K,j}(t)k^j(t) \right)^{-\frac{1}{\sigma^j}} \left( y^j(t) \right)^{\frac{1}{\sigma^j}} = R(t), \quad (32b)$$

where we denote by  $k^j(t) \equiv K^j(t)/L^j(t)$  the capital-labor ratio for sector  $j = H, N$ , and  $y^j(t) \equiv Y^j(t)/L^j(t)$  refers to value added per hours worked.

Demand for inputs can be rewritten in terms of their respective cost in value added; for labor, we have  $s_L^j(t) = \gamma^j \left( \frac{A^j(t)}{y^j(t)} \right)^{\frac{\sigma^j-1}{\sigma^j}}$ . Applying the same logic for capital and denoting

<sup>22</sup>Since the profit function is a linear function of the technology utilization rate, i.e.,  $\tilde{\Pi}^j(t) = u^{Z,j}(t)\Pi^j(t)$ ,  $u^{Z,j}(t)$  does not show up in the first-order conditions shown in (32).



the ratio of labor to capital income share by  $S^j(t) \equiv \frac{s_L^j(t)}{1-s_L^j(t)}$ , we have:

$$S^j(t) \equiv \frac{s_L^j(t)}{1-s_L^j(t)} = \frac{\gamma^j}{1-\gamma^j} \left( \frac{B^j(t)u^{K,j}(t)K^j(t)^j(t)}{A^j(t)L^j(t)} \right)^{\frac{1-\sigma^j}{\sigma^j}}. \quad (33)$$

When technological change is assumed to be Hicks-neutral, productivity increases uniformly across inputs, i.e.,  $\hat{A}^j(t) = \hat{B}^j(t)$ . Hence a change in  $B^j(t)/A^j(t)$  on the RHS of eq. (33) has no impact on sectoral LISs which are only indirectly affected through changes in  $u^{K,j}(t)k^j(t)$ . Therefore, if sector  $j$  decides to use less capital, its LIS  $s_L^j(t)$  declines because capital and labor are gross complements in production, i.e.,  $\sigma^j < 1$ , as evidence suggests. By contrast, when technological change is factor-biased, an increase in capital relative to labor efficiency (i.e., a rise in  $B^j(t)/A^j(t)$ ) impinges on the sectoral LIS directly and indirectly through changes in capital use  $\tilde{k}^j(t) = u^{K,j}(t)k^j(t)$ . The measure of capital-utilization-adjusted-FBTC in sector  $j$  is:  $\text{FBTC}_{adjK}^j(t) = (B^j(t)/A^j(t))^{\frac{1-\sigma^j}{\sigma^j}}$ . Utilization-adjusted technological change is biased toward labor when  $\text{FBTC}_{adjK}^j(t)$  increases.

Finally, aggregating over the two sectors gives us the resource constraint for capital:

$$K^H(t) + K^N(t) = K(t). \quad (34)$$

### 3.3 Technology Frontier

While households choose capital and technology utilization rates, firms within each sector  $j = H, N$  decide about the split of capital-utilization-adjusted TFP, denoted by  $Z^j(t) = u^{Z,j}(t)\bar{Z}^j$  where  $\bar{Z}^j$  is normalized to one, between labor- and capital-augmenting efficiency  $\tilde{A}^j(t)$  and  $\tilde{B}^j(t)$ . Following Caselli and Coleman [2006] and Caselli [2016], we assume that firms choose a mix of  $\tilde{A}^j(t)$  and  $\tilde{B}^j(t)$  along a technology frontier (which is assumed to take a CES form):

$$\left[ \gamma_Z^j \left( \tilde{A}^j(t) \right)^{\frac{\sigma_Z^j-1}{\sigma_Z^j}} + \left( 1 - \gamma_Z^j \right) \left( \tilde{B}^j(t) \right)^{\frac{\sigma_Z^j-1}{\sigma_Z^j}} \right]^{\frac{\sigma_Z^j}{\sigma_Z^j-1}} \leq Z^j(t), \quad (35)$$

where  $Z^j(t) > 0$  is the height of the technology frontier,  $0 < \gamma_Z^j < 1$  is the weight of labor efficiency in TFP and  $\sigma_Z^j > 0$  corresponds to the elasticity of substitution between labor- and capital-augmenting productivity. Firms choose labor and capital efficiency,  $\tilde{A}^j$  and  $\tilde{B}^j$ , along the technology frontier described by eq. (35) that minimize the unit cost function. The unit cost minimization requires that (see Online Appendix K.2):

$$\frac{\gamma_Z^j}{1-\gamma_Z^j} \left( \frac{\tilde{A}^j(t)}{\tilde{B}^j(t)} \right)^{\frac{\sigma_Z^j-1}{\sigma_Z^j}} = \frac{s_L^j(t)}{1-s_L^j(t)} \equiv S^j(t). \quad (36)$$

Solving (36) for the LIS in sector  $j$  leads to  $s_L^j = \gamma_Z^j \left( \frac{\tilde{A}^j}{\tilde{B}^j} \right)^{\frac{\sigma_Z^j-1}{\sigma_Z^j}}$ . Inserting this equality into the log-linearized version of the technology frontier (35) enables us to express the percentage deviation of utilization-adjusted TFP in sector  $j$  relative to its initial steady-state

as a factor-income-share-weighted sum of the percentage deviation of labor- and capital-augmenting efficiency:

$$\hat{Z}^j(t) = s_L^j \hat{A}^j(t) + (1 - s_L^j) \hat{B}^j(t). \quad (37)$$

While the technological frontier imposes a structure on the mapping between the utilization-adjusted TFP and factor-augmenting efficiency, as described by (37), it has the advantage to ensure a consistency between the theoretical and the empirical approach where we used the utilization-adjusted-Solow residual to measure technological change, the latter being driven by factor-augmenting productivity shifts which can occur at the same rate or at different rate, thus leading technological change to be biased toward one factor of production, in the latter case.

### 3.4 Government

The final agent in the economy is the government. Government spending includes expenditure on non-traded goods,  $G^N$ , home- and foreign-produced traded goods,  $G^H$  and  $G^F$ , respectively. The government finances public spending by raising lump-sum taxes,  $T$ . As a result, Ricardian equivalence obtains and the time path of taxes is irrelevant for the real allocation. We may thus assume without loss of generality that government budget is balanced at each instant:

$$G(t) \equiv P^N(t)G^N(t) + P^H(t)G^H(t) + G^F(t) = T(t). \quad (38)$$

### 3.5 Model Closure and Equilibrium

To fully describe the equilibrium, we impose goods market clearing conditions for non-traded and home-produced traded goods:

$$Y^N(t) = C^N(t) + J^N(t) + G^N(t) + C^{K,N}(t)K^N(t) + C^{Z,N}(t), \quad (39a)$$

$$Y^H(t) = C^H(t) + J^H(t) + G^H(t) + X^H(t) + C^{K,H}(t)K^H(t) + C^{Z,H}(t), \quad (39b)$$

where  $X^H$  stands for exports of home-produced goods; exports are assumed to be a decreasing function of terms of trade,  $P^H$ :<sup>23</sup>

$$X^H(t) = \varphi_X (P^H(t))^{-\phi_X}, \quad (40)$$

where  $\varphi_X > 0$  is a scaling parameter, and  $\phi_X$  is the elasticity of exports w.r.t.  $P^H$ .

In order to account for the dynamic adjustment of  $G(t)$  (see Fig. 2(a)), we assume that the deviation of government spending relative to its initial value as a percentage of initial GDP is governed by the law of motion:

$$dG(t)/Y = e^{-\xi t} - (1 - g) e^{-\chi t}. \quad (41)$$

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<sup>23</sup>Domestic exports are the sum of the foreign demand for the domestically produced tradable consumption goods and investment inputs denoted by  $C^{F,*}$  and  $J^{F,*}$ , and we assume that the rest of the world have similar preferences with potentially different elasticities (i.e.  $\phi^* \neq \phi$  and  $\phi_J^* \neq \phi_J$ ) between foreign and domestic tradable goods. Since we abstract from trend labor-augmenting technological change, foreign prices remain fixed so that domestic exports are decreasing in the terms of trade,  $P^H(t)$ .

where  $dG(t) = G(t) - G$  is the deviation of government consumption relative to the initial steady-state,  $g > 0$  parametrizes the magnitude of the exogenous fiscal shock,  $\xi > 0$  and  $\chi > 0$  are (positive) parameters which are set in order to capture the hump-shaped endogenous response of  $G(t)$ . We assume that the rise in government consumption is split into non-traded,  $\omega_{GN}$ , home-produced traded goods,  $\omega_{GH} = \frac{P^H G^H}{G}$ , and foreign-produced traded goods,  $\omega_{GF}$ . Formally we have  $dG(t)/Y = \sum_{g=F,H,N} \omega_{Gg} dG(t)/Y$ . In line with the evidence we document in Appendix E,  $\omega_{GN}$  refers to the non-tradable content of government consumption as well as the intensity of the government spending shock in non-traded goods.

As detailed in subsection 4.2, we estimate the dynamic adjustment of  $\hat{A}^j(t)$  and  $\hat{B}^j(t)$  which are required to explain the variation in the ratio of labor to the capital income share  $S^j(t)$  in sector  $j$ , as described by eq. (33), and also satisfy the shifts along the technology frontier (37). To achieve a perfect match with the data, we specify the law of motion for labor- and capital-augmenting efficiency:

$$\hat{A}^j(t) = e^{-\xi_A^j t} - (1 - a^j) e^{-\chi_A^j t}, \quad (42a)$$

$$\hat{B}^j(t) = e^{-\xi_B^j t} - (1 - b^j) e^{-\chi_B^j t}, \quad (42b)$$

and choose  $a^j$  ( $b^j$ ) to reproduce the impact response of labor- (capital-) augmenting technological change while  $\xi_A^j > 0$  ( $\xi_B^j > 0$ ) and  $\chi_A^j > 0$  ( $\chi_B^j > 0$ ) are chosen to reproduce the shape of factor-augmenting productivity together with their cumulative change following a shock to government consumption we infer from (33) and (37).

The adjustment of the open economy toward the steady state is described by a dynamic system which comprises six equations that are functions of  $K(t)$ ,  $Q(t)$ ,  $G(t)$ ,  $A^j(t)$ ,  $B^j(t)$ :

$$\dot{K}(t) = \Upsilon (K(t), Q(t), G(t), A^H(t), B^H(t), A^N(t), B^N(t)), \quad (43a)$$

$$\dot{Q}(t) = \Sigma (K(t), Q(t), G(t), A^H(t), B^H(t), A^N(t), B^N(t)), \quad (43b)$$

where  $j = H, N$ . The first dynamic equation corresponds to the non-traded goods market clearing condition (39a) and the second dynamic equation corresponds to (27e) which equalizes the rates of return on domestic equities and foreign bonds,  $r^*$ , once we have substituted appropriate first-order conditions.

Linearizing the dynamic equations (43a)-(43b) in the neighborhood of the steady-state, inserting the law of motion of government consumption (41) and factor-augmenting efficiency (42a)-(42b) leads to a system of first-order linear differential equations which can be solved by applying standard methods (see Buiter [1984] who presents the continuous time adaptation of the method of Blanchard and Kahn [1980]):

$$K(t) - \tilde{K} = X_1(t) + X_2(t), \quad Q(t) - \tilde{Q} = \omega_2^1 X_1(t) + \omega_2^2 X_2(t), \quad (44)$$

where we denote the negative eigenvalue by  $\nu_1$ , the positive eigenvalue by  $\nu_2$ , and  $\omega_2^i$  is the element of the eigenvector associated with the eigenvalue  $\nu_i$  (with  $i = 1, 2$ ) and  $X_1(t)$

and  $X_2(t)$  are solutions which characterize the trajectory of  $K(t)$  and  $Q(t)$ . See Online Appendix P which details the solution method.

Using the properties of constant returns to scale in production, identities  $P_C(t)C(t) = \sum_g P^g(t)C^g(t)$  and  $P_J(t)J(t) = \sum_g P^g(t)J^g(t)$  (with  $g = F, H, N$ ) along with market clearing conditions (39), the current account equation (20) can be rewritten as a function of the trade balance:

$$\dot{N}(t) = r^*N(t) + P^H(t)X^H(t) - M^F(t), \quad (45)$$

where  $M^F(t) = C^F(t) + J^F(t)$  stands for imports of foreign-produced consumption and investment goods. Eq. (45) can be written as a function of state and control variables, i.e.,  $\dot{N}(t) \equiv r^*N(t) + \Xi(K(t), Q(t), G(t), A^H(t), B^H(t), A^N(t), B^N(t))$ . Linearizing around the steady state, substituting the solutions for  $K(t)$  and  $Q(t)$  given by (44), solving and invoking the transversality condition leads to the intertemporal solvency condition:

$$(N_0 - N) + \frac{\omega_N^1}{\nu_1 - r^*} + \frac{\omega_N^{2,G}}{\xi + r^*} + \sum_{X^j} \frac{\omega_N^{2,X^j}}{\xi_X^j + r^*} = 0, \quad (46)$$

where  $N_0$  is the initial stock of traded bonds,  $X^j = A^j, B^j$  with  $j = H, N$ ;  $\omega_N^1, \omega_N^{2,G}, \omega_N^{2,X^j}$ , are terms which are functions of parameters, eigenvalues and eigenvectors. The assumption  $\beta = r^*$  requires the joint determination of the transition and the steady-state since the constancy of the marginal utility of wealth implies that the intertemporal solvency condition (46) depends on eigenvalues' and eigenvectors' elements, see e.g., Turnovsky [1997].<sup>24</sup>

## 4 Quantitative Analysis

In this section, we take the model to the data. For this purpose we solve the model numerically. Therefore, first we discuss parameter values before turning to the effects of an exogenous temporary increase in government consumption.

### 4.1 Calibration

At the steady-state, utilization rates for technology,  $u^{Z,j}$ , and capital,  $u^{K,j}$ , collapse to one so that  $\tilde{Y}^j = Y^j$  and  $\tilde{K}^j = K^j$ . We consider an initial steady-state with Hicks-neutral technological change and normalize  $A^j = B^j = Z^j$  to 1. To ensure that the steady-state is invariant when  $\sigma^j$  is changed, we normalize (31) by choosing the initial steady-state in a model with Cobb-Douglas production functions as the normalization point. Denoting the LIS in sector  $j$  in the Cobb-Douglas economy by  $\theta^j$ , the normalized version of the CES production function reads as follows:

$$\frac{Y^j}{\bar{Y}^j} = \left[ \theta^j \left( \frac{L^j}{\bar{L}^j} \right)^{\frac{\sigma^j-1}{\sigma^j}} + (1 - \theta^j) \left( \frac{K^j}{\bar{K}^j} \right)^{\frac{\sigma^j-1}{\sigma^j}} \right]^{\frac{\sigma^j}{\sigma^j-1}},$$

<sup>24</sup>Eq. (46) determines the steady-state change in the net foreign asset position following a temporary fiscal expansion as the assumption  $\beta = r^*$  implies that temporary policies have permanent effects.

where quantities in the CES economy are divided by their Cobb-Douglas counterparts denoted with a bar. Because we consider the initial steady-state with Cobb-Douglas production functions as the normalization point, we have to calibrate the model with  $\sigma^j = 1$  to the data. To calibrate the reference model, we estimated a set of ratios and parameters for the eighteen OECD economies in our dataset. Our reference period for the calibration corresponds to the period 1970-2015. Table 7 summarizes our estimates of the ratios and estimated parameters for all countries in our sample.

We first calibrate the model to a representative OECD country to assess the model performance when we allow for time-varying technological change and contrast the model's predictions when we shut down technological change. Because the evidence we have documented in section 2 shows that technology shifts vary considerably across countries, later, we move a step further and calibrate the model to country-specific data to quantify the contribution of technological change to international differences in sectoral government spending multipliers. To capture the key properties of a typical OECD economy, we take unweighted average values of ratios which are shown in the last line of Table 7. Among the 32 parameters that the model contains, 22 have empirical counterparts while the remaining 10 parameters, i.e.,  $\varphi$ ,  $\iota$ ,  $\varphi^H$ ,  $\iota^H$ ,  $\vartheta$ ,  $\delta_K$ ,  $\xi_1^H$ ,  $\xi_1^N$ ,  $\chi_1^H$ ,  $\chi_1^N$  together with initial conditions  $(N_0, K_0)$  must be endogenously calibrated to match ratios  $1 - \alpha_C$ ,  $1 - \alpha_J$ ,  $\alpha^H$ ,  $\alpha_J^H$ ,  $\frac{L^N}{L}$ ,  $\omega_J$ ,  $R/P^H$ ,  $R/P^N$ ,  $Y^H$ ,  $Y^N$ ,  $v_{NX} = \frac{NX}{P^H Y^H}$  with  $NX = P^H X^H - C^F - I^F - G^F$ , as summarized in Table 1. We choose the model period to be one year and set the world interest rate,  $r^*$ , which is equal to the subjective time discount rate,  $\beta$ , to 3%, in line with the average of our estimates shown in the last line of Table 6.

The degree of labor mobility captured by  $\epsilon$  is set to 0.83, in line with the average of our estimates shown in the last line of column 17 of Table 7. Estimated values of  $\epsilon$  range from a low of about 0.1 for Ireland and Norway to a high of 2.3 for South Korea and 2.4 for the United States.<sup>25</sup>

Building on our panel data estimates, the elasticity of substitution  $\phi$  between traded and non-traded goods is set to 0.77 in the baseline calibration since this value corresponds to the average of estimates shown in the last line of column 16 of Table 7. This value is close to the value estimated by Mendoza [1995] who reports an estimate of 0.74.<sup>26</sup> The weight of consumption in non-tradables  $1 - \varphi$  is set to target a non-tradable content in total consumption expenditure (i.e.,  $1 - \alpha_C$ ) of 56%, in line with the average of our estimates (see the last line of column 2 of Table 7). Following Backus et al. [1994], we set the elasticity of substitution,  $\rho$ , in consumption between home- and foreign-produced traded goods (inputs)

<sup>25</sup>To estimate  $\epsilon$ , we run the regression in panel format on annual data of the percentage change in the labor share of sector  $j$  on the percentage change in the relative share of value added paid to workers in sector  $j$ . The Online Appendix J.2 details our empirical strategy and shows our panel data estimations of  $\epsilon$ .

<sup>26</sup>The Online Appendix J.2 details our empirical strategy and shows our panel data estimations of  $\phi$  by running the regression of the logged share of non-tradables in consumption expenditure on the logged ratio of the price of non-tradables to the consumption price index.

to 1.5. The weight of consumption in home-produced traded goods  $\varphi^H$  is set to target a home content of consumption expenditure in tradables (i.e.  $\alpha^H$ ) of 66%, in line with the average of our estimates shown in the last line of column 8.

We choose a value of one for the elasticity of intertemporal substitution for consumption,  $\sigma_C$ , which is a typical choice in the business cycle literature. We set the Frisch elasticity of labor supply to 1 which is halfway between the large values of  $\sigma_L$  reported by Peterman [2016] and low values reported by Fiorito and Zanella [2012]. The weight of labor supply to the non-traded sector,  $1 - \vartheta$ , is set to target a share of non-tradables in total hours worked of 62% (see the last line of column 5 of Table 7).

We now describe the calibration of production-side parameters. We assume that physical capital depreciates at a rate  $\delta_K = 7.8\%$  to target an investment-to-GDP ratio of 24% (see column 14 of Table 7). In line with mean values shown in columns 11 and 12 of Table 7, the shares of labor income in traded and non-traded value added,  $\theta^H$  and  $\theta^N$ , are set to 0.63 and 0.69, respectively, which leads to an aggregate LIS of 66% (see the last line of column 13 of Table 7). We set the elasticity of substitution,  $\phi_J$ , between  $J^T$  and  $J^N$  to 1, in line with the empirical findings documented by Bems [2008] for OECD countries. Further, the weight of non-traded investment ( $1 - \varphi_I$ ) is set to target a non-tradable content of investment expenditure of 69% (see the last line of column 3 of Table 7). Likewise for consumption goods, following Backus et al. [1994], we set the elasticity of substitution,  $\rho_J$ , in investment between home- and foreign-produced traded inputs to 1.5. The weight of home-produced traded investment  $\iota^H$  is set to target a home content of investment expenditure in tradables (i.e.  $\alpha_J^H$ ) of 43% (see column 9 of Table 7). We choose the value of parameter  $\kappa$  so that the elasticity of  $I/K$  with respect to Tobin's  $q$ , i.e.,  $Q/P_J$ , is equal to the value implied by estimates in Eberly, Rebelo, and Vincent [2008]. The resulting value of  $\kappa$  is equal to 17.<sup>27</sup>

As shown in columns 4 and 10 of Table 7, the non-tradable,  $\omega_{GN}$ , and the home-produced tradable content,  $\omega_{GH}$ , of government spending averages 80% and 18%, respectively. The import content of government spending is lower at  $\omega_{GF} = 2\%$ . We set government consumption on non-traded goods and home-produced traded goods, i.e.,  $G^N$  and  $G^H$ , so as to target both the non-tradable and home-tradable share of government spending, together with government spending as a share of GDP of 19% (see column 15 Table 7).

We choose initial conditions so that trade is initially balanced. Since net exports are nil, the investment-to-GDP ratio,  $\omega_J$ , and government spending as a share of GDP,  $\omega_G$ , implies a consumption-to-GDP ratio of  $\omega_C = 57\%$ . It is worth mentioning that the tradable content of GDP is endogenously determined by the tradable content of consumption,  $\alpha_C$ , investment,  $\alpha_J$ , and government expenditure,  $\omega_{GT}$ , along with  $\omega_C$ ,  $\omega_J$ , and  $\omega_G$ . More

<sup>27</sup>Eberly, Rebelo, and Vincent [2008] run the regression  $I/K = \alpha + \beta \cdot \ln(q)$  and obtain a point estimate for  $\beta$  of 0.06. In our model, the steady-state elasticity of  $I/K$  with respect to Tobin's  $q$  is  $1/\kappa$ . Equating  $1/\kappa$  to 0.06 gives a value for  $\kappa$  of 17.

Table 1: Baseline Parameters (Representative OECD Economy)

Definition	Value		Reference
	CD	CES/Restricted	
Period of time	year	year	data frequency
<b>A. Preferences</b>			
Subjective time discount rate, $\beta$	3%	3%	equal to the world interest rate
Intertemporal elasticity of substitution for consumption, $\sigma_C$	1	1	standard
Intertemporal elasticity of substitution for labor, $\sigma_L$	1	1	standard
Elasticity of substitution between $C^T$ and $C^N$ , $\phi$	0.77	0.77	our estimates (EU KLEMS [2011], [2017] and OECD STAN [2011], [2017] databases)
Elasticity of substitution between $J^T$ and $J^N$ , $\phi_J$	1	1	Bems [2008]
Elasticity of substitution between $C^H$ and $C^F$ , $\rho$	1.5	1.5	Backus, Kehoe and Kydland [1994]
Elasticity of substitution between $J^H$ and $J^F$ , $\rho_J$	1.5	1.5	Backus, Kehoe and Kydland [1994]
Elasticity of labor supply across sectors, $\epsilon$	0.83	0.83	our estimates (EU KLEMS [2011], [2017] and OECD STAN [2011], [2017] databases)
<b>B. Non-tradable share</b>			
Weight of consumption in non-traded goods, $1 - \varphi$	0.54	0.54	set to target $1 - \alpha_C = 56\%$ (United Nations, COICOP [2017])
Weight of labor supply to the non-traded sector, $1 - \vartheta$	0.60	0.60	set to target $L^N/L = 62\%$ (EU KLEMS [2011], [2017] and OECD STAN [2011], [2017] databases)
Weight of non-traded investment, $1 - \iota$	0.69	0.69	set to target $1 - \alpha_J = 69\%$ (OECD Input-Output database [2017])
Non-tradable content of government expenditure, $\omega_{GN}$	0.80	0.80	our estimates (Input-Output dataset, WIOD [2013])
<b>C. Home share</b>			
Weight of consumption in home traded goods, $\varphi^H$	0.73	0.73	set to target $\alpha^H = 66\%$ (United Nations, Comtrade [2017])
Weight of home traded investment, $\iota^H$	0.52	0.52	set to target $\alpha^H = 43\%$ (United Nations, Comtrade [2017])
Home traded content of government expenditure, $\omega_{GH}$	0.18	0.18	our estimates (Input-Output dataset, WIOD [2013])
Export price elasticity, $\phi_X$	1.7	1.7	Imbs and Mejean [2015]
<b>C. Production</b>			
Labor income share in the non-traded sector, $\theta^N$	0.69	0.69	our estimates (EU KLEMS [2011], [2017] and OECD STAN [2011], [2017] databases)
Labor income share in the traded sector, $\theta^H$	0.63	0.63	our estimates (EU KLEMS [2011], [2017] and OECD STAN [2011], [2017] databases)
Elasticity of substitution between $K^H$ and $L^H$ , $\sigma^H$	1	0.69	our estimates (EU KLEMS [2011], [2017] and OECD STAN [2011], [2017] databases)
Elasticity of substitution between $K^N$ and $L^N$ , $\sigma^N$	1	0.72	our estimates (EU KLEMS [2011], [2017] and OECD STAN [2011], [2017] databases)
<b>D. GDP demand components</b>			
Physical capital depreciation rate, $\delta_K$	7.8%	7.8%	set to target $\omega_J = 24\%$ (Source: OECD Economic Outlook Database)
Parameter governing capital adjustment cost, $\kappa$	17	17	set to match the elasticity $I/K$ to Tobin's $q$ (Eberly et al. [2008])
Government spending as a ratio of GDP, $\omega_G$	19%	19%	our estimates (Source: OECD Economic Outlook Database)
<b>E. Utilization adjustment costs</b>			
Parameter governing capital utilization cost, $\xi_2^H$	0.27	$\infty$	set to reproduce IRF for $u^{K,H}(t)$
Parameter governing capital utilization cost, $\xi_2^N$	0.03	$\infty$	set to reproduce IRF for $u^{K,N}(t)$
Parameter governing technology utilization cost, $\chi_2^H$	0.80	$\infty$	set to reproduce IRF for $u^{Z,H}(t)$
Parameter governing technology utilization cost, $\chi_2^N$	2.85	$\infty$	set to reproduce IRF for $u^{Z,N}(t)$
<b>F. Government Spending Shock</b>			
Exogenous fiscal shock, $g$	1%	1%	To generate $dG(0)/Y = 1\%$
Persistence and shape of endogenous response of $G$ , $\xi$	0.430	0.430	set to target $\int_0^5 [dG(\tau)/Y] e^{-r^* \tau} d\tau = g'$ and $\dot{G}(1) = 0$
Persistence and shape of endogenous response of $G$ , $\chi$	0.439	0.439	set to target $\int_0^5 [dG(\tau)/Y] e^{-r^* \tau} d\tau = g'$ and $\dot{G}(1) = 0$

Notes: 'CD' refers to the Cobb-Douglas model which is taken as the reference point; 'CES' refers to the baseline model where production functions are of the CES type with  $\sigma^j < 1$  and  $\xi_2^j < \infty$ ,  $\chi_2^j < \infty$ ; 'restricted' refers to the restricted model with Cobb-Douglas production functions (i.e.,  $\sigma^j = 1$ ) and technological change is shut down by letting  $\xi_2^j$  and  $\chi_2^j$  tend toward infinity.



precisely, dividing the traded goods market clearing condition by GDP,  $Y$ , leads to an expression that allows us to calculate the tradable content of GDP:

$$P^H Y^H / Y = \omega_C \alpha_C + \omega_J \alpha_J + \omega_{GT} \omega_G = 36\%, \quad (47)$$

where  $\omega_C = 57\%$ ,  $\alpha_C = 44\%$ ,  $\omega_J = 24\%$ ,  $\alpha_J = 31\%$ ,  $\omega_{GT} = 20\%$ , and  $\omega_G = 19\%$ . According to (47), the ratios we target for demand components generates a tradable content of GDP of 36% as found in the data (see the last line of column 1 of Table 7). Finally, building on structural estimates of the price elasticities of aggregate exports documented by Imbs and Mejean [2015], we set the export price elasticity,  $\phi_X$ , to 1.7 in the baseline calibration (see the last line of last column of Table 7). Because trade is balanced, export as a share of GDP,  $\omega_X = P^H X^H / Y$ , is endogenously determined by the import content of consumption,  $1 - \alpha^H$ , and investment expenditure,  $1 - \alpha_J^H$ , along with  $\omega_C$  and  $\omega_J$ .

Since the model with Cobb-Douglas production functions is the normalization point, when we calibrate the model with CES production functions,  $\varphi$ ,  $\iota$ ,  $\varphi^H$ ,  $\iota^H$ ,  $\vartheta$ ,  $\delta_K$ ,  $N_0$ ,  $K_0$ , are endogenously set to target  $1 - \bar{\alpha}_C$ ,  $1 - \bar{\alpha}_J$ ,  $\bar{\alpha}^H$ ,  $\bar{\alpha}_J^H$ ,  $\bar{L}^N / \bar{L}$ ,  $\bar{\omega}_J$ ,  $\bar{v}_{NX}$ ,  $\bar{K}$ , respectively, where a bar indicates that the ratio is obtained from the Cobb-Douglas economy. Drawing on Antràs [2004], we estimate the elasticity of substitution between capital and labor for tradables and non-tradables and set  $\sigma^H$  and  $\sigma^N$ , to 0.64 and 0.80 (see the last line of columns 18 and 19 of Table 7).

## 4.2 Government Spending Shock and Technology: Calibration

**Endogenous response of government consumption to exogenous fiscal shock.** In order to capture the endogenous response of government spending to an exogenous fiscal shock, we assume that the dynamic adjustment of government consumption is governed by eq. (41). In the quantitative analysis, we set  $g = 0.01$  so that government consumption increases by 1% of initial GDP. To calibrate  $\xi$  and  $\chi$  that parametrize the shape of the dynamic adjustment of government consumption along with its persistence, we proceed as follows. Because  $G(t)$  peaks after one year, we have  $\dot{G}(1)/Y = -[\xi e^{-\xi} - \chi(1-g)e^{-\chi}] = 0$ . In addition, the cumulative response of government consumption over a six-year horizon is  $\int_0^5 [dG(\tau)/Y] e^{-r^* \tau} d\tau = g'$  with  $g' = 5.5$  percentage point of GDP. We choose  $\xi = 0.430$  and  $\chi = 0.439$ . Left-multiplying eq. (41) by  $\omega_{Gg}$  (with  $g = H, N$ ) gives the dynamic adjustment of sectoral government consumption to an exogenous fiscal shock:

$$\omega_{Gg} \frac{dG(t)}{Y} = \omega_{Gg} \left[ e^{-\xi t} - (1-g)e^{-\chi t} \right], \quad (48)$$

where  $\omega_{Gg}$  is the fraction of government consumption in good  $g$ . To determine (48), we assume that the parameters that govern the persistence and shape of the response of sectoral government consumption are identical across sectors, while the sectoral intensity of the government spending shock is constant over time and thus corresponds to the share of



government final consumption expenditure on good  $j$ .<sup>28</sup>

**Capital and technology utilization adjustment costs.** We turn to the calibration of parameters which govern the capital and technology adjustment cost functions described by (19a) and (19b), respectively. Evaluating first-order conditions (27f)-(27g) at the steady-state leads to  $\xi_1^j = \frac{R}{P^j}$  and thus  $\xi_1^j$  is endogenously pinned down by the initial steady-state value of the ratio of the capital rental rate to the value added deflator,  $P^j$ . It gives us  $\xi_1^H = 0.11$  and  $\xi_1^N = 0.09$ . Denoting  $R^j(t) = R(t)u^{Z,j}(t)$  and log-linearizing (27f)-(27g) leads to:

$$\hat{u}^{K,j}(t) = \frac{\xi_1^j}{\xi_2^j} \left( \hat{R}^j(t) - \hat{P}^j(t) \right). \quad (49)$$

According to eq. (49), it is profitable to increase the capital utilization rate when the real capital cost goes up while the parameter  $\xi_2^j$  determines the magnitude of the adjustment in  $u^{K,j}(t)$ . We choose a value for the parameter  $\xi_2^j$  so as to account for the empirical response of the capital utilization rate to government shock found in the data, see Fig. 8 in Online Appendix D. As reported in Table 1, we choose a value for  $\xi_2^H$  of 0.27 and a value for  $\xi_2^N$  of 0.03.<sup>29</sup> The same logic applies to pin down the parameters governing the endogenous response of the technology utilization rate in sector  $j$  to a shock to government consumption. Evaluating first-order conditions (27h)-(27i) at the steady-state leads to  $\chi_1^j = Y^j$  and thus  $\chi_1^j$  is endogenously pinned down by the initial steady-state of value added at constant prices in sector  $j$ ,  $Y^j$ . We obtain  $\chi_1^H = 0.84$  and  $\chi_1^N = 1.19$ . Log-linearizing (27h)-(27i) leads to:

$$\hat{u}^{Z,j}(t) = \frac{\chi_1^j}{\chi_2^j} \hat{Y}^j(t). \quad (50)$$

According to eq. (50), the technology utilization rate is pro-cyclical; intuitively, since  $Y^j(t) = \frac{W^j(t)L^j(t)+R(t)\hat{K}^j(t)}{P^j(t)}$ , it is profitable to increase the technology rate when the real cost of producing goes up. The parameter  $\chi_2^j$  determines the magnitude of the response of the technology utilization rate  $u^{Z,j}(t)$ . We choose a value for  $\chi_2^j$  in order to reproduce the empirical response of capital-utilization-adjusted-TFP,  $Z^j(t)$ . It gives us  $\chi_2^H = 0.8$  for the traded sector and  $\chi_2^N = 2.85$  for the non-traded sector.

**Factor-augmenting efficiency.** Since our VAR evidence documented in subsection 2.5 reveals that technological change is factor-biased, we need to set the dynamics for factor-augmenting efficiency,  $B^j(t)$  and  $A^j(t)$ . We first derive the change in capital relative to labor efficiency, by log-linearizing (33) which describes the demand for labor relative to capital:

$$\left( \hat{B}^j(t) - \hat{A}^j(t) \right) = \frac{\sigma^j}{1 - \sigma^j} \hat{S}^j(t) - \hat{k}^j(t) - \hat{u}^{K,j}(t), \quad (51)$$

<sup>28</sup> Assuming that the intensity of the non-traded sector in the government spending shock collapses to the non-tradable content of government consumption is in line with the evidence documented in Online Appendix E, especially in the short-run.

<sup>29</sup> Eq. (49) can be solved for  $\xi_2^j = \frac{\xi_1^j}{\hat{u}^{K,j}(t)} \left( \hat{R}^j(t) - \hat{P}^j(t) \right)$ . Plugging empirical IRF from VAR estimations and calculating the mean returns a value of 0.22 for  $\xi_2^H$  and 0.05 for  $\xi_2^N$ .

all variables being expressed in percentage deviation from the initial steady-state. Given the adjustment of relative capital efficiency inferred from (51), we have to determine the dynamics of  $B^j(t)$  and  $A^j(t)$  consistent with the technology frontier. Using the fact that factor-augmenting productivity has a symmetric time-varying component denoted by  $u^{Z,j}(t)$  such that  $\tilde{A}^j(t) = u^{Z,j}(t)A^j(t)$  and  $\tilde{B}^j(t) = u^{Z,j}(t)B^j(t)$ , eq. (51) and the log-linearized version of the technology frontier (37) can be solved for labor- and capital-augmenting productivity:

$$\hat{A}^j(t) = - \left(1 - s_L^j\right) \left[ \left(\frac{\sigma^j}{1 - \sigma^j}\right) \hat{S}^j(t) - \hat{k}^j(t) - \hat{u}^{K,j}(t) \right], \quad (52a)$$

$$\hat{B}^j(t) = s_L^j \left[ \left(\frac{\sigma^j}{1 - \sigma^j}\right) \hat{S}^j(t) - \hat{k}^j(t) - \hat{u}^{K,j}(t) \right]. \quad (52b)$$

Plugging estimated values for  $\sigma^j$  and empirically estimated responses for  $s_L^j(t)$ ,  $k^j(t)$ ,  $u^{K,j}(t)$  into above equations enables us to recover the dynamics for  $A^j(t)$  and  $B^j(t)$  consistent with the demand of factors of production (33) and the technology frontier (37). In this regard, the route taken to infer  $\hat{A}^j(t)$  and  $\hat{B}^j(t)$  from (52a)-(52b) amounts to conducting a wedge analysis.

Once we have determined the underlying dynamic process for labor and capital efficiency by using (52a)-(52b), we have to choose values for exogenous parameters  $x^j$  (for  $x = a, b$ ),  $\xi_X^j$  (for  $X = A, B$ ) within sector  $j = H, N$ , which are consistent with the continuous time paths (42). Setting  $t = 0$  into (42a)-(42b) yields  $a^j = \hat{A}^j(0)$ , and  $b^j = \hat{B}^j(0)$  and we choose  $a^j$  and  $b^j$  so as to reproduce impact responses of factor-augmenting productivity in sector  $j$ . Making use of the time series generated by (52a) and (52b) gives us  $a^H = -0.006$ ,  $b^H = 0.010$ ,  $a^N = -0.017$ ,  $b^N = 0.037$ . Next, we choose values for  $\xi_X^j$  and  $\chi_X^j$  so as to reproduce the shape of the dynamic adjustment of sectoral factor-augmenting efficiency recovered by using (52) together with its cumulative change over a six-year period. It gives us  $\xi_A^H = 0.395$  and  $\chi_A^H = 0.430$ ,  $\xi_B^H = 0.460$  and  $\chi_B^H = 0.393$  for the traded sector,  $\xi_A^N = 0.350$  and  $\chi_A^N = 0.316$ ,  $\xi_B^N = 0.300$  and  $\chi_B^N = 0.383$  for the non-traded sector.

### 4.3 Government Spending Shock and Technology: Model Performance

In this subsection, we analyze the role of technology in shaping the size of sectoral fiscal multipliers in an open economy following an exogenous temporary increase in government consumption by 1% of GDP. In our baseline calibration, we assume that capital and technology utilization rates,  $u^{K,j}(t)$  and  $u^{Z,j}(t)$ , respond endogenously to the government spending shock, and allow for time-varying FBTC in sector  $j$  driven by the dynamic adjustment of labor- and capital-augmenting efficiency whilst sectoral goods are produced from CES production functions. To gauge the quantitative implications of technology for fiscal transmission, we contrast our results with those obtained in a model with Cobb-Douglas production functions where we shut down the endogenous response of capital and technology utilization

by letting  $\xi_2^j$  and  $\chi_2^j$  tend towards infinity and impose  $\xi_A^j = \xi_B^j = \chi_A^j = \chi_B^j = 0$  so that  $u^{K,j}(t) = u^{Z,j}(t) = A^j(t) = B^j(t) = 1$ .

In Table 2, we report the simulated impact (i.e., at  $t = 0$ ) and six-year cumulative (i.e., at  $t = 0, \dots, 5$ ) effects. Cumulative effects are expressed in present discounted value terms.<sup>30</sup> While columns 1 and 4 show impact and (present discounted value of) cumulative responses from local projection for comparison purposes, columns 2 and 5 show results for the baseline model. We contrast the benchmark results with those shown in columns 3 and 6 for impact and cumulative effects, respectively, which are obtained in the restricted model.

**Adjustment in government consumption.** As shown in Fig. 6(a), the black line with squares lies within the confidence bounds and therefore the endogenous response of government spending to an exogenous fiscal shock that we generate theoretically by specifying the law of motion (41) reproduces well the dynamic adjustment of  $G(t)/Y$  estimated from the local projection. As can be seen in the first row of panel A of Table 2, the baseline (and the restricted) model generates a present discounted value of the cumulative change in government consumption of 5.46 ppt of GDP (see columns 5-6), close to our estimation of 5.51 ppt (see column 4).

**Restricted model.** We first consider the scenario with Cobb-Douglas production functions, i.e.,  $\sigma^j = 1$ , we let  $\xi_2^j$  and  $\chi_2^j$  tend toward infinity, and impose  $\xi_A^j = \xi_B^j = \chi_A^j = \chi_B^j = 0$ . Results for the restricted model are reported in columns 3 and 6 of Table 2. Because the capital and technology utilization rates remain fixed, sectoral TFPs are unchanged, as can be seen in panel D. Because the elasticity of value added w.r.t. inputs (i.e.,  $\theta^j$ ) is fixed, the fraction of value added paid to workers, i.e., the LIS, does not change over time, as shown in panel E.

We start with the aggregate effects. By producing a negative wealth effect, a balanced-budget government spending shock leads agents to supply more labor, which in turn increases real GDP. As shown in panel A of Table 2, a rise in government consumption by 1% of GDP generates an increase in total hours worked by 0.63% and a rise in real GDP by 0.42% on impact, the latter value being almost three times smaller what we estimate empirically (i.e., 1.18%, see column 1).

Panel B Table 2 shows the distribution of the rise in total hours worked across sectors. Hours worked increase by 0.54 ppt of total hours worked in the non-traded sector and by 0.09 ppt only in the traded sector. Formally, the rise in non-traded hours worked can be broken into two components, i.e.,  $\alpha_L^N \hat{L}^N = \alpha_L^N \hat{L}(t) + d\nu^{L,N}(t)$ . According to this decomposition, the non-traded sector receives a fraction (equal to  $\alpha_L^N = 63\%$  in the data)

<sup>30</sup>The percentage deviation of each macroeconomic variable  $X(t)$  relative to its initial steady-state is denoted with a hat, i.e.,  $\hat{X}(t) = dX(t)/X$ . We calculate the present discounted value of the percentage deviation relative to the initial steady-state as follows  $\int_0^t \hat{X}(\tau) e^{-r^* \tau} d\tau$ .

of the increase in total hours worked and also benefits from labor reallocation because the rise in government spending is biased toward non-tradables. As shown in the third row of panel B, the demand shock raises the labor share of non-tradables by 0.14 ppt of total hours worked on impact, close to what we estimate empirically. In this regard, it is worth mentioning that we allow for barriers to mobility which avoids the model overestimating the reallocation of labor. If we had imposed perfect mobility of labor and exogenous terms of trade, the labor share of non-tradables would have increased by 0.77 ppt of total hours worked while traded hours worked would have declined dramatically (by  $\alpha_L^H \hat{L}^H(0) = -0.71$  ppt of total hours worked).<sup>31</sup> Conversely, by increasing the demand for labor in the traded sector and hampering the reallocation of labor toward the non-traded sector, both the appreciation in the terms of trade and workers' mobility costs mitigate the rise in  $\nu^{L,N}(t)$  and thus allow the model to generate an increase in  $L^H$ . However, these two elements are not sufficient on their own to reproduce the cumulative value of labor reallocation along the transitional path. As shown in column 6, the restricted model shutting down technological change predicts a cumulative change in  $\nu^{L,N}(t)$  by 0.71 ppt of total hours worked, a value which is more than two times smaller what is estimated empirically (i.e., 1.68 ppt of total hours worked).

We turn to the distribution of the rise in real GDP across sectors displayed by Panel C of Table 2. The first row of panel C reveals that  $\tilde{Y}^H$  falls by 0.04 ppt of GDP and non-traded value added rises by 0.46 ppt of GDP. The decline in traded value added caused by the capital outflow experienced by this sector is at odds with the evidence as we find empirically that  $\tilde{Y}^H$  rises by 0.33 ppt of GDP on impact (see column 1). The restricted model also understates the rise in  $\tilde{Y}^N$  both on impact (0.46 ppt against 0.85 ppt in the data) and along the transitional path (2.41 ppt against 4.88 ppt in the data). Whilst the restricted model underpredicts  $\hat{Y}^N(t)$ , it produces a cumulative change in  $\nu^{Y,N}(t)$  by 1.07 ppt of GDP, in contradiction with our empirical findings indicating that the value added share of non-tradables is essentially unchanged (see the third row of column 4). According to the decomposition of non-traded value added, i.e.,  $\nu^{Y,N} \hat{Y}^N(t) = \nu^{Y,N} \hat{Y}_R(t) + d\nu^{Y,N}(t)$ , since the model overstates the rise in  $\nu^{Y,N}(t)$  both on impact and in cumulative terms (see column 3 and 6), the underestimation of the increase in  $\tilde{Y}^N(t)$  is caused by the underestimation of real GDP growth as TFP remains fixed.

**Baseline model.** The performance of the model increases when capital and technology utilization rate are allowed to respond endogenously to the government spending shock and firms bias technological change toward production factors. Quantitative results are shown in column 2 for impact effects and column 5 for (the present discounted value of the)

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<sup>31</sup>Columns 3 and 6 of Table 13 in Online Appendix L show numerical results for a model assuming Cobb-Douglas production frictions, abstracting from technological change, imposing perfect mobility of labor across sectors (i.e., we let  $\epsilon$  tend toward infinity) and exogenous terms of trade (i.e., we let  $\rho$  and  $\rho_J$  tend toward infinity).

Table 2: Impact and Cumulative Effects of an Increase in Government Consumption by 1% of GDP

	LP $t = 0$		Impact Responses		LP $t = 0..5$		Cumulative Responses	
	Data		CES-TECH	CD	Data	CES-TECH	CD	
	(1)		(2)	(3)	(4)	(5)	(6)	
<b>A. Aggregate Multipliers</b>								
Gov. spending, $dG(t)$	1.00		1.00	1.00	5.51	5.46	5.46	
Total hours worked, $dL(t)$	0.91		0.97	0.63	6.37	5.61	3.34	
Real GDP, $dY_R(t)$	1.18		1.07	0.42	7.74	7.36	2.14	
<b>B. Sectoral Labor</b>								
Traded labor, $dL^H(t)$	0.21		0.18	0.09	0.73	0.69	0.47	
Non-traded labor, $dL^N(t)$	0.71		0.78	0.54	5.64	4.92	2.87	
Labor share of non-tradables, $dv^{L,N}(t)$	0.13		0.16	0.14	1.68	1.26	0.71	
<i>Decomposition</i>								
Caused by $d\omega^{Y,N}(t)$			0.14	0.14		0.31	0.71	
Caused by cap. deep. differential			0.01	0.00		-0.24	0.00	
Caused by FBTC differential			0.01	0.00		1.20	0.00	
<b>C. Sectoral Value Added</b>								
Traded VA, $dY^H(t)$	0.33		0.20	-0.04	2.86	3.12	-0.28	
Non-traded VA, $dY^N(t)$	0.85		0.88	0.46	4.88	4.24	2.41	
Non-traded VA share, $dv^{Y,N}(t)$	0.16		0.20	0.20	-0.01	-0.41	1.07	
<i>Decomposition</i>								
Caused by TFP differential			0.03	0.00		-0.47	0.00	
Caused by labor reallocation			0.16	0.14		1.30	0.72	
Caused by capital reallocation			0.01	0.06		-1.23	0.35	
<b>D. Technology</b>								
Traded technology utilization, $du^{Z,H}(t)$	0.21		0.28	0.00	6.04	4.35	0.00	
Non-Traded technology utilization, $du^{Z,N}(t)$	0.37		0.41	0.00	2.15	1.97	0.00	
Traded TFP, $dTFP^H(t)$	0.66		0.34	0.00	7.82	5.50	0.00	
Non-Traded TFP, $dTFP^N(t)$	0.36		0.49	0.00	0.28	2.37	0.00	
<b>E. Redistributive effects</b>								
Traded LIS, $ds_L^H(t)$	0.14		0.12	0.00	-2.23	-3.08	0.00	
<i>Decomposition</i>								
Caused by capital deepening			-0.09			0.96		
Caused by FBTC			0.21			-4.04		
Non-traded LIS, $ds_L^N(t)$	0.25		0.25	0.00	3.07	2.99	0.00	
<i>Decomposition</i>								
Caused by capital deepening			-0.04			-0.58		
Caused by FBTC			0.29			3.57		

Notes: Impact ( $t = 0$ ) and cumulative ( $t = 0..5$ ) effects of an exogenous temporary increase in government consumption by 1% of GDP. Panels A,B,C,D,E show the deviation in percentage relative to steady-state for aggregate and sectoral variables. Sectoral value added and value added share are expressed in percent of initial GDP while sectoral labor and labor shares are expressed in percent of initial total hours worked; responses of sectoral LISs are measured in percent of value added of the corresponding sector. Columns 2 and 5 labelled 'CES-TECH' show predictions of the baseline model while columns 3 and 6 labelled 'CD' shows predictions of the restricted version of the model. In the restricted model, we impose  $\sigma^j = 1$  so that production functions are Cobb-Douglas, let  $\xi_2^j, \chi_2^j$  tend toward infinity so that the capital and technology utilization rate collapses to one, and set  $\xi_A^j, \chi_A^j, \xi_B^j, \chi_B^j$  to zero so that labor- and capital-augmenting technological rate remains fixed. In columns 1 and 4, we report point estimates from the VAR model. Since there is a (slight) discrepancy between the response of aggregate real GDP (total hours worked) and the sum of the responses of traded and non-traded value added (hours worked), columns 1 and 4 report the sum of responses of  $Y^H$  and  $Y^N$  ( $L^H$  and  $L^N$ , resp.) to ensure consistency between aggregate and sectoral responses. We also report the reconstructed response of non-traded TFP and capital-utilization adjusted non-traded TFP in order to be consistent with macroeconomic identities. We reconstruct the empirical response of  $TFP^N(t)$  and  $Z^N(t)$  because we found a discrepancy between empirically estimated and reconstructed responses. To reconstruct the empirical responses of  $TFP^N(t)$  and  $Z^N(t)$ , we use empirical responses of aggregate and traded TFP which are both statistically significant to recover the dynamic response of  $\hat{TFP}^N(t)$  (by using eq. (67)) and we plug the latter together with the response of  $\hat{u}^{K,N}(t)$  to recover  $\hat{Z}^N(t)$ . Inspection of Fig. 5(d) and Fig. 5(e) gives a sense of the discrepancy between the estimated vs. reconstructed response of capital-utilization adjusted non-traded TFP and non-traded TFP.

cumulative effects over a six-year horizon.

As can be seen in panel A of Table 2, the baseline model does a good job in reproducing the aggregate effects of a shock to government consumption. More specifically, total hours worked increase by 0.97%, close to the rise by 0.91% we estimate empirically. Like in the data, we find a government spending multiplier above one as real GDP increases by 1.07% on impact against 1.18% in the data. Along the transitional path, the baseline model produces a cumulative change in  $L(t)$  and real GDP of 5.61% and 7.36% (vs. 6.37% and 7.74% in the data), respectively, thus generating a government spending multiplier on labor and real GDP of 1.03 and 1.35 on average the first six years close to the multipliers of 1.16 and 1.40 we estimate empirically. Three factors amplify the rise in total hours worked and in real GDP compared with the restricted model. First, in face of a higher real capital cost, both sectors, especially traded firms, increase the capital utilization rate,  $u^{K,j}(t)$ . By raising the demand for labor and the use of capital input, higher capital utilization amplifies the rise in  $L(t)$  and  $\tilde{Y}_R(t)$ . Second, because the rise in government spending puts upward pressure on the unit cost for producing, it is optimal to increase the technology utilization rate in both sectors.<sup>32</sup> Whilst the rise in aggregate TFP increases directly real GDP, it also raises  $\tilde{Y}_R(t)$  by increasing the wage rate which encourages agents to supply more labor. Third, as discussed below, the rise in  $L(t)$  (and thereby in real GDP) is amplified because the production technology becomes more labor intensive in the non-traded sector which accounts for two-third of total hours worked.

The responses of technology to an exogenous shock to government consumption are shown in panel D. The first two rows of panel D reveal that the capital-utilization adjusted TFP increases by 0.28% and 0.41%, respectively, in the traded and the non-traded sector, close to what we estimate empirically (i.e., 0.21% and 0.37%) on impact. Whilst technology improvements are similar across sectors on impact, the cumulative effect over a six-year horizon reveals that the increase in the efficiency in the use of inputs is much more pronounced in the traded than in the non-traded sector because the cost of adjusting technology is lower in the traded than in the non-traded sector.<sup>33</sup> The combined effect of a higher capital and technology utilization rate pushes up traded and non-traded TFP by 5.5% and 2.37% in cumulative terms, respectively.<sup>34</sup>

Panel E of Table 2 shows that our model reproduces well the adjustment in the sectoral LISs both on impact and over a six-year horizon. Log-linearizing (33) and using the fact that  $\hat{s}_L^j(t) = \hat{S}^j(t) (1 - s_L^j)$ , shows that the response of the LIS in sector  $j$  is driven by

<sup>32</sup>Aggregate TFP increases by 0.43%, a value close to what we estimate empirically (i.e., 0.5%) on impact.

<sup>33</sup>Although the model understates the cumulative change in the capital-utilization-adjusted-traded TFP (4.35% against 6.04% in the data), the model reproduces well the cumulative change in traded value added (3.12 ppt against 2.86 ppt of GDP in the data).

<sup>34</sup>The model overstates the rise in non-traded TFP as it somewhat overpredicts the increase in the non-traded capital utilization rate.



capital deepening and FBTC:

$$ds_L^j(t) = s_L^j \left(1 - s_L^j\right) \frac{1 - \sigma^j}{\sigma^j} \tilde{k}^j(t) + s_L^j \left(1 - s_L^j\right) \text{FBTC}_{adjK}^j, \quad (53)$$

where  $\tilde{k}^j(t) = u^{K,j}(t)k^j(t)$  and  $\text{FBTC}_{adjK}^j = \left(\frac{B^j(t)}{A^j(t)}\right)^{\frac{1-\sigma^j}{\sigma^j}}$ . The first term on the RHS captures the effect of capital deepening while the second term reflects the impact of utilization-adjusted-FBTC in sector  $j$ . When we shut down technological change and assume  $\sigma^j < 1$  (as evidence suggests), both sectors experience a fall in  $\tilde{k}^j(t)$  which lowers  $s_L^j(t)$ , in contradiction with our evidence.<sup>35</sup> Conversely, as long as we allow for time-varying FBTC, the ability of the model to reproduce the dynamics of sectoral LISs substantially increases. As shown in column 5, technological change biased toward capital in the traded sector generates a cumulative decline in  $s_L^H(t)$  by -4.04% while technological change biased toward capital in the non-traded sector increases  $s_L^N(t)$  by 3.57%. Because FBTC now shifts capital toward the traded sector which becomes more capital intensive, the traded LIS increases through the capital deepening channel by 0.96%. Conversely, the non-traded sector experiences a capital outflow which lowers  $s_L^N(t)$  by -0.58% through the capital deepening channel. In both cases, the FBTC channel more than offsets the capital deepening channel so that the baseline model predicts a cumulative fall in the traded LIS by -3.08% (-2.28% in the data) and a cumulative increase in the non-traded LIS by 2.99% (3.07% in the data).

While FBTC is key to reproducing the dynamics of sectoral LISs, it also increases the performance of the model in reproducing the shift of labor across sectors. Panel B of Table 2 reveals that the baseline model with endogenous technology reproduces well the adjustment in traded and non-traded hours worked both on impact (column 2 vs. column 1) and over a six-year horizon (column 5 vs. column 4). As mentioned above, the restricted model understates the expansionary effect of a government spending shock on sectoral hours worked by shutting down the capital and technology utilization rates together with FBTC. Conversely, as shown in column 5, the baseline model reproduces well the cumulative rise in traded and non-traded hours worked which amounts to 0.69 ppt and 4.92 ppt of total hours worked. The reason is twofold. First, the model allowing for technological change can account for the increase in total hours worked, each sector receiving a share (equal to their labor compensation share  $\alpha_L^j$ ) of  $\hat{L}(t)$ . Second, because non-traded firms bias technological change toward labor and traded firms bias technological change toward capital, technology further tilts the demand of labor toward non-tradables which amplifies the shift of labor toward the non-traded sector as detailed below.

In the last three rows of panel B, we break down the change in the labor share of non-tradables into three components by log-linearizing its equilibrium value described by eq.

<sup>35</sup>For reasons of space, we do not show results for the model assuming CES production functions and abstracting from technological change. This restricted model predicts a fall in both  $\tilde{k}^H(t)$  and  $\tilde{k}^N(t)$  which generates a (present value) cumulative decline in  $s_L^H(t)$  by -0.6% and in  $s_L^N(t)$  by -0.12%, both computed over six-year horizon, see column 6 of panel E of Table 13 in Online Appendix L.



(6) and by making use of (53) (see Online Appendix K.2):

$$\begin{aligned}
d\nu^{L,N}(t) &= \alpha_L(1 - \alpha_L) \frac{\epsilon}{1 + \epsilon} \frac{\hat{\omega}^{Y,N}(t)}{1 - \omega^{Y,N}} \\
&+ \alpha_L(1 - \alpha_L) \frac{\epsilon}{1 + \epsilon} \left[ (1 - s_L^N) \left( \frac{1 - \sigma^N}{\sigma^N} \right) \hat{k}^N(t) - (1 - s_L^H) \left( \frac{1 - \sigma^H}{\sigma^H} \right) \hat{k}^H(t) \right] \\
&+ \frac{\epsilon}{1 + \epsilon} \left[ (1 - s_L^N) \text{FBTC}_{adjK}^N(t) - (1 - s_L^H) \text{FBTC}_{adjK}^H(t) \right], \tag{54}
\end{aligned}$$

The first term on the RHS of (54) measures the change in  $\nu^{L,N}(t)$  driven by the rise in the value added share of non-tradables at current prices denoted by  $\omega^{Y,N}(t)$ . This term measures the change in  $\nu^{L,N}(t)$  driven by the biasedness of the demand shock toward non-tradables which increases  $\omega^{Y,N}(t)$ . The second and third term on the RHS of eq. (54) measures the change in  $\nu^{L,N}(t)$  driven by the rise in the non-traded relative to the traded LIS. As shown in eq. (53), both capital deepening and FBTC impinge on the LIS. The second term on the RHS of (54) measures the change in  $\nu^{L,N}(t)$  driven by sector differences in capital deepening. The third term on the RHS of (54) captures the change in  $\nu^{L,N}(t)$  driven by the differential in the utilization-adjusted-FBTC (scaled by LIS) between non-tradables and tradables. Focusing on cumulative changes, the decomposition shown in column 6 of panel B for the restricted model reveals that the rise in  $\nu^{L,N}(t)$  by 0.71 ppt of total hours worked is only driven by the biasedness of the demand shock toward non-tradables. When we turn to the decomposition of  $d\nu^{L,N}(t)$  for the baseline model shown in column 5, we find that technological change biased toward labor in the non-traded sector and biased toward capital in the traded sector generates on its own a cumulative reallocation of labor of 1.2 ppt of total hours worked toward the non-traded sector. The biasedness of the demand shock toward non-tradables further increases  $\nu^{L,N}(t)$  by 0.31 ppt of total hours worked. Conversely, by increasing the productivity of labor in the traded sector, the shift of capital toward this sector lowers  $\nu^{L,N}(t)$  by -0.24 ppt of total hours worked. The sum of these three effects results in a cumulative increase in the labor share of non-tradables by 1.26 ppt of total hours worked (1.68 ppt in the data). Importantly, FBTC contributes 69% on its own to the change in  $\nu^{L,N}(t)$  on average the first six years.<sup>36</sup> As shown in the second row, because traded firms use more intensively physical capital than non-traded firms which increases the productivity of labor in the traded sector, capital deepening mitigates the impact of FBTC on  $\nu^{L,N}(t)$ .

We turn to the adjustment in sectoral value added at constant prices shown in panel C of Table 2. While FBTC influences labor reallocation and the responses of sectoral hours

<sup>36</sup>Because each factor contributing to  $d\nu^{L,N}(t)$  may exert either a positive or a negative impact on the labor share of non-tradables, we calculate the contribution of each factor  $k$  as follows:

$$\text{Contribution of } k \text{ to } d\nu^{L,N}(t) = \frac{d\nu_k^{L,N}(t)}{\sum_k |d\nu_k^{L,N}(t)|},$$

where  $k$  is either  $\omega^{Y,N}(t)$ , the capital deepening differential between non-tradables and tradables, or the FBTC differential between non-tradables and tradables.

worked, the variations in sectoral value added are mostly influenced by changes in sectoral TFPs. As can be seen in the first row of panel C, the restricted model abstracting from endogenous technological change predicts a fall in  $Y^H$  both on impact and over a six-year horizon which enters in sharp contradiction with our estimates shown in columns 1 and 4. In contrast, by letting traded firms to use more intensively installed capital and existing technology, the baseline model generates an increase in  $\tilde{Y}^H(t)$  by 0.20 ppt of GDP on impact (0.33 ppt in the data) and 3.12 ppt of GDP over a six-year horizon (2.86 ppt in the data), as can be seen in columns 2 and 5. As discussed below, by allowing for sectoral differences in technology improvement, the baseline model can account for the distribution of the government spending multiplier on real GDP across sectors. More specifically, we estimate empirically a government spending multiplier of 0.52 ( $= 2.86/5.51$ ) for tradables and 0.89 ( $= 4.88/5.51$ ) for non-tradables on average over a six-year horizon while the baseline model generates a multiplier of 0.57 ( $= 3.12/5.46$ ) for tradables and 0.78 ( $= 4.24/5.46$ ) for non-tradables, respectively.

