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# LABOR MARKET EFFECTS OF TECHNOLOGY SHOCKS BIASED TOWARD THE TRADED SECTOR* 

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#### Abstract

Motivated by recent evidence pointing at an increasing contribution of asymmetric shocks across sectors to economic fluctuations, we explore the sectoral composition effects of technology shocks biased toward the traded sector. Using a panel of seventeen OECD countries over the period 1970-2013, our VAR evidence reveals that a permanent increase in traded relative to non-traded TFP lowers the traded hours worked share by shifting labor toward the non-traded sector, and has an expansionary effect on the labor income share in both sectors. Our quantitative analysis shows that the open economy version of the neoclassical model can reproduce the reallocation and redistributive effects we document empirically once we allow for technological change biased toward labor together with additional specific elements. Calibrating the model to country-specific data, the model can account for the cross-country dispersion in the reallocation and redistributive effects we document empirically once we let factor-biased technological change vary across sectors and between countries. Finally, we document evidence which supports our hypothesis of factor-biased technological change as we find empirically that countries where capital-intensive industries contribute more to the increase in traded TFP are those where capital relative to labor efficiency increases.


Keywords: Sectoral technology shocks; factor-augmenting efficiency; Open economy; Labor reallocation across sectors; CES production function; Labor income share.
JEL Classification: E22; F11; F41; F43;

[^0]
## 1 Introduction

A common feature shared by OECD countries is that technological change takes place at uneven rates across sectors and generates a change in the sectoral composition of labor and output which can be amplified in an open economy (see e.g., Swiecki [2017], Kehoe et al. [2018]). Evidence documented by Alvarez-Cuadrado et al. [2018] also reveals that the reallocation of economic activity across sectors is associated with a secular decline in sectoral labor income shares (LIS henceforth) in industrialized economies. While the consequences of sector-specific technological change are now well documented by the structural change literature, less is known about the reallocation and redistributive effects triggered by an acceleration in technological change concentrated in traded industries. By using a panel of seventeen OECD countries over the period 1970-2013, the purpose of this paper is to document and rationalize a set of evidence on the labor market effects of technology shocks biased toward the traded sector. We find that the degree of substitutability across goods, imperfect mobility of labor (IML henceforth), trade openness and factor-biased technological change (FBTC henceforth) are key to reproducing quantitatively the decline in the share of tradables in total hours worked and the responses of LIS we document empirically.

The pioneering work of Gali [1999] has sparked a broad literature investigating the labor market effects of technology shocks. ${ }^{1}$ Because variations in aggregate TFP can be driven by movements that are both common across sectors and sector-specific, shocks to aggregate TFP can be broken down into symmetric and asymmetric technology shocks across sectors. Using U.S. data, Foerster et al. [2011], Garìn et al. [2018] find that decreased volatility of symmetric shocks and constant volatility of sectoral shocks led to a greater contribution of asymmetric shocks across sectors in the post-1984 period. Since exporting firms are far more productive than non-exporting firms, a natural way to allow for asymmetric technology shocks is to make the distinction between a traded vs. a non-traded sector.

Because asymmetric shocks provide incentives for labor reallocation, we expect fluctuations of relative productivity of tradables and the traded goods-sector share of total hours worked to be more correlated over the post-1984 period than from 1970 to 1983. To explore this hypothesis, we plot in Fig. 1(a) the detrended (logged) ratio of traded to non-traded TFP (displayed by the blue line) and the detrended labor share of tradables (displayed by the black line) for the United States. The correlation is essentially zero over 1970-1983 and stands at -0.67 from 1984 to 2013. The United Kingdom for which the great moderation occurs in the post-1992 period, see Benati [2008], has also experienced a sharp increase in the correlation between the relative productivity and the labor share of tradables which

[^1]has doubled, passing from -0.38 from 1970-1992 to -0.76 over the post-1992 period. As can be seen in Fig. 1(c), the pre-financial crisis period is characterized by an acceleration in technological change concentrated in traded industries and a fall in the labor share of tradables while the other way around is true after 2008. Like the U.K, a reallocation of labor toward the traded sector accompanies the fall in the relative productivity of tradables in Ireland and Spain in the aftermath of the financial crisis, as can be seen in Fig. 1(d) and Fig. 1(e). The growing importance of asymmetric technology shocks across sectors and the subsequent shift of labor between industries is not limited to the aforementioned countries. For the whole sample shown in Fig. 1(b), the correlation between the relative productivity and the labor share of tradables is 0.23 over 1973-1992 and stands at -0.58 from 1993 to 2013.
< Please insert Fig. 1 about here >
By adopting the identification scheme of technology shocks proposed by Gali [1999], we document a set of VAR evidence which confirms the empirical facts we describe above. Our estimates reveal that the contribution of identified asymmetric technology shocks across sectors to the forecast error variance of aggregate TFP growth has increased dramatically over time and stands at about $40 \%$ over 1993-2013 while asymmetric technology shocks play a negligible role during the pre-1993 period. When we estimate the effects of technology shocks biased toward the traded sector, we find that the relative price of non-tradables appreciates disproportionately (i.e., more than the productivity differential) and labor shifts toward non-traded industries. To rationalize the labor reallocation effects that we document empirically, we put forward a two-sector open economy model with flexible prices. We find quantitatively that the model can account for the magnitude of the decline in the labor share of tradables once it contains a combination of three elements: high substitutability between home- and foreign-produced traded goods, IML and FBTC.

These three specific features are necessary to mitigate the labor reallocation movement caused by the combined effect of financial openness and a low value for the elasticity between traded and non-traded consumption goods. The gross complementarity between tradables and non-tradables, corroborated by existing as well as our own estimates, plays the same role as in the structural change literature, see e.g., Ngai and Pissarides [2007]. Intuitively, while both consumption in traded and non-traded goods increase, the biasedness of the technology shock toward tradables generates an excess supply for traded goods and an excess demand for non-traded goods. By producing a disproportionate appreciation in the relative price of non-tradables, the gross complementary between traded and non-traded goods increases the share of non-tradables in total expenditure which provides incentives for shifting productive resources toward the non-traded sector, especially labor. Our quantitative analysis reveals that the model considerably overstates the reallocation of labor across sectors however and
thus the decline in the labor share of tradables. One reason to this is that we consider an open economy setup where the access to foreign borrowing significantly biases labor demand toward the non-traded sector. ${ }^{2}$

To mitigate labor reallocation, we first allow for endogenous TOT. As a result of high substitutability between home- and foreign-produced traded goods, the decline in the relative price of home-produced traded goods caused by the excess supply for traded goods has a positive impact on hiring by traded firms, thus curbing the decline in the labor share of tradables. The second key element is IML. In line with our evidence indicating that the labor reallocation process is associated with mobility costs, we allow for limited substitutability in hours worked across sectors which further hampers labor reallocation. Even with the two aforementioned ingredients, the model still overstates the shift of labor toward the non-traded sector and does not replicate well the responses of sectoral hours worked. The third and pivotal element is FBTC which is recovered from our estimation of redistributive effects, i.e., the responses of LIS.

Our evidence reveals that the LIS increases in both sectors which implies that technological change is not Hicks-neutral but rather biased toward labor. Intuitively, when technological change is Hicks-neutral, the LIS is a function of the capital-labor ratio only. The gross complementarity between capital and labor in production found in the data (see e.g., Klump et al. [2007], Herrendorf et al. [2015], Oberfield and Raval [2014], Chirinko and Mallick [2017]) and corroborated by our own estimates implies that the LIS and the capital-labor ratio move in the same direction. Because a technology shock biased toward the traded sector drives capital out of the traded sector while labor is subject to mobility costs, the capital-labor ratio falls dramatically, thus driving down the traded LIS under the assumption of Hicks-neutral technological change. Since the non-traded capital-labor ratio is unresponsive to the shock, this assumption also implies that the non-traded LIS should remain unchanged, in contradiction with our evidence. To account for the rise in LISs that we estimate empirically, we assume that capital relative to labor efficiency increases which in turn biases technological change toward labor within each sector. ${ }^{3}$ By lowering the demand for capital, the model augmented with FBTC captures very well the dynamics of the sectoral LIS and capital-labor ratios. While the model can account for the redistributive effects when we allow for sectoral FBTC, the differential in FBTC between sectors increases

[^2]the performance of the model with IML and endogenous TOT in reproducing the labor reallocation effects we document empirically. Because technological change is more biased toward labor in the traded than in the non-traded sector, traded firms are encouraged to hire more workers which mitigates the shift of labor toward the non-traded sector.

When we explore international differences, we find that the substitutability across goods, labor mobility costs and the differential in FBTC between sectors matter in determining the sectoral composition effects of technology shocks biased toward tradables. More specifically, our estimates show that the labor share of tradable falls less in countries where labor mobility costs are higher and/or technological change is more biased toward labor in the traded than in the non-traded sector. We also find that the value added share of nontradables declines by a smaller amount in countries where the gross complementary between traded and non-traded goods is higher as the reallocation of productive resources toward the non-traded sector is amplified.

We also take advantage of the panel data dimension of our dataset to further investigate about the redistributive effects and the relevancy of our assumption of FBTC. When estimating the redistributive effects at a country level, the responses of the LIS display a wide cross-country dispersion, i.e., the LIS may fall or rise by a magnitude which varies considerably between OECD countries. In line with Caselli [2016], we construct time series for sectoral FBTC and estimate its dynamic adjustment following a technology shock biased toward tradables. We detect a strong and positive cross-country relationship between the responses of LIS and FBTC. Once calibrated to country-specific data, numerical results show that the model can account for international differences in the redistributive and reallocation effects once we let FBTC vary across countries. First, the model imposing Hicks-neutral technological change generates a shift of capital across sectors which is not sufficient on its own to account for the cross-country dispersion in the responses of sectoral LIS. Second, because the differential in FBTC across sectors influences sectoral labor demand and thus labor reallocation, the model imposing Hicks-neutral technological change has some difficulty in reproducing the cross-country dispersion in the decline in the labor share of tradables. Importantly, our assumption of FBTC is supported by the evidence we document empirically as we find that countries where TFP gains are concentrated in capital (labor) intensive industries also experience a rise in capital (labor) relative to labor (capital) efficiency, in line with Acemoglu's [2003] model assumptions.

Related Literature. Our paper fits into several different literatures as we bring several different and distinct threads in the existing literature together. First, our setup includes several key features which have been put forward by the literature investigating the ability of the neoclassical model to rationalize the response of hours worked to a positive productivity shock. Like Collard and Dellas [2007], the open economy dimension of our setup
greatly enhances the flexible price model's ability to account for the labor market effects of technology shocks through the TOT deterioration. In contrast to the work by Collard and Dellas where an elasticity of substitution between home- and foreign-produced traded goods smaller than one is key to generating a decline in total hours worked, we find that the TOT channel increases the ability of our model to account for the dynamics of sectoral hours worked once home- and foreign-produced traded goods are gross substitutes, in line with the international RBC literature, see e.g., Backus et al. [1994]. ${ }^{4}$ Like Cantore et al. [2014], [2017], we put forward FBTC to account for the responses of hours worked to a technology shock. The authors show that technology shocks biased toward capital allow the RBC model to generate a negative response of hours worked while we find that technology shocks are biased toward labor (for the whole sample and the U.S. as well). The reason for this discrepancy lies in the fact that aggregate technology shocks are a combination of symmetric and asymmetric technology shocks, the former shock being biased toward capital and the latter biased toward labor.

The contribution of asymmetric technology shocks across sectors to economic fluctuations has received attention only very recently. Using U.S. data over 1961-2008 and distinguishing between a consumption and an investment sector, Chen and Wemy [2015] find that technology shocks biased toward the capital-producing sector explain more than $50 \%$ of TFP fluctuations. In the same vein, our evidence reveals that the contribution of technology shocks biased toward the traded sector to TFP fluctuations stands at 40\% in OECD countries over 1993-2013. Drawing on the pioneering work by Long and Plosser [1983] and revitalized later by Horvath [2000], Holly and Petrella [2012] quantify the contribution of industry specific shocks to aggregate fluctuations by considering input-output linkages. Differently, we explore the sectoral composition effects driven by a shock to TFP taking place at uneven rates across sectors and uncover the key role of heterogenous substitutability across sectoral goods and FBTC in the same spirit as the structural change literature, see e.g., Ngai and Pissarides [2007] and Alvarez-Cuadrado et al. [2018], respectively. Our results also stress the role of imperfect substitutability across sectoral hours worked as reallocative shocks generate a shift of labor which is subject to mobility costs, which echo the findings documented by Garìn et al. [2018].

Sectoral reallocation has also received a lot of attention in the open economy literature, e.g., Benigno and Fornaro [2014], Kehoe and Ruhl [2009], Arrellano et al. [2018]. The former work finds that large capital inflows episodes have contributed to shifting labor out of the traded sector while the latter two works show that sudden stops do the opposite. Similarly to Benigno and Fornaro [2014], financial openness amplifies the incentives to shift labor toward the non-traded sector. While Arrellano et al. [2018] assume a default risk to

[^3]rationalize the shift of labor toward the traded sector during the recent sovereign debt crisis in Europe, such a labor reallocation is the result of declining TFP in tradables relative to non-tradables in our model. Our framework is close to the model laid out by Kehoe and Ruhl's [2009] who depart from the small open economy setup by assuming that the country is large enough to influence the price of its export goods and find, like us, that the TOT play a key role in shaping the labor movement across sectors.

The remainder of the paper is organized as follows. In section 2, we investigate empirically the sectoral composition and redistributive effects of a technology shock biased toward the traded sector. In section 3, we develop an open economy version of the neoclassical model with tradables and non-tradables. In section 4, we report the results of our numerical simulations and assess the ability of the model to account for the evidence on the reallocation and redistributive effects of a technology shock which increases permanently traded relative to non-traded TFP. In section 5 , we summarize our main results and present our conclusions. The Online Appendix provides the main steps to solve the model, shows analytical results from a restricted version of the full model, contains empirical results and robustness checks.

## 2 Evidence on the Effects of Technology Shocks Biased toward Tradables

In this section, we document evidence on the reallocation and redistributive effects driven by a technology shock biased toward the traded sector. Before estimating the sectoral composition effects, we identify symmetric vs. asymmetric technology shocks and assess the contribution of the latter to the variance of aggregate TFP growth. We denote below the percentage deviation from initial steady-state (or the rate of change) with a hat.

### 2.1 Identification of Asymmetric Technology Shocks

To shed some light on the transmission of technology shocks biased toward the traded sector, we estimate a structural VAR model in panel format on annual data. Following Gali [1999], permanent productivity shocks are identified by assuming that technology shocks are the only source of movements in long-run productivity. ${ }^{5}$ While the bulk of the literature restricts attention on the effects of aggregate technology shocks on total hours worked, we aim at exploring the sectoral composition effects caused by asymmetric technology shocks across sectors. Since exporting firms are far more productive than non-exporting firms, a natural way to allow for asymmetric technology shocks is to make the distinction between a traded

[^4](indexed by the superscript $H$ ) vs. non-traded sector (indexed by the superscript $N$ ). In order to gain further insight about the relationship between our analysis and the literature pioneered by Gali [1999], it is convenient to write down the sectoral decomposition of the percentage deviation of aggregate TFP relative to its initial steady-state, denoted by $\hat{Z}_{i t}^{A}$ (see Online Appendix B):
\[

$$
\begin{equation*}
\hat{Z}_{i t}^{A}=\nu_{i}^{Y, H} \hat{Z}_{i t}^{H}+\left(1-\nu_{i}^{Y, H}\right) \hat{Z}_{i t}^{N} \tag{1}
\end{equation*}
$$

\]

where subscripts $i$ and $t$ denote the country and the year. Eq. (1) states that the percentage deviation of aggregate TFP relative to its initial steady-state is equal to the weighted sum of the percentage deviation of TFP relative to initial steady-state in the traded and the nontraded sector, denoted by $\hat{Z}_{i t}^{H}$ and $\hat{Z}_{i t}^{N}$, respectively, where the weight $\nu_{i}^{Y, j}($ with $j=H, N)$ is the share of sectoral value added in GDP.

According to eq. (1), variations in aggregate TFP, $\hat{Z}_{i t}^{A}$, can be the result of movements in TFP which are common across sectors so that shocks are symmetric across sectors. Changes in aggregate TFP can also be driven by technology shocks which are asymmetric across sectors so that the shifts in sectoral TFPs take place at uneven rates across sectors. The shock is reallocative as it modifies relative prices and generates a shift of labor across sectors. To investigate whether a shock to aggregate TFP is evenly or unevenly distributed across sectors, we first identify a shock to aggregate TFP, denoted by $\varepsilon_{i t}^{Z A}$, by estimating a VAR model with two lags in panel format (17 OECD countries) on annual data (19702013) that includes aggregate TFP and total hours worked, both in rate of growth. ${ }^{6}$ To identify aggregate technology shocks, we impose restrictions on the long-run cumulative matrix such that only shocks to aggregate TFP increase permanently $Z_{i t}^{A}$. In the second step, we consider a VAR model which includes identified technology shocks, $\varepsilon_{i t}^{Z A}$, ordered first, the rate of growth of traded, non-traded and aggregate TFP, and adopt a Cholesky decomposition. Next, we plot in Fig. 2(a) the responses for $Z_{i t}^{H}$ shown in the blue line and $Z_{i t}^{N}$ shown in the black line following a $1 \%$ permanent increase in $Z_{i t}^{A}$ in the long-run. Estimates show that aggregate technology shocks are not evenly distributed since traded TFP increases significantly more than non-traded TFP.

Above VAR evidence can be mapped into the sectoral decomposition of aggregate TFP by rearranging eq. (1) as follows:

$$
\begin{equation*}
\hat{Z}_{i t}^{A}=\hat{Z}_{i t}^{N}+\nu_{i}^{Y, H}\left(\hat{Z}_{i t}^{H}-\hat{Z}_{i t}^{N}\right) \tag{2}
\end{equation*}
$$

According to our estimates, an aggregate technology shock which raises $Z_{i t}^{A}$ by $1 \%$ in the long-run gives rise to an increase in $Z_{i t}^{N}$ by $0.8 \%$ augmented by a productivity differential between tradables and non-tradables of $0.4 \%$ (weighted by $\nu_{i}^{Y, H}$ ). The RHS of eq. (2)

[^5]paves the way for the identification of symmetric and asymmetric technology shocks across sectors. When the technology shock is symmetric, sectoral and aggregate TFP increases by the same amount and the last term vanishes. On the contrary, when the shock is asymmetric, both the ratio $Z_{i t}^{H} / Z_{i t}^{N}$ and $Z_{i t}^{A}$ are permanently increased.

To identify symmetric and asymmetric technology shocks, we consider a VAR model which includes the ratio of traded to non-traded TFP, $Z_{i t}^{H} / Z_{i t}^{N}$, aggregate TFP, $Z_{i t}^{A}$, and hours worked, $L_{i t}$, all variables entering the VAR model in growth rates. We impose long-run restrictions such that both symmetric and asymmetric technology shocks increase permanently $Z_{i t}^{A}$ (see the black line in Fig. 2(b) and Fig. 2(c)) while only asymmetric technology shocks increase permanently $Z_{i t}^{H} / Z_{i t}^{N}$ in the long-run (see the blue line in Fig. 2(b) and Fig. 2(c)). Once we have identified symmetric and asymmetric technology shocks across sectors, we can gauge their contribution to aggregate TFP growth by computing a forecast error variance decomposition (FEVD). To explore whether the contribution of shocks to $Z^{H} / Z^{N}$ has changed over time, we estimate the VAR model over two sub-periods, i.e., 1970-1992 and 1993-2013, respectively. Columns 1, 4, 7 of Table 1 report the share of the forecast error variance of aggregate TFP growth attributable to the shock to the ratio of sectoral TFP, $Z_{i t}^{H} / Z_{i t}^{N}$, over the whole period and over two sub-periods. As shown in column 4, the contribution of shocks to $Z_{i t}^{H} / Z_{i t}^{N}$ is negligible over 1970-1992 and stands at about $40 \%$ over 1993-2013 (see column 7).
$<$ Please insert Fig. 2 and Table 1 about here >
Whilst we explore empirically the effects of a technology shock which increases permanently traded relative to non-traded TFP, we augment the productivity differential with weights in order to get a reference point. The reason is that a model with no frictions and exogenous TOT predicts an appreciation in the relative price of non-tradables by $1 \%$ following a weighted productivity differential of $1 \%$ and any departure from this equality reveals the presence of frictions and/or endogenous TOT. In line with the literature pioneered by Balassa [1964] and Samuelson [1964], we construct a weighted productivity differential index between tradables and non-tradables (see Online Appendix H.9):

$$
\begin{equation*}
\hat{Z}_{i t}=a_{i} \hat{Z}_{i t}^{H}-b_{i} \hat{Z}_{i t}^{N}, \tag{3}
\end{equation*}
$$

where $a=\left[\left(1-\alpha_{J}\right)+\alpha_{J} \frac{s_{I}^{H}}{s_{L}^{N}}\right]^{-1}$, and $b=a \frac{s_{L}^{H}}{s_{L}^{N}}$ are country-specific and time-invariant weights which are functions of the LIS in sector $j, s_{L}^{j}$, and the tradable share in total investment expenditure, $\alpha_{J}$, both averaged over 1970-2013. In the rest of the paper, for simplicity purposes, we refer to $Z=\left(Z^{H}\right)^{a} /\left(Z^{N}\right)^{b}$ as the ratio of traded to non-traded TFP. To identify technology shocks biased toward the traded sector, we consider a vector $\hat{X}_{i t}=\left[\hat{Z}_{i t}, \hat{V}_{i t}\right]$ where $\hat{Z}_{i t}$ and $\hat{V}_{i t}$ denote the TFP growth differential and variables of interest detailed later, respectively. Omitting both country and time fixed effects for expositional convenience, the structural moving average representation reads $\hat{X}_{i t}=A(L) \varepsilon_{i t}=\sum_{k=0}^{\infty} A_{k} \varepsilon_{i t-k}$
where $\varepsilon_{i t}=\left[\varepsilon_{i t}^{Z}, \varepsilon_{i t}^{V}\right]$ is the vector of structural shocks. To identify $\varepsilon_{i t}^{Z}$, we use the restriction that the unit root in the ratio of sectoral TFP originates exclusively from technology shocks biased toward the traded sector which implies that the upper triangular elements of the long-run cumulative matrix $A(1)$ must be zero. Denoting the vector of reduced-form innovations by $\eta_{i t}$ with a variance-covariance matrix given by $\Sigma$, we estimate the reduced form of the VAR model $B(L)^{-1} \hat{X}_{i t}=\eta_{i t}$ by panel OLS regression with country and time fixed effects. The matrices $B_{k}$ and $\Sigma$ are assumed to be invariant across time and countries and all VARs have two lags. The structural shocks can be recovered from $\varepsilon_{i t}=A(1)^{-1} B(1) \eta_{i t}$ where the matrix $A(1)$ is computed as the Cholesky decomposition of $B(1) \Sigma B(1)^{\prime}$.

### 2.2 Data Construction

Before presenting the VAR model specification, we briefly discuss the dataset we use. Our sample contains annual observations and consists of a panel of 17 OECD countries. The baseline period is running from 1970 to 2013. Table 2 provides a list of countries and data sources while more details can be found in Online Appendix D. We use the EU KLEMS [2011], [2017] and OECD STAN [2011], [2017] databases which provide domestic currency series of value added in current and constant prices, labor compensation and hours worked at an industry level. All quantities are scaled by the working age population.
< Please insert Table 2 about here >
Since our primary objective is to investigate the sectoral composition effects, we describe below how we construct time series at a sectoral level. Our sample covers eleven 1-digit ISIC-rev. 3 industries which are split into traded and non-traded sectors by adopting the classification by De Gregorio et al. [1994]. Agriculture, hunting, forestry and fishing; Mining and quarrying; Total manufacturing; Transport, storage and communication are classified as traded industries. Following Jensen and Kletzer [2006], we updated the classification by De Gregorio et al. [1994] by treating Financial intermediation as a traded industry. Electricity, gas and water supply; Construction; Wholesale and retail trade; Hotels and restaurants; Real estate, renting and business services; Community, social and personal services are classified as non-traded industries. ${ }^{7}$

Once industries have been classified as traded or non-traded, series for sectoral value added in current (constant) prices are constructed by adding value added in current (constant) prices for all sub-industries $k$ in sector $j=H, N$, i.e., $P_{i t}^{j} Y_{i t}^{j}=\sum_{k} P_{k, i t}^{j} Y_{k, i t}^{j}\left(\bar{P}_{i t}^{j} Y_{i t}^{j}=\right.$ $\sum_{k} \bar{P}_{k, i t}^{j} Y_{k, i t}^{j}$ where the bar indicates that prices $P^{j}$ are those of the base year), from which we construct price indices (or sectoral value added deflators), $P_{i t}^{j}$. Normalizing base year

[^6]price indices $\bar{P}^{j}$ to 1 , the relative price of non-tradables, $P_{i t}$, is defined as the ratio of the non-traded value added deflator to the traded value added deflator (i.e., $P_{i t}=P_{i t}^{N} / \mathrm{P}_{i t}^{H}$ ). The relative price of home-produced traded goods (or the TOT, denoted by $P_{i t}^{H}$ ) is constructed as the ratio of the traded value added deflator $\left(\mathrm{P}_{i t}^{H}\right)$ to the price deflator of imported goods and services $\left(P_{i t}^{F}\right)$. The same logic applies to constructing series for hours worked $\left(L^{j}=\sum_{k} L_{k, i t}^{j}\right)$ and labor compensation in the traded and the non-traded sectors which allow us to construct sectoral wages, $W_{i t}^{j}$. As shall be useful later, total labor compensation is the sum of labor compensation across sectors, i.e., $W_{i t} L_{i t}=\sum_{j} W_{i t}^{j} L_{i t}^{j}$ where $W$ and $L$ are the aggregate nominal wage and total hours worked, respectively. We also construct hours worked and valued added shares of sector $j$ (at constant prices), denoted by $\nu_{i t}^{L, j}$ and $\nu_{i t}^{Y, j} .{ }^{8}$ Finally, as will be useful when estimating the redistributive effects, we calculate the LIS for each sector $j$, denoted by $s_{L, i t}^{j}$, as the ratio of labor compensation to valued added at current prices in sector $j$.

Like Chang and Hong [2006], we use sectoral TFPs, $Z^{j}$, to approximate technical change. Sectoral TFPs are constructed as Solow residuals from constant-price (domestic currency) series of value added, $Y_{i t}^{j}$, capital stock, $K_{i t}^{j}$, and hours worked, $L_{i t}^{j} .{ }^{9}$

$$
\begin{equation*}
\hat{Z}_{i t}^{j}=\hat{Y}_{i t}^{j}-s_{L, i}^{j} \hat{L}_{i t}^{j}-\left(1-s_{L}^{j}\right) \hat{K}_{i t}^{j}, \tag{4}
\end{equation*}
$$

where $s_{L, i}^{j}$ is the LIS in sector $j$ averaged over the period 1970-2013. To obtain series for sectoral capital stock, we first compute the overall capital stock by adopting the perpetual inventory approach, using constant-price investment series taken from the OECD's Annual National Accounts. Following Garofalo and Yamarik [2002], we split the gross capital stock into traded and non-traded industries by using sectoral valued added shares. Once we have a measure for technological change for the traded and the non-traded sector, we can construct the productivity differential index as defined in eq. (3).

### 2.3 VAR Specification

In order to explore empirically the sectoral composition effects of asymmetric technology shocks across sectors, we consider three specifications. The choice of variables is motivated in part by the variables discussed in the quantitative analysis. All variables enter the VAR model in growth rate (denoted by a hat).

Estimation of sectoral composition effects. To investigate the sectoral composition effects of a technology shock, we consider a VAR model that includes (in rate of change)

[^7]value added at constant prices in sector $j, \hat{Y}_{i t}^{j}$, hours worked in sector $j, \hat{L}_{i t}^{j}$, and the real consumption wage in sector $j, \hat{W}_{C, i t}^{j}$ where $W_{C, i t}^{j}$ is defined as the sectoral nominal wage $W_{i t}^{j}$ divided by the consumption price index $P_{C, i t}$. Our vector of endogenous variables, is given by: $x_{i t}^{S, j}=\left[\hat{Z}_{i t}, \hat{Y}_{i t}^{j}, \hat{L}_{i t}^{j}, \hat{W}_{C, i t}^{j}\right]$ with $j=H, N$, where $\hat{Z}_{i t}$ is the productivity growth differential (3). To insure a clear mapping between the responses of sectoral hours worked, $\hat{L}_{i t}^{j}$, and the response of total hours worked, $\hat{L}_{i t}$, we multiply $\hat{L}_{i t}^{j}$ by the labor compensation share of the corresponding sector since in an economy where labor is imperfectly mobile across sectors, we have $\hat{L}_{i t}=\alpha_{L, i} \hat{L}_{i t}^{H}+\left(1-\alpha_{L, i}\right) \hat{L}_{i t}^{N}$ where $\alpha_{L, i}$ is the labor compensation share of tradables $\frac{W_{i t}^{H} L_{i t}^{H}}{W_{i t} L_{i t}}$ averaged over 1970-2013. The same logic applies to real GDP, denoted by $Y_{R, i t}$, i.e., $\hat{Y}_{R, i t}=\nu_{i}^{Y, H} \hat{Y}_{i t}^{H}+\left(1-\nu_{i}^{Y, H}\right) \hat{Y}_{i t}^{N}$, except that the weight is the sectoral value added share (averaged over 1970-2013).

Estimation of reallocation effects. To estimate the magnitude of the reallocation effects caused by an asymmetric technology shock, we consider a VAR model where we divide quantities and wages of sector $j=H, N$ by their aggregate counterpart (in rate of change): $x_{i t}^{R, j}=\left[\hat{Z}_{i t}, \hat{Y}_{i t}^{j}-\hat{Y}_{i t}, \hat{L}_{i t}^{j}-\hat{L}_{i t}, \hat{W}_{i t}^{j}-\hat{W}_{i t}\right]$. Since the sum of labor flows between sectors cancel out when the percentage change in the labor share is multiplied by the labor compensation share, i.e., $\sum_{j} \alpha_{L, i}^{j} \cdot\left(\hat{L}_{i t}^{j}-\hat{L}_{i t}\right)=0$, we perform this scaling to express the response of the labor share to the shock to a productivity differential, $d \nu_{i t}^{L, j}$, in total hours worked units:

$$
\begin{equation*}
d \nu_{i t}^{L, j}=\alpha_{L, i}^{j} \cdot\left(\hat{L}_{i t}^{j}-\hat{L}_{i t}\right), \quad j=N, H . \tag{5}
\end{equation*}
$$

Eq. (5) captures the change in sectoral hours worked if total hours worked remained fixed and thus captures the change in $L_{i t}^{j}$ driven by labor reallocation only. The same logic applies to the construction of the response of the value added share at constant prices of sector $j$ which reads as follows: $d \nu_{i t}^{Y, j}=\nu_{i}^{Y, j}\left(\hat{Y}_{i t}^{j}-\hat{Y}_{R, i t}\right)$ (with $\left.j=H, N\right)$; $d \nu_{i t}^{Y, j}$ captures the change in sectoral value added (in GDP units) driven by the reallocation of productive resources. As shall be useful later, we derive below an expression of the change in the value added share of tradables which shows that $d \nu_{i t}^{Y, H}$ can be brought about by a high productivity growth in this sector relative to average, and/or a labor inflow, and/or a capital inflow (see Online Appendix C):

$$
\begin{equation*}
d \nu_{i t}^{Y, H}=\nu_{i}^{Y, H}\left[\left(\hat{Z}_{i t}^{H}-\hat{Z}_{i t}^{A}\right)+\left(\hat{L}_{i t}^{H}-\hat{L}_{i t}\right)+\left(1-s_{L, i}^{H}\right)\left(\hat{k}_{i t}^{H}-\hat{k}_{i t}\right)\right], \tag{6}
\end{equation*}
$$

where $\hat{Z}^{A}$ is aggregate TFP growth (see eq. (2)), $k^{H}=K^{H} / L^{H}$ stands for the traded capital-labor ratio and $k=K / L$ is the aggregate capital-labor ratio.

Transmission mechanism. To shed some light on the transmission mechanism of asymmetric technology shocks, we also investigate the relative price effects; we consider the following VAR model: $x_{i t}^{P}=\left[\hat{Z}_{i t}, \hat{Y}_{i t}^{H}-\hat{Y}_{i t}^{N}, \hat{P}_{i t}\right]$ since changes in relative prices are associated with changes in the ratio of sectoral quantities. When investigating the response
of the TOT to a technology shock, we replace $\hat{P}_{i t}$ with $\hat{P}_{i t}^{H}$ in the VAR model. ${ }^{10}$

### 2.4 Sectoral Composition Effects: VAR Evidence

We generated impulse response functions which summarize the responses of variables to a $1 \%$ permanent increase in traded relative to non-traded TFP (see eq. (3)). Fig. 3 displays the estimated effects of a technology shock. The horizontal axis measures time after the shock in years and the vertical axis measures percentage deviations from trend. In each case, the solid line represents the point estimate, while the shaded area indicates $90 \%$ confidence bounds obtained by bootstrap sampling. Point estimates are shown in Table 3 on impact (i.e., $t=0$ ), and in the long-run (i.e., at a 10-year horizon).

Adjustment of sectoral TFP. As displayed by the solid blue line in Fig. 3(a), the relative productivity of tradables rises by $0.9 \%$ on impact and increases gradually to reach $1 \%$ after 10 years. As reported in panel A of Table 3, while TFP of tradables increases by $0.72 \%$, its rise is not large enough to raise $Z$ by $0.9 \%$ on impact and thus TFP of nontradables must decline by $0.17 \%$. Fig. 3(e) shows that traded TFP grows over time while $Z^{N}$ remains fairly constant. ${ }^{11}$

Sectoral composition effects. The second and third column of Fig. 3 show the output and labor distributional effects of a $1 \%$ permanent increase in TFP in tradables relative to non-tradables. As reported in panel B of Table 3 , the asymmetric technology shock gives rise to an increase in traded value added by $0.24 \%$ of GDP on impact whilst non-traded value added is virtually unchanged. While Fig. 3(b) shows that $Y^{H}$ is growing over time, as can be seen in the solid blue line of Fig. $3(\mathrm{f})$, the rise in the value added share of tradables (i.e., $\nu^{Y, H}$ ) stabilizes at $0.14 \%$ of GDP after two years, thus indicating that the discrepancy in sectoral value added responses between sectors remains constant over time. However, the rise in $\nu^{Y, H}$ is less than if capital and labor were immobile across sectors since productive resources, especially labor, shifts toward the non-traded sector. As displayed in the third column of Fig. 3, non-traded hours worked rise by $0.1 \%$ of total hours worked on impact and by $0.15 \%$ in the long-run while hours worked do not respond at any horizon in the traded sector (see panel B of Table 3). As can be seen in the dashed blue line in Fig.

[^8]3(f), labor shifts away from the traded sector as the labor share of tradables, $\nu^{L, H}$, declines by $0.05 \%$ of total hours worked in the long-run.

Incentives for labor reallocation. The evidence documented in the last column of Fig. 3 enables us to shed some light on the transmission mechanism. As displayed by the black line in Fig. 3(d), a shock to the productivity differential generates an excess demand for non-traded goods which appreciates the relative price of non-tradables by $0.99 \%$ (see panel C of Table 3). Because the magnitude of the appreciation in the relative price is larger than the productivity differential we estimate on impact (i.e., $0.90 \%$ ), the share of non-tradables increases which has an expansionary effect on hiring in the non-traded sector.

Our VAR evidence in Fig. 3 are in line with the class of neoclassical models where productivity gains vary across sectors and the elasticity of substitution between consumption goods is smaller than one. Intuitively, the low substitutability between traded and non-traded goods allows non-traded firms to set higher prices which more than offsets their productivity disadvantage. The more than proportional appreciation in the relative price of non-tradables generates a shift of productive resources toward the non-traded sector which mitigates the rise in the value added share of tradables (and the decline in the value added share of non-tradables).

Factors hampering labor reallocation. As displayed by the blue line in Fig. 3(d), a $1 \%$ permanent increase in TFP of tradables relative to non-tradables leads to a significant deterioration in the TOT which fall by more than $0.4 \%$ (see panel C of Table 3). By making home-produced traded goods cheaper, lower prices have a positive effect on hiring in the traded sector because home- and foreign-produced traded goods are highly substitutable as evidence suggests, see Bajzik et al. [2019]. Through this channel, the TOT deterioration hampers the outflow of workers experienced by the traded sector. Fig. 3(h) reveals that the shift of labor toward the non-traded sector is further mitigated by the presence of labor mobility costs. Such mobility costs give rise to a positive wage differential for non-tradables by $0.06 \%$ in the long-run (see panel D of Table 3), as displayed by the black line, and a fall in the relative wage of tradables by $0.12 \%$, as shown by the blue line.

Labor mobility costs and substitutability across goods: A cross-country analysis. To gauge the role of labor mobility costs in determining the extent of the shifts of labor across sectors, we take advantage of the panel data dimension of our sample and estimate the same VAR model, i.e., $x_{t}^{R, j}$ which includes the sectoral shares and the wage differential, but for one country at a time. We expect countries with a higher degree of labor mobility to experience a greater decline in the labor share of tradables. ${ }^{12}$ To explore the cross-country relationship between changes in the labor share of tradables and the magnitude of workers'

[^9]costs of switching sectors, we need a measure of the degree of labor mobility. In the lines of Horvath [2000], we estimate the elasticity of labor supply across sectors for each country $i$ denoted by $\epsilon_{i}$ by running a regression in panel format on annual data of the percentage change in the labor share of sector $j$ on the percentage change in the relative share of value added paid to workers in sector $j$; see Online Appendix F. 3 for further details about the derivation of the testable equation and the empirical strategy. While country fixed effects included in the regression capture psychological and geographical mobility costs which are assumed to be the same for all periods, the elasticity of labor supply across sectors, $\epsilon_{i}$, captures the utility loss caused by sector-specific human capital which may not be perfectly transferable across sectors (see e.g., Lee and Wolpin [2006], Dix-Carneiro [2014]). Higher values of $\epsilon$ imply that workers experience lower labor mobility costs. In the first column of Fig. 4, we plot impact responses of the labor share of tradables to a $1 \%$ permanent increase in the relative productivity of tradables on the vertical axis against our measure of the degree of labor mobility, $\epsilon_{i}$, on the horizontal axis. In line with our hypothesis, Fig. 4(a) shows that the labor share of tradables declines more in countries where labor mobility costs are lower (i.e., $\epsilon$ takes higher values).

As demonstrated by Ngai and Pissarides [2007], a low elasticity of substitution between traded and non-traded goods leads to a shift of productive resources to sectors with low TFP growth which in turn mitigates the decline in their value added share. Because less productive resources shift toward the non-traded sector as the elasticity of substitution between traded and non-traded goods, $\phi$, takes higher values, we should observe a larger decline in the value added share of non-tradables in countries where the substitutability between the two goods is higher. In the second column of Fig. 4, we plot impact responses of the value added share of non-tradables against the elasticity of substitution between traded and non-traded goods, $\phi_{i}$, we estimate empirically for each country; see Online Appendix F. 2 for further details about the empirical strategy to estimate $\phi_{i}$. In accordance with Ngai and Pissarides model's predictions, the trend line reveals that the value added share of non-tradables declines more in countries where $\phi$ is higher.
$<$ Please insert Fig. 3-5 about here $>$
< Please insert Table 3 about here >

### 2.5 Redistributive Effects: VAR Evidence

While so far our VAR evidence reveals that labor moves toward the non-traded sector following a technology shock biased toward the traded sector, we now analyze its implications for capital reallocation along with LIS. ${ }^{13}$ To explore empirically the redistributive effects,

[^10]we consider a VAR specification, $x_{i t}^{L I S, j}=\left[\hat{Z}_{i t}, \hat{s}_{L, i t}^{j}, \hat{k}_{i t}^{j}\right]$, which includes the LIS, denoted by $s_{L}^{j}$, and the capital-labor ratio, $k^{j} \equiv K^{j} / L^{j}$, both in rate of growth.

Capital reallocation and LIS. The first and second column of Fig. 5 shows the dynamic responses of capital-labor ratios and LIS, respectively, while panel E of Table 3 displays point estimates on impact and the long-run. The last term of the decomposition described by eq. (6) indicates that the change in the value added share is driven by the change in capital intensity relative to average which collapses to $\left(1-\alpha_{L}\right)\left(\hat{k}^{H}-\hat{k}^{N}\right)$ when LIS are symmetric across sectors. ${ }^{14}$ When $\hat{k}^{H}-\hat{k}^{N}$ is negative, the traded sector experiences a capital outflow. Our VAR evidence reveals that $k^{H}$ falls significantly by $0.08 \%$ of the aggregate capital stock while $k^{N}$ is almost unaffected because the rise in non-traded hours worked offsets the capital inflow. ${ }^{15}$ If production functions were Cobb-Douglas, the shift of capital would have no impact on sectoral LISs. However, as displayed in the second column of Fig. 5, the LIS increases in both sectors. More precisely, as reported in panel E of Table $3, s_{L}^{H}$ increases by more than $0.09 \%$ of traded value added on impact while $s_{L}^{N}$ increases gradually up to $0.07 \%$ of non-traded value added in the long-run. This finding suggests that sectoral goods are produced from CES production functions which is corroborated by our estimates discussed below indicating that the elasticity of substitution between capital and labor in production is smaller than one.

FBTC hypothesis. The positive and significant response of the LIS in the traded sector together with the fall in $k^{H}$ calls into question the assumption of Hicks-neutral technological change (HNTC henceforth). The reason is that when capital and labor are gross complements in production, as our evidence and those documented by the existing literature on the subject suggests, see e.g., Klump et al. [2007], Herrendorf et al. [2015], Oberfield and Raval [2014], Chirinko and Mallick [2017], the decline in $k^{H}$ drives down $s_{L}^{H}$, in contradiction with our empirical findings. A natural candidate to reconcile theory with our evidence is FBTC. When capital and labor are gross complements, an increase in capital relative to labor efficiency biases technological change toward labor which raises the LIS. To test this hypothesis, we first construct time series for FBTC by drawing on Caselli and Coleman [2006] and Caselli [2016]. Assuming that production functions display constant returns to scale and using the fact that factors are paid their marginal product, the ratio of labor to capital income share for country $i$ at time $t$, denoted by $S_{i t}^{j}=\frac{s_{L, i t}^{j}}{1-s_{L, i t}^{J}}$, is equal to

[^11]the ratio of the elasticity of output w.r.t. input, i.e., $S_{i t}=\frac{\hat{Y}_{i t}^{j} / \hat{L}_{i t}^{j}}{\hat{Y}_{i t}^{j} / \hat{K}_{i t}^{j}}$. Totally differentiating this equality and denoting the elasticity of substitution between capital and labor in sector $j$ by $\sigma^{j}$, leads to an expression which enables us to make inference of FBTC: ${ }^{16}$
\[

$$
\begin{equation*}
\mathrm{FBTC}_{i t}^{j}=\hat{S}_{i t}^{j}-\left(\frac{\sigma_{i}^{j}}{1-\sigma_{i}^{j}}\right) \hat{k}_{i t}^{j} \tag{7}
\end{equation*}
$$

\]

An increase in $\mathrm{FBTC}^{j}$ means that technological change is biased toward labor. As shall be clear later in section $3.2, \mathrm{FBTC}_{i t}^{j}$ is a function of $\sigma^{j}$ and factor-augmenting technological change. When $\sigma^{j}<1$, the rise in $\mathrm{FBTC}_{i t}^{j}$ is driven by an increase in capital relative to labor efficiency.

To get estimates of $\sigma^{j}$ at a sectoral level, following Antràs [2004], we run the regression of logged real value added per hours worked (per unit of capital) on the logged real wage (real capital cost) in this sector with country-specific linear trends over 1970-2013. Since all variables display unit root process, we use the fully modified OLS (FMOLS) procedure for cointegrated panel proposed by Pedroni [2000] to estimate the cointegrating relationship. Columns 17 and 18 of Table 4 report estimates for $\sigma^{H}$ and $\sigma^{N}$ we use to recover FBTC from (7). FMOLS estimated values for the whole sample, i.e., $\sigma^{H}=0.687$ and $\sigma^{N}=0.716$, reveal that capital and labor are gross complements in both sectors. Turning to estimates at a country level, in three cases, we obtain inconsistent (i.e., negative values) or not statistically significant estimates for $\sigma^{j}$ when considering the demand of labor and we replace them with those obtained from the demand of capital. ${ }^{17}$ Once we have values for $\sigma^{j}$, we plug time series for $k^{j}$ and $s_{L}^{j}$ into the RHS of eq. (7) to recover time series for FBTC in sector $j$. Next, we estimate a simple VAR model that includes the productivity differential, $\hat{Z}_{i t}$, and $\mathrm{FBTC}_{i t}^{j}$. The third column of Fig. 5 shows the responses of FBTC following a $1 \%$ permanent increase in the relative productivity of tradables. Our estimates reveal that FBTC recovered from (7) increases significantly in the traded sector and thus technological change is biased toward labor which is consistent with the rise in $s_{L}^{H}$ we estimate empirically. While technological change is also biased toward labor in the non-traded sector, the rise in $\mathrm{FBTC}^{N}$ is not statistically significant. For both sectors, point estimates are associated with wide confidence bounds which may suggest that the direction of FBTC varies substantially across countries, thus explaining why FBTC does not increase significantly in the nontraded sector. We explore this assumption below.

Cross-country redistributive effects and FBTC. To investigate whether FBTC

[^12]varies across countries, we re-estimate the VAR model, i.e., $\left[\hat{Z}_{t}, \mathrm{FBTC}_{t}^{j}\right]$, for one country at a time and plot impact responses of sectoral LIS on the vertical axis against estimated responses of FBTC on the horizontal axis in the last column of Fig.5. ${ }^{18}$ Evidence reveals that the responses of sectoral LISs for the whole sample mask a wide cross-country dispersion as the share of value added paid to workers rises only in a subset of countries and the magnitude of the change varies considerably between OECD economies. Importantly, we detect a strong and positive cross-country relationship between the change in the LIS and FBTC in both sectors. Focusing first on the traded sector, countries which lie in the north-east of the scatter-plot such as Australia, Canada, Norway, Netherlands, and the U.S. where the LIS increases also experience technological change biased toward labor. Since all of these countries display an elasticity $\sigma^{H}$ smaller than one, capital relative to labor efficiency increases. Conversely, in countries such as Belgium, Denmark, France, Japan, Ireland which lie in the south-west part of the scatter-plot, the decline in the LIS is associated with technological change biased toward capital. Except for Japan where $\sigma^{H}>1$, labor relative to capital efficiency increases in the aforementioned countries. Turning to the non-traded sector, technological change is biased toward labor in Australia, Denmark, Germany, and the U.S. which generates an increase in the LIS as a result of a rise in capital to labor efficiency, except in Denmark where capital and labor are substitutes. Conversely, technological change is biased toward capital in Austria, Belgium, France, Italy, and the U.K. which has a negative impact on the LIS. In conclusion, asymmetric technology shocks are not associated with HNTC but rather FBTC since capital and labor efficiency increase at uneven rates.

Within and Between Sector Decomposition of LIS. Because both sectors are made up of several industries, the LIS of the broad sector can be affected by changes in the LIS within sub-sectors (keeping the value added share of each sub-sector fixed) and changes in the value added share of those sub-sectors (keeping the LIS of each sub-sector fixed). Because we put forward FBTC at the forefront, we implicitly assume that the response of the LIS to the shock to a productivity differential is mostly driven by the within-effect. To explore empirically our assumption, let us write out the within-between decomposition of the percentage deviation of the LIS relative to its initial steady-state for sector $j$ across industries $k$ (see Online Appendix G.4):

$$
\begin{equation*}
\hat{s}_{L}^{j, t}=\underbrace{\sum_{k} \bar{\alpha}^{k, j} \hat{s}_{L, t}^{k, j}}_{\text {Within Effect }}+\underbrace{\sum_{k} \bar{\alpha}^{k, j} \hat{\omega}_{t}^{k, j}}_{\text {Between Effect }}, \tag{8}
\end{equation*}
$$

where $\bar{\alpha}^{k, j}$ refers to the labor compensation share averaged over 1970-2013 and $\omega_{t}^{k, j}$ is the share of value added of sub-sector $k$ in the value added of sector $j$ at current prices. We

[^13]estimate the same VAR model as above for one country at a time, i..e., $\left[\hat{Z}_{t}, \operatorname{LIS}_{D, t}^{j}, \hat{k}_{t}^{j}\right]$ where $\mathrm{LIS}_{D, t}^{j}$ is now constructed in accordance with the first term of eq. (8) when $D=$ Within or alternatively the second term when $D=$ Between. Next, we calculate the contribution of the within-component to the response of the LIS of the broad sector for each country and find that on average, $2 / 3(80 \%)$ of the impact response of the LIS in tradables (nontradables) can be attributed to the within-effect. Henceforth, the response of the LIS of the broad sector is primarily driven by the responses of LISs within sub-sectors and thus results from FBTC.

## Cross-country labor reallocation effects and differential in FBTC across sec-

 tors. While the last column of Fig. 5 shows that FBTC varies greatly across countries, Fig. 4(c) also reveals that the differential in FBTC between tradables and non-tradables displays a significant cross-country dispersion. We expect the differential in FBTC across sectors to influence labor reallocation as FBTC impinges on sectoral labor demand. Indeed, in Fig. 4(c), we detect a positive cross-country relationship indicating that the response of the labor share of tradables to a shock to the relative productivity of tradables (vertical axis) is increasing in the differential in FBTC between tradables and non-tradables (horizontal axis). Intuitively, when technological change is more biased toward labor in tradables than in non-tradables, it has an expansionary effect on hiring by traded firms which mitigates the fall in the labor share of tradables and may increase it like in Canada. Conversely, in Denmark and Germany, technological change is more biased toward labor in non-tradables which amplifies the decline in the labor share of tradables. As we shall see it when discussing numerical results, the assumption of FBTC increases the ability of our model to account for the labor reallocation effects we document empirically.
## 3 A Semi-Small Open Economy Model with Tradables and Non-Tradables

We consider a semi-small open economy that is populated by a constant number of identical households and firms that have perfect foresight and live forever. The country is assumed to be semi-small in the sense that it is price-taker in international capital markets, and thus faces a given world interest rate, $r^{\star}$, but is large enough on world good markets to influence the price of its export goods. The open economy produces a traded good which can be exported, consumed or invested and imports consumption and investment goods. Besides the home-produced traded good, denoted by the superscript $H$, a non-traded sector produces a good, denoted by the superscript $N$, for domestic absorption only. The foreign good is chosen as the numeraire. Time is continuous and indexed by $t$.

### 3.1 Households

At each instant the representative household consumes traded and non-traded goods denoted by $C^{T}$ and $C^{N}$, respectively, which are aggregated by means of a CES function:

$$
\begin{equation*}
C(t)=\left[\varphi^{\frac{1}{\phi}}\left(C^{T}(t)\right)^{\frac{\phi-1}{\phi}}+(1-\varphi)^{\frac{1}{\phi}}\left(C^{N}(t)\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}, \tag{9}
\end{equation*}
$$

where $0<\varphi<1$ is the weight of the traded good in the overall consumption bundle and $\phi$ corresponds to the elasticity of substitution between traded goods and non-traded goods. The traded consumption index $C^{T}$ is defined as a CES aggregator of home-produced traded goods, $C^{H}$, and foreign-produced traded goods, $C^{F}$ :

$$
\begin{equation*}
C^{T}=\left[\left(\varphi^{H}\right)^{\frac{1}{\rho}}\left(C^{H}\right)^{\frac{\rho-1}{\rho}}+\left(1-\varphi^{H}\right)^{\frac{1}{\rho}}\left(C^{F}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \tag{10}
\end{equation*}
$$

where $0<\varphi^{H}<1$ is the weight of the home-produced traded good and $\rho$ corresponds to the elasticity of substitution between home- and foreign-produced traded goods.

The consumption-based price index $P_{C}$ is a function of traded and non-traded prices:

$$
\begin{equation*}
P_{C}(t)=\left[\varphi\left(P^{T}(t)\right)^{1-\phi}+(1-\varphi)\left(P^{N}(t)\right)^{1-\phi}\right]^{\frac{1}{1-\phi}} \tag{11}
\end{equation*}
$$

where the price index for traded goods is a function of the TOT, $P^{H}$ :

$$
\begin{equation*}
P^{T}(t)=\left[\varphi^{H}\left(P^{H}(t)\right)^{1-\rho}+\left(1-\varphi^{H}\right)\right]^{\frac{1}{1-\rho}} \tag{12}
\end{equation*}
$$

The representative household supplies labor to the traded and non-traded sectors denoted by $L^{H}$ and $L^{N}$, respectively. To rationalize the sectoral wage differential which accompanies an asymmetric technology shock across sectors, we assume that workers experience a utility loss when shifting hours worked from one sector to another. More specifically, in the lines of Horvath [2000], we consider that hours worked in the traded and the non-traded sectors are imperfect substitutes and aggregated by means of a CES function:

$$
\begin{equation*}
L(t)=\left[\vartheta^{-1 / \epsilon}\left(L^{H}(t)\right)^{\frac{\epsilon+1}{\epsilon}}+(1-\vartheta)^{-1 / \epsilon}\left(L^{N}(t)\right)^{\frac{\epsilon+1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon+1}} \tag{13}
\end{equation*}
$$

where $0<\vartheta<1$ parametrizes the weight attached to the supply of hours worked in the traded sector and $\epsilon$ is the elasticity of substitution between sectoral hours worked. The advantage of our modelling of IML is twofold. First, the formulation (13) lends itself easily to the estimation of the deep parameter $\epsilon$ for each country of our sample and thus serves our purpose which is to assess quantitatively the ability of the model to account for our evidence. Second, the case of perfect mobility of labor (PML henceforth) is nested under the assumption that $\epsilon$ tends towards infinity which makes our results directly comparable with those obtained in the special case where workers no longer experience switching costs. The aggregate wage index $W$ (.) associated with the above defined labor index (13) is:

$$
\begin{equation*}
W(t)=\left[\vartheta\left(W^{H}(t)\right)^{\epsilon+1}+(1-\vartheta)\left(W^{N}(t)\right)^{\epsilon+1}\right]^{\frac{1}{\epsilon+1}}, \tag{14}
\end{equation*}
$$

where $W^{H}(t)$ and $W^{N}(t)$ are wages paid in the traded and the non-traded sectors, respectively.

The representative agent is endowed with one unit of time, supplies a fraction $L(t)$ as labor, and consumes the remainder $l(t) \equiv 1-L(t)$ as leisure. At any instant of time, households derive utility from their consumption and experience disutility from working. Assuming that the felicity function is additively separable in consumption and labor, the representative household maximizes the following objective function:

$$
\begin{equation*}
U=\int_{0}^{\infty}\left\{\frac{C(t)^{1-\frac{1}{\sigma_{C}}}}{1-\frac{1}{\sigma_{C}}}-\frac{L(t)^{1+\frac{1}{\sigma_{L}}}}{1+\frac{1}{\sigma_{L}}}\right\} e^{-\beta t} \mathrm{~d} t \tag{15}
\end{equation*}
$$

where $\beta$ is the discount rate, $\sigma_{C}>0$ the intertemporal elasticity of substitution for consumption, and $\sigma_{L}>0$ is the Frisch elasticity of (aggregate) labor supply.

Factor income is derived by supplying labor $L(t)$ at a wage rate $W(t)$, and capital $K(t)$ at a rental rate $R(t)$. In addition, households accumulate internationally traded bonds, $N(t)$, that yield net interest rate earnings of $r^{\star} N(t)$. Households' flow budget constraint states that real disposable income (on the RHS of the equation below) can be saved by accumulating traded bonds, consumed, $P_{C} C$, or invested, $P_{J} J$ :

$$
\begin{equation*}
\dot{N}(t)+P_{C}(t) C(t)+P_{J}(t) J(t)=r^{\star} N(t)+R(t) K(t)+W(t) L(t), \tag{16}
\end{equation*}
$$

where $P_{J}$ is the investment price index defined below and $J$ is total investment.
The investment good is (costlessly) produced using inputs of the traded good and the non-traded good according to a constant-returns-to-scale function which is assumed to take a CES form:

$$
\begin{equation*}
J(t)=\left[\varphi_{J}^{\frac{1}{\phi_{J}}}\left(J^{T}(t)\right)^{\frac{\phi_{J}-1}{\phi_{J}}}+\left(1-\varphi_{J}\right)^{\frac{1}{\phi_{J}}}\left(J^{N}(t)\right)^{\frac{\phi_{J}-1}{\phi_{J}}}\right]^{\frac{\phi_{J}}{\phi_{J}-1}} \tag{17}
\end{equation*}
$$

where $0<\varphi_{J}<1$ is the weight of the investment traded input and $\phi_{J}$ corresponds to the elasticity of substitution between investment traded goods and investment non-traded goods. The index $J^{T}(t)$ is defined as a CES aggregator of home-produced traded inputs, $J^{H}(t)$, and foreign-produced traded inputs, $J^{F}(t)$ :

$$
\begin{equation*}
J^{T}(t)=\left[\left(\iota_{H}\right)^{\frac{1}{\rho_{J}}}\left(J^{H}(t)\right)^{\frac{\rho_{J}-1}{\rho_{J}}}+\left(1-\iota^{H}\right)^{\frac{1}{\rho_{J}}}\left(J^{F}(t)\right)^{\frac{\rho_{J}-1}{\rho_{J}}}\right]^{\frac{\rho_{J}}{\rho_{J}-1}} \tag{18}
\end{equation*}
$$

where $0<\iota^{H}<1$ is the weight of the home-produced traded good and $\rho_{J}$ corresponds to the elasticity of substitution between home-produced traded inputs and foreign-produced traded inputs. The investment-based price index $P_{J}(t)$ is a function of traded and nontraded prices:

$$
\begin{equation*}
P_{J}(t)=\left[\iota\left(P_{J}^{T}(t)\right)^{1-\phi_{J}}+(1-\iota)\left(P^{N}(t)\right)^{1-\phi_{J}}\right]^{\frac{1}{1-\phi_{J}}}, \tag{19}
\end{equation*}
$$

where the price index for traded investment goods is a function of the TOT:

$$
\begin{equation*}
P_{J}^{T}(t)=\left[\iota^{H}\left(P^{H}(t)\right)^{1-\rho_{J}}+\left(1-\iota^{H}\right)\right]^{\frac{1}{1-\rho_{J}}} . \tag{20}
\end{equation*}
$$

Installation of new investment goods involves convex costs, assumed quadratic. Thus, total investment $J$ differs from effectively installed new capital:

$$
\begin{equation*}
J(t)=I(t)+\frac{\kappa}{2}\left(\frac{I(t)}{K(t)}-\delta_{K}\right)^{2} K(t), \tag{21}
\end{equation*}
$$

where the parameter $\kappa>0$ governs the magnitude of adjustment costs to capital accumulation. Aggregate investment, $I(t)$, gives rise to capital accumulation according to the dynamic equation:

$$
\begin{equation*}
\dot{K}(t)=I(t)-\delta_{K} K(t), \tag{22}
\end{equation*}
$$

where $0 \leq \delta_{K}<1$ is a fixed depreciation rate.
Households choose consumption, worked hours and investment in physical capital by maximizing lifetime utility (15) subject to (16) and (22) together with (21). Denoting by $\lambda$ and $Q^{\prime}$ the co-state variables associated with (16) and (22), the first-order conditions characterizing the representative household's optimal plans are:

$$
\begin{gather*}
C(t)=\left(P_{C}(t) \lambda\right)^{-\sigma_{C}},  \tag{23a}\\
L(t)=(W(t) \lambda)^{\sigma_{L}},  \tag{23b}\\
\frac{I(t)}{K(t)}=\frac{1}{\kappa}\left(\frac{Q(t)}{P_{J}(t)}-1\right)+\delta_{K},  \tag{23c}\\
\dot{\lambda}(t)=\lambda(t)\left(\beta-r^{\star}\right),  \tag{23d}\\
\dot{Q}(t)=\left(r^{\star}+\delta_{K}\right) Q(t)-\left\{R(t)+P_{J}(t) \frac{\kappa}{2}\left(\frac{I(t)}{K(t)}-\delta_{K}\right)\left(\frac{I(t)}{K(t)}+\delta_{K}\right)\right\}, \tag{23e}
\end{gather*}
$$

and the transversality conditions $\lim _{t \rightarrow \infty} \lambda N(t) e^{-\beta t}=0, \lim _{t \rightarrow \infty} Q(t) K(t) e^{-\beta t}=0$ where $Q(t)=Q^{\prime}(t) / \lambda$. In an open economy model with a representative agent having perfect foresight, a constant rate of time preference and perfect access to world capital markets, we impose $\beta=r^{\star}$ in order to generate an interior solution. Setting $\beta=r^{\star}$ into (23d) implies that $\lambda$ is constant over time. When new information about the technology shock arrives, $\lambda$ jumps (to fulfill the intertemporal solvency condition determined later) and remains constant afterwards.

Solving (23c) for investment, i.e., $\frac{I(t)}{K(t)}=\frac{1}{\kappa}\left(\frac{Q(t)}{P_{J}(t)}-1\right)+\delta_{K}$, leads to a positive relationship between investment and Tobin's $q$ which is defined as the shadow value to the firm of installed capital, $Q(t)$, divided by its replacement cost, $P_{J}(t)$. For the sake of clarity, we drop the time argument below provided this causes no confusion.

Applying Shephard's lemma (or the envelope theorem) yields the following demand for the home- and the foreign-produced traded good for consumption and investment, respectively:

$$
\begin{array}{lll}
C^{H}=\varphi\left(\frac{P^{T}}{P_{C}}\right)^{-\phi} \varphi^{H}\left(\frac{P^{H}}{P^{T}}\right)^{-\rho} C, & C^{F}=\varphi\left(\frac{P^{T}}{P_{C}}\right)^{-\phi}\left(1-\varphi^{H}\right)\left(\frac{1}{P^{T}}\right)^{-\rho} C, \\
J^{H}=\iota\left(\frac{P_{J}^{T}}{P_{J}}\right)^{-\phi_{J}} \iota^{H}\left(\frac{P^{H}}{P_{J}^{T}}\right)^{-\rho_{J}} J, & J^{F}=\iota\left(\frac{P_{J}^{T}}{P_{J}}\right)^{-\phi_{J}}\left(1-\iota^{H}\right)\left(\frac{1}{P_{J}^{T}}\right)^{-\rho_{J}} J, \tag{24b}
\end{array}
$$

and the demand for non-traded consumption and investment goods, respectively:

$$
\begin{equation*}
C^{N}=(1-\varphi)\left(\frac{P^{N}}{P_{C}}\right)^{-\phi} C, \quad J^{N}=(1-\iota)\left(\frac{P^{N}}{P_{J}}\right)^{-\phi_{J}} J . \tag{25}
\end{equation*}
$$

The substitutability across goods has important implications for the sectoral composition effects of asymmetric technology shocks across sectors. First, rearranging the first equality of eq. (25) reveals that the share of non-traded goods in aggregate consumption expenditure, i.e., $1-\alpha_{C}=\frac{P^{N} C^{N}}{P_{C} C}=(1-\varphi)\left(\frac{P^{N}}{P_{C}}\right)^{1-\phi}$, is increasing in non-traded prices when $\phi<1$ as evidence suggests. Conversely, the home content of consumption and investment expenditure in tradables, i.e., $\alpha^{H}=\frac{P^{H} C^{H}}{P^{T} C^{T}}=\varphi^{H}\left(\frac{P^{T}}{P^{H}}\right)^{\rho-1}$ and $\alpha_{J}^{H}=\frac{P^{H} J^{H}}{P_{J}^{T} J^{T}}=\iota^{H}\left(\frac{P_{J}^{T}}{P^{H}}\right)^{\rho_{J}-1}$, increases as the TOT (i.e., $P^{H}$ ) decline since home- and foreign-produced traded goods are high substitutes, see e.g., Bajzik et al. [2019]. These parameters, $\phi, \rho$ and $\rho_{J}$, will play an important role in the transmission mechanism of an increase in the relative productivity of tradables by affecting the share of expenditure in good $j$ and thus sectoral labor demand.

Given the aggregate wage index, we can derive the allocation of aggregate labor supply to the traded and the non-traded sector:

$$
\begin{equation*}
L^{H}=\vartheta\left(\frac{W^{H}}{W}\right)^{\epsilon} L, \quad L^{N}=(1-\vartheta)\left(\frac{W^{N}}{W}\right)^{\epsilon} L, \tag{26}
\end{equation*}
$$

where $\epsilon$ determines the percentage change in the share of hours worked in sector $j, L^{j} / L$, following a rise in the relative wage, $W^{j} / W$, by $1 \%$. As the elasticity of labor supply across sectors, $\epsilon$, takes higher values, workers experience lower mobility costs and thus more labor shifts from one sector to another. Denoting by $\alpha_{L}$ the labor compensation share of tradables, labor income from supplying hours worked in the traded sector is $W^{H} L^{H}=\alpha_{L} W L$.

### 3.2 Firms

Each sector consists of a large number of identical firms which use labor, $L^{j}$, and physical capital, $K^{j}$, according to a constant returns to scale technology described by a CES production function:

$$
\begin{equation*}
Y^{j}(t)=\left[\gamma^{j}\left(A^{j}(t) L^{j}(t)\right)^{\frac{\sigma^{j}-1}{\sigma j}}+\left(1-\gamma^{j}\right)\left(B^{j}(t) K^{j}(t)\right)^{\frac{\sigma^{j}-1}{\sigma j}}\right]^{\frac{\sigma^{j}}{\sigma^{j}-1}}, \tag{27}
\end{equation*}
$$

where $\gamma^{j}$ and $1-\gamma^{j}$ are the weight of labor and capital in the production technology, respectively, $\sigma^{j}$ is the elasticity of substitution between capital and labor in sector $j=H, N$, $A^{j}(t)$ and $B^{j}(t)$ are labor- and capital-augmenting efficiency.

Firms lease the capital from households and hire workers. They face two cost components: a capital rental cost equal to $R$, and the wage rates in the traded and non-traded sector equal to $W^{H}$ and $W^{N}$, respectively. Both sectors are assumed to be perfectly competitive and thus choose capital and labor by taking prices as given. Since capital can move freely between the two sectors, the value of marginal products in the traded and the
non-traded sectors equalize while costly labor mobility implies a wage differential across sectors:

$$
\begin{gather*}
P^{j}(t) \gamma^{j}\left(A^{j}(t)\right)^{\frac{\sigma^{j}-1}{\sigma^{j}}}\left(y^{j}(t)\right)^{\frac{1}{\sigma^{j}}}=W^{j},  \tag{28a}\\
P^{j}(t)\left(1-\gamma^{j}\right)\left(B^{j}(t)\right)^{\frac{\sigma^{j}-1}{\sigma^{j}}}\left(k^{j}(t)\right)^{-\frac{1}{\sigma^{j}}}\left(y^{j}(t)\right)^{\frac{1}{\sigma^{j}}}=R, \tag{28b}
\end{gather*}
$$

where we denote by $k^{j}(t) \equiv K^{j}(t) / L^{j}(t)$ the capital-labor ratio for sector $j=H, N$, and $y^{j}(t) \equiv Y^{j}(t) / L^{j}(t)$ refers to value added per hours worked described by

$$
\begin{equation*}
y^{j}(t)=\left[\gamma^{j}\left(A^{j}(t)\right)^{\frac{\sigma^{j}-1}{\sigma{ }^{j}}}+\left(1-\gamma^{j}\right)\left(B^{j}(t) k^{j}(t)\right)^{\frac{\sigma^{j}-1}{\sigma^{j}}}\right]^{\frac{\sigma^{j}}{\sigma^{j}-1}} . \tag{29}
\end{equation*}
$$

Demand for inputs can be rewritten in terms of their respective cost in value added; for labor, we have $s_{L}^{j}(t)=\gamma^{j}\left(\frac{A^{j}(t)}{y^{j}(t)}\right)^{\frac{\sigma^{j}-1}{\sigma j}}$. Applying the same logic for capital and denoting by $S^{j}(t) \equiv \frac{s_{L}^{j}(t)}{1-s_{L}^{j}(t)}$ the ratio of labor to capital income share, we have:

$$
\begin{equation*}
S^{j}(t) \equiv \frac{s_{L}^{j}(t)}{1-s_{L}^{j}(t)}=\frac{\gamma^{j}}{1-\gamma^{j}}\left(\frac{B^{j}(t) k^{j}(t)}{A^{j}(t)}\right)^{\frac{1-\sigma^{j}}{\sigma j}} \tag{30}
\end{equation*}
$$

When technological change is assumed to be Hicks-neutral, productivity increases uniformly across inputs, i.e., $\hat{A}^{j}(t)=\hat{B}^{j}(t)$, and thus relative capital efficiency, $B^{j}(t) / A^{j}(t)$, in eq. (30) vanishes. As a result, sectoral LISs are no longer directly affected by productivity growth but indirectly through changes in sectoral capital-labor ratios. Since evidence reveals that capital and labor are gross complements in production, i.e., $\sigma^{j}<1$, sectoral LISs are increasing in $k^{j}$. By contrast, when technological change is factor-biased, an increase in capital relative to labor efficiency, $B^{j}(t) / A^{j}(t)$, impinges on the sectoral LIS directly and indirectly through changes in $k^{j}(t)$.

Finally, aggregating over the two sectors gives us the resource constraint for capital:

$$
\begin{equation*}
K^{H}+K^{N}=K \tag{31}
\end{equation*}
$$

### 3.3 Technology Frontier

Eq. (30) can be used to determine the direction and the extent of the change in relative capital efficiency which is consistent with observed changes in $S^{j}$ and $k^{j}$. In order to be consistent with our empirical strategy, we need to specify a technology frontier which determines how TFP in sector $j$ is split between capital and labor efficiency for a given change in relative capital efficiency inferred from (30). A natural way to map $A^{j}$ and $B^{j}$ into $Z^{j}$ is to assume that besides optimally choosing factor inputs, firms also optimally choose the technology of production. Following Caselli and Coleman [2006] and Caselli [2016], the menu of possible choices of the technology of production is represented by a set of possible $\left(A^{j}, B^{j}\right)$ pairs which are chosen along a technology frontier which is assumed to
take a Cobb-Douglas form: ${ }^{19}$

$$
\begin{equation*}
Z^{j}=\left(A^{j}(t)\right)^{\alpha^{j}(t)}\left(B^{j}(t)\right)^{1-\alpha^{j}(t)}, \tag{32}
\end{equation*}
$$

where $Z^{j}$ measures the height of the technology frontier and $\alpha^{j}$ is a positive parameter which determines the weight of labor-augmenting technological change. Firms choose labor and capital efficiency, $A^{j}$ and $B^{j}$, along the technology frontier described by eq. (32) that minimize the unit cost function denoted by $c^{j}$. The unit cost for producing is minimized when the contribution of higher capital efficiency exactly offsets the change in $c^{j}$ caused by lower labor efficiency, i.e., $\left(1-s_{L}^{j}\right) \hat{B}^{j}=-s_{L}^{j} \hat{A}^{j}$. Since along the same technology frontier, a fall in $\alpha^{j} \hat{A}^{j}$ must be compensated by a rise by $\left(1-\alpha^{j}\right) \hat{B}^{j}$ to keep $Z^{j}$ constant, the optimal trade-off between $A^{j}$ and $B^{j}$ that minimizes the unit cost is such that the weight of capital efficiency $1-\alpha^{j}$ collapses to its contribution, $1-s_{L}^{j}$, to the decline in the unit cost. The weight of labor and capital efficiency into the technology frontier which minimizes the unit cost for producing is thus strictly equal to the share of labor and capital cost in value added, respectively, i.e., $\alpha^{j}(t)=s_{L}^{j}(t)$, see Online Appendix J.6.

Inserting the optimal choice of technology pair $\left(A^{j}, B^{j}\right)$ into (32) leads to a relationship between TFP and labor- and capital-augmenting productivity:

$$
\begin{equation*}
Z^{j}(t)=\left(A^{j}\right)^{s_{L}^{j}(t)}\left(B^{j}\right)^{1-s_{L}^{j}(t)} . \tag{33}
\end{equation*}
$$

where the weight $s_{L}^{j}$ is time-varying since the production function (27) takes a CES form with $\sigma^{j} \neq 1$. While the technological frontier imposes a structure on the mapping between TFP and factor-augmenting efficiency, as described by (33), it has the advantage to insure a consistency between the theoretical and the empirical approach where technological shifts can be Hicks-neutral or factor-biased.

### 3.4 Model Closure and Equilibrium

To fully describe the equilibrium, we impose goods market clearing conditions for nontraded and home-produced traded goods:

$$
\begin{equation*}
Y^{N}(t)=C^{N}(t)+J^{N}(t), \quad Y^{H}(t)=C^{H}(t)+J^{H}(t)+X^{H}(t), \tag{34}
\end{equation*}
$$

where $X^{H}$ stands for exports of home-produced goods; exports are assumed to be a decreasing function of TOT, $P^{H}$ :

$$
\begin{equation*}
X^{H}(t)=\varphi_{X}\left(P^{H}(t)\right)^{-\phi_{X}}, \tag{35}
\end{equation*}
$$

where $\varphi_{X}>0$ is a scaling parameter, and $\phi_{X}$ is the elasticity of exports w.r.t. the TOT.
Log-linearizing (33) shows that sectoral TFPs dynamics are driven by the dynamics of labor- and capital-augmenting efficiency, i.e., $\hat{Z}^{j}(t)=s_{L}^{j} \hat{A}^{j}(t)+\left(1-s_{L}^{j}\right) \hat{B}^{j}(t)$. We

[^14]drop the time index below to denote steady-state values. The adjustment of $A^{j}(t)$ and $B^{j}(t)$ toward their long-run level expressed in percentage deviation from initial steadystate follows a continuous time autoregressive path:
\[

$$
\begin{equation*}
\hat{A}^{j}(t)-\hat{A}^{j}=\bar{a}^{j} e^{-\xi^{j} t}, \quad \hat{B}^{j}(t)-\hat{B}^{j}=\bar{b}^{j} e^{-\xi^{j} t} \tag{36}
\end{equation*}
$$

\]

where $\bar{a}^{j}$ and $\bar{b}^{j}$ are parameters, and $\xi^{j}>0$ measures the speed at which productivity closes the gap with its long-run level; $\hat{A}^{j}$ and $\hat{B}^{j}$ are steady-state changes in labor and capital efficiency in percentage; inserting (36) into the log-linearized version of the technology frontier allows us to recover the dynamics of TFP in sector $j:{ }^{20}$

$$
\begin{equation*}
\hat{Z}^{j}(t)-\hat{Z}^{j}=\bar{z}^{j} e^{-\xi^{j} t} \tag{37}
\end{equation*}
$$

where $\bar{z}^{j}=s_{L}^{j} \bar{a}^{j}+\left(1-s_{L}^{j}\right) \bar{b}^{j}$ and $\hat{Z}^{j}=s_{L}^{j} \hat{A}^{j}+\left(1-s_{L}^{j}\right) \hat{B}^{j}$ is the steady-state change in $Z^{j}$ in percentage.

The adjustment of the open economy towards the steady state is described by a dynamic system which comprises six equations that form a separate subsystem in $K, Q, A^{H}, B^{H}$, $A^{N}, B^{N}:{ }^{21}$

$$
\begin{gather*}
\dot{K}(t)=\Upsilon\left(K(t), Q(t), A^{H}(t), B^{H}(t), A^{N}(t), B^{N}(t)\right),  \tag{38a}\\
\dot{Q}(t)=\Sigma\left(K(t), Q(t), A^{H}(t), B^{H}(t), A^{N}(t), B^{N}(t)\right),  \tag{38b}\\
\dot{A}^{j}(t)=-\xi^{j}\left(A^{j}(t)-\tilde{A}^{j}\right), \quad \dot{B}^{j}(t)=-\xi^{j}\left(B^{j}(t)-\tilde{B}^{j}\right) . \tag{38c}
\end{gather*}
$$

where $j=H, N$. The first dynamic equation corresponds to the non-traded goods market clearing condition (34) and the second dynamic equation corresponds to (23e) which equalizes the rates of return on domestic equities and foreign bonds, $r^{\star}$, once we have substituted appropriate first-order conditions. Equations (38c) are the law of motion of labor and capital efficiency, respectively, in sector $j$. Linearizing (38a)-(38b) around the steady-state and denoting by $\omega_{k}^{i}$ the $k t h$ element of eigenvector $\omega^{i}$ related to eigenvalue $\nu_{i}$, the general solution that characterizes the adjustment toward the new steady-state can be written as follows: $V(t)-V=\sum_{i=1}^{6} \omega^{i} D_{i} e^{\nu_{i} t}$ where $V$ is the vector of state and control variables. Denoting the positive eigenvalue by $\nu_{2}>0$, we set $D_{2}=0$ to eliminate explosive paths and determine the five arbitrary constants $D_{i}$ (with $i=1, \ldots, 6, i \neq 2$ ) by using the five initial conditions, i.e., $K(0)=K_{0}, A^{j}(0)=A_{0}^{j}$, and $B^{j}(0)=B_{0}^{j}$ for $j=H, N$.

