Investigating the effect of temporal and spatial flexibility on the performance of one-way electric carsharing systems

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Abstract

One-way electric carsharing systems provide an environmentally friendly option for facilitating urban mobility needs. However, the management of one-way electric carsharing systems presents operational challenges stemming from the need to relocate cars in order to strike an optimum balance between demand and supply. As a result, the cost associated with vehicle relocation operations represents a significant proportion of the total operating cost. In the context of electric carsharing systems, the problem of vehicle relocation is further exacerbated by the car battery charging requirements. The introduction of temporal and spatial flexibility regarding the pick-up and drop-off of vehicles provides the means of improving the efficiency of one-way electric carsharing systems. However, the literature currently lacks models that can be used to investigate the effect of temporal and spatial flexibility on the performance of one-way electric carsharing systems. In this paper, we are introducing an integrated modeling and solution framework for investigating the effect of temporal and/or spatial flexibility, and different options for processing trip requests to the profitability and utilization of one-way electric carsharing systems. The application of the proposed framework to a realistic size system suggests that spatial flexibility has a stronger effect on the system performance than temporal flexibility. Furthermore, both spatial and temporal flexibility can increase the profitability of the system by serving more customers with fewer vehicle relocation needs.

Keywords: one-way carsharing, electric carsharing systems, vehicle relocation optimization, integer programming, network flow, spatial and temporal flexibility

1. Introduction

In 2014, more than 54% of the world population was living in urban areas. By 2060 it is expected this ratio will go up to 66% (UN, 2014). In order to deal with increased population with limited resources, local and regional governments try to decrease private-use vehicles with disincentives such as congestion-pricing and increased taxation. Although these measures encourage some city dwellers to choose more societal-friendly options, there is still a significant proportion of trips that unavoidably require driving a vehicle. One of the environmental-friendly mobility options is to provide on-demand car transportation by using a shared-use vehicle system (e.g. bikesharing, carsharing) (Shaheen et al., 2015).

Carsharing is a more accessible and efficient car rental concept usually for a short period of time. Different than the conventional car rental, in a carsharing system, users have self-service access to vehicles scattered around the city throughout the day. Thanks to rapid improvements in mobile communication technologies, users are provided with a contactless card and a dedicated app that allows them to locate and access vehicles. When someone becomes a member of the service, he/she gains access to vehicles (in some cases located in multiple cities) without any time limitation in a short period of time.

Carsharing systems can be classified according to their operating practices. *Station-based* systems allow users to pick-up and drop-off cars to/from designated parking spots. In *round-trip* systems each vehicle has a designated

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parking spot and vehicles should be returned to their pick-up locations after their use. More flexible *one-way* systems allow users to pick-up and drop-off vehicles from/to different stations. *Free-floating* systems allow users to park the rented vehicles to any legal parking spots in a designated area. Carsharing systems can also be classified according to vehicle engine types, *electric* or *non-electric*. Although the former option with its zero emissions is more environmentally friendly, it also needs investment in charging facilities and consideration of battery levels when making rental decisions. There is also a classification for reservation opportunities. Some carsharing systems allow only *last-minute reservations*, reserving vehicles for a period of time in the near future, usually between 15 minutes and an hour, whereas some other systems allow to have *advance reservations* for the distant future, which could be a couple of hours to weeks before rental. In the former option, users are allowed to lock vehicles with an app for a period of time whereas in the latter option users are provided with an option to have reservations in the future for a period of time.

Owing to technological advancements and the potential of carsharing systems to generate economic, societal, and environmental benefits, the carsharing services market is expanding significantly both in terms of fleet size and membership. An industry report by Frost and Sullivan, predicts that due to technological advancement, market consolidation, and government initiatives, the global carsharing market will increase from "over seven million users and 112,000 vehicles in 2015 to over 36 million users and 427,000 vehicles in 2025, at an annual combined growth rate of 16.4% and 14.3% respectively" (Leveque, 2016).

Carsharing has various benefits to the environment and society as a whole. By eliminating the cost of owning a car, it reduces personal transportation cost, increases mobility to disadvantaged groups and encourages people to prefer more environmental-friendly alternatives. Carsharing systems are also contributing to the integration of the public transport system by filling the gap between the flexibility offered by privately owned car transportation and fixed conventional public transport services. Therefore, carsharing systems have the potential to decrease car ownership, which in turn could decrease the need for parking space, traffic congestion, vehicle kilometers traveled and greenhouse gas (GHG) emissions (Crane et al., 2012). A study conducted in North America (Shaheen and Chan, 2015) among 9,500 members of round-trip carsharing systems found that due to the use of the services offered by carsharing systems, quarter of their members sold a vehicle and another quarter delayed the purchase of a vehicle. This change in car ownership behavior has potential to remove 9-13 privately owned vehicles for every carsharing vehicle. The knock-off effect of car ownership has potential to reduce vehicle miles traveled by 27-43%, and GHG emissions by 34-41%. Studies conducted in Europe and Australia on round-trip systems found that one carsharing vehicle replaces respectively 4-10 and 7-10 privately owned cars (Shaheen et al., 2015; Martin et al., 2010). The predicted significant increase in the use of carsharing systems coupled with their impact on private car ownership provide tangible evidence of the potential of carsharing systems to generate significant economic, societal, and environmental benefits.

Although users prefer to have more flexible one-way or free-floating options over their round-trip counterparts (Le Vine and Polak, 2019), operators are usually inclined to provide simple-to-operate round-trip systems. Specifically, vehicle and personnel *relocation operations* for achieving spatial-temporal demand-supply balance have a significant effect on the economic and operational performance of one-way electric carsharing systems (Boyacı et al., 2015, 2017). Relocation operations are usually leading to higher operating costs, lower vehicle utilization rates, and larger fleets. One approach that can be used to reduce the operating cost of one-way carsharing systems is to incentivize the users of one-way carsharing systems to perform part of the required relocation operations (Barth et al., 2004). This means that potential users of the system may be offered to use vehicle pick-up and drop-off locations and/or times that are not necessarily their first choice (Correia et al., 2014). The introduction of temporal and/or spatial flexibility provides operators of one-way electric carsharing systems, the opportunity to better match the temporal and spatial profile of demand with the available supply, leading to a potential reduction of the required vehicle relocation operations and consequently to the number of personnel needed to perform them. The increased spatio-temporal flexibility of demand provides the means of better aligning the supply of carsharing vehicles with the requested carsharing services, leading to more efficient utilization of the fleet. It is worth noting that the positive impact of the spatio-temporal flexibility of demand is not on the expense of the total system cost since the users of the system are compensated through the provision of incentives, to modify their demand patterns.

The positive effects of the spatial and/or temporal flexibility of demand on the performance of app-based flexible public transport systems (Tong et al., 2017), and ridesharing systems (Stiglic et al., 2015; Zhao et al., 2018b) have been studied. However, the literature lacks models that can investigate the effect of temporal and/or spatial flexibility on the performance of one-way electric carsharing system, while providing a realistic representation of car and personnel relocation, and car charging operations. In this paper, we are focusing on the effect of the spatio-temporal flexibility of demand on the performance of one-way electric carsharing systems to close the identified literature gap. In particular, we investigate the effect of introducing flexibility, regarding the pick-up time and/or the location of picking-up and/or dropping-off the vehicles of one-way electric carsharing systems, under different reservation options, on the system profitability. The emphasis and primary focus of our work is to develop a modeling and solution framework that will provide operators of one-way electric carsharing systems the capability to consider simultaneously the temporal and spatial flexibility of reservations and relocation operations. The proposed framework will help decision-makers to assess the impact of different levels of flexibility on the performance of their system, under different regimes for serving the demand for one-way carsharing trips.

We organize the rest of the paper as follows: Section 2 covers the most recent literature review that is relevant to our research. We conclude this section with the rationale for the need and the added value of this research. Section 3 presents the modeling and solution framework. We describe all relevant assumptions, models and approaches used in the development of the framework in this section. Section 4 provides the implementation details and results of the computational experiments. Section 5 discusses the concluding remarks and provides future research directions.

2. Literature review

The optimization of decisions regarding the organization, planning, and operations of one-way carsharing system has attracted considerable research attention. The decisions regarding the management of one-way carsharing systems fall into three major interrelated categories according to the time horizon of their implementation, the frequency of their update, and the size of the capital investments needed. Examples of strategic one-way carsharing decisions include the optimization of the number, location and parking spaces of parking stations. Indicative tactical decisions include the determination of the fleet size, the number of required relocation personnel, while operational decisions include the assignment of vehicles to trip-requests, the assignment of personnel to vehicles to be relocated, etc. (Boyacı et al., 2015, 2017).

A major factor differentiating the station based one-way carsharing systems over their round-trip counterparts is its requirement to dynamically rebalance the number of cars required to be available at each station due to the spatio-temporal asymmetry of the demand. A special category of one-way carsharing systems is the systems that are using fleets of electric cars. An important determinant of the operational context of the one-way electric carsharing systems is the availability of sufficient battery charge to accommodate the various trip requests. In this operating context, an extra degree of complexity is added to the vehicle relocation decisions which stems from the need to determine feasible vehicle-to-trip assignments. In this work, we develop a framework for the optimization of operational decisions (e.g. accepting/rejecting demand requests, relocating vehicles and personnel, and assigning trip requests to vehicles) of one-way electric carsharing systems that provide spatial and temporal flexibility to its customers. Therefore, in what follows, we are focusing our attention to previous work that has modeled operational decisions of one-way (electric) carsharing systems. The list of papers reviewed is also provided in Table 1. For a comprehensive review of carsharing papers, readers are referred to the survey papers Jorge and Correia (2013), Brandstätter et al. (2016), Ferrero et al. (2018) and Laporte et al. (2018). For a review of modeling efforts related more specifically to the vehicle relocation problem in one-way carsharing systems, the interested readers are referred to Illgen and Höck (2019). Given the fact that our proposed optimization framework introduces the concept of spatio-temporal flexibility in one-way electric carsharing systems, we are also reviewing literature related to the use of the concept of spatial and/or temporal flexibility in flexible transport systems regardless of their one-way electric carsharing characteristics.

Nair and Miller-Hooks (2011) formulated the operational problem as a mixed integer stochastic optimization problem. The model aims to find the ideal vehicle locations that serve a number of demand requests of the near future with minimum operational cost. The model was tested on a system with 94 vehicles and 14 stations operating in Singapore.

Bruglieri et al. (2014) formulated a vehicle relocation problem in a station-based one-way electric carsharing system. They proposed to use bikes to travel between stations then repositioning vehicles between stations. They considered the projected travel demand in their analysis and assume all demand must be served. They tested their model on a system with 30 electric vehicles.

Correia et al. (2014) proposed a mathematical model to evaluate the impact of user flexibility and sharing vehicle stock information with the users in a one-way carsharing system. They tested three different scenarios: inflexible

Table 1: List of referenced articles

article	one-way trips	electric vehicles	relocation operations	personnel movements	temporal and spatial flexibility
Nair and Miller-Hooks (2011)	✓	×	✓	×	×
Bruglieri et al. (2014)	✓	✓	✓	✓	×
Correia et al. (2014)	✓	×	×	×	✓
Nourinejad et al. (2015)	✓	×	✓	✓	×
Repoux et al. (2015)	✓	×	✓	✓	×
Boyacı et al. (2017)	✓	✓	✓	✓	×
Gambella et al. (2018)	✓	✓	✓	✓	×
Xu et al. (2018)	✓	✓	✓	✓	×
Zhao et al. (2018a)	✓	✓	✓	✓	×
Ströhle et al. (2019)	×	×	×	×	✓
Bruglieri et al. (2019)	✓	✓	✓	✓	×
Zhang et al. (2019)	✓	✓	✓	×	×
Stiglic et al. (2015)	N/A	N/A	N/A	N/A	✓
Tong et al. (2017)	N/A	N/A	N/A	N/A	✓
Zhao et al. (2018b)	N/A	N/A	N/A	N/A	✓
proposed framework	✓	✓	✓	✓	✓

users that use only closest stations to their origin and destination, flexible users that are content to use second and third closest stations as alternatives without the vehicle and empty spot information and flexible users with adequate system information. Their model has not considered relocation decisions, flexibility in pick-up times and the discounts in case of offering alternative trip options. They tested the algorithm on a system with 116 potential locations with station sizes up to 35 spots. Results show that the flexibility with and without option increase the profit by around 3 and 5 fold compared to the scenario without flexibility.

Nourinejad et al. (2015) dealt with vehicle and staff relocations with two integrated multiple traveling salesman formulations. They used a mathematical formulation to solve instances with 40 demand requests. For larger instances, a heuristic approach was developed by decomposing the first formulation to two formulations and solving them sequentially.

Repoux et al. (2015) developed a discrete-event simulator for the analyses of the algorithms in a station-based one-way carsharing system. It uses limited information (instead of the full information) and the idea of partial floating (if the stations are full vehicles are allowed to park outside designated parking spots). They implemented an optimization framework that decides on relocations to perform and the personnel to do relocations in separate mathematical models. The framework was tested on a system with 59 stations and 59 to 118 vehicles.

Boyacı et al. (2017) developed and solved a multi-objective mixed integer linear programming (MMILP) model coupled with a discrete simulation to dynamically optimize throughout a day vehicle and personnel relocation decisions for one-way electric carsharing systems. The proposed framework involved a station clustering model, a vehicle operations optimization model, and a personnel flow model. The station clustering model was used at a pre-processing stage to reduce the complexity of the model by performing relocations between clusters of stations as opposed to individual stations. The proposed optimization-simulation framework was integrated in an iterative fashion. Initially, vehicle assignments to trips, and vehicle and personnel relocation decisions are optimized assuming that all electric cars have sufficient battery to serve any trip request. Following the optimization cycle, simulation is used to check the feasibility of the car assignments to trips based on the available level of battery charge. Infeasibilities detected during the simulation cycle are fed back into the optimization component, and the car assignments and relocation personnel flows are re-optimized. The proposed framework was used to investigate the performance of one-way electric carsharing operations using data from the carsharing system in Nice, France. Two different trip assignment regimes were used namely, a trip assignment in the presence of full information regarding the trip requests for the entire day (system with advance reservations), and a system where the trips are assigned by having information for one trip request at a time.

Gambella et al. (2018) considered the charging level of the vehicles in the mathematical model. The first model is used to find the set of served trips and relocation operations while considering the detailed personnel movements. The model also decides on the optimal locations at the beginning of the day. The secondary model is utilized to find the best relocation strategy during the night when the system is not available to users. It was found that the mathematical model is computationally impractical and as a result, a few heuristic approaches are used instead. The system was tested on a system with 14 stations and 20 electric vehicles for a 10 hour period (i.e. 40 intervals of 15 minutes).

Xu et al. (2018) considered the fleet sizing and trip pricing problems together while considering vehicle and personnel relocation operations and elastic demand. They proposed a mixed integer non-convex programming problem formulation. They used an outer approximation method to solve the problem optimally. They tested the algorithm on a system with 27 stations with 500 spots each and 5000 electric vehicles. Since they used pricing and elastic demand, the number of the aggregated served trip went up to 268 among 39,787 potential requests (when the system is free of charge).

Zhao et al. (2018a) dealt with the vehicle and staff relocations simultaneously. They proposed a mathematical model working on discrete time-space. They did not take station capacities into consideration and assume all requests are served. They minimize the cost of vehicles, personnel, and relocations in the objective function and use Lagrangean relaxation to find fast near-optimal solutions. They tested the algorithm on instances with 30 stations, 60 trip requests and stations, and 30 relocation personnel.

Bruglieri et al. (2019) considered relocation operations and use Adaptive Large Neighborhood Search Algorithm. This algorithm does not decide on accepting/rejecting demand but instead assumes every demand requested is served. They aimed to minimize the number of relocation personnel operating the system. The algorithm was tested on instances with up to 100 pick-up and drop-off requests in total. Their further analyses have shown that in all small and in most of the medium-sized instances, they found the optimal solutions whereas in the large instances, the average optimality gap was around 6%.

Zhang et al. (2019), investigated the assignment of vehicles and the use of relays in electric carsharing systems as a means of coping with the electric car range constraints and increasing vehicle utilization. The electric vehicle assignment problem considers simultaneously routing and charging decisions in order to ensure that the battery level of the electric vehicles is sufficient to accommodate most of the trip requests. The electric vehicle assignment model with relays was used to extend the range of the trips that can be served (longer trips) by considering simultaneously car routing and battery charging decisions. The proposed models do not consider relocation personnel decisions. The proposed models were used to investigate one-way electric carsharing operations in the Jiading and Pudong districts in Shanghai, China.

The concept of spatial and temporal flexibility of demand has been introduced in the literature of flexible public transport systems as a means of i) increasing the number of matched participants, and reducing the system-wide driving distance of ridesharing systems (Stiglic et al., 2015); ii) increasing vehicle utilization, and increasing the number of satisfied requests (Tong et al., 2017); iii) reducing cost, and improving quality of app-based transport services (Zhao et al., 2018b); and iv) decreasing fleet size requirements in round-trip carsharing systems (Ströhle et al., 2019).