According to the decomposition of the percentage deviation of sectoral value added,  $\nu^{Y,j}\hat{Y}^j(t) = \nu^{Y,j}\hat{Y}_R^j(t) + d\nu^{Y,j}(t)$ , each sector receives a fraction (equal to  $\nu^{Y,j}$ ) of real GDP growth while the change in the value added share,  $d\nu^{Y,j}(t)$ , indicates whether real GDP growth is symmetrically (i.e.,  $d\nu^{Y,j}(t) = 0$ ) or asymmetrically distributed across sectors. As shown in the third row of column 4, the value added share of non-tradables remains unchanged and thus real GDP growth is distributed across sectors in accordance with their value added share. To quantify the role of technology in driving the distribution of the government spending multiplier across sectors, we break down analytically the change in the value added share of non-tradables into three components (see Online Appendix K.1)<sup>37</sup>

$$\begin{aligned} d\nu^{Y,N}(t) &= \nu^{Y,H} (1 - \nu^{Y,H}) \left( \text{T}\hat{\text{F}}\text{P}^H(t) - \text{T}\hat{\text{F}}\text{P}^N(t) \right) + \nu^{Y,H} (1 - \nu^{Y,H}) \left( \hat{L}^H(t) - \hat{L}^N(t) \right) \\ &+ \nu^{Y,H} (1 - \nu^{Y,H}) \left[ (1 - s_L^H) \hat{k}^H(t) - (1 - s_L^L) \hat{k}^N(t) \right]. \end{aligned} \quad (55)$$

The first term on the RHS of (55) measures the change in  $\nu^{Y,N}(t)$  driven by the TFP differential. The second and the third term on the RHS of (55) captures the change in the value added share of non-tradables driven by labor and capital reallocation. The last three rows of panel C of Table 2 provides a quantitative decomposition of the cumulative change in the value added share of non-tradables. As can be seen in column 6, when technological change is shut down, both labor and capital shifts toward the non-traded sector, increasing  $\nu^{Y,N}(t)$  by 1.07 ppt of GDP. In contrast, in the baseline model, because technological change is biased toward labor in the non-traded sector and biased toward capital in the traded sector, more labor shifts toward the non-traded sector while capital shifts away from this sector. As displayed by the last three rows of column 5, the capital outflow caused

<sup>37</sup>We use the fact that  $d\nu^{Y,N}(t) = (1 - \nu^{Y,H}) \nu^H \left( \hat{Y}^N(t) - \hat{Y}^H(t) \right)$ , and  $\hat{Y}^j(t) = \text{T}\hat{\text{F}}\text{P}^j(t) + \hat{L}^j(t) + (1 - s_L^j) \hat{k}^j(t)$ .

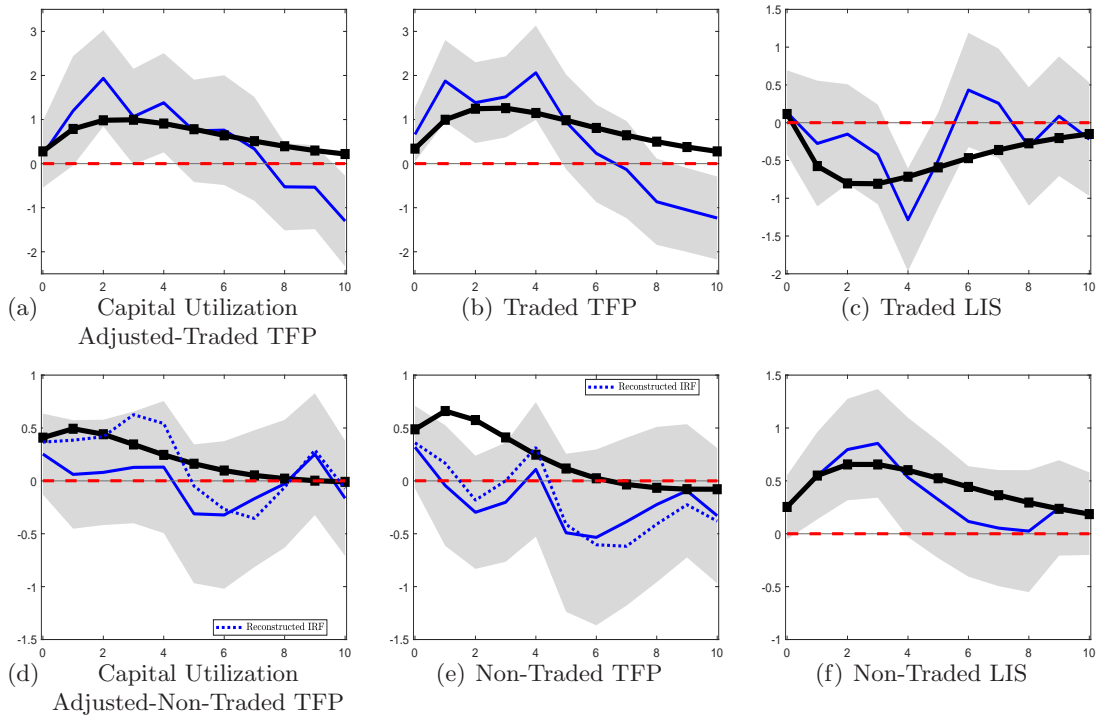


Figure 5: Theoretical vs. Empirical Responses Following Unanticipated Government Spending Shock: Technology Effects. *Notes:* Solid blue line displays point estimate of VAR with shaded areas indicating 90% confidence bounds; in the dotted blue line, we reconstruct the empirical response of  $TFP^N(t)$  and  $Z^N(t)$  because we found a substantial discrepancy between the empirically estimated and reconstructed responses. In the latter case, we use empirical responses of aggregate and traded TFP which are both statistically significant to reconstruct the dynamic responses of  $TFP^N(t)$  (by using eq. (67)) and we plug the latter together with the response of  $\hat{u}^{K,N}(t)$  to recover  $\hat{Z}^N(t)$ . The thick solid black line with squares displays model predictions in the baseline scenario with capital and technology utilization together with FBTC while the dashed red line shows predictions of a model with Cobb-Douglas production functions and abstracting from capital and technology utilization.

by the FBTC differential almost offsets the labor inflow, and the TFP differential causes a cumulative fall in  $\nu^{Y,N}(t)$  by -0.47 ppt of GDP.

While in Table 2, we restrict our attention to impact and cumulative responses, in Fig. 5 and Fig. 6, we contrast theoretical (displayed by solid black lines with squares) with empirical (displayed by solid blue lines) dynamic responses. Empirical responses display the point estimate obtained from local projection, with the shaded area indicating the 90% confidence bounds. We also contrast theoretical responses from the baseline model with the predictions of the restricted model where we shut down the response of technology as shown in the dashed red lines.

We start with the adjustment of technology displayed by Fig. 5. Because higher government consumption increases the demand for traded and non-traded goods, both sectors find it profitable to raise the efficiency in the use of inputs to meet higher demand for sectoral goods. While the demand shock is biased toward non-traded goods, Fig. 5(a) and Fig. 5(d) show that technology improvements along the transitional path (i.e.,  $\hat{Z}^j(t) > 0$ ) are much more pronounced in the traded than in the non-traded sector because the former sector experiences a lower adjustment cost of technology. Since the demand of capital rises in both sectors which puts upward pressure on the real capital rental rates, it is profitable to use the stock of capital more intensively (i.e.,  $u^{K,j}(t)$  rises) which results in higher sectoral TFPs

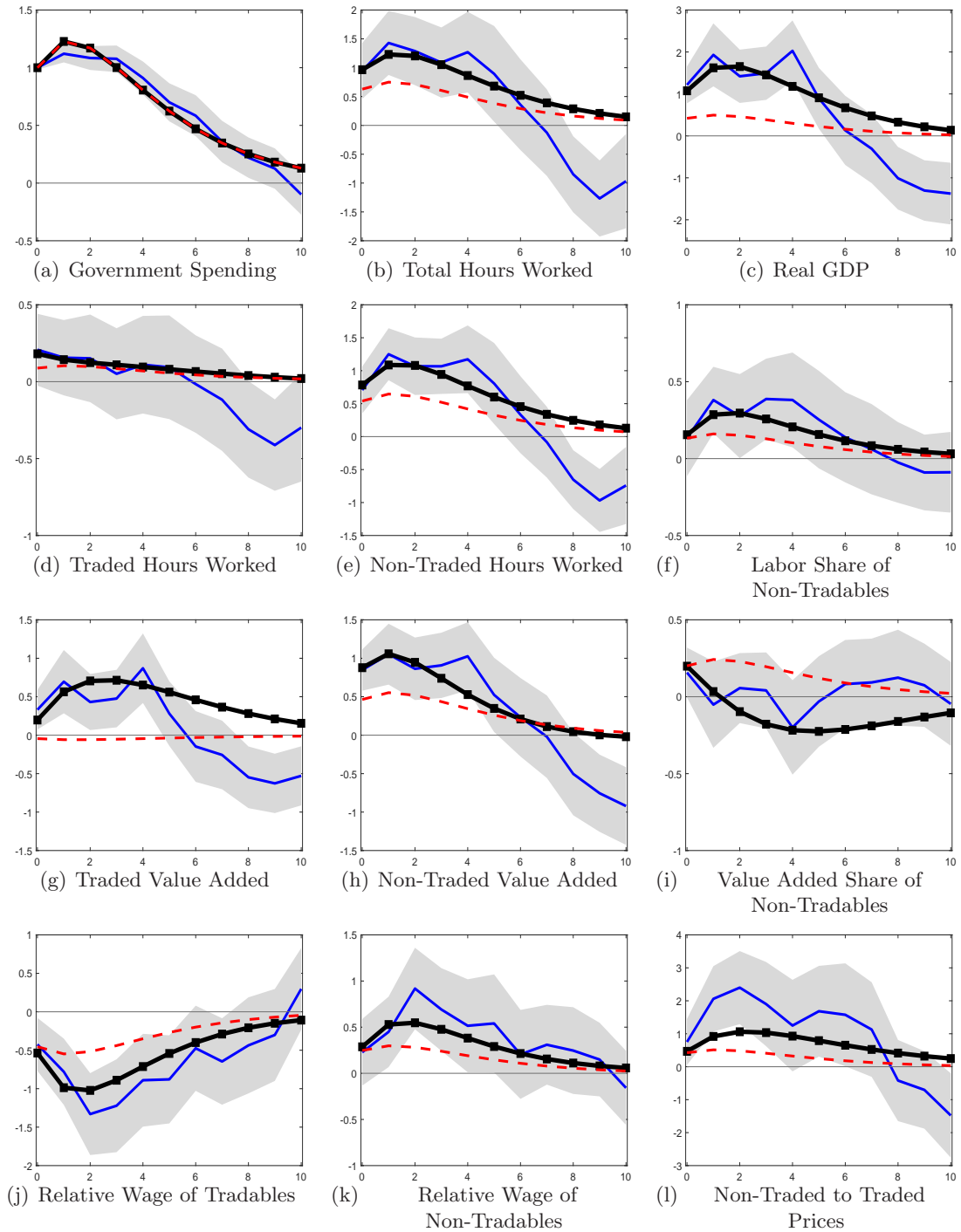


Figure 6: Theoretical vs. Empirical Responses Following Unanticipated Government Spending Shock: Labor and Output Effects. *Notes:* Solid blue line displays point estimate of VAR with shaded areas indicating 90% confidence bounds; the thick solid black line with squares displays model predictions in the baseline scenario with capital and technology utilization together with FBTC while the dashed red line shows predictions of a model with Cobb-Douglas production functions and abstracting from capital and technology utilization.

(i.e.,  $\text{TFP}^j(t) = \hat{Z}^j(t) + (1 - s_L^j) \hat{u}^{K,j}(t) > 0$ ), as displayed by Fig. 5(b) and Fig. 5(e). Besides technology improvements, firms change the mix of labor- and capital-augmenting efficiency. Because traded firms bias technological change toward capital, the traded LIS falls below trend as shown in Fig. 5(c). Conversely, non-traded firms bias technological change toward labor which increases the non-traded LIS as displayed by Fig. 5(f).

As shown in Fig. 6, both the shift in the technology frontier and the change in the mix of labor- and capital-augmenting efficiency along the technology frontier increase the ability of the two-sector open economy model to account for the VAR evidence. Following a rise in government consumption shown in Fig. 6(a), the baseline model is able to capture the dynamics of total hours worked and real GDP once we allow for an endogenous response of sectoral TFP, as can be seen in Fig. 6(b) and Fig. 6(c). Intuitively, technology improvements and a higher labor intensity of production result in a higher wage rate which encourages agents to supply more labor. The combined effect of the rise in aggregate TFP and higher labor supply amplifies the increase in real GDP.

We turn to the second row of Fig. 6 which shows the distribution of the rise in total hours worked between the traded and the non-traded sector. Fig. 6(f) reveals that the baseline model reproduces well the rise in labor share of non-tradables because non-traded (traded) firms use labor (capital) more intensively which amplifies the reallocation of hours worked toward this sector. By contrast, as shown in the dashed red line, the biasedness of the demand shock toward non-traded goods is not sufficient on its own to account for the adjustment in the labor share. As it stands out in Fig. 6(d) and Fig. 6(e), the model tracks well the dynamics of traded and non-traded hours worked once we allow non-traded firms to bias technological change toward labor and let traded technology become more capital intensive.

The third row of Fig. 6 shows the distribution of the rise in real GDP across sectors. As displayed by the dashed red lines in Fig. 6(i), because the restricted model overstates the shift of productive resources toward the non-traded sector and generates an increase in the value added share of non-tradables, it generates a decline in  $\tilde{Y}^H(t)$  in contradiction with our evidence (see Fig. 6(g)). Conversely, as shown in Fig. 6(g), the baseline model is able to generate the hump-shaped dynamics of traded value added which is driven by the endogenous rise in the capital and technology utilization rates. Because it generates an expansionary effect on real GDP in line with the evidence, only the baseline model can account for the dynamics of  $\tilde{Y}^N(t)$  shown in Fig. 6(h).

While in section 2, we focus on the adjustment of quantities and stress the role of technology, in the fourth row of Fig. 6, we assess the ability of our model to account for the behavior of relative wages  $\tilde{W}^j(t)/\tilde{W}(t)$  and the relative price of non-tradables. As shown in Fig. 6(j) and Fig. 6(k), non-traded firms pay higher wages relative to traded

firms to encourage workers to shift hours worked toward the non-traded sector. Because technological change is biased toward capital in the traded sector and toward labor in the non-traded sector, the adjustment in relative wages is more pronounced in the baseline model, in line with the evidence. As can be seen in Fig. 6(1), because the demand shock is biased toward non-tradables, the relative price of non-tradables appreciates. The appreciation in  $P^N(t)/P^H(t)$  is more pronounced in the baseline model because the TFP differential mitigates the increase in non-traded relative to traded value added.

#### 4.4 Government Spending Shock and Technology: Cross-Country Differences

We now move a step further and calibrate our model to country-specific data. Our objective is to assess the impact of international differences in the adjustment of technology following a fiscal shock on sectoral fiscal multipliers. To isolate the pure role of technological change, we control for international differences in the biasedness of the demand shock toward non-tradables by assuming that the intensity of the non-traded sector in the government spending shock,  $\omega_{GN}$ , is symmetric across countries.

**Calibration to country-specific data.** To conduct our cross-country analysis, we calibrate our model to match key ratios of the 18 OECD economies in our sample, as summarized in Table 7, while  $\epsilon$ ,  $\phi$ ,  $\sigma^j$ ,  $\phi_X$  are set in accordance with estimates shown in the last five columns of the table. We also set  $\beta = r^*$  in line with our estimates for each OECD country shown in the first column of Table 6.

As discussed in subsection 4.1, we consider the initial steady-state with Cobb-Douglas production functions as the normalization point and calibrate the reference model to the data;  $\varphi$ ,  $\iota$ ,  $\varphi^H$ ,  $\iota^H$ ,  $\vartheta$ ,  $\delta_K$  together with initial conditions need to be endogenously calibrated to target  $1 - \alpha_C$ ,  $1 - \alpha_J$ ,  $\alpha^H$ ,  $\alpha_J^H$ ,  $\frac{L^N}{L}$ ,  $\omega_J$ , and  $v_{NX} = 0$  (see subsection 4.1); we also choose values for the LIS,  $\theta^j$ , in accordance with our estimates shown in columns 11 and 12 of Table 7;  $\omega_{GN}$  and  $\omega_G$  are chosen to match the non-tradable content of government spending and the share of government spending in GDP (see columns 4 and 15, respectively, of Table 7). The remaining parameters, i.e.,  $\sigma_L$ ,  $\sigma_C$ ,  $\rho$ ,  $\phi_J$ ,  $\rho_J$ ,  $\kappa$  take the same values as those summarized in Table 1.

The high uncertainty surrounding estimates of responses of sectoral capital utilization rates at a country level leads us to abstract from endogenous capital utilization so that technology improvement collapses to variations of TFP. Because we aim at quantifying precisely sectoral government spending multipliers, we treat technological progress as an exogenous process and proceed as follows. Because we abstract from capital utilization and treat technological change as exogenous, we remove the tilde notation. Letting  $\xi_2^j$  tend toward infinity (which implies  $\hat{u}^{K,j}(t) = 0$ ) and log-linearizing (33) leads to  $\left(\hat{B}^j(t) - \hat{A}^j(t)\right) = \frac{\sigma^j}{1-\sigma^j} \hat{S}^j(t) - \hat{k}^j(t)$ . Making use of the log-linearized version of the

technology frontier given by (37), i.e.,  $\widehat{\text{TFP}}^j(t) = s_L^j \hat{A}^j(t) + (1 - s_L^j) \hat{B}^j(t)$  where technology improvements are now captured by changes in sectoral TFP. Solving for  $\hat{A}^j(t)$  and  $\hat{B}^j(t)$  leads to  $\hat{A}^j(t) = \widehat{\text{TFP}}^j(t) - (1 - s_L^j) \left[ \left( \frac{\sigma^j}{1 - \sigma^j} \right) \hat{S}^j(t) - \hat{k}^j(t) \right]$  and  $\hat{B}^j(t) = \widehat{\text{TFP}}^j(t) + s_L^j \left[ \left( \frac{\sigma^j}{1 - \sigma^j} \right) \hat{S}^j(t) - \hat{k}^j(t) \right]$ . We plug country-specific estimates of  $\sigma^j$  and country-specific estimated responses for  $s_L^j(t)$ ,  $k^j(t)$ ,  $\widehat{\text{TFP}}^j(t)$  into the above equations to recover the dynamics for  $A^j(t)$  and  $B^j(t)$  and choose parameters  $\bar{a}^j$ ,  $\bar{b}^j$ ,  $\xi_X^j$ ,  $\chi_X^j$  for  $X = A, B$  within sector  $j = H, N$  so as to reproduce the dynamic adjustment of sectoral factor-augmenting efficiency, as described by eq. (42a)-(42b), specific to each country; in addition, we ensure that the cumulative change in  $A^j(t)$ ,  $B^j(t)$ ,  $\widehat{\text{TFP}}^j(t)$  over a six-year horizon reproduces the cumulative change we estimate empirically.

As for a representative OECD economy, the endogenous response of government consumption to an exogenous fiscal shock is governed by the continuous time path (41) and we choose parameters  $\xi$  and  $\chi$  to reproduce the empirical response of  $G(t)$  we estimate for each country of our sample. In calibrating the model to country-specific data, we assume that the biadness of the shock to government consumption is symmetric across countries while technological change is country-specific. We thus set  $\omega_{GN} = 80\%$  and  $\omega_{GH} = 18\%$  for each OECD country. Once the model is calibrated, we estimate numerically the effects of an exogenous temporary increase in government consumption by 1% of GDP.

Columns 1-3 of Table 3 show numerical results when we simulate the baseline model with CES production functions and technological change. Columns 5-7 of Table 3 show the excess of the government spending multiplier driven by technological change which is computed as the difference between the government spending multiplier in the baseline model and the government spending multiplier in the restricted model with no technological change. Column 4 shows the TFP differential between tradables and non-tradables for Panel A and the FBTC differential between non-tradables and tradables (where  $\text{FBTC}^j(t)$  is scaled by the capital income share  $1 - s_L^j$ ) for panel B. Each figure is computed as the ratio of the present discounted value of the cumulative change of the corresponding quantity divided by the present discounted value of the cumulative change of government consumption, both calculated over a six-year period. Therefore, each cell indicates the average annual rise in value added or hours worked following a rise in government spending by 1% of GDP the first six years.

**Cross-country differences in government spending multiplier on non-traded value added.** How international differences in the response of technology to a government spending shock modify the government spending multiplier on sectoral value added? For convenience, we repeat the decomposition of the rise in non-traded value added following a rise in government consumption:

$$\nu^{Y,N} \hat{Y}^N(t) = \nu^{Y,N} \hat{Y}_R(t) + d\nu^{Y,N}(t). \quad (56)$$



When  $d\nu^{Y,N}(t) = 0$ , real GDP growth is symmetrically distributed across sectors. If  $d\nu^{Y,N}(t) > 0$ , the government spending shock benefits disproportionately the non-traded sector. Column 1 of Panel A of Table 3 shows the (cross-country) average value of the government spending multiplier on real GDP.<sup>38</sup> The government spending multiplier averages about one the first six years. As shown in column 2 of Table 3, the government spending multiplier on non-traded value added is equal to 0.6 ppt of GDP while the value added share of non-tradables declines very slightly by 0.05 ppt of GDP as can be seen in column 3. The rationale behind the insignificant change in  $\nu^{Y,N}(t)$  lies in the TFP differential between tradables and non-tradables of 0.12% shown in column 4 which offsets the positive impact of the biasedness of the government spending shock toward non-tradables on  $d\nu^{Y,N}(t)$ .<sup>39</sup>

Column 5 of Table 3 shows the excess of the government spending multiplier on real GDP caused by technological change. On average, technological change increases the government spending multiplier by 0.64 ppt of GDP. Fig. 7(a) plots the excess (or the reduction) of the government spending multiplier driven by technological change over a six-year horizon against the excess of traded over non-traded TFP. One-third of OECD countries which are positioned in the south-west experience a decline in traded relative to non-traded TFP which averages 0.92% (see column 4 of Table 3). For these economies, technological change lowers the government spending multiplier by -1 ppt of GDP (see the row before the last in column 5) because these economies experience a decline in aggregate TFP. Conversely, countries positioned in the north-east of Fig. 7(a) experience a positive TFP differential which averages 0.64% and these economies also have a government spending multiplier which is 1.5 ppt of GDP larger on average (see the last row of column 5). The reason is that when technological change is concentrated in traded industries, evidence indicates that aggregate TFP rises.

Interestingly, the excess (driven by technological) change) of the government spending multiplier on non-traded value added shown in column 6 remains similar whether traded TFP increases or declines relative to non-traded TFP. The reason is intuitive. In countries which experience a fall in traded relative to non-traded TFP (as shown in the north-west of Fig. 7(c)), the rise in  $\nu^{Y,N}(t)$  by 0.79 ppt of GDP more than offsets lower  $\hat{Y}_R(t)$ . The corollary is that technological change lowers considerably traded value added in these OECD economies. More specifically, using the formula  $\nu^{Y,H}\hat{Y}^H(t) = \nu^{Y,H}\hat{Y}_R(t) - d\nu^{Y,N}(t)$  where  $\nu^{Y,H} = 0.36$ , allowing for technological change reduces the multiplier on traded value added by 1.16 ppt of GDP. In countries where  $TFP^H(t)/TFP^N(t)$  increases (as shown in the south-east of Fig. 7(c)), additional real GDP growth more than offsets the fall in

<sup>38</sup>Table 11 of Online Appendix K.1 shows the government spending multiplier on real GDP over a six-year period for each OECD country.

<sup>39</sup>In accordance with eq. (3) which decomposes the change in the value added share of non-tradables at constant prices, we scale the TFP differential by  $(1 - \nu^{Y,H})\nu^{Y,H}$  so that the figure gives the change in the value added share of non-tradables in ppt of GDP driven by the change in the relative TFP of tradables.

Table 3: Numerically Computed Values of Government Spending Multiplier on Non-Tradables

<b>A-Value Added</b>	Baseline Model			Excess Baseline Model over Restricted model			
	$\hat{Y}_R(t)$ (1)	$\nu^{Y,N}\hat{Y}^N(t)$ (2)	$d\nu^{Y,N}(t)$ (3)	TFP diff (4)	$\hat{Y}_R(t)$ (5)	$\nu^{Y,N}\hat{Y}^N(t)$ (6)	$d\nu^{Y,N}(t)$ (7)
Mean	1.03	0.60	-0.05	0.12	0.64	0.18	-0.22
TFP diff < 0				-0.92	-1.01	0.15	0.79
TFP diff > 0				0.64	1.47	0.19	-0.73
<b>B-Hours</b>	Baseline Model			Excess Baseline Model over Restricted model			
	$\hat{L}(t)$ (1)	$\alpha^{L,N}\hat{L}^N(t)$ (2)	$d\nu^{L,N}(t)$ (3)	FBTC diff (4)	$\hat{L}(t)$ (5)	$\alpha^{L,N}\hat{L}^N(t)$ (6)	$d\nu^{L,N}(t)$ (7)
Mean	0.68	0.55	0.10	0.08	0.02	0.05	0.03
FBTC diff < 0				-0.54	-0.19	-0.27	-0.13
FBTC diff > 0				0.69	0.24	0.36	0.19

**Notes:** Columns 1-4 show numerical results when we simulate the baseline model with CES production functions and technological change. Columns 1-2 show the government spending multiplier on real GDP and non-traded value added (panel A), on total hours worked and non-traded hours worked (panel B). Column 3 shows the change in the value added (panel A) and labor share (panel B) of non-tradables. Column 4 shows the response of the TFP differential between tradables and non-tradables (panel A) and FBTC differential between non-tradables and tradables (panel B) to a shock to government consumption. We compute numerically the responses of real GDP/hours worked, non-traded value added/hours worked, value added/labor share of non-tradables to a 1% temporary increase in government consumption and calculate the government spending multiplier as the ratio of the present discounted value of the cumulative change of the corresponding quantity to the present discounted of the cumulative change of government consumption over a six-year horizon. To ensure consistency, the TFP/FBTC differential is expressed in present discounted cumulative change divided by the present discounted cumulative change of government consumption. Columns 5-6 show the excess of the government spending multiplier in the baseline model over a model with Cobb-Douglas production functions and shutting down technological change (panel A) or FBTC (panel B). Column 7 shows the excess of the change in the value added (panel A) and labor share (panel B) of non-tradables in the baseline model over the restricted model.

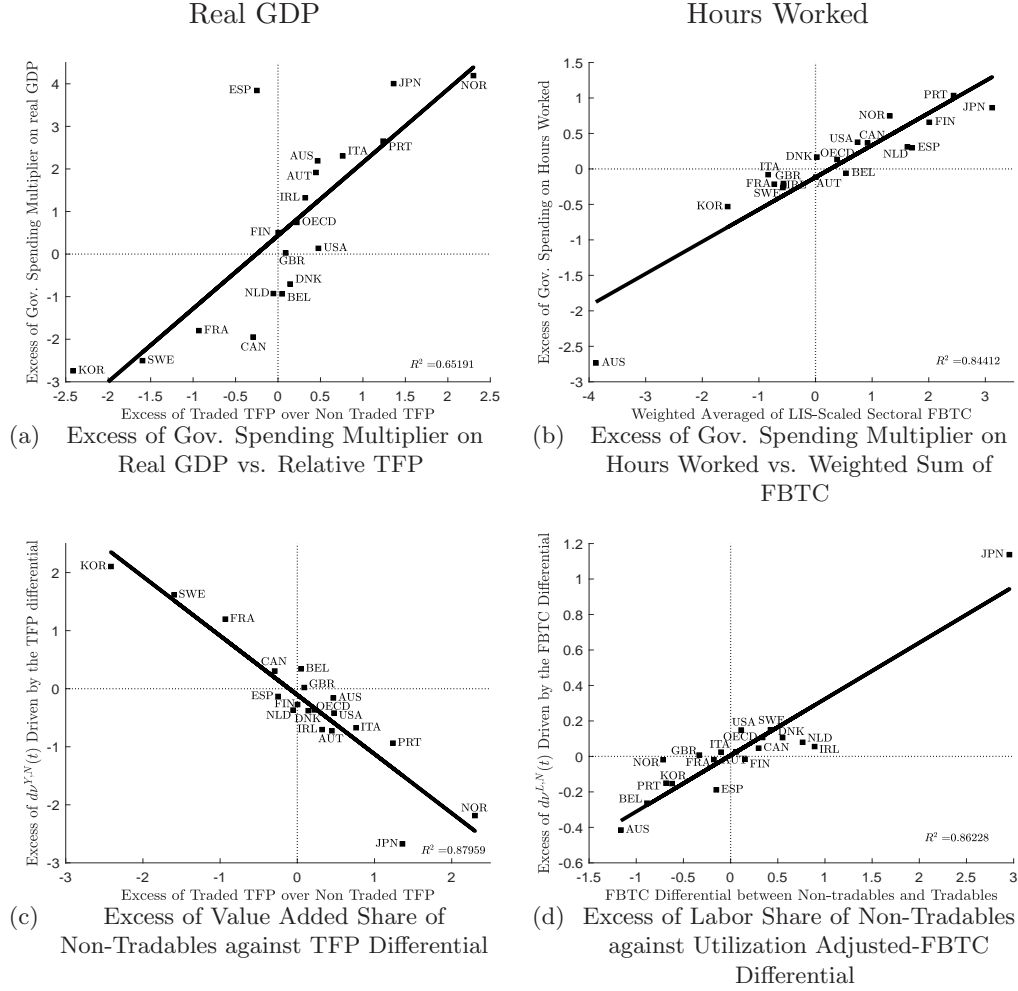
$\nu^{Y,N}(t)$  by -0.73 ppt of GDP. By using the formula above, the multiplier on traded value added is increased by 1.28 ppt of GDP through the technology channel. In conclusion, while on average government spending shocks bias technological change toward traded industries which ensures that real GDP growth is symmetrically distributed across sectors, international differences in responses of  $\text{TFP}^H(t)/\text{TFP}^N(t)$  result in a wide cross-country heterogeneity in  $\nu^{Y,H}\hat{Y}^H(t)$  while  $\nu^{Y,N}\hat{Y}^N(t)$  displays a smaller cross-country dispersion because  $\nu^{Y,N}(t)$  increases when  $\hat{Y}_R(t)$  gets smaller.

**Cross-country differences in government spending multipliers on non-traded hours worked.** We now explore the role of international differences in technology in driving cross-country differences in the government spending multiplier on non-traded hours worked which can be broken down into two components:

$$\alpha_L^N \hat{L}^N(t) = \alpha_L^N \hat{L}(t) + d\nu^{L,N}(t), \quad (57)$$

where  $\alpha_L^N$  is the labor compensation share of non-tradables. When  $d\nu^{L,N}(t) = 0$ , the rise in total hours worked caused by a shock to government consumption is distributed symmetrically across sectors, i.e., in accordance with their labor compensation share. If  $d\nu^{L,N}(t) > 0$ , then non-traded hours worked increase disproportionately relative to traded hours worked as labor shifts toward the non-traded sector.

As can be seen in column 1 of Panel B of Table 3, the government spending multiplier on total hours worked averages 0.68 over a six-year horizon. Column 2 reveals that non-traded hours worked increase by 0.55 ppt of total worked which account for more than 80%



**Figure 7: Government Spending Multiplier and Technology: Cross-Country Analysis.**  
**Notes:** Fig. 7(a) plots the excess of the government spending multiplier on real GDP (vertical axis) in the baseline model over a model with Cobb-Douglas production functions shutting down technological change against the (scaled) excess of traded over non-traded TFP (horizontal axis), i.e.,  $\nu^{Y,H} (1 - \nu^{Y,H}) (\hat{\text{TFP}}^H(t) - \hat{\text{TFP}}^N(t))$ . Fig. 7(b) plots the excess of the government spending multiplier on total hours worked in the baseline model over a model with Cobb-Douglas production functions with time-varying sectoral TFPs while shutting down sectoral FBTC against the weighted sum of sectoral FBTC adjusted with the capital income share, i.e.,  $\alpha_L^N (1 - s_L^N) \hat{\text{FBTC}}^N(t) - \alpha_L^H (1 - s_L^H) \hat{\text{FBTC}}^H(t)$ . Fig. 7(c) plots the excess of the change in the value added share of non-tradables,  $d\nu^{Y,N}(t)$ , in the baseline model over a model shutting down technological change (vertical axis), against the (scaled) excess of traded TFP relative to non-traded TFP (horizontal axis), i.e.,  $\nu^{Y,H} (1 - \nu^{Y,H}) (\hat{\text{TFP}}^H(t) - \hat{\text{TFP}}^N(t))$ . Fig. 7(d) plots the excess of the change in the labor share of non-tradables,  $d\nu^{L,N}(t)$ , in the baseline model over a model imposing Hicks-neutral technological change (vertical axis) against the differential in the utilization-adjusted FBTC scaled by the capital income share between non-tradables and tradables (horizontal axis).

of the rise in  $L(t)$ . The bulk of the labor growth is concentrated in the non-traded sector because this sector accounts for almost two-third of total hours worked and also benefits from a shift of labor as captured by a rise in  $\nu^{L,N}(t)$  by 0.10 ppt of total hours worked. The reallocation of hours worked toward the non-traded sector is driven by the biasedness of the demand shock toward non-tradables together with technological change biased toward non-tradables.

Column 4 displays the adjusted differential in FBTC (scaled by the capital income share) between non-tradables and tradables.<sup>40</sup> Column 5 shows that the excess of the rise in total hours worked in a model which allows for FBTC compared with a model which imposes Hicks-neutral technological change averages 0.02 ppt of total hours worked. As can be seen in column 6, FBTC amplifies the rise in non-traded hours worked by 0.05 ppt of total hours worked, mostly due to the reallocation of labor toward the non-traded sector which amounts to 0.03 ppt of total hours worked. These low figures mask a wide cross-country dispersion however. In Fig. 7(b), we plot the excess of total hours worked caused by FBTC (on the vertical axis) against the weighted sum of FBTC in the traded and the non-traded sector (on the horizontal axis), i.e.,  $\sum_{j=H,N} \alpha_L^j (1 - s_L^j) \widehat{\text{FBTC}}^j(t)$ .<sup>41</sup> The scatter-plot shows that there exists a strong and positive cross-country relationship between the weighted sum of sectoral FBTC and the rise in total hours worked following a government spending shock. More specifically, in (the seven OECD) countries where technological change is biased toward capital, i.e.,  $\sum_{j=H,N} \alpha_L^j (1 - s_L^j) \widehat{\text{FBTC}}^j(t) < 0$ , the rise in total hours worked is 0.6 ppt lower than the increase in  $L(t)$  in a model abstracting from FBTC. Conversely, in countries positioned in the north-east part of Fig. 7(b) where technological change is biased toward labor, the rise in total hours worked is amplified by 0.4 ppt on average.

As can be seen in the last two rows of column 4 of Panel B of Table 3, the FBTC differential between non-tradables and tradables varies widely across countries.<sup>42</sup> Column 6 shows that in countries where technological change is biased toward capital in the non-traded sector (relative to the traded sector), the rise in non-traded hours worked is lowered by 0.27 ppt of total hours worked. Conversely, in countries where technological change is (relatively) biased toward labor in the non-traded sector (relative to the traded sector), the rise in non-traded hours worked is amplified by 0.36 ppt of total hours worked. As

<sup>40</sup>The adjusted FBTC differential reads  $\alpha_L^H \alpha_L^N \left[ (1 - s_L^N) \widehat{\text{FBTC}}^N(t) - (1 - s_L^H) \widehat{\text{FBTC}}^H(t) \right]$ . In accordance with eq. 54, we scale the FBTC differential with  $\alpha_L^H \alpha_L^N$  so that the figure gives directly the change in the labor share of non-tradables in ppt of total hours worked driven by the difference in FBTC between the non-traded and the traded sector.

<sup>41</sup>It is worth mentioning that the weighted sum of FBTC in the traded and the non-traded sector is strongly correlated with non-traded FBTC.

<sup>42</sup>While on the Fig. 7(b), we consider aggregate FBTC, i.e.,  $\sum_{j=H,N} \alpha_L^j (1 - s_L^j) \widehat{\text{FBTC}}^j(t)$ , to measure its impact on the rise in total hours worked, in Fig. 7(d), we consider the FBTC differential as we are interested in determining its impact on the difference in the government spending multiplier as captured by  $d\nu^{L,N}(t)$ .

shown in eq. (57), the reduction or the amplification of the rise in  $L^N(t)$  is the result of the reduction or the amplification of the rise in total hours worked together with the reduction or the amplification of the rise in the labor share of non-tradables. Column 7 reveals that in countries where technological change is (relatively) biased toward capital,  $\nu^{L,N}(t)$  is reduced by 0.13 ppt of total hours worked (compared with a model shutting down FBTC) while  $\nu^{L,N}(t)$  is amplified by 0.19 ppt of total hours worked in countries where technological change is (relatively) biased toward labor. Therefore, on average, half of the reduction or the amplification of the government spending multiplier on non-traded hours worked is the result of the change in  $\nu^{L,N}(t)$  (which measures the reallocation of labor toward the non-traded sector).

Fig. 7(d) plots the average change (over a six-year horizon) in the labor share of non-tradables caused by FBTC (vertical axis) against the adjusted differential in FBTC between non-tradables and tradables (horizontal axis). Inspection of Fig. 7(d) reveals that technological change is more biased toward labor in the non-traded than in the traded sector in about half of the countries. For these nine economies positioned in the north-east, technological change amplifies the shift of labor toward the non-traded sector and thus further increases the government spending multiplier on non-traded hours worked. Conversely, in the remaining nine OECD countries positioned in the south-west, a shock to government consumption leads non-traded firms to bias technological change toward capital which shifts labor toward the traded sector and thus reduces the rise in non-traded hours worked.

## 5 Conclusion

This paper contributes to the literature related to the effects of a government spending shock both empirically and theoretically. From an empirical point of view, we use a panel of eighteen OECD countries over the period 1970-2015 and document evidence pointing at the key role of technological change in determining the size of sectoral fiscal multipliers. First, we find empirically a government spending multiplier higher than one over a six-year horizon and 39% of the rise in real GDP is driven by the endogenous increase in aggregate TFP. Second, the value added share of non-tradables (at constant prices) remains unresponsive to the demand shock at any horizon. According to our evidence, real GDP growth is distributed uniformly across sectors because technology improvement is concentrated in traded industries which neutralizes the impact of the biasedness of the spending shock toward non-tradables on the value added share of non-tradables. Third, 88% of the rise in total hours worked is concentrated in the non-traded sector. Our empirical findings reveal that the disproportionate increase in non-traded hours worked is driven by FBTC as non-traded firms bias technological change toward labor and traded firms bias technological

change toward capital. Fourth, our hypothesis of FBTC accords well with the redistributive effects we document empirically as our estimates reveal that the non-traded LIS increases whilst the traded LIS declines.

To rationalize our evidence, we develop a semi-small open economy with tradables and non-tradables, in the lines of Kehoe and Ruhl [2009], which contains three sets of key features. First, we allow for labor mobility costs and endogenous terms of trade to account for the frictions on factor movements between the traded and non-traded sector. Second, to account for real GDP growth and its distribution across sectors, drawing on Bianchi et al. [2019], we assume that each sector can choose to use more intensively the capital stock and existing technologies. We put forward FBTC as a third key ingredient. Adapting the methodology of Caselli and Coleman [2006] to our model with capital and labor, we allow firms to change the mix of labor- and capital-augmenting technological change at each point of time.

To quantify the role of technology in determining the size of government spending multipliers and their distribution across sectors, we contrast the predictions of the baseline model with those of a restricted model where technological change is shut down and sectoral goods are produced from Cobb-Douglas production functions. Our quantitative analysis shows that a model abstracting from technological change cannot generate the rise in real GDP and in total hours worked we estimate empirically, generates a disproportionate increase in non-traded relative to traded value added in contradiction with our evidence, understates the rise in non-traded hours worked and cannot account for the dynamics of sectoral LISs. Conversely, letting technological change respond endogenously to the government spending shock, the model can account for the evidence once we let the decision on technology improvement vary across sectors and allow firms to change the factor intensity of production over time.

We also take advantage of the panel data dimension of our sample to quantify the role of technology in driving international differences in government spending multipliers. We calibrate the semi-small open economy to country-specific data and isolate the pure effect of technology by assuming that the intensity of the non-traded sector in the government spending shock is symmetric across countries. We compute the aggregate and sectoral government spending multipliers over a six-year horizon in the baseline and the restricted model where technological change is shut down which allows us to calculate the excess or the reduction of value added growth through the technology channel. While traded relative to non-traded TFP increases in two-third of the OECD economies in response to a government spending shock, non-traded relative to traded TFP rises in one-third of the countries. Technological change amplifies real GDP growth which disproportionately benefits the traded sector in the first group of countries whilst the decline in aggregate TFP

reduces real GDP growth in the second group of countries where non-traded value added increases disproportionately. Importantly, while the increase in the government spending multiplier on non-traded value added driven by technological change which remains stable in both groups of countries, the multiplier on traded value added displays a wide dispersion, exceeding one in the first group and moving into negative values in the second group.

Turning to labor, we compute the aggregate and sectoral government spending multipliers on hours worked over a six-year horizon in the baseline and the restricted model where technological change is assumed to be Hicks-neutral which allows us to calculate the excess or the reduction of labor growth caused by FBTC. We find that a government spending shock leads firms to bias technological toward labor in two-third of OECD countries which increases labor growth by 0.4 ppt. Conversely, in the remaining countries where technological change is biased toward capital, the rise in total hours worked is reduced by 0.6 ppt. FBTC also varies between sectors which affects the distribution of labor growth between the traded and non-traded sector. In half of the countries, technological change is more biased toward labor in the non-traded than in the traded sector which increases the government spending multiplier on non-traded hours worked by 0.36 ppt of total hours worked. Conversely, in the remaining half of OECD countries, technological change is more biased toward capital in the non-traded than in the traded sector which lowers the government spending multiplier on non-traded hours worked by 0.27 ppt of total hours worked. In both cases, half of the excess or reduction in the multiplier on non-traded hours worked is caused by the reallocation of labor toward or away from the non-traded sector.

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## A Sectoral Decomposition of Real GDP

We consider an open economy which produces domestic traded goods, denoted by a superscript  $H$ , and non-traded goods, denoted by a superscript  $N$ . The foreign-produced traded good is the numeraire and its price is normalized to 1. We consider an initial steady-state where prices are those at the base year so that initially real GDP, denoted by  $Y_R$ , and the value added share at constant prices, denoted by  $\nu^{Y,j}$ , collapses to nominal GDP (i.e.,  $Y$ ) and the value added share at current prices, respectively. Before moving forward, it is worth mentioning that whilst in the model and the quantitative analysis, we add a tilde when value added is inclusive of the technology utilization rate since we allow for endogenous utilization of existing technologies, we do not need to make this distinction in the data and  $Y_t^j$  refers to value added inclusive of technology improvement below.

Summing value added at constant prices across sectors gives real GDP:

$$Y_{R,t} = P^H Y_t^H + P^N Y_t^N, \quad (58)$$

where  $P^H$  and  $P^N$  stand for the price of home-produced traded goods and non-traded goods, respectively, which are kept fixed since we consider value added at constant prices.

Log-linearizing (58), and denoting the percentage deviation from initial steady-state by a hat leads to:

$$\hat{Y}_{R,t} = \nu^{Y,H} \hat{Y}_t^H + \nu^{Y,N} \hat{Y}_t^N, \quad (59)$$

where  $\nu^{Y,j} = \frac{P^j Y^j}{Y}$  is the value added share of home-produced traded goods evaluated at the initial steady-state. **Eq. (59) corresponds to eq. (1) in the main text.** We drop the time index below as long as it does not cause confusion.

Subtracting real GDP growth from both sides of (59) leads to the sum of the change in the value added share denoted by  $d\nu_t^{Y,j}$ :

$$0 = d\nu_t^{Y,H} + d\nu_t^{Y,N}. \quad (60)$$

The change in the value added share is computed as the excess (measured in ppt of GDP) of value added growth at constant prices in sector  $j = H, N$  over real GDP growth:

$$d\nu_t^{Y,j} = \nu^{Y,j} \left( \hat{Y}_t^j - \hat{Y}_{R,t} \right). \quad (61)$$

Capital  $K^j$  can be freely reallocated across sectors while labor  $L^j$  is subject to mobility costs which creates a sectoral wage differential. We denote the capital rental cost by  $R$  and the wage rate in sector  $j$  by  $W^j$  (with  $j = H, N$ ). Under assumption of perfect competition in product and input markets, factors of production are paid their marginal product in both sectors:

$$P^j \frac{\partial Y^j}{\partial L^j} = W^j, \quad (62a)$$

$$P^j \frac{\partial Y^j}{\partial K^j} = R. \quad (62b)$$

Assuming constant returns to scale in production and making use of (62), the log-linearized version of the production function reads:

$$\hat{Y}_t^j = \text{TFP}_t^j + s_L^j \hat{L}_t^j + (1 - s_L^j) \hat{K}_t^j, \quad (63)$$

where  $s_L^j$  and  $\text{TFP}^j$  are the labor income share and total factor productivity in sector  $j$ , respectively, and  $k^j \equiv K^j/L^j$  stands for the capital-labor ratio.

We derive below an expression of the deviation of real GDP relative to initial steady-state. Since we assume perfect capital mobility, the resource constraint for capital reads as follows  $K = K^H + K^N$ . Totally differentiating, multiplying both sides by the capital rental cost  $R$ , and dividing by GDP leads to:

$$(1 - s_L) \hat{K}_t = \nu^{Y,H} (1 - s_L^H) \hat{K}_t^H + (1 - \nu^{Y,H}) (1 - s_L^N) \hat{K}_t^N. \quad (64)$$

The same logic applies to labor except that we assume imperfect mobility of labor across sectors. In this case, the percentage deviation of total hours worked relative to its initial steady-state is defined as the weighted sum of the percentage deviation of sectoral hours worked relative to initial steady-state, i.e.,  $\hat{L}_t = \alpha_L \hat{L}_t^H + (1 - \alpha_L) \hat{L}_t^N$ , where  $\alpha_L = \frac{W^H L^H}{WL}$  is the labor compensation share for tradables. Multiplying both sides by total compensation of employees,  $WL$ , and dividing by GDP leads to:

$$s_L \hat{L}_t = \nu^{Y,H} s_L^H \hat{L}_t^H + (1 - \nu^{Y,H}) s_L^N \hat{L}_t^N. \quad (65)$$

Plugging (63) into (59) and making use of (64)-(65) allows us to express the change in real GDP in terms of aggregate TFP changes and accumulation of inputs:

$$\hat{Y}_{R,t} = \text{TFP}_t + s_L \hat{L}_t + (1 - s_L) \hat{K}_t, \quad (66)$$

where the percentage deviation of aggregate TFP relative to its initial steady-state is equal to the weighted sum of the percentage deviation of TFP relative to initial steady-state in the traded and the non-traded sector

$$\widehat{\text{TFP}}_t = \nu^{Y,H} \widehat{\text{TFP}}_t^H + (1 - \nu^{Y,H}) \widehat{\text{TFP}}_t^N. \quad (67)$$

**Eq. (67) corresponds to eq. (2) in the main text.**

Considering the non-traded sector (i.e., setting  $j = N$ ) and plugging (66) into (61) shows that the change in the value added share at constant prices of non-tradables can be brought about by a TFP growth differential, a labor and/or a capital inflow. Formally, the decomposition of the change in the value added share of the non-tradables reads:

$$d\nu_t^{Y,N} = (1 - \nu^{Y,H}) \left[ \left( \widehat{\text{TFP}}_t^N - \widehat{\text{TFP}}_t \right) + \left( \widehat{L}_t^N - \widehat{L}_t \right) + (1 - s_L^N) \widehat{k}_t^N - (1 - s_L) \widehat{k}_t \right], \quad (68)$$

where  $k^N \equiv K^N/L^N$  is the capital-labor ratio in the non traded sector,  $k \equiv K/L$  is the aggregate capital-labor ratio, and we used the fact that changes in sectoral value added and aggregate real GDP, as described by (63) and (66), respectively, can be rewritten as follows:

$$\widehat{Y}_t^j = \widehat{\text{TFP}}_t^j + \widehat{L}_t^j + \left( 1 - s_L^j \right) \widehat{k}_t^j, \quad (69a)$$

$$\widehat{Y}_{R,t} = \widehat{\text{TFP}}_t + \widehat{L}_t + (1 - s_L) \widehat{k}_t. \quad (69b)$$

Plugging the sectoral decomposition of the deviation of aggregate TFP relative to its initial steady-state described by eq. (67) into the change in the value added share of non-tradables at constant prices (68) leads to the decomposition of the change in the value added share of non-tradables which reads:

$$\begin{aligned} d\nu_t^{Y,N} = & - (1 - \nu^{Y,H}) \nu^{Y,H} \left( \widehat{\text{TFP}}_t^H - \widehat{\text{TFP}}_t^N \right) + (1 - \nu^{Y,H}) \left[ \left( \widehat{L}_t^N - \widehat{L}_t \right) \right. \\ & \left. + (1 - s_L^N) \widehat{k}_t^N - (1 - s_L) \widehat{k}_t \right]. \end{aligned} \quad (70)$$

**Eq. (70) corresponds to eq. (3) in the main text.**

## B Sectoral Decomposition of Hours Worked

In this section, we detail the steps of derivation of the relationship between the labor share of non-tradables and the responses of LISs. While  $Y_t^j$  refers to value added inclusive of technology improvement below, we make the distinction between the capital stock  $K_t^j$  and the capital stock inclusive of capital utilization  $\tilde{K}_t^j$  by adding a tilde.

In an economy where labor is imperfectly mobile across sectors, the percentage deviation of total hours worked relative to its initial steady-state (i.e.,  $\widehat{L}_t$ ) following a shock to government consumption is equal to the weighted sum of the percentage deviation of sectoral hours worked relative to initial steady-state (i.e.,  $\widehat{L}_t^j$ ):

$$\widehat{L}_t = \alpha_L \widehat{L}_t^H + (1 - \alpha_L) \widehat{L}_t^N. \quad (71)$$

where  $\alpha_L (1 - \alpha_L)$  is the labor compensation share of tradables (non-tradables). **Eq. (71) corresponds to eq. (4) in the main text.** Note that we use interchangeably  $\alpha_L = \alpha_L^H$  and  $1 - \alpha_L = \alpha_L^N$ .

If we subtract the share of  $\widehat{L}_t$  received by each sector from the change in sectoral hours worked, we obtain the change in the labor share of sector  $j$ , denoted by  $\nu^{L,j}$ , which measures the contribution of the reallocation of labor across sectors to the change in hours worked in sector  $j$ :<sup>43</sup>

$$d\nu_t^{L,j} = \alpha_L^j \left( \widehat{L}_t^j - \widehat{L}_t \right) \quad j = H, N. \quad (72)$$

**Eq. (72) corresponds to eq. (5) in the main text.** The differential between the responses of sectoral and total hours worked on the RHS of eq. (72) can be viewed as the change in labor in sector  $j$  if  $L$  remained fixed and thus reflects higher employment in this sector resulting from the reallocation of labor.

Both the traded and non-traded sectors use physical capital (inclusive of capital utilization),  $\tilde{K}^j = u^{K,j} K^j$ , and labor,  $L^j$ , according to constant returns to scale production functions which are assumed to take a CES form:

$$Y_t^j = \left[ \gamma^j \left( A_t^j L_t^j \right)^{\frac{\sigma^j - 1}{\sigma^j}} + (1 - \gamma^j) \left( B_t^j \tilde{K}_t^j \right)^{\frac{\sigma^j - 1}{\sigma^j}} \right]^{\frac{\sigma^j}{\sigma^j - 1}}, \quad (73)$$

<sup>43</sup>While the two measures are equivalent in level, we differentiate between  $\nu^{L,j}$  and  $\alpha_L$  since the change in the labor share is calculated by keeping  $W^j/W$  constant.

where  $\gamma^j$  and  $1 - \gamma^j$  are the weight of labor and capital in the production technology,  $\sigma^j$  is the elasticity of substitution between capital and labor in sector  $j = H, N$ ,  $A^j$  and  $B^j$  are labor- and capital-augmenting efficiency. Both sectors face two cost components: a capital rental cost equal to  $R$ , and a labor cost equal to the wage rate, i.e.,  $W^H$  in the traded sector and  $W^N$  in the non-traded sector.

Both sectors are assumed to be perfectly competitive and thus choose capital and labor by taking prices as given:

$$\max_{\tilde{K}_t^j, L_t^j} \Pi_t^j = \max_{K_t^j, L_t^j} \left\{ P_t^j Y_t^j - W_t^j L_t^j - R_t \tilde{K}_t^j \right\}. \quad (74)$$

Since capital can move freely between the two sectors, the value of marginal products in the traded and non-traded sectors equalizes while costly labor mobility implies a wage differential across sectors:

$$P_t^j \gamma^j \left( A_t^j \right)^{\frac{\sigma^j-1}{\sigma^j}} \left( L_t^j \right)^{-\frac{1}{\sigma^j}} \left( Y_t^j \right)^{\frac{1}{\sigma^j}} \equiv W_t^j, \quad (75a)$$

$$P_t^j (1 - \gamma^j) \left( B_t^j \right)^{\frac{\sigma^j-1}{\sigma^j}} \left( \tilde{k}_t^j \right)^{-\frac{1}{\sigma^j}} \left( y_t^j \right)^{\frac{1}{\sigma^j}} \equiv R_t, \quad (75b)$$

where we denote by  $\tilde{k}_t^j \equiv \tilde{K}_t^j / L_t^j$  the capital-labor ratio for sector  $j = H, N$ , and  $y_t^j \equiv Y_t^j / L_t^j$  value added per hours worked described by

$$y_t^j = \left[ \gamma^j \left( A_t^j \right)^{\frac{\sigma^j-1}{\sigma^j}} + (1 - \gamma^j) \left( B_t^j \tilde{k}_t^j \right)^{\frac{\sigma^j-1}{\sigma^j}} \right]^{\frac{\sigma^j}{\sigma^j-1}}. \quad (76)$$

Denoting the LIS in sector  $j$  by  $s_L^j$ , and pre-multiplying both sides of (75a) by  $L^j$  and dividing by value added at current prices in sector  $j$ ,  $P^j Y^j$  leads to the labor income share:

$$s_L^j = \gamma^j \left( \frac{A^j}{y^j} \right)^{\frac{\sigma^j-1}{\sigma^j}}. \quad (77)$$

Multiplying both sides of (75b) by  $K^j$  and dividing by value added at current prices in sector  $j$  leads to the capital income share:

$$1 - s_L^j = (1 - \gamma^j) \left( \frac{B^j \tilde{k}^j}{y^j} \right)^{\frac{\sigma^j-1}{\sigma^j}}. \quad (78)$$

Dividing eq. (77) by eq. (78), the ratio of the labor to the capital income share denoted by  $S^j = \frac{s_L^j}{1-s_L^j}$  reads as follows:

$$S^j = \frac{\gamma^j}{1 - \gamma^j} \left( \frac{B_t^j \tilde{K}_t^j}{A_t^j L_t^j} \right)^{\frac{1-\sigma^j}{\sigma^j}}. \quad (79)$$

Let us denote:

$$\text{FBTC}^j = \left( \frac{B_t^j u_t^{K,j}}{A_t^j} \right)^{\frac{1-\sigma^j}{\sigma^j}}, \quad (80a)$$

$$\text{FBTC}_{adjK}^j = \left( \frac{B_t^j}{A_t^j} \right)^{\frac{1-\sigma^j}{\sigma^j}}. \quad (80b)$$

Using the definition of the LIS, the demand for labor by firms in sector  $j$  (75a) can be rewritten as follows:

$$s_{L,t}^j \frac{P_t^j Y_t^j}{L_t^j} = W_t^j. \quad (81)$$

Aggregating across sectors and dividing by GDP at current prices, i.e.,  $\sum_j s_{L,t}^j \frac{P_t^j Y_t^j}{Y_t} = \frac{W_t L_t}{Y_t}$ , leads to aggregate demand for labor:

$$s_{L,t} \frac{P_t Y_t}{L_t} = W_t. \quad (82)$$

Dividing the demand for labor in sector  $j$  (81) by aggregate labor demand (82):

$$\frac{W_t^j L_t^j}{W_t L_t} = \frac{s_{L,t}^j}{s_{L,t}} \omega_t^{Y,j}, \quad (83)$$

Table 4: Sample Range for Empirical and Numerical Analysis

Country	Code	Period	Obs.
Australia	(AUS)	1970 - 2015	46
Austria	(AUT)	1970 - 2015	46
Belgium	(BEL)	1970 - 2015	46
Canada	(CAN)	1970 - 2015	46
Denmark	(DNK)	1970 - 2015	46
Spain	(ESP)	1970 - 2015	46
Finland	(FIN)	1970 - 2015	46
France	(FRA)	1970 - 2015	46
Great Britain	(GBR)	1970 - 2015	46
Ireland	(IRL)	1970 - 2015	46
Italy	(ITA)	1970 - 2015	46
Japan	(JPN)	1974 - 2015	41
Korea	(KOR)	1970 - 2015	46
Netherlands	(NLD)	1970 - 2015	46
Norway	(NOR)	1970 - 2015	46
Portugal	(PRT)	1970 - 2015	46
Sweden	(SWE)	1970 - 2015	46
United States	(USA)	1970 - 2015	46
Total number of obs.			823
Main data sources		EU KLEMS & OECD STAN	

Notes: Column 'period' gives the first and last observation available. Obs. refers to the number of observations available for each country.

where  $\omega_t^{Y,j} = \frac{P_t^j Y_t^j}{Y_t}$  stands for the value added share of sector  $j$  at current prices. Drawing on Horvath [2000], we generate imperfect mobility of labor by assuming that sectoral hours worked are imperfect substitutes which gives rise to a labor share in sector  $j$  which is elastic to the relative wage:

$$\frac{L_t^j}{L_t} = \vartheta^j \left( \frac{W_t^j}{W_t} \right)^\epsilon, \quad (84)$$

where  $\vartheta^j$  stands for the weight attached to labor supply in sector  $j = H, N$  and  $\epsilon$  is the elasticity of labor supply across sectors which captures the degree of labor mobility. Plugging labor supply to sector  $j$  (84) into (83), the equilibrium labor share in sector  $j$  reads as follows:

$$\frac{L_t^j}{L_t} = (\vartheta^j)^{\frac{1}{1+\epsilon}} \left( \frac{s_{L,t}^j}{s_{L,t}} \right)^{\frac{\epsilon}{1+\epsilon}} \left( \omega_t^{Y,j} \right)^{\frac{\epsilon}{1+\epsilon}}. \quad (85)$$

Eq. (85) corresponds to eq. (6) in the main text.

## C Data Description for Empirical Analysis

**Coverage:** Our sample consists of a panel of 18 countries: Australia (AUS), Austria (AUT), Belgium (BEL), Canada (CAN), Denmark (DNK), Finland (FIN), France (FRA), Ireland (IRL), Italy (ITA), Japan (JPN), Korea (KOR), the Netherlands (NLD), Norway (NOR), Portugal (PRT), Spain (ESP), Sweden (SWE), the United Kingdom (GBR) and the United States (USA). The baseline period is running from 1970 to 2015, except for Japan (1974-2015). Table 4 summarizes our dataset.

**Sources:** Our primary sources for sectoral data are the OECD and EU KLEMS databases. We use data from EU KLEMS ([2011], [2017]) March 2011 and July 2017 releases. The EU KLEMS dataset covers all countries of our sample, with the exceptions of Canada and Norway. For these two countries, sectoral data are taken from the Structural Analysis (STAN) database provided by the OECD ([2011], [2017]). For both EU KLEMS and STAN databases, the March 2011 release provides data for eleven 1-digit ISIC-rev.3 industries over the period 1970-2007 while the July 2017 release provides data for thirteen 1-digit-rev.4 industries over the period 1995-2015.

The construction of time series for sectoral variables over the period 1970-2015 involves two steps. First, we identify tradable and non-tradable sectors. We adopt the classification proposed by De Gregorio et al. [1994]. Following Jensen and Kletzer [2006], we have updated this classification by treating the financial sector as a traded industry. We map the ISIC-rev.4 classification into the ISIC-rev.3 classification in accordance with the concordance Table 5. Once industries have been classified as traded or non-traded, for any macroeconomic variable  $X$ , its sectoral counterpart  $X^j$  for  $j = H, N$  is constructed by adding the  $X_k$  of all sub-industries  $k$  classified in sector  $j = H, N$  as follows  $X^j = \sum_{k \in j} X_k$ . Second, series for tradables and non-tradables variables from EU KLEMS



[2011] and OECD [2011] databases (available over the period 1970-2007) are extended forward up to 2015 using annual growth rate estimated from EU KLEMS [2017] and OECD [2017] series (available over the period 1995-2015).