Using the properties of constant returns to scale in production, identities $P_{C}(t) C(t)=$ $\sum_{g} P^{g}(t) C^{g}(t)$ and $P_{J}(t) J(t)=\sum_{g} P^{g}(t) J^{g}(t)$ (with $g=F, H, N$ ) along with market clearing conditions (34), the current account equation (16) can be rewritten as a function of the balance of trade (last two terms on the RHS of the equation below):

$$
\begin{equation*}
\dot{N}(t)=r^{\star} N(t)+P^{H}(t) X^{H}(t)-M^{F}(t) \tag{39}
\end{equation*}
$$

[^15]where $M^{F}(t)=C^{F}(t)+J^{F}(t)$ stands for imports of foreign-produced consumption and investment goods. Eq. (39) can be written as a function of state and control variables, i.e., $\dot{N}(t) \equiv r^{\star} N(t)+\Xi\left(K(t), Q(t), A^{H}(t), B^{H}(t), A^{N}(t), B^{N}(t)\right)$. Linearizing around the steady-state, inserting the solutions for $K(t), Q(t)$ together with (38c), solving and invoking the transversality condition, yields the solution for traded bonds:
\[

$$
\begin{equation*}
N(t)-N=\sum_{i=1, i \neq 2}^{6} \Phi_{N}^{i} e^{\nu_{i} t} \tag{40}
\end{equation*}
$$

\]

where $\Phi_{N}^{i}=\frac{E_{i} D_{i}}{r^{\star}-\nu_{i}}$ with $E_{i}=\Xi_{K} \omega_{1}^{i}+\Xi_{Q} \omega_{2}^{i}+\Xi_{A^{H}} \omega_{3}^{i}+\Xi_{B^{H}} \omega_{4}^{i}+\Xi_{A^{N}} \omega_{5}^{i}+\Xi_{B^{N}} \omega_{6}^{i}$; partial derivatives of $\Xi$ w.r.t. $K, Q, A^{j}, B^{j}$, are evaluated at the steady-state. Eq. (40) gives the trajectory for $N(t)$ consistent with the intertemporal solvency condition:

$$
\begin{equation*}
N-N_{0}=\sum_{i=1, i \neq 2}^{6} \Phi_{N}^{i} . \tag{41}
\end{equation*}
$$

## 4 Quantitative Analysis

In this section, we take the model to the data. For this purpose we solve the model numerically. ${ }^{22}$ Therefore, first we discuss parameter values before turning to the effects of a technology shock biased toward the traded sector.

### 4.1 Normalization of CES Production Functions

Since we compare numerical results obtained in a model assuming CES production functions with those in a model considering a Cobb-Douglas form for production functions, we normalize (27) so that the steady-state is invariant when the elasticity of substitution is changed, see e.g., Cantore et al. [2014], Herrendorf et al. [2015]. To perform this normalization, we assume that at the initial steady-state, technological change is Hicks-neutral, i.e., $A^{j}=B^{j}=Z^{j}$, so that eq. (29) reduces to:

$$
\begin{equation*}
y^{j}=Z^{j}\left[\gamma^{j}+\left(1-\gamma^{j}\right)\left(k^{j}\right)^{\frac{\sigma^{j}-1}{\sigma \overline{ }}}\right]^{\frac{\sigma^{j}}{\sigma J-1}}, \tag{42}
\end{equation*}
$$

and the LIS is a function of the capital-labor ratio only:

$$
\begin{equation*}
s_{L}^{j}=\gamma^{j}\left(Z^{j} / y^{j}\right)^{\frac{\sigma^{j}-1}{\sigma^{j}}} . \tag{43}
\end{equation*}
$$

We choose the initial steady-state in a model with Cobb-Douglas production functions as the normalization point and denote the LIS in sector $j$ by $\theta^{j} ; \bar{k}^{j}$ and $\bar{y}^{j}$ are the steady-state quantities from the Cobb-Douglas case. The objective of the normalization is to choose $\gamma^{j}$ in (43) so as to maintain the steady-state LIS in sector $j$ at $\theta^{j}$ and to set parameter $Z^{j}$ in (42) to maintain the steady-state level of $y^{j}$ equal to the Cobb-Douglas value $\bar{y}^{j}$. Equating

[^16]$y^{j}$ and $k^{j}$ to $\bar{y}^{j}$ and $\bar{k}^{j}$, respectively, eqs. (42) and (43) can be solved for $\gamma^{j}$ and $Z^{j}$. Next we plug back these two expressions into the CES production function in an intensive form described by (42) which leads to the normalized version of the CES production function where quantities in the CES economy are divided by their Cobb-Douglas counterparts denoted with a bar:
$$
Y^{j} / \bar{Y}^{j}=\left[\theta^{j}\left(L^{j} / \bar{L}^{j}\right)^{\frac{\sigma^{j}-1}{\sigma j}}+\left(1-\theta^{j}\right)\left(K^{j} / \bar{K}^{j}\right)^{\frac{\sigma^{j}-1}{\sigma j}}\right]^{\frac{\sigma^{j}}{\sigma^{j}-1}}
$$
where $\theta^{j}$ is the LIS in the reference model with Cobb-Douglas production functions.

### 4.2 Calibration

Because we consider the initial steady-state with Cobb-Douglas production functions as the normalization point, we have to calibrate the model with $\sigma^{j}=1$ to the data. To calibrate the reference model, we estimated a set of parameters so that the initial steadystate is consistent with the key empirical properties of a representative OECD economy. Our sample covers the seventeen OECD economies in our dataset. Our reference period for the calibration corresponds to the period 1970-2013. Since we calibrate a two-sector model with tradables and non-tradables and differentiate between home- and foreign-produced traded goods, we pay particular attention to the adequacy of the non-tradable and home tradable content of the model to the data. Table 4 summarizes our estimates of the ratios and estimated parameters for all countries in our sample. ${ }^{23}$
$<$ Please insert Table 4 and 5 about here $>$
To capture the key properties of a typical OECD economy which is chosen as the baseline scenario, we take unweighted average values of ratios which are shown in the last line of Table 4. Among the 24 parameters that the model contains, 18 have empirical counterparts while the remaining 6 parameters, i.e., $\varphi, \iota, \varphi^{H}, \iota^{H}, \vartheta, \delta_{K}$ together with initial conditions $\left(N_{0}, K_{0}\right)$ must be endogenously calibrated to match ratios $1-\alpha_{C}, 1-\alpha_{J}, \alpha^{H}, \alpha_{J}^{H}, \frac{L^{N}}{L}, \omega_{J}$, and $v_{N X}=\frac{N X}{P^{H} Y^{H}}$ with $N X=P^{H} X^{H}-C^{F}-I^{F}$, as summarized in Table 5. More details about the calibration procedure can be found in Online Appendix L. We choose the model period to be one year and set the world interest rate, $r^{\star}$, which is equal to the subjective time discount rate, $\beta$, to $4 \%$.

The degree of labor mobility is measured by the elasticity of labor supply across sectors, $\epsilon$, which is set to 1.6 to allow the model to replicate the long-run wage differential we document empirically for tradables and non-tradables (see Fig. 3(h)). As summarized in column 16 of Table 4, our panel data estimates of $\epsilon$ over the period 1970-2013 range from a low of 0.01 for Norway to a high of 3.2 for the U.S. and thus a value of 1.6 is halfway

[^17]between these two estimates. ${ }^{24}$
Following Stockman and Tesar [1995], we choose a value for the elasticity of substitution $\phi$ between traded and non-traded goods of 0.44 which is the value commonly used in the international RBC literature. This value falls in the range of our panel data estimates for the whole sample which vary between 0.66 and 0.33 depending on whether the testable equation includes or not a country-specific linear time trend. ${ }^{25}$ The weight of consumption in nontradables $1-\varphi$ is set to target a non-tradable content in total consumption expenditure (i.e., $1-\alpha_{C}$ ) of $53 \%$, in line with the average of our estimates shown in the last line of column 2 of Table 4. Following Backus et al. [1994], we set the elasticity of substitution, $\rho$, in consumption between home- and foreign-produced traded goods (inputs) to $1.5 .{ }^{26}$ The weight of consumption in home-produced traded goods $\varphi^{H}$ is set to target a home content of consumption expenditure in tradables (i.e. $\alpha^{H}$ ) of $77 \%$, in line with the average of our estimates shown in the last line of column 8.

While an elasticity of intertemporal substitution (IES) around one is a typical choice in the business cycle literature, we choose a value for the IES of two in line with estimates documented by Gruber [2013]. As is well known (and demonstrated analytically in Online Appendix K), when $\sigma_{C}=1$, the wealth and substitution effect cancel out and total hours worked remain unresponsive to a technology shock. A value of two mitigates the negative impact of the wealth effect on labor supply and enables us to generate a positive response of total hours worked to the shock on impact in line with our evidence. ${ }^{27}$ Based on estimates of the macro Frisch elasticity of labor supply documented by Peterman [2016], we set $\sigma_{L}$ to 1.6 ; this value enables us to generate the increase in total hours worked by $0.09 \%$ we estimate empirically on impact (see Fig. 10(a) in Online Appendix E.2). ${ }^{28}$ The weight of labor supply to the non-traded sector, $1-\vartheta$, is set to 0.6 to target a share of non-tradables in total hours worked of $63 \%$, in line with the average of our estimates shown in the last

[^18]line of column 5 of Table 4.
We now describe the calibration of production-side parameters. We assume that physical capital depreciates at a rate $\delta_{K}=9.3 \%$ to target an investment-GDP ratio of $24 \%$ (see column 13 of Table 4). In line with mean values shown in columns 10 and 11 of Table 4 , the shares of labor income in traded and non-traded value added, $\theta^{H}$ and $\theta^{N}$, are set to 0.63 and 0.68 , respectively, which leads to an aggregate LIS of $66 \%$ (see the last line of column 12 of Table 4). We consider an initial steady-state with HNTC and normalize $A^{j}=B^{j}=Z^{j}$ to 1 . We set the elasticity of substitution, $\phi_{J}$, between $J^{T}$ and $J^{N}$ to 1 , in line with the empirical findings documented by Bems [2008] for OECD countries. Further, the weight of non-traded investment $\left(1-\varphi_{I}\right)$ is set to target a non-tradable content of investment expenditure of $62 \%$, in line with our estimates shown in the last line of column 3 of Table 4. Like for consumption goods, following Backus et al. [1994], we set the elasticity of substitution, $\rho_{J}$, in investment between home- and foreign-produced traded inputs to 1.5 . The weight of home-produced traded investment $\iota^{H}$ is set to 0.62 to target a home content of investment expenditure in tradables (i.e. $\alpha_{J}^{H}$ ) of $51 \%$ (see column 9 of Table 4). We choose the value of parameter $\kappa$ so that the elasticity of $I / K$ with respect to Tobin's q , i.e., $Q / P_{J}$, is equal to the value implied by estimates in Eberly, Rebelo, and Vincent [2008]. The resulting value of $\kappa$ is equal to $17 .{ }^{29}$

We set government spending on non-traded goods $G^{N}$ and traded goods $G^{T}$ so as to yield a non-tradable share of government spending, $\omega_{G^{N}}$, of $90 \%$, and government spending as a share of GDP, $\omega_{G}$, of $20 \%$. While Comtrade database from the United Nations [2017] enables us to breakdown imports into capital and consumption goods, we cannot differentiate between government and households purchases. We assume that the government does not import goods from abroad and thus set $G^{F}=0$.

We choose initial conditions so that trade is initially balanced. Since net exports are nil, the investment-to-GDP ratio, $\omega_{J}$, and government spending as a share of GDP, $\omega_{G}$, implies a consumption-to-GDP ratio of $\omega_{C}=56 \%$. It is worth mentioning that the tradable content of GDP is endogenously determined by the tradable content of consumption, $\alpha_{C}$, investment, $\alpha_{J}$, and government expenditure, $\omega_{G^{T}}$, along with $\omega_{C}, \omega_{J}$, and $\omega_{G}$. More precisely, dividing the traded goods market clearing condition by GDP, $Y$, leads to an expression that allows us to calculate the tradable content of GDP:

$$
\begin{equation*}
P^{H} Y^{H} / Y=\omega_{C} \alpha_{C}+\omega_{J} \alpha_{J}+\omega_{G^{T}} \omega_{G}=38 \% \tag{44}
\end{equation*}
$$

where $\omega_{C}=56 \%, \alpha_{C}=47 \%, \omega_{J}=24 \%, \alpha_{J}=38 \%, \omega_{G^{T}}=10 \%$, and $\omega_{G}=20 \%$. According to (44), the ratios we target for demand components generates a tradable content of GDP of $38 \%$ close to $39 \%$ found in the data, as reported in the last line of column 1

[^19]of Table $4 .{ }^{30}$ Finally, building on structural estimates of the price elasticities of aggregate exports documented by Imbs and Mejean [2015], we set the export price elasticity, $\phi_{X}$, to 1.7 in the baseline calibration (see the last line of last column of Table 4). Since we set initial conditions so that the economy starts with balanced trade, export as a share of GDP, $\omega_{X}=P^{H} X^{H} / Y$, is endogenously determined by the import content of consumption, $1-\alpha^{H}$, and investment expenditure, $1-\alpha_{J}^{H}$, along with $\omega_{C}$ and $\omega_{J}$.

Since the model with Cobb-Douglas production functions is the normalization point, when we calibrate the model with CES production functions, $\varphi, \iota, \varphi^{H}, \iota^{H}, \vartheta, \delta_{K}, N_{0}$ and $K_{0}$ are endogenously set to target $1-\bar{\alpha}_{C}, 1-\bar{\alpha}_{J}, \bar{\alpha}^{H}, \bar{\alpha}_{J}^{H}, \bar{L}^{N} / \bar{L}, \bar{\omega}_{J}, \bar{v}_{N X}, \bar{K}$, respectively, where a bar indicates that the ratio is obtained from the Cobb-Douglas economy. ${ }^{31}$ Drawing on Antràs [2004], we estimate the elasticity of substitution between capital and labor for tradables and non-tradables in panel data and set $\sigma^{H}$ and $\sigma^{N}$, to 0.69 and 0.72 (see the last line of columns 17 and 18 of Table 4).

As the baseline scenario, we take the model with CES production functions, IML, endogenous TOT. While in our baseline calibration we set $\sigma^{j}<1, \epsilon=1.6, \rho=\rho_{J}=1.5$, we also conduct a sensitivity analysis with respect to these parameters by setting $\sigma^{j}=1$ and letting $\epsilon$ or $\rho$ tend toward infinity in order to contrast our results with those obtained when imposing PML or exogenous TOT. In our baseline scenario, we also consider FBTC and thus we contrast the model's predictions with those when we impose HNTC.

### 4.3 Factor-Augmenting Efficiency and Sectoral TFP Dynamics

Since our VAR evidence documented in subsection 2.5 reveals that technological change is factor-biased, we first need to set the dynamics for factor efficiency, $B^{j}(t)$ and $A^{j}(t)$. We first derive the change in capital relative to labor efficiency, by log-linearizing (30) which describes the demand for factors of production:

$$
\begin{equation*}
\left(\hat{B}^{j}(t)-\hat{A}^{j}(t)\right)=\frac{\sigma^{j}}{1-\sigma^{j}} \hat{S}^{j}(t)-\hat{k}^{j}(t), \tag{45}
\end{equation*}
$$

all variables being expressed in percentage deviation from the initial steady-state. Next, given the adjustment of relative capital efficiency inferred from (45), we have to determine the dynamics of $B^{j}(t)$ and $A^{j}(t)$ consistent with the dynamics of sectoral TFP we estimate empirically. Log-linearizing the technology frontier (33) in the neighborhood of the initial steady-state allows to map labor- and capital-augmenting technological change into sectoral TFP dynamics, i.e., $\hat{Z}^{j}(t)=s_{L}^{j} \hat{A}^{j}(t)+\left(1-s_{L}^{j}\right) \hat{B}^{j}(t)$. The latter equation together with

[^20](45) can be solved for labor and capital-augmenting efficiency: ${ }^{32}$
\[

$$
\begin{gather*}
\hat{A}^{j}(t)=\hat{Z}^{j}(t)-\left(1-s_{L}^{j}\right)\left[\left(\frac{\sigma^{j}}{1-\sigma^{j}}\right) \hat{S}^{j}(t)-\hat{k}^{j}(t)\right]  \tag{46a}\\
\hat{B}^{j}(t)=\hat{Z}^{j}(t)+s_{L}^{j}\left[\left(\frac{\sigma^{j}}{1-\sigma^{j}}\right) \hat{S}^{j}(t)-\hat{k}^{j}(t)\right] \tag{46~b}
\end{gather*}
$$
\]

Plugging empirically estimated responses for $s_{L}^{j}(t)$ and $k^{j}(t)$ (see Fig. 5), $Z^{j}(t)$ (see Fig. $3(\mathrm{e})$ ) into the above equations enables us to recover the dynamics for $A^{j}(t)$ and $B^{j}(t)$ consistent with the demand of factors of production (45) and adjustment of sectoral TFPs. In this regard, the route taken to infer $\hat{A}^{j}(t)$ and $\hat{B}^{j}(t)$ from (46a)-(46b) shares some similarities with a wedge analysis. It is worth mentioning that, as a result of general equilibrium effects, inserting estimated responses of $s_{L}^{j}(t), k^{j}(t), Z^{j}(t)$ into (46) does not necessarily imply that the model will reproduce well the responses of $s_{L}^{j}$ and $k^{j}$ since these macroeconomic variables also depend on the dynamics of labor- and capital-augmenting efficiency of the other sector together with $Q(t)$ and $K(t)$.

Once we have determined the underlying dynamic process for labor and capital efficiency by using (46), we have to choose values for exogenous parameters $\bar{a}^{j}, \bar{b}^{j}$, and $\xi^{j}$, which are consistent with the continuous time autoregressive paths (36). We choose $\bar{a}^{j}, \bar{b}^{j}$ by setting $t=0$ into (36) which yields $\bar{a}^{j}=-\left(\hat{A}^{j}-\hat{A}^{j}(0)\right)$, and $\bar{b}^{j}=-\left(\hat{B}^{j}-\hat{B}^{j}(0)\right)$. Making use of the time series generated by (46a) and (46b) gives us $\bar{a}^{H}=-0.029840, \bar{b}^{H}=-0.202769$, $\bar{a}^{N}=0.234035, \bar{b}^{N}=-0.500629$. By using the fact that $\bar{z}^{j}=s_{L}^{j} \bar{a}^{j}+\left(1-s_{L}^{j}\right) \bar{b}^{j}$ (see eq. (37)), we have $\bar{z}^{H}=-0.093566$ and $\bar{z}^{N}=0.000164$ for the parameters governing the gap which must be fulfilled when sectoral TFP converges toward its long-run equilibrium. To determine the value for the speed of adjustment of sectoral TFP, we solve (37) for $\xi^{j}$, i.e., $\xi^{j}=-\frac{1}{t} \ln \left(\frac{\hat{Z}^{j}(t)-\hat{Z}^{j}}{\bar{z}^{j}}\right)$; setting $t=3$ leads to $\xi^{H}=0.570885$ for the traded sector and $\xi^{N}=1.166821$ for the non-traded sector which gives us the best fit of the response of $\hat{Z}^{j}(t)$ estimated empirically. Once we have the dynamic paths for $\hat{Z}^{H}(t)$ and $\hat{Z}^{N}(t)$, we can compute the dynamics for the shock to the TFP differential between tradables and non-tradables (see eq. (3)):

$$
\begin{equation*}
\hat{Z}(t)=a \hat{Z}^{H}(t)-b \hat{Z}^{N}(t) \tag{47}
\end{equation*}
$$

where $\hat{Z}=a \hat{Z}^{H}-b \hat{Z}^{N}$ is normalized to $1 \%$ in the long-run.

### 4.4 Reallocation and Redistributive Effects: Model Performance

In this subsection, we analyze the role of FBTC, TOT and IML in shaping the reallocation and redistributive effects in an open economy in response to a $1 \%$ permanent increase in TFP of tradables relative to TFP of non-tradables. In Table 6, we report the simulated

[^21]impact (i.e., at $t=0$ ) and long-run (i.e., at $t=10$ ) effects. While columns 1 and 6 show impact and long-run responses from our VAR model for comparison purposes, columns 2 and 7 show results for the baseline model where we consider endogenous TOT, IML, and FBTC. We contrast the benchmark results with those shown in the next three columns for impact and long-run effects, respectively, which are obtained when one or two key elements included in the baseline model are shut down. While in Table 6, we restrict our attention to impact and long-run responses, in Fig. 6 and 7, we contrast theoretical (displayed by solid black lines with squares) with empirical (displayed by solid blue lines) responses. In each panel, the responses display the point estimate of the VAR model, with the shaded area indicating the $90 \%$ confidence bounds. We also contrast theoretical responses from the baseline model when we allow for FBTC with the model's predictions when we impose HNTC shown in the dashed red line.

Adjustment in sectoral TFP. We start with the shock to the productivity differential between tradables and non-tradables and the adjustment of sectoral TFPs shown in the first column of Fig. 6. As can be seen in Fig. 6(a), the dynamics of the productivity differential that we generate theoretically by specifying the law of motions (37) and (47) reproduces the dynamic adjustment from the VAR model very well as the black line with squares and the blue line can merely be differentiated. The productivity differential is mostly driven by the adjustment in $Z^{H}(t)$ while $Z^{N}(t)$ remains constant, in line with our VAR estimates, as shown in Fig. 6(e).
$<$ Please insert Table 6, Fig. 6 and Fig. 7 about here $>$
We now assess the ability of the baseline model with IML, TOT, CES productions functions and FBTC to account for our evidence on the reallocation and redistributive effects summarized in Table 6 and next we explore the role of each ingredient in driving the results.

Baseline model. To begin with, as can be seen in panel A of Table 6, the baseline model is able to account for the sectoral composition effects we estimate empirically. First, as in the data, the traded sector drives real GDP growth since $Y^{H}$ and $Y^{N}$ increases by $0.22 \%$ and $0.01 \%$ of GDP, respectively, close to our VAR evidence ( $0.24 \%$ and $0.01 \%$, resp.). Conversely, the non-traded sector drives the rise in total hours worked as $L^{H}$ remains unresponsive on impact and $L^{N}$ rises by $0.11 \%$ of total hours worked, in line with our empirical findings ( $-0.01 \%$ and $0.10 \%$, resp.). As displayed in column 7 , the baseline model also reproduces well the adjustment in sectoral hours worked in the long-run. As can be seen in panel C, incentives for increasing $L^{N}$ are brought about by an appreciation in the relative price of non-tradables (i.e., $0.97 \%$ at $t=0$ and $1.08 \%$ at $t=10$ ) which is larger than the productivity differential, in accordance with our estimates $0.99 \%$ at $t=0$ and $1.06 \%$ at $t=10$ ), as a result of $\phi<1$. While consumption in traded goods increases,
a technology shock biased toward tradables generates an excess supply for traded goods which deteriorates the TOT by $0.27 \%$ on impact ( $0.41 \%$ in the data) and $0.37 \%$ in the long-run ( $0.44 \%$ in the data).

As a result of low substitutability between traded and non-traded goods, the appreciation in $P$ increases the share of non-tradables in total consumption expenditure and drives labor out of the traded sector. The model generates a decline in the share of tradables in total hours worked (i.e., $\nu^{L, H}$ ) by the same amount that is estimated empirically (i.e., $0.04 \%$ of total hours worked) as can be seen in panel B. Because workers experience labor mobility costs, non-traded firms pay higher wages to encourage them to shift, thus producing a positive wage differential for non-tradables and a negative wage differential for tradables, close to our estimates, especially in the long-run (see column 7).

In addition to producing a labor outflow, the large appreciation in $P$ also drives capital out of the traded sector. Since labor is subject to mobility costs, the capital-labor ratio, $k^{H}$, falls substantially by $0.13 \%$ on impact and $0.12 \%$ in the long-run (see panel D). According to the decomposition of the change in the value added share of tradables, $\nu^{Y, H}$, shown in eq. (6), if capital and labor were immobile across sectors, the change in $\nu^{Y, H}$ in the long-run would collapse to $\nu^{Y, H}\left(1-\nu^{Y, H}\right)\left(\hat{Z}^{H}-\hat{Z}^{N}\right)$ where we made use of (1). Since $\nu^{Y, H}=0.4$ approximately and the productivity differential is $1 \%$ in the long-run, a back of the envelope calculation indicates that $\nu^{Y, H}$ would increase by $0.24 \%$ of GDP. ${ }^{33}$ As can be seen in the last line of panel $\mathrm{B}, \nu^{Y, H}$ increases by $0.16 \%(0.14 \%$ in the data) of GDP only as a result of the reallocation of productive resources toward the non-traded sector.

Turning to the adjustment of sectoral variables toward the steady-state shown by the solid black line with squares in Fig. 6, the dynamics of relative prices and the sectoral wage differential which materializes after one year are captured fairly well by the baseline model. As can be seen in the fourth column of Fig. 6, the increase in the productivity differential over times further appreciates the relative price of non-tradables and amplifies the TOT deterioration. The time-increasing appreciation in $P$ has an expansionary effect on $L^{N}$ as displayed in Fig. 6(g). While labor keeps on shifting toward the non-traded sector as can be seen in the lower part of Fig. 6(f), the rise in the productivity of tradables prevents the value added share of tradables from declining (see the upper part of Fig. 6(f)). As shown in Fig. 6(b), the baseline model also reproduces well the dynamics for traded output, $Y^{H}$, while it underestimates the rise in $Y^{N}$ (shown in the lower part of the figure) which is not statistically significant however.

Implications of FBTC. In columns 3 and 8 of Table 4 , we consider the same model as in the baseline scenario except that we impose HNTC. Overall, on impact, the model assuming HNTC performs as well as the baseline model, except for the reallocation of labor

[^22]and the responses of LIS. To have a clearer picture of the performance of both models, it is useful to start with Fig. 7 which shows the dynamics for $k^{j}$ and sectoral LIS. The conclusion that emerges is that a model imposing HNTC fails to account for the shifts of capital across sectors as it understates the fall in $k^{H}$ during the adjustment and predicts a rise in $k^{N}$ (instead of a decline); most importantly, a model abstracting from FBTC cannot account for the rise in LIS. Conversely, as shown in the black line with squares in Fig. 7(a) and $7(\mathrm{c})$, the combined effect of the rise in capital relative to labor efficiency and the gross complementarity between capital and labor in production generates an expansionary effect on labor demand in both sectors which amplifies the decline in $k^{H}$ and generates a declining path for $k^{N}$. As can be seen in Fig. 7(b) and 7(d), technological change biased toward labor dominates and overturns the negative impact on LIS caused by the fall in $k^{j}$, which leads the model to reproduce well the time-varying responses of the share of value added paid to workers, $s_{L}^{j}$.

As can be seen in the third column of Fig. 6, a model imposing HNTC tends to understate the responses of sectoral hours worked over time as shown in the dashed red lines in Fig. 6(c) and 6(g). Conversely, by raising labor demand in both sectors, technological change biased toward labor increases the performance of the model in reproducing the dynamics for sectoral hours worked, as shown in the black lines with squares. A model imposing HNTC also overstates the decline in the labor share of tradables shown in the lower part of Fig. 6(f) while the model assuming FBTC tracks well the fall in $\nu^{L, H}$. The reason is that technological change is more biased toward labor in the traded than in the non-traded sector which has a positive impact on labor demand in the former sector and thus hampers the shift of labor toward the non-traded sector. ${ }^{34}$

Quantitatively the discrepancy in models' predictions between assumptions of FBTC and HNTC is less striking for reallocation than for redistributive effects since LISs mostly depend on FBTC (directly and indirectly through changes in $k^{j}$ ) while the reallocation effects depends on the differential in FBTC between sectors along with other key channels such as substitutability across goods and the degree of labor mobility. As we shall see below, the differential in FBTC between the traded and non-traded sector plays an important role, however, when it comes to explaining international differences in the reallocation effects caused by technology shocks biased toward the traded sector.

Implications of endogenous TOT. A second key element of the model is endogenous TOT. Columns 5 and 10 of Table 6 shows results when we impose HNTC, $\sigma^{j}=1$, together with perfect substitutability between home- and foreign-produced traded goods (i.e., we let

[^23]$\rho \rightarrow \infty) .{ }^{35}$ When imposing exogenous TOT, the labor share of tradables falls by $0.12 \%$ of total hours worked, a decline which is three times larger what is estimated empirically. The reason is that keeping the relative price of home-produced traded goods fixed leads the model to overstate the demand boom for non-tradables, as reflected in an appreciation in the relative price of non-tradables by $1.1 \%$ above what is estimated empirically ( $0.99 \%$ in the data, see panel C of Table 6). Because the model imposing $\rho \rightarrow \infty$ overstates considerably the shift of labor between sectors, it fails to account for the responses of sectoral value added and hours worked shown in panel A.

Implications of labor mobility costs. IML also plays a key role in driving the reallocation effects as can be seen in columns 4 and 9 which show results of a model imposing HNTC, $\sigma^{j}=1$, and PML. A direct implication of abstracting from labor mobility costs is that the model cannot account for the sectoral wage differential which materializes after one year (see panel B). As expected, when we contrast VAR evidence reported in column 1 with numerical results for the restricted model displayed in column 4, the assumption of PML leads to a decline in the labor share of tradables $(0.09 \%)$ which is more than twice as large as that estimated empirically ( $0.04 \%$ in the data, see panel B). By overestimating the labor inflow in the non-traded sector, the model overpredicts the rise in $L^{N}$ (see panel A) and mitigates the excess demand for non-traded goods. As a result, the model predicts an appreciation in the relative price of non-tradables (see panel C) by $0.89 \%$ below what is estimated empirically ( $0.99 \%$ ).

### 4.5 Redistributive and Reallocation Effects across Countries: Model vs. Data

We now move a step further and calibrate our model to country-specific data. Our objective is to assess the ability of our baseline model to account for the cross-country dispersion in the reallocation and redistributive effects we estimate empirically and to shed some light on the role of FBTC. ${ }^{36}$

Calibration to country-specific data. To conduct this analysis, we calibrate our model to match key ratios of the 17 OECD economies in our sample, as summarized in Table 4, while $\epsilon, \phi, \sigma^{j}, \phi_{X}$ are set in accordance with estimates shown in the last five columns of Table 4. As discussed in subsection 4.1, we consider the initial steady-state with Cobb-Douglas production functions (i.e., $\sigma^{j}$ is set to 1 ) as the normalization point and calibrate the reference model to the data; $\varphi, \iota, \varphi^{H}, \iota^{H}, \vartheta, \delta_{K}$ together with initial conditions need to be endogenously calibrated to target $1-\alpha_{C}, 1-\alpha_{J}, \alpha^{H}, \alpha_{J}^{H}, \frac{L^{N}}{L}, \omega_{J}$, and $v_{N X}=0$ (see subsection 4.2); we also choose values for the LIS in the Cobb-Douglas

[^24]case, $\theta^{j}$, in accordance with our estimates shown in columns 10 and 11 of Table $4 ; \omega_{G^{N}}$ and $\omega_{G}$ are chosen to match the non-tradable content of government spending and the share of government spending in GDP, respectively, which are displayed in columns 4 and 14 of Table 4. The remaining parameters, i.e., $\sigma_{L}, \sigma_{C}, \rho, \phi_{J}, \rho_{J}, \kappa$ take the same values as those summarized in Table 5. To compute FBTC, we proceed as in subsection 4.3 except that to estimate (46a)-(46b), we use country-specific estimates of $\sigma^{j}$ and country-specific estimated responses of $s_{L}^{j}(t), k^{j}(t), Z^{j}(t) \cdot{ }^{37}$ Once we have recovered time series for FBTC in sector $j=H, N$ for each country, we choose parameters $\bar{a}^{j}$ and $\bar{b}^{j}$ by setting $t=0$ into (36) and we choose parameter $\xi^{j}$ by choosing time $t$ in eq. (37) which gives the best fit of sectoral TFP dynamics to the data. Once the model is calibrated, we estimate numerically the labor reallocation and redistributive effects of a $1 \%$ permanent increase in traded relative to non-traded TFP.
$<$ Please insert Fig. 8 about here $>$
Redistributive effects across countries. We first assess the ability of the model to account for the cross-country dispersion in the responses of LIS we estimate empirically. In the first row of Fig 8, we plot impact responses of the ratio of factor income shares, $S^{j}$, we estimate empirically (vertical axis) against impact responses of $S^{j}$ we compute numerically (horizontal axis). ${ }^{38}$ To have a sense of the importance of FBTC in driving the cross-country redistributive effects, we contrast the model's predictions when we impose HNTC which are displayed in red triangles with the model's predictions when assuming FBTC shown in black squares. It is worth mentioning that $\hat{S}^{j}(t)=\frac{\hat{s}_{L}^{j}(t)}{1-s_{L}^{j}}$ and thus the response of $S^{j}$ is similar to that of the LIS which is scaled by one less the LIS (i.e., the capital income share). As it stands out, a model imposing HNTC cannot account for international differences in the responses of sectoral LIS. The correlation between empirical and theoretical estimates of impact responses of LIS on impact is -0.42 for the traded sector. While the correlation between the data and HNTC model's predictions for the impact response of $s_{L}^{N}$ improves as it stands at 0.29 , the model performance still remains low. Intuitively, the shifts of capital between sectors generated by a model imposing HNTC are not large enough on their own to reproduce the cross-country dispersion in the responses of LIS. Conversely, by influencing sectoral LISs directly and indirectly through the shifts of capital, the model assuming FBTC is able to generate a wide cross-country dispersion in the responses of LIS which fits well the data as the correlation between model's predictions and the data is 0.99 for the traded sector and 0.97 for the non-traded sector. For example, FBTC in

[^25]tradables varies on impact from $-3.3 \%$ for Belgium to $1.8 \%$ for Canada approximately and this results in an heterogeneity in responses of $S^{j}(-3.2 \%$ and $1.1 \%$, resp.) close to our estimates. While the same picture emerges for non-tradables, international differences are less as FBTC varies between $-1.2 \%$ for Belgium up to $0.4 \%$ for Denmark which results in a decline in the ratio of factor income share (i.e., $S^{N}$ ) by $-1.1 \%$ in the former country and an increase by $0.5 \%$ in the latter economy, respectively.

Reallocation effects across countries. In the second row of Fig. 8, we plot impact responses of the value added share of non-tradables and the labor share of tradables, respectively, we estimate empirically (vertical axis) against impact responses of the same variables we compute numerically (horizontal axis). Black squares show model's predictions when we allow for FBTC while red triangles shows model's predictions when we impose HNTC. The red trend line shows the fit of the model to the data when imposing HNTC and the black trend line shows the model fit when we assume FBTC. Inspection of trend lines in Fig. 8(c) reveals that both models (i.e., with either HNTC or FBTC) reproduce well the cross-country dispersion in the responses of the value added share of non-tradables. This finding suggests that international differences in the responses of sectoral value added shares are mostly driven by international differences in the elasticity of substitution between traded and non-traded goods (i.e., $\phi$ ) and sectoral TFP shocks (which we allow to vary across countries).

Conversely, we reach a different conclusion in Fig. 8(d) which plots responses of the labor share of tradables we estimate empirically against those we compute numerically. As is evident from trend lines, the ability of the model to account for the cross-country dispersion in the responses of the labor share is higher when we allow for FBTC (as shown in the black trend line). The correlation between numerical and empirical estimates stands at 0.76 with FBTC and falls to 0.35 when we shut down this feature. Intuitively, a sectoral differential in FBTC modifies sectoral labor demand and thus either amplifies or mitigates the shift of labor across two sectors in a way that increases the ability of the baseline model to account for the cross-country dispersion in the reallocation effects. One most prominent example is Germany which experiences technological change biased toward capital in the traded sector and technological change biased toward labor in the non-traded sector. The former lowers labor demand in the traded sector while the latter stimulates labor demand in the non-traded sector. The shift of labor toward the non-traded sector is thus amplified which allows the baseline model to generate a decline in the labor share of tradables by $0.12 \%$ close to our estimates (i.e., $-0.15 \%$ ). Conversely, a model imposing HNTC produces a decline in $\nu^{L, H}$ which is more than three times smaller what we estimate empirically.

### 4.6 A Test for FBTC Hypothesis

So far, we have put forward international differences in FBTC as an explanation of the crosscountry redistributive and reallocation effects. To provide some support for our hypothesis of FBTC, we draw on Acemoglu's [2003] model. In Acemoglu's setup, capital-augmenting technological change is the result of research efforts by capital intensive firms and laboraugmenting technological change is the result of research efforts by labor intensive firms. Because asymmetric technology shocks across sectors are caused by higher productivity of tradables in most of the countries of the sample, we restrict our attention to the traded sector below. ${ }^{39}$

To implement our empirical strategy, we proceed as follows. We identify technology shocks biased toward the traded sector, $\epsilon_{t}^{Z}$, for each country in our sample by estimating a VAR model which includes aggregate variables $x_{t}^{A}=\left[\hat{Z}_{t}, \hat{Y}_{R, t}, \hat{L}_{t}, \hat{W}_{C, t}\right]$. For each industry $k$, we estimate the VAR model which includes the identified shock to the productivity differential, $\epsilon_{t}^{Z}$, TFP in industry $k$ denoted by $Z^{H, k}$ and the ratio of traded to non-traded TFP, i.e., $x_{t}^{Z, k}=\left[\epsilon_{t}^{Z}, \hat{Z}_{t}^{H, k}, \hat{Z}_{t}\right]$, and adopt a Cholesky decomposition. Then, we generate impulse response functions in order to recover the percentage change in TFP in industry $k$ in the traded sector, denoted by $\hat{Z}_{t}^{H, k}$, triggered by the productivity differential, $\hat{Z}$, normalized to one percent in the long-run. The percentage deviation of TFP of tradables relative to initial steady-state is a weighted average of industry's TFP changes, i.e., $\hat{Z}_{t}^{H}=$ $\sum_{k} \nu^{Y, H, k} \hat{Z}_{t}^{H, k}$ where $\nu^{Y, H, k}$ is the share of industry $k$ 's value added in traded value added at constant prices. Substituting the linearized version of the technology frontier (37) for each industry $k$ leads to:

$$
\begin{equation*}
\hat{Z}_{t}^{H}=\sum_{k} \nu^{Y, H, k}\left[s_{L}^{H, k} \hat{A}_{t}^{H, k}+\left(1-s_{L}^{H, k}\right) \hat{B}_{t}^{H, k}\right] \tag{48}
\end{equation*}
$$

Drawing on Acemoglu's [2003] model, HNTC corresponds to a situation where all industries have the same LIS so that $s_{L}^{H, k}$ collapses to the LIS of the broad sector, $s_{L}^{H}$; in this situation, eq. (48) reduces to

$$
\begin{equation*}
\hat{\bar{Z}}_{t}^{H}=\sum_{k} \nu^{Y, H, k}\left[s_{L}^{H} \hat{A}_{t}^{H, k}+\left(1-s_{L}^{H}\right) \hat{B}_{t}^{H, k}\right], \tag{49}
\end{equation*}
$$

where a bar above $Z^{H}$ on the LHS of (49) refers to traded TFP if LISs were identical across traded industries. When $s_{L}^{H, k}=s_{L}^{H}$ for all industries $k$ of the traded sector, we have $\hat{A}_{t}^{H, k}=\hat{B}_{t}^{H, k}$ for each industry $k$ so that technological change in the traded sector is Hicks-neutral. Subtracting (49) from (48) leads to a measure of the deviation from HNTC:

$$
\begin{equation*}
\hat{Z}_{t}^{H}-\hat{\bar{Z}}_{t}^{H}=\sum_{k} \nu^{Y, H, k}\left[\left(1-s_{L}^{H, k}\right)-\left(1-s_{L}^{H}\right)\right]\left(\hat{B}_{t}^{H, k}-\hat{A}_{t}^{H, k}\right) . \tag{50}
\end{equation*}
$$

[^26]Like Acemoglu [2003], we assume that industries which are more capital (labor) intensive only perform capital- (labor-) augmenting technological change so that the change in TFP in traded industry $k$ we estimate empirically reduces to the change in capital (labor) efficiency. These assumptions can be summarized as follows:

$$
\left\{\begin{align*}
& \hat{Z}_{t}^{H, k}=\hat{B}_{t}^{H, k} \text { if }  \tag{51}\\
& \hat{Z}_{t}^{H, k}=-\hat{A}_{t}^{H, k}\left(1-s_{L}^{H, k}\right)>\left(1-s_{L}^{H}\right), \\
& \text { if } \quad s_{L}^{H, k}>s_{L}^{H}
\end{align*}\right.
$$

where $s_{L}^{H}$ is the LIS averaged across all industries in the traded sector. It is worth mentioning that the minus in front of $\hat{A}_{t}^{H, k}$ (see the second line of (51)) allows us to differentiate graphically countries where labor-intensive industries contribute more to the TFP growth in the traded sector from those where a greater part of $\hat{Z}^{H}$ can be attributed to capitalintensive industries. More precisely, if labor-intensive industries contribute more to TFP growth in the traded sector, then the measure of the deviation from Hicks neutral technological change is negative. Conversely, the measure (50) turns out to be positive for countries where capital-intensive industries contribute more to $\hat{Z}^{H}$.
$<$ Please insert Fig. 9 about here $>$
Next, we contrast deviation from HNTC from empirical estimates with the measure (50) computed numerically. To construct the latter measure, we make inference of $\hat{A}^{H}$ and $\hat{B}^{H}$ by using (46a) and (46b), respectively and we further assume that capital-augmenting technological change is identical across capital-intensive industries and thus $\hat{B}_{t}^{H, k}=\hat{B}_{t}^{H}$. The same logic applies for labor-intensive industries, i.e., $\hat{A}_{t}^{H, k}=\hat{A}_{t}^{H}$. Like for empirical estimates, we add a minus for labor-augmenting technological change in order to differentiate labor- from capital-augmenting technological change graphically. In Fig. 9, we plot the measure of the deviation from HNTC (50) estimated empirically (on the horizontal axis) against the measure estimated numerically (on the vertical axis). The left panel of Fig. 9 contrasts empirical with numerical estimates of (50) on impact (i.e., $t=0$ ) when we allow for two lags in the VAR model (to estimate $\hat{Z}_{t}^{H, k}$ ) while the right panel compares both measures by allowing for one lag. ${ }^{40}$ If technology shocks were Hicks neutral, all countries should be positioned at point $(0,0)$. By contrast, we find that capital- and labor-efficiency increases at uneven rates. More specifically, countries positioned in the north-east of the scatter-plot are those where TFP changes in the traded sector are mostly driven by capitalintensive industries while those located in the south-west are those where labor-intensive industries contribute more to $\hat{Z}_{t}^{H}$. Importantly, we detect a positive cross-country relationship which is robust to the number of lags included in the VAR model. ${ }^{41}$ Such a finding

[^27]reveals that in line with Acemoglu model's assumptions, in countries where capital-intensive industries contribute more to TFP growth in the traded sector, capital relative labor efficiency increases so that technological change favors the use of labor (as long as $\sigma_{i}^{H}<1$ ). Conversely, in countries where TFP gains are concentrated on labor-intensive industries, labor relative to capital efficiency rises which biases technological change toward capital (as long as $\sigma_{i}^{H}<1$ ).

## 5 Conclusion

Motivated by the evidence documented by Foerster et al. [2011] and Garìn et al. [2018], we explore the reallocation and redistributive effects caused by asymmetric technology shocks across sectors in an open economy setup. To conduct this analysis, we use a panel of 17 OECD countries over 1970-2013 and adopt the identification approach of technology shocks proposed by Gali [1999]. Since we consider an open economy, we differentiate between a traded and non-traded sector. We find empirically that traded TFP increases significantly more than non-traded TFP following an aggregate TFP shock and the contribution of shocks to the productivity differential between tradables and non-tradables to the forecast error variance of aggregate TFP growth has increased dramatically over time and stands at more than $40 \%$ over 1993-2013. When we estimate the effects of a permanent increase in traded relative to non-traded TFP, our evidence reveals that the share of tradables in real GDP increases while the labor share of tradables declines as labor shifts toward the non-traded sector.

To rationalize our VAR evidence, we put forward an open economy version of the neoclassical model with tradables and non-tradables. Our quantitative analysis reveals that the low substitutability between traded and non-traded goods in consumption and financial openness leads the model to substantially overstate the decline in the labor share of tradables. To account for the magnitude of the reallocation effects we document empirically, we consider three key elements. Like Kehoe and Ruhl [2009], we allow for endogenous TOT. Since domestically and foreign-produced traded goods are gross substitutes, the TOT deterioration stimulates hiring in the traded sector and thus curbs the shift of labor toward the non-traded sector. The second element is IML which is modelled in the lines of Horvath [2000] by assuming imperfect substitutability across sectoral hours worked. The presence of labor mobility costs generates a sectoral wage differential in accordance with our estimates and strengthens the TOT channel by further hampering labor reallocation across sectors.

We put forward FBTC as a third key ingredient. Adapting the methodology of Caselli and Coleman [2006] to our setup, we use the demand of inputs and our estimates of the elasticity of substitution between capital and labor to construct times series for FBTC. Our VAR estimates reveal that technological change is biased toward labor in both sectors
following a shock to traded relative to non-traded TFP which is consistent with the rise in sectoral LIS found in the data. Once we include the three aforementioned elements, the model reproduces well the reallocation and redistributive effects we estimate empirically for the whole sample. Our VAR estimates also reveal that the responses of sectoral LIS display a wide cross-country dispersion. Importantly, we detect empirically a strong and positive cross-country relationship between the responses of sectoral LIS and factor-biased technological shifts. When we calibrate the model to country-specific data, our model can account for the cross-country dispersion in the responses of sectoral LIS once we allow sectoral FBTC to vary across countries. Turning to the reallocation effects, we find that the ability of the model to account for international differences in the decline in the labor share of tradables increases significantly once when we let FBTC vary across sectors and between countries. Finally, in line with Acemoglu's [2003] model assumptions, our empirical results show that in countries where capital-intensive (labor-intensive resp.) industries contribute more to the rise in TFP in the traded sector, capital relative to labor efficiency increases. Because capital and labor are gross complements for most of the sectors/countries, technological change is biased toward labor (capital resp.) in these OECD economies.

We view our analysis of asymmetric technology shocks across sectors as a step toward a better understanding of the labor market effects of aggregate technology shocks which can be viewed as a combination of symmetric and asymmetric technology shocks across sectors. By providing incentives for hiring by non-traded firms, and all the more so in countries where technological change is biased toward labor, asymmetric technology shocks have a positive impact on total hours worked because the non-traded sector accounts for two-third of labor in OECD countries. By contrast, by reducing dramatically the relative price of non-tradables (as a result the gross complementarity between traded and non-traded goods) and thus the demand for labor in the non-traded sector, symmetric technology shocks exert a negative impact on total hours worked. In this regard, we may expect the increasing importance of asymmetric technology shocks to increase the response of hours worked to technology shock over time.

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Figure 1: Relative Productivity and Labor Share of Tradables (1993-2013) Notes: In Fig. 1, we plot the detrended ratio of TFP of tradables to TFP of non-tradables (or the relative productivity of tradables) shown in the blue line, against the detrended labor share of tradables shown in the black line. TFP of tradables, $Z_{t}^{H}$, and TFP of non-tradables, $Z_{t}^{N}$, are the Solow residual. The labor share of tradables is calculated as the ratio of hours worked in the traded sector to total hours worked. Detrended relative productivity of tradables is computed as the difference between the logarithm of actual time series $Z_{t}^{H} / Z_{t}^{N}$ and the trend of (logged) relative productivity of tradables. The trend of logged relative productivity of tradables is obtained by applying a Hodrick-Prescott filter with a smoothing parameter of $\lambda=100$ (as we use annual data) to the (logged) time series $Z_{t}^{H} / Z_{t}^{N}$. Detrended labor share of tradables is computed as the difference between actual time series for $L_{t}^{H} / L_{t}$ and the trend of the labor share of tradables, the latter being obtained by applying a Hodrick-Prescott filter with a smoothing parameter of $\lambda=100$. Sample: United States, United Kingdom, Ireland, Spain. For the last three economies, we restrict attention to the period 1993-2013 as the great moderation starts in the post-1992 period in European countries, see González Cabanillas and Ruscher [2008]. Fig. 1(b) plots detrended relative productivity of tradables and detrended labor share of tradables for the whole sample where sectoral TFP and the labor share of tradables are calculated as the working age population weighted sum of the seventeen OECD countries. Sample: 17 OECD countries, 1970-2013, annual data.


Figure 2: Symmetric and Asymmetric Technology Shocks across Sectors. Notes: In Fig. 2(a), we plot the responses of traded and non-traded TFP, $Z_{t}^{H}$ (shown in the blue line) and $Z_{t}^{N}$ (shown in the black line), to identified shock to aggregate TFP, $Z_{t}^{A}$. To identify empirically aggregate technology shocks, we consider a VAR model which includes aggregate TFP, $Z_{t}^{A}$, and total hours worked, $L_{t}$, both in rate of growth, and assume that only shocks to aggregate TFP increases permanently $Z_{t}^{A}$. Once the aggregate technology shock is identified, we plug the shock into a VAR model which also includes $Z_{t}^{H}, Z_{t}^{N}$ and $Z_{t}^{A}$; we estimate the VAR model by adopting a Cholesky decomposition and generate responses of $Z_{t}^{H}$ and $Z_{t}^{N}$ following a $1 \%$ permanent increase in $Z_{t}^{A}$. Fig. 2(b) and Fig. 2(c) show the effects of symmetric and asymmetric technology shocks where the blue line and the black line display the responses for $Z_{t}^{H} / Z_{t}^{N}$ and $Z_{t}^{A}$, respectively. To identify these two long-run shocks, we consider a VAR model which includes the ratio of traded to non-traded TFP, $Z_{t}^{H} / Z_{t}^{N}$, aggregate TFP, $Z_{t}^{A}$, and hours worked, $L_{t}$, all variables entering the VAR model in growth rates. We impose long-run restrictions such that asymmetric technology shocks across sectors increase permanently both $Z_{t}^{H} / Z_{t}^{N}$ and $Z_{t}^{A}$ while symmetric technology shocks only increase permanently $Z_{t}^{A}$. Shaded area indicates the 90 percent confidence bounds obtained by bootstrap sampling; sample: 17 OECD countries, 1970-2013, annual data.

Table 1: The Share of the FEV of Aggregate TFP Growth Attributable to Asymmetric Technology Shocks across Sectors in \%

| Horizon | FEVD for $Z^{A}$ |  |  |  |  |  |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $1970-2013$ |  |  | $1970-1992$ |  |  | $1993-2013$ |  |  |
|  | $Z^{H} / Z^{N}$ | $Z^{A}$ | $L$ | $Z^{H} / Z^{N}$ | $Z^{A}$ | $L$ | $Z^{H} / Z^{N}$ | $Z^{A}$ | $L$ |
|  | $(1)$ | $(2)$ | $(3)$ | $(4)$ | $(5)$ | $(6)$ | $(7)$ | $(8)$ | $(9)$ |
| 0 | 19.065 | 68.970 | 11.964 | 2.589 | 87.143 | 10.268 | 41.187 | 50.816 | 7.997 |
| 5 | 17.634 | 68.902 | 13.464 | 2.961 | 81.842 | 15.197 | 39.878 | 50.487 | 9.635 |
| 10 | 17.632 | 68.897 | 13.472 | 2.960 | 81.804 | 15.236 | 39.878 | 50.487 | 9.635 |

Notes: FEV: Forecast Error Variance. The number in columns 1-9 denotes the fraction of the total forecast error variance of aggregate TFP growth $\hat{Z}_{t}^{A}$ attributable to identified asymmetric technology shocks across sectors (shock to $Z_{t}^{H} / Z_{t}^{N}$, see columns $1,4,7$ ), symmetric technology shocks across sectors (shock to $Z_{t}^{A}$ leaving unaffected $Z_{t}^{H} / Z_{t}^{N}$, see columns $2,5,8$ ), and a third shock to which we do not attach any structural interpretation (shock to $L_{t}$, see columns 3,6,9). We consider a forecast horizon of $1,5,10$ years and compute the FEVs in the three-variable VAR model which includes $Z^{H} / Z^{N}, Z^{A}$, and $L$, all in growth rate. Sample: 17 OECD countries, 1970-2013, annual data.

Table 2: Sample Range for Empirical and Numerical Analysis

| Country | Code | Period | Obs. |
| :--- | :--- | :---: | :---: |
| Australia | (AUS) | $1970-2013$ | 44 |
| Austria | (AUT) | $1970-2013$ | 44 |
| Belgium | (BEL) | $1970-2013$ | 44 |
| Canada | (CAN) | $1970-2013$ | 44 |
| Germany | (DEU) | $1970-2013$ | 44 |
| Denmark | (DNK) | $1970-2013$ | 44 |
| Spain | (ESP) | $1970-2013$ | 44 |
| Finland | (FIN) | $1970-2013$ | 44 |
| France | (FRA) | $1970-2013$ | 44 |
| Great Britain | (GBR) | $1970-2013$ | 44 |
| Ireland | (IRL) | $1970-2013$ | 44 |
| Italy | (ITA) | $1970-2013$ | 44 |
| Japan | (JPN) | $1974-2013$ | 40 |
| Netherlands | (NLD) | $1970-2013$ | 44 |
| Norway | (NOR) | $1970-2013$ | 44 |
| Sweden | (SWE) | $1970-2013$ | 44 |
| United States | (USA) | $1970-2013$ | 44 |
| Total number of obs. |  |  | 744 |
| Main data sources |  | EU KLEMS \& OECD STAN |  |

Notes: Column 'period' gives the first and last observation available. Obs. refers to the number of observations available for each country.


Figure 3: Sectoral Effects of a Permanent Increase in Traded Relative to Non-Traded TFP. Notes: Exogenous increase of TFP in tradables relative to non-tradables (3) by 1\%. Horizontal axes indicate years. Vertical axes measure percentage deviation from trend in GDP units (sectoral value added, sectoral value added share), percentage deviation from trend in total hours worked units (sectoral hours worked, sectoral hours worked share), percentage deviation from trend (sectoral TFPs, relative price of non-tradables, TOT, relative wage). Shaded areas indicate the 90 percent confidence bounds obtained by bootstrap sampling. The blue line shows the response for tradables while the black line line shows the response for non-tradables. Sample: 17 OECD countries, 1970-2013, annual data.

Table 3: Sectoral Composition Effects of a Technology Shock Biased toward Tradables: Point Estimates

| Variables | $\begin{array}{c}\text { Impact } \\ (t=0)\end{array}$ | $\begin{array}{c}\text { Long-run } \\ (t=10) \\ (1)\end{array}$ | $\begin{array}{c}\text { Impact } \\ (t=0)\end{array}$ | $\begin{array}{c}\text { Long-run } \\ (t=10)\end{array}$ |
| :--- | :---: | :---: | :---: | :---: |
|  | Tradables |  |  |  |$)$

Notes: * denote significance at $10 \%$ level. Standards errors are bootstrapped with 10000 replications.


Figure 4: Cross-Country Effects of a Permanent Increase in Traded Relative to Non-Traded TFP. Notes: Figure 4 plots impact responses of the labor share of tradables and the value added share of nontradables to a $1 \%$ permanent increase in the relative productivity of tradables against three key estimated parameters. Impact responses shown in the vertical axis are obtained by running a VAR model for one country at a time and are expressed in percentage point. Horizontal axis in Fig. 4(a) and Fig. 4(b) displays the elasticity of labor supply across sectors, $\epsilon$ (which captures the degree of labor mobility across sectors) and the elasticity of substitution between traded and non-traded goods, $\phi$, respectively. Panel data estimates for $\epsilon$ and $\phi$ are taken from column 16 and 15 of Table 4, respectively. Horizontal axis in Fig. 4(c) displays the differential in FBTC between tradables and non-tradables whose estimates are obtained by running the VAR model $\left[\hat{Z}_{t}, \mathrm{FBTC}_{t}^{j}\right]$ for one country at at a time where time series for $\mathrm{FBTC}_{t}^{j}$ are constructed by using (7).


Figure 5: Redistributive Effects of a Permanent Increase in Traded Relative to Non-Traded TFP. Notes: Exogenous increase of TFP in tradables relative to non-tradables by $1 \%$. The first two columns show the responses of LIS and capital-labor ratios for tradables and non-tradables. Horizontal axes indicate years. Vertical axes measure deviations from trend in percentage of value added for the LIS, and percentage deviation from trend in capital stock units for the capital-labor ratio. The third column plots the response of FBTC in sector $j=H, N$ which is obtained by running a simple VAR $\left[\hat{Z}_{i t}, \mathrm{FBTC}_{i t}^{j}\right]$ where time series for $\mathrm{FBTC}_{i t}^{j}$ are computed by making use of eq. (7). Results for baseline specification are displayed by solid lines with shaded area indicating 90 percent confidence bounds obtained by bootstrap sampling. The last column plots impact (i.e., at time $t=0$ ) responses of the ratio of factor income shares, $\hat{S}_{t}^{j}=\frac{\hat{s}_{L, t}^{j}}{1-s_{L}^{j}}$, on the vertical axis against FBTC in sector $j=H, N$ on the horizontal axis. Impact and long-run responses are obtained by running a VAR model for each country and are expressed in percent. Sample: 17 OECD countries, 1970-2013, annual data.
Table 4: Data to Calibrate the Two-Sector Model (1970-2013)

| Countries | Non-tradable share |  |  |  |  |  | Home share |  |  | Labor Share |  | Aggregate ratios |  |  | Elasticities |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | GDP <br> (1) | Cons. (2) | Inv. <br> (3) | Gov. <br> (4) | Labor (5) | Lab. comp. (6) | Imp./Y <br> (7) | Cons. (8) | Inv. (9) | $\begin{gathered} \mathrm{LIS}^{H} \\ (10) \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{LIS}^{N} \\ (11) \\ \hline \end{gathered}$ | $\begin{gathered} s_{L} \\ (12) \end{gathered}$ | $\begin{aligned} & I / Y \\ & (13) \end{aligned}$ | $\begin{gathered} G / Y \\ (14) \end{gathered}$ | $\begin{gathered} \phi \\ (15) \end{gathered}$ | $\begin{gathered} \epsilon \\ (16) \end{gathered}$ | $\begin{gathered} \sigma^{H} \\ (17) \\ \hline \end{gathered}$ | $\begin{gathered} \sigma^{N} \\ (18) \end{gathered}$ | $\begin{gathered} \phi_{X} \\ (19) \\ \hline \end{gathered}$ |
| AUS | 0.60 | 0.53 | 0.62 | 0.88 | 0.64 | 0.63 | 0.07 | 0.86 | 0.64 | 0.59 | 0.67 | 0.64 | 0.27 | 0.18 | 0.40 | 0.37 | 0.47 | 0.53 | 1.64 |
| AUT | 0.61 | 0.54 | 0.60 | 0.88 | 0.60 | 0.61 | 0.14 | 0.70 | 0.44 | 0.68 | 0.68 | 0.68 | 0.25 | 0.18 | 1.52 | 1.10 | 0.77 | 1.30 | 1.70 |
| BEL | 0.62 | 0.53 | 0.58 | 0.89 | 0.64 | 0.63 | 0.25 | 0.32 | 0.19 | 0.66 | 0.67 | 0.67 | 0.23 | 0.22 | 1.24 | 0.61 | 0.83 | 1.07 | n.a. |
| CAN | 0.62 | 0.53 | 0.68 | 0.90 | 0.67 | 0.65 | 0.10 | 0.81 | 0.28 | 0.54 | 0.62 | 0.59 | 0.22 | 0.21 | 0.75 | 0.39 | 0.48 | 0.67 | 2.25 |
| DEU | 0.61 | 0.52 | 0.64 | 0.91 | 0.60 | 0.57 | 0.09 | 0.81 | 0.52 | 0.76 | 0.64 | 0.69 | 0.23 | 0.19 | 0.58 | 1.01 | 0.64 | 0.99 | 1.47 |
| DNK | 0.65 | 0.51 | 0.62 | 0.94 | 0.66 | 0.67 | 0.12 | 0.72 | 0.40 | 0.65 | 0.70 | 0.68 | 0.21 | 0.25 | 1.08 | 0.29 | 0.42 | 1.28 | n.a. |
| ESP | 0.60 | 0.55 | 0.79 | 0.87 | 0.60 | 0.62 | 0.08 | 0.81 | 0.42 | 0.60 | 0.66 | 0.63 | 0.24 | 0.16 | 1.39 | 1.02 | 1.03 | 0.48 | 1.87 |
| FIN | 0.57 | 0.50 | 0.65 | 0.89 | 0.58 | 0.60 | 0.09 | 0.83 | 0.51 | 0.65 | 0.74 | 0.70 | 0.25 | 0.20 | 0.85 | 0.43 | 0.76 | 0.79 | 1.62 |
| FRA | 0.66 | 0.49 | 0.64 | 0.91 | 0.64 | 0.66 | 0.08 | 0.82 | 0.61 | 0.72 | 0.69 | 0.70 | 0.23 | 0.22 | 0.89 | 1.40 | 0.87 | 0.92 | 1.66 |
| GBR | 0.59 | 0.57 | 0.60 | 0.94 | 0.65 | 0.61 | 0.09 | 0.78 | 0.60 | 0.70 | 0.74 | 0.72 | 0.20 | 0.19 | 0.00 | 0.60 | 0.60 | 0.56 | 1.51 |
| IRL | 0.52 | 0.54 | 0.60 | 0.87 | 0.58 | 0.60 | 0.14 | 0.72 | 0.29 | 0.51 | 0.69 | 0.60 | 0.22 | 0.18 | 1.35 | 0.22 | 0.74 | 0.63 | n.a. |
| ITA | 0.60 | 0.48 | 0.64 | 0.91 | 0.58 | 0.58 | 0.07 | 0.87 | 0.68 | 0.74 | 0.67 | 0.70 | 0.22 | 0.18 | 0.72 | 1.66 | 0.84 | 0.47 | 1.72 |
| JPN | 0.61 | 0.57 | 0.58 | 0.90 | 0.61 | 0.63 | 0.04 | 0.90 | 0.90 | 0.60 | 0.66 | 0.64 | 0.29 | 0.16 | 1.05 | 0.87 | 1.16 | 0.64 | 1.60 |
| NLD | 0.63 | 0.54 | 0.60 | 0.90 | 0.67 | 0.67 | 0.16 | 0.65 | 0.15 | 0.61 | 0.74 | 0.69 | 0.22 | 0.22 | 0.52 | 0.22 | 0.91 | 0.44 | n.a. |
| NOR | 0.54 | 0.46 | 0.67 | 0.89 | 0.62 | 0.64 | 0.08 | 0.85 | 0.53 | 0.44 | 0.63 | 0.54 | 0.26 | 0.20 | 0.89 | 0.01 | 0.63 | 0.56 | 1.78 |
| SWE | 0.62 | 0.56 | 0.47 | 0.92 | 0.65 | 0.64 | 0.10 | 0.74 | 0.65 | 0.67 | 0.74 | 0.71 | 0.24 | 0.25 | 0.51 | 0.53 | 0.61 | 0.38 | 1.78 |
| USA | 0.66 | 0.59 | 0.56 | 0.89 | 0.70 | 0.66 | 0.05 | 0.89 | 0.77 | 0.62 | 0.62 | 0.62 | 0.22 | 0.16 | 0.82 | 3.22 | 0.77 | 0.88 | 1.45 |
| OECD | 0.61 | 0.53 | 0.62 | 0.90 | 0.63 | 0.63 | 0.10 | 0.77 | 0.51 | 0.63 | 0.68 | 0.66 | 0.24 | 0.20 | 0.44 | 1.60 | 0.69 | 0.72 | 1.70 |



 labor in sector $j=H, N$; estimates of the elasticity of exports w.r.t. terms of trade, $\phi_{X}$, are taken from Imbs and Mejean [2015].
Table 5: Baseline Parameters (Representative OECD Economy)

| Definition | Value |  | Reference |
| :---: | :---: | :---: | :---: |
|  | OECD | Sensitivity |  |
| Period of time | year | year | data frequency |
| A.Preferences |  |  |  |
| Subjective time discount rate, $\beta$ | $4 \%$ | $4 \%$ | equal to the world interest rate |
| Intertemporal elasticity of substitution for consumption, $\sigma_{C}$ | 2 | 2 | Gruber [2013] |
| Intertemporal elasticity of substitution for labor, $\sigma_{L}$ | 1.6 | 1.6 | Peterman [2016] |
| Elasticity of substitution between $C^{T}$ and $C^{N}, \phi$ | 0.44 | 0.44 | Stockman and Tesar [1995] |
| Elasticity of substitution between $J^{T}$ and $J^{N}, \phi_{J}$ | 1 | 1 | Bems [2008] |
| Elasticity of substitution between $C^{H}$ and $C^{F}, \rho$ | 1.5 | $\infty$ | Backus, Kehoe and Kydland [1994] |
| Elasticity of substitution between $J^{H}$ and $J^{F}, \rho_{J}$ | 1.5 | $\infty$ | Backus, Kehoe and Kydland [1994] |
| Elasticity of labor supply across sectors, $\epsilon$ | 1.6 | 1.6 | our estimates (EU KLEMS [2011], [2017] and OECD STAN [2011], [2017] databases) |
| B.Non-tradable share |  |  |  |
| Weight of consumption in non-traded goods, $1-\varphi$ | 0.49 | 0.49 | set to target $1-\alpha_{C}=53 \%$ (United Nations, COICOP [2017]) |
| Weight of labor supply to the non-traded sector, $1-\vartheta$ | 0.6 | 0.6 | set to target $L^{N} / L=63 \%$ (KLEMS [2011]) |
| Weight of non-traded investment, $1-\iota$ | 0.62 | 0.62 | set to target $\alpha_{J}=62 \%$ (OECD Input-Output database [2017]) |
| Non-tradable content of government expenditure, $\omega_{G^{N}}$ | 0.9 | 0.9 | our estimates (COFOG, OECD [2017]) |
| Labor income share in the non-traded sector, $\theta^{N}$ | 0.68 | 0.68 | our estimates (EU KLEMS [2011], [2017] and OECD STAN [2011], [2017] databases) |
| Labor income share in the traded sector, $\theta^{H}$ | 0.63 | 0.63 | our estimates (EU KLEMS [2011], [2017] and OECD STAN [2011], [2017] databases) |
| TFP index, $Z^{j}$ | 1 | 1 | Initial steady-state value |
| C.Home share |  |  |  |
| Weight of consumption in home traded goods, $\varphi^{H}$ | 0.84 | 0.84 | set to target $\alpha^{H}=77 \%$ (United Nations, Comtrade [2017]) |
| Weight of home traded investment, $\iota^{H}$ | 0.62 | 0.62 | set to target $\alpha_{J}^{H}=51 \%$ (United Nations, Comtrade [2017]) |
| Export price elasticity, $\phi_{X}$ | 1.7 | 1.7 | Imbs and Mejean [2015] |
| D.GDP demand components |  |  |  |
| Physical capital depreciation rate, $\delta_{K}$ | 9.3\% | 9.3\% | set to target $\omega_{J}=24 \%$ (Source: OECD Economic Outlook Database) |
| Parameter governing capital adjustment cost, $\kappa$ | 17 | 17 | set to match the elasticity $I / K$ to Tobin's q (Eberly et al. [2008]) |
| Government spending as a ratio of GDP, $\omega_{G}$ E.Technology shock | 20\% | 20\% | our estimates (Source: OECD Economic Outlook Database) |
| E.Technology shock |  |  |  |
| Exogenous shock to productivity differential, $\hat{Z}$ | 1\% | 1\% | to generate $\hat{Z}=1 \%$ |
| Scaling parameter for $\hat{Z}^{H}(0), \bar{z}^{H}$ | -0.0936 | -0.0936 | set to target $\hat{Z}^{H}(0)$ |
| Scaling parameter for $\hat{Z}^{N}(0), \bar{z}^{N}$ | 0.0002 | 0.0002 | set to target $\hat{Z}^{N}(0)$ |
| Speed of adjustment of $Z^{H}, \xi^{H}$ | 0.5709 | 0.5709 | set to target $-\frac{1}{3} \ln \left(\frac{\hat{Z}^{H}(3)-\hat{Z}^{H}}{\bar{z}^{H}}\right)$ |
| Speed of adjustment of $Z^{N}, \xi^{N}$ | 1.1668 | 1.1668 | set to target $-\frac{1}{3} \ln \left(\frac{\hat{Z}^{N}(3)-\hat{Z}^{N}}{\bar{z}^{N}}\right)$ |

Table 6: Impact and Long-Run Effects of a 1\% Permanent Increase in Traded relative to Non-Traded TFP

|  | VAR $t=0$ | Impact Theoretical Responses |  |  |  | VAR $t=10$ | Long-run Theoretical Responses |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Data | Bench | CES | PML \& TOT | IML | Data | Bench | CES | PML \& TOT | IML |
|  |  | FBTC | HNTC | $(\epsilon \rightarrow \infty)$ | $(\rho \rightarrow \infty)$ |  | FBTC | HNTC | $(\epsilon \rightarrow \infty)$ | $(\rho \rightarrow \infty)$ |
|  | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) | (10) |
| A.Sectoral Labor and Output |  |  |  |  |  |  |  |  |  |  |
| Traded labor, $d L^{H}(0)$ | -0.01 | -0.00 | -0.03 | -0.05 | -0.10 | 0.01 | 0.01 | -0.02 | -0.04 | -0.02 |
| Non-traded labor, $d L^{N}(0)$ | 0.10 | 0.11 | 0.12 | 0.14 | 0.14 | 0.15 | 0.15 | 0.13 | 0.15 | 0.15 |
| Traded output, $d Y^{H}(0)$ | 0.24 | 0.22 | 0.22 | 0.21 | 0.14 | 0.27 | 0.27 | 0.28 | 0.27 | 0.59 |
| Non-traded output, $d Y^{N}(0)$ | 0.01 | 0.01 | 0.00 | 0.02 | 0.06 | 0.06 | 0.02 | 0.03 | 0.05 | 0.17 |
| B.Reallocation Effects |  |  |  |  |  |  |  |  |  |  |
| Non-traded wage, $d\left(W^{N} / W\right)(0)$ | 0.01 | 0.04 | 0.06 | 0.00 | 0.12 | 0.06 | 0.04 | 0.06 | 0.00 | 0.12 |
| Traded wage, $d\left(W^{H} / W\right)(0)$ | -0.02 | -0.07 | -0.11 | 0.00 | -0.20 | -0.12 | -0.07 | -0.11 | -0.00 | -0.20 |
| Labor share of tradables, $d \nu^{L, H}(0)$ | -0.04 | -0.04 | -0.06 | -0.09 | -0.12 | -0.05 | -0.05 | -0.06 | -0.08 | -0.07 |
| Output share of tradables, $d \nu^{Y, H}(0)$ | 0.13 | 0.13 | 0.13 | 0.12 | 0.06 | 0.14 | 0.16 | 0.16 | 0.15 | 0.15 |
| C.Sectoral Prices |  |  |  |  |  |  |  |  |  |  |
| Relative price, $d P(0)$ | $0.99$ | $0.97$ | 1.00 | $0.89$ | $1.10$ | $1.06$ | 1.08 | 1.10 | 0.99 | 1.11 |
| $\text { TOT, } d P^{H}(0)$ | $-0.41$ | -0.27 | -0.29 | -0.27 | 0.00 | -0.44 | -0.37 | -0.40 | -0.38 | 0.00 |
| D.Redistributive effects |  |  |  |  |  |  |  |  |  |  |
| Traded capital-labor, $d k^{H}(0)$ | -0.08 | -0.13 | -0.06 | -0.03 | -0.08 | -0.14 | -0.12 | -0.02 | 0.01 | -0.02 |
| Non-traded capital-labor, $d k^{N}(0)$ | -0.01 | 0.03 | -0.01 | -0.04 | 0.07 | -0.04 | -0.04 | 0.03 | 0.01 | 0.07 |
| Traded LIS, $d s_{L}^{H}(0)$ | 0.09 | 0.08 | -0.01 | 0.00 | 0.00 | 0.10 | 0.10 | -0.01 | 0.00 | 0.00 |
| Non-traded LIS, $d s_{L}^{N}(0)$ | 0.01 | 0.02 | -0.00 | 0.00 | 0.00 | 0.07 | 0.07 | 0.00 | 0.00 | 0.00 | Notes: Impact and steady-state effects of a $1 \%$ permanent increase in traded relative to non-traded TFP. Panels A,B,C,D show the initial deviation in percentage relative to

steady-state for sectoral variables. Responses of relative wages and relative prices are percentage deviation from initial steady-state. Sectoral value added and value added share are expressed in percent of initial GDP while sectoral labor and labor shares are expressed in percent of initial total hours worked; changes in capital-labor ratios are expressed in percent of the aggregate stock of capital while responses of sectoral LIS are measured in percent of value added of the corresponding sector; $\epsilon$ is the elasticity of labor supply across sectors and captures the degree of labor mobility; when we let $\epsilon$ tend toward infinity, the special case of PML is obtained so that wages equalize across sectors; $\rho$ is the elasticity of substitution between home- and foreign-produced traded goods. When we let $\rho$ tend toward infinity, TOT remain fixed. In our baseline calibration (labelled 'Bench' n columns 2 and 7 ), we set $\epsilon=1.6, \phi=0.44, \sigma_{L}=1.6, \kappa=17, \rho=\rho_{J}=1.5, \phi_{X}=1.7, \sigma^{H}=0.69, \sigma^{N}=0.72$ and allow for labor- and capital-augmenting technological change nferred from (46a)-(46b). In columns 3 and 8 , we consider the same model as in the baseline but impose HNTC, i.e., $\hat{A}^{j}(t)=\hat{B}^{j}(t)=\hat{Z}^{j}(t)$. In columns $4,5,9,10$ we consider Cobb-Douglas production functions. Because CES and Cobb-Douglas production functions deliver the same results when imposing HNTC, we omit the results when we let $\epsilon$, $\rho$ tend toward infinity while assuming CES production functions.


Figure 6: Sectoral Effects a Permanent Technology Shock Biased Toward Tradables: Model vs. Data Notes: Solid blue lines display point estimate of VAR model with shaded area indicating $90 \%$ confidence bounds; solid black lines with squares display baseline model's predictions, i.e., when we allow for $\operatorname{IML}(\epsilon=1.6)$, endogenous TOT ( $\rho=\rho_{J}=1.5$ ), gross complementarity between capital and labor in production (i.e., $\sigma^{H}=0.687$, $\sigma^{N}=0.716$ ), and technological change biased toward labor, i.e., $\mathrm{FBTC}^{H}=0.58 \%$ and $\mathrm{FBTC}^{N}=0.36 \%$ in the long-run; dashed red lines show predictions of the same model when we impose HNTC.


Figure 7: Redistributive Effects of a Permanent Technology Shock Biased Toward Tradables: Model vs. Data. Notes: Solid blue lines display point estimate of VAR model with shaded area indicating $90 \%$ confidence bounds; solid black line with squares display model's predictions in the baseline scenario with IML across sectors $(\epsilon=1.6)$, endogenous TOT ( $\rho=\rho_{J}=1.5$ ), gross complementarity between capital and labor in production (i.e., $\sigma^{H}=0.687, \sigma^{N}=0.716$ ), and technological change biased toward labor, i.e., $\mathrm{FBTC}^{H}=0.58 \%$ and $\mathrm{FBTC}^{N}=0.36 \%$ in the long-run); dashed red lines show predictions of the same model except for technological change which is assumed to be Hicks-neutral (i.e., $\left.\hat{B}^{j}(t)=\hat{A}^{j}(t)\right)$.


Figure 8: Cross-Country Relationship under FBTC and HNTC hypothesis: Model vs. Data. Notes: Fig. 8 plots impact (i.e., at time $t=0$ ) responses of LIS and sectoral shares estimated empirically on the vertical axis against the responses of the corresponding variables computed numerically. In each panel, we contrast the predictions from a model imposing HNTC shown in red triangles with the predictions of the baseline model assuming FBTC displayed in black squares. The red trend line shows the fit of the model to the data when imposing HNTC while the black trend line shows the fit of the model to the data when assuming FBTC. For the first row, we do not show the trend line obtained from a model imposing HNTC as the trend line would be vertical as a result of the low performance of the model.


Figure 9: Deviation from Hicks-Neutral Technology Shocks: Model vs. Data. Notes: According to Acemoglu's [2003] model assumptions, capital- (labor-) intensive industries perform capital- (labor-) augmenting technological change. In Fig. 9, we investigate whether in countries where capital relative to labor efficiency increases, capital-intensive industries contribute more to TFP changes on impact. To perform this investigation, we compute a measure of the deviation from Hicks neutral technological change. This index is a weighted average of TFP shocks within each industry; each industry's TFP shock is weighted by the product of its valued added share and the difference between this industry's capital income share and the broad sector's capital income share. On the horizontal axis, we report estimated values of our measure of deviation from HNTC. When this measure takes positive (negative) values, relative capital (labor) efficiency increases. The vertical axis shows the same measure computed numerically. According to Acemoglu's [2003] model, if capital income shares were equal across industries, then technological change would be Hicks-neutral so that capital and labor efficiency would increase at the same speed and all observations would be positioned at point $(0,0)$.

## A VAR Identification: Empirical Strategy

In this section we detail the empirical approach we adopt to identify permanent technology shocks and next we provide a short survey of the literature and motivate the choice of our method described below.

