Serendipity? The Inspiration of Medieval Masons in Cathedral Floor-plan Design

Chaoran Wang [a] [*]and Michael Andrew Hann [b]

[a] Chaoran Wang
[b] Michael Andrew Hann
*Corresponding author e-mail: chaoran.wang@lancaster.ac.uk

Abstract: Cathedrals as very large religious buildings spread worldwide, but in medieval times were peculiar to Europe. It seems that the design of cathedrals often had various symbolic implications. Usually, cathedral floor plans are considered as the first step in the whole construction process, and are related to other parts of the cathedral's construction. Previous literature suggested that complicated geometrical and numerical proportions were found in the measurements. The masons, including the master masons, were not likely to have had a sophisticated understanding of mathematical and geometrical systems. This paper suggests some simple geometric methods which may used in the design of the whole cathedral floor plan, based on the square and its derivative constructions, including golden-section rectangles, $\sqrt{2}$ and other root rectangles and sacred cut squares. It is argued that simple methods were adopted by the masons in the planning and design process associated with cathedrals in medieval times.

Keywords: Cathedral, Proportion, Floor plans, Medieval masons

1. Introduction

Cathedral designs have long been studied by scholars from different cultural backgrounds. Complicated geometrical and numerical proportions have been found in the measurements (Wu, 2002, p.1). However, evidence shows that the masons in the Middle-Ages (5th - 15th centuries) were not likely to have had a sophisticated understanding of mathematical and geometrical systems (McCague, 2003, p.11). This paper suggests some simple methods which may have been used in the design of cathedral floor plans, based on the use of square derivative constructions of various kinds.

Prior to this, a brief review of background information as well as the scope and the source of the case study samples are described. Attention then switches to reviewing existing analytical methods for various floor plans. Further related geometric principles such as the combinational use of the golden-section rectangle, $\sqrt{2}$ and other root rectangles and sacred-cut square are dealt with. Examples with illustrations are also given to help explain the stages of analysis. The last section presents an overview of findings.

2. The selection of cases

This case study is based on the floor plans of twenty well known British cathedrals selected from 'The cathedrals of Britain' listed on the BBC history archive website (http://www.bbc.co.uk) [accessed 12 May 2016].

Twenty samples were selected from the following twenty-seven cathedrals: Bradford Cathedral (location: Bradford, years built: 1400 – 1965); Cardiff Metropolitan Cathedral (location: Cardiff, years built: 1839–1842); Chester Cathedral (location: Chester, years built: 1541); Christ Church Cathedral (location: Oxford, years built 1160–1200); Clifton Cathedral (location: Bristol, years built: 1970-1973); Coventry Cathedral (location: Coventry, years built: 1956–1962); Durham Cathedral (location: Durham, years built: 1093–1133); Ely Cathedral (location: Ely, years built: 1083–1375); Exeter Cathedral (location: Exeter, years built: 1112-1400); Liverpool's Anglican Cathedral (location: Liverpool, Years built:1904–1978); Lincoln Cathedral (location: Lincoln, years built: 1185–1311); Liverpool Roman Catholic Cathedral (location: Liverpool, years built:1962–1967); Norwich Cathedral (location: Norwich, years built: 1096–1145); Peterborough Cathedral (location: Peterborough, years built: 1118–1237); Ripon Cathedral (location: Ripon, years built: 1160–1547); St. Magnus Cathedral (location: Kirkwall, years built: 1137-1468); Salisbury Cathedral (location: Salisbury, years built: 1220–1320); St. Paul's Cathedral (location: London ,years built: 1675–1720); St. Michael's of Coventry (ruins) (location: Coventry ,years built: the late 14th century and early 15th century, ruined in 1940); St. Albans Cathedral (location: St Albans, years built: 1077–1893); St. Giles' Cathedral (location: Edinburgh, years built: 1883); Southwark Cathedral (location: London , years built: 1106–1897); Westminster Cathedral (location: London , years built: 1895–1903); Winchester Cathedral (location: Winchester , Ground breaking: 1079); Worcester Cathedral (location: Worcester , years built: 1084-1504); Wells Cathedral (location: Wells , years built: 1176–1490) and York Minster (location: York, years built: 637).

Each cathedral's official website was visited and the floor plan image was downloaded. However, in the data collection process, the floor plan images of Bradford Cathedral; Cardiff Metropolitan Cathedral; Clifton Cathedral; Liverpool's Anglican Cathedral; Liverpool Catholic cathedral and St Magnus Cathedral could not be found, and Coventry Cathedral was rebuilt next to the ruins. Due to the lack of images (sources) as well as the ambiguities of reconstructions after the ruins, these seven cathedrals were excluded from the case study selections.

It was on this basis, therefore that the twenty sample cathedrals were selected. Floor plan images were reproduced in Illustrator with each original image taken from the official website. The aim was to ensure maximum clarity and the minimum distortion of each image.

3. Methodology

Geometry has been applied widely in architectural designs, and evidence can be found in church or cathedral plans in the Roman, Medieval and Renaissance periods (Williams, 1997, p.9) across much of Europe. Although no one declared that geometry was applied rigidly (Wu, 2002, p.7), arithmetical as well as geometrical considerations were found to be of relevance in most cathedrals between parts and the whole (Wu, 2002, p.8; Duvernoy, 2015, p.23). The broad use of geometry in cathedral designs is due to a range of reasons. Gothic designs always had symbolic meanings (Davis and Neagley 2000, p.18), often associated with elementary shapes. Simple geometric manipulation can generate great complexity (Neagley, 1992, p.2). The geometric solution is the simplest method (Yeomans, 2011, p.3) as only simple tools such as compasses and straight edges were needed (Jacobson, 1986, p.5). It is also an efficient, economic and practical way (Neagley 1992, p.13) to set up an overall building plan using a logical process (Cohen, 2008, p.22). Furthermore, the harmony proportion system generated by geometrical constructions is easy to repeat (McCague, 2003, p.6). For example, proportions governing the cathedral plan are also found often to govern the facade as well (Wu, 2002, p.28).

The next two sub-sections review basic types of geometric shapes and proportions as well as the relationship between them in the design of cathedral floor plans.

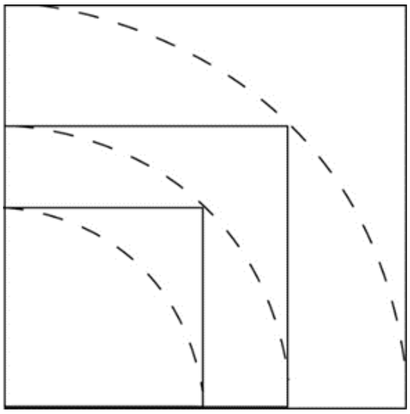
3.1 Square structures in cathedral floor-plan design

The square and its derivatives govern the geometry of medieval designs (Bucher, 1972, p.37), as it is standardised, easy to repeat and has no irrational numbers (Bucher, 1972, p.37). The geometric construction of a single square, square grids and square rotations were all of importance as was a square inscribed within a square, known as 'quadrature' (Hiscock, 2007, p.181). When a small square is inscribed in a large square, with the diagonal length of the small square equal to the side length of the large square, the figure is known as an *ad quadratum* figure (Calter, 2008, p. 96), which will be referred to later in this section.

Vitruvius stated that the first step to set up a building is to fix the north-south axis (Yeomans, 2011, p.26). The same procedure exists in cathedral plan designs as most cathedrals/churches were designed based on longitudinal axis and a perpendicular axis (Wu, 2002, p.38). All the other structures are centred on these axes (Wu, 2002, p.38). Similarly, the architecture in Roman design is found to be highly axial and centred (Watts and Watts, 1986, p.138).