Table 5: Summary of Sectoral Classifications

Sector	ISIC-rev.4 Classification (sources: EU KLEMS [2017] and OECD ([2017]))		ISIC-rev.3 Classification (sources: EU KLEMS [2011] and OECD ([2011]))	
	Industry	Code	Industry	Code
Tradables ( $H$ )	Agriculture, Forestry and Fishing	A	Agriculture, Hunting, Forestry and Fishing	AtB
	Mining and Quarrying	B	Mining and Quarrying	C
	Total Manufacturing	C	Total Manufacturing	D
	Transport and Storage	H	Transport, Storage and Communication	I
	Information and Communication	J		
	Financial and Insurance Activities	K	Financial Intermediation	J
Non Tradables ( $N$ )	Electricity, Gas and Water Supply	D-E	Electricity, Gas and Water Supply	E
	Construction	F	Construction	F
	Wholesale and Retail Trade, Repair of Motor Vehicles and Motorcycles	G	Wholesale and Retail Trade	G
	Accommodation and Food Service Activities	I	Hotels and Restaurants	H
	Real Estate Activities	L	Real Estate, Renting and Business Services	K
	Professional, Scientific, Technical, Administrative and Support Service Activities	M-N		
	Community Social and Personal Services	O-U	Community Social and Personal Services	LtQ

Relevant to our work, the EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases provide data, for each industry and year, on value added at current and constant prices, permitting the construction of sectoral deflators of value added, as well as details on labor compensation and hours worked data, allowing the construction of sectoral wage rates. All quantity variables are scaled by the working age population (15-64 years old). Source: OECD ALFS Database for the working age population (data coverage: 1970-2015). Normalizing base year price indices  $\bar{P}^j$  to 1, we describe below the construction for the sectoral data employed in the main text (mnemonics are given in parentheses):

- **Sectoral value added**,  $Y^j$ : sectoral value added at constant prices in sector  $j = H, N$  (VA\_QI). Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases.
- **Sectoral value added share**,  $\nu^{Y,j}$ : ratio of value added at constant prices in sector  $j$  to GDP at constant prices, i.e.,  $Y^j/(Y^H + Y^N)$  for  $j = H, N$ .
- **Relative price of non-tradables**,  $P$ : ratio of the non-traded value added deflator to the traded value added deflator, i.e.,  $P = P^N/P^H$ . The sectoral value added deflator  $P^j$  for sector  $j = H, N$  is calculated by dividing value added at current prices (VA) by value added at constant prices (VA\_QI) in sector  $j$ . Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases.
- **Terms of trade**,  $P^H/P^F$ : ratio of the traded value added deflator to price deflator of imports of goods and services, i.e.,  $P^H/P^F$ . The traded value added deflator  $P^H$  is calculated by dividing value added at current prices (VA) by value added at constant prices (VA\_QI) in sector  $H$ . Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) for  $P^H$  and OECD National Accounts Database for  $P^F$ .
- **Sectoral hours worked**,  $L^j$ : total hours worked by persons engaged in sector  $j$  (H\_EMP). Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases.
- **Sectoral labor share**,  $\nu^{L,j}$ : ratio of hours worked in sector  $j$  to total hours worked, i.e.,  $L^j/(L^H + L^N)$  for  $j = H, N$ .
- **Sectoral nominal wage**,  $W^j$ : ratio of the labor compensation (compensation of employees plus compensation of self-employed) in sector  $j = H, N$  (LAB) to total hours worked by persons engaged (H\_EMP) in that sector. Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases.
- **Relative wage**,  $W^j/W$ : ratio of the nominal wage in the sector  $j$  to the aggregate nominal wage  $W$ .
- **Labor income share (LIS)**,  $s_L^j$ : ratio of labor compensation (compensation of employees plus compensation of self-employed) in sector  $j = H, N$  (LAB) to value added at current prices (VA) of that sector. Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases.



We detail below the data construction for aggregate variables (mnemonics are in parentheses). For all variables, the reference period is running from 1970 to 2015:

- **Government spending**,  $G$ : government final consumption expenditure (CGV). Source: OECD Economic Outlook Database [2017].
- **Gross domestic product**,  $Y_R$ : real gross domestic product. By construction, real GDP is the sum of traded and non-traded value added at constant prices. Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases.
- **Total hours worked**,  $L$ : total hours worked by persons engaged (H\_EMP). By construction, total hours worked is the sum of traded and non-traded hours worked. Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases.
- **Real consumption wage**,  $W_C = W/P_C$ : nominal aggregate wage divided by the consumer price index (CPI). Source: OECD Prices and Purchasing Power Parities Database [2017] for the consumer price index. The nominal aggregate wage is calculated by dividing labor compensation (LAB) by total hours worked by persons engaged (H\_EMP). Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases.
- **Aggregate total factor productivity**, TFP : Aggregate TFP is constructed as a Solow residual from constant-price domestic currency series of GDP, capital, LIS  $s_L$ , and total hours worked. In Appendix D, we detail the procedure to construct time series for the aggregate capital stock. The aggregate LIS,  $s_L$ , is the ratio of labor compensation (compensation of employees plus compensation of self-employed) (LAB) to GDP at current prices (VA) in sector averaged over the period 1970-2015 (except Japan: 1974-2015). Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases.

## D Construction of Capital Utilization Adjusted TFP Time Series: Imbs [1999] Construction

We construct time-varying capital utilization series using the procedure discussed in Imbs [1999] to construct our own series of utilization-adjusted TFP. We assume perfectly competitive factor and product markets. Both the traded and non-traded sectors use physical capital,  $K^j$ , and labor,  $L^j$ , according to constant returns to scale production functions which are assumed to take a CES form:

$$Y_t^j = \left[ \gamma^j \left( A_t^j L_t^j \right)^{\frac{\sigma^j-1}{\sigma^j}} + (1-\gamma^j) \left( B_t^j u_t^{K,j} K_t^j \right)^{\frac{\sigma^j-1}{\sigma^j}} \right]^{\frac{\sigma^j}{\sigma^j-1}}. \quad (86)$$

We denote the capital utilization rate by  $u_t^{K,j}$ . Because more intensive capital use depreciates the capital more rapidly, we assume the following relationship between capital use and depreciation:

$$\delta_{K,t}^j = \delta_K \left( u_t^{K,j} \right)^{\phi_K}, \quad (87)$$

where  $\delta_K$  is the capital depreciation rate and  $\phi_K$  is the parameter which must be determined. At the steady-state, we have  $u^{K,j} = 1$  and thus capital depreciation collapses to  $\delta_K$  which is assumed to be symmetric across sectors. Firms also choose  $A^j$  and  $B^j$  along the technology frontier that we assume to be Cobb-Douglas:

$$Z_t^j = \left( A_t^j \right)^{s_{L,t}^j} \left( B_t^j \right)^{1-s_{L,t}^j}. \quad (88)$$

Note that both  $A^j$  and  $B^j$  in (86) include technology utilization. Thus in contrast to the model's notations,  $Y^j$  stands for value added at constant prices and thus is inclusive of technology utilization. While in the main text, we assume that the technology frontier (35) is CES and above we assume it is Cobb-Douglas, it leads to the same outcome, i.e.,  $\hat{Z}_t^j = s_L^j \hat{A}_t^j + (1-s_L^j) \hat{B}_t^j$ , see eq. (37).

Denoting the capital rental cost by  $R_t = P_{J,t} (\delta_{K,t} + r^*)$ , and the labor cost by  $W_t^j$ , firms choose the capital stock, capital utilization and labor so as the maximize profit:

$$\Pi_t^j = P_t^j Y_t^j - W_t^j L_t^j - R_t K_t^j. \quad (89)$$

Profit maximization leads to first order conditions on  $K^j$ ,  $u^{K,j}$ ,  $L^j$ :

$$P_t^j (1-\gamma^j) \left( B_t^j u_t^{K,j} \right)^{\frac{\sigma^j-1}{\sigma^j}} \left( K_t^j \right)^{-\frac{1}{\sigma^j}} \left( Y_t^j \right)^{\frac{1}{\sigma^j}} = R_t, \quad (90a)$$

$$P_t^j (1-\gamma^j) \left( B_t^j K_t^j \right)^{\frac{\sigma^j-1}{\sigma^j}} \left( u_t^{K,j} \right)^{-\frac{1}{\sigma^j}} \left( Y_t^j \right)^{\frac{1}{\sigma^j}} = P_{J,t} \delta_K \phi_K \left( u_t^{K,j} \right)^{\phi_K-1} K^j, \quad (90b)$$

$$P_t^j \gamma^j \left( A_t^j \right)^{\frac{\sigma^j-1}{\sigma^j}} \left( L_t^j \right)^{-\frac{1}{\sigma^j}} \left( Y_t^j \right)^{\frac{1}{\sigma^j}} = W_t^j. \quad (90c)$$

Multiplying both sides of the first equality by  $K^j$  and dividing by sectoral value added leads to the capital income share:

$$1 - s_{L,t}^j = (1 - \gamma^j) \left( \frac{B_t^j u_t^{K,j} K_t^j}{Y_t^j} \right)^{\frac{\sigma^j - 1}{\sigma^j}}. \quad (91)$$

By using the definition of the capital income share above and inserting the expression for the capital rental cost, first-order conditions can be rewritten as follows:

$$(1 - s_L^j) \frac{P_t^j Y_t^j}{P_{J,t} K_t^j} = (\delta_{K,t} + r^*), \quad (92a)$$

$$(1 - s_L^j) \frac{P_t^j Y_t^j}{P_{J,t} K_t^j} = \delta_{K,t} \phi_K, \quad (92b)$$

$$s_{L,t}^j \frac{P_t^j Y_t^j}{L_t^j} = W_t^j. \quad (92c)$$

Evaluating (92a) and (92b) at the steady-state and rearranging terms leads to:

$$(r^* + \delta_K) = \delta_K \phi_K, \quad (93)$$

which allows us to pin down  $\phi_K$ . We let the capital depreciation rate  $\delta_K$  and the real interest rate  $r^*$  (long-run interest rate minus CPI inflation rate) vary across countries to compute  $\phi_K$ .

In the line of Garofalo and Yamarik [2002], we use the value added share at current prices to allocate the aggregate capital stock to sector  $j$ :

$$K_t^j = \omega_t^{Y,j} K_t, \quad (94)$$

where  $K_t$  is the aggregate capital stock at constant prices and  $\omega_t^{Y,j} = \frac{P_t^j Y_t^j}{P_t Y_{R,t}}$  is the value added share of sector  $j = H, N$  at current prices. The methodology by Garofalo and Yamarik [2002] is based on the assumption of perfect mobility of capital across sectors and a small discrepancy in the LIS across sectors, i.e.,  $s_L^H \simeq s_L^N$ . Inserting (94) into (92a)-(92b), first order conditions on  $K^j$  and  $u^{K,j}$  now read as follows:

$$(1 - s_{L,t}^j) \frac{P_t Y_{R,t}}{P_{J,t} K_t} = (\delta_{K,t} + r^*), \quad (95a)$$

$$(1 - s_{L,t}^j) \frac{P_t Y_{R,t}}{P_{J,t} K_t} = \delta_{K,t} \phi_K. \quad (95b)$$

Solving (95b) for  $u_t^{K,j}$  leads to:

$$u_t^{K,j} = \left[ \frac{(1 - s_{L,t}^j) P_t Y_{R,t}}{\delta_K \phi_K P_{J,t} K_t} \right]^{\frac{1}{\phi_K}}, \quad (96)$$

where  $\phi_K = \frac{r^* + \delta_K}{\delta_K}$  (see eq. (93)). Dropping the time index to denote the steady-state value, the capital utilization rate is:

$$u^{K,j} = \left[ \frac{(1 - s_L^j) P Y_R}{\delta_K \phi_K P_J K} \right]^{\frac{1}{\phi_K}}. \quad (97)$$

Dividing (96) by (97) leads to the capital utilization rate relative to its steady-state:

$$\frac{u_t^{K,j}}{u^{K,j}} = \left[ \frac{(1 - s_{L,t}^j) P_t Y_{R,t} P_J K}{(1 - s_L^j) P Y_R P_{J,t} K_t} \right]^{\frac{1}{\phi_K}}, \quad (98)$$

We denote total factor productivity in sector  $j = H, N$  by  $\text{TFP}^j$  which is defined as follows:

$$\text{TFP}_t^j = \frac{Y_t^j}{\left[ \gamma^j (L_t^j)^{\frac{\sigma^j - 1}{\sigma^j}} + (1 - \gamma^j) (K_t^j)^{\frac{\sigma^j - 1}{\sigma^j}} \right]^{\frac{\sigma^j}{\sigma^j - 1}}}. \quad (99)$$

Log-linearizing (99), the Solow residual is:

$$T\hat{F}P_t^j = \hat{Y}_t^j - s_L^j \hat{L}_t^j - (1 - s_L^j) \hat{K}_t^j. \quad (100)$$

Log-linearizing the production function (86) shows that the Solow residual can alternatively be decomposed into utilization-adjusted TFP and capital utilization correction:

$$\widehat{\text{TFP}}_t^j = \widehat{Z}_t^j + (1 - s_L^j) \widehat{u}_t^{K,j}, \quad (101)$$

where utilization-adjusted TFP denoted by  $Z^j$  is equal to:

$$\widehat{Z}_t^j = s_L^j \widehat{A}_t^j + (1 - s_L^j) \widehat{B}_t^j. \quad (102)$$

**Construction of time series for sectoral capital stock,  $K_t^j$ .** To construct the series for the sectoral capital stock, we proceed as follows. We first construct time series for the aggregate capital stock for each country in our sample. To construct  $K_t$ , we adopt the perpetual inventory approach. The inputs necessary to construct the capital stock series are a i) capital stock at the beginning of the investment series,  $K_{1970}$ , ii) a value for the constant depreciation rate,  $\delta_K$ , iii) real gross capital formation series,  $I_t$ . Real gross capital formation is obtained from OECD National Accounts Database [2017] (data in millions of national currency, constant prices). We construct the series for the capital stock using the law of motion for capital in the model:

$$K_{t+1} = I_t + (1 - \delta_K) K_t. \quad (103)$$

for  $t = 1971, \dots, 2015$ . The value of  $\delta_K$  is chosen to be consistent with the ratio of capital depreciation to GDP observed in the data and averaged over 1970-2015:

$$\frac{1}{46} \sum_{t=1970}^{2015} \frac{\delta_K P_{J,t} K_t}{Y_t} = \frac{CFC}{Y}, \quad (104)$$

where  $P_{J,t}$  is the deflator of gross capital formation series,  $Y_t$  is GDP at current prices, and  $CFC/Y$  is the ratio of consumption of fixed capital at current prices to nominal GDP averaged over 1970-2015. Deflator of gross capital formation, GDP at current prices and consumption of fixed capital are taken from the OECD National Account Database [2017]. The second column of Table 6 shows the value of the capital depreciation rate obtained by using the formula (104). The capital depreciates rate averages to 5%.

To have data on the capital stock at the beginning of the investment series, we use the following formula:

$$K_{1970} = \frac{I_{1970}}{g_I + \delta_K}, \quad (105)$$

where  $I_{1970}$  corresponds to the real gross capital formation in the base year 1970,  $g_I$  is the average growth rate from 1970 to 2015 of the real gross capital formation series. The system of equations (103), (104) and (105) allows us to use data on investment to solve for the sequence of capital stocks and for the depreciation rate,  $\delta_K$ . There are 47 unknowns:  $K_{1970}$ ,  $\delta_K$ ,  $K_{1971}$ , ..., and  $K_{2015}$ , in 47 equations: 45 equations (103), where  $t = 1971, \dots, 2015$ , (104), and (105). Solving this system of equations, we obtain the sequence of capital stocks and a calibrated value for depreciation,  $\delta_K$ . Following Garofalo and Yamarik [2002], the gross capital stock is then allocated to traded and non-traded industries by using the sectoral value added share, see eq. (94).

**Construction of time series for sectoral TFPs.** Sectoral TFPs,  $\text{TFP}_t^j$ , at time  $t$  are constructed as Solow residuals from constant-price (domestic currency) series of value added,  $Y_t^j$ , capital stock,  $K_t^j$ , and hours worked,  $L_t^j$ , by using eq. (100). The LIS in sector  $j$ ,  $s_L^j$ , is the ratio labor compensation (compensation of employees plus compensation of self-employed) to nominal value added in sector  $j = H, N$ , averaged over the period 1970-2015 (except Japan: 1974-2015). Data for the series of constant price value added (VA\_QI), current price value added (VA), hours worked (H\_EMP) and labor compensation (LAB) are taken from the EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases.

**Construction of time series for real interest rate,  $r^*$ .** The real interest rate is computed as the real long-term interest rate which is the nominal interest rate on 10 years government bonds minus the rate of inflation which is the rate of change of the Consumption Price Index (CPI). Sources: OECD Economic Outlook Database [2017] for the long-term interest rate on government bonds and OECD Prices and Purchasing Power Parities Database [2017] for the CPI. Data coverage: 1970-2015 except for IRL (1990-2015) and KOR (1983-2015). The first column of Table 6 shows the value of the real interest rate which averages 3% over the period 1970-2015.

**Construction of time series for capital utilization,  $u_t^{K,j}$ .** To construct time series for the capital utilization rate,  $u_t^{K,j}$ , we proceed as follows. We use time series for the real interest rate,  $r^*$  and for the capital depreciation rate,  $\delta_K$  to compute  $\phi = \frac{r^* + \delta_K}{\delta_K}$  (see eq. (93)). Once we have calculated  $\phi$  for each country, we use time series for the LIS in sector  $j$ ,  $s_L^j$ , GDP at current prices,  $P_t Y_{R,t} = Y_t$ , the deflator for investment,  $P_{J,t}$ , and times series for the aggregate capital stock,  $K_t$

Table 6: Data on Real Interest Rate ( $r^*$ ) and Fixed Capital Depreciation Rate ( $\delta_K$ )

Country	$r^*$	$\delta_K$
AUS	0.029	0.058
AUT	0.030	0.040
BEL	0.033	0.041
CAN	0.032	0.100
DNK	0.046	0.062
ESP	0.020	0.036
FIN	0.025	0.048
FRA	0.032	0.043
GBR	0.025	0.031
IRL	0.035	0.042
ITA	0.025	0.029
JPN	0.017	0.050
KOR	0.052	0.061
NLD	0.030	0.035
NOR	0.027	0.102
PRT	0.023	0.038
SWE	0.031	0.026
USA	0.026	0.069
OECD	0.030	0.050

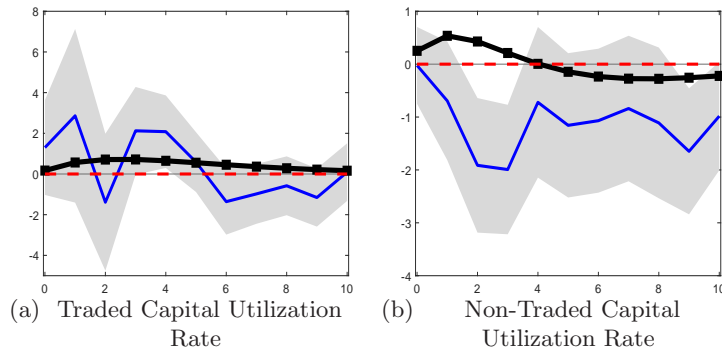


Figure 8: Theoretical vs. Empirical Responses Following Unanticipated Government Spending Shock: Capital Utilization Rate. *Notes:* Solid blue line displays point estimate of VAR with shaded areas indicating 90% confidence bounds; the thick solid black line with squares displays model predictions in the base-line scenario with capital and technology utilization together with FBTC while the dashed red line shows predictions of a model with Cobb-Douglas production functions and abstracting from capital and technology utilization.

to compute time series for  $u_t^{K,j}$  by using the formula (96). Fig. 8 plots empirical responses of the capital utilization rate for the traded and the non-traded sector shown in blue lines. Black lines with squares plots theoretical responses for  $u_t^{K,H}$  and  $u_t^{K,N}$ . The confidence bounds indicate that none of the responses are statistically significant. The reason is that there exists a wide cross-country dispersion in the movement of the capital utilization rates across countries in terms of both direction and magnitude. As shown in Fig. 8(a), our model reproduces well the adynamic adjustment of the capital utilization rate for tradables while Fig. 8(b) indicates that the model tends to somewhat overstate the response of  $u_t^{K,N}$ , especially in the short-term.

**Construction of time series for utilization-adjusted TFP,  $Z_t^j$ .** According to (101), capital utilization-adjusted sectoral TFP expressed in percentage deviation relative to the steady-state reads:

$$\begin{aligned} \hat{Z}_t^j &= T\hat{F}P_t^j - (1 - s_L^j) \hat{u}_t^{K,j}, \\ \ln Z_t^j - \ln \bar{Z}_t^j &= \left( \ln TFP_t^j - \ln \bar{T}FP_t^j \right) - \left( 1 - s_L^j \right) \left( \ln u_t^{K,j} - \ln \bar{u}_t^{K,j} \right). \end{aligned} \quad (106)$$

The percentage deviation of variable  $X_t$  from initial steady-state is denoted by  $\hat{X}_t = \ln X_t - \ln \bar{X}_t$  where we let the steady-state varies over time; the time-varying trend  $\ln \bar{X}_t$  is obtained by applying a HP filter with a smoothing parameter of 100 to logged time series. To compute  $T\hat{F}P_t^j$ , we take the log of  $TFP_t^j$  and subtract the trend component extracted from a HP filter applied to logged  $TFP_t^j$ , i.e.,  $\ln TFP_t^j - \ln \bar{T}FP_t^j$ . The same logic applies to  $u_t^{K,j}$ . Once we have computed the percentage

deviation  $\ln Z_t^j - \ln \bar{Z}_t^j$ , we reconstruct time series for  $\ln Z_t^j$ :

$$\ln Z_t^j = \left( \ln Z_t^j - \ln \bar{Z}_t^j \right) + \ln \bar{Z}_t^j. \quad (107)$$

The construction of time series of logged sectoral TFP,  $\ln \text{TFP}_t^j$ , capital utilization-adjusted sectoral TFP,  $\ln Z_t^j$ , is consistent with the movement of capital utilization along the business cycle.

## E Construction of Non-Traded Demand Components

In this section, we detail the construction of time series for non-traded government consumption,  $G_t^N$ , non-traded consumption,  $C_t^N$ , and non-traded investment,  $J_t^N$ . We use the World Input-Output Databases ([2013], [2016]). The 2013 release provides data for eleven 1-digit ISIC-rev.3 industries over the period 1995-2011 while the 2016 release provides data for thirteen 1-digit-rev.4 industries over the period 2000-2014. As sectoral data are classified using identical ISIC revisions in both the EU KLEMS and WIOD datasets, we map the WIOD ISIC-rev.4 classification (the 2016 release) into the WIOD ISIC-rev.3 classification (the 2013 release) in accordance with the concordance Table 5. Consistent with the methodology we used to extend series taken from the EU KLEMS ([2011], [2017]), time series for traded and non-traded variables from the WIOD [2013] dataset (available over the period 1995-2011) are extended forward up to 2014 using annual growth rate estimated from WIOD [2016] series (available over the period 2000-2014). Coverage: 1995-2014 except for NOR (2000-2014).

To compute non-traded demand components, we have to overcome two difficulties. While the input-output WIOD dataset gives purchases of non-traded goods and services from the private sector, data also includes purchases of imported goods and services. Whereas consumption and investment expenditure can be split into traded and non-traded expenditure, this split does not exist for government spending for most of the countries in our sample. We detail below how we overcome the two aforementioned difficulties.

To begin with, the non traded and the home-produced traded goods markets must clear such that:

$$Y^N = C^N + J^N + G^N + X^N - M^N, \quad (108a)$$

$$Y^H = C^H + J^H + G^H + X^H - M^H, \quad (108b)$$

where  $Y^j$  is value added at constant prices in sector  $j = H, N$ ,  $C^j$  consumption in good  $j$ ,  $J^j$  investment in good  $j$ ,  $G^j$  government consumption in good  $j$  and  $X^j$  stands for exports. Imports (by households, firms, and the government) in good  $j$  denoted by  $M^j$  can be broken into three components:

$$M^N = C^{N,F} + J^{N,F} + G^{N,F}, \quad (109a)$$

$$M^H = C^{H,F} + J^{H,F} + G^{H,F}, \quad (109b)$$

where  $C^{H,F}$ ,  $J^{H,F}$  and  $G^{H,F}$  are foreign-produced traded good for consumption, investment and government spending respectively, and  $C^{N,F}$ ,  $J^{N,F}$  and  $G^{N,F}$  denote consumption, investment and government spending domestic demand for non-traded goods produced by the rest of the world respectively. Next, each demand component  $C^j$ ,  $J^j$ ,  $G^j$  of sector  $j = H, N$  can be split into a domestic demand for home-produced good (denoted by  $C^{j,D}$ ,  $J^{j,D}$ ,  $G^{j,D}$ ) and a domestic demand for foreign-produced good (denoted by  $C^{j,F}$ ,  $J^{j,F}$ ,  $G^{j,F}$ ) by the rest of the world. This decomposition yields the following identities:

$$C^N = C^{N,D} + C^{N,F}, \quad (110a)$$

$$J^N = J^{N,D} + J^{N,F}, \quad (110b)$$

$$G^N = G^{N,D} + G^{N,F}, \quad (110c)$$

$$C^H = C^{H,D} + C^{H,F}, \quad (110d)$$

$$J^H = J^{H,D} + J^{H,F}, \quad (110e)$$

$$G^H = G^{H,D} + G^{H,F}. \quad (110f)$$

We denote total imports by  $M$  which consist of imports of consumption goods by households and the government and imports of capital goods by firms:

$$M = M^N + M^H. \quad (111)$$

Total exports to the rest of the world include exports of non-traded and traded goods:

$$X = X^N + X^H. \quad (112)$$

Obviously, we are aware that non-traded goods are not subject to international trade but we use this terminology to avoid confusion between the model's annotations and the data.

Combining (108a) and (108b) and using (111)-(112) leads to the standard accounting identity between the sum of sectoral value added and final expenditure:

$$\begin{aligned} P^H Y^H + P^N Y^N &= P_C C + P_J J + G + P_X X - P_M M, \\ Y &= P_C C + P_J J + G + N X, \end{aligned} \quad (113)$$

where we normalize  $P_G$  to one in (113) to be consistent with the model's annotations. Dividing (113) by GDP implies that consumption expenditure, investment expenditure, government spending, and net exports as a share of GDP must sum to one:

$$1 = \omega_C + \omega_J + \omega_G + \omega_{NX}. \quad (114)$$

We focus first on components of government spending. We use the accounting identity (108a) to compute time series for  $G^N$ :

$$P^N G^N = P^N Y^N - P^N C^N - P^N J^N - P^N X^N + P^N M^N \quad (115)$$

We divide both sides by nominal GDP, i.e.,  $P^H Y^H + P^N Y^N = Y$ . The LHS of eq. (115) divided by nominal GDP reads:

$$\begin{aligned} \frac{P^N G^N}{Y} &= \frac{P^N G^N}{G} \frac{G}{Y}, \\ &= \omega_{G^N} \omega_G. \end{aligned} \quad (116)$$

Making use of (115)-(116), we can calculate time series for  $\omega_{G^N}$  as follows:

$$\omega_{G^N} = \frac{1}{\omega_G} \left[ \frac{P^N Y^N}{Y} - \frac{P^N C^N - P^N J^N - P^N X^N + P^N M^N}{Y} \right]. \quad (117)$$

While in the model, we assume that non-traded industries do not trade with the rest of the world, the definition of a non-traded industry in the data is based on an arbitrary rule. Industries whose the sum of exports plus imports in percentage of GDP is lower than 20% are treated as non-tradables; since these industries trade, we have to split  $G^N$  into  $G^{N,D}$  and  $G^{N,F}$  so as to calculate time series for  $\omega_{G^{N,D}}$ . According to (109a), total imports of non-traded goods and services include imports by households, firms and the government, i.e.,  $M^N = C^{N,F} + J^{N,F} + G^{N,F}$ . Thus,  $G^{N,F} = M^N - C^{N,F} - J^{N,F}$ , from which we get  $\omega_{G^{N,F}} = G^{N,F}/G$ . By using (110c),  $G^{N,D}$  can be computed as  $G^{N,D} = G^N - G^{N,F}$ . This allows us to recover the share of non-traded government consumption which excludes imports:  $\omega_{G^{N,D}} = \omega_{G^N} - \omega_{G^{N,F}}$ . Next, government spending on foreign-produced traded goods  $G^{H,F}$  can be calculated by using the definition of imports of final traded goods and services:  $M^F = C^{H,F} + J^{H,F} + G^{H,F}$ , where  $C^{H,F}$  and  $J^{H,F}$  are consumption and investment in home-produced traded goods. Rearranging the last equation give  $G^{H,F} = M^F - C^{H,F} - J^{H,F}$ . It follows that  $\omega_{G^{H,F}} = G^{H,F}/G$ . Once we have time series for  $G^{N,D}$ ,  $G^{N,F}$ ,  $G^{H,F}$ , we can recover time series for government spending in home-produced traded goods,  $G^{H,D}$  by using the accounting identity which says that total government spending is equal to the sum of four components:  $G = G^{N,D} + G^{N,F} + G^{H,D} + G^{H,F}$ . Dividing both sides by  $G$  gives:

$$\begin{aligned} 1 &= \omega_{G^{N,D}} + \omega_{G^{N,F}} + \omega_{G^{H,F}} + \omega_{G^{H,D}}, \\ 1 &= \omega_{G^{N,D}} + \omega_{G^F} + \omega_{G^{H,D}}, \\ \omega_{G^{H,D}} &= 1 - \omega_{G^{N,D}} - \omega_{G^F}, \end{aligned} \quad (118)$$

where  $\omega_{G^{N,F}} + \omega_{G^{H,F}} = \omega_{G^F}$  is the import content of government spending

Since data taken from WIOD allows to differentiate between domestic demand for home- and foreign-produced goods, we are able to construct time series for the home content of consumption and investment in traded goods as follows:

$$\alpha^H = \frac{P^H C^{H,D}}{P^T C^T} = \frac{(P^T C^T - C^{H,F})}{P^T C^T}, \quad (119a)$$

$$\alpha^H_J = \frac{P^H J^{H,D}}{P^T J^T} = \frac{(P^T J^T - J^{H,F})}{P^T J^T}. \quad (119b)$$

To compute time series for non-traded consumption,  $C^{N,D}$ , and non-traded investment,  $J^{N,D}$ , we make use of imports of final consumption and investment goods, and then we divide by total consumption and investment expenditure, respectively, to obtain their non-tradable content:

$$1 - \alpha_C = \frac{P^N C^{N,D}}{P_C C} = \frac{P^N (C^N - C^{N,F})}{P_C C}, \quad (120a)$$

$$1 - \alpha_J = \frac{P^N J^{N,D}}{P_J J} = \frac{P^N (J^N - J^{N,F})}{P_J J}. \quad (120b)$$



We obtain data on GDP and its demand components (consumption, investment, government spending, exports and imports) from the World Input-Output Databases ([2013], [2016]) for all years between 1995 and 2014 and all 1-digit ISIC rev.3 and rev.4 industries. Indexing the sector with a superscript  $j = H, N$  and indexing the origin of demand of goods and services with a superscript  $k = D, F$  where  $D$  refers to domestic demand of home-produced goods and services and  $F$  refers to domestic demand of foreign-produced goods and services, we provide below details about data construction:

- Consumption  $C^{j,k}$  for  $j = H, N$  and  $k = D, F$ : total consumption expenditure (at current prices) by households and by non-profit organizations serving households on good  $j$  produced by firms from country  $k$ . Data coverage: 1995-2014 except for NOR (2000-2014).
- Investment  $I^{j,k}$  for  $j = H, N$  and  $k = D, F$ : total gross fixed capital formation plus changes in inventories and valuables (at current prices) on good  $j$  produced by firms from country  $k$ . Data coverage: 1995-2014 except for NOR (2000-2014).
- Government spending  $G^{j,k}$  for  $j = H, N$  and  $k = D, F$ : total consumption expenditure (at current prices) by government on good  $j$  produced by firms from country  $k$ . Data coverage: 1995-2014 except for NOR (2000-2014).
- Exports  $X^{j,k}$  for  $j = H, N$  and  $k = D, F$ : total exports (at current prices) of final and intermediate good  $j$  produced by firms from country  $k$ . Data coverage: 1995-2014 except for NOR (2000-2014).
- Imports  $M^{j,k}$  for  $j = H, N$  and  $k = D, F$ : total imports (at current prices) of final and intermediate good  $j$  produced by firms from country  $k$ . Data coverage: 1995-2014 except for NOR (2000-2014).

Finally, when we use (115) to obtain the time series for  $G^N$ , the valuation of output  $Y^N$  and imports  $M^N$  include taxes and subsidies on products and trade and transport margins respectively. These adjustments are necessary to achieve consistency and to balance resources and uses.

**Response of non-traded government consumption to government spending shock.** World Input-Output Databases ([2013], [2016]) allow us to get time series for total government spending with a breakdown of  $G_{it}$  by components  $G_{it}^{j,D}$  and  $G_{it}^{j,F}$  for  $j = H, N$ . All the obtained series are available at current prices which allows us to compute the non-tradable content of government consumption. To compute time series for non-traded government consumption at constant prices, we can take two routes. First, we can use time series for the non-tradable content of government spending,  $\omega_{GN,D,it}$ , obtained from WIOD dataset and then we construct time series for  $G_{it}^{N,D}$  at constant prices by multiplying time series for real government final consumption expenditure,  $G_{it}$ , with the time-varying non-tradable content of government consumption,  $\omega_{GN,D,it}$ . Second, we can alternatively construct time series for  $G_{it}^N$  at constant prices by multiplying time series for real government final consumption expenditure,  $G_{it}$ , with  $\omega_{GN,D,i}$  averaged over 1995-2014. This alternative is guided by the data as the non-tradable content of government spending is somewhat erratic. In Fig. 9, we plot empirical responses of non-traded government consumption at constant prices to an exogenous increase in aggregate government consumption by 1% of GDP shown in the blue line and contrast them with theoretical responses shown in black lines with squares. To compute the theoretical response, we proceed as follows. Denoting by  $\omega_{GN}$  the non-traded content of government spending, by  $\omega_{GH}$  the home component of the traded content of government spending, we have:

$$G(t) = \omega_{GN}G(t) + \omega_{GH}G(t) + \omega_{GF}G(t), \quad (121)$$

where  $\omega_{GF}$  is the imported content of government spending. Using the dynamic adjustment of  $dG(t)/Y$  described by eq. (30) and assuming that  $\omega_{Gj}$  is fixed over time, the endogenous response of the content of government spending in good  $j = H, F, N$  to an exogenous shock to aggregate government consumption reads:

$$\frac{dG^j(t)}{Y} = \omega_{Gj}e^{-\xi t} - (1 - g)e^{-\chi t}. \quad (122)$$

As can be seen in Fig. 9(a) which plots the response of  $G^N$  (in real terms) constructed by using the time-varying non-traded content of government spending, the theoretical response shown in the black line with squares and derived from (122) accounts reasonably well for the empirical IRF. Since the empirical response of  $G^N$  to an exogenous shock to government consumption is erratic, we alternatively construct time series for  $G^N$  by assuming that  $\omega_{GN}$  is constant over time and corresponds to its average over 1995-2014. As shown in Fig. 9(b), the theoretical response derived from (122) replicates very well the empirical response the first four years and somewhat overstates the empirical response afterwards. However, the empirical response is not statistically significant after six years. In Fig. 9(c), we compare the model's prediction shown in the solid black line with



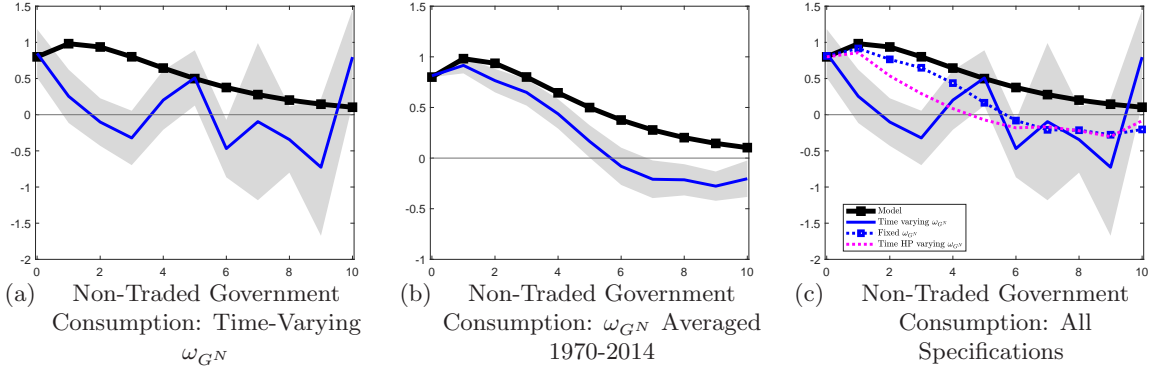


Figure 9: Empirical vs. Theoretical Responses of Non-Traded Government Consumption following a Shock to Aggregate Government Consumption. *Notes:* The solid blue line shows the response of aggregate and sectoral variables to an exogenous increase in government final consumption expenditure by 1% of GDP. In Fig. 9(a), we use time series for the non-traded content of government spending,  $\omega_{GN,it}$ , obtained from WIOD dataset and then we construct time series for  $G_{it}^N$  at constant prices by multiplying time series for real government final consumption expenditure,  $G_{it}$ , with  $\omega_{GN,it}$ . In Fig. 9(b), we estimate the response of  $G^N$  at constant prices whose time series are obtained by calculating the product of time series for real government final consumption expenditure,  $G_{it}$ , with the non-traded content of government consumption averaged over 1995-2014. In Fig. 9(c), we compare the model's prediction shown in the solid black line with squares with the empirical responses with time-varying and fixed  $\omega_{GN}$  shown in the solid blue line and the dotted blue line with squares, respectively. In the dotted magenta line line, we allow the non-traded content of government consumption to vary across time and smooth its adjustment by applying a HP filter to  $\omega_{GN,it}$  with a smoothing parameter of 100. Shaded areas indicate the 90 percent confidence bounds. The black line with squares shows the theoretical response of  $G^N$ . Sample: 18 OECD countries, 1970-2014 (except for NOR (2000-2014)), annual data.

squares with the empirical responses with time-varying and fixed  $\omega_{GN}$  shown in the dotted blue line with squares and the solid blue line, respectively. In the dotted magenta line line, we allow the non-traded content of government consumption to vary across time and smooth its adjustment by applying a HP filter to  $\omega_{GN,it}$  with a smoothing parameter of 100.

## F Construction of Time Series for FBTC

In this section, we detail the methodology to construct time series for capital-utilization-adjusted-FBTC in sector  $j = H, N$ . We choose the initial steady-state in a model with Cobb-Douglas production functions as the normalization point. When we calibrate the model with Cobb-Douglas production functions to the data, the ratios we target are averaged values over 1970-2015.

The starting point is the ratio of the labor to the capital income share in sector  $j$  given by eq. (79) which can be solved for capital-utilization-adjusted-FBTC in sector  $j$ :

$$\text{FBTC}_{t,adjK}^j \equiv \left( \frac{B_t^j}{A_t^j} \right)^{\frac{1-\sigma^j}{\sigma^j}} = S_t^j \frac{1-\gamma^j}{\gamma^j} \left( k_t^j \right)^{-\frac{1-\sigma^j}{\sigma^j}} \left( u_t^{K,j} \right)^{-\frac{1-\sigma^j}{\sigma^j}}, \quad (123)$$

where  $u_t^{K,j}$  is constructed by using the formula (96).

Since we normalize CES production functions so that the relative weight of labor and capital is consistent with the labor and capital income share in the data, solving for  $\gamma^j$  leads to:

$$\gamma^j = \left( \frac{\bar{A}^j}{\bar{y}^j} \right)^{\frac{1-\sigma^j}{\sigma^j}} \bar{s}_L^j, \quad (124a)$$

$$1 - \gamma^j = \left( \frac{\bar{B}^j \bar{u}^{K,j} \bar{k}^j}{\bar{y}^j} \right)^{\frac{1-\sigma^j}{\sigma^j}} \left( 1 - \bar{s}_L^j \right). \quad (124b)$$

Dividing (124a) by (124b) leads to:

$$\bar{S}^j = \frac{\gamma^j}{1 - \gamma^j} \left( \frac{\bar{S}^j \bar{u}^{K,j} \bar{k}^j}{\bar{k}^j} \right)^{\frac{1-\sigma^j}{\sigma^j}}, \quad (125)$$

where variables with a bar are averaged values of the corresponding variables over 1970-2015.

The methodology adopted to calculate  $\gamma^j$  amounts to using averaged values as the normalization point to compute time series for FBTC. Dividing (123) by (125) yields:

$$\left( \frac{B_t^j / \bar{B}^j}{A_t^j / \bar{A}^j} \right)^{\frac{1-\sigma^j}{\sigma^j}} = \frac{S_t^j}{\bar{S}^j} \left( \frac{k_t^j}{\bar{k}^j} \right)^{-\frac{1-\sigma^j}{\sigma^j}} \left( \frac{u_t^{K,j}}{\bar{u}^{K,j}} \right)^{-\frac{1-\sigma^j}{\sigma^j}}. \quad (126)$$

**Eq. (126) corresponds to eq. (11) in the main text.** To construct time series for  $FTBC_{t,adjK}^j$ , we plug estimates for the elasticity of substitution between capital and labor,  $\sigma^j$ , and time series for the ratio of the labor to the capital income share,  $S_t^j$ , the capital-labor ratio,  $k_t^j$ , and the capital utilization rate,  $u_t^{K,j}$ , in sector  $j = H, N$ . Next we divide yearly data by averaged values of the corresponding variable over 1970-2015. In Appendix J.3, we detail the empirical strategy to estimate the elasticity of substitution between capital and labor  $\sigma^j$ .

## G Construction of Unit Cost for Producing and Relative Labor Cost

In this section, we detail how we construct time series for the real unit cost for producing. Dividing (75a) by (75b) leads to a positive relationship between the relative cost of labor and the capital-labor ratio in sector  $j$ :

$$\frac{W^j}{R} = \frac{\gamma^j}{1 - \gamma^j} \left( \frac{B^j}{A^j} \right)^{\frac{1 - \sigma^j}{\sigma^j}} \left( \frac{\tilde{K}^j}{L^j} \right)^{\frac{1}{\sigma^j}}, \quad (127)$$

where  $\tilde{K}^j = u^{K,j} K^j$ . We manipulate (127) To to determine the conditional demands for both inputs:

$$L^j = \tilde{K}^j \left( \frac{\gamma^j}{1 - \gamma^j} \right)^{\sigma^j} \left( \frac{B^j}{A^j} \right)^{1 - \sigma^j} \left( \frac{W^j}{R} \right)^{-\sigma^j}, \quad (128a)$$

$$\tilde{K}^j = L^j \left( \frac{1 - \gamma^j}{\gamma^j} \right)^{\sigma^j} \left( \frac{B^j}{A^j} \right)^{\sigma^j - 1} \left( \frac{W^j}{R} \right)^{\sigma^j}. \quad (128b)$$

Inserting eq. (128a) (eq. (128b) resp.) in the CES production function (73) and solving for  $L^j$  ( $\tilde{K}^j$  resp.) leads to the conditional demand for labor (capital resp.):

$$\gamma^j (A^j L^j)^{\frac{\sigma^j - 1}{\sigma^j}} = (Y^j)^{\frac{\sigma^j - 1}{\sigma^j}} (\gamma^j)^{\sigma^j} \left( \frac{W^j}{A^j} \right)^{1 - \sigma^j} (X^j)^{-1}, \quad (129a)$$

$$(1 - \gamma^j) (B^j \tilde{K}^j)^{\frac{\sigma^j - 1}{\sigma^j}} = (Y^j)^{\frac{\sigma^j - 1}{\sigma^j}} \left( \frac{R}{B^j} \right)^{\sigma^j} (X^j)^{\frac{\sigma^j}{1 - \sigma^j}}, \quad (129b)$$

where  $X^j$  is given by:

$$X^j = (\gamma^j)^{\sigma^j} (A^j)^{\sigma^j - 1} (W^j)^{1 - \sigma^j} + (1 - \gamma^j)^{\sigma^j} (B^j)^{\sigma^j - 1} R^{1 - \sigma^j}. \quad (130)$$

Total cost is equal to the sum of the labor and capital cost:

$$C^j = W^j L^j + R \tilde{K}^j. \quad (131)$$

Inserting conditional demand for inputs (129) into total cost (131), we find that  $C^j$  is homogenous of degree one with respect to value added:

$$C^j = c^j Y^j, \quad \text{with} \quad c^j = (X^j)^{\frac{1}{1 - \sigma^j}}, \quad (132)$$

where the unit cost for producing is:

$$c^j = \left[ (\gamma^j)^{\sigma^j} \left( \frac{W^j}{A^j} \right)^{1 - \sigma^j} + (1 - \gamma^j)^{\sigma^j} \left( \frac{R}{B^j} \right)^{1 - \sigma^j - 1} \right]^{\frac{1}{1 - \sigma^j}}. \quad (133)$$

Because we investigate how firms adjust technology when the unit cost for producing is modified, we construct a technology adjusted unit cost for producing, denoted by  $UC^j$ :

$$UC^j = \left[ (\gamma^j)^{\sigma^j} (W^j)^{1 - \sigma^j} + (1 - \gamma^j)^{\sigma^j} R^{1 - \sigma^j} \right]^{\frac{1}{1 - \sigma^j}}, \quad (134)$$

where  $\gamma^j$  is described by eq. (124a). To ensure that  $0 < \gamma^j < 1$ , we normalize  $\bar{A}^j = \bar{B}^j = \bar{y}^j = 1$  ( $y^j$  is a volume index and thus this assumption has no impact) so that  $\gamma^j = s_L^j$  averaged over 1970-2015. To construct time series for  $UC^j$ , we insert our estimates of the elasticity of substitution between capital and labor,  $\sigma^j$ , shown in columns 18 and 19 of Table 7, and we plug time series for the wage rate in sector  $j$  and the capital rental rate. While we assume perfect mobility of capital, the capital rental rate in the traded sector may temporarily deviate from the capital rental rate in

the non-traded sector in the data. The property of constant returns to scale in production implies that value added is exhausted by the payment of factors, i.e.,  $P^j Y^j = W^j L^j + R^j \tilde{K}^j$ . Solving the latter equality for the capital rental rate leads to  $R^j = \frac{P^j Y^j - W^j L^j}{u^{K,j} \tilde{K}^j}$  where  $P^j Y^j$  is value added at current prices,  $W^j L^j$  is labor compensation,  $\tilde{K}^j$  is the stock of capital at constant prices, and  $u^{K,j}$  the capital utilization rate in sector  $j$  (see eq. (96)).

Firms choose the optimal level of value added by equating sectoral prices to the ratio of the unit cost for producing to the capital-utilization-adjusted TFP:

$$P^j = \frac{UC^j}{u^{Z,j}}. \quad (135)$$

Eq. (135) shows that in face of a rise the unit cost for producing  $UC^j$ , firms can increase prices,  $P^j$ , or can achieve some technology improvement (i.e.,  $u^{Z,j}$  increases), or both. Firms will increase the technology utilization rate by a larger amount in sectors/countries where the cost of adjusting technology is lower.

## H Technology Frontier and FBTC

Following Caselli and Coleman [2006] and Caselli [2016], the menu of possible choices of production functions is represented by a set of possible  $(A^j, B^j)$  pairs. These pairs are chosen along the technology frontier which is assumed to take a CES form:

$$\left[ \gamma_Z^j (A^j(t))^{\frac{\sigma_Z^j - 1}{\sigma_Z^j}} + (1 - \gamma_Z^j) (B^j(t))^{\frac{\sigma_Z^j - 1}{\sigma_Z^j}} \right]^{\frac{\sigma_Z^j}{\sigma_Z^j - 1}} \leq Z^j(t), \quad (136)$$

where  $Z^j > 0$  is the height of the technology frontier,  $0 < \gamma_Z^j < 1$  is the weight of labor efficiency in TFP and  $\sigma_Z^j > 0$  corresponds to the elasticity of substitution between labor and capital efficiency. Totally differentiating (136) leads to

$$\begin{aligned} 0 &= \gamma_Z^j (A^j(t))^{\frac{\sigma_Z^j - 1}{\sigma_Z^j}} \hat{A}^j(t) + (1 - \gamma_Z^j) (B^j(t))^{\frac{\sigma_Z^j - 1}{\sigma_Z^j}} \hat{B}^j(t), \\ \frac{\hat{B}^j(t)}{\hat{A}^j(t)} &= -\frac{\gamma_Z^j}{1 - \gamma_Z^j} \left( \frac{B^j(t)}{A^j(t)} \right)^{\frac{1 - \sigma_Z^j}{\sigma_Z^j}}. \end{aligned} \quad (137)$$

Firms choose  $A^j$  and  $B^j$  along the technology frontier so that minimizes the unit cost function described by (133) subject to (136) which holds as an equality. Differentiating (133) w.r.t.  $A^j$  and  $B^j$  (while keeping  $W^j$  and  $R$  fixed) leads to:

$$\hat{c}^j(t) = -(\gamma^j)^{\sigma^j} \left( \frac{W^j(t)}{A^j(t)} \right)^{1 - \sigma^j} (c^j(t))^{\sigma^j - 1} \hat{A}^j(t) - (1 - \gamma^j)^{\sigma^j} \left( \frac{R(t)}{B^j(t)} \right)^{1 - \sigma^j} (c^j(t))^{\sigma^j - 1} \hat{B}^j(t). \quad (138)$$

Setting (138) to zero and inserting (136), the cost minimization leads to the following optimal choice of technology:

$$\begin{aligned} \left( \frac{\gamma^j}{1 - \gamma^j} \right)^{\sigma^j} \left( \frac{W^j(t)}{R(t)} \right)^{1 - \sigma^j} \left( \frac{B^j(t)}{A^j(t)} \right)^{1 - \sigma^j} &= \frac{\gamma_Z^j}{1 - \gamma_Z^j} \left( \frac{B^j(t)}{A^j(t)} \right)^{\frac{1 - \sigma_Z^j}{\sigma_Z^j}}, \\ \left( \frac{B^j(t)}{A^j(t)} \right)^{\frac{1 - \sigma^j}{\sigma^j}} \left( \frac{B^j(t)}{A^j(t)} \right)^{-\left( \frac{1 - \sigma_Z^j}{\sigma_Z^j} \right) \frac{1}{\sigma^j}} &= \left( \frac{\gamma_Z^j}{1 - \gamma_Z^j} \right)^{\frac{1}{\sigma^j}} \left( \frac{1 - \gamma^j}{\gamma^j} \right) \left( \frac{W^j(t)}{R(t)} \right)^{-\frac{1 - \sigma^j}{\sigma^j}}, \\ \left( \frac{B^j(t)}{A^j(t)} \right)^{\frac{1 - \sigma^j}{\sigma^j} \left[ 1 - \left( \frac{1 - \sigma_Z^j}{1 - \sigma^j} \right) \frac{1}{\sigma^j} \right]} &= \left( \frac{\gamma_Z^j}{1 - \gamma_Z^j} \right)^{\frac{1}{\sigma^j}} \left( \frac{1 - \gamma^j}{\gamma^j} \right) \left( \frac{W^j(t)}{R(t)} \right)^{-\frac{1 - \sigma^j}{\sigma^j}}, \end{aligned} \quad (139)$$

where  $\text{FBCT}_{adjK}^j = \left( \frac{B^j(t)}{A^j(t)} \right)^{\frac{1 - \sigma^j}{\sigma^j}}$  is capital-utilization-adjusted-FBTC in sector  $j$ . According to eq. (139), when we let  $\sigma_Z^j$  tend toward one so that the technology frontier is Cobb-Douglas, relative capital efficiency  $B^j(t)/A^j(t)$  in sector  $j$  is decreasing in the wage-to-capital-rental-rate ratio  $W^j(t)/R(t)$ . Thereby, in face of a rise in  $W^j/R$ , firms increase  $A^j$  and thus lower  $B^j/A^j$ . Intuitively, it is optimal for firms to bias factor efficiency toward the most expensive factor. However, when  $A^j$  and  $B^j$  are gross complements in technology production, the rise in  $A^j$  will require more units of

$B^j$  (which may increase disproportionately) which may result in an increase in  $B^j/A^j$  if the gross complementarity between  $A^j$  and  $B^j$  is high enough, i.e.,

$$\frac{1 - \sigma_Z^j}{\sigma_Z^j} > 1 - \sigma^j. \quad (140)$$

When the inequality (140) holds, then a rise in  $W^j(t)/R(t)$  may increase  $\text{FBTC}_{adjK}^j$  instead of decreasing it.

Denoting  $d^j = \frac{1}{\sigma^j} \ln \left( \frac{\gamma_Z^j}{1 - \gamma_Z^j} \right) + \ln \left( \frac{1 - \gamma^j}{\gamma^j} \right)$  and  $\Omega^j(t) = \left( \frac{W^j(t)}{R(t)} \right)^{\frac{1 - \sigma^j}{\sigma^j}}$ , and taking log of both sides of eq. (139) leads to:

$$\ln \text{FBTC}_{adjK}^j(t) = e^j + (\delta^j)^{-1} \ln \Omega^j(t), \quad (141)$$

where  $e^j = \frac{d^j}{\delta^j}$  with  $\delta^j = \left[ \left( \frac{1 - \sigma_Z^j}{1 - \sigma^j} \right) \frac{1}{\sigma_Z^j} - 1 \right] \geq 0$ . Our objective is to estimate the responses of  $\text{FBTC}_{adjK}^j(t)$  and  $\Omega^j(t)$  to an increase in government consumption by 1% of GDP and to investigate whether the response of capital-utilization-adjusted FBTC in sector  $j$  moves in opposite direction relative to the response of the adjusted wage-to-capital-rental-rate ratio  $(\delta^j)^{-1} \ln \Omega^j(t)$ . When  $\sigma_Z^j = 1$ ,  $\delta^j$  collapses to minus one so that  $\ln \text{FBTC}_{adjK}^j(t)$  and  $\ln \Omega^j(t)$  should move in opposite direction. However, our estimates reveal that both co-move which suggest that  $\sigma_Z^j$  takes values much lower than one because  $\delta^j$  turns out to be positive, which thus generates a positive relationship between  $\ln \text{FBTC}_{adjK}^j(t)$  and  $(\delta^j)^{-1} \ln \Omega^j(t)$ . To explore empirically the relationship between capital-utilization-adjusted FBTC and the adjusted wage-to-capital-rental-rate ratio, we have to estimate the elasticity of substitution between labor- and capital-augmenting efficiency  $\sigma_Z^j$  to compute the value of  $\delta^j$  in order to scale the response of  $\Omega^j(t) = \left( \frac{W^j(t)}{R(t)} \right)^{\frac{1 - \sigma^j}{\sigma^j}}$ .

To pin down the value of  $\sigma_Z^j$ , we proceed as follows. Using the fact that  $(\gamma^j)^{\sigma^j} \left( \frac{W^j(t)}{A^j(t)} \right)^{1 - \sigma^j} (c^j(t))^{\sigma^j - 1} = s_L^j(t)$ , eq. (138) can be rewritten as  $-s_L^j \hat{A}^j(t) - (1 - s_L^j) \hat{B}^j(t) = \hat{c}^j(t)$ . Setting this equality to zero and inserting (137) leads to:

$$\frac{\gamma_Z^j}{1 - \gamma_Z^j} \left( \frac{B^j(t)}{A^j(t)} \right)^{\frac{1 - \sigma_Z^j}{\sigma_Z^j}} = \frac{s_L^j(t)}{1 - s_L^j(t)} \equiv S^j(t). \quad (142)$$

**Eq. (142) corresponds to eq. (36) in the main text.** Demand for inputs can be rewritten in terms of their respective cost in value added; for labor, we have  $s_L^j(t) = \gamma^j \left( \frac{A^j(t)}{y^j(t)} \right)^{\frac{\sigma^j - 1}{\sigma^j}}$ . Applying the same logic for capital and denoting the ratio of labor to capital income share by  $S^j(t) \equiv \frac{s_L^j(t)}{1 - s_L^j(t)}$ , we have:

$$S^j(t) \equiv \frac{s_L^j(t)}{1 - s_L^j(t)} = \frac{\gamma^j}{1 - \gamma^j} \left( \frac{B^j(t) u^{K,j}(t) K^j(t) k^j(t)}{A^j(t) L^j(t)} \right)^{\frac{1 - \sigma^j}{\sigma^j}}. \quad (143)$$

Making use of (142) to eliminate  $B^j/A^j$  from eq. (143) and solving leads to:

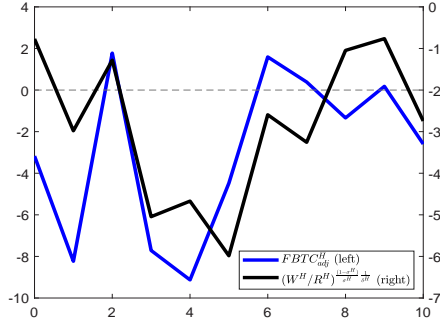
$$u^{K,j}(t) k^j(t) = (S^j(t))^{\frac{\sigma^j}{1 - \sigma^j} - \frac{\sigma_Z^j}{1 - \sigma_Z^j}} \left( \frac{1 - \gamma^j}{\gamma^j} \right)^{\frac{\sigma^j}{1 - \sigma^j}} \left( \frac{\gamma_Z^j}{1 - \gamma_Z^j} \right)^{\frac{\sigma_Z^j}{1 - \sigma_Z^j}}. \quad (144)$$

Denoting  $f^j = \frac{\sigma^j}{1 - \sigma^j} \ln \left( \frac{1 - \gamma^j}{\gamma^j} \right) + \frac{\sigma_Z^j}{1 - \sigma_Z^j} \ln \left( \frac{\gamma_Z^j}{1 - \gamma_Z^j} \right)$  and taking log of both sides of eq. (144) leads to:

$$\ln [u^{K,j}(t) k^j(t)] = f^j + \zeta^j \ln S^j(t). \quad (145)$$

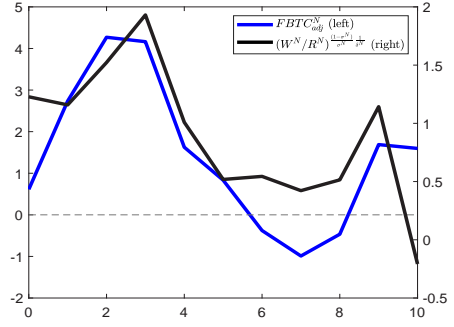
where  $\zeta^j = \left[ \frac{\sigma^j}{1 - \sigma^j} - \frac{\sigma_Z^j}{1 - \sigma_Z^j} \right]$ . We add error terms on the RHS of eq. (145) and run the regression of the logged capital-labor ratio inclusive of the capital utilization rate, i.e.,  $\ln [u^{K,j}(t) k^j(t)]$ , on the logged ratio of labor to capital income share,  $\ln S^j(t)$ , by allowing for country fixed effects, time dummies and country-specific linear time trend. Since all variables display unit root process, we estimate cointegrating relationships by using the fully modified OLS (FMOLS) procedure for cointegrated panel proposed by Pedroni [2000]. Panel data estimations return a value of  $\zeta^H = -0.6$  for the traded sector and a value of  $\zeta^N = -0.59$  for the non-traded sector. By using panel data estimations of the elasticity of substitution between capital and labor in the traded and non-traded sector, i.e.,  $\sigma^H = 0.638$  and  $\sigma^N = 0.799$ , we use the formula  $\sigma_Z^j = \frac{\frac{\sigma^j}{1 - \sigma^j} - \zeta^j}{1 + \frac{\sigma^j}{1 - \sigma^j} - \zeta^j}$  to infer the value

Traded Capital-Utilization-Adjusted  
FBTC vs. Adjusted-Wage-to-Capital-Cost



(a)  $FBTC_{adjK}^H(t)$  against  
 $\left(\frac{W^H(t)}{R^H(t)}\right) \left(\frac{1-\sigma^H}{\sigma^H}\right) \frac{1}{\delta^H}$

Non-Traded Capital-Utilization-Adjusted  
FBTC vs. Adjusted-Wage-to-Capital-Cost



(b)  $FBTC_{adjK}^N(t)$  against  
 $\left(\frac{W^N(t)}{R^N(t)}\right) \left(\frac{1-\sigma^N}{\sigma^N}\right) \frac{1}{\delta^N}$

Figure 10: Utilization-Adjusted FBTC and Adjusted-Wage-to-Capital Cost following a Government Spending Shock. Notes: Fig. 10 explores the factors leading firms to adjust their technology following an exogenous increase in government consumption by 1% of GDP. Fig. 10(a) and Fig. 10(b) plot the response of capital-utilization-adjusted FBTC in sector  $j$  to the government spending shock shown in the blue line against the response of the adjusted-wage-to-capital-rental-rate ratio in sector  $j$ , i.e.,  $\Omega^j(t) = \left(\frac{W^j(t)}{R^j(t)}\right)^{\frac{1-\sigma^j}{\sigma^j}}$ , where the response of  $\Omega^j(t)$  is augmented by the inverse of  $\delta^j = \left[1 - \left(\frac{1-\sigma_Z^j}{1-\sigma^j}\right) \frac{1}{\sigma_Z^j}\right]$ . The response of  $(\delta^j)^{-1} \ln \Omega^j(t)$  is shown in the black line. Sample: 18 OECD countries, 1970-2015, annual data.

for the elasticity of substitution between labor- and capital-augmenting efficiency which is equal to  $\sigma_Z^H = 0.65$  for the traded sector and  $\sigma_Z^N = 0.72$  for the non-traded sector. The values for  $\sigma^j$  and  $\sigma_Z^j$  implies that the inequality (140) holds so that  $\delta^j > 0$  for both the traded and the non-traded sector, i.e.,  $\delta^H = 0.515$  and  $\delta^N = 0.905$ . Because  $\delta^j$  is positive, capital-utilization-adjusted-FBTC in sector  $j$  is positively related to the wage-to-capital-rental-rate ratio. Since government spending shocks are biased toward non-tradables and thus triggers a reallocation of labor towards the non-traded sector, the existence of labor mobility costs lead non-traded firms to pay higher wages to encourage workers to shift (i.e.,  $W^N/R$  rises), while  $W^H/R$  falls in the traded sector. Whilst it is optimal for firms to bias factor efficiency toward the most expensive factor, the complementarity between labor- and capital-augmenting productivity leads to an increase in  $FBTC_{adjK}^N(t)$  in the non-traded sector and a fall in  $FBTC_{adjK}^H(t)$  in the traded sector.