Let $\hat{X}_{i t}=\left(\hat{Z}_{i t}, \hat{Y}_{i t}\right)$ be a vector of $n$ variables for $i=1, \ldots, N$ and $t=1, \ldots, T$ where $\hat{Z}_{i t}$ denotes the first difference of the variable that is assumed to respond (in the long run) exclusively to permanent technology shocks biased toward the traded sector and $\hat{Y}_{i t}$ includes the first difference of the $(n-1)$ variables of interest. The structural moving average representation for $\hat{X}_{i t}$ (omitting country fixed effects and time fixed effects without loss of generality) reads:

$$
\begin{equation*}
\hat{X}_{i t}=A(L) \varepsilon_{i t} \tag{52}
\end{equation*}
$$

where $A(L)=\sum_{k=0}^{\infty} A_{k} L^{k}, L$ is the backshift operator and $\varepsilon_{i t}=\left(\varepsilon_{i t}^{z}, \varepsilon_{i t}^{y}\right)^{\prime}$ is the vector of structural shocks with $\varepsilon_{i t}^{z}$ denoting the technology shock (to be identified) and $\varepsilon_{i t}^{y}$ the other non-technology structural shocks (unidentified). The structural shocks are uncorrelated, have zero mean and, by normalization, unit variance, i.e. $E\left[\varepsilon_{i t} \varepsilon_{i t}^{\prime}\right]=I_{n}$. To identify the shock of interest $\varepsilon_{i t}^{z}$, we use the restriction that the unit root in the variable $Z_{i t}$ originates exclusively from technology shocks biased toward the traded sector. Hence, non technology shocks $\varepsilon_{i t}^{y}$ have no long-run effect on the ratio of TFP in the home traded good sector to TFP in the non-traded sector. Our identification restriction implies that the upper triangular elements of the long-run impact matrix $A(1)$ must be zero. The reduced form moving average representation associated with (52) is:

$$
\begin{equation*}
\hat{X}_{i t}=B(L) \eta_{i t}, \tag{53}
\end{equation*}
$$

with $B(L)=I_{n}+\sum_{k=1}^{\infty} B_{k} L^{k}$ and where $\eta_{i t}$ is a vector of white noise processes with $E\left[\eta_{i t} \eta_{i t}^{\prime}\right]=\Sigma$. Given representations (52) and (53), the structural economic shocks $\varepsilon_{i t}$ are related to reduced form residuals $\eta_{i t}$ through $\eta_{i t}=A_{0} \varepsilon_{i t}$, which implies $\Sigma=A_{0} A_{0}^{\prime}$ and $A_{j}=B_{j} A_{0}$. The VMA representation (53) is obtained by estimating its reduced form $\operatorname{VAR}(p)$ :

$$
\begin{equation*}
C(L) \hat{X}_{i t}=\eta_{i t} \tag{54}
\end{equation*}
$$

with $C(L)=I_{n}-\sum_{k=1}^{p} C_{k} L^{k}$ and $B(L)=(C(L))^{-1}$. System (54) is estimated by panel OLS regression with country fixed effects and time fixed effects. We assume that the matrices $C_{k}$ and $\Sigma$ are invariant across time and countries. To ensure that differences across VAR models (presented below) are not driven by differences in selected lags $p$, we set $p=2$ in all reported results. In order to recover the identifying restriction from the estimated VAR model (54), the matrix $A(1)$ is computed as the Cholesky decomposition of $B(1) \Sigma B(1)^{\prime}$, i.e. $A(1) A(1)^{\prime}=B(1) \Sigma B(1)^{\prime}$. The structural shocks can then be recovered using $\varepsilon_{i t}=A(1)^{-1} B(1) \eta_{i t}$ where we used the fact that $A_{0}=A(1)^{-1} B(1)$.

While we adopt the identification of permanent technology shocks pioneered by Gali [1999], and assume that per capita hours worked enter the VAR model in growth rate, Christiano, Eichenbaum, and Vigfusson [2004] argue for using per capita hours in log-levels rather than in growth rates and find the opposite to Gali's result, say hours worked rise after a positive technology shock. We conducted unit root tests in panel data and find that all variables entering the VAR model are integrated of order one, see Technical Appendix G.1. Several papers have questioned Galí's identifying assumption that technology shocks are the only shocks that increase permanently labor productivity. First, Mertens and Ravn [2010] find that permanent changes in income tax rates induce permanent changes in hours worked as well as in labor productivity which leads to a violation of the standard long-run identification strategy for technology shocks. Second, Francis, Owyang, Rousch, DiCecio [2014] identify the technology shock as that associated with the maximum forecast-error variance share in labor productivity at a long, finite horizon, and find that hours worked decline. One advantage of this method is that it let other shocks influence labor productivity after a certain horizon of time. Like Chang and Hong [2006], we measure technological change with TFP and this measure should mitigate the effects of other shocks. Finally, Basu, Fernald and Kimball [2006] find that when technology improves, utilization falls so that TFP initially rises less than technology does. The authors construct a measure of aggregate technological change controlling for varying utilization of capital and labor. To adjust the annual Solow residual for the utilization of inputs, they use observed hours per worker as a proxy. While we have the data to estimate growth in factor utilization for each sector/country in our sample, the estimation equation (see equation 18 in Basu et al. [2006]) requires the use of instruments since there is a potential correlation between input growth and the standard Solow residual which we would lead us too far. Reassuringly, since we focus on the ratio of sectoral TFP, not adjusting sectoral TFP time series for factor utilization should not pose a problem. Chodorow-Reich, Karabarbounis, and Kekre [2019] have estimated the utilization rate for tradables and non-tradables by using Greek data and find that the movements are highly correlated.

## B Sectoral Decomposition of Aggregate TFP

We consider an open economy which produces domestic traded goods, denoted by a superscript $H$, and non-traded goods, denoted by a superscript $N$. The foreign-produced traded good is the numeraire and its price is normalized to 1 . We consider an initial steady-state where prices are those at the base year so that initially real GDP, denoted by $Y_{R}$, and the value added share at constant prices, denoted by $\nu^{Y, j}$, collapse to nominal GDP (i.e., $Y$ ) and the value added share at current prices, respectively.

Summing value added at constant prices across sectors gives real GDP:

$$
\begin{equation*}
Y_{R, t}=P^{H} Y_{t}^{H}+P^{N} Y_{t}^{N}, \tag{55}
\end{equation*}
$$

where $P^{H}$ and $P^{N}$ stand for the price of home-produced traded goods and non-traded goods, respectively, which are kept fixed since we consider value added at constant prices.

Log-linearizing (55), and denoting the percentage deviation from initial steady-state by a hat leads to:

$$
\begin{equation*}
\hat{Y}_{R, t}=\nu^{Y, H} \hat{Y}_{t}^{H}+\left(1-\nu^{Y, H}\right) \hat{Y}_{t}^{N} \tag{56}
\end{equation*}
$$

where $\nu^{Y, H}=\frac{p^{H} Y^{H}}{Y}$ is the value added share of home-produced traded goods evaluated at the initial steady-state. We drop the time index below as long as it does not cause confusion.

Capital $K^{j}$ can be freely reallocated across sectors while labor $L^{j}$ is subject to mobility costs which creates a sectoral wage differential. We denote the capital rental cost by $R$ and the wage rate in sector $j$ by $W^{j}$ (with $j=H, N$ ). Under assumption of perfect competition in product and input markets, factors of production are paid their marginal product in both sectors:

$$
\begin{align*}
P^{j} \frac{\partial Y^{j}}{\partial L^{j}} & =W^{j}  \tag{57a}\\
P^{j} \frac{\partial Y^{j}}{\partial K^{j}} & =R \tag{57b}
\end{align*}
$$

Assuming constant returns to scale in production and making use of (57), the log-linearized versions of the production function reads:

$$
\begin{equation*}
\hat{Y}^{j}=\hat{Z}^{j}+s_{L}^{j} \hat{L}^{j}+\left(1-s_{L}^{j}\right) \hat{K}^{j}, \tag{58}
\end{equation*}
$$

where $s_{L}^{j}$ and $Z^{j}$ are the labor income share and TFP in sector $j$, respectively.
Using the fact that $W L=W^{H} L^{H}+W^{N} L^{N}$, and $R K=R K^{H}+R K^{N}$, dividing both sides of these identities by GDP enables us to express the aggregate labor income share, $s_{L}$, and capital income share, $1-s_{L}$, as a weighted sum of sectoral factor income shares:

$$
\begin{gather*}
s_{L}=\nu^{Y, H} s_{L}^{H}+\left(1-\nu^{Y, H}\right) s_{L}^{N}  \tag{59a}\\
1-s_{L}=\nu^{Y, H}\left(1-s_{L}^{H}\right)+\left(1-\nu^{Y, H}\right)\left(1-s_{L}^{N}\right) \tag{59b}
\end{gather*}
$$

Since we assume perfect capital mobility, the resource constraint for capital reads as follows $K=$ $K^{H}+K^{N}$. Totally differentiating, multiplying both sides by the capital rental cost $R$, and dividing by GDP leads to:

$$
\begin{equation*}
\left(1-s_{L}\right) \hat{K}=\nu^{Y, H}\left(1-s_{L}^{H}\right) \hat{K}^{H}+\left(1-\nu^{Y, H}\right)\left(1-s_{L}^{N}\right) \hat{K}^{N} . \tag{60}
\end{equation*}
$$

The same logic applies to labor except that we assume imperfect mobility of labor across sectors. In this case, the percentage deviation of total hours worked relative to its initial steady-state is defined as the weighted sum of the percentage deviation of sectoral hours worked relative to initial steady-state, i.e., $\hat{L}=\alpha_{L} \hat{L}^{H}+\left(1-\alpha_{L}\right) \hat{L}^{N}$, where $\alpha_{L}=\frac{W^{H} L^{H}}{W L}$ is the labor compensation share for tradables. Multiplying both sides by total compensation of employees, $W L$, and dividing by GDP leads to:

$$
\begin{equation*}
s_{L} \hat{L}=\nu^{Y, H} s_{L}^{H} \hat{L}^{H}+\left(1-\nu^{Y, H}\right) s_{L}^{N} \hat{L}^{N} . \tag{61}
\end{equation*}
$$

Inserting (58) into (56):

$$
\begin{aligned}
\hat{Y}_{R}= & {\left[\nu^{Y, H} \hat{Z}^{H}+\left(1-\nu^{Y, H}\right) \hat{Z}^{N}\right]+\left[\nu^{Y, H} s_{L}^{H} \hat{L}^{H}+\left(1-\nu^{Y, H}\right) s_{L}^{N} \hat{L}^{N}\right] } \\
& +\left[\nu^{Y, H}\left(1-s_{L}^{H}\right) \hat{K}^{H}+\left(1-\nu^{Y, H}\right)\left(1-s_{L}^{N}\right) \hat{K}^{N}\right] .
\end{aligned}
$$

Next plugging (60) and (61) into the above equation and denoting aggregate TFP by $Z^{A}$ leads to:

$$
\begin{equation*}
\hat{Y}_{R}=\hat{Z}^{A}+s_{L} \hat{L}+\left(1-s_{L}\right) \hat{K}, \tag{62}
\end{equation*}
$$

where we set

$$
\begin{equation*}
\hat{Z}^{A}=\nu^{Y, H} \hat{Z}^{H}+\left(1-\nu^{Y, H}\right) \hat{Z}^{N} . \tag{63}
\end{equation*}
$$

Eq. (63) corresponds to eq. (1) in the main text.

## C Construction of Sectoral Shares

In this section, we provide more details about the construction of sectoral shares. Dropping the country index $i$, in an economy where labor is imperfectly mobile across sectors, the percentage deviation of total hours worked relative to its initial steady-state (i.e., $\hat{L}_{t}$ ) following a technology shock is equal to the weighted sum of the percentage deviation of sectoral hours worked relative to initial steady-state (i.E., $\hat{L}_{t}^{j}$ ):

$$
\begin{equation*}
\hat{L}_{t}=\alpha_{L} \hat{L}_{t}^{H}+\left(1-\alpha_{L}\right) \hat{L}_{t}^{N} . \tag{64}
\end{equation*}
$$

where $\alpha_{L}$ is the labor compensation share of tradables. If we subtract the share of higher total hours worked received by each sector from the change in sectoral hours worked, we obtain the change in the labor share of sector $j$, denoted by $\nu^{L, j}$, which measures the contribution of the reallocation of labor across sectors to the change in hours worked in sector $j:^{42}$

$$
\begin{equation*}
d \nu_{t}^{L, j}=\alpha_{L}^{j} \cdot\left(\hat{L}_{t}^{j}-\hat{L}_{t}\right) \quad j=H, N . \tag{65}
\end{equation*}
$$

The differential between the responses of sectoral and total hours worked on the RHS of eq. (65) can be viewed as the change in labor in sector $j$ if $L$ remained fixed and thus reflects higher employment in this sector resulting from the reallocation of labor. Eq. (65) corresponds to eq. (5) in the main text.

If we subtract the share of higher real GDP received by each sector from the change in sectoral value added in GDP units, we obtain the change in the value added share at constant prices of sector $j$, denoted by $\nu_{t}^{Y, j}$, which reads as follows:

$$
\begin{equation*}
d \nu_{t}^{Y, j}=\nu^{Y, j}\left(\hat{Y}_{t}^{j}-\hat{Y}_{R, t}\right) \tag{66}
\end{equation*}
$$

where $Y_{R}$ is real GDP. A rise in the value added share at constant prices of sector $j$ can be brought about by a high productivity growth relative to average, and/or a labor inflow, and/or a greater capital intensity. Formally, the decomposition of the change in the value added share of sector $j$ reads:

$$
\begin{equation*}
d \nu_{t}^{Y, j}=\nu^{Y, j}\left[\left(\hat{Z}_{t}^{j}-\hat{Z}_{t}^{A}\right)+\left(\hat{L}_{t}^{j}-\hat{L}_{t}\right)+\left(1-s_{L}^{j}\right)\left(\hat{k}_{t}^{j}-\hat{k}_{t}\right)\right], \tag{67}
\end{equation*}
$$

where $Z^{A}$ is aggregate TFP growth defined by eq. (63) and $s_{L}^{j}$ is the LIS in sector $j ; k^{j}=K^{j} / L^{j}$ stands for the capital-labor ratio in sector $j$ and $k=K / L$ is the aggregate capital-labor ratio where $K=K^{H}+K^{N}$ and $L=L\left(L^{H}, L^{N}\right)$ (since we assume IML and sectoral hours worked are aggregated by means of a CES function). Eq. (67) corresponds to eq. (6) in the main text.

To obtain (67), we proceed as follows. First, the percentage change in real GDP is a weighted sum of the percentage change in sectoral value added at constant prices: $\hat{Y}_{R}=\nu^{Y, H} \hat{Y}^{H}+\left(1-\nu^{Y, H}\right) \hat{Y}^{N}$. Subtracting the percentage change in real GDP from both sides, changes in sectoral value added shares cancel out:

$$
\begin{equation*}
0=\nu^{Y, H}\left(\hat{Y}_{t}^{H}-\hat{Y}_{R, t}\right)+\left(1-\nu^{Y, H}\right)\left(\hat{Y}_{t}^{N}-\hat{Y}_{R, t}\right)=d \nu_{t}^{Y, H}+d \nu_{t}^{Y, N} \tag{68}
\end{equation*}
$$

Second, we use the fact that the percentage change in real GDP and the percentage change in sectoral value added can be rewritten as $\hat{Y}_{R}=\hat{Z}^{A}+\hat{L}+\left(1-s_{L}\right) \hat{k}$ and $\hat{Y}^{j}=\hat{Z}^{j}+\hat{L}^{j}+\left(1-s_{L}^{j}\right) \hat{k}^{j}$, respectively. Inserting these equations into the sectoral decomposition of the percentage change in real GDP and making use of (59b), we find that:

$$
\begin{align*}
0= & \nu^{Y, H}\left[\left(\hat{Z}_{t}^{H}-\hat{Z}_{t}^{A}\right)+\left(\hat{L}_{t}^{H}-\hat{L}_{t}\right)+\left(1-s_{L}^{H}\right)\left(\hat{k}_{t}^{H}-\hat{k}_{t}\right)\right] \\
& +\left(1-\nu^{Y, H}\right)\left[\left(\hat{Z}_{t}^{N}-\hat{Z}_{t}^{A}\right)+\left(\hat{L}_{t}^{N}-\hat{L}_{t}\right)+\left(1-s_{L}^{N}\right)\left(\hat{k}_{t}^{N}-\hat{k}_{t}\right)\right] \tag{69}
\end{align*}
$$

From (69) and (68), we have (67).
Because we assume perfect mobility of capital across sectors, we have $K=K^{H}+K^{N}$. Loglinearizing the resource constant for capital and denoting $\alpha_{K}=R K^{H} / R K=K^{H} / K$ the share of traded capital into the aggregate capital stock, leads to:

$$
\begin{equation*}
\hat{K}_{t}=\alpha_{K} \hat{K}_{t}^{H}+\left(1-\alpha_{K}\right) \hat{K}_{t}^{N} . \tag{70}
\end{equation*}
$$

Subtracting (64) from (70) and assuming that $\alpha_{K} \simeq \alpha_{L}$ leads to:

$$
\hat{K}_{t}-\hat{L}_{t}=\hat{k}_{t}=\alpha_{L} \hat{k}_{t}^{H}+\left(1-\alpha_{L}\right) \hat{k}_{t}^{N}
$$

[^29]where $k^{j}=K^{j} / L^{j}$. Subtracting $\hat{k}_{t}$ from $\hat{k}_{t}^{H}$ by using the above equation leads to:
\[

$$
\begin{equation*}
\hat{k}_{t}^{H}-\hat{k}_{t}=\left(1-\alpha_{L}\right)\left(\hat{k}_{t}^{H}-\hat{k}_{t}^{N}\right) . \tag{71}
\end{equation*}
$$

\]

Assumption $\alpha_{K} \simeq \alpha_{L}$ amounts to assuming that the LIS in sector $j$ is close to the aggregate LIS which is defined as a value added weighted average of sectoral LIS. For the whole sample, we have $s_{L}^{H}=0.63$ and $s_{L}^{N}=0.69$ while the aggregate LIS stands at 0.66 which makes assumption $\alpha_{K} \simeq \alpha_{L}$ reasonable.

## D Data Description for Empirical Analysis

Coverage: Our sample consists of a panel of 17 countries: Australia (AUS), Austria (AUT), Belgium (BEL), Canada (CAN), Denmark (DNK), Finland (FIN), France (FRA), Germany (DEU), Ireland (IRL), Italy (ITA), Japan (JPN), the Netherlands (NLD), Norway (NOR), Spain (ESP), Sweden (SWE), the United Kingdom (GBR) and the United States (USA). The baseline period is running from 1970 to 2013, except for Japan (1974-2013). Although sectoral data are available over the period 1970-2015 (see below), our preferred time span is 1970-2013. The reason is that all quantity variables entering the VAR model are scaled by the working age population for which data are spotty for last years, making it impractical to work with it for periods that extend after 2013.

Sources: Our primary sources for sectoral data are the OECD and EU KLEMS databases. We use data from EU KLEMS ([2011], [2017]) March 2011 and July 2017 releases. The EU KLEMS dataset covers all countries of our sample, with the exceptions of Canada and Norway. For these two countries, sectoral data are taken from the Structural Analysis (STAN) database provided by the OECD ([2011], [2017]). For both EU KLEMS and STAN databases, the March 2011 release provides data for eleven 1-digit ISIC-rev. 3 industries over the period 1970-2007 while the July 2017 release provides data for thirteen 1-digit-rev. 4 industries over the period 1995-2013.

The construction of time series for sectoral variables over the period 1970-2013 involves two steps. First, we identify tradable and non-tradable sectors. To do so, we adopt the classification proposed by De Gregorio et al. [1994]. Following Jensen and Kletzer [2006], we have updated this classification by treating the financial sector as a traded industry. We map the ISIC-rev. 4 classification into the ISIC-rev. 3 classification in accordance with the concordance Table 7. Once industries have been classified as traded or non-traded, for any macroeconomic variable $X$, its sectoral counterpart $X^{j}$ for $j=H, N$ is constructed by adding the $X_{k}$ of all sub-industries $k$ classified in sector $j=H, N$ as follows $X^{j}=\sum_{k \in j} X_{k}$. Second, series for tradables and non-tradables variables from EU KLEMS [2011] and OECD [2011] databases (available over the period 1970-2007) are extended forwards up to 2013 using annual growth rate estimated from EU KLEMS [2017] and OECD [2017] series (available over the period 1995-2013).

Table 7: Summary of Sectoral Classifications

| Sector | ISIC-rev.4 Classification(sources: EU KLEMS [2017] and OECD ([2017]) |  | ISIC-rev. 3 Classification |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Industry | Code | Industry | Code |
| Tradables $(H)$ | Agriculture, Forestry and Fishing Mining and Quarrying <br> Total Manufacturing <br> Transport and Storage <br> Information and Communication <br> Financial and Insurance Activities | $\begin{gathered} \hline \mathrm{A} \\ \mathrm{~B} \\ \mathrm{C} \\ \mathrm{H} \\ \mathrm{~J} \\ \mathrm{~K} \end{gathered}$ | Agriculture, Hunting, Forestry and Fishing Mining and Quarrying <br> Total Manufacturing Transport, Storage and Communication <br> Financial Intermediation | $\begin{gathered} \text { AtB } \\ \text { C } \\ \text { D } \\ \text { I } \end{gathered}$ |
| Non <br> Tradables $(N)$ | Electricity, Gas and Water Supply Construction <br> Wholesale and Retail Trade, Repair of Motor Vehicles and Motorcycles Accommodation and Food Service Activities Real Estate Activities Professional, Scientific, Technical, Administrative and Support Service Activities Community Social and Personal Services | D-E F G I L M-N O-U | Electricity, Gas and Water Supply Construction <br> Wholesale and Retail Trade <br> Hotels and Restaurants <br> Real Estate, Renting and Business Services <br> Community Social and Personal Services | E F <br> G <br> H <br> K <br> LtQ |

Relevant to our work, the EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases provide data, for each industry and year, on value added at current and constant prices, permitting the construction of sectoral deflators of value added, as well as details on labor compensation and hours worked data, allowing the construction of sectoral wage rates. All quantity variables are scaled by the working age population (15-64 years old). Source: OECD ALFS Database for the working age population (data coverage: 1970-2013). We describe below the construction for the sectoral data employed in the main text (mnemonics are given in parentheses):

- Sectoral value added, $Y^{j}$ : sectoral value added at constant prices in sector $j=H, N$ (VA_QI). Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases.
- Relative value added, $Y^{H} / Y^{N}$ : ratio of traded value added to non-traded value added at constant prices.
- Sectoral value added share, $\nu^{Y, j}$ : ratio of value added at constant prices in sector $j$ to GDP at constant prices, i.e., $Y^{j} /\left(Y^{H}+Y^{N}\right)$ for $j=H, N$.
- Relative price of non-tradables, $P$ : ratio of the non-traded value added deflator to the traded value added deflator, i.e., $P=P^{N} / P^{H}$. The sectoral value added deflator $P^{j}$ for sector $j=H, N$ is calculated by dividing value added at current prices (VA) by value added at constant prices (VA_QI) in sector $j$. EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases.
- Terms of Trade, $P^{H} / P^{F}$ : ratio of the traded value added deflator to price deflator of imports of goods and services, i.e., $P^{H} / P^{F}$. The traded value added deflator $P^{H}$ is calculated by dividing value added at current prices (VA) by value added at constant prices (VA_QI) in sector $H$. Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) for $P^{H}$ and OECD National Accounts Database for $P^{F}$.
- Sectoral hours worked, $L^{j}$ : total hours worked by persons engaged in sector $j$ (H_EMP).EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases.
- Relative hours worked, $L^{H} / L^{N}$ : ratio of hours worked in the traded sector to hours worked in the non-traded sector.
- Sectoral labor share, $\nu^{L, j}$ : ratio of hours worked in sector $j$ to total hours worked, i.e., $L^{j} /\left(L^{H}+L^{N}\right)$ for $j=H, N$.
- Sectoral real consumption wage, $W_{C}^{j}$ : nominal wage in sector $j$ divided by the consumer price index (CPI), i.e. $W_{C}^{j}=W^{j} / P_{C}$. Source: OECD Prices and Purchasing Power Parities for the consumer price index. The sectoral nominal wage $W^{j}$ for sector $j=H, N$ is calculated by dividing labor compensation in sector $j$ (LAB) by total hours worked by persons engaged (H_EMP) in that sector. Labor compensation is total labor costs that include compensation of employees and labor income of the self-employed and other entrepreneurs. Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases.
- Relative wage: ratio of the nominal wage in the sector $j$ to the aggregate nominal wage $W$, i.e., $W^{j} / W$.
- Labor income share (LIS), $s_{L}^{j}$ : ratio of labor compensation in sector $j=H, N$ (LAB) to value added at current prices (VA) of that sector. Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases.
- Capital-labor ratio, $k^{j}$ : ratio of capital stock in sector $j=H, N$ to total hours worked by persons engaged in that sector (H_EMP). Aggregate capital stocks are estimated from the perpetual inventory approach by using real gross capital formation from OECD Economic Outlook Database (data in millions of national currency, constant prices) and assuming a depreciation rate of $5 \%$. Following Garofalo and Yamarik [2002], the capital stock is then allocated to traded and non-traded industries by using sectoral output shares $K^{j}=\omega^{Y, j} K$ where $\omega^{Y, j}$ is the value added share of sector $j$ at current prices, see Appendix G.7. Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases.
- Relative productivity of tradables, $Z$ : labor share-adjusted ratio of traded $\mathrm{TFP} Z^{H}$ to the non-traded TFP $Z^{N}$, i.e. $Z=\left(Z^{H}\right)^{a} /\left(Z^{N}\right)^{b}$ where $a=\left[\left(1-\alpha_{J}\right)+\alpha_{J} \frac{s_{L}^{H}}{s_{L}^{N}}\right]^{-1}$, and $b=a \frac{s_{L}^{H}}{s_{L}^{N}}$. Sectoral TFPs $Z^{j}$ for $j=H, N$ are constructed as Solow residuals from constantprice domestic currency series of value added (VA_QI), capital, LIS $s_{L}^{j}$, and hours worked (H_EMP) in sector $j . s_{L}^{j}$ is the ratio of the compensation of employees (LAB) to value added (VA) in sector $j=H, N$, averaged over the period 1970-2013 (except Japan: 1974-2013). Sources: EU EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases. $\alpha_{J}$ is the tradable share in total investment expenditure averaged over the period 1970-2013. Source: OECD Input-Output database [2017].
In the following, we provide details on data construction for aggregate variables (mnemonics are in parentheses):
- Gross domestic product, $Y_{R}$ : real gross domestic product (GDPV). Source: OECD Economic Outlook Database. Data coverage: 1970-2013.
- Total hours worked, $L$ : total hours worked by persons engaged (H_EMP). Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases.
- Real consumption wage, $W_{C}=W / P_{C}$ : nominal aggregate wage divided by the consumer price index (CPI). Source: OECD Prices and Purchasing Power Parities for the consumer price index. The nominal aggraget wage is calculated by dividing labor compensation (LAB) by total hours worked by persons engaged (H_EMP). Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases.


## E Data for Calibration

## E. 1 Non-Tradable Content of GDP and its Components

Table 4 shows the non-tradable content of GDP, consumption, investment, government spending, labor and labor compensation (columns 1 to 6 ). In addition, it gives information about the sectoral labor income shares (columns 10 and 11). The home content of consumption and investment expenditure in tradables together with the ratio of final goods imports to GDP are reported in columns 7 to 9 . Columns 12 to 14 display the labor income share, investment-to-GDP ratio and government spending in \% of GDP, respectively, for the whole economy. Our sample covers the 17 OECD countries mentioned in Section B. Our reference period for the calibration corresponds to the period 1970-2013. The choice of this period has been dictated by data availability. In the following, statistics for the sample as a whole represent (unweighted) averages of the corresponding variables.

To calculate the non-tradable share of value added (column 1), labor (column 5) and labor compensation (column 6), we split the eleven industries into traded and non-traded sectors by adopting the classification proposed by De Gregorio et al. [1994] and updated by Jensen and Kletzer [2006]. Details about data construction for sectoral output and sectoral labor are provided above. We calculate the non-tradable share of labor compensation as the ratio of labor compensation in the non-traded sector (i.e., $W^{N} L^{N}$ ) to overall labor compensation (i.e., $W L$ ). Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases. Data coverage: 1970-2013 for all countries (except Japan: 1974-2013). The non-tradable content of GDP, labor and labor compensation, shown in columns 1, 5 and 6 of Table 4, average to $60 \%, 63 \%$ and $63 \%$ respectively.

To split consumption expenditure (at current prices) into consumption in traded and non-traded goods, we made use of the Classification of Individual Consumption by Purpose (COICOP) published by the United Nations (Source: United Nations [2017]). Among the twelve items, the following ones are treated as consumption in traded goods: "Food and Non-Alcoholic Beverages", "Alcoholic Beverages Tobacco and Narcotics", "Clothing and Footwear", "Furnishings, Household Equipment and Routine Maintenance of the House" and "Transport". The remaining items are treated as consumption in non-traded goods: "Housing, Water, Electricity, Gas and Fuels", "Health", "Communications", "Recreation and Culture", "Education", "Restaurants and Hotels". Because the item "Miscellaneous Goods and Services" is somewhat problematic, we decided to consider it as both tradable ( $50 \%$ ) and non-tradable ( $50 \%$ ) with equal shares. Data coverage: AUS (1970-2013), AUT (1995-2013), BEL (1995-2013), CAN (1981-2013), DEU (1995-2013), DNK (1970-2013), ESP (19952013), FIN (1975-2013), FRA (1970-2013), GBR (1995-2013), IRL (1995-2013), ITA (1995-2013), JPN (1994-2013), NLD (1995-2013), NOR (1970-2013), SWE (1993-2013) and USA (1970-2013). The non-tradable share of consumption shown in column 2 of Table 4 averages to $53 \%$.

To calculate the non-tradable share of investment expenditure, we follow the methodology proposed by Burstein et al. [2004] who treat "Total Construction" as non-tradable investment and "Transport Equipment", "ICT Equipment", "Cultivated Biological Resources", "Intellectual Property Product" as tradable investment expenditure. The item "Other machinery and equipment and weapon system" is considered as both tradable ( $50 \%$ ) and non-tradable ( $50 \%$ ) with equal shares. Source: OECD Input-Output database [2017]. Data coverage: AUS (1970-2013), AUT (1995-2013), BEL (1995-2013), CAN (1970-2013), DEU (1995-2013), DNK (1970-2013), ESP (1995-2013), FIN (1980-2013), FRA (1978-2013), GBR (1997-2013), IRL (1995-2013), ITA (1995-2013), JPN (19942013), NLD (1995-2013), NOR (1970-2013), SWE (1993-2013) and USA (1970-2013). non-tradable share of investment shown in column 3 of Table 4 averages to $62 \%$, in line with estimates provided by Burstein et al. [2004] and Bems [2008].

Sectoral government expenditure data (at current prices) are taken from the OECD General Government Accounts database (Source: COFOG, OECD [2017]). The following four items pertaining to "Economic Affairs" are treated as traded: "Fuel and Energy", "Agriculture, Forestry, Fishing, and Hunting", "Mining, Manufacturing, and Construction", "Transport and Communications". Items treated as non-traded are: "General Public Services", "Defence", "Public Order and Safety", "Environment Protection", "Housing and Community Amenities", "Health", "Recreation, Culture and Religion", "Education" and "Social Protection". Data coverage: AUS (1998-2013),

AUT (1995-2013), BEL (1995-2013), DEU (1995-2013), DNK (1995-2013), ESP (1995-2013), FIN (1990-2013), FRA (1995-2013), GBR (1995-2013), IRL (1995-2013), ITA (1995-2013), JPN (20052013), NLD (1995-2013), NOR (1995-2013), SWE (1995-2013) and USA (1970-2013). Data are not available for CAN. Thus, for this country, when we calibrate the model to each OECD country, we choose a non-tradable content of government expenditure that is given by the unweighed average, i.e., 0.90 , as can be seen in column 4 of Table 4.

To compute the home content of consumption and investment expenditure in tradables, we use the Comtrade database from the United Nations. There are three basic classes of goods in SNA in the categories of classification of Broad Economic Categories (BEC): capital goods, intermediate goods and consumption goods. Since we focus on sectoral value added and its final use, we exclude intermediate goods. The sum of capital and consumption goods imports as a share of GDP averages $10.4 \%$ as can be seen in column 7 of Table 4 . When we calibrate the model to a representative OECD economy, we assume that trade is initially balanced. This assumption is roughly consistent with the data which indicate that exports of consumption and capital goods as a share of GDP average $10.8 \%$. Excluding trade on intermediate goods, the Comtrade database enables us to construct time series for the content of imports in consumption goods, $C^{F} / M^{F}$, and investment goods, $J^{F} / M^{F}$. Since we can compute consumption and investment goods as a share of GDP, i.e., $C^{F} / Y$ and $J^{F} / Y$, we can determine the import content of consumption and investment expenditure in tradables, by using the following decomposition:

$$
\begin{align*}
1-\alpha^{H} & =\frac{C^{F}}{P^{H} C^{H}}=\frac{C^{F}}{Y} \frac{1}{\omega_{C} \alpha_{C}}  \tag{72a}\\
1-\alpha_{J}^{H} & =\frac{J^{F}}{P_{J}^{H} J^{H}}=\frac{J^{F}}{Y} \frac{1}{\omega_{J} \alpha_{J}} \tag{72b}
\end{align*}
$$

where $\omega_{C}=1-\omega_{J}-\omega_{G}$ with $\omega_{J}$ and $\omega_{G}$ shown in columns 13 and 14 of Table 4 ; the tradable content of consumption expenditure, $\alpha_{C}$, can be calculated by using column 2 which gives $1-\alpha_{C}$. Once we have computed $1-\alpha^{H}$ and $1-\alpha_{J}^{H}$, we can compute the home content of consumption and investment expenditure in tradables which are shown in columns 8 and 9 . The home content of consumption expenditure in tradables, $\alpha^{H}$, averages $77 \%$ while the home content of investment expenditure in tradables, $\alpha_{J}^{H}$, averages $51 \%$. Source: United Nations Comtrade database [2017]. Data coverage: 1998-2013 for all countries.

The labor income share for sector $j$ denoted by $s_{L}^{j}$ is calculated as the ratio of labor compensation of sector $j$ to value added of sector $j$ at current prices. Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases. Data coverage: 1970-2013 for all countries (except Japan: 1974-2013). As shown in columns 10 and 11 of Table $4, s_{L}^{H}$ and $s_{L}^{N}$ average 0.63 and 0.68 , respectively.

Column 12 of Table 4 gives the aggregate labor income share which averages 0.66 in our sample. Columns 13 and 14 of Table 4 display gross capital formation and final consumption expenditure of general government as a share of GDP, respectively. Source: OECD National Accounts Database. Data coverage: 1970-2013 for all countries.

Columns from 15 to 19 of Table 4 display estimates of the elasticity of substitution between tradables and non-tradables in consumption, $\phi$, the elasticity of labor supply across sectors, $\epsilon$, the elasticity of substitution between capital and labor in the traded and the non-traded sector, i.e., $\sigma^{H}$ and $\sigma^{N}$, respectively, the elasticity of exports w.r.t. the terms of trade, $\phi_{X}$. In subsections E. 4 and E.5, we detail the empirical strategy to estimate these parameters, except for the price elasticity of exports shown in the last column of Table 4 whose estimates are taken from Imbs and Mejean [2015].

Because data source and construction are heterogenous across variables as a result of different nomenclatures, Table 8 provides a summary of the classification adopted to split value added and its demand components as well into traded and non-traded goods.

## E. 2 Response of Hours Worked to Asymmetric and Symmetric Technology Shocks across Sectors

We explore empirically below the response of total hours worked to the asymmetric technology shock because this variable receives a lot of attention in the literature pioneered by Gali [1999]. We consider the VAR model which includes aggregate variables such a real GDP, total hours worked, the real consumption wage in addition to the productivity differential ordered first. Interestingly, a shock to the ratio of traded to non-traded TFP increases significantly hours worked. While it is beyond the scope of this article, we estimated the response of hours worked to a symmetric technology shock across sectors (i.e., a shock to $Z^{A}$ leaving unchanged the ratio $Z^{H} / Z^{N}$ and we find empirically that hours worked decline substantially, see Fig. 10(b). The discrepancy in the response of hours worked between symmetric and asymmetric technology shocks is caused by the reallocation incentives we focus on in this work. While a technology shock biased toward the traded
Table 8: Construction of Variables and Data Sources

| Variable | Countries covered | Period | Construction and aggregation | Database |
| :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & \begin{array}{l} \text { Value added } Y^{H} \\ \text { (constant prices) } \end{array} \& Y^{N} \\ & \hline \end{aligned}$ | AUS, AUT, BEL, CAN, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, NLD, NOR, SWE, USA | $\begin{gathered} \hline 1970-2013 \\ \text { (JPN: 74-13) } \\ \hline \end{gathered}$ | T: Agriculture, Mining, Manufacturing, Transport, Finance Intermediation $N$ : Electricity, Construction, Trade, Hotels, Real Estate, Personal Services | $\begin{gathered} \text { EU KLEMS } \\ \& \text { STAN } \\ \hline \end{gathered}$ |
| $\begin{aligned} & \text { Value added } P^{H} Y^{H} \& P^{N} Y^{N} \\ & \text { (current prices) } \end{aligned}$ | AUS, AUT, BEL, CAN, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, NLD, NOR, SWE, USA | $\begin{gathered} \text { 1970-2013 } \\ \text { (JPN: 74-13) } \\ \hline \end{gathered}$ | T: Agriculture, Mining, Manufacturing, Transport, Finance Intermediation $N$ : Electricity, Construction, Trade, Hotels, Real Estate, Personal Services | $\begin{gathered} \text { EU KLEMS } \\ \text { \& STAN } \\ \hline \end{gathered}$ |
| Labor $L^{H} \& L^{N}$ (total hours worked by persons engaged) | AUS, AUT, BEL, CAN, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, NLD, NOR, SWE, USA | $\begin{gathered} 1970-2013 \\ \text { (JPN: 74-13) } \\ \hline \end{gathered}$ | T: Agriculture, Mining, Manufacturing, Transport, Finance Intermediation N: Electricity, Construction, Trade, Hotels, Real Estate, Personal Services | $\begin{aligned} & \text { EU KLEMS } \\ & \& \text { STAN } \\ & \hline \end{aligned}$ |
| Labor compensation $L A B^{H}$ $\& L A B^{N}$ (current prices) | AUS, AUT, BEL, CAN, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, NLD, NOR, SWE, USA | $\begin{gathered} 1970-2013 \\ \text { (JPN: 74-13) } \\ \hline \end{gathered}$ | T: Agriculture, Mining, Manufacturing, Transport, Finance Intermediation $N$ : Electricity, Construction, Trade, Hotels, Real Estate, Personal Services | $\begin{gathered} \text { EU KLEMS } \\ \text { \& STAN } \\ \hline \end{gathered}$ |
| Relative Output $Y^{H} / Y^{N}$ | AUS, AUT, BEL, CAN, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, NLD, NOR, SWE, USA | $\begin{gathered} \text { 1970-2013 } \\ \text { (JPN: 74-13) } \\ \hline \end{gathered}$ | Value added at constant prices in tradables $\left(Y^{H}\right)$ over value added at constant prices in non-tradables $\left(Y^{N}\right)$ | authors' calculations |
| $\begin{aligned} & \text { Output Share } \\ & \nu^{Y, H} \& \nu^{Y, N} \\ & \hline \end{aligned}$ | AUS, AUT, BEL, CAN, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, NLD, NOR, SWE, USA | $\begin{gathered} \text { 1970-2013 } \\ \text { (JPN: 74-13) } \\ \hline \end{gathered}$ | Value added at constant prices $\left(Y^{j}\right)$ over value added at constant prices in total economy $\left(Y^{H}+Y^{N}\right)$ | authors <br> calculations |
| Relative Labor $L^{H} / L^{N}$ | AUS, AUT, BEL, CAN, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, NLD, NOR, SWE, USA | $\begin{gathered} \text { 1970-2013 } \\ \text { (JPN: 74-13) } \\ \hline \end{gathered}$ | Total hours worked by persons engaged in tradables $\left(L^{H}\right)$ over total hours worked by persons engaged in non-tradables $\left(L^{N}\right)$ | $\begin{gathered} \text { authors' } \\ \text { calculations } \end{gathered}$ |
| $\begin{aligned} & \text { Labor Share } \\ & \nu^{L, H} \& \nu^{L, N} \\ & \hline \end{aligned}$ | AUS, AUT, BEL, CAN, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, NLD, NOR, SWE, USA | $\begin{gathered} \text { 1970-2013 } \\ \text { (JPN: 74-13) } \\ \hline \end{gathered}$ | Total hours worked by persons engaged ( $L^{j}$ ) over total hours worked by persons engaged in total economy ( $L^{H}+L^{N}$ ) | authors' calculations |
| $\begin{aligned} & \text { Price } P^{H} \& P^{N} \\ & \text { (value added deflator) } \end{aligned}$ | AUS, AUT, BEL, CAN, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, NLD, NOR, SWE, USA | $\begin{gathered} \text { 1970-2013 } \\ \text { (JPN: 74-13) } \\ \hline \end{gathered}$ | Value added at current prices ( $P^{j} Y^{j}$ ) over value added at constant prices ( $Y^{j}$ ) |  |
| Relative Price $P$ (index $1995=100$ ) | AUS, AUT, BEL, CAN, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, NLD, NOR, SWE, USA | $\begin{gathered} \text { 1970-2013 } \\ \text { (JPN: 74-13) } \\ \hline \end{gathered}$ | Value added deflator of non-traded goods $\left(P^{N}\right)$ over value added deflator of traded goods $\left(P^{H}\right)$ | $\begin{gathered} \text { authors' } \\ \text { calculations } \end{gathered}$ |
| $\begin{aligned} & \text { Wage } W^{H} \& W^{N} \\ & \text { (nominal and per hour) } \end{aligned}$ | AUS, AUT, BEL, CAN, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, NLD, NOR, SWE, USA | $\begin{gathered} 1970-2013 \\ \text { (JPN: 74-13) } \end{gathered}$ | Labor compensation ( $L A B^{j}$ ) over total hours worked by persons engaged ( $L^{j}$ ) | authors' calculations |
| Wage $W$ (nominal and per hour) | AUS, AUT, BEL, CAN, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, NLD, NOR, SWE, USA | $\begin{gathered} 1970-2013 \\ (J P N: ~ 74-13) \\ \hline \end{gathered}$ | Labor compensation ( $L A B$ ) over total hours worked by persons engaged ( $L$ ) |  |
| $\begin{aligned} & \text { Wage } W^{H} / P_{C} \& W^{N} / P_{C} \\ & \text { (real and per hour) } \end{aligned}$ | AUS, AUT, BEL, CAN, ESP, DNK, FIN, FRA, GBR, IRL, ITA, JPN, NLD, NOR, SWE, USA | $\begin{gathered} \text { 1970-2013 } \\ \text { (JPN: 74-13) } \end{gathered}$ | Nominal wage ( $W^{j}$ ) divided by the consumer price index ( $P_{C}$ ) | authors' calculations |
| $\begin{aligned} & \text { Wage } W / P_{C} \\ & \text { (real and per hour) } \end{aligned}$ | AUS, AUT, BEL, CAN, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, NLD, NOR, SWE, USA | $\begin{gathered} \text { 1970-2013 } \\ \text { (JPN: 74-13) } \\ \hline \end{gathered}$ | Nominal wage ( $W$ ) divided by the consumer price index ( $P_{C}$ ) | $\begin{gathered} \text { authors' } \\ \text { calculations } \end{gathered}$ |
| Relative Wage $\Omega$ (index $1995=100$ ) | AUS, AUT, BEL, CAN, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, NLD, NOR, SWE, USA | $\begin{gathered} \text { 1970-2013 } \\ \text { (JPN: } 74-13 \text { ) } \\ \hline \end{gathered}$ | Nominal wage in non-tradables ( $W^{N}$ ) over nominal wage in tradables ( $W^{H}$ ) |  |
| Labor income shares $s_{L}^{H} \& s_{L}^{N}$ | AUS, AUT, BEL, CAN, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, NLD, NOR, SWE, USA | $\begin{gathered} 1970-2013 \\ \text { (JPN: 74-13) } \\ \hline \end{gathered}$ | Labor compensation ( $L A B^{j}$ ) over value added at current prices ( $P^{j} Y^{j}$ ) | authors calculations |
| (constant prices) <br> Capital stock $K$ | AUS, AUT, BEL, CAN, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, NLD, NOR, SWE, USA | $\begin{gathered} 1970-2013 \\ \text { (JPN: 74-13) } \\ \hline \end{gathered}$ | Computed from the perpetual inventory approach using data of aggregate investment in constant prices (depreciation rate: 5\%) | authors' calculations |
| Sectoral capital stocks $K^{H} \& K^{N}$ | AUS, AUT, BEL, CAN, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, NLD, NOR, SWE, USA | $\begin{gathered} \text { 1970-2013 } \\ \text { (JPN: 74-13) } \\ \hline \end{gathered}$ | Computed from Garofalo and Yamarik [2002] using sectoral current prices output shares $\left(P^{j} Y^{j} /\left(P^{H} Y^{H}+P^{N} Y^{N}\right)\right)$ | authors' calculations |
| $\begin{aligned} & \text { Capital-labor ratios } \\ & k^{H} \& k^{N} \\ & \hline \end{aligned}$ | AUS, AUT, BEL, CAN, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, NLD, NOR, SWE, USA | $\begin{gathered} 1970-2013 \\ \text { (JPN: 74-13) } \\ \hline \end{gathered}$ | Capital stock ( $K^{j}$ ) over total hours worked by persons engaged ( $L^{j}$ ) | authors' calculations |
| $\begin{aligned} & \text { Sectoral TFPs } Z^{H} \& Z^{N} \\ & \text { (index } 1995=100 \text { ) } \end{aligned}$ | AUS, AUT, BEL, CAN, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, NLD, NOR, SWE, USA | $\begin{gathered} 1970-2013 \\ \text { (JPN: } 74-13 \text { ) } \\ \hline \end{gathered}$ | Computed as Solow residuals from $\log Z^{j}=\log Y^{j}-s_{L}^{j} \log L^{j}-\left(1-s_{L}^{j}\right) \log K^{j}$ where is $s_{L}^{j}$ the labor share in value added averaged over 1970-2013 | $\begin{gathered} \text { authors' } \\ \text { calculations } \end{gathered}$ |
| Relative TFP Z | AUS, AUT, BEL, CAN, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, NLD, NOR, SWE, USA | $\begin{gathered} \text { 1970-2013 } \\ \text { (JPN: 74-13) } \end{gathered}$ | TFP in tradables $\left(Z^{H}\right)$ over TFP in non-tradables $\left(Z^{N}\right)$ adjusted by labor shares ( $\theta^{H}$ and $\theta^{N}$ ) and the tradable share in investment ( $\alpha_{J}$ ) | authors' calculations |
| Consumer Price Index $P_{C}$ (index $1995=100$ ) | AUS, AUT, BEL, CAN, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, NLD, NOR, SWE, USA | 1970-2013 | Consumer prices, all items index | OECD Prices |
| Gross domestic product $Y_{R}$ (constant prices) | AUS, AUT, BEL, CAN, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, NLD, NOR, SWE, USA | 1970-2013 | Gross domestic product (GDPV) | OECD Outlook |
| $\begin{aligned} & \text { Investment } \\ & P^{H} I^{H} \& P^{N} I^{N} \\ & \text { (current prices) } \\ & \hline \end{aligned}$ | AUS ( $70-13$ ), AUT ( $95-13$ ), BEL ( $95-13$ ), CAN ( $70-13$ ), ESP (95-13), FIN (80-13), FRA (78-13), GBR (97-13), JPN (94-13), NLD (95-13), NOR (70-13), SWE (93-13), | $\begin{aligned} & 5-13), \text { DNK }(70-13), \\ & 13), \text { ITA }(95-13) \\ & 0-13) \end{aligned}$ | T: Transport, ICT, Biological Resources, Property Product, Other $N$ : Construction, Other <br> (Other: defined as $50 \%$ tradable and $50 \%$ non-tradable) | OECD Input-Output |
| $\begin{aligned} & \text { Consumption } \\ & P^{H} C^{H} \& P^{N} C^{N} \\ & \text { (current prices) } \\ & \hline \end{aligned}$ | AUS (70-13), AUT (95-13), BEL (95-13), CAN (81-13), <br> ESP (95-13), FIN (75-13), FRA (70-13), GBR (95-13), <br> JPN (94-13), NLD (95-13), NOR (70-13), SWE (93-13), | $\begin{aligned} & 5-13), \text { DNK }(70-13), \\ & 13), \text { ITA }(95-13), \\ & 0-13) \end{aligned}$ | T: Food, Beverages, Clothing, Furnishings, Transport, Miscellaneous $N$ : Housing, Health, Communications, Recreation, Education, Restaurants and Miscellaneous (defined as $50 \%$ tradable and $50 \%$ non-tradable) | COICOP |
| $\begin{aligned} & \text { Government spending } \\ & P^{H} G^{H} \& P^{N} G^{N} \\ & \text { (current prices) } \\ & \hline \end{aligned}$ | AUS (98-13), AUT (95-13), BEL (95-13), DEU (95-13), FIN (90-13), FRA (95-13), GBR (95-13), IRL (95-13), NLD (95-13), NOR (95-13), SWE (95-13), USA (70-13 | 5-13), ESP (95-13), <br> 13), JPN (05-13), | T: Energy, Agriculture, Manufacturing, Transport <br> N: Public Services, Defence, Safety, Education, Health, Housing, Recreation Environment, Social Protection | COFOG |



Figure 10: Dynamic Adjustment of Hours Worked: Empirical vs. Theoretical IRF. Notes: Fig. 10(a) contrasts the empirical response of total hours worked shown in the blue line with the baseline model's prediction with FBTC displayed by the black line with squares. The dashed red line shows the theoretical response from the reference model for the calibration with Cobb-Douglas production functions and HNTC. Fig. 10(b) shown empirical responses of total hours worked to identified symmetric (black line) and asymmetric (blue line) technology shocks across sectors. Sample: 17 OECD countries, 1970-2013, annual data.
sector appreciates the relative price of non-tradables and has an expansionary effect on hiring by non-traded firms, a symmetric technology shock across sectors depreciates the relative price of nontradables which lowers the share of non-tradables in expenditure and thus exerts a negative impact on labor demand by non-traded firms. Since the labor share of non-tradables is two-third, more hiring in this sector increases total hours worked while less incentives to hire in this sector lower total hours worked. In this regard, the gross complementarity between traded and non-traded goods and the gross substitutability between home- and foreign-produced traded goods play a pivotal role in the response of total hours worked to aggregate technology shocks. In addition, as mentioned in the main text, aggregate technology shocks are a combination of asymmetric and symmetric technology shocks whose contribution varies over time, and thus the response of hours worked is most likely to increase over time because the contribution of asymmetric technology shocks increases.

Empirical and theoretical impulse response functions are contrasted in Fig. 10(a). Empirical responses are displayed by solid blue lines and theoretical responses from the baseline model with FBTC are displayed by the solid black lines with squares. The dashed red line shows model's predictions when we consider Cobb-Douglas production functions which correspond to the normalization point (since we normalize CES production functions by taking the steady-state in a Cobb-Douglas economy as the reference point). We set $\sigma_{L}$ to 1.6 in order to let the reference model with CobbDouglas production functions reproduce the impact response of total hours worked. While the impact response is almost identical, the baseline model reproduces well the dynamics of total hours worked while the model with Cobb-Douglas production functions somewhat understates the growing time profile of total hours worked.

## E. 3 Technology Shock Biased toward Tradables

Once the model has been calibrated to reproduce the key features of a representative OECD economy, we have to generate shocks to sectoral TFP which are in line with the data. To determine the dynamic adjustment of $Z^{j}$ following a long-run permanent increase in $\hat{Z}$ by $1 \%$, we first estimate the VAR model that includes government final consumption expenditure, real GDP, total hours worked, and the real consumption wage and identify technology shocks as shocks that increase permanently the weighted ratio of traded relative to non-traded TFP. Then, we consider a VAR model in panel format on annual data that includes identified technology shocks, $\epsilon_{i t}^{Z}$, ordered first, TFP in the traded sector, $Z_{i t}^{H}$, TFP in the non-traded sector, $Z_{i t}^{H}$, and the ratio of TFP, $Z_{i t}$, where all variables are measured in growth rate. We estimate the VAR model $x_{i, t}^{Z}=\left[\epsilon_{i t}^{Z}, \hat{Z}_{i t}^{H}, \hat{Z}_{i t}^{N}, \hat{Z}_{i t}\right]$ and adopt a Cholesky decomposition. While the weights $a$ and $b$ are assumed to be constant over time, we find a slight discrepancy in the estimated technology shock biased toward the traded sector because $\hat{Z}_{t}$ slightly differs from the weighted average $a \hat{Z}_{t}^{H}-b \hat{Z}_{t}^{N}$. We thus take the following route. We compute $\hat{Z}_{t}^{N}$ at various horizons by using the following formula $\hat{Z}_{t}^{N}=\frac{a Z_{t}^{H}-\hat{Z}_{t}}{b}$ so that the asymmetric technology shock is equal to the labor share-adjusted TFP differential at each point of time. It is worth mentioning that the difference between the actual and rescaled response of non-traded TFP is negligible.

To replicate the dynamics of sectoral TFP we estimate empirically, we assume that the adjustment of labor and capital efficiency follows a continuous time autoregressive path:

$$
\begin{equation*}
\hat{A}^{j}(t)-\hat{A}^{j}=\bar{a}^{j} e^{-\xi^{j} t}, \quad \hat{B}^{j}(t)-\hat{B}^{j}=\bar{b}^{j} e^{-\xi^{j} t} \tag{73}
\end{equation*}
$$



Figure 11: Dynamic Adjustment of Sectoral TFP following a $1 \%$ Permanent Increase in Traded relative to non-traded TFP: Empirical vs. Theoretical IRF. Notes: The empirical responses of TFP in the traded sector (i.e., $Z^{H}$ ) and non-traded sector (i.e., $Z^{N}$ ) to the identified (in the baseline VAR model) technology shock biased toward the traded sector are displayed by solid blue lines with shaded area indicating the 90 percent confidence bounds obtained by bootstrap sampling; the model's prediction is shown in the solid black line with squares. Sample: 17 OECD countries, 1970-2013, annual data.
where $\xi^{j}>0$ measures the speed at which productivity closes the gap with its long-run level. When parameters $\bar{a}^{j}$ or $\bar{b}^{j}$ take negative values, productivity undershoots its new steady-state value on impact. Log-linearizing the technology frontier (32) around the initial steady-state leads to:

$$
\begin{equation*}
\hat{Z}^{j}(t)=s_{L}^{j} \hat{A}^{j}(t)+\left(1-s_{L}^{j}\right) \hat{B}^{j}(t), \tag{74}
\end{equation*}
$$

where $s_{L}^{j}$ is the LIS in sector $j$ at the initial steady-state. Inserting (73) into (74) and using the fact the $\hat{Z}^{j}=s_{L}^{j} \hat{A}^{j}+\left(1-s_{L}^{j}\right) \hat{B}^{j}$ in the long+run enables us to map the dynamics for labor and capital efficiency into the law of motion for sectoral TFP:

$$
\begin{equation*}
\hat{Z}^{j}(t)-\hat{Z}^{j}=\bar{z}^{j} e^{-\xi^{j} t} \tag{75}
\end{equation*}
$$

where $\bar{z}^{j}=s_{L}^{j} \bar{a}^{j}+\left(1-s_{L}^{j}\right) \bar{b}^{j}$. We choose $\bar{a}^{j}, \bar{b}^{j}$ by setting $t=0$ into (73) which yields $\bar{a}^{j}=$ $-\left(\hat{A}^{j}-\hat{A}^{j}(0)\right)$, and $\bar{b}^{j}=-\left(\hat{B}^{j}-\hat{B}^{j}(0)\right)$. Making use of the time series generated by (46a) and (46b) gives us $\bar{a}^{H}=-0.029840, \bar{b}^{H}=-0.202769, \bar{a}^{N}=0.234035, \bar{b}^{N}=-0.500629$. To determine the value for the speed of adjustment of sectoral TFP, we solve (75) for $\xi^{j}$ :

$$
\begin{equation*}
\xi^{j}=-\frac{1}{t} \ln \left(\frac{\hat{Z}^{j}(t)-\hat{Z}^{j}}{\bar{z}^{j}}\right) . \tag{76}
\end{equation*}
$$

We choose time $t$ for which we calculate $\xi^{j}$ that gives us the best fit of the response of $\hat{Z}^{j}(t)$ estimated empirically. Setting $t=3$ leads to $\xi^{H}=0.570885$ for the traded sector and $\xi^{N}=1.166821$ for the non-traded sector which gives us the best fit of the response of $\hat{Z}^{j}(t)$ estimated empirically.

Given the values for $\bar{z}^{j}, \xi^{j}$ and $\hat{Z}^{j}$, we can compute the transitional path for $\hat{Z}^{j}(t)$ by using (75) and thus the dynamics for the productivity differential (47) where we assume that weights $a$ and $b$ are constant over time. In Fig. 11, we contrast empirical shown in blue lines with theoretical responses displayed by the solid black lines with squares. We may notice that the law of motion (75) we impose to capture the dynamic adjustment of sectoral TFPs allows us to reproduce very well responses of $Z^{j}(t)$ we estimate empirically. When we calibrate the model to country-specific data, we adopt the same approach as for the calibration to a representative economy.

## E. 4 Estimates of $\epsilon$ and $\phi$ : Empirical strategy

Table 9 shows our estimates of the elasticity of labor supply across sectors, $\epsilon$, while Table 10 shows our estimates of the elasticity of substitution in consumption between traded and non-traded goods, $\phi$. We present our empirical strategy to estimate these two parameters. More details can be found in Appendix F. 3 and F.2, respectively.

Elasticity of labor supply across sectors. Drawing on Horvath [2000], we derive a testable equation by combining optimal rules for labor supply and labor demand and estimate $\epsilon$ by running the regression of the worker inflow in sector $j=H, N$ of country $i$ at time $t$ arising from labor reallocation across sectors computed as $\hat{L}_{i, t}^{j}-\hat{L}_{i, t}$ on the relative labor's share percentage changes in sector $j, \hat{\beta}_{i, t}^{j}$ :

$$
\begin{equation*}
\hat{L}_{i, t}^{j}-\hat{L}_{i, t}=f_{i}+f_{t}+\gamma_{i} \hat{\beta}_{i, t}^{j}+\nu_{i, t}^{j}, \tag{77}
\end{equation*}
$$

Table 9: Estimates of Elasticity of Labor Supply across Sectors ( $\epsilon$ )

| Country | Elasticity of labor supply across Sectors ( $\epsilon$ ), eq. (77) |
| :---: | :---: |
| AUS | $\underset{(3.20)}{0.375^{a}}$ |
| AUT | ${\underset{(3.00)}{1.103^{a}}}^{a}$ |
| BEL | ${ }_{(3.57)}^{0.610^{a}}$ |
| CAN | $\underbrace{0.390^{a}}_{(4.12)}$ |
| DEU | ${\underset{(3.52)}{1.012^{a}}}^{2}$ |
| DNK | $0_{(2.50)}^{0.286^{a}}$ |
| ESP | $1_{(3.73)}$ |
| FIN | $0_{(4.39)}^{0.431}{ }^{\text {a }}$ |
| FRA | $1_{(2.83)}^{1.400^{a}}$ |
| GBR | ${\underset{(3.91)}{0.601}}^{a}$ |
| IRL | $0_{(3.74)}^{0.216^{a}}$ |
| ITA | $\underset{(3.01)}{1.664^{a}}$ |
| JPN | $0_{(3.55)}{ }^{2}$ |
| NLD | $\underbrace{0.219^{b}}_{(2.05)}$ |
| NOR | $\underset{(0.34)}{0.011}$ |
| SWE | $\underset{(4.28)}{0.534^{a}}$ |
| USA | ${ }_{(1.83)}^{3.222^{c}}$ |
| Countries | 17 |
| Observations | 1456 |
| Data coverage | 1971-2013 |
| Country fixed effects | yes |
| Time trend | no |

Notes: ${ }^{a},{ }^{b}$ and ${ }^{c}$ denote significance at $1 \%, 5 \%$ and $10 \%$ levels. Heteroskedasticity and autocorrelation consistent t-statistics are reported in parentheses.
where $\nu_{i, t}^{j}$ is an i.i.d. error term; country fixed effects are captured by country dummies, $f_{i}$, and common macroeconomic shocks by year dummies, $f_{t}$. The LHS term of (77) is calculated as the difference between changes (in percentage) in hours worked in sector $j, \hat{L}_{i, t}^{j}$, and in total hours worked, $\hat{L}_{i, t}$. The RHS term $\beta^{j}$ corresponds to the fraction of labor's share of value added accumulating to labor in sector $j$. Denoting by $P_{t}^{j} Y_{t}^{j}$ value-added at current prices in sector $j=H, N$ at time $t, \beta_{t}^{j}$ is computed as $\frac{s_{L}^{j} P_{t}^{j} Y_{t}^{j}}{\sum_{j=H}^{N} s_{L}^{j} P_{t}^{j} Y_{t}^{j}}$ where $s_{L}^{j}$ is the LIS in sector $j=H, N$ defined as the ratio of the compensation of employees to value added in the $j$ th sector, averaged over the period 1970-2013. Because hours worked are aggregated by means of a CES function, total hours percentage change $\hat{L}_{i, t}$ is calculated as a weighted average of sectoral hours worked percentage changes, i.e., $\hat{L}_{t}=\sum_{j=H}^{N} \beta_{t-1}^{j} \hat{L}_{t}^{j}$. The parameter we are interested in, the degree of substitutability of hours worked across sectors, is given by $\epsilon_{i}=\gamma_{i} /\left(1-\gamma_{i}\right)$. In the regressions that follow, the parameter $\gamma_{i}$ is assumed to be different across countries when estimating $\epsilon$ for each economy ( $\gamma_{i} \neq \gamma_{i^{\prime}}$ for $i \neq i^{\prime}$ ). To construct $\hat{L}^{j}$ and $\hat{\beta}^{j}$ we combine raw data on hours worked $L^{j}$, nominal value added $P^{j} Y^{j}$ and labor compensation $W^{j} L^{j}$. All required data are taken from the EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases. The sample includes the 17 OECD countries mentioned above over the period 1971-2013 (except for Japan: 1975-2013). Table 9 reports empirical estimates that are consistent with $\epsilon>0$. All values are statistically significant at $10 \%$, except for Norway. Overall, we find that $\epsilon$ ranges from a low of 0.01 for NOR to a high of 3.222 for USA.

Elasticity of substitution between traded and non-traded goods in consumption. To estimate the elasticity of substitution in consumption, $\phi$, between traded and non-traded goods, we derive a testable equation by rearranging the optimal rule for optimal demand for non-traded goods, i.e., $C_{t}^{N}=(1-\varphi)\left(\frac{P_{t}^{N}}{P_{C, t}}\right)^{-\phi} C_{t}$, since time series for consumption in non-traded goods are too short. More specifically, we derive an expression for the non-tradable content of consumption expenditure by using the market clearing condition for non-tradables and construct time series for $1-\alpha_{C, t}$ by using time series for non-traded value added and demand components of GDP while
keeping the non-tradable content of investment and government expenditure fixed, in line with the evidence documented by Bems [2008] for the share of non-traded goods in investment and building on our own evidence for the non-tradable content of government spending. After verifying that the (logged) share of non-tradables and the (logged) ratio of non-traded prices to the consumption price index are both integrated of order one and cointegrated, we run the regression by adding country and time fixed effects by using a FMOLS estimator. We consider two variants, one including a country-specific time trend and one without the time trend. We provide more details below.

Multiplying both sides of $C_{t}^{N}=(1-\varphi)\left(\frac{P_{t}^{N}}{P_{C, t}}\right)^{-\phi} C_{t}$ by $P^{N} / P_{C}$ leads to the non-tradable content of consumption expenditure:

$$
\begin{equation*}
1-\alpha_{C, t}=\frac{P_{t}^{N} C_{t}^{N}}{P_{C, t} C_{t}}=(1-\varphi)\left(\frac{P_{t}^{N}}{P_{C, t}}\right)^{1-\phi} \tag{78}
\end{equation*}
$$

Because time series for non-traded consumption display a short time horizon for most of the countries of our sample while data for sectoral value added and GDP demand components are available for all of the countries of our sample over the period running from 1970 to 2013, we construct time series for the share of non-tradables by using the market clearing condition for non-tradables:

$$
\begin{equation*}
\frac{P_{t}^{N} C_{t}^{N}}{P_{C, t} C_{t}}=\frac{1}{\omega_{C, t}} \cdot\left[\frac{P_{t}^{N} Y_{t}^{N}}{Y_{t}}-\left(1-\alpha_{J}\right) \cdot \omega_{J, t}-\omega_{G^{N}} \cdot \omega_{G, t}\right] . \tag{79}
\end{equation*}
$$

Since the time horizon is too short at a disaggregated level (for $I^{j}$ and $G^{j}$ ) for most of the countries, we draw on the evidence documented by Bems [2008] which reveals that $1-\alpha^{J}=\frac{P^{N} J^{N}}{P^{J} J}$ is constant over time; we further assume that $\frac{P^{N} G^{N}}{G}=\omega_{G^{N}}$ is constant as well in line with our evidence. We thus recover time series for the share of non-tradables by using time series for the non-traded value added at current prices, $P_{t}^{N} Y_{t}^{N}$, GDP at current prices, $Y_{t}$, consumption expenditure, gross fixed capital formation, $I_{t}$, government spending, $G_{t}$ while keeping the non-tradable content of investment and government expenditure, $1-\alpha_{J}$, and $\omega_{G^{N}}$, fixed.

Once we have constructed time series for $1-\alpha_{C, t}=\frac{P_{t}^{N} C_{t}^{N}}{P_{C, t} C_{t}}$ by using (79), we take the logarithm of both sides of (78) and run the regression of the logged share of non-tradables on the logged ratio of non-traded prices to the consumption price index:

$$
\begin{equation*}
\ln \left(1-\alpha_{C, i t}\right)=f_{i}+f_{t}+\alpha_{i} \cdot t+(1-\phi) \ln \left(P^{N} / P_{C}\right)_{i t}+\mu_{i t} \tag{80}
\end{equation*}
$$

where $f_{i}$ captures the country fixed effects, $f_{t}$ are time dummies, and $\mu_{i t}$ are the i.i.d. error terms. Because parameter $\varphi$ in (78) may display a trend over time, we add country-specific trends, as captured by $\alpha_{i} t$. It is worth mentioning that $P^{N}$ is the value added deflator of non-tradables.

Data for non-traded value added at current prices, $P_{t}^{N} Y_{t}^{N}$ and GDP at current prices, $Y_{t}$, are taken from EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases (data coverage: 1970-2013 for all countries, except Japan: 1974-2013). To construct time series for consumption, investment and government expenditure as a percentage of nominal GDP, i.e., $\omega_{C, t}, \omega_{J, t}$ and $\omega_{G, t}$ respectively, we use data at current prices obtained from the OECD Economic Outlook Database (data coverage: 1970-2013). Sources, construction and data coverage of time series for the share of non-tradables in investment $\left(1-\alpha_{J}\right)$ and in government spending ( $\omega_{G^{N}}$ ) are described in depth in section D; $P^{N}$ is the value added deflator of non-tradables. Data are taken from EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases (data coverage: 1970-2013 for all countries, except Japan: 1974-2013). Finally, data for the consumer price index $P_{C, t}$ are obtained from the OECD Prices and Purchasing Power Parities database (data coverage: 1970-2013).

Since both sides of (80) display trends, we ran unit root and then cointegration tests. Having verified that these two assumptions are empirically supported, we estimate the cointegrating relationships by using fully modified OLS (FMOLS) procedure for cointegrated panel proposed by Pedroni [2000], [2001]. FMOLS estimates of (80) are reported in Table 10. When we include a country-specific time trend, the vast majority ( 15 out of 17 ) of the FMOLS estimated coefficients are positive; yet, only ten out of seventeen are statistically significant: AUS, AUT, CAN, DEU, DNK, ESP, IRL, JPN, NOR, USA. We thus also run the same regression as in eq. (80) by ignoring country-specific time trends. We replace inconsistent (i.e., negative or no statistically significant) estimates for $\phi$ when adding a country-specific time trend with those obtained when we excluded the country-specific time trend. Except for GBR for which estimates are negative in both cases and BEL for which estimates are not statistically significant, one out of the two regressions leads to consistent estimates for the elasticity of substitution. For the countries mentioned below, estimates for $\phi$ obtained with a time trend are replaced with those when we drop the time trend: $\phi=0.852$ $(t=8.97)$ for FIN, $\phi=0.885(t=2.76)$ for FRA, $\phi=0.723(t=5.54)$ for ITA, $\phi=0.526(t=2.89)$ for NLD and $\phi=0.5123(t=2.59)$ for SWE. For BEL, we take the estimate obtained when we remove country-specific time trend (i.e. $\phi=1.236$ ) since the $t$-stat is close to the threshold of $10 \%$.

Table 10: Elasticity of Substitution between Tradables and Non-Tradables ( $\phi$ )

| Country | Elasticity of substitution |
| :---: | :---: |
|  | between $C^{T}$ and $C^{N}(\phi)$, eq. (77) |
| AUS | $\underset{(2.25)}{0.396^{b}}$ |
| AUT | ${\underset{(6.35)}{1.518^{a}}}^{a}$ |
| BEL | $\underset{(1.29)}{1.236^{\star}}$ |
| CAN | $\underbrace{0.748^{a}}_{(4.32)}$ |
| DEU | $\underset{(2.79)}{0.577^{a}}$ |
| DNK | ${\underset{(3.77)}{1.083^{a}}}^{a}$ |
| ESP | $\underset{(2.19)}{1.387^{b}}$ |
| FIN | $\underset{(8.97)}{0.852^{a \star}}$ |
| FRA | $\underset{(2.76)}{0.885^{a \star}}$ |
| GBR | 0 |
| IRL | ${\underset{(3.70)}{1.352^{a}}}^{a}$ |
| ITA | $\underset{(5.54)}{0.723^{a \star}}$ |
| JPN | $1_{(5.12)}^{1.052^{a}}$ |
| NLD | $\underset{(2.89)}{0.526^{a}}$ |
| NOR | $0_{(3.33)}^{0.891^{a}}$ |
| SWE | $\underset{(2.59)}{0.513^{a}}$ |
| USA | $\begin{gathered} 0.821^{a} \\ \hline \end{gathered}$ |
| Whole Sample | $\underset{(12.03)}{0.662^{a}} / \underset{(6.05)}{0.333^{a \star}}$ |
| Countries | 17 |
| Observations | 739 |
| Data coverage | 1970-2013 |
| Country fixed effects | yes |
| Time trend | yes |

Notes: ${ }^{a},{ }^{b}$ and ${ }^{c}$ denote significance at $1 \%, 5 \%$ and $10 \%$ levels. Heteroskedasticity and autocorrelation consistent t-statistics are reported in parentheses. The superscript * indicates that the estimate is obtained in a regression without a country-specific linear time trend.

For GBR, the estimate is negative whether there is a time trend in the regression or not and thus we set $\phi$ to zero for the rest of the analysis for this country. Table 10 shows estimates for $\phi$ for each country. We add the superscript * when estimates come from regression (80) without countryspecific linear time trend. The last line of Table 4 reveals that $\phi$ stands at 0.66 when adding a time trend while the estimate for the parameter is twice as small when dropping the time trend. The unweighted average of these two estimates, say 0.49 , is close to the value of $\phi$ which is commonly set in the international RBC literature and taken from Stockman and Tesar [1995] who find a value for $\phi$ of 0.44 . One point merits comments. When running eq. (80), data for the RHS variable, i.e. $P^{N} / P_{C}$, have a good coverage for all countries of our sample. Indeed, we are able to cover our baseline period 1970-2013 for this variable (except for JPN: 1974-2013). By contrast, the LHS variable is constructed by using the share of non-tradables in investment $\left(1-\alpha_{J}\right)$ and in government spending ( $\omega_{G^{N}}$ ), averaged over the period 1995-2013 (due to data availability). In light of these limitations, we also run eq. (80) for the overlap period 1995-2013. Over this period of time, we have a balanced panel and time series of a reasonable length. Using again the FMOLS estimator, we obtain $\phi=0.474$ for the whole sample. As a robustness check, we also used the DOLS estimator with one lead/lag which gives a value of 0.415 . The unweighted average of these two estimates is $\phi=0.445$ for the whole sample, in accordance with the estimated value of 0.44 documented by Stockman and Tesar [1995].

## E. 5 Estimates of $\sigma^{j}$ : Empirical strategy

To estimate the elasticity of substitution between capital and labor, $\sigma^{j}$, we draw on Antràs [2004]. We allow labor- and capital-augmenting technological change to grow at constant rate:

$$
\begin{align*}
& A_{t}^{j}=A_{0}^{j} e^{a^{j} t}  \tag{81a}\\
& B_{t}^{j}=B_{0}^{j} e^{b^{j} t} \tag{81b}
\end{align*}
$$

where $a^{j}$ and $b^{j}$ denote the constant growth rates of labor- and capital-augmenting technical progress and $A_{0}^{j}$ and $B_{0}^{j}$ are the initial levels of technology. Inserting first (81a) and (81b) into the demand for labor and capital (28a)-(28b), taking logarithm and rearranging gives:

$$
\begin{align*}
\ln \left(Y_{t}^{j} / L_{t}^{j}\right) & =\alpha_{1}+\left(1-\sigma^{j}\right) a^{j} t+\sigma_{j} \ln \left(W_{t}^{j} / P_{t}^{j}\right),  \tag{82a}\\
\ln \left(Y_{t}^{j} / K_{t}^{j}\right) & =\alpha_{2}+\left(1-\sigma^{j}\right) b^{j} t+\sigma_{j} \ln \left(R_{t} / P_{t}^{j}\right), \tag{82b}
\end{align*}
$$

where $\alpha_{1}=\left[\left(1-\sigma^{j}\right) \ln A_{0}^{j}-\sigma^{j} \ln \gamma^{j}\right]$ and $\alpha_{2}=\left[\left(1-\sigma^{j}\right) \ln B_{0}^{j}-\sigma^{j} \ln \left(1-\gamma^{j}\right)\right]$ are constants. Above equations describe firms' demand for labor and capital respectively.

We estimate the elasticity of substitution between capital and labor in sector $j=H, N$ from firstorder conditions (82a)-(82b) in panel format on annual data. Adding an error term and controlling for country fixed effects, we explore empirically the following equations:

$$
\begin{align*}
\ln \left(Y_{i t}^{j} / L_{i t}^{j}\right) & =\alpha_{1 i}+\lambda_{1 i} t+\sigma_{i}^{j} \ln \left(W_{i t}^{j} / P_{i t}^{j}\right)+u_{i t},  \tag{83a}\\
\ln \left(Y_{i t}^{j} / K_{i t}^{j}\right) & =\alpha_{2 i}+\lambda_{2 i} t+\sigma_{i}^{j} \ln \left(R_{i t} / P_{i t}^{j}\right)+v_{i t}, \tag{83b}
\end{align*}
$$

where $i$ and $t$ index country and time and $u_{i t}$ and $v_{i t}$ are i.i.d. error terms. Country fixed effects are represented by dummies $\alpha_{1 i}$ and $\alpha_{2 i}$, and country-specific trends are captured by $\lambda_{1 i}$ and $\lambda_{2 i}$. Since all variables display unit root process, we estimate cointegrating relationships by using the fully modified OLS (FMOLS) procedure for cointegrated panel proposed by Pedroni [2000].

Estimation of (83a) and (83b) requires data for each sector $j=H, N$ on sectoral value added at constant prices $Y^{j}$, sectoral hours worked $L^{j}$, sectoral capital stock $K^{j}$, sectoral value added deflator $P^{j}$, sectoral wage rate $W^{j}$ and capital rental cost $R$. Data for sectoral value added $Y^{H}$ and $Y^{N}$, hours worked $L^{H}$ and $L^{N}$, value added price deflators $P^{H}$ and $P^{N}$, and, nominal wages $W^{H}$ and $W^{N}$ are taken form the EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases. To construct the national stock of capital $K$, we use the perpetual inventory method with a fixed depreciation rate of $5 \%$ and the time series of constant-price investment from the OECD Economic Outlook Database. Next, following Garofalo and Yamarik [2002], the capital stock is allocated to traded and non-traded industries by using sectoral output shares. Finally, we measure the aggregate rental price of capital $R$ as the ratio of capital income to capital stock. Capital income is derived as nominal value added minus labor compensation. For all aforementioned variables, the sample includes includes the 17 OECD countries over the period 1970-2013 (except for Japan: 1974-2013).

While we take the demand for labor as our baseline model (i.e. eq. (83a), Table 11 provides FMOLS estimates of $\sigma^{j}$ for the demand of labor and capital. The bulk (3 out of 34) of the FMOLS estimated coefficients from eq. (83a) are positive and statistically significant. One estimated coefficient is negative ( $\sigma^{H}$ for IRL) while estimates of $\sigma^{N}$ for ITA and SWE are positive but not statistically significant. As in Antràs [2004], we alternatively run the regression of the ratio of value added to capital stock at constant prices on the real capital cost $R / P^{j}$ in sector $j$, i.e., eq. (83b). We then replace inconsistent estimates for $\sigma^{j}$ obtained from labor demand with those obtained from the demand of capital. Columns 17-18 of table 4 report estimates for $\sigma^{H}$ and $\sigma^{N}$.