The square is 'the first and the most central design shape' (Bucher, 1972, p.37) in Gothic plan design, as most cathedral plans use the crossing square as a centre and all the other plan parts were generated from it (Wu, 2002, p.129). Examples can be found in various cathedral/church plans. The transept is invariably the centre of the plan (Neagley, 1992, p.401). The Saint-Urbain and Saint-Ouen plans were, for example, generated from the centre of their transepts (Davis and Neagley, 2000, p.177).

The gnomon is a way to add to or subtract from a square to make it larger or smaller (Watts and Watts, 1992, p.309). It is a construction created by drawing arcs which are centred at the corners of the square and the arc radius equals to the length of the square diagonals (Watts and Watts, 1992, p.309). This is illustrated in Figure 1a. This progression has been applied in the plan of Muckross Friary (location: Killarney, Ireland, year built: 1448) (Figure 1b) (Yeomans, 2011, p.37).



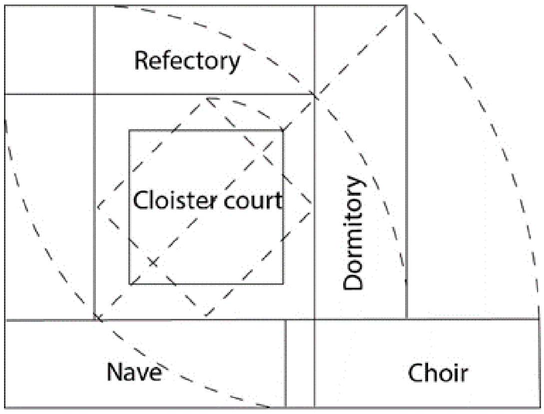


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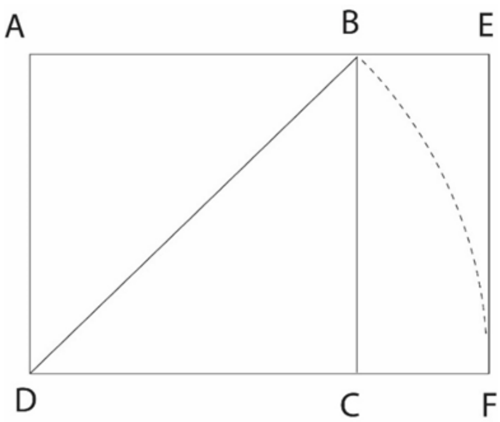
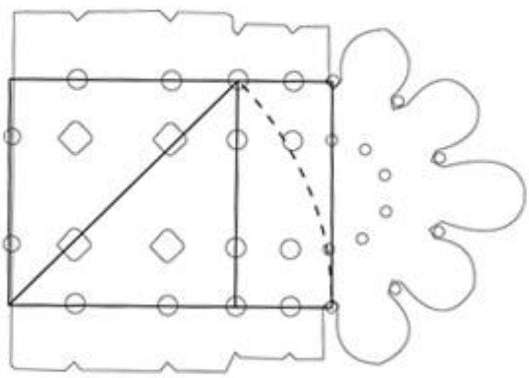
Figure 1. The gnomonic progression of a square and its application in Mukross Friary (location: Killarney, Ireland, year built: 1448) floor plan. Source: Drawn from Yeomans, 2011, p.37.

A $\sqrt{2}$ rectangle is also commonly seen in cathedral floor plan construction. A $\sqrt{2}$ rectangle can be constructed based simply on a square and its diagonal. As shown in Figure 2a, ABCD is a square with the edge length of 1 unit, so that the length of DB is $\sqrt{2}$. Connect DB and make DF=DB, the resultant rectangle AEFD is a root-two rectangle (Hambidge, 1926, p.18; Hann, 2012, p.39; Elam, 2001, p.34), which provides a 1: $\sqrt{2}$ proportion between its length and width.

A $\sqrt{2}$ rectangle existed in the plan of Notre-Dame de Remis (east end of Notre-Dame Cathedral) (location: Paris, France, years built: 1163-1250) (Figure 2b)(Wu, 2002, p.152).The plan of the Basilica of San Lorenzo (location: Florence, Italy, year built: 1470) (Figure 2c) (Cohen, 2008, p.37); the floor plans of Rouen, Saint-Ouen Cathedral (location: Rouen, France, years built: 1318-1537) (Figure 2d) (Davis and Neagley, 2000, p.167); the plan of the Pantheon (location: Roman, Italy, years built: 118-128) (Figure 2e) (Williams, 1997, p.108) as well as the ground plan of Medici Chapel (location: Florence, Italy, years built: 16th-17th century), all included $\sqrt{2}$ rectangles (Figure 2f) (Williams, 1997, p.108).





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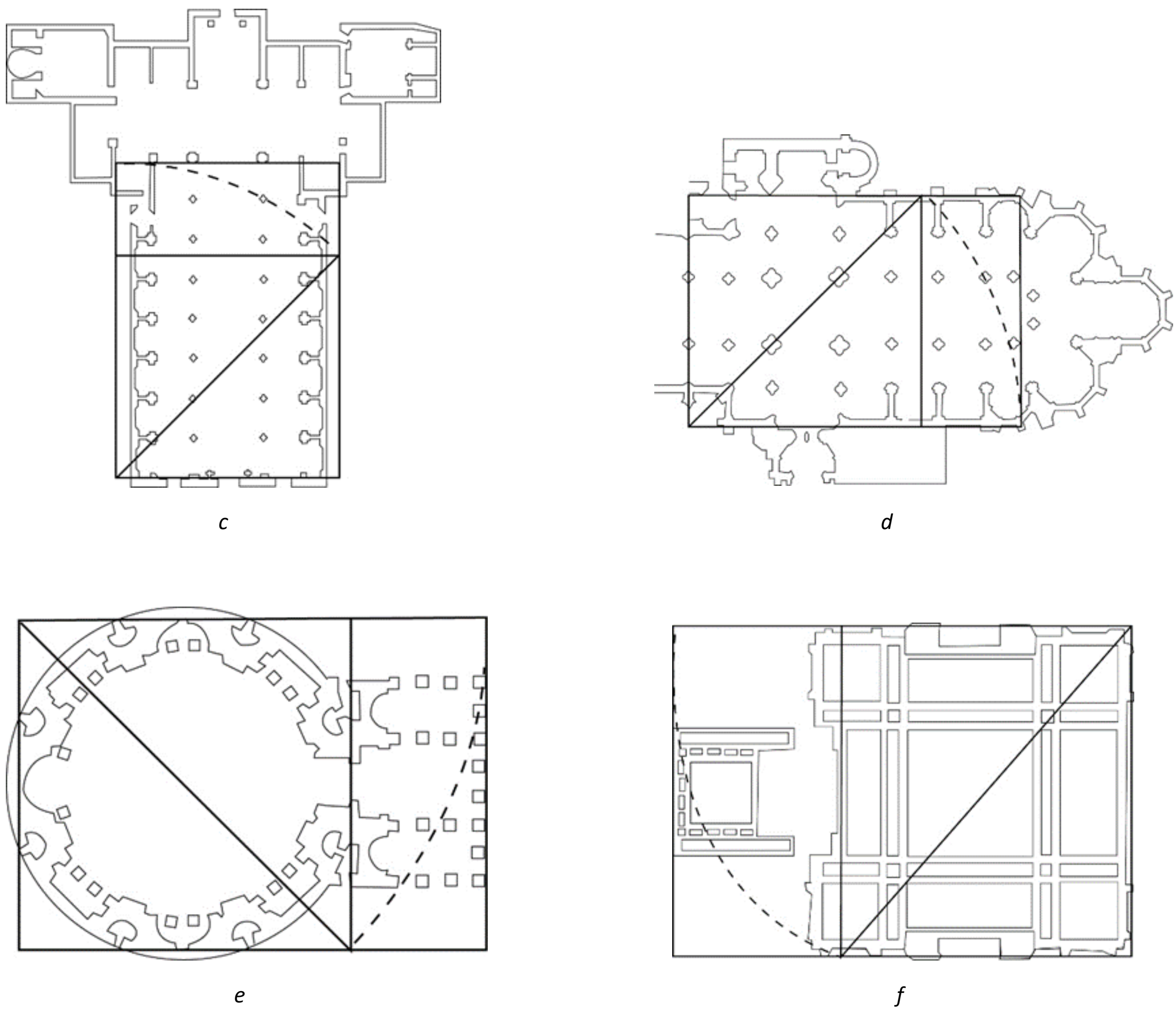

Figure 2. The construction of a root-two rectangle and over floor plans. Source: Drawn from Wu, 2002, p.152; Cohen, 2008, p.37; Davis and Neagley,2000, p.167; Williams, 1997, p.108.