To estimate the dynamic adjustment of capital-utilization-adjusted-FBTC in sector  $j$ , we construct time series by using (123) and generate the response of  $FBTC_{adjK}^j(t)$  by using local projection where the shock is identified by running a VAR model including government consumption, real GDP, total hours worked, the real consumption wage and aggregate TFP. To construct  $\Omega^j(t) = \left(\frac{W^j(t)}{R^j(t)}\right)^{\frac{1-\sigma^j}{\sigma^j}}$ , we divide the wage rate  $W^j(t)$  by the capital rental rate in sector  $j$  calculated as follows  $R^j(t) = \frac{P^j(t)Y^j(t) - W^j(t)L^j(t)}{u^{K,j}(t)K^j(t)}$  and we use estimates of  $\sigma^j$  shown in the last line of columns 18 and 19 of Table 7. Fig. 10(a) and Fig. 10(b) plot the response of capital-utilization adjusted FBTC in sector  $j$  to the government spending shock shown in the blue line against the response of the adjusted wage-to-capital-rental-rate ratio in sector  $j$ . As displayed by the black line in Fig. 10(a), the adjusted wage-to-capital-rental-rate ratio scaled by  $\frac{1}{\delta^H}$  falls following a rise in government spending. If the technology frontier were of the Cobb-Douglas type, traded firms would increase capital- relative to labor-augmenting efficiency. However, our estimates reveal that the technology frontier is of the CES type with an elasticity of substitution between  $A^H$  and  $B^H$  smaller than one. The gross complementarity between  $A^H$  and  $B^H$  in technology is high enough to encourage firms to increase  $A^H$  relative to  $B^H$  and generate a fall in  $FBTC_{adjK}^H(t)$  which reflects the fact that the traded sector biases technological change toward capital. As displayed by the black line in Fig. 10(b), the adjusted wage-to-capital-rental-rate ratio scaled by  $\frac{1}{\delta^N}$  increases following a rise in government spending. While the higher wages lead non-traded firms to bias technological change toward the most expensive factor, say labor, the gross complementarity between  $A^N$  and  $B^N$  is high enough to generate an increase in the relative capital efficiency. Therefore, as displayed by the blue line in Fig. 10(b), technological change is biased toward labor labor as reflected into a rise in  $FBTC_{adjK}^N(t)$ . In both the traded and the non-traded sector, the correlation between the two impulse response functions is high as it stands at 0.6 in the traded sector and 0.7 in the non-traded sector.

# I Isolating the Role of Technology

In order to shed some light on the role of the technology channel for fiscal transmission and guide our quantitative analysis, we estimate the VAR model in panel format on annual data. We consider a structural model with  $k = 2$  lags in the following form:

$$AZ_{i,t} = \sum_{k=1}^2 B_k Z_{i,t-k} + \epsilon_{i,t}, \quad (146)$$

where subscripts  $i$  and  $t$  denote the country and the year, respectively,  $Z_{i,t}$  is the vector of endogenous variables,  $A$  is a matrix that describes the contemporaneous relation among the variables collected in vector  $Z_{i,t}$ ,  $B_k$  is a matrix of lag specific own- and cross-effects of variables on current observations, and the vector  $\epsilon_{i,t}$  contains the structural disturbances which are uncorrelated with each other.

Because the VAR model cannot be estimated in its structural form, we pre-multiply (146) by  $A^{-1}$  which gives the reduced form of the VAR model:

$$Z_{i,t} = \sum_{k=1}^2 A^{-1} B_k Z_{i,t-k} + e_{i,t}, \quad (147)$$

where  $A^{-1} B_k$  and  $e_{it} = A^{-1} \epsilon_{it}$  are estimated by using a panel OLS regression with country fixed effects and country specific linear trends. To identify the VAR model and recover the government spending shocks, we need assumptions on the matrix  $A$  as the reduced form of the VAR model that we estimate contains fewer parameters than the structural VAR model shown in eq. (146). Like Blanchard and Perotti [2002], we base the identification scheme on the assumption that discretionary government spending is subject to certain decision and implementation lags that prevent government spending from responding to current output developments. This amounts to a recursive identification with government spending shocks ordered first which implies that matrix  $A$  in eq. (146) is lower-triangular:

$$\begin{pmatrix} 1 & 0 & 0 \\ a_{21} & 1 & 0 \\ a_{31} & a_{32} & 1 \end{pmatrix} \begin{pmatrix} g_{i,t} \\ \text{tech}_{i,t} \\ \text{sec}_{i,t} \end{pmatrix} = \sum_{k=1}^2 B_k \begin{pmatrix} g_{i,t-k} \\ \text{tech}_{i,t-k} \\ \text{sec}_{i,t-k} \end{pmatrix} + \begin{pmatrix} \epsilon_{i,t}^G \\ \epsilon_{i,t}^Z \\ \epsilon_{i,t}^S \end{pmatrix}, \quad (148)$$

where the VAR model includes three variables, i.e., (logged) government consumption, (logged) technology variables, and (logged) sectoral variables where all quantities are divided by population.

The term  $a_{31}$  in the structural form (148) of the VAR model captures the direct or standard effect of government consumption on sectoral variables. The term  $a_{21}$  captures the effect of government consumption on technology variables and the term  $a_{32}$  captures the effect of technological change on sectoral variables. The multiplicative term  $a_{21} \times a_{32}$  thus measures the technology channel of government spending on impact. This decomposition paves the way to isolate the technology channel in the data in the same way as we would do in a model. Our objective is to determine the hypothetical response of sectoral variables if technology remained unresponsive to the government spending shock at all forecast horizons. When we compare the actual response of the sectoral variable with its hypothetical response, we have a measure of the importance of the technology channel in the transmission of government spending shocks. To determine the hypothetical response of sectoral variables if the technology channel were shut down, we follow the same methodology as Bachmann and Sims [2012]. Intuitively, this method amounts to creating a sequence of technology shocks so as to zero out the response of technology to a rise in government spending. Given this sequence, we can compute the modified impulse response function of sectoral variables as if technology were unresponsive to government spending shocks.

In the main text, we consider three variants of the VAR model:

- In the first variant, we consider a VAR model which includes (logged) government consumption, the (logged) ratio of traded to non-traded TFP, the (logged) value added share of non-tradables, i.e.,  $[g_{it}, \text{tfp}_{it}^H - \text{tfp}_{it}^N, \ln \nu_{it}^{Y,N}]$ . Estimates are shown in the first column of Fig. 3.
- In the second variant, we consider a VAR model which includes (logged) government consumption, the (logged) ratio of non-traded to traded capital-income-hare-adjusted-FBTC, the (logged) ratio of non-traded to non-traded LIS, i.e.,  $[g_{it}, (1 - s_L^N) \text{fbtc}_{adj,it}^N - (1 - s_L^H) \text{fbtc}_{adj,it}^H, \ln (s_{L,it}^N / s_{L,it}^H)]$ . Estimates are shown in the second column of Fig. 3.
- In the third variant, we consider a VAR model which includes (logged) government consumption, the (logged) ratio of non-traded to aggregate LIS, the (logged) adjusted labor share of non-tradables, i.e.,  $[g_{it}, \ln (s_{L,it}^N / s_{L,it}), \ln \left[ \frac{L_{it}^N}{L_{it}} \left( \omega_{it}^{Y,N} \right)^{\frac{\epsilon_{it}}{1+\epsilon_{it}}} \right]]$  where  $s_L$  is the aggregate LIS,  $\epsilon$



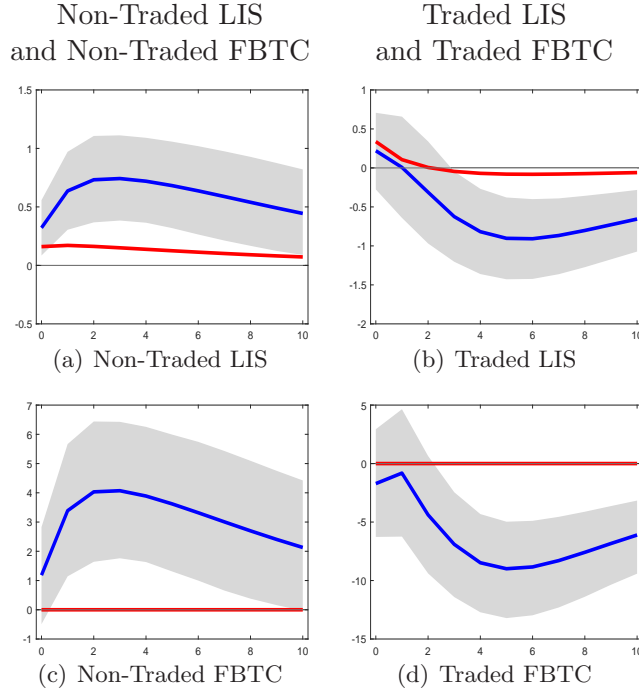


Figure 11: Dynamic Adjustment to Government Spending Shocks: Isolating the Technology Channel. *Notes:* Fig. 4 plots the dynamic adjustment of sectoral LISs to a 1% exogenous increase in government by isolating the pure technology effect. We estimate a VAR model including logged government consumption, logged utilization adjusted-FBTC in sector  $j$ , i.e.,  $\ln \text{FBTC}_{adjK,it}^j$ , and the (logged) LIS in sector  $j$ . The blue line shows the actual dynamic adjustment when we let technological change responds to the government spending shock while the red line shows the hypothetical dynamic adjustment of variables if FBTC were unresponsive to the demand shock at all horizons. While the first row shows the responses of LISs, the second row shows the dynamic adjustment of utilization-adjusted-FBTC in sector  $j$  scaled by the capital income share in this sector. Sample: 18 OECD countries, 1970-2015, annual data.

is the elasticity of labor supply across sectors whose estimated values are taken from column 17 of Table 7,  $\omega^{Y,N}$  is the value added share of non-tradables at current prices. We augment the labor share of non-tradables with  $\epsilon$  and  $\omega^{Y,N}$ , in accordance with eq. (6), in order to control the effects of international differences in labor mobility costs and in the biasedness of the demand shock toward non-tradables. Estimates are shown in the third column of Fig. 3.

- In the fourth variant, we estimate a VAR model including government consumption, utilization adjusted-FBTC in sector  $j$ , i.e.,  $\ln \text{FBTC}_{adjK,it}^j$ , and the LIS in sector  $j$ . The results are not included in the main text for reasons of space. Estimates are shown in Fig. 11.

The first row of Fig. 11 displays the dynamic adjustment of the non-traded and traded LIS while the second row shows the dynamic adjustment of capital-utilization-adjusted FBTC in the non-traded and traded sector. The blue line shows actual responses of variables while the red line shows responses of the variable when FBTC is shut down. As is clear from Fig. 11, the rise in the non-traded LIS is driven by technological change biased toward labor, as captured by an increase in  $\ln \text{FBTC}_{adjK,it}^N$ . Conversely, the decline in the traded LIS is brought about by technological change biased toward capital, as reflected into a fall in  $\ln \text{FBTC}_{adjK,it}^H$ .

## J Data for Calibration

### J.1 Non-Tradable Content of GDP and its Demand Components

Table 7 shows the non-tradable content of GDP, consumption, investment, government spending, labor and labor compensation (columns 1 to 6). In addition, it gives information about the sectoral labor income shares (columns 11 and 12). The home content of consumption and investment expenditure in tradables and the home content of government spending are reported in columns 8 to 10. Column 7 shows the ratio of exports to GDP. Columns 11 and 12 shows the labor income share in the traded and non-traded sector. Columns 13 to 15 display the aggregate labor income share, investment-to-GDP ratio and government spending in % of GDP, respectively, for the whole economy. Our sample covers the 18 OECD countries mentioned in section A. The reference period for the calibration of labor variables is 1970-2015 while the reference period for demand components is 1995-2014 due to data availability, as detailed below. When we calibrate the model to a representative economy, we use the last line which shows the (unweighted) average of the corresponding



variable.

**Aggregate ratios.** Columns 13 to 15 show the aggregate labor income share,  $s_L$ , the investment-to-GDP ratio,  $\omega_J$  and government spending as a share of GDP,  $\omega_G$ . The aggregate labor income share is calculated as the ratio of labor compensation (compensation of employees plus compensation of self-employed) to GDP at current prices. Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases. Data coverage: 1970-2015 (1974-2015 for JPN). To calculate  $\omega_J$ , we use time series for gross capital formation at current prices and GDP at current prices, both obtained from the OECD National Accounts Database [2017]. Data coverage: 1970-2015 for all countries. To calculate  $\omega_G$ , we use time series for final consumption expenditure of general government (at current prices) and GDP (at current prices). Source: OECD National Accounts Database [2017]. Data coverage: 1970-2015 for all countries. We consider a steady-state where trade is initially balanced and we calculate the consumption-to-GDP ratio,  $\omega_C$  by using the accounting identity between GDP and final expenditure:

$$\omega_C = 1 - \omega_J - \omega_G. \quad (149)$$

As displayed by the last line of Table 7, investment expenditure (see column 14) and government spending (see column 15) as a share of GDP average to 24% and 19%, respectively, while the aggregate labor income share averages to 66% (see column 13).

**Non-traded demand components.** Columns 2 to 4 show non-tradable content of consumption (i.e.,  $1 - \alpha_C$ ), investment (i.e.,  $1 - \alpha_J$ ), and government spending (i.e.,  $\omega_{GN}$ ), respectively. These demand components have been calculated by adopting the methodology described in eqs. (120a)-(120b), and eq. (117). Sources: World Input-Output Databases ([2013], [2016]). Data coverage: 1995-2014 except for NOR (2000-2014). The non-tradable share of consumption, investment and government spending shown in column 2 to 4 of Table 7 averages to 56%, 69% and 80%, respectively.

In the empirical analysis, we use data from EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases for constructing sectoral value added over the period running from 1970 to 2015. Since the demand components for non-tradables are computed over 1995-2014 by using the WIOD dataset, to ensure that the value added is equal to the sum of its demand components, we have calculated the non-tradable content of value added shown in column 1 of Table 7 as follows:

$$\begin{aligned} \omega^{Y,N} &= \frac{P^N Y^N}{Y}, \\ &= \omega_C (1 - \alpha_C) + \omega_J (1 - \alpha_J) + \omega_{GN} \omega_G, \end{aligned} \quad (150)$$

where  $1 - \alpha_C$  and  $1 - \alpha_J$  are the non-tradable content of consumption and investment expenditure shown in columns 2 and 3,  $\omega_{GN}$  is the non-tradable content of government spending shown in column 4,  $\omega_C$  and  $\omega_J$  are consumption- and investment-to-GDP ratios, and  $\omega_G$  is government spending as a share of GDP.

**Non-tradable content of hours worked and labor compensation.** To calculate the non-tradable share of labor shown in column 5 and labor compensation shown in column 6, we split the eleven industries into traded and non-traded sectors by adopting the classification proposed by De Gregorio et al. [1994] and updated by Jensen and Kletzer [2006]. Details about data construction for sectoral output and sectoral labor are provided above. We calculate the non-tradable share of labor compensation as the ratio of labor compensation in the non-traded sector (i.e.,  $W^N L^N$ ) to overall labor compensation (i.e.,  $WL$ ). Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases. Data coverage: 1970-2015 for all countries (except Japan: 1974-2015). The non-tradable content of labor and labor compensation, shown in columns 5 and 6 of Table 7, average to 62% and 63% respectively.

**Home content of consumption and investment expenditure in tradables.** Columns 8 to 9 of Table 7 show the home content of consumption and investment in tradables, denoted by  $\alpha^H$  and  $\alpha_J^H$  in the model. These shares are obtained from time series calculated by using the formulas (119a)-(119b). Sources: World Input-Output Databases ([2013], [2016]). Data coverage: 1995-2014 except for NOR (2000-2014). Column 10 shows the content of government spending in home-produced traded goods. Taking data from the WIOD dataset, time series for  $\omega_{GH}$  are constructed by using the formula (118). Data coverage: 1995-2014 except for NOR (2000-2014). As shown in the last line of columns 8 and 9, the home content of consumption and investment expenditure in traded goods averages to 66% and 43%, respectively, while the share of government spending in home-produced traded goods averages 19%. Since the non-tradable content of government spending averages 80% (see column 4), the import content of government spending is 1% only.

Since we set initial conditions so that the economy starts with balanced trade, export as a share of GDP,  $\omega_X$ , shown in column 7 of Table 7 is endogenously determined by the import content of consumption,  $1 - \alpha^H$ , investment expenditure,  $1 - \alpha_J^H$ , and government spending,  $\omega_{GF}$ , along with the consumption-to-GDP ratio,  $\omega_C$ , the investment-to-GDP ratio,  $\omega_J$ , and government spending as a share of GDP,  $\omega_G$ . More precisely, dividing the current account equation at the steady-state by GDP,  $Y$ , leads to an expression that allows us to calculate the GDP share of exports of final goods

and services produced by the home country:

$$\omega_X = \frac{P^H X^H}{Y} = \omega_C \alpha_C (1 - \alpha^H) + \omega_J \alpha_J (1 - \alpha^H) + \omega_G \omega_{GF}, \quad (151)$$

$\omega_{GF} = 1 - \omega_{GN,D} - \omega_{GH,D}$ . The last line of column 7 of Table 7 shows that the export to GDP ratio averages 13%.

**Sectoral labor income shares.** The labor income share for the traded and non-traded sector, denoted by  $s_L^H$  and  $s_L^N$ , respectively, are calculated as the ratio of labor compensation of sector  $j$  to value added of sector  $j$  at current prices. Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases. Data coverage: 1970-2015 for all countries (except Japan: 1974-2015). As shown in columns 11 and 12 of Table 7,  $s_L^H$  and  $s_L^N$  averages to 63% and 69%, respectively.

**Estimated elasticities.** Columns from 16 to 20 of Table 7 display estimates of the elasticity of substitution between tradables and non-tradables in consumption,  $\phi$ , the elasticity of labor supply across sectors,  $\epsilon$ , the elasticity of substitution between capital and labor in the traded and the non-traded sector, i.e.,  $\sigma^H$  and  $\sigma^N$ , respectively, the elasticity of exports w.r.t. the terms of trade,  $\phi_X$ . In subsections J.2 and J.3, we detail the empirical strategy to estimate these parameters, except for the price elasticity of exports shown in the last column of Table 7 whose estimates are taken from Imbs and Mejean [2015].

## J.2 Estimates of $\epsilon$ and $\phi$ : Empirical Strategy

Table 8 shows our estimates of the elasticity of labor supply across sectors,  $\epsilon$ , while Table 9 shows our estimates of the elasticity of substitution in consumption between traded and non-traded goods,  $\phi$ . We present our empirical strategy to estimate these two parameters. The derivation of equations we explore empirically is detailed in the technical appendix of the working paper by Bertinelli, Cardi and Restout [2020].

**Elasticity of labor supply across sectors.** Drawing on Horvath [2000], we derive a testable equation by combining optimal rules for labor supply and labor demand and estimate  $\epsilon$  by running the regression of the worker inflow in sector  $j = H, N$  of country  $i$  at time  $t$  arising from labor reallocation across sectors computed as  $\hat{L}_{i,t}^j - \hat{L}_{i,t}$  on the relative labor's share percentage changes in sector  $j$ ,  $\hat{\beta}_{i,t}^j$ :

$$\hat{L}_{i,t}^j - \hat{L}_{i,t} = f_i + f_t + \gamma_i \hat{\beta}_{i,t}^j + \nu_{i,t}^j, \quad (152)$$

where  $\nu_{i,t}^j$  is an i.i.d. error term; country fixed effects are captured by country dummies,  $f_i$ , and common macroeconomic shocks by year dummies,  $f_t$ . The LHS term of (152) is calculated as the difference between changes (in percentage) in hours worked in sector  $j$ ,  $\hat{L}_{i,t}^j$ , and in total hours worked,  $\hat{L}_{i,t}$ . The RHS term  $\beta^j$  corresponds to the fraction of labor's share of value added accumulating to labor in sector  $j$ . Denoting by  $P_t^j Y_t^j$  value added at current prices in sector  $j = H, N$  at time  $t$ ,  $\beta_t^j$  is computed as  $\frac{s_L^j P_t^j Y_t^j}{\sum_{j=H}^N s_L^j P_t^j Y_t^j}$  where  $s_L^j$  is the LIS in sector  $j = H, N$  defined as the ratio of the compensation of employees to value added in the  $j$ th sector, averaged over the period 1970-2015. Because hours worked are aggregated by means of a CES function, percentage change in total hours worked,  $\hat{L}_{i,t}$ , is calculated as a weighted average of sectoral hours worked percentage changes, i.e.,  $\hat{L}_t = \sum_{j=H}^N \beta_{t-1}^j \hat{L}_t^j$ . The parameter we are interested in, say the degree of substitutability of hours worked across sectors, is given by  $\epsilon_i = \gamma_i / (1 - \gamma_i)$ . In the regressions that follow, the parameter  $\gamma_i$  is assumed to be different across countries when estimating  $\epsilon_i$  for each economy ( $\gamma_i \neq \gamma_{i'}$  for  $i \neq i'$ ). To construct  $\hat{L}^j$  and  $\hat{\beta}^j$  we combine raw data on hours worked  $L^j$ , nominal value added  $P^j Y^j$  and labor compensation  $W^j L^j$ . All required data are taken from the EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases. The sample includes the 18 OECD countries mentioned above over the period 1971-2015 (except for Japan: 1975-2015). Table 8 reports empirical estimates that are consistent with  $\epsilon > 0$ . All values are statistically significant at 10%, except for Norway. Overall, we find that  $\epsilon$  ranges from a low of 0.023 for NOR to a high of 2.439 for USA. Since the estimated value for  $\epsilon$  is not statistically significant for Norway, we run the same regression as in eq. (152) but use the output instead of value added to construct  $\hat{\beta}^j$ . We find a value of 0.13, as reported in column 17 of Table 7, and this estimated value is statistically significant.

**Elasticity of substitution between traded and non-traded goods in consumption.** To estimate the elasticity of substitution in consumption,  $\phi$ , between traded and non-traded goods, we derive a testable equation by rearranging the optimal rule for optimal demand for non-traded goods, i.e.,  $C_t^N = (1 - \varphi) \left( \frac{P_t^N}{P_{C,t}} \right)^{-\phi} C_t$ , since time series for consumption in non-traded goods are too short. More specifically, we derive an expression for the non-tradable content of consumption expenditure by using the market clearing condition for non-tradables and construct time series for  $1 - \alpha_{C,t}$  by using time series for non-traded value added and demand components of GDP while keeping the non-tradable content of investment and government expenditure fixed, in line with the

Table 7: Data to Calibrate the Two-Sector Model

Countries	Non-tradable share						Home share				Labor Share		Aggregate ratios			Elasticities				
	GDP (1)	Cons. (2)	Inv. (3)	Gov. (4)	Labor (5)	Lab. comp. (6)	$X^H$ (7)	$C^H$ (8)	$I^H$ (9)	$G^H$ (10)	$LIS^H$ (11)	$LIS^N$ (12)	$LIS$ (13)	$I/Y$ (14)	$G/Y$ (15)	$\phi$ (16)	$\epsilon$ (17)	$\sigma^H$ (18)	$\sigma^N$ (19)	$\phi_X$ (20)
AUS	0.62	0.58	0.76	0.54	0.65	0.64	0.09	0.76	0.49	0.44	0.58	0.67	0.64	0.27	0.18	0.45	0.41	0.55	0.67	1.22
AUT	0.64	0.56	0.60	0.91	0.61	0.61	0.18	0.56	0.42	0.05	0.68	0.68	0.68	0.25	0.18	1.27	1.10	0.94	1.43	1.77
BEL	0.65	0.53	0.63	0.97	0.65	0.63	0.22	0.44	0.20	0.00	0.66	0.67	0.67	0.23	0.22	1.22	0.60	0.74	1.10	1.81
CAN	0.62	0.49	0.66	0.95	0.67	0.65	0.14	0.68	0.31	0.05	0.54	0.63	0.60	0.22	0.21	0.55	0.39	1.09	0.90	2.53
DNK	0.68	0.60	0.73	0.83	0.66	0.67	0.16	0.49	0.22	0.15	0.64	0.70	0.68	0.21	0.25	1.04	0.28	0.57	1.17	n.a.
ESP	0.64	0.59	0.77	0.65	0.60	0.63	0.11	0.73	0.38	0.33	0.60	0.66	0.63	0.24	0.16	1.42	0.95	1.10	0.90	1.93
FIN	0.66	0.57	0.73	0.81	0.59	0.61	0.12	0.67	0.41	0.16	0.64	0.74	0.70	0.25	0.20	0.51	0.42	1.14	0.84	1.62
FRA	0.70	0.57	0.76	0.98	0.64	0.66	0.10	0.71	0.46	0.00	0.72	0.69	0.70	0.23	0.22	0.98	1.39	0.75	0.82	1.67
GBR	0.65	0.58	0.70	0.81	0.66	0.61	0.12	0.66	0.42	0.19	0.69	0.74	0.72	0.20	0.19	0.00	0.61	0.40	0.46	1.54
IRL	0.62	0.50	0.74	0.87	0.58	0.60	0.21	0.48	0.19	0.12	0.50	0.69	0.60	0.22	0.18	1.27	0.09	0.71	0.64	n.a.
ITA	0.64	0.55	0.64	0.98	0.58	0.58	0.09	0.79	0.60	0.01	0.73	0.67	0.70	0.22	0.18	0.31	1.65	0.86	0.50	1.60
JPN	0.67	0.66	0.71	0.61	0.61	0.63	0.04	0.85	0.82	0.39	0.60	0.66	0.64	0.29	0.16	0.88	0.79	0.94	0.90	1.46
KOR	0.56	0.55	0.67	0.31	0.49	0.52	0.09	0.81	0.59	0.67	0.72	0.82	0.77	0.31	0.12	0.59	2.27	0.43	0.80	1.35
NLD	0.66	0.52	0.69	0.97	0.67	0.67	0.18	0.55	0.25	0.00	0.61	0.74	0.69	0.22	0.22	0.83	0.22	1.08	0.61	n.a.
NOR	0.62	0.53	0.63	0.86	0.62	0.64	0.14	0.66	0.49	0.11	0.43	0.63	0.54	0.25	0.20	1.01	0.13	0.42	0.84	1.88
PRT	0.60	0.54	0.71	0.67	0.55	0.58	0.15	0.65	0.33	0.31	0.73	0.61	0.66	0.25	0.16	0.30	0.59	0.53	0.46	2.12
SWE	0.67	0.56	0.60	0.97	0.65	0.65	0.15	0.63	0.38	0.01	0.67	0.74	0.71	0.24	0.25	0.49	0.53	0.65	0.52	1.81
USA	0.67	0.69	0.61	0.63	0.70	0.66	0.06	0.83	0.68	0.37	0.62	0.62	0.62	0.22	0.16	0.78	2.44	0.71	1.10	1.16
OECD	0.64	0.56	0.69	0.80	0.62	0.63	0.13	0.66	0.43	0.18	0.63	0.69	0.66	0.24	0.19	0.77	0.83	0.64	0.80	1.70

Notes: Columns 1-6 show the GDP share of non-tradables, the non-tradable content of consumption, investment and government expenditure, the share of non-tradables in labor, and the non-tradable content of labor compensation. Column 7 gives the ratio of exports of final goods and services to GDP; columns 8 and 9 show the home share of consumption and investment expenditure in tradables and column 10 shows the content of government spending in home-produced traded goods;  $LIS^j$  stands for the labor income share in sector  $j = H, N$  while  $LIS$  refers to the aggregate  $LIS$ ;  $I/Y$  is the investment-to-GDP ratio and  $G/Y$  is government spending as a share of GDP;  $\phi$  is the elasticity of substitution between traded and non-traded goods in consumption;  $\epsilon$  is the elasticity of labor supply across sectors;  $\sigma^j$  is the elasticity of substitution between capital and labor in sector  $j = H, N$ ; estimates of the elasticity of exports w.r.t. terms of trade,  $\phi_X$ , are taken from Imbs and Mejean [2015].

Table 8: Estimates of Elasticity of Labor Supply across Sectors ( $\epsilon$ )

Country	Elasticity of labor supply across Sectors ( $\epsilon$ ), eq. (152)
AUS	0.412 <sup>a</sup> (2.83)
AUT	1.102 <sup>b</sup> (2.49)
BEL	0.602 <sup>a</sup> (2.97)
CAN	0.388 <sup>a</sup> (3.42)
DNK	0.277 <sup>b</sup> (2.05)
ESP	0.948 <sup>a</sup> (3.08)
FIN	0.425 <sup>a</sup> (3.61)
FRA	1.389 <sup>b</sup> (2.36)
GBR	0.610 <sup>a</sup> (3.31)
IRL	0.090 <sup>b</sup> (2.22)
ITA	1.651 <sup>b</sup> (2.53)
JPN	0.793 <sup>a</sup> (2.94)
KOR	2.267 <sup>a</sup> (2.79)
NLD	0.218 <sup>c</sup> (1.73)
NOR	0.023 (0.62)
PRT	0.586 <sup>a</sup> (3.48)
SWE	0.527 <sup>a</sup> (3.53)
USA	2.439 <sup>c</sup> (1.79)
Countries	18
Observations	806
Data coverage	1971-2015
Country fixed effects	yes
Time dummies	yes
Time trend	no

Notes: <sup>a</sup>, <sup>b</sup> and <sup>c</sup> denote significance at 1%, 5% and 10% levels. Heteroskedasticity and autocorrelation consistent t-statistics are reported in parentheses.

evidence documented by Bems [2008] for the share of non-traded goods in investment and building on our own evidence for the non-tradable content of government spending. After verifying that the (logged) share of non-tradables and the (logged) ratio of non-traded prices to the consumption price index are both integrated of order one and cointegrated, we run the regression by adding country and time fixed effects by using a FMOLS estimator. We consider two variants, one including a country-specific time trend and one without the time trend. We provide more details below.

Multiplying both sides of  $C_t^N = (1 - \varphi) \left( \frac{P_t^N}{P_{C,t}} \right)^{-\phi} C_t$  by  $P^N/P_C$  leads to the non-tradable content of consumption expenditure:

$$1 - \alpha_{C,t} = \frac{P_t^N C_t^N}{P_{C,t} C_t} = (1 - \varphi) \left( \frac{P_t^N}{P_{C,t}} \right)^{1-\phi}. \quad (153)$$

Because time series for non-traded consumption display a short time horizon for most of the countries of our sample while data for sectoral value added and GDP demand components are available for all of the countries of our sample over the period running from 1970 to 2015, we construct time series for the share of non-tradables by using the market clearing condition for non-tradables:

$$\frac{P_t^N C_t^N}{P_{C,t} C_t} = \frac{1}{\omega_{C,t}} \left[ \frac{P_t^N Y_t^N}{Y_t} - (1 - \alpha_J) \omega_{J,t} - \omega_{G^N} \omega_{G,t} \right]. \quad (154)$$

Since the time horizon is too short at a disaggregated level (for  $I^j$  and  $G^j$ ) for most of the countries, we draw on the evidence documented by Bems [2008] which reveals that  $1 - \alpha^J = \frac{P^N J^N}{P^J J}$  is constant over time; we further assume that  $\frac{P^N G^N}{G} = \omega_{G^N}$  is constant as well in line with our evidence. We thus recover time series for the share of non-tradables by using time series for the non-traded value added at current prices,  $P_t^N Y_t^N$ , GDP at current prices,  $Y_t$ , consumption expenditure, gross fixed capital formation,  $I_t$ , government spending,  $G_t$  while keeping the non-tradable content of investment and government expenditure,  $1 - \alpha_J$ , and  $\omega_{G^N}$ , fixed.

Once we have constructed time series for  $1 - \alpha_{C,t} = \frac{P_t^N C_t^N}{P_{C,t} C_t}$  by using (154), we take the logarithm of both sides of (153) and run the regression of the logged share of non-tradables on the logged ratio of non-traded prices to the consumption price index:

$$\ln(1 - \alpha_{C,it}) = f_i + f_t + \alpha_i .t + (1 - \phi) \ln(P^N/P_C)_{it} + \mu_{it}, \quad (155)$$

where  $f_i$  captures the country fixed effects,  $f_t$  are time dummies, and  $\mu_{it}$  are the i.i.d. error terms. Because parameter  $\varphi$  in (153) may display a trend over time, we add country-specific trends, as captured by  $\alpha_i .t$ . It is worth mentioning that  $P^N$  is the value added deflator of non-tradables.

Data for non-traded value added at current prices,  $P_t^N Y_t^N$  and GDP at current prices,  $Y_t$ , are taken from EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases (data coverage: 1970-2015 for all countries, except Japan: 1974-2015). To construct time series for consumption, investment and government expenditure as a percentage of nominal GDP, i.e.,  $\omega_{C,t}$ ,  $\omega_{J,t}$  and  $\omega_{G,t}$ , respectively, we use data at current prices obtained from the OECD Economic Outlook [2017] Database (data coverage: 1970-2015). Sources, construction and data coverage of time series for the share of non-tradables in investment ( $1 - \alpha_J$ ) and in government spending ( $\omega_{G^N}$ ) are described in depth in Appendix E (see eq. (117));  $P^N$  is the value added deflator of non-tradables. Data are taken from EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases (data coverage: 1970-2015 for all countries, except for Japan: 1974-2015). Finally, data for the consumer price index  $P_{C,t}$  are obtained from the OECD Prices and Purchasing Power Parities [2017] database (data coverage: 1970-2015).

Since both sides of (155) display trends, we ran unit root and then cointegration tests. Having verified that these two assumptions are empirically supported, we estimate the cointegrating relationships by using the fully modified OLS (FMOLS) procedure for cointegrated panel proposed by Pedroni [2000], [2001]. FMOLS estimates of (155) are reported in Table 9. When we include a country-specific time trend, the vast majority (16 out of 18) of the FMOLS estimated coefficients are positive; yet, twelve out of seventeen are statistically significant, including AUS, AUT, CAN, DNK, ESP, FIN, IRL, ITA, JPN, KOR, NOR, USA. We thus also run the same regression as in eq. (155) by ignoring country-specific time trends. We replace inconsistent (i.e., negative or no statistically significant) estimates for  $\phi$  when adding a country-specific time trend with those obtained when we excluded the country-specific time trend. Except for GBR for which estimates are negative in both cases, one out of the two regressions leads to consistent estimates for the elasticity of substitution. For the countries mentioned below, estimates for  $\phi$  obtained with a time trend are replaced with those when we drop the time trend:  $\phi = 1.221$  ( $t = 1.68$ ) for BEL,  $\phi = 0.978$  ( $t = 2.10$ ) for FRA,  $\phi = 0.826$  ( $t = 6.25$ ) for NLD,  $\phi = 0.299$  ( $t = 2.61$ ) for PRT and  $\phi = 0.487$  ( $t = 2.49$ ) for SWE. For GBR, the estimated value is negative whether there is a time trend in the regression or not and thus we set  $\phi$  to zero for the rest of the analysis for this country. Table 9 shows estimates for  $\phi$  for each country. All values are statistically significant at 10%. Overall, we find that  $\phi$  ranges from a low of 0.299 for PRT to a high of 1.417 for ESP.

Table 9: Elasticity of Substitution between Tradables and Non-Tradables ( $\phi$ )

Country	eq. (155)	Time trend
AUS	0.447 <sup>b</sup> (2.36)	yes
AUT	1.275 <sup>a</sup> (5.60)	yes
BEL	1.221 <sup>c</sup> (1.68)	no
CAN	0.546 <sup>a</sup> (3.71)	yes
DNK	1.039 <sup>a</sup> (2.72)	yes
ESP	1.417 <sup>b</sup> (2.54)	yes
FIN	0.509 <sup>a</sup> (2.82)	yes
FRA	0.978 <sup>b</sup> (2.10)	yes
GBR	0	
IRL	1.273 <sup>a</sup> (3.55)	yes
ITA	0.314 <sup>a</sup> (2.68)	yes
JPN	0.884 <sup>a</sup> (4.01)	yes
KOR	0.592 <sup>b</sup> (2.15)	yes
NLD	0.826 <sup>a</sup> (6.25)	no
NOR	1.006 <sup>a</sup> (4.72)	yes
PRT	0.299 <sup>a</sup> (2.61)	no
SWE	0.487 <sup>b</sup> (2.49)	no
USA	0.777 <sup>a</sup> (3.32)	yes
Countries	18	
Observations	824	
Data coverage	1970-2015	
Country fixed effects	yes	
Time dummies	yes	

Notes: <sup>a</sup>, <sup>b</sup> and <sup>c</sup> denote significance at 1%, 5% and 10% levels. Heteroskedasticity and autocorrelation consistent t-statistics are reported in parentheses.



### J.3 Estimates of $\sigma^j$ : Empirical strategy

To estimate the elasticity of substitution between capital and labor,  $\sigma^j$ , we draw on Antràs [2004]. We let labor- and capital-augmenting technological change grow at a constant rate:

$$A_t^j = A_0^j e^{a^j t}, \quad (156a)$$

$$B_t^j = B_0^j e^{b^j t}, \quad (156b)$$

where  $a^j$  and  $b^j$  denote the constant growth rate of labor- and capital-augmenting technical progress and  $A_0^j$  and  $B_0^j$  are initial levels of technology. Inserting first (156a) and (156b) into the demand for labor and capital, taking logarithm and rearranging gives:

$$\ln(Y_t^j/L_t^j) = \alpha_1 + (1 - \sigma^j) a^j t + \sigma_j \ln(W_t^j/P_t^j), \quad (157a)$$

$$\ln(Y_t^j/K_t^j) = \alpha_2 + (1 - \sigma^j) b^j t + \sigma_j \ln(R_t/P_t^j), \quad (157b)$$

where  $\alpha_1 = [(1 - \sigma^j) \ln A_0^j - \sigma^j \ln \gamma^j]$  and  $\alpha_2 = [(1 - \sigma^j) \ln B_0^j - \sigma^j \ln(1 - \gamma^j)]$  are constants. Above equations describe firms' demand for labor and capital respectively.

We estimate the elasticity of substitution between capital and labor in sector  $j = H, N$  from first-order conditions (157a)-(157b) in panel format on annual data. Adding an error term and controlling for country fixed effects, we explore empirically the following equations:

$$\ln(Y_{it}^j/L_{it}^j) = \alpha_{1i} + \lambda_{1i}t + \sigma_i^j \ln(W_{it}^j/P_{it}^j) + u_{it}, \quad (158a)$$

$$\ln(Y_{it}^j/K_{it}^j) = \alpha_{2i} + \lambda_{2i}t + \sigma_i^j \ln(R_{it}/P_{it}^j) + v_{it}, \quad (158b)$$

where  $i$  and  $t$  index country and time and  $u_{it}$  and  $v_{it}$  are i.i.d. error terms. Country fixed effects are represented by dummies  $\alpha_{1i}$  and  $\alpha_{2i}$ , and country-specific trends are captured by  $\lambda_{1i}$  and  $\lambda_{2i}$ . Since all variables display unit root process, we estimate cointegrating relationships by using the fully modified OLS (FMOLS) procedure for cointegrated panel proposed by Pedroni [2000].

Estimation of (158a) and (158b) requires data for each sector  $j = H, N$  on sectoral value added at constant prices  $Y^j$ , sectoral hours worked  $L^j$ , sectoral capital stock  $K^j$ , sectoral value added deflator  $P^j$ , sectoral wage rate  $W^j$  and capital rental cost  $R$ . Data for sectoral value added  $Y^H$  and  $Y^N$ , hours worked  $L^H$  and  $L^N$ , value added price deflators  $P^H$  and  $P^N$ , and, nominal wages  $W^H$  and  $W^N$  are taken from the EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases. To construct the national stock of capital  $K$ , we use the perpetual inventory method with a fixed depreciation rate taken from Table 6 and the time series of constant prices investment from the OECD Economic Outlook [2017] Database. Next, following Garofalo and Yamarik [2002], the capital stock is allocated to traded and non-traded industries by using sectoral output shares. Finally, we measure the aggregate rental price of capital  $R$  as the ratio of capital income to capital stock. Capital income is derived as nominal value added minus labor compensation. For all aforementioned variables, the sample includes the 18 OECD countries over the period 1970-2015 (except for Japan: 1974-2015).

While we take the demand for labor as our baseline model (i.e. eq. (158a)), Table 10 provides FMOLS estimates of  $\sigma^j$  for the demand of labor and capital. The bulk (32 out of 36) of the FMOLS estimated coefficients from eq. (158a) are positive and statistically significant. One estimated coefficient is negative ( $\sigma^H$  for IRL) while estimates of  $\sigma^H$  for Finland and Portugal and  $\sigma^N$  for Japan are positive but not statistically significant. To deal with this issue, we run again the same regression by dropping time dummies which gives consistent estimates for  $\sigma^H$  for Finland, Portugal and for  $\sigma^N$  for Japan. However, the estimate for  $\sigma^H$  is still negative for Ireland. As in Antràs [2004], we alternatively run the regression of the ratio of value added to capital stock at constant prices on the real capital cost  $R/P^j$  in sector  $j$ , i.e., eq. (158b). We then replace inconsistent estimates for  $\sigma^j$  obtained from labor demand with those obtained from the demand of capital. Columns 19-20 of Table 7 report estimates for  $\sigma^H$  and  $\sigma^N$ .

## K Numerical Decomposition of Government Spending Multiplier on Non-Tradables

In this section, we calibrate the model to country-specific data and compute numerically the government spending multiplier on non-traded value added and hours worked, and next we detail the steps of the decomposition of the changes in the value added and labor share of non-tradables in order to compute numerically the contribution of technology to their responses.



Table 10: FMOLS Estimates of the Sectoral Elasticity of Substitution between Capital and Labor ( $\sigma^j$ )

Country	Tradables ( $\sigma^H$ )			Non-Tradables ( $\sigma^N$ )		
	ln( $Y^H/K^H$ )	ln( $Y^H/L^H$ )	ln( $Y^H/L^H$ )	ln( $Y^N/K^N$ )	ln( $Y^N/L^N$ )	ln( $Y^N/L^N$ )
Dependent var.	ln( $R/P^H$ )	ln( $W^H/P^H$ )	ln( $W^H/P^H$ )	ln( $R/P^N$ )	ln( $W^N/P^N$ )	ln( $W^N/P^N$ )
Explanatory var.	ln( $R/P^H$ )	ln( $W^H/P^H$ )	ln( $W^H/P^H$ )	ln( $R/P^N$ )	ln( $W^N/P^N$ )	ln( $W^N/P^N$ )
AUS	0.339 <sup>a</sup> (2.62)	0.550 <sup>a</sup> (7.19)	0.417 <sup>a</sup> (3.34)	0.490 <sup>a</sup> (3.38)	0.669 <sup>a</sup> (12.19)	0.522 <sup>a</sup> (5.64)
AUT	0.511 <sup>a</sup> (3.62)	0.936 <sup>a</sup> (15.78)	0.909 <sup>a</sup> (6.36)	0.004 (0.02)	1.432 <sup>a</sup> (9.48)	1.341 <sup>a</sup> (12.82)
BEL	0.079 (0.44)	0.739 <sup>a</sup> (9.51)	0.839 <sup>a</sup> (10.12)	-0.067 (-0.83)	1.098 <sup>a</sup> (8.42)	1.113 <sup>a</sup> (7.42)
CAN	-0.028 (-0.20)	1.091 <sup>a</sup> (5.66)	0.510 <sup>a</sup> (3.13)	0.867 <sup>a</sup> (5.88)	0.902 <sup>a</sup> (12.28)	0.669 <sup>a</sup> (7.90)
DNK	-0.063 (-0.46)	0.574 <sup>a</sup> (5.04)	0.447 <sup>a</sup> (5.05)	0.407 <sup>a</sup> (7.13)	1.168 <sup>a</sup> (6.64)	1.253 <sup>a</sup> (7.10)
ESP	0.410 <sup>a</sup> (3.63)	1.098 <sup>a</sup> (10.43)	0.996 <sup>a</sup> (11.09)	0.272 <sup>b</sup> (2.07)	0.900 <sup>a</sup> (4.54)	0.485 <sup>a</sup> (3.27)
FIN	0.105 (0.60)	0.100 (0.34)	1.141 <sup>a</sup> (3.24)	0.409 <sup>a</sup> (6.51)	0.841 <sup>a</sup> (8.36)	0.847 <sup>a</sup> (8.77)
FRA	0.103 (0.73)	0.753 <sup>a</sup> (7.55)	1.003 <sup>a</sup> (5.94)	0.095 <sup>a</sup> (2.73)	0.815 <sup>a</sup> (3.99)	0.910 <sup>a</sup> (3.76)
GBR	-0.041 (-0.24)	0.398 <sup>a</sup> (5.32)	0.617 <sup>a</sup> (8.04)	0.126 (1.24)	0.464 <sup>a</sup> (3.49)	0.644 <sup>a</sup> (3.33)
IRL	0.714 <sup>a</sup> (11.15)	-0.017 (-0.09)	-0.130 (-0.71)	0.635 <sup>a</sup> (3.97)	0.983 <sup>a</sup> (4.32)	0.609 <sup>a</sup> (2.67)
ITA	0.512 <sup>a</sup> (2.65)	0.860 <sup>a</sup> (11.90)	0.892 <sup>a</sup> (9.28)	0.333 <sup>b</sup> (2.08)	0.503 <sup>a</sup> (4.15)	0.246 (1.37)
JPN	0.793 <sup>a</sup> (7.69)	0.936 <sup>a</sup> (4.43)	1.202 <sup>a</sup> (7.82)	0.502 <sup>a</sup> (6.13)	0.290 (1.43)	0.899 <sup>a</sup> (3.63)
KOR	0.279 <sup>a</sup> (5.45)	0.432 <sup>a</sup> (5.85)	0.636 <sup>a</sup> (7.57)	0.513 <sup>a</sup> (12.42)	0.795 <sup>a</sup> (6.83)	0.827 <sup>a</sup> (6.85)
NLD	0.436 <sup>a</sup> (4.85)	1.075 <sup>a</sup> (9.41)	0.970 <sup>a</sup> (6.70)	0.170 <sup>a</sup> (7.74)	0.610 <sup>a</sup> (6.62)	0.422 <sup>a</sup> (3.38)
NOR	0.797 <sup>a</sup> (4.36)	0.421 <sup>a</sup> (3.07)	0.664 <sup>a</sup> (3.98)	0.674 <sup>a</sup> (12.70)	0.840 <sup>a</sup> (6.57)	0.588 <sup>a</sup> (5.24)
PRT	-0.066 <sup>b</sup> (-2.14)	0.175 (1.31)	0.525 <sup>b</sup> (2.52)	0.308 (1.19)	0.455 <sup>a</sup> (9.02)	0.495 <sup>a</sup> (10.58)
SWE	0.086 (0.34)	0.648 <sup>a</sup> (14.52)	0.604 <sup>a</sup> (8.36)	0.054 (0.61)	0.519 <sup>a</sup> (4.54)	0.116 (0.53)
USA	-0.081 (-0.58)	0.706 <sup>a</sup> (3.51)	0.758 <sup>a</sup> (10.31)	0.400 <sup>a</sup> (5.47)	1.098 <sup>a</sup> (7.90)	0.854 <sup>a</sup> (4.83)
Whole sample	0.271 <sup>a</sup> (10.49)	0.638 <sup>a</sup> (28.46)	0.722 <sup>a</sup> (26.44)	0.344 <sup>a</sup> (18.96)	0.799 <sup>a</sup> (28.46)	0.713 <sup>a</sup> (23.36)
Countries	18	18	18	18	18	18
Observations	824	824	824	824	824	824
Fixed effects	yes	yes	yes	yes	yes	yes
Time dummies	yes	yes	no	yes	yes	no
Time trend	yes	yes	yes	yes	yes	yes

Notes: <sup>a</sup>, <sup>b</sup> and <sup>c</sup> denote significance at 1%, 5% and 10% levels. Heteroskedasticity and autocorrelation consistent t-statistics are reported in parentheses. Data coverage: 1970-2015.

## K.1 Government Spending Multiplier on Non-Traded Value Added

A rise in government spending generates a deviation of sectoral value added relative to its initial steady-state value in percentage by  $\hat{Y}^j(t)$ . Adding and subtracting the percentage deviation of real GDP relative to its initial steady-state, i.e.,  $\hat{Y}^j(t) = \hat{Y}_R(t) + (\hat{Y}^j(t) - \hat{Y}_R(t))$ , and multiplying both sides by  $\nu^{Y,j}$  allows us to decompose the change in sectoral value added as follows:

$$\nu^{Y,j} \hat{Y}^j(t) = \nu^{Y,j} \hat{Y}_R(t) + d\nu^{Y,j}(t), \quad (159)$$

where  $d\nu^{Y,j}(t) = \nu^{Y,j} (\hat{Y}^j(t) - \hat{Y}_R(t))$  is the change in the value added share of sector  $j$ . According to the first term on the RHS of eq. (159), each sector receives a share  $\nu^{Y,j}$  of the percentage deviation of real GDP relative to its initial steady-state. Because the value added share of non-tradables is 64% on average across OECD countries, the non-traded sector receives a higher fraction of real GDP growth. If the intensity of sector  $j$  in the government spending shock is higher than its value added share,  $\nu^{Y,j}$ , then  $d\nu^{Y,j}(t) > 0$  which further increases value added at constant prices. Conversely, if the TFP of sector  $j$  falls relative to the TFP of the other sector, then the TFP differential exerts a negative impact on  $d\nu^{Y,j}(t)$ .

We have calibrated the open economy model with CES production functions and FBTC to country-specific data and we have computed numerically the change in non-traded value added at constant prices and the change in the value added share of non-tradables following an exogenous and temporary increase in government consumption by 1% of GDP. When we calibrate the open economy model to country-specific data, we shut-down the capital utilization and technology utilization rate as point estimates from estimation of the VAR model at a country level are not accurate enough to enable us to calibrate the capital and technology utilization rate and thus to generate the dynamic path of sectoral TFP we estimate empirically. We thus assume that sectoral TFP is governed by an exogenous process and we generate a dynamic adjustment of sectoral TFP which matches empirical responses. More specifically, we assume that labor- and capital-augmenting efficiency is governed by the law of motion given by eq. (42a) and eq. (42b), respectively. To pin down the value of parameters  $a^j$ ,  $b^j$ ,  $\xi_A^j$ ,  $\xi_B^j$ ,  $\chi_A^j$ ,  $\chi_B^j$ , we have to uncover the dynamic process underlying factor-augmenting productivity by adopting the same procedure as in the subsection 4.2:

$$\hat{A}^j(t) = \text{TFP}^j(t) - (1 - s_L^j) \left[ \left( \frac{\sigma^j}{1 - \sigma^j} \right) \hat{S}^j(t) - \hat{k}^j(t) \right], \quad (160a)$$

$$\hat{B}^j(t) = \text{TFP}^j(t) + s_L^j \left[ \left( \frac{\sigma^j}{1 - \sigma^j} \right) \hat{S}^j(t) - \hat{k}^j(t) \right], \quad (160b)$$

where we abstract from the capital utilization rate. We plug country-specific estimates of  $\sigma^j$  and country-specific estimated responses for  $s_L^j(t)$ ,  $k^j(t)$ ,  $\text{TFP}^j(t)$  into the above equations to recover the dynamics for  $A^j(t)$  and  $B^j(t)$ . Impact estimates, i.e., setting  $t = 0$  into (160a) and (160b), enable us to pin down the values for  $a^j$  and  $b^j$  since  $\hat{A}^j(0) = a^j$  and  $\hat{B}^j(0) = b^j$  (set  $t = 0$  into (42a)-(42b)). Next, we choose  $\xi_A^j$ ,  $\xi_B^j$ ,  $\chi_A^j$ ,  $\chi_B^j$  to match empirical IRFs (160a)-(160b). We also calibrate the endogenous response of government consumption to an exogenous temporary increase in government consumption by 1% of GDP by adopting the same procedure as in subsection 4.2 but for one country at a time. We also calibrate the model to a representative OECD country. We assume that the process of capital utilization-adjusted sectoral TFP is exogenous and allow for endogenous capital utilization since our estimates are accurate enough to generate a dynamic adjustment of sectoral TFP which matches what we estimate empirically.

We have computed numerically the responses of real GDP, non-traded value added, value added share of non-tradables (at constant prices) to a temporary increase in government consumption by 1% of GDP and then calculate government spending multipliers. Columns 1-4 of Table 11 show numerical results when we simulate the baseline model with CES production functions and technological change. While column 5 shows the response of the TFP differential between tradables and non-tradables to a shock to government consumption, columns 6-7 show the excess of the government spending multiplier on real GDP

and on non-traded value added in the baseline model over a model with Cobb-Douglas production functions where technological change is shut down. Column 8 shows the excess of the change in the value added share of non-tradables in the baseline model over the restricted model. Column 9 shows the contribution of the excess of  $d\nu^{Y,N}(t)$  to the excess of government spending multiplier on non-tradables  $\nu^{Y,N}\hat{Y}^N(t)$ .

The objective of our quantitative analysis is threefold:

- First, we compute numerically the government spending multiplier on real GDP and on non-traded value added at constant prices in a model where production functions are CES and sectoral TFP together with FBTC are time-varying. More specifically, we calculate the government spending multiplier as the ratio of the present discounted value of the cumulative change of value added to the present discounted of the cumulative change of government consumption over a six-year horizon:

$$\text{Government Spending Multiplier on Real GDP} = \frac{\int_0^t \hat{Y}_R(\tau) e^{-r^*\tau} d\tau}{\omega_G \int_0^t \hat{G}(\tau) e^{-r^*\tau} d\tau}, \quad (161a)$$

$$\text{Government Spending Multiplier on } Y^N = \frac{\nu^{Y,N} \int_0^t \hat{Y}^N(\tau) e^{-r^*\tau} d\tau}{\omega_G \int_0^t \hat{G}(\tau) e^{-r^*\tau} d\tau}. \quad (161b)$$

Columns 1-2 of Table 11 show the government spending multiplier on real GDP and on non-traded value added.

- Second, we compute numerically the change in the value added share of non-tradables:

$$\text{Average Change in Value Added Share} = \frac{\int_0^t d\nu^{Y,N}(\tau) e^{-r^*\tau} d\tau}{\omega_G \int_0^t \hat{G}(\tau) e^{-r^*\tau} d\tau}. \quad (162)$$

Column 3 of Table 11 shows the change in the value added share of non-tradables. Column 4 shows the contribution of  $d\nu^{Y,N}(t)$  to the government spending multiplier on non-tradables  $\nu^{Y,N}\hat{Y}^N(t)$ . We calculate the contribution of  $d\nu^{Y,N}(t)$  to  $\nu^{Y,N}\hat{Y}^N(t)$  as follows:

$$\text{Contribution of } d\nu^{Y,N}(t) = \frac{d\nu^{Y,N}(t)}{|\nu^{Y,N}\hat{Y}_R(t)| + |d\nu^{Y,N}(t)|}. \quad (163)$$

When we calculate the mean of the contribution of the change in the value added share of non-tradables to the government spending multiplier on non-traded value added, we use the absolute value of  $d\nu^{Y,N}(t)$ , i.e.,  $\text{mean} = \frac{1}{18} \sum_1^{18} \frac{|d\nu^{Y,N}(t)|}{|\nu^{Y,N}\hat{Y}_R(t)| + |d\nu^{Y,N}(t)|}$  where 18 is the number of OECD countries in our sample.

- Third, we compute numerically the government spending multiplier on real GDP, i.e., eq. (161a), on non-traded value added, i.e., eq. (161b), and the change in the value added share of non-tradables, i.e., eq. (162), in the baseline model and in a restricted version of the baseline model where we consider Cobb-Douglas production functions and we keep sectoral TFPs fixed. Columns 6-8 of Table 11 show the excess of the government spending multiplier on real GDP and on non-traded value added in the baseline model over a model with Cobb-Douglas production functions and shutting down technological change. Column 8 shows the excess of the change in the value added share of non-tradables in the baseline model over the restricted model. Column 9 shows the contribution of the excess of  $d\nu^{Y,N}(t)$  to the excess of government spending multiplier on non-tradables  $\nu^{Y,N}\hat{Y}^N(t)$ .

Column 1 of Table 11 shows the average value of the government spending multiplier on real GDP over a six-year period. The first eighteen rows show  $\hat{Y}_R(t)$  for the eighteen countries of our sample while the row 'OECD' shows the value when we calibrate the model to a representative OECD economy. The row 'Mean' shows the average value across the eighteen countries of our sample. Overall, the government spending multiplier averages one the first six years. If the demand shock were evenly distributed across sectors and if sectoral TFPs were fixed, then the non-traded sector should receive a fraction of the rise

in real GDP equivalent to its share in real GDP equal to 64%. As shown in column 2 of Table 11, the government spending multiplier on non-traded value added is equal to 0.6% of GDP instead of 0.65% since as can be seen in column 3, the value added share of non-tradables falls by 0.05% of GDP as a result of a TFP differential between tradables and non-tradables of 0.12%, as displayed by column 5. However, the change in the value added share of non-tradables is very small, if not insignificant following an increase in government consumption by 1% of GDP. The rationale behind this result is that rise in traded TFP relative to non-traded offsets the effect of the biasedness of the government spending shock toward non-tradables.

Column 6 of Table 11 shows the excess of the government spending multiplier on real GDP caused by technological change, i.e., we calculate the difference between the government spending multiplier in the baseline model with CES production functions and technological change and the government spending multiplier in the restricted model with no technological change. On average, technological change increases the government spending multiplier by 0.64% of GDP which in turn raises the size of the government spending multiplier on non-traded value added. However, as can be seen in column 7, the non-traded sector receives a fraction of the rise in real GDP which is smaller than its value added share since government spending multiplier increases by 0.18% of GDP only. The reason to this is that traded TFP increases relative to non-traded TFP which lowers the value added share of non-tradable compared with a model keeping sectoral TFP fixed. As can be seen in column 8, the value added share of non-tradables falls by 0.22% of GDP. On average, the excess of the change (driven by technological change) in the value added share of non-tradables contributes 41% to the excess (driven by technological change) of the government spending multiplier on non-traded value added over a model shutting down technological change.

The change in the value added share of non-tradables (at constant prices) in percentage point of GDP at time  $t$  is (see eq. (61)):

$$d\nu^{Y,N}(t) = (1 - \nu^{Y,H}) \left( \hat{Y}^N(t) - \hat{Y}^R(t) \right). \quad (164)$$

Plugging the log-linearized sectoral decomposition of real GDP, i.e.,  $\hat{Y}^R(t) = \nu^{Y,H} \hat{Y}^H(t) + \nu^{Y,N} \hat{Y}^N(t)$ , given by eq. (59), into (164) allows us to rewrite the change in the value added share of non-tradables as a function of the value added growth differential between non-tradables and tradables:

$$d\nu^{Y,N}(t) = (1 - \nu^{Y,H}) \nu^{Y,H} \left( \hat{Y}^N(t) - \hat{Y}^H(t) \right). \quad (165)$$

Plugging the log-linearized version of the CES production function, i.e.,  $\hat{Y}^j(t) = \text{TFP}^j(t) + \hat{L}^j(t) + (1 - s_L^j) \hat{k}^j(t)$ , given by eq. (69a), into (165) leads to the decomposition of the change in the value added share of non-tradables into three components:

$$\begin{aligned} d\nu^{Y,N}(t) = & \underbrace{-(1 - \nu^{Y,H}) \nu^{Y,H} \left( \text{TFP}^H(t) - \text{TFP}^N(t) \right)}_{\text{TFP diff}} + \underbrace{(1 - \nu^{Y,H}) \nu^{Y,H} \left( \hat{L}^N(t) - \hat{L}^H(t) \right)}_{\text{Lab diff}} \\ & + \underbrace{(1 - \nu^{Y,H}) \nu^{Y,H} \left[ (1 - s_L^N) \hat{k}^N(t) - (1 - s_L^H) \hat{k}^H(t) \right]}_{\text{Cap diff}}. \end{aligned} \quad (166)$$

**Eq. (166) corresponds to eq. (55) in the main text.** The first term on the RHS of eq. (166) measures the change in  $\nu^{Y,N}(t)$  driven by the the TFP differential between tradables and non-tradables. The second and third term on the RHS of eq. (166) captures the contribution of labor and capital reallocation to the change in the value added share of non-tradables.