## F Data Description

In this section, we present some additional information about the data we use in the empirical and numerical analysis and the empirical strategy adopted to estimate key parameters. First, we provide details on the construction of sectoral TFP. Then, we describe empirical strategies to estimate four parameters involved in our quantitative analysis: the elasticity of substitution in consumption between traded and non-traded goods, $\phi$, the degree of substitutability of hours worked across sectors, $\epsilon$, the elasticity of substitution between capital and labor in production, $\sigma^{H}$ and $\sigma^{N}$.

## F. 1 Construction of Sectoral TFPs

Sectoral TFPs (in $\log$ ) $Z_{t}^{j}$ at time $t$ are constructed as Solow residuals from constant-price (domestic currency) series of value added $Y_{t}^{j}$ and capital stock $K_{t}^{j}$, and labor input $L_{t}^{j}$ :

$$
\begin{equation*}
\log Z_{t}^{j}=\log Y_{t}^{j}-s_{L}^{j} \log L_{t}^{j}-\left(1-s_{L}^{j}\right) \log K_{t}^{j} \tag{84}
\end{equation*}
$$

Table 11: FMOLS Estimates of the Sectoral Elasticity of Substitution between Capital and Labor ( $\sigma^{j}$ )

| Country | Tradables $\left(\sigma^{H}\right)$ |  | non-tradables $\left(\sigma^{N}\right)$ |  |
| :---: | :---: | :---: | :---: | :---: |
| Dependent variable | $\ln \left(Y^{H} / K^{H}\right)$ | $\ln \left(Y^{H} / L^{H}\right)$ | $\ln \left(Y^{N} / K^{N}\right)$ | $\ln \left(Y^{N} / L^{N}\right)$ |
| Explanatory variable | $\ln \left(R / P^{H}\right)$ | $\ln \left(W^{H} / P^{H}\right)$ | $\ln \left(R / P^{N}\right)$ | $\ln \left(W^{N} / P^{N}\right)$ |
| AUS | ${ }_{(6.67)}^{0.607^{a}}$ | ${\underset{(3.79)}{0.474^{a}}}^{2}$ | $\underset{(4.03)}{0.459^{a}}$ | ${\underset{(5.69)}{ } 0.529^{a}}^{a}$ |
| AUT | ${\underset{(2.65)}{0.235^{a}}}^{(2)}$ | ${\underset{(6.04)}{0.774^{a}}}^{a}$ | $\underset{(1.22)}{0.105}$ | ${ }_{(13.04)}$ |
| BEL | $\underset{(3.01)}{0.389^{a}}$ | $\underset{(8.89)}{0.829^{a}}$ | ${ }_{(7.37)}^{0.266^{a}}$ | $\underset{(7.10)}{1.069^{a}}$ |
| CAN | ${\underset{(3.99)}{0.595^{a}}}^{a}$ | $\underbrace{0.480^{a}}_{(2.94)}$ | ${ }_{(8.62)}^{0.855^{a}}$ | ${ }_{(7.65)}^{0.668^{a}}$ |
| DEU | $\begin{gathered} -0.123 \\ (-0.68) \end{gathered}$ | ${ }_{(8.56)}^{0.642^{a}}$ | ${\underset{(8.88)}{0.512^{a}}}^{a}$ | ${ }_{(6.97)}^{0.987^{a}}$ |
| DNK | $\underset{(1.84)}{0.267^{c}}$ | $\underset{(4.32)}{0.417^{a}}$ | ${ }_{(7.83)}^{0.502^{a}}$ | $1.282^{a}$ |
| ESP | $\underset{(7.11)}{0.747^{a}}$ | ${ }_{(10.62)}^{1.033^{a}}$ | ${ }_{(3.65)}^{0.682^{a}}$ | ${\underset{(3.35)}{0.476^{a}}}^{a}$ |
| FIN | $\underset{(2.90)}{0.249^{a}}$ | $\underset{(1.98)}{0.764^{b}}$ | ${ }_{(6.64)}^{0.560^{a}}$ | $\underset{(8.30)}{0.794^{a}}$ |
| FRA | $\underbrace{0.267^{a}}_{(4.82)}$ | $\underbrace{0.870^{a}}_{(4.82)}$ | ${\underset{(11.04)}{0.294^{a}}}^{a}$ | $0_{(4.21)}^{0.916^{a}}$ |
| GBR | $\underset{(0.95)}{0.242}$ | $\underbrace{0.603^{a}}_{(6.42)}$ | $\begin{aligned} & 0.008 \\ & (0.08) \end{aligned}$ | $\underset{(2.68)}{0.561^{a}}$ |
| IRL | $\underset{(18.46)}{0.737^{a}}$ | $\underset{(-0.50)}{-0.125}$ | ${ }_{(5.73)}^{0.762^{a}}$ | ${ }_{(3.16)}^{0.627^{a}}$ |
| ITA | $0_{(3.82)}^{0.506^{a}}$ | ${ }_{(8.80)}^{0.837^{a}}$ | ${\underset{(3.23)}{0.471}}^{a}$ | $0.259$ |
| JPN | ${ }_{(8.16)}^{0.622^{a}}$ | ${ }_{(6.73)}^{1.164^{a}}$ | $\underset{(7.97)}{0.417^{a}}$ | $\underset{(2.47)}{0.635^{b}}$ |
| NLD | ${ }_{(5.13)}^{0.645^{a}}$ | $0_{(5.98)}^{0.910^{a}}$ | ${\underset{(9.14)}{0.287^{a}}}^{a}$ | $\underset{(3.74)}{0.444^{a}}$ |
| NOR | $\underbrace{0.798^{a}}_{(4.60)}$ | $\underbrace{0.629^{a}}_{(4.39)}$ | ${\underset{(10.17)}{0.653^{a}}}^{a}$ | $\underbrace{0.556^{a}}_{(4.72)}$ |
| SWE | $\underset{(0.35)}{0.052}$ | ${ }_{(8.56)}^{0.607^{a}}$ | ${ }_{(6.71)}^{0.378^{a}}$ | $\underset{(0.95)}{0.194}$ |
| USA | ${ }_{(6.85)}$ | $\begin{gathered} 0.766^{a} \\ (9.51) \\ \hline \end{gathered}$ | $0_{(6.64)}^{0.723^{a}}$ | $\begin{gathered} 0.876^{a} \\ \hline \end{gathered}$ |
| Whole Sample | $\begin{gathered} 0.489^{a} \\ (19.56) \\ \hline \end{gathered}$ | $\begin{aligned} & 0.687^{a} \\ & (24.70) \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.467^{a} \\ & (26.42) \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.716^{a} \\ & (21.16) \\ & \hline \end{aligned}$ |
| Countries | 17 | 17 | 17 | 17 |
| Observations | 745 | 745 | 745 | 745 |
| Data coverage | 1970-2013 | 1970-2013 | 1970-2013 | 1970-2013 |
| Country fixed effects | yes | yes | yes | yes |
| Time trend | yes | yes | yes | yes |

Notes: ${ }^{a},{ }^{b}$ and ${ }^{c}$ denote significance at $1 \%, 5 \%$ and $10 \%$ levels. Heteroskedasticity and autocorrelation consistent t-statistics are reported in parentheses.
where $s_{L}^{j}$ is the LIS in sector $j$ averaged over period 1970-2013 (1974-2013 for Japan). Data for the series of constant-price value-added (VA_QI) and hours worked (H_EMP) are taken from EU KLEMS database. The sectoral LIS is calculated as the ratio of labor compensation in sector $j$ (LAB) to value added at current prices (VA). Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases.

To construct the series for the sectoral capital stock, we proceed as follows. Capital stocks are estimated from the perpetual inventory approach. In order to apply this method, we need $(i)$ real gross capital formation series, (ii) the initial capital stock in the base year, which is set to be 1970 and (iii) the rate of depreciation of the existing capital stock. Real gross capital formation is obtained from OECD National Accounts Database [2017] (data in millions of national currency, constant prices). Consistent with the neoclassical growth model, the initial capital stock, $K_{1970}$, is computed using the following formula:

$$
K_{1970}=\frac{I_{1970}}{g_{I}+\delta_{K}},
$$

where $I_{1970}$ corresponds to the real gross capital formation in the base year 1970, $g_{I}$ is the average growth rate from 1970 to 2013 of the real gross capital formation series and $\delta$ is the depreciation rate which is assumed to be $5 \%$ (see Hall and Jones [1999]). The capital stock is obtained by using the standard capital accumulation equation: $K_{t+1}=(1-\delta) K_{t}+I_{t}$ for $t=1970, \ldots, 2013$ and where $K_{t}$ is the capital stock at the beginning of period $t$. Following Garofalo and Yamarik [2002], the gross capital stock is then allocated to traded and non-traded industries by using the sectoral value added share:

$$
K_{t}^{j}=\omega_{t}^{Y, j} K_{t}
$$

where $\omega_{t}^{Y, j}$ is the value added share of sector $j$ at current prices.
Finally, the productivity differential variable is computed as the difference in the labor shareadjusted TFP growth between the traded sector and the non-traded sector:

$$
\begin{equation*}
\hat{Z}_{t}=a \hat{Z}_{t}^{H}-b \hat{Z}_{t}^{N} \tag{85}
\end{equation*}
$$

where $a=\left[\left(1-\alpha_{J}\right)+\alpha_{J}\left(s_{L}^{H} / s_{L}^{N}\right)\right]^{-1}, b=a\left(s_{L}^{H} / s_{L}^{N}\right)$, with $\alpha_{J}$ the tradable share in total investment expenditure.

## F. 2 Estimates of $\phi$ : Empirical Strategy

In this section, we detail our empirical strategy to estimate the elasticity of substitution between traded and non-traded goods $\phi$. Estimates of the elasticity of substitution $\phi$ documented by the existing literature are rather diverse. The cross-section studies report an estimate of $\phi$ ranging from 0.44 to 0.74, see e.g., Stockman and Tesar [1995] and Mendoza [1995], respectively. ${ }^{43}$ The literature adopting the Generalized Method of Moments and the cointegration methods, see e.g. Ostry and Reinhart [1992] and Cashin and Mc Dermott [2003], respectively, reports a value in the range $[0.75,1.50]$ for developing countries and in the range $[0.63,3.50]$ for developed countries. Since estimates for $\phi$ display a sharp dispersion across empirical studies, we conduct an empirical analysis in order to estimate this parameter for each country in our sample.

## F.2.1 Empirical Strategy

To estimate $\phi$, we adopt the following strategy. At each instant of time, the representative household consumes traded and non-traded goods denoted by $C^{T}$ and $C^{N}$, respectively, which are aggregated by means of a CES function:

$$
\begin{equation*}
C=\left[\varphi^{\frac{1}{\phi}}\left(C^{T}\right)^{\frac{\phi-1}{\phi}}+(1-\varphi)^{\frac{1}{\phi}}\left(C^{N}\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \tag{86}
\end{equation*}
$$

where $0<\varphi<1$ is the weight of the traded good in the overall consumption bundle and $\phi$ corresponds to the elasticity of substitution between traded goods and non-traded goods. The index $C^{T}$ is defined as a CES aggregator of home-produced traded goods, $C^{H}$, and foreign produced traded goods, $C^{F}$ :

$$
\begin{equation*}
C^{T}=\left[\left(\varphi^{H}\right)^{\frac{1}{\rho}}\left(C^{H}\right)^{\frac{\rho-1}{\rho}}+\left(1-\varphi_{H}\right)^{\frac{1}{\rho}}\left(C^{F}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \tag{87}
\end{equation*}
$$

[^30]where $0<\varphi_{H}<1$ is the weight of the home-produced traded good and $\rho$ corresponds to the elasticity of substitution between home- and foreign produced traded goods.

Applying Shephard's lemma (or the envelope theorem) yields the following demand for traded and non-traded goods:

$$
\begin{gather*}
C^{T}=\varphi\left(\frac{P^{T}}{P_{C}}\right)^{-\phi} C,  \tag{88a}\\
C^{N}=(1-\varphi)\left(\frac{P^{N}}{P_{C}}\right)^{-\phi} C . \tag{88b}
\end{gather*}
$$

Multiplying both sides of (88b) by $P^{N} / P_{C}$ leads to the non-tradable content of consumption expenditure:

$$
\begin{equation*}
1-\alpha_{C}=\frac{P^{N} C^{N}}{P_{C} C}=(1-\varphi)\left(\frac{P^{N}}{P_{C}}\right)^{1-\phi} \tag{89}
\end{equation*}
$$

The market clearing for non-tradables reads:

$$
\begin{equation*}
Y^{N}=C^{N}+J^{N}+G^{N} \tag{90}
\end{equation*}
$$

Multiplying both sides by $P^{N}$ and dividing by GDP at current prices, $Y=P^{H} Y^{H}+P^{N} Y^{N}$, leads to:

$$
\begin{equation*}
\frac{P^{N} Y^{N}}{Y}=\frac{P^{N} C^{N}}{Y}+\frac{P^{N} J^{N}}{P^{J} J} \cdot \frac{P^{J} J}{Y}+\frac{P^{N} G^{N}}{G} \cdot \frac{G}{Y} \tag{91}
\end{equation*}
$$

We denote the investment-to-GDP ratio by $\omega_{J}=\frac{P^{J} J}{Y}$ and the share of government spending in GDP by $\omega_{G}=\frac{G}{Y}$. Building on the evidence documented by Bems [2008], we assume that $1-\alpha_{J}=\frac{P^{N} J^{N}}{P^{J} J}$ is constant over time; we further assume that $\frac{P^{N} G^{N}}{G}=\omega_{G^{N}}$ is constant as well in line with our evidence. Under these assumptions and by using the fact that $\frac{P^{N} C^{N}}{Y}=\left(1-\alpha_{C}\right) \omega_{C}$, eq. (91) can be solved for the share of non-tradables into consumption expenditure:

$$
\begin{equation*}
\frac{P_{t}^{N} C_{t}^{N}}{P_{C, t} C_{t}}=\frac{1}{\omega_{C, t}} \cdot\left[\frac{P_{t}^{N} Y_{t}^{N}}{Y_{t}}-\left(1-\alpha_{J}\right) \omega_{J, t}-\omega_{G^{N}} \omega_{G, t}\right] \tag{92}
\end{equation*}
$$

where the shares $1-\alpha_{J}$ and $\omega_{G^{N}}$ are kept constant over time whilst we let the shares $\frac{P_{t}^{N} Y_{t}^{N}}{Y_{t}}, \omega_{C, t}$, $\omega_{J, t}, \omega_{G, t}$ vary across time.

Once we have constructed time series for $1-\alpha_{C, t}=\frac{P_{t}^{N} C_{t}^{N}}{P_{C, t} C_{t}}$ by using (92), we take the logarithm of both sides of (89) and we run the regression of the logged share of non-tradables on the logged ratio of non-traded prices to the consumption price index:

$$
\begin{equation*}
\ln \left(1-\alpha_{C}\right)_{i, t}=f_{i}+f_{t}+\alpha_{i} t+(1-\phi) \ln \left(P^{N} / P_{C}\right)_{i, t}+\mu_{i, t} \tag{93}
\end{equation*}
$$

where $f_{i}$ captures the country fixed effects, $f_{t}$ are time dummies, and $\mu_{i t}$ are the i.i.d. error terms. Because parameter $\varphi$ in (89) may display a trend over time, we add country-specific trends, as captured by $\alpha_{i} t$. It is worth mentioning that $P^{N}$ is the value added deflator of non-tradables.

## F.2.2 Data Construction and Source

We provide more details below on the construction of data employed to estimate equation (93):

- Non-Traded value added, $P^{N} Y^{N}$ : value added at current prices in sector $N$ (VA). Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases. Data coverage: 1970-2013 except for JPN 1974-2013.
- Nominal GDP, $Y$ : value added at current prices in total economy (VA), i.e. $Y=P^{H} Y^{H}+$ $P^{N} Y^{N}$. Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases. Data coverage: 1970-2013 except for JPN 1974-2013.
- Share of consumption expenditure in total GDP, $\omega_{C}$ : final consumption expenditure of households at current prices over gross domestic product (expenditure approach) at current prices. Source: OECD National Accounts Database [2017]. Data coverage: 1970-2013.
- Share of investment expenditure in total GDP, $\omega_{J}$ : gross fixed capital formation at current prices over gross domestic product (expenditure approach) at current prices. Source: OECD National Accounts Database [2017]. Data coverage: 1970-2013.
- Share of government spending in total GDP, $\omega_{G}$ : final consumption expenditure of general government at current prices over gross domestic product (expenditure approach) at current prices. Source: OECD National Accounts Database [2017]. Data coverage: 1970-2013.
- Share of non-tradables in total investment expenditure, $1-\alpha_{J}$ : investment expenditure on non-tradables at current prices over total investment expenditure at current prices. Source: OECD Input-output database [2017]. Data coverage: AUS (1970-2013), AUT (1995-2013), BEL (1995-2013), CAN (1970-2013), DEU (1995-2013), DNK (1970-2013), ESP (1995-2013), FIN (1980-2013), FRA (1978-2013), GBR (1997-2013), IRL (1995-2013), ITA (1995-2013), JPN (1994-2013), NLD (1995-2013), NOR (1970-2013), SWE (1993-2013) and USA (19702013).
- Share of non-tradables in total government spending, $\omega_{G^{N}}$ : government spending on nontradables at current prices over total government spending at current prices. Source: COFOG, OECD [2017]. Data coverage: AUS (1998-2013), AUT (1995-2013), BEL (1995-2013), DEU (1995-2013), DNK (1995-2013), ESP (1995-2013), FIN (1990-2013), FRA (1995-2013), GBR (1995-2013), IRL (1995-2013), ITA (1995-2013), JPN (2005-2013), NLD (1995-2013), NOR (1995-2013), SWE (1995-2013) and USA (1970-2013). Data are not available for CAN. For this country, we choose $\omega_{G^{N}}=0.90$ which corresponds to the cross-country unweighed average.
- Sectoral value added price deflator, $P^{N}$ : value added at current prices (VA) over value added at constant prices (VA_QI) in sector $N$. Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases. Data coverage: 1970-2013 except for JPN 1974-2013.
- Consumer price index, $P_{C}$ : consumer price index (all items, Index, 2010=100). Source: OECD Prices and Purchasing Power Parities. Data coverage: 1970-2013.

We use data described above to construct time series for $\left(1-\alpha_{C}\right)_{i, t}$ and $\left(P^{N} / P_{C}\right)_{i, t}$. When estimating equation (93), all variables are converted into index $2010=100$ and are expressed in log levels.

## F.2.3 Empirical Results

Since the two variables of interest in regression (93) display trends, we first run panel unit root tests, see Table 12. By and large, all tests, with the exception of LLC, for the variable $\ln \left(1-\alpha_{C}\right)$, show that non stationarity is pervasive, making it clear that pursuing a cointegration analysis is appropriate.

Table 12: Panel Unit Root Tests (p-values)

|  | LLC <br> $($ t-stat $)$ | Breitung <br> $($ t-stat $)$ | IPS <br> $($ W-stat $)$ | MW <br> $($ ADF $)$ | Hadri <br> $\left(Z_{\mu}\right.$-stat $)$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $\ln \left(1-\alpha_{C}\right)$ | 0.011 | 0.941 | 0.992 | 0.991 | 0.000 |
| $\ln \left(P^{N} / P_{C}\right)$ | 0.077 | 0.950 | 0.886 | 0.833 | 0.000 |

Notes: For all tests, except for Hadri [2000], the null of a unit root is not rejected if p-value $\geq 0.05$ at a $5 \%$ significance level. For Hadri [2000], the null of stationarity is rejected if p -value $\leq 0.05$ at a $5 \%$ significance level.

We thus implement the seven Pedroni's [2004] tests of the null hypothesis of no cointegration, see Table 13. All panel tests, with the exception of non-parametric $\nu$ statistic, reject the null hypothesis of no cointegration between $\ln \left(1-\alpha_{C}\right)$ and the relative price $\ln \left(P^{N} / P_{C}\right)$ at the $5 \%$ significance level. In particular, the group-mean parametric t-stat test suggest the existence of a cointegration relationship between the variables of interest at $1 \%$ significance level. In small samples, Pedroni's [2004] simulations reveal that the group-mean parametric t-stat is the most powerful. Based on this result, the null hypothesis of no cointegration is strongly rejected at the $1 \%$ level

Table 14 shows estimates of $\phi$ when running regression (93) where the dependent variable is the $\log$ of $\left(1-\alpha_{C}\right)$. The regressor is the (logged) price of non-tradables in terms of the consumer price index $\left(P^{N} / P_{C}\right)$. The sample covers all countries we are interested in. For the whole sample, the FMOLS estimate gives a significant value of $\phi$ of 0.662 . This estimated coefficient is statistically significant. The majority ( 10 out of 17 ) of the individual FMOLS estimated coefficients are positive and statistically significant. Two estimated coefficients are negative (GBR and SWE), although none of them are statistically significant. Focusing only on countries with positive statistically significant estimates, we find that $\phi$ varies from a low of 0.396 for AUS to a high of 1.518 for AUT.

## F. 3 Estimates of $\epsilon$ : Empirical Strategy

In this section, we detail our empirical strategy to estimate the elasticity of labor supply across sectors, $\epsilon$, which captures the degree of labor mobility across sectors.

Table 13: Panel Cointegration Tests (p-values)

| Dependent variable | $\ln \left(1-\alpha_{C}\right)$ |
| :--- | :---: |
| Explanatory variable | $\ln \left(P^{N} / P_{C}\right)$ |
| Panel tests |  |
| Non-parametric $\nu$ | 0.034 |
| Non-parametric $\rho$ | 0.015 |
| Non-parametric $t$ | 0.000 |
| Parametric $t$ | 0.005 |
| Group-mean tests |  |
| Non-parametric $\nu$ | 0.227 |
| Non-parametric $t$ | 0.001 |
| Parametric $t$ | 0.009 |
| Notes: the null hypothesis of no cointe- |  |
| gration is rejected if the p-value is below |  |
| 0.05 (0.10 resp.) at $5 \%$ (10\% resp.) sig- |  |
| nificance level. |  |

Table 14: FMOLS Estimates of $\phi$

| Country | $\hat{\phi}_{i}^{F M O L S}$ |
| :---: | :---: |
| AUS | $\underset{(2.25)}{0.396^{b}}$ |
| AUT | ${ }_{(6.35)}^{1.518^{a}}$ |
| BEL | $\underset{(0.63)}{0.612}$ |
| CAN | $0_{(4.32)}^{0.748^{a}}$ |
| DEU | $\underset{(2.79)}{0.577^{a}}$ |
| DNK | $1_{(3.77)}$ |
| ESP | $\underbrace{1.387^{b}}_{(2.19)}$ |
| FIN | $\underset{(1.16)}{0.225}$ |
| FRA | $\underset{(1.38)}{0.353}$ |
| GBR | $\begin{aligned} & -0.267 \\ & (-0.87) \end{aligned}$ |
| IRL | $1_{(3.70)}^{1.352^{a}}$ |
| ITA | $\underset{(1.60)}{0.284}$ |
| JPN | $1_{(5.12)}$ |
| NLD | $\begin{aligned} & 0.389 \\ & (0.93) \end{aligned}$ |
| NOR | ${ }_{(3.33)}^{0.891^{a}}$ |
| SWE | $\begin{gathered} -0.173 \\ (-0.73) \end{gathered}$ |
| USA | ${ }_{(3.73)}^{0.821^{a}}$ |
| Whole Sample | $\begin{aligned} & 0.662^{a} \\ & (12.03) \\ & \hline \end{aligned}$ |
| Countries | 17 |
| Observations | 739 |
| Data coverage | 1970-2013 |
| Country fixed effects | yes |
| Time trend | yes |

Notes: ${ }^{a},{ }^{b}$ and ${ }^{c}$ denote significance at $1 \%$, $5 \%$ and $10 \%$ levels. Heteroskedasticity and autocorrelation consistent t-statistics are reported in parentheses.

## F.3.1 Limited Substitutability of Hours Worked across Sectors and the Derivation of the Testable Equation

The economy consists of $M$ distinct sectors, indexed by $j=0,1, \ldots, M$ each producing a different good. Along the lines of Horvath [2000], the aggregate labor index is assumed to take the form:

$$
\begin{equation*}
L=\left[\int_{0}^{M}\left(\vartheta^{j}\right)^{-\frac{1}{\epsilon}}\left(L^{j}\right)^{\frac{\epsilon+1}{\epsilon}} d j\right]^{\frac{\epsilon}{\epsilon+1}} \tag{94}
\end{equation*}
$$

The agent seeks to maximize her labor income

$$
\begin{equation*}
\int_{0}^{M} W^{j} L^{j} d j=X \tag{95}
\end{equation*}
$$

for given utility loss; $L^{j}$ is labor supply to sector $j, W^{j}$ the wage rate in sector $j$ and $X$ total labor income. The form of the aggregate labor index (94) implies that there exists an aggregate wage index $W($.$) , whose expression will be determined later. Thus equation (95) can be rewritten as$ follows:

$$
\begin{equation*}
\int_{0}^{M} W^{j} L^{j} d j=W L \tag{96}
\end{equation*}
$$

Writing down the Lagrangian and denoting by $\mu$ the Lagrangian multiplier to the constraint, the first-order reads as:

$$
\begin{equation*}
\left(\vartheta^{j}\right)^{-\frac{1}{\epsilon}}\left(L^{j}\right)^{\frac{1}{\epsilon}} L^{-\frac{1}{\epsilon}}=\mu W^{j} . \tag{97}
\end{equation*}
$$

Left-multiplying both sides of eq. (97) by $L^{j}$, summing over the $M$ sectors and using eqs. (94) and (96) implies that $\mu=\frac{1}{W}$. Plugging the expression for the Lagrangian multiplier into (97) and rearranging terms leads to optimal labor supply $L^{j}$ to sector $j$ :

$$
\begin{equation*}
L^{j}=\vartheta^{j}\left(\frac{W^{j}}{W}\right)^{\epsilon} L \tag{98}
\end{equation*}
$$

We assume that within each sector, there is a large number of identical firms which produces $Y^{j}$ by using labor $L^{j}$ and capital $K^{j}$ according to constant returns to scale in production. The representative firm faces two cost components: a capital rental cost equal to $R$, and sectoral wage $W^{j}$ and $W^{N}$, respectively. Since each sector is assumed to be perfectly competitive, the representative firm chooses capital and labor by taking prices as given:

$$
\begin{equation*}
\max _{K^{j}, L^{j}} \Pi^{j}=\max _{K^{j}, L^{j}}\left\{P^{j} Y^{j}-W^{j} L^{j}-R K^{j}\right\} . \tag{99}
\end{equation*}
$$

Since that the production function displays constant returns to scale and using the fact that factors are paid their marginal product, the demand for labor and capital are: $\partial Y^{j} / \partial L^{j}=W^{j} / P^{j}$ and $\partial Y^{j} / \partial K^{j}=R / P^{j}$, respectively; denoting the LIS in sector $j$ by $s_{L}^{j}$, the demand for capital and labor can be rewritten as follows: $\hat{Y}^{j} / \hat{L}^{j}=s_{L}^{j}$ and $\hat{Y}^{j} / \hat{K}^{j}=1-s_{L}^{j}$ which leads to:

$$
\begin{gather*}
s_{L}^{j} \frac{P^{j} Y^{j}}{L^{j}}=W^{j},  \tag{100a}\\
\left(1-s_{L}^{j}\right) \frac{P^{j} Y^{j}}{K^{j}}=R . \tag{100b}
\end{gather*}
$$

Inserting labor demand (100a) into labor supply to sector $j$ (98) and solving leads the share of sector $j$ in aggregate labor:

$$
\begin{equation*}
\frac{L^{j}}{L}=\left(\vartheta^{j}\right)^{\frac{1}{\epsilon+1}}\left(\frac{s_{L}^{j} P^{j} Y^{j}}{\int_{0}^{M} s_{L}^{j} P^{j} Y^{j} d j}\right)^{\frac{\epsilon}{\epsilon+1}} \tag{101}
\end{equation*}
$$

where we combined (96) and (100a) to rewrite the aggregate wage as follows:

$$
\begin{equation*}
W=\frac{\int_{0}^{M} s_{L}^{j} P^{j} s_{L}^{j} d j}{L} \tag{102}
\end{equation*}
$$

We denote by $\beta^{j}$ the fraction of labor's share of value added accumulating to labor in sector $j$ :

$$
\begin{equation*}
\beta^{j}=\frac{s_{L}^{j} P^{j} Y^{j}}{\sum_{j=1}^{M} s_{L}^{j} P^{j} Y^{j}} . \tag{103}
\end{equation*}
$$

Using (103), the labor share in sector $j$ (101) can be rewritten as follows:

$$
\begin{equation*}
\frac{L^{j}}{L}=\left(\vartheta^{j}\right)^{\frac{1}{\epsilon+1}}\left(\beta^{j}\right)^{\frac{\epsilon}{\epsilon+1}} \tag{104}
\end{equation*}
$$

Introducing a time subscript and taking logarithm, eq. (104) reads as:

$$
\begin{equation*}
\ln \left(\frac{L^{j}}{L}\right)_{t}=\frac{1}{\epsilon+1} \ln \vartheta^{j}+\frac{\epsilon}{\epsilon+1} \ln \beta_{t}^{j} \tag{105}
\end{equation*}
$$

Totally differentiating (105) and denoting the rate of change of the variable with a hat, we find that the change in hours worked in sector $j$ caused by labor reallocation across sectors is driven by the change in the fraction $\beta^{j}$ of the labor's share of aggregate output accumulating to labor in sector $j$ :

$$
\begin{equation*}
\hat{L}_{t}^{j}-\hat{L}_{t}=\gamma \hat{\beta}_{t}^{j} \tag{106}
\end{equation*}
$$

where $\gamma=\frac{\epsilon}{\epsilon+1}$.
We use panel data to estimate (106). Including country fixed effects captured by country dummies, $f_{i}$, and common macroeconomic shocks by year dummies, $f_{t}$, (106) can be rewritten as follows:

$$
\begin{equation*}
\hat{L}_{i t}^{j}-\hat{L}_{i t}=f_{i}+f_{t}+\gamma_{i} \hat{\beta}_{i t}^{j}+\nu_{i t}^{j} \tag{107}
\end{equation*}
$$

where $\gamma_{i}=\frac{\epsilon_{i}}{\epsilon_{i}+1}$ and $\beta_{i t}^{j}$ is given by (103); $j$ indexes the sector, $i$ the country, and $t$ indexes time. The LHS and RHS variables are defined as follows:

$$
\begin{equation*}
\hat{L}_{i t}=\sum_{j=1}^{M} \beta_{i, t-1}^{j} \hat{L}_{i, t}^{j} . \tag{108}
\end{equation*}
$$

and

$$
\begin{equation*}
\beta_{i t}^{j}=\frac{s_{L, i}^{j} P^{j} Y_{i t}^{j}}{\sum_{j=1}^{M} s_{L, i}^{j} P_{i t}^{j} Y_{i t}^{j}}, \tag{109}
\end{equation*}
$$

where $s_{L, i}^{j}$ is the LIS in sector $j$ in country $i$ which is averaged over 1970-2013. When exploring empirically (107), the coefficient $\gamma$ is alternatively assumed to be identical, i.e., $\gamma_{i}=\gamma$, or to vary across countries. The LHS term of (107), i.e., $\hat{L}_{i t}^{j}-\hat{L}_{i t}$, gives the percentage change in hours worked in sector $j$ driven by the pure reallocation of labor across sectors.

To determine (108) we proceed as follows. Approximate changes in aggregate labor with differentials, we get:

$$
\begin{equation*}
\mathrm{d} L_{t} \equiv L_{t}-L_{t-1}=\left(L_{t-1}^{H}\right)^{\frac{1}{\epsilon}}\left(L_{t-1}\right)^{-\frac{1}{\epsilon}} \mathrm{~d} L_{t}^{H}+\left(L_{t-1}^{N}\right)^{\frac{1}{\epsilon}}\left(L_{t-1}\right)^{-\frac{1}{\epsilon}} \mathrm{~d} L_{t}^{N} . \tag{110}
\end{equation*}
$$

Expressing (110) in percentage changes and inserting $\left(\frac{L^{j}}{L}\right)^{\frac{\epsilon+1}{\epsilon}}=\beta^{j}$, we have:

$$
\begin{align*}
\hat{L}_{t} \equiv \frac{L_{t}-L_{t-1}}{L_{t-1}} & =\left(\frac{L_{t-1}^{H}}{L_{t-1}}\right)^{\frac{\epsilon+1}{\epsilon}} \hat{L}_{t}^{H}+\left(\frac{L_{t-1}^{N}}{L_{t-1}}\right)^{\frac{\epsilon+1}{\epsilon}} \hat{L}_{t}^{N} \\
& =\beta_{t-1}^{H} \hat{L}_{t}^{H}+\beta_{t-1}^{N} \hat{L}_{t}^{N} . \tag{111}
\end{align*}
$$

According to eq. (111), the percentage change in total hours worked, $\hat{L}_{t}$, can be approximated by a weighted average of changes in sectoral hours worked $\hat{L}_{t}^{j}$ (in percentage), the weight being equal to $\beta_{t-1}^{j}$.

## F.3.2 Data Description

Data are taken from EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases. EU KLEMS data provide yearly information for the period 1970-2013 (except for JPN: 1974-2013) for 15 countries of our sample (AUS, AUT, BEL, DEU, DNK, ESP, FIN, FRA, GBR, IRL, ITA, JPN, NLD, SWE and USA). For CAN and NOR, annual sectoral data stems from the STAN database. To classify hours worked and value added as traded or non-traded, we adopt the classification described in Appendix D. We provide more details below about the data used to estimate equation (107):

- Sectoral hours worked, $L^{j}(j=H, N)$ : total hours worked by persons engaged in sector $j$ (H_EMP). Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases.
- Sectoral value added, $P^{j} Y^{j}(j=H, N)$ : value added at current prices in millions of national currency in sector $j$ (VA). Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases.
- Sectoral labor income share, $s_{L}^{j}(j=H, N)$ : labor compensation in sector $j$ (LAB) over value added at current prices (VA) averaged over the period 1970-2013 (1974-2013 for JPN). Sources: EU KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases.
By combining $s_{L}^{j}$ and $P^{j} Y^{j}$, we can construct time series $\beta^{j}$ as defined by (109).

Table 15: Estimates of the Elasticity of Labor Supply across Sectors ( $\epsilon$ )

| Country | $\hat{\epsilon}_{i}$ |
| :---: | :---: |
| AUS | $0_{(3.20)}^{0.375^{a}}$ |
| AUT | $1_{(3.00)}^{1.103^{a}}$ |
| BEL | ${ }_{(3.57)}^{0.610^{a}}$ |
| CAN | $\underbrace{0.390^{a}}_{(4.12)}$ |
| DEU | ${ }_{(3.52)}^{1.012^{a}}$ |
| DNK | $\underbrace{0.286^{a}}_{(2.50)}$ |
| ESP | $1_{(3.73)}^{1.015^{a}}$ |
| FIN | ${\underset{(4.39)}{0.431^{a}}}^{a}$ |
| FRA | ${ }_{(2.40)^{a}}$ |
| GBR | ${\underset{(3.91)}{0.601}}^{a}$ |
| IRL | $0_{(3.74)}$ |
| ITA | ${\underset{(3.01)}{1.664^{a}}}^{a}$ |
| JPN | $0_{(3.55)}$ |
| NLD | $\underset{(2.05)}{0.219^{b}}$ |
| NOR | $\underset{(0.34)}{0.011}$ |
| SWE | $\underset{(4.28)}{0.534^{a}}$ |
| USA | ${ }_{(1.83)}^{3.222^{c}}$ |
| Countries | 17 |
| Observations | 1456 |
| Data coverage | 1971-2013 |
| Country fixed effects | yes |
| Time trend | no |
| Notes: ${ }^{a},{ }^{b}$ and ${ }^{c}$ denote significance at $1 \%, 5 \%$ and $10 \%$ levels. Heteroskedasticity and autocorrelation consistent t-statistics are reported in parentheses. |  |

## F.3.3 Panel Data Estimates of $\epsilon$

The parameter we are interested in, the degree of substitutability of hours worked across sectors, is given by $\epsilon_{i}=\gamma_{i} /\left(1-\gamma_{i}\right)$. In the regression below, coefficient $\gamma_{i}$ is assumed to be different across countries, i.e., $\gamma_{i} \neq \gamma_{i^{\prime}}$ for $i \neq i^{\prime}$. The sample is running from 1971 to 2013.

Empirical results reported in Table 15 are consistent with $\epsilon>0$. Among the 17 countries, we find that 16 have statistically significant (at the $10 \%$ level) estimates of $\epsilon$. We find that the degree of substitutability of hours worked across sectors ranges from a low of 0.01 for NOR to a high of 3.222 for USA.

## F. 4 Sectoral Elasticity of Substitution between Capital and Labor

Our empirical results have established that the effects of a permanent rise in TFP in tradables relative to non-tradables on sectoral LIS depends on the elasticity of substitution between capital and labor. We detail below the estimation strategy of the elasticity of substitution between capital and labor, $\sigma^{j}$, for sector $j=H, N$.

## F.4.1 Empirical Strategy

We assume CES productions::

$$
\begin{equation*}
Y_{t}^{j}=\left[\gamma^{j}\left(A_{t}^{j} L_{t}^{j}\right)^{\frac{\sigma^{j}-1}{\sigma^{j}}}+\left(1-\gamma^{j}\right)\left(B_{t}^{j} K_{t}^{j}\right)^{\frac{\sigma^{j}-1}{\sigma^{j}}}\right]^{\frac{\sigma^{j}}{\sigma^{j}-1}} \tag{112}
\end{equation*}
$$

where $\sigma^{j}$ is the constant elasticity of substitution between capital and labor in sector $j=H, N, \gamma^{j}$ is the weight of labor in the production technology, $A_{t}^{j}$ and $B_{t}^{j}$ denote the level of efficiency of labor and capital, respectively. Variations over time of $A_{t}^{j}$ and $B_{t}^{j}$ capture labor- and capital-augmenting
technological change. Note that we allow factors efficiency to differ across sectors, i.e. $\hat{A}^{H} \neq \hat{A}^{N}$ and $\hat{B}^{H} \neq \hat{B}^{N}$. When assuming biased-technological change, the identification of the parameter of interest, $\sigma^{j}$, turns to be problematic as the elasticity and biased technical change cannot be simultaneously identified given time series of output, inputs and factors shares. To circumvent this problem, we assume that labor- and capital-augmenting technological changes grow at constant rate:

$$
\begin{align*}
& A_{t}^{j}=A_{0}^{j} e^{a^{j} t}  \tag{113a}\\
& B_{t}^{j}=B_{0}^{j} e^{b^{j} t} \tag{113b}
\end{align*}
$$

where $a^{j}$ and $b^{j}$ denote the constant growth rates of labor- and capital-augmenting technical progress and $A_{0}^{j}$ and $B_{0}^{j}$ are the initial levels of technology.

We assume perfect mobility of capital across sectors and define the capital rental cost by $R$ with $R=R^{H}=R^{N}$. Labor is imperfectly mobile across sectors and the wage rate in sector $j=H, N$ is denoted $W^{j}$. Profit maximization by firms in a competitive framework implies the first-order conditions:

$$
\begin{gather*}
P_{t}^{j} \gamma^{j}\left(A_{t}^{j}\right)^{\frac{\sigma^{j}-1}{\sigma^{j}}}\left(L_{t}^{j}\right)^{-\frac{1}{\sigma^{j}}}\left(Y_{t}^{j}\right)^{\frac{1}{\sigma j}}=W_{t}^{j},  \tag{114a}\\
P_{t}^{j}\left(1-\gamma^{j}\right)\left(B_{t}^{j}\right)^{\frac{\sigma^{j}-1}{\sigma^{j}}}\left(K_{t}^{j}\right)^{-\frac{1}{\sigma^{j}}}\left(Y_{t}^{j}\right)^{\frac{1}{\sigma^{j}}}=R_{t}, \tag{114b}
\end{gather*}
$$

where $P^{j}$ is value added price deflator in sector $j$. Taking logarithm of (114a)-(114b) and rearranging gives:

$$
\begin{align*}
\ln \left(Y_{t}^{j} / L_{t}^{j}\right) & =\alpha_{1}+\left(1-\sigma^{j}\right) a^{j} t+\sigma_{j} \ln \left(W_{t}^{j} / P_{t}^{j}\right),  \tag{115a}\\
\ln \left(Y_{t}^{j} / K_{t}^{j}\right) & =\alpha_{2}+\left(1-\sigma^{j}\right) b^{j} t+\sigma_{j} \ln \left(R_{t} / P_{t}^{j}\right), \tag{115b}
\end{align*}
$$

where $\alpha_{1}=\left[\left(1-\sigma^{j}\right) \ln A_{0}^{j}-\sigma^{j} \ln \gamma^{j}\right]$ and $\alpha_{2}=\left[\left(1-\sigma^{j}\right) \ln A_{0}^{j}-\sigma^{j} \ln \left(1-\gamma^{j}\right)\right]$ are constants. These equations represent the first-order conditions (FOC) with respect to labor and capital and can be interpreted as describing the firms' demand for labor and capital respectively. We estimate the elasticity of substitution between capital and labor in sector $j=H, N$ from FOCs (115a)-(115b) in panel format on annual data. Adding an error term and controlling for country fixed effects yields our testable regressions:

$$
\begin{align*}
\ln \left(Y_{i t}^{j} / L_{i t}^{j}\right) & =\alpha_{1 i}+\lambda_{1 i} t+\sigma_{i}^{j} \ln \left(W_{i t}^{j} / P_{i t}^{j}\right)+u_{i t},  \tag{116a}\\
\ln \left(Y_{i t}^{j} / K_{i t}^{j}\right) & =\alpha_{2 i}+\lambda_{2 i} t+\sigma_{i}^{j} \ln \left(R_{i t} / P_{i t}^{j}\right)+v_{i t} \tag{116b}
\end{align*}
$$

where $i$ and $t$ index country and time and $u_{i t}$ and $v_{i t}$ are i.i.d. error terms. Country fixed effects are represented by dummies $\alpha_{1 i}$ and $\alpha_{2 i}$, and country-specific trends are captured by $\lambda_{1 i}$ and $\lambda_{2 i}$.

To estimate the elasticity of substitution between capital and labor for tradables and nontradables, we follow closely the approach suggested by Antràs [2004] which derives alternative specifications based on factor demand functions. ${ }^{44}$ This approach possesses three particulary attractive properties. First, the econometric specifications allow for biased technological change. The choice of the specification determines the type of technological change which can be captured within the framework of econometric estimation. For instance, in case of the FOC for labor, capital biased technological change drops out. Therefore, labor-augmenting technological change can be identified, together with $\sigma^{j}$, from equation (115a). Second, it allows for a clear treatment of the nonstationary nature of the data involved in the estimation. Regressions (116a) and (116b) feature two trends governed by $a^{j}$ and $b^{j}$ and several variables which potentially follow nonstationary processes $\left(Y_{i t}^{j}\right) L_{i t}^{j}$, $Y_{i t}^{j} / K_{i t}^{j}, W_{i t}^{j} / P_{i t}^{j}$ and $\left.R_{i t} / P_{i t}^{j}\right)$. Following Antràs [2004], we tackle this nonstationary issue by applying the fully modified OLS (FMOLS) procedure for the cointegrated panel proposed by Pedroni ([2000], [2001]) to equations (116a) and (116b). FMOLS is a nonparametric approach to adjust for the effects of endogenous regressors and serial correlation. Another econometric problem when estimating (116a) and (116b) is the potential endogeneity of regressors. As shown by Pedroni ([2000], [2001]), using FMOLS technique can address this issue too as this estimator is also extremely accurate in panels with heterogeneous serial correlation and endogenous regressors. Third, employing Monte Carlo experiments, León-Ledesma et al. [2010] compare the different approaches for estimating the elasticity of substitution between capital and labor (single equation based on FOCs, system,

[^31]linear, non linear and normalization). Their evidence suggests that provided that the true value of $\sigma$ is below 1.3, estimates of both the elasticity of substitution and technical change are close to their true values when the FOC with respect to labor is used (eq. (115a)). Below we report sectoral elasticities well below unity when using the FOCs. The panel estimates of $\sigma^{H}$ and $\sigma^{N}$ obtained from the FOC with respect to labor (capital resp.) are 0.687 and 0.716 ( 0.489 and 0.467 resp.). Our results thus lend credence to the use of specifications (116a) and (116b) based on the FOCs as a way to obtain precise estimates of the elasticity of substitution between capital and labor at the sectoral level. In addition, results of León-Ledesma et al. [2010] show that the capital FOC (eq. (116b)) performs worse than labor FOC (eq. (116a)) as estimates of $\sigma$ are sensitive to measurement errors and endogeneity in the capital stock. Consequently, in the following, when presenting our own estimates of $\sigma^{j}$ for both sectors, the labor demand equation, i.e. regression (116a), is preferred.

An alternative way to recover the CES production parameters is the supply-side system method (see Klump et al. [2007] and León-Ledesma et al. [2010]). This approach consists of the joint nonlinear estimation of a three-equation system combining the CES production function (equation (112) in $\log$ form) together with the first-order conditions for the optimal choices of labor and capital, i.e. FOCs (115a)-(115b). Despite system approach's appealing features, we stick to the single-equation methodology developed by Antràs [2004] because, in our context, this estimation method has several advantages over the three-equation system advocated by León-Ledesma et al. [2010]. First, the supply-side system method has the disadvantage that it does rely on non-linear estimations, so the results are obtained numerically and sensitive to the choice of initial values (especially in the nonnormalized system). By contrast, we estimate equations (116a) and (116b) with the FMOLS approach which avoids such numerical computations. Second, estimation of the three-equation system involves the estimation of a large number of parameters which may affect estimation accuracy. Instead, the single-equation is a more parsimonious specification as it reduces considerably the number of estimated coefficients and thus is particularly well suited when estimating the elasticity of substitution at the sectoral level.

## F.4.2 Data Description

Estimation of equations (116a) and (116b) requires data for each sector $j=H, N$ on value added at constant prices, $Y^{j}$, hours worked, $L^{j}$, capital stock, $K^{j}$, value added deflator, $P^{j}$, wage rate, $W^{j}$ and capital rental cost, $R$. We describe below the time series we use in estimating $\sigma^{j}$ (codes in EU KLEMS/STAN are reported in parentheses):

- Sectoral value added, $Y^{j}(j=H, N)$ : value added at constant prices in sector $j$ (VA_QI). Sources: KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases. Data coverage: 1970-2013 except for JPN 1974-2013.
- Sectoral hours worked, $L^{j}$ : total hours worked by persons engaged in sector $j$ (H_EMP). Sources: KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases. Data coverage: 1970-2013 except for JPN 1974-2013.
- Sectoral capital stock, $K^{j}$ : aggregate capital stocks are estimated from the perpetual inventory approach by using real gross capital formation from OECD National Accounts Database [2017] (data in millions of national currency, constant prices) and assuming a depreciation rate of $5 \%$. Following Garofalo and Yamarik [2002], the capital stock is then allocated to traded and non-traded industries by using the sectoral value added share, i.e., $K^{j}=\omega_{t}^{Y, j} K$ where $\omega_{t}^{Y, j}$ is the value added share at current prices. Sources: KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases. Data coverage: 1970-2013 except for JPN 1974-2013.
- Sectoral value added price deflator, $P^{j}$ : value added at current prices (VA) over value added at constant prices (VA_QI) in sector $j$. Sources: KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases. Data coverage: 1970-2013 except for JPN 1974-2013.
- Sectoral nominal wage, $W^{j}$ : labor compensation in sector $j(\mathrm{LAB})$ over total hours worked by persons engaged (H_EMP) in that sector. Labor compensation is total labor costs that include compensation of employees and labor income of the self-employed and other entrepreneurs. Sources: KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases. Data coverage: 1970-2013 except for JPN 1974-2013.
- Aggregate rental price of capital, $R$ : capital income over capital stock $K$ in the total economy. Capital income is derived as nominal value added (VA) minus labor compensation (LAB). Sources: KLEMS ([2011], [2017]) and OECD STAN ([2011], [2017]) databases. Data coverage: 1970-2013 except for JPN 1974-2013.
The data construction merits further discussion. First, sectoral wages do not equalize $\left(W^{H} \neq\right.$ $W^{N}$ ) while the sectoral rental costs of capital equalize ( $R^{H}=R^{N} \equiv R$ ). These choices are consistent with our theoretical model in which physical capital is perfectly mobile across sectors and the
presence of mobility costs implies that both sectors do not pay the same wage. Second, when calculating sectoral wages, labor compensation includes total labor costs (wages, salaries and all other costs of employing labour which are borne by the employer) as well as the income of self-employed. Treating all self-employed income as labor income allows us to obtain a consistent measurement of the labor share (Gollin [2002]). As a robustness check, we also split self-employed income into capital and labor income based on the assumption that the labor income of the self-employed has the same mix of labor and capital income as the rest of the economy (in other words, total labor compensation comprises the labor compensation of employees and the self-employed income scaled by the labor share of employees only). This adjustment turns out to have only a marginal effect on the estimates of $\sigma^{j}$ (results available upon request).


## F.4.3 Empirical Results

Table 16 reports a summary of the panel unit root tests we performed on each of the series involved in the estimation of cointegrating equations. As is clear from Table 16, except for the LLC test applied to the variable $\ln \left(W^{N} / P^{N}\right)$, for none of the eight series do the LLC, Breitung, IPS and Madalla-Wu tests reject the hypothesis of a unit root at the $5 \%$ level of significance. ${ }^{45}$ As a robustness check, we also consider the test developed by Hadri of the null that the time series for each cross section is stationary against the alternative of a unit root in the panel data. We reach the same conclusion and conclude that all eight series are nonstationary and integrated of order one.

Table 16: Panel Unit Root Tests (p-values)

|  | LLC <br> (t-stat) | Breitung <br> (t-stat) | IPS <br> (W-stat) | MW <br> $($ ADF $)$ | Hadri <br> $\left(Z_{\mu}\right.$-stat) |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $\ln \left(Y^{H} / L^{H}\right)$ | 0.990 | 1.000 | 1.000 | 0.970 | 0.000 |
| $\ln \left(Y^{N} / L^{N}\right)$ | 0.286 | 1.000 | 0.607 | 0.225 | 0.000 |
| $\ln \left(Y^{H} / K^{H}\right)$ | 0.998 | 1.000 | 0.999 | 0.981 | 0.000 |
| $\ln \left(Y^{N} / K^{N}\right)$ | 0.960 | 0.999 | 0.990 | 0.994 | 0.000 |
| $\ln \left(W^{H} / P^{H}\right)$ | 0.636 | 1.000 | 0.758 | 0.735 | 0.000 |
| $\ln \left(W^{N} / P^{N}\right)$ | 0.006 | 0.209 | 0.716 | 0.643 | 0.000 |
| $\ln \left(R / P^{H}\right)$ | 0.866 | 1.000 | 0.679 | 0.498 | 0.000 |
| $\ln \left(R / P^{N}\right)$ | 0.999 | 0.999 | 0.791 | 0.218 | 0.000 |
| Notes: For all tests, except for Hadri [2000], the null of a unit root is not rejected if |  |  |  |  |  |
| p-value $\geq 0.05$ at a $5 \%$ significance level. For Hadri $[2000]$, the null of stationarity |  |  |  |  |  |
| is rejected if p-value $\leq 0.05$ at a $5 \%$ significance level. |  |  |  |  |  |

Table 17 presents the results from parametric and non parametric cointegration tests developed by Pedroni ([1999], [2004]). All statistics hinge on testing the stationarity of the residuals of equations (116a) and (116b). As is apparent from Table 17 the results are conclusive: for at least five of the seven tests the null hypothesis of no cointegration between $\ln \left(Y^{j} / L^{j}\right)\left(\ln \left(Y^{j} / K^{j}\right)\right.$ resp.) and $\ln \left(W^{j} / P^{j}\right)\left(\ln \left(R^{/} P^{j}\right)\right.$ resp.) is rejected for all four specifications at the $10 \%$ significance level. As pointed out by Pedroni [2004], the group-mean parametric t-test is more powerful than other tests in finite samples. Based on the statistic parametric $t$ (reported in the last row), the null hypothesis of zero cointegrating vectors is clearly rejected at the $10 \%$ significance level for any of the four specifications.

Table 18 summarizes FMOLS estimates elasticity of substitution between capital and labor for the tradables and non-tradables sectors. Results for the labor (capital resp.) demand equation are presented in columns 2 and 4 (columns 1 and 3 resp.). ${ }^{46}$ As noted previously, on the basis of the extensive Monte Carlo simulations provided by León-Ledesma et al. [2010], the FOC for labor specification (equation (116a)) is preferred to the FOC for capital specification (equation (116b)) because in the former case the elasticity of substitution is estimated quite precisely. To ease discussion, we therefore restrict the discussion to the results obtained with labor demand equation. For the whole sample, the FMOLS estimate of $\sigma^{H}$ from regression (116a) (see column 2) gives a value of 0.687 . The estimated coefficient is statistically different from zero with a t -statistic of 24.70 . Furthermore, the null hypothesis of a panel unit elasticity is strongly rejected at the $5 \%$ significance level. However, there is substantial evidence of parameter heterogeneity across countries inside the sample. The vast majority (16 out of 17) of the individual FMOLS estimated coefficients $\sigma^{H}$

[^32]Table 17: Panel Cointegration Tests (p-values)

| Dependent variable | $\ln \left(Y^{H} / L^{H}\right)$ | $\ln \left(Y^{H} / K^{H}\right)$ | $\ln \left(Y^{N} / L^{N}\right)$ | $\ln \left(Y^{N} / K^{N}\right)$ |
| :--- | :---: | :---: | :---: | :---: |
| Explanatory variable | $\ln \left(W^{H} / P^{H}\right)$ | $\ln \left(R / P^{H}\right)$ | $\ln \left(W^{N} / P^{N}\right)$ | $\ln \left(R / P^{N}\right)$ |
|  | Eq. (116a) | Eq. $(116 \mathrm{~b})$ | Eq. $(116 \mathrm{a})$ | Eq. $(116 \mathrm{~b})$ |
| Panel tests | 0.000 |  |  |  |
| Non-parametric $\nu$ | 0.000 | 0.021 | 0.170 | 0.030 |
| Non-parametric $\rho$ | 0.000 | 0.055 | 0.073 | 0.010 |
| Non-parametric $t$ | 0.000 | 0.043 | 0.050 | 0.002 |
| Parametric $t$ |  |  | 0.054 | 0.003 |
| Group-mean tests | 0.010 | 0.420 | 0.145 | 0.012 |
| Non-parametric $\nu$ | 0.000 | 0.147 | 0.059 | 0.001 |
| Non-parametric $t$ | 0.000 | 0.065 | 0.064 | 0.001 |
| Parametric $t$ |  |  |  |  |

Notes: the null hypothesis of no cointegration is rejected if the p-value is below $0.05(0.10$ $\overline{\text { resp.) }}$ at $5 \%$ ( $10 \%$ resp.) significance level.
are positive. The only exception is IRL for which $\sigma^{H}$ is estimated to be negative. Although the estimated value for IRL is not statistically different from zero, this negative value is difficult to justify by economic theory. In order to avoid inconsistent estimates of $\sigma^{H}$, we replace the negative value IRL with the one obtained when using the demand for capital (see column 1), namely we set $\sigma_{I R L}^{H}=0.737$. Focusing only on countries with positive FMOLS estimates of $\sigma^{H}$, we find that all have statistically significant coefficients at a standard threshold, ranging from a low of 0.417 (DNK) to a high of 1.164 (JPN). Overall, out the 16 positive estimates in column 2, 14 are lower than one (exceptions are ESP and JPN with $\sigma^{H}=1.033$ and $\sigma^{H}=1.164$ respectively); out these 14 estimates, 8 are significantly below one at the $5 \%$ level: for AUT, BEL, ESP, FIN, FRA, ITA, JPN and NLD the null hypothesis of a unit elasticity is rejected at the $5 \%$ significance level. Columns 3 and 4 show FMOLS estimates for the non-traded sector. For labor demand (column 4), we find $\sigma^{N}=0.716$ in the entire panel. This value is significantly different than zero and lower than one at the $1 \%$ level. The estimates range from 0.194 (SWE) to 1.298 (AUT). The vast majority ( 15 out of 17) of the individual FMOLS estimated coefficients are statistically significant except for ITA and SWE. Note also that the coefficient $\sigma^{N}$ is found to be larger than one in only three countries (AUT, BEL and DNK). Among these three countries, the null hypothesis of a unit elasticity is not rejected at the $5 \%$ significance level in BEL and DNK. Finally, for 10 out of the 17 countries, the results lead to a rejection of the null hypothesis of a unit elasticity of substitution in the non-traded sector at the $5 \%$ significance level (AUS, AUT, CAN, ESP, FIN, GBR, ITA, NLD, NOR and SWE)

Overall, we find that, controlling for biased technological change, the elasticity of substitution between capital and labor for traded and non-traded sectors is lower than one, implying that capital and labor are less substitutable than a Cobb-Douglas production function. This result is consistent with previous estimates found in the literature (see Antràs [2004], Klump et al. [2007] and LeónLedesma et al. [2010] among others).

## G More VAR Results and Robustness Check

In this section, we provide more VAR results and conduct several robustness checks. Because in the main text, all variables enter in growth rate, Appendix G. 1 shows panel unit tests for all variables considered in the empirical analysis. For reason of space, in the main text, we report results of selected sectoral variables and do not show aggregate effects. Appendix G. 2 shows aggregate effects of a technology shock biased toward the traded sector and also reports results for all variables and all VAR models mentioned in the main text. Due to data availability, we use annual data for eleven 1-digit ISIC-rev. 3 industries that we classify as tradables or non-tradables. Because at this level of disaggregation, the classification is somewhat ambiguous as some sub-industries could be classified as tradables while other sub-industries are treated as non-tradables, Appendix G. 3 investigates the sensitivity of our empirical results to the classification of industries as tradables or non-tradables. Since the traded and the non-traded sector are made up of sub-sectors, we explore in Appendix G. 4 whether our results for the LIS are not driven by changes in value added shares of sub-sectors. In the main text, we compute the LIS like Gollin [2002], i.e., labor compensation is defined as the sum of compensation of employees plus compensation self-employed. Since there exists alternative ways in constructing labor compensation, we explore empirically in Appendix G. 5 whether the evidence on redistributive effects we document in the main text are robust to alternative measures of the LIS. In Appendix G.6, we address a potential concern related to the fact that various VAR models

Table 18: FMOLS Estimates of the Sectoral Elasticity of Substitution between Capital and Labor ( $\sigma^{j}$ )

| Country <br> Dependent variable <br> Explanatory variable | Tradables ( $\sigma^{H}$ ) |  | Non-Tradables $\left(\sigma^{N}\right)$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  | $\ln \left(Y^{H} / K^{H}\right)$ | $\ln \left(Y^{H} / L^{H}\right)$ | $\ln \left(Y^{N} / K^{N}\right)$ | $\ln \left(Y^{N} / L^{N}\right)$ |
|  | $\ln \left(R / P^{H}\right)$ | $\ln \left(W^{H} / P^{H}\right)$ | $\ln \left(R / P^{N}\right)$ | $\ln \left(W^{N} / P^{N}\right)$ |
|  | (1) | (2) | (3) | (4) |
| AUS | $0_{(6.67)}^{0.607^{a}}$ | ${\underset{(3.79)}{0.474^{a}}}^{1}$ | $\underset{(4.03)}{0.459^{a}}$ | $0_{(5.69)}^{0.529^{a}}$ |
| AUT | ${\underset{(2.65)}{0.235^{a}}}^{a}$ | $\underbrace{0.774^{a}}_{(6.04)}$ | $\underset{(1.22)}{0.105}$ | ${ }_{(13.04)}$ |
| BEL | $\underset{(3.01)}{0.389^{a}}$ | $0_{(8.89)}^{0.829^{a}}$ | ${ }_{(7.37)}^{0.266^{a}}$ | $\underset{(7.10)}{1.069^{a}}$ |
| CAN | ${\underset{(3.99)}{0.595^{a}}}^{(3)}$ | ${\underset{(2.94)}{0.480^{a}}}^{a}$ | ${ }_{(8.62)}^{0.855^{a}}$ | $0_{(7.65)}^{0.668^{a}}$ |
| DEU | $\underset{(-0.68)}{-0.123}$ | ${\underset{(8.56)}{0.642^{a}}}^{(8)}$ | ${\underset{(8.88)}{0.512^{a}}}^{a}$ | ${\underset{(6.97)}{0.987^{a}}}^{a}$ |
| DNK | $\underset{(1.84)}{0.267^{c}}$ | ${\underset{(4.32)}{0.417^{a}}}^{a}$ | ${ }_{(7.83)}^{0.502^{a}}$ | ${ }_{(6.74)}^{1.282^{a}}$ |
| ESP | $0_{(7.11)}^{0.747^{a}}$ | ${ }_{(10.62)}^{1.033^{a}}$ | ${ }_{(3.65)}^{0.682^{a}}$ | ${ }_{(3.35)}^{0.476^{a}}$ |
| FIN | $\underbrace{0.249^{a}}_{(2.90)}$ | $\underset{(1.98)}{0.764^{b}}$ | $\underbrace{0.560^{a}}_{(6.64)}$ | $\underbrace{0.794^{a}}_{(8.30)}$ |
| FRA | $\underbrace{0.267^{a}}_{(4.82)}$ | $\underbrace{0.870^{a}}_{(4.82)}$ | ${\underset{(11.04)}{0.294^{a}}}^{a}$ | $0_{(4.21)}^{0.916^{a}}$ |
| GBR | $\underset{(0.95)}{0.242}$ | ${ }_{(6.42)}^{0.603^{a}}$ | $\underset{(0.08)}{0.008}$ | $\underset{(2.68)}{0.561^{a}}$ |
| IRL | $\underset{(18.46)}{0.737^{a}}$ | $\underset{(-0.50)}{-0.125}$ | ${ }_{(5.73)}^{0.762^{a}}$ | $\underset{(3.16)}{0.627^{a}}$ |
| ITA | $\underbrace{0.506^{a}}_{(3.82)}$ | $\underset{(8.80)}{0.837^{a}}$ | ${\underset{(3.23)}{0.471}}^{a}$ | $\begin{aligned} & 0.259 \\ & (1.51) \end{aligned}$ |
| JPN | ${ }_{(8.16)}^{0.622^{a}}$ | ${ }_{(6.73)}^{1.164^{a}}$ | $\underset{(7.97)}{0.417^{a}}$ | $\underset{(2.47)}{0.635^{b}}$ |
| NLD | ${ }_{(5.13)}^{0.645^{a}}$ | ${ }_{(5.98)}^{0.910^{a}}$ | ${\underset{(9.14)}{0.287^{a}}}^{a}$ | ${\underset{(3.74)}{0.444^{a}}}^{a}$ |
| NOR | $0_{(4.60)}^{0.798^{a}}$ | ${ }_{(4.39)}^{0.629^{a}}$ | $\underset{(10.17)}{0.653^{a}}$ | $0_{(4.72)}^{0.556^{a}}$ |
| SWE | $\underset{(0.35)}{0.052}$ | ${ }_{(8.56)}^{0.607^{a}}$ | ${ }_{(6.71)}^{0.378^{a}}$ | $\underset{(0.95)}{0.194}$ |
| USA | ${ }_{(6.85)}^{1.485^{a}}$ | ${ }_{(9.51)}^{0.766^{a}}$ | $\underbrace{0.723^{a}}_{(6.64)}$ | $\begin{gathered} 0.876^{a} \\ \hline \end{gathered}$ |
| Whole Sample | $\begin{gathered} 0.489^{a} \\ (19.56) \\ \hline \end{gathered}$ | $\begin{aligned} & 0.687^{a} \\ & (24.70) \\ & \hline \end{aligned}$ | $\begin{aligned} & 0.467^{a} \\ & (26.42) \\ & \hline \end{aligned}$ | $\begin{gathered} 0.716^{a} \\ (21.16) \\ \hline \end{gathered}$ |
| Countries | 17 | 17 | 17 | 17 |
| Observations | 745 | 745 | 745 | 745 |
| Data coverage | 1970-2013 | 1970-2013 | 1970-2013 | 1970-2013 |
| Country fixed effects | yes | yes | yes | yes |
| Time trend | yes | yes | yes | yes |

Notes: ${ }^{a},{ }^{b}$ and ${ }^{c}$ denote significance at $1 \%, 5 \%$ and $10 \%$ levels. Heteroskedasticity and autocorrelation consistent t-statistics are reported in parentheses.
could identify different structural technology shocks. Finally, since we split the gross capital stock into traded and non-traded industries by using sectoral valued added shares, in Appendix G.7, we conduct a robustness check by taking time series for sectoral capital stock from KLEMS.

## G. 1 Panel Unit Root Tests

When estimating alternative VAR specifications, all variables enter in growth rates. In order to support our assumption of $I(1)$ variables, we ran panel unit root tests displayed in Table 19. We consider five panel unit root tests among the most commonly used in the literature: Levin, Lin and Chu ([2002], hereafter LLC), Breitung [2000], Im, Pesaran and Shin ([2003], hereafter IPS), Maddala and Wu ([1999], hereafter MW) and Hadri [2000]. All tests, with the exception of Hadri [2000], consider the null hypothesis of a unit root against the alternative that some members of the panel are stationary. Additionally, they are designed for cross sectionally independent panels. LLC and IPS are based on the use of the Augmented Dickey-Fuller test (ADF hereafter) to each individual series of the form $\Delta x_{i, t}=\alpha_{i}+\rho_{i} x_{i, t-1}+\sum_{j=1}^{q_{i}} \theta_{i, j} \Delta x_{i, t-j}+\varepsilon_{i, t}$, where $\varepsilon_{i, t}$ are assumed to be i.i.d. (the lag length $q_{i}$ is permitted to vary across individual members of the panel). Under the homogenous alternative the coefficient $\rho_{i}$ in LLC is required to be identical across all units ( $\rho_{i}=\rho, \forall i$ ). IPS relax this assumption and allow for $\rho_{i}$ to be individual specific under the alternative hypothesis. MW propose a Fisher type test based on the p-values from individual unit root statistics (ADF for instance). Like IPS, MW allow for heterogeneity of the autoregressive root $\rho_{i}$ under the alternative. We also apply the pooled panel unit root test developed by Breitung [2000] which does not require bias correction factors when individual specific trends are included in the ADF type regression. This is achieved by an appropriate variable transformation. As a sensitivity analysis, we also employ the test developed by Hadri [2000] which proposes a panel extension of the Kwiatkowski et al. [1992] test of the null that the time series for each cross section is stationary against the alternative of a unit root in the panel data. Breitung' and Hadri's tests, like LLC's test, are pooled tests against the homogenous alternative. ${ }^{47}$

Table 19: Panel Unit Root Tests (p-values)

|  | LLC <br> (t-stat) | Breitung <br> (t-stat) | IPS <br> (W-stat) | MW <br> $($ ADF $)$ | Hadri <br> $\left(Z_{\mu}\right.$-stat $)$ |
| :--- | :---: | :---: | :---: | :---: | :---: |
| $\ln (Z)$ | 0.977 | 0.000 | 0.999 | 0.999 | 0.000 |
| $\ln \left(Y_{R}\right)$ | 0.979 | 0.999 | 0.959 | 0.941 | 0.000 |
| $\ln (L)$ | 0.173 | 0.996 | 0.941 | 0.950 | 0.000 |
| $\ln \left(W / P_{C}\right)$ | 0.000 | 1.000 | 0.294 | 0.002 | 0.000 |
| $\ln \left(W^{H} / W\right)$ | 0.910 | 0.094 | 0.882 | 0.945 | 0.000 |
| $\ln \left(W^{N} / W\right)$ | 0.232 | 0.971 | 0.415 | 0.349 | 0.000 |
| $\ln \left(Y^{H} / Y_{R}\right)$ | 0.472 | 0.924 | 0.827 | 0.859 | 0.000 |
| $\ln \left(L^{H} / L\right)$ | 1.000 | 0.012 | 0.998 | 0.999 | 0.000 |
| $\ln \left(Y^{N} / Y_{R}\right)$ | 0.252 | 0.109 | 0.549 | 0.500 | 0.000 |
| $\ln \left(L^{N} / L\right)$ | 0.885 | 1.000 | 0.998 | 0.996 | 0.000 |
| $\ln \left(Y^{H} / Y^{N}\right)$ | 0.451 | 0.882 | 0.819 | 0.858 | 0.000 |
| $\ln \left(P^{N} / P^{H}\right)$ | 0.692 | 0.000 | 0.961 | 0.992 | 0.000 |
| $\ln \left(P^{H} / P^{F}\right)$ | 0.380 | 0.358 | 0.476 | 0.590 | 0.000 |
| $\ln \left(s_{L}^{H}\right)$ | 0.145 | 0.312 | 0.142 | 0.081 | 0.000 |
| $\ln \left(k^{H}\right)$ | 0.995 | 0.479 | 0.997 | 0.999 | 0.000 |
| $D^{H}$ | 0.223 | 0.483 | 0.261 | 0.227 | 0.000 |
| $\ln \left(s_{L}^{N}\right)$ | 0.999 | 0.186 | 0.988 | 0.943 | 0.000 |
| $\ln \left(k^{N}\right)$ | 0.701 | 0.887 | 0.900 | 0.936 | 0.000 |
| $D^{N}$ | 0.999 | 0.820 | 0.982 | 0.945 | 0.000 |
| Notes. LLC and Breitung are the t-statistics of Levin et al 22002$]$ | Breitung |  |  |  |  |

Notes: LLC and Breitung are the t-statistics of Levin et al. [2002] and Breitung [2000] respectively. IPS is the $W_{t b a r}$ test proposed by Im et al. [2003]. MW (ADF) is the Maddala and Wu's [1999] $P$ test based on Augmented Dickey-Fuller p-values. Hadri is the Hadri's [2000] $Z_{\mu}$ test. For all tests, except for Hadri [2000], the null of a unit root is not rejected if p-value $\geq 0.05$ at a $5 \%$ significance level. For Hadri [2000], the null of stationarity is rejected if p-value $\leq 0.05$ at a $5 \%$ significance level. In all tests and for all variables, we allow for individual deterministic trends and fixed effects. $D^{j}$ is defined as $D^{j}=\left(B^{j} / A^{j}\right)^{\left(1-\sigma^{j}\right) / \sigma^{j}}$ for $j=H, N$.

As noted above, IPS and MW tests allow for heterogeneity of the autoregressive root, accordingly,

[^33]Table 20: Aggregate and Sectoral Effects of a 1\% Permanent Increase in Traded relative to Non-Traded TFP: Point Estimates

| Variables | A.Aggregate |  | B.Tradables |  | C.Non-Tradables |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | Impact <br> ( $t=0$ ) <br> (1) | $\begin{aligned} & \text { Long-run } \\ & (t=10) \end{aligned}$ <br> (2) | Impact <br> ( $t=0$ ) <br> (3) | Long-run $(t=10)$ <br> (4) | Impact $(t=0)$ <br> (5) | Long-run $(t=10)$ <br> (6) |
| Relative Prod. | 0.895* | 1.000* | 0.934* | 1.000* | 0.879* | 1.000* |
| Value Added | 0.246* | 0.337* | 0.223* | 0.259* | 0.011 | 0.061 |
| Labor | 0.088* | 0.156 | -0.009 | 0.009 | 0.097* | 0.154* |
| Real Wage | 0.095* | 0.235* | 0.095 | 0.095 | 0.090 | 0.295* |

Notes: Horizon measured in year units. ${ }^{*}$ denote significance at $10 \%$ level. Standard errors are bootstrapped with 10000 replications.
we will focus intensively on these tests when testing for unit roots. In all cases, except for the MW test applied to $W / P_{C}$, the null hypothesis of a unit root against the alternative of trend stationarity cannot be rejected at conventional significance levels, suggesting that the set of variables of interest are integrated of order one. When considering the Hadri's test for which the null hypothesis implies stationary against the alternative of a unit root in the panel data, we reach the same conclusion and conclude again that all series are nonstationary. Taken together, unit root tests applied to our variables of interest show that non stationarity is pervasive, suggesting that all variables should enter in the VAR models in first difference.

## G. 2 Aggregate and Sectoral Effects: VAR Evidence

In the main text, we concentrate on the reallocation and redistributive effects of asymmetric technology shocks across sectors. We provide below the results for the full set of aggregate and sectoral effects of technology shocks biased toward the traded sector.

To explore the magnitude of the aggregate effects empirically, we consider a VAR model that includes in the baseline specification the rate of growth of the technology index biased toward the traded sector, $\hat{Z}_{i t}$, real GDP, $\hat{Y}_{R, i t}$, total hours worked, $\hat{L}_{i t}$, the real consumption wage denoted by $W_{C, i t}$. Our vector of endogenous variables, is given by: $x_{i t}=\left[\hat{Z}_{i t}, \hat{Y}_{R, i t}, \hat{L}_{i t}, \hat{W}_{C, i t}\right]$. All data for aggregate variables are obtained from the OECD Economic Outlook. For real GDP, we use the volumes reported by the OECD. We use hours worked to measure labor. ${ }^{48}$ All quantities are scaled by the working age population and expressed in rate of growth. The real consumption wage is the ratio of the nominal aggregate wage, $W_{i t}$, to the consumption price index, $P_{C, i t}$. The nominal wage is obtained by calculating the ratio of labor compensation to the number of hours worked. Details of data construction and the source of variables used in our estimation are provided in Appendix D.

Table 20 displays point estimates on impact and in the long-run. The dynamic effects of a technology shock biased toward the traded sector on aggregate variables are shown in Fig. 12. The top left panel shows that productivity in tradables relative to non-tradables increases by $0.9 \%$ while the productivity differential increases gradually to reach $1 \%$ after 10 years. The technology shock increases real GDP on impact by $0.25 \%$. Higher productivity in tradables relative to non-tradables also increases significantly hours worked by $0.09 \%$ on impact and generates an initial increase in the real consumption wage by $0.1 \%$.

The sectoral effects of a technology shock are displayed in Fig. 13 while point estimates are reported in Table 20. The responses of sectoral value added and hours worked enable us to explore empirically the breakdown of changes in real GDP and labor into the traded and non-traded sector. Whilst higher productivity of tradables has a significant expansionary effect on traded value added which increases by $0.22 \%$ GDP on impact and $0.26 \%$ in the long-run, non-traded value added is unresponsive at any horizon. Conversely, the non-traded sector experiences a significant increase in hours worked on impact by $0.10 \%$ of total hours worked while hours worked remain fairly unchanged in the traded sector.

## G. 3 Robustness Check: Sectoral Classification

Objective. This subsection explores the robustness of our findings to the classification of the eleven 1-digit ISIC-rev. 3 industries as tradables or non-tradables. When we conduct the robustness analysis, we modify the baseline classification in a number of ways to ensure that some industries

[^34]

Figure 12: Effects of Technology Shock Biased toward the Traded Sector on Aggregate Variables. Notes: Exogenous increase of TFP in tradables relative to non-tradables adjusted with labor income shares by $1 \%$. Aggregate variables include GDP (constant prices), total hours worked, and the real consumption wage. Horizontal axes indicate years. Vertical axes measure percentage deviation from trend. Results for baseline specification are displayed by solid lines with shaded area indicating 90 percent confidence bounds obtained by bootstrap sampling; sample: 17 OECD countries, 1970-2013, annual data.


Figure 13: Sectoral Effects of Technology Shock Biased toward the Traded Sector. Notes: Exogenous increase of TFP in tradables relative to non-tradables adjusted with labor income shares by $1 \%$. Horizontal axes indicate years. Vertical axes measure percentage deviation from trend in GDP units (sectoral value added), percentage deviation from trend in total hours worked units (sectoral hours worked), and percentage deviation from trend (real wages). Results for baseline specification are displayed by solid lines with shaded area indicating 90 percent confidence bounds obtained by bootstrap sampling; sample: 17 OECD countries, 1970-2013, annual data.
with specific characteristics are not driving the results. There are a few sectors which may display some ambiguity related to their tradability, including "Hotels and Restaurants", "Financial Intermediation" and "Real Estate, Renting and Business Services". The reason is twofold. Some sectors such as "Hotels and Restaurants", "Financial Intermediation" have experienced a large increase in tradability over the last fifty years. Since we adopt a VAR methodology, we need a long time horizon for each country which constrain us to use a less detailed sectoral disaggregation so that the sample starts from 1970 otherwise, the sample would start in 1995 for most of the countries in our sample. The lower level of sectoral disaggregation implies that "Financial Intermediation" and "Real Estate, Renting and Business Services" are made up of sub-sectors which display a high heterogeneity in terms of tradability. The most prominent example is "Real Estate, Renting and Business Services" which includes "Real Estate Activities" which displays a very low tradability and "Professional, Scientific, Technical, Administrative, and Support Service Activities" which displays a high level of tradability. Since tradability of sectors varies across time and across subsectors, we perform a sensitivity analysis with respect to the classification for the three aforementioned sectors.