A square and its diagonal can produce a $\sqrt{2}$ rectangle (shown in Figure 2a). A $\sqrt{2}$ rectangle and its diagonal can produce a $\sqrt{3}$ rectangle etc. Figure 3a shows the relationship between square and the root rectangles $\sqrt{2}$ (AEFD), $\sqrt{3}$ (AGHD), $\sqrt{4}$ (AIJD) and $\sqrt{5}$ (AKLD) rectangles.

Root rectangles can be found in some of the cathedral floor plans. The floor plan of Xavier de Vigge Biaundo in San Francisco (built in 1697) is one of the examples (Figure 3b). The construction of the nave is based on the root proportions generated by the square, the side width of which equals the transept width (Schuetz-Miller, 2006, p.368).

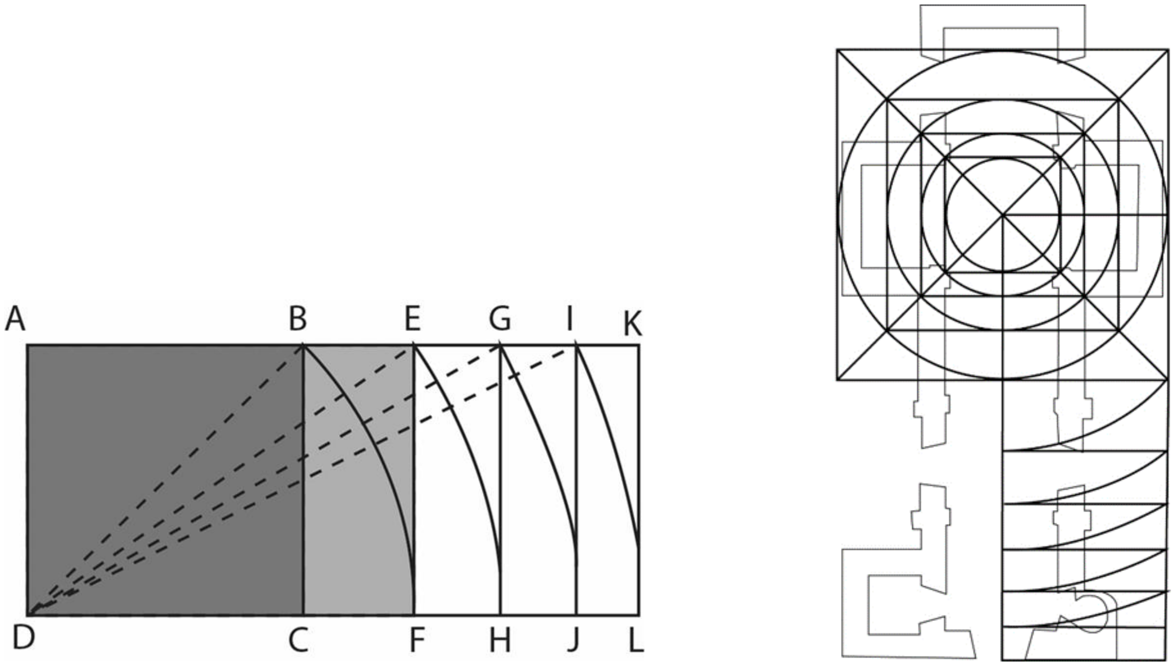

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Figure 3. Root rectangles and their application in floor plan of Xavier de Vigge Biaundo (location: San Francisco, USA, year built: 1697). Source: Drawn from Schuetz-Miller, 2006, p.368.

A golden-section rectangle can be drawn from the diagonal line of half a square. As shown in Figure 4a, ABCD is a square. Bisect CD, connect the middle point of DC and B, use the distance as a radius, and draw an arch between BF that inserts DC's extension at F, then draw a line EF parallel to AD and continue AB to E, the resulting rectangle AEFD is a golden-section rectangle. The small rectangle BEFC is a golden-section rectangle also since it has the same angles and proportions as rectangle AEFD. These two rectangles are called similar polygons (Rich, 1963, p.92). Also rectangle BEFC is known as the reciprocal of rectangle AEFD (Hambidge, 1926, p.30; Hann, 2012, p.111).

If two golden-section rectangles are interlocking together, a $\sqrt{5}$ rectangle is then generated (shown in Figure 4b). As shown in the figure, ABCD is a square, the side length of which is 1 unit. Rectangle EBCH and rectangle AFGD are two interlocking golden-section rectangles. EFGH is a $\sqrt{5}$ rectangle.

The golden-section proportion has been found in the plan of Salisbury and Reims (Hiscock, 2000, p.180). Also, in the Saint-Urbain (located: Troyes Aube France, years built: 1262-1905) floor plan, the golden-section rectangle constructed is based on the square of the cross, which defines the position of the outer wall of the cathedral (Figure 4c) (Davis and Neagley, 2000, p.167). In the case of Saint-Ouen (located: Rouen France, years built: 1318-1537), the golden-section rectangles generated from the square over the crossing define the width of the aisle in the north, south and west directions (Figure 4d) (Davis and Neagley, 2000, p.170).