Stiglic et al. (2015) introduced the notion of spatial flexibility in ridesharing systems through the use of meeting points as an alternative to the actual origin-destination of trips. The meeting points are places where riders can be picked-up and/or dropped off within a pre-specified distance from their actual origin/destination. They formulated the problem as a one-to-many matching problem and develop an algorithm for solving the problem under Euclidean driving and walking distance and constant speed assumptions. Through a simulation study, they demonstrated that spatial flexibility in the form of meeting points can significantly improve the matching of drivers with riders and reduce the system-wide driving distance.

Tong et al. (2017) incorporated spatial and temporal flexibility to jointly optimize assignments of passengers to vehicles and bus routes. The time flexibility was introduced as a time window at both ends of the trip, while the spatial flexibility was introduced as an acceptable radius around the origin and destination within which the riders can be picked-up or dropped off. They formulated the resulting customized bus service design problem as a multi-commodity network flow optimization model. Sensitivity analysis regarding the length of the preferred time windows was performed. It was found that under loose time windows, the flexible bus service was able to serve more passengers.

Zhao et al. (2018b), studied the effect of using spatial flexibility in the form of flexible pick-up and drop-off locations of app-based transportation services. Under this approach, the pick-up and drop-off locations of the customers are repositioned through communication between the driver and the customer. The resulting problem was modeled as a pick-up and delivery problem with time windows. Numerical experiments suggested properly repositioning the pick-up and drop-off locations of the passengers can reduce the cost of offering the services and improve the matching of vehicles and passengers.

Ströhle et al. (2019) considered temporal and spatial flexibility in round-trip carsharing systems. They proposed a mathematical model and solved it to optimality for weekly instances. They observed that the spatial flexibility of 1km can decrease the fleet size (12%) two times more than the temporal flexibility of 4 hours (4%). They also observed synergy happens when users accept both temporal and spatial flexibility to a range together. They also evaluated the cost of flexibility to customers with the help of over 1500 responds.

Our literature review (summarized in Table 1) revealed that the consideration of spatial and temporal flexibility in designing ridesharing systems has the potential to improve the system performance. However, the current literature lacks models that can investigate the effect of temporal and/or spatial flexibility on the performance of one-way electric carsharing system while providing a realistic representation of the car and personnel relocation, and car charging operations. This work aims to close this gap by introducing a framework for optimizing one-way electric carsharing operations by incorporating spatial and temporal flexibility of demand. Specifically, the proposed framework contributes to the current state of the art by introducing:

- A new optimization model that incorporates the spatial and temporal flexibility offered to users. The proposed model optimizes jointly relocation personnel and vehicle flows of the carsharing system. According to the proposed modeling approach, the charging level feasibility of the carsharing vehicles is checked, in addition to simulation, by an optimization model. This allows for the exploration of the entire solution space.
- An exact algorithm to generate clusters of carsharing stations using asymmetric travel times. The proposed algorithm improves the computational performance of the proposed platform without compromising the accuracy of the model results.
- A new regime for serving demand in batches. According to this regime, it is not necessary for the operator of the carsharing system to know all the demand at once.
- An application of the proposed framework using realistic problem instances in order to demonstrate its capabilities.

3. Methodology

In this work, we are considering a reservation-based one-way (station based) electric carsharing system with a user incentive-based flexibility of pick-up time, and the origin and destination of trips. In the proposed system, the demand is known in advance regardless of the way that the users are informed as to how they will be served. The travel time between the carsharing stations is assumed to be deterministic, constant, and asymmetric throughout the day.

- The relocation time between any two stations equal to the travel time plus a fixed amount of time that is required to plug-off and on the vehicles.
- Each station has a fixed capacity of parking spots with standard chargers. As soon as vehicles arrive at the stations, either the users or relocation personnel connect vehicles to dedicated chargers.
- The relocation operations between stations are executed throughout the day by a dedicated team of relocation personnel. Each personnel works during one of the predefined working shifts. They travel between stations with (*driving*) or without (*moving*) vehicles. The speed of *driving* (i.e. personnel traveling with a vehicle between two stations) and *moving* (i.e. personnel traveling without a vehicle between two stations) operations can be different.
- The framework decides on the starting and ending nodes of each personnel and their detailed itinerary for their shifts. Although it can be altered by additional constraints, the current framework uses the optimal starting locations for each personnel that maximizes the profit.
- The solution framework models the system with short and equal time intervals (e.g. 15 minutes). The vehicles and the relocation personnel cannot have more than one state during any time interval.

- The solution framework offers alternative trips that may differ from their requested and preferred choice. The alternative trips can start (and end) earlier or later than the requested time, originate from and/or destined to different stations. For simplicity, we assume that the trip length and duration are not changing when an alternative is offered. Original and alternative requests have the same trip lengths and driving distances. The operator offers alternative trips with a discount depending on the difference between the start times', and the distances between original and destination stations' of alternative and original trip requests. There is an incentive for the operator to offer users their original requests. For simplicity and due to lack of user behavior data regarding temporal and spatial flexibility in this paper, we use a predefined range of (pick-up) time and (origin and destination station) distance values for all users. If the flexibility regarding the users' behavior is provided for each request, the framework can be used without substantial changes to solve a problem using the flexibility range provided for each individual request.
- The framework maximizes the profit. The revenue of the system comes from serving the requested trips. Fuel cost of relocations and personnel hiring cost are deducted from the revenue to calculate the profit. Although it can be altered by additional constraints, the number of personnel assigned to each shift is decided by the solution framework. Vehicle depreciation and station operating related costs are not taken into consideration since both the vehicle and station related properties are taken as input to the model.
- In order to reduce computational complexity, the stations are partitioned into clusters and a representative node is introduced to each cluster. Instead of modeling relocation operations from origin to destination stations directly, we model relocations in three separate "imaginary" steps: (1) from the origin station to origin cluster node, (2) from the origin cluster node to destination cluster node, and (3) from the destination cluster node to destination station. Each step takes a non-negative number of time steps. These clusters are decided by utilizing a mathematical model and they never underestimate the real travel times.
- The travel times between two stations are not set to be symmetric. Traveling from stations i to l is not equal to traveling from station l to i. This property is taken into consideration in every step of the solution framework.
- In order to make the solution framework more efficient, different approaches are used to solve the vehicle assignment problem. The proposed framework offers the capability to use either simulation and/or optimization for the vehicle assignment problem. After solving the Flexible Operational Optimization Model (FOOM), the faster approach (i.e. simulation) that do not explore the entire solution space is used first. If a satisfactory solution is not found, we utilize a slower approach (i.e. optimization) that explores the entire solution space for the sake of efficiency.

The complete flow chart of the solution framework is given in Fig. 1. We start the problem with raw input data. Raw data include information on asymmetric driving and moving travel times between each station pair, and the total number of trips originating from and destined to each station used by the Asymmetric Station Clustering Model (ASCM, see Section 3.1). The number of parking spots at each station, initial number of vehicles parked at each station, shift start and end times, and detailed trip information (i.e. origin and destination stations, pick-up and drop-off times, trip distance) for the instance that is going to be analyzed in the current instance is used by the FOOM (see Section 3.2). We use the asymmetric distance matrices for both *driving* (i.e. relocating a carsharing vehicle) and *moving* (i.e. relocation of a personnel without a vehicle) operations in ASCM. Since the locations of stations are the same for all instances, this model is solved twice, for *driving* and *moving* operations separately, and the same clustering information is used for all the instances.

After clusters are decided by the ASCM, the FOOM is solved with the input from raw data and ASCM. The FOOM regards each personnel and vehicle movements as unit flows. It solves a parallel network flow problem that models vehicles and each personnel shift in separate networks as unit flows (see Fig. 2). The FOOM passes all identical unit flows of vehicles and personnel between stations and station cluster nodes to the Personnel Flow Model (PFM) and (if needed) to the Joint (Personnel and Vehicle) Flow Model (JFM). Note that the FOOM regards every vehicle and personnel (of each shift) identical. In other words, the FOOM does not differentiate among vehicles and among personnel of each shift. As a result, the solution of the FOOM corresponds to more than one feasible vehicle and personnel assignments. Note that, for the electric carsharing systems, some or all of these assignments are not necessarily charging level feasible solutions, i.e. vehicles may reach charging levels lower than the level they are allowed to serve. In other words, by using the solution of the FOOM, at least one feasible solution can be generated for *non-electric* carsharing systems, but this does not mean that any solution created by the FOOM is a feasible solution

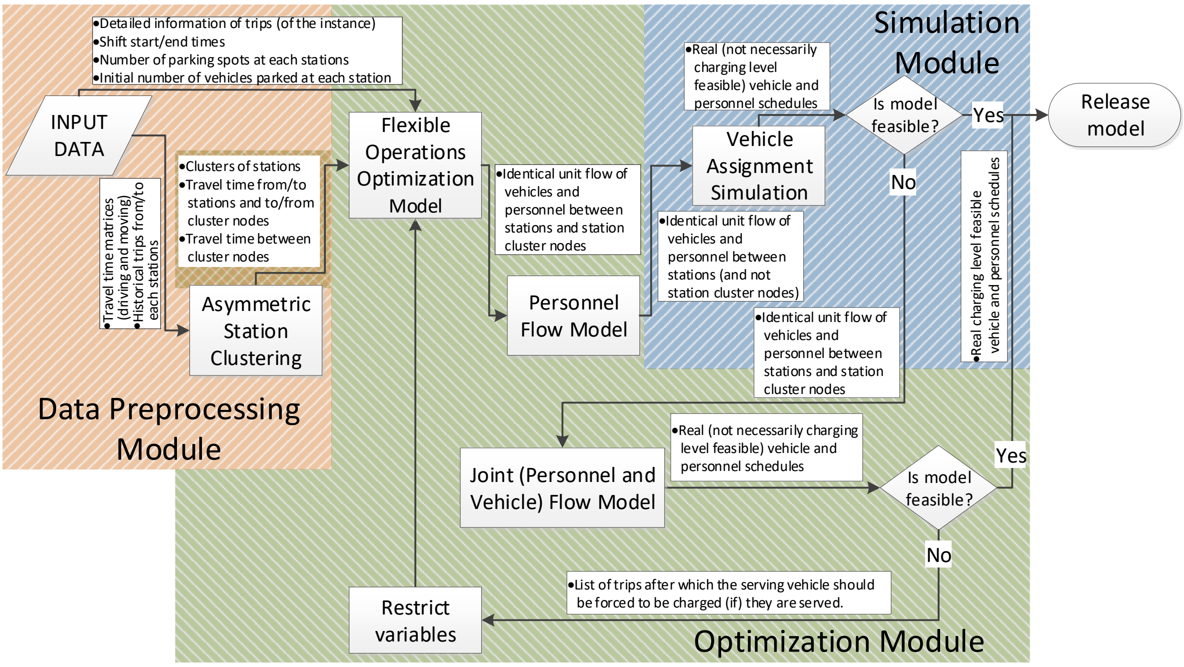

Figure 1: Proposed solution framework

for an *electric* carsharing system.

The next step after solving the FOOM is finding personnel and vehicle flows. This problem is solved with two different methods in the solution framework to improve the efficiency of the solution process. The PFM (see 3.4) considers only the flows of personnel and creates complete relocation assignments from the constituent elements of relocation flows, i.e. from origin to destination through origin and destination cluster nodes. The solution of the PFM identifies all relocation operations between two stations with and without vehicles. Then, the Vehicle Assignment Simulation (VAS, see 3.5) takes identical flows of vehicles and personnel between stations (not station cluster nodes) and assigns vehicles and personnel to each unit flow, by utilizing a discrete event simulation that considers myopic assignment policies. In other words, the VAS converts unit vehicle and personnel flows to a real, but not necessarily a charging level feasible, schedule. If the schedule is a charging level feasible schedule, then the result is released. If it is not feasible and there is at least one vehicle with an infeasible charging level, then we use the JFM (see 3.3). The JFM takes the identical unit flows of vehicles and personnel between stations and station cluster nodes generated by the latest solution of the FOOM as input and decides on the personnel flows and vehicle assignments simultaneously within a single mathematical model. Since it explores a larger solution space than the previous approach, it takes longer to solve than the PFM and VAS combined. However, since it explores the entire solution space in one step, it may return a charging level feasible solution when the other approach cannot. If it finds out there is a charging level feasible solution, the solution is released and the algorithm terminates. If there does not exist any feasible solutions, the JFM releases the trips which were attempted by the vehicles with infeasible charging levels. The FOOM takes as input the list of trips that will not be possible to serve unless the serving vehicles are further charged and requires the vehicles that will serve these trips to be charged after serving these trips. We resolve the FOOM and this loop continues until a charging level feasible solution is released.

In the proposed solution framework, besides VAS and JFM, we have experimented with a Vehicle Assignment Model (VAM, see Appendix A). However preliminary experiments have shown that the running times for the same instances of the VAM and JFM are almost the same. Therefore, we did not integrate VAM in the solution framework.

In what follows we are providing the mathematical description[1] of the constituent modules of the proposed framework.

[1]In all of the mathematical model presentations, we follow these rules to be consistent: a) Sets are presented with the uppercase italic letters.

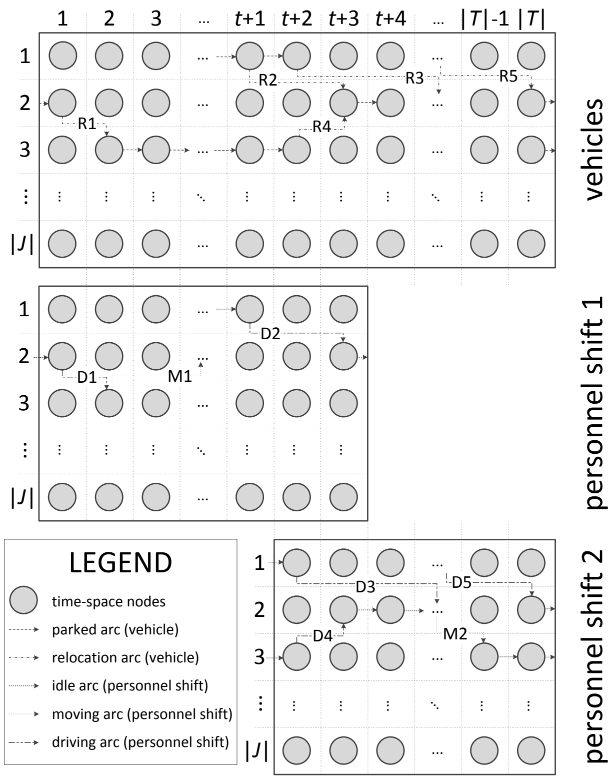

Figure 2: Time-space network figure from Boyacı et al. (2017). Parallel time-space networks for vehicles and personnel shift for the set of stations (J) and time intervals (T). Each network has a separate set of flows. In the vehicle network, parked and relocation (R) flows are used. In the personnel networks, we see flows for idle, moving (M) or driving (D) personnel. There is a corresponding relocation flow in the vehicle network for every driving flow that exists in one of the personnel networks.

3.1. Asymmetric Station Clustering Model (ASCM)

The ASCM is used to cluster stations to simplify relocation operations in the FOOM. If the time interval and node sets are represented with T and N simultaneously, allowing relocation operations throughout the day between any station pairs requires $O(|T||N|^2)$ variables for relocation operations only. In order to use fewer relocation variables, we introduce imaginary cluster nodes and make relocation operations in three steps (Boyacı et al., 2017). When there is a relocation from node j to l, the relocation is done (1) from node j to the cluster node of j, then (2) from the cluster node of j to cluster node of l, and finally (3) from the cluster node of l to node l. If the cluster set is represented with M, this approach decreases the number of variables for relocation to $O(|T||M|^2 + |T||N|)$. Although the clustering approach slightly overestimates the relocation times between a few node pairs, it helps us to build a solution framework easier to solve.

Note that the proposed clustering approach differs from the approach introduced in Boyacı et al. (2017), in terms of two important characteristics affecting the accuracy of the results. First, the proposed approach uses asymmetric travel times to represent more realistically the traffic conditions of the underlying transport network. Second, the proposed approach uses an exact algorithm as opposed to the heuristic algorithm used in Boyacı et al. (2017).

3.1.1. Set and indices

$j, l \in J$ nodes (stations)
$m, n \in M$ clusters

b) Variables and indices are presented with the lowercase italic letters. c) Parameters are presented either with the all-lowercase words followed by indices inside the brackets or the capitalized first letters of the parameters with indices in super and subscripts.