Literature on Tradedness of Industries. While we treat "Real Estate, Renting and Business Services" and "Hotels and Restaurants" as non-tradables, Jensen and Kletzer [2006] find that "Professional, scientific and technical activities" included in the former sector is highly tradable whilst evidence collected by Piton [2017] who calculates the degree of openness for 18 industries over 1995-2014 reveals that "Foods and Accommodation" included in the latter sector displays significant tradability as well. Thus, in the following, we pay particular attention of the sensitivity of our results when either "Real Estate, Renting and Business Services" (red line) or "Hotels and Restaurants" (yellow line) is classified as tradable instead of non-tradable. Moreover, Jensen and Kletzer [2006] find that the subsectors included in "Financial Intermediation" vary substantially in terms of tradability. Accordingly, we also conduct a robustness check w.r.t. this subsector which includes "Financial Intermediation" (black line) into the traded goods sector.

Empirical Strategy. In order to address these issues, we re-estimate the various VAR specifications for different classifications in which one of the three aforementioned industries initially marked as tradable or non-tradable is classified as non-tradable or tradable resp, all other industries staying in their original sector. In doing so, the classification of only one industry is altered, allowing us to see if the results are sensitive to the inclusion of a particular industry in the traded or the non-traded sector. The baseline and the three alternative classifications considered in this exercise are shown in Table 21.

Table 21: Robustness Check: Classification of Industries as Tradables or Non-Tradables

|  | KLEMS code | Classification |  |  |  |
| :--- | :---: | :---: | :---: | :---: | :---: |
|  |  | Baseline | $\# 1$ | $\# 2$ | $\# 3$ |
| Agriculture, Hunting, Forestry and Fishing | AtB | H | H | H | H |
| Mining and Quarrying | C | H | H | H | H |
| Total Manufacturing | D | H | H | H | H |
| Electricity, Gas and Water Supply | E | N | N | N | N |
| Construction | F | N | N | N | N |
| Wholesale and Retail Trade | G | N | N | N | N |
| Hotels and Restaurants | H | N | H | N | N |
| Transport, Storage and Communication | I | H | H | H | H |
| Financial Intermediation | J | H | H | H | N |
| Real Estate, Renting and Business Services | K | N | N | H | N |
| Community Social and Personal Services | LtQ | N | N | N | N |
| Color line in Fig. 14 to 17 |  | blue | yellow | red | black |

Notes: H stands for the Traded sector and N for the non-traded sector.

Results We start with the analysis of the sensitivity of aggregate effects of a technology shock to the classification of industries as tradables or non-tradables. As displayed in Fig. 14, the conclusions for aggregate effects are not sensitive to sector classification. When contrasting the effects across their magnitude, treating "Real Estate, Renting and Business Services" as tradables tends to amplify the positive response of real GDP. Conversely, treating "Hotels and Restaurants" as tradables merely modifies the results quantitatively.

We investigate below the robustness of our results related to the effects of a technology shock biased toward the traded sector on the sectoral composition and redistributive effects. Fig. 15 and Fig. 16 contrast sectoral and reallocation effects of higher productivity of tradables relative to non-tradables according to the classification of industries. First, as displayed in the red line ('Real estate, renting and business services' classified as H), more labor shifts toward the non-traded sector while the relative wage of the traded sector increases instead of declining. With the exception of the latter finding, all of our conclusions hold. In Fig. 17, we investigate whether our conclusion for


Figure 14: Sensitivity of the Effects of Technology Shock Biased toward the Traded Sector on Aggregate Variables to the Classification of Industries as Tradable or Non-Tradable. Notes: Exogenous increase of TFP in tradables relative to non-tradables adjusted with labor income shares by $1 \%$. Aggregate variables include GDP (constant prices), total hours worked and the real consumption wage. Horizontal axes indicate years. Vertical axes measure percentage deviation from trend in output units (GDP), percentage deviation from trend in labor units (total hours worked) and percentage deviation from trend (real consumption wage). Results for baseline specification are displayed by solid lines with shaded area indicating 90 percent confidence bounds obtained by bootstrap sampling. The yellow line and the red line show results when 'Hotels and restaurants' and 'Real Estate, renting and business services' are treated as tradables, respectively. The black line shows results when 'Financial intermediation' is classified as tradables. Sample: 17 OECD countries, 1970-2013, annual data.
redistributive effects (i.e., for sectoral LIS) is robust to the classification of industries. Across all scenarios, LISs in both sectors increase, except when treating "Real Estate, Renting and Business Services" (as displayed by the red line) as tradables. While sectoral LISq do not change when treating "Real Estate, Renting and Business Services" as a traded industry, the capital-labor ratio in the traded and non-traded sector falls which implies that technological change is biased toward labor in both sectors, otherwise LIS would decline. Across all scenarios in Fig. 17, the discrepancy in the estimated effect is not statistically significant.

## G. 4 Breaking Down Sectoral LIS into a Within- and Between-Effect

In the main text, we document evidence which reveals that LISs increase in both the traded and the non-traded sector. Because both sectors are made up of several industries, the change in the LIS of the broad sector is driven by changes in LIS within sub-sectors (keeping the value added share of the sub-sectors fixed) and also by changes in the value added share of those sub-sectors (keeping the LIS of each sub-sector fixed). We break down below the change in the LIS of the broad sector $j=H, N$ into a within- and a between-effect.

To explore empirically the contribution of the change in the LIS of each sub-sector to the change in the LIS of sector $j$, we proceed as follows. As shall be useful,

$$
\begin{gather*}
W_{t}^{j} L_{t}^{j}=\sum_{k} W_{t}^{k, j} L_{t}^{k, j}  \tag{117a}\\
\frac{W_{t}^{j} L_{t}^{j}}{P_{t}^{j} Y_{t}^{j}}=\sum_{k} \frac{W_{t}^{k, j} L_{t}^{k, j}}{P_{t}^{k, j} Y_{t}^{k, j} \cdot \omega_{t}^{Y, k, j}}  \tag{117b}\\
s_{L, t}^{j}=\sum_{k} s_{L, t}^{k, j} \cdot \omega_{t}^{Y, k, j} \tag{117c}
\end{gather*}
$$

where we denote by $\omega_{t}^{Y, k, j}=\frac{P_{t}^{k, j} Y_{t}^{k, j}}{P_{t}^{j} Y_{t}^{j}}$ the share of value added of sub-sector $k$ in sector $j$ in the


Figure 15: Sensitivity of the Effects of Technology Shock Biased toward the Traded Sector on Sectoral Variables to the Classification of Industries as Tradable or Non-Tradable. Notes: Exogenous increase of TFP in tradables relative to non-tradables adjusted with labor income shares by 1\%. Horizontal axes indicate years. Vertical axes measure percentage deviation from trend in GDP units (sectoral output, sectoral value added shares), percentage deviation from trend in total hours worked units (sectoral hours worked, sectoral labor shares), and percentage deviation from trend (sectoral real consumption wages and sectoral relative wages). Results for baseline specification are displayed by solid lines with shaded area indicating 90 percent confidence bounds obtained by bootstrap sampling. The yellow line and the red line show results when 'Hotels and restaurants' and 'Real Estate, renting and business services' are treated as tradables, respectively. The black line shows results when 'Financial intermediation' is classified as tradables. Sample: 17 OECD countries, 1970-2013, annual data.


Figure 16: Sensitivity of the Effects of Unanticipated Technology Shock Biased toward the Traded Sector on Sectoral Variables to the Classification of Industries as Tradable or Non-tradable. Notes: Exogenous increase of TFP in tradables relative to non-tradables adjusted with labor income shares by $1 \%$. Horizontal axes indicate years. Vertical axes measure deviations from trend (ratio of traded value added to non-traded value added and ratio of hours worked of tradables to hours worked of non-tradables), percentage deviation from trend (relative price of non-tradables, terms of trade and relative wage). Results for baseline specification are displayed by solid lines with shaded area indicating 90 percent confidence bounds obtained by bootstrap sampling. The yellow line and the red line show results when 'Hotels and restaurants' and 'Real Estate, renting and business services' are treated as tradables, respectively. The black line shows results when 'Financial intermediation' is classified as tradables. Sample: 17 OECD countries, 1970-2013, annual data.


Figure 17: Sensitivity of the Effects of Unanticipated Technology Shock Biased toward the Traded Sector on Sectoral Variables to the Classification of Industries as Tradable or non-tradable. Notes: Exogenous increase of TFP in tradables relative to non-tradables adjusted with labor income shares by $1 \%$. Horizontal axes indicate years. Vertical axes measure deviations from trend (ratio of labor compensation to value added at current prices) and percentage deviation from trend in capital stock units (ratio of capital to labor). Results for baseline specification are displayed by solid lines with shaded area indicating 90 percent confidence bounds obtained by bootstrap sampling. The yellow line and the red line show results when 'Hotels and restaurants' and 'Real Estate, renting and business services' are treated as tradables, respectively. The black line shows results when 'Financial intermediation' is classified as tradables. Sample: 17 OECD countries, 1970-2013, annual data.

$$
\begin{align*}
d s_{L, t}^{j} & =\sum_{k} \tilde{\omega}^{Y, k, j} d s_{L, t}^{k, j}+\tilde{s}_{L}^{k, j} d \omega_{t}^{k, j} \\
\frac{d s_{L, t}^{j}}{\tilde{s}_{L, t}^{j}} & =\sum_{k} \tilde{\omega}^{Y, k, j} \frac{\tilde{s}_{L}^{k, j}}{\tilde{s}_{L}^{j}} \frac{d s_{L, t}^{k, j}}{\tilde{s}_{L}^{k, j}}+\sum_{k} \tilde{s}_{L}^{k, j} \frac{\tilde{\omega}^{Y, k, j}}{\tilde{s}_{L}^{j}} \frac{d \omega_{t}^{Y, k, j}}{\tilde{\omega}^{Y, k, j}} \\
\hat{s}_{L, t}^{j} & =\sum_{k} \alpha_{L}^{k, j}\left(\hat{s}_{L, t}^{k, j}+\hat{\omega}_{t}^{Y, k, j}\right) \tag{118}
\end{align*}
$$

where $\alpha_{L}^{k, j}$ is the labor compensation share of sub-sector $k$ in sector $j$, i.e.,

$$
\begin{equation*}
\alpha_{L}^{k, j}=\tilde{\omega}^{Y, k, j} \frac{\tilde{s}_{L}^{k, j}}{\tilde{s}_{L}^{j}}=\frac{W^{k, j} L^{k, j}}{W^{j} L^{j}} \tag{119}
\end{equation*}
$$

From eq. (118), we are able to write the within-between decomposition for each sector $j$ across industries $k$ :

$$
\begin{equation*}
\hat{s}_{L}^{j, t}=\underbrace{\sum_{k} \bar{\alpha}^{k, j} \hat{s}_{L, t}^{k, j}}+\underbrace{\sum_{k} \bar{\alpha}^{k, j} \hat{\omega}_{t}^{k, j}} \tag{120}
\end{equation*}
$$

Within Effect Between Effect
where $\bar{\alpha}^{k, j}$ refers to the labor compensation share averaged over 1970-2013. Eq. (120) shows that the response of the LIS in sector $j$ can be decomposed into a within-effect (keeping the value added share constant) and a between-effect (keeping the LIS constant). In accordance with (120), we first construct time series for the LIS of the broad sector $j=H, N$ as if the value added share remained constant over 1970-2013:

$$
\begin{equation*}
\operatorname{LIS}_{w i t h i n}^{j}=\sum_{k} \bar{\alpha}^{k, j} \hat{S}_{L, t}^{k, j} . \tag{121}
\end{equation*}
$$

Eq. (121) corresponds to the within-effect. We estimate the same VAR as in the main text, i.e., $\left[\hat{Z}_{i t},\left(\operatorname{LIS}_{\text {within }}^{j}\right)_{i t}, \hat{k}_{i t}^{j}\right]$ where variables enter the VAR model in growth rates, except that the LIS is constructed in accordance with eq. (121). The response of LIS ${ }_{w i t h i n}^{j}$ to a shock to a productivity differential will allow us to calculate the rise in the LIS if the value added share remained constant.

Next, we construct time series for the LIS of the broad sector $j=H, N$ as if the LIS in sub-sector $k$ remained constant over 1970-2013:

$$
\begin{equation*}
\operatorname{LIS}_{\text {between }}^{j}=\sum_{k} \bar{\alpha}^{k, j} \hat{\omega}_{t}^{k, j} \tag{122}
\end{equation*}
$$

Eq. (122) corresponds to the between-effect. We estimate the same VAR as in the main text, i.e., $\left[\hat{Z}_{i t},\left(\operatorname{LIS}_{\text {between }}^{j}\right)_{i t}, \hat{k}_{i t}^{j}\right]$ where variables enter the VAR model in growth rates, except that the LIS is constructed in accordance with eq. (122). The response of $\operatorname{LIS}_{\text {between }}^{j}$ to a shock to a productivity differential will allow us to calculate the rise in the LIS driven by changes in value added shares of sub-sectors. Once we have estimated the responses of (121) and (122), we then sum the responses:

$$
\begin{equation*}
\mathrm{LIS}_{\text {rescaled }}^{j}=\mathrm{LIS}_{\text {within }}^{j}+\mathrm{LIS}_{\text {between }}^{j} . \tag{123}
\end{equation*}
$$

We refer below to (123) as the responses of re-scaled LIS of sector $j$. Importantly, equation (123) allows us to also gauge the contribution of each component to the re-scaled LIS variation by calculating $\mathrm{LIS}_{\text {within }}^{j}-\mathrm{LIS}_{\text {rescaled }}^{j}$ and $\mathrm{LIS}_{\text {between }}^{j}-\mathrm{LIS}_{\text {rescaled }}^{j}$.

Fig. 18 shows the responses of variables of interest to a $1 \%$ permanent increase in traded relative to non-traded TFP. For each sector $j=H, N$, the blue line shows the dynamic adjustment of the LIS $\left(\hat{s}_{L}^{j, t}\right)$ after the technology shock while the dashed red line and the dotted green line display the within effect and the between effect respectively. The sum of the two components, the re-scaled LIS (eq. (123)), is displayed by the black line. While according to (123), the sum of the within- and between-effect should be, by construction, equal to the response of $s_{L}^{j}$, our results show that the discrepancy between the blue line (i.e. the empirical response of $s_{L}^{j}$ ) and the black line (corresponding to the response of the re-scaled LIS of sector $j$ ) is reassuringly small along the dynamic adjustment. For tradables, the observed increase in the labor share is mostly driven by the between effect at impact only, i.e. at time the shock occurs, the increase in $s_{L}^{H}$ is due to changing value added shares of industries. Afterwards, and critical for our analysis, the increase in the LIS in the traded sector is predominantly explained by the within component. On average, more than $60 \%$ of the LIS increase in sector $H$ can be attributed to the rise in LIS in sub-sectors. Turning to the non-traded sector, the contribution of the within-effect is lower but remains significant at roughly $30 \%$. But as we shall see below, this conclusion is deceptive. The reason is that for the within-effect, the LIS falls in some countries and rise in others so that on aggregate, the LIS is unresponsive on


Figure 18: Decomposition of Effects of Unanticipated Technology Shock Biased toward the Traded Sector on LIS. Notes: Exogenous increase of TFP in tradables relative to non-tradables adjusted with labor income shares by $1 \%$. Horizontal axes indicate years. Vertical axes measure percentage deviations from trend. In both panels the solid blue line shows the response of the labor share in sector $j, \hat{s}_{L, i t}^{j}$, to identified technology shock biased toward the traded sector. The dashed red line displays the adjustment of within component (eq. (121)) while the dotted green line displays the the adjustment of between component (eq. (122)). The black line represents the response of the sum of the two components (see (123)).
impact. To address this problem, we have to estimate the within effect at a country level. In other words, we cannot draw any conclusion from the decomposition (123) when considering the whole sample.

According to the evidence displayed in the main text, the responses of sectoral LIS for the whole sample masks a wide cross-country dispersion since the LIS increases in half of the countries approximately and falls in the remaining sample. Accordingly, to gauge the contribution of the within-effect in a consistent way, we have re-estimated the VAR models $\left[\hat{Z}_{i t}, \hat{s}_{L, i t}^{j}, \hat{k}_{i t}^{j}\right]$ and $\left[\hat{Z}_{i t},\left(\operatorname{LIS}_{w i t h i n}^{j}\right)_{i t}, \hat{k}_{i t}^{j}\right]$ for one country at a time and plot responses of sectoral LIS on the vertical axis against estimated responses of $\operatorname{LIS}_{w i t h i n}^{j}$ on the horizontal axis in Fig. 19. Impact (long-run resp.) responses, i.e. at time $t=0(t=10$ resp.) are displayed in the first (second resp.) row of Fig. 19. In each panel, we obtain a strong and positive cross-country relationship between the change in the LIS and that of the within-effect. ${ }^{49}$ Focusing on impact responses in sector $H, 15$ countries (out of 17) lie in the north-east or south-west of the scatter plot, indicating that short-run changes in $s_{L}^{H}$ and $\operatorname{LIS}_{\text {within }}^{H}$ have the same sign (the two exceptions are CAN and NLD for which the impact response of $s_{L}^{H}$ is positive while the impact response of $\operatorname{LIS}_{\text {within }}^{H}$ is negative). In the long-term, essentially the same picture emerges in the traded sector as the direction of the response of $s_{L}^{H}$ collapses to the direction of the response of $\operatorname{LIS}_{\text {within }}^{H}$ for 14 countries out of 17 (exceptions are CAN, DNK and NOR). For the non-traded sector, we reach the same conclusion: at impact and in the long-run, for all countries (with the notable exception of JPN at time $t=10$ ), the sign of the empirical response of $s_{L}^{H}$ is consistent with that of the within-effect $\operatorname{LIS}_{w i t h i n}^{H}$.

Finally, Table 22 reports the decomposition from eq. (123) and shows the contribution of the within-effect to the re-scaled LIS change in both sectors, at different time horizon. The results summarized in Table 22 show that, on average, about $60 \%$ of either short- and long-run changes in the LIS in tradables after an increase in traded relative to non-traded TFP can be attributed to the within-effect. The contribution of the within effect stands at $80 \%$ on impact and $66 \%$ in the long-run. Overall, these results confirm that the response of the LIS in sector $j=H, N$ to a technology shock is mostly explained by the responses of LISs in sub-sectors rather than the change in the value added composition.

## G. 5 Alternative Calculations of LIS

When exploring empirically the redistributive effects of a technology shock biased toward the traded sector, an issue is the way the share of labor in total income is constructed. Gollin [2002] pointed out that the treatment of self-employment income affects the measurement of the LIS. In particular, it is unclear how the income of proprietors (self-employed) should be allocated to labor income or to capital revenue. Here in this paper, our preferred measure (called benchmark bench hereafter) is to treat all the income of self-employed as labor income. Although this choice overstates the measure of the LIS, it has the virtue of being simple and transparent. Moreover data involved in the construction of this calculation of the LIS are comparable across industries and readily available

[^35]Sector $H$

(a) $\operatorname{Impact}(t=0)$

(c) Long-Run $(t=10)$

## Sector $N$


(b) $\operatorname{Impact}(t=0)$

(d) Long-Run $(t=10)$

Figure 19: Cross-Country Effects of Technology Shock Biased toward the Traded Sector on LIS and LIS within . Notes: Exogenous increase of TFP in tradables relative to non-tradables adjusted with labor income shares by $1 \%$. Horizontal axes report responses of $\operatorname{LIS}_{\text {within }}^{j}$ obtained by running the VAR $\left[\hat{Z}_{i t}, \operatorname{LIS}_{\text {within }, i t}^{j}, \hat{k}_{i t}^{j}\right]$. Vertical axes show responses of $\hat{s}_{L, i t}^{j}$ obtained from the VAR $\left[\hat{Z}_{i t}, \hat{s}_{L, i t}^{j}, \hat{k}_{i t}^{j}\right]$. Sample: 17 OECD countries, 1970-2013, annual data. Impact (long-run resp.) responses, i.e. at time $t=0$ ( $t=10$ resp.) are displayed in the first (second resp.) row.

Table 22: Contribution of the Within-component to the re-scaled LIS Variation (in \%)

| Country | Sector $H$ |  | Sector $N$ |  |
| :--- | :---: | :---: | :---: | :---: |
|  | Impact | Long-Run | Impact | Long-Run |
| AUS | 23.73 | 41.60 | 81.88 | 62.09 |
| AUT | 81.42 | 53.48 | 89.37 | 24.43 |
| BEL | 99.34 | 89.13 | 66.98 | 45.15 |
| CAN | 10.87 | 38.58 | 64.47 | 58.99 |
| DEU | 90.10 | 92.80 | 93.30 | 68.93 |
| DNK | 78.62 | 12.23 | 87.49 | 79.36 |
| ESP | 76.35 | 52.47 | 58.94 | 63.31 |
| FIN | 87.18 | 44.53 | 83.24 | 37.68 |
| FRA | 97.62 | 83.43 | 79.19 | 77.04 |
| GBR | 43.57 | 37.40 | 97.79 | 79.70 |
| IRL | 73.12 | 39.89 | 77.68 | 69.23 |
| ITA | 90.12 | 63.32 | 96.75 | 88.13 |
| JPN | 89.85 | 94.44 | 96.62 | 43.21 |
| NLD | 12.80 | 89.39 | 49.85 | 91.25 |
| NOR | 44.81 | 15.74 | 87.60 | 86.89 |
| SWE | 74.58 | 90.59 | 64.09 | 53.81 |
| USA | 63.71 | 70.74 | 90.83 | 90.93 |
| Mean | 66.93 | 59.40 | 80.36 | 65.89 |

Notes: each entry in the table gives, for each sector $j=$ $H, N$ and horizon $t=0,10$, the share of re-scaled LIS change attributed to the within-component, i.e. $100 \times$ $\left(\operatorname{LIS}_{\text {within }}^{j} / \operatorname{LIS}_{\text {rescaled }}^{j}\right)$.
for all countries of our sample. Specifically, the LIS in sector $j=H, N$ is constructed as follows:

$$
\begin{equation*}
s_{L}^{j, b e n c h}=\frac{W_{e m p l}^{j} L_{e m p l}^{j}+I n c_{s e l f}^{j}}{P^{j} Y^{j}} \tag{124}
\end{equation*}
$$

where $W_{e m p l}^{j} L_{\text {empl }}^{j}$ is the labor compensation of employees, $I n c_{\text {self }}$ is total income of self-employed and $P^{j} Y^{j}$ is the valued added at current prices in sector $j$. Note that labor compensation of
employees includes total labor costs: wages, salaries and all other costs of employing labor which are borne by the employer whilst $I n c_{\text {self }}$ comprises both labor and capital income components, noted $W_{\text {self }}^{j} L_{\text {self }}^{j}$ and $R_{\text {self }}^{j} K_{\text {self }}^{j}$ respectively such that $I n c_{\text {self }}^{j}=W_{\text {self }}^{j} L_{\text {self }}^{j}+R_{\text {self }}^{j} K_{\text {self }}^{j}$.

As a first alternative measure of the LIS, we use only employees compensation as a measure of labor income. This LIS measure, denoted by $s_{L}^{j, 1}$, is constructed as follows:

$$
\begin{equation*}
s_{L}^{j, 1}=\frac{W_{e m p l}^{j} L_{e m p l}^{j}}{P^{j} Y^{j}} \tag{125}
\end{equation*}
$$

Measure (125) omits the income of the self-employed, i.e. this income is totally counted as capital income.

As a second alternative measure, we split self-employed income into capital and labor income based on the assumption that the labor income of the self-employed has the same mix of labor and capital income as the rest of the economy. In other words, total labor compensation comprises labor compensation of employees, $W_{\text {empl }}^{j} L_{\text {empl }}^{j}$, and the self-employed income scaled by the LIS of employees only, i.e. $\operatorname{Inc} c_{\text {self }}^{j} \times s_{L}^{j, 1}$. With this adjustment, the LIS, denoted by $s_{L}^{j, 2}$, is constructed as follows:

$$
\begin{equation*}
s_{L}^{j, 2}=\frac{W_{e m p l}^{j} L_{e m p l}^{j}+I n c_{s e l f}^{j} \times s_{L}^{j, 1}}{P^{j} Y^{j}} \tag{126}
\end{equation*}
$$

Finally, the last alternative to compute the LIS relies upon the assumption that self-employed earn the same hourly compensation as employees. Thus, we use the hourly wage earned by employees $W_{\text {empl }}^{j}$ as a shadow price of labor of self-employed workers. The LIS, denoted by $s_{L}^{j, 3}$, is constructed as follows:

$$
\begin{equation*}
s_{L}^{j, 3}=\frac{W_{e m p l}^{j} \times\left(L_{e m p l}^{j}+L_{\text {self }}^{j}\right)}{P^{j} Y^{j}} \tag{127}
\end{equation*}
$$

In Fig. 20 we display the results of this sensitivity analysis with respect to the construction of the labor income share. To do so, we measure the effects of an exogenous increase in TFP of tradables relative to non-tradables by $1 \%$ on LIS and capital-labor ratio in sector $j=H, N$ by contrasting the impulse response functions of the two variables when the LIS is measured as either $s_{L}^{j, b e n c h}$ (blue line), or $s_{L}^{j, 1}$ (red line), or $s_{L}^{j, 2}$ (green line), or $s_{L}^{j, 3}$ (black line). The IRFs are obtained for each specification by running the VAR model $\left[\hat{Z}_{i t}, \hat{s}_{L, i t}^{H}, \hat{k}_{i t}^{H}\right]$ and $\left[\hat{Z}_{i t}, \hat{s}_{L, i t}^{N}, \hat{k}_{i t}^{N}\right]$. As Fig. 20 shows, the responses of LIS and capital-labor ratios for the four specifications are qualitatively similar. In panels (a) and (c), the IRFs obtained with the three alternative measures of $s_{L}^{j}$ are well within the confidence interval (for the benchmark specification $s_{L}^{j, b e n c h}$ ) for all horizons. Overall, our main findings regarding the response of $s_{L}^{j}$ and $k^{j}$ for $j=H, N$ to an increase in TFP of tradables to non-tradables are robust and unsensitive to the way the share of labor in total income is constructed in the data.

## G. 6 Identified Technology Shocks across Alternative VAR Specifications

We address a potential concern related to the fact that the technology shock may display noticeable differences across alternative VAR specifications. Such differences could potentially make the comparison of the effects of a technology shock across sectors difficult. Because in the quantitative analysis we base our calibration on one unique technology shock, such differences could potentially undermine the comparison of theoretical with empirical responses. Before summarizing the results of our robustness exercises, it is worth mentioning that, in line with the current practice, to facilitate the interpretation of our results, we normalize the shock to a productivity differential to $1 \%$ in the long-run. Such a normalization thus makes the responses of economic variables directly comparable quantitatively across VAR models. However, even if the magnitude and the shape of the technology shock is similar across VAR specifications, different VAR models could pickup different structural technology shocks, i.e., underlying sectoral TFPs responses could differ across VAR specifications. In order to investigate the extent of the discrepancy in the estimated responses caused by potentially different technology shocks across VAR specifications, we identify the technology shock in the baseline VAR model which includes aggregate variables and augment all VAR models with the identified technology shock ordered first. Reassuringly, the discrepancy in estimated responses turns out to be quite moderate in all robustness exercises.

We conduct below an elaborate investigation of the potential discrepancy in the estimated effects caused by considering alternative VAR models. To perform such an analysis, we proceed as follows. Once we have identified the technology shock in the first VAR model that includes aggregate variables $x_{i t}=\left[Z_{i t}, Y_{i t}, L_{i t}, W_{C, i t}\right]$, we augment each VAR model with the identified technology shock, ordered first. More precisely, we run the following VAR specifications:

$$
\text { - } x_{i t}^{R, j}=\left[\hat{Z}_{i t}, \hat{Y}_{i t}^{j}-\hat{Y}_{i t}, \hat{L}_{i t}^{j}-\hat{L}_{i t}, \hat{W}_{i t}^{j}-\hat{W}_{i t}\right] \text { for } j=H, N,
$$



Figure 20: Effects of Technology Shock Biased toward the Traded Sector on LISs and Capital-Labor Ratios. Notes: Exogenous increase of TFP in tradables relative to non-tradables adjusted with labor income shares by $1 \%$. Sectoral variables include labor income shares and capital-labor ratios. Horizontal axes indicate years. Vertical axes measure deviations from trend (ratio of labor compensation to value added at current prices) and percentage deviation from trend in capital stock units (ratio of capital to labor). Results for baseline specification (eq. (124)) are displayed by solid lines with shaded area indicating 90 percent confidence bounds obtained by bootstrap sampling; sample: 17 OECD countries, 1970-2013, annual data. The red line reports results for specification (125) when $s_{L}^{j}=s_{L}^{j, 1}$. The green and black lines shows results for specifications (126) and (127) respectively, i.e. $s_{L}^{j}=s_{L}^{j, 2}$ for the green line and $s_{L}^{j}=s_{L}^{j, 3}$ for the black line.

- $x_{i t}^{P}=\left[\epsilon_{i t}^{Z}, \hat{Z}_{i t}, \hat{Y}_{i t}^{H}-\hat{Y}_{i t}^{N}, \hat{P}_{i t}^{N}-\hat{P}_{i t}^{H}\right]$ and $x_{i t}^{P}=\left[\epsilon_{i t}^{Z}, \hat{Z}_{i t}, \hat{Y}_{i t}^{H}-\hat{Y}_{i t}^{N}, \hat{P}_{i t}^{H}-\hat{P}_{i t}^{F}\right]$,
- $x_{i t}^{L I S}=\left[\epsilon_{i t}^{Z}, \hat{Z}_{i t}, \hat{s}_{L, i t}^{j}, \hat{k}_{i t}^{j}\right]$ for $j=H, N$,
where $\epsilon_{i t}^{Z}$ is the the identified technology shock estimated in the baseline VAR model, i.e. $x_{i t}^{A}=$ $\left[\hat{Z}_{i t}, \hat{Y}_{R, i t}, \hat{L}_{i t}, \hat{W}_{C, i t}\right]$. Then, we contrast the responses for the baseline model with those for augmented VAR models.

Fig. 21 and Fig. 22 compare the results in the main text displayed by the solid blue line with those for the same VAR model augmented with the identified technology shock. As shown in the first row of Fig. 21 and Fig. 22, across all VAR specifications, the endogenous response of the productivity differential is quite similar, if not identical, whether the baseline VAR model is augmented (solid black line) or not with the identified technology shock (solid blue line). Turning to the sectoral composition effects displayed in Fig. 21, all of the conclusions mentioned in the main text hold since the solid black line lies for all variables within the original confidence bounds of those obtained when the VAR model is not augmented with the identified technology shock. We may notice some slight differences for the relative wage in tradables and non-tradables, but the discrepancy is not statistically significant, except in the short-run. In Fig. 22, one can observe that the IRFs fall within the confidence interval for all horizons and all variables, with the exceptions of the terms of trade and the relative wage of non-tradables (but only in the sort-run for the latter). Overall, reassuringly, this robustness exercise shows that our different VAR models identify similar structural technological shocks and it turns out that differences are statistically negligible.

## G. 7 Robustness Check to Series for Physical Capital

In the main text, due to data availability, we construct time series for sectoral capital by computing the overall capital stock by adopting the perpetual inventory approach and then by splitting the gross capital stock into traded and non traded industries by using sectoral valued added shares. In this Appendix, we investigate whether the effects on $k^{j}$ we estimate empirically are not driven by our assumption about the construction of time series for sectoral capital stock. To conduct this robustness check, we take time series for sectoral capital stock from KLMES and. To conduct this robustness check, we contrast below empirically the responses of $k^{j}$ when sectoral capital stocks are measured by adopting the Garofalo and Yamarik's [2002] methodology (our benchmark) with those obtained by using sectoral data on $K^{j}$ provided by EU KLEMS [2011], [2017] databases. In both cases, we explore empirically the VAR model $\left[\hat{Z}_{i t}, \hat{s}_{L, i t}^{j}, \hat{k}_{i t}^{j}\right]$. Due to data availability, our results in the latter case include a sample of nine OECD countries which provide time series on sectoral capital of reasonable length. To be consistent, our benchmark also includes these nine countries only.

The methodology by Garofalo and Yamarik's [2002] is based on the assumption of perfect mobility of capital across sectors and a small discrepancy in the LIS across sectors, i.e., $s_{L}^{H} \simeq s_{L}^{N}$. The assumption of perfect capital mobility implies that the marginal revenue product of capital must equalize across sectors:

$$
\begin{equation*}
P_{t}^{H} s_{L}^{H} \frac{Y_{t}^{H}}{K_{t}^{H}}=P_{t}^{N} s_{L}^{N} \frac{Y_{t}^{N}}{K_{t}^{N}} . \tag{128}
\end{equation*}
$$

Using the resource constraint for capital, $K=K^{H}+K^{N}$, dividing the numerator and the denominator in the LHS of (128) by GDP, $Y$, and denoting by $\omega_{t}^{Y, j}=\frac{P_{t}^{j} Y_{t}^{j}}{Y_{t}}$ the share of value added of sector $j$ in GDP at current prices (at time $t$ ), eq. (128) can be solved for the $K^{H} / K$ :

$$
\begin{equation*}
\frac{K_{t}^{H}}{K_{t}}=\frac{\omega_{t}^{Y, H} s_{L}^{H}}{s_{L}^{N}\left(1-\omega_{t}^{Y, H}\right)+s_{L}^{H} \omega_{t}^{Y, H}} . \tag{129}
\end{equation*}
$$

Assuming that $s_{L}^{H} \simeq s_{L}^{N}$ leads to the rule we apply to split the aggregate stock of capital into tradables and non tradables:

$$
\begin{equation*}
\frac{K_{t}^{H}}{K_{t}^{H}}=\omega_{t}^{Y, H} . \tag{130}
\end{equation*}
$$

In the baseline, we adopt the methodology of Garofalo and Yamarik [2002] to split the national gross capital stock into traded and non-traded industries by using sectoral value added shares at current prices. Let $\omega^{Y, j}$ be the share of sector $j^{\prime}$ 's value added (at current prices) $P^{j} Y^{j}$ for $j=H, N$ in overall output (at current prices) $Y \equiv P^{H} Y^{H}+P^{N} Y^{N}$, the allocation of the national capital stock to sector $j$ is given by the rule:

$$
\begin{equation*}
K_{G Y}^{j}=\omega^{Y, j} K=\frac{P^{j} Y^{j}}{Y} K \tag{131}
\end{equation*}
$$



Figure 21: Assessing Differences Caused by Potentially Identifying Different Technology Shocks across VAR Models. Notes: Exogenous $1 \%$ increase in TFP of tradables relative to non-tradables. Horizontal axes indicate years. Vertical axes measure percentage deviation from trend in GDP units (sectoral value added shares), percentage deviation from trend in hours worked units (sectoral labor shares), and percentage deviation from trend (sectoral relative wages). Results for baseline specification are displayed by solid blue lines with shaded area indicating 90 percent confidence bounds obtained by bootstrap sampling. The solid black line reports the results for the same VAR model which is augmented with the identified technology shock obtained in the baseline VAR model $x_{i t}^{A}=\left[\hat{Z}_{i t}, \hat{Y}_{R, i t}, \hat{L}_{i t}, \hat{W}_{C, i t}\right]$. Sample: 17 OECD countries, 1970-2013, annual data.


Figure 22: Assessing Differences Caused by Potentially Identifying Different Technology Shocks across VAR Models. Notes: Exogenous $1 \%$ increase in TFP of tradables relative to non-tradables adjusted. Horizontal axes indicate years. Vertical axes measure deviations from trend (ratio of traded value added to non-traded value added and ratio of labor compensation to value added at current prices), percentage deviation from trend (relative price of non-tradables, terms of trade) and percentage deviation from trend in capital stock units (ratio of capital to labor). Results for baseline specification are displayed by solid blue lines with shaded area indicating 90 percent confidence bounds obtained by bootstrap sampling. The solid black line reports the results for the same VAR model which is augmented with the identified technology shock obtained in the baseline VAR model $x_{i t}^{A}=\left[\hat{Z}_{i t}, \hat{Y}_{R, i t}, \hat{L}_{i t}, \hat{W}_{C, i t}\right]$. Sample: 17 OECD countries, 1970-2013, annual data.
where we denote the sectoral stock of capital obtained with the decomposition by Garofalo and Yamarik [2002] by $K_{G Y}^{j}$. National capital stocks are estimated from the perpetual inventory approach. Following Garofalo and Yamarik [2002], the gross capital stock is then allocated to traded and non-traded industries by using sectoral value added shares according to eq. (131). Once the series for $K_{G Y}^{j}$ are obtained, we can construct the sectoral capital-labor ratios, $k_{G Y}^{j}=K_{G Y}^{j} / L^{j}$, and sectoral TFPs, $Z_{G Y}^{j}$, which are constructed as the Solow residual.

As a robustness check, we alternatively take capital stock series from the EU KLEMS [2011] and [2017] databases which provide disaggregated capital stock data (at constant prices) at the 1-digit ISIC-rev. 3 level for up to 11 industries, but only for nine countries of our sample over the entire period 1970-2013 (AUS, CAN, DNK, ESP, FIN, GBR, ITA, NLD and the USA). ${ }^{50}$ For future reference, we denote the sectoral stock of capital and TFP by $K_{K L}^{j}$ and $T F P_{K L}^{j}$, respectively, when we take sectoral data from the EU KLEMS [2011], [2017] databases.

Before presenting VAR estimates from the sensitivity analysis with respect to the calculation of sectoral capital stocks, we show pairwise correlations between selected variables ( $K^{j}, k^{j}$ and $Z^{j}$ for $j=H, N$ along with the identified structural productivity shock $\epsilon^{Z}$ ) constructed with the Garofalo and Yamarik [2002] methodology or alternatively with the direct use of the EU KLEMS [2011] and [2017] databases. We focus on the full available sample period 1970-2013 for 9 OECD countries (AUS, CAN, DNK, ESP, FIN, GBR, ITA, NLD and the USA). Table 23 provides the summary results for pairwise correlations. Series for all variables are positively and highly correlated, the average pairwise correlation is 0.885 and the correlation coefficients range from a low 0.755 for the identified technological shock $\epsilon^{Z}$ to a high of 0.983 for $K^{N}$. These results are suggestive, but of course not dispositive, that Garofalo and Yamarik's [2002] approach provides consistent estimates of the capital stock at the sectoral level.

Table 23: Sectoral Capital Stocks: Correlations for Selected Variables

| Variable | $\left(K_{G Y}^{H}, K_{K L}^{H}\right)$ | $\left(K_{G Y}^{N}, K_{K L}^{N}\right)$ | $\left(k_{G Y}^{H}, k_{K L}^{H}\right)$ | $\left(k_{G Y}^{N}, k_{K L}^{N}\right)$ | $\left(Z_{G Y}, Z_{K L}\right)$ | $\left(\epsilon_{G Y}^{Z}, \epsilon_{K L}^{Z}\right)$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| correlation | 0.907 | 0.983 | 0.906 | 0.789 | 0.973 | 0.755 |

Notes: subscripts "GY" and "KL" refer to the two methods to construct sectoral capital stocks. $K^{j}$ is the capital stock in sector $j=H, N, k^{j}$ is the capital-labor ratio in sector $j=H, N, Z=\left(Z^{H}\right)^{a} /\left(Z^{N}\right)^{b}$ is the labor share-adjusted TFP ratio between traded and non-traded sectors with $a=\left[\left(1-\alpha_{J}\right)+\alpha_{J}\left(s_{L}^{H} / s_{L}^{N}\right)\right]^{-1}\left(\alpha_{J}\right.$ being the tradable share in total investment expenditure) and $b=a\left(s_{L}^{H} / s_{L}^{N}\right)$ and $\epsilon^{Z}$ is the identified technology shock obtained by running the VAR including aggregate variables, i.e., $x_{i t}^{A}=\left[\hat{Z}_{i t}, \hat{Y}_{i t}, \hat{L}_{i t}, \hat{W}_{C, i t}\right]$. Sample: 9 OECD countries (AUS, CAN, DNK, ESP, FIN, GBR, ITA, NLD and the USA), annual data; 1970-2013.

Next, Fig. 23 plots identified shocks to the productivity differential, $\epsilon^{Z}$, obtained with the two measures of Solow residuals constructed from sectoral capital stocks by adopting the two alternative methods. We detect very small differences between the two sets of data for all considered countries. Next, in Fig. 24 we plot estimated shocks $\epsilon^{Z}$ using KLEMS data on the vertical axis against estimated shocks $\epsilon^{Z}$ using the method of Garofalo and Yamarik [2002] on the horizontal axis. In line with results presented above, the scatter-plot shows a strong positive correlation. Also reported in Fig. 24 is a regression line, whose slope coefficient and standard error are 1.040 and 0.020 respectively, implying that the estimated coefficient is significant at the $1 \%$ level (the R -squared is $0.878)$.

Finally, we compare the responses of $k^{j}$ for the baseline method to split the national gross capital stock into tradables and non-tradables with those obtained from the alternative approach where we take data on sectoral capital from KLEMS [2011], [2017] databases. We estimate the effects of a $1 \%$ permanent increase in TFP of tradables relative to non-tradables on the capital-labor ratio in sector $j=H, N$ and contrast the IRFs whether the sectoral capital stock is measured by $K_{G Y}^{j}$ (blue line) or by $K_{K L}^{j}$ (black line). In both cases, we estimate the VAR model which includes the LIS and the capital-labor ratios, i.e., $x_{i t}^{L I S, H}=\left[\hat{Z}_{i t}, \hat{s}_{L, i t}^{H}, \hat{k}_{i t}^{H}\right]$ and $x_{i t}^{L I S, N}=\left[\hat{Z}_{i t}, \hat{s}_{L, i t}^{N}, \hat{k}_{i t}^{N}\right]$. As shown in Fig. 25 , the responses of capital-labor ratios for the two methods are qualitatively similar since the solid black line lies within the original confidence bounds of those obtained when $K^{j}$ is constructed with the use of the methodology of Garofalo and Yamarik [2002]. In particular, one can observe that the discrepancy in the results is small and not statistically significant at conventional level. Overall, our main findings regarding the response of $k^{j}$ for $j=H, N$ to an increase in TFP in tradables to non-tradables are robust and unsensitive to the way the sectoral capital stocks are constructed in the data.

[^36]

Figure 23: Identified Technology Shock $\epsilon^{Z}$ from Garofalo-Yamarik or KLEMS methodology: Time-series Evidence. Notes: "GY" refers to the case where we use methodology of Garofalo and Yamarik [2002] to split the national gross capital stock into traded and non-traded industries. "KLEMS" refers to the case where we use the EU KLEMS [2011] and [2017] databases to construct sectoral capital stocks series. The identified technology shock $\epsilon^{Z}$ is obtained by running the VAR including aggregate variables, i.e., $x_{i t}=\left[\hat{Z}_{i t}, \hat{Y}_{i t}, \hat{L}_{i t}, \hat{W}_{C, i t}\right]$ (sample: 9 OECD countries, 1970-2013, 2 lags).


Figure 24: Identified Technology Shock $\epsilon^{Z}$ : Cross-Country Comparisons between GarofaloYamarik and KLEMS methodologies. Notes: subscript "GY" refers to the case where we use methodology of Garofalo and Yamarik [2002] to split the national gross capital stock into traded and non-traded industries. Subscript "KLEMS" refers to the case where we use the EU KLEMS [2011] and [2017] databases to construct sectoral capital stocks series.


Figure 25: Effects of Technology Shock Biased toward the Traded Sector on Capital-Labor Ratios. Notes: Effects of a $1 \%$ permanent increase in TFP of tradables relative to non-tradables. Horizontal axes indicate years. Vertical axes measure percentage deviation from trend in capital stock units. Results for baseline specification (i.e. we use methodology of Garofalo and Yamarik [2002] to construct the sectoral capital stocks $K^{H}$ and $K^{N}$ ) are displayed by blue lines with shaded area indicating 90 percent confidence bounds obtained by bootstrap sampling; sample: 9 OECD countries, 1970-2013, annual data. The black line reports results when we use the EU KLEMS [2011] and [2017] databases to construct sectoral capital stocks series $K^{j}$.

## H Semi-Small Open Economy Model

This Appendix puts forward an open economy version of the neoclassical model with tradables and non-tradables, imperfect mobility of labor across sectors, capital adjustment costs and endogenous terms of trade. This section illustrates in detail the steps we follow in solving this model. We assume that production functions take a Cobb-Douglas form since this economy is the reference model for our calibration as we normalize CES productions by assuming that the initial steady state of the Cobb-Douglas economy is the normalization point.

Households supply labor, $L$, and must decide on the allocation of total hours worked between the traded sector, $L^{H}$, and the non-traded sector, $L^{N}$. They consume both traded, $C^{T}$, and non-traded goods, $C^{N}$. Traded goods are a composite of home-produced traded goods, $C^{H}$, and foreignproduced foreign (i.e., imported) goods, $C^{F}$. Households also choose investment which is produced using inputs of the traded, $J^{T}$, and the non-traded good, $J^{N}$. As for consumption, input of the traded good is a composite of home-produced traded goods, $J^{H}$, and foreign imported goods, $J^{F}$. The numeraire is the foreign good whose price, $P^{F}$, is thus normalized to one.

## H. 1 Households

At each instant of time, the representative household consumes traded and non-traded goods denoted by $C^{T}$ and $C^{N}$, respectively, which are aggregated by means of a CES function:

$$
\begin{equation*}
C=\left[\varphi^{\frac{1}{\phi}}\left(C^{T}\right)^{\frac{\phi-1}{\phi}}+(1-\varphi)^{\frac{1}{\phi}}\left(C^{N}\right)^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}} \tag{132}
\end{equation*}
$$

where $0<\varphi<1$ is the weight of the traded good in the overall consumption bundle and $\phi$ corresponds to the elasticity of substitution between traded goods and non-traded goods. The index $C^{T}$ is defined as a CES aggregator of home-produced traded goods, $C^{H}$, and foreign-produced traded goods, $C^{F}$ :

$$
\begin{equation*}
C^{T}=\left[\left(\varphi^{H}\right)^{\frac{1}{\rho}}\left(C^{H}\right)^{\frac{\rho-1}{\rho}}+\left(1-\varphi_{H}\right)^{\frac{1}{\rho}}\left(C^{F}\right)^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}} \tag{133}
\end{equation*}
$$

where $0<\varphi_{H}<1$ is the weight of the home-produced traded good in the overall traded consumption bundle and $\rho$ corresponds to the elasticity of substitution between home-produced traded goods goods and foreign-produced traded goods.

As in De Cordoba and Kehoe [2000], the investment good is produced using inputs of the traded good and the non-traded good according to a constant-returns-to-scale function which is assumed to take a CES form:

$$
\begin{equation*}
J \equiv J\left(J^{T}, J^{N}\right)=\left[\iota^{\frac{1}{\phi_{J}}}\left(J^{T}\right)^{\frac{\phi_{J}-1}{\phi_{J}}}+(1-\iota)^{\frac{1}{\phi_{J}}}\left(J^{N}\right)^{\frac{\phi_{J}-1}{\phi_{J}}}\right]^{\frac{\phi_{J}}{\phi_{J}-1}} \tag{134}
\end{equation*}
$$

where $\iota$ is the weight of the investment traded input $(0<\iota<1)$ and $\phi_{J}$ corresponds to the elasticity of substitution in investment between traded and non-traded inputs. The index $J^{T}$ is defined as a CES aggregator of home-produced traded inputs, $J^{H}$, and foreign-produced traded inputs, $J^{F}$ :

$$
\begin{equation*}
J^{T}=\left[\left(\iota_{H}\right)^{\frac{1}{\rho_{J}}}\left(J^{H}\right)^{\frac{\rho_{J}-1}{\rho_{J}}}+\left(1-\iota_{H}\right)^{\frac{1}{\rho_{J}}}\left(J^{F}\right)^{\frac{\rho_{J}-1}{\rho_{J}}}\right]^{\frac{\rho_{J}}{\rho_{J}-1}}, \tag{135}
\end{equation*}
$$

where $0<\iota_{H}<1$ is the weight of the home-produced traded in input in the overall traded investment bundle and $\rho_{J}$ corresponds to the elasticity of substitution between home- and foreign-produced traded inputs.

Following Horvath [2000], we assume that hours worked in the traded and the non-traded sectors are aggregated by means of a CES function:

$$
\begin{equation*}
L=\left[\vartheta^{-1 / \epsilon}\left(L^{H}\right)^{\frac{\epsilon+1}{\epsilon}}+(1-\vartheta)^{-1 / \epsilon}\left(L^{N}\right)^{\frac{\epsilon+1}{\epsilon}}\right]^{\frac{\epsilon}{\epsilon+1}} \tag{136}
\end{equation*}
$$

where $0<\vartheta<1$ is the weight of labor supply to the traded sector in the labor index $L($.$) and \epsilon$ measures the ease with which hours worked can be substituted for each other and thereby captures the degree of labor mobility across sectors.

The representative household chooses consumption, decides on labor supply, and investment that maximizes his/her lifetime utility:

$$
\begin{equation*}
U=\int_{0}^{\infty}\left\{\frac{1}{1-\frac{1}{\sigma_{C}}} C(t)^{1-\frac{1}{\sigma_{C}}}-\frac{1}{1+\frac{1}{\sigma_{L}}} L(t)^{1+\frac{1}{\sigma_{L}}}\right\} e^{-\beta t} \mathrm{~d} t \tag{137}
\end{equation*}
$$

subject to the flow budget constraint:

$$
\begin{equation*}
\dot{N}(t)=r^{\star} N(t)+R(t) K(t)+W(t) L(t)-T(t)-P_{C}\left(P^{T}(t), P^{N}(t)\right) C(t)-P_{J}\left(P^{T}(t), P^{N}(t)\right) J(t) \tag{138}
\end{equation*}
$$

and capital accumulation which evolves as follows:

$$
\begin{equation*}
\dot{K}(t)=I(t)-\delta_{K} K(t), \tag{139}
\end{equation*}
$$

where $I$ is investment and $0 \leq \delta_{K}<1$ is a fixed depreciation rate. The first term on the RHS of (138) $r^{\star} N(t)+R(t) K(t)+W(t) L(t)-T(t)$ is the representative household's real disposable income while the second term on the RHS, i.e., $P_{C}\left(P^{T}(t), P^{N}(t)\right) C(t)+P_{J}\left(P_{J}^{T}(t), P^{N}(t)\right) J(t)$, corresponds to consumption and investment expenditure including capital installation costs. More specifically, we assume that capital accumulation is subject to increasing and convex cost of net investment, $I(t)-\delta_{K} K(t):$

$$
\begin{equation*}
J(t)=I(t)+\Psi(I(t), K(t)) K(t) \tag{140}
\end{equation*}
$$

where $\Psi($.$) is increasing (i.e., \Psi^{\prime}()>$.0 ), convex (i.e., $\Psi^{\prime \prime}()>$.0 ), is equal to zero at $\delta_{K}$ (i.e., $\Psi\left(\delta_{K}\right)=0$ ), and has first partial derivative equal to zero as well at $\delta_{K}$ (i.e., $\Psi^{\prime}\left(\delta_{K}\right)=0$ ). We suppose the following functional form for the adjustment cost function:

$$
\begin{equation*}
\Psi(I(t), K(t))=\frac{\kappa}{2}\left(\frac{I(t)}{K(t)}-\delta_{K}\right)^{2} \tag{141}
\end{equation*}
$$

Using (141), partial derivatives of total investment expenditure are:

$$
\begin{align*}
\frac{\partial J(t)}{\partial I(t)} & =1+\kappa\left(\frac{I(t)}{K(t)}-\delta_{K}\right)  \tag{142a}\\
\frac{\partial J(t)}{\partial K(t)} & =-\frac{\kappa}{2}\left(\frac{I(t)}{K(t)}-\delta_{K}\right)\left(\frac{I(t)}{K(t)}+\delta_{K}\right) \tag{142b}
\end{align*}
$$

Denoting the co-state variables associated with (138) and (139) by $\lambda$ and $Q^{\prime}$, respectively, the first-order conditions characterizing the representative household's optimal plans are:

$$
\begin{gather*}
C(t)=\left(P_{C}(t) \lambda\right)^{-\sigma_{C}},  \tag{143a}\\
L(t)=(W(t) \lambda)^{\sigma_{L}},  \tag{143b}\\
Q(t)=P_{J}(t)\left[1+\kappa\left(\frac{I(t)}{K(t)}-\delta_{K}\right)\right],  \tag{143c}\\
\dot{\lambda}(t)=\lambda\left(\beta-r^{\star}\right),  \tag{143d}\\
\dot{Q}(t)=\left(r^{\star}+\delta_{K}\right) Q(t)-\left\{R(t)+P_{J}(t) \frac{\kappa}{2}\left(\frac{I(t)}{K(t)}-\delta_{K}\right)\left(\frac{I(t)}{K(t)}+\delta_{K}\right)\right\}, \tag{143e}
\end{gather*}
$$

and the transversality conditions $\lim _{t \rightarrow \infty} \bar{\lambda} N(t) e^{-\beta t}=0$ and $\lim _{t \rightarrow \infty} Q(t) K(t) e^{-\beta t}=0$; to derive (143c) and (143e), we used the fact that $Q(t)=Q^{\prime}(t) / \lambda(t)$.

Given the above consumption indices, we can derive appropriate price indices. With respect to the general consumption index, we obtain the consumption-based price index $P_{C}$ :

$$
\begin{equation*}
P_{C}=\left[\varphi\left(P^{T}\right)^{1-\phi}+(1-\varphi)\left(P^{N}\right)^{1-\phi}\right]^{\frac{1}{1-\phi}} \tag{144}
\end{equation*}
$$

where the price index for traded goods is:

$$
\begin{equation*}
P^{T}=\left[\varphi_{H}\left(P^{H}\right)^{1-\rho}+\left(1-\varphi_{H}\right)\right]^{\frac{1}{1-\rho}} \tag{145}
\end{equation*}
$$

Given the consumption-based price index (144), the representative household has the following demand of traded and non-traded goods:

$$
\begin{gather*}
C^{T}=\varphi\left(\frac{P^{T}}{P_{C}}\right)^{-\phi} C,  \tag{146a}\\
C^{N}=(1-\varphi)\left(\frac{P^{N}}{P_{C}}\right)^{-\phi} C . \tag{146b}
\end{gather*}
$$

Given the price indices (144) and (145), the representative household has the following demand of home-produced traded goods and foreign-produced traded goods:

$$
\begin{gather*}
C^{H}=\varphi\left(\frac{P^{T}}{P_{C}}\right)^{-\phi} \varphi_{H}\left(\frac{P^{H}}{P^{T}}\right)^{-\rho} C,  \tag{147a}\\
C^{F}=\varphi\left(\frac{P^{T}}{P_{C}}\right)^{-\phi}\left(1-\varphi_{H}\right)\left(\frac{1}{P_{T}}\right)^{-\rho} C . \tag{147b}
\end{gather*}
$$

As will be useful later, the percentage change in the consumption price index is a weighted average of percentage changes in the price of traded and non-traded goods in terms of foreign goods:

$$
\begin{gather*}
\hat{P}_{C}=\alpha_{C} \hat{P}^{T}+\left(1-\alpha_{C}\right) \hat{P}^{N}  \tag{148a}\\
\hat{P}^{T}=\alpha_{H} \hat{P}^{H} \tag{148b}
\end{gather*}
$$

where $\alpha_{C}$ is the tradable content of overall consumption expenditure and $\alpha^{H}$ is the home-produced goods content of consumption expenditure on traded goods:

$$
\begin{align*}
\alpha_{C} & =\varphi\left(\frac{P^{T}}{P_{C}}\right)^{1-\phi}  \tag{149a}\\
1-\alpha_{C} & =(1-\varphi)\left(\frac{P^{N}}{P_{C}}\right)^{1-\phi}  \tag{149b}\\
\alpha^{H} & =\varphi_{H}\left(\frac{P^{H}}{P^{T}}\right)^{1-\rho}  \tag{149c}\\
1-\alpha^{H} & =\left(1-\varphi_{H}\right)\left(\frac{1}{P^{T}}\right)^{1-\rho} \tag{149d}
\end{align*}
$$

Given the CES aggregator functions above, we can derive the appropriate price indices for investment. With respect to the general investment index, we obtain the investment-based price index $P_{J}$ :

$$
\begin{equation*}
P_{J}=\left[\iota\left(P_{J}^{T}\right)^{1-\phi_{J}}+(1-\iota)\left(P^{N}\right)^{1-\phi_{J}}\right]^{\frac{1}{1-\phi_{J}}} \tag{150}
\end{equation*}
$$

where the price index for traded goods is:

$$
\begin{equation*}
P_{J}^{T}=\left[\iota^{H}\left(P^{H}\right)^{1-\rho_{J}}+\left(1-\iota^{H}\right)\right]^{\frac{1}{1-\rho_{J}}} . \tag{151}
\end{equation*}
$$

Given the investment-based price index (150), we can derive the demand for inputs of the traded good and the non-traded good:

$$
\begin{gather*}
J^{T}=\iota\left(\frac{P_{J}^{T}}{P_{J}}\right)^{-\phi_{J}} J,  \tag{152a}\\
J^{N}=(1-\iota)\left(\frac{P^{N}}{P_{J}}\right)^{-\phi_{J}} J . \tag{152b}
\end{gather*}
$$

Given the price indices (150) and (151), we can derive the demand for inputs of home-produced traded goods and foreign-produced traded goods:

$$
\begin{gather*}
J^{H}=\iota\left(\frac{P_{J}^{T}}{P_{J}}\right)^{-\phi_{J}} \iota^{H}\left(\frac{P^{H}}{P_{J}^{T}}\right)^{-\rho_{J}} J,  \tag{153a}\\
J^{F}=\iota\left(\frac{P_{J}^{T}}{P_{J}}\right)^{-\phi_{J}}\left(1-\iota^{H}\right)\left(\frac{1}{P_{J}^{T}}\right)^{-\rho_{J}} J . \tag{153b}
\end{gather*}
$$

As will be useful later, the percentage change in the investment price index is a weighted average of percentage changes in the price of traded and non-traded inputs in terms of foreign inputs:

$$
\begin{gather*}
\hat{P}_{J}=\alpha_{J} \hat{P}_{J}^{T}+\left(1-\alpha_{J}\right) \hat{P}^{N},  \tag{154a}\\
\hat{P}_{J}^{T}=\alpha_{J}^{H} \hat{P}^{H}, \tag{154b}
\end{gather*}
$$

where $\alpha_{J}$ is the tradable content of overall investment expenditure and $\alpha_{J}^{H}$ is the home-produced goods content of investment expenditure on traded goods:

$$
\begin{gather*}
\alpha_{J}=\iota\left(\frac{P_{J}^{T}}{P_{J}}\right)^{1-\phi_{J}}  \tag{155a}\\
1-\alpha_{J}=(1-\iota)\left(\frac{P^{N}}{P_{J}}\right)^{1-\phi_{J}}  \tag{155b}\\
\alpha_{J}^{H}=\iota^{H}\left(\frac{P^{H}}{P_{J}^{T}}\right)^{1-\rho_{J}}  \tag{155c}\\
1-\alpha_{J}^{H}=\left(1-\iota^{H}\right)\left(\frac{1}{P_{J}^{T}}\right)^{1-\rho_{J}} . \tag{155d}
\end{gather*}
$$

The aggregate wage index, $W(t)$, associated with the labor index defined above (136) is:

$$
\begin{equation*}
W=\left[\vartheta\left(W^{H}\right)^{\epsilon+1}+(1-\vartheta)\left(W^{N}\right)^{\epsilon+1}\right]^{\frac{1}{\epsilon+1}} \tag{156}
\end{equation*}
$$

where $W^{H}$ and $W^{N}$ are wages paid in the traded and the non-traded sectors, respectively.
Given the aggregate wage index, we can derive the allocation of aggregate labor supply to the traded and the non-traded sector:

$$
\begin{gather*}
L^{H}=\vartheta\left(\frac{W^{H}}{W}\right)^{\epsilon} L,  \tag{157a}\\
L^{N}=(1-\vartheta)\left(\frac{W^{N}}{W}\right)^{\epsilon} L . \tag{157b}
\end{gather*}
$$

As will be useful later, the percentage change in the aggregate wage index is a weighted average of percentage changes in sectoral wages:

$$
\begin{equation*}
\hat{W}=\alpha_{L} \hat{W}^{H}+\left(1-\alpha_{L}\right) \hat{W}^{N}, \tag{158}
\end{equation*}
$$

where $\alpha_{L}$ is the tradable content of aggregate labor compensation:

$$
\begin{align*}
\alpha_{L} & =\vartheta\left(\frac{W^{H}}{W}\right)^{1+\epsilon}  \tag{159a}\\
1-\alpha_{L} & =(1-\vartheta)\left(\frac{W^{N}}{W}\right)^{1+\epsilon} . \tag{159b}
\end{align*}
$$

## H. 2 Firms

Both the traded and non-traded sectors use physical capital, $K^{j}$, and labor, $L^{j}$, according to constant returns to scale production functions $Y^{j}=Z^{j} F^{j}\left(K^{j}, L^{j}\right)$ which are assumed to take a CobbDouglas form:

$$
\begin{equation*}
Y^{j}=Z^{j}\left(L^{j}\right)^{\theta^{j}}\left(K^{j}\right)^{1-\theta^{j}}, \quad j=H, N \tag{160}
\end{equation*}
$$

where $\theta^{j}$ is the labor income share in sector $j$ and $Z^{j}$ corresponds to the total factor productivity. Both sectors face two cost components: a capital rental cost equal to $R$, and a labor cost equal to the wage rate, i.e., $W^{H}$ in the traded sector and $W^{N}$ in the non-traded sector.

Both sectors are assumed to be perfectly competitive and thus choose capital and labor by taking prices as given:

$$
\begin{equation*}
\max _{K^{j}, L^{j}} \Pi^{j}=\max _{K^{j}, L^{j}}\left\{P^{j} Y^{j}-W^{j} L^{j}-R K^{j}\right\} \tag{161}
\end{equation*}
$$

Since capital can move freely between the two sectors, the value of marginal products in the traded and non-traded sectors equalizes while costly labor mobility implies a wage differential across sectors:

$$
\begin{gather*}
P^{H} Z^{H}\left(1-\theta^{H}\right)\left(k^{H}\right)^{-\theta^{H}}=P^{N} Z^{N}\left(1-\theta^{N}\right)\left(k^{N}\right)^{-\theta^{N}} \equiv R,  \tag{162a}\\
P^{H} Z^{H} \theta^{H}\left(k^{H}\right)^{1-\theta^{H}} \equiv W^{H},  \tag{162b}\\
P^{N} Z^{N} \theta^{N}\left(k^{N}\right)^{1-\theta^{N}} \equiv W^{N}, \tag{162c}
\end{gather*}
$$

where $k^{j} \equiv K^{j} / L^{j}$ denotes the capital-labor ratio for sector $j=H, N$.
The resource constraint for capital is:

$$
\begin{equation*}
K^{H}+K^{N}=K . \tag{163}
\end{equation*}
$$

## H. 3 Short-Run Solutions

## Consumption and Labor

Before linearizing, we have to determine short-run solutions. First-order conditions (143a) and (143b) can be solved for consumption and aggregate labor supply which of course must hold at any point of time:

$$
\begin{equation*}
C=C\left(\bar{\lambda}, P^{N}, P^{H}\right), \quad L=L\left(\bar{\lambda}, W^{H}, W^{N}\right), \tag{164}
\end{equation*}
$$

with partial derivatives given by

$$
\begin{gather*}
\hat{C}=-\sigma_{C} \hat{\bar{\lambda}}-\sigma_{C} \alpha_{C} \alpha^{H} \hat{P}^{H}-\sigma_{C}\left(1-\alpha_{C}\right) \hat{P}^{N},  \tag{165a}\\
\hat{L}=\sigma_{L} \hat{\bar{\lambda}}+\sigma_{L}\left(1-\alpha_{L}\right) \hat{W}^{N}+\sigma_{L} \alpha_{L} \hat{W}^{H}, \tag{165b}
\end{gather*}
$$

where we used (158) and (148).
Inserting first the solution for consumption (164) into (146a)-(147b) allows us to solve for $C^{N}$, $C^{H}$, and $C^{F}$ :

$$
\begin{equation*}
C^{N}=C^{N}\left(\bar{\lambda}, P^{N}, P^{H}\right), \quad C^{H}=C^{H}\left(\bar{\lambda}, P^{N}, P^{H}\right), \quad C^{F}=C^{F}\left(\bar{\lambda}, P^{N}, P^{H}\right) \tag{166}
\end{equation*}
$$

with partial derivatives given by

$$
\begin{align*}
\hat{C}^{N} & =-\phi \hat{P}^{N}+\left(\phi-\sigma_{C}\right) \hat{P}_{C}-\sigma_{C} \hat{\bar{\lambda}}, \\
& =-\left[\left(1-\alpha_{C}\right) \phi+\alpha_{C} \sigma_{C}\right] \hat{P}^{N}+\left(\phi-\sigma_{C}\right) \alpha_{C} \alpha^{H} \hat{P}^{H}-\sigma_{C} \hat{\bar{\lambda}}  \tag{167a}\\
\hat{C}^{H} & \left.=-\left[\rho\left(1-\alpha^{H}\right)+\phi\left(1-\alpha_{C}\right) \alpha^{H}+\sigma_{C} \alpha_{C} \alpha^{H}\right] \hat{P}^{H}+\left(1-\alpha_{C}\right)\left(\phi-\sigma_{C}\right) \hat{P}^{N}-\sigma_{C}{ }_{C} \hat{\bar{x}} 167 \mathrm{~b}\right) \\
\hat{C}^{F} & =\alpha^{H}\left[\rho-\phi\left(1-\alpha_{C}\right)-\sigma_{C} \alpha_{C}\right] \hat{P}^{H}+\left(1-\alpha_{C}\right)\left(\phi-\sigma_{C}\right) \hat{P}^{N}-\sigma_{C} \hat{\bar{\lambda}} . \tag{167c}
\end{align*}
$$

Inserting first the solution for labor (164) into (157a)-(158) allows us to solve for $L^{H}$ and $L^{N}$ :

$$
\begin{equation*}
L^{H}=L^{H}\left(\bar{\lambda}, W^{H}, W^{N}\right), \quad L^{N}=L^{N}\left(\bar{\lambda}, W^{H}, W^{N}\right) \tag{168}
\end{equation*}
$$

with partial derivatives given by:

$$
\begin{gather*}
\hat{L}^{H}=\left[\epsilon\left(1-\alpha_{L}\right)+\sigma_{L} \alpha_{L}\right] \hat{W}^{H}-\left(1-\alpha_{L}\right)\left(\epsilon-\sigma_{L}\right) \hat{W}^{N}+\sigma_{L} \hat{\bar{\lambda}},  \tag{169a}\\
\hat{L}^{N}=\left[\epsilon \alpha_{L}+\sigma_{L}\left(1-\alpha_{L}\right)\right] \hat{W}^{N}-\alpha_{L}\left(\epsilon-\sigma_{L}\right) \hat{W}^{H}+\sigma_{L} \hat{\bar{\lambda}} . \tag{169b}
\end{gather*}
$$

## Sectoral Wages and Capital-Labor Ratios

Plugging the short-run solutions for $L^{H}$ and $L^{N}$ given by (168) into the resource constraint for capital (163), the system of four equations consisting of (162a)-(162c) together with (163) can be solved for sectoral wages $W^{j}$ and sectoral capital-labor ratios $k^{j}$. Denoting by $\xi^{N} \equiv K^{N} / K$ the share of non-traded capital in the aggregate stock of physical capital and log-differentiating (162a)-(162c) together with (163) yields in matrix form:

$$
\begin{align*}
& \left(\begin{array}{cccc}
-\theta^{H} & \theta^{N} & 0 & 0 \\
\left(1-\theta^{H}\right) & 0 & -1 & 0 \\
0 & \left(1-\theta^{N}\right) & 0 & -1 \\
\left(1-\xi^{N}\right) & \xi^{N} & \Psi_{W^{H}} & \Psi_{W^{N}}
\end{array}\right)\left(\begin{array}{c}
\hat{k}^{H} \\
\hat{k}^{N} \\
\hat{W}^{H} \\
\hat{W}^{N}
\end{array}\right) \\
= & \left(\begin{array}{c}
\hat{P}^{N}-\hat{P}^{H}-\hat{Z}^{H}+\hat{Z}^{N} \\
-\hat{P}^{H}-\hat{Z}^{H} \\
-\hat{P}^{N}-\hat{Z}^{N} \\
\hat{K}-\Psi_{\bar{\lambda}} \hat{\bar{\lambda}}
\end{array}\right) \tag{170}
\end{align*}
$$

where we set:

$$
\begin{align*}
\Psi_{W^{j}} & =\left(1-\xi^{N}\right) \frac{L_{W^{j}}^{H} W^{j}}{L^{H}}+\xi^{N} \frac{L_{W^{j}}^{N} W^{j}}{L^{N}}  \tag{171a}\\
\xi^{N} & \equiv \frac{k^{N} L^{N}}{K}  \tag{171b}\\
\Psi_{\bar{\lambda}} & =\left(1-\xi^{N}\right) \sigma_{L}+\xi^{N} \sigma_{L}=\sigma_{L} \tag{171c}
\end{align*}
$$

The short-run solutions for sectoral wages and capital-labor ratios are:

$$
\begin{equation*}
W^{j}=W^{j}\left(\bar{\lambda}, K, P^{N}, P^{H}, Z^{H}, Z^{N}\right), \quad k^{j}=k^{j}\left(\bar{\lambda}, K, P^{N}, P^{H}, Z^{H}, Z^{N}\right) \tag{172}
\end{equation*}
$$

Inserting first sectoral wages (172), sectoral hours worked (168) can be solved as functions of the shadow value of wealth, the capital stock, the price of non-traded goods in terms of foreign goods, $P^{N}$, and the terms of trade:

$$
\begin{equation*}
L^{j}=L^{j}\left(\bar{\lambda}, K, P^{N}, P^{H}, Z^{H}, Z^{N}\right) . \tag{173}
\end{equation*}
$$

Finally, plugging solutions for sectoral labor (173) and sector capital-labor ratios (172), production functions (160) can be solved for sectoral value added:

$$
\begin{equation*}
Y^{j}=Y^{j}\left(\bar{\lambda}, K, P^{N}, P^{H}, Z^{H}, Z^{N}\right) \tag{174}
\end{equation*}
$$

where

$$
\begin{equation*}
\hat{Y}^{j}=\hat{Z}^{j}+\sum_{X} \frac{\partial L^{j}}{\partial X} \frac{X}{L^{j}}+\left(1-\theta^{j}\right) \frac{\partial k^{j}}{\partial X} \frac{X}{k^{j}}, \tag{175}
\end{equation*}
$$

where $X=\bar{\lambda}, K, P^{N}, P^{H}, Z^{H}, Z^{N}$.
The Return on Domestic Capital, $R$
The return on domestic capital is:

$$
\begin{equation*}
R=P^{H} Z^{H}\left(1-\theta^{H}\right)\left(k^{H}\right)^{-\theta^{H}} \tag{176}
\end{equation*}
$$

Inserting first the short-run static solution for the capital-labor ratio $k^{H}$ given by (172), eq. (176) can be solved for the return on domestic capital:

$$
\begin{equation*}
R=R\left(\bar{\lambda}, K, P^{N}, P^{H}, Z^{H}, Z^{N}\right) \tag{177}
\end{equation*}
$$

where partial derivatives are

$$
\begin{equation*}
\hat{R}=\hat{Z}^{j}+\sum_{X} \frac{\partial k^{H}}{\partial X} \frac{X}{k^{H}} \tag{178}
\end{equation*}
$$

where $X=\bar{\lambda}, K, P^{N}, P^{H}, Z^{H}, Z^{N}$.
Optimal Investment Decision, $I / K$
Eq. (143c) can be solved for the investment rate:

$$
\begin{equation*}
\frac{I}{K}=v\left(\frac{Q}{P_{I}\left(P^{T}, P^{N}\right)}\right)+\delta_{K} \tag{179}
\end{equation*}
$$

where

$$
\begin{equation*}
v(.)=\frac{1}{\kappa}\left(\frac{Q}{P_{J}}-1\right) \tag{180}
\end{equation*}
$$

with

$$
\begin{gather*}
v_{Q}=\frac{\partial v(.)}{\partial Q}=\frac{1}{\kappa} \frac{1}{P_{J}}>0  \tag{181a}\\
v_{P^{H}}=\frac{\partial v(.)}{\partial P^{H}}=-\frac{1}{\kappa} \frac{Q}{P_{J}} \frac{\alpha_{J} \alpha_{J}^{H}}{P^{H}}<0  \tag{181b}\\
v_{P^{N}}=\frac{\partial v(.)}{\partial P^{N}}=-\frac{1}{\kappa} \frac{Q}{P_{J}} \frac{\left(1-\alpha_{J}\right)}{P^{N}}<0 . \tag{181c}
\end{gather*}
$$

Inserting (179) into (140), investment including capital installation costs can be rewritten as follows:

$$
\begin{align*}
J & =K\left[\frac{I}{K}+\frac{\kappa}{2}\left(\frac{I}{K}-\delta_{K}\right)^{2}\right] \\
& =K\left[v(.)+\delta_{K}+\frac{\kappa}{2}(v(.))^{2}\right] \tag{182}
\end{align*}
$$

Eq. (182) can be solved for investment including capital installation costs:

$$
\begin{equation*}
J=J\left(K, Q, P^{N}, P^{H}\right) \tag{183}
\end{equation*}
$$

where

$$
\begin{gather*}
J_{K}=\frac{\partial J}{\partial K}=\frac{J}{K}  \tag{184a}\\
J_{X}=\frac{\partial J}{\partial X}=\kappa v_{X}(1+\kappa v(.))>0 \tag{184b}
\end{gather*}
$$

with $X=Q, P^{H}, P^{N}$.
Substituting (184) into (152b), (153a), and (153b) allows us to solve for the demand of nontraded, home-produced traded, and foreign inputs:

$$
\begin{equation*}
J^{N}=J^{N}\left(K, Q, P^{N}, P^{H}\right), \quad J^{H}=J^{H}\left(K, Q, P^{N}, P^{H}\right), \quad J^{F}=J^{F}\left(K, Q, P^{N}, P^{H}\right) \tag{185}
\end{equation*}
$$

with partial derivatives given by

$$
\begin{align*}
\hat{J}^{N} & =-\alpha_{J} \phi_{J} \hat{P}^{N}+\phi_{J} \alpha_{J} \alpha_{J}^{H} \hat{P}^{H}+\hat{J}, \\
& =\frac{Q}{P_{J}} \frac{(1+\kappa v(.))}{J} \hat{Q}-\left[\alpha_{J} \phi_{J}+\frac{Q}{P_{J}} \frac{(1+\kappa v(.))}{J}\left(1-\alpha_{J}\right)\right] \hat{P}^{N} \\
& +\alpha_{J} \alpha_{J}^{H}\left[\phi_{J}-\frac{Q}{P_{J}} \frac{(1+\kappa v(.))}{J}\right] \hat{P}^{H}+\hat{K},  \tag{186a}\\
\hat{J}^{H} & =-\left[\rho_{J}\left(1-\alpha_{J}^{H}\right)+\alpha_{J}^{H} \phi_{J}\left(1-\alpha_{J}\right)\right] \hat{P}^{H}+\phi_{J}\left(1-\alpha_{J}\right) \hat{P}^{N}+\hat{J}, \\
& =-\left\{\left[\rho_{J}\left(1-\alpha_{J}^{H}\right)+\alpha_{J}^{H} \phi_{J}\left(1-\alpha_{J}\right)\right]+\alpha_{J} \alpha_{J}^{H} \frac{Q}{P_{J}} \frac{(1+\kappa v(.))}{J}\right\} \hat{P}^{H} \\
& +\left(1-\alpha_{J}\right)\left[\phi_{J}-\frac{Q}{P_{J}} \frac{(1+\kappa v(.))}{J}\right] \hat{P}^{N}+\frac{Q}{P_{J}} \frac{(1+\kappa v(.))}{J} \hat{Q}+\hat{K},  \tag{186b}\\
\hat{J}^{F} & =\alpha_{J}^{H}\left[\rho_{J}-\phi_{J}\left(1-\alpha_{J}\right)\right] \hat{P}^{H}+\phi_{J}\left(1-\alpha_{J}\right) \hat{P}^{N}+\hat{J}, \\
& =\alpha_{J}^{H}\left\{\left[\rho_{J}-\phi_{J}\left(1-\alpha_{J}\right)\right]-\alpha_{J} \frac{Q}{P_{J}} \frac{(1+\kappa v(.))}{J}\right\} \hat{P}^{H} \\
& +\left(1-\alpha_{J}\right)\left[\phi_{J}-\frac{Q}{P_{J}} \frac{(1+\kappa v(.))}{J}\right] \hat{P}^{N}+\frac{Q}{P_{J}} \frac{(1+\kappa v(.))}{J} \hat{Q}+\hat{K}, \tag{186c}
\end{align*}
$$

where use has been made of (184), i.e.,

$$
\begin{aligned}
\hat{J}= & \hat{K}+\frac{Q}{P_{J}} \frac{(1+\kappa v(.))}{J} \hat{Q}-\frac{Q}{P_{J}} \frac{(1+\kappa v(.))}{J}\left(1-\alpha_{J}\right) \hat{P}^{N} \\
& -\alpha_{J} \alpha_{J}^{H} \frac{Q}{P_{J}} \frac{(1+\kappa v(.))}{J} \hat{P}^{H} .
\end{aligned}
$$

## H. 4 Market Clearing Conditions

Finally, we have to solve for the relative price of non-traded goods and the terms of trade.
Market Clearing Condition for Non-Tradables
The role of the price of non-tradables in terms of foreign goods is to clear the non-traded goods market:

$$
\begin{equation*}
Y^{N}=C^{N}+G^{N}+J^{N} \tag{187}
\end{equation*}
$$

Inserting solutions for $C^{N}, J^{N}, Y^{N}$ given by (166), (183), (174), respectively, the non-traded goods market clearing condition (187) can be rewritten as follows:

$$
\begin{equation*}
Y^{N}\left(\bar{\lambda}, K, P^{N}, P^{H}, Z^{H}, Z^{N}\right)=C^{N}\left(\bar{\lambda}, P^{N}, P^{H}\right)+G^{N}+J^{N} \tag{188}
\end{equation*}
$$

Eq. (188) can be solved for the relative price of non-tradables:

$$
\begin{equation*}
P^{N}=\Psi^{N}\left(K, Q, P^{H}, Z^{H}, Z^{N}, \bar{\lambda}\right) \tag{189}
\end{equation*}
$$

with partial derivatives given by:

$$
\begin{align*}
\Psi_{K}^{N} & =\frac{\partial \Psi^{N}}{\partial K}=-\frac{\left(Y_{K}^{N}-J_{K}^{N}\right)}{\Delta^{N}}<0  \tag{190a}\\
\Psi_{Q}^{N} & =\frac{\partial \Psi^{N}}{\partial Q}=\frac{J_{Q}^{N}}{\Delta^{N}}>0  \tag{190b}\\
\Psi_{P^{H}}^{N} & =\frac{\partial \Psi^{N}}{\partial P^{H}}=-\frac{\left(Y_{P^{H}}^{N}-C_{P^{H}}^{N}-J_{P^{H}}^{N}\right)}{\Delta^{N}}>0  \tag{190c}\\
\Psi_{Z^{H}}^{N} & =\frac{\partial \Psi^{N}}{\partial Z^{H}}=-\frac{Y_{Z^{H}}^{N}}{\Delta^{N}}>0  \tag{190d}\\
\Psi_{Z^{N}}^{N} & =\frac{\partial \Psi^{N}}{\partial Z^{N}}=-\frac{Y_{Z^{N}}^{N}}{\Delta^{N}}<0, \tag{190e}
\end{align*}
$$

where we set

$$
\begin{equation*}
\Delta^{N}=\left(Y_{P^{N}}^{N}-C_{P^{N}}^{N}-J_{P^{N}}^{N}\right)>0 . \tag{191}
\end{equation*}
$$

## Market Clearing Condition for Home-Produced Traded Goods

The role of the price of home-produced goods in terms of foreign-produced goods or the terms of trade is to clear the home-produced traded goods market:

$$
\begin{equation*}
Y^{H}=C^{H}+G^{H}+J^{H}+X^{H} \tag{192}
\end{equation*}
$$

where $X^{H}$ stands for exports which are negatively related to the terms of trade:

$$
\begin{equation*}
X^{H}=\varphi_{X}\left(P^{H}\right)^{-\phi_{X}} \tag{193}
\end{equation*}
$$

where $\phi_{X}$ is the elasticity of exports with respect to the terms of trade.
Inserting solutions for $C^{H}, J^{H}, Y^{H}$ given by (166), (183), (174), respectively, the traded goods market clearing condition (192) can be rewritten as follows:

$$
\begin{equation*}
Y^{H}\left(\bar{\lambda}, K, P^{N}, P^{H}, Z^{H}, Z^{N}\right)=C^{H}\left(\bar{\lambda}, P^{N}, P^{H}\right)+G^{H}+J^{H}+X^{H}\left(P^{H}\right) . \tag{194}
\end{equation*}
$$

Eq. (194) can be solved for the terms of trade:

$$
\begin{equation*}
P^{H}=\Psi^{H}\left(K, Q, P^{N}, Z^{H}, Z^{N}, \bar{\lambda}\right), \tag{195}
\end{equation*}
$$

with partial derivatives given by:

$$
\begin{align*}
\Psi_{K}^{H} & =\frac{\partial \Psi^{H}}{\partial K}=-\frac{\left(Y_{K}^{H}-J_{K}^{H}\right)}{\Delta^{H}}<0  \tag{196a}\\
\Psi_{Q}^{H} & =\frac{\partial \Psi^{H}}{\partial Q}=\frac{J_{Q}^{H}}{\Delta^{H}}>0  \tag{196b}\\
\Psi_{P^{N}}^{H} & =\frac{\partial \Psi^{H}}{\partial P^{N}}=-\frac{\left(Y_{P^{N}}^{H}-C_{P^{N}}^{H}-J_{P^{N}}^{H}\right)}{\Psi^{N}}>0  \tag{196c}\\
\Psi_{Z^{H}}^{H} & =\frac{\partial \Psi^{H}}{\partial Z^{H}}=-\frac{Y_{Z^{H}}^{H}}{\Delta^{H}}<0  \tag{196d}\\
\Psi_{Z^{N}}^{H} & =\frac{\partial \Psi^{H}}{\partial Z^{N}}=-\frac{Y_{Z^{N}}^{H}}{\Delta^{H}}>0 \tag{196e}
\end{align*}
$$

where we set

$$
\begin{equation*}
\Delta^{H}=\left(Y_{P^{H}}^{H}-C_{P^{H}}^{H}-J_{P^{H}}^{H}-X_{P^{H}}^{H}\right)>0, \tag{197}
\end{equation*}
$$

where $X_{P^{H}}^{H}=\frac{\partial X^{H}}{\partial P^{H}}<0$.