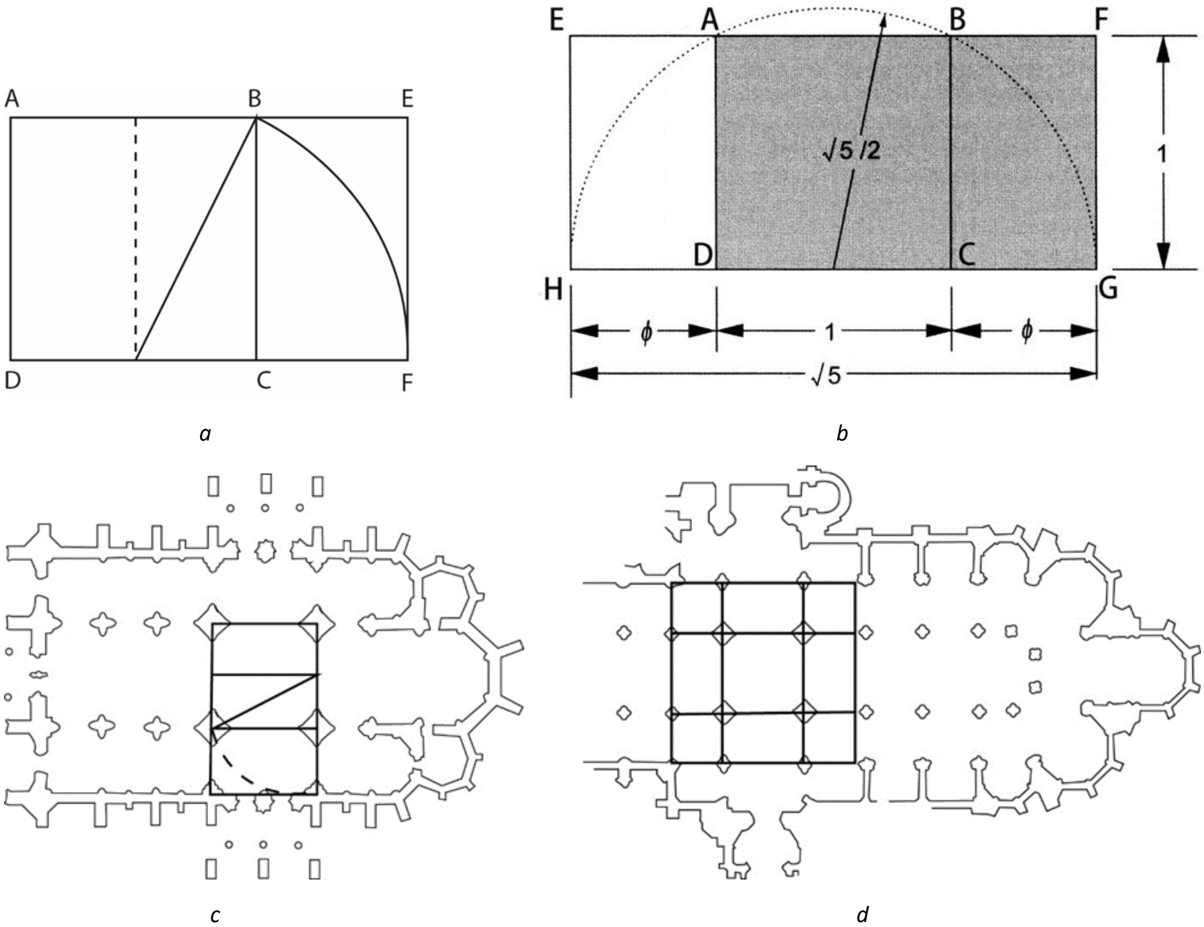

Figure 4. A golden-section rectangle, the construction of a root-five rectangle and its application in Saint-Urbain (located: Troyes Aube, France, years built: 1262-1905) and Saint-Ouen (located: Rouen, France, years built: 1318-1537) floor plans. Source: Drawn from Birkett and Jurgenson, 2001, p.264; Davis and Neagley, 2000, p.167 and Davis and Neagley, 2000, p.170.

The Brunes star (Figure 5a) is shown when a square is divided into four equal parts with the addition of half and full diagonals (Hann, 2012, p.42). The intersecting points which are illustrated as black dots in Figure 5a are highly symmetrical and the parallel lines passing through those points divide the square into various equal sections (Figure 5a) (Hann, 2012, p.43).

The Brunes star is commonly seen in cathedral floor design. For example, Schuetz-Miller found that the Brunes star existed in Xavier de Vigge Biaundo Biaundo (located: San Francisco, USA, year built: 1697) (Schuetz-Miller, 2006, p.367), which is shown in Figure 5b.

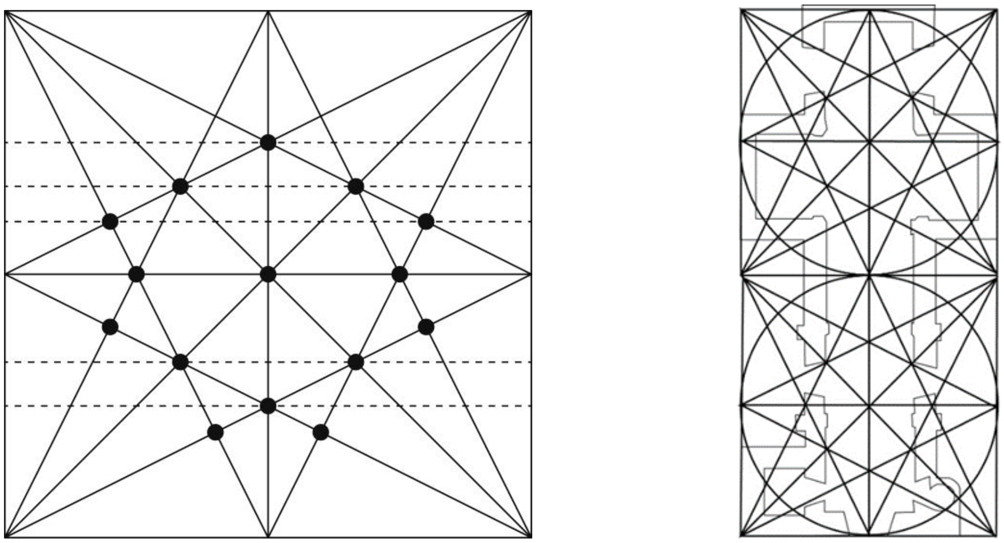

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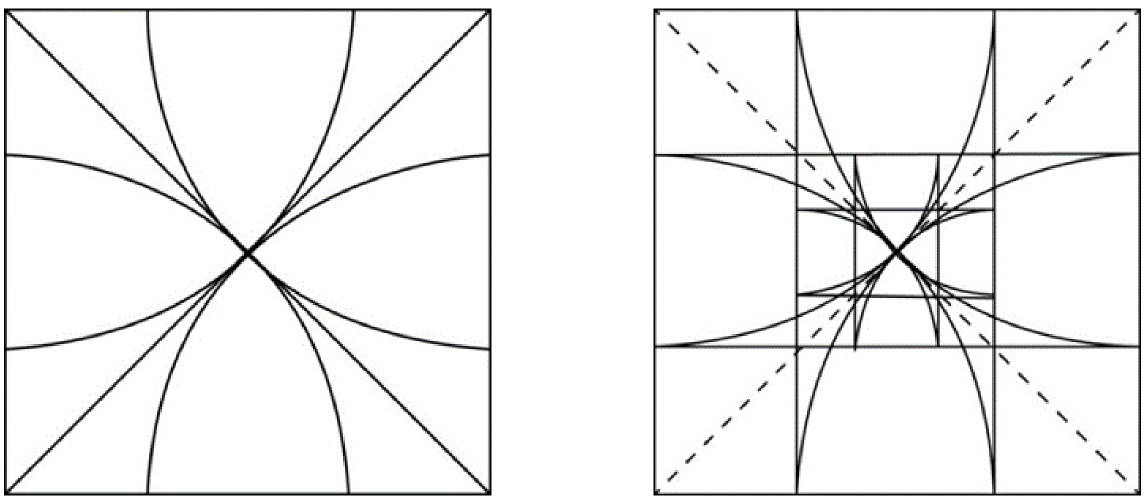
Figure 5. The Brunes star and its application in Xavier de Vigge Biaundo (located: San Francisco, USA, year built: 1697) Source: Drawn from Hann, 2012, p.43; Schuetz-Miller, 2006, p.367.

The 'sacred cut' refers to a construction of four arcs that are centred on the four corners of a square, with the half diagonal length of the square as the radius in each case, and those arcs intersecting each other in the centre of the square (Figure 6a) (Watts and Watts, 1992, p.309).When four lines are drawn which touch the arcs while parallel to the square edges, a nine-unit-grid is yielded (Figure 6b) (Wightman, 1997, p.68). The inner square can be sacred cut again and this process can be continued.

This type of division has been found in ancient Roman Gardens (Watts and Watts, 1986, p.137), paintings (Watts and Watts, 1987, p.274) and cathedral floor-plan constructions. For example, the sacred-cut division exists in Durham cathedral (location: Durham, UK, years built: 1093-1133) (part and whole) plans (McCague, 2003, p.13) as shown in Figure 6c and d.