3.1.2. Parameters

t_{jl} travel time from node j to l

w_{jl} weight of relocations from node j to l expressing the popularity (preference of users to use this station)

3.1.3. Variables

x_j^m 1 if node j is in cluster m, 0 otherwise

u_j approximated travel time from node j to its own cluster

$\overline{u}_j$ approximated travel time to node j from its own cluster

v^{mn} approximated travel time from cluster node m to n

d_{jl} approximated travel time from node j to node l

3.1.4. Formulation

$$\min \sum_{j,l} w_{jl}(d_{jl} - t_{jl}) \tag{1}$$

subject to

$$\sum_{m} x_j^m = 1 \qquad \forall j \tag{2}$$

$$d_{jl} \leq u_j + v^{mn} + \overline{u}_l + \mathrm{M}(2 - x_j^m - x_l^n) \qquad \forall j, l \neq j \in J; m, n \in M \tag{3}$$

$$d_{jl} \geq u_j + v^{mn} + \overline{u}_l + \mathrm{M}(x_j^m + x_l^n - 2) \qquad \forall j, l \neq j \in J; m, n \in M \tag{4}$$

$$d_{jl} \geq t_{jl} \qquad \forall j, l \tag{5}$$

$$\sum_{j} x_j^m \geq 1 \qquad \forall m \tag{6}$$

$$x_j^m \in \{0, 1\};\ u_j, \overline{u}_j, v^{mn}, d_{jl} \in \{0\} \cup \mathbb{Z}^+ \qquad \forall j, l, m, n \tag{7}$$

The clustering problem is formulated by Eqs. 1-7. The formulation aims to minimize the weighted error of relocation operations through clustering nodes. Inputs of the model are the number of clusters and the travel time between each station pairs. The solution of the model determines the clusters, and the approximated travel times (1) from nodes to clusters, (2) between clusters and (3) from clusters to nodes. Please note that the approximated traveling times through clusters cannot be shorter than the actual travel times. Therefore, this model prevents the underestimation of travel times between node pairs.

Objective 1 minimizes the weighted difference between the estimated travel time (t_{jl}) and the real travel time (d_{jl}) between every node pairs. By introducing weights in the objective function, we aim to incorporate the "popularity" of the station in the clustering decision. In the computational experiments, the weight used for every pair (w_{jl}) is equal to the total number of trips originating from and destined to nodes j and l. We assume, if a station is used more frequently by the users, it is more likely that it is involved in relocation operations.

Constraints 2 require that every node should belong to one and only one cluster.

Constraints 3 and 4 calculate the approximated travel time d_{jl} between every node pairs j and l. Constraints 3 state that if nodes j and l belong to clusters m and n respectively, then travel time d_{jl} should be less than or equal to the summation of travel times (1) from node j to its cluster node, (2) from cluster node m to cluster node n and (3) from the cluster node of l to node l. If node j does not belong to cluster m or node l does not belong to cluster n, the big-M part of the constraints make the inequalities not binding. Similarly, constraints 4 binds the value of d_{jl} from above. Constraints 3 and 4 ensure that $d_{jl} = u_j + \overline{u}_l + v^{mn}$ if nodes j and l belong to clusters m and n respectively.

Constraints 5 require that between every node pairs, approximated travel time (d_{jl}) to be at least as long as the real travel time (t_{jl}). These constraints prevent any approximated travel times to be smaller than the real travel times.

Constraints 6 postulate every cluster should at least have one node assigned to it. They are redundant constraints added to the model to have shorter solution times.

Constraints 7 set the domains of each variable. All the variables except x_j^m can take any non-negative integer values whereas x_j^m is defined as a binary variable.

3.2. *Flexible Operations Optimization Model (FOOM)*

The FOOM formulates the operational planning of a carsharing system as an advanced parallel network flow problem (Fig. 2). The model considers accepting/rejecting each demand request, decides on the offered trip, which is either exactly the same as the requested demand or different but within the predefined range of temporal and spatial flexibility, to the user, and determines the relocations with and without vehicles throughout the operational time of the system. In order to increase the efficiency of the model, relocations with and without vehicles are done through two different sets of clustering nodes (see Section 3.1), and each vehicle and personnel at each shift are considered as unit flows on different layers of the parallel networks. The unique flows for each vehicle and personnel are decided by different methods including the JFM given in Section 3.3. The FOOM uses an approach similar to the Operations Optimization Model described in Boyacı et al. (2017). However, there are significant differences between the two models. This new model allows the operator to offer alternative trips different than the original requests with a discount. Furthermore, it utilizes asymmetric travel times between stations during relocations and utilizes relocation personnel better. Furthermore, the current formulation makes a core realistic assumption regarding the availability of the relocation personnel. Specifically in the previous formulation (see Boyacı et al., 2017) an assumption was made requiring a relocation personnel to wait one time interval (15 minutes) before its assignment to a new relocation task. In this version, we have lifted this constraint and therefore the relocation personnel can be assigned as soon as the previous request is completed. As a result, the utilization of the relocation personnel is improved.

3.2.1. *Sets and indices*

$i \in I$	demand
$j, l \in J$	stations
$t, u, w \in T$	time intervals
$s \in S$	working shifts
$b, d \in B$	station clusters
$e \in E$	start time options (of any demand)
$\bar{o}, o \in O$	origin/destination options (of any demand)
a	trip (4-tuple of demand, and start time, origin and destination options)

3.2.2. *Parameters*

$\text{start}(s)/\text{end}(s)$	start/end time intervals of shift s
$\text{start}(i,e)/\text{end}(i,e)$	pick-up/drop-off time of start time option e of demand i
$\text{origin}(i,e,\bar{o})$	origin option $\bar{o}$ of demand i
$\text{dest}(i,e,o)$	destination option o of demand i
$\text{c.end}(i,e)$	last charging time interval of option e of demand i
$\text{d.end}(t,b,d)$	time interval at which the driving starting at t reaches from cluster b to d
$\text{m.end}(t,b,d)$	time interval at which the moving starting at t reaches from cluster b to d
$\text{d.from}(j)$	time interval difference between the driving starting from station j and reaching to its cluster node
$\text{m.from}(j)$	time interval difference between the moving starting from station j and reaching to its cluster node
$\text{d.to}(j)$	time interval difference between the driving reaching to station j starting from its cluster node
$\text{m.to}(j)$	time interval difference between the moving reaching to station j starting from its cluster node
PC_s	cost of relocation personnel for shift s
$\overline{\text{RC}}_j/\text{RC}_j$	intra-relocation cost from/to station j
$\widetilde{\text{RC}}_{bd}$	inter-relocation cost from cluster b to d
SG	safety gap (i.e. number of time intervals vehicles forced to be kept at the station after drop-off)
I_c	set of trip options requiring charging
P_i	gross revenue of serving demand i
$\tilde{\text{D}}_i^e/\text{D}_i^o/\overline{\text{D}}_i^{\bar{o}}$	discount of demand i if it is served by {start time option e} / {origin option $\bar{o}$} / {drop-off option o}
$a_i/a_e/a_o/a_{\bar{o}}$	demand / {start time option} / {origin option} / {destination option} of tuple a
C	set of trips that need charging
$C(a)$	set of recently added trips to the model that are served by the same vehicle which serves trip a

3.2.3. Variables

- z_i 1 if demand i is served; 0 otherwise
- $\tilde{z}_i^e$ 1 if demand i's start time option e is served; 0 otherwise
- $\overline{z}_i^{e\bar{o}} / z_i^{eo}$ 1 if demand i's {origin option $\bar{o}$} / {destination option o} with time option e is served; 0 otherwise
- $\hat{z}_i^{e\bar{o}o}$ 1 if demand i's time option e, origin option $\bar{o}$, and destination option o is served and this trip is in the forced charging set (C); 0 otherwise
- v_s number of personnel used from shift s
- n_j^t number of vehicles at station j at time interval t before other decisions
- $\tilde{n}_j^t$ number of vehicles that should stay at station j at time interval t for safety or charging reasons
- m_{sj}^t number of personnel of shift s at station j at time interval t before other decisions
- r_{sj}^t number of vehicles relocated by a personnel from shift s to station j finishing at time interval t
- $\overline{r}_{sj}^t$ number of vehicles relocated by a personnel from shift s from station j starting at time interval t
- p_{sj}^t number of personnel of shift s moved to station j finishing at time interval t without a vehicle
- $\overline{p}_{sj}^t$ number of personnel of shift s moved from station j starting at time interval t without a vehicle
- $\tilde{r}_{sbd}^{t(u)}$ number of vehicles relocated by the personnel of shift s from cluster b to cluster d starting at time interval t and finishing at time interval u
- $\tilde{p}_{sbd}^{t(u)}$ number of personnel of shift s moved from cluster b to cluster d starting at time interval t and finishing at time interval u

3.2.4. Formulation

$$\min \underbrace{\overbrace{\sum_i \mathrm{P}_i z_i}^{\text{gross revenue}} - \overbrace{\sum_{i,e} \tilde{\mathrm{D}}_i^e \tilde{z}_i^e}^{\text{start time discount}} - \overbrace{\sum_{i,o,e} \mathrm{D}_i^o z_i^{eo}}^{\text{drop-off discounts}} - \overbrace{\sum_{i,o,e} \overline{\mathrm{D}}_i^{\bar{o}} \overline{z}_i^{e\bar{o}}}^{\text{pick-up discounts}}}_{\text{net revenue}}$$
$$- \left(\underbrace{\overbrace{\sum_{j,t} \overline{\mathrm{RC}}_j \overline{r}_{sj}^t}^{\text{station}\to\text{cluster}} + \overbrace{\sum \widetilde{\mathrm{RC}}_{bd} \tilde{r}_{sbd}^{t(u)}}^{\text{cluster}\to\text{cluster}} + \overbrace{\sum_{j,t} \mathrm{RC}_j r_{sj}^t}^{\text{cluster}\to\text{station}}}_{\text{relocation cost}} \right) - \underbrace{\sum_s \mathrm{PC}_s v_s}_{\text{personnel cost}} \quad (8)$$

subject to

$$\sum_{e \in E_i} \tilde{z}_i^e = z_i \qquad \sum_{o \in O_i^e} z_i^{eo} = \tilde{z}_i^e \qquad \sum_{o \in \overline{O}_i^e} \overline{z}_i^{e\bar{o}} = \tilde{z}_i^e \qquad \forall i \text{ and } i, e \quad (9)$$

$$n_j^{t+1} = n_j^t - \sum_{\substack{i,e:\mathrm{start}(i,e)=t \\ \bar{o}:\mathrm{origin}(i,e,\bar{o})=j}} \overline{z}_i^{e\bar{o}} - \sum_s \overline{r}_{sj}^t + \sum_{\substack{i,e:\mathrm{end}(i,e)=t \\ o:\mathrm{dest}(i,e,o)=j}} z_i^{eo} + \sum_s r_{sj}^t \qquad \forall j, t \backslash t_{\mathrm{last}} \quad (10)$$

$$n_j^t \geq \sum_{\substack{i,e:\mathrm{start}(i,e)=t \\ \bar{o}:\mathrm{origin}(i,e,\bar{o})=j}} \overline{z}_i^{e\bar{o}} + \sum_s \overline{r}_{sj}^t + \tilde{n}_j^t \qquad n_j^t \leq \mathrm{CAP}_j^t \qquad \forall j, t \quad (11)$$

$$0 \leq n_j^{t_{\mathrm{last}}} - \sum_{\substack{i,e:\mathrm{start}(i,e)=t_{\mathrm{last}} \\ o:\mathrm{origin}(i,e,\bar{o})=j}} \overline{z}_i^{e\bar{o}} - \overline{r}_{sj}^{t_{\mathrm{last}}} + \sum_{\substack{i,e:\mathrm{end}(i,e)=t_{\mathrm{last}} \\ \mathrm{dest}(i,e,o)=j}} z_i^{eo} + r_{sj}^{t_{\mathrm{last}}} \leq CAP_j^{t_{\mathrm{last}}} \qquad \forall j \quad (12)$$

$$\sum_j m_{sj}^{\mathrm{start}(s)} = v_s \qquad \sum_j m_{sj}^{\mathrm{end}(s)} = v_s \qquad \forall s \quad (13)$$

$$m_{sj}^{t+1} = m_{sj}^t - \overline{p}_{sj}^t - \overline{r}_{sj}^t + p_{sj}^t + r_{sj}^t \qquad \forall s, j, t \backslash t_{\mathrm{last}} \quad (14)$$

$$\sum_d \tilde{r}_{sbd}^{t(u)} = \sum_{j \in J_b} \overline{r}_{sj}^{t-\mathrm{d.from}(j)} \qquad \sum_d \tilde{r}_{sdb}^{u(t)} = \sum_{j \in J_b} r_{sj}^{t+\mathrm{d.to}(j)} \qquad \forall s, b, t \quad (15)$$

$$\sum_{d} \tilde{p}^{t(u)}_{sbd} = \sum_{j \in \bar{J}_b} \overline{p}^{t-\mathrm{m.from}(j)}_{sj} \qquad \sum_{d} \tilde{p}^{u(t)}_{sdb} = \sum_{j \in \bar{J}_b} p^{t+\mathrm{m.to}(j)}_{sj} \qquad \forall s, b, t \quad (16)$$

$$\tilde{n}^t_j \geq \sum_{\substack{i,e,\bar{o},o: \\ \mathrm{dest}(i,e,o)=j \\ \mathrm{end}(i,e) \leq t \leq \mathrm{end}(i,e)+\mathrm{SG}}} z^{eo}_i \qquad \tilde{n}^t_j \geq \sum_{\substack{(i,e,\bar{o},o) \in C: \\ \mathrm{dest}(i,e,o)=j \\ \mathrm{end}(i,e) \leq t \leq \mathrm{c.end}(i,e)}} \hat{z}^{e\bar{o}o}_i \qquad \forall j, t \quad (17)$$

$$\hat{z}^{e\bar{o}o}_i \geq z_i + \tilde{z}^e_i + \overline{z}^{e\bar{o}}_i + z^{eo}_i - 3 \qquad (i, e, o, \bar{o}) \in C \quad (18)$$

$$z_i, \tilde{z}^e_i, z^{eo}_i, \overline{z}^{e\bar{o}}_i, \hat{z}^{e\bar{o}o}_i \in \{0, 1\} \qquad \forall i, e, o, \bar{o} \quad (19)$$

$$v_s, n^t_j, \tilde{n}^t_j, m^t_{sj}, r^t_{sj}, \overline{r}^t_{sj}, p^t_{sj}, \overline{p}^t_{sj}, \tilde{p}^{t(u)}_{sbd}, \tilde{r}^{t(u)}_{sbd} \in \{0\} \cup \mathbb{Z}^+ \qquad \forall s, j, t, b, d \quad (20)$$

The FOOM is formulated by Eqs. 8-20. The formulation aims to minimize the cost of operating the system. The model inputs are (i) the properties of the system (e.g. driving and moving duration between stations, relocation personnel shifts), (ii) the cluster sets created by the ASCM (see Section 3.1), and (iii) demand requests to be served (with alternative start time, origin and destination options). The model output includes the most profitable set of demand (options) to be served and the relocation operations that need to be executed. This model regards each vehicle and personnel as unit flows in separate layers of parallel networks (Fig. 2). The model does not directly check the charging levels of each electric vehicle but eventually becomes charging level feasible with additional constraints proposed by the JFM (see Section 3.3).

Objective 8 maximizes the profit. We assume that every alternative start time, pick-up or drop-off option offered by the system discount the price of the service. The net revenue earned from a single trip is equal to the gross revenue earned from this trip minus the discounts occurring as a result of offering the user start time, and pick-up and drop-off stations different than the user's original choices. The second part of the objective function contains the cost of relocation operations and the personnel hiring. Since the relocation operations are assumed to be done in three separate steps, (1) from station j to the cluster node of j, then (2) from the cluster node of j to cluster node of l, and finally (3) from the cluster node of l to station l, the cost of relocation is also calculated in three separate steps. Personnel cost is calculated by multiplying the unit cost of personnel times the number of personnel working at each shift.

Constraints 9 are used to connect demand (z_i) and option($\tilde{z}^e_i, \overline{z}^{e\bar{o}}_i, z^{eo}_i$) variables. Constraints 9a ensure that if demand i is served, only one of the start time option (e) is selected. If demand i is not served, then none of the start time options is served. We use similar approaches for the other two constraints. Constraints 9b and c ensure that if one of the start time options is served ($\tilde{z}^e_i$), one of the origin ($\overline{z}^{e\bar{o}}_i$) and destination ($z^{eo}_i$) options are served.

Constraints 10 ensure vehicle flow conservation is satisfied. The number of vehicles at station j at the beginning of time interval $t+1$ is equal to the number of vehicles at station j at the beginning of time interval t plus the net number of vehicles arriving at the station at time interval t. The net number of vehicles arriving at the station is equal to the number of vehicles arriving from rentals or relocation operations minus the number of vehicles departing for rentals or relocation operations from station j at time interval t. The flow diagram located on the left of Fig. 3 illustrates this relationship.