## H. 5 Solving the Model

In our model, there are three state variables, namely $K, Z^{H}, Z^{N}$, and one control variable, $Q$. To solve the model, we have to express all variables in terms of state and control variables. Plugging first eq. (195) into (189) allows us to solve for the relative price of non-tradables:

$$
\begin{equation*}
P^{N}=P^{N}\left(K, Q, Z^{H}, Z^{N}, \bar{\lambda}\right), \tag{198}
\end{equation*}
$$

where partial derivatives (with respect to $X=K, Q, Z^{H}, Z^{N}$ ) are given by

$$
\begin{equation*}
P_{X}^{N}=\frac{\partial P^{N}}{\partial X}=\frac{\Psi_{X}^{N}+\Psi_{P_{H}}^{N} \Psi_{X}^{H}}{\Delta^{N}+\Psi_{P^{H}}^{N} \Psi_{P^{N}}^{H}}, \tag{199}
\end{equation*}
$$

with $P_{K}^{N}<0, P_{Q}^{N}>0, P_{Z^{H}}^{N} \gtrless 0, P_{Z^{N}}^{N}<0$.
Plugging first eq. (198) into (195) allows us to solve for the terms of trade:

$$
\begin{equation*}
P^{H}=P^{H}\left(K, Q, Z^{H}, Z^{N}, \bar{\lambda}\right) \tag{200}
\end{equation*}
$$

where partial derivatives (with respect to $X=K, Q, Z^{H}, Z^{N}$ ) are given by

$$
\begin{equation*}
P_{X}^{H}=\frac{\partial P^{H}}{\partial X}=\Psi_{X}^{H}+\Psi_{P^{N}}^{H} P_{X}^{N} \tag{201}
\end{equation*}
$$

with $P_{K}^{H}<0, P_{Q}^{H}>0, P_{Z^{H}}^{H}<0, P_{Z^{N}}^{H} \lessgtr 0$.
Substituting solutions for the relative price of non-tradables (198) and the terms of trade (200) into solutions for consumption (166), sectoral output (174), the return on domestic capital (177), and the optimal investment decision (179) yields:

$$
\begin{align*}
C^{j} & =C^{j}\left(K, Q, Z^{H}, Z^{N}, \bar{\lambda}\right)  \tag{202a}\\
Y^{j} & =Y^{j}\left(K, Q, Z^{H}, Z^{N}, \bar{\lambda}\right)  \tag{202b}\\
R & =R\left(K, Q, Z^{H}, Z^{N}, \bar{\lambda}\right)  \tag{202c}\\
v & =v\left(K, Q, Z^{H}, Z^{N}, \bar{\lambda}\right) \tag{202d}
\end{align*}
$$

Remembering that the non-traded input $J^{N}$ used to produce the capital good is equal to $(1-\iota)\left(\frac{P^{N}}{P_{J}}\right)^{-\phi_{J}} J$ (see eq. (152b)) with $J=I+\frac{\kappa}{2}\left(\frac{I}{K}-\delta_{K}\right)^{2} K$, using the fact that $J^{N}=$ $Y^{N}-C^{N}-G^{N}$ and inserting $I=\dot{K}+\delta_{K}$, the capital accumulation equation reads as follows:

$$
\begin{equation*}
\dot{K}=\frac{Y^{N}-C^{N}-G^{N}}{(1-\iota)\left(\frac{P^{N}}{P_{J}}\right)^{-\phi_{J}}}-\delta_{K} K-\frac{\kappa}{2}\left(\frac{I}{K}-\delta_{K}\right)^{2} K \tag{203}
\end{equation*}
$$

Inserting short-run solutions for non-traded output (202b) and for consumption in non-tradables (202a), substituting optimal investment decision (202d) into the physical capital accumulation equation (203), and plugging the short-run solution for the return on domestic capital (202c) into the dynamic equation for the shadow value of capital stock (143e), the dynamic system reads as follows: ${ }^{51}$

$$
\begin{align*}
\dot{K} \equiv \Upsilon\left(K, Q, Z^{H}, Z^{N}\right)= & \frac{Y^{N}\left(K, Q, Z^{H}, Z^{N}\right)-C^{N}\left(K, Q, Z^{H}, Z^{N}\right)-G^{N}}{(1-\iota)\left\{\frac{P^{N}(.)}{P_{J}\left[P^{H}(.), P^{N}(.)\right]}\right\}^{-\phi_{J}}} \\
& -\delta_{K} K-\frac{K}{2 \kappa}\left\{\frac{Q}{P_{J}\left[P^{H}(.), P^{N}(.)\right]}-1\right\}^{2},  \tag{204a}\\
\dot{Q} \equiv \Sigma\left(K, Q, Z^{H}, Z^{N}\right)= & \left(r^{\star}+\delta_{K}\right) Q-\left[R\left(K, Q, Z^{H}, Z^{N}\right)\right. \\
& \left.+P_{J}\left[P^{H}(.), P^{N}(.)\right] \frac{\kappa}{2} v(.)\left(v(.)+2 \delta_{K}\right)\right], \tag{204b}
\end{align*}
$$

where $P^{N}($.$) and P^{H}($.$) are given by (198) and (200).$
To facilitate the linearization, it is useful to break down the capital accumulation into two components:

$$
\begin{equation*}
\hat{K}=J-\delta_{K} K-\frac{\kappa}{2}\left(\frac{I}{K}-\delta_{K}\right)^{2} K \tag{205}
\end{equation*}
$$

The first component is $J$. Using the fact that $J=\frac{J^{N}}{(1-\iota)\left(\frac{P^{N}}{P_{J}}\right)^{-\phi_{J}}}$ and log-linearizing gives:

$$
\begin{equation*}
\hat{J}=\hat{J}^{N}+\phi_{J} \alpha_{J} \hat{P}^{N}-\phi_{J} \alpha_{J} \alpha_{J}^{H} \hat{P}^{H} \tag{206}
\end{equation*}
$$

where we used the fact that $\hat{P}_{J}=\alpha_{J} \alpha_{J}^{H} \hat{P}^{H}+\left(1-\alpha_{J}\right) \hat{P}^{N}$. Using (205) and the fact that $J^{N}=$ $Y^{N}-C^{N}-G^{N}$, linearizing (205) in the neighborhood of the steady-state gives:

$$
\begin{align*}
\dot{K} & =\frac{J}{J^{N}}\left[d Y^{N}(t)-d C^{N}(t)\right]+\phi_{J} \frac{J}{P^{N}} \alpha_{J} d P^{N}(t) \\
& -\phi_{J} \frac{J}{P^{H}} \alpha_{J} \alpha_{J}^{H} d P^{H}(t)-\delta_{K} d K(t), \tag{207}
\end{align*}
$$

where $J=I=\delta_{K} K$ in the long-run.
As will be useful, let us denote by $\Upsilon_{K}, \Upsilon_{Q}$, and $\Upsilon_{Z^{j}}$ the partial derivatives evaluated at the steady-state of the capital accumulation equation w.r.t. $K, Q$, and $Z^{j}$, respectively. Using (202) and (207), these elements of the Jacobian matrix are given by:

$$
\begin{align*}
\Upsilon_{K} & \equiv \frac{\partial \dot{K}}{\partial K}=\frac{J}{J^{N}}\left(Y_{K}^{N}-C_{K}^{N}\right)+\alpha_{J} \phi_{J} J\left(\frac{P_{K}^{N}}{P^{N}}-\alpha_{J}^{H} \frac{P_{K}^{H}}{P^{H}}\right)-\delta_{K} \gtrless 0  \tag{208a}\\
\Upsilon_{Q} & \equiv \frac{\partial \dot{K}}{\partial Q}=\frac{J}{J^{N}}\left(Y_{Q}^{N}-C_{Q}^{N}\right)+\alpha_{J} \phi_{J} J\left(\frac{P_{Q}^{N}}{P^{N}}-\alpha_{J}^{H} \frac{P_{Q}^{H}}{P^{H}}\right)>0  \tag{208b}\\
\Upsilon_{Z^{j}} & \equiv \frac{\partial \dot{K}}{\partial Z^{j}}=\frac{J}{J^{N}}\left(Y_{Z^{j}}^{N}-C_{Z^{j}}^{N}\right)+\alpha_{J} \phi_{J} J\left(\frac{P_{Z^{j}}^{N}}{P^{N}}-\alpha_{J}^{H} \frac{P_{Z^{j}}^{H}}{P^{H}}\right), \tag{208c}
\end{align*}
$$

where $J=\delta_{K} K$ in the long run.
Let us denote by $\Sigma_{K}, \Sigma_{Q}$, and $\Sigma_{Z^{j}}$ the partial derivatives evaluated at the steady-state of the dynamic equation for the marginal value of an additional unit of capital w.r.t. $K, Q$, and $Z^{j}$, respectively:

$$
\begin{align*}
\Sigma_{K} & \equiv \frac{\partial \dot{Q}}{\partial K}=-R_{K}-P_{J} \kappa v_{K} \delta_{K}>0  \tag{209a}\\
\Sigma_{Q} & \equiv \frac{\partial \dot{Q}}{\partial Q}=\left(r^{\star}+\delta_{K}\right)-P_{J} \kappa v_{Q} \delta_{K}=r^{\star}>0  \tag{209b}\\
\Sigma_{Z^{j}} & \equiv \frac{\partial \dot{Q}}{\partial Z^{j}}=-R_{Z^{j}}-P_{J} \kappa v_{Z^{j}} \delta_{K} \tag{209c}
\end{align*}
$$

[^37]Assuming that the saddle-path stability condition is fulfilled, and denoting the negative eigenvalue by $\nu_{1}$ and the positive eigenvalue by $\nu_{2}$, the general solutions for $K$ and $Q$ are:

$$
\begin{equation*}
K(t)-\tilde{K}=D_{1} e^{\nu_{1} t}+D_{2} e^{\nu_{2} t}, \quad Q(t)-\tilde{Q}=\omega_{2}^{1} D_{1} e^{\nu_{1} t}+\omega_{2}^{2} D_{2} e^{\nu_{2} t} \tag{210}
\end{equation*}
$$

where $K_{0}$ is the initial capital stock and $\left(1, \omega_{2}^{i}\right)^{\prime}$ is the eigenvector associated with eigenvalue $\nu_{i}$ :

$$
\begin{equation*}
\omega_{2}^{i}=\frac{\nu_{i}-\Upsilon_{K}}{\Upsilon_{Q}} \tag{211}
\end{equation*}
$$

Because $\nu_{1}<0, \Upsilon_{K}>0$ and $\Upsilon_{Q}>0$, we have $\omega_{2}^{1}<0$, regardless of sectoral capital intensities, which implies that the shadow value of investment and the stock physical capital move in opposite direction along a stable path (i.e., $D_{2}=0$ ).

## H. 6 Current Account Equation and Intertemporal Solvency Condition

To determine the current account equation, we use the following identities and properties:

$$
\begin{gather*}
P_{C} C=P^{H} C^{H}+C^{F}+P^{N} C^{N},  \tag{212a}\\
P_{J} J=P^{H} J^{H}+J^{F}+P^{N} J^{N},  \tag{212b}\\
T=G=P^{H} G^{H}+G^{F}+P^{N} G^{N},  \tag{212c}\\
W L+R K=\left(W^{H} L^{H}+R K^{H}\right)+\left(W^{N} L^{N}+R K^{N}\right)=P^{H} Y^{H}+P^{N} Y^{N}, \tag{212d}
\end{gather*}
$$

where (212d) follows from Euler theorem. Using (212d), inserting (212a)-(212c) into (138) and invoking market clearing conditions for non-traded goods (187) and home-produced traded goods (192) yields:

$$
\begin{align*}
\dot{N} & =r^{\star} N+P^{H}\left(Y^{H}-C^{H}-G^{H}-J^{H}\right)-\left(C^{F}+J^{F}+G^{F}\right), \\
& =r^{\star} N+P^{H} X^{H}-M^{F}, \tag{213}
\end{align*}
$$

where $X^{H}=Y^{H}-C^{H}-G^{H}-J^{H}$ stands for exports of home goods and we denote by $M^{F}$ imports of foreign consumption and investment goods:

$$
\begin{equation*}
M^{F}=C^{F}+G^{F}+J^{F} . \tag{214}
\end{equation*}
$$

Substituting first solutions for $P^{N}$ and $P^{H}$ given by (198) and (200), respectively, into (185) and (193) allows us to express the demand for input of foreign-produced traded goods, $J^{F}$, and exports of home goods, $X^{H}$ :

$$
\begin{align*}
J^{F} & =J^{F}\left(K, Q, Z^{H}, Z^{N}, \bar{\lambda}\right)  \tag{215a}\\
X^{H} & =X^{H}\left(K, Q, Z^{H}, Z^{N}, \bar{\lambda}\right) \tag{215b}
\end{align*}
$$

Inserting (215a)-(215b) into(213) allows us to write the current account equation as follows:

$$
\begin{align*}
\dot{N} & \equiv r^{\star} N+\Xi\left(K, Q, Z^{H}, Z^{N}\right) \\
& =r^{\star} N+P^{H}\left(K, Q, Z^{H}, Z^{N}\right) X^{H}\left(K, Q, Z^{H}, Z^{N}\right)-M^{F}\left(K, Q, Z^{H}, Z^{N}\right) \tag{216}
\end{align*}
$$

Let us denote by $\Xi_{K}, \Xi_{Q}$, and $\Xi_{Z^{j}}$ the partial derivatives evaluated at the steady-state of the dynamic equation for the current account w.r.t. $K, Q$, and $Z^{j}$, respectively:

$$
\begin{align*}
\Xi_{K} & \equiv \frac{\partial \dot{N}}{\partial K}=\left(1-\phi_{X}\right) X^{H} P_{K}^{H}-M_{K}^{F}  \tag{217a}\\
\Xi_{Q} & \equiv \frac{\partial \dot{N}}{\partial Q}=\left(1-\phi_{X}\right) X^{H} P_{Q}^{H}-M_{Q}^{F}  \tag{217b}\\
\Xi_{Z^{j}} & \equiv \frac{\partial \dot{N}}{\partial Z^{j}}=\left(1-\phi_{X}\right) X^{H} P_{Z^{j}}^{H}-M_{Z^{j}}^{F} \tag{217c}
\end{align*}
$$

where we used the fact that $P^{H} X^{H}=\varphi_{X}\left(P^{H}\right)^{1-\phi_{X}}$ (see eq. (193)).
Linearizing (216) in the neighborhood of the steady-state, making use of (217a) and (217b), inserting solutions for $K(t)$ and $Q(t)$ given by (210) and solving yields the general solution for the net foreign asset position:

$$
\begin{equation*}
N(t)=\tilde{N}+\left[\left(N_{0}-\tilde{N}\right)-\Psi_{1} D_{1}-\Psi_{2} D_{2}\right] e^{r^{\star} t}+\Psi_{1} D_{1} e^{\nu_{1} t}+\Psi_{2} D_{2} e^{\nu_{2} t} \tag{218}
\end{equation*}
$$

where $N_{0}$ is the initial stock of traded bonds and we set

$$
\begin{align*}
E_{i} & =\Xi_{K}+\Xi_{Q} \omega_{2}^{i}  \tag{219a}\\
\Psi_{i} & =\frac{E_{i}}{\nu_{i}-r^{\star}} \tag{219b}
\end{align*}
$$

Invoking the transversality condition leads to the linearized version of the nations's intertemporal solvency condition:

$$
\begin{equation*}
\tilde{N}-N_{0}=\Psi_{1}\left(\tilde{K}-K_{0}\right) \tag{220}
\end{equation*}
$$

where $K_{0}$ is the initial stock of physical capital.

## H. 7 Derivation of the Accumulation Equation of Non Human Wealth

Remembering that the stock of financial wealth $A(t)$ is equal to $N(t)+Q(t) K(t)$, differentiating w.r.t. time, i.e., $\dot{A}(t)=\dot{N}(t)+\dot{Q}(t) K(t)+Q(t) \dot{K}(t)$, plugging the dynamic equation for the marginal value of capital (143e), inserting the accumulation equations for physical capital (139) and traded bonds (138), yields the accumulation equation for the stock of financial wealth or the dynamic equation for private savings:

$$
\begin{equation*}
\dot{A}(t)=r^{\star} A(t)+W(t) L(t)-T(t)-P_{C}(t) C(t) \tag{221}
\end{equation*}
$$

where we assume that the government levies lump-sum taxes, $T$, to finance purchases of foreignproduced, home-produced and non-traded goods, i.e., $T=G=\left(G^{F}+P^{H}(.) G^{H}+P^{N}(.) G^{N}\right)$.

We first determine short-run solutions for aggregate labor supply and aggregate wage index. Inserting first short-run solutions for the relative price of non-tradables (198) and the terms of trade (200) into (156) allows us to solve for sectoral wages, $W^{j}=W^{j}\left(K, Q, Z^{H}, Z^{N}, \bar{\lambda}\right)$. Then inserting sectoral wages into (156) and (164) allows us to solve for aggregate wage, aggregate labor supply and consumption:

$$
\begin{align*}
W & =W\left(K, Q, Z^{H}, Z^{N}, \bar{\lambda}\right)  \tag{222a}\\
L & =L\left(K, Q, Z^{H}, Z^{N}, \bar{\lambda}\right)  \tag{222b}\\
C & =C\left(K, Q, Z^{H}, Z^{N}, \bar{\lambda}\right) . \tag{222c}
\end{align*}
$$

Inserting short-run solutions for the relative price of non-tradables (198) and the terms of trade (200) into (156) into (144) and (212c) allows us to solve for the consumption price index and government spending:

$$
\begin{align*}
G & =G\left(K, Q, Z^{H}, Z^{N}, \bar{\lambda}\right),  \tag{223a}\\
P_{C} & =P_{C}\left(K, Q, Z^{H}, Z^{N}, \bar{\lambda}\right) \tag{223b}
\end{align*}
$$

where partial derivatives are $G_{X}=P_{X}^{H} G^{H}+P_{X}^{N} G^{N}$ with $X=K, Q, Z^{j}(j=H, N)$ and

$$
\begin{equation*}
\frac{\partial P_{C}}{\partial X}=\alpha_{C} \alpha^{H} \frac{P_{C}}{P^{H}} P_{X}^{H}+\left(1-\alpha_{C}\right) \frac{P_{C}}{P^{N}} P_{X}^{N} \tag{224}
\end{equation*}
$$

with $X=K, Q, Z^{j}$
Inserting (222a)-(222c) into (213) allows us to write the current account equation as follows:

$$
\begin{align*}
\dot{A} & \equiv r^{\star} A+\Lambda\left(K, Q, Z^{H}, Z^{N}\right) \\
& =r^{\star} A+W\left(K, Q, Z^{H}, Z^{N}\right) L\left(K, Q, Z^{H}, Z^{N}\right)-G\left(K, Q, Z^{H}, Z^{N}\right) \\
& -P_{C}\left[P^{H}(.), P^{N}(.)\right] C\left(K, Q, Z^{H}, Z^{N}\right) \tag{225}
\end{align*}
$$

where $P^{N}$ and $P^{H}$ are given by (198) and (200), respectively.
Let us denote by $\Lambda_{K}, \Lambda_{Q}$, and $\Lambda_{Z^{j}}$ the partial derivatives evaluated at the steady-state of the dynamic equation for the non human wealth w.r.t. $K, Q$, and $Z^{j}$, respectively:

$$
\begin{align*}
\Lambda_{K} & \equiv \frac{\partial \dot{A}}{\partial K}=\left(W_{K} L+W L_{K}\right)-G_{K}-\left(\frac{\partial P_{C}}{\partial K} C+P_{C} C_{K}\right)  \tag{226a}\\
\Lambda_{Q} & \equiv \frac{\partial \dot{A}}{\partial Q}=\left(W_{Q} L+W L_{Q}\right)-G_{Q}-\left(\frac{\partial P_{C}}{\partial Q} C+P_{C} C_{Q}\right)  \tag{226b}\\
\Lambda_{Z^{j}} & \equiv \frac{\partial \dot{A}}{\partial Z^{j}}=\left(W_{Z^{j}} L+W L_{Z^{j}}\right)-G_{Z^{j}}-\left(\frac{\partial P_{C}}{\partial Z^{j}} C+P_{C} C_{Z^{j}}\right) . \tag{226c}
\end{align*}
$$

Linearizing (225) in the neighborhood of the steady-state, making use of (226a) and (226b), inserting solutions for $K(t)$ and $Q(t)$ given by (210) and solving yields the general solution for the stock of non human wealth:

$$
\begin{equation*}
A(t)=\tilde{A}+\left[\left(A_{0}-\tilde{A}\right)-\Delta_{1} D_{1}-\Delta_{2} D_{2}\right] e^{r^{\star} t}+\Delta_{1} D_{1} e^{\nu_{1} t}+\Delta_{2} D_{2} e^{\nu_{2} t} \tag{227}
\end{equation*}
$$

where $A_{0}$ is the initial stock of financial wealth and we set

$$
\begin{align*}
M_{i} & =A_{K}+A_{Q} \omega_{2}^{i},  \tag{228a}\\
\Delta_{i} & =\frac{M_{i}}{\nu_{i}-r^{\star}} . \tag{228b}
\end{align*}
$$

The linearized version of the representative household's intertemporal solvency condition is:

$$
\begin{equation*}
\tilde{A}-A_{0}=\Delta_{1}\left(\tilde{K}-K_{0}\right) \tag{229}
\end{equation*}
$$

where $A_{0}$ is the initial stock of non human wealth.

## H. 8 The Steady-State

Below, we characterize the whole steady-state and use tilde to denote long-run values. Setting $\dot{N}=\dot{K}=\dot{Q}=0$ into (138), (139) and (143e), and inserting short-run static solutions for $k^{N}, Y^{N}$ and $Y^{H}, C^{j}$ derived above, the steady-state can be summarized by four equations:

$$
\begin{gather*}
Z^{H}\left(1-\theta^{H}\right)\left[k^{H}\left(\tilde{K}, \tilde{P}^{H}, \tilde{P}^{N}, Z^{H}, Z^{N}, \bar{\lambda}\right)\right]^{-\theta^{H}}=P_{J}\left(\tilde{P}^{H}, \tilde{P}^{N}\right)\left(r^{\star}+\delta_{K}\right),  \tag{230a}\\
Y^{N}\left(\tilde{K}, \tilde{P}^{H}, \tilde{P}^{N}, Z^{H}, Z^{N}, \bar{\lambda}\right)=C^{N}\left(\tilde{P}^{H}, \tilde{P}^{N}, \bar{\lambda}\right)+\left(1-\alpha_{J}\right) P_{J}\left(\tilde{P}^{H}, \tilde{P}^{N}\right) \delta_{K} \tilde{K}+G^{N},  \tag{230b}\\
Y^{H}\left(\tilde{K}, \tilde{P}^{H}, \tilde{P}^{N}, Z^{H}, Z^{N}, \bar{\lambda}\right)=C^{H}\left(\tilde{K}, \tilde{Q}, Z^{H}, Z^{N} \bar{\lambda}\right)+\alpha_{J} \alpha_{J}^{H} P_{J}\left(\tilde{P}^{H}, \tilde{P}^{N}\right) \delta_{K} \tilde{K}+G^{H}+X^{H}\left(\tilde{P}^{H}\right),  \tag{230c}\\
r^{\star} \tilde{N}+\tilde{P}^{H} X^{H}\left(\tilde{P}^{H}\right)-M^{F}\left(\tilde{K}, \tilde{P}^{H}, \tilde{P}^{N}, \bar{\lambda}\right)  \tag{230d}\\
\tilde{N}-N_{0}=\Psi_{1}\left(\tilde{K}-K_{0}\right) \tag{230e}
\end{gather*}
$$

These five equations jointly determine $\tilde{P}^{N}, \tilde{P}^{H}, \tilde{K}, \tilde{N}$ and $\bar{\lambda}$.

## H. 9 Solving for the Steady-State Value of the Price of Non-Tradables in terms of Tradables

In this subsection, we show that when investment is both traded and non-traded, a technology shock biased toward the traded sector must be consistently measured by the log-linearized version of the expression below:

$$
\begin{equation*}
\left[\frac{Z^{H}}{\left(Z^{N}\right)^{\frac{\theta^{H}}{\theta^{N}}}}\right]^{\frac{1}{\left(1-\alpha_{J}\right)+\alpha_{J}\left(\frac{\theta^{H}}{\theta^{N}}\right)}} \tag{231}
\end{equation*}
$$

Note that the LIS is denoted by $s_{L}^{j}$ in the main text. To differentiate the LIS in the CES economy from the LIS in the Cobb-Douglas economy, the LIS in the latter model is denoted by $\theta^{j}$. To obtain (231), we first repeat the first-order conditions from the firm's maximization problem:

$$
\begin{gather*}
P^{H} Z^{H}\left(1-\theta^{H}\right)\left(k^{H}\right)^{-\theta^{H}}=P^{N} Z^{N}\left(1-\theta^{N}\right)\left(k^{N}\right)^{-\theta^{N}} \equiv R,  \tag{232a}\\
P^{H} Z^{H} \theta^{H}\left(k^{H}\right)^{1-\theta^{H}} \equiv W^{H}  \tag{232b}\\
P^{N} Z^{N} \theta^{N}\left(k^{N}\right)^{1-\theta^{N}} \equiv W^{N} \tag{232c}
\end{gather*}
$$

Combining the return on domestic capital with the return on labor leads to the sectoral capital-labor ratio:

$$
\begin{equation*}
k^{j}=\left(\frac{1-\theta^{j}}{\theta^{j}}\right) \frac{W^{j}}{R} . \tag{233}
\end{equation*}
$$

Dividing (232c) by (232b) and eliminating the capital-labor ratio by using (233) yields:

$$
\frac{P^{N} Z^{N} \theta^{N}\left[\left(\frac{1-\theta^{N}}{\theta^{N}}\right) \frac{W^{N}}{R}\right]^{1-\theta^{N}}}{P^{H} Z^{H} \theta^{H}\left[\left(\frac{1-\theta^{H}}{\theta^{H}}\right) \frac{W^{H}}{R}\right]^{1-\theta^{H}}}=\frac{W^{N}}{W^{H}}
$$

Rearranging terms and isolating $P^{N} / P^{H}$ leads to:

$$
\begin{equation*}
\frac{P^{N}}{P^{H}}=\frac{Z^{H}}{Z^{N}} \frac{\left(\theta^{H}\right)^{\theta^{H}}\left(1-\theta^{H}\right)^{1-\theta^{H}}}{\left(\theta^{N}\right)^{\theta^{N}}\left(1-\theta^{N}\right)^{1-\theta^{N}}} \frac{\left(W^{N}\right)^{\theta^{N}}}{\left(W^{H}\right)^{\theta^{H}}} \frac{R^{1-\theta^{N}}}{R^{1-\theta^{T}}} \tag{234}
\end{equation*}
$$

Substituting (233) into (232b) leads to:

$$
\begin{align*}
W^{H} & =P^{H} Z^{H} \theta^{H}\left[\left(\frac{1-\theta^{H}}{\theta^{H}}\right) \frac{W^{H}}{R}\right]^{1-\theta^{H}} \\
& =\left(P^{H}\right)^{\frac{1}{\theta^{H}}}\left(Z^{H}\right)^{\frac{1}{\theta^{H}}}\left(\Psi^{H}\right)^{\frac{1}{\theta^{H}}} R^{-\frac{1-\theta^{H}}{\theta^{H}}} \tag{235}
\end{align*}
$$

where we set

$$
\begin{equation*}
\Psi^{j}=\left(\theta^{j}\right)^{\theta^{j}}\left(1-\theta^{j}\right)^{1-\theta^{j}}, j=H, N . \tag{236}
\end{equation*}
$$

Multiplying and dividing the RHS of eq. (234) by $\left(W^{N}\right)^{\theta^{N}}$, using (236), substituting (235), eq. (234) can be rewritten as follows:

$$
\begin{align*}
P^{N} & =\frac{Z^{H}}{Z^{N}} \frac{\Psi^{H}}{\Psi^{N}}\left(\frac{W^{N}}{W^{H}}\right)^{\theta^{N}}\left(W^{H}\right)^{\theta^{N}-\theta^{H}} \frac{R^{1-\theta^{N}}}{R^{1-\theta^{T}}}, \\
& =\left(P^{H}\right)^{\frac{\theta^{N}}{\theta^{H}}} \frac{\left(Z^{H}\right)^{\frac{\theta^{N}}{\theta^{H}}}}{Z^{N}} \frac{\left(\Psi^{H}\right)^{\frac{\theta^{N}}{\theta^{H}}}}{\Psi^{N}}\left(\frac{W^{N}}{W^{H}}\right)^{\theta^{N}} R^{\frac{\theta^{H}-\theta^{N}}{\theta^{H}}} . \tag{237}
\end{align*}
$$

Totally differentiating eq. (230a), i.e., $R=P_{J}\left(P^{H}, P^{N}\right)\left(r^{\star}+\delta_{K}\right)$, yields:

$$
\begin{equation*}
\hat{R}=\alpha_{J} \alpha_{J}^{H} \hat{P}^{H}+\left(1-\alpha_{J}\right) \hat{P}^{N} . \tag{238}
\end{equation*}
$$

Totally differentiating eq. (237) and inserting (238), the relative price of non-tradables in rate of change reads as follows:

$$
\begin{align*}
\hat{P}^{N} & =\left(\frac{\theta^{N}}{\theta^{H}}\right) \hat{Z^{H}}-\hat{Z}^{N}+\left(\frac{\theta^{N}}{\theta^{H}}\right) \hat{P}^{H}+\theta^{N}\left(\hat{W}^{N}-\hat{W}^{H}\right) \\
& +\left(\frac{\theta^{H}-\theta^{N}}{\theta^{H}}\right)\left[\alpha_{J} \alpha_{J}^{H} \hat{P}^{H}+\left(1-\alpha_{J}\right) \hat{P}^{N}\right], \\
\hat{P}^{N}\left[\theta^{H}+\left(\theta^{N}-\theta^{H}\right)\left(1-\alpha_{J}\right)\right] & =\theta^{N} \hat{Z}^{H}-\theta^{H} \hat{Z}^{N}+\theta^{N} \theta^{H}\left(\hat{W}^{N}-\hat{W}^{H}\right) \\
& +\left[\theta^{N}+\left(\theta^{H}-\theta^{N}\right) \alpha_{J} \alpha_{J}^{H}\right] \hat{P}^{H}, \\
\hat{P}^{N}\left[\left(1-\alpha_{J}\right)+\alpha_{J} \frac{\theta^{H}}{\theta^{N}}\right] & =\left(\hat{Z}^{H}-\frac{\theta^{H}}{\theta^{N}} \hat{Z}^{N}\right)+\theta^{H}\left(\hat{W}^{N}-\hat{W}^{H}\right) \\
& +\left[1+\left(\frac{\theta^{H}-\theta^{N}}{\theta^{N}}\right) \alpha_{J} \alpha_{J}^{H}\right] \hat{P}^{H} . \tag{239}
\end{align*}
$$

The change in the relative price of non-tradables in percentage is thus given by:

$$
\begin{align*}
\hat{P}^{N} & =\frac{\left(\hat{Z}^{H}-\frac{\theta^{H}}{\theta^{N}} \hat{Z}^{N}\right)}{\left[\left(1-\alpha_{J}\right)+\alpha_{J} \frac{\theta^{H}}{\theta^{N}}\right]}+\frac{\theta^{H}\left(\hat{W}^{N}-\hat{W}^{H}\right)}{\left[\left(1-\alpha_{J}\right)+\alpha_{J} \frac{\theta^{H}}{\theta^{N}}\right]} \\
& +\frac{\left[1+\left(\frac{\theta^{H}-\theta^{N}}{\theta^{N}}\right) \alpha_{J} \alpha_{J}^{H}\right] \hat{P}^{H}}{\left[\left(1-\alpha_{J}\right)+\alpha_{J} \frac{\theta^{H}}{\theta^{N}}\right]} . \tag{240}
\end{align*}
$$

To calculate the change in price of non-traded goods relative to traded goods, we subtract $\hat{P}^{T}=$ $\alpha^{H} \hat{P}^{H}$ from both sides of (240) by assuming that $\alpha^{H} \simeq \alpha_{J}^{H}$ :

$$
\begin{equation*}
\hat{P}^{N}-\hat{P}^{T}=\frac{\left(\hat{Z}^{H}-\frac{\frac{\theta}{}^{H}}{\theta^{N}} \hat{Z}^{N}\right)}{\left[\left(1-\alpha_{J}\right)+\alpha_{J} \frac{\theta^{H}}{\theta^{N}}\right]}+\frac{\theta^{H}\left(\hat{W}^{N}-\hat{W}^{H}\right)}{\left[\left(1-\alpha_{J}\right)+\alpha_{J} \frac{\theta^{H}}{\theta^{N}}\right]}+\frac{\left(1-\alpha^{H}\right) \hat{P}^{H}}{\left[\left(1-\alpha_{J}\right)+\alpha_{J} \frac{\theta^{H}}{\theta^{N}}\right]} . \tag{241}
\end{equation*}
$$

Eq. (241) shows that sector $j$ 's TFP must be adjusted with sectoral labor income shares, $\theta^{j}$, along with the tradable content of investment expenditure, $\alpha_{J}$. Thus, denoting by:

$$
\begin{gather*}
a=\frac{1}{\left[\left(1-\alpha_{J}\right)+\alpha_{J} \frac{\theta^{H}}{\theta^{N}}\right]},  \tag{242a}\\
b=a \frac{\theta^{H}}{\theta^{N}}, \tag{242b}
\end{gather*}
$$

the measure of the technology bias toward tradables is given by:

$$
\begin{equation*}
\frac{\left(Z^{H}\right)^{a}}{\left(Z^{H}\right)^{b}} \tag{243}
\end{equation*}
$$

It is worth mentioning that:

- if the country is small on world goods market, then the terms of trade are fixed, i.e., $\hat{P}^{H}=0$, or if the country does not import consumption and investment goods, i.e., $\alpha^{H}=1$, the last term on the RHS of eq. (241) vanishes;
- if we assume perfect mobility of labor across sectors, then sectoral wages grow that the same speed, i.e., $\hat{W}^{N}=\hat{W}^{H}$, and thus the second term on the RHS of eq. (241) vanishes;
- if we consider a small open economy model with perfect mobility of labor across sectors, then a labor share adjusted productivity differential of $1 \%$, i.e., $\frac{\left(\hat{Z}^{H}-\frac{\theta^{H}}{\theta^{N}} \hat{Z}^{N}\right)}{\left[\left(1-\alpha_{J}\right)+\alpha_{J} \frac{\theta^{H}}{\theta^{N}}\right]}=1 \%$, appreciates the price of non-traded goods relative to traded goods by $1 \%$ in the long-run, i.e., $\hat{P}^{N}-\hat{P}^{T}=1 \%$.


## I Solving for Permanent Technology Shocks

In this section, we provide the main steps for the derivation of formal solutions following a permanent technology shock biased toward the traded sector.

## I. 1 Sectoral Technology Shocks

In line with our empirical findings, we assume that total factor productivity in sector $j, Z^{j}(t)$, evolves according to the following dynamic equation:

$$
\begin{equation*}
Z^{j}(t)=\tilde{Z}^{j}+z^{j} e^{-\xi^{j} t} \tag{244}
\end{equation*}
$$

where $\tilde{Z}^{j}$ and $\tilde{Z}_{0}^{j}$ are the new and initial steady-state values of TFP in sector $j ; z^{j}=\tilde{Z}_{0}^{j} \bar{z}^{j}$ is a parameter whose significance will be detailed below; $\xi^{j}$ is a positive parameter which governs the speed at which sector $j^{\prime}$ TFP converges toward its new long-run level. To be consistent with our VAR specification, we express (244) in percentage deviation from initial steady-state:

$$
\begin{align*}
\hat{Z}^{j}(t) & =\frac{Z^{j}(t)-\tilde{Z}_{0}^{j}}{\tilde{Z}_{0}^{j}} \\
& =\hat{\tilde{Z}}^{j}+\bar{z}^{j} e^{-\xi^{j} t} \tag{245}
\end{align*}
$$

where $\hat{\tilde{Z}}^{j}$ is the percentage deviation of sector $j$ ' TFP relative to its initial value:

$$
\begin{equation*}
\hat{\tilde{Z}}^{j}=\frac{\tilde{Z}_{1}^{j}-\tilde{Z}_{0}^{j}}{\tilde{Z}_{0}^{j}} \tag{246}
\end{equation*}
$$

Setting $t=0$ into (245) yields:

$$
\begin{equation*}
\hat{Z}^{j}(0)=\hat{\tilde{Z}}^{j}+\bar{z}^{j} \tag{247}
\end{equation*}
$$

Since our VAR evidence indicates that TFP in both sectors rise initially and increase monotonically toward their long-run levels, the parameter $\bar{z}^{j}$ will take negative values as $Z^{j}$ undershoots its statestate value on impact. Differentiating (244) with respect to time leads to:

$$
\begin{align*}
\dot{Z}^{j}(t) & =-\xi^{j} z^{j} e^{-\xi^{j} t} \\
& =-\xi^{j}\left(Z^{j}(t)-\tilde{Z}^{j}\right) \tag{248}
\end{align*}
$$

where $\xi^{j}$ measures the speed at which $Z^{j}$ closes the gap with its long-run level.
As shown in section H.9, the 'true' measure of the technology bias toward tradables is given by $\frac{\left(Z^{H}(t)\right)^{a}}{\left(Z^{H}(t)\right)^{b}}$. In the quantitative analysis, we consider permanent changes in sectoral TFP, $\tilde{Z}^{j}$, so that the labor share-adjusted TFP differential is $1 \%$ in the long run:

$$
\begin{equation*}
a \hat{\tilde{Z}}^{H}-b \hat{\tilde{Z}}^{N}=1 \% . \tag{249}
\end{equation*}
$$

## I. 2 Formal Solutions for $K(t)$ and $Q(t)$

Using (204a), (204b), and (248), the adjustment of the open economy towards the steady-state is described by a dynamic system which comprises four equations:

$$
\begin{gather*}
\dot{K}=\Upsilon\left(K(t), Q(t), Z^{H}(t), Z^{N}(t)\right),  \tag{250a}\\
\dot{Q}=\Sigma\left(K(t), Q(t), Z^{H}(t), Z^{N}(t)\right),  \tag{250b}\\
\dot{Z}^{H}(t)=-\xi^{H}\left(Z^{H}(t)-\tilde{Z}^{H}\right),  \tag{250c}\\
\dot{Z}^{N}(t)=-\xi^{N}\left(Z^{N}(t)-\tilde{Z}^{N}\right) . \tag{250d}
\end{gather*}
$$

The linearized system can be written in a matrix form:

$$
\left(\begin{array}{c}
\dot{K}(t)  \tag{251}\\
\dot{Q}(t) \\
\dot{Z}^{H}(t) \\
\dot{Z}^{N}(t)
\end{array}\right)=\left(\begin{array}{cccc}
\Upsilon_{K} & \Upsilon_{Q} & \Upsilon_{Z^{H}} & \Upsilon_{Z^{N}} \\
\Sigma_{K} & \Sigma_{Q} & \Sigma_{Z^{H}} & \Sigma_{Z^{N}} \\
0 & 0 & -\xi^{H} & 0 \\
0 & 0 & 0 & -\xi^{N}
\end{array}\right)\left(\begin{array}{c}
K(t)-\tilde{K} \\
Q(t)-\tilde{Q} \\
Z^{H}(t)-\tilde{Z}^{H} \\
Z^{N}(t)-\tilde{Z}^{N}
\end{array}\right)
$$

where the coefficients of the Jacobian matrix, $\Upsilon_{X}$ and $\Sigma_{X}$ with $X=K, Q, Z^{H}, Z^{N}$, are given by (208) and (209).

Denoting by $\nu_{i}$ the eigenvalue (with $i=1,2,3,4$ ), the characteristic polynomial is:

$$
\begin{equation*}
\left(\xi^{N}+\nu_{i}\right)\left(\xi^{H}+\nu_{i}\right)\left[\left(\nu_{i}\right)^{2}-\nu_{i}\left(\Upsilon_{K}+\Sigma_{K}\right)-\left(\Upsilon_{Q} \Sigma_{K}+\Upsilon_{K} \Sigma_{Q}\right)\right]=0 \tag{252}
\end{equation*}
$$

where $\Upsilon_{K}+\Sigma_{K}=r^{\star}$. The characteristic polynomial has three negative roots and one postive root:

$$
\begin{equation*}
\nu_{4}=-\xi^{N}<\nu_{3}=-\xi^{H}<\nu_{1}<0<r^{\star}<\nu_{2}, \tag{253}
\end{equation*}
$$

where inequality $\xi^{N}>\xi^{H}$ follows from the calibration.
We denote by $\omega_{j}^{i}$ the $j$ th element of eigenvector $\omega^{i}$ related to eigenvalue $\nu_{i}$, calculated as $\left(\nu_{i} I_{4 \times 4}-J\right) \omega^{i}=0$ (where $J$ is the Jacobian matrix given by $(251)$ ). The general solution that characterize the adjustment toward the new steady-state can be written as follows:

$$
\begin{gather*}
K(t)-\tilde{K}=\sum_{i=1}^{4} \omega_{1}^{i} D_{i} e^{\nu_{i} t}  \tag{254a}\\
Q(t)-\tilde{Q}=\sum_{i=1}^{4} \omega_{2}^{i} D_{i} e^{\nu_{i} t}  \tag{254b}\\
Z^{H}(t)-\tilde{Z}^{H}=D_{3} e^{\nu_{3} t}  \tag{254c}\\
Z^{N}(t)-\tilde{Z}^{N}=D_{4} e^{\nu_{4} t} \tag{254d}
\end{gather*}
$$

where we normalized $\omega_{1}^{1}, \omega_{1}^{2}, \omega_{3}^{3}$, and $\omega_{4}^{4}$ to 1 . To allow the dynamic system to converge toward the new long-run equilibrium, we eliminate explosive paths and set $D_{2}=0 . D_{i}$ is an arbitrary constant which is determined by initial conditions:

$$
\begin{gather*}
K(0)-\tilde{K}=D_{1}+\omega_{1}^{3} D_{3}+\omega_{1}^{4} D_{4}  \tag{255a}\\
Z^{H}(0)-\tilde{Z}^{H}=D_{3}=z^{H}  \tag{255b}\\
Z^{N}(0)-\tilde{Z}^{N}=D_{4}=z^{N} \tag{255c}
\end{gather*}
$$

where $K(0)=K_{0}$ is the initial capital stock, $Z^{H}(0)=\tilde{Z}_{0}^{H}$ and $Z^{N}(0)=\tilde{Z}_{0}^{N}$ are initial sectoral TFP; setting $t=0$ into (244) and using (255a), we thus have

$$
\begin{gather*}
D_{1}=K_{0}-\tilde{K}-\omega_{1}^{3} z^{H}-\omega_{1}^{4} z^{N},  \tag{256a}\\
D_{3}=z^{H}  \tag{256b}\\
D_{4}=z^{N} . \tag{256c}
\end{gather*}
$$

## I. 3 Formal Solution for the Net Foreign Asset Position, $N(t)$

To determine the formal solution for the net foreign asset position, we first linearize the current account equation (216) in the neighborhood of the steady-state

$$
\begin{equation*}
\dot{N}(t)=r^{\star}(N(t)-\tilde{N})+\sum_{X} \Xi_{X}(X(t)-\tilde{X}) \tag{257}
\end{equation*}
$$

where $X=K, Q, Z^{H}, Z^{N}$, and substitute the solutions for $K(t)$ and $Q(t)$ along with dynamic equations of sectoral TFP described by (254), remembering that $D_{2}=0$ :

$$
\begin{equation*}
\dot{N}(t)=r^{\star}(N(t)-\tilde{N})+\sum_{i=1,3,4} E_{i} D_{i} e^{\nu_{i} t} \tag{258}
\end{equation*}
$$

where

$$
\begin{gather*}
E_{1}=\Xi_{K}+\Xi_{Q} \omega_{2}^{1}  \tag{259a}\\
E_{3}=\Xi_{K} \omega_{1}^{3}+\Xi_{Q} \omega_{2}^{3}+\Xi_{Z^{H}}  \tag{259b}\\
E_{4}=\Xi_{K} \omega_{1}^{4}+\Xi_{Q} \omega_{2}^{4}+\Xi_{Z^{N}} \tag{259c}
\end{gather*}
$$

Solving the differential equation (259) for $N(t)$ yields the general solution for the net foreign asset position:

$$
\begin{equation*}
N(t)-\tilde{N}=\left[\left(N_{0}-\tilde{N}\right)+\sum_{i=1,3,4} \Phi_{N}^{i}\right] e^{r^{\star} t}-\sum_{i=1,3,4} \Phi_{N}^{i} e^{\nu_{i} t} \tag{260}
\end{equation*}
$$

where we set $\Phi_{N}^{i}=\frac{E_{i} D_{i}}{r^{\star}-\nu_{i}}$.
Invoking the transversality condition, one obtains the 'stable' solution for the stock of net foreign assets so that $N(t)$ converges toward its steady-state value $\tilde{N}$ :

$$
\begin{equation*}
N(t)-\tilde{N}=\sum_{i=1,3,4} \Phi_{N}^{i} e^{\nu_{i} t} \tag{261}
\end{equation*}
$$

Eq. (261) gives the trajectory for $N(t)$ consistent with the intertemporal solvency condition:

$$
\begin{equation*}
\tilde{N}-N_{0}=\sum_{i=1,3,4} \Phi_{N}^{i} \tag{262}
\end{equation*}
$$

Differentiating (261) w.r.t. time gives the trajectory for the current account along the transitional path when sectoral TFP follows the temporal path given by eq. (248):

$$
\begin{equation*}
\dot{N}(t)=\nu_{i} \sum_{i=1,3,4} \Phi_{N}^{i} e^{\nu_{i} t} \tag{263}
\end{equation*}
$$

## I. 4 Formal Solution for the Stock of Non Human Wealth, $A(t)$

To determine the formal solution for the stock of non human wealth, we first linearize the current account equation (225) in the neighborhood of the steady-state

$$
\begin{equation*}
\dot{A}(t)=r^{\star}(A(t)-\tilde{A})+\sum_{X} \Lambda_{X}(X(t)-\tilde{X}) \tag{264}
\end{equation*}
$$

where $X=K, Q, Z^{H}, Z^{N}$, and substitute the solutions for $K(t)$ and $Q(t)$ along with dynamic equations of sectoral TFP described by (254), remembering that $D_{2}=0$ :

$$
\begin{equation*}
\dot{A}(t)=r^{\star}(A(t)-\tilde{A})+\sum_{i=1,3,4} M_{i} D_{i} e^{\nu_{i} t} \tag{265}
\end{equation*}
$$

where

$$
\begin{gather*}
M_{1}=\Lambda_{K}+\Lambda_{Q} \omega_{2}^{1},  \tag{266a}\\
M_{3}=\Lambda_{K} \omega_{1}^{3}+\Lambda_{Q} \omega_{2}^{3}+\Lambda_{Z^{H}},  \tag{266b}\\
M_{4}=\Lambda_{K} \omega_{1}^{4}+\Lambda_{Q} \omega_{2}^{4}+\Lambda_{Z^{N}} . \tag{266c}
\end{gather*}
$$

Solving the differential equation (265) for $A(t)$ yields the general solution for the stock of non human wealth:

$$
\begin{equation*}
A(t)-\tilde{A}=\left[\left(A_{0}-\tilde{A}\right)+\sum_{i=1,3,4} \Phi_{A}^{i}\right] e^{r^{\star} t}-\sum_{i=1,3,4} \Phi_{A}^{i} e^{\nu_{i} t} \tag{267}
\end{equation*}
$$

where we set $\Phi_{A}^{i}=\frac{M_{i} D_{i}}{r^{\star}-\nu_{i}}$.
Invoking the transversality condition, one obtains the 'stable' solution for the stock of non human wealth so that $A(t)$ converges toward its steady-state value $\tilde{A}$ :

$$
\begin{equation*}
A(t)-\tilde{A}=\sum_{i=1,3,4} \Phi_{A}^{i} e^{\nu_{i} t} \tag{268}
\end{equation*}
$$

Eq. (268) gives the trajectory for $A(t)$ consistent with the intertemporal solvency condition:

$$
\begin{equation*}
\tilde{A}-A_{0}=\sum_{i=1,3,4} \Phi_{A}^{i} . \tag{269}
\end{equation*}
$$

Differentiating (269) w.r.t. time gives the trajectory for private savings (equal to national savings as we abstract from public debet) along the transitional path when sectoral TFP follows the temporal path given by eq. (248):

$$
\begin{equation*}
\dot{A}(t)=\nu_{i} \sum_{i=1,3,4} \Phi_{A}^{i} e^{\nu_{i} t} \tag{270}
\end{equation*}
$$

## I. 5 Formal Solution for $Q(t) K(t)$

To determine the dynamics of investment, we first derive the formal solution for the shadow value of the capital stock, $Q(t) K(t)$. We thus linearize $Q(t) K(t)$ in the neighborhood of the steady-state:

$$
\begin{equation*}
Q(t) K(t)-P_{J} \tilde{K}=P_{J}(K(t)-\tilde{K})+\tilde{K}(Q(t)-\tilde{K}), \tag{271}
\end{equation*}
$$

where we used the fact that $\tilde{Q}=P_{J}$ in the long-run. Substitute the solutions for $K(t)$ and $Q(t)$ along with dynamic equations of sectoral TFP described by (254), remembering that $D_{2}=0$ :

$$
\begin{equation*}
Q(t) K(t)-P_{J} \tilde{K}=\sum_{i=1,3,4} S_{i} D_{i} e^{\nu_{i} t} \tag{272}
\end{equation*}
$$

where $S_{1}=P_{J} \omega_{1}^{i}+\tilde{K} \omega_{2}^{i}$. Totally differentiating (272) w.r.t. time gives the trajectory for private investment along the transitional path when sectoral TFP follows the temporal path given by eq. (248):

$$
\begin{equation*}
Q(t) \dot{\bar{K}} K(t)=\nu_{i} \sum_{i=1,3,4} S_{i} D_{i} e^{\nu_{i} t} . \tag{273}
\end{equation*}
$$

Since $N(t)=A(t)-Q(t) K(t)$, we thus have:

$$
\begin{equation*}
\dot{N}(t)=\dot{A}(t)-Q(t) \dot{K}(t) \tag{274}
\end{equation*}
$$

where expressions for the current account, national savings and private investment are given by (263), (270), and (273), respectively.

## J Semi-Small Open Economy Model with CES Production Functions

This section extends the model laid out in section H to CES production functions and factor biased technological change. Since first order conditions from households' maximization problem detailed in subsection H. 1 remain identical, we do not repeat them and emphasize the main changes caused by the assumption of CES production functions.

## J. 1 Firms

Both the traded and non-traded sectors use physical capital, $K^{j}$, and labor, $L^{j}$, according to constant returns to scale production functions which are assumed to take a CES form:

$$
\begin{equation*}
Y^{j}=\left[\gamma^{j}\left(A^{j} L^{j}\right)^{\frac{\sigma^{j}-1}{\sigma j}}+\left(1-\gamma^{j}\right)\left(B^{j} K^{j}\right)^{\frac{\sigma^{j}-1}{\sigma^{j}}}\right]^{\frac{\sigma^{j}}{\sigma^{j}-1}} \tag{275}
\end{equation*}
$$

where $\gamma^{j}$ and $1-\gamma^{j}$ are the weight of labor and capital in the production technology, $\sigma^{j}$ is the elasticity of substitution between capital and labor in sector $j=H, N, A^{j}$ and $B^{j}$ are labor- and capital-augmenting efficiency. Both sectors face two cost components: a capital rental cost equal to $R$, and a labor cost equal to the wage rate, i.e., $W^{H}$ in the traded sector and $W^{N}$ in the non-traded sector.

## First-Order Conditions

Both sectors are assumed to be perfectly competitive and thus choose capital and labor by taking prices as given:

$$
\begin{equation*}
\max _{K^{j}, L^{j}} \Pi^{j}=\max _{K^{j}, L^{j}}\left\{P^{j} Y^{j}-W^{j} L^{j}-R K^{j}\right\} . \tag{276}
\end{equation*}
$$

Since capital can move freely between the two sectors, the value of marginal products in the traded and non-traded sectors equalizes while costly labor mobility implies a wage differential across sectors:

$$
\begin{gather*}
P^{H}\left(1-\gamma^{H}\right)\left(B^{H}\right)^{\frac{\sigma^{H}-1}{\sigma^{H}}}\left(k^{H}\right)^{-\frac{1}{\sigma^{H}}}\left(y^{H}\right)^{\frac{1}{\sigma^{H}}}=P^{N}\left(1-\gamma^{N}\right)\left(B^{N}\right)^{\frac{\sigma^{N}-1}{\sigma^{N}}}\left(k^{N}\right)^{-\frac{1}{\sigma^{N}}}\left(y^{N}\right)^{\frac{1}{\sigma^{N}}} \equiv R,  \tag{277a}\\
P^{H} \gamma^{H}\left(A^{H}\right)^{\frac{\sigma^{H}-1}{\sigma^{H}}}\left(L^{H}\right)^{-\frac{1}{\sigma^{H}}}\left(Y^{H}\right)^{\frac{1}{\sigma^{H}}} \equiv W^{H}  \tag{277b}\\
P^{N} \gamma^{N}\left(A^{N}\right)^{\frac{\sigma^{N}-1}{\sigma^{N}}}\left(L^{N}\right)^{-\frac{1}{\sigma^{N}}}\left(Y^{N}\right)^{\frac{1}{\sigma^{N}}} \equiv W^{N}, \tag{277c}
\end{gather*}
$$

where we denote by $k^{j} \equiv K^{j} / L^{j}$ the capital-labor ratio for sector $j=H, N$, and $y^{j} \equiv Y^{j} / L^{j}$ value added per hours worked described by

$$
\begin{equation*}
y^{j}=\left[\gamma^{j}\left(A^{j}\right)^{\frac{\sigma^{j}-1}{\sigma^{j}}}+\left(1-\gamma^{j}\right)\left(B^{j} k^{j}\right)^{\frac{\sigma^{j}-1}{\sigma j}}\right]^{\frac{\sigma^{j}}{\sigma^{j}-1}} \tag{278}
\end{equation*}
$$

The resource constraint for capital is:

$$
\begin{equation*}
K^{H}+K^{N}=K \tag{279}
\end{equation*}
$$

## Some Useful Results

Multiplying both sides of (277b)-(277c) by $L^{j}$ and dividing by sectoral value added leads to the labor income share:

$$
\begin{equation*}
s_{L}^{j}=\gamma^{j}\left(\frac{A^{j}}{y^{j}}\right)^{\frac{\sigma^{j}-1}{\sigma^{j}}} \tag{280}
\end{equation*}
$$

Multiplying both sides of (277a) by $K^{j}$ and dividing by sectoral value added leads to the capital income share:

$$
\begin{equation*}
1-s_{L}^{j}=\left(1-\gamma^{j}\right)\left(\frac{B^{j} k^{j}}{y^{j}}\right)^{\frac{\sigma^{j}-1}{\sigma^{j}}} \tag{281}
\end{equation*}
$$

Dividing eq. (280) by eq. (281), the ratio of the labor to the capital income share denoted by $S^{j}=\frac{s_{L}^{j}}{1-s_{L}^{j}}$ reads as follows:

$$
\begin{equation*}
S^{j}=\frac{\gamma^{j}}{1-\gamma^{j}}\left(\frac{B^{j} K^{j}}{A^{j} L^{j}}\right)^{\frac{1-\sigma^{j}}{\sigma^{j}}} \tag{282}
\end{equation*}
$$

Dividing (277b)-(277c) by (277a) leads to a positive relationship between the relative cost of labor and the capital-labor ratio in sector $j$ :

$$
\begin{equation*}
\frac{W^{j}}{R}=\frac{\gamma^{j}}{1-\gamma^{j}}\left(\frac{B^{j}}{A^{j}}\right)^{\frac{1-\sigma^{j}}{\sigma^{j}}}\left(\frac{K^{j}}{L^{j}}\right)^{\frac{1}{\sigma^{j}}} . \tag{283}
\end{equation*}
$$

To determine the conditional demands for both inputs, we make use of (283) which leads to:

$$
\begin{align*}
L^{j} & =K^{j}\left(\frac{\gamma^{j}}{1-\gamma^{j}}\right)^{\sigma^{j}}\left(\frac{B^{j}}{A^{j}}\right)^{1-\sigma^{j}}\left(\frac{W^{j}}{R}\right)^{-\sigma^{j}}  \tag{284a}\\
K^{j} & =L^{j}\left(\frac{1-\gamma^{j}}{\gamma^{j}}\right)^{\sigma^{j}}\left(\frac{B^{j}}{A^{j}}\right)^{\sigma^{j}-1}\left(\frac{W^{j}}{R}\right)^{\sigma^{j}} \tag{284b}
\end{align*}
$$

Inserting eq. (284b) (eq. (284a) resp.) in the CES production function and solving for $L^{j}$ ( $K^{j}$ resp.) leads to the conditional demand for labor (capital resp.):

$$
\begin{equation*}
L^{j}=Y^{j}\left(A^{j}\right)^{\sigma^{j}-1}\left(\frac{\gamma^{j}}{W^{j}}\right)^{\sigma}\left(X^{j}\right)^{\frac{\sigma^{j}}{1-\sigma^{j}}}, \quad K^{j}=Y^{j}\left(B^{j}\right)^{\sigma^{j}-1}\left(\frac{1-\gamma^{j}}{R}\right)^{\sigma^{j}}\left(X^{j}\right)^{\frac{\sigma^{j}}{1-\sigma^{j}}} \tag{285}
\end{equation*}
$$

where $X^{j}$ is given by:

$$
\begin{equation*}
X^{j}=\left(\gamma^{j}\right)^{\sigma^{j}}\left(A^{j}\right)^{\sigma^{j}-1}\left(W^{j}\right)^{1-\sigma^{j}}+\left(1-\gamma^{j}\right)^{\sigma^{j}}\left(B^{j}\right)^{\sigma^{j}-1} R^{1-\sigma^{j}} \tag{286}
\end{equation*}
$$

Total cost is equal to the sum of the labor and capital cost:

$$
\begin{equation*}
C^{j}=W^{j} L^{j}+R K^{j} . \tag{287}
\end{equation*}
$$

Inserting conditional demand for inputs (284) into total cost (287), we find $C^{j}$ is homogenous of degree one with respect to the level of production

$$
\begin{equation*}
C^{j}=c^{j} Y^{j}, \quad \text { with } \quad c^{j}=\left(X^{j}\right)^{\frac{1}{1-\sigma^{j}}} . \tag{288}
\end{equation*}
$$

Using the fact that $\left(c^{j}\right)^{1-\sigma^{j}}=X^{j}$, conditional demand for labor (284a) can be rewritten as $L^{j}=Y^{j}\left(A^{j}\right)^{\sigma^{j}-1}\left(\frac{\gamma^{j}}{W^{j}}\right)\left(c^{j}\right)^{\sigma^{j}}$ which gives the labor share denoted by $s_{L}^{j}$ :

$$
\begin{gather*}
s_{L}^{j}=\frac{W^{j} L^{j}}{P^{j} Y^{j}}=\left(\gamma^{j}\right)^{\sigma^{j}}\left(\frac{W^{j}}{A^{j}}\right)^{1-\sigma^{j}}\left(c^{j}\right)^{\sigma^{j}-1}  \tag{289a}\\
1-s_{L}^{j}=\frac{R K^{j}}{P^{j} Y^{j}}=\left(1-\gamma^{j}\right)^{\sigma^{j}}\left(\frac{R}{B^{j}}\right)^{1-\sigma^{j}}\left(c^{j}\right)^{\sigma^{j}-1} . \tag{289b}
\end{gather*}
$$

## J. 2 Short-Run Solutions

## Sectoral Wages and Capital-Labor Ratios

Plugging the short-run solutions for $L^{H}$ and $L^{N}$ given by (168) into the resource constraint for capital (279), the system of four equations consisting of (277a)-(277c) together with (279) can be solved for sectoral wages $W^{j}$ and sectoral capital-labor ratios $k^{j}$. Log-differentiating (277a)-(277c) together with (279) yields in matrix form:

$$
\begin{align*}
\left(\begin{array}{cccc}
-\left(\frac{s_{L}^{H}}{\sigma^{H}}\right. & \left(\frac{s_{L}^{N}}{\sigma^{N}}\right) & 0 & 0 \\
\left(\frac{1-s_{L}^{H}}{\sigma^{H}}\right) & 0 & -1 & 0 \\
0 & \left(\frac{1-s_{L}^{N}}{\sigma^{N}}\right) & 0 & -1 \\
\frac{K^{H}}{K} & \frac{K^{N}}{K} & \Psi_{W^{H}} & \Psi_{W^{N}}
\end{array}\right)\left(\begin{array}{c}
\hat{k}^{H} \\
\hat{k}^{N} \\
\hat{W}^{H} \\
\hat{W}^{N}
\end{array}\right) \\
=\left(\begin{array}{c}
\hat{P}^{N}-\hat{P}^{H}-\left(\frac{\sigma^{H}-s_{L}^{H}}{\sigma^{H}}\right) \hat{B}^{H}+\left(\frac{\sigma^{N}-s_{L}^{N}}{\sigma^{N}}\right) \hat{B}^{N}--\left(\frac{s_{L}^{H}}{\sigma^{H}}\right) \hat{A}^{H}+\left(\frac{s_{L}^{N}}{\sigma^{N}}\right) \hat{A}^{N} \\
-\hat{P}^{H}-\left[\frac{\left(\sigma^{H}-1\right)+s_{L}^{H}}{\sigma^{H}}\right] \hat{A}^{H}-\left(\frac{1-s_{L}^{H}}{\sigma^{H}}\right) \hat{B}^{H} \\
\\
\quad-\hat{P}^{N}-\left[\frac{\left(\sigma^{N}-1\right)+s_{L}^{N}}{\sigma^{N}}\right] \hat{A}^{N}-\left(\frac{1-s_{L}^{N}}{\sigma^{N}}\right) \hat{B}^{N} \\
\hat{K}-\Psi_{\bar{\lambda}} \hat{\bar{\lambda}}
\end{array}\right), \tag{290}
\end{align*}
$$

where we set:

$$
\begin{align*}
\Psi_{W^{j}} & =\frac{K^{H}}{K} \frac{L_{W^{j}}^{H} W^{j}}{L^{H}}+\frac{K^{N}}{K} \frac{L_{W^{j}}^{N} W^{j}}{L^{N}}  \tag{291a}\\
\Psi_{\bar{\lambda}} & =\frac{K^{H}}{K} \sigma_{L}+\frac{K^{N}}{K} \sigma_{L}=\sigma_{L} \tag{291b}
\end{align*}
$$

The short-run solutions for sectoral wages and capital-labor ratios are:

$$
\begin{equation*}
W^{j}=W^{j}\left(\bar{\lambda}, K, P^{N}, P^{H}, A^{H}, A^{N}, B^{H}, B^{N}\right), \quad k^{j}=k^{j}\left(\bar{\lambda}, K, P^{N}, P^{H}, A^{H}, A^{N}, B^{H}, B^{N}\right) . \tag{292}
\end{equation*}
$$

Inserting first sectoral wages (292), sectoral hours worked (289a) can be solved as functions of the shadow value of wealth, the capital stock, the price of non-traded goods in terms of foreign goods, $P^{N}$, and the terms of trade:

$$
\begin{equation*}
L^{j}=L^{j}\left(\bar{\lambda}, K, P^{N}, P^{H}, A^{H}, A^{N}, B^{H}, B^{N}\right) \tag{293}
\end{equation*}
$$

Totally differentiating output per hours worked (278) leads to:

$$
\begin{equation*}
\hat{y}^{j}=s_{L}^{j} \hat{A}^{j}+\left(1-s_{L}^{j}\right) \hat{B}^{j}+\left(1-s_{L}^{j}\right) \hat{k}^{j}, \tag{294}
\end{equation*}
$$

where $s_{L}^{j}$ and $1-s_{L}^{j}$ are the labor and capital income share, respectively, described by eqs. (280)(281). Plugging solutions for sectoral capital-labor ratios (292) into (294) allows us to solve for sectoral value added per hours worked:

$$
\begin{equation*}
y^{j}=y^{j}\left(\bar{\lambda}, K, P^{N}, P^{H}, A^{H}, A^{N}, B^{H}, B^{N}\right) \tag{295}
\end{equation*}
$$

Using the fact that $Y^{j}=y^{j} L^{j}$, differentiating, inserting (295) and solutions for sectoral labor (293) and sectoral capital-labor ratios (292), one obtains the solutions for sectoral value added:

$$
\begin{equation*}
Y^{j}=Y^{j}\left(\bar{\lambda}, K, P^{N}, P^{H}, A^{H}, A^{N}, B^{H}, B^{N}\right) . \tag{296}
\end{equation*}
$$

## The Return on Domestic Capital, $R$

The return on domestic capital is:

$$
\begin{equation*}
R=P^{N}\left(1-\gamma^{N}\right)\left(B^{N}\right)^{\frac{\sigma^{N}-1}{\sigma^{N}}}\left(k^{N}\right)^{-\frac{1}{\sigma^{N}}}\left(y^{N}\right)^{\frac{1}{\sigma^{N}}} . \tag{297}
\end{equation*}
$$

Differentiating (297) and making use of (294) leads to:

$$
\begin{equation*}
\hat{R}=\hat{P}^{N}-\frac{s_{L}^{N}}{\sigma^{N}} \hat{k}^{N}+\frac{s_{L}^{N}}{\sigma^{N}} \hat{A}^{N}+\left(\frac{\sigma^{N}-s_{L}^{N}}{\sigma^{N}}\right) \hat{B}^{N} . \tag{298}
\end{equation*}
$$

Inserting the short-run static solution for the capital-labor ratio $k^{N}$ given by (292), eq. (297) can be solved for the return on domestic capital:

$$
\begin{equation*}
R=R\left(\bar{\lambda}, K, P^{N}, P^{H}, A^{H}, A^{N}, B^{H}, B^{N}\right) \tag{299}
\end{equation*}
$$

## Market Clearing Condition for Non-Tradables

The role of the price of non-tradables in terms of foreign goods is to clear the non-traded goods market:

$$
\begin{equation*}
Y^{N}=C^{N}+G^{N}+J^{N} \tag{300}
\end{equation*}
$$

Inserting solutions for $C^{N}, J^{N}, Y^{N}$ given by (166), (183), (296), respectively, the non-traded goods market clearing condition (300) can be rewritten as follows:

$$
\begin{equation*}
Y^{N}\left(\bar{\lambda}, K, P^{N}, P^{H}, A^{H}, A^{N}, B^{H}, B^{N}\right)=C^{N}\left(\bar{\lambda}, P^{N}, P^{H}\right)+G^{N}+J^{N}\left(K, Q, P^{N}, P^{H}\right) . \tag{301}
\end{equation*}
$$

Eq. (301) can be solved for the relative price of non-tradables:

$$
\begin{equation*}
P^{N}=\Psi^{N}\left(K, Q, P^{H}, A^{H}, A^{N}, B^{H}, B^{N}, \bar{\lambda}\right) \tag{302}
\end{equation*}
$$

with partial derivatives given by:

$$
\begin{align*}
\Psi_{K}^{N} & =\frac{\partial \Psi^{N}}{\partial K}=-\frac{\left(Y_{K}^{N}-J_{K}^{N}\right)}{\Delta^{N}}<0  \tag{303a}\\
\Psi_{Q}^{N} & =\frac{\partial \Psi^{N}}{\partial Q}=\frac{J_{Q}^{N}}{\Delta^{N}}>0  \tag{303b}\\
\Psi_{P^{H}}^{N} & =\frac{\partial \Psi^{N}}{\partial P^{H}}=-\frac{\left(Y_{P^{H}}^{N}-C_{P^{H}}^{N}-J_{P^{H}}^{N}\right)}{\Delta^{N}}>0  \tag{303c}\\
\Psi_{Z^{H}}^{N} & =\frac{\partial \Psi^{N}}{\partial A^{j}}=-\frac{Y_{A^{j}}^{N}}{\Delta^{N}}>0  \tag{303d}\\
\Psi_{Z^{N}}^{N} & =\frac{\partial \Psi^{N}}{\partial B^{j}}=-\frac{Y_{B^{j}}^{N}}{\Delta^{N}}<0 \tag{303e}
\end{align*}
$$

where we set

$$
\begin{equation*}
\Delta^{N}=\left(Y_{P^{N}}^{N}-C_{P^{N}}^{N}-J_{P^{N}}^{N}\right)>0 \tag{304}
\end{equation*}
$$

## Market Clearing Condition for Home-Produced Traded Goods

The role of the price of home-produced traded goods in terms of foreign-produced goods or the terms of trade is to clear the home-produced traded goods market:

$$
\begin{equation*}
Y^{H}=C^{H}+G^{H}+J^{H}+X^{H} \tag{305}
\end{equation*}
$$

where $X^{H}$ stands for exports which are negatively related to the terms of trade:

$$
\begin{equation*}
X^{H}=\varphi_{X}\left(P^{H}\right)^{-\phi_{X}} \tag{306}
\end{equation*}
$$

where $\phi_{X}$ is the elasticity of exports with respect to the terms of trade.
Inserting solutions for $C^{H}, J^{H}, Y^{H}$ given by (166), (183), (296), respectively, the traded goods market clearing condition (305) can be rewritten as follows:
$Y^{H}\left(\bar{\lambda}, K, P^{N}, P^{H}, A^{H}, A^{N}, B^{H}, B^{N}\right)=C^{H}\left(\bar{\lambda}, P^{N}, P^{H}\right)+G^{H}+J^{H}\left(K, Q, P^{N}, P^{H}\right)+X^{H}\left(P^{H}\right)$.
Eq. (307) can be solved for the terms of trade:

$$
\begin{equation*}
P^{H}=\Psi^{H}\left(K, Q, P^{N}, A^{H}, A^{N}, B^{H}, B^{N}, \bar{\lambda}\right) \tag{308}
\end{equation*}
$$

with partial derivatives given by:

$$
\begin{align*}
\Psi_{K}^{H} & =\frac{\partial \Psi^{H}}{\partial K}=-\frac{\left(Y_{K}^{H}-J_{K}^{H}\right)}{\Delta^{H}}<0  \tag{309a}\\
\Psi_{Q}^{H} & =\frac{\partial \Psi^{H}}{\partial Q}=\frac{J_{Q}^{H}}{\Delta^{H}}>0  \tag{309b}\\
\Psi_{P^{H}}^{N} & =\frac{\partial \Psi^{H}}{\partial P^{N}}=-\frac{\left(Y_{P^{N}}^{H}-C_{P^{N}}^{H}-J_{P^{N}}^{H}\right)}{\Psi^{N}}>0  \tag{309c}\\
\Psi_{A^{j}}^{H} & =\frac{\partial \Psi^{H}}{\partial A^{j}}=-\frac{Y_{A^{j}}^{H}}{\Delta^{H}}<0  \tag{309d}\\
\Psi_{B^{j}}^{H} & =\frac{\partial \Psi^{H}}{\partial B^{j}}=-\frac{Y_{B^{j}}^{H}}{\Delta^{H}}>0 \tag{309e}
\end{align*}
$$

where we set

$$
\begin{equation*}
\Delta^{H}=\left(Y_{P^{H}}^{H}-C_{P^{H}}^{H}-J_{P^{H}}^{H}-X_{P^{H}}^{H}\right)>0, \tag{310}
\end{equation*}
$$

where $X_{P^{H}}^{H}=\frac{\partial X^{H}}{\partial P^{H}}<0$.