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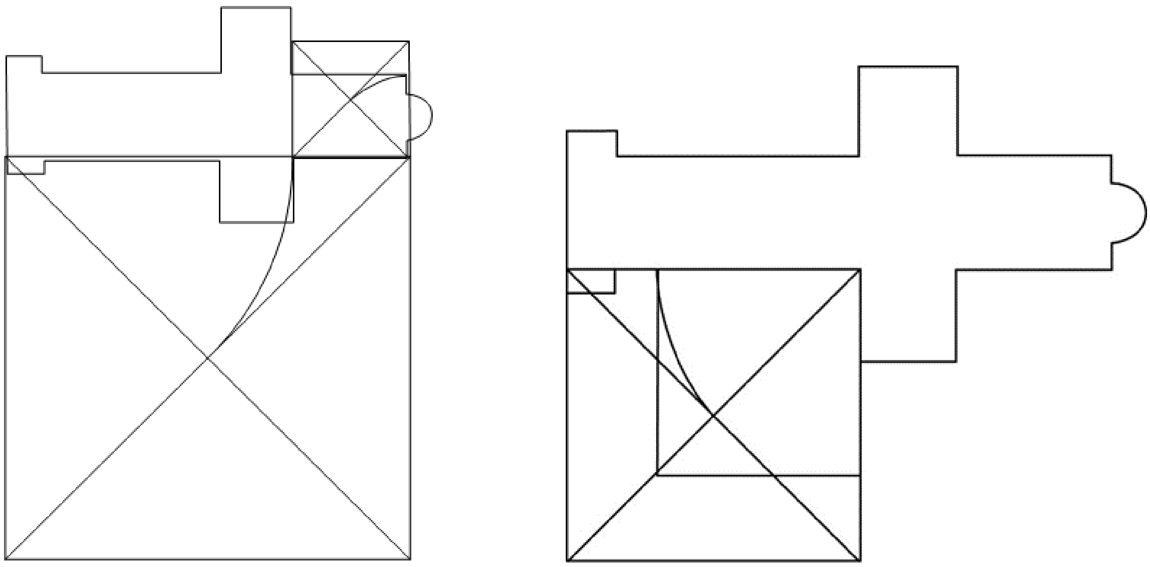

c d

Figure 6: 'Sacred cut' construction; endless division and its application in Durham Cathedral (location: Durham, UK, years built: 1093-1133) plan. Source: Drawn from Watts and Watts, 1992, p309, Wightman, 1997, p.68, McCague, 2003, p13-14.

Bucher pointed out that the rotation of squares is the 'practical and aesthetic key for Gothic architecture' (Bucher, 1972, p.43). This text below reviews the geometric constructions in cathedral plans which were generated by square and square rotation. Two equal-sided squares, with one tipped 45 degrees while over laid on the other one, generate an 8-point star (Figure 7a). The overlapping area of the two squares creates a regular octagon (coloured in grey in Figure 7a).

This lay-out exists in the floor plan of St.-Maclou (location: Rouen, France, years built: 1436-1521), the inner octagon generated by square rotation marks the edges of the east end of the plan (Figure 7b) (Neagley, 1992, p.405).

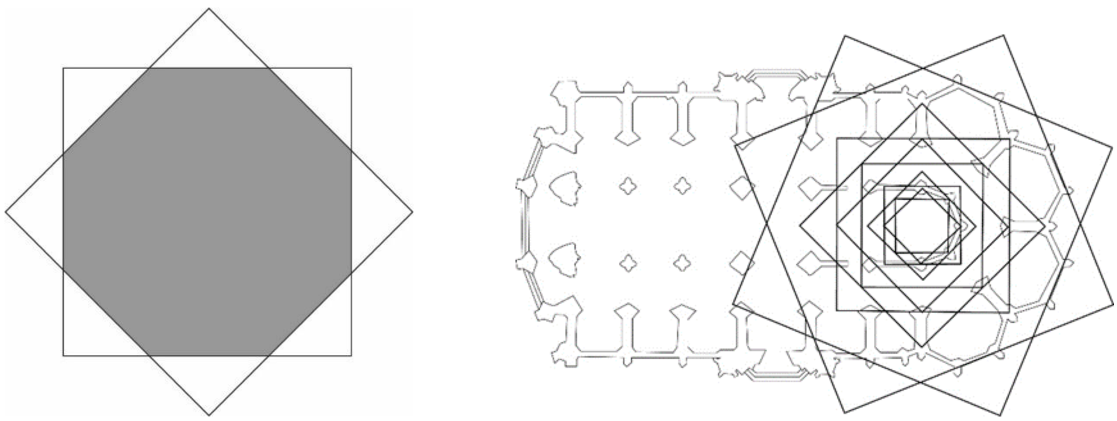

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Figure 7. An eight-point star generated by square rotation and its application in the floor plan of St. Maclou (location: Rouen, France, years built: 1436-1521). Source: Drawn from Neagley, 1992, p.405.

Another square rotation structure in cathedral architecture plans is a square divided by a smaller square, which is set to 45 degrees inside it, and the corners of the small square touch the middle points of the large square (Calter, 2008, p.96). This is known as the *ad quadratum* figure. This process can be continued endlessly and a $\sqrt{2}/2$ proportion exists between the side lengths of the adjacent squares (Figure 8a). The $\sqrt{2}/2$ proportion between a square side's lengths can also be achieved by constructing squares inscribed in circles, which is shown in Figure 8b (Birkett and Jurgenson, 2001, p.256). In practice, two interlocking '*ad quadratum*' figures were drawn in the plan of Saint-Urbain (location: Troyes, Aube France, years built: 1262-1905) (Figure 8c), one of the inner squares marks the edge of the east end of the plan (Davis and Neagley, 2000, p.165). Furthermore, the *ad quadratum* figure was believed to define the length of the main section of Dombauhütte plan (location: Köln, Germany, years built: 1248-1473) (Anderson, 2008, p.454) (Figure 8d).