## K.2 Government Spending Multiplier on Non-Traded Hours Worked

A rise in government spending generates a deviation of sectoral hours worked relative to its initial steady-state value in percentage by  $\hat{L}^j(t)$ . Adding and subtracting the change

Table 11: Numerically Computed Values of Government Spending Multiplier on Non-Traded Value Added

Country	Baseline Model			Excess Baseline Model over Restricted model					
	$\hat{Y}_R(t)$ (1)	$\nu^{Y,N}\hat{Y}^N(t)$ (2)	$d\nu^{Y,N}(t)$ (3)	Contribution (4)	TFP diff (5)	$\hat{Y}_R(t)$ (6)	$\nu^{Y,N}\hat{Y}^N(t)$ (7)	$d\nu^{Y,N}(t)$ (8)	Contribution (9)
AUS	2.54	1.58	0.00	0.00	0.47	2.19	1.20	-0.16	-0.11
AUT	2.30	0.92	-0.54	-0.27	0.45	1.92	0.49	-0.72	-0.37
BEL	-0.56	0.15	0.51	0.58	0.05	-0.93	-0.26	0.34	0.36
CAN	-1.60	-0.48	0.52	0.34	-0.29	-1.95	-0.91	0.30	0.20
DNK	-0.28	-0.45	-0.26	-0.58	0.14	-0.70	-0.86	-0.38	-0.44
ESP	4.16	2.68	0.02	0.01	-0.25	3.84	2.33	-0.13	-0.05
FIN	0.96	0.53	-0.11	-0.15	0.00	0.50	0.06	-0.27	-0.45
FRA	-1.35	0.40	1.35	0.59	-0.93	-1.80	-0.06	1.20	0.49
GBR	0.49	0.53	0.22	0.41	0.09	0.03	0.04	0.02	0.55
IRL	1.68	0.49	-0.55	-0.35	0.32	1.32	0.11	-0.70	-0.46
ITA	2.70	1.28	-0.46	-0.21	0.76	2.30	0.82	-0.67	-0.31
JPN	4.34	0.32	-2.55	-0.47	1.36	4.00	-0.02	-2.67	-0.50
KOR	-2.30	1.09	2.36	0.65	-2.41	-2.74	0.60	2.10	0.59
NLD	-0.50	-0.55	-0.22	-0.40	-0.05	-0.93	-0.98	-0.37	-0.38
NOR	4.51	0.75	-1.99	-0.43	2.30	4.19	0.36	-2.19	-0.47
PRT	3.01	1.07	-0.74	-0.29	1.24	2.65	0.66	-0.94	-0.37
SWE	-2.03	0.43	1.79	0.57	-1.60	-2.50	-0.07	1.62	0.49
USA	0.44	0.01	-0.28	-0.49	0.48	0.14	-0.33	-0.42	-0.82
OECD	1.13	0.55	-0.18	-0.20	0.22	0.74	0.11	-0.37	-0.44
Mean	1.03	0.60	-0.05	-0.03	0.12	0.64	0.18	-0.22	0.41
TFP diff < 0					-0.92	-1.01	0.15	0.79	
TFP diff > 0					0.64	1.47	0.19	-0.73	

Notes: Columns 1-4 show numerical results when we simulate the baseline model with CES production functions and technological change. Columns 1-2 show the government spending multiplier on real GDP and on non-traded value added. Column 3 shows the change in the value added share of non-tradables. Column 4 shows the contribution of  $d\nu^{Y,N}(t)$  to the government spending multiplier on non-tradables  $\nu^{Y,N}\hat{Y}^N(t)$ . Column 5 shows the response of the TFP differential between tradables and non-tradables to a shock to government consumption. Columns 6-7 show the excess of the government spending multiplier on real GDP and on non-traded value added in the baseline model over a model with Cobb-Douglas production functions and shutting down technological change. Column 8 shows the excess of the change in the value added share of non-tradables in the baseline model over the restricted model. Column 9 shows the contribution of the excess of  $d\nu^{Y,N}(t)$  to the excess of government spending multiplier on non-tradables  $\nu^{Y,N}\hat{Y}^N(t)$ .

in total hours worked, i.e.,  $\hat{L}^j(t) = \hat{L}(t) + (\hat{L}^j(t) - \hat{L}(t))$ , and multiplying both sides by the labor compensation share of sector  $j$ , i.e.,  $\alpha_L^j$ , allows us to decompose the change in sectoral hours worked as follows:

$$\alpha_L^j \hat{L}^j(t) = \alpha_L^j \hat{L}(t) + d\nu^{L,j}(t), \quad (167)$$

where  $d\nu^{L,j}(t) = \alpha_L^j (\hat{L}^j(t) - \hat{L}(t))$  is the change in the labor share of sector  $j$ . According to (167), each sector receives a fraction  $\alpha_L^j$  of the rise in total hours worked as captured by the first term on the RHS of eq. (167). If the demand shock is biased toward non-tradables and/or technological change is more biased toward labor in the non-traded than in the traded sector, then labor shifts toward the non-traded sector, i.e.,  $d\nu^{L,N}(t) > 0$ , as captured by the second term on the RHS of eq. (167), which further increases the response of non-traded hours worked, i.e.,  $\alpha_L^N \hat{L}^N(t)$ .

We have computed numerically the responses of total hours worked, non-traded hours worked, the labor share of non-tradables to a temporary increase in government consumption by 1% of GDP and then we have calculated the government spending multiplier on hours worked. Columns 1-4 of Table 12 show numerical results when we simulate the baseline model with CES production functions and technological change. While column 5 shows the response of the FBTC differential between non-tradables and tradables (where sectoral FBTC is scaled by the capital income share) to a shock to government consumption, columns 6-7 show the excess of the government spending multiplier on total hours worked and on non-traded hours worked in the baseline model over a model with Cobb-Douglas production functions with time-varying sectoral TFP where technological change is Hicks-neutral instead of being factor-biased. Column 8 shows the excess of the change in the labor share of non-tradables in the baseline model over the restricted model. Column 9 shows the contribution of the excess of  $d\nu^{L,N}(t)$  to the excess of government spending multiplier on non-traded hours worked  $\alpha_L^N \hat{L}^N(t)$ .

The objective of our quantitative analysis is threefold:

- First, we compute numerically the government spending multiplier on total hours worked and on non-traded hours worked in a model where production functions are CES and sectoral TFP together with FBTC are time-varying. More specifically, we calculate the government spending multiplier as the ratio of the present discounted value of the cumulative change of hours worked to the present discounted of the cumulative change of government consumption over a six-year horizon:

$$\text{Government Spending Multiplier on } L: = \frac{\int_0^t \hat{L}(\tau) e^{-r^* \tau} d\tau}{\omega_G \int_0^t \hat{G}(\tau) e^{-r^* \tau} d\tau}, \quad (168a)$$

$$\text{Government Spending Multiplier on } L^N: = \frac{\alpha_L^N \int_0^t \hat{L}^N(\tau) e^{-r^* \tau} d\tau}{\omega_G \int_0^t \hat{G}(\tau) e^{-r^* \tau} d\tau}. \quad (168b)$$

Columns 1-2 of Table 12 show the government spending multiplier on total and non-traded hours worked.

- Second, we compute numerically the change in the labor share of non-tradables:

$$\text{Average Change in Labor Share:} = \frac{\int_0^t d\nu^{L,N}(\tau) e^{-r^* \tau} d\tau}{\omega_G \int_0^t \hat{G}(\tau) e^{-r^* \tau} d\tau}. \quad (169)$$

Column 3 of Table 12 shows the change in the labor share of non-tradables. Column 4 shows the contribution of  $d\nu^{L,N}(t)$  to the government spending multiplier on non-traded hours worked  $\alpha_L^N \hat{L}^N(t)$ . We calculate the contribution of  $d\nu^{L,N}(t)$  to  $\alpha_L^N \hat{L}^N(t)$  as follows:

$$\text{Contribution of } d\nu^{L,N}(t): = \frac{d\nu^{L,N}(t)}{|\alpha_L^N \hat{L}(t)| + |d\nu^{L,N}(t)|}. \quad (170)$$

When we calculate the mean of the contribution of the change in the labor share of non-tradables to the government spending multiplier on non-traded hours worked, we



use the absolute value of  $d\nu^{L,N}(t)$ , i.e.,  $\text{mean} = \frac{1}{18} \sum_1^{18} \frac{|d\nu^{L,N}(t)|}{|\alpha_L^N \hat{L}(t)| + |d\nu^{L,N}(t)|}$  where 18 is the number of OECD countries in our sample.

- Third, we compute numerically the government spending on total hours worked, i.e., eq. (168a), non-traded hours worked, i.e., eq. (168b), and the change in the labor share of non-tradables, i.e., eq. (169), in the baseline model and in a restricted version of the baseline model where we consider Cobb-Douglas production functions where we allow for time-varying sector TFPs with Hicks-neutral technological change (no FBTC). Columns 6-8 of Table 12 show the excess of the government spending multiplier on total hours worked and on non-traded hours worked in the baseline model over a model with Cobb-Douglas production functions with no FBTC. Column 8 shows the excess of the change in the labor share of non-tradables in the baseline model over the restricted model. Column 9 shows the contribution of the excess of  $d\nu^{L,N}(t)$  to the excess of government spending multiplier on non-tradables  $\alpha_L^N \hat{L}^N(t)$ .

Column 1 of Table 12 shows the average value of the government spending multiplier on total hours worked over a six-year period. The first eighteen rows show  $\hat{L}(t)$  for the eighteen countries of our sample while the row 'OECD' shows the value when we calibrate the model to a representative OECD economy. The row 'Mean' shows the average value across the eighteen countries of our sample. Overall, the government spending multiplier on total hours worked averages 0.68 the first six years, i.e., an increase in government consumption by 1% of GDP raises total hours worked by 0.68% per year on average. If the demand shock were evenly distributed across sectors and if FBTC were unresponsive to the government spending shock, then the non-traded sector should receive a fraction of the rise in total hours worked equivalent to its share in labor compensation equal to 66%. A back of the envelope calculation gives a rise in non-traded hours worked by 0.45% of total hours worked. As shown in column 2 of Table 12, the government spending multiplier on non-traded hours worked is equal to 0.55% of total hours worked instead of 0.43% since as can be seen in column 3, the labor share of non-tradables increases by 0.10% of GDP as a result of a FBTC differential between non-tradables and tradables of 0.08%, as displayed by column 5. Intuitively, because technological change is biased toward labor in the non-traded sector and biased toward capital in the traded sector, the shock to government consumption increases labor demand in the non-traded sector which strengthens the impact of the biasedness of the demand shock toward non-tradables on the labor share of non-tradables.

Column 6 of Table 12 shows the excess of the government spending multiplier on total hours worked caused by FBTC, i.e., we calculate the difference between the government spending multiplier in the baseline model with CES production functions and technological change and the government spending multiplier in the restricted model with Cobb-Douglas production functions and Hicks-neutral technological change. If we consider the cross-country average, FBTC only slightly further increases total hours worked (i.e., by 0.02%) while it somewhat amplifies the rise in non-traded hours worked, i.e., by 0.05% of total hours worked (see columns 7 and 8). When we differentiate between countries with a negative and a positive FBTC differential between non-tradables and tradables, the conclusion is different. For countries where technological change is more biased toward labor in the non-traded than in the traded sector (i.e., as shown in the last row), the rise in total hours worked is amplified by 0.24% (see column 6) compared with a model shutting down FBTC. The government spending multiplier on non-traded hours worked is increased by 0.36% of total hours worked (see column 7) because the non-traded sector receives a fraction  $\alpha_L^N$  of  $\hat{L}(t)$  and also experiences a labor inflow caused by the rise in labor demand in the non-traded sector triggered by FBTC. More specifically, in countries where the FBTC differential between non-tradables and tradables is positive, the labor share of non-tradables rises by 0.19% (see column 8). For these economies, the rise in  $\nu^{L,N}(t)$  accounts for 32% (see column 9) of the amplification effect caused by FBTC on the government spending multiplier on non-traded hours worked.

We detail below the steps of derivation of the decomposition of the change in the labor share of non-tradables. The optimal decision of labor supply to sector  $j = H, N$  is  $L^j(t) = \vartheta \left( \frac{\tilde{W}^j(t)}{\tilde{W}(t)} \right)^\epsilon L(t)$  where  $\epsilon$  is the elasticity of labor supply across sectors. Dividing the labor



Table 12: Numerically Computed Values of Government Spending Multiplier on Non-Traded Labor

Country	Baseline Model			Excess Baseline Model over Restricted model					
	$L(t)$ (1)	$\alpha^{L,N} \hat{L}^N(t)$ (2)	$d\mu^{L,N}(t)$ (3)	Contribution (4)	Scaled FBTC diff (5)	$L(t)$ (6)	$\alpha^{L,N} \hat{L}^N(t)$ (7)	$d\mu^{L,N}(t)$ (8)	Contribution (9)
AUS	-2.72	-2.10	-0.32	-0.15	-1.16	-2.73	-2.20	-0.41	-0.19
AUT	0.40	0.23	-0.02	-0.07	0.06	-0.11	-0.05	0.03	0.26
BEL	0.63	0.30	-0.11	-0.21	-0.89	-0.06	-0.30	-0.26	-0.87
CAN	1.05	0.92	0.22	0.24	0.30	0.37	0.29	0.05	0.16
DNK	1.06	0.91	0.17	0.19	0.55	0.17	0.22	0.11	0.48
ESP	0.98	0.37	-0.26	-0.29	-0.15	0.30	-0.01	-0.19	-0.50
FIN	1.32	0.95	0.05	0.05	0.15	0.66	0.44	-0.02	-0.03
FRA	0.28	0.46	0.26	0.58	-0.18	-0.21	-0.17	-0.02	-0.11
GBR	0.52	0.46	0.12	0.25	-0.33	-0.21	-0.13	0.01	0.05
IRL	0.38	0.33	0.06	0.19	0.89	-0.24	-0.11	0.06	0.25
ITA	0.59	0.62	0.25	0.40	-0.10	-0.08	-0.03	0.02	0.32
JPN	2.07	2.40	0.98	0.42	2.95	0.86	1.74	1.14	0.66
KOR	0.19	-0.38	-0.49	-0.81	-0.62	-0.53	-0.46	-0.15	-0.33
NLD	1.29	1.02	0.11	0.11	0.76	0.31	0.30	0.08	0.27
NOR	1.30	0.91	0.03	0.03	-0.71	0.75	0.49	-0.02	-0.04
PRT	2.00	1.17	0.06	0.05	-0.68	1.03	0.43	-0.15	-0.21
SWE	-0.11	0.22	0.30	0.80	0.42	-0.26	-0.03	0.15	0.45
USA	1.01	1.03	0.35	0.35	0.11	0.38	0.40	0.15	0.37
OECD	0.78	0.70	0.19	0.27	0.34	0.13	0.19	0.11	0.55
Mean	0.68	0.55	0.10	0.29	0.08	0.02	0.05	0.03	0.31
FBTC diff < 0					-0.54	-0.19	-0.27	-0.13	-0.21
FBTC diff > 0					0.69	0.24	0.36	0.19	0.32

Notes: Columns 1-4 show numerical results when we simulate the baseline model with CES production functions and technological change. Columns 1-2 show the government spending multiplier on total hours worked and on non-traded hours worked. Column 3 shows the change in the labor share of non-tradables. Column 4 shows the contribution of  $d\mu^{L,N}(t)$  to the government spending multiplier on non-traded hours worked  $\alpha^{L,N} \hat{L}^N(t)$ . Column 5 shows the response of the FBTC differential between non-tradables and tradables (where sectoral FBTC is scaled by the capital income share) to a shock to government consumption. We compute numerically the responses of total hours worked, non-traded hours worked, labor share of non-tradables, FBTC differential to a temporary increase in government consumption by 1% of GDP and then calculate government spending multipliers as the ratio of the present discounted value of the cumulative change of the corresponding quantity to the present discounted of the cumulative change of government consumption over a six-year horizon. Columns 6-7 show the excess of the government spending multiplier on total hours worked and on non-traded hours worked in the baseline model over a model with Cobb-Douglas production functions where we allow for time-varying sectoral TFPs but shut down FBTC. Column 8 shows the excess of the change in the labor share of non-tradables in the baseline model over the restricted model. Column 9 shows the contribution of the excess of  $d\mu^{L,N}(t)$  to the excess of government spending multiplier on non-traded hours worked  $\alpha^{L,N} \hat{L}^N(t)$ .

supply decision to the non-traded sector by the labor supply decision to the traded sector leads to a relationship between the optimal allocation of hours worked across sectors and the wage differential:

$$\frac{L^H(t)}{L^N(t)} = \left( \frac{1 - \vartheta}{\vartheta} \right) \left( \frac{\tilde{W}^N(t)}{\tilde{W}^H(t)} \right)^\epsilon. \quad (171)$$

According to the definition of the LIS in sector  $j$ , labor compensation is a fraction  $s_{L,t}^j$  of the value added at current prices of sector  $j = H, N$ , i.e.,  $\tilde{W}^j(t)L^j(t) = s_{L,t}^j P^j(t) \tilde{Y}^j(t)$  where a tilde indicates that the variable includes the technology utilization rate  $u^{Z,j}(t)$ . Dividing traded by non-traded labor compensation, inserting (171) to eliminate the ratio of sectoral wages and solving leads to the ratio of non-traded to traded hours worked:

$$\frac{L^N(t)}{L^H(t)} = \left( \frac{1 - \vartheta}{\vartheta} \right)^{\frac{1}{1+\epsilon}} \left( \frac{s_L^N(t)}{s_L^H(t)} \right)^{\frac{\epsilon}{1+\epsilon}} \left( \frac{P^N \tilde{Y}^N(t)}{P^H \tilde{Y}^H(t)} \right)^{\frac{\epsilon}{1+\epsilon}}. \quad (172)$$

Denoting the share of non-traded value added at current prices in GDP at current prices by  $\tilde{\omega}^{Y,N}(t) = \frac{P^N \tilde{Y}^N(t)}{\tilde{Y}(t)}$  where  $\tilde{Y}(t)$  is GDP at current prices and log-linearizing (172) in the neighborhood of the steady-state leads to:

$$\hat{L}^N(t) - \hat{L}^H(t) = \frac{\epsilon}{1 + \epsilon} (\hat{s}_L^N(t) - \hat{s}_L^H(t)) + \frac{\epsilon}{1 + \epsilon} \frac{\hat{\omega}^{Y,N}(t)}{1 - \omega^{Y,N}}, \quad (173)$$

where we omit the tilde when the variable is evaluated at the steady-state since the technology utilization rate collapses to one.

Plugging the log-linearized version of the sectoral decomposition of the response of total hours worked (71), i.e.,  $\hat{L}(t) = \alpha_L \hat{L}^H(t) + (1 - \alpha_L) \hat{L}^N(t)$ , into the the change in the labor share of non-tradables (72), i.e.,  $d\nu^{L,N}(t) = (1 - \alpha_L) (\hat{L}^N(t) - \hat{L}(t))$ , allows us to express the change in the labor share of non-tradables in terms of the growth differential in hours worked between non-tradables and tradables:

$$d\nu^{L,N}(t) = (1 - \alpha_L) \alpha_L (\hat{L}^N(t) - \hat{L}^H(t)). \quad (174)$$

Plugging (173) into (174) allows us to break down the change in the labor share of non-tradables into two components:

$$d\nu^{L,N}(t) = (1 - \alpha_L) \alpha_L \frac{\epsilon}{1 + \epsilon} \left[ \frac{\hat{\omega}^{Y,N}(t)}{1 - \omega^{Y,N}} + (\hat{s}_L^N(t) - \hat{s}_L^H(t)) \right], \quad (175)$$

where  $\alpha_L = \alpha_L^H$  and  $1 - \alpha_L = \alpha_L^N$ . Eq. (175) states the the change in  $\nu^{L,N}(t)$  can be driven by the change in the value added share of non-tradables  $\hat{\omega}^{Y,N}(t)$  and the differential in the responses of LIS between non-tradables and tradables as captured by the last term in brackets. The last term impinges on  $\nu^{L,N}(t)$  only if the production function is of the CES type otherwise the LIS remains fixed.

Since we aim at quantifying the role of technology and the change in LIS can be driven by the variation in the capital-labor ratio  $\tilde{k}^j(t) = u^{K,j}(t)k^j(t)$  including the capital utilization rate  $u^{K,j}(t)$  on the one hand and capital-utilization adjusted FBTC in sector  $j$  on the other, we turn to the decomposition of the change in the sectoral LIS. The starting point is the the ratio of the labor to the capital income share:

$$S^j(t) \equiv \frac{s_L^j(t)}{1 - s_L^j(t)} = \frac{\gamma^j}{1 - \gamma^j} \left( \frac{B^j(t)}{A^j(t)} \right)^{\frac{1-\sigma^j}{\sigma^j}} \left( \tilde{k}^j(t) \right)^{\frac{1-\sigma^j}{\sigma^j}}. \quad (176)$$

Log-linearizing (176) in the neighborhood of the steady-state and using the fact that  $\hat{s}_L^j(t) = (1 - s_L^j) \hat{S}^j(t)$ , the percentage deviation of the LIS in sector  $j$  relative to the initial steady-state is:

$$\hat{s}_L^j(t) = (1 - s_L^j) \left[ \frac{1 - \sigma^j}{\sigma^j} (\hat{B}^j(t) - \hat{A}^j(t)) + \frac{1 - \sigma^j}{\sigma^j} \hat{k}^j(t) \right]. \quad (177)$$

Table 13: Impact and Cumulative Effects of an Increase in Government Consumption by 1% of GDP in Two Restricted Versions of the Model

	LP $t = 0$			LP $t = 0..5$		
	Data	CES	CD-PML	Data	CES	CD-PML
	(1)	(2)	(3)	(4)	(5)	(6)
<b>A. Aggregate Multipliers</b>						
Gov. spending, $dG(t)$	1.00	1.00	1.00	5.51	5.46	5.46
Total hours worked, $dL(t)$	0.91	0.60	0.17	6.37	3.18	1.04
Real GDP, $dY_R(t)$	1.18	0.40	0.11	7.74	2.04	0.74
<b>B. Sectoral Labor</b>						
Traded labor, $dL^H(t)$	0.21	0.06	-0.71	0.73	0.32	-3.77
Non-traded labor, $dL^N(t)$	0.71	0.54	0.88	5.64	2.87	4.81
Labor share of non-tradables, $d\nu^{L,N}(t)$	0.13	0.15	0.77	1.68	0.79	4.11
<b>C. Sectoral Value Added</b>						
Traded VA, $dY^H(t)$	0.33	-0.05	-0.75	2.86	-0.28	-3.96
Non-traded VA, $dY^N(t)$	0.85	0.45	0.86	4.88	2.32	4.70
Non-traded VA share, $d\nu^{Y,N}(t)$	0.16	0.19	0.79	-0.01	1.03	-4.22
<b>D. Technology</b>						
Traded technology utilization, $du^{Z,H}(t)$	0.21	0.00	0.00	6.04	0.00	0.00
Non-Traded technology utilization, $du^{Z,N}(t)$	0.37	0.00	0.00	2.15	0.00	0.00
Traded TFP, $dTFP^H(t)$	0.66	0.00	0.00	7.82	0.00	0.00
Non-Traded TFP, $dTFP^N(t)$	0.36	0.00	0.00	0.28	0.00	0.00
<b>E. Redistributive effects</b>						
Traded LIS, $ds_L^H(t)$	0.14	-0.11	0.00	-2.23	-0.60	0.00
Non-traded LIS, $ds_L^N(t)$	0.25	-0.02	0.00	3.07	-0.12	0.00

**Eq. (178) corresponds to eq. (53) in the main text.** Note that when we calibrate the model to country-specific data, we shut down the capital utilization rate and thus  $\tilde{k}^j(t)$  collapses to  $k^j(t)$  since  $u^{K,j}(t) = 1$ .

Denoting capital-utilization adjusted FBTC in sector  $j$  by  $\text{FBTC}_{adjK}^j(t) = \left(\frac{B^j(t)}{A^j(t)}\right)^{\frac{1-\sigma^j}{\sigma^j}}$  and plugging (177) into (175) enables us to decompose the change in the labor share of non-tradables into three effects:

$$\begin{aligned}
d\nu^{L,N}(t) &= \alpha_L (1 - \alpha_L) \frac{\epsilon}{1 + \epsilon} \frac{\hat{\omega}^{Y,N}(t)}{1 - \omega^{Y,N}} \\
&+ \alpha_L (1 - \alpha_L) \frac{\epsilon}{1 + \epsilon} \left[ (1 - s_L^N) \left(\frac{1 - \sigma^N}{\sigma^N}\right) \hat{k}^N(t) - (1 - s_L^H) \left(\frac{1 - \sigma^H}{\sigma^H}\right) \hat{k}^H(t) \right] \\
&+ \alpha_L (1 - \alpha_L) \frac{\epsilon}{1 + \epsilon} \left[ (1 - s_L^N) \text{FBTC}_{adjK}^N(t) - (1 - s_L^H) \text{FBTC}_{adjK}^H(t) \right]. \quad (178)
\end{aligned}$$

**Eq. (178) corresponds to eq. (54) in the main text.** According to (178), the change in the labor share of non-tradables is driven by the variation in the value added share of non-tradables at current prices as captured by  $\hat{\omega}^{Y,N}(t)$  (first term on the RHS of eq. (178)), by the shift of capital toward the non-traded sector as captured by the second term on the RHS, and by the differential in utilization-adjusted FBTC between non-tradables and tradables as captured by the last term on the RHS of eq. (178).

## L More Numerical Results

Since in the text, we cannot show all scenarios for reasons of space, in this section, we show more numerical results for two restricted versions of the model. Columns 3 and 6 of Table 13 display impact and cumulative effects (over a six-year horizon) for a model imposing Cobb-Douglas production functions, abstracting from capital and utilization rates, and imposing perfect mobility of labor and exogenous terms of trade. Columns 2 and 5 show results for a restricted version of the baseline model where we shut down capital and technology utilization rates by letting  $\xi_2^j$  and  $\chi_2^j$  tend toward infinity and we abstract from FBTC.

## M More Empirical Results and Robustness Checks

In this section, we conduct some robustness checks. Due to data availability, we use annual data for eleven 1-digit ISIC-rev.3 industries that we classify as tradables or non-tradables. At this level of disaggregation, the classification is somewhat ambiguous because some broad sectors are made-up of heterogenous sub-industries, a fraction being tradables and the remaining industries being non-tradables. Since we consider a sample of 18 OECD countries and a period running from 1970 to 2015, the classification of some sectors may vary across time and countries. Industries such as 'Transport and Communication', 'Finance Intermediation' classified as tradables, 'Hotels and Restaurants' classified as non-tradables display intermediate levels of tradability which may vary considerably across countries but also across time. Subsection M.1 deals with this issue and conducts a robustness check to investigate the sensitivity of our empirical results to the classification of industries as tradables or non-tradables.

Another concern is related to the presence of anticipation effects. As argued by Ramey [2011], Blanchard and Perotti's [2002] approach to identifying government spending shocks in VAR models may lead to incorrect timing of the identified fiscal shocks. If the fiscal shock is anticipated in advance, agents may have modified their decisions before the rise in government spending actually materializes. Consequently, when the fiscal shock is anticipated, and thus VAR approach captures the shocks too late, it misses the initial changes in variables that occur as soon as the news is learned. Subsection M.2 conducts an investigation of the potential presence of anticipation effects by using a dataset constructed by Born, Juessen and Müller [2013] which contains one year-ahead OECD forecasts for government spending.

In the main text, we compute the labor income share in the lines of Gollin [2002], i.e., labor compensation is defined as the sum of compensation of employees plus compensation of self-employed. Since there exists alternative ways in constructing labor compensation, we explore empirically in subsection M.3 whether the results we document in the main text are robust to alternative measures of the labor income share.

Our dataset covers eleven industries which are classified as tradables or non tradables. The traded sector is made up of five industries and the non-traded sector of six industries. In subsection M.4, we conduct our empirical analysis at a more disaggregate level. The objective is twofold. First, we investigate whether all industries classified as tradables or non-tradables behave homogeneously or heterogeneously. Second, we explore empirically which industry drives the responses of broad sectors following a rise in government spending by 1% of GDP.

A close empirical analysis to ours is that performed by Cardi, Restout and Claeys (CRC henceforth) [2020] who investigate the sectoral and reallocation effects of an exogenous and temporary increase in government consumption. We compare our empirical findings shown in Fig. 2 with CRC [2020] estimates. In contrast to CRC [2020], we identify shocks to government consumption once for all by estimating one unique VAR model instead of different VAR models which can potentially pickup different structural government spending shocks and we trace the dynamic effects by adopting the local projection method, all variables responding to the same government spending shock. We consider a sample of 18 OECD countries over 1970-2015 instead of 16 OECD countries over 1970-2007. In subsection M.5, we report significant differences between our own findings and estimates documented by CRC [2020] and show that both the sample and the empirical strategy matters.<sup>44</sup>

### M.1 Sectoral Classification

This section explores the robustness of our findings to the classification of the eleven 1-digit ISIC-rev.3 industries as tradables or non tradables.

Following De Gregorio et al. [1994], we define the tradability of an industry by con-

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<sup>44</sup>CRC [2020] document evidence pointing at a significant increase in the value added and the labor share of non-tradables. We find that the value added share of non-tradables is unresponsive and show that CRC's [2020] finding stems from using a different sample and adopting a one step VAR approach which lead technological change and labor income shares to be unresponsive.

structuring its openness to international trade given by the ratio of total trade (imports + exports) to gross output. Data for trade and output are provided by the World Input-Output Databases ([2013], [2016]). Table 14 gives the openness ratio (averaged over 1995-2014) for each industry in all countries of our sample. Unsurprisingly, "Agriculture, Hunting, Forestry and Fishing", "Mining and Quarrying", "Total Manufacturing" and "Transport, Storage and Communication" exhibit high openness ratios (0.54 in average if "Mining and Quarrying", due to its relatively low weight in GDP, is not considered). These four sectors are consequently classified as tradables. At the opposite, "Electricity, Gas and Water Supply", "Construction", "Wholesale and Retail Trade" and "Community Social and Personal Services" are considered as non tradables since the openness ratio in this group of industries is low (0.07 in average). For the three remaining industries "Hotels and Restaurants", "Financial Intermediation", "Real Estate, Renting and Business Services" the results are less clearcut since the average openness ratio amounts to 0.18 which is halfway between the two aforementioned averages. In the benchmark classification, we adopt the standard classification of De Gregorio et al. [1994] by treating "Real Estate, Renting and Business Services" and "Hotels and Restaurants" as non traded industry. Given the dramatic increase in financial openness that OECD countries have experienced since the end of the eighties, we allocate "Financial Intermediation" to the traded sector. This choice is also consistent with the classification of Jensen and Kletzer [2006] who categorize "Finance and Insurance" as tradable. They use locational Gini coefficients to measure the geographical concentration of different sectors and classify sectors with a Gini coefficient below 0.1 as non-tradable and all others as tradable (the authors classify activities that are traded domestically as potentially tradable internationally).

Table 14: Openness Ratios for individual, average over period 1995-2014

	Agri.	Minig	Manuf.	Elect.	Const.	Trade	Hotels	Trans.	Finance	Real Est.	Comm.
AUS	0.242	0.721	0.643	0.007	0.005	0.025	0.255	0.247	0.054	0.051	0.054
AUT	0.344	2.070	1.152	0.178	0.075	0.135	0.241	0.491	0.302	0.221	0.043
BEL	1.198	13.374	1.414	0.739	0.067	0.186	0.389	0.536	0.265	0.251	0.042
CAN	0.304	0.821	0.966	0.098	0.002	0.030	0.338	0.211	0.169	0.121	0.038
DNK	0.470	0.786	1.329	0.214	0.014	0.092	0.021	0.832	0.138	0.131	0.027
ESP	0.386	4.699	0.680	0.021	0.003	0.044	0.008	0.206	0.130	0.149	0.022
FIN	0.228	2.899	0.796	0.117	0.006	0.094	0.131	0.280	0.153	0.256	0.021
FRA	0.280	3.632	0.815	0.049	0.004	0.048	0.001	0.224	0.068	0.070	0.014
GBR	0.360	0.853	0.958	0.017	0.010	0.024	0.148	0.209	0.233	0.147	0.041
IRL	0.298	1.384	1.127	0.154	0.013	0.652	0.772	0.285	1.014	0.988	0.049
ITA	0.300	4.130	0.603	0.041	0.013	0.087	0.035	0.150	0.095	0.082	0.012
JPN	0.158	3.923	0.293	0.004	0.000	0.067	0.021	0.159	0.034	0.020	0.005
KOR	0.175	18.603	0.507	0.012	0.003	0.213	0.029	0.388	0.071	0.114	0.052
NLD	0.988	1.496	1.259	0.082	0.076	0.106	0.011	0.562	0.245	0.405	0.114
NOR	0.391	0.837	0.995	0.146	0.024	0.097	0.009	0.354	0.146	0.143	0.058
PRT	0.351	2.954	0.880	0.050	0.005	0.067	0.105	0.378	0.125	0.114	0.026
SWE	0.294	2.263	0.969	0.119	0.020	0.163	0.019	0.392	0.274	0.256	0.026
USA	0.207	0.541	0.428	0.012	0.001	0.055	0.003	0.109	0.066	0.052	0.008
OECD	0.388	3.666	0.879	0.114	0.019	0.121	0.141	0.334	0.199	0.198	0.036
<i>H/N</i>	<i>H</i>	<i>H</i>	<i>H</i>	<i>N</i>	<i>N</i>	<i>N</i>	<i>N</i>	<i>H</i>	<i>H</i>	<i>N</i>	<i>N</i>

Notes: the complete designations for each industry are as follows (EU KLEMS codes are given in parentheses). "Agri.": "Agriculture, Hunting, Forestry and Fishing" (AtB), "Minig": "Mining and Quarrying" (C), "Manuf.": "Total Manufacturing" (D), "Elect.": "Electricity, Gas and Water Supply" (E), "Const.": "Construction" (F), "Trade": "Wholesale and Retail Trade" (G), "Hotels": "Hotels and Restaurants" (H), "Trans.": "Transport, Storage and Communication" (I), "Finance": "Financial Intermediation" (J), "Real Est.": "Real Estate, Renting and Business Services" (K), "Comm.": "Community Social and Personal Services" (LtQ). The openness ratio is the ratio of total trade (imports + exports) to gross output (source: World Input-Output Databases ([2013], [2016])).

We conduct below a sensitivity analysis with respect to the three industries ("Real Estate, Renting and Business Services", "Hotels and Restaurants" and "Financial Intermediation") which display some ambiguity in terms of tradability to ensure that the benchmark classification does not drive the results. In order to address this issue, we re-estimate the dynamic responses to a government spending shock for the main variables of interest using local projections for different classifications in which one of the three above industries initially marked as tradable (non tradable resp.) is classified as non tradable (tradable resp.), all other industries staying in their original sector. In doing so, the classification of only



one industry is altered, allowing us to see if the results are sensitive to the inclusion of a particular industry in the traded or the non traded sector.

As an additional robustness check, we also exclude the industry "Community Social and Personal Services" from the non-tradable industries' set. This robustness analysis is based on the presumption that among the industries provided by the EU KLEMS and STAN databases, this industry is government-dominated. While this exercise is interesting on its own as it allows us to explore the size of the impact of a government spending shock on the business sector, we also purge for the potential and automatic link between non-traded value added and public spending because government purchases (to the extent that the government is the primary purchaser of goods from this industry) account for a significant part of non-traded value added.<sup>45</sup> The baseline and the four alternative classifications considered in this exercise are shown in Table 15. The last line provides the matching between the color line (when displaying IRFs below) and the classification between tradables and non tradables.

Table 15: Robustness check: Classification of Industries as Tradables or Non Tradables

	KLEMS code	Classification				
		Baseline	#1	#2	#3	#4
Agriculture, Hunting, Forestry and Fishing	AtB	H	H	H	H	H
Mining and Quarrying	C	H	H	H	H	H
Total Manufacturing	D	H	H	H	H	H
Electricity, Gas and Water Supply	E	N	N	N	N	N
Construction	F	N	N	N	N	N
Wholesale and Retail Trade	G	N	N	N	N	N
Hotels and Restaurants	H	N	N	N	<b>H</b>	N
Transport, Storage and Communication	I	H	H	H	H	H
Financial Intermediation	J	H	<b>N</b>	H	H	H
Real Estate, Renting and Business Services	K	N	N	<b>H</b>	N	N
Community Social and Personal Services	LtQ	N	N	N	N	<b>neither H or N</b>
Color line in Figure 12		blue	red	black	green	yellow

Notes: H stands for the Traded sector and N for the Non traded sector.

Fig. 12 shows the responses of variables of interest to an exogenous increase in government consumption by one percent of GDP. The solid blue line shows results for the baseline classification while the responses for the alternative classifications are shown in the four colored lines. In each panel, the shaded area corresponds to the 90% confidence bounds for the baseline.

The first row of Fig. 12 reports the effects of an exogenous increase in government consumption by 1% of GDP on the main aggregate variables, namely, government spending, total hours, real GDP and TFP. For government spending and TFP, the responses are remarkably similar across the baseline and alternative classifications. We can notice that the expansionary effect of an exogenous increase in government consumption on total hours is mitigated when the public sector is excluded (classification #4 and the yellow line) but the shape of the dynamic adjustment of the two variables is similar to the benchmark classification and the alternative IRFs lie within the confidence bounds of the baseline classification. The second and third row of Fig. 12 contrast the responses of sectoral hours worked ( $L^j$ ), sectoral value added ( $Y^j$ ), the value added share of non-tradables ( $\nu^{Y,N}$ ), the labor share of non-tradables ( $\nu^{L,N}$ ), the ratio of non-traded to traded LIS, i.e.,  $s_L^N/s_L^H$ , the ratio of traded to non-traded TFP, i.e.,  $TFP^H/TFP^N$  for the baseline classification with those obtained for alternative classifications of industries as tradables or non-tradables. Alternative responses are fairly close to those estimated for the baseline classification as they lie within the confidence interval (for the baseline classification) for all the selected horizons (8 years). With regard to the responses of utilization-adjusted TFPs shown in the fourth row, the dynamic adjustment of  $Z^j(t)$  displays a similar pattern across the baseline and alternative classifications: utilization-adjusted TFP increases in the traded sector while it is essentially unchanged in the non-traded sector. The response of utilization-adjusted FBTC in sector  $N$  is similar across all classifications. The dynamic adjustment of the

<sup>45</sup>This exercise has been conducted by Cardi et al. [2020] and Beetsma and Giuliodori [2008], among others, in order to deal with the potential endogeneity of government purchases with respect to output.

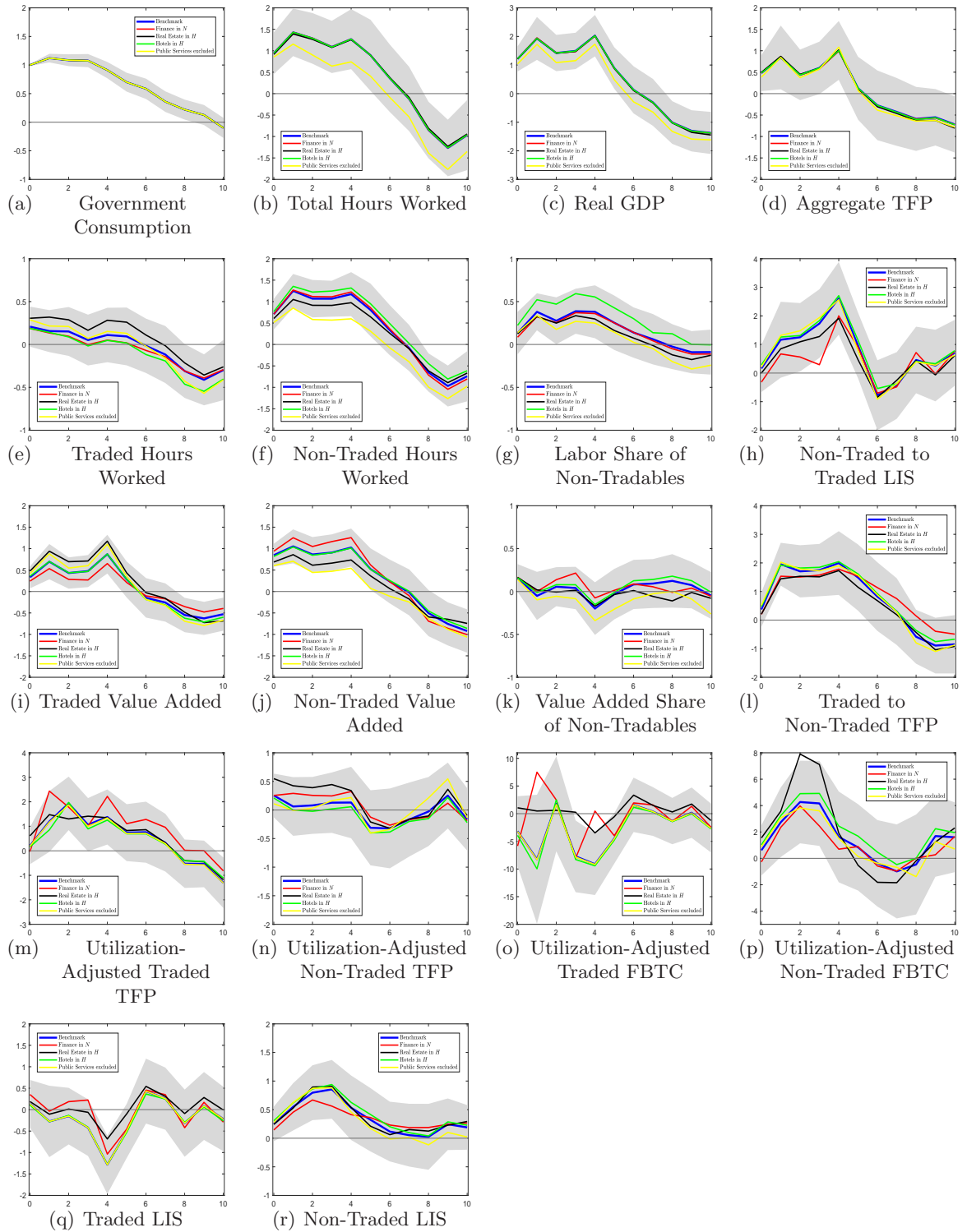


Figure 12: Sensitivity of the Effects of a Government Spending Shock to the Classification of Industries as Tradable or Non-Tradable. *Notes:* Effects of an exogenous increase in government consumption by 1% of GDP obtained by means of local projections à la Jordà [2005]. Results for the baseline specification are displayed by the blue line with shaded area indicating 90 percent confidence bounds. The green line and the black line show results when "Hotels and Restaurants" and "Real Estate, Renting and Business Services" are treated as tradables, respectively. The red line shows results when "Financial Intermediation" is classified as non-tradables. The yellow line displays results when "Community Social and Personal Services" is not considered. Sample: 18 OECD countries, 1970-2015, annual data.

utilization-adjusted FBTC in sector  $H$  displays some differences across the baseline and the four alternative classifications: the decline is less pronounced when the industry "Real Estate, Renting and Business Services" is treated as tradables (classification #2 and the black line) and the IRF is more erratic when the public sector is excluded (classification #4 and the yellow line). However, in both cases, the IRF lies well within the confidence interval for almost all the selected horizons. In addition, in the last row of Fig. 12, we investigate whether our conclusion for redistributive effects (i.e., for sectoral labor income



shares) is robust to the classification of industries. Across all scenarios, LISs in both sectors exhibit a similar dynamic adjustment following an increase in government spending. One can notice that the discrepancy in the estimated effect between the benchmark and the alternative classifications is not statistically significant. In conclusion, our main findings hold and remain insensitive to the classification of one specific industry as tradable or non-tradable. In this regard, the specific treatment of "Hotels and Restaurants", "Real Estate, Renting and Business Services", "Financial Intermediation" or "Community Social and Personal Services" does not drive the results.

## M.2 Controlling for Potential Anticipation Effects

We now address concern one potential issue related to the anticipation effects. When the fiscal shock is anticipated, and the VAR approach captures the shocks too late, it misses the initial changes in variables that occur as soon as the news is learned. We conduct below an investigation of the potential presence of anticipation effects, using two alternative measures of forecasts for government spending. We also re-estimate the effects of a government spending shock by controlling for anticipation effects. The first measure is provided by Born, Juessen and Müller [2013] and stems from the OECD forecasts, while the second is taken from a dataset constructed by Fioramanti et al. [2016] where forecasts are performed by the European Commission. We use two alternative datasets as the former contains observations from 1986 to 2014 for all countries, while the latter provides a longer time horizon but for a restricted set of countries.

Drawing on previous studies, we conduct three robustness exercises to explore the potential implications of anticipations effects:

- First, we replace in the local-projection regressions the SVAR identified government spending shock  $\epsilon_{it}^G$  with the forecast error  $FE_{it}^G$  computed as the difference between actual series and forecast series of the government spending growth rate:  $FE_{it}^G = \Delta g_{it} - for_{it}^G$ , where  $\Delta g_{it}$  is the actual government consumption growth and  $for_{it}^G$  is the previous period's forecast. The idea is simply to purge actual government spending growth of what professional forecasters project spending growth to be.
- A second way to deal with the potentially anticipated government spending shocks is to augment the SVAR specification we used to estimate the identified government spending shock  $\epsilon_{it}^G$  with the forecasts for government spending growth  $for_{it}^G$ . This allows us to identify the unanticipated shock to government spending in the presence of fiscal foresight. Drawing on Beetsma and Giuliodori [2008], we estimate two VAR models: 1) we extend our baseline model with government spending growth forecast  $for_{it}^G$  we used earlier and the vector of variables in the VAR now reads  $[g_{it}, for_{it}^G, y_{R,it}, l_{it}, w_{C,it}, TFP_{it}]$  implying that  $for_{it}^G$  is treated as an endogenous variable and 2) we augment the baseline VAR model with  $for_{it}^G$  as an exogenous variable, i.e.  $\epsilon_{it}^G$  is identified in a VARX model in which the vector of endogenous variables is  $[g_{it}, y_{R,it}, l_{it}, w_{C,it}, TFP_{it}]$  and  $for_{it}^G$  is an exogenous variable. The first approach is attractive because it accounts automatically for any effects that expectations might have on the others aggregate variables and for the determinants of the expectations themselves. However, this method increases the number of estimated parameters within the VAR structure. Given the data limitations on the variable  $for_{it}^G$ , we complement the first approach 1) with the second exercise to get more parsimonious VAR models.
- The third robustness test repeats the previous exercise by considering an alternative measure of forecasts: the forecast for the budget balance-GDP ratio which we denote by  $for_{it}^{bbr}$ . The year-ahead forecasts are taken from the Commission's Fall forecasts, see Fioramanti et al. [2016] for details of construction of  $for_{it}^{bbr}$ .

In the following, we conduct an investigation of the potential presence of anticipation effects by performing the three robustness exercises mentioned above. To perform the first two robustness tests, we use a dataset constructed by Born, Juessen and Müller [2013]

that contains time series for forecasts for government spending growth from the OECD.<sup>46</sup> The data availability for the variable  $for_{it}^G$  is: AUS (1997-2014), BEL (1999-2014), CAN (1986-2014), DNK (1997-2010), ESP (1997-2014), FIN (1997-2014), FRA (1986-2014), GBR (1986-2014), IRL (1997-2014), ITA (1986-2014), JPN (1986-2014), KOR (1998-2014), NLD (1997-2014), NOR (1997-2014), PRT (1997-2014), SWE (1997-2014) and USA (1986-2014). No data are available for Austria. Regarding the general government balance to GDP ratio forecast (one year ahead) used in the third exercise, time series are available for AUT (1995-2014), BEL (1971-2014), DNK (1977-2014), ESP (1987-2014), FIN (1995-2014), FRA (1970-2014), GBR (1974-2014), IRL (1974-2014), ITA (1970-2014), NLD (1970-2014), PRT (1987-2014) and SWE (1995-2014). Note that, the European Commission provide forecasts for the budget balance-GDP ratio only for European countries. Accordingly, non-European countries of our panel (AUS, CAN, JPN, KOR, NOR and USA) are not considered in the third robustness test.

Figure 13 shows IRFs when the SVAR identified government spending shock  $\epsilon_{it}^G$  is replaced by the forecast error  $FE_{it}^G$  in the local-projection regressions. The blue line reports the results for the baseline case (i.e. the fiscal shock is  $\epsilon_{it}^G$ ), while the red line displays the results when the fiscal shock considered is  $FE_{it}^G$ . In both cases, the local-projection regressions are estimated over the period running from 1986 to 2014 to obtain comparable results. Overall, we find that the response of the vast majority of variables, with the notable exception of the utilization-adjusted TFP in the non-traded sector, is consistent with the baseline. We can notice some quantitative differences however. When using the forecast error  $FE_{it}^G$  as a fiscal shock, the rise in both total hours worked and non-traded hours worked is more pronounced. The response of the labor share of non-tradables,  $\nu^{L,N}$ , positive and statistically significant, also increases more than in the baseline case. Thus the larger increase in non-traded hours worked is driven by the greater rise in total hours worked and also the higher reallocation of labor toward the non-traded sector. Like in the baseline, the value added share of non-tradables,  $\nu^{Y,N}$ , remains unresponsive to the fiscal shock whether it is measured with  $\epsilon_{it}^G$  or  $FE_{it}^G$ . One may be concerned with the negative response at impact of the utilization-adjusted TFP in the non-traded sector when we use the forecast error  $FE_{it}^G$  as a measure of the fiscal shock (-0.98), however this fall is not statistically significant. Finally, we may also note that, for utilization-adjusted TFP in the traded sector and also for traded and non-traded FBTC, the responses lie within the confidence interval for almost all horizons.

As mentioned above, in the second exercise, the forecasts for government spending growth  $for_{it}^G$  are used directly to control for anticipations effects. For that purpose, the baseline VAR model that allows us to identify structural fiscal shocks is modified by allowing the latter variable to enter the vector of variables as an endogenous variable (the VAR case in the sequel) or as an exogenous variable (the VARX case). Fig. 14 shows results for the baseline ( $for_{it}^G$  is not considered) together with the VAR and VARX cases to control for potential fiscal foresight. Overall, it turns out that differences are moderate and anticipation effects thus play a limited quantitative role in the dynamic adjustment to a government spending shock. The impulse response functions for the two alternatives are, qualitatively, similar to those under the baseline shown in the blue line. Quantitatively, despite some differences, for almost all variables and all horizons considered, the IRFs are within the confidence interval.

The final exercise we consider amounts to repeating the previous analysis by replacing the forecast variable  $for_{it}^G$  with the forecast of the budget balance-GDP ratio  $for_{it}^{bbr}$ . Fig. 15 plots the estimated impulse response for this robustness test. Overall, the results are qualitatively and quantitatively similar to those obtained in the baseline and thus do not deserve further comments.

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<sup>46</sup>We thank Gernot Müller for providing this dataset to us.

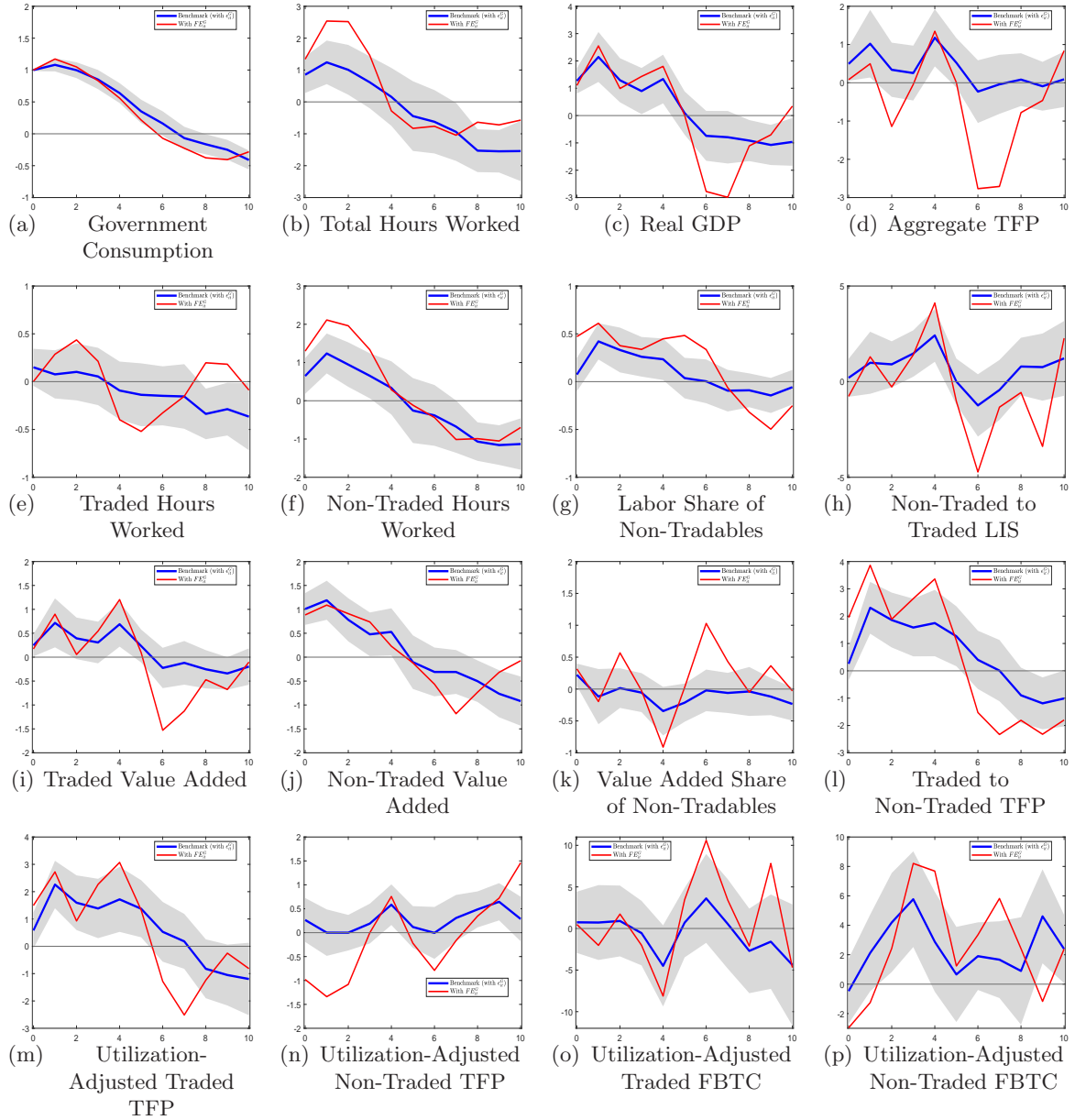


Figure 13: Effects of an Unanticipated Government Spending Shock: Controlling for Anticipation Effects with Forecast Errors ( $FE_{it}^G$ ). *Notes:* Effects of an exogenous increase in government consumption by 1% of GDP obtained by means of local projections à la Jordà [2005]. Results for the baseline specification (i.e. the government spending shock  $\epsilon_{it}^G$  is identified in a SVAR model) are displayed by the blue line with shaded area indicating 90 percent confidence bounds. The red line shows results when the forecast error  $FE_{it}^G$  is used as a measure of the government spending shock in local projections. Sample: 17 OECD countries (AUS, BEL, CAN, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, KOR, NLD, NOR, PRT, SWE and USA), 1986-2014, annual data (367 observations).

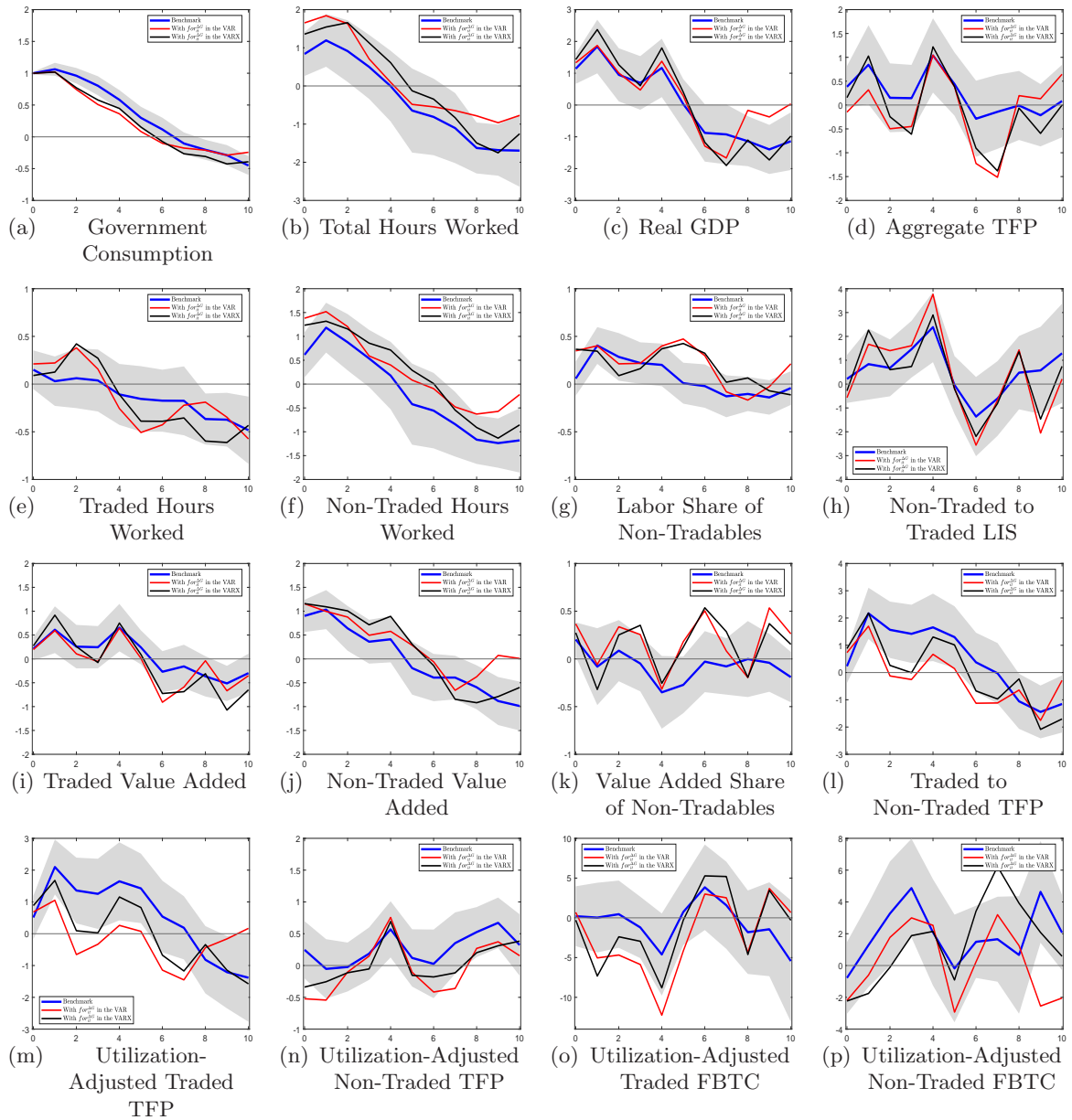


Figure 14: Effects of an Unanticipated Government Spending Shock: Controlling for Anticipation Effects with the Forecast for Government Spending Growth ( $for_{it}^G$ ). Notes: Effects of an exogenous increase in government consumption by 1% of GDP obtained by means of local projections à la Jordà [2005]. Results for the baseline specification are displayed by the blue line with shaded area indicating 90 percent confidence bounds. The red line shows results when the identified spending shock  $\epsilon_{it}^G$  is estimated in the baseline VAR model augmented with the forecast for government spending growth  $for_{it}^G$ . The black line shows results when the identified spending shock  $\epsilon_{it}^G$  is estimated in a VARX model that includes the forecast for government spending growth  $for_{it}^G$  as an exogenous variable. Sample: 17 OECD countries (AUS, BEL, CAN, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, KOR, NLD, NOR, PRT, SWE and USA), 1986-2014, annual data (367 observations).