## J. 3 Solving the Model

In our model, there are five state variables, namely $K, A^{H}, A^{N}, B^{H}, B^{N}$, and one control variable, $Q$. To solve the model, we have to express all variables in terms of state and control variables. Plugging first eq. (308) into (302) allows us to solve for the relative price of non-tradables:

$$
\begin{equation*}
P^{N}=P^{N}\left(K, Q, A^{H}, A^{N}, B^{H}, B^{N}, \bar{\lambda}\right), \tag{311}
\end{equation*}
$$

where partial derivatives (with respect to $X=K, Q, Z^{H}, Z^{N}$ ) are given by

$$
\begin{equation*}
P_{X}^{N}=\frac{\partial P^{N}}{\partial X}=\frac{\Psi_{X}^{N}+\Psi_{P_{H}}^{N} \Psi_{X}^{H}}{\Delta^{N}+\Psi_{P^{H}}^{N} \Psi_{P^{N}}^{H}} \tag{312}
\end{equation*}
$$

with $P_{K}^{N}<0, P_{Q}^{N}>0, P_{Z^{H}}^{N} \gtrless 0, P_{Z^{N}}^{N}<0$.
Plugging first eq. (311) into (308) allows us to solve for the terms of trade:

$$
\begin{equation*}
P^{H}=P^{H}\left(K, Q, A^{H}, A^{N}, B^{H}, B^{N}, \bar{\lambda}\right), \tag{313}
\end{equation*}
$$

where partial derivatives (with respect to $X=K, Q, Z^{H}, Z^{N}$ ) are given by

$$
\begin{equation*}
P_{X}^{H}=\frac{\partial P^{H}}{\partial X}=\Psi_{X}^{H}+\Psi_{P^{N}}^{H} P_{X}^{N} \tag{314}
\end{equation*}
$$

with $P_{K}^{H}<0, P_{Q}^{H}>0, P_{A^{H}}^{H}<0, P_{B^{H}}^{H}<0 P_{A^{N}}^{H} \lessgtr 0, P_{B^{N}}^{H} \lessgtr 0$.
Substituting solutions for the relative price of non-tradables (311) and the terms of trade (313) into solutions for consumption (166), sectoral value added (296), the return on domestic capital (299), and the optimal investment decision (179) yields:

$$
\begin{align*}
C^{j} & =C^{j}\left(K, Q, A^{H}, A^{N}, B^{H}, B^{N}, \bar{\lambda}\right)  \tag{315a}\\
Y^{j} & =Y^{j}\left(K, Q, A^{H}, A^{N}, B^{H}, B^{N}, \bar{\lambda}\right)  \tag{315b}\\
R & =R\left(K, Q, A^{H}, A^{N}, B^{H}, B^{N}, \bar{\lambda}\right)  \tag{315c}\\
v & =v\left(K, Q, A^{H}, A^{N}, B^{H}, B^{N}, \bar{\lambda}\right) . \tag{315d}
\end{align*}
$$

Remembering that the non-traded input $J^{N}$ used to produce the capital good is equal to $(1-\iota)\left(\frac{P^{N}}{P_{J}}\right)^{-\phi_{J}} J$ (see eq. (152b)) with $J=I+\frac{\kappa}{2}\left(\frac{I}{K}-\delta_{K}\right)^{2} K$, using the fact that $J^{N}=$ $Y^{N}-C^{N}-G^{N}$ and inserting $I=\dot{K}+\delta_{K}$, the capital accumulation equation reads as follows:

$$
\begin{equation*}
\dot{K}=\frac{Y^{N}-C^{N}-G^{N}}{(1-\iota)\left(\frac{P^{N}}{P_{J}}\right)^{-\phi_{J}}}-\delta_{K} K-\frac{\kappa}{2}\left(\frac{I}{K}-\delta_{K}\right)^{2} K . \tag{316}
\end{equation*}
$$

Inserting short-run solutions for non-traded output (315b) and for consumption in non-tradables (315a), substituting optimal investment decision (315d) into the physical capital accumulation equation (316), and plugging the short-run solution for the return on domestic capital (315c) into the
dynamic equation for the shadow value of capital stock (143e), the dynamic system reads as follows: ${ }^{52}$

$$
\begin{align*}
\dot{K} \equiv \Upsilon\left(K, Q, A^{H}, A^{N}, B^{H}, B^{N}\right)= & \frac{Y^{N}\left(K, Q, A^{H}, A^{N}, B^{H}, B^{N}\right)-C^{N}\left(K, Q, A^{H}, A^{N}, B^{H}, B^{N}\right)-G^{N}}{(1-\iota)\left\{\frac{P^{N}(.)}{P_{J}\left[P^{H}(.), P^{N}(.)\right]}\right\}^{-\phi_{J}}} \\
& -\delta_{K} K-\frac{K}{2 \kappa}\left\{\frac{Q}{P_{J}\left[P^{H}(.), P^{N}(.)\right]}-1\right\}^{2},  \tag{317a}\\
\dot{Q} \equiv \Sigma\left(K, Q, A^{H}, A^{N}, B^{H}, B^{N}\right)= & \left(r^{\star}+\delta_{K}\right) Q-\left[R\left(K, Q, A^{H}, A^{N}, B^{H}, B^{N}\right)\right. \\
& \left.+P_{J}\left[P^{H}(.), P^{N}(.)\right] \frac{\kappa}{2} v(.)\left(v(.) 2 \delta_{K}\right)\right], \tag{317b}
\end{align*}
$$

where $P^{N}($.$) and P^{H}($.$) are given by (311) and (313).$

## J. 4 Current Account Equation and Intertemporal Solvency Condition

Following the same steps as in subsection H.6, the current account reads as:

$$
\begin{equation*}
\dot{N}=r^{\star} N+P^{H} X^{H}-M^{F}, \tag{318}
\end{equation*}
$$

where $X^{H}=Y^{H}-C^{H}-G^{H}-J^{H}$ stands for exports of home goods and we denote by $M^{F}$ imports of foreign consumption and investment goods:

$$
\begin{equation*}
M^{F}=C^{F}+G^{F}+J^{F} \tag{319}
\end{equation*}
$$

Substituting first solutions for $P^{N}$ and $P^{H}$ given by (311) and (313), respectively, into (185) and (306) allows us to express the demand for input of foreign-produced traded goods, $J^{F}$, and exports of home goods, $X^{H}$ :

$$
\begin{align*}
J^{F} & =J^{F}\left(K, Q, A^{H}, A^{N}, B^{H}, B^{N}, \bar{\lambda}\right)  \tag{320a}\\
X^{H} & =X^{H}\left(K, Q, A^{H}, A^{N}, B^{H}, B^{N}, \bar{\lambda}\right) \tag{320b}
\end{align*}
$$

Inserting (320a)-(320b) into(318) allows us to write the current account equation as follows:

$$
\begin{align*}
\dot{N} \equiv & r^{\star} N+\Xi\left(K, Q, A^{H}, A^{N}, B^{H}, B^{N}\right) \\
= & r^{\star} N+P^{H}\left(K, Q, A^{H}, A^{N}, B^{H}, B^{N}\right) X^{H}\left(K, Q, A^{H}, A^{N}, B^{H}, B^{N}\right) \\
& -M^{F}\left(K, Q, A^{H}, A^{N}, B^{H}, B^{N}\right) \tag{321}
\end{align*}
$$

## J. 5 Dynamics of Factor-Augmenting Efficiency

We further specify a dynamic adjustment for $\hat{A}^{j}(t)$ and $\hat{B}^{j}(t)$ similar to that described by eq. (245), i.e.,

$$
\begin{align*}
A^{j}(t) & =\tilde{A}^{j}+a^{j} e^{-\xi^{j} t}  \tag{322a}\\
B^{j}(t) & =\tilde{B}^{j}+b^{j} e^{-\xi^{j} t} \tag{322b}
\end{align*}
$$

where $a^{j}\left(b^{j}\right)$ will take negative values as $A^{j}\left(B^{j}\right)$ undershoots its state-state value on impact., parameter $\xi^{j}$ measures the speed at which $A^{j}$ and $B^{j}$ close the gap with its respective long-run level; we assume that the speed of adjustment $\xi^{j}$ corresponds to the speed of adjustment of sectoral TFP, $Z^{j}$; since the paths of factor biased technological change are expressed in percentage deviation relative to initial steady-state, we have:

$$
\begin{gather*}
\hat{A}^{j}=\frac{\tilde{A}^{j}-\tilde{A}_{0}^{j}}{\tilde{A}_{0}^{j}},  \tag{323a}\\
\hat{A}^{j}(t)=\frac{\hat{B}^{j}=\frac{\tilde{B}^{j}-\tilde{B}_{0}^{j}}{\tilde{B}_{0}^{j}}(t)-\tilde{A}_{0}^{j}}{\tilde{A}_{0}^{j}},  \tag{323b}\\
\hat{B}^{j}(t)=\frac{B^{j}(t)-\tilde{B}_{0}^{j}}{\tilde{B}_{0}^{j}},
\end{gather*}
$$

where $\tilde{A}^{j}$ and $\tilde{B}^{j}$ are the final steady-state levels of labor and capital efficiency.
In percentage deviation relative to initial steady-state, the adjustment in factor-biased technological change is assumed to be described by the following set of dynamic equations

$$
\begin{align*}
& \hat{A}^{j}(t)=\hat{A}^{j}+\bar{a}^{j} e^{-\xi^{j} t},  \tag{324a}\\
& \hat{B}^{j}(t)=\hat{B}^{j}+\bar{b}^{j} e^{-\xi^{j} t} \tag{324b}
\end{align*}
$$

[^38]where $\bar{a}^{j}=a^{j} / \tilde{A}_{0}^{j}$ and $\bar{b}^{j}=b^{j} / \tilde{B}_{0}^{j}$. Differentiating (324) with respect to time leads to:
\[

$$
\begin{align*}
\dot{A}^{j}(t) & =-\xi^{j}\left(A^{j}(t)-\tilde{A}^{j}\right),  \tag{325a}\\
\dot{B}^{j}(t) & =-\xi^{j}\left(B^{j}(t)-\tilde{B}^{j}\right) . \tag{325b}
\end{align*}
$$
\]

## J. 6 The Technology Frontier

While we relax the assumption of Hicks-neutral technological change, we have to relate the changes in labor and capital efficiency, i.e., $\hat{A}^{j}(t)$ and $\hat{B}^{j}(t)$, respectively, to the percentage deviation of TFP in sector $j$, i.e., $\hat{Z}^{j}(t)$, in order to be consistent with our empirical strategy. A natural way to map $A^{j}$ and $B^{j}$ into $Z^{j}$ is to assume that besides optimally choosing factor inputs, firms also optimally choose the production function. Following Caselli and Coleman [2006] and Caselli [2016], the menu of possible choices of production functions is represented by a set of possible $\left(A^{j}, B^{j}\right)$ pairs. These pairs are chosen along the technology frontier which is assumed to take a Cobb-Douglas form:

$$
\begin{equation*}
\left(A^{j}(t)\right)^{\alpha^{j}(t)}\left(B^{j}(t)\right)^{1-\alpha^{j}(t)} \leq Z^{j}(t) \tag{326}
\end{equation*}
$$

where $Z^{j}>0$ is the height of the technology frontier and $\alpha^{j}(t)$ is a time-varying positive parameter which determines the weight of labor-augmenting technological change.

Firms choose $A^{j}$ and $B^{j}$ along the technology frontier described by eq. (326) that minimizes the cost function (see (286)-(288)) described by:

$$
\begin{equation*}
c^{j}(t) \equiv\left[\left(\gamma^{j}\right)^{\sigma^{j}}\left(\frac{W^{j}(t)}{A^{j}(t)}\right)^{1-\sigma^{j}}+\left(1-\gamma^{j}\right)^{\sigma^{j}}\left(\frac{R(t)}{B^{j}(t)}\right)^{1-\sigma^{j}}\right]^{\frac{1}{1-\sigma^{j}}} \tag{327}
\end{equation*}
$$

subject to (326) which holds as an equality. Differentiating (327) and next (326) to eliminate $\hat{B}^{j}(t)$ (keeping $\hat{Z}^{j}$ fixed) leads to:

$$
\begin{align*}
\hat{c}^{j}(t) & =-\left(\gamma^{j}\right)^{\sigma^{j}}\left(\frac{W^{j}(t)}{A^{j}(t)}\right)^{1-\sigma^{j}}\left(c^{j}(t)\right)^{\sigma^{j}-1} \hat{A}^{j}(t)-\left(1-\gamma^{j}\right)^{\sigma^{j}}\left(\frac{R(t)}{B^{j}(t)}\right)^{1-\sigma^{j}}\left(c^{j}(t)\right)^{\sigma^{j}-1} \hat{B}^{j}(t), \\
& =-s_{L}^{j}(t) \hat{A}^{j}(t)-\left(1-s_{L}^{j}(t)\right) \hat{B}^{j}(t, \\
& =-s_{L}^{j}(t) \hat{A}^{j}(t)+\left(1-s_{L}^{j}\right) \frac{\alpha^{j}(t)}{1-\alpha^{j}(t)} \hat{A}^{j}(t), \tag{328}
\end{align*}
$$

where we used the fact that $\left(\gamma^{j}\right)^{\sigma^{j}}\left(\frac{W^{j}(t)}{A^{j}(t)}\right)^{1-\sigma^{j}}\left(c^{j}(t)\right)^{\sigma^{j}-1}=s_{L}^{j}(t)$ (see eq. (289a)), and $\left(1-\gamma^{j}\right)^{\sigma^{j}}$ $\left(\frac{R(t)}{B^{j}(t)}\right)^{1-\sigma^{j}}\left(c^{j}(t)\right)^{\sigma^{j}-1}=1-s_{L}^{j}(t)$ (see eq. (289b)), together with $\hat{B}^{j}(t)=-\frac{\alpha^{j}}{1-\alpha^{j}} \hat{A}^{j}(t)$. Setting the above equation to zero to perform the cost minimization and solving leads to:

$$
\begin{equation*}
\alpha^{j}(t)=s_{L}^{j}(t), \tag{329}
\end{equation*}
$$

where $s_{L}^{j}$ is described by (280). The intuition behind equality (329) is straightforward. Firms choose parameters $A^{j}$ and $B^{j}$ along the technology frontier described by eq. (326) that minimizes the unit cost function (327). More specifically, firms intend to choose the optimal trade-off between $A^{j}$ and $B^{j}$ that minimizes $c^{j}$. Variations in $A^{j}$ and $B^{j}$ modify the unit cost for producing in proportion to the share of labor and capital cost in value added, i.e., $\hat{c}^{j}=-s_{L}^{j} \hat{A}^{j}-\left(1-s_{L}^{j}\right) \hat{B}^{j}$. The unit cost for producing is minimized when the contribution of higher capital efficiency exactly offsets lower labor efficiency, i.e., $\left(1-s_{L}^{j}\right) \hat{B}^{j}=-s_{L}^{j} \hat{A}^{j}$. Since along the same technology frontier, a fall in $\alpha^{j} \hat{A}^{j}$ must be compensated by a rise by $\left(1-\alpha^{j}\right) \hat{B}^{j}$ to keep $Z^{j}$ constant, the optimal trade-off that minimizes the unit cost is that the weight of capital efficiency $1-\alpha^{j}$ is equivalent to its contribution to the decline in the unit cost, $1-s_{L}^{j}$. The weight of labor and capital efficiency into the technology frontier which minimizes the unit cost for producing are thus strictly equal to the shares of labor and capital cost in value added.

Inserting the optimal choice of $\left(A^{j}, B^{j}\right)$ pair along the technology frontier and assuming that $D^{j}=Z^{j}$, one obtains a relationship between total factor productivity and labor- and capitalaugmenting productivity:

$$
\begin{equation*}
Z^{j}(t)=\left(A^{j}(t)\right)^{s_{L}^{j}(t)}\left(B^{j}(t)\right)^{1-s_{L}^{j}(t)} \tag{330}
\end{equation*}
$$

We assume Hicks-neutral technological change at the initial steady-state, i.e., $A^{j}=B^{j}=Z^{j}$. Log-linearizing eq. (330) in the neighborhood of the initial steady-state leads to:

$$
\begin{aligned}
\ln Z^{j}(t)-\ln \tilde{Z}_{0}^{j}= & \tilde{s}_{L, 0}^{j}\left(\ln A^{j}(t)-\ln \tilde{A}_{0}^{j}\right)+\left(1-\tilde{s}_{L, 0}^{j}\right)\left(\ln B^{j}(t)-\ln \tilde{B}_{0}^{j}\right) \\
& +\ln \tilde{A}_{0}^{j}\left(s_{L}^{j}(t)-\tilde{s}_{L, 0}^{j}\right)+\ln \tilde{B}_{0}^{j}\left[\left(1-s_{L}^{j}(t)\right)-\left(1-\tilde{s}_{L, 0}^{j}\right)\right], \\
= & \tilde{s}_{L, 0}^{j}\left(\ln A^{j}(t)-\ln \tilde{A}_{0}^{j}\right)+\left(1-\tilde{s}_{L, 0}^{j}\right)\left(\ln B^{j}(t)-\ln \tilde{B}_{0}^{j}\right),
\end{aligned}
$$

where the last two terms cancel out as a result of our assumption that initially $\tilde{A}_{0}^{j}=\tilde{B}_{0}^{j}=\tilde{Z}_{0}^{j}$. Denoting by a hat the deviation in percentage from initial steady-state, the above equation simply reads as follows:

$$
\begin{equation*}
\hat{Z}^{j}(t)=\tilde{s}_{L, 0}^{j} \hat{A}^{j}(t)+\left(1-\tilde{s}_{L, 0}^{j}\right) \hat{B}^{j}(t) \tag{331}
\end{equation*}
$$

Log-linearizing (282) in the neighborhood of the initial steady-state leads to:

$$
\begin{equation*}
\hat{B} j(t)-\hat{A}^{j}(t)=\left(\frac{\sigma^{j}}{1-\sigma^{j}}\right) \hat{S}^{j}(t)-\hat{k}^{j}(t) \tag{332}
\end{equation*}
$$

The system consisting of the technology frontier (331) and the demand for factors of production (332) can be solved for $\hat{A}^{j}(t)$ and $\hat{B}^{j}(t)$ which leads to (46a)-(46b) in the main text.

As shown in section H.9, the 'true' measure of the technology bias toward tradables is given by $\frac{\left(Z^{H}(t)\right)^{a}}{\left(Z^{H}(t)\right)^{b}}$. In the quantitative analysis, we consider permanent changes in sectoral TFP, $\tilde{Z}^{j}$, so that the labor share-adjusted TFP differential is $1 \%$ in the long run:

$$
\begin{equation*}
a \hat{\tilde{Z}}^{H}-b \hat{\tilde{Z}}^{N}=1 \%, \tag{333}
\end{equation*}
$$

where $\hat{Z}^{j}$ is given by eq. (331).

## Graphical Representation of the Technology Frontier

The technology frontier plots the set of labor and capital efficiency in $\left(\ln A^{j}, \ln B^{j}\right)$-space for given $Z^{j}$. Log-linearizing eq. (326) leads to:

$$
\begin{equation*}
\frac{\partial \ln B^{j}(t)}{\partial \ln A^{j}(t)}=-\frac{\alpha^{j}(t)}{1-\alpha^{j}(t)}<0 \tag{334}
\end{equation*}
$$

Raising the weight of labor-augmenting technological change leads to a steeper technology frontier. The technology frontier has an intercept along the vertical axis of $\frac{\ln Z^{j}}{1-s_{L}^{j}}$ while an intercept along the horizontal axis of $\frac{\ln Z^{j}}{s_{L}^{j}}$.

Totally differentiating the unit cost function leads to:

$$
\begin{equation*}
\frac{\partial \ln B^{j}(t)}{\partial \ln A^{j}(t)}=-\left(\frac{\gamma^{j}}{1-\gamma^{j}}\right)^{\sigma^{j}}\left(\frac{W^{j}}{R} \frac{B^{j}}{A^{j}}\right)^{1-\sigma^{j}}<0 \tag{335}
\end{equation*}
$$

The unit cost function is downward-sloping in the $\left(\ln A^{j}, \ln B^{j}\right)$-space; the unit cost function is convex as long as $\sigma^{j}<1$. From the differentiation of the unit cost function, we have:

$$
\begin{align*}
\hat{c}^{j}= & \left(\gamma^{j}\right)^{\sigma^{j}}\left(\frac{W^{j}}{A^{j}}\right)^{1-\sigma^{j}}\left(c^{j}\right)^{\sigma^{j}-1}\left(\hat{W}^{j}-\hat{A}^{j}\right) \\
& +\left(1-\gamma^{j}\right)^{\sigma^{j}}\left(\frac{R}{B^{j}}\right)^{1-\sigma^{j}}\left(c^{j}\right)^{\sigma^{j}-1}\left(\hat{R}-\hat{B}^{j}\right),  \tag{336}\\
= & s_{L}^{j}\left(\hat{W}^{j}-\hat{A}^{j}\right)+\left(1-s_{L}^{j}\right)\left(\hat{R}-\hat{B}^{j}\right), \tag{337}
\end{align*}
$$

it is straightforward to see that the when $\sigma^{j}<1$, a rise in $W^{j}$ or in $R$ causes the cost function to shift downward in the $\left(\ln A^{j}, \ln B^{j}\right)$-space. In deriving (337), we made use of (289a)-(289b).

Firms will choose a $\left(\ln A^{j}, \ln B^{j}\right)$ pair by equating the slope of the unit cost function to the slope of the technology frontier, i.e.,

$$
\begin{align*}
\left(\frac{\gamma^{j}}{1-\gamma^{j}}\right)^{\sigma^{j}}\left(\frac{W^{j}(t)}{R(t)} \frac{B^{j}(t)}{A^{j}(t)}\right)^{1-\sigma^{j}} & =\frac{\alpha^{j}(t)}{1-\alpha^{j}(t)} \\
\left(\frac{\gamma^{j}}{1-\gamma^{j}}\right)^{\sigma^{j}}\left(\frac{W^{j}(t)}{R(t)} \frac{B^{j}(t)}{A^{j}(t)}\right)^{1-\sigma^{j}} & =\frac{\alpha^{j}(t)}{1-\alpha^{j}(t)} \\
S^{j}(t) & =\frac{\alpha^{j}(t)}{1-\alpha^{j}(t)} \tag{338}
\end{align*}
$$

where $S^{j}=\frac{s_{L}^{j}}{1-s_{L}^{j}}$; we have inserted (283) to get the second line of (338), and we have substituted (282) to get the last line. According to (338), as production becomes more labor intensive, i.e., $S^{j}$ increases, the economy moves along the steeper part of the unit cost for producing, and it is optimal for firms to increase the weight of labor-augmenting technological change. Graphically, as the economy, the technology frontier rotates clockwise and thus firms choose to reduce $A^{j}$ and increase $B^{j}$, for given $Z^{j}$. If we consider an increase in $Z^{j}$ associated with a rise in $S^{j}$, the technology frontier shifts upward and becomes steeper.

## J. 7 CES Technology Frontier

In this subsection, we investigate the implications of assuming a more general form for the technology frontier. As we shall see it, a CES or a Cobb-Douglas form for the technology frontier leads to the same results for our analysis. We assume that firms in sector $j$ choose labor and capital efficiency along the technology frontier which is assumed to take a CES form:

$$
\begin{equation*}
\left[\gamma_{Z}^{j}\left(A^{j}\right)^{\frac{\sigma_{Z}^{j}-1}{\sigma_{Z}^{j}}}+\left(1-\gamma_{Z}^{j}\right)\left(B^{j}\right)^{\frac{\sigma_{Z}^{j}-1}{\sigma_{Z}^{j}}}\right]^{\frac{\sigma_{Z}^{j}}{\sigma_{Z}^{\sigma_{Z}-1}}} \leq Z^{j}(t) \tag{339}
\end{equation*}
$$

where $Z^{j}>0$ is the height of the technology frontier, $0<\gamma_{Z}^{j}<1$ is the weight of labor efficiency in TFP and $\sigma_{Z}^{j}>0$ corresponds to the elasticity of substitution between labor and capital efficiency. Performing the minimization of the unit cost for producing (327) subject to the technology frontier (339) leads to:

$$
\begin{equation*}
\frac{\gamma_{Z}^{j}}{1-\gamma_{Z}^{j}}\left(\frac{A^{j}}{B^{j}}\right)^{\frac{\sigma_{Z}^{j}-1}{\sigma_{Z}^{j}}}=\frac{s_{L}^{j}}{1-s_{L}^{j}}, \tag{340}
\end{equation*}
$$

where we used the fact that $\left(\gamma^{j}\right)^{\sigma^{j}}\left(\frac{W^{j}(t)}{A^{j}(t)}\right)^{1-\sigma^{j}}\left(c^{j}(t)\right)^{\sigma^{j}-1}=s_{L}^{j}(t)$ (see eq. (289a)), and $\left(1-\gamma^{j}\right)^{\sigma^{j}}$ $\left(\frac{R(t)}{B^{j}(t)}\right)^{1-\sigma^{j}}\left(c^{j}(t)\right)^{\sigma^{j}-1}=1-s_{L}^{j}(t)$ (see eq. (289b)). When $\sigma_{Z}^{j}=1$, eq. (340) collapses to (329), i.e., $\gamma^{j}=\alpha^{j}=s_{L}^{j}$. We explore below the implications of $\sigma_{Z}^{j} \neq 1$. As shall be useful later, we solve eq. (340) for $s_{L}^{j}$ :

$$
\begin{align*}
s_{L}^{j} & =\frac{\gamma_{Z}^{j}\left(A^{j}\right)^{\frac{\sigma_{Z}^{j}-1}{\sigma_{Z}^{j}}}}{\gamma_{Z}^{j}\left(A^{j}\right)^{\frac{\sigma_{Z}^{j}-1}{\sigma_{Z}^{j}}}+\left(1-\gamma_{Z}^{j}\right)\left(B^{j}\right)^{\frac{\sigma_{Z}^{j}-1}{\sigma_{Z}^{j}}}}, \\
& =\gamma_{Z}^{j}\left(\frac{A^{j}}{Z^{j}}\right)^{\frac{\sigma_{Z}^{j}-1}{\sigma_{Z}^{j}}}, \tag{341}
\end{align*}
$$

where we made use of (339) to obtain the last line.
Log-linearizing (339) in the neighborhood of the initial steady-state and making use of eq. (341) leads to:

$$
\begin{align*}
\hat{Z}^{j}(t) & =\gamma_{Z}^{j}\left(\frac{A_{0}^{j}}{Z_{0}^{j}}\right)^{\frac{\sigma_{Z}^{j}-1}{\sigma_{Z}^{j}}} \hat{A}^{j}(t)+\left(1-\gamma_{Z}^{j}\right)\left(\frac{B_{0}^{j}}{Z_{0}^{j}}\right)^{\frac{\sigma_{Z}^{j}-1}{\sigma_{Z}^{j}}} \hat{B}^{j}(t) \\
& =s_{L, 0}^{j} \hat{A}^{j}(t)+\left(1-s_{L, 0}^{j}\right) \hat{B}^{j}(t) \tag{342}
\end{align*}
$$

Eq. (342) is identical to (331) obtained in the Cobb-Douglas case. Solving eq. (342) and the log-linearized version of the demand for factors of production (332) leads to the solutions for $\hat{A}^{j}(t)$ and $\hat{B}^{j}(t)$ described by (46a)-(46b) in the main text obtained by assuming a Cobb-Douglas for the technology frontier.

## K Effects of Technology Shocks Biased Toward Tradables: Inspecting the Mechanism

In this section, we solve the model analytically by abstracting from physical capital. This enables us to derive a number of analytical results which emphasize the role of IML across sectors and endogenous TOT in driving the transmission mechanism of technology shocks.

Both sectors use labor as the sole input in a constant returns to scale technology, i.e., $Y^{j}=A^{j} L^{j}$ with $j=T, N$. We set the productivity index in non-tradables to 1 , i.e., $A^{N}=1$. Because there is a difficulty in reallocating labor, sectoral wages do not equalize:

$$
\begin{equation*}
P^{H} A^{H}=W^{H}, \quad \text { and } \quad P^{N}=W^{N} \tag{343}
\end{equation*}
$$

The key equations characterizing optimal household behavior are given by first-order conditions described by (23a)-(23b), (24a) and (25), (26). The market clearing conditions for non-traded and home-produced traded goods read as:

$$
\begin{equation*}
L^{N}=C^{N}, \quad \text { and } \quad Y^{H}=C^{H}+X^{H} \tag{344}
\end{equation*}
$$

where exports, $X^{H}$, are governed by eq. (35). The current equation equation can be rewritten as $\dot{N}=r^{\star} N+P^{H} X^{H}-C^{F}$. To be able to derive useful analytical expressions which emphasize the distinct role of these two features, it is necessary to recourse to a number of assumptions. First, we solve the model by assuming that productivity in tradables increases once for and all to its new long-run level, i.e., $\frac{A^{H}-A_{0}^{H}}{A_{0}^{H}}=a$. This assumption implies that the dynamics toward the final steady-state degenerate and the intertemporal solvency condition reduces to:

$$
\begin{equation*}
\tilde{N}=N_{0} \tag{345}
\end{equation*}
$$

Aggregation of market clearing conditions (344) leads to the standard equality between GDP, $Y$, and final expenditure, $P_{C} C-r^{\star} N_{0}$. To keep analytical expressions simple, we assume that the country starts with a zero net foreign asset position, i.e., $N_{0}=0$. This assumption implies that the consumption-to-GDP ratio, $\omega_{C}$, is equal to one, since trade is initially balanced:

$$
\begin{equation*}
P^{H} X^{H}=C^{F} \tag{346}
\end{equation*}
$$

For later use, we denote $\omega_{X}=\frac{P^{H} X^{H}}{Y}$ the ratio of exports to GDP and $\alpha_{L}=\frac{P^{H} A^{H} L^{H}}{Y}$ the home tradable content of GDP which is equivalent to the labor compensation share of the home-produced traded goods sector. ${ }^{53}$ Even under these assumptions, the model remains analytically untractable. Since our objective is to disentangle the role of IML across sectors and endogenous TOT, we explore below two polar cases. We first solve the model by allowing for IML across sectors while assuming that home- and foreign-produced traded goods are perfect substitutes, i.e., we let $\rho$ tend toward infinity. Next, we consider a semi-small open economy with endogenous TOT by imposing perfect mobility of labor across sectors, i.e., we let $\epsilon$ tend toward infinity.

## K. 1 IML across Sectors

We solve the model by assuming that home- and foreign-produced traded goods are perfect substitutes. When we let $\rho$ tend toward infinity into (10), we have $C^{T}=C^{H}+C^{F}$ and $P^{T}=1$ so that consumption in tradables reduces to:

$$
\begin{equation*}
C^{T}=\varphi P_{C}^{\phi} C \tag{347}
\end{equation*}
$$

Since the traded good is the numeraire, the price of non-tradables $P^{N}$ is equivalent to $P$. The market clearing condition for tradables (344) reads now as:

$$
\begin{equation*}
Y^{H}=C^{T} \tag{348}
\end{equation*}
$$

under assumption $N_{0}=0$. Inserting first (23b) into (26), (23a) into (25) and (347), and substituting the resulting expressions into the market clearing condition for non-tradables (344) and tradables (348), the steady-state can be reduced to two equations:

$$
\begin{gather*}
(1-\vartheta) P^{\epsilon} W^{-\left(\epsilon-\sigma_{L}\right)} \bar{\lambda}^{\sigma_{L}}=(1-\varphi) P^{-\phi} P_{C}^{\left(\phi-\sigma_{C}\right)} \bar{\lambda}^{-\sigma_{C}},  \tag{349a}\\
r^{\star} N_{0}+\vartheta\left(A^{H}\right)^{1+\epsilon} W^{-\left(\epsilon-\sigma_{L}\right)} \bar{\lambda}^{\sigma_{L}}=\varphi P_{C}^{\left(\phi-\sigma_{C}\right)} \bar{\lambda}^{-\sigma_{C}}, \tag{349b}
\end{gather*}
$$

which jointly determine the relative price of non-tradables, $P$, and the shadow value of wealth, $\bar{\lambda}$. Under assumption $N_{0}=0$, we have $\omega_{C}=\frac{P_{C} C}{Y}=1$ so that $\alpha_{L}=\alpha_{C}$.

A rise in the productivity index of tradables, $A^{H}$, produces a positive wealth effect reflected by a decline in the shadow value of wealth: ${ }^{54}$

$$
\begin{equation*}
\hat{\bar{\lambda}}=-\frac{\left\{(1+\epsilon)\left[\left(\sigma_{L}+\sigma_{C}\right)+\alpha_{C}\left(\phi-\sigma_{C}\right)\right]+\alpha_{C}\left(\epsilon-\sigma_{L}\right)(1-\phi)\right\}}{\left(\sigma_{L}+\sigma_{C}\right)(\epsilon+\phi)}<0 \tag{350}
\end{equation*}
$$

[^39]where the negative sign of the RHS term follows from evidence which suggests that $\phi<1$. The positive wealth effect described by (350) encourages agents to work less and increase consumption expenditure. Because the rise in real expenditure is spread over the two goods while productivity in non-tradables is unchanged, an excess demand arises in the non-traded goods market, which in turn causes the relative price of non-tradables to appreciate: ${ }^{55}$
\[

$$
\begin{equation*}
\hat{P}=\frac{1+\epsilon}{\phi+\epsilon} a>0 \tag{351}
\end{equation*}
$$

\]

Eqs. (350) and (351) show that the degree of labor mobility across sectors influence both the extent of the decline in $\bar{\lambda}$ and the magnitude of the appreciation in the relative price of non-tradables. More specifically, as $\epsilon$ takes higher values, it can be shown analytically that the shadow value of wealth falls less while the relative price of non-tradables appreciates by a lower amount. Intuitively, following an increase in $A^{H}$, more labor shifts toward the non-traded sector as the degree of labor mobility increases which results in a smaller excess of demand in the non-traded goods market so that the relative price appreciates less. Since non-traded wages increase by a smaller amount as well, the positive wealth effect is mitigated. In the situation of perfect mobility of labor across sectors, we have $\lim _{\epsilon \rightarrow \infty} \hat{P}=a$, and $\lim _{\epsilon \rightarrow \infty} \hat{\bar{\lambda}}=-\frac{\left[\left(\sigma_{L}+\sigma_{C}\right)+\alpha_{L}\left(1-\sigma_{C}\right)\right]}{\sigma_{L}+\sigma_{C}} a<0$.

Because labor shifts away from the traded to the non-traded sector, the share of non-tradables in labor, $\nu^{L, N}$, increases. To see it formally, totally differentiate (26) together with (343), and substitute (351):

$$
\begin{equation*}
\hat{\nu}^{L, N}=\alpha_{L}\left(1-\alpha_{L}\right) \frac{\epsilon(1-\phi)}{\epsilon+\phi} a \gtrless 0 . \tag{352}
\end{equation*}
$$

Eq. (352) shows that both elasticity of substitution between traded and nontraded goods, $\phi$, and the elasticity of labor supply across sectors, $\epsilon$, matter in determining the response of the share of non-tradables in labor. Since evidence documented by the literature overwhelmingly suggest that $\phi<1$, a technology shock biased toward the traded sector leads to a reallocation of labor toward the non-traded sector. Intuitively, when $\phi<1$, the appreciation in the relative price, $P$, raises expenditure in non-tradables relative to tradables and thus boosts labor demand in the non-traded sector. Thus, $\nu^{L, N}$ increases in line with our empirical findings documented in section 2 . As the elasticity of labor supply across sectors (i.e., $\epsilon$ ) takes higher values, workers are more willing to shift their hours worked toward the non-traded sector and thus $\nu^{L, N}$ increases more, as long as $\phi<1$.

While the traded sector experiences a labor outflow, the share of tradables in real GDP, $\nu^{Y, H}$, unambiguously rises. In the data, the response of the sectoral output share is calculated as the growth differential in GDP units between traded value added at constant prices and real GDP, i.e., $\hat{\nu}^{Y, H}=\alpha_{L}\left(\hat{Y}^{H}-\hat{Y}_{R}\right)$. Totally differentiating real GDP and inserting the resulting expression reveals that change in the share of tradables in real GDP is positively related to the appreciation in the relative price of non-tradables:

$$
\begin{equation*}
\hat{\nu}^{Y, H}=\alpha_{L}\left(1-\alpha_{L}\right) \phi \hat{P} \tag{353}
\end{equation*}
$$

where $\hat{P}$ is given by eq. (351). Because a higher degree of labor mobility across sectors mitigates the excess demand in the non-traded goods market, and thus the appreciation in the relative price, $P$, the share of tradables in real GDP increases less as more labor shifts away from the traded sector.

How do hours worked and the real consumption wage react to a technology shock biased toward the traded sector? Higher productivity in tradables increases both traded and non traded wages and thus raises the aggregate wage by an amount given by $\hat{W}=\alpha_{L} \hat{A}^{H}+\left(1-\alpha_{L}\right) \hat{P}$. Totally differentiating $W_{C}=W / P_{C}$ and inserting (351) gives the response of the real consumption wage, i.e., $\hat{W}_{C}=\alpha_{C} \hat{A}^{H}>0$. Thus the percentage change in the real consumption wage is independent of the degree of labor mobility across sectors. By raising the aggregate wage and reducing the shadow value of wealth, a technology shock biased toward the traded sector exerts two opposite effects on labor supply. Totally differentiating (23b), i.e., $L=(W \lambda)^{\sigma_{L}}$, and inserting (350) along with (351) shows that total hours worked remain unaffected when $\sigma_{C}=1$ :

$$
\begin{equation*}
\hat{L}=-\frac{\sigma_{L} \alpha_{L}\left(1-\sigma_{C}\right)}{\sigma_{L}+\sigma_{C}} a \leq 0 . \tag{354}
\end{equation*}
$$

When $\sigma_{C}=1$, the rise in leisure triggered by the wealth effect following a technology shock is exactly offset by the fall in leisure resulting from the substitution effect caused by a higher wage. When $\sigma_{C}>1$, the curvature of the utility function derived from consumption is less so that the marginal utility of consumption declines less rapidly. Henceforth, the impact of the wealth effect on leisure is mitigated and the substitution effect dominates. Hence, a technology shock increases labor supply when $\sigma_{C}>1$. It is worth noting that the elasticity of labor supply across sectors has no impact on the response of total hours worked as a rise in $\epsilon$ lowers the extent of the wealth and substitution effect by the same magnitude.

[^40]
## K. 2 Endogenous TOT

We now shed some light on the implications of endogenous TOT. We solve the model by assuming that workers do not experience a utility loss when shifting hours worked from one sector to another. When we let $\epsilon$ tend toward infinity into (13), we have:

$$
\begin{equation*}
L=L^{H}+L^{N} . \tag{355}
\end{equation*}
$$

Because workers are devote their whole time to the sector that pays highest wages, both sectors must pay the same wage; thus eqs. (343) reduce to:

$$
\begin{equation*}
W=P^{H} A^{H}=P^{N} \tag{356}
\end{equation*}
$$

Totally differentiating (356) reveals that the price of non-traded goods in TOTd goods, $\hat{P}=\hat{P}^{N}$ $\hat{P}^{H}$, appreciates by the same amount as $\hat{A}^{H}$, like in a model where TOT are exogenous. Differently, as long as home- and foreign-produced traded goods are imperfect substitutes, such an appreciation is achieved through a smaller appreciation in the relative price of non-tradables, $\hat{P}^{N}$, and a decline in the TOT. As we shall see below, the fall in the relative price of tradables plays a key role by mitigating the shift of labor toward the non-traded sector as the decrease in $P^{H}$ encourages households to substitute home-produced traded goods for non-traded goods.

When the TOT are endogenous, two additional parameters determine the response of the open economy to a technology shock: the export price elasticity, $\phi_{X}$, and the elasticity of substitution between home- and foreign- produced traded goods. We assume that both parameters are larger than one. The assumption of $\phi_{X}>1$ is supported by evidence documented by Mejean and Imbs [2015] which indicates that $\phi_{X}>1$ for the vast majority of the OECD countries.

Inserting first appropriate optimal decisions into (344), (346), (355), and (356), and differentiating leads to the response of the TOT to a technology shock biased toward the traded sector: ${ }^{56}$

$$
\begin{equation*}
\hat{P}^{H}=-\frac{\chi^{H}}{\left[\left(\phi_{X}-1\right)+\alpha^{H} \rho\right]\left(\sigma_{L}+\sigma_{C}\right)+\left(1-\alpha^{H}\right) \chi^{H}} a<0 \tag{357}
\end{equation*}
$$

where

$$
\begin{equation*}
\chi^{H}=\sigma_{C} \alpha_{C}\left(\sigma_{L}+1\right)+\left(1-\alpha_{C}\right) \phi\left(\sigma_{L}+\sigma_{C}\right)>0 \tag{358}
\end{equation*}
$$

As shown in eq. (357), for the TOT to decline, the export price elasticity, $\phi_{X}$, must be larger than one. Intuitively, a technology shock produces a positive wealth effect which encourages agents to consume more. Because imports increase, for trade to be balanced, the value of exports in terms of foreign-produced goods, i.e, $P^{H} X^{H}=\varphi_{X}\left(P^{H}\right)^{1-\phi_{X}}$, must increase; when $\phi_{X}>1$, the fall in $P^{H}$ improves the balance of trade.

Because the decline in TOT mitigates the rise in traded wages, $W^{H}$, the marginal utility of wealth declines less than that if the TOT were exogenous. ${ }^{57}$ Inserting first (35) and (24a) together with (23a) into (346), differentiating and inserting (357) shows that the marginal utility of wealth unambiguously declines:
$\hat{\bar{\lambda}}=-\left\{\frac{\left[\left(\phi_{X}-1\right)+\alpha^{H}\left(\rho-\sigma_{C}\right)\right]\left[\alpha_{L}+\sigma_{L}+\left(1-\alpha_{C}\right) \sigma_{C}\right]+\left(\sigma_{C}-\phi\right)\left(1-\alpha_{C}\right) \alpha^{H}\left(\sigma_{L}+\sigma_{C}\right)}{\left[\left(\phi_{X}-1\right)+\alpha^{H} \rho\right]\left(\sigma_{L}+\sigma_{C}\right)+\left(1-\alpha^{H}\right) \chi^{H}}\right\} a<0$.

[^41]Then inserting first (24a) and (26) into the market clearing condition for home-produced traded goods (344), making use of the balanced trade condition to eliminate $X^{H}$ and differentiating leads to:

$$
\alpha_{L} \hat{A}^{H}+\sigma_{L} \hat{\bar{\lambda}}+\sigma_{L}\left(\hat{P}^{H}+\hat{A}^{H}\right)=\hat{C}-\omega_{X} \hat{P}^{H}
$$

where we eliminated $\alpha_{L} \hat{L}^{H}$ from the above equation by using the first equation. Totally differentiate (23a) and the market clearing condition for non-tradables by inserting first (25) and (26) in order to eliminate the $\hat{P}^{N}$ from the above equation; totally differentiating the balanced trade condition (346) to eliminate $\hat{\bar{\lambda}}$; collecting terms leads to (357).
${ }^{57}$ The change in the equilibrium value of the marginal utility of wealth can be rewritten in terms of $\lim _{\rho \rightarrow \infty} \hat{\bar{\lambda}}$, i.e.,
$\hat{\bar{\lambda}}=\left\{\frac{\left[\left(\phi_{X}-1\right)+\alpha^{H} \rho\right]\left(\sigma_{L}+\omega_{C} \sigma_{C}\right)}{\left[\left(\phi_{X}-1\right)+\alpha^{H} \rho\right]\left(\sigma_{L}+\sigma_{C}\right)+\left(1-\alpha^{H}\right) \chi^{H}} \lim _{\rho \rightarrow \infty} \hat{\bar{\lambda}}+\frac{\alpha^{H}\left[\alpha_{C} \sigma_{C}\left(\sigma_{L}+\omega_{C}\right)+\left(1-\alpha_{C}\right) \phi\left(\sigma_{L}+\omega_{C} \sigma_{C}\right)\right]}{\left[\left(\phi_{X}-1\right)+\alpha^{H} \rho\right]\left(\sigma_{L}+\sigma_{C}\right)+\left(1-\alpha^{H}\right) \chi^{H}} a\right\}$.
Since the term in front of $\lim _{\rho \rightarrow \infty} \hat{\bar{\lambda}}$ is positive and smaller than one, while the second term on the RHS is positive, the marginal utility of wealth declines less when the TOT deteriorate.

Eq. (359) shows that the marginal utility of wealth unambiguously declines as long as export price elasticity, $\phi_{X}$, is larger than one, and households are willing to substitute home- for foreignproduced traded goods, i.e., if $\rho>1$. Intuitively, when the export price elasticity is larger than one, the TOT decline which provides incentives to substitute home- for foreign-produced traded goods. If $\left(\phi_{X}-1\right)+\alpha^{H}\left(\rho-\sigma_{C}\right)>0$, the fall in the relative price of tradables exerts a negative impact on imports. For trade to be balanced, the shadow value of wealth must decrease to increase imports of foreign-produced traded goods.

Because the positive wealth effect encourages households to consume more, the demand for nontraded goods increases. Since productivity of non-tradables remains unchanged, an excess demand shows up which appreciates the price of non-traded goods in terms of foreign-produced traded goods: ${ }^{58}$
$\hat{P}^{N}=\frac{\left\{\left[\left(\phi_{X}-1\right)+\alpha^{H}\left(\rho-\sigma_{C}\right)+\alpha^{H}\left(1-\alpha_{C}\right)\left(\sigma_{C}-\phi\right)\right]\left(\sigma_{L}+\omega_{C} \sigma_{C}\right)+\alpha^{H} \alpha_{C} \omega_{C} \sigma_{C}\left(\sigma_{C}-1\right)\right\}}{\left[\left(\phi_{X}-1\right)+\alpha^{H} \rho\right]\left(\sigma_{L}+\sigma_{C}\right)+\left(1-\alpha^{H}\right) \chi^{H}} a>0$.
It can be shown analytically that $\hat{P}^{N}<\hat{A}^{H}=a$ and thus the relative price of non-traded goods appreciates less than that if the TOT were exogenous. The reason is that the decline in TOT boosts consumption in home-produced traded goods which in turn mitigates the increase in demand for nontradables. For labor to be shifted toward the non-traded sector, the elasticity of substitution between traded and non-traded goods must be smaller than $1 .{ }^{59}$ As long as $\phi<1$, the share of non-tradables in labor, $\nu^{N, L}$, increases. To show it formally, we totally differentiate the resource constraint for labor (355) and use the resulting expression to eliminate $\hat{L}$ from $\hat{\nu}^{L, N}=\left(1-\alpha_{L}\right)\left(\hat{L}^{N}-\hat{L}\right)$, i.e., $\hat{\nu}^{L, N}=\left(1-\alpha_{L}\right) \alpha_{L}\left(\hat{L}^{N}-\hat{L}^{H}\right)$. Computing responses in hours worked in the non-traded and the traded sector, the change in share of non-tradables in labor is: ${ }^{60}$

$$
\begin{equation*}
\hat{\nu}^{L, N}=\left(1-\alpha_{L}\right) \alpha_{L} \frac{\left[\left(\phi_{X}-1\right)+\alpha^{H} \rho\right]\left(\sigma_{L}+\sigma_{C}\right)(1-\phi)}{\left[\left(\phi_{X}-1\right)+\alpha^{H} \rho\right]\left(\sigma_{L}+\sigma_{C}\right)+\left(1-\alpha^{H}\right) \chi^{H}} a>0 . \tag{361}
\end{equation*}
$$

While hours worked are reallocated toward the non-traded sector, the extent of the labor shifts are smaller than that if the TOT remained fixed. More precisely, by hampering the boom for nontradables, the adjustment in the TOT curbs the rise in the labor share of non-tradables. ${ }^{61}$ Because less labor shifts toward the non-traded sector when the TOT decline, the share of tradables in real GDP, $\nu^{Y, H}$, increases by a larger amount. To compute the growth differential in GDP units between traded value added at constant prices and real GDP, use the fact that $\hat{Y}_{R}=\alpha_{L} \hat{Y}^{H}+\left(1-\alpha_{L}\right) \hat{Y}^{N}$ to eliminate $\hat{Y}_{R}$ from $\alpha_{L}\left(\hat{Y}^{H}-\hat{Y}_{R}\right)$, and substitute $\hat{Y}^{N}=\hat{C}^{N}$ and $\hat{Y}^{H}=\hat{C}^{T}-\left(1-\alpha^{H}\right) \hat{P}^{H}$, we get: ${ }^{62}$

$$
\begin{equation*}
\hat{\nu}^{Y, H}=\left(1-\alpha_{L}\right) \alpha_{L}\left[\phi a-\left(1-\alpha^{H}\right)(1-\phi) \hat{P}^{H}\right]>0 \tag{362}
\end{equation*}
$$

where $\hat{P}^{H}$ is given by eq. (357). Since $\lim _{\rho \rightarrow \infty} \hat{P}^{H}=0$, we have $\lim _{\rho \rightarrow \infty} \hat{\nu}^{Y, H}=\phi a>0$. As long as home- and foreign-produced traded goods are imperfect substitutes, i.e., $\rho<\infty$, the decline in

[^42]the TOT increases exports and $C^{H}$ which in turn mitigates the reallocation of labor toward the non-traded sector and thus amplifies the rise in the share of tradables in real GDP.

## L Calibration Procedure

In this section, we provide more details about the calibration to a representative OECD economy and to data from 17 OECD countries. Appendix E presents the source and construction of data.

## L. 1 Initial Steady-State

Since we consider CES production functions and we compare the results with Cobb-Douglas production functions, we have to normalize the CES productions so that the steady-state is invariant when the elasticity of substitution $\sigma^{j}$ is changed. Our strategy is to choose the initial steady-state in a model with Cobb-Douglas production functions as the normalization point and set parameters in the CES economy so as to target the ratios of the Cobb-Douglas economy. Because we consider the initial steady-state with Cobb-Douglas production functions as the normalization point, we have to calibrate the model with Cobb-Douglas production functions to the data.

Normalizing total factor productivity (TFP henceforth) for the non-traded sector $Z^{N}$ to 1 , the calibration reduces to 24 parameters: $r^{\star}, \beta, \sigma_{C}, \sigma_{L}, \epsilon, \vartheta, \phi, \rho, \varphi, \varphi^{H}, \phi_{J}, \rho_{J}, \iota, \iota^{H}, \varphi_{X}, \phi_{X}, \kappa, \delta_{K}$, $\theta^{H}, \theta^{N}, Z^{H}, \omega_{G}\left(=\frac{G}{Y}\right), \omega_{G^{N}}\left(=\frac{P^{N} G^{N}}{G}\right), \omega_{G^{H}}\left(=\frac{P^{H} G^{H}}{G^{T}}\right)$, and initial conditions $N_{0}, K_{0}$.

Since we focus on the long-run equilibrium, the tilde is suppressed for the purposes of clarity.

$$
\begin{align*}
& C=\left(P_{C} \bar{\lambda}\right)^{-\sigma_{C}},  \tag{363a}\\
& L=(W \bar{\lambda})^{\sigma_{L}},  \tag{363b}\\
& C^{N}=(1-\varphi)\left(\frac{P}{P_{C}}\right)^{-\phi} C,  \tag{363c}\\
& C^{H}=\varphi\left(\frac{P^{T}}{P_{C}}\right)^{-\phi} \varphi_{H}\left(\frac{P^{H}}{P^{T}}\right)^{-\rho} C,  \tag{363d}\\
& C^{F}=\varphi\left(\frac{P^{T}}{P_{C}}\right)^{-\phi}\left(1-\varphi_{H}\right)\left(\frac{1}{P^{T}}\right)^{-\rho} C \text {, }  \tag{363e}\\
& L^{N}=(1-\vartheta)\left(\frac{W^{N}}{W}\right)^{\epsilon} L,  \tag{363f}\\
& L^{H}=\vartheta\left(\frac{W^{H}}{W}\right)^{\epsilon} L  \tag{363g}\\
& I^{N}=\left(1-\varphi_{J}\right)\left(\frac{P}{P_{J}}\right)^{-\phi_{J}} I,  \tag{363h}\\
& I^{H}=\iota\left(\frac{P_{J}^{T}}{P_{J}}\right)^{-\phi_{J}} \iota_{H}\left(\frac{P^{H}}{P_{J}^{T}}\right)^{-\rho_{J}} I,  \tag{363i}\\
& I^{F} \iota\left(\frac{P_{J}^{T}}{P_{J}}\right)^{-\phi_{J}} \varphi_{H}\left(\frac{1}{P_{J}^{T}}\right)^{-\rho_{J}} I,  \tag{363j}\\
& I=\delta_{K} K,  \tag{363k}\\
& \frac{G}{Y}=\omega_{G},  \tag{3631}\\
& P^{H} Z^{H}\left(1-\theta^{H}\right)=P_{J}\left(r^{\star}+\delta_{K}\right),  \tag{363m}\\
& P^{H} Z^{H}\left(1-\theta^{H}\right)\left(k^{H}\right)^{-\theta^{H}}=P^{N} Z^{N}\left(1-\theta^{N}\right)\left(k^{N}\right)^{-\theta^{N}},  \tag{363n}\\
& P^{H} Z^{H} \theta^{H}\left(k^{H}\right)^{1-\theta^{H}}=W^{H},  \tag{363o}\\
& P^{N} Z^{N} \theta^{N}\left(k^{N}\right)^{1-\theta^{N}}=W^{N},  \tag{363p}\\
& k^{H} L^{H}+k^{N} L^{N}=K,  \tag{363q}\\
& Z^{N} L^{N}\left(k^{N}\right)^{1-\theta^{N}}=C^{N}+G^{N}+I^{N},  \tag{363r}\\
& X^{H}=\varphi_{X}\left(P^{H}\right)^{-\phi_{X}},  \tag{363s}\\
& Z^{H} L^{H}\left(k^{H}\right)^{1-\theta^{H}}=C^{H}+X^{H}+I^{H}+G^{H},  \tag{363t}\\
& r^{\star} N+P^{H} X^{H}-M^{F}=0, \tag{363u}
\end{align*}
$$

and the intertemporal solvency condition

$$
\begin{equation*}
N-N_{0}=\Psi_{1}\left(K-K_{0}\right), \tag{363v}
\end{equation*}
$$

where we used the fact that at the steady-state $I^{j}=J^{j}$ (with $j=T, N$ ), and we also have

$$
\begin{gather*}
G^{N}=\left(\omega_{G^{N}} / P^{N}\right) G,  \tag{364a}\\
G^{H}=\left[\left(1-\omega_{G^{N}}\right) \omega_{G^{H}} / P^{H}\right] G,  \tag{364b}\\
G^{F}=\left(1-\omega_{G^{N}}\right)\left(1-\omega_{G^{H}}\right) G,  \tag{364c}\\
P_{C}=\left[\varphi\left(P^{T}\right)^{1-\phi}+(1-\varphi)\left(P^{N}\right)^{1-\phi}\right]^{\frac{1}{1-\phi}},,  \tag{364d}\\
P^{T}=\left[\varphi_{H}\left(P^{H}\right)^{1-\rho}+\left(1-\varphi_{H}\right)\right]^{\frac{1}{1-\rho}},  \tag{364e}\\
P_{J}=\left[\iota\left(P_{J}^{T}\right)^{1-\phi_{J}}+(1-\iota)\left(P^{N}\right)^{1-\phi_{J}}\right]^{\frac{1}{1-\phi_{J}}},  \tag{364f}\\
P_{J}^{T}=\left[\iota_{H}\left(P^{H}\right)^{1-\rho_{J}}+\left(1-\iota_{H}\right)\right]^{\frac{1}{1-\rho_{J}}},  \tag{364~g}\\
W=\left[\vartheta\left(W^{H}\right)^{\epsilon+1}+(1-\vartheta)\left(W^{N}\right)^{\epsilon+1}\right]^{\frac{1}{\epsilon+1}},  \tag{364h}\\
M^{F}=C^{F}+I^{F}+G^{F},  \tag{364i}\\
Y^{H}=Z^{H} L^{H}\left(k^{H}\right)^{1-\theta^{H}},  \tag{364j}\\
Y^{N}=Z^{N} L^{N}\left(k^{N}\right)^{1-\theta^{N}},  \tag{364k}\\
Y=P^{H} Y^{H}+P^{N} Y^{N} . \tag{3641}
\end{gather*}
$$

Using (364), the system (363) jointly determines the following 22 variables $C, L, C^{N}, C^{H}, C^{F}, L^{N}$, $L^{H}, I^{N}, I^{H}, I^{F}, I, G, k^{H}, k^{N}, W^{H}, W^{N}, K, P^{N}, X^{H}, P^{H}, N, \bar{\lambda}$.

Before going any further, it is worth mentioning that in accordance with the empirical findings documented by Bems [2008] for OECD countries, we choose an elasticity of substitution between $J^{N}$ and $J^{T}$ of 1, i.e.,

$$
\begin{equation*}
J=\left(\frac{J^{T}}{\alpha_{J}}\right)^{\alpha_{J}}\left(\frac{J^{N}}{1-\alpha_{J}}\right)^{1-\alpha_{J}} \tag{365}
\end{equation*}
$$

where $\alpha_{J}=\frac{P_{J}^{T} J^{T}}{P_{J} J}$ and $1-\alpha_{J}=\frac{P^{N} J_{J}^{N}}{P_{J} J}$ are investment expenditure shares on tradables and nontradables, respectively, which are fixed parameters. The investment price index, $P_{J}=P_{J}\left(P_{J}^{T}, P^{N}\right)$, associated with aggregator function (365) is:

$$
\begin{equation*}
P_{J}=\left(P_{J}^{T}\right)^{\alpha_{J}}\left(P^{N}\right)^{1-\alpha_{J}} \tag{366}
\end{equation*}
$$

Some of the values of parameters can be taken directly from data, but others need to be endogenously calibrated to fit a set of an average OECD economy features. Among the 24 parameters, 5 parameters, i.e., $\varphi^{H}, \iota^{H}, \varphi, \iota, \vartheta, \delta_{K}$ together with initial conditions $\left(N_{0}, K_{0}\right)$ must be set in order to match key properties of a typical OECD economy. More precisely, the parameters $\varphi^{H}, \iota^{H}, \varphi, \iota$, $\vartheta, \delta_{K}$ together with the set of initial conditions are set to target $\alpha^{H}, \alpha_{J}^{H}, \alpha_{C}, \alpha_{L}, \omega_{J}, v_{N}$.

We denote by $\nu^{Y, H}$ the GDP share of home-produced traded goods, $v_{G^{j}}=G^{j} / P^{j} Y^{j}$ and $v_{J^{j}}=P_{J}^{j} J^{j} / P^{j} Y^{j}$ the ratio of government spending and investment expenditure on good $j$ to output in sector $j$, respectively, $v_{N}=\frac{r^{\star} N}{P^{H} Y^{H}}$ the ratio of interest receipts from traded bonds holding to traded output, $\omega_{X}=\frac{P^{H} X^{H}}{Y}$ the ratio of exports to GDP, $\omega_{G}$ the ratio of government spending to GDP, and $\omega_{J}=\frac{P_{J} J}{Y}$ the ratio of investment expenditure to GDP. The steady-state can be reduced to the following five equations:

$$
\begin{gather*}
\frac{\nu^{Y, H}}{1-\nu^{Y, H}} \frac{\left(1+v_{N}-v_{J^{H}}+v_{G^{H}}\right)}{\left(1-v_{J^{N}}-v_{G^{N}}\right)}=\frac{\varphi}{1-\varphi}\left(\frac{P^{T}}{P^{N}}\right)^{\phi},  \tag{367a}\\
\frac{\nu^{Y, H}}{1-\nu^{Y, H}}=\frac{\left(P^{H}\right)^{\frac{1+\epsilon}{\theta^{H}}}}{\left(P^{N}\right)^{\frac{1+\epsilon}{\theta^{N}}}}\left(P_{J}\right)\left(\frac{\theta^{H}-\theta^{N}}{\theta^{H} \theta^{N}}\right)(1+\epsilon)  \tag{367b}\\
\nu^{Y, H}=\omega_{C} \alpha_{C} \alpha^{H}+\omega_{J} \alpha_{J} \alpha_{J}^{H}+\omega_{G^{H}}\left(1-\omega_{G^{N}}\right) \omega_{G}+\omega_{X},  \tag{367c}\\
\left(1-\theta^{H}\right) \nu^{Y, H}+\left(1-\theta^{N}\right)\left(1-\nu^{Y, H}\right)=P_{J}\left(r^{\star}+\delta_{K}\right) \frac{K}{Y},  \tag{367d}\\
v_{N}=v_{N_{0}}+\frac{r^{\star} \Psi_{1}}{\nu^{Y, H}}\left(\frac{K}{Y}-v_{K_{0}}\right), \tag{367e}
\end{gather*}
$$

where $v_{K_{0}}=\frac{K_{0}}{Y}$ and $\Pi$ is a term composed of parameters described by:

$$
\begin{align*}
\Pi & \equiv \frac{\left(Z^{H}\right)^{\frac{1+\epsilon}{\theta^{H}}}}{\left(Z^{N}\right)^{\frac{1+\epsilon}{\theta N}}} \frac{\vartheta}{1-\vartheta}\left(r^{\star}+\delta_{K}\right)^{\left(\frac{\theta^{H}-\theta^{N}}{\theta^{H} \theta^{N}}\right)(1+\epsilon)} \\
& \times \frac{\left[\left(\theta^{H}\right)^{\epsilon \theta^{H}}\left(1-\theta^{H}\right)^{\left(1-\theta^{H}\right)(1+\epsilon)}\right]^{1 / \theta^{H}}}{\left[\left(\theta^{N}\right)^{\epsilon \theta^{N}}\left(1-\theta^{N}\right)^{\left(1-\theta^{N}\right)(1+\epsilon)}\right]^{1 / \theta^{N}}} . \tag{368}
\end{align*}
$$

The system (367) consisting of five equations determine $\nu^{Y, H}, P^{N}, P^{H}, K / Y$, and $v_{N}$. The five equations (367a)-(367e) described the goods market equilibrium for tradables relative to non-tradables, the labor market equilibrium, the goods market equilibrium for the home-produced traded goods market equilibrium, the resource constraint for capital, the intertemporal solvency condition, respectively.

It is worth noting that $\varphi_{X}$ is a free parameter which does not play any role in this calibration strategy since the ratio of exports to GDP is determined residually by $v_{N}, \nu^{Y, H}, \omega_{C}, \alpha_{C}, \alpha^{H}, \omega_{J}$, $\alpha_{J}, \alpha_{J}^{H}$. To see it formally, use the current account equation in the long-run and divide both sides by GDP; one obtains:

$$
\begin{equation*}
\omega_{X}=-v_{N} \nu^{Y, H}+\omega_{C} \alpha_{C}\left(1-\alpha^{H}\right)+\omega_{J} \alpha_{J}\left(1-\alpha_{J}^{H}\right) . \tag{369}
\end{equation*}
$$

While $\varphi_{X}$ does not play any role in the calibration strategy with Cobb-Douglas production functions, this parameter is necessary to target $\omega_{X}$ when we allow for CES production functions since the steady-state with Cobb-Douglas production functions is chosen as the normalization point.

Left-multiplying the home-produced traded goods market equilibrium (363t) by $P^{H}$, eliminating $P^{H} X^{H}$ by using the current account equation (363u), i.e., $P^{H} X^{H}=M^{F}-r^{\star} N$, leads to the goods market equilibrium for tradables:

$$
\begin{equation*}
P^{H} Y^{H}=P^{T} C^{T}+P_{J}^{T} J^{T}+G^{T}-r^{\star} N . \tag{370}
\end{equation*}
$$

Let multiplying (363r) by $P^{N}$, dividing the market clearing condition for tradables (370) by the market clearing condition for the non-traded good (363r) and equating the resulting expression with the demand of tradables in terms of non-tradables for consumption obtained by calculating the ratio of $P^{T} C^{T}=P^{H} C^{H}+C^{F}$ using (363d)-(363e) to (363c), i.e., $\frac{P^{T} C^{T}}{P^{N} C^{N}}=\frac{\varphi}{1-\varphi}\left(\frac{P^{N}}{P^{T}}\right)^{\phi-1}$, leads to the goods market equilibrium (367a). The derivation of the labor market equilibrium requires more steps. As mentioned below, we assume that the aggregator function for inputs of the investment good is Cobb-Douglas since data suggest that $\phi_{J}=1$. In this case, the investment price index simplifies as (366). First, combining (363m) and (363n) leads to:

$$
\begin{equation*}
\frac{\left(k^{H}\right)^{1-\theta^{H}}}{\left(k^{N}\right)^{1-\theta^{N}}}=\frac{\left[P^{H} Z^{H}\left(1-\theta^{H}\right)\right]^{\frac{1-\theta^{H}}{\theta^{H}}}}{\left[P^{N} Z^{N}\left(1-\theta^{N}\right)\right]^{\frac{1-\theta^{N}}{\theta_{N}}}}\left[P_{J}\left(r^{\star}+\delta_{K}\right)\right]^{\frac{1-\theta^{H}}{\theta^{H}}}-\frac{1-\theta^{N}}{\theta^{N}} . \tag{371}
\end{equation*}
$$

Dividing (363g) by (363f) leads to the supply of hours worked in the traded sector relative to the non-traded sector, i.e., $\frac{L^{H}}{L^{N}}=\frac{\vartheta}{1-\vartheta}\left(\frac{W^{H}}{W^{N}}\right)^{\epsilon}$. Dividing (363o) by (363p) leads to the relative wage of tradables, i.e., $\frac{W^{H}}{W^{N}}=\frac{P^{H} Z^{H} \theta^{H}\left(k^{H}\right)^{1-\theta^{H}}}{P^{N} Z^{N} \theta^{N}\left(k^{N}\right)^{1-\theta^{N}}}$. Inserting the latter expression into the former and using the production functions for the traded sector and non-traded sectors which imply $L^{H}=\frac{Y^{H}}{Z^{H}\left(k^{H}\right)^{1-\theta^{H}}}$ and $L^{N}=\frac{Y^{N}}{Z^{N}\left(k^{N}\right)^{1-\theta^{N}}}$, one obtains:

$$
\frac{Y^{H}}{Y^{N}}=\frac{\vartheta}{1-\vartheta}\left(\frac{Z^{H}}{Z^{N}}\right)^{\epsilon+1}\left(\frac{P^{H}}{P^{N}}\right)^{\epsilon}\left(\frac{\theta^{H}}{\theta^{N}}\right)^{\epsilon}\left[\frac{\left(k^{H}\right)^{1-\theta^{H}}}{\left(k^{N}\right)^{1-\theta^{N}}}\right]^{1+\epsilon}
$$

Left-multiplying the above expression by $\frac{P^{H} Y}{P^{N} Y}$, inserting (371), and collecting terms leads to the labor market equilibrium (367b) while we set $\Pi$ to eq. (368) in order to write the equation in compact form. To determine (367c), use the fact that $K^{j}=k^{j} L^{j}$, multiply both sides of (363q) by $\frac{R}{Y}$ where $R=P_{J}\left(r^{\star}+\delta_{K}\right)$ is the capital rental cost; we get:

$$
\frac{R K^{H}}{P^{H} Y^{H}} \frac{P^{H} Y^{H}}{Y}+\frac{R K^{N}}{P^{N} Y^{N}} \frac{P^{N} Y^{N}}{Y}=\frac{R K}{Y} .
$$

Using the fact that the capital income share $\frac{R K^{j}}{P^{j} Y^{j}}$ in sector $j$ is equal to $\left(1-\theta^{j}\right)$, one obtains the resource constraint for capital described by eq. (367c). Multiplying both sides of (363t) by $\frac{P^{H}}{Y}$, and using (363d)-(363e) and (363i)-(363j) leads to (367c):

$$
\begin{aligned}
\nu^{Y, H}= & \frac{P^{H} C^{H}}{C^{F}} \frac{C^{F}}{Y}+\frac{P^{H} J^{H}}{J^{F}} \frac{J^{F}}{Y}+\frac{P^{H} G^{H}}{Y}+\frac{P^{H} X^{H}}{Y}, \\
= & \frac{\varphi^{H}}{1-\varphi^{H}}\left(P^{H}\right)^{1-\rho}\left(1-\alpha^{H}\right) \alpha_{C} \omega_{C}+\frac{\iota^{H}}{1-\iota^{H}}\left(P^{H}\right)^{1-\rho_{J}}\left(1-\alpha_{J}^{H}\right) \alpha_{J} \omega_{J} \\
& +\omega_{G^{H}}\left(1-\omega_{G^{N}}\right) \omega_{G}+\omega_{X} .
\end{aligned}
$$

Finally, to get (367e), multiply both sides of (363v) by $\frac{r^{*}}{P^{H} Y^{H}}$, denote the ratio of interest receipts from the initial stock of traded bonds to traded output by $v_{N_{0}}=\frac{r^{\star} N_{0}}{P^{H} Y^{H}}$ and the ratio of the initial capital stock to GDP by $v_{K_{0}}=\frac{K_{0}}{Y}$ leads to eq. (367e) that describes the intertemporal solvency condition.

Because the ratios we wish to target are different from the macroeconomic aggregates, i.e., $\nu^{Y, H}, P^{N}, P^{H}, K / Y, v_{N}$, that are jointly determined by the system of equations (367), we have to relate the latter ratios to the former. First, the price of home-produced traded goods in terms of foreign-produced traded goods, $P^{H}$, determines the home content of consumption and investment expenditure in tradables by setting $\varphi^{H}$ and $\varphi_{J}^{H}$ :

$$
\begin{equation*}
\alpha^{H}=\frac{\varphi^{H}\left(P^{H}\right)^{1-\rho}}{\varphi^{H}\left(P^{H}\right)^{1-\rho}+\left(1-\varphi^{H}\right)}, \quad \text { and } \quad \alpha_{J}^{H}=\frac{\iota^{H}\left(P^{H}\right)^{1-\rho_{J}}}{\iota^{H}\left(P^{H}\right)^{1-\rho_{J}}+\left(1-\iota^{H}\right)} . \tag{372}
\end{equation*}
$$

Second, the price of non-traded goods in terms of foreign-produced traded goods, $P^{N}$, determines the home tradable content of consumption expenditure by setting $\varphi$ :

$$
\begin{equation*}
\alpha_{C}=\frac{\varphi\left(P^{H}\right)^{1-\phi}}{\varphi^{H}\left(P^{H}\right)^{1-\phi}+\left(1-\varphi^{H}\right)\left(P^{N}\right)^{1-\phi}} . \tag{373}
\end{equation*}
$$

Third, the ratio $K / Y$ along with the relative price of tradables, $P^{H}$, and the relative price of nontradables, $P^{N}$ (see (364g) and (366)), determine the investment-to-GDP ratio $P_{J} I / Y$ by setting $\delta_{K}$ (see eq. (363k)):

$$
\begin{equation*}
\frac{P_{J} I}{Y}=P_{J} \frac{\delta_{K} K}{Y} \tag{374}
\end{equation*}
$$

The ratio of net interest receipts from traded bonds holding to traded output, i.e., $v_{N}$, determines the ratio of next exports to traded output, i.e. $v_{N X}=\frac{N X}{P^{H} Y^{H}}$ with $N X=P^{H} X^{H}-M^{F}$; dividing both sides of the current account equation (363u) leads to:

$$
\begin{equation*}
v_{N X}=-v_{N} \tag{375}
\end{equation*}
$$

Finally, we show below that $\nu^{Y, H}$ is related to the share of tradables $L^{H} / L$ which we target by setting $\vartheta$. To do so, using the definition of the aggregate wage index (364h), the ratio of the aggregate wage to the non-traded wage can be rewritten as follows:

$$
\begin{aligned}
\left(\frac{W}{W^{H}}\right)^{\epsilon+1} & =\frac{\vartheta\left(W^{H}\right)^{\epsilon+1}+(1-\vartheta)\left(W^{N}\right)^{\epsilon+1}}{\left(W^{H}\right)^{\epsilon+1}} \\
& =\vartheta+(1-\vartheta)\left(\frac{W^{N}}{W^{H}}\right)^{\epsilon+1}
\end{aligned}
$$

and by solving, we get

$$
\begin{equation*}
\frac{W}{W^{H}}=\left[\vartheta+(1-\vartheta)\left(\frac{W^{N}}{W^{H}}\right)^{\epsilon+1}\right]^{\frac{1}{\epsilon+1}} \tag{376}
\end{equation*}
$$

Since $\theta^{j}$ is the labor income share in sector $j$, the ratio of the non-traded wage to the traded wage can be written as follows:

$$
\begin{equation*}
\frac{W^{N}}{W^{H}}=\frac{\theta^{N}}{\theta^{N}}\left(\frac{1-\nu^{Y, H}}{\nu^{Y, H}}\right) \frac{L^{H}}{L^{N}} \tag{377}
\end{equation*}
$$

Dividing (363g) by (363f) leads to a positive relationship between the supply of hours worked to the traded sector relative to the non-traded sector and the traded wage relative to the non-traded wage, i.e., $\frac{L^{H}}{L^{N}}=\frac{\vartheta}{1-\vartheta}\left(\frac{W^{H}}{W^{N}}\right)^{\epsilon}$. Substituting the latter equation, eq. (377) can be solved for $W^{N} / W^{H}$, i.e.,

$$
\begin{equation*}
\frac{W^{N}}{W^{H}}=\left[\frac{\vartheta}{1-\vartheta} \frac{\theta^{N}}{\theta^{H}}\left(\frac{1-\nu^{Y, H}}{\nu^{Y, H}}\right)\right]^{\frac{1}{\epsilon+1}} . \tag{378}
\end{equation*}
$$

Additionally, since $\alpha_{L}=\frac{W^{H} L^{H}}{W L}=\vartheta\left(\frac{W^{H}}{W}\right)^{\epsilon+1}$, the share of traded hours worked in total hours worked is governed by the following optimal rule:

$$
\begin{align*}
\frac{L^{H}}{L} & =\vartheta\left(\frac{W^{H}}{W}\right)^{\epsilon} \\
& =\vartheta\left(\frac{W}{W^{H}}\right)^{-\epsilon} . \tag{379}
\end{align*}
$$

Inserting (378) into (376) and plugging the resulting expression into (379) gives us a relationship between the share of tradables in employment and the share of tradables in GDP, $\nu^{Y, H}$ :

$$
\begin{align*}
\frac{L^{H}}{L} & =\vartheta\left[\vartheta+(1-\vartheta)\left(\frac{W^{N}}{W^{H}}\right)^{\epsilon+1}\right]^{-\frac{\epsilon}{\epsilon+1}} \\
& =\vartheta^{\frac{1}{\epsilon+1}}\left[1+\frac{\theta^{N}}{\theta^{H}}\left(\frac{1-\nu^{Y, H}}{\nu^{Y, H}}\right)\right]^{-\frac{\epsilon}{\epsilon+1}} \tag{380}
\end{align*}
$$

According to (380), given $\nu^{Y, H}$, setting $\vartheta$ allows us to target the ratio $L^{H} / L$ found in the data.

## L. 2 Calibration to a Representative OECD Economy

To calibrate our model, we estimated a set of parameters so that the initial steady state is consistent with the key empirical properties of a representative OECD economy. This section provides more details about how we calibrate the model to match the key empirical properties of a representative OECD economy. Because we consider an open economy setup with traded and non-traded goods, we calculate the non-tradable content of GDP, employment, consumption, gross fixed capital formation, government spending, labor compensation, for all countries in our sample, as summarized in Table 4. Since we assume that home- and foreign-produced traded goods are imperfect substitutes, we calculate the home content of consumption and investment expenditure in tradables on the one hand, and between purchases of home-produced goods from the home and the rest of the world (i.e., exports) on the other hand. To capture the key properties a typical OECD economy which is chosen as the baseline scenario, we take unweighted average values shown in the last line of Table 4. Columns 12-14 of Table 4 also report government spending and investment as a share of GDP along with the aggregate labor income share.

We first describe the parameters that are taken directly from the data; we start with the preference parameters shown in panel A of Table 5:

- One period in the model is a year.
- The world interest rate, $r^{\star}$, equal to the subjective time discount rate, $\beta$, is set to $4 \%$.
- We set the intertemporal elasticity of substitution for consumption, $\sigma_{C}$, to 2 in line with estimates documented by Gruber [2013]. While this value is higher than that usually used in the international RBC literature (i.e., $\sigma_{C}=1$ ), we choose this value to reduce the impact of the wealth effect on labor supply and generate a positive response of total hours worked.
- Next, we turn to the Frisch elasticity of labor supply. We set the intertemporal elasticity of substitution for labor supply $\sigma_{L}$ to 1.6 , in line with the evidence reported by Peterman [2016] who find a value for the macro Frisch elasticity of 1.5 and 1.75 for the population aged between 20 and 55 , and between 20 and 60 . This value of 1.6 enables us to generate an initial increase in total hours worked by $0.09 \%$ we estimate empirically following a $1 \%$ permanent increase in TFP of tradables relative to non-tradables, see Fig 10(a).
- The elasticity of labor supply across sectors, $\epsilon$, which captures the degree of labor mobility is set to 1.6. Our estimates display a wide dispersion across countries as they range from a low of 0.01 for Norway to a high of 3.2 for the United States, see Table 9. This value of 1.6 is halfway between the lowest and highest estimate for the degree of labor mobility across sectors . ${ }^{63}$
- We set the elasticity of substitution between traded and non-traded goods, $\phi$, to 0.44 , in line with estimates by Stockman and Tesar [1995]. Because this parameter plays a key role in the quantitative analysis, we have estimated this parameter by running the regression of the share of non-tradables in consumption expenditure on the ratio of non-traded prices to CPI. We explore empirically two variants of the testable equation by including or not a country-specific

[^43]linear time trend which captures the fact that the preference for consumption in non-tradables may vary over time (see Appendix F.2). As can be seen in the last row for Table 10 which reports estimates for the whole sample, we find that $\phi$ stands at 0.66 or 0.33 depending on whether a country-specific linear time trend is included or not. A value of 0.44 falls in the range of these estimates. ${ }^{64}$

- We set the elasticity of substitution, $\phi_{J}$, in investment between traded and non-traded inputs to 1 , in line with the empirical findings documented by Bems [2008] for OECD countries.
- Following Backus, Kehoe and Kydland [1994], we set the elasticity of substitution, $\rho\left(\rho_{J}\right)$, in consumption (investment) between home- and foreign-produced traded goods (inputs) to 1.5 .