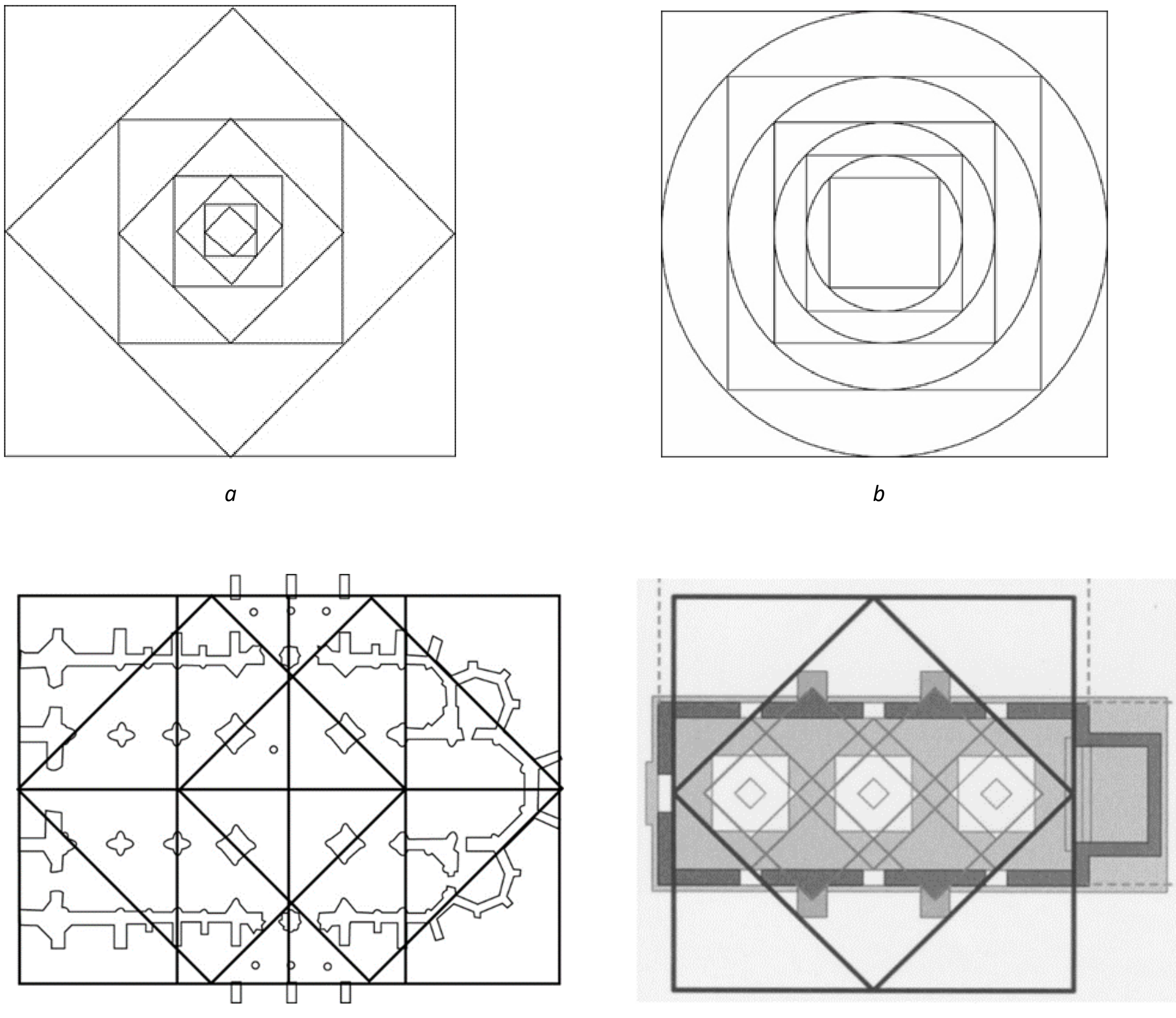

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Figure 8. The ad quadratum and its application in Saint-Urbain's floor plan (location: Troyes, Aube France, years built: 1262-1905) and in Dombauhütte (location: Köln, Germany, years built: 1248-1473) floor plan. Source: Drawn from Birkett and Jurgenson, 2001, p.256, Davis and Neagley, 2000, p.165 and Anderson, 2008, p.454.

A modular system was found in several cathedral plan designs. A modular system uses 'multiples (or divisions) of a standard length…to establish its dimension' (Wu, 2002, p.58). Le Corbusier defined the modular as 'a tool of linear or optical measurements, similar to music script' (Cohen, 2014, p.1). The square is the most common shape used as a module to define grids in building structures (Bucher, 1972, p.37). There are various ways to define the module in practice. For example, the wall thickness of Ely Cathedral (Ely, Britain) has been taken as one unit in the analysis (Hiscock, 2000, p.194). While in the plan of St. Gall (location: Switzerland, years built: the 8th century), the square generated by the crossing of the church are considered as a unit (Figure 9a) (Bucher, 1972, p.37).

In the plan of Dombauhütte (location: Köln, Germany, years built: 1248-1473), a grid takes the smallest square in the floor plan as a unit and the whole grid is rotated by 45 degrees and taken to define the modular structure (Figure 9b)(Anderson, 2008, p.448).

The plan design of the Basilica of San Lorenzo (location: Florence Italy, year built: 1470) is hypothesised to be based on the division and sub-divisions of two 65 x 65 squares to define the 'chapels, crossing square transept arms and nave'(Figure 9c) (Cohen, 2008, p.32).

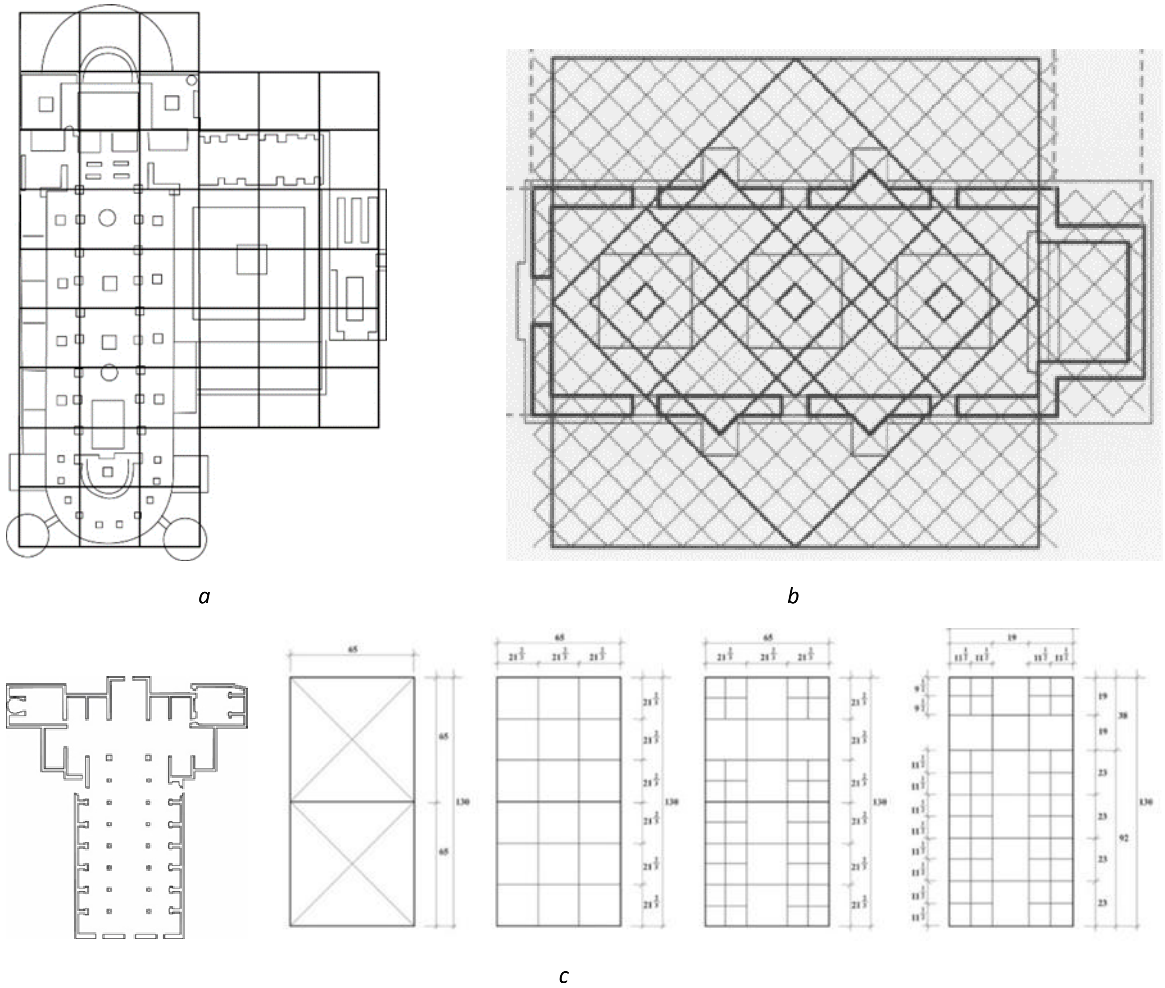

Figure 9. The modular construction of St. Gall (location: Switzerland, years built: the 8th century), Dombauhütte plan (location: Köln, Germany, years built: 1248-1473) and the Basilica of San Lorenzo (location: Florence Italy, year built: 1470) plan division. Source: Drawn from Bucher, 1972, p.37; Anderson, 2008, p.454, Cohen, 2008, p.32.

3.2 Other polygons in cathedral floor-plan design

Apart from square and square-derived constructions, polygons such as triangles and pentagons can be found used in the architectural floor plan as underlying guidelines.