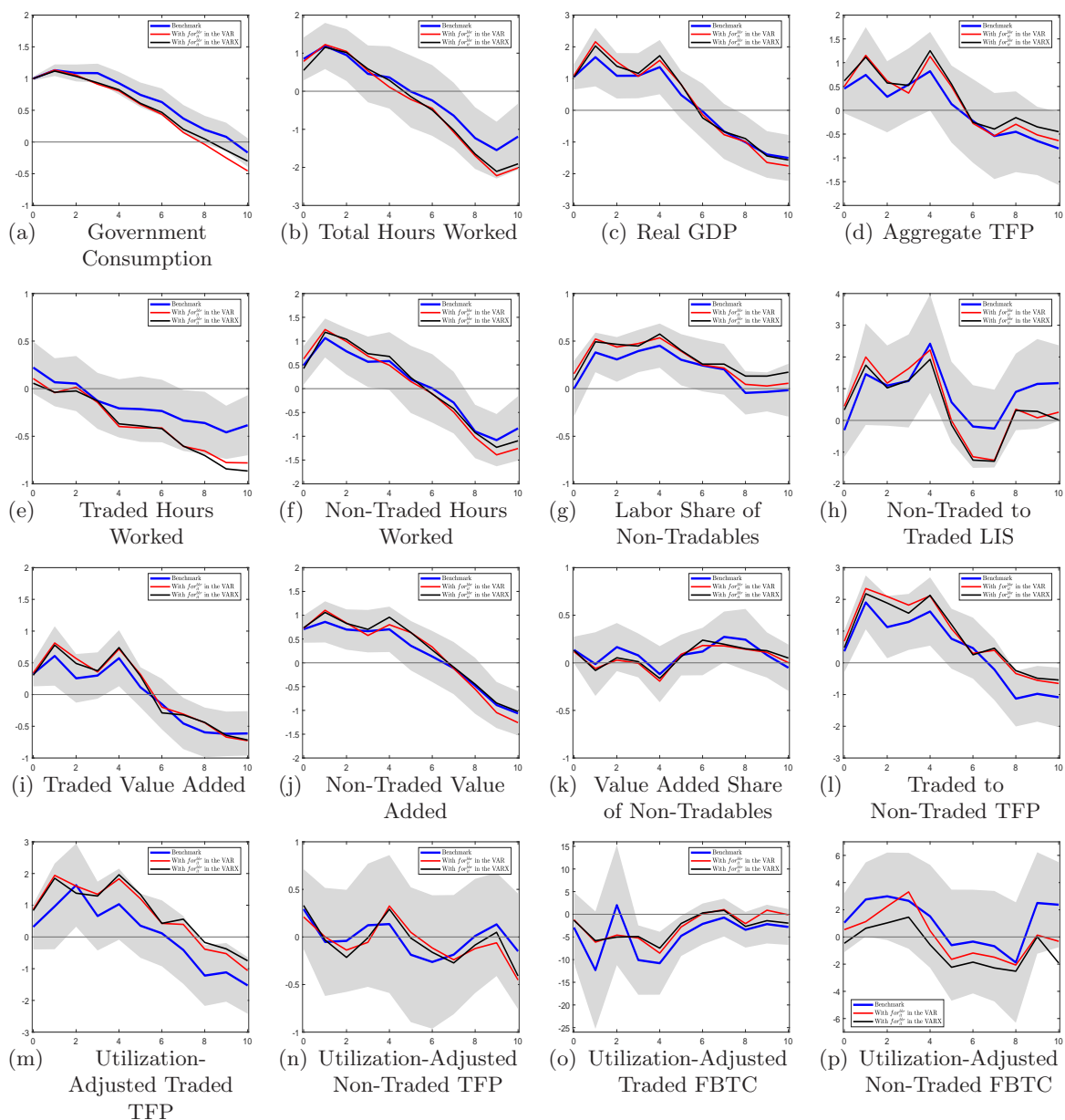


Figure 15: Effects of an Unanticipated Government Spending Shock: Controlling for Anticipation Effects with the Budget Balance-GDP Ratio ( $for_{it}^{bbr}$ ). Notes: Effects of an exogenous increase in government consumption by 1% of GDP obtained by means of local projections à la Jordà [2005]. Results for the baseline specification are displayed by the blue line with shaded area indicating 90 percent confidence bounds. The red line shows results when the identified spending shock  $\epsilon_{it}^G$  is estimated in the baseline VAR model augmented with the forecast for the budget balance-GDP ratio  $for_{it}^{bbr}$ . The black line shows results when the identified spending shock  $\epsilon_{it}^G$  is estimated in a VARX model that includes the forecast for the budget balance-GDP ratio  $for_{it}^{bbr}$  as an exogenous variable. Sample: 12 OECD countries (AUT, BEL, DNK, ESP, FIN, FRA, GBR, IRL, ITA, NLD, PRT and SWE), 1970-2014, annual data (415 observations).

### M.3 Alternative Measures of the Labor Income Share

When exploring empirically the effects of an exogenous increase in government spending on the labor income share, an issue is the way the share of labor in total income is constructed. Gollin [2002] pointed out that the treatment of self-employment income affects the measurement of the LIS. In particular, it is unclear how the income of proprietors (self-employed) should be allocated to labor income or to capital revenue. In the main text, our preferred measure (called benchmark *bench* hereafter) is to treat all the income of self-employed as labor income. Although this choice overstates the measure of the LIS, it has the virtue of being simple and transparent. Moreover data involved in the construction of this calculation of the LIS are comparable across industries and directly available for all countries of our sample. Specifically, the LIS in sector  $j = H, N$  is constructed as follows:

$$s_L^{j,bench} = \frac{W_{empl}^j L_{empl}^j + Inc_{self}^j}{P^j Y^j}, \quad (179)$$

where  $W_{empl}^j L_{empl}^j$  is the labor compensation of employees,  $Inc_{self}$  is total income of self-employed and  $P^j Y^j$  is the valued added at current prices in sector  $j$ . Note that labor compensation of employees includes total labor costs: wages, salaries and all other costs of employing labor which are born by the employer whilst  $Inc_{self}$  comprises both labor and capital income components, noted  $W_{self}^j L_{self}^j$  and  $R_{self}^j K_{self}^j$  respectively such that  $Inc_{self}^j = W_{self}^j L_{self}^j + R_{self}^j K_{self}^j$ .

As a first alternative measure of the LIS, we use only employees compensation as a measure of labor income. This LIS measure, denoted by  $s_L^{j,1}$ , is constructed as follows:

$$s_L^{j,1} = \frac{W_{empl}^j L_{empl}^j}{P^j Y^j}. \quad (180)$$

The measure described by eq. (180) omits the income of the self-employed, i.e. this income being entirely counted as capital income.

As a second alternative measure, we split self-employed income into capital and labor income based on the assumption that the labor income of the self-employed has the same mix of labor and capital income as the rest of the economy. In other words, total labor compensation comprises labor compensation of employees,  $W_{empl}^j L_{empl}^j$ , and the self-employed income scaled by the LIS of employees only, i.e.  $Inc_{self}^j \times s_L^{j,1}$ . With this adjustment, the LIS, denoted by  $s_L^{j,2}$ , is constructed as follows:

$$s_L^{j,2} = \frac{W_{empl}^j L_{empl}^j + Inc_{self}^j \times s_L^{j,1}}{P^j Y^j}. \quad (181)$$

The third alternative to compute the LIS relies upon the assumption that self-employed earn the same hourly compensation as employees. Thus, we use the hourly wage earned by employees  $W_{empl}^j$  as a shadow price of labor of self-employed workers. The LIS, denoted by  $s_L^{j,3}$ , is constructed as follows:

$$s_L^{j,3} = \frac{W_{empl}^j \times (L_{empl}^j + L_{self}^j)}{P^j Y^j}. \quad (182)$$

Finally, and following Bridgman [2018], the labor income share is adjusted for capital depreciation. In that case, the LIS, denoted by  $s_L^{j,4}$ , is constructed as follows:

$$s_L^{j,4} = \frac{W_{empl}^j L_{empl}^j}{P^j Y^j - CFC^j} \quad (183)$$

where  $CFC^j$  is the Consumption of Fixed Capital (current prices) in sector  $j$ . In the benchmark and the first three alternatives, the "gross" labor income share treats depreciation as a return to capital. By contrast, the construction  $s_L^{j,4}$  is a "net" income distribution indicator as it adjusts value added for depreciation.



In Fig. 16 we display the results of this sensitivity analysis with respect to the construction of the labor income share. We measure the effects to an exogenous increase in government spending by one percent of GDP on aggregate and sectoral variables of interest by contrasting the impulse response functions of the variables when the LIS is measured as either  $s_L^{j,bench}$  (blue line), or  $s_L^{j,1}$  (red line), or  $s_L^{j,2}$  (black line), or  $s_L^{j,3}$  (green line), or  $s_L^{j,4}$  (yellow line). In addition, the last row of the figure presents the IRFs of the labor income share  $j = H, N$ . Fig. 16 demonstrates that all the measures of  $s_L^j$  being compared imply essentially identical IRFs to an increase in government spending. For a large set of variables ( $G, L, TFP, Y_R, L^H, L^N, Y^H, Y^N, L^N/L, Y^N/Y, s_L^N/s_L^H$  and  $TFP^H/TFP^N$ ) the IRFs for the five specifications are qualitatively and quantitatively similar, if not identical. With regard to the responses of utilization-adjusted TFP and FBTC, the IRFs obtained from the use of four alternative measures of  $s_L^j$  display the same dynamic adjustment and are well within the confidence interval (for the benchmark specification  $s_L^{j,bench}$ ) for all horizons. Finally, the responses of the labor income share  $s_L^j$  in both sectors for the four specifications are also very close to the IRF obtained in the benchmark. Overall, our main findings are robust and insensitive to the method adopted to construct the labor income share.

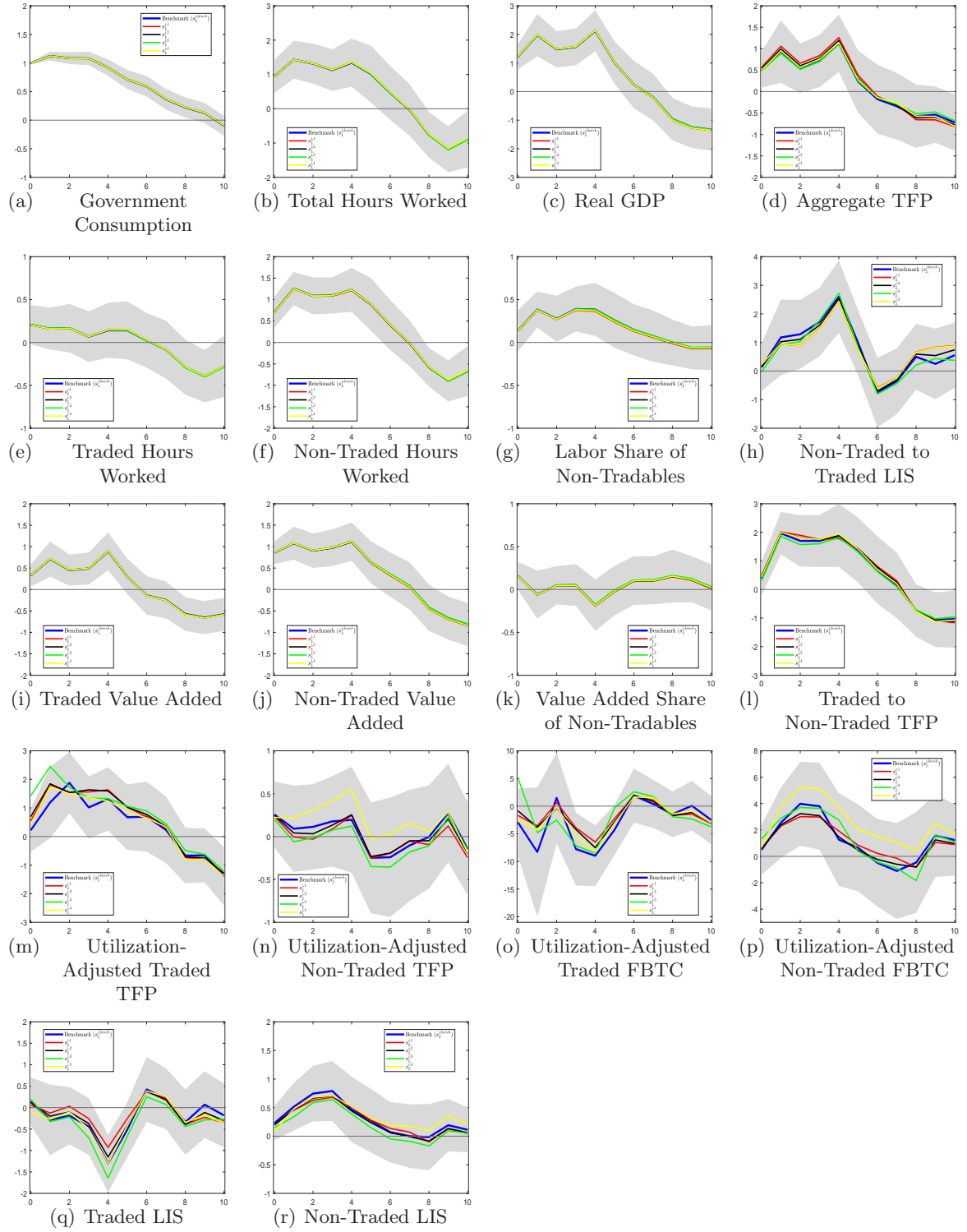


Figure 16: Sensitivity of the Effects of an Unanticipated Government Spending Shock to the Construction of the LIS. *Notes:* Effects of an exogenous increase in government consumption by 1% of GDP obtained by means of local projections à la Jordà [2005]. Results for the baseline specification ( $s_L^j = s_L^{j,bench}$ ) are displayed by the blue line with shaded area indicating 90 percent confidence bounds. The red (black, green and yellow resp.) line reports results when  $s_L^j = s_L^{j,1}$  ( $s_L^j = s_L^{j,2}$ ,  $s_L^j = s_L^{j,3}$  and  $s_L^j = s_L^{j,4}$  resp.). Sample: 18 OECD countries, 1970-2015, annual data.

## M.4 Effects of Exogenous Government Spending Shock on Sub-Sectors

**Empirical analysis at a disaggregate sectoral level.** Our dataset covers eleven industries which are classified as tradables or non-tradables. The traded sector is made up of five industries and the non-traded sector of six industries. To conduct a decomposition of the sectoral effects at a sub-sector level, we estimate the responses of sub-sectors to the same identified government spending shock by adopting the two-step approach detailed in the main text. More specifically, indexing countries with  $i$ , time with  $t$ , sectors with  $j$ , and sub-sectors with  $k$ , we first identify the shock to government consumption by estimating the VAR model which includes aggregate variables:  $[g_{i,t}, y_{i,t}^{k,j}, l_{i,t}^j, w_{C,i,t}^j, \text{tfp}_{i,t}^A]$  where low-case letters indicate that the variable is logged (all quantities are divided by the working age population) and next we estimate the dynamic effects by using the Jordà's [2005] single-equation method. The local projection method amounts to running a series of regression of each variable of interest on a structural identified shock for each horizon  $h = 0, 1, 2, \dots$ :

$$x_{i,t+h}^{k,j} = \alpha_{i,h}^{k,j} + \alpha_{t,h}^{k,j} + \beta_{i,h}^{k,j} \cdot t + \psi_h^{k,j}(L) z_{i,t-1} + \gamma_h^{k,j} \cdot \epsilon_{i,t}^G + \nu_{i,t+h}^{k,j}, \quad (184)$$

where  $x^{k,j} = L^{k,j}, L^{k,j}/L, Y^{k,j}, Y^{k,j}/Y_R, \text{TFP}^{k,j}$  (variables are logged). To express the results in meaningful units, i.e., in GDP units or total hours worked units, we multiply the responses of value added at constant prices and value added share at constant prices of sub-sector  $k$  by its share in GDP (at current prices), i.e.,  $\omega^{Y,k} = \frac{P^{k,j} Y^{k,j}}{P Y_R}$  where  $Y_R$  is real GDP, and we multiply the responses of hours worked and labor share of sub-sector  $k$  by its labor compensation share, i.e.,  $\alpha^{L,k} = \frac{W^{k,j} L^{k,j}}{W L}$ . We detail below the mapping between the responses of broad sector's variables and responses of variables in sub-sector  $k$  of one broad sector  $j$ .

The response of  $\ln L^{k,j}$  to a shock to government consumption is the percentage deviation of hours worked in sub-sector  $k \in j$  relative to initial steady-state:  $\ln L_t^{k,j} - \ln L^{k,j} \simeq \frac{dL_t^{k,j}}{L^{k,j}} = \hat{L}_t^{k,H}$  where  $L^{k,j}$  is the initial steady-state. We assume that hours worked of the broad sector is an aggregate of sub-sector hours worked which are imperfect substitutes. Therefore, the response of hours worked in the broad sector  $\hat{L}_t^j$  is a weighted average of the responses of hours worked  $\frac{W^{k,j} L^{k,j}}{W^j L^j} \hat{L}_t^{k,j}$  where  $\frac{W^{k,j} L^{k,j}}{W^j L^j}$  is the share of labor compensation of sub-sector  $k$  in labor compensation of the broad sector  $j$ :

$$\begin{aligned} \hat{L}_t^j &= \sum_{k \in j} \frac{W^{k,j} L^{k,j}}{W^j L^j} \hat{L}_t^{k,j}, \\ \frac{W^j L^j}{W L} \hat{L}_t^j &= \sum_{k \in j} \frac{W^{k,j} L^{k,j}}{W L} \hat{L}_t^{k,j}, \\ \alpha^{L,j} \hat{L}_t^j &= \sum_{k \in j} \alpha^{L,k} \hat{L}_t^{k,j}, \end{aligned} \quad (185)$$

where  $\sum_j \sum_k \alpha^{L,k} = 1$ . Above equation breaks down the response of hours worked in broad sector  $j$  into the responses of hours worked in sub-sectors  $k \in j$  weighted by their labor compensation share  $\alpha^{L,k} = \frac{W^{k,j} L^{k,j}}{W L}$  averaged over 1970-2015. In multiplying  $\hat{L}_t^{k,j}$  by  $\alpha^{L,k}$ , we express the response of hours worked in sub-sector  $k \in j$  in percentage point of total hours worked.

We turn to the normalization of the response of value added at constant prices in sub-sector  $k$ . The value added at constant prices of sector  $j$  is a weighted average of value added of sub-sector  $k \in j$ , i.e.,  $P^j Y_t^j = \sum_{k \in j} P^{k,j} Y_t^{k,j}$  where prices are those at the base year. Log-linearizing  $P^j Y_t^j = \sum_{k \in j} P^{k,j} Y_t^{k,j}$  in the neighborhood of the steady-state leads

to:

$$\begin{aligned}
P^j Y^j \frac{dY_t^j}{Y^j} &= \sum_{k \in j} P^{k,j} Y^{k,j} \frac{dY_t^{k,j}}{Y^{k,j}}, \\
\frac{P^j Y^j}{PY_R} \hat{Y}_t^j &= \sum_{k \in j} \frac{P^{k,j} Y^{k,j}}{PY_R} \hat{Y}_t^{k,j}, \\
\omega^{Y,j} \hat{Y}_t^j &= \sum_{k \in j} \omega^{Y,k} \hat{Y}_t^{k,j},
\end{aligned} \tag{186}$$

where  $\omega^{Y,k} = \frac{P^{k,j} Y^{k,j}}{PY_R}$  averaged over 1970-2015 is the value added share at current prices of sub-sector  $k \in j$  which collapses (at the initial steady-state) to the value added share at constant prices as prices at the base year are prices at the initial steady-state. Note that  $\sum_j \sum_{k \in j} \omega^{Y,k} = 1$ . In multiplying the response of value added at constant prices in sub-sector  $k \in j$  by its value added share  $\omega^{Y,k,j}$ , we express the response of value added at constant prices in sub-sector  $k \in j$  in percentage point of GDP.

The response of TFP in the broad sector  $j$  is a weighted average of responses  $\text{TFP}_t^{k,j}$  of TFP in sub-sector  $k \in j$  where the weight collapses to the value added share of sub-sector  $k$ :

$$\begin{aligned}
\text{TFP}_t^j &= \sum_{k \in j} \frac{P^{k,j} Y^{k,j}}{P^j Y^j} \hat{\text{TFP}}_t^{k,j}, \\
\frac{P^j Y^j}{PY_R} \text{TFP}_t^j &= \sum_{k \in j} \frac{P^{k,j} Y^{k,j}}{PY_R} \hat{\text{TFP}}_t^{k,j}, \\
\omega^{Y,j} \text{TFP}_t^j &= \sum_{k \in j} \omega^{Y,k} \hat{\text{TFP}}_t^{k,j},
\end{aligned} \tag{187}$$

where  $\omega^{Y,k} = \frac{P^{k,j} Y^{k,j}}{PY_R}$  averaged over 1970-2015.

**Empirical results.** The first and third columns of Fig. 17 show results for sub-sectors classified in the traded sector. Overall, all traded industries behave as the broad traded sector. More specifically, as shown in Fig. 17(a), hours worked increase slightly in the short-run in traded sub-sectors. As can be seen in Fig. 17(c), the labor share falls in all traded industries, except in Mining, while Manufacturing contributes the most to the decline in the traded goods-sector share of total hours worked. As shown in Fig. 17(e), value added at constant prices increase in all traded industries in the short-run. Fig. 17(g) shows that Manufacturing experiences the greatest decline in its value added share which somewhat balance out with the increase experienced by industries such as Finance, Mining, Transport and Communication. Importantly, Fig. 17(i) shows that all traded industries experience an increase in their TFP. This result lends some credence to our classification of traded industries and also reveals that the rise in TFP in the traded sector is driven by a rise in TFP within each sub-sector.

The second and fourth columns show results for sub-sectors classified in the non-traded sector. As shown in Fig. 17(b), except for 'Hotels and Restaurants', 'Electricity, Gas, Water Supply', all non-traded sub-sectors experience a significant rise in hours worked. As can be seen in Fig. 17(d), the significant rise in the labor share of non-tradables is driven by the 'Community Social and Personal Services' (i.e., the public sector which also includes health and education services) and next by 'Construction' together with 'Real Estate, Renting and Business Services'. Fig. 17(f) reveals that value added at constant prices increases in most of non-traded industries although the responses are somewhat muted in 'Hotels and Restaurants' and 'Electricity, Gas, Water Supply'. Fig. 17(h) shows that the value added share of non-tradables increases in the public sector and Construction while it declines in all remaining non-traded industries, especially in 'Wholesale and Retail Trade'. Fig. 17(j) reveals that the responses of non-traded TFP are muted across all non-traded industries, in accordance with its response at the broad sector level. Finally, Fig. 17(k) shows that the responses of the LIS in traded sub-sectors are fairly muted except for the LIS of 'Manufacturing' which declines significantly. When we turn to the non-traded

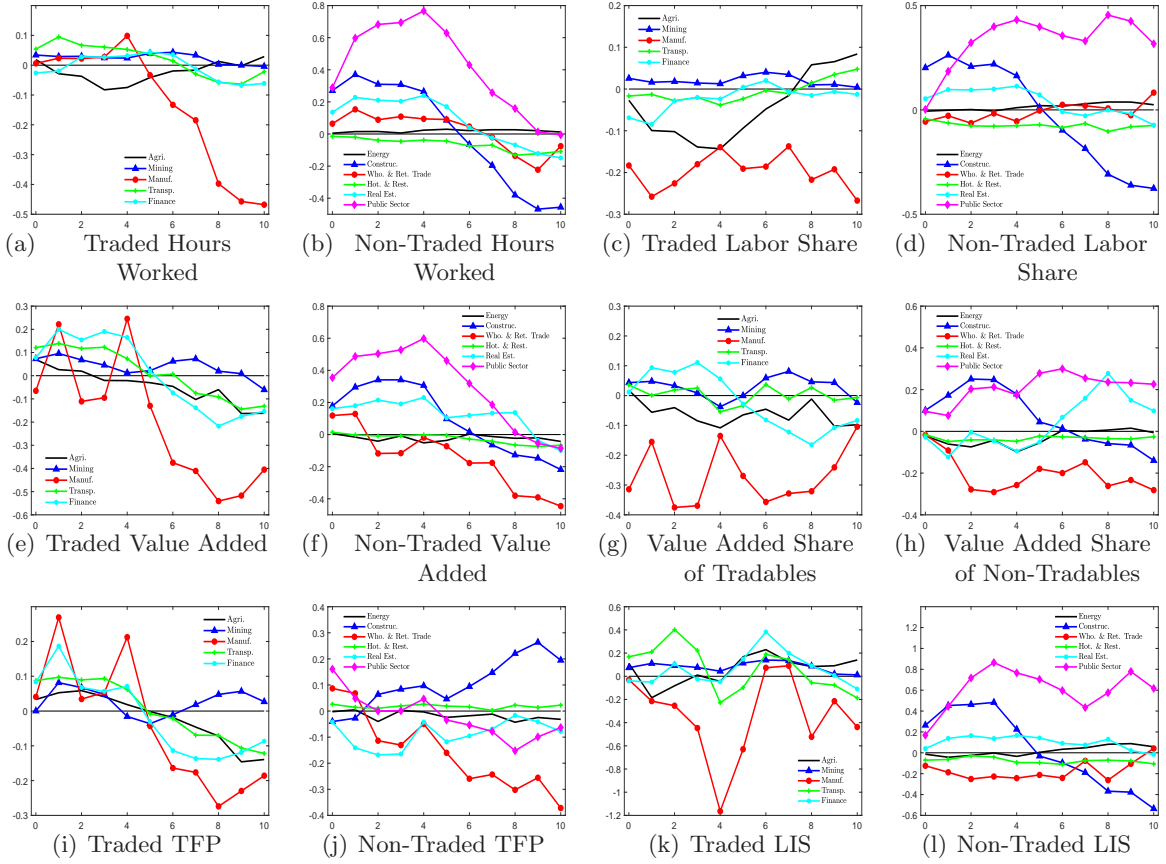


Figure 17: Effects of Exogenous Government Spending Shock on Sub-Sectors<sub>Notes</sub>: Because the traded and non-traded sector are made up of industries, we conduct a decomposition of the sectoral effects at a sub-sector level following an exogenous increase in government consumption final expenditure by 1% of GDP. To quantify the contribution of each industry to the change in the sectoral variable of the corresponding broad sector, we estimate the responses of each sub-sector variable to the identified government spending shock by using the Jordà's [2005] single-equation method. To express the results in meaningful units, i.e., in GDP units or total hours worked units, we multiply the responses of value added at constant prices and value added share at constant prices of sub-sector  $k$  by its share in GDP (at current prices), and we multiply the responses of hours worked and labor share of sub-sector  $k$  by its labor compensation share. The first/third columns show results for sub-sectors classified in the traded sector. The black line shows results for 'Agriculture', the blue line with triangles for 'Mining and Quarrying', the red line with circles for 'Manufacturing', the green line with a plus for 'Transport and Communication', and the cyan line with a circle for 'Financial Intermediation'. The second/fourth columns show results for sub-sectors classified in the non-traded sector. The black line shows results for 'Electricity, Gas and Water Supply', the blue line with triangles for 'Construction', the red line with circles for 'Wholesale and Retail Trade', the green line with a plus for 'Hotels and Restaurants', the cyan line with a circle for 'Real Estate, Renting, and Business Services', and the line in magenta with diamond for 'Community Social and Personal Services'. Horizontal axes indicate years. Vertical axes measure percentage deviation from trend in GDP units (sectoral value added, sectoral value added share, labor income share), percentage deviation from trend in total hours worked units (sectoral hours worked, labor share), percentage deviation from trend (sectoral TFPs). Sample: 18 OECD countries, 1970-2015, annual data.

sector, Fig. 17(l) shows that the rise in the non-traded LIS is driven by increases of the LIS in 'Community Social and Personal Services' (i.e., the public sector which also includes health and education services), 'Construction' together with 'Real Estate, Renting and Business Services'.

## M.5 Comparison with CRC's [2020] Empirical Findings: Sample and Empirical Strategy

In this subsection, we run a robustness check with regard to the sample/period and the empirical strategy. A close empirical analysis to ours is that performed by Cardi, Restout and Claeys (henceforth CRC) [2020]. Like CRC [2020], we estimate a panel VAR on annual data and investigate the effects of a government spending shock (identified by adopting Blanchard and Perotti's [2002] method) on both traded and non-traded value added and hours worked. Yet, our empirical analysis differs in three major respects:

- **Sample.** First, regarding the sample, we use a panel of 18 OECD economies over 1970-2015 while CRC [2020] consider a sample of 16 OECD economies over 1970-2007. Note that we consider the sixteen OECD countries included in the CRC's

[2020] sample plus Korea and Portugal.

- **Empirical strategy.** Second, our empirical strategy differs along one major dimension. CRC [2020] estimate the effects of a shock to government consumption on sectoral variables by estimating a VAR model including government consumption (ordered first), sectoral value added at constant prices, sectoral hours worked, sectoral wage rate or a VAR model where sectoral variables are divided by their aggregate counterpart. In contrast to CRC [2020], we adopt a two-step approach where we identify the structural shock by using a Cholesky decomposition in which government spending is ordered before the other variables and trace out the dynamic effects by using Jordà's [2005] projection method. The advantage of our two-step approach is twofold. First, we estimate one unique VAR model and thus identify one unique structural shock. Second, the local project method has the advantage that it does not impose the dynamic restrictions implicitly embedded in VARs and can accommodate non-linearities in the response function.
- **Variables.** Third, CRC [2020] restrict their attention to the effects of a government spending shock on hours worked and value added while in the present paper, we explore empirically the impact of a rise in government consumption on both labor income shares and technological change as measured by TFP, capital-utilization adjusted TFP, capital-utilization adjusted FBTC. Importantly, we show that the endogenous response of technological change to a shock to government consumption can rationalize the differences in sectoral fiscal multipliers.

CRC [2020] estimate different VAR models while we estimate one unique VAR model which includes government consumption, real GDP, total hours worked, the real consumption wage and aggregate TFP where all quantities are divided by the working age population and all variables are logged. CRC [2020] explore the size of sectoral fiscal multipliers empirically by estimating a VAR model,  $[g_{it}, y_{it}^j, l_{it}^j, w_{C,it}^j]$ , which includes government consumption, value added at constant prices in sector  $j$ , hours worked in sector  $j$ , the real consumption wage in sector  $j$ , where all sectoral quantities are divided by population and all variables are logged. To estimate the change in the value added share of sector  $j$  and the response of the labor share in sector  $j$ , CRC [2020] estimate a VAR model where sectoral quantities are divided by their aggregate counterpart. While CRC [2020] do not estimate the effects of a rise in government spending on sectoral TFP, we also run a VAR model which includes government consumption (ordered first), traded TFP, non-traded TFP to investigate the extent to which the sample and the method matter in determining the response of technology. As mentioned above, our two-step method has the advantage to estimate the dynamic adjustment of sectoral variables to one unique government spending shock by adopting the local projection method which imposes fewer dynamic restrictions than those implicitly embedded in VARs.

In Fig. 18, we compare our empirical findings with those obtained when using the same sample (i.e., 16 OECD countries over 1970-2007) and/or adopting the same methodology as CRC [2020]. In the black line with squares, we report the results obtained in the main text; more specifically, we adopt the two-step approach detailed in length in the main text (i.e., we identify one unique government spending shock and trace out the dynamic responses of variables by using the local projection method) and consider a sample of 18 OECD countries over 1970-2015. The dashed black line with stars shows results when we re-estimate the effects of a shock to government consumption by considering 16 OECD countries over the period 1970-2007, as considered by CRC [2020], and still adopt the two-step approach. Our conclusions remain unchanged. The most notable quantitative difference is the response of the non-traded LIS shown in Fig. 18(1) which reveals that the increase in  $s_L^N$  is twice as less as in the baseline scenario.

The solid blue line shows results of [2020] when we consider a sample of 16 OECD countries over the period 1970-2007 and estimate the government spending shock and the response of sectoral variables by considering different VAR models and thus potentially identifying different structural shocks, i.e., some VAR models could potentially identify a



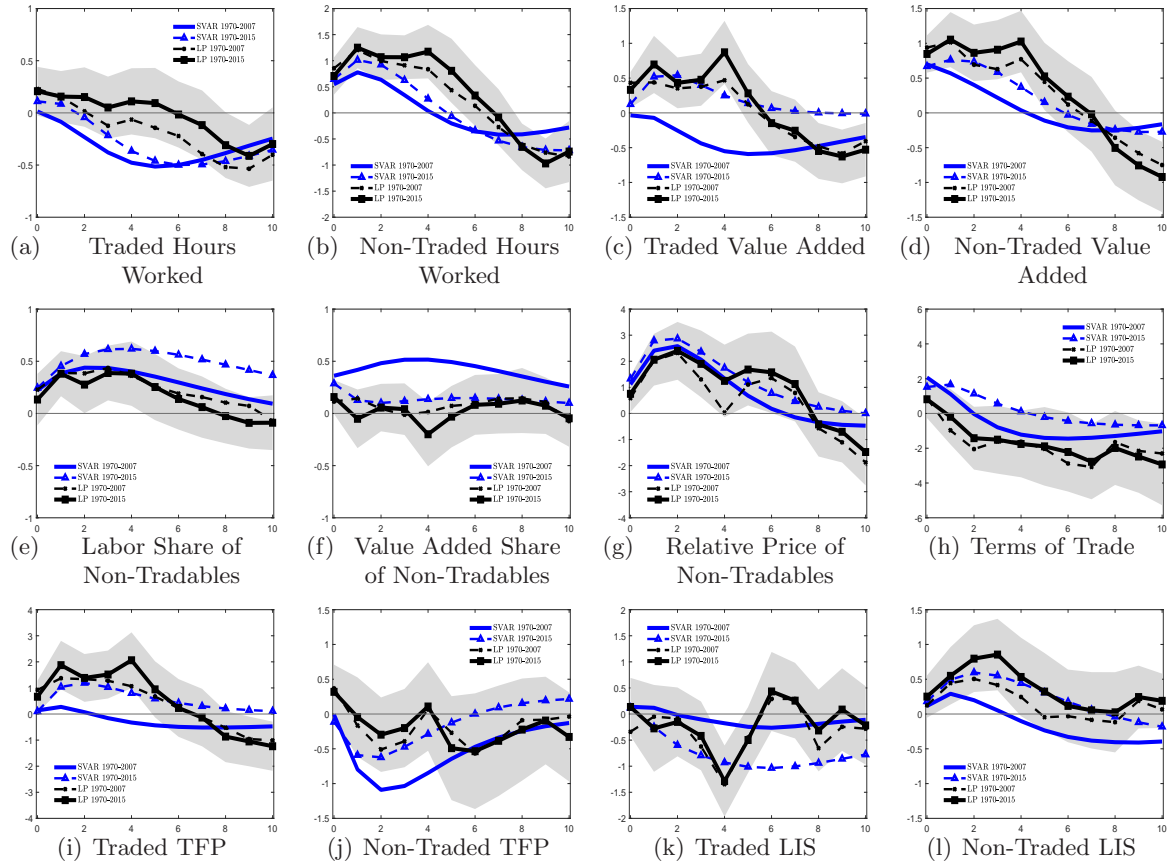


Figure 18: Sectoral Effects of a Shock to Government Consumption: Robustness Check

The solid black line with squares shows the response of sectoral variables to an exogenous increase in government final consumption expenditure by 1% of GDP. Shaded areas indicate the 90 percent confidence bounds. While in the baseline scenario shown in the black line with squares, the period is running from 1970 to 2015 and the sample includes 18 OECD countries, the dashed black line shows the effects before the Great Recession, i.e., over the period 1970-2007 and for 16 OECD countries. In both cases, we estimate the dynamic responses to a shock to government consumption by adopting a two-step method. In the first step, the government spending shock is identified by estimating a VAR model that includes real government final consumption expenditure, real GDP, total hours worked, the real consumption wage, and aggregate TFP. In the second step, we estimate the effects by using Jordà's [2005] single-equation method. Horizontal axes indicate years. Vertical axes measure percentage deviation from trend in GDP units (sectoral value added, sectoral value added share, labor income share), percentage deviation from trend in total hours worked units (sectoral hours worked, labor share), percentage deviation from trend (sectoral TFPs, relative prices). The blue lines show the effects of shock to government consumption when we identify and generate empirical IRFs by running a VAR model which includes sectoral variables. The solid blue line shows the sectoral effects when estimating the VAR model over 1970-2007 for 16 OECD countries while the dashed blue line with triangles shows estimates when we estimate the same VAR model over 1970-2015 for 18 OECD countries. The solid blue line shows results obtained by Cardì, Restout and Claeys [2020]. Sample: 18 OECD countries, 1970-2015, annual data.

shock to government spending which is more biased toward non-tradables. To investigate the extent of the role of the sample in driving the empirical findings, we show results in the dashed blue line with triangles when we consider a sample of 18 OECD countries over 1970-2015. As shown in the first row of Fig. 18, adopting a VAR methodology instead of local projection method tends to mitigate the rise in hours worked and in value added at constant prices. As shown in the solid blue line in Fig. 18(f), a shock to government consumption has a strong and significant impact on the value added share which suggests that the shock is strongly biased toward non-tradables. We can notice that traded TFP remains unchanged while non-traded TFP declines as can be seen in Fig. 18(i) and Fig. 18(j), respectively, when considering 16 OECD countries over 1970-2007 (see the solid blue line). The fall in non-traded TFP is not large enough to overturn the positive impact on  $\nu^{Y,N}(t)$  driven by the biasedness of the government spending shock toward non-tradables.

In contrast, when we consider the sample of 18 OECD countries over 1970-2015, the rise in the value added share of non-tradables vanishes, as shown in the dashed blue line. The reason is that as shown in Fig. 18(i), traded TFP increases which neutralizes the impact of the biasedness of the spending shock toward non-tradables. Because traded TFP increases less when adopting the VAR methodology (see both the solid and dashed blue lines in Fig. 18(i)), the terms of trade appreciate more in the short-run as can be seen in Fig. 18(h). As displayed by the blue lines in Fig. 18(g) the relative price of non-tradables appreciates more when we estimate the dynamic effects by using a VAR instead of local projection which suggests that the VAR methodology picks up government spending shocks which are more strongly biased toward non-tradables.

The labor share of non-tradables increases significantly in the VAR approach, regardless of the sample, as can be seen in the blue lines in Fig. 18(e). We can notice that the rise in  $\nu^{L,N}(t)$  is more pronounced when we consider the sample of 18 OECD countries over 1970-2015, as can be seen in the dashed blue line. The reason is that the responses of LISs in the traded and non-traded sector shown in Fig. 18(k) and Fig. 18(l) are muted at any horizon when considering 16 OECD countries over 1970-2007 (see the solid blue line) while considering a sample of 18 OECD countries over 1970-2015 leads  $s_L^H$  to decline and  $s_L^N$  to increase significantly, thus amplifying the rise in the demand for labor in the non-traded relative to the traded sector.

In conclusion, we can notice some important differences with the empirical findings reported by CRC [2020]. First, we find that the empirical strategy adopted by CRC [2020] tends to lead the authors to pick up a shock to government consumption which is more strongly biased toward non-tradables, thus leading the value added share of non-tradables to increase more. Second, the sample also plays a role. It appears that both traded TFP and the non-traded labor income share increase less when we consider a sample of 16 OECD countries over period 1970-2007 than in a sample of 18 OECD countries over 1970-2015. Third, both the VAR method and the sample tend to generate a fall in non-traded TFP and understate the rise in traded TFP which can rationalize the smaller increase in both traded and non-traded value added. Fourth, it stands out that the non-traded LIS increases much less when we consider a sample of 16 OECD countries over 1970-2007 instead of a sample of 18 OECD countries over 1970-2015. Because non-traded firms do not bias technological change toward labor, non-traded hours worked increase much less in CRC [2020].

## N Semi-Small Open Economy Model

This Appendix puts forward an open economy version of the neoclassical model with tradables and non-tradables, imperfect mobility of labor across sectors, capital adjustment costs and endogenous terms of trade. This section illustrates in detail the steps we follow in solving this model. We assume that production functions take a Cobb-Douglas form since this economy is the reference model for our calibration as we normalize CES productions by assuming that the initial steady state of the Cobb-Douglas economy is the normalization point.

Households supply labor,  $L$ , and must decide on the allocation of total hours worked between the traded sector,  $L^H$ , and the non-traded sector,  $L^N$ . They consume both traded,

$C^T$ , and non-traded goods,  $C^N$ . Traded goods are a composite of home-produced traded goods,  $C^H$ , and foreign-produced foreign (i.e., imported) goods,  $C^F$ . Households also choose investment which is produced using inputs of the traded,  $J^T$ , and the non-traded good,  $J^N$ . As for consumption, input of the traded good is a composite of home-produced traded goods,  $J^H$ , and foreign imported goods,  $J^F$ . The numeraire is the foreign good whose price,  $P^F$ , is thus normalized to one. While households choose the utilization of the stock of physical and intangible capital, firms decide about the mix of labor- and capital-augmenting productivity.

## N.1 Households

At each instant of time, the representative household consumes traded and non-traded goods denoted by  $C^T$  and  $C^N$ , respectively, which are aggregated by means of a CES function:

$$C = \left[ \varphi^{\frac{1}{\phi}} (C^T)^{\frac{\phi-1}{\phi}} + (1-\varphi)^{\frac{1}{\phi}} (C^N)^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}}, \quad (188)$$

where  $0 < \varphi < 1$  is the weight of the traded good in the overall consumption bundle and  $\phi$  corresponds to the elasticity of substitution between traded goods and non-traded goods. The index  $C^T$  is defined as a CES aggregator of home-produced traded goods,  $C^H$ , and foreign-produced traded goods,  $C^F$ :

$$C^T = \left[ (\varphi_H)^{\frac{1}{\rho}} (C^H)^{\frac{\rho-1}{\rho}} + (1-\varphi_H)^{\frac{1}{\rho}} (C^F)^{\frac{\rho-1}{\rho}} \right]^{\frac{\rho}{\rho-1}}, \quad (189)$$

where  $0 < \varphi_H < 1$  is the weight of the home-produced traded good in the overall traded consumption bundle and  $\rho$  corresponds to the elasticity of substitution between home-produced traded goods and foreign-produced traded goods.

As in De Cordoba and Kehoe [2000], the investment good is produced using inputs of the traded good and the non-traded good according to a constant-returns-to-scale function which is assumed to take a CES form:

$$J = \left[ \iota^{\frac{1}{\phi_J}} (J^T)^{\frac{\phi_J-1}{\phi_J}} + (1-\iota)^{\frac{1}{\phi_J}} (J^N)^{\frac{\phi_J-1}{\phi_J}} \right]^{\frac{\phi_J}{\phi_J-1}}, \quad (190)$$

where  $\iota$  is the weight of the investment traded input ( $0 < \iota < 1$ ) and  $\phi_J$  corresponds to the elasticity of substitution in investment between traded and non-traded inputs. The index  $J^T$  is defined as a CES aggregator of home-produced traded inputs,  $J^H$ , and foreign-produced traded inputs,  $J^F$ :

$$J^T = \left[ (\iota_H)^{\frac{1}{\rho_J}} (J^H)^{\frac{\rho_J-1}{\rho_J}} + (1-\iota_H)^{\frac{1}{\rho_J}} (J^F)^{\frac{\rho_J-1}{\rho_J}} \right]^{\frac{\rho_J}{\rho_J-1}}, \quad (191)$$

where  $0 < \iota_H < 1$  is the weight of the home-produced traded in input in the overall traded investment bundle and  $\rho_J$  corresponds to the elasticity of substitution between home- and foreign-produced traded inputs.

Following Horvath [2000], we assume that hours worked in the traded and the non-traded sectors are aggregated by means of a CES function:

$$L = \left[ \vartheta^{-1/\epsilon} (L^H)^{\frac{\epsilon+1}{\epsilon}} + (1-\vartheta)^{-1/\epsilon} (L^N)^{\frac{\epsilon+1}{\epsilon}} \right]^{\frac{\epsilon}{\epsilon+1}}, \quad (192)$$

where  $0 < \vartheta < 1$  is the weight of labor supply to the traded sector in the labor index  $L(\cdot)$  and  $\epsilon$  measures the ease with which hours worked can be substituted for each other and thereby captures the degree of labor mobility across sectors.

The representative agent is endowed with one unit of time, supplies a fraction  $L(t)$  as labor, and consumes the remainder  $1 - L(t)$  as leisure. At any instant of time, households derive utility from their consumption and experience disutility from working. Assuming that

the felicity function is additively separable in consumption and labor, the representative household maximizes the following objective function:

$$U = \int_0^\infty \left\{ \frac{1}{1 - \frac{1}{\sigma_C}} C(t)^{1 - \frac{1}{\sigma_C}} - \frac{\gamma}{1 + \frac{1}{\sigma_L}} L(t)^{1 + \frac{1}{\sigma_L}} \right\} e^{-\beta t} dt, \quad (193)$$

where  $\beta > 0$  is the discount rate,  $\sigma_C > 0$  the intertemporal elasticity of substitution for consumption, and  $\sigma_L > 0$  the Frisch elasticity of (aggregate) labor supply.

We assume that the households own the physical capital stock and choose the level of capital utilization  $u^{K,j}(t)$ . The households also own the intangible stock of capital  $\bar{Z}^j(t)$  and choose the level of utilization of existing technology  $u^{Z,j}(t)$ , i.e.,  $Z^j(t) = u^{Z,j}(t)\bar{Z}^j$ . We further assume that the technology utilization rate is Hicks-neutral. In the sequel, we normalize the stock of knowledge,  $\bar{Z}^j$ , to one as we abstract from endogenous choices on the stock of knowledge. Households lease capital services (the product of utilization and physical capital) to firms in sector  $j$  at rental rate  $R(t)$  augmented with the technology utilization rate, i.e.,  $R(t)u^{Z,j}(t)$ . Thus capital income received by households reads  $\sum_j u^{Z,j}(t)R(t)u^{K,j}(t)K^j(t)$ . Households supply labor services to firms in sector  $j$  at a wage rate  $W^j(t)$  augmented with  $u^{Z,j}(t)$ , i.e.,  $u^{Z,j}(t)W^j(t)$ . Thus labor income received by households reads  $\sum_j u^{Z,j}(t)W^j(t)L^j(t)$ . In addition, households accumulate internationally traded bonds,  $N(t)$ , that yield net interest rate earnings of  $r^*N(t)$ . Denoting lump-sum taxes by  $T(t)$ , households' flow budget constraint states that real disposable income can be saved by accumulating traded bonds, consumed,  $P_C(t)C(t)$ , invested,  $P_J(t)J(t)$ , and covers the capital and technology utilization costs:

$$\begin{aligned} \dot{N}(t) &= r^*N(t) + [\alpha_K(t)u^{K,H}(t)u^{Z,H}(t) + (1 - \alpha_K(t))u^{K,N}(t)u^{Z,N}(t)] R(t)K(t) \\ &+ [\alpha_L(t)u^{Z,H}(t) + (1 - \alpha_L(t))u^{Z,N}(t)] W(t)L(t) - T(t) - P_C(t)C(t) - P_J(t)J(t) \\ &- P^H(t)C^{K,H}(t)\alpha_K(t)K(t) - P^N(t)C^{K,N}(t)(1 - \alpha_K(t))K(t) \\ &- P^H(t)C^{Z,H}(t) - P^N(t)C^{Z,N}(t), \end{aligned} \quad (194)$$

where we denote the share of traded capital in the aggregate capital stock by  $\alpha_K(t) = K^H(t)/K(t)$  and the labor compensation share of tradables by  $\alpha_L(t) = \frac{W^H(t)L^H(t)}{W(t)L(t)}$  defined below.

The role of the capital utilization rate is to mitigate the effect of a rise in the capital cost. Symmetrically, the role of the technology utilization rate is to dampen the effects of increased costs of factors of production. We let the function  $C^{K,j}(t)$  and  $C^{Z,j}(t)$  denote the adjustment costs associated with the choice of capital and technology utilization rates which are increasing and convex functions of utilization rates  $u^{K,j}(t)$  and  $u^{Z,j}(t)$ :

$$C^{K,j}(t) = \xi_1^j (u^{K,j}(t) - 1) + \frac{\xi_2^j}{2} (u^{K,j}(t) - 1)^2, \quad (195a)$$

$$C^{Z,j}(t) = \chi_1^j (u^{Z,j}(t) - 1) + \frac{\chi_2^j}{2} (u^{Z,j}(t) - 1)^2, \quad (195b)$$

where  $\xi_2^j > 0$ ,  $\chi_2^j$  are free parameters; as  $\xi_2 \rightarrow \infty$ ,  $\chi_2 \rightarrow \infty$ , utilization is fixed at unity;  $\xi_1^j$ ,  $\chi_1^j$  must be restricted so that the optimality conditions are consistent with the normalization of steady state utilization of 1.

Capital accumulation evolves as follows:

$$\dot{K}(t) = I(t) - \delta_K K(t), \quad (196)$$

where  $I$  is investment and  $0 \leq \delta_K < 1$  is a fixed depreciation rate. We assume that capital accumulation is subject to increasing and convex cost of net investment:

$$J(t) = I(t) + \Psi(I(t), K(t)) K(t), \quad (197)$$

where  $\Psi(\cdot)$  is increasing (i.e.,  $\Psi'(\cdot) > 0$ ), convex (i.e.,  $\Psi''(\cdot) > 0$ ), is equal to zero at  $\delta_K$  (i.e.,  $\Psi(\delta_K) = 0$ ), and has first partial derivative equal to zero as well at  $\delta_K$  (i.e.,  $\Psi'(\delta_K) = 0$ ). We suppose the following functional form for the adjustment cost function:

$$\Psi(I(t), K(t)) = \frac{\kappa}{2} \left( \frac{I(t)}{K(t)} - \delta_K \right)^2. \quad (198)$$

Using (198), partial derivatives of total investment expenditure are:

$$\frac{\partial J(t)}{\partial I(t)} = 1 + \kappa \left( \frac{I(t)}{K(t)} - \delta_K \right), \quad (199a)$$

$$\frac{\partial J(t)}{\partial K(t)} = -\frac{\kappa}{2} \left( \frac{I(t)}{K(t)} - \delta_K \right) \left( \frac{I(t)}{K(t)} + \delta_K \right). \quad (199b)$$

Denoting the co-state variables associated with (194) and (196) by  $\lambda$  and  $Q'$ , respectively, the first-order conditions characterizing the representative household's optimal plans are:

$$(C(t))^{-\frac{1}{\sigma_C}} = P_C(t)\lambda(t), \quad (200a)$$

$$\gamma(L(t))^{\frac{1}{\sigma_L}} = \lambda(t) [\alpha_L(t)u^{Z,H}(t) + (1 - \alpha_L(t))u^{Z,N}(t)]W(t), \quad (200b)$$

$$Q(t) = P_J(t) \left[ 1 + \kappa \left( \frac{I(t)}{K(t)} - \delta_K \right) \right], \quad (200c)$$

$$\dot{\lambda}(t) = \lambda(\beta - r^*), \quad (200d)$$

$$\begin{aligned} \dot{Q}(t) = (r^* + \delta_K)Q(t) - \left\{ [\alpha_K(t)u^{K,H}(t)u^{Z,H}(t) + (1 - \alpha_K(t))u^{K,N}(t)u^{Z,N}(t)] \right. \\ \left. - P^H(t)C^{K,H}(t)\alpha_K(t) - P^N(t)C^{K,N}(t)(1 - \alpha_K(t)) + P_J(t)\frac{\kappa}{2} \left( \frac{I(t)}{K(t)} - \delta_K \right) \left( \frac{I(t)}{K(t)} + \delta_K \right) \right\}, \end{aligned} \quad (200e)$$

$$R(t)u^{Z,H}(t) = P^H(t) [\xi_1^H + \xi_2^H (u^{K,H}(t) - 1)], \quad (200f)$$

$$R(t)u^{Z,N}(t) = P^N(t) [\xi_1^N + \xi_2^N (u^{K,N}(t) - 1)], \quad (200g)$$

$$R(t)u^{K,H}(t)K^H(t) + W^H(t)L^H(t) = P^H(t) [\chi_1^H + \chi_2^H (u^{Z,H}(t) - 1)], \quad (200h)$$

$$R(t)u^{K,N}(t)K^N(t) + W^N(t)L^N(t) = P^N(t) [\chi_1^N + \chi_2^N (u^{Z,N}(t) - 1)], \quad (200i)$$

and the transversality conditions  $\lim_{t \rightarrow \infty} \bar{\lambda}N(t)e^{-\beta t} = 0$  and  $\lim_{t \rightarrow \infty} Q(t)K(t)e^{-\beta t} = 0$ ; to derive (200c) and (200e), we used the fact that  $Q(t) = Q'(t)/\lambda(t)$ .

Given the above consumption indices, we can derive appropriate price indices. With respect to the general consumption index, we obtain the consumption-based price index  $P_C$ :

$$P_C = \left[ \varphi (P^T)^{1-\phi} + (1 - \varphi) (P^N)^{1-\phi} \right]^{\frac{1}{1-\phi}}, \quad (201)$$

where the price index for traded goods is:

$$P^T = \left[ \varphi_H (P^H)^{1-\rho} + (1 - \varphi_H) \right]^{\frac{1}{1-\rho}}. \quad (202)$$

Given the consumption-based price index (201), the representative household has the following demand of traded and non-traded goods:

$$C^T = \varphi \left( \frac{P^T}{P_C} \right)^{-\phi} C, \quad (203a)$$

$$C^N = (1 - \varphi) \left( \frac{P^N}{P_C} \right)^{-\phi} C. \quad (203b)$$

Given the price indices (201) and (202), the representative household has the following demand of home-produced traded goods and foreign-produced traded goods:

$$C^H = \varphi \left( \frac{P^T}{P_C} \right)^{-\phi} \varphi_H \left( \frac{P^H}{P^T} \right)^{-\rho} C, \quad (204a)$$

$$C^F = \varphi \left( \frac{P^T}{P_C} \right)^{-\phi} (1 - \varphi_H) \left( \frac{1}{P^T} \right)^{-\rho} C. \quad (204b)$$

As will be useful later, the percentage change in the consumption price index is a weighted average of percentage changes in the price of traded and non-traded goods in

terms of foreign goods:

$$\hat{P}_C = \alpha_C \hat{P}^T + (1 - \alpha_C) \hat{P}^N, \quad (205a)$$

$$\hat{P}^T = \alpha_H \hat{P}^H, \quad (205b)$$

where  $\alpha_C$  is the tradable content of overall consumption expenditure and  $\alpha^H$  is the home-produced goods content of consumption expenditure on traded goods:

$$\alpha_C = \varphi \left( \frac{P^T}{P_C} \right)^{1-\phi}, \quad (206a)$$

$$1 - \alpha_C = (1 - \varphi) \left( \frac{P^N}{P_C} \right)^{1-\phi}, \quad (206b)$$

$$\alpha^H = \varphi_H \left( \frac{P^H}{P^T} \right)^{1-\rho}, \quad (206c)$$

$$1 - \alpha^H = (1 - \varphi_H) \left( \frac{1}{P^T} \right)^{1-\rho}. \quad (206d)$$

Given the CES aggregator functions above, we can derive the appropriate price indices for investment. With respect to the general investment index, we obtain the investment-based price index  $P_J$ :

$$P_J = \left[ \iota (P_J^T)^{1-\phi_J} + (1 - \iota) (P^N)^{1-\phi_J} \right]^{\frac{1}{1-\phi_J}}, \quad (207)$$

where the price index for traded goods is:

$$P_J^T = \left[ \iota^H (P^H)^{1-\rho_J} + (1 - \iota^H) \right]^{\frac{1}{1-\rho_J}}. \quad (208)$$

Given the investment-based price index (207), we can derive the demand for inputs of the traded good and the non-traded good:

$$J^T = \iota \left( \frac{P_J^T}{P_J} \right)^{-\phi_J} J, \quad (209a)$$

$$J^N = (1 - \iota) \left( \frac{P^N}{P_J} \right)^{-\phi_J} J. \quad (209b)$$

Given the price indices (207) and (208), we can derive the demand for inputs of home-produced traded goods and foreign-produced traded goods:

$$J^H = \iota \left( \frac{P_J^T}{P_J} \right)^{-\phi_J} \iota^H \left( \frac{P^H}{P_J^T} \right)^{-\rho_J} J, \quad (210a)$$

$$J^F = \iota \left( \frac{P_J^T}{P_J} \right)^{-\phi_J} (1 - \iota^H) \left( \frac{1}{P_J^T} \right)^{-\rho_J} J. \quad (210b)$$

As will be useful later, the percentage change in the investment price index is a weighted average of percentage changes in the price of traded and non-traded inputs in terms of foreign inputs:

$$\hat{P}_J = \alpha_J \hat{P}_J^T + (1 - \alpha_J) \hat{P}^N, \quad (211a)$$

$$\hat{P}_J^T = \alpha_J^H \hat{P}^H, \quad (211b)$$

where  $\alpha_J$  is the tradable content of overall investment expenditure and  $\alpha_J^H$  is the home-



produced goods content of investment expenditure on traded goods:

$$\alpha_J = \iota \left( \frac{P_J^T}{P_J} \right)^{1-\phi_J}, \quad (212a)$$

$$1 - \alpha_J = (1 - \iota) \left( \frac{P_J^N}{P_J} \right)^{1-\phi_J}, \quad (212b)$$

$$\alpha_J^H = \iota^H \left( \frac{P_J^H}{P_J^T} \right)^{1-\rho_J}, \quad (212c)$$

$$1 - \alpha_J^H = (1 - \iota^H) \left( \frac{1}{P_J^T} \right)^{1-\rho_J}. \quad (212d)$$

Before deriving the allocation of hours worked across sectors, it is convenient to rewrite the optimal decision for aggregate labor supply described by eq. (200b). As shall be useful, we denote sectoral wages including technology utilization rates with a tilde, i.e.,  $\tilde{W}^j(t) = u^{Z,j}(t)W^j(t)$ . Multiplying both sides of (200b) by  $L(t)$  and denoting by  $\tilde{W}(t)$  the aggregate wage index inclusive of technology utilization leads to:

$$\begin{aligned} \gamma(L(t))^{\frac{1}{\sigma_L}+1} &= \lambda(t) [\alpha_L(t)u^{Z,H}(t) + (1 - \alpha_L(t))u^{Z,N}(t)] W(t)L(t), \\ \gamma(L(t))^{\frac{1}{\sigma_L}+1} &= \lambda(t) [W^H(t)u^{Z,H}(t)L^H(t) + W^N(t)u^{Z,N}(t)L^N(t)], \\ \gamma(L(t))^{\frac{1}{\sigma_L}+1} &= \lambda(t) [\tilde{W}^H(t)L^H(t) + \tilde{W}^N(t)L^N(t)], \end{aligned}$$

where we used the definition of the labor compensation share of tradables and non-tradables, i.e.,  $\alpha_L(t)W(t)L(t) = W^H(t)L^H(t)$  and  $(1 - \alpha_L(t))W(t)L(t) = W^N(t)L^N(t)$ . Dividing both sides of the above equation by  $L(t)$  and using the definition of the aggregate wage index which includes technology utilization rates enables us to rewrite eq. (200b) as follows:

$$\gamma(L(t))^{\frac{1}{\sigma_L}} = \lambda(t)\tilde{W}(t). \quad (213)$$

The aggregate wage index,  $\tilde{W}(t)$ , associated with the labor index defined above (192) is:

$$\tilde{W}(t) = \left[ \vartheta \left( \tilde{W}^H(t) \right)^{\epsilon+1} + (1 - \vartheta) \left( \tilde{W}^N(t) \right)^{\epsilon+1} \right]^{\frac{1}{\epsilon+1}}, \quad (214)$$

where  $\tilde{W}^H(t) = u^{Z,H}(t)W^H(t)$  and  $\tilde{W}^N(t) = u^{Z,N}(t)W^N(t)$  are wages paid in the traded and the non-traded sectors, respectively.

Given the aggregate wage index, we can derive the allocation of aggregate labor supply to the traded and the non-traded sector:

$$L^H = \vartheta \left( \frac{\tilde{W}^H(t)}{\tilde{W}(t)} \right)^{\epsilon} L(t), \quad (215a)$$

$$L^N = (1 - \vartheta) \left( \frac{\tilde{W}^N(t)}{\tilde{W}(t)} \right)^{\epsilon} L(t). \quad (215b)$$

As will be useful later, log-linearizing the aggregate wage index in the neighborhood of the initial steady-state leads to:

$$\hat{\tilde{W}}(t) = \alpha_L \hat{\tilde{W}}^H(t) + (1 - \alpha_L) \hat{\tilde{W}}^N(t), \quad (216)$$

where  $\alpha_L$  is the tradable content of aggregate labor compensation:

$$\alpha_L = \vartheta \left( \frac{W^H}{W} \right)^{1+\epsilon}, \quad (217a)$$

$$1 - \alpha_L = (1 - \vartheta) \left( \frac{W^N}{W} \right)^{1+\epsilon}. \quad (217b)$$

Note that because we log-linearize in the neighborhood of the steady-state, the labor compensation share,  $\tilde{\alpha}_L$ , inclusive of the technology utilization rate collapses to the technology utilization adjusted labor compensation share,  $\alpha_L$ .