Constraints 11a require that the number of vehicles at station j at the beginning of time interval t is more than the sum of the number of demand ($\overline{z}^{e\bar{o}}_i$) and driving operations ($\overline{r}^t_{sj}$) destined from this station, and the number of vehicles forced to be charged ($\tilde{n}^t_j$) at this station during time interval t. With these constraints, we ensure that any order of driving operations and rentals departing from this station is feasible. Constraints 11b restrict the number of vehicles at the station with the capacity of the station.

Constraints 12 are introduced to the model to restrict the number of vehicles at each station at the end of the last time intervals. Number of vehicles at the beginning of the last time interval plus the number of vehicles arriving at the station from rentals or relocation operations minus the number of vehicles leaving the station for rentals or relocation operations should be between 0 and the capacity of the station.

Constraints 13 are used to keep track of the number of personnel assigned to each shift. Constraints 13a and b require that the sum of the number of personnel from shift s (m^t_{sj}) at all stations at the beginning and end of time intervals of the same shift should be equal to the number of personnel assigned (v_s) to this shift.

Constraints 14 are flow conservation equations for personnel. The number of personnel at station j at the beginning of time interval $t+1$ from shift s is equal to the number of personnel at station j at the beginning of time interval t plus

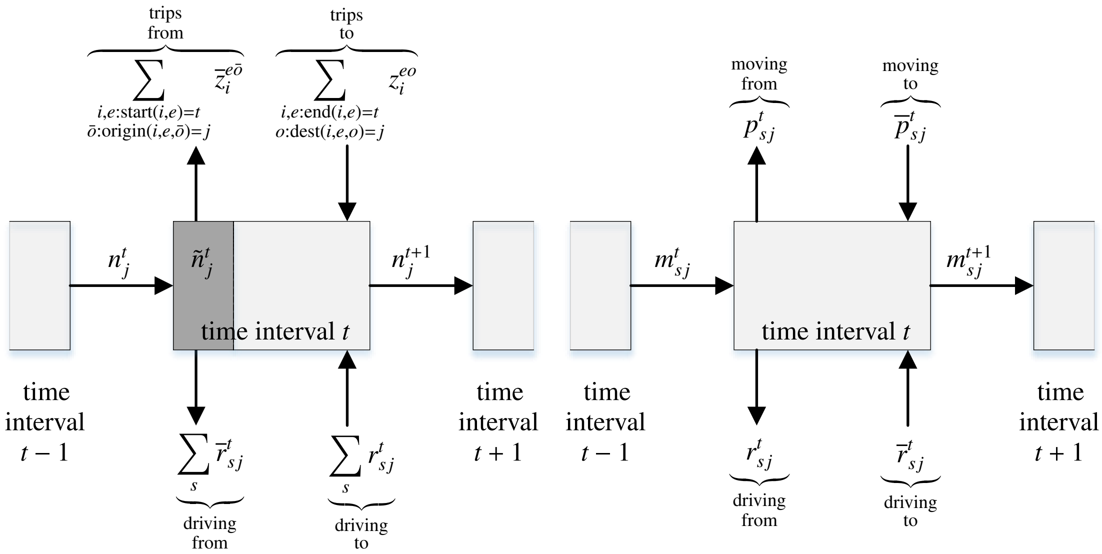

Figure 3: Flow diagram of vehicles (left) and personnel shift s (right) in station j at time interval t.

the net number of personnel arriving at the station from shift s. The net number of personnel arriving at the station is equal to the number of personnel arriving at the station with (r_{sj}^t) or without (p_{sj}^t) vehicles minus the number of personnel departing from the station with ($\overline{r}_{sj}^t$) or without vehicles ($\overline{p}_{sj}^t$). The flow diagram located on the right of Fig. 3 illustrates this relationship.

Constraints 15 and 16 are required to have feasible relocation arcs that form feasible relocation operations when they are joined. They are simply flow conservation equations of relocation flows for the cluster nodes. Constraints 15a and 16a require that the number of relocation flows with and without vehicle originating from the cluster node b at time interval t should be equal to the sum of the relocation flows arriving to cluster node b at time interval t from the nodes of cluster b. Similarly, constraints 15b and 16b require that the number of relocation flows with and without vehicle destined to b at time interval t should be equal to the sum of the relocation flows departing from cluster node b at time interval t to the nodes of cluster b.

Constraints 17 are required to force vehicles to stay at the stations for either safety or charging purposes. Constraints 17a consider that the rented vehicles could be returned to their destination station later than the reserved end time. To ensure the quality of service, we do not allow users to rent these returned vehicles for a while. We keep a spot with the returned vehicle at the destination station starting from the end time interval of the demand for safety gap (SG) many time intervals. With these constraints, even if the vehicle is returned later than the reservation end time, as long as it is within safety gap limits, it does not leave any accepted rentals without vehicles. Constraints 17b ensure that if any trips are in the forced charging set C, the vehicle that is serving one of these exact trips should be kept for a while at the station until it is charged enough to cover the energy consumption of the trip. Note that, constraints 18 force the variable $\hat{z}_i^{e\bar{o}o}$ to be 1 only if exactly the same demand is served with the same options. In other words, if the demand i is not served with start time option e, origin option $\bar{o}$ or destination option o which was forced to be charged in one of the previous iterations, then any vehicles will not be forced to stay and charged at the station after serving the trip.

Constraints 19 and 20 define the domains of each variable. Variables related to demand and their options are set to be binary. The rest of the variables that keep track of the number of vehicles and personnel at each node, and the number of relocation operations executed are all set to be non-negative integers.

Constraints 10-17 are similar to those used in Boyacı et al. (2017). However, the implementation of spatial and temporal flexibility, and the introduction of the asymmetric station clustering procedure require the modification of the modeling approach. Specifically, in constraints 10-12, instead of demand, we use origin and destination options of the demand requests. Similarly, in constraints 15 and 16, the superscripts of r_{sj}^t ($\overline{r}_{sj}^t$) and p_{sj}^t ($\overline{p}_{sj}^t$) are changing according to the travel times from (to) these nodes to (from) their cluster nodes. There are also additional unique constraints in this proposed model (e.g. 9, 18) that are not proposed by Boyacı et al. (2017).

3.3. *Joint (Personnel and Vehicle) Flow Model (JFM)*

The JFM considers both the movements of personnel and vehicles, and charging levels of the vehicles at the same time for all shifts together. Although it is possible and easier to solve personnel and vehicle flow problems separately, dealing with both problems within a single model provides better results in finding charging level feasible solutions. For given vehicle and personnel flows, taking both the personnel assignment and vehicle flow decisions at the same time with the JFM explores the entire solution space whereas solving each problem sequentially discards part of the solution space.

3.3.1. *Sets and indices*

$k \in K$	(individual) vehicles
$q \in Q$	(individual) personnel
$s \in S$	working shifts
$j \in J$	stations
$t \in T$	time intervals
$b, d \in B$	clusters
$f \in F$	(relocation and rental) flows

3.3.2. *Parameters*

n^t_j / m^t_{sj}	number of vehicles/personnel at station j at the beginning of time interval t
r^t_{sj} / p^t_{sj}	number of personnel finishing driving/moving to station j at time interval t
$\overline{r}^t_{sj} / \overline{p}^t_{sj}$	number of personnel starting driving/moving from station j at time interval t
$\tilde{r}^{t(u)}_{sbd} / \tilde{p}^{t(u)}_{sbd}$	number of personnel driving/moving from cluster node b to d starting at time interval t
v_s	number of personnel from shift s
origin(f)/dest(f)	origin/destination node of flow f
start(f)/end(f)	start/end time interval of flow f
c.end(f)	charging end time interval of flow f
dur(f)/dis(f)	duration/distance of flow f
shift(f)	shift of relocation flow f
type(f)	type of flow f (D: driving, M: moving, S: rental, R: relocation = D∪M, V: vehicle = D∪S)
$J_b / \overline{J}_b$	set of stations of driving/moving cluster b
d.from(j)/m.from(j)	number of time intervals needed to drive/move from station j to its cluster node
d.to(j)/m.to(j)	number of time intervals needed to drive/move to station j from its cluster node
MC_f	minimum charging level required to serve flow f
$\Delta\mathrm{C}_f$	percent of charging level used to serve flow f
CR_j	charging rate of station j (percent charged per time interval)
C	set of flows that need charging

3.3.3. *Variables*

$\tilde{x}^t_{jk} / x^t_{sjq}$	1, if {vehicle k} / {personnel q of shift s} is at station j before any decisions at time interval t; 0 otherwise
$\tilde{y}_{kf} / y_{qf}$	1, if flow f is served by {vehicle k} / {personnel q}; 0 otherwise
c^t_k	charging level of vehicle k before any decisions at time interval t
$\tilde{c}_{kf}$	1, if flow f served by vehicle k is assumed to spend no charge (infeasible charging); 0 otherwise

3.3.4. Formulation

$$\min \sum_{k,f} \tilde{c}_{kf} \tag{21}$$

subject to

$$\sum_{q} x^t_{sjq} = m^t_{sj} \qquad \sum_{j} x^t_{sjq} \leq 1 \qquad \forall s, j, t \text{ and } s, q, t \tag{22}$$

$$\sum_{k} \tilde{x}^t_{jk} = n^t_j \qquad \sum_{j} \tilde{x}^t_{jk} \leq 1 \qquad \forall j, t \text{ and } k, t \tag{23}$$

$$x^{t+1}_{sjq} = x^t_{sjq} - \sum_{\substack{q,f:\text{start}(f)=t,\text{origin}(f)=j,\\ \text{shift}(f)=s,\text{type}(f)=\text{D}}} y_{qf} + \sum_{\substack{q,f:\text{end}(f)=t,\text{dest}(f)=j,\\ \text{shift}(f)=s,\text{type}(f)=\text{D}}} y_{qf} \qquad \forall s, j, q, t \backslash t_{\text{last}} \tag{24}$$

$$\tilde{x}^{t+1}_{jk} = \tilde{x}^t_{jk} - \sum_{f:\text{start}(f)=t,\text{origin}(f)=j,\text{type}(f)=\text{V}} \tilde{y}_{kf} + \sum_{f:\text{end}(f)=t,\text{dest}(f)=j,\text{type}(f)=\text{V}} \tilde{y}_{kf} \qquad \forall j, k, t \backslash t_{\text{last}} \tag{25}$$

$$\sum_{\substack{q,f:\text{start}(f)=t,\text{origin}(f)=j,\\ \text{shift}(f)=s,\text{type}(f)=\text{D}}} y_{qf} = \overline{r}^t_{sj} \qquad \sum_{\substack{q,f:\text{end}(f)=t,\text{dest}(f)=j,\\ \text{shift}(f)=s,\text{type}(f)=\text{D}}} y_{qf} = r^t_{sj} \qquad \forall s, j, t \tag{26}$$

$$\sum_{\substack{q,f:\text{start}(f)=t,\text{origin}(f)=j,\\ \text{shift}(f)=s,\text{type}(f)=\text{M}}} y_{qf} = \overline{p}^t_{sj} \qquad \sum_{\substack{q,f:\text{end}(f)=t,\text{dest}(f)=j,\\ \text{shift}(f)=s,\text{type}(f)=\text{M}}} y_{qf} = p^t_{sj} \qquad \forall s, j, t \tag{27}$$

$$\sum_{\substack{q,f:\text{origin}(f)\in J_b,\text{dest}(f)\in J_d,\\ \text{shift}(f)=s,\text{type}(f)=\text{D},\\ \text{start}(f)=t-\text{d.from}(\text{origin}(f))\\ \text{end}(f)=u+\text{d.to}(\text{dest}(f))}} y_{qf} = \tilde{r}^{t(u)}_{sbd} \qquad \sum_{\substack{q,f:\text{origin}(f)\in \overline{J}_b,\text{dest}(f)\in \overline{J}_d,\\ \text{shift}(f)=s,\text{type}(f)=\text{M},\\ \text{start}(f)=t-\text{m.from}(\text{origin}(f))\\ \text{end}(f)=u+\text{m.to}(\text{dest}(f))}} y_{qf} = \tilde{p}^{t(u)}_{sbd} \qquad \forall s, b, d, t \tag{28}$$

$$\sum_{k,f:\text{start}(f)=t,\text{origin}(f)=j,\text{type}(f)=\text{D}} \tilde{y}_{kf} = \overline{r}^t_{sj} \qquad \sum_{k,f:\text{end}(f)=t,\text{dest}(f)=j,\text{type}(f)=\text{D}} \tilde{y}_{kf} = r^t_{sj} \qquad \forall s, j, t \tag{29}$$

$$\sum_{k} \tilde{y}_{kf} = 1 \qquad \sum_{k} \tilde{y}_{kf} = \sum_{q} y_{qf} \qquad f : \text{type}(f) = \text{S and type}(f) = \text{D} \tag{30}$$

$$c^{t+1}_k \leq c^t_k + \sum_{j} \text{CR}_j \tilde{x}^t_{jk} - \sum_{f:\text{end}(f)=t,\text{type}(f)=\text{V}} \left[\Delta \text{C}_f + \text{CR}_j\right] \left(\tilde{y}_{kf} - \tilde{c}_{kf}\right) \qquad \forall k, t \backslash t_{\text{last}} \tag{31}$$

$$c^t_k \geq \text{MC}_f \tilde{y}_{kf} \qquad \forall k, t, f : \text{start}(f) = t, \text{type}(f) = \text{V} \tag{32}$$

$$\sum_{j} \tilde{x}^t_{jk} + \sum_{\substack{k,f:\text{start}(f)>t\\ \text{end}(f)\leq t}} \tilde{y}_{kf} \leq 1 \qquad \sum_{\substack{k,f:\text{start}(f)\geq t\\ \text{end}(f)\leq t}} \tilde{y}_{kf} \leq 1 \qquad \forall k, t \tag{33}$$

$$\tilde{c}_{kf} \leq y_{kf} \qquad \forall f, k : \text{type}(f) = \text{S} \tag{34}$$

$$\tilde{x}^t_{jk} \geq \tilde{y}_{kf} \qquad \forall f \in C, k, j = \text{dest}(f), \text{end}(f) \leq t \leq \text{c.end}(f) \tag{35}$$

$$x^t_{sjq}, y_{qf} \in \{0, 1\} \qquad \forall s, j, t, q, f : \text{type}(f) = \text{R} \tag{36}$$

$$\tilde{x}^t_{jk}, \tilde{c}_{kf} \in \{0, 1\}, c^t_k \in [0, 1] \qquad \tilde{y}_{kf} \in \{0, 1\} \qquad \forall j, t, k, f : \text{type}(f) = \text{V and } k, f : \text{type}(f) = \text{V} \tag{37}$$

The JFM converts unit personnel and vehicle flows into feasible assignments for each individual personnel and vehicles. Note that, any solution created by the FOOM corresponds to at least one feasible solution that assigns each personnel and vehicle to unique flows. However, these solutions are not necessarily charging level feasible for the vehicles. That is the reason, in this model, our aim is to minimize the number of infeasible charging levels occurring in the objective with Objective 21.

Constraints 22a and 23a postulate that the number of personnel and vehicles at each node j during each time interval t stated by the FOOM should be equal to the number of individual personnel and vehicles assigned to the same nodes respectively. Note that constraints 22a also iterate over the shifts since the FOOM differentiates personnel according to their shifts. Constraints 22b and 23b ensure that any individual personnel and vehicles cannot be in more than one stations during the same time interval respectively.

Constraints 24 and 25 are flow conservation equations for individual personnel and vehicles respectively. These constraints ensure that if personnel q from shift s (vehicle k) is at station j at time interval $t + 1$, either the personnel (vehicle) was there during time interval t and not depart for any relocation (and trip) starting at time t or has arrived from another relocation (and trip) ending at time interval t.

Constraints 26, 27 and 28 ensure that every relocation flow is covered by one and only one personnel. As stated before, relocation operations are modeled with three separate flows instead of one in the Flexible Operations Optimization Model: A relocation with (without) vehicle from station j to l is modeled as (1) from station j to its cluster node B_j ($\overline{B}_j$), (2) from cluster node B_j ($\overline{B}_j$) to cluster node of station l, B_l ($\overline{B}_l$) and (3) from cluster node B_l ($\overline{B}_l$) to station l. Constraints 26a (26b) deal with the first (third) step of these flows and postulate that the number of relocation operations with vehicles from (to) station j starting (ending) at time interval t executed by a personnel from shift s stated by the FOOM should be equal to the total number of individual personnel from shift s assigned to exactly the same number of relocation operations with vehicles starting (ending) at time interval t from (to) station j. Constraints 27a (27b) postulate exactly the same restrictions for the relocation operations without vehicles for the personnel flows. Constraints 28a and 28b deal with the second step of relocations with and without vehicles respectively. Constraints 28a (28b) postulate the number of relocation operations with (without) vehicles from stations that belong to cluster b to stations that belong to cluster d, reaching to cluster b at time interval t should be equal to the number of individual relocation personnel assigned to these operations.