We carry on with the non-tradable content of consumption, investment and government expenditure, employment, along with sectoral labor income shares shown in the last line of Table 4 that reports the average of our estimates while panel B of Table 5 displays the value of parameters we choose to calibrate the model:

- The weight of consumption in non-tradables $1-\varphi$ is set to 0.49 to target a non-tradable content in total consumption expenditure (i.e. $1-\alpha_{C}$ ) of $53 \%$.
- In order to target a non-tradable content of hours worked of $63 \%$ which corresponds to the 17 OECD countries' unweighted average shown in the last line of Table 4, we set the weight of labor supply to the traded sector in the labor index $L(),. 1-\vartheta$, to 0.6.
- We choose a value for the weight of non-traded inputs in the investment aggregator function $J(),. 1-\iota$, of 0.62 which allows us to obtain a non-tradable content of investment expenditure of $62 \%$.
- In accordance with our estimate shown in the last line of Table 4, we choose a non-tradable content of government spending, $\omega_{G^{N}}=\frac{P^{N} G^{N}}{G}$, of $90 \%$; by construction, we have a share of government consumption on tradables in total government spending, $\omega_{G^{H}}=1-\omega_{G^{N}}$, of $10 \%$.
- Columns 10 and 11 of Table 4 give the LIS of the traded and the non-traded sector for the seventeen OECD countries in our sample. LISs $\theta^{H}$ and $\theta^{N}$ average respectively to 0.63 and 0.68 . These average values reveal that the non-traded sector is relatively more labor intensive than the traded sector. It is worth mentioning that our estimates of 0.63 and 0.68 for $\theta^{H}$ and $\theta^{N}$, respectively, are consistent with an aggregate labor income share of $66 \%$, as shown in column 12 of Table 4. Formally, the aggregate labor income share, denoted by $s_{L}$, is a value-weighted average of the sectoral labor income shares, i.e., $s_{L}=\frac{\theta^{H} P^{H} Y^{H}}{Y}+\frac{\theta^{N} P^{N} Y^{N}}{Y}$.
- We assume that initially, traded firms are as much productive as non-traded firms and thus normalize $Z^{j}$ to 1 .

We describe below the choice of parameters displayed in panel C of Table 5 which target the home content of expenditure in tradables:

- In order to target a home content of consumption expenditure in tradables of $77 \%$ which corresponds to the 17 OECD countries' unweighted average shown in the last line of Table 4, we set the weight of home-produced traded goods in the consumption aggregator function for tradables $C^{T}(),. \varphi^{H}$, to 0.84 .
- We choose a value for the weight of home-produced traded inputs in the traded investment aggregator function $J^{T}(),. \iota^{H}$, of 0.62 which allows us to obtain a home content of investment expenditure in tradables of $51 \%$.
- Since data availability does not enable us to differentiate between government expenditure in home- and foreign-produced traded goods, we assume that the government does not import goods and services from abroad, and thus set $\omega_{G^{H}}=\frac{P^{H} G^{H}}{G^{T}}$ to 1 and $\omega_{G^{F}}=0$.
- Building on structural estimates of the price elasticities of aggregate exports documented by Imbs and Mejean [2015], we set the export price elasticity, $\phi_{X}$, to 1.7 in the baseline calibration.

We describe below the choice of parameters displayed in panel D of Table 5 characterizing macroeconomic variables such as investment, government spending and the balance of trade of a typical OECD economy:

- As shown in the last line of column 14 of Table 4, government spending as a percentage of GDP averages $20 \%$ and thus we set $\omega_{G}=\frac{G}{Y}$ to 0.2 .

[^44]- In order to target an investment-to-GDP ratio, $\omega_{J}=\frac{P_{J} I}{Y}$, of $24 \%$ as shown in the last line of column 13 of Table 4, we set the rate of physical capital depreciation, $\delta_{K}$, to $9.3 \%$.
- We choose the value of parameter $\kappa$ so that the elasticity of $I / K$ with respect to Tobin's q, i.e., $Q / P_{J}$, is equal to the value implied by estimates in Eberly, Rebelo, and Vincent [2008]. The resulting value of $\kappa$ is equal to $17 .{ }^{65}$
- Finally, we choose initial values for $N_{0}$ and $K_{0}$ for the ratio of net exports to traded output to be nil at the initial steady-state, i.e., $v_{N X} \simeq 0$.

Investment- and government spending-to-GDP ratios along with balanced trade endogenously determine the consumption-to-GDP ratio. More precisely, since GDP is equal to the sum if its demand components, remembering that at the steady-state $I=J$, we thus have the following accounting identity, $Y=P_{C} C+P_{J} I+G+N X$. Dividing both sides by $Y$ and remembering that net exports are nil, i.e., $N X=0$, the consumption-to-GDP ratio denoted by $\omega_{C}=\frac{P_{C} C}{Y}$ is thus equal to $56 \%$ :

$$
\begin{equation*}
\omega_{C}=\frac{P_{C} C}{Y}=1-\left(\omega_{J}+\omega_{G}+\frac{N X}{Y}\right)=56 \% \tag{381}
\end{equation*}
$$

where $\omega_{J}=\frac{P_{J} I}{Y}=24 \%, \omega_{G}=\frac{G}{Y}=20 \%$, and $N X=0$.
It is worth mentioning that the tradable content of GDP is endogenously determined by the tradable content of consumption, $\alpha_{C}$, of investment, $\alpha_{J}$, and of government expenditure, $\omega_{G^{T}}$, along with the consumption-to-GDP ratio, $\omega_{C}$, the investment-to-GDP ratio, $\omega_{J}$, and government spending as a share of GDP, $\omega_{G}$. More precisely, dividing the traded good market clearing condition (370) by GDP, $Y$, leads to an expression that allows us to calculate the tradable content of GDP:

$$
\begin{equation*}
\frac{P^{H} Y^{H}}{Y}=\omega_{C} \alpha_{C}+\omega_{J} \alpha_{J}+\omega_{G} \omega_{G}=38 \% \tag{382}
\end{equation*}
$$

where $\omega_{C}=56 \%, \alpha_{C}=47 \%, \omega_{J}=24 \%, \alpha_{J}=38 \%, \omega_{G^{T}}=10 \%$, and $\omega_{G}=20 \%$. According to (382), the values we target for the non-tradable content of consumption, investment and government spending along with the consumption-, investment-, and government spending-to-GDP ratios are roughly consistent with a tradable content of GDP of $39 \%$ found in the data, as reported in the last line of column 1 of Table 4. The cause of the slight discrepancy in the estimated tradable content of GDP is that nomenclatures for valued added by industry and for expenditure in consumption, investment, government expenditure by items are different. Reassuringly, the GDP share of tradables (39\%) is close to that calculated by using demand components (38\%).

Since we set initial conditions so that the economy starts with balanced trade, export as a share of GDP, $\omega_{X}$, is endogenously determined by the import content of consumption, $1-\alpha^{H}$, of investment, $1-\alpha_{J}^{H}$, and of government expenditure in tradables, $1-\omega_{G^{H}}$, along with the consumption-to-GDP ratio, $\omega_{C}$, and the investment-to-GDP ratio, $\omega_{J}$, and government spending as a share of GDP, $\omega_{G}$. More precisely, dividing the zero current account equation (363u) by GDP, $Y$, leads to an expression that allows us to calculate the GDP share of exports of final goods and services produced by the home country:

$$
\begin{equation*}
\frac{P^{H} X^{H}}{Y}=\omega_{C} \alpha_{C}\left(1-\alpha^{H}\right)+\omega_{J} \alpha_{J}\left(1-\alpha_{J}^{H}\right)+\left(1-\omega_{G^{H}}\right)\left(1-\omega_{G^{N}}\right) \omega_{G}=10.4 \%, \tag{383}
\end{equation*}
$$

where $\omega_{C}=56 \%, 1-\alpha^{H}=23 \%, \omega_{J}=24 \%, 1-\alpha_{J}^{H}=49 \%, 1-\omega_{G^{H}}=0$; in line with our evidence reported in column 7 of Table 4, the ratios we target enable us to reproduce the imports to GDP ratio of $10 \%$, keeping in mind that we consider trade on final goods.

In order to capture the dynamic adjustment of productivity in tradables relative to non-tradables, we assume that the response of sectoral TFP in percent is governed by the following dynamic equation: ${ }^{66}$

$$
\begin{equation*}
\hat{Z}^{j}(t)=\hat{\tilde{Z}}^{j}+\bar{z}^{j} e^{-\xi^{j} t} \tag{384}
\end{equation*}
$$

where $\hat{\tilde{Z}}^{j}$ is the percentage steady-state change in sectoral TFP; $\bar{z}^{j}$ and $\xi^{j}>0$ parametrize the initial increase in sectoral TFP and the speed at which $Z^{j}$ reaches its new steady-state level, respectively. More precisely, $\bar{z}^{j}$ takes negative values when sectoral TFP undershoots its steady-state level. The 'true' measure of the technology bias toward tradables denoted by $Z$ is given by $Z(t)=\frac{\left(Z^{H}(t)\right)^{a}}{\left(Z^{H}(t)\right)^{b}}$ with $a=\frac{1}{\left[\left(1-\alpha_{J}\right)+\alpha_{J} \frac{\theta^{H}}{\theta^{N}}\right]}$ and $b=a \frac{\theta^{H}}{\theta^{N}}($ see $(242))$. We present below the parameters related to endogenous responses of sectoral TFPs to an exogenous shock to a productivity differential which are summarized in panel E of Table 5:

[^45]- In the quantitative analysis, we consider permanent changes in sectoral TFP, $\tilde{Z}^{j}$, so that the labor share-adjusted TFP differential is $1 \%$ in the long run:

$$
\begin{equation*}
\hat{Z}=a \hat{\tilde{Z}}^{H}-b \hat{\tilde{Z}}^{N}=1 \% \tag{385}
\end{equation*}
$$

- We estimate a simple VAR model $\left[\epsilon^{Z}, \hat{Z}, \hat{Z}^{H}, Z^{N}\right]$ where $\epsilon^{Z}$ is the shock to a productivity differential which is identified by considering the baseline VAR model which includes aggregate variables. When we generate IRFs for traded and non-traded TFP, we find a slight discrepancy in the estimated technology shock biased toward the traded sector because $\hat{Z}(t)$ slightly differs from the weighted average $a \hat{\tilde{Z}}^{H}(t)-b \hat{\tilde{Z}}^{N}(t)$. We thus take the following route. We compute $\hat{Z}^{N}(t)$ at various horizons by using the following formula $\hat{Z}^{N}(t)=\frac{a Z^{H}(t)-\hat{Z}(t)}{b}$ (see eq. (385).
- To reproduce the initial response of sectoral TFP we estimate empirically, we choose $\bar{z}^{j}$ by setting $t=0$ into (384):

$$
\begin{equation*}
\bar{z}^{j}=-\left(\hat{\tilde{Z}}^{j}-\hat{Z}^{j}(0)\right), \tag{386}
\end{equation*}
$$

where $\hat{\tilde{Z}}^{j}$ corresponds to steady-state change in percentage of TFP in sector $j=H, N$ and $\hat{Z}^{j}(0)$ is the initial response of TFP in sector $j$. Eq. (386) gives us $\bar{z}^{H}=-0.0936$ and $\bar{z}^{N}=0.0002$.

- To reproduce the shape IRFs of sectoral TFPs, we first solve (384) for $\xi^{j}$ :

$$
\begin{equation*}
\xi^{j}=-\frac{1}{t} \ln \left(\frac{\hat{Z}^{j}(t)-\hat{\tilde{Z}}^{j}}{\bar{z}^{j}}\right) . \tag{387}
\end{equation*}
$$

We choose time $t$ so that $\xi^{j}$ gives us the best fit of the response of $\hat{Z}^{j}(t)$ estimated empirically. For both sectors, we take $t=3$ which gives us $\xi^{H}=0.5709$ and to $\xi^{N}=1.1668$.

- Given values for $\bar{z}^{j}, \xi^{j}$ and $\hat{\tilde{Z}}^{j}$, we can compute the transitional path for $\hat{Z}^{j}(t)$ by using (384) and thus the adjustment of the productivity of tradables relative to non-tradables by using (385), assuming that the weights $a$ and $b$ are constant over time.


## L. 3 Calibration Procedure with CES Production Functions

The production functions are assumed to take a CES form which we repeat for convenience:

$$
\begin{equation*}
Y^{j}(t)=\left[\gamma^{j}\left(A^{j}(t) L^{j}(t)\right)^{\frac{\sigma^{j}-1}{\sigma^{j}}}+\left(1-\gamma^{j}\right)\left(B^{j}(t) K^{j}(t)\right)^{\frac{\sigma^{j}-1}{\sigma^{j}}}\right]^{\frac{\sigma^{j}}{\sigma^{j}-1}} \tag{388}
\end{equation*}
$$

where $A^{j}$ and $B^{j}$ are labor- and capital-augmenting productivity, and $\sigma^{j}$ the elasticity of substitution between capital and labor in production.

Compared with a model imposing Cobb-Douglas production functions, the model assuming CES form for production technology has 8 additional parameters, i.e., $\sigma^{H}, \sigma^{N}, \gamma^{H}, \gamma^{N}, A^{H}, B^{H}$, $A^{N}, B^{N}$. Given that we assume Hicks-neutral technological change at the initial steady-state, i.e., $A^{j}=B^{j}=Z^{j}$, and sectoral TFP are set to one, it leaves us with 4 additional (compared with subsection L.1) parameters only. Among these four parameters, two can be taken from the data. Following Antràs [2004], we run the regression of (logged) valued added per hours worked on (logged) real wage over 1970-2013 in panel data while letting the coefficient in front of $W^{j} / P^{j}$ vary across countries, see section F.4. We take unweighed average values shown in the last line of columns 17-18 of Table 4 and set $\sigma^{H}=0.69$ and $\sigma^{N}=0.72$. We normalize CES production functions because, as underlined by León-Ledesma et al. [2010], the normalization allows CES production functions featuring different elasticity of substitution to share the a common baseline point.

We assume Hicks-neutral technological change at the initial steady-state, i.e., $A_{0}^{j}=B_{0}^{j}=Z_{0}^{j}$, so that eq. (388) now reads as follows:

$$
\begin{equation*}
y_{0}^{j}=Z_{0}^{j}\left[\gamma^{j}+\left(1-\gamma^{j}\right)\left(k_{0}^{j}\right)^{\frac{\sigma^{j}-1}{\sigma^{j}}}\right]^{\frac{\sigma^{j}}{\sigma^{j}-1}} \tag{389}
\end{equation*}
$$

and the labor income share is given by

$$
\begin{equation*}
s_{L, 0}^{j}=\gamma^{j}\left(\frac{Z_{0}^{j}}{y_{0}^{j}}\right)^{\frac{\sigma^{j}-1}{\sigma^{j}}} \tag{390}
\end{equation*}
$$

The steady-state of a semi-small open economy with CES production functions is described by the following set of equations:

$$
\begin{gather*}
y^{H}=Z^{H}\left[\gamma^{H}+\left(1-\gamma^{H}\right)\left(k^{H}\right)^{\frac{\sigma^{H}-1}{\sigma^{H}}}\right]^{\frac{\sigma^{H}}{\sigma^{H}-1}},  \tag{391a}\\
y^{N}=Z^{N}\left[\gamma^{N}+\left(1-\gamma^{N}\right)\left(k^{N}\right)^{\frac{\sigma^{N}-1}{\sigma^{N}}}\right]^{\frac{\sigma^{N}}{\sigma^{N}-1}},  \tag{391b}\\
s_{L}^{H}=\gamma^{H}\left(\frac{Z^{H}}{y^{H}}\right)^{\frac{\sigma^{H}-1}{\sigma^{H}}},  \tag{391c}\\
s_{L}^{N}=\gamma^{N}\left(\frac{Z^{N}}{y^{N}}\right)^{\frac{\sigma^{N}-1}{\sigma^{N}}},  \tag{391d}\\
1-s_{L}^{H}=\left(1-\gamma^{H}\right)\left(\frac{Z^{H} k^{H}}{y^{H}}\right)^{\frac{\sigma^{H}-1}{\sigma^{H}}},  \tag{391e}\\
1-s_{L}^{N}=\left(1-\gamma^{N}\right)\left(\frac{Z^{N} k^{N}}{y^{N}}\right)^{\frac{\sigma^{N}-1}{\sigma^{N}}},  \tag{391f}\\
\frac{\nu^{Y, H}}{1-\nu^{Y, H}} \frac{\left(1+v_{N}-v_{J^{H}}+v_{G^{H}}\right)}{\left(1-v_{J^{N}}-v_{G^{N}}\right)}=\frac{\varphi^{1-\varphi}}{1-\varphi^{\prime}}\left(\frac{P^{T}}{P^{N}}\right)^{\phi},  \tag{391g}\\
\frac{\nu^{Y, H}}{1-\nu^{Y, H}}=\frac{v^{\prime}}{1-\vartheta}\left(\frac{\gamma^{H}}{\gamma^{N}}\right)^{\epsilon}\left(\frac{P^{H}}{P^{N}}\right)^{1+\epsilon} \frac{\left(Z^{H}\right)\left(\frac{\sigma^{H}-1}{\sigma^{H}}\right) \epsilon}{\left(Z^{N}\right)\left(\frac{\sigma^{N}-1}{\sigma^{N}}\right) \epsilon} \frac{\left(y^{H}\right)^{\frac{\epsilon}{\sigma^{H}}+1}}{\left(y^{N}\right)^{\frac{\epsilon}{\sigma^{N}+1}}},  \tag{391h}\\
\nu^{Y, H}=\omega_{C} \alpha_{C} \alpha^{H}+\omega_{J} \alpha_{J} \alpha_{J}^{H}+\omega_{G^{H}}\left(1-\omega_{G^{N}}\right) \omega_{G}+\omega_{X},  \tag{391i}\\
\left(1-\theta^{H}\right) \nu^{Y, H}+\left(1-\theta^{N}\right)\left(1-\nu^{Y, H}\right)=P_{J}\left(r^{\star}+\delta_{K}\right) \frac{K}{Y},  \tag{391j}\\
v_{N}=v_{N_{0}}+\frac{r^{\star} \Psi_{1}}{\nu^{Y, H}\left(\frac{K}{Y}-v_{K_{0}}\right),} \tag{391k}
\end{gather*}
$$

where $v_{N_{0}}=\frac{r^{\star} N_{0}}{Y}, v_{K_{0}}=\frac{K_{0}}{Y}$. The system (391) consisting of eleven equations determine $y^{H}, y^{N}$, $s_{L}^{H}, s_{L}^{N}, k^{H}, k^{N}, \nu^{Y, H}, P^{N}, P^{H}, K / Y$, and $v_{N}$. The five equations $(391 \mathrm{~g})-(391 \mathrm{k})$ stand for the goods market equilibrium for tradables relative to non-tradables, the labor market equilibrium, the goods market equilibrium for the home-produced traded goods market equilibrium, the resource constraint for capital, the intertemporal solvency condition, respectively.

While these last five equations have been derived in subsection L.1, one equation deserves attention as the assumption of CES production functions modifies the derivation of the labor market equilibrium. Dividing (363g) by (363f) leads to the supply of hours worked in the traded sector relative to the non-traded sector, i.e., $\frac{L^{H}}{L^{N}}=\frac{\vartheta}{1-\vartheta}\left(\frac{W^{H}}{W^{N}}\right)^{\epsilon}$. Dividing (277b) by (277c) leads to the relative wage of tradables, i.e.,

$$
\frac{W^{H}}{W^{N}}=\frac{\gamma^{H}}{\gamma^{N}} \frac{\left(Z^{H}\right)^{\frac{\sigma^{H}-1}{\sigma^{H}}}}{\left(Z^{N}\right)^{\frac{\sigma^{N}-1}{\sigma^{N}}}} \frac{\left(y^{H}\right)^{\frac{1}{\sigma^{H}}}}{\left(y^{N}\right)^{\frac{1}{\sigma^{N}}}}
$$

Inserting the latter expression into the labor supply equation and using the fact that $L^{H}=\frac{Y^{H}}{y^{H}}$ and $L^{N}=\frac{Y^{N}}{y^{N}}$, one obtains:

$$
\frac{Y^{H}}{Y^{N}}=\frac{\vartheta}{1-\vartheta}\left(\frac{\gamma^{H}}{\gamma^{N}}\right)^{\epsilon}\left(\frac{P^{H}}{P^{N}}\right)^{\epsilon} \frac{\left(Z^{H}\right)^{\left(\frac{\sigma^{H}-1}{\sigma^{H}}\right) \epsilon}}{\left(Z^{N}\right)^{\left(\frac{\sigma^{N}-1}{\sigma^{N}}\right) \epsilon}} \frac{\left(y^{H}\right)^{\frac{\epsilon}{\sigma^{H}+1}}}{\left(y^{N}\right)^{\frac{\epsilon}{\sigma^{N}+1}}}
$$

Left-multiplying the above expression by $\frac{P^{H} Y}{P^{N} Y}$, and collecting terms leads to the labor market equilibrium (391h).

We choose the initial steady-state in a model with Cobb-Douglas production functions described in section L. 1 as the normalization point; $\bar{k}^{j}$ and $\bar{y}^{j}$ are the steady-state quantities from the CobbDouglas case. The objective of the normalization is to choose $\gamma^{j}$ in eq. (390), so as to maintain the steady-state sectoral labor income share at $\theta^{j}$, and to choose $Z^{j}$ in eq. (389) so as to maintain the sectoral steady-state output level equal to the Cobb-Douglas value $\bar{y}^{j}$. Let us remind that $\theta^{j}$ is the labor income share in the baseline model with Cobb-Douglas production functions; equating $y_{0}^{j}$ and
$k_{0}^{j}$ to $\bar{y}^{j}$ and $\bar{k}^{j}$, respectively, eqs. (389) and (390) can be solved for parameter $\gamma^{j}$

$$
\begin{equation*}
\gamma^{j}=\theta^{j}\left[\theta^{j}+\left(1-\theta^{j}\right)\left(\bar{k}^{j}\right)^{\frac{1-\sigma^{j}}{\sigma^{j}}}\right]^{-1} \tag{392}
\end{equation*}
$$

and parameter $Z^{j}$

$$
\begin{equation*}
Z^{j}=\bar{y}^{j}\left[\gamma^{j}+\left(1-\gamma^{j}\right)\left(\bar{k}^{j}\right)^{\frac{\sigma^{j}-1}{\sigma^{j}}}\right]^{\frac{\sigma^{j}}{1-\sigma^{j}}} \tag{393}
\end{equation*}
$$

Making use of (372) and (373), we set $\varphi^{H}, \iota^{H}$, and $\varphi$ to target $\bar{\alpha}^{H}, \bar{\alpha}_{J}^{H}$, and $\bar{\alpha}_{C}$ :

$$
\begin{align*}
\varphi^{H} & =\frac{\bar{\alpha}^{H}}{\bar{\alpha}^{H}+\left(1-\bar{\alpha}^{H}\right)\left(\bar{P}^{H}\right)^{1-\rho}}  \tag{394a}\\
\iota^{H} & =\frac{\bar{\alpha}_{J}^{H}}{\bar{\alpha}_{J}^{H}+\left(1-\bar{\alpha}_{J}^{H}\right)\left(\bar{P}^{H}\right)^{1-\rho_{J}}}  \tag{394b}\\
\varphi & =\frac{\bar{\alpha}_{C}}{\bar{\alpha}_{C}+\left(1-\bar{\alpha}_{C}\right)\left(\frac{\bar{P}^{N}}{P^{H}}\right)^{\phi-1}} \tag{394c}
\end{align*}
$$

We choose $\vartheta$ so as to target the tradable content of labor compensation $\bar{\alpha}_{L}$ :

$$
\begin{equation*}
\vartheta=\frac{\bar{\alpha}_{L}}{\bar{\alpha}_{L}+\left(1-\bar{\alpha}_{L}\right)\left(\frac{\bar{W}^{H}}{\bar{W}^{N}}\right)^{1+\epsilon}} \tag{395}
\end{equation*}
$$

Using the fact that $\omega_{X}=\frac{P^{H} X^{H}}{Y}$ with $X^{H}=\varphi_{X}\left(P^{H}\right)^{-\phi_{X}}$, we set $\varphi_{X}$ to target an export-to-GDP ratio $\bar{\omega}_{X}$ :

$$
\begin{equation*}
\varphi_{X}=\bar{Y} \bar{\omega}_{X}\left(\bar{P}^{H}\right)^{\phi_{X}-1} \tag{396}
\end{equation*}
$$

We choose $\delta_{K}$ so as to target an investment-to-GDP ratio $\bar{\omega}_{J}$ :

$$
\begin{equation*}
\delta_{K}=\frac{\bar{\omega}_{J} \bar{Y}}{\bar{P}_{J} \bar{K}} . \tag{397}
\end{equation*}
$$

We set $N_{0}$ so as to target $\bar{\omega}_{C}$ or alternatively balanced net exports (which imply $v_{N}=0$ ) by using the accounting identity between GDP and the sum of demand components:

$$
\begin{equation*}
N_{0}=\bar{Y}\left(\frac{\bar{\omega}_{C}+\bar{\omega}_{J}+\omega_{G}-1}{r^{\star}}\right) . \tag{398}
\end{equation*}
$$

Finally, we choose $K_{0}$ to target $\bar{K}$ by using the intertemporal solvency condition:

$$
\begin{equation*}
K_{0}=\bar{K}+\left(\frac{N_{0}-\bar{N}}{\bar{\Psi}_{1}}\right) \tag{399}
\end{equation*}
$$

## L. 4 Calibration Procedure with CES Production Functions and Factor Biased Technological Change

In this subsection, we provide more details about how we determine the direction and the magnitude of factor biased technological change. We begin with the approach adopted in the main text and then we contrast the results with those obtained by following an alternative method.

## Estimating Empirically Factor Biased Technological Change

To calibrate the dynamic responses of $A^{j}$ and $B^{j}$, we proceed as follows. To start with, we repeat the ratio of factor income share for convenience:

$$
\begin{equation*}
S^{j}=\frac{\gamma^{j}}{1-\gamma^{j}}\left(\frac{B^{j} K^{j}}{A^{j} L^{j}}\right)^{\frac{1-\sigma^{j}}{\sigma^{j}}} \tag{400}
\end{equation*}
$$

Since we normalize CES production function (388) so that the relative weight of labor and capital is consistent with the labor and capital income share in the data, solving for $\gamma^{j}$ leads to:

$$
\begin{gather*}
\gamma^{j}=\left(\frac{\tilde{A}_{0}^{j}}{\tilde{y}_{0}^{j}}\right)^{\frac{1-\sigma^{j}}{\sigma^{j}}} \tilde{s}_{L, 0}^{j},  \tag{401a}\\
1-\gamma^{j}=\left(\frac{\tilde{B}_{0}^{j} \tilde{k}_{0}^{j}}{\tilde{y}_{0}^{j}}\right)^{\frac{1-\sigma^{j}}{\sigma j}}\left(1-\tilde{s}_{L, 0}^{j}\right) . \tag{401b}
\end{gather*}
$$

$$
\begin{equation*}
\tilde{S}_{0}^{j}=\frac{\gamma^{j}}{1-\gamma^{j}}\left(\frac{\tilde{B}_{0}^{j} \tilde{k}_{0}^{j}}{\tilde{A}_{0}^{j}}\right)^{\frac{1-\sigma^{j}}{\sigma^{j}}} \tag{402}
\end{equation*}
$$

Dividing (400) by (402) and solving for relative capital efficiency leads to:

$$
\begin{equation*}
\left(\frac{B^{j}(t) / \tilde{B}_{0}^{j}}{A^{j}(t) / \tilde{A}_{0}^{j}}\right)=\left(\frac{k^{j}(t)}{\tilde{k}_{0}^{j}}\right)^{-1}\left(\frac{S^{j}(t)}{\tilde{S}_{0}^{j}}\right)^{\frac{\sigma^{j}}{1-\sigma^{j}}} . \tag{403}
\end{equation*}
$$

Since initially we assume Hicks-neutral technological change at the initial steady-state, we have $\tilde{A}_{0}^{j}=\tilde{B}_{0}^{j}=\tilde{Z}_{0}^{j}$. The technology frontier is described by

$$
\begin{equation*}
\frac{Z^{j}(t)}{\tilde{Z}_{0}^{j}}=\left(\frac{A^{j}(t)}{\tilde{A}_{0}^{j}}\right)^{s_{L}^{j}(t)}\left(\frac{B^{j}(t)}{\tilde{B}_{0}^{j}}\right)^{1-s_{L}^{j}(t)} \tag{404}
\end{equation*}
$$

Log-linearizing, the system (403)-(404) can be solved for labor and capital productivity:

$$
\begin{gather*}
\hat{A}^{j}(t)=\hat{Z}^{j}(t)-\left(1-\tilde{s}_{L, 0}^{j}\right)\left[\left(\frac{\sigma^{j}}{1-\sigma^{j}}\right) \hat{S}^{j}(t)-\hat{k}^{j}(t)\right],  \tag{405a}\\
\hat{B}^{j}(t)=\hat{Z}^{j}(t)+\tilde{s}_{L, 0}^{j}\left[\left(\frac{\sigma^{j}}{1-\sigma^{j}}\right) \hat{S}^{j}(t)-\hat{k}^{j}(t)\right] \tag{405b}
\end{gather*}
$$

where $\hat{S}^{j}(t)=\frac{\hat{s}^{j}(t)}{1-\tilde{s}_{L, o}^{j}}$. To recover the dynamics of $A^{j}$ and $B^{j}$, we first estimate two VAR models; the first VAR model includes the productivity differential, $Z$, the labor income share in sector $j$, and the capital-labor ratio in sector $j$, i.e., $\left[\hat{Z}, \hat{s}_{L}^{j}, \hat{k}^{j}\right]$; the second VAR model includes the technology shock (identified from the estimation of the baseline VAR model including aggregate variables), sectoral TFPs, and the productivity differential. Next, we generate IRFs and plug estimated responses of $Z^{j}, k^{j}, s_{L}^{j}$ into (405a)-(405b) which allows us to make inference of the underlying process of $A^{j}$ and $B^{j}$ in the data. As discussed below, four situations may emerge.

Differentiating (403) leads to:

$$
\begin{equation*}
\hat{B}^{j}(t)-\hat{A}^{j}(t)=\left(\frac{\sigma^{j}}{1-\sigma^{j}}\right) \hat{S}^{j}(t)-\hat{k}^{j}(t) \tag{406}
\end{equation*}
$$

While eq. (406) gives us the excess of capital productivity growth over labor productivity growth, the system of equations which comprises (405a)-(405b) allows us to determine the changes in labor capital efficiency:

$$
\begin{gather*}
\hat{A}^{j}(t)=\hat{Z}^{j}(t)-\left(1-\tilde{s}_{L, 0}^{j}\right)\left(\hat{B}^{j}(t)-\hat{A}^{j}(t)\right)  \tag{407a}\\
\hat{B}^{j}(t)=\hat{Z}^{j}(t)+\tilde{s}_{L, 0}^{j}\left(\hat{B}^{j}(t)-\hat{A}^{j}(t)\right) \tag{407b}
\end{gather*}
$$

Eqs. (407a)-(407b) show that four situations can emerge:

- When the productivity differential between capital and labor $\left(\hat{B}^{j}-\hat{A}^{j}\right)>0$ exceeds $\frac{\hat{Z}^{j}}{1-s_{L}^{j}}>$ 0 , we have $\hat{A}^{j}<0\left(\right.$ and $\left.\hat{B}^{j}>0\right)$.
- When the decline in relative capital efficiency $-\left(\hat{B}^{j}-\hat{A}^{j}\right)>0$ exceeds $\frac{\hat{Z}^{j}}{s_{L}^{j}}>0$, we have $\hat{B}^{j}<0\left(\right.$ and $\left.\hat{A}^{j}>0\right)$.
- When the productivity differential between capital and labor falls between $-\frac{\hat{Z}^{j}}{s_{L}^{j}}$ and $\frac{\hat{Z}^{j}}{1-s_{L}^{j}}$, we have $\hat{B}^{j}>0$ and $\hat{A}^{j}>0$ :
- if $\left(\frac{\sigma^{j}}{1-\sigma^{j}}\right) \hat{S}^{j}(t)>\hat{k}^{j}(t)$, relative capital efficiency increases;
- if $\left(\frac{\sigma^{j}}{1-\sigma^{j}}\right) \hat{S}^{j}(t)<\hat{k}^{j}(t)$, relative capital efficiency declines.

We further specify a dynamic adjustment for $\hat{A}^{j}(t)$ and $\hat{B}^{j}(t)$ similar to that described by eq. (384), i.e.,

$$
\begin{equation*}
\hat{A}^{j}(t)=\hat{A}^{j}+\bar{a}^{j} e^{-\xi^{j} t}, \quad \hat{B}^{j}(t)=\hat{B}^{j}+\bar{b}^{j} e^{-\xi^{j} t} \tag{408}
\end{equation*}
$$

Traded Sector

(a) FBTS on Impact

(c) Factor Biased Technological Shock in
the Long-Run

(b) Factor Biased Technological Shock on Impact

(d) Factor Biased Technological Shock in the Long-Run

Figure 26: Empirically vs. Numerically Estimated Factor Biased Technology Shock in the Home-Produced Traded Goods and Non-Traded Goods Sector. Notes: Figure 27 plots impact (i.e., at time $t=0$ ) and long-run (i.e., at time $t=10$ ) responses of $\mathrm{FBTC}_{i t}^{j}$ estimated numerically (by using (46a)-(46b)) on the horizontal axis against those estimated empirically (by using (7) to construct time series for FBTC and then estimating a VAR $\left.\left[\hat{Z}_{i t}, \operatorname{FBTC}_{i t}^{j}\right]\right)$ on the vertical exis.
where we assume that the speed of adjustment $\xi^{j}$ corresponds to the speed of adjustment of sectoral TFP, $Z^{j}$ (i.e., $\xi^{H}=0.5709$ and $\xi^{N}=1.1668$, see subsection L.2). We choose $\bar{a}^{j}, \bar{b}^{j}$ by setting $t=0$ into (408), i.e., $\bar{a}^{j}=-\left(\hat{\tilde{A}}^{j}-\hat{A}^{j}(0)\right)$, and $\bar{b}^{j}=-\left(\hat{\tilde{B}}^{j}-\hat{B}^{j}(0)\right)$ which gives us $\bar{a}^{H}=-0.029840$, $\bar{b}^{H}=-0.202769, \bar{a}^{N}=0.234035, \bar{b}^{N}=-0.500629$.

## Contrasting Numerical vs. Empirical Estimates of FBTC

One alternative approach to that described above amounts to constructing time series for FBTC by using eq. (403), i.e.,

$$
\begin{align*}
\operatorname{FBTC}^{j}(t) & \equiv\left(\frac{B^{j}(t) / \tilde{B}_{0}^{j}}{A^{j}(t) / \tilde{A}_{0}^{j}}\right)^{\frac{1-\sigma^{j}}{\sigma J}}, \\
& =\left(\frac{k^{j}(t)}{\tilde{k}_{0}^{j}}\right)^{-\frac{1-\sigma^{j}}{\sigma j}}\left(\frac{S^{j}(t)}{\tilde{S}_{0}^{j}}\right) . \tag{409}
\end{align*}
$$

Using time series for sectoral capital ratios, $k^{j}$, labor income share, $s_{L}^{j}$, along with our estimates of $\sigma^{j}$, one can make inference on FBTC which we have denoted by $\mathrm{FBTC}_{i t}^{j}$. Then, we estimate a simple VAR model [ $\left.\hat{Z}_{i t}, \mathrm{FBTC}_{i t}^{j}\right]$ by adopting the identification approach by Gali [1999]. Fig. 26 plots impact and long-run responses of FBTC estimated empirically on the vertical axis against FBTC computed numerically by using (46a)-(46b). Overall, differences between the two approaches are quantitatively small. While both methods should identical, computation of FBTC by using (46a)-(46b) slightly improves the fit to the data.

## M More Numerical Results

In this section, we provide additional numerical results which are not included in the main text for reasons of space. In subsection M.1, we conduct a sensitivity analysis with respect to the degree of labor mobility across sectors which displays a wide cross-country dispersion across countries, exogenous vs. endogenous TOT, FBTC vs. HNTC, capital installation costs vs. no installation costs, Cobb-Douglas economy vs. CES economy. In section M.2, we plot the technology frontier, we contrast empirical with theoretical responses of the ratio of factor income shares, we contrast plot the responses of the ratio of factor income shares against the responses of FBTC and contrast the cross-country relationship from empirical results with the cross-country relationship from numerical results. Finally, we plot the long-run measure of the deviation from HNTC estimated empirically against the measure estimated numerically. We consider VAR estimates with two lags and one lag.

## M. 1 Effects of Asymmetric Technology Shocks across Sectors: Sensitivity Analysis

Table 24 reports impact effects for additional scenarios which are not shown in the main text for reason of space. Column 1 reports the data, i.e., impact effects from VAR models specified in the main text whilst column 2 shows results for the baseline model with capital adjustment costs, IML, endogenous TOT, and FBTC.

Columns 3 and 4 show results when the production function is of the CES or Cobb-Douglas ('CD') type while technological change is Hicks neutral. When contrasting the results with those shown in column 2 from our baseline model with CES production functions and FBTC, as can be seen in panel C, the model with FBTC performs better in reproducing the change in the labor share of tradables. As a result, the baseline model also reproduces better the responses of sectoral labor and sectoral value added. The performance of the baseline model is also much higher for the responses of the sectoral LIS.

Column 7 shows results when the elasticity of aggregate labor supply, $\sigma_{L}$, and the IES for consumption, $\sigma_{C}$, are set to one while we assume Cobb-Douglas production functions. In this case, the negative impact of the wealth effect on labor supply is higher which implies that labor supply becomes unresponsive to the technology shock. Consequently, we need to impose a higher value for $\sigma_{C}$ to generate the rise in total hours worked observed in the data.

Turning to the implications of labor mobility costs, our estimates of the elasticity of labor supply across sectors display a wide dispersion across countries and we therefore conduct a sensitivity analysis with respect to this parameter. We set $\epsilon$ to 0.22 (for Ireland) and 3.2 (for the United States) and in both cases we assume Cobb-Douglas production functions and thus abstract from FBTC. While the introduction of a difficulty in reallocating labor across sectors improves significantly the performance of the model, columns 5 and 6 allow us to gauge the extent to which different values for $\epsilon$ affect differently 'High mobility' and 'Low mobility' economies. As shown analytically in Appendix K.1, a rise in the degree of labor mobility exerts two opposite effects on sectoral labor shares: while workers are more willing to shift across sectors, the relative price of non-tradables appreciates less which mitigates the incentive for labor reallocation. We find numerically that raising the elasticity of labor supply across sectors, $\epsilon$, from 0.22 to 3.2 amplifies the decline in the labor share of tradables and thus the increase the share of non-tradables in labor. Quantitatively, the increase in $\nu^{L, N}$ in 'High mobility' economies is more than three-fold that obtained in the group of low mobility.

Columns 8 and 9 display results for two restricted versions of the model. In both cases we assume Cobb-Douglas production functions and thus the results are comparable with those shown in column 4. While in column 8, we allow for endogenous TOT but abstract from capital adjustment costs ( $\kappa=0$ ), in column 9 , we allow for capital adjustment costs but impose exogenous TOT by considering a small open economy. As it stands out, capital adjustment costs do not play a key role in allowing the model to match the evidence, as can be seen by contrasting the results shown in column 9 with those displayed in column 4 where we consider Cobb-Douglas production functions. On the contrary endogenous TOT play a pivotal role (column 8) since imposing exogenous TOT leads the model to considerably overstate the shift of labor across sectors.

Columns 10-13 show results from a model imposing perfect mobility of labor across sectors. Column 10-11 show results when allowing for CES production functions while column 11 further assumes FBTC. The last two columns show results when we impose Cobb-Douglas production functions. We further restrict the model in column 13 by imposing exogenous TOT. Overall, across all scenarios, the model imposing perfect mobility of labor substantially overstates the reallocation of labor toward the non-traded sector, even when we allow for endogenous TOT and FBTC (i.e., column 10).
Table 24: Impact Responses to a Technology Shock Biased toward Tradables (in \%)






## M. 2 Cross-Country Analysis

In this subsection, we provide additional numerical results which are not included in the main text for reasons of space. Before discussing the results, let us mention that there exists a linear mapping between the response of the ratio of labor to capital income share, denoted by $\hat{S}^{j}(t)$, and the response of the LIS, $\hat{s}_{L}^{j}(t)$, since $\hat{S}^{j}(t)=\frac{\hat{s}_{L}^{j}(t)}{1-s_{L}^{j}}$ where $1-s_{L}^{j}$ is the capital income share in sector $j$ averaged over 1970-2013 for each country in our sample (except Japan: 1974-2013).

## M.2. 1 Technology Frontier

Fig. 27 plots impact (fist column) and long-run changes (second column) in labor efficiency on the horizontal axis against capital efficiency on the vertical axis in the traded (first row) and non-traded sector (second row). Following a technology shock biased toward tradables of $1 \%$, TFP in the traded sector increases so that the technology frontier for tradables moves away from the origin and within each country. Traded firms must choose new levels for labor and capital efficiency, i.e., $\left(A^{j}, B^{j}\right)$, along the technology frontier. Each square in Fig. 27(b) represents this choice for each country and thus allows us to plot a trend line which describes the world technology frontier for tradables in the long-run. Along this world technology frontier, we identify four situations. Firms can choose to increase capital relative to labor efficiency by such an amount that the latter declines, like Japan, the Netherlands, Norway, Spain, the U.K., the United States (see the north-western part of Fig. 27(b)). Conversely, firms can choose to reduce $B^{j} / A^{j}$ to an extent that results in a decline in capital efficiency, such as in Belgium, Denmark, Finland, France, Germany (see the south-eastern part of Fig. 27(b)). Between these two extremes, some OECD countries choose to increase both capital and labor efficiency, the rise in the former being larger or smaller than the latter. Australia, Canada, Italy, and Sweden raises $B^{j} / A^{j}$ whilst Austria and Ireland lower relative capital efficiency. While the same picture emerges for the non-traded sector as can be seen in Fig. 27(d), non-traded TFP declines and thus the technology frontier moves downward. The reason is that we normalize the productivity differential between tradables and nontradables to $1 \%$ and data suggest that such a productivity differential can be achieved only by lowering $Z^{N}$. The world technology frontier for non-tradables indicates that in twelve out the seventeen countries, non-traded firms decide to increase capital relative to labor efficiency. This rise in $B^{N} / A^{N}$ is pronounced in Belgium and Germany (because $\sigma^{j}$ is close to 1 ), and to a lesser extent in Canada, France and the United States as labor efficiency declines whilst capital efficiency increases. Since recovering time series for capitaland labor-augmenting technological change involves the elasticity of substitution between capital and labor, i.e., $\left(\hat{B}^{j}(t)-\hat{A}^{j}(t)\right)=\frac{\sigma^{j}}{1-\sigma^{j}} \hat{S}^{j}(t)-\hat{k}^{j}(t)$, when $\sigma^{j}$ takes values close to one, such as for Belgium and Germany, the ratio $\frac{\sigma^{j}}{1-\sigma^{j}}$ takes large values which results in very high values for $\hat{B}^{j}(t)-\hat{A}^{j}(t)$ for these two economies for $j=N$. Conversely, only a few countries such as Denmark, Italy, Norway, Sweden decide to lower capital efficiency while increasing labor efficiency. Other countries such as Austria lower $A^{N}$ and $B^{N}$ while $B^{N} / A^{N}$ increases. By contrast, the U.K lowers labor and capital efficiency and reduces $B^{N} / A^{N}$.

## M.2.2 Responses of LIS across Countries: Model vs. Data

In the main text, we infer changes in labor and capital efficiency following a productivity differential between tradables and non-tradables of $1 \%$ in the long-run, i.e., $\hat{A}^{j}(t)$ and $\hat{B}^{j}(t)$, by using eqs. (46a)-(46b). More precisely, we determine the shifts in $A^{j}(t)$ and $B^{j}(t)$ along the optimal technology frontier (33) which are consistent with the equality between the ratio of labor to capital income share for country $i$ at time $t$, i.e., $S_{i t}^{j}=\frac{s_{L, i t}^{j}}{1-s_{L, i t}^{j}}$, and the ratio of the elasticity of output w.r.t. labor to that w.r.t. capital, i.e., $\frac{\hat{Y}_{i t}^{j}, \hat{L}_{i t}^{j}}{\hat{Y}_{i t}^{j} / \hat{K}_{i t}^{j}}$. Because we plug responses of TFP, LIS, and the capital-labor ratio in sector $j$ estimated empirically into (46a)-(46b), at first sight, our model should be able to reproduce perfectly well the


Figure 27: Technology Frontier in the Home-Produced Traded Goods and Non-Traded Goods Sector. Notes: Figure 27 plots impact (i.e., at time $t=0$ ) and long-run (i.e., at time $t=10$ ) changes in labor and capital efficiency in sector $j$ on the horizontal and the vertical axis, respectively. Changes in $\hat{A}^{j}(t)$ and $\hat{B}^{j}(t)$ in percentage from initial steady-state are computed numerically by using eqs. (46a)-(46b).


Figure 28: Numerically vs. Empirically Estimated Responses of Ratio of Labor to Capital Income Share. Notes: Fig. 28 contrasts numerically computed with empirically estimated responses of the ratio of the labor to capital income share, $S^{j}=\frac{s_{L}^{j}}{1-s_{L}^{j}}$, on impact (i.e., at time $t=0$ ) and in the long-run (i.e., at time $t=10$ ). Numerically computed responses of LIS in sector $j$ are those obtained by calibrating and simulating the semi-small open economy model with imperfect mobility of labor and CES production functions given shocks to $B^{j}$ and $A^{j}$ in sector $j$ derived from (46a)-(46b). Empirical responses of LIS are obtained by estimating a VAR model which reads as follows $\left[\hat{Z}_{i t}, \hat{s}_{L, i t}^{j}, \hat{k}_{i t}^{j}\right]$. Sample: annual Data, 17 OECD countries, 1970-2007.
responses of $S^{j}$ estimated empirically. However, we have to bear in mind that changes in $A^{j}$ and $B^{j}$ also impinge on $P^{H}$ and $P^{N}$ which in turn have a feedback effect on $k^{j}$ and $s_{L}^{j}$ since we consider a general equilibrium model. Moreover, there is some uncertainty regarding the response of $k^{j}$ whose construction requires a number of assumptions. We thus contrast in Fig. 28 the response of $S^{j}$ estimated empirically with that computed numerically when we simulate the semi-small open economy model laid out in Appendix J while changes in $A^{j}$ and $B^{j}$ are calibrated by plugging estimated values for $\sigma^{j}$ and estimated responses from the VAR model for $Z^{j}, k^{j}, s_{L}^{j}$ into eqs. (46a)-(46b).

In Fig. 28, we plot estimated responses of $S^{j}$ in sector $j$ from the VAR model (vertical axis) against numerically computed responses of $S^{j}$ (horizontal axis). While the first row shows impact responses (i.e., at time $t=0$ ), long-run responses (i.e., at time $t=10$ ) are displayed in the second row. Strikingly, as can be seen in Fig. 28(a)-28(b), the model reproduces very well the responses of the LIS on impact in both sectors, the correlation coefficient being equal to 0.99 for tradables and 0.94 non-tradables as well. In the longrun, the correlation coefficient is lower at 0.93 for tradables and 0.80 for non-tradables. For tradables, our model has some difficulty to replicate the response of $S^{H}$ for Austria at $t=10$. Turning to non-tradables, the model has some difficulties to account for the responses for Japan at $t=10$ and Ireland at any horizon. With the exception of these countries (mostly in the long-run), our model does a good job in replicating the evidence for redistributive effects.


Figure 29: Cross-Country Relationship from Simulated Responses vs. Cross-Country Relationship from VAR Estimates. Notes: Fig. 29 plots impact and long-run responses of LIS responses in the traded and the non-traded sector against FBTC following a $1 \%$ permanent increase in the relative productivity of tradables. Horizontal axes report numerical and empirical estimates of FBTC. Vertical axes report responses of LIS from the baseline model (black squares) and the VAR model (blue circles). The solid blue line shows the cross-country relationship from VAR estimates whilst the solid black line displays the cross-country relationship from numerical estimates. Time series for FBTC are constructed by using (7) and responses of FBTC are estimated empirically by running a simple VAR model $\left[\hat{Z}_{i t}, \mathrm{FBTC}_{i t}\right]$ for one country at at time; responses of LIS are obtained by estimating a VAR model which reads as follows $\left[\hat{Z}_{i t}, \hat{s}_{L, i t}^{j}, \hat{k}_{i t}^{j}\right]$. The system consisting of (46a)-(46b) enable us to compute numerically FBTC as shocks to $A^{j}$ and $B^{j}$ consistent with empirical responses of $Z^{j}, s_{L}^{j}$ and $k^{j}$ and making use of panel data estimates of $\sigma^{j}$ taken from columns of Table 4. Numerically computed responses of LIS in sector $j$ are those obtained by calibrating and simulating the semi-small open economy model with IML and CES production functions given shocks to $B^{j}$ and $A^{j}$ in sector $j$.

## M.2.3 Cross-Country Relationship between LIS and FBTC: Model vs. Data for Long-run Responses

In the main text, we contrast empirical with numerical estimates of impact responses of LIS by considering alternatively a model imposing HNTC or a model allowing for FBTC. In Fig 29, we plot impact and long-run responses of LIS on the vertical axis against responses of FBTC on the horizontal axis. Simulated responses are shown in black squares while empirical responses are displayed in blue circles. We restrict attention to the baseline model which allows for FBTC. The conclusion that emerges is that the model can produce a positive cross-country relationship shown in the black line which is similar to that found in the data shown in the blue line at any horizon and in both sectors.

## M.2.4 Deviation from Hicks-Neutral Technological Change in the Long-Run

In the main text, to save space, we restrict attention to impact effects and plot the measure of the deviation from HNTC (50) estimated empirically (on the horizontal axis) against the measure estimated numerically (on the vertical axis). The left panel of Fig. 30 contrasts empirical with numerical estimates of (50) in the long-run (i.e., $t=10$ ) when we allow for


Figure 30: Deviation from Hicks-Neutral Technology Shocks: Data vs. Model. Notes: In Fig. 9 , we investigate whether in countries where technology shifts are biased toward capital, capital intensive industries contribute more to TFP changes in the long-run. To perform this investigation, we compute a measure of the deviation from Hicks neutral technological change. This index is a weighted average of TFP shocks within each industry which are weighted by the valued added share of this industry along with the difference between this industry's capital income share and average capital income share. Drawing on Acemoglu's [2003] model, capital- (labor-) intensive industries perform capital- (labor-) augmenting technological change. In this setup, if capital income shares were equal across industries, then technological change should be Hicks-neutral. On the horizontal axis, we report estimated values of our measure of deviation from Hicks neutral technological change. When this measure takes positive (negative) values, technology shocks are biased toward capital (labor). Then, we compute the same measure numerically which is shown on the vertical axis.
two lags in the VAR model while the right panel compares both measures by allowing for one lag. Like for impact responses, we find that countries where capital relative to labor efficiency increases are those where capital-intensive industries contribute more to traded TFP shifts. More specifically, economies which lie in the south-west are those where TFP gains are concentrated toward labor-intensive industries and these countries experience an increase in labor relative to capital efficiency. Conversely, countries which lie in the north-east are those where TFP shifts are concentrated toward capital-intensive industries (horizontal axis) and these economies experience an increase in capital relative to labor efficiency (vertical axis). Conversely,

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[^0]:    *We thank participants at the Conference Theories and Methods of Macroeconomics in Paris, 15-16th March 2018, the Asian Meeting of Econometric Society in Seoul, June 21-23rd 2018, the North American Meeting of the Econometric Society in Seattle, June 27-30th 2019, the European Economic Association Meeting in Manchester, August 26-30th 2019, for valuable comments. We are also grateful to Vudayagi Balasubramanyam, David Peel, William Tayler, Kwok Tong Soo for suggestions and comments at an early stage of the paper. Obviously, any remaining shortcomings are our own.
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[^1]:    ${ }^{1}$ There exists a broad literature investigating the implications of neutral technology shocks, see Ramey [2016] (section 5) for a review of the literature. We provide a short survey of the literature in Online Appendix A. While Gali [1999] uses labor productivity, like Chang and Hong [2006], we measure technological change with TFP.

[^2]:    ${ }^{2}$ In a closed economy, Ngai and Pissarides [2007] show that the elasticity of substitution between goods and services must be smaller than one to generate a secular decline in the share of goods in total employment. An economy which is financially open finds it optimal to borrow abroad following a technology shock. Since traded goods can be imported while non-traded goods must be produced domestically, access to foreign borrowing may lead to an increase in the share of consumption expenditure in tradables even if traded and non-traded goods are gross substitutes.
    ${ }^{3}$ Technically, we adapt the methodology by Caselli and Coleman [2006] and make inference about changes in capital relative to labor efficiency consistent with the demand for factors of production and estimated redistributive effects; next, we recover capital- and labor-augmenting technological change so that their underlying process are consistent with the sectoral TFP shock that we estimate empirically by assuming that firms choose labor and capital efficiency along a technology frontier.

[^3]:    ${ }^{4}$ As shown by Collard and Dellas, low trade elasticities require a significant decline in the TOT that discourage output expansion. As output will not expand much while workers are more productive, employment declines.

[^4]:    ${ }^{5}$ In Online Appendix A, we detail the identification strategy and take stock about the potential limits of our approach. While Christiano, Eichenbaum, and Vigfusson [2004] argue for using per capita hours in log-levels rather than in growth rates, we follow Gali [1999] in choosing to let all variables enter the VAR model in growth rates after conducting panel unit root tests which indicate that all time series are integrated of order one. Results are relegated to Online Appendix G.1.

[^5]:    ${ }^{6}$ Table 2 provides a list of countries included in the analysis. Data sources and variable definitions are detailed in the next section.

[^6]:    ${ }^{7}$ Because "Financial Intermediation" and "Real Estate, Renting and Business Services" are made up of sub-sectors which display a high heterogeneity in terms of tradability and "Hotels and Restaurants" has experienced a large increase in tradability over the last fifty years, we perform a sensitivity analysis with respect to the classification for the three aforementioned sectors in Online Appendix G.3. Treating "Financial Intermediation" as non-tradables or classifying "Hotels and Restaurants" or "Real Estate, Renting and Business Services" as tradables does not affect our main results.

[^7]:    ${ }^{8}$ We consider an open economy which produces a traded and a non-traded good while the foreign good is the numeraire and its price is normalized to 1. Real GDP, $Y_{R}(t)$, is equal to the sum of traded and non-traded value added at constant prices, i.e., $Y_{R}(t)=P^{H} Y^{H}(t)+P^{N} Y^{N}(t)$ where prices at the initial steady-state are those at the base year so that real GDP collapses to nominal GDP, $Y$, initially; henceforth, the value added share at current prices also collapses to the value added share at constant prices initially.
    ${ }^{9}$ Basu, Fernald and Kimball [2006] adjust the annual Solow residual with factor utilization. Correcting for unobserved input utilization can avoid understate TFP changes when technology improves because utilization falls. Since we focus on the ratio of sectoral TFP, not adjusting sectoral TFP time series for factor utilization should not pose a problem.

[^8]:    ${ }^{10}$ Because we consider alternative VAR models, one might be concerned by the fact that identified technology shocks display substantial differences across VAR specifications. To address this issue, we ran a number of robustness checks by augmenting each VAR model with the same identified technology shock, ordered first. In the quantitative analysis, we take the VAR model which includes the relative productivity of tradables, $Z_{i t}$, real GDP, $Y_{R, i t}$, total hours worked, $L_{i t}$, the real consumption wage, $W_{C, i t}$, i.e., $x_{i t}^{A}=\left[\hat{Z}_{i t}, \hat{Y}_{R, i t}, \hat{L}_{i t}, \hat{W}_{C, i t}\right]$ as our benchmark model to calibrate the technology shock. Augmenting each VAR model with the technology shock identified for this benchmark specification, we find that the responses lie within the confidence bounds and thus differences are not statistically significant. Results can be found in Online Appendix G.6.
    ${ }^{11}$ To determine the responses of $Z^{j}$, we adopt a two-step method. In the first step, we estimate the baseline VAR model, i.e., $x_{i t}^{A}=\left[\hat{Z}_{i t}, \hat{Y}_{i t}, \hat{L}_{i t}, \hat{W}_{C, i t}\right]$, to identify the technology shock biased toward the traded sector denoted by $\varepsilon_{i t}^{Z}$. In the second step, we estimate a VAR model which includes identified technology shock ordered first, sectoral TFP and the relative TFP of tradables, i.e., $x_{i, t}^{Z}=\left[\varepsilon_{i t}^{Z}, \hat{Z}_{i t}^{H}, \hat{Z}_{i t}^{N}, \hat{Z}_{i t}\right]$. See Online Appendix E. 3 for further details.

[^9]:    ${ }^{12}$ Because estimates of the elasticity between home- and foreign-produced traded goods are not available for all countries of our sample, we cannot plot the responses of sectoral labor shares against this parameter as we do for the degree of labor mobility.

[^10]:    ${ }^{13}$ We compute the LIS like Gollin [2002], i.e., labor compensation is defined as the sum of compensation of employees plus compensation of self-employed. Since there exists alternative ways in constructing labor compensation, we have conducted a robustness check. We find that our results are not affected by the alternative definitions of the LIS, see Online Appendix G.5.

[^11]:    ${ }^{14}$ Assuming perfect mobility of capital, $\hat{K}^{H}-\hat{K}$ collapses to $\left(1-\alpha_{K}\right)\left(\hat{K}^{H}-\hat{K}^{N}\right)$ where $\alpha_{K}=K^{H} / K$. Since $\hat{L}^{H}-\hat{L}=\left(1-\alpha_{L}\right)\left(\hat{L}^{H}-\hat{L}^{N}\right)$, we have $\hat{k}^{H}-\hat{k} \simeq\left(1-\alpha_{L}\right)\left(\hat{k}^{H}-\hat{k}^{N}\right)$ if $\alpha_{L} \simeq \alpha_{K}$. The latter assumption holds when $s_{L}^{H} \simeq s_{L}^{N}$.
    ${ }^{15}$ Due to limited data availability, in the line of Garofalo and Yamarik [2002], we split the aggregate capital stock into tradables and non-tradables in accordance with their value added share. In Online Appendix G.7, we estimate the same VAR model by using databases which provide disaggregated capital stock data (at constant prices) at the 1-digit ISIC-rev. 3 level for nine countries of our sample over the entire period 19702013. The Garofalo and Yamarik's [2002] methodology we adopt in this paper gives very similar results to those obtained when using disaggregated capital stock data.

[^12]:    ${ }^{16}$ Since $\hat{S}_{i t}^{j}$ can be rewritten as $\frac{\hat{s}_{L, i t}^{j}}{1-s_{L, i}^{j}}$ and thus the rate of growth of the ratio of the labor to capital income share is proportional to the percentage change in the LIS, $\hat{s}_{L}^{j}(t)$, we refer interchangeably to the LIS or the ratio of factor income share as long as it does not cause confusion.
    ${ }^{17}$ Online Appendix E. 5 provides more details about our empirical strategy to estimate $\sigma^{j}$. While the bulk of the FMOLS estimated coefficients are positive and statistically significant, the estimated value of $\sigma^{H}$ is negative for Ireland while estimates of $\sigma^{N}$ are not statistically significant for Italy and Sweden. As in Antràs [2004], we alternatively run the regression of the ratio of value added to capital stock at constant prices on the real capital cost $R / P^{j}$ in sector $j$ and replace inconsistent estimates for $\sigma^{j}$ obtained from labor demand with those obtained from the demand of capital.

[^13]:    ${ }^{18}$ In Online Appendix M.2, we plot long-run responses of $S_{t}^{j}$ against long-run responses of $\mathrm{FBTC}_{t}^{j}$ and detect a strong positive cross-country relationship.

[^14]:    ${ }^{19}$ In Online Appendix J.7, we alternatively assume that labor and capital efficiency are aggregated by means of a CES function and find that the same results hold.

[^15]:    ${ }^{20}$ Since we assume HNTC at the initial steady-state, i.e., $Z^{j}=A^{j}=B^{j}$, log-linearization of (33) implies that the terms involving changes in labor and capital income shares cancel out.
    ${ }^{21}$ See Appendix J which details the steps to obtain the dynamic system (38).

[^16]:    ${ }^{22}$ Technically, the assumption $\beta=r^{\star}$ requires the joint determination of the transition and the steady state since the constancy of the marginal utility of wealth implies that the intertemporal solvency condition (41) depends on eigenvalues and eigenvectors' elements, see e.g., Turnovsky [1997].

[^17]:    ${ }^{23}$ Government spending on traded $G^{T}$ and non-traded goods $G^{N}$ are considered for calibration purposes. As pointed out below, we assume that the government purchases from the rest of the world is zero so that $G^{T}=P^{H} G^{H}$. The market clearing condition for the home-produced traded good and the non-traded good at the steady-state are $Y^{H}=C^{H}+I^{H}+G^{H}+X^{H}$ and $Y^{N}=C^{N}+I^{N}+G^{N}$.

[^18]:    ${ }^{24}$ To estimate $\epsilon$, we closely follow Horvath [2000]. We first derive a testable equation by combining first-order conditions for labor supply and labor demand. Details of derivation of the equation we explore empirically can be found in Online Appendix F.3. Then we run the regression of the growth rate of hours worked in sector $j$ arising from labor reallocation across sectors on the relative share of value added paid to workers in sector $j$, see Online Appendix E.4. In contrast to Horvath [2000], we use value added instead of output to be consistent with our setup where we abstract from intermediate inputs in our model.
    ${ }^{25}$ As detailed in Online Appendix E.4, to obtain estimates of $\phi$, we run the regression of the logged share of non-tradables in consumption expenditure on the logged ratio of the price of non-tradables to the consumption price index. We obtain a value for $\phi$ of 0.66 and 0.33 , respectively, when adding or not a country-specific time trend.
    ${ }^{26}$ In line with the bulk of the international RBC literature, we set $\rho=1.5$ instead of choosing a value of 3 as suggested by Bajzik et al. [2019].
    ${ }^{27}$ When we restrict attention to the period 1970-2007, we find that total hours worked are unresponsive to asymmetric technology shocks across sectors and thus a value of 1 for the IES squares well with our evidence over this period. Since we find that total hours worked increase significantly following a shock to a productivity differential, the positive response is caused by the period 2007-2013. During this period, the value for the IES has increased sharply, as suggested by the empirical study by Cundy [2018] who report a value of 2.8 for the IES between 2009 and 2014. A rationale behind the rise in total hours worked following an increase in traded relative to non-traded TFP is that during and/or in the aftermath of the Great Recession, the relative productivity of tradables and total hours worked have been reduced dramatically, thus reinforcing the positive relationship between the two variables.
    ${ }^{28}$ The estimates of the macro Frisch elasticity of labor supply documented by Peterman [2016] vary between 1.5 and 1.75 for the population aged between 20 and 55 , and between 20 and 60 , respectively.

[^19]:    ${ }^{29}$ Eberly, Rebelo, and Vincent [2008] run the regression $I / K=\alpha+\beta \cdot \ln (q)$ and obtain a point estimate for $\beta$ of 0.06 . In our model, the steady-state elasticity of $I / K$ with respect to Tobin's q is $1 / \kappa$. Equating $1 / \kappa$ to 0.06 gives a value for $\kappa$ of 17 .

[^20]:    ${ }^{30}$ The cause of the slight discrepancy in the estimated tradable content of GDP is due to different nomenclatures for valued added by industry and for expenditure in consumption, investment, government expenditure.
    ${ }^{31}$ More details can be found in Online Appendix L.3.

[^21]:    ${ }^{32}$ In Online Appendix L.4, we contrast $\mathrm{FBTC}^{j}$ estimated empirically (in subsection 2.5) with FBTC in sector $j$ recovered from (46a)-(46b), i.e., $\mathrm{FBTC}^{j}=\frac{1-\sigma^{j}}{\sigma^{j}}\left(\hat{B}^{j}(t)-\hat{A}^{j}(t)\right)$. The cross-country correlation between the two series is 0.95 and 0.94 for tradables and non-tradables on impact, and 0.90 and 0.97 in the long-run.

[^22]:    ${ }^{33}$ Note that $a$ and $b$ are close to 1 for the whole sample and thus $1 \%=a \hat{Z}^{H}-b \hat{Z}^{N} \simeq \hat{Z}^{H}-\hat{Z}^{N}$.

[^23]:    ${ }^{34}$ It is worth mentioning that a model imposing HNTC reproduces better the long-run dynamics for the sectoral wage differential shown in Fig. 6(h) since the value of $\epsilon$ is chosen to replicate the wage differential in a model imposing HNTC which corresponds to the reference point used to normalize CES production functions.

[^24]:    ${ }^{35}$ It is worth mentioning that when we impose HNTC, assuming CES or Cobb-Douglas production functions do not make any difference even quantitatively.
    ${ }^{36}$ We mostly focus on sectoral shares and do not contrast the responses of sectoral hours worked since the former are influenced by a few parameter such as $\phi, \epsilon$, FBTC, while the latter variables are also affected by additional parameters such as $\sigma_{L}$ for which we do not have estimates at a country level.

[^25]:    ${ }^{37}$ Like for a representative OECD economy, we estimate the VAR model $x_{t}^{L I S, j}=\left[\hat{Z}_{t}, \hat{s}_{L, t}^{j}, \hat{k}_{t}^{j}\right]$, but for one country at a time and generate the responses of $s_{L}^{j}$ and $k^{j}$; we also estimate the VAR model which includes aggregate variables, i.e., $x_{t}^{A}=\left[\hat{Z}_{t}, \hat{Y}_{R, t}, \hat{L}_{t}, \hat{W}_{C, t}\right]$, to identify the technology shock; next we plug the identified technology shock into the VAR model $x_{t}^{Z}=\left[\varepsilon_{t}^{Z}, \hat{Z}_{t}^{H}, \hat{Z}_{t}^{N}, \hat{Z}_{t}\right]$, adopt a Choleski decomposition and generate responses of $Z^{j}$.
    ${ }^{38}$ To save space, we plot the long-run responses of LIS against the long-run responses of FBTC in Online Appendix M. 2 and find a similar conclusion in the long-run.

[^26]:    ${ }^{39}$ Since the home-produced traded goods sector is highly intensive in $R \& D$, whilst the non-traded sector displays a low R\&D intensity, the Acemoglu's setup is less relevant for non-traded industries. More specifically, the evidence documented by Galindo-Rueda and Verger [2016] for manufacturing and nonmanufacturing activities reveals that industries we classify as tradables (except for financial and insurance activities which are classified as low R\&D intensity industries) display high intensity in R\&D.

[^27]:    ${ }^{40}$ There is substantial uncertainty surrounding point estimates when estimating the VAR model at a country level given the relatively small number of observations available per country. Since the magnitude of the responses of TFP at a country/industry level may vary substantially with the number of lags, we find it appropriate to show estimates with one or two lags.
    ${ }^{41}$ In Online Appendix M.2, we plot the measure of the deviation from HNTC (50) estimated empirically against the measure estimated numerically in the long-run and find that our conclusion holds at a longer horizon.

[^28]:    Acemoglu, Daron (2003) Labor- and Capital-Augmenting Technical Change. Journal of the European Economic Association, 1(1), pages 1-37.
    Alvarez-Cuadrado, Francisco, Ngo Van Long and Markus Poschke (2018) Capital-Labor Substitution, Structural Change and the Labor income Share. Journal of Economic Dynamics and Control, 87, pages 206-231.
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    Bajzik, Jozef, Tomas Havranek, Zuzana Irsova, and Jiri Schwarz (2019) The Elasticity of Substitution between Domestic and Foreign Goods: A Quantitative Survey. EconStor Preprints 200207, ZBW.
    Balassa, Bela (1964) The Purchasing Power Parity Doctrine: a Reappraisal. Journal of Political Economy, 72, pp. 584-596.

[^29]:    ${ }^{42}$ While the two measures are equivalent in level, we differentiate between $\nu^{L, j}$ and $\alpha_{L}$ since the change in the labor share is calculated by keeping $W^{j} / W$ constant.

[^30]:    ${ }^{43}$ While the sample used by Stockman and Tesar [1995] covers 30 countries (including 17 developing and 13 industrialized), Mendoza [1995] uses exactly the same data set in his estimation but includes only the 13 industrialized countries. Note that the estimate of $\phi$ has been obtained by using the cross sectional dataset by Kravis, Heston and Summers for the year 1975.

[^31]:    ${ }^{44}$ It is worth noting that Antràs [2004] derives six econometric functional forms to estimate $\sigma$ : FOC with respect to labor (eq. (115a)), FOC with respect to capital (eq. (115b)), a combination of both FOCs and the remaining three are the reciprocal thereof. However, we focus on the first two because only the use of the FOCs permits the identification of growth rate of labor- and capital-augmenting technological change while the third specification captures the overall technological bias.

[^32]:    ${ }^{45}$ As IPS and MW allow for heterogeneity of the autoregressive root, we prefer these tests over the LLC test for which the autoregressive coefficient is required to be identical across all units.
    ${ }^{46}$ To conserve space we only report in Table 18 the results for the elasticity of substitution $\sigma^{H}$ and $\sigma^{N}$. The estimates of the parameters $\lambda_{1}$ and $\lambda_{2}$, that is estimates of the growth rate of labor- and capital-augmenting technological change are available from the authors upon request.

[^33]:    ${ }^{47}$ In all aforementioned tests and for all variables of interest, we allow for individual deterministic trends and country-fixed effects. Conclusions of unit root tests are robust whether there are individual trends in regressions or not. Appropriate lag length $q_{i}$ is determined according to the Akaike criterion.

[^34]:    ${ }^{48}$ Alternatively we use the number of employees as a measure of labor. All results remain almost unchanged.

[^35]:    ${ }^{49}$ Slope coefficients of regression lines shown in Fig. 19 range from 0.74 to 1.17 while R-squared fall in the range $[0.59 ; 0.84]$.

[^36]:    ${ }^{50}$ IRL and NOR do not provide disaggregated capital stock series. In efforts to have a balanced panel and time series of a reasonable length, AUT (1976-2013), BEL (1995-2013), DEU (1991-2013), FRA (19782013), JPN (1974-2007) and SWE (1993-2007) are removed from the sample, which leaves us with 9 OECD countries over the period 1970-2013.

[^37]:    ${ }^{51}$ We omit the shadow value of wealth from short-run solutions for clarity purposes as $\lambda$ remains constant over time.

[^38]:    ${ }^{52}$ We omit the shadow value of wealth from short-run solutions for clarity purposes as $\lambda$ remains constant over time.

[^39]:    ${ }^{53}$ Using the fact that $Y=W L$ and $P^{H} A^{H}=W^{H}$, we have $\alpha_{L}=\frac{W^{H} L^{H}}{W^{H}}$.
    ${ }^{54}$ Totally differentiating (349a) leads to: $\hat{P}=\frac{-\left(\sigma_{L}+\sigma_{C}\right) \hat{\lambda}+\alpha_{L}\left(\epsilon-\sigma_{L}\right) \hat{A}^{H}}{\Psi^{N}}$ with $\Psi^{N}=\left[\epsilon \alpha_{L}+\sigma_{L}\left(1-\alpha_{L}\right)\right]+$ $\left[\alpha_{C} \phi+\left(1-\alpha_{C}\right) \sigma_{C}\right]>0$. Totally differentiating (349b) and using the above equation to eliminate $\hat{P}$ yields (350).

[^40]:    ${ }^{55}$ Totally differentiating (349a) and plugging (350) leads to (351).

[^41]:    ${ }^{56}$ Insert first (23b), (356) and the market clearing condition for non-tradables (344) into (355) and differentiate:

    $$
    \alpha_{L} \hat{L}^{H}=\sigma_{L} \bar{\lambda}+\sigma_{L}\left(\hat{P}^{H}+\hat{A}^{H}\right)-\left(1-\alpha_{L}\right) \hat{C}^{N}
    $$

[^42]:    ${ }^{58}$ Totally differentiating (356) and substituting (357) and rearranging terms leads to (360).
    ${ }^{59}$ To see it formally, insert first (25) and (26) into the market clearing condition for non-traded goods (344), eliminate $P^{N}$ by using (356), totally differentiate and insert (357); one obtains:

    $$
    \hat{L}^{N}=\frac{\left[\left(\phi_{X}-1\right)+\alpha^{H} \rho\right] \alpha_{C}\left[\sigma_{L}\left(\sigma_{C}-\phi\right)+\omega_{C} \sigma_{C}(1-\phi)\right]}{\left[\left(\phi_{X}-1\right)+\alpha^{H} \rho\right]\left(\sigma_{L}+\sigma_{C}\right)+\left(1-\alpha^{H}\right) \chi^{H}} a>0
    $$

    where the positive sign of the above equation follows from assumption $\phi<1$ and $\sigma_{C} \simeq 1$.
    ${ }^{60}$ To compute the change in hours worked in the traded sector, divide both sides of the market clearing condition of the home-produced traded good (344) by $X^{H}$ and use the balanced trade condition (346) to eliminate $X^{H}$ on the RHS of the equation, i.e., $\frac{Y^{H}}{X^{H}}=1+\frac{P^{H} C^{H}}{C^{F}}$. Inserting first (35) and (24a), and totally differentiating leads to: $\hat{Y}^{H}=-\left[\phi_{X}+\alpha^{H}(\rho-1)\right] \hat{P}^{H}$ where we used the fact that $\omega_{C} \alpha_{C}=\alpha_{L}$. Using the fact that $\hat{L}^{H}=\hat{Y}^{H}-\hat{A}^{H}$, substituting (357) and rearranging terms leads to the percentage change in hours worked in the traded sector:

    $$
    \hat{L}^{H}=-\frac{\left[\left(\phi_{X}-1\right)+\alpha^{H} \rho\right]\left\{\left(\sigma_{L}+\omega_{C} \sigma_{C}\right)\left(1-\alpha_{C}\right)\left(\sigma_{C}-\phi\right)+\left(1-\sigma_{C}\right)\left[\sigma_{L}+\omega_{C} \sigma_{C}\left(1-\alpha_{C}\right)\right]\right\}}{\left[\left(\phi_{X}-1\right)+\alpha^{H} \rho\right]\left(\sigma_{L}+\omega_{C} \sigma_{C}\right)+\left(1-\alpha^{H}\right) \chi^{H}} a<0
    $$

    where the negative sign of the above equation holds for $\phi<1$ and as long as $\sigma_{C}$ takes values close to one. Subtracting $L^{H}$ from $L^{N}$ and multiply by $\left(1-\alpha_{L}\right) \alpha_{L}$ leads to (361).
    ${ }^{61}$ To see it formally, let $\rho$ tend toward infinity into (361) and apply l'Hôpital's rule; we get $\lim _{\rho \rightarrow \infty} \hat{\nu}^{L, N}=$ $\left(1-\alpha_{L}\right) \alpha_{L}(1-\phi) a$; since $\frac{\left[\left(\phi_{X}-1\right)+\alpha^{H} \rho\right]\left(\sigma_{L}+\sigma_{C}\right)}{\left[\left(\phi_{X}-1\right)+\alpha^{H} \rho\right]\left(\sigma_{L}+\sigma_{C}\right)+\left(1-\alpha^{H}\right) \chi^{H}}<1$, then eq. (361) is a scaled-down of $\lim _{\rho \rightarrow \infty} \hat{\nu}^{L, N}$.
    ${ }^{62}$ Eliminate $X^{H}$ from $Y^{H}=C^{H}+X^{H}$ by using the fact that $X^{H}=C^{F} / P^{H}$, totally differentiate and make use of the fact that $\hat{C}^{T}=\alpha^{H} \hat{C}^{H}+\left(1-\alpha^{H}\right) \hat{C}^{F}$.

[^43]:    ${ }^{63}$ Appendix F. 3 presents the empirical strategy and contains the details of derivation of the relationship we explore empirically.

[^44]:    ${ }^{64}$ We derive a testable equation by combining the demand for non-traded goods and the market clearing condition for non-tradables. Details of derivation of the equation we explore empirically can be found in section F.2.

[^45]:    ${ }^{65}$ Eberly, Rebelo, and Vincent [2008] run the regression $I / K=\alpha+\beta \cdot \ln (q)$ and obtain a point estimate for $\beta$ of 0.06 . In our model, the steady-state elasticity of $I / K$ with respect to Tobin's $q$ is $1 / \kappa$. Equating $1 / \kappa$ to 0.06 gives a value for $\kappa$ of 17 .
    ${ }^{66}$ More technical details can be found in section I.1.