The equilateral triangle was found to define some cathedral floor plans. Part of the plan of Sant'lvo (location: Rome, Italy, year built: 1660) shows a hexagon inscribed in an equilateral triangle and a circle (Figure 10a). According to Smyth-Pinney (2000, p.315), the centre of the hexagon and the circle were constructed first. Then the lobes were added, which are indicated as three small equilateral triangles attached to the hexagon sides in Figure 10a.

A 3-4-5 triangle was found in the floor plan of Temple A of the Asklepieion at Kos (location: Kos, Greece, year built: 350 B.C.E.) (Senseney, 2007, p.577). After inscribing the main constructions in a circle, a 3-4-5 triangle marks the width of the plan (Figure 10b).

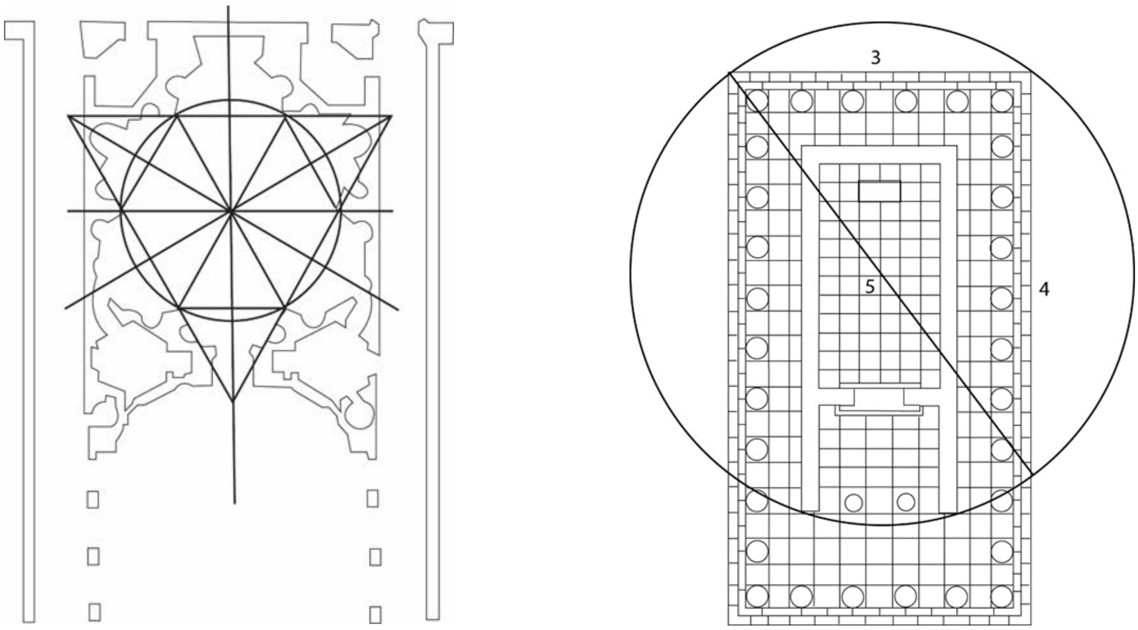

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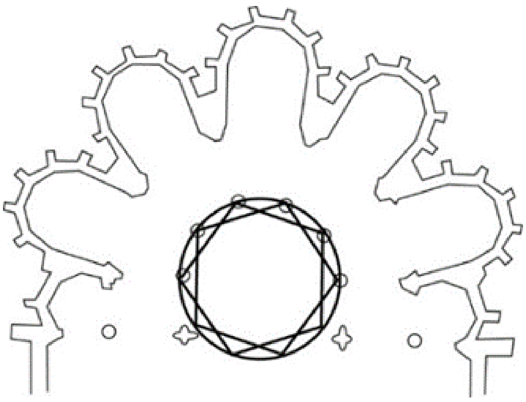
Figure 10. Triangle structures in San'lvo plan (location: Rome, Italy, year built: 1660) and in the floor plan of Temple of the Asklepieion (location: Kos, Greece, year built: 350 B.C.E.). Source: Drawn from Smyth-Pinney 2000, p.315, Senseney, 2007, p.577.

Pentagons are sometimes seen in cathedral plan structure. A regular pentagon is a polygon which has five equal sides and five equal interior angles (Calter, 2008, p.136). Pentagons are closely associated with the number five and contain a golden-section triangle. Rotational pentagons can be found in the semi-circle constructions of several cathedrals. For example, in the plan of Saint-Quentin (location: France, years built: the 12th-the 16th century), two rotated pentagons mark the position of chancel (Figure 11a) and the chapels (Figure 11b) (Wu, 2002, p.132).



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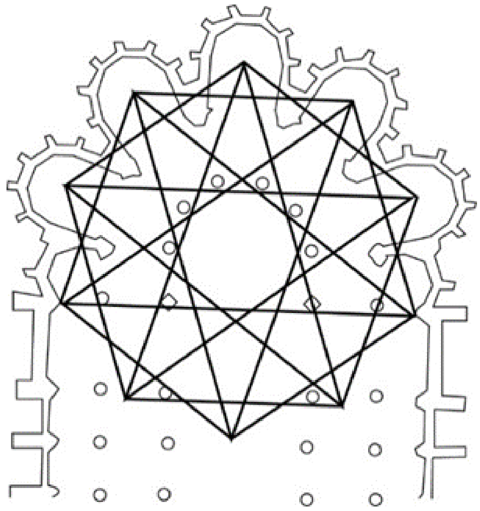

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Figure 11. Pentagons in the plan and in the chapel of Saint-Quentin (location: France, years built: the 12th-the 16th century). Source: Drawn from Wu, 2002, p.144.

Moreover, some plans are constructed based on circles. Figure 12 shows an example.

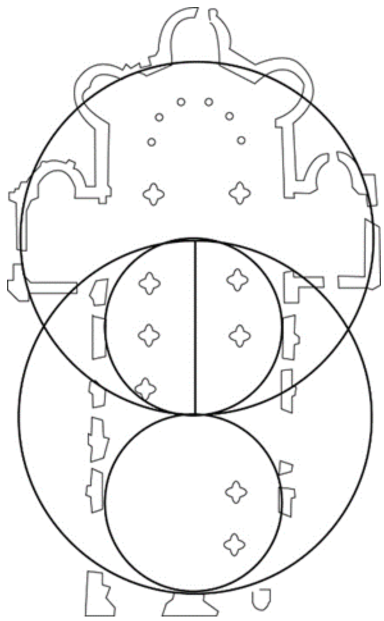

Figure 12. Circles in St. Etienne plan (location: Nevers, France, years built: 1063-1097). Source: Drawn from Wu, 2002, p.50.

From the above literature, various shapes (equilateral triangles, squares and regular pentagons), shape combinations (ad quadratum, triangle on triangle, pentagon on pentagon) and proportions derived from regular shapes (the 3-4-5 triangle, golden-section triangle; golden-section rectangle, root rectangles), and space division methods such as proportional division of root two and golden-section rectangles, the Brunes star, the sacred-cut square and the modular (grid-type) systems have been found in floor plan analysis. Often, the square over the crossing defines the width of the aisle.

Furthermore, the golden-section proportion has been found in the plan of Salisbury (Hiscock, 2000, p.180), Reims (Hiscock, 2000, p.180), Saint-Urbain (Davis and Neagley, 2000, p.167) and Saint-Ouen floor plans (Davis and Neagley, 2000, p.170). In most cases, the golden-section rectangle is constructed based on the square of the cross, and usually the golden-section rectangles generated from the square over the crossing define the width of the aisle.

Therefore, the hypothesis of this research is that in many cases the whole cathedral floor plan can be constructed based on a square (used particularly as the central square of the cross) and its derivative constructions, while combining various basic shapes and simple geometric constructions without any further measurement. Furthermore, common proportions provided by simple geometric operations were found to exist between the parts and whole of a floor plan.