Constraints 29a (29b) ensure that the number of relocation operations with vehicles starting (ending) at time interval t from (to) station j stated by the FOOM is equal to the total number of separate vehicles assigned to the flows starting (ending) at time interval t from (to) station j.

Constraints 30a postulate that each accepted rental should be assigned to one and only one vehicle flows. Similarly, constraints 30b ensure that the total number of relocation flows with vehicles assigned to any individual personnel should be equal to the total number of separate vehicle flows assigned to the same flows. Note that constraints 30a and 30b hold for every flow from the set of trips (S) and relocation operations with vehicles (D) respectively.

Constraints 31 are used to keep track of the charging level of each vehicle. The charging level of a vehicle at the beginning of time interval $t + 1$ should be less than or equal to the charging level of the same vehicle at the beginning time interval t, plus the charging rate of the station that the vehicle is being charged (if it is parked at a station), minus the charging level spend for the latest flow the vehicle is serving. The constraints are holding with inequality rather than equality since the vehicle battery cannot be charged more than its capacity. Note that, we subtract the charging rate if a flow is served since variable $\tilde{x}^t_{jk}$ shows the state of the vehicle before any decisions including flows. If a vehicle is serving a flow, it cannot be charged during the period it starts serving the flow. We also subtract $\tilde{c}_{kf}$ from $\tilde{y}_{kf}$. If a flow cannot be served because of the charging level at the end of the flow, $\tilde{c}_{kf}$ gets the value of 1 which shows us we need to force the vehicle to charge its batteries that serve flow f after serving in the next FOOM. Last but not least, although in reality, the charging levels of the vehicles drop continuously, we deduce the charging level of the vehicles at the end of the flows. Since we are comparing the charging levels at the beginning of the flows with the minimum charging levels needed to serve the flows, dropping charging levels at the end of the flows is an acceptable assumption.

Constraints 32 ensure that the vehicle serving flow f has enough energy in its battery. As stated before, the charging levels of the vehicles are checked only at the beginning of the flows.

Constraints 33a and 33b are redundant cuts added to the model to decrease the solution space and time eventually. Constraints 33a state that the same vehicle cannot be in two places at the same time. It cannot stay at different nodes while serving different flows happening at the same time. If flow f is served by vehicle k, at the first time interval of flow f vehicle is at the origin node of flow f. That is the reason, in constraints 33a, the second summation starts from the time interval after the start time interval of the flows. Constraints 33b state that every vehicle can serve at most one flow at each time interval.

As stated before, $\tilde{c}_{kf}$ gets the value of 1 if, at the beginning of flow f, the charging level of vehicle k is not enough to serve the same flow. With constraints 34, we set the relationship between variables $\tilde{y}_{kf}$ and $\tilde{c}_{kf}$ for the same flow f and vehicle k. If vehicle k is not serving flow f (i.e. $\tilde{y}_{kf} = 0$), then the variable $\tilde{c}_{kf}$, which controls if the vehicle k serve flow f without enough charging level, cannot have value of 1.

At every iteration, the FOOM keeps track of flows that require forced charging after they are being served. With constraints 35, we satisfy these restrictions. If the flow is in the forced charging set (C) then the vehicle serving flow f is forced to stay at the destination station and charge itself after serving f. If the vehicle is in C, it is required to stay at the destination station from flow end time end(f) to charging end time c.end(f).

Constraints 36 and 37 set the domains for the variables used for the individual personnel and vehicles respectively. All the variables except $\tilde{c}_{kf}$ are binary. $\tilde{c}_{kf}$ represents the percent charging level of vehicles at each time interval and can take any values between 0 and 1.

3.4. *Personnel Flow Model (PFM)*

Although the JFM always gives the assignment set that minimizes the number of trips that are served by vehicles with infeasible charging levels, solving the problem by using the JFM is more time consuming than solving personnel and vehicle flow problems separately. In this section, we will describe the PFM that creates assignments for personnel only, and do not consider vehicles and their charging levels at all.

The PFM is similar to the model formulated by Boyacı et al. (2017). However, since in the clustering model we are considering a travel time between nodes and their clusters, to reflect this requirement, appropriate changes have been made in the proposed formulation. The personnel flow problem can be modeled as follows by using the personnel flow related indices, parameters, and variables defined in the JFM in Section 3.3.

$$\min \sum_{s,f,k} \text{dis}(f) y_{qf} \tag{38}$$

subject to

(22), (24), (26), (27), (28), (36).

Since the PFM is not considering vehicles and their charging levels, we use the distances of complete relocation flows formed. Although this approach does not necessarily increase the chance of having charging level feasible assignments for the vehicles in the next stage of the solution, minimizing the distance traveled in relocation operations consumes less energy and eventually keeps on requirements that the sum of the charging levels of vehicles are higher.

Note that, this model can also be solved separately for each shift s without the loss of generality since each relocation operation and as a result, the flows generated from them are assigned to the shift executing them.

3.5. *Vehicle Assignment Simulation (VAS)*

VAS is a fast discrete-event simulation approach which uses a simple policy to assign vehicles to trip requests. When there are more than one vehicles which are located at the origin station of a trip or driving request, the simulation assigns the vehicle with the highest charging level. The assignment policy orders the requests in decreasing order of charging level need and the vehicles in decreasing order of charging levels. Subsequently, the requests and vehicles are assigned to each other in the same order. This myopic approach returns a charging level feasible solution in most of the cases. However, it is not exploring the entire solution space, therefore it may miss a feasible solution especially in cases that vehicles are more intensively utilized.

The algorithm works as follows. We first create vehicle objects for each vehicle (k) and vehicle sets (K_j^t) for each station and time interval. These sets are initialized with the initial number of vehicles (n_j^0) given as input to the model. In other words, we place a given number of vehicles to each station set for the initial time interval. The algorithm iterates according to the increasing order of time intervals, followed by the random order of stations. At each iteration, the algorithm first identifies the flows starting at the current time interval from the current station. The flows are ordered in decreasing order of their minimum charging level requirements (MC_f). The vehicles belonging to the vehicle set of the current time interval and station are ordered in the decreasing order of their charging levels (c_k^t). Subsequently, the algorithm matches each flow with each vehicle on one-to-one basis. Since the simulation takes the FOOM and PFM outputs as input, the number of flows cannot exceed the number of vehicles. However, the charging level of the vehicles might not be suitable to serve certain flows. If the charging level of any vehicles is lower than the minimum charging level requirement of the assigned flows, the solution being created by the VAS is charging level infeasible, and the algorithm terminates. If there are feasible charging level assignments of vehicles to flows, the algorithm continues by updating the charging levels of the vehicles and locations that are assigned to flows. At the end of each iteration, the vehicles that are not assigned to any flows are transferred to the next time interval. The charging levels of these vehicles are also updated according to the charging rate of the stations (CR_j). If all time intervals and stations have been successfully iterated, the algorithm returns the pairs of vehicles and flows that are assigned to these vehicles. More details regarding the structure and implementation of the VAS are provided by the pseudocode given in Algorithm 1. The algorithm uses as input the accepted rental and relocation flows (F), time intervals (T), stations (J) and initial number of vehicles at each stations (n_j^0) and returns the vehicles (k) and flow pairs that are assigned to these vehicles (y_{kf}).

This algorithm improves the efficiency of the simulation module introduced in Boyacı et al. (2017), through i) the removal of personnel objects and activities from the simulation, and ii) the consideration of trips and relocations of vehicles as identical activities with different charging level requirements. Both of these changes decrease the running time of the simulation by decreasing the number of calculations performed at each iteration.

Input: $f \in F, t \in T, j \in J, n_j^0$
Output: y_{kf}
`// Initialization`
Define vehicle objects k
Define vehicle object sets K_j^t $j \in J, t \in T$
Assign vehicles to set K_j^0 such that $|K_j^0| = n_j^0$ $\forall j$ `// initialize vehicle locations`
Set $c_k^0 = 1$ $\forall k$ `// initialize charging levels`
`// Iteration`
foreach $t \in T$ **do**
    foreach $j \in J$ **do**
        $F^\star = f : \mathrm{start}(f) = t$ and $\mathrm{origin}(f) = j$ `// filter flows`
        Sort $F^\star$ in the decreasing order of MC_f `// sort selected flows`
        $K^\star = K_j^t$ `// filter vehicles`
        Sort $K^\star$ in the decresing order of c_k^t `// sort selected vehicles`
        `// vehicles assigned to flows`
        for $i \leftarrow 1$ **to** $|F^\star|$ **do**
            $f = F^\star_{(i)}$ and $k = K^\star_{(i)}$ `// current vehicle and flow`
            $y_{kf} = 1$ `// assign vehicle to flow`
            if $c_k^t < MC_f$ **then**
                return $\emptyset$ `// infeasible charging level`
            end
            $c_k^{\mathrm{end}(f)} = c_k^t - \Delta \mathrm{C}_f$ `// update vehicle's charging level`
            $K_j^t \leftarrow K_j^t \setminus \{k\}$ and $K_j^{\mathrm{end}(f)} \leftarrow K_j^{\mathrm{end}(f)} \cup \{k\}$ `// update vehicle's location`
        end
        `// vehicles not assigned to flows`
        for $i \leftarrow |F^\star| + 1$ **to** $|K^\star|$ **do**
            $k = K^\star_{(i)}$ `// current vehicle`
            $K_j^t \leftarrow K_j^t \setminus \{k\}$ and $K_j^{t+1} \leftarrow K_j^{t+1} \cup \{k\}$ `// move vehicle to next time interval`
            $c_k^{t+1} = \min\{c_k^t + \mathrm{CR}_j, 1\}$ `// update vehicle's charging levels`
        end
    end
end
return y_{kf} $\forall f$ and k

Algorithm 1: Pseudocode for the Vehicle Assignment Simulation (VAS).

4. Model application

We applied the solution framework represented in Fig. 1. We use the data gathered from the one-way electric carsharing system of Auto Bleue operating in Nice, France (Auto Bleue, 2018). Auto Bleue is operating a one-way system since 2014. We use the anonymized one-way rental and publicly available station location data. In our analyses, we considered only 60 stations located in the city center and excluded the 6 stations in the suburbs (Fig. 4). This was dictated by the fact that these 6 stations were rarely used or not allowed to be used for one-way trips in the data. In order to stress the system resource usage with higher demand levels, we aggregated data from individual days, and added spatial and temporal flexibility for different options in our runs.

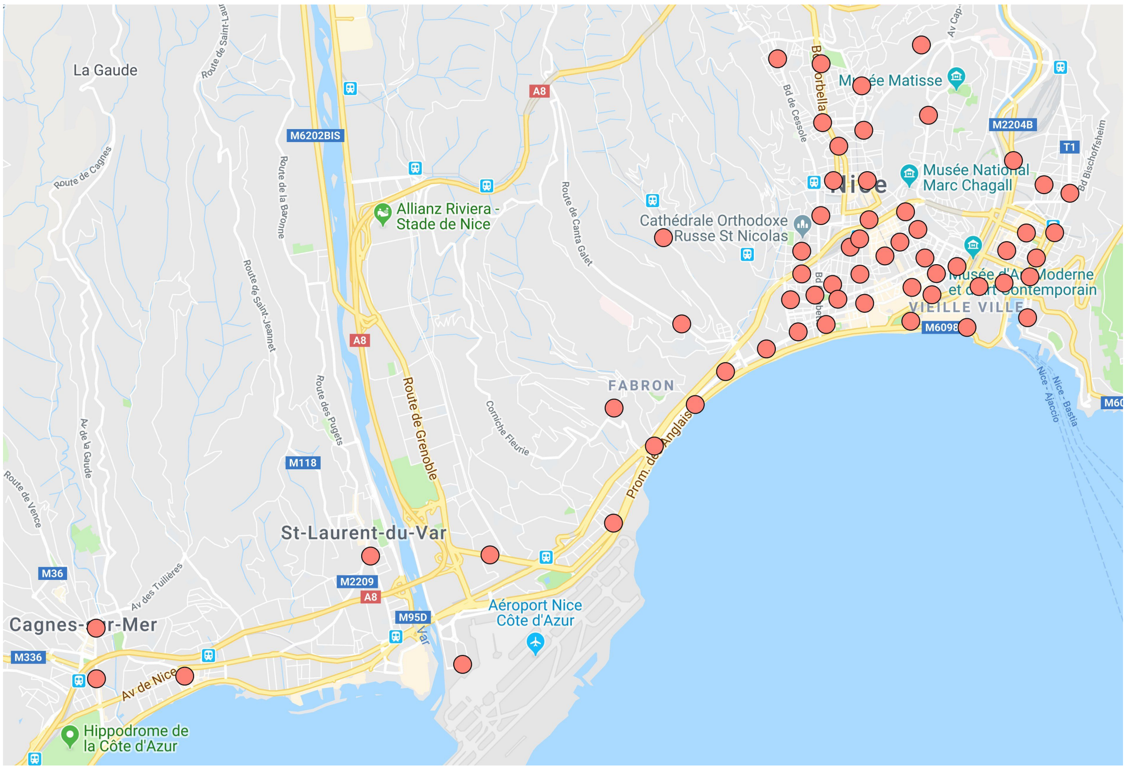

Figure 4: Locations of 60 stations considered in the analyses.

We implemented the model with C# in .NET environment. IBM ILOG Cplex 12.8 is used to solve MILPs. Experiments are conducted on a workstation with Intel Xeon E5-2640 v3 processor.

The proposed solution framework assumes the states of the system components change in discrete times, i.e. time intervals. In our experiments, we set the duration of each time intervals to 15 minutes. A vehicle or a personnel can have only one state during a time interval. The relocation personnel is assumed to either drive a car (driving) or cycle a bike (moving) between stations. We use the driving and cycling distances that are provided by Google Maps API (Google, 2018) and assumed driving and moving speeds are 30 and 15 km/h respectively. Please note that, although the same model could be used for different relocation speeds throughout the day for any station pairs, we keep the same speed for the simplicity of our analyses. We use 4 different personnel shifts: 6:00-09:45, 09:45-13:45, 13:45-17:45 and 17:45-21:45. The other important parameters we used in our analyses are given in Table 2.

4.1. Station clustering

We started our analyses with the station clustering for both driving and moving operations. Since there are 60 stations and 64 time intervals (from 6:00 to 22:00 with 15 minute intervals), without clustering, we need to deal with over 450,000 ($2\times60\times59\times64$) relocation variables. Using station clustering (see Section 3.1), the relocation operations

Table 2: Important parameter values used in the analyses.

relocation personnel cost (€/shift):	72
relocation (fuel) cost (€/km):	0.02
charging duration per distance (sec/km):	240
vehicle range (km):	120
minimum trip start charging level	40%
min-max (mean) trip distance (km):	3-40 (18)
min-max (mean) trip duration (hour):	.2-10 (.8)
rental fee (€/hour):	18
spatial flexibility discount (€/km):	4
temporal flexibility discount (€/hour):	9
minimum time between pick-up and drop-offs (min):	30

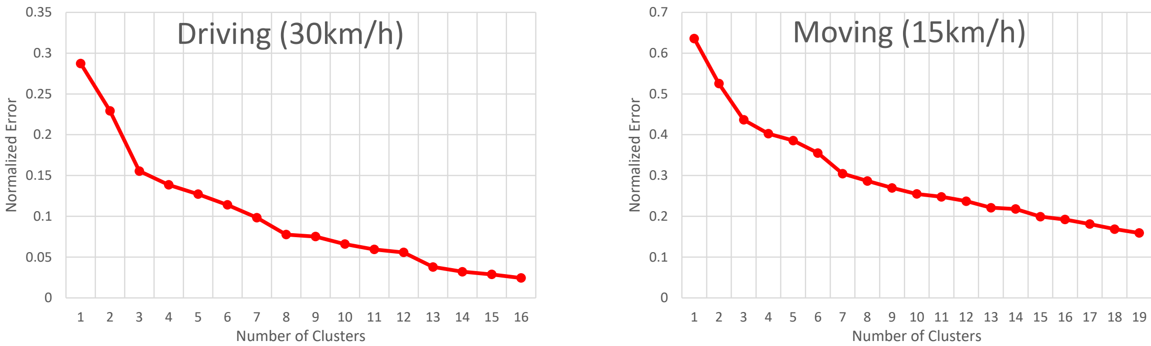

Figure 5: Normalized error emerging from using different number of clusters for driving (left) and moving (right) operations.

are modeled through cluster nodes, therefore the number of relocation arcs and eventually variables is decreased and the model can be solved for real-world instances.

In clustering, we use the mathematical model presented in Section 3.1. We solve the mathematical model for different number of clusters and relocation operations. We generate different clusters for driving and moving operations separately since the relocation time between each pair of stations differs according to the type of relocation operation.

In order to decrease the size of the clustering model, we use a preprocessing step to identify the stations that will take the same number of time intervals to travel to all other stations. In other words, we identify stations that should be at the same cluster in an optimal solution. In the mathematical model, we consider these station groups as the same stations. This approach decreases the problem size for driving from 60 stations to 29 station groups. Unfortunately, the same approach does not contribute to the reduction of the problem size for the moving operations.