## N.2 Firms

We denote the value added in sector  $j = H, N$  by  $Y^j$ . When we add a tilde, it means that value added is inclusive of the technology utilization rate. Both the traded and non-traded sectors use physical capital inclusive of capital utilization,  $\tilde{K}^j(t) = u^{K,j}(t)K^j(t)$ , and labor,  $L^j$ , according to constant returns to scale production functions which are assumed to take a Cobb-Douglas form. We allow for labor- and capital-augmenting productivity denoted by  $\tilde{A}^j(t)$  and  $\tilde{B}^j(t)$ . We assume that factor-augmenting productivity has a symmetric time-varying component denoted by  $u^{Z,j}(t)$  such that  $\tilde{A}^j(t) = u^{Z,j}(t)A^j(t)$  and  $\tilde{B}^j(t) = u^{Z,j}(t)B^j(t)$ . Since factor-augmenting productivity has no impact when considering Cobb-Douglas production function, we assume  $\bar{Z}^j = (A^j)^{\theta^j} (B^j)^{1-\theta^j}$  and as mentioned above, we normalize  $\bar{Z}^j$  to one. The production function of sector  $j$  reads as follows:

$$\begin{aligned}\tilde{Y}^j(t) &= u^{Z,j}(t)Y^j(t), \\ &= u^{Z,j}(t) (L^j(t))^{\theta^j} \left(\tilde{K}^j(t)\right)^{1-\theta^j},\end{aligned}\quad (218)$$

where  $\theta^j$  is the labor income share in sector  $j$ .

Firms face two cost components: a capital rental cost equal to  $\tilde{R}^j(t) = R(t)u^{Z,j}(t)$ , and a labor cost equal to the wage rate  $\tilde{W}^j(t) = W^j(t)u^{Z,j}(t)$ , both inclusive of technology utilization. Both sectors are assumed to be perfectly competitive and thus choose capital services and labor by taking prices as given:

$$\max_{\tilde{K}^j(t), L^j(t)} \Pi^j(t) = \max_{\tilde{K}^j(t), L^j(t)} \left\{ P^j(t)\tilde{Y}^j(t) - \tilde{W}^j(t)L^j(t) - \tilde{R}^j(t)\tilde{K}^j(t) \right\}. \quad (219)$$

The first order conditions of the firm problem are:

$$P^j(t)\theta^j u^{Z,j}(t) \left(\tilde{k}^j(t)\right)^{1-\theta^j} \equiv \tilde{W}^j(t), \quad (220a)$$

$$P^j(t) (1 - \theta^j) u^{Z,j}(t) \left(\tilde{k}^j(t)\right)^{-\theta^j} = \tilde{R}^j(t), \quad (220b)$$

where  $\tilde{k}^j(t) = \frac{\tilde{K}^j(t)}{L^j(t)}$  is the capital-labor ratio in sector  $j$ . By using the definition of  $\tilde{W}^j(t)$  and  $\tilde{R}^j(t)$ , the technology utilization rate vanishes from first-order conditions. Since capital can move freely between the two sectors, the value of marginal products in the traded and non-traded sectors equalizes while costly labor mobility implies a wage differential across sectors:

$$P^H(t) (1 - \theta^H) (u^{K,H}(t)k^H(t))^{-\theta^H} = P^N(t) (1 - \theta^N) (u^{K,N}(t)k^N(t))^{-\theta^N} \equiv R(t), \quad (221a)$$

$$P^H(t)\theta^H (u^{K,H}(t)k^H(t))^{1-\theta^H} \equiv W^H(t), \quad (221b)$$

$$P^N(t)\theta^N (u^{K,N}(t)k^N(t))^{1-\theta^N} \equiv W^N(t). \quad (221c)$$

The resource constraint for capital is:

$$K^H(t) + K^N(t) = K(t). \quad (222)$$

## N.3 Solving the Model

### Consumption and Labor

Before linearizing, we have to determine short-run solutions. First-order conditions (200a) and (200b) can be solved for consumption and aggregate labor supply which of course must hold at any point of time:

$$C = C(\bar{\lambda}, P^N, P^H), \quad L = L(\bar{\lambda}, \tilde{W}^H, \tilde{W}^N), \quad (223)$$

with partial derivatives given by

$$\hat{C} = -\sigma_C \hat{\lambda} - \sigma_C \alpha_C \alpha^H \hat{P}^H - \sigma_C (1 - \alpha_C) \hat{P}^N, \quad (224a)$$

$$\hat{L} = \sigma_L \hat{\lambda} + \sigma_L (1 - \alpha_L) \hat{W}^N + \sigma_L \alpha_L \hat{W}^H, \quad (224b)$$

where we have used (205) and (216).

Inserting first the solution for consumption (223) into (203b), (204a), (204b) enables us to solve for  $C^N$ ,  $C^H$ , and  $C^F$ :

$$C^N = C^N(\bar{\lambda}, P^N, P^H), \quad C^H = C^H(\bar{\lambda}, P^N, P^H), \quad C^F = C^F(\bar{\lambda}, P^N, P^H), \quad (225)$$

with partial derivatives given by

$$\begin{aligned} \hat{C}^N &= -\phi \hat{P}^N + (\phi - \sigma_C) \hat{P}_C - \sigma_C \hat{\lambda}, \\ &= -[\alpha_C \phi + (1 - \alpha_C) \sigma_C] \hat{P}^N + (\phi - \sigma_C) \alpha_C \alpha^H \hat{P}^H - \sigma_C \hat{\lambda}, \end{aligned} \quad (226a)$$

$$\hat{C}^H = -[\rho(1 - \alpha^H) + \phi(1 - \alpha_C) \alpha^H + \sigma_C \alpha_C \alpha^H] \hat{P}^H + (1 - \alpha_C) (\phi - \sigma_C) \hat{P}^N - \sigma_C \hat{\lambda} \quad (226b)$$

$$\hat{C}^F = \alpha^H [\rho - \phi(1 - \alpha_C) - \sigma_C \alpha_C] \hat{P}^H + (1 - \alpha_C) (\phi - \sigma_C) \hat{P}^N - \sigma_C \hat{\lambda}. \quad (226c)$$

Inserting first the solution for labor (223) into (215a)-(215b) allows us to solve for  $L^H$  and  $L^N$ :

$$L^H = L^H(\bar{\lambda}, W^H, W^N, u^{Z,H}, u^{Z,N}), \quad L^N = L^N(\bar{\lambda}, W^H, W^N, u^{Z,H}, u^{Z,N}), \quad (227)$$

with partial derivatives given by:

$$\begin{aligned} \hat{L}^H &= [\epsilon(1 - \alpha_L) + \sigma_L \alpha_L] (\hat{W}^H + \hat{u}^{Z,H}) - (1 - \alpha_L) (\epsilon - \sigma_L) (\hat{W}^N + \hat{u}^{Z,N}) \\ &\quad + \sigma_L \hat{\lambda}, \end{aligned} \quad (228a)$$

$$\begin{aligned} \hat{L}^N &= [\epsilon \alpha_L + \sigma_L (1 - \alpha_L)] (\hat{W}^N + \hat{u}^{Z,N}) - \alpha_L (\epsilon - \sigma_L) (\hat{W}^H + \hat{u}^{Z,H}) \\ &\quad + \sigma_L \hat{\lambda}. \end{aligned} \quad (228b)$$

### Sectoral Wages and Capital-Labor Ratios

Plugging the short-run solutions for  $L^H$  and  $L^N$  given by (227) into the resource constraint for capital (222), the system of four equations consisting of (221a)-(221c) together with (222) can be solved for sectoral wages  $W^j$  and sectoral capital-labor ratios  $k^j$ . Log-differentiating (221a)-(221c) together with (222) yields in matrix form:

$$\begin{aligned} &\begin{pmatrix} -\theta^H & \theta^N & 0 & 0 \\ (1 - \theta^H) & 0 & -1 & 0 \\ 0 & (1 - \theta^N) & 0 & -1 \\ \alpha_K & 1 - \alpha_K & \Psi_{W^H} & \Psi_{W^N} \end{pmatrix} \begin{pmatrix} \hat{k}^H \\ \hat{k}^N \\ \hat{W}^H \\ \hat{W}^N \end{pmatrix} \\ &= \begin{pmatrix} \hat{P}^N - \hat{P}^H + \theta^H \hat{u}^{K,H} - \theta^N \hat{u}^{K,N} \\ -\hat{P}^H - (1 - \theta^H) \hat{u}^{K,H} \\ -\hat{P}^N - (1 - \theta^N) \hat{u}^{K,N} \\ \hat{K} - \Psi_{\bar{\lambda}} \hat{\lambda} - \sum_j \Psi_{u^{Z,j}} \hat{u}^{Z,j} \end{pmatrix}, \end{aligned} \quad (229)$$

where we set:

$$\Psi_{W^j} W^j = \alpha_K \frac{L^H W^j}{L^H} + (1 - \alpha_K) \frac{L^N W^j}{L^N}, \quad (230a)$$

$$\Psi_{u^{Z,j}} u^{Z,j} = \alpha_K \frac{L^H u^{Z,j}}{L^H} + (1 - \alpha_K) \frac{L^N u^{Z,j}}{L^N}, \quad (230b)$$

$$\Psi_{\bar{\lambda}} \bar{\lambda} = \alpha_K \sigma_L + (1 - \alpha_K) \sigma_L = \sigma_L. \quad (230c)$$

where  $u^{Z,j} = 1$  at  $\alpha_K \equiv K^H/K$  stands for the share of traded capital in the aggregate stock of physical capital.

The short-run solutions for sectoral wages and capital-labor ratios are:

$$W^j = W^j(\bar{\lambda}, K, P^N, P^H, u^{K,H}, u^{K,N}, u^{Z,H}, u^{Z,N}), \quad (231a)$$

$$k^j = k^j(\bar{\lambda}, K, P^N, P^H, u^{K,H}, u^{K,N}, u^{Z,H}, u^{Z,N}). \quad (231b)$$

Inserting first sectoral wages (231a), sectoral hours worked (227) can be solved as functions of the shadow value of wealth, the capital stock, the price of non-traded goods in terms of foreign goods,  $P^N$ , the terms of trade, capital and technology utilization rates:

$$L^j = L^j(\bar{\lambda}, K, P^N, P^H, u^{K,H}, u^{K,N}, u^{Z,H}, u^{Z,N}). \quad (232)$$

Log-linearizing the production function (218), i.e.,  $Y^j = (L^j)^{\theta^j} (\tilde{K}^j)^{1-\theta^j}$  where  $\tilde{K}^j = u^{K,j} K^j$ , using the fact that  $k^j = K^j/L^j$ , leads to:

$$\hat{Y}^j = \hat{L}^j + (1 - \theta^j) (\hat{k}^j + \hat{u}^{K,j}). \quad (233)$$

Plugging solutions for sectoral hours worked (232) and sectoral capital-labor ratios (231b) enables us to solve for technology utilization adjusted sectoral value added:

$$Y^j = Y^j(\bar{\lambda}, K, P^N, P^H, u^{K,H}, u^{K,N}, u^{Z,H}, u^{Z,N}). \quad (234)$$

By using the fact that  $K^j = k^j L^j$  and inserting solutions for  $k^j$  and  $L^j$  described by (231b) and (232) enables us to solve for the sectoral capital stocks:

$$K^j = K^j(\bar{\lambda}, K, P^N, P^H, u^{K,H}, u^{K,N}, u^{Z,H}, u^{Z,N}). \quad (235)$$

### Capital and Technology Utilization Rates, $u^{K,j}(t)$ and $u^{Z,j}(t)$

Inserting firm's optimal decisions for capital (220b), i.e.,  $P^j(t) (1 - \theta^j) u^{Z,j}(t) (\tilde{k}^j(t))^{-\theta^j} = R(t)u^{Z,j}(t)$  into optimal choices for capital utilization (200f)-(200g), and invoking the Euler theorem which leads to  $W^H L^H + R u^{K,H} K^H = P^H Y^H$  to rewrite optimal choices for technology utilization (200h)-(200i), we have:

$$(1 - \theta^H) u^{Z,H}(t) (u^{K,H}(t) k^H(t))^{-\theta^H} = \xi_1^H + \xi_2^H (u^{K,H}(t) - 1), \quad (236a)$$

$$R(t) u^{Z,N}(t) = P^N(t) [\xi_1^N + \xi_2^N (u^{K,N}(t) - 1)], \quad (236b)$$

$$Y^H(t) = \chi_1^H + \chi_2^H (u^{Z,H}(t) - 1), \quad (236c)$$

$$Y^N(t) = \chi_1^N + \chi_2^N (u^{Z,N}(t) - 1), \quad (236d)$$

Log-linearizing optimal decisions on capital and technology utilization rates described by (236a)-(236d) leads to in a matrix form:

$$\begin{pmatrix} \begin{bmatrix} \frac{\xi_2^H}{\xi_1^H} + \theta^H + \theta^H \frac{k^{K,H}}{k^H} \\ \theta^N \frac{k^{K,H}}{k^N} \\ -\frac{Y^H}{Y^H} \\ -\frac{Y^N}{Y^N} \end{bmatrix} & \begin{bmatrix} \theta^H \frac{k^{K,N}}{k^H} \\ \left[ \frac{\xi_2^N}{\xi_1^N} + \theta^N + \theta^N \frac{k^{K,N}}{k^N} \right] \\ -\frac{Y^H}{Y^H} \\ -\frac{Y^N}{Y^N} \end{bmatrix} & \begin{bmatrix} \left[ \theta^H \frac{k^{Z,H}}{k^H} - 1 \right] \\ \theta^N \frac{k^{Z,H}}{k^N} \\ \left[ \frac{\chi_2^H}{\chi_1^H} - \frac{Y^H}{Y^H} \right] \\ -\frac{Y^N}{Y^N} \end{bmatrix} & \begin{bmatrix} \theta^H \frac{k^{Z,N}}{k^H} \\ \left[ \theta^N \frac{k^{Z,N}}{k^N} - 1 \right] \\ -\frac{Y^H}{Y^H} \\ \left[ \frac{\chi_2^N}{\chi_1^N} - \frac{Y^N}{Y^N} \right] \end{bmatrix} \end{pmatrix} \begin{pmatrix} \hat{u}^{K,H} \\ \hat{u}^{K,N} \\ \hat{u}^{Z,H} \\ \hat{u}^{Z,N} \end{pmatrix} \\ = \begin{pmatrix} -\theta^H \frac{k^H}{k^H} dK - \theta^H \frac{k^H}{k^H} dP^H - \theta^H \frac{k^H}{k^H} dP^N - \theta^H \frac{k^H}{k^H} d\bar{\lambda} \\ -\theta^N \frac{k^N}{k^N} dK - \theta^N \frac{k^N}{k^N} dP^H - \theta^N \frac{k^N}{k^N} dP^N - \theta^N \frac{k^N}{k^N} d\bar{\lambda} \\ \frac{Y^H}{Y^H} dK + \frac{Y^H}{Y^H} dP^H + \frac{Y^H}{Y^H} dP^N + \frac{Y^H}{Y^H} d\bar{\lambda} \\ \frac{Y^N}{Y^N} dK + \frac{Y^N}{Y^N} dP^H + \frac{Y^N}{Y^N} dP^N + \frac{Y^N}{Y^N} d\bar{\lambda} \end{pmatrix}. \quad (237)$$

The short-run solutions for capital and technology utilization rates are:

$$u^{K,j} = u^{K,j}(\bar{\lambda}, K, P^N, P^H), \quad (238a)$$

$$u^{Z,j} = u^{Z,j}(\bar{\lambda}, K, P^N, P^H). \quad (238b)$$

### Intermediate Solutions for $k^j, W^j, L^j, Y^j, K^j$

Plugging back solutions for capital and technology utilization rates (238a)-(238b) into (231a), (231b), (232), (234), (235) leads to intermediate solutions for sectoral wages, sectoral capital-labor ratios, sectoral hours worked, sectoral value added, and sectoral capital stocks:

$$W^j, k^j, L^j, Y^j, K^j (\bar{\lambda}, K, P^N, P^H). \quad (239)$$

### Optimal Investment Decision, $I/K$

Eq. (212d) can be solved for the investment rate:

$$\frac{I}{K} = v \left( \frac{Q}{P_I (P^T, P^N)} \right) + \delta_K, \quad (240)$$

where

$$v(\cdot) = \frac{1}{\kappa} \left( \frac{Q}{P_J} - 1 \right), \quad (241)$$

with

$$v_Q = \frac{\partial v(\cdot)}{\partial Q} = \frac{1}{\kappa} \frac{1}{P_J} > 0, \quad (242a)$$

$$v_{P^H} = \frac{\partial v(\cdot)}{\partial P^H} = -\frac{1}{\kappa} \frac{Q}{P_J} \frac{\alpha_J \alpha_J^H}{P^H} < 0, \quad (242b)$$

$$v_{P^N} = \frac{\partial v(\cdot)}{\partial P^N} = -\frac{1}{\kappa} \frac{Q}{P_J} \frac{(1 - \alpha_J)}{P^N} < 0. \quad (242c)$$

Inserting (240) into (197), investment including capital installation costs can be rewritten as follows:

$$\begin{aligned} J &= K \left[ \frac{I}{K} + \frac{\kappa}{2} \left( \frac{I}{K} - \delta_K \right)^2 \right], \\ &= K \left[ v(\cdot) + \delta_K + \frac{\kappa}{2} (v(\cdot))^2 \right]. \end{aligned} \quad (243)$$

Eq. (243) can be solved for investment including capital installation costs:

$$J = J(K, Q, P^N, P^H), \quad (244)$$

where

$$J_K = \frac{\partial J}{\partial K} = \frac{J}{K}, \quad (245a)$$

$$J_X = \frac{\partial J}{\partial X} = K v_X (1 + \kappa v(\cdot)) > 0, \quad (245b)$$

with  $X = Q, P^H, P^N$ .

Substituting (244) into (209b), (217a), and (217b) allows us to solve for the demand for non-traded, home-produced traded, and foreign-produced traded inputs:

$$J^N = J^N(K, Q, P^N, P^H), \quad J^H = J^H(K, Q, P^N, P^H), \quad J^F = J^F(K, Q, P^N, P^H), \quad (246)$$

with partial derivatives given by

$$\begin{aligned}
\hat{J}^N &= -\alpha_J \phi_J \hat{P}^N + \phi_J \alpha_J \alpha_J^H \hat{P}^H + \hat{J}, \\
&= \frac{QK}{P_J J} \frac{(1 + \kappa v(\cdot))}{\kappa} \hat{Q} - \left[ \alpha_J \phi_J + \frac{QK}{P_J J} \frac{(1 + \kappa v(\cdot))}{\kappa} (1 - \alpha_J) \right] \hat{P}^N \\
&+ \alpha_J \alpha_J^H \left[ \phi_J - \frac{QK}{P_J J} \frac{(1 + \kappa v(\cdot))}{\kappa} \right] \hat{P}^H + \hat{K}, \tag{247a}
\end{aligned}$$

$$\begin{aligned}
\hat{J}^H &= -[\rho_J (1 - \alpha_J^H) + \alpha_J^H \phi_J (1 - \alpha_J)] \hat{P}^H + \phi_J (1 - \alpha_J) \hat{P}^N + \hat{J}, \\
&= -\left\{ [\rho_J (1 - \alpha_J^H) + \alpha_J^H \phi_J (1 - \alpha_J)] + \alpha_J \alpha_J^H \frac{QK}{P_J J} \frac{(1 + \kappa v(\cdot))}{\kappa} \right\} \hat{P}^H \\
&+ (1 - \alpha_J) \left[ \phi_J - \frac{QK}{P_J J} \frac{(1 + \kappa v(\cdot))}{\kappa} \right] \hat{P}^N + \frac{QK}{P_J J} \frac{(1 + \kappa v(\cdot))}{\kappa} \hat{Q} + \hat{K}, \tag{247b}
\end{aligned}$$

$$\begin{aligned}
\hat{J}^F &= \alpha_J^H [\rho_J - \phi_J (1 - \alpha_J)] \hat{P}^H + \phi_J (1 - \alpha_J) \hat{P}^N + \hat{J}, \\
&= \alpha_J^H \left\{ [\rho_J - \phi_J (1 - \alpha_J)] - \alpha_J \frac{QK}{P_J J} \frac{(1 + \kappa v(\cdot))}{\kappa} \right\} \hat{P}^H \\
&+ (1 - \alpha_J) \left[ \phi_J - \frac{QK}{P_J J} \frac{(1 + \kappa v(\cdot))}{\kappa} \right] \hat{P}^N + \frac{QK}{P_J J} \frac{(1 + \kappa v(\cdot))}{\kappa} \hat{Q} + \hat{K}, \tag{247c}
\end{aligned}$$

where use has been made of (245), i.e.,

$$\begin{aligned}
\hat{J} &= \hat{K} + \frac{QK}{P_J J} \frac{(1 + \kappa v(\cdot))}{\kappa} \hat{Q} - \frac{QK}{P_J J} \frac{(1 + \kappa v(\cdot))}{\kappa} (1 - \alpha_J) \hat{P}^N \\
&- \alpha_J \alpha_J^H \frac{QK}{P_J J} \frac{(1 + \kappa v(\cdot))}{\kappa} \hat{P}^H.
\end{aligned}$$

#### N.4 Market Clearing Conditions

Finally, we have to solve for the relative price of non-traded goods and the terms of trade.

##### Market Clearing Condition for Non-Tradables

The role of the price of non-tradables in terms of foreign-produced traded goods is to clear the non-traded goods market:

$$u^{Z,N}(t)Y^N(t) = C^N(t) + G^N(t) + J^N(t) + C^{K,N}(t)K^N(t) + C^{Z,N}(t). \tag{248}$$

Inserting solutions for  $C^N$ ,  $J^N$ ,  $Y^N$  given by (225), (226a), (221), respectively, the non-traded goods market clearing condition (248) can be rewritten as follows:

$$\begin{aligned}
&u^{Z,N}(\bar{\lambda}, K, P^N, P^H) Y^N(\bar{\lambda}, K, P^N, P^H) = C^N(\bar{\lambda}, P^N, P^H) + G^N \\
&+ J^N(K, Q, P^N, P^H) + C^{K,N}[u^{K,N}(\bar{\lambda}, K, P^N, P^H)] K^N(\bar{\lambda}, K, P^N, P^H) \\
&+ C^{Z,N}[u^{Z,N}(\bar{\lambda}, K, P^N, P^H)]. \tag{249}
\end{aligned}$$

Linearizing (249) leads to:

$$Y^N du^{Z,N}(t) + dY^N(t) = dC^N(t) + dG^N(t) + dJ^N(t) + K^N \xi_1^N du^{K,N}(t) + \chi_1^N du^{Z,N}(t), \tag{250}$$

where the terms  $Y^N du^{Z,N}(t)$  and  $\chi_1^N du^{Z,N}(t)$  cancel out because eq. (236d) evaluated at the steady-state implies  $Y^N = \chi_1^N$ .

##### Market Clearing Condition for Home-Produced Traded Goods

The role of the price of home-produced goods in terms of foreign-produced goods or the terms of trade is to clear the home-produced traded goods market:

$$u^{Z,H}(t)Y^H(t) = C^H(t) + G^H(t) + J^H(t) + X^H(t) + C^{K,H}(t)K^H(t) + C^{Z,H}(t), \tag{251}$$

where  $X^H$  stands for exports which are negatively related to the terms of trade:

$$X^H = \varphi_X (P^H)^{-\phi_X}, \tag{252}$$

where  $\phi_X$  is the elasticity of exports with respect to the terms of trade.



Inserting solutions for  $C^H$ ,  $J^H$ ,  $Y^H$  given by (225), (226a), (221), respectively, the traded goods market clearing condition (251) can be rewritten as follows:

$$\begin{aligned} & u^{Z,H}(\bar{\lambda}, K, P^N, P^H) Y^H(\bar{\lambda}, K, P^N, P^H) = C^H(\bar{\lambda}, P^N, P^H) + G^H \\ & + J^H(K, Q, P^N, P^H) + C^{K,H}[u^{K,H}(\bar{\lambda}, K, P^N, P^H)] K^H(\bar{\lambda}, K, P^N, P^H) \\ & + C^{Z,H}[u^{Z,H}(\bar{\lambda}, K, P^N, P^H)]. \end{aligned} \quad (253)$$

Linearizing (253) leads to:

$$Y^H du^{Z,H}(t) + dY^H(t) = dC^H(t) + dG^H(t) + dJ^H(t) + dX^H(t) + K^H \xi_1^H du^{K,H}(t) + \chi_1^H du^{Z,H}(t), \quad (254)$$

where the terms  $Y^H du^{Z,H}(t)$  and  $\chi_1^H du^{Z,H}(t)$  cancel out because eq. (236c) evaluated at the steady-state implies  $Y^H = \chi_1^H$ .

### Sectoral Government Spending

We assume that the rise in government consumption is broken into non-traded and traded goods, and into home- and foreign-produced traded goods in accordance with their respective shares,  $\omega_{GN} = P^N G^N / G$  and  $\omega_{GH} = \frac{P^H G^H}{G}$ . Formally, we have:

$$dG(t) = \omega_{GN} dG(t) + \omega_{GH} dG(t) + \omega_{GF} dG(t). \quad (255)$$

Linearizing the allocation of total government consumption across sectoral goods (measured at constant prices) leads to:

$$dG(t) = P^N dG^N(t) + P^H dG^H(t) + dG^F(t). \quad (256)$$

Eqs. (255)-(256) can be solved for sectoral government consumption:

$$G^N, G^H, G^F(G(t)), \quad (257)$$

where partial derivatives are given by

$$G_G^N = \frac{\partial G^N}{\partial G} = \frac{\omega_{GN}}{P^N}, \quad (258a)$$

$$G_G^H = \frac{\partial G^H}{\partial G} = \frac{\omega_{GH}}{P^H}, \quad (258b)$$

$$G_G^F = \frac{\partial G^F}{\partial G} = \omega_{GF}. \quad (258c)$$

### Solving for Relative Prices

As shall be useful below, we write out a number of useful notations:

$$\Psi_{PN}^N = Y_{PN}^N - C_{PN}^N - J_{PN}^N - K^N \xi_1^N u_{PN}^{K,N}, \quad (259a)$$

$$\Psi_{PH}^N = Y_{PH}^N - C_{PH}^N - J_{PH}^N - K^N \xi_1^N u_{PH}^{K,N}, \quad (259b)$$

$$\Psi_K^N = Y_K^N - J_K^N - K^N \xi_1^N u_K^{K,N}, \quad (259c)$$

$$\Psi_\lambda^N = Y_\lambda^N - C_\lambda^N - K^N \xi_1^N u_\lambda^{K,N}, \quad (259d)$$

$$\Psi_{PN}^H = Y_{PN}^H - C_{PN}^H - J_{PN}^H - K^H \xi_1^H u_{PN}^{K,H}, \quad (259e)$$

$$\Psi_{PH}^H = Y_{PH}^H - C_{PH}^H - J_{PH}^H - X_{PH}^H - K^H \xi_1^H u_{PH}^{K,H}, \quad (259f)$$

$$\Psi_K^H = Y_K^H - J_K^H - K^H \xi_1^H u_K^{K,H}, \quad (259g)$$

$$\Psi_\lambda^H = Y_\lambda^H - C_\lambda^H - K^H \xi_1^H u_\lambda^{K,H}. \quad (259h)$$

Linearized versions of market clearing conditions described by eq. (250) and eq. (254) can be rewritten in a matrix form:

$$\begin{aligned} & \begin{pmatrix} \Psi_{PN}^N & \Psi_{PH}^N \\ \Psi_{PN}^H & \Psi_{PH}^H \end{pmatrix} \begin{pmatrix} dP^N \\ dP^H \end{pmatrix} \\ & = \begin{pmatrix} -\Psi_K^N dK + J_Q^N dQ + G_G^N dG - \Psi_\lambda^N d\bar{\lambda} \\ -\Psi_K^H dK + J_Q^H dQ + G_G^H dG - \Psi_\lambda^H d\bar{\lambda} \end{pmatrix}. \end{aligned} \quad (260)$$

The short-run solutions for capital and technology utilization rates are:

$$P^N = P^N(\bar{\lambda}, K, Q, G), \quad (261a)$$

$$P^H = P^H(\bar{\lambda}, K, Q, G). \quad (261b)$$

## N.5 Solving the Model

In our model, there is one state variable, namely the capital stock  $K$ , and one control variable, namely the shadow price of the capital stock  $Q$ . To solve the model, we have to express all variables in terms of state and control variables. Plugging back solutions for the relative price of non-tradables (261a) and the terms of trade (261b) into (225), (246), (239), capital and technology utilization rates (238a)-(238b) leads to solutions for sectoral consumption, sectoral inputs for capital goods, sectoral wages, sectoral capital-labor ratios, sectoral hours worked, sectoral value added, sectoral capital stocks:

$$C^j, J^j, W^j, k^j, L^j, Y^j, K^j, v, u^{K,j}, u^{Z,j}(\bar{\lambda}, K, Q, G). \quad (262)$$

The technology-utilization-adjusted-return on domestic capital is:

$$R(t) = P^H(t) (1 - \theta^H) (u^{K,H}(t)k^H(t))^{-\theta^H}. \quad (263)$$

Log-linearizing (263) in the neighborhood of the initial steady-state leads to:

$$\hat{R}(t) = \hat{P}^H(t) - \theta^H (\hat{u}^{K,H}(t) + \hat{k}^H(t)). \quad (264)$$

Inserting the solution for the terms of trade,  $P^H$ , the capital utilization rate,  $u^{K,H}$ , and the capital-labor ratio  $k^H$  described by (262), eq. (263) can be solved for the return on domestic capital:

$$R = R(\bar{\lambda}, K, Q, G). \quad (265)$$

Remembering that the non-traded input  $J^N$  used to produce the capital good is described by  $(1 - \iota) \left(\frac{P^N}{P_J}\right)^{-\phi_J} J$  (see eq. (209b)) with  $J = I + \frac{\kappa}{2} \left(\frac{I}{K} - \delta_K\right)^2 K$ , using the fact that  $J^N = Y^N - C^N - G^N$  and inserting  $I = \dot{K} + \delta_K K$ , the capital accumulation equation reads as follows:

$$\dot{K}(t) = \frac{Y^N(t) - C^N(t) - G^N(t) - C^{K,N}(t)K^N(t) - C^{Z,N}(t)}{(1 - \iota) \left(\frac{P^N(t)}{P_J(t)}\right)^{-\phi_J}} - \delta_K K(t) - \frac{\kappa}{2} \left(\frac{I(t)}{K(t)} - \delta_K\right)^2 K(t). \quad (266)$$

We drop the time index below so as to write equations in a more compact form. Inserting first solutions for non-traded output, consumption in non-tradables, demand for non-traded input, non-traded capital and technology utilization rates described by eq. (262) together with optimal investment decision (245a) into the physical capital accumulation equation (266), and plugging the short-run solution for the return on domestic capital (265) into the dynamic equation for the shadow value of capital stock (200e), the dynamic system reads as follows:

$$\begin{aligned} \dot{K} &\equiv \Upsilon(K, Q, G), \\ &= \frac{Y^N(K, Q, G) - C^N(K, Q, G) - G^N(G) - C^{K,N}[u^{K,N}(K, Q, G)]K^N - C^{Z,N}[u^{Z,N}(K, Q, G)]}{(1 - \iota) \left[\frac{P^N(K, Q, G)}{P_J(K, Q, G)}\right]^{-\phi_J}} \\ &\quad - \delta_K K - \frac{K}{2\kappa} \left[\frac{Q}{P_J(K, Q, G)} - 1\right]^2, \end{aligned} \quad (267a)$$

$$\begin{aligned} \dot{Q} &\equiv \Sigma(K, Q, G) \\ &= (r^* + \delta_K) Q - \left[ \frac{R(K, Q, G)}{K} \sum_{j=H,N} u^{K,j}(K, Q, G) u^{Z,j}(K, Q, G) K^j(K, Q, G) \right. \\ &\quad - \sum_{j=H,N} P^j(K, Q, G) C^{K,j}[u^{K,j}(K, Q, G)] \frac{K^j(K, Q, G)}{K} \\ &\quad \left. + P_J [P^H(\cdot), P^N(\cdot)] \frac{\kappa}{2} v(\cdot) (v(\cdot) + 2\delta_K) \right]. \end{aligned} \quad (267b)$$

To ease the linearization, it is useful to break down the capital accumulation into two components:

$$\hat{K} = J - \delta_K K - \frac{\kappa}{2} \left( \frac{I}{K} - \delta_K \right)^2 K. \quad (268)$$

The first component is  $J$ . Using the fact that  $J = \frac{J^N}{(1-\iota)\left(\frac{P^N}{P^H}\right)^{-\phi_J}}$  and log-linearizing gives:

$$\hat{J} = \hat{J}^N + \phi_J \alpha_J \hat{P}^N - \phi_J \alpha_J \alpha_J^H \hat{P}^H \quad (269)$$

where we used the fact that  $\hat{P}_J = \alpha_J \alpha_J^H \hat{P}^H + (1 - \alpha_J) \hat{P}^N$ . Using (268) and the fact that  $J^N = Y^N - C^N - G^N - C^{K,N} K^N - C^{Z,N}$ , linearizing (268) in the neighborhood of the steady-state gives:

$$\begin{aligned} \dot{K} &= \frac{J}{J^N} [dY^N(t) - dC^N(t) - dG^N(t) - K^N \xi_1^N du^{K,N}(t)] + \phi_J \frac{J}{P^N} \alpha_J dP^N(t) \\ &\quad - \phi_J \frac{J}{P^H} \alpha_J \alpha_J^H dP^H(t) - \delta_K dK(t), \end{aligned} \quad (270)$$

where  $J = I = \delta_K K$  in the long-run and we used the fact that  $Y^N du^{Z,N}(t)$  and  $\chi_1^N du^{Z,N}(t)$  cancel out.

Let us denote by  $\Upsilon_K$ ,  $\Upsilon_Q$ , and  $\Upsilon_G$  the partial derivatives evaluated at the steady-state of the capital accumulation equation w.r.t.  $K$ ,  $Q$ , and  $G$ , respectively. Using (262) and (270), these elements of the linearized capital accumulation equation are given by:

$$\Upsilon_K \equiv \frac{\partial \dot{K}}{\partial K} = \frac{J}{J^N} \left( Y_K^N - C_K^N - K^N \xi_1^N u_K^{K,N} \right) + \alpha_J \phi_J J \left( \frac{P_K^N}{P^N} - \alpha_J^H \frac{P_K^H}{P^H} \right) - \delta_K \quad (271a)$$

$$\Upsilon_Q \equiv \frac{\partial \dot{K}}{\partial Q} = \frac{J}{J^N} \left( Y_Q^N - C_Q^N - K^N \xi_1^N u_Q^{K,N} \right) + \alpha_J \phi_J J \left( \frac{P_Q^N}{P^N} - \alpha_J^H \frac{P_Q^H}{P^H} \right) > 0, \quad (271b)$$

$$\Upsilon_G \equiv \frac{\partial \dot{K}}{\partial G} = \frac{J}{J^N} \left( Y_G^N - G_G^N - C_G^N - K^N \xi_1^N u_G^{K,N} \right) + \alpha_J \phi_J J \left( \frac{P_G^N}{P^N} - \alpha_J^H \frac{P_G^H}{P^H} \right), \quad (271c)$$

where  $J = \delta_K K$  in the long run.

Let us denote by  $\Sigma_K$ ,  $\Sigma_Q$ , and  $\Sigma_G$  the partial derivatives evaluated at the steady-state of the dynamic equation for the marginal value of an additional unit of capital w.r.t.  $K$ ,  $Q$ , and  $G$ , respectively:

$$\begin{aligned} \Sigma_K \equiv \frac{\partial \dot{Q}}{\partial K} &= - \left\{ R_K - \frac{R}{K} + \frac{R}{K} \sum_{j=H,N} \left[ K^j u_K^{K,j} + K^j u_K^{Z,j} \right] \right. \\ &\quad + \frac{R}{K} [K_K^H + K_K^N] - \sum_{j=H,N} \frac{P^j K^j}{K} \xi_1^j u_K^{K,j} \\ &\quad \left. + P_J \kappa v_K \delta_K \right\} > 0, \end{aligned} \quad (272a)$$

$$\begin{aligned} \Sigma_Q \equiv \frac{\partial \dot{Q}}{\partial Q} &= (r^* + \delta_K) - \left\{ R_Q + \frac{R}{K} \sum_{j=H,N} \left[ K^j u_Q^{K,j} + K^j u_Q^{Z,j} \right] \right. \\ &\quad + \frac{R}{K} (K_Q^H + K_Q^N) - \sum_{j=H,N} \frac{P^j K^j}{K} \xi_1^j u_Q^{K,j} \\ &\quad \left. + P_J \kappa v_Q \delta_K \right\} > 0, \end{aligned} \quad (272b)$$

$$\begin{aligned} \Sigma_G \equiv \frac{\partial \dot{Q}}{\partial G} &= - \left\{ R_G + \frac{R}{K} \sum_{j=H,N} \left[ K^j u_G^{K,j} + K^j u_G^{Z,j} \right] \right. \\ &\quad + \frac{R}{K} (K_G^H + K_G^N) - \sum_{j=H,N} \frac{P^j K^j}{K} \xi_1^j u_G^{K,j} \\ &\quad \left. + P_J \kappa v_G \delta_K \right\} > 0. \end{aligned} \quad (272c)$$

Assuming that the saddle-path stability condition is fulfilled, and denoting the negative eigenvalue by  $\nu_1$  and the positive eigenvalue by  $\nu_2$ , the general solutions for  $K$  and  $Q$  are:

$$K(t) - K = D_1 e^{\nu_1 t} + D_2 e^{\nu_2 t}, \quad Q(t) - Q = \omega_2^1 D_1 e^{\nu_1 t} + \omega_2^2 D_2 e^{\nu_2 t}, \quad (273)$$

where  $K_0$  is the initial capital stock and  $(1, \omega_2^i)'$  is the eigenvector associated with eigenvalue  $\nu_i$ :

$$\omega_2^i = \frac{\nu_i - \Upsilon_K}{\Upsilon_Q}. \quad (274)$$

Because  $\nu_1 < 0$ ,  $\Upsilon_K > 0$  and  $\Upsilon_Q > 0$ , we have  $\omega_2^1 < 0$ , regardless of sectoral capital intensities, which implies that the shadow value of investment and the stock physical capital move in opposite direction along a stable path (i.e.,  $D_2 = 0$ ).

## N.6 Current Account Equation and Intertemporal Solvency Condition

To determine the current account equation, we use the following identities and properties:

$$P_C(t)C(t) = P^H(t)C^H(t) + C^F(t) + P^N(t)C^N(t), \quad (275a)$$

$$P_J(t)J(t) = P^H(t)J^H(t) + J^F(t) + P^N(t)J^N(t), \quad (275b)$$

$$T(t) = G(t) = P^H(t)G^H(t) + G^F(t) + P^N(t)G^N(t), \quad (275c)$$

$$\tilde{W}(t)L(t) + \tilde{R}(t)\tilde{K}(t) = \sum_{j=H,N} u^{Z,j}(t) \left( W^j(t)L^j(t) + R^j(t)\tilde{K}^j(t) \right) = \sum_{j=H,N} P^j(t)\tilde{Y}^j(t), \quad (275d)$$

where (275d) follows from Euler theorem. Using (275d), inserting (275a)-(275c) into (194) and invoking market clearing conditions for non-traded goods (248) and home-produced traded goods (251) yields:

$$\begin{aligned} \dot{N}(t) &= r^* N(t) + P^H(t) \left( \tilde{Y}^H(t) - C^H(t) - G^H(t) - J^H(t) - C^{K,H}(t)K^H(t) - C^{Z,H}(t) \right) \\ &\quad - (C^F(t) + J^F(t) + G^F(t)), \\ &= r^* N(t) + P^H(t)X^H(t) - M^F(t), \end{aligned} \quad (276)$$

where  $X^H = Y^H - C^H - G^H - J^H - C^{K,H}K^H - C^{Z,H}$  stands for exports of home-produced traded goods and we denote imports of foreign consumption and investment goods by  $M^F$ :

$$M^F(t) = C^F(t) + G^F(t) + J^F(t). \quad (277)$$

Inserting (262) into (276) and the solution for  $P^H$  described by eq. (261b) into  $X^H = X^H(P^H)$  leads to:

$$\begin{aligned} \dot{N}(t) &\equiv r^* N(t) + \Xi(K(t), Q(t), G(t)), \\ &= r^* N(t) + P^H(K(t), Q(t), G(t)) X^H [P^H(K(t), Q(t), G(t))] - M^F(K(t), Q(t), G(t)) \end{aligned} \quad (278)$$

Let us denote by  $\Xi_K$ ,  $\Xi_Q$ , and  $\Xi_{Z^j}$  the partial derivatives evaluated at the steady-state of the dynamic equation for the current account w.r.t.  $K$ ,  $Q$ , and  $Z^j$ , respectively:

$$\Xi_K \equiv \frac{\partial \dot{N}}{\partial K} = (1 - \phi_X) X^H P_K^H - M_K^F, \quad (279a)$$

$$\Xi_Q \equiv \frac{\partial \dot{N}}{\partial Q} = (1 - \phi_X) X^H P_Q^H - M_Q^F, \quad (279b)$$

$$\Xi_G \equiv \frac{\partial \dot{N}}{\partial G} = (1 - \phi_X) X^H P_G^H - M_G^F. \quad (279c)$$

where we used the fact that  $P^H X^H = \varphi_X (P^H)^{1-\phi_X}$  (see eq. (252)).

Linearizing (278) in the neighborhood of the steady-state, making use of (279a) and (279b), inserting solutions for  $K(t)$  and  $Q(t)$  given by (273) and solving yields the general solution for the net foreign asset position:

$$N(t) = N + [(N_0 - N) - \Psi_1 D_1 - \Psi_2 D_2] e^{r^* t} + \Psi_1 D_1 e^{\nu_1 t} + \Psi_2 D_2 e^{\nu_2 t}, \quad (280)$$

where  $N_0$  is the initial stock of traded bonds and we set

$$E_i = \Xi_K + \Xi_Q \omega_2^i, \quad (281a)$$

$$\Psi_i = \frac{E_i}{\nu_i - r^*}. \quad (281b)$$

Invoking the transversality condition leads to the linearized version of the nations's intertemporal solvency condition:

$$N - N_0 = \Psi_1 (K - K_0), \quad (282)$$

where  $K_0$  is the initial stock of physical capital.

## N.7 Derivation of the Accumulation Equation of Non Human Wealth

The stock of financial wealth  $A(t)$  is equal to  $N(t) + Q(t)K(t)$ ; differentiating w.r.t. time, i.e.,  $\dot{A}(t) = \dot{N}(t) + \dot{Q}(t)K(t) + Q(t)\dot{K}(t)$ , plugging the dynamic equation for the marginal value of capital (200e), inserting the accumulation equations for physical capital (196) and for traded bonds (194), yields the accumulation equation for the stock of financial wealth or the dynamic equation for private savings:

$$\dot{A}(t) = r^* A(t) + \sum_{j=H,N} u^{Z,j}(t) W^j(t) L^j(t) - T(t) - P_C(t) C(t) - \sum_{j=H,N} P^j(t) C^{Z,j}(t), \quad (283)$$

where we assume that the government levies lump-sum taxes,  $T$ , to finance purchases of foreign-produced, home-produced traded goods and non-traded goods, i.e.,  $T = G = G^F + P^H G^H + P^N G^N$ .

Inserting short-run solutions for the relative price of non-tradables (261a) and the terms of trade (261b) into  $G = G^F + P^H G^H + P^N G^N$  and (201) allows us to solve government spending and the consumption price index:

$$G = G(K, Q, G, \bar{\lambda}), \quad (284a)$$

$$P_C = P_C(K, Q, G, \bar{\lambda}), \quad (284b)$$

where partial derivatives are  $G_X = P_X^H G^H + P_X^N G^N$  with  $X = K, Q, G$  and

$$\frac{\partial P_C}{\partial X} = \alpha_C \alpha^H \frac{P_C}{P^H} P_X^H + (1 - \alpha_C) \frac{P_C}{P^N} P_X^N, \quad (285)$$

where  $X = K, Q, G$ .

Inserting first short-run solutions for consumption and labor together with solutions for technology utilization rates given by eq. (262), substituting solutions for government spending and the consumption price index described by (284a)-(284b) leads to:

$$\begin{aligned} \dot{A} &\equiv r^* A + \Lambda(K, Q, G), \\ &= r^* A + \sum_{j=H,N} u^{Z,j}(K, Q, G) W^j(K, Q, G) L^j(K, Q, G) - G(K, Q, G) \\ &\quad - P_C(K, Q, G) C(K, Q, G) - \sum_{j=H,N} P^j(K, Q, G) C^{Z,j}(K, Q, G). \end{aligned} \quad (286)$$

Let us denote by  $\Lambda_K$ ,  $\Lambda_Q$ , and  $\Lambda_G$  the partial derivatives evaluated at the steady-state of the dynamic equation for the non human wealth w.r.t.  $K$ ,  $Q$ , and  $G$ , respectively, which

are given by:

$$\begin{aligned}\Lambda_K &\equiv \frac{\partial \dot{A}}{\partial K} = \sum_{j=H,N} \left( W_K^j L^j + W^j L_K^j + W^j L^j u_K^{Z,j} \right) - G_K - \left( \frac{\partial P_C}{\partial K} C + P_C C_K \right) \\ &\quad - \sum_{j=H,N} P^j \xi_1^j u_K^{Z,j},\end{aligned}\tag{287a}$$

$$\begin{aligned}\Lambda_Q &\equiv \frac{\partial \dot{A}}{\partial Q} = \sum_{j=H,N} \left( W_Q^j L^j + W^j L_Q^j + W^j L^j u_Q^{Z,j} \right) - G_Q - \left( \frac{\partial P_C}{\partial Q} C + P_C C_Q \right) \\ &\quad - \sum_{j=H,N} P^j \xi_1^j u_Q^{Z,j},\end{aligned}\tag{287b}$$

$$\begin{aligned}\Lambda_G &\equiv \frac{\partial \dot{A}}{\partial G} = \sum_{j=H,N} \left( W_G^j L^j + W^j L_G^j + W^j L^j u_G^{Z,j} \right) - G_G - \left( \frac{\partial P_C}{\partial G} C + P_C C_G \right) \\ &\quad - \sum_{j=H,N} P^j \xi_1^j u_G^{Z,j}.\end{aligned}\tag{287c}$$

Linearizing (286) in the neighborhood of the steady-state, making use of (287a) and (287b), inserting solutions for  $K(t)$  and  $Q(t)$  given by (273) and solving yields the general solution for the stock of non human wealth:

$$A(t) = A + [(A_0 - A) - \Delta_1 D_1 - \Delta_2 D_2] e^{r^* t} + \Delta_1 D_1 e^{\nu_1 t} + \Delta_2 D_2 e^{\nu_2 t},\tag{288}$$

where  $A_0$  is the initial stock of financial wealth and we set

$$M_i = \Lambda_K + \Lambda_Q \omega_2^i,\tag{289a}$$

$$\Delta_i = \frac{M_i}{\nu_i - r^*}.\tag{289b}$$

The linearized version of the representative household's intertemporal solvency condition is:

$$A - A_0 = \Delta_1 (K - K_0),\tag{290}$$

where  $A_0$  is the initial stock of non human wealth.

## O Semi-Small Open Economy Model with CES Production Functions

This section extends the model laid out in section N to CES production functions and factor biased technological change. Since first order conditions from households' maximization problem detailed in subsection N.1 remain identical, we do not repeat them and emphasize the main changes caused by the assumption of CES production functions.

### O.1 Firms

We denote technology adjusted value added in sector  $j = H, N$  by  $Y^j$ . When we add a tilde, it means that value added is inclusive of the technology utilization rate, i.e.,  $\tilde{Y}^j(t) = u^Z(t) Y^j(t)$ . We allow for labor- and capital-augmenting productivity denoted by  $\tilde{A}^j(t)$  and  $\tilde{B}^j(t)$ . We allow for labor- and capital-augmenting efficiency denoted by  $\tilde{A}^j(t)$  and  $\tilde{B}^j(t)$ . We assume that factor-augmenting productivity has a symmetric time-varying component denoted by  $u^{Z,j}(t)$  such that  $\tilde{A}^j(t) = u^{Z,j}(t) A^j(t)$  and  $\tilde{B}^j(t) = u^{Z,j}(t) B^j(t)$ . Both the traded and non-traded sectors use physical capital inclusive of capital utilization,  $\tilde{K}^j(t) = u^{K,j}(t) K^j(t)$ , and labor,  $L^j$ , according to constant returns to scale production functions which are assumed to take a CES form:

$$\tilde{Y}^j(t) = \left[ \gamma^j \left( \tilde{A}^j(t) L^j(t) \right)^{\frac{\sigma^j-1}{\sigma^j}} + (1 - \gamma^j) \left( \tilde{B}^j(t) \tilde{K}^j(t) \right)^{\frac{\sigma^j-1}{\sigma^j}} \right]^{\frac{\sigma^j}{\sigma^j-1}},\tag{291}$$

where  $\gamma^j$  and  $1 - \gamma^j$  are the weight of labor and capital in the production technology,  $\sigma^j$  is the elasticity of substitution between capital and labor in sector  $j = H, N$ . Firms lease the capital from households and hire workers. They face two cost components: a capital rental cost equal to  $\tilde{R}^j(t) = R(t)u^{Z,j}(t)$ , and a labor cost equal to the wage rate  $\tilde{W}^j(t) = W^j(t)u^{Z,j}(t)$ , both inclusive of technology utilization.

### First-Order Conditions

Firms lease the capital from households and hire workers. They face two cost components: a capital rental cost equal to  $\tilde{R}^j(t) = R(t)u^{Z,j}(t)$ , and a labor cost equal to the wage rate  $\tilde{W}^j(t) = W^j(t)u^{Z,j}(t)$ , both inclusive of technology utilization. Both sectors are assumed to be perfectly competitive and thus choose capital services and labor by taking prices as given:

$$\begin{aligned} \max_{\tilde{K}^j, L^j} \tilde{\Pi}^j &= \max_{\tilde{K}^j, L^j} \left\{ P^j \tilde{Y}^j - \tilde{W}^j L^j - \tilde{R}^j \tilde{K}^j \right\}, \\ &= \max_{\tilde{K}^j, L^j} u^{Z,j}(t) \left\{ P^j Y^j - W^j L^j - R \tilde{K}^j \right\}, \\ &= \max_{\tilde{K}^j, L^j} u^{Z,j} \Pi^j, \end{aligned} \quad (292)$$

where technology-utilization-adjusted CES production function reads:

$$Y^j(t) = \left[ \gamma^j (A^j(t)L^j(t))^{\frac{\sigma^j-1}{\sigma^j}} + (1 - \gamma^j) (B^j(t)\tilde{K}^j(t))^{\frac{\sigma^j-1}{\sigma^j}} \right]^{\frac{\sigma^j}{\sigma^j-1}}. \quad (293)$$

Since capital can move freely between the two sectors, the value of marginal products in the traded and non-traded sectors equalizes while costly labor mobility implies a wage differential across sectors:

$$P^H (1 - \gamma^H) (B^H)^{\frac{\sigma^H-1}{\sigma^H}} (u^{K,H}k^H)^{-\frac{1}{\sigma^H}} (y^H)^{\frac{1}{\sigma^H}} = P^N (1 - \gamma^N) (B^N)^{\frac{\sigma^N-1}{\sigma^N}} (u^{K,N}k^N)^{-\frac{1}{\sigma^N}} (y^N)^{\frac{1}{\sigma^N}} \equiv R, \quad (294a)$$

$$P^H \gamma^H (A^H)^{\frac{\sigma^H-1}{\sigma^H}} (L^H)^{-\frac{1}{\sigma^H}} (Y^H)^{\frac{1}{\sigma^H}} \equiv W^H, \quad (294b)$$

$$P^N \gamma^N (A^N)^{\frac{\sigma^N-1}{\sigma^N}} (L^N)^{-\frac{1}{\sigma^N}} (Y^N)^{\frac{1}{\sigma^N}} \equiv W^N, \quad (294c)$$

where we denote by  $k^j \equiv K^j/L^j$  the capital-labor ratio for sector  $j = H, N$ , and  $y^j \equiv Y^j/L^j$  value added per hours worked described by

$$y^j = \left[ \gamma^j (A^j)^{\frac{\sigma^j-1}{\sigma^j}} + (1 - \gamma^j) (B^j u^{K,j} k^j)^{\frac{\sigma^j-1}{\sigma^j}} \right]^{\frac{\sigma^j}{\sigma^j-1}}. \quad (295)$$

The resource constraint for capital is:

$$K^H + K^N = K. \quad (296)$$

### Some Useful Results

Multiplying both sides of (294b)-(294c) by  $L^j$  and dividing by sectoral value added leads to the labor income share:

$$s_L^j = \gamma^j \left( \frac{A^j}{y^j} \right)^{\frac{\sigma^j-1}{\sigma^j}}. \quad (297)$$

Multiplying both sides of (294a) by  $K^j$  and dividing by sectoral value added leads to the capital income share:

$$1 - s_L^j = (1 - \gamma^j) \left( \frac{B^j u^{K,j} k^j}{y^j} \right)^{\frac{\sigma^j-1}{\sigma^j}}. \quad (298)$$

Dividing eq. (297) by eq. (298), the ratio of the labor to the capital income share denoted by  $S^j = \frac{s_L^j}{1-s_L^j}$  reads as follows:

$$S^j = \frac{\gamma^j}{1 - \gamma^j} \left( \frac{B^j u^{K,j} K^j}{A^j L^j} \right)^{\frac{1-\sigma^j}{\sigma^j}}. \quad (299)$$



### Unit Cost for Producing

Dividing (294b)-(294c) by (294a) leads to a positive relationship between the wage-to-capital-rental-rate ratio and the capital-labor ratio in sector  $j$ :

$$\frac{W^j}{R} = \frac{\gamma^j}{1 - \gamma^j} \left( \frac{B^j}{A^j} \right)^{\frac{1-\sigma^j}{\sigma^j}} \left( \frac{\tilde{K}^j}{L^j} \right)^{\frac{1}{\sigma^j}}. \quad (300)$$

To determine the conditional demands for both inputs, we solve (300) for hours worked and next for capital:

$$L^j = \tilde{K}^j \left( \frac{\gamma^j}{1 - \gamma^j} \right)^{\sigma^j} \left( \frac{B^j}{A^j} \right)^{1-\sigma^j} \left( \frac{W^j}{R} \right)^{-\sigma^j}, \quad (301a)$$

$$\tilde{K}^j = L^j \left( \frac{1 - \gamma^j}{\gamma^j} \right)^{\sigma^j} \left( \frac{B^j}{A^j} \right)^{\sigma^j - 1} \left( \frac{W^j}{R} \right)^{\sigma^j}. \quad (301b)$$

Eq. (301a) can be rewritten as follows:

$$\gamma^j (A^j L^j)^{\frac{\sigma^j - 1}{\sigma^j}} = (\gamma^j)^{\sigma^j} (1 - \gamma^j)^{-\sigma^j} \left( \frac{W^j}{A^j} \right)^{1-\sigma^j} \left( \frac{R}{B^j} \right)^{\sigma^j - 1} (1 - \gamma^j) (B^j \tilde{K}^j)^{\frac{\sigma^j - 1}{\sigma^j}}.$$

Plugging the above equation into (293) leads to:

$$(1 - \gamma^j) (B^j \tilde{K}^j)^{\frac{\sigma^j - 1}{\sigma^j}} = (Y^j)^{\frac{\sigma^j - 1}{\sigma^j}} (X^j)^{-1} (1 - \gamma^j)^{\sigma^j} \left( \frac{R}{B^j} \right)^{1-\sigma^j} \quad (302)$$

where we set

$$X^j = (\gamma^j)^{\sigma^j} (A^j)^{\sigma^j - 1} \left( \frac{W^j}{A^j} \right)^{1-\sigma^j} + (1 - \gamma^j)^{\sigma^j} \left( \frac{R}{B^j} \right)^{1-\sigma^j}. \quad (303)$$

Eq. (301b) can be rewritten as follows:

$$(1 - \gamma^j) (B^j \tilde{K}^j)^{\frac{\sigma^j - 1}{\sigma^j}} = (\gamma^j)^{-\sigma^j} (1 - \gamma^j)^{\sigma^j} \left( \frac{R}{B^j} \right)^{1-\sigma^j} \left( \frac{W^j}{A^j} \right)^{\sigma^j - 1} \gamma^j (A^j L^j)^{\frac{\sigma^j - 1}{\sigma^j}}.$$

Plugging the above equation into (293) leads to:

$$\gamma^j (A^j L^j)^{\frac{\sigma^j - 1}{\sigma^j}} = (Y^j)^{\frac{\sigma^j - 1}{\sigma^j}} (X^j)^{-1} (\gamma^j)^{\sigma^j} (A^j)^{\sigma^j - 1} \left( \frac{W^j}{A^j} \right)^{1-\sigma^j}, \quad (304)$$

where  $X^j$  is given by (303).

Eq. (304) can be solved for the conditional demand for labor and eq. (302) can be solved for the conditional demand for capital (inclusive of capital utilization):

$$L^j = Y^j (A^j)^{\sigma^j - 1} \left( \frac{\gamma^j}{W^j} \right)^{\sigma^j} (X^j)^{\frac{\sigma^j}{1-\sigma^j}}, \quad (305a)$$

$$\tilde{K}^j = Y^j (B^j)^{\sigma^j - 1} \left( \frac{1 - \gamma^j}{R} \right)^{\sigma^j} (X^j)^{\frac{\sigma^j}{1-\sigma^j}}, \quad (305b)$$

where  $X^j$  is given by (302).

Total cost is equal to the sum of the labor and capital cost:

$$\text{Cost}^j = W^j L^j + R \tilde{K}^j. \quad (306)$$

Inserting conditional demand for inputs (318) into total cost (306) leads to:

$$\begin{aligned} \text{Cost}^j &= Y^j (X^j)^{\frac{\sigma^j}{1-\sigma^j}} (\gamma^j)^{\sigma^j} (A^j)^{\sigma^j - 1} \left( \frac{W^j}{A^j} \right)^{1-\sigma^j} + (1 - \gamma^j)^{\sigma^j} \left( \frac{R}{B^j} \right)^{1-\sigma^j}, \\ &= Y^j (X^j)^{\frac{1}{1-\sigma^j}}. \end{aligned}$$

The above equation shows that  $\text{Cost}^j$  is homogenous of degree one with respect to the level of production

$$\text{Cost}^j = c^j Y^j, \quad \text{with} \quad c^j = (X^j)^{\frac{1}{1-\sigma^j}}, \quad (307)$$

where

$$c^j \equiv \left[ (\gamma^j)^{\sigma^j} \left( \frac{W^j}{A^j} \right)^{1-\sigma^j} + (1-\gamma^j)^{\sigma^j} \left( \frac{R}{B^j} \right)^{1-\sigma^j} \right]^{\frac{1}{1-\sigma^j}}. \quad (308)$$

When we include technology utilization, eqs. (306)-(307) can be rewritten as follows:

$$\begin{aligned} \tilde{\text{Cost}}^j &= \tilde{W}^j L^j + \tilde{R}^j \tilde{K}^j, \\ &= c^j \tilde{Y}^j, \end{aligned} \quad (309)$$

where the unit cost for producing, denoted by  $c^j$ , inclusive of the technology utilization rate reads:

$$c^j = \left[ (\gamma^j)^{\sigma^j} \left( \frac{\tilde{W}^j}{\tilde{A}^j} \right)^{1-\sigma^j} + (1-\gamma^j)^{\sigma^j} \left( \frac{\tilde{R}^j}{\tilde{B}^j} \right)^{1-\sigma^j} \right]^{\frac{1}{1-\sigma^j}}, \quad (310)$$

where  $\tilde{W}^j = u^{Z,j} W^j$ ,  $\tilde{R}^j = u^{Z,j} R$ ,  $\tilde{A}^j = u^{Z,j} A^j$ ,  $\tilde{B}^j = u^{Z,j} B^j$ . Because the unit cost for producing is homogeneous of degree one, denoting the technology-utilization-adjusted unit cost by  $\text{UC}^j$  enables us to rewrite total cost described by eq. (309) as follows:

$$\tilde{\text{Cost}}^j = \frac{\text{UC}^j}{u^{Z,j}} \tilde{Y}^j, \quad (311)$$

where

$$\text{UC}^j = \left[ (\gamma^j)^{\sigma^j} \left( \frac{\tilde{W}^j}{\tilde{A}^j} \right)^{1-\sigma^j} + (1-\gamma^j)^{\sigma^j} \left( \frac{\tilde{R}^j}{\tilde{B}^j} \right)^{1-\sigma^j} \right]^{\frac{1}{1-\sigma^j}}. \quad (312)$$

Using the fact that  $(c^j)^{1-\sigma^j} = X^j$ , conditional demand for labor (318) can be rewritten as  $L^j = Y^j (\gamma^j)^{\sigma^j} \left( \frac{W^j}{A^j} \right)^{1-\sigma^j} (c^j)^{\sigma^j}$  which gives the labor share denoted by  $s_L^j$ :

$$s_L^j = \frac{W^j L^j}{P^j Y^j} = (\gamma^j)^{\sigma^j} \left( \frac{W^j}{A^j} \right)^{1-\sigma^j} (c^j)^{\sigma^j-1}, \quad (313a)$$

$$1 - s_L^j = \frac{R \tilde{K}^j}{P^j Y^j} = (1-\gamma^j)^{\sigma^j} \left( \frac{R}{B^j} \right)^{1-\sigma^j} (c^j)^{\sigma^j-1}, \quad (313b)$$

where we used the fact that  $P^j = c^j$ .