When we apply the clustering model, we use objective function weights (see Objective 1) proportional to the number of relocations happening between station pairs. The assignment of weights to the pairs of stations proportional to their demand for relocation operations leads to the reduction of error due to clustering. However, since the relocation operations are decisions generated by the Flexible Operations Optimization Model, it is not possible to find how many relocations are happening between every station pairs. In order to define this number, we assume that the number of relocations happening between stations (or station groups) should be directly proportional to the demand originating from and destined to these two stations (or station groups). We normalize the weights by dividing all original weights by the number of all relocation operations.

After applying the preprocessing and setting the solution time to 24 hours, we run the algorithm for different number of clusters. The weighted error emerging from the clustering approach for using different number of clusters is given in Fig. 5.

As it is stated above, we are using the clustering approach to have a more efficient solution framework while keeping the accuracy within an acceptable range. Using too many clusters reduces the overestimation error associated with the duration of relocation operations results in a more accurate model albeit with more demanding computational difficulties. In contrast, the use of too few clusters leads to a higher overestimation of the duration of relocations but to an easier problem to solve. After analyzing the relationship between the number of clusters and the associated error (see Fig. 5) we decided that 13 clusters for driving and 15 clusters for moving provide us acceptable efficiency and accuracy levels for the application of our framework to the problem under consideration. The clustering results for the selected number of clusters are demonstrated in Fig. 6. The left and right maps present 13 and 15 clusters used for driving and moving operations respectively. Beyond these points, there is a diminishing rate of improvements in the solutions. The matrices located on the top-left of the maps present the number of intervals to travel from the cluster nodes representing stations on the first column to the cluster nodes representing stations on the first row. Most of the travel from/to cluster nodes to/from stations approximated to take no time. There are only a few nodes (two in driving and eight in moving cases) from/to which it takes a positive number of time intervals to travel. These nodes are shown in the figure with numbers on the upper-left of the nodes. The first number shows the number of time intervals required to travel from this node to the cluster node. Similarly, the second number shows the number of time intervals required to travel to this node from its cluster node. These two variables are represented with u_j and $\overline{u}_j$ in the ASCM (see Section 3.1) respectively.

We measured the error for all our experiments using 13 and 15 clusters for the relocation operations with and without vehicles. The mean error (emerging from the overestimation of the relocation times) among all instances was found to be 2.13%. The distribution of the errors was right-skewed. The median of the errors was 1.39%. There were only a few (around 3.2%) instances with error over 10% and most of the instances (around 87.7%) have an error under 4%. Considering the computational efficiency gains and the small size of the error introduced by the proposed clustering procedure, we consider its results to be acceptable. The error can be decreased (and even completely eliminated) by using more clusters.

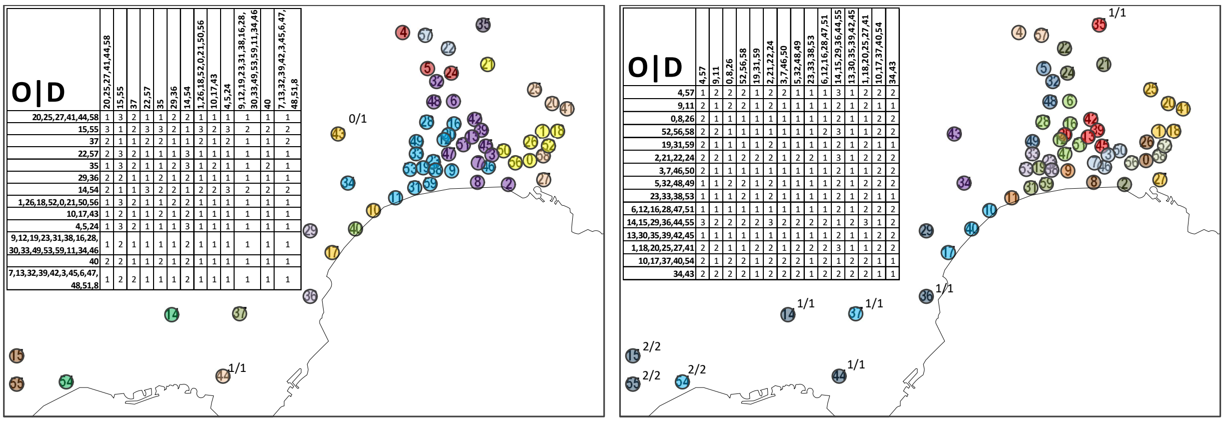

Figure 6: Clusters for driving (left) and moving (right) generated by the ASCM (see Section 3.1). The matrices on the left top show the number of time intervals needed to travel from cluster node of stations on the first column to the cluster node of stations on the first row. If the travel time from/to any node to/from its cluster is different than zero, these nodes contain a value pair on the top-left of their representations. The number pairs show the number of time intervals needed to travel from/to this node to/from its cluster node.

In the subsequent sections, we demonstrate the capabilities of the proposed framework to analyze:

- The effect of spatial and temporal flexibility
- The effect of service type
- The effect of spatial and temporal flexibility discount level
- The decisions and performance of an instance

We would like to reiterate that these findings are relevant to the system under consideration and the incentive values assumed. The objective of these analyses is to demonstrate the capabilities of the proposed framework, to investigate the introduction of temporal and spatial flexibility into one-way electric carsharing systems, and service types. It is clear that the acceptability of the alternative trip offers to those originally requested by the users and the associated incentives depend entirely on the user behavior. User behavior modeling has not been introduce to this paper. However, the outcomes of a user behavior study in terms of the acceptability of alternative trips and incentives can be used as an input to our model. Instead of the assumed values used in this paper, values reflecting the mutual behavior of the users (as defined by user acceptability models) can be used in our model.

4.2. The effect of spatial and temporal flexibility

We first analyze the effects of the spatial and temporal flexibility level offered to the users. We assumed in these experiments that the users are offered alternative trips originating from and destined to different stations, starting at different times than originally requested by the users. We limit the alternative offers to the users to be within a predefined range of the original requests origin and destination stations, and pick-up times. Travel times and driving distances requested by the users are assumed to stay the same. If a user starts a trip earlier or later than the original trip, he/she finishes the trip earlier or later with the same offset. In these instances, we assume the decision on demand is done with perfect information: The entire set of demand requests are known to the solution framework in the process of decision (see Service Type 1 in Section 4.3).

We use 5 different temporal flexibility levels (i.e. 0, 15, 30, 45 and 60 minutes earlier or later) for the time between original and offered pick-up times. It is worth noting here that the temporal and spatial flexibility values used in this section are not the outcome of a user behavior study and as such do not represent the potential behavior of the users of one-way carsharing systems. These values are used parametrically to cover a wide range of potential temporal and spatial flexibility values in order to demonstrate the use of the proposed framework for studying the effect of temporal and spatial flexibility of trips on the system performance. A discount is applied to the price of the service if the offered trip that is different than the original request is chosen to be served by the operator. Serving the trip not at the time of the original request decreases the revenue earned from this trip. Similarly, five different spatial flexibility levels (i.e. 0, 0.25, 0.5, 0.75 and 1km) are used for the distance between the requested and offered origin and destination stations. We limited the distance flexibility to be half of the original trip length: A station more than half of the original travel distance away from the original origin and destination stations are not offered as alternative stations. We conduct the experiments for 3 different demand levels (300, 400 and 500 requests/day) and repeated the experiments with 20 different demand sets.

Figs. 7 and 8 demonstrate the 3 important service measures of one-way systems: demand served, profit earned and personnel count. We create these figures from 20 different instances by running with different flexibility levels. For every instance, we take the runs with no flexibility option as the base case for every instance and depict the average improvements from these base cases to show percent improvements.

For the system under consideration, we observe for the same level of demand, the spatial flexibility has a significant effect on the percentage increase in the number of trips served (top row in Fig. 7 and top-left in Fig. 8). In the base cases of 300, 400 and 500 demand instances, the systems have served on average 207.25, 250.1 and 278.2 customers respectively. With the increase in spatial flexibility, we see 20-30% increase in the number of demand served. This effect is apparent when the spatial flexibility is beyond 250 meters. This is because the system has only 3 station pairs that are within 250 meters distance. Temporal flexibility has also an effect but not as strong as the spatial flexibility. In combination with spatial flexibility increase, the increase in demand has also a positive effect on the percentage of trips served in addition to the base case, i.e. instances with no flexibility. In other words, as the number of demand request increases in the system, the percent increase gained with spatial flexibility also increases. Instances with 300 demand increase the number of demand served by 20%, whereas the instances with 400 and 500 demand increase the number of demand served by 25 and 30% respectively.

The profitability of the system (middle row in Fig. 7 and top-right in Fig. 8) also increases with the increase in spatial flexibility. Compared to the base case with no flexibility, we observe over 10% improvement in the revenue when the spatial flexibility is increased to 1000 meters. Temporal flexibility has also a slight effect but it is nowhere near to the effect of the spatial flexibility. In combination with spatial flexibility increase, the increase in demand increases the profits slightly. This effect is not as strong as the one we see in the percentage increase in the number of trips served but it is apparent that the instances with more demand gains proportionally more. The instances with 500 demand increase their profit over 10% on average with the maximum flexibility level whereas the instances with 300 and 400 demand have a profit increase just under 10%.

We also analyze the number of personnel (bottom row in Fig. 7 and bottom-left in Fig. 8) since it is an additional challenge for one-way systems that the operators do not want to deal with. In those figures, we depict the percent

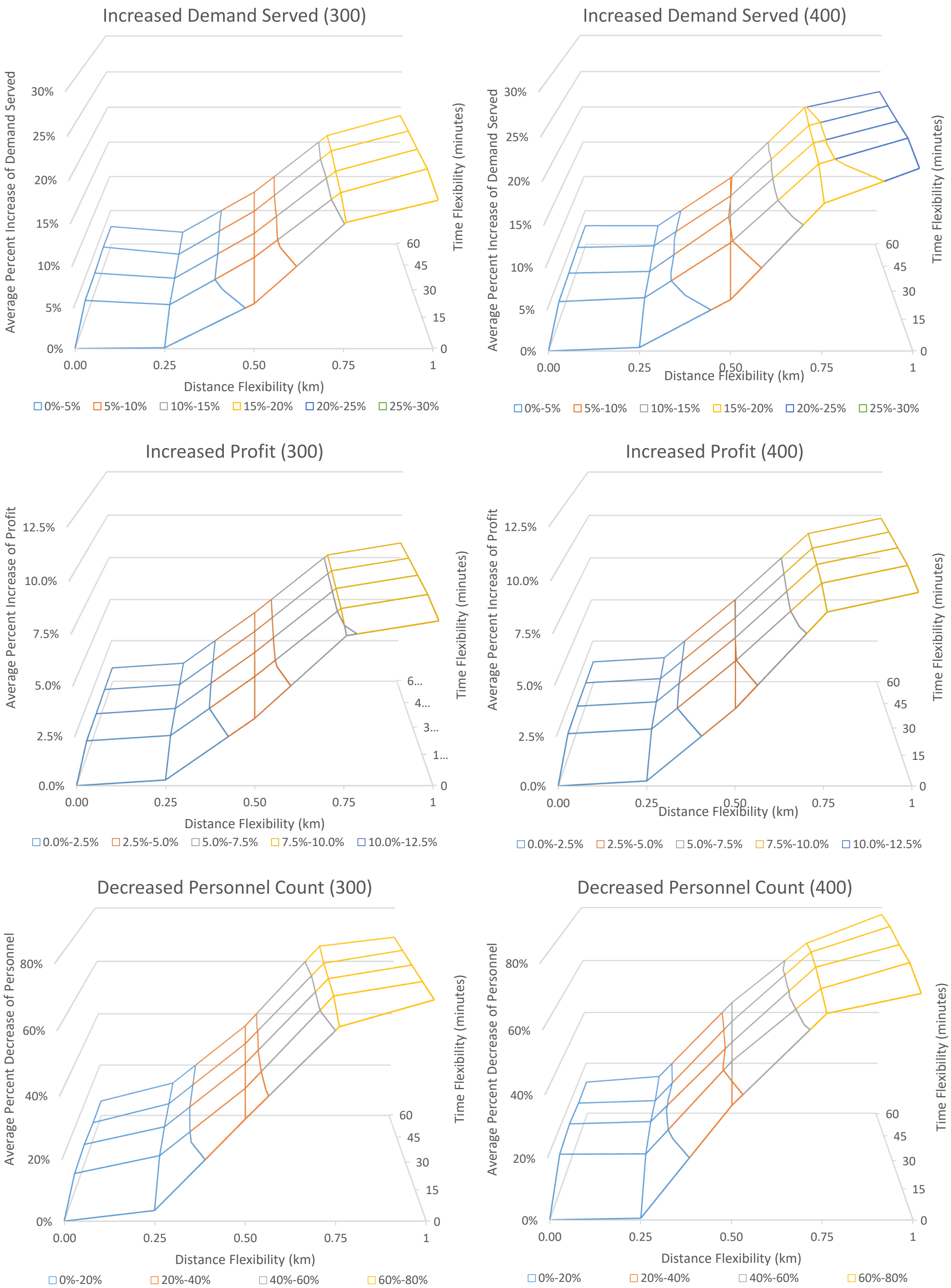

Figure 7: Effects of temporal and spatial flexibility on the number of demand served (top), profit (middle) and personnel count (bottom) for the instances with 300 (left) and 400 (right) demand requests.

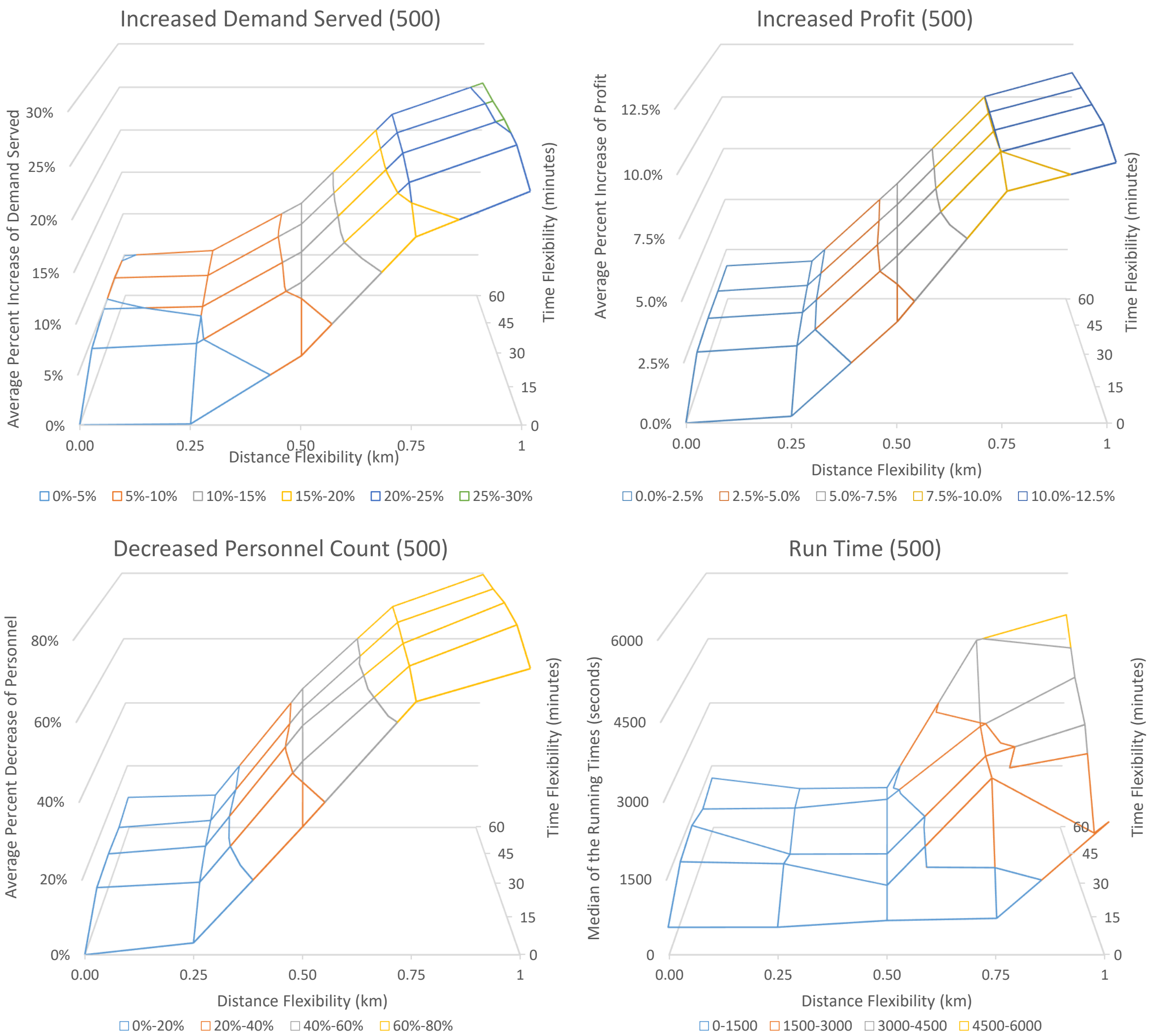

Figure 8: Effects of temporal and spatial flexibility on the number of demand served (top-left), profit (top-right), personnel count (bottom-left) and running times (bottom-right) for the instances with 500 demand requests.

decrease of personnel cost from the base instance with no flexibility. Since we are using standard personnel shifts in our instances, the personnel cost is directly proportional to the number of personnel in the system. In the base instances of 300, 400 and 500 demand requests, on average there are 4.85, 6.2 and 7.6 personnel respectively. In both 3 demand levels, we have observed that the spatial flexibility of 1000 meters decreases the personnel costs by almost 80%. Again the temporal flexibility has a slight effect but it is not as strong as the spatial flexibility.

Finally yet importantly, we analyze the total running times of every instance. The instances with 300 and 400 demand on average were solved in 273 and 412 seconds respectively. We have not observed a significant relationship between the running time, and the predefined range of spatial and temporal flexibility in these instances. The instances with 500 demand on average took 4844 seconds to solve. Median of the solution times was 1218 seconds. After further investigation, we have realized that the mean was influenced by a few runs of high temporal and spatial flexibility. When we analyze all the instances with 500 demand in detail (bottom-right in Fig. 8), we have seen that increasing distance flexibility increases the computational time by a factor of two on average.

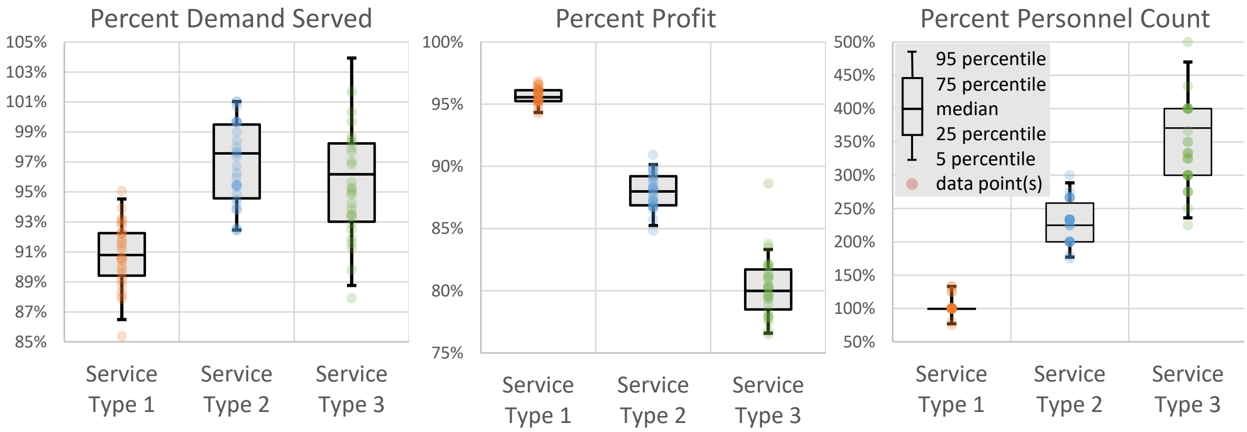

Figure 9: Effects of the service type on the number of demand served (left), profit (middle) and personnel count (right) for the instances with 400 demand. Service Type 0 (i.e. system with no charging needs and no safety gaps that processes the entire demand all at once) is taken as the base case and changes of other three service types are reported in these three graphs.

4.3. The effect of service type

In the second set of analyses, we look for the effect of the 3 different service type configurations compared to the base configuration. In Service Type 1, we assume that the solution framework processes the entire demand all at once and decided accordingly. In other words, the system works with perfect information. In Service Type 2 and 3, the system and eventually the solution framework processes the demand in batches. At every iteration, 5% of the demand requests are known to the system. The solution framework has to inform the users if it will be able to serve the requested demand or not. In Service Type 3, we assume that the system informs the users by providing the exact pick-up time, and origin and destination stations. On the other hand, in Service Type 2, the users are only informed if they are going to be served or not. They do not learn the pick-up time, and origin and destination stations until the entire set of demand requests are processed. If there is no temporal or spatial flexibility introduced to the system, Service Type 2 and 3 return the same results since there will be only one option to serve a demand request. However, when there is flexibility introduced, systems providing Service Type 2 may use the opportunity of delaying decisions and may profit more or serve more requests. With these analyses, we want to demonstrate if there is any benefit in delaying these decisions. Fig. 9 depicts the effect of different service types to 30 different instances with 400 demand, 500 meters of spatial flexibility and 30 minutes of temporal flexibility. We compare all the runs with the instances run under the setting of the Service Type 0. Service Type 0 assumes vehicles do not need any charging and removes the safety gap between consecutive rentals.

The graph in the left of Fig. 9 shows that introducing the safety gap to the model prevents us to serve around 10% more demand if the entire set of demand requests are known in advance. However, surprisingly, Type 2 and Type 3 Service configurations serve more demand. This is most probably because of the limited information about the incoming demand. The system operating with Service Type 1 accepts to serve fewer but more profitable demand requests whereas, since the systems operating with Service Type 2 and 3 give their decisions about each demand request without knowing the incoming demand requests in the future, they choose more but less profitable demand requests.

The graph in the middle of Fig. 9 compares the percent profits of each service type compared to the Type 0. This figure shows that introducing the safety gap decreases the profit by around 4% on average. In addition, when the system is utilizing a configuration as Service Type 3, it loses around 15% of its profit. However, if the time and location information is delayed to the end of the decision process (see Service Type 2), 8% of this loss can be recovered.

The graph on the right of Fig. 9 shows how many personnel are used at each system configuration compared to the Type 0. As expected, the Service Type 0 uses the least personnel on average although there are instances of Service Type 1 using fewer personnel. We see the number of personnel more than doubles when the system is served by Service Type 2. The number of personnel in Service Type 3 is the highest among the configurations. It needs on average 2.5 more personnel than the Type 0 case. The systems that are operated by Service Type 2 and 3 serving more demand but also using more personnel to maintain the service level of the system.

We also compare the average running times of the instances under different service types. The average running times of instances under service type 0, 1, 2 and 3 were 332, 413, 14395 and 15379 seconds respectively. As expected the first two service type runs take extremely short times to run compared to the last two service type runs. Service type 0 and 1 require instances to be solved with the input of the entire demand only once. On the other hand, service type 2 and 3 require instances to receive the demand in (20 different) batches and the entire solution framework (excluding the data preprocessing module) is run for every batch separately. That is the main reason the former two service types' running times are in the order of minutes whereas the latter two service types' running times are in the order of hours. However, please note that the last two methods iterate 20 times in our experiments and it takes on average around 12 minutes to run each iteration.

4.4. The effect of discount level

In this section, we investigate the effect of the discount offered on the profitability and service level of the system. Similar to the first set of analysis, we assume that the users are offered alternative trips originating from and destined to different stations, starting at different times than the original trips requested by the users. However, in this case, we fix the maximum distance between the requested and offered stations to 1km and the trip start time to 60 minutes. We examine the effect of spatial and temporal flexibility discounts on average demand served, the average percentage change in profit, by increasing and decreasing the spatial and temporal flexibility discounts by 10%, 20% and 30% over their initially assumed values. In other words we work with seven different spatial (2.8, 3.2, 3.6, 4, 4.4, 4.8, 5.2€/km) and temporal (6.3, 7.2, 8.1, 9, 9.9, 10.8, 11.7€/hour) flexibility discounts. We conduct the experiments for the demand level of 400 for the Service Type 1. We performed repeated experiments using 30 different demand sets.

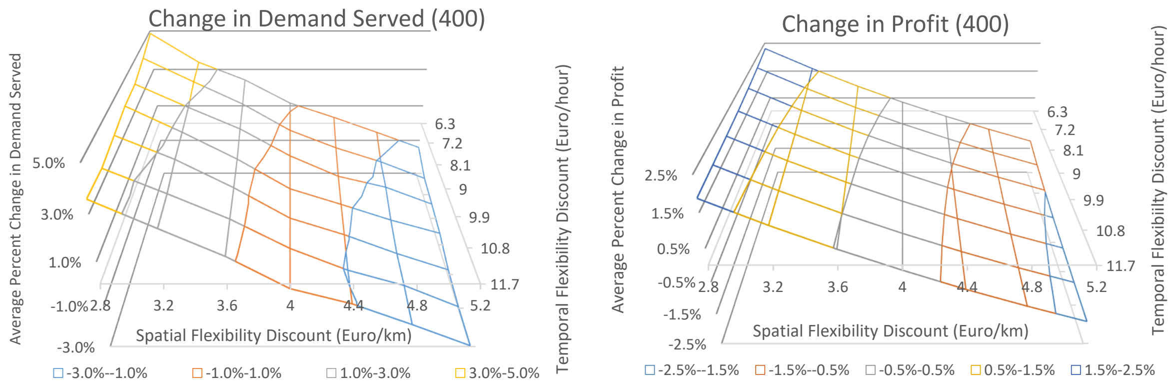

Figure 10: Effects of the temporal and spatial flexibility discount level on the number of demand served (left) and the profit (right).

Fig. 10 summarizes two important performance indicators: demand served (left) and profit (right). In Fig. 11 we also include the change in the discount (top-left), time deviation (top-right), and the average distance between requested and offered origin (bottom-left) and destination (bottom-right) stations per trip. In all figures, average percent changes of the new instances compared to the base instances, i.e. 4€/km spatial flexibility discount and 9€/hour temporal flexibility discount.

As expected, we observe an increase in the number of demand served and profit as the spatial and temporal flexibility discount decreases and vice versa. We observe that the spatial flexibility discount is considerably more effective in improving the profitability and the service level of the system. A 30% decrease in the spatial flexibility discount increases the number of demand served by 4% and the profitability of the system by 2% on average. On the other hand, a 30% decrease in the temporal flexibility discount resulted in on average 1.5% and 0.4% increase in the number of demand served and profitability of the system respectively. When both the spatial and temporal flexibility discounts are decreased by 30%, the number of demand served and profit were increased by over 4.9% and 2.3% respectively. Please note, in the base scenario, the average profit and number of demand served were 4703€ and 305.6 respectively.

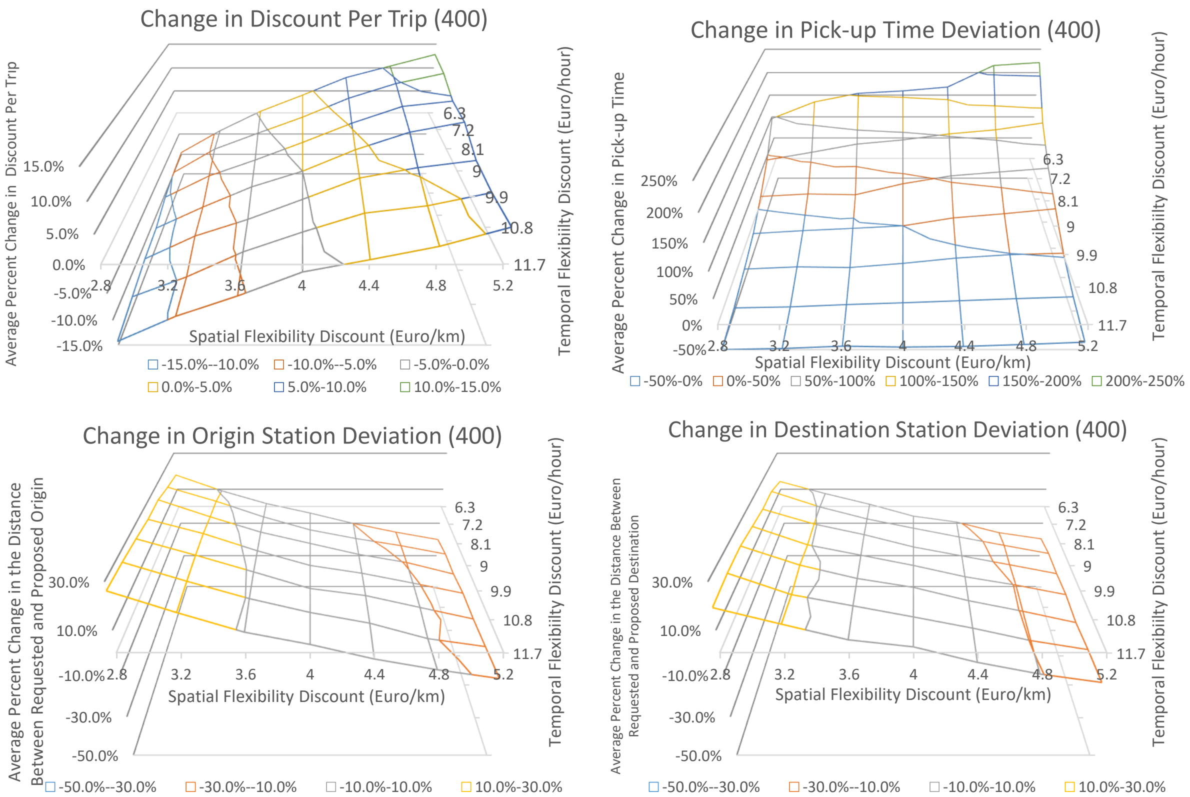

Figure 11: Effects of the temporal and spatial flexibility discount levels on the flexibility discount per trip (top-left), the average change in the pick-up time (top-right), and the average distance between the requested and proposed origin (bottom-left) and destination stations (bottom-right).

The analysis of discount changes per trip (top-left chart in Fig. 11), suggests that, although the change in the discount of spatial and temporal flexibility affects the discount per trip inversely, the highest discount per trip is seen when the spatial flexibility discount is maximum and the temporal flexibility discount is minimum. It is worth noting that for the same scenario there is a two-fold increase in the average difference between requested and proposed pick-up times (top-right chart in Fig. 11). On the other hand, the average distance between requested and proposed origin and destination stations (bottom charts in Fig. 11) is decreasing by almost 20%. These findings suggest that the temporal flexibility allows the operator to serve more profitable trips with higher discounts than the spatial flexibility. In other words, the model moves trips to different time intervals instead of serving them using different origin and destination stations. Obviously, the former is profitable only if the spatial flexibility discount is increased by 30% and the temporal flexibility discount is increased by 30% at the same time. In the base scenario, the average discount is measured as just over 1€/trip.

As it is highlighted in the previous paragraph, the decrease of the temporal flexibility discount results in shifting trips to time intervals (see top-right chart in Fig. 11). Specifically, each 10% decrease in the temporal flexibility discount increases the change in the pick-up time by 39%, 88%, and 164% respectively. On the other hand, a 10% increase in the temporal flexibility discount decreases the change in the pick-up time by 15%. A similar effect is seen when the spatial flexibility (see bottom charts in Fig. 11) discount is decreased. Each 10% decrease in this discount increases the average distance between the requested and proposed origin (destination) stations by 4.5% (6.6%), 10.5% (14.7%) and 18.7% (22.8%) respectively. On the other hand, a 10% increase in the spatial flexibility discount decreases the average distance between requested and proposed stations by 5.1%. Please note that, in the base scenario the percentage of trips that are served at their requested pick-up time, origin and destination stations are 98.1%, 79.2%, and 83.6% respectively.