## O.2 Short-Run Solutions

### Sectoral Wages and Capital-Labor Ratios

Plugging the short-run solutions for  $L^H$  and  $L^N$  given by (227) into the resource constraint for capital (296), the system of four equations consisting of (294a)-(294c) together with (296) can be solved for sectoral wages  $W^j$  and sectoral capital-labor ratios  $k^j$ . Log-differentiating (294a)-(294c) together with (296) yields in matrix form:

$$\begin{aligned} & \begin{pmatrix} -\left(\frac{s_L^H}{\sigma^H}\right) & \left(\frac{s_L^N}{\sigma^N}\right) & 0 & 0 \\ \left(\frac{1-s_L^H}{\sigma^H}\right) & 0 & -1 & 0 \\ 0 & \left(\frac{1-s_L^N}{\sigma^N}\right) & 0 & -1 \\ \frac{K^H}{K} & \frac{K^N}{K} & \Psi_{WH} & \Psi_{WN} \end{pmatrix} \begin{pmatrix} \hat{k}^H \\ \hat{k}^N \\ \hat{W}^H \\ \hat{W}^N \end{pmatrix} \\ & = \begin{pmatrix} \hat{P}^N - \hat{P}^H - \left(\frac{\sigma^H - s_L^H}{\sigma^H}\right) \hat{B}^H + \left(\frac{\sigma^N - s_L^N}{\sigma^N}\right) \hat{B}^N - \left(\frac{s_L^H}{\sigma^H}\right) \hat{A}^H + \left(\frac{s_L^N}{\sigma^N}\right) \hat{A}^N + \frac{s_L^H}{\sigma^H} \hat{u}^{K,H} - \frac{s_L^N}{\sigma^N} \hat{u}^{K,N} \\ -\hat{P}^H - \left[\frac{(\sigma^H-1)+s_L^H}{\sigma^H}\right] \hat{A}^H - \left(\frac{1-s_L^H}{\sigma^H}\right) \hat{B}^H - \left(\frac{1-s_L^H}{\sigma^H}\right) \hat{u}^{K,H} \\ -\hat{P}^N - \left[\frac{(\sigma^N-1)+s_L^N}{\sigma^N}\right] \hat{A}^N - \left(\frac{1-s_L^N}{\sigma^N}\right) \hat{B}^N - \left(\frac{1-s_L^N}{\sigma^N}\right) \hat{u}^{K,N} \\ \hat{K} - \Psi_{\lambda} \hat{\lambda} - \Psi_{u^{Z,H}} \hat{u}^{Z,H} \end{pmatrix} \quad (314) \end{aligned}$$

where we set:

$$\Psi_{W^j} = \frac{K^H L_{W^j}^H W^j}{K L^H} + \frac{K^N L_{W^j}^N W^j}{K L^N}, \quad (315a)$$

$$\Psi_{u^{z,j}} = \frac{K^H L_{u^{z,j}}^H u^{z,j}}{K L^H} + \frac{K^N L_{u^{z,j}}^N u^{z,j}}{K L^N}, \quad (315b)$$

$$\Psi_{\bar{\lambda}} = \frac{K^H}{K} \sigma_L + \frac{K^N}{K} \sigma_L = \sigma_L. \quad (315c)$$

The short-run solutions for sectoral wages and capital-labor ratios are:

$$W^j = W^j(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N, u^{K,H}, u^{K,N}), \quad (316a)$$

$$k^j = k^j(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N, u^{K,H}, u^{K,N}). \quad (316b)$$

Inserting first sectoral wages (316), sectoral hours worked (313a) can be solved as functions of the shadow value of wealth, the capital stock, the price of non-traded goods in terms of foreign goods,  $P^N$ , and the terms of trade:

$$L^j = L^j(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N, u^{K,H}, u^{K,N}). \quad (317)$$

Totally differentiating output per hours worked (295) leads to:

$$\hat{y}^j = s_L^j \hat{A}^j + (1 - s_L^j) [\hat{B}^j + \hat{u}^{K,j} + \hat{k}^j], \quad (318)$$

where  $s_L^j$  and  $1 - s_L^j$  are the labor and capital income share, respectively, described by eqs. (297)-(298). Plugging solutions for sectoral capital-labor ratios (316) into (318) allows us to solve for sectoral value added per hours worked:

$$y^j = y^j(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N, u^{K,H}, u^{K,N}), \quad (319)$$

where for example the change in technology utilization adjusted value added per hour worked for tradables reads

$$\begin{aligned} dy^H &= \left[ \frac{y^H}{A^H} s_L^H + \frac{y^H}{k^H} (1 - s_L^H) k_{A^H}^H \right] dA^H \\ &+ \left[ \frac{y^H}{B^H} s_L^H + \frac{y^H}{k^H} (1 - s_L^H) k_{B^H}^H \right] dB^H \\ &+ \left[ \frac{y^H}{u^{K,H}} s_L^H + \frac{y^H}{k^H} (1 - s_L^H) k_{u^{K,H}}^H \right] du^{K,H} \\ &+ \frac{y^H}{k^H} (1 - s_L^H) dk^H. \end{aligned}$$

Using the fact that  $Y^j = y^j L^j$ , inserting solutions for  $y^j$  (319) and  $L^j$  (317), and differentiating, one obtains the solutions for sectoral value added:

$$Y^j = Y^j(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N, u^{K,H}, u^{K,N}). \quad (320)$$

Using the fact that  $K^j = k^j L^j$ , inserting solutions for  $k^j$  (316b) and  $L^j$  (317), differentiating, one obtains the solutions for the sectoral capital stock:

$$K^j = K^j(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N, u^{K,H}, u^{K,N}). \quad (321)$$

### Capital and Technology Utilization Rates, $u^{K,j}(t)$ and $u^{Z,j}(t)$

Inserting firm's optimal decisions for capital (294a), i.e.,  $P^j (1 - \gamma^j) (B^j)^{\frac{\sigma^j - 1}{\sigma^j}} (u^{K,j} k^j)^{-\frac{1}{\sigma^j}} (y^j)^{\frac{1}{\sigma^j}} = R$  into optimal choices for capital utilization (200f)-(200g), and invoking the Euler theorem which leads to  $W^H L^H + R u^{K,H} K^H = P^H Y^H$  to rewrite optimal choices for technology

utilization (200h)-(200i), we have:

$$\frac{R(t)u^{Z,H}(t)}{P^H(t)} = (1 - \gamma^H) u^{Z,H}(t) (B^H(t))^{\frac{\sigma^H-1}{\sigma^H}} (u^{K,H}(t)k^H(t))^{-\frac{1}{\sigma^H}} (y^H(t))^{\frac{1}{\sigma^H}} = \xi_1^H + \xi_2^H (u^{K,H}(t) - 1), \quad (322a)$$

$$\frac{R(t)u^{Z,N}(t)}{P^N(t)} = (1 - \gamma^N) (B^N(t))^{\frac{\sigma^N-1}{\sigma^N}} (u^{K,N}(t)k^N(t))^{-\frac{1}{\sigma^N}} (y^N(t))^{\frac{1}{\sigma^N}} [\xi_1^N + \xi_2^N (u^{K,N}(t) - 1)], \quad (322b)$$

$$Y^H(t) = \chi_1^H + \chi_2^H (u^{Z,H}(t) - 1), \quad (322c)$$

$$Y^N(t) = \chi_1^N + \chi_2^N (u^{Z,N}(t) - 1). \quad (322d)$$

Log-linearizing optimal decisions on capital and technology utilization rates described by (322a)-(322d) leads to in a matrix form:

$$= \begin{pmatrix} \left[ \left( \frac{\xi_2^H}{\xi_1^H} + \frac{s_L^H}{\sigma^H} \right) + \frac{s_L^H k^H u^{K,H}}{\sigma^H k^H} \right] & \frac{s_L^H k^H u^{K,N}}{\sigma^H k^H} & \left[ \frac{s_L^H k^H u^{Z,H}}{\sigma^H k^H} - 1 \right] & \frac{s_L^H k^H u^{Z,N}}{\sigma^H k^H} \\ \frac{s_L^N k^N u^{K,H}}{\sigma^N k^N} & \left[ \left( \frac{\xi_2^N}{\xi_1^N} + \frac{s_L^N}{\sigma^N} \right) + \frac{s_L^N k^N u^{K,N}}{\sigma^N k^N} \right] & \frac{s_L^N k^N u^{Z,H}}{\sigma^N k^N} & \left[ \frac{s_L^N k^N u^{Z,N}}{\sigma^N k^N} - 1 \right] \\ -\frac{Y^H u^{K,H}}{Y^H} & -\frac{Y^H u^{K,N}}{Y^H} & \left[ \frac{\chi_2^H}{\chi_1^H} - \frac{Y^H u^{Z,H}}{Y^H} \right] & -\frac{Y^H u^{Z,N}}{Y^H} \\ -\frac{Y^N u^{K,H}}{Y^N} & -\frac{Y^N u^{K,N}}{Y^N} & -\frac{Y^N u^{Z,H}}{Y^N} & \left[ \frac{\chi_2^N}{\chi_1^N} - \frac{Y^N u^{Z,N}}{Y^N} \right] \end{pmatrix} \begin{pmatrix} \hat{u}^{K,H} \\ \hat{u}^{K,N} \\ \hat{u}^{Z,H} \\ \hat{u}^{Z,N} \end{pmatrix} \\ = \begin{pmatrix} -\frac{s_L^H k^H u^{X,H}}{\sigma^H k^H} dX^H + \frac{s_L^H}{\sigma^H} \left[ \frac{1}{A^H} - \frac{k^H}{k^H} \right] dA^H + \left[ \left( \frac{\sigma^H - s_L^H}{\sigma^H} \right) \frac{1}{B^H} - \frac{s_L^H k^H u^{B,H}}{\sigma^H k^H} \right] dB^H \\ -\frac{s_L^N k^N u^{X,N}}{\sigma^N k^N} dX^N + \frac{s_L^N}{\sigma^N} \left[ \frac{1}{A^N} - \frac{k^N}{k^N} \right] dA^N + \left[ \left( \frac{\sigma^N - s_L^N}{\sigma^N} \right) \frac{1}{B^N} - \frac{s_L^N k^N u^{B,N}}{\sigma^N k^N} \right] dB^N \\ \frac{Y^H u^{X,H}}{Y^H} dX^H + \frac{Y^H u^{A,H}}{Y^H} dA^H + \frac{Y^H u^{B,H}}{Y^H} dB^H \\ \frac{Y^N u^{X,N}}{Y^N} dX^N + \frac{Y^N u^{A,N}}{Y^N} dA^N + \frac{Y^N u^{B,N}}{Y^N} dB^N \end{pmatrix}, \quad (32)$$

where  $X^H = P^H, P^N, K, \bar{\lambda}, A^N, B^N$  and  $X^N = P^H, P^N, K, \bar{\lambda}, A^H, B^H$ .

The short-run solutions for capital and technology utilization rates are:  
labela12.116

$$u^{K,j} = u^{K,j} (\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N), \quad (324a)$$

$$u^{Z,j} = u^{Z,j} (\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N). \quad (324b)$$

### Intermediate Solutions for $k^j, W^j, L^j, Y^j, K^j$

Plugging back solutions for capital and technology utilization rates (324a)-(324b) into (316a), (316b), (317), (319), (320), (321) leads to intermediate solutions for sectoral wages, sectoral capital-labor ratios, sectoral hours worked, sectoral value added, and sectoral capital stocks:

$$W^j, k^j, L^j, Y^j, K^j (\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N). \quad (325)$$

### Market Clearing Condition for Non-Tradables

The role of the price of non-tradables in terms of foreign-produced traded goods is to clear the non-traded goods market:

$$u^{Z,N}(t)Y^N(t) = C^N(t) + G^N(t) + J^N(t) + C^{K,N}(t)K^N(t) + C^{Z,N}(t). \quad (326)$$

Inserting solutions for  $C^N, J^N, Y^N$  given by (225), i.e.,  $C^N(\bar{\lambda}, P^N, P^H)$ , (246), i.e.,  $J^N = J^N(K, Q, P^N, P^H)$ , (325), i.e.,  $Y^N = Y^N(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N)$ , the non-traded goods market clearing condition (326) can be rewritten as follows:

$$u^{Z,N}Y^N(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N) = C^N(\bar{\lambda}, P^N, P^H) + G^N + J^N(K, Q, P^N, P^H) \\ + C^{K,N} [u^{K,N}(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N)] K^N(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N) \\ + C^{Z,N}(u^{Z,N}). \quad (327)$$

Linearizing (327) leads to:

$$Y^N du^{Z,N}(t) + dY^N(t) = dC^N(t) + dG^N(t) + dJ^N(t) + K^N \xi_1^N du^{K,N}(t) + \chi_1^N du^{Z,N}(t), \quad (328)$$

where the terms  $Y^N du^{Z,N}(t)$  and  $\chi_1^N du^{Z,N}(t)$  cancel out because eq. (236d) evaluated at the steady-state implies  $Y^N = \chi_1^N$ .

### Market Clearing Condition for Home-Produced Traded Goods

The role of the price of home-produced traded goods in terms of foreign-produced traded goods or the terms of trade is to clear the home-produced traded goods market:

$$u^{Z,H}(t)Y^H(t) = C^H(t) + G^H(t) + J^H(t) + X^H(t) + P^H(t)C^{K,H}(t)K^H(t) + P^H(t)C^{Z,H}(t), \quad (329)$$

where  $X^H$  stands for exports which are negatively related to the terms of trade:

$$X^H = \varphi_X (P^H)^{-\phi_X}, \quad (330)$$

where  $\phi_X$  is the elasticity of exports with respect to the terms of trade.

Inserting solutions for  $C^H$ ,  $J^H$ ,  $Y^H$  given by (225), (226a), (325), respectively, the traded goods market clearing condition (329) can be rewritten as follows:

$$\begin{aligned} u^{Z,H}Y^H(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N) &= C^H(\bar{\lambda}, P^N, P^H) + G^H + J^H(K, Q, P^N, P^H) \\ &+ C^{K,H}[u^{K,H}(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N)]K^H(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N) \\ &+ C^{Z,H}(u^{Z,H}). \end{aligned} \quad (331)$$

Linearizing (331) leads to:

$$Y^H du^{Z,H}(t) + dY^H(t) = dC^H(t) + dG^H(t) + dJ^H(t) + dX^H(t) + K^H \xi_1^H du^{K,H}(t) + \chi_1^H du^{Z,H}(t), \quad (332)$$

where the terms  $Y^H du^{Z,H}(t)$  and  $\chi_1^H du^{Z,H}(t)$  cancel out because eq. (236c) evaluated at the steady-state implies  $Y^H = \chi_1^H$ .

### Solving for Relative Prices

As shall be useful below, we write out a number of useful notations:

$$\Psi_{P^N}^N = Y_{P^N}^N - C_{P^N}^N - J_{P^N}^N - K^N \xi_1^N u_{P^N}^{K,N}, \quad (333a)$$

$$\Psi_{P^H}^N = Y_{P^H}^N - C_{P^H}^N - J_{P^H}^N - K^N \xi_1^N u_{P^H}^{K,N}, \quad (333b)$$

$$\Psi_K^N = Y_K^N - J_K^N - K^N \xi_1^N u_K^{K,N}, \quad (333c)$$

$$\Psi_{A^j}^N = Y_{A^j}^N - K^N \xi_1^N u_{A^j}^{K,N}, \quad (333d)$$

$$\Psi_{B^j}^N = Y_{B^j}^N - K^N \xi_1^N u_{B^j}^{K,N}, \quad (333e)$$

$$\Psi_\lambda^N = Y_\lambda^N - C_\lambda^N - K^N \xi_1^N u_\lambda^{K,N}, \quad (333f)$$

$$\Psi_{P^N}^H = Y_{P^N}^H - C_{P^N}^H - J_{P^N}^H - K^H \xi_1^H u_{P^N}^{K,H}, \quad (333g)$$

$$\Psi_{P^H}^H = Y_{P^H}^H - C_{P^H}^H - J_{P^H}^H - X_{P^H}^H - K^H \xi_1^H u_{P^H}^{K,H}, \quad (333h)$$

$$\Psi_K^H = Y_K^H - J_K^H - K^H \xi_1^H u_K^{K,H}, \quad (333i)$$

$$\Psi_{A^j}^H = Y_{A^j}^H - K^H \xi_1^H u_{A^j}^{K,H}, \quad (333j)$$

$$\Psi_{B^j}^H = Y_{B^j}^H - K^H \xi_1^H u_{B^j}^{K,H}, \quad (333k)$$

$$\Psi_\lambda^H = Y_\lambda^H - C_\lambda^H - K^H \xi_1^H u_\lambda^{K,H}. \quad (333l)$$

Linearized versions of market clearing conditions described by eq. (328) and eq. (332) can be rewritten in a matrix form:

$$\begin{aligned} &\begin{pmatrix} \Psi_{P^N}^N & \Psi_{P^H}^N \\ \Psi_{P^N}^H & \Psi_{P^H}^H \end{pmatrix} \begin{pmatrix} dP^N \\ dP^H \end{pmatrix} \\ &= \begin{pmatrix} -\Psi_K^N dK + J_Q^N dQ + G_G^N dG - \sum_{j=H}^N \Psi_{A^j}^N dA^j - \sum_{j=H}^N \Psi_{B^j}^N dB^j - \Psi_\lambda^N d\bar{\lambda} \\ -\Psi_K^H dK + J_Q^H dQ + G_G^H dG - \sum_{j=H}^N \Psi_{A^j}^H dA^j - \sum_{j=H}^N \Psi_{B^j}^H dB^j - \Psi_\lambda^H d\bar{\lambda} \end{pmatrix} \end{aligned} \quad (334)$$

The short-run solutions for capital and technology utilization rates are:

$$P^N = P^N(\bar{\lambda}, K, Q, G, A^H, A^N, B^H, B^N), \quad (335a)$$

$$P^H = P^H(\bar{\lambda}, K, Q, G, A^H, A^N, B^H, B^N). \quad (335b)$$

### O.3 Solving the Model

In our model, there is five state variables, namely the capital stock  $K$ , labor-augmenting productivity,  $A^H$ ,  $A^N$ , Capital-augmenting productivity,  $B^H$ ,  $B^N$ , and one control variable, namely the shadow price of the capital stock  $Q$ . To solve the model, we have to express all variables in terms of state and control variables. Plugging back solutions for the relative price of non-tradables (335a) and the terms of trade (335b) into consumption in sectoral goods (225), investment inputs (246), sectoral output (325), capital and technology utilization rates (324a)-(324b) leads to solutions for sectoral consumption, sectoral inputs for capital goods, sectoral wages, sectoral capital-labor ratios, sectoral hours worked, sectoral value added, sectoral capital stocks:

$$C^j, J^j, W^j, k^j, L^j, Y^j, K^j, v, u^{K,j}, u^{Z,j} (\bar{\lambda}, K, Q, G, A^H, A^N, B^H, B^N). \quad (336)$$

The return on domestic capital is:

$$R = P^H (1 - \gamma^H) (B^H)^{\frac{\sigma^H - 1}{\sigma^H}} (u^{K,H} k^H)^{-\frac{1}{\sigma^H}} (y^H)^{\frac{1}{\sigma^H}}. \quad (337)$$

Differentiating (337) and making use of (318) leads to:

$$\hat{R} = \hat{P}^H - \frac{s_L^H}{\sigma^H} (\hat{k}^H + \hat{u}^{K,H}) + \frac{s_L^H}{\sigma^H} \hat{A}^N + \left( \frac{\sigma^H - s_L^H}{\sigma^H} \right) \hat{B}^H. \quad (338)$$

Inserting the short-run static solutions for the capital-labor ratio  $k^H$  and the capital utilization rate (336), eq. (337) can be solved for the return on domestic capital:

$$R = R(\bar{\lambda}, K, P^N, P^H, A^H, A^N, B^H, B^N). \quad (339)$$

Remembering that the non-traded input  $J^N$  used to produce the capital good is described by  $(1 - \iota) \left( \frac{P^N}{P_J} \right)^{-\phi_J} J$  (see eq. (209b)) with  $J = I + \frac{\kappa}{2} \left( \frac{I}{K} - \delta_K \right)^2 K$ , using the fact that  $J^N = Y^N - C^N - G^N$  and inserting  $I = \dot{K} + \delta_K$ , the capital accumulation equation reads as follows:

$$\dot{K} = \frac{Y^N - C^N - G^N - C^{K,N} K^N - C^{Z,N}}{(1 - \iota) \left( \frac{P^N}{P_J} \right)^{-\phi_J}} - \delta_K K - \frac{\kappa}{2} \left( \frac{I}{K} - \delta_K \right)^2 K. \quad (340)$$

Inserting first solutions for non-traded output, consumption in non-tradables, demand for non-traded input, non-traded capital and technology utilization rates described by eq. (336) together with optimal investment decision (245a) into the physical capital accumulation equation (340), and plugging the short-run solution for the return on domestic capital (339) into the dynamic equation for the shadow value of capital stock (200e), the dynamic

system reads as follows:

$$\begin{aligned}\dot{K} &\equiv \Upsilon(K, Q, G, A^j, B^j), \\ &= \frac{Y^N(K, Q, G, A^j, B^j) - C^N(K, Q, G, A^j, B^j) - G^N(G)}{(1-\iota) \left[ \frac{P^N(K, Q, G, A^j, B^j)}{P_J(K, Q, G, A^j, B^j)} \right]^{-\phi_J}} \\ &\quad - \frac{C^{K,N} [u^{K,N}(K, Q, G, A^j, B^j)] K^N - C^{Z,N} [u^{Z,N}(K, Q, G, A^j, B^j)]}{(1-\iota) \left[ \frac{P^N(K, Q, G, A^j, B^j)}{P_J(K, Q, G, A^j, B^j)} \right]^{-\phi_J}} \\ &\quad - \delta_K K - \frac{K}{2\kappa} \left[ \frac{Q}{P_J(K, Q, G, A^j, B^j)} - 1 \right]^2, \tag{341a}\end{aligned}$$

$$\begin{aligned}\dot{Q} &\equiv \Sigma(K, Q, G, A^j, B^j), \\ &= (r^* + \delta_K) Q - \left[ \frac{R(K, Q, G, A^j, B^j)}{K} \right. \\ &\quad \times \sum_{j=H,N} u^{K,j}(K, Q, G, A^j, B^j) u^{Z,j}(K, Q, G, A^j, B^j) K^j(K, Q, G, A^j, B^j) \\ &\quad \left. - \sum_{j=H,N} P^j(K, Q, G, A^j, B^j) C^{K,j} [u^{K,j}(K, Q, G, A^j, B^j)] \frac{K^j(K, Q, G, A^j, B^j)}{K} \right] \tag{341b}\end{aligned}$$

$$+ P_J [P^H(\cdot), P^N(\cdot)] \frac{\kappa}{2} v(\cdot) (v(\cdot) + 2\delta_K) \Big]. \tag{341c}$$

Let us denote by  $\Upsilon_X$ , the partial derivative evaluated at the steady-state of the capital accumulation equation w.r.t.  $X = K, Q, G, A^j, B^j$ . Partial derivatives evaluated at the steady-state are described by (271a)-(271c) together with:

$$\Upsilon_{A^j} \equiv \frac{\partial \dot{K}}{\partial A^j} = \frac{J}{J^N} \left( Y_{A^j}^N - C_{A^j}^N - K^N \xi_1^N u_{A^j}^{K,N} \right) + \alpha_J \phi_J J \left( \frac{P_{A^j}^N}{P^N} - \alpha_J^H \frac{P_{A^j}^H}{P^H} \right), \tag{342a}$$

$$\Upsilon_{B^j} \equiv \frac{\partial \dot{K}}{\partial B^j} = \frac{J}{J^N} \left( Y_{B^j}^N - C_{B^j}^N - K^N \xi_1^N u_{B^j}^{K,N} \right) + \alpha_J \phi_J J \left( \frac{P_{B^j}^N}{P^N} - \alpha_J^H \frac{P_{B^j}^H}{P^H} \right) \tag{342b}$$

Let us denote by  $\Sigma_X$ , the partial derivative evaluated at the steady-state of the dynamic equation for the marginal value of an additional unit of capital w.r.t.  $X = K, Q, G, A^j, B^j$ . Partial derivatives evaluated at the steady-state are described by (272a)-(272c) together with:

$$\begin{aligned}\Sigma_{A^j} &\equiv \frac{\partial \dot{Q}}{\partial A^j} = - \left\{ R_{A^j} + \frac{R}{K} \sum_{j=H,N} \left[ K^j u_{A^j}^{K,j} + K^j u_{A^j}^{Z,j} \right] \right. \\ &\quad + \frac{R}{K} (K_{A^j}^H + K_{A^j}^N) - \sum_{j=H,N} \frac{P^j K^j}{K} \xi_1^j u_{A^j}^{K,j} \\ &\quad \left. + P_J \kappa v_{A^j} \delta_K \right\} > 0. \tag{343a}\end{aligned}$$

$$\begin{aligned}\Sigma_{B^j} &\equiv \frac{\partial \dot{Q}}{\partial B^j} = - \left\{ R_{B^j} + \frac{R}{K} \sum_{j=H,N} \left[ K^j u_{B^j}^{K,j} + K^j u_{B^j}^{Z,j} \right] \right. \\ &\quad + \frac{R}{K} (K_{B^j}^H + K_{B^j}^N) - \sum_{j=H,N} \frac{P^j K^j}{K} \xi_1^j u_{B^j}^{K,j} \\ &\quad \left. + P_J \kappa v_{B^j} \delta_K \right\} > 0. \tag{343b}\end{aligned}$$

#### O.4 Current Account Equation and Intertemporal Solvency Condition

Following the same steps as in subsection N.6, the current account reads as:

$$\dot{N}(t) = r^* N(t) + P^H(t) X^H(t) - M^F(t), \tag{344}$$



where  $X^H = Y^H - C^H - G^H - J^H - C^{K,H}K^H - C^{Z,H}$  stands for exports of home goods and we denote by  $M^F$  imports of foreign consumption and investment goods:

$$M^F(t) = C^F(t) + G^F(t) + J^F(t). \quad (345)$$

Inserting (325) into (344) and the solution for  $P^H$  described by eq. (335b) into  $X^H = X^H(P^H)$  leads to:

$$\begin{aligned} \dot{N} &\equiv r^*N + \Xi(K, Q, G, A^H, A^N, B^H, B^N), \\ &= r^*N + P^H(K, Q, G, A^H, A^N, B^H, B^N) X^H(K, Q, G, A^H, A^N, B^H, B^N) \\ &\quad - M^F(K, Q, G, A^H, A^N, B^H, B^N). \end{aligned} \quad (346)$$

Let us denote by  $\Xi_X$  the partial derivative evaluated at the steady-state of the dynamic equation for the current account w.r.t. to  $X = K, Q, G, A^j, B^j$ . Partial derivatives evaluated at the steady-state are described by (279a)-(279c) together with:

$$\Xi_{A^j} \equiv \frac{\partial \dot{N}}{\partial A^j} = (1 - \phi_X) X^H P_{A^j}^H - M_{A^j}^F, \quad (347a)$$

$$\Xi_{B^j} \equiv \frac{\partial \dot{N}}{\partial B^j} = (1 - \phi_X) X^H P_{B^j}^H - M_{B^j}^F. \quad (347b)$$

## O.5 The Technology Frontier

Since we relax the assumption of Hicks-neutral technological change, we have to relate changes in labor- and capital-augmenting efficiency, i.e.,  $\hat{A}^j(t)$  and  $\hat{B}^j(t)$ , respectively, to the percentage deviation of capital-utilization-adjusted TFP in sector  $j$ , i.e.,  $\hat{Z}^j(t)$ , in order to be consistent with our empirical strategy. A natural way to map  $\hat{A}^j$  and  $\hat{B}^j$  into  $Z^j$  is to assume that besides optimally choosing factor inputs, firms also optimally choose the technology of production function. Following Caselli and Coleman [2006] and Caselli [2016], the menu of possible choices of production functions is represented by a set of possible  $(\hat{A}^j, \hat{B}^j)$  pairs. We assume that firms in sector  $j$  choose labor and capital efficiency along the technology frontier which is assumed to take a CES form:

$$\left[ \gamma_Z^j (u^{Z,j}(t) A^j(t))^{\frac{\sigma_Z^j - 1}{\sigma_Z^j}} + (1 - \gamma_Z^j) (u^{Z,j}(t) B^j(t))^{\frac{\sigma_Z^j - 1}{\sigma_Z^j}} \right]^{\frac{\sigma_Z^j}{\sigma_Z^j - 1}} \leq Z^j(t), \quad (348)$$

where  $Z^j > 0$  is the height of the technology frontier,  $0 < \gamma_Z^j < 1$  is the weight of labor efficiency in TFP and  $\sigma_Z^j > 0$  corresponds to the elasticity of substitution between labor and capital efficiency. Using the fact that  $Z^j(t) = u^{Z,j}(t) \bar{Z}^j$  and totally differentiating (348) leads to

$$\begin{aligned} 0 &= \gamma_Z^j (A^j(t))^{\frac{\sigma_Z^j - 1}{\sigma_Z^j}} \hat{A}^j(t) + (1 - \gamma_Z^j) (B^j(t))^{\frac{\sigma_Z^j - 1}{\sigma_Z^j}} \hat{B}^j(t), \\ \frac{\hat{B}^j(t)}{\hat{A}^j(t)} &= - \frac{\gamma_Z^j}{1 - \gamma_Z^j} \left( \frac{B^j(t)}{A^j(t)} \right)^{\frac{1 - \sigma_Z^j}{\sigma_Z^j}}. \end{aligned} \quad (349)$$

Eq. (349) measures the number of capital-augmenting efficiency units the firm must give up to increase labor-augmenting productivity by one unit.

Firms choose  $A^j$  and  $B^j$  along the technology frontier so as to minimize the unit cost function (308) which we repeat for convenience:

$$c^j \equiv \left[ (\gamma^j)^{\sigma^j} \left( \frac{W^j(t)}{A^j(t)} \right)^{1 - \sigma^j} + (1 - \gamma^j)^{\sigma^j} \left( \frac{R(t)}{B^j(t)} \right)^{1 - \sigma^j} \right]^{\frac{1}{1 - \sigma^j}}, \quad (350)$$

subject to (348) which holds as an equality. Differentiating (133) w.r.t.  $A^j(t)$  and  $B^j(t)$  (while keeping  $W^j$  and  $R$  fixed) and setting the expression to zero leads to:

$$\begin{aligned}\hat{c}^j(t) &= -(\gamma^j)^{\sigma^j} \left(\frac{W^j(t)}{A^j(t)}\right)^{1-\sigma^j} (c^j(t))^{\sigma^j-1} \hat{A}^j(t) - (1-\gamma^j)^{\sigma^j} \left(\frac{R(t)}{B^j(t)}\right)^{1-\sigma^j} (c^j(t))^{\sigma^j-1} \hat{B}^j(t), \\ \frac{\hat{B}^j(t)}{\hat{A}^j(t)} &= -\left(\frac{\gamma^j}{1-\gamma^j}\right)^{\sigma^j} \left(\frac{W^j(t)}{R(t)}\right)^{1-\sigma^j} \left(\frac{B^j(t)}{A^j(t)}\right)^{1-\sigma^j}.\end{aligned}\quad (351)$$

Eq. (351) measures the number of capital-augmenting efficiency units the firm must give up following an increase in labor-augmenting productivity by one unit to keep the unit cost for producing unchanged.

Performing the cost minimization of (350) subject to (348) amounts to equating (349) with (351) which leads to the following optimal choice of technology:

$$\begin{aligned}\left(\frac{\gamma^j}{1-\gamma^j}\right)^{\sigma^j} \left(\frac{W^j(t)}{R(t)}\right)^{1-\sigma^j} \left(\frac{B^j(t)}{A^j(t)}\right)^{1-\sigma^j} &= \frac{\gamma_Z^j}{1-\gamma_Z^j} \left(\frac{B^j(t)}{A^j(t)}\right)^{\frac{1-\sigma_Z^j}{\sigma_Z^j}}, \\ \left(\frac{B^j(t)}{A^j(t)}\right)^{\frac{1-\sigma^j}{\sigma^j}} \left(\frac{B^j(t)}{A^j(t)}\right)^{-\left(\frac{1-\sigma_Z^j}{\sigma_Z^j}\right)\frac{1}{\sigma^j}} &= \left(\frac{\gamma_Z^j}{1-\gamma_Z^j}\right)^{\frac{1}{\sigma^j}} \left(\frac{1-\gamma^j}{\gamma^j}\right) \left(\frac{W^j(t)}{R(t)}\right)^{-\frac{1-\sigma^j}{\sigma^j}}\end{aligned}\quad (352)$$

Eq. (352) states that it is optimal for firms to bias factor efficiency toward the most expensive factor as long as  $\frac{1-\sigma_Z^j}{\sigma_Z^j} < 1 - \sigma^j$ .

Using the fact that  $(\gamma^j)^{\sigma^j} \left(\frac{W^j(t)}{A^j(t)}\right)^{1-\sigma^j} (c^j(t))^{\sigma^j-1} = s_L^j(t)$  (see eq. (313a)), eq. (351) can be rewritten as  $-s_L^j \hat{A}^j(t) - (1-s_L^j) \hat{B}^j(t) = -\hat{c}^j(t)$ . Setting this equality to zero and making use of (349) leads to:

$$\frac{\gamma_Z^j}{1-\gamma_Z^j} \left(\frac{B^j(t)}{A^j(t)}\right)^{\frac{1-\sigma_Z^j}{\sigma_Z^j}} = \frac{s_L^j(t)}{1-s_L^j(t)} \equiv S^j(t).\quad (353)$$

Eq. (353) can be solved for  $s_L^j(t)$ :

$$\begin{aligned}s_L^j(t) &= \frac{\gamma_Z^j (A^j(t))^{\frac{\sigma_Z^j-1}{\sigma_Z^j}}}{\gamma_Z^j (A^j(t))^{\frac{\sigma_Z^j-1}{\sigma_Z^j}} + (1-\gamma_Z^j) (B^j(t))^{\frac{\sigma_Z^j-1}{\sigma_Z^j}}}, \\ &= \gamma_Z^j \left(\frac{A^j(t)}{\bar{Z}^j}\right)^{\frac{\sigma_Z^j-1}{\sigma_Z^j}},\end{aligned}\quad (354)$$

where we made use of (348) to obtain the last line and we used the fact that  $Z^j(t) = u^{Z,j}(t) \bar{Z}^j$ .

Log-linearizing (348) in the neighborhood of the initial steady-state and making use of eq. (354) leads to:

$$\begin{aligned}0 &= \gamma_Z^j \left(\frac{A^j}{\bar{Z}^j}\right)^{\frac{\sigma_Z^j-1}{\sigma_Z^j}} \hat{A}^j(t) + (1-\gamma_Z^j) \left(\frac{B^j}{\bar{Z}^j}\right)^{\frac{\sigma_Z^j-1}{\sigma_Z^j}} \hat{B}^j(t), \\ 0 &= s_L^j \hat{A}^j(t) + (1-s_L^j) \hat{B}^j(t),\end{aligned}\quad (355)$$

where we used the fact that  $\bar{Z}^j$  is constant.

Log-linearizing (299) in the neighborhood of the initial steady-state leads to:

$$\hat{B}^j(t) - \hat{A}^j(t) = \left(\frac{\sigma^j}{1-\sigma^j}\right) \hat{S}^j - \left(\hat{u}^{K,j}(t) + \hat{k}^j(t)\right).\quad (356)$$

The system which comprises eq. (355) and eq. (356) can be solved for the percentage deviation of factor-augmenting efficiency relative to the initial steady-state:

$$\hat{A}^j(t) = - \left(1 - s_L^j\right) \left[ \left(\frac{\sigma^j}{1 - \sigma^j}\right) \hat{S}^j(t) - \hat{k}^j(t) - \hat{u}^{K,j}(t) \right], \quad (357a)$$

$$\hat{B}^j(t) = s_L^j \left[ \left(\frac{\sigma^j}{1 - \sigma^j}\right) \hat{S}^j(t) - \hat{k}^j(t) - \hat{u}^{K,j}(t) \right]. \quad (357b)$$

Eq. (357a) and eq. (357b) correspond to eq. (52a) and eq. (52b) in the main text.

## P Solving for Temporary Government Spending Shocks

### P.1 Setting the Dynamics of Government Shock and Factor-Augmenting Efficiency

Because the endogenous response of government spending to an exogenous fiscal shock is hump-shaped, we assume that government consumption as a percentage of GDP evolves according to the following dynamic equation:

$$\frac{dG(t)}{Y} = e^{-\xi t} - (1 - g) e^{-\chi t}, \quad (358)$$

where  $dG(t) = G(t) - G$  is the deviation of government consumption relative to the initial steady-state,  $g > 0$  parametrizes the magnitude of the exogenous fiscal shock,  $\xi > 0$  and  $\chi > 0$  are (positive) parameters which are set in order to capture the non-monotonic endogenous response of  $G(t)$ . We assume that the rise in government consumption is split between non-traded and traded goods and government consumption in traded goods is split between government consumption in home-produced traded goods and foreign-produced traded goods. Denoting the non-tradable content of government spending by  $\omega_{GN}$  and the home traded goods content of government spending by  $\omega_{GH} = \frac{P^H G^H}{G}$ , formally we have:

$$\frac{dG(t)}{Y} = \omega_{GN} \frac{dG(t)}{Y} + \omega_{GH} \frac{dG(t)}{Y} + \omega_{GF} \frac{dG(t)}{Y}, \quad (359)$$

where  $\omega_{GF} = \frac{G^F}{G}$  is the import content of government spending. In line with the evidence we document in Appendix E,  $\omega_{GN}$  refers to the non-tradable content of government consumption as well as the intensity of the government spending shock in non-traded goods.

We further specify a dynamic adjustment for  $A^j(t)$  and  $B^j(t)$ :

$$\frac{dA^j(t)}{A^j} = e^{-\xi_A^j t} - (1 - a^j) e^{-\chi_A^j t}, \quad (360a)$$

$$\frac{dB^j(t)}{B^j} = e^{-\xi_B^j t} - (1 - b^j) e^{-\chi_B^j t}, \quad (360b)$$

where  $a^j$  ( $b^j$ ) parameterizes the impact response of labor- (capital-) augmenting technological change;  $\xi_A^j > 0$  ( $\xi_B^j > 0$ ) and  $\chi_A^j > 0$  ( $\chi_B^j > 0$ ) are (positive) parameters which are set in order to reproduce the dynamic adjustment of labor-augmenting (capital-augmenting) technological change.

### P.2 Solving for Temporary Government Spending Shocks

Linearizing (341a)-(341c) in the neighborhood of the steady-state, we get in a matrix form:

$$\begin{pmatrix} \dot{K}(t) \\ \dot{Q}(t) \end{pmatrix} = \begin{pmatrix} \Upsilon_K & \Upsilon_Q \\ \Sigma_K & \Sigma_Q \end{pmatrix} \begin{pmatrix} dK(t) \\ dQ(t) \end{pmatrix} + \begin{pmatrix} \Upsilon_G dG(t) + \sum_{j=H}^N \Upsilon_{A^j} dA^j(t) + \sum_{j=H}^N \Upsilon_{B^j} dB^j(t) \\ \Sigma_G dG(t) + \sum_{j=H}^N \Sigma_{A^j} dA^j(t) + \sum_{j=H}^N \Sigma_{B^j} dB^j(t) \end{pmatrix}, \quad (361)$$

where the coefficients of the Jacobian matrix are partial derivatives evaluated at the steady-state, e.g.,  $\Upsilon_X = \frac{\partial \Upsilon}{\partial X}$  with  $X = K, Q$ , and the direct effects of an exogenous change in

government spending on  $K$  and  $Q$  are described by  $\Upsilon_G = \frac{\partial \Upsilon}{\partial G}$  and  $\Sigma_G = \frac{\partial \Sigma}{\partial G}$ , also evaluated at the steady-state.

As shall be useful below to write the solutions in a compact form, let us define the following terms:

$$\Phi_1^G = [(\Upsilon_K - \nu_2) \Upsilon_G + \Upsilon_Q \Sigma_G], \quad (362a)$$

$$\Phi_2^G = [(\Upsilon_K - \nu_1) \Upsilon_G + \Upsilon_Q \Sigma_G], \quad (362b)$$

$$\Phi_1^{Aj} = [(\Upsilon_K - \nu_2) \Upsilon_{Aj} + \Upsilon_Q \Sigma_{Aj}], \quad (362c)$$

$$\Phi_2^{Aj} = [(\Upsilon_K - \nu_1) \Upsilon_{Aj} + \Upsilon_Q \Sigma_{Aj}], \quad (362d)$$

$$\Phi_1^{Bj} = [(\Upsilon_K - \nu_2) \Upsilon_{Bj} + \Upsilon_Q \Sigma_{Bj}], \quad (362e)$$

$$\Phi_2^{Bj} = [(\Upsilon_K - \nu_1) \Upsilon_{Bj} + \Upsilon_Q \Sigma_{Bj}]. \quad (362f)$$

We denote by  $V = (V^1, V^2)$  the matrix of eigenvectors with  $V^{i,j} = (1, \omega_2^i)$  and we denote by  $V^{-1}$  the inverse matrix of  $V$ . Let us define:

$$\begin{pmatrix} X_1(t) \\ X_2(t) \end{pmatrix} \equiv V^{-1} \begin{pmatrix} dK(t) \\ dQ(t) \end{pmatrix}. \quad (363)$$

Differentiating w.r.t. time, one obtains:

$$\begin{aligned} \begin{pmatrix} \dot{X}_1(t) \\ \dot{X}_2(t) \end{pmatrix} &= \begin{pmatrix} \nu_1 & 0 \\ 0 & \nu_2 \end{pmatrix} \begin{pmatrix} X_1(t) \\ X_2(t) \end{pmatrix} + V^{-1} \begin{pmatrix} \Upsilon_G dG(t) + \sum_{j=H}^N \Upsilon_{Aj} dA^j(t) + \sum_{j=H}^N \Upsilon_{Bj} dB^j(t) \\ \Sigma_G dG(t) + \sum_{j=H}^N \Sigma_{Aj} dA^j(t) + \sum_{j=H}^N \Sigma_{Bj} dB^j(t) \end{pmatrix}, \\ &= \begin{pmatrix} \nu_1 X_1(t) \\ \nu_2 X_2(t) \end{pmatrix} + \frac{1}{\nu_1 - \nu_2} \begin{pmatrix} \Phi_1^G dG(t) + \sum_{j=H}^N \Phi_1^{Aj} dA^j(t) + \sum_{j=H}^N \Phi_1^{Bj} dB^j(t) \\ -\Phi_2^G dG(t) - \sum_{j=H}^N \Phi_2^{Aj} dA^j(t) - \sum_{j=H}^N \Phi_2^{Bj} dB^j(t) \end{pmatrix}. \end{aligned} \quad (364)$$

As will be useful below, in order to express solutions in a compact form, we set:

$$\Gamma_1^G = -\frac{\Phi_1^G Y}{\nu_1 - \nu_2} \frac{1}{\nu_1 + \xi}, \quad (365a)$$

$$\Gamma_2^G = -\frac{\Phi_2^G Y}{\nu_1 - \nu_2} \frac{1}{\nu_2 + \xi}, \quad (365b)$$

$$\Theta_1^G = (1 - g) \frac{\nu_1 + \xi}{\nu_1 + \chi}, \quad (365c)$$

$$\Theta_2^G = (1 - g) \frac{\nu_2 + \xi}{\nu_2 + \chi}. \quad (365d)$$

$$(365e)$$

and for  $X^j = A^j, B^j$ :

$$\Gamma_1^{X^j} = -\frac{\Phi_1^{X^j} X^j}{\nu_1 - \nu_2} \frac{1}{\nu_1 + \xi_X^j}, \quad (366a)$$

$$\Gamma_2^{X^j} = -\frac{\Phi_2^{X^j} X^j}{\nu_1 - \nu_2} \frac{1}{\nu_2 + \xi_X^j}, \quad (366b)$$

$$\Theta_1^{X^j} = (1 - x^j) \frac{\nu_1 + \xi}{\nu_1 + \chi_X^j}, \quad (366c)$$

$$\Theta_2^{X^j} = (1 - x^j) \frac{\nu_2 + \xi}{\nu_2 + \chi_X^j}, \quad (366d)$$

$$(366e)$$

where  $x^j = a^j, b^j$ .

Solving for  $X_1(t)$  gives:

$$\begin{aligned}
X_1(t) &= e^{\nu_1 t} \left\{ X_1(0) + \frac{\Phi_1^G}{\nu_1 - \nu_2} \int_0^t dG(\tau) e^{-\nu_1 \tau} d\tau + \sum_{X^j} \frac{\Phi_1^{X^j}}{\nu_1 - \nu_2} \int_0^t dX^j(\tau) e^{-\nu_1 \tau} d\tau \right\}, \\
&= e^{\nu_1 t} \left\{ X_1(0) + \frac{\Phi_1^G Y}{\nu_1 - \nu_2} \int_0^t \left[ e^{-(\xi + \nu_1)\tau} - (1-g) e^{-(\chi + \nu_1)\tau} \right] d\tau \right. \\
&\quad \left. + \sum_{X^j} \frac{\Phi_1^{X^j} X^j}{\nu_1 - \nu_2} \int_0^t \left[ e^{-(\xi_X^j + \nu_1)\tau} - (1-x^j) e^{-(\chi_X^j + \nu_1)\tau} \right] d\tau \right\}, \\
&= e^{\nu_1 t} X_1(0) + \frac{\Phi_1^G Y}{\nu_1 - \nu_2} \left[ \left( \frac{e^{\nu_1 t} - e^{-\xi t}}{\nu_1 + \xi} \right) - (1-g) \left( \frac{e^{\nu_1 t} - e^{-\chi t}}{\nu_1 + \chi} \right) \right] \\
&\quad + \sum_{X^j} \frac{\Phi_1^{X^j} X^j}{\nu_1 - \nu_2} \left[ \left( \frac{e^{\nu_1 t} - e^{-\xi_X^j t}}{\nu_1 + \xi_X^j} \right) - (1-x^j) \left( \frac{e^{\nu_1 t} - e^{-\chi_X^j t}}{\nu_1 + \chi_X^j} \right) \right], \\
&= e^{\nu_1 t} \left[ X_1(0) - \Gamma_1^G (1 - \Theta_1^G) - \sum_{X^j} \Gamma_1^{X^j} (1 - \Theta_1^{X^j}) \right] \\
&\quad + \Gamma_1^G \left( e^{-\xi t} - \Theta_1^G e^{-\chi t} \right) + \sum_{X^j} \Gamma_1^{X^j} \left( e^{-\xi_X^j t} - \Theta_1^{X^j} e^{-\chi_X^j t} \right), \tag{367}
\end{aligned}$$

where  $\Gamma_1^G$  and  $\Theta_1^G$  are given by (365a) and (365c), respectively, and  $\Gamma_1^{X^j}$  and  $\Theta_1^{X^j}$  are given by (366a) and (366c), respectively.

Solving for  $X_2(t)$  gives:

$$X_2(t) = e^{\nu_2 t} \left\{ X_2(0) - \frac{\Phi_2^G}{\nu_1 - \nu_2} \int_0^t dG(\tau) e^{-\nu_2 \tau} d\tau - \sum_{X^j} \frac{\Phi_2^{X^j}}{\nu_1 - \nu_2} \int_0^t dX^j(\tau) e^{-\nu_2 \tau} d\tau \right\}. \tag{368}$$

Because  $\nu_2 > 0$ , for the solution to converge to the steady-state, the term in brackets must be nil when we let  $t$  tend toward infinity:

$$\begin{aligned}
X_2(0) &= \frac{\Phi_2^G Y}{\nu_1 - \nu_2} \left[ \frac{1}{\xi + \nu_2} - (1-g) \frac{1}{\chi + \nu_2} \right] + \sum_{X^j} \frac{\Phi_2^{X^j} X^j}{\nu_1 - \nu_2} \left[ \frac{1}{\xi_X^j + \nu_2} - (1-x^j) \frac{1}{\chi_X^j + \nu_2} \right], \\
&= -\Gamma_2^G (1 - \Theta_2^G) - \sum_{X^j} \Gamma_2^{X^j} (1 - \Theta_2^{X^j}), \tag{369}
\end{aligned}$$

where  $\Gamma_2^G$  and  $\Theta_2^G$  are given by (365b) and (365d), respectively, and  $\Gamma_2^{X^j}$  and  $\Theta_2^{X^j}$  are given by (366b) and (366d), respectively.

Inserting first  $X_2(0)$ , the 'stable' solution for  $X_2(t)$ , i.e., consistent with convergence toward the steady-state when  $t$  tends toward infinity, is thus given by:

$$\begin{aligned}
X_2(t) &= e^{\nu_2 t} \left\{ \frac{\Phi_2^G Y}{\nu_1 - \nu_2} \left[ \frac{e^{-(\xi + \nu_2)t}}{\xi + \nu_2} - (1-g) \frac{e^{-(\chi + \nu_2)t}}{\chi + \nu_2} \right] \right. \\
&\quad \left. + \sum_{X^j} \frac{\Phi_2^{X^j} X^j}{\nu_1 - \nu_2} \left[ \frac{e^{-(\xi_X^j + \nu_2)t}}{\xi_X^j + \nu_2} - (1-x^j) \frac{e^{-(\chi_X^j + \nu_2)t}}{\chi_X^j + \nu_2} \right] \right\}, \\
&= -\Gamma_2^G \left( e^{-\xi t} - \Theta_2^G e^{-\chi t} \right) - \sum_{X^j} \Gamma_2^{X^j} \left( e^{-\xi_X^j t} - \Theta_2^{X^j} e^{-\chi_X^j t} \right). \tag{370}
\end{aligned}$$

Using the definition of  $X_i(t)$  (with  $i = 1, 2$ ) given by (363), we can recover the solutions for  $K(t)$  and  $Q(t)$ :

$$K(t) - \tilde{K} = X_1(t) + X_2(t), \tag{371a}$$

$$Q(t) - \tilde{Q} = \omega_2^1 X_1(t) + \omega_2^2 X_2(t). \tag{371b}$$

Setting  $t = 0$  into (371a) gives  $X_1(0) = (K(0) - K) - X_2(0)$  where  $X_2(0)$  is described by eq. (369); the solution (367) for  $X_1(t)$  can be rewritten as follows:

$$\begin{aligned} X_1(t) &= e^{\nu_1 t} \left[ (K(0) - K) + \Gamma_2^G (1 - \Theta_2^G) - \Gamma_1^G (1 - \Theta_1^G) + \sum_{X^j} \Gamma_2^{X^j} (1 - \Theta_2^{X^j}) - \sum_{X^j} \Gamma_1^{X^j} (1 - \Theta_1^{X^j}) \right] \\ &+ \Gamma_1^G \left( e^{-\xi t} - \Theta_1^G e^{-\chi t} \right) + \sum_{X^j} \Gamma_1^{X^j} \left( e^{-\xi_X^j t} - \Theta_1^{X^j} e^{-\chi_X^j t} \right). \end{aligned} \quad (372)$$

Linearizing the current account equation (344) around the steady-state:

$$\begin{aligned} \dot{N}(t) &= r^* dN(t) + \Xi_K dK(t) + \Xi_Q dQ(t) + \Xi_G dG(t) + \sum_{X^j} \Xi_{X^j} dX^j(t), \\ &= (\Xi_K + \Xi_Q \omega_2^1) X_1(t) + (\Xi_K + \Xi_Q \omega_2^2) X_2(t) \\ &+ \Xi_G Y \left[ e^{-\xi t} - (1 - g) e^{-\chi t} \right] + \sum_{X^j} X^j \left[ e^{-\xi_X^j t} - (1 - x^j) e^{-\chi_X^j t} \right]. \end{aligned} \quad (373)$$

Setting  $N_1 = \Xi_K + \Xi_Q \omega_2^1$ ,  $N_2 = \Xi_K + \Xi_Q \omega_2^2$ , inserting solutions for  $K(t)$  and  $Q(t)$  given by (371), solving and invoking the transversality condition, yields the solution for traded bonds:

$$\begin{aligned} dN(t) &= e^{r^* t} (N_0 - N) + \frac{\omega_N^1}{\nu_1 - r^*} \left( e^{r^* t} - e^{\nu_1 t} \right) \\ &+ \frac{N_1 \Gamma_1^G}{\xi + r^*} \left[ \left( e^{r^* t} - e^{-\xi t} \right) - \Theta_1^{G,j} \left( e^{r^* t} - e^{-\chi t} \right) \right] \\ &+ \sum_{X^j} \frac{N_1 \Gamma_1^{X^j}}{\xi_X^j + r^*} \left[ \left( e^{r^* t} - e^{-\xi_X^j t} \right) - \Theta_1^{X^j,j} \left( e^{r^* t} - e^{-\chi_X^j t} \right) \right] \\ &- \frac{N_2 \Gamma_2^G}{\xi + r^*} \left[ \left( e^{r^* t} - e^{-\xi t} \right) - \Theta_2^{G,j} \left( e^{r^* t} - e^{-\chi t} \right) \right] \\ &- \sum_{X^j} \frac{N_2 \Gamma_2^{X^j}}{\xi_X^j + r^*} \left[ \left( e^{r^* t} - e^{-\xi_X^j t} \right) - \Theta_2^{X^j,j} \left( e^{r^* t} - e^{-\chi_X^j t} \right) \right] \\ &+ \frac{\Xi_G Y}{\xi + r^*} \left[ \left( e^{r^* t} - e^{-\xi t} \right) - \Theta^{G,j} \left( e^{r^* t} - e^{-\chi t} \right) \right] \\ &+ \sum_{X^j} \frac{\Xi_{X^j} X^j}{\xi_X^j + r^*} \left[ \left( e^{r^* t} - e^{-\xi_X^j t} \right) - \Theta^{X^j,j} \left( e^{r^* t} - e^{-\chi_X^j t} \right) \right], \end{aligned}$$

where

$$\begin{aligned} \omega_N^1 &= N_1 \left[ (K_0 - K) + \Gamma_2^G (1 - \Theta_2^G) - \Gamma_1^G (1 - \Theta_1^G) \right. \\ &\left. + \sum_{X^j} \Gamma_2^{X^j} (1 - \Theta_2^{X^j}) - \sum_{X^j} \Gamma_1^{X^j} (1 - \Theta_1^{X^j}) \right]. \end{aligned} \quad (374)$$

and

$$\Theta^{G,j} = (1 - g) \frac{\xi + r^*}{\chi + r^*}, \quad (375a)$$

$$\Theta_1^{G,j} = \Theta_1^G \frac{\xi + r^*}{\chi + r^*}, \quad (375b)$$

$$\Theta_2^{G,j} = \Theta_2^G \frac{\xi + r^*}{\chi + r^*}, \quad (375c)$$

$$\Theta^{X^j,j} = (1 - x^j) \frac{\xi_X^j + r^*}{\chi_X^j + r^*}, \quad (375d)$$

$$\Theta_1^{X^j,j} = \Theta_1^{X^j} \frac{\xi_X^j + r^*}{\chi_X^j + r^*}, \quad (375e)$$

$$\Theta_2^{X^j,j} = \Theta_2^{X^j} \frac{\xi_X^j + r^*}{\chi_X^j + r^*}. \quad (375f)$$

By rearranging terms, we get

$$\begin{aligned}
dN(t) &= e^{r^*t} \left[ (N_0 - N) \frac{\omega_N^1}{\nu_1 - r^*} + \frac{N_1 \Gamma_1^G}{\xi + r^*} (1 - \Theta_1^{G,j'}) + \sum_{X^j} \frac{N_1 \Gamma_1^{X^j}}{\xi_X^j + r^*} (1 - \Theta_1^{X^j,j'}) \right. \\
&\quad - \frac{N_2 \Gamma_2^G}{\xi + r^*} (1 - \Theta_2^{G,j'}) - \sum_{X^j} \frac{N_2 \Gamma_2^{X^j}}{\xi_X^j + r^*} (1 - \Theta_2^{X^j,j'}) \\
&\quad \left. + \frac{\Xi_G Y}{\xi + r^*} (1 - \Theta_2^{G,j'}) + \sum_{X^j} \frac{\Xi_{X^j} X^j}{\xi_X^j + r^*} (1 - \Theta_2^{X^j,j'}) \right] \\
&\quad - \frac{\omega_N^1}{\nu_1 - r^*} e^{\nu_1 t} - \frac{N_1 \Gamma_1^G}{\xi + r^*} (e^{-\xi t} - \Theta_1^{G,j'} e^{-\chi t}) - \sum_{X^j} \frac{N_1 \Gamma_1^{X^j}}{\xi_X^j + r^*} (e^{-\xi_X^j t} - \Theta_1^{G,j'} e^{-\chi_X^j t}) \\
&\quad + \frac{N_2 \Gamma_2^G}{\xi + r^*} (e^{-\xi t} - \Theta_2^{G,j'} e^{-\chi t}) + \sum_{X^j} \frac{N_2 \Gamma_2^{X^j}}{\xi_X^j + r^*} (e^{-\xi_X^j t} - \Theta_2^{G,j'} e^{-\chi_X^j t}) \\
&\quad - \frac{\Xi_G Y}{\xi + r^*} (e^{-\xi t} - \Theta_2^{G,j'} e^{-\chi t}) - \sum_{X^j} \frac{\Xi_{X^j} X^j}{\xi_X^j + r^*} (e^{-\xi_X^j t} - \Theta_2^{G,j'} e^{-\chi_X^j t}).
\end{aligned}$$

Invoking the transversality condition, to ultimately remain solvent, the open economy must satisfy the following condition:

$$(N_0 - N) + \frac{\omega_N^1}{\nu_1 - r^*} + \frac{\omega_N^{2,G}}{\xi + r^*} + \sum_{X^j} \frac{\omega_N^{2,X^j}}{\xi_X^j + r^*} = 0, \quad (376)$$

where

$$\omega_N^{2,G} = N_1 \Gamma_1^G (1 - \Theta_1^{G,j'}) - N_2 \Gamma_2^G (1 - \Theta_2^{G,j'}) + \Xi_G Y (1 - \Theta_2^{G,j'}), \quad (377a)$$

$$\omega_N^{2,X^j} = N_1 \Gamma_1^{X^j} (1 - \Theta_1^{X^j,j'}) - N_2 \Gamma_2^{X^j} (1 - \Theta_2^{X^j,j'}) + \Xi_{X^j} X^j (1 - \Theta_2^{X^j,j'}). \quad (377b)$$

The convergent path for the net foreign asset position is:

$$\begin{aligned}
dN(t) &= \frac{\omega_N^1}{\nu_1 - r^*} e^{\nu_1 t} - \frac{N_1 \Gamma_1^G}{\xi + r^*} (e^{-\xi t} - \Theta_1^{G,j'} e^{-\chi t}) - \sum_{X^j} \frac{N_1 \Gamma_1^{X^j}}{\xi_X^j + r^*} (e^{-\xi_X^j t} - \Theta_1^{G,j'} e^{-\chi_X^j t}) \\
&\quad + \frac{N_2 \Gamma_2^G}{\xi + r^*} (e^{-\xi t} - \Theta_2^{G,j'} e^{-\chi t}) + \sum_{X^j} \frac{N_2 \Gamma_2^{X^j}}{\xi_X^j + r^*} (e^{-\xi_X^j t} - \Theta_2^{G,j'} e^{-\chi_X^j t}) \\
&\quad - \frac{\Xi_G Y}{\xi + r^*} (e^{-\xi t} - \Theta_2^{G,j'} e^{-\chi t}) - \sum_{X^j} \frac{\Xi_{X^j} X^j}{\xi_X^j + r^*} (e^{-\xi_X^j t} - \Theta_2^{G,j'} e^{-\chi_X^j t}). \quad (378)
\end{aligned}$$



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