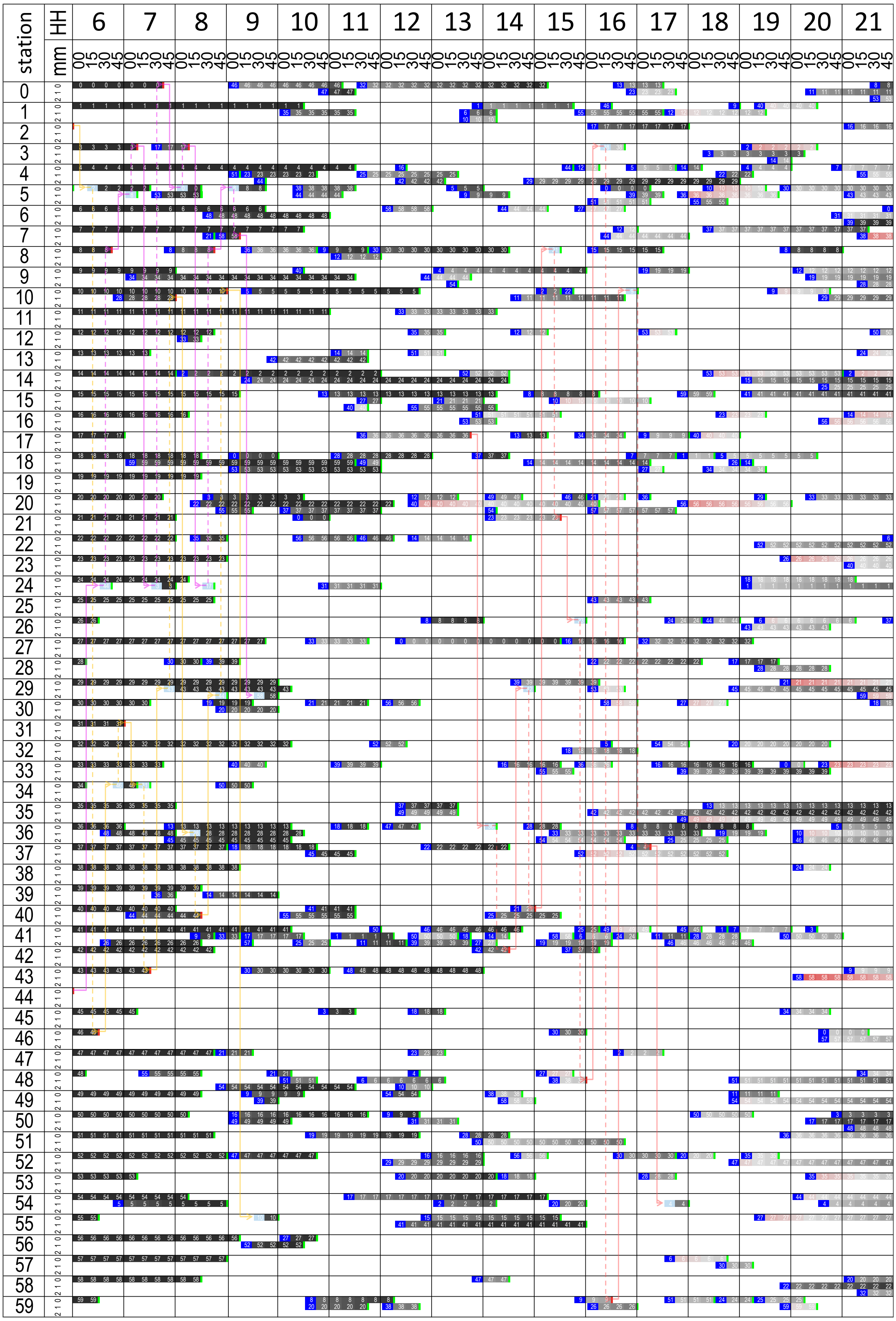

Figure 12: Schedule of vehicles, stations and relocation personnel for the selected instance with 500 demand (see Fig. 13 for the color coding).

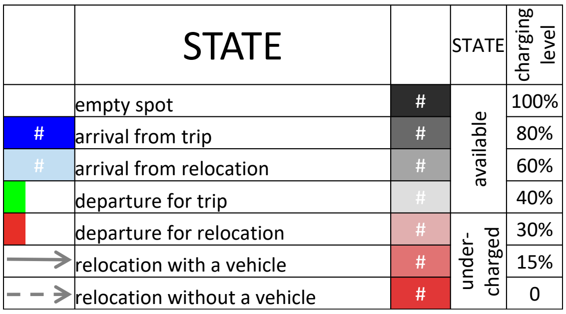

Figure 13: Legend of Fig. 12.

4.5. *Detailed decision and performance analyses of an instance*

The proposed framework also provides the operator, very specific information regarding which demand to accept/reject and offer as an alternative and how to schedule the relocation personnel. The aim of this analysis is to show the capabilities of the solution framework in making operational decisions.

In this section, we analyze one of the instances with 500 demand requests with 1000 meters spatial and 60 minutes temporal flexibility. Among 500 demand requests, the operational framework decides to serve 352 of them with three personnel. The aim of this analysis is to demonstrate the capabilities of the proposed framework to support operational decisions.

Fig. 12 shows the state of each station throughout the day. It also presents the vehicle and personnel movements in detail. The first and the second columns of the figure show the station and spot IDs for each station. The first and second rows of the figure show the starting hour and minute of the time interval. Every station in the system has three parking spots. At the beginning of the day, there is one vehicle available at each station. In this specific instance, the number of personnel used is three. Two of these personnel were assigned to the shift very early in the morning whereas the last personnel was working on the afternoon shift between 13:45-17:45. The movements of the personnel with (straight line) and without vehicle (dashed line) between stations are also shown in this figure. If the spot is occupied by a vehicle, its ID can be seen inside the color coded boxes. The states of the vehicles are shown with different colors. If a vehicle is staying at a station, it is represented with either gray-black or gray-red colors. The former shows the vehicle has at least 40% charging level and is available for a client to pick-up. As the vehicle charging level gets higher, the color gets darker. The latter shows that the vehicle is currently undercharged and cannot be picked-up by a user. As the vehicle charging level gets lower, the intensity of the red color increases. A vehicle can be picked-up from or dropped-off to a spot either by a personnel for a relocation operation (red or light blue) or a user for a trip (green or blue). The complete color coding for the figure is also given in Fig. 13.

It can be seen from Fig. 12 that most of the vehicles have a charging level sufficient for a trip specifically at the beginning of the day. Towards the end of the day, we start to see more vehicles starting to get undercharged. Furthermore, we see that there are a few stations with high vehicle occupations, e.g. stations 5, 9, 20 and 36, whereas most of the stations have never used more than one spot, e.g. stations 2, 11, 17, 19, 21, 45. Increasing the capacities of the former set of stations could help the system to serve more demand whereas if there is a need to remove a few spots, decreasing the sizes of the stations from the latter set could not affect the performance of the system in general.

Fig. 14 shows how the solution framework uses the spatial and temporal flexibility. The x and y axes show the distance between the requested and proposed origin and destination stations respectively. The different colors of the bubbles show if the demand is served on time (blue), 15 minutes earlier (gray) or 15 minutes later (orange) than the requested time. The figure shows that among the 352 served demand, 341 of them are served on time, 4 of them are served 15 minutes earlier and 7 of them are served 15 minutes later than requested. 230 demand served at their original pick-up time, from the requested origin and destination stations. All 4 demand served 15 minutes earlier served from the exact origin and destination stations. Among 7 demand served later, 5 of them are served from the exact origin and destination stations. The other two demand are dropped-off to stations different than the requested destination station. 111 demand requests are served on time from different origin and/or destination stations. Among these demand requests, only 11 of them are served from both different origin and different destination stations. Although this is the result of a single instance, it shows that although it is available, over 65% of the demand is served at the requested time and stations. The solution framework does not prefer to use the flexibility option in most of the cases since it costs to the system.

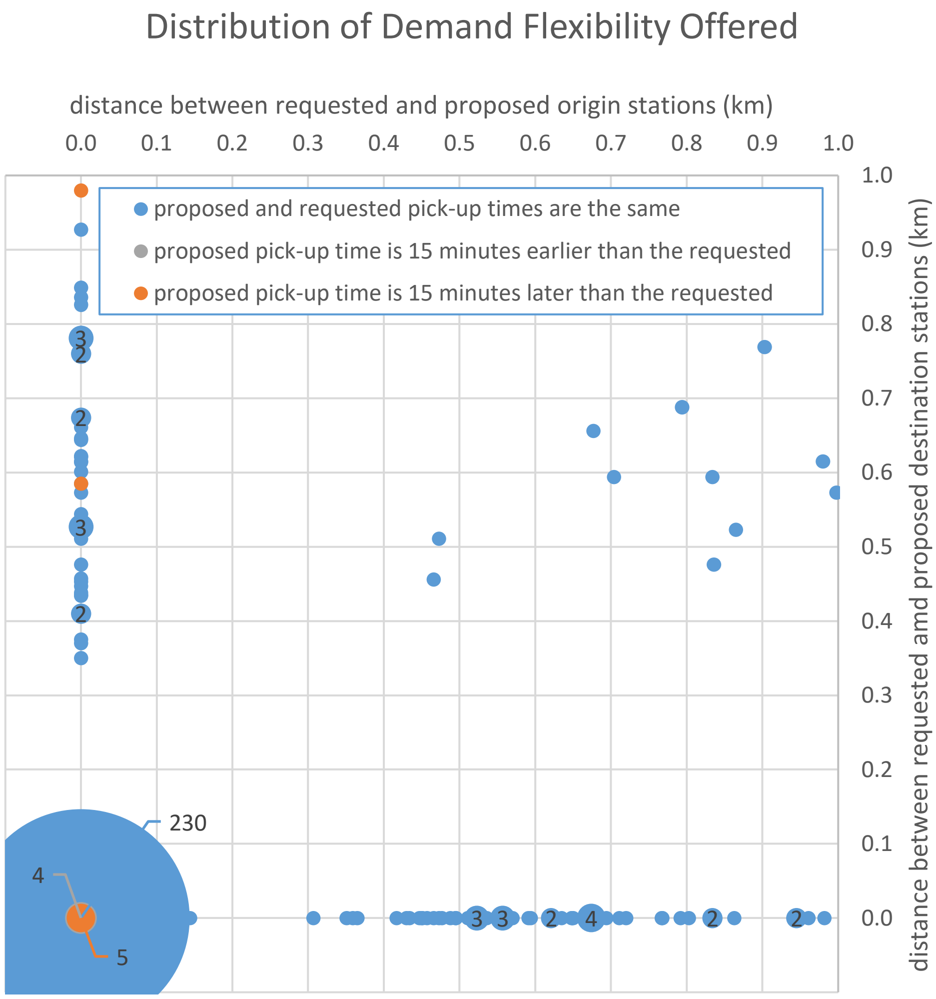

Figure 14: Bubble chart presenting the distance and time differences between the requested and served trip start times, and origin and destination stations.

Fig. 15 shows how each vehicle was utilized during the day. We see the vehicle IDs on the x-axis. The vehicles are ordered according to their total utilization time. The number of trips served by each vehicle is also added to the figure, on top of the columns showing the total utilization of the vehicles. The average utilization of the vehicles were 45% and most of the vehicles utilized around this value. There are a few vehicles which are extremely over or underutilized. Vehicle number 31 is the most utilized vehicle with a utilization level over 75%. However, it is also interesting that it serves only 3 trips. Further looking into data shows that one of the trips (from station 50 to station 6 starting at 13:30 and ending at 21:00) that the vehicle serves takes 7.5 hours which makes almost 47% of the vehicle's utilization. Vehicle 9 serves the maximum number of trips. It has served 10 trips and relocated only once.

5. Concluding Remarks and Future Work

In this paper, we have introduced an integrated modeling and computational framework for analyzing the effect of spatial and/or temporal flexibility and reservation processing type on the performance of one-way electric carsharing

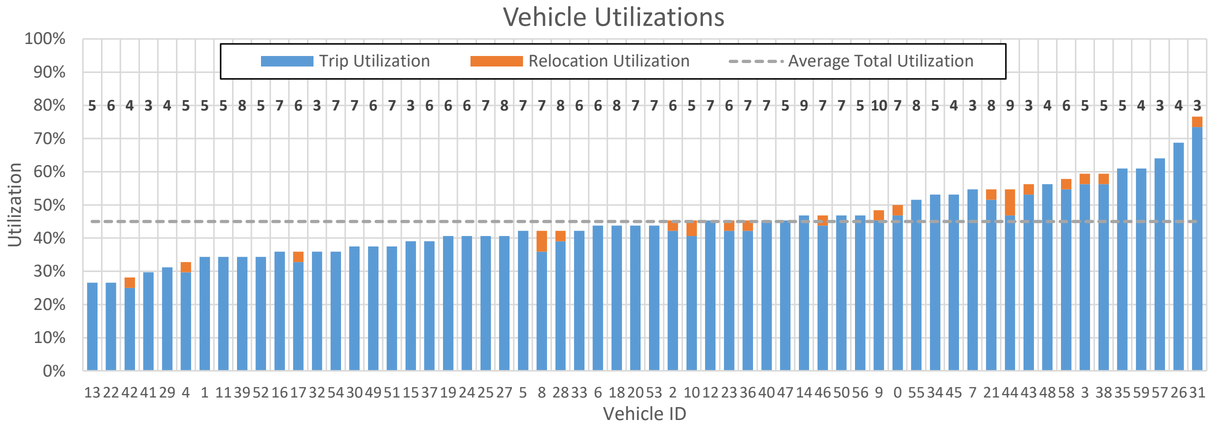

Figure 15: Utilization of vehicles. Numbers on top of the columns show the number of trips served by each vehicle.

systems. The proposed framework consists of the following modules: preprocessing, optimization and simulation modules which model the one-way electric carsharing operations realistically in terms of the vehicle and personnel relocation, and battery charging requirements. The proposed framework was applied using the operational characteristics of a real system in Nice France, and by parametrically changing the spatial and temporal flexibility attributes of the requested trips.

The proposed system provides the capability to analyze the effect of spatial and temporal flexibility, discount level, and the effect of different demand service types on the performance of one-way electric carsharing systems. In addition, the proposed framework is capable to optimize operational decisions that maximize the total profitability of one-way (electric) carsharing systems. For the specific case study analyzed, we observe that the spatial flexibility has a more significant effect than the temporal flexibility. Furthermore, for the system that we analyze, we observe that if the spatial flexibility is set to be 1000 meters, the system can increase its profit around 10% by serving over 25% more demand and hiring almost 80% fewer relocation personnel. It was also found that decreasing the discount level of the spatial and temporal flexibility by 30% over the initially assumed values results in a 2.5% profit increase and in a 5% increase in served demand. As a caveat, we would like to reiterate here that the results discussed here reflect the operational conditions, input data, and assumptions made regarding the temporal and spatial flexibility of trips and the values of the associated incentives, and they are not generalizable. However, the parametric analysis performed sheds light on the potential impacts of spatial and temporal flexibility on the performance of one-way electric carsharing systems. In a nutshell, the objective of the performed numerical experiments was to demonstrate the capabilities of the proposed framework to analyze the impact of spatial and temporal flexibility on system performance.

In the course of our research, we have identified a number of issues that provide a fertile ground of further advancing the state of the art regarding the optimization of operational decisions of one-way electric carsharing systems. Specifically, the consideration of last-minute reservations in addition to advanced reservations considered in the proposed framework will provide the opportunity to use the proposed framework in operational environments that reflect this characteristic. In addition, the use of a feedback loop between the flexible operations optimization model and the asymmetric clustering model for updating the weights used in the clustering model will further improve the clustering procedure. Furthermore, the consideration of the charging level of each individual vehicle as an endogenous variable in the mathematical model will eliminate the need to check the charging level feasibility. This approach coupled with the development of efficient solution algorithms will provide an alternative framework for analyzing operational decisions of one-way electric carsharing systems. Finally, the incorporation of uncertainty in modeling carsharing systems will enhance the robustness of the operational decisions.

In our analysis, we have changed parametrically the spatial and/or temporal flexibility and the associated incentives. We have implicitly assumed that the users will accept the proposed modifications of their original requests based on the proposed incentives. However, in reality, the acceptable level and value of flexibility may differ according to the user and trip types. Integrating behavioral models to the proposed framework could provide more accurate assumptions that may lead to more accurate decisions. To make these models more applicable to real-life systems, integrating behavioral models to operational decisions should also be a research priority.

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Appendix A. Vehicle Assignment Model (VAM)

Although the VAS returns charging level feasible vehicle assignments in most of the instances, it is not exploring the entire solution space and may not be able to find an existing feasible solution. In those instances, we can use the JFM to find a feasible solution. However, an intermediary model which is faster than JFM but more accurate than VAS could be useful. For this reason, we developed another model that uses the relocations created by the PFM instead of deciding everything in one model as JFM does.

The VAM assigns vehicles to already settled flows. It takes all the rental flows from the FOOM and all driving flows from the PFM, and finds flows for each vehicle. When the flow assignment is being done, the VAM keeps track of the charging levels of each vehicle. The objective function aims to minimize the number of infeasible flows existing in the model. These flows that are identified by the VAM are added to set C. The FOOM is solved again with the updates in C to find a charging level feasible solution.

$$\min \sum_{k,f} \tilde{c}_{kf} \tag{A.1}$$

subject to

$$\sum_{k} \tilde{y}_{kf} = 1 \qquad \forall f \in F \tag{A.2}$$

(23), (25), (31) − (35), (37)

The VAM aims to serve the trip and relocation flows identified by the FOOM and PFM respectively while minimizing the number of infeasible flows (Objective A.1). Different than the flows in the PFM or JFM, all the flows including the relocation flows in this model are all originated from and destined to stations. They are not from and/or to cluster nodes. Strictly speaking, the flow the VAM deals with are real complete flows between two stations. That is the reason, we introduce constraints A.2 to the model instead of using Constraints 29 and 30. Note that, the set F in this new model is composed of complete vehicle flows only. That is the reason, constraints A.2 should hold for all flows.

Unfortunately, our experiments have shown that the VAM is solved on average less than one minute faster than the JFM for the same instances. Although its accuracy is slightly lower than JFM, we decided it is better to remove the VAM from the solution process and use the JFM if the VAS cannot produce a feasible solution.