4. The application of the proposed analytical methods in cathedral floor plans

Chester Cathedral (location: Chester UK, year built: 1541) [Accessed 15 May 2016] has been selected to indicate how each of the sample cathedral plans was examined. The names of important parts of the cathedral are shown in Figure 13 (www.chestercathedral.com) [Accessed 18 May 2016].

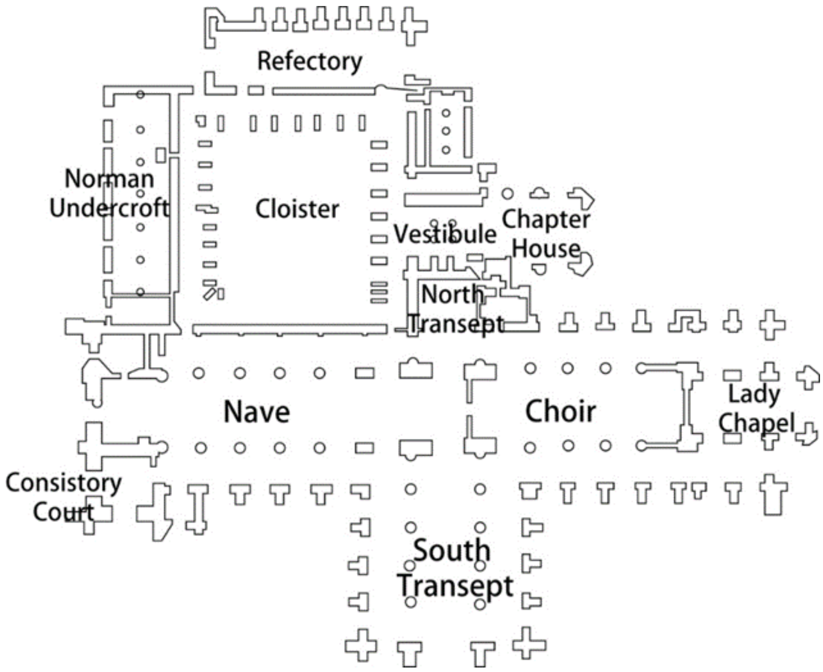

Figure 13. The floor plan of Chester Cathedral (location: Chester UK, year built: 1541). Source: Drawn from chestercathedral.com [Accessed 18 May 2016]

The first step is to identify the central square which has the same width as the Nave and Choir. Golden-section rectangles can be drawn based on the square in four directions (shown in Figure 14). These golden-section rectangles mark the key stones which define the south transept, the north transept, the Nave and the Choir. When the edges of the four overlapping golden-section rectangles are extended, a square which marks the centre area of the cross is generated (which is coloured in grey in Figure 15). This square can be considered as a unit and can be duplicated. As shown in Figure

15, the distance between the south transect and the refectory is four units. One unit on both east and west side of the central unit marks the edge of the Lady Chapel and the Nave. The golden-section rectangles can be constructed on the east and west units to define the east and west ends of the cathedral (shown in Figure 15). Furthermore, a root-two rectangle can be constructed based on the square unit which covers the area of the North transept and the Vestibule (Figure 15). The edge of this root-two rectangle defines the Chapter House.

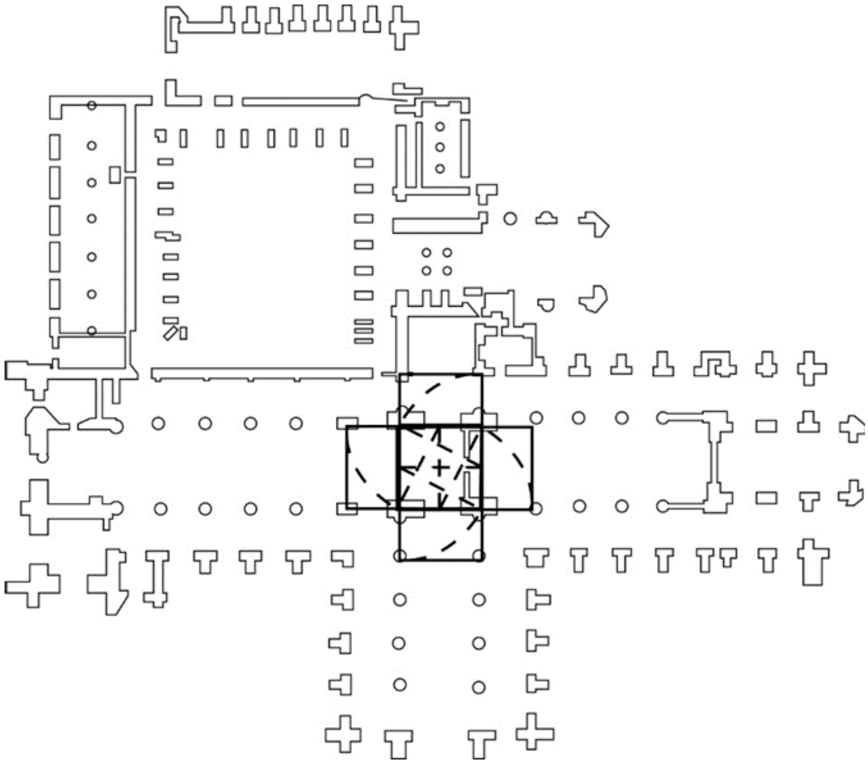

Figure 14. The golden-section rectangles in Chester Cathedral plan

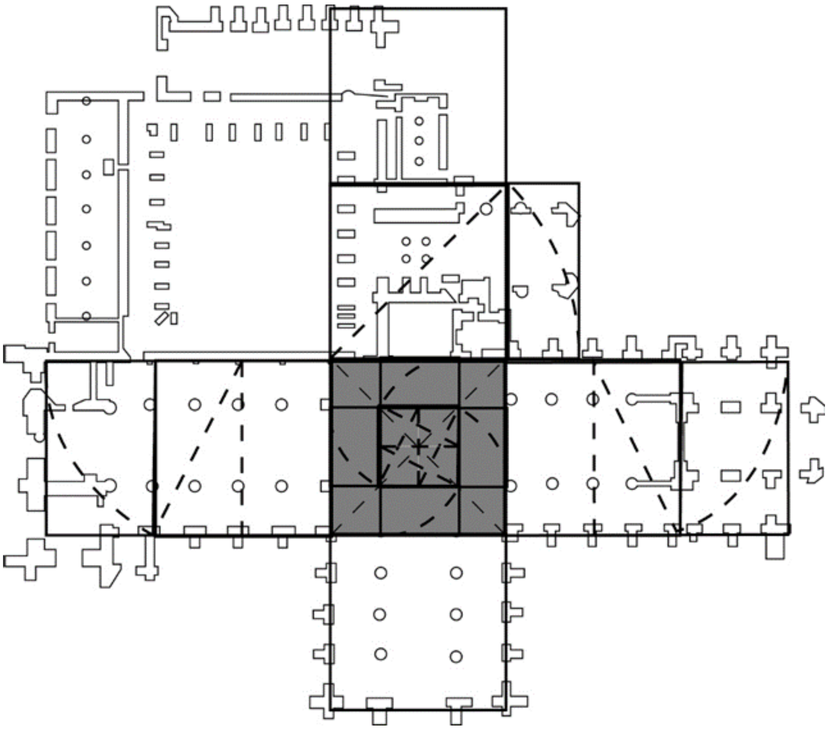

Figure 15. The modular nature of Chester Cathedral plan

A sacred-cut square can be drawn to cover the area of the Cloister and the Norman Undercroft (Figure 16). The wall between the Cloister and the Norman Undercroft is against a sacred-cut division line. When the sacred cut is conducted again in the middle square, the inner Cloister garden is defined by a division line. Furthermore, the inner garden and the refectory are all designed based on the sacred cut of the corner square (Figure 16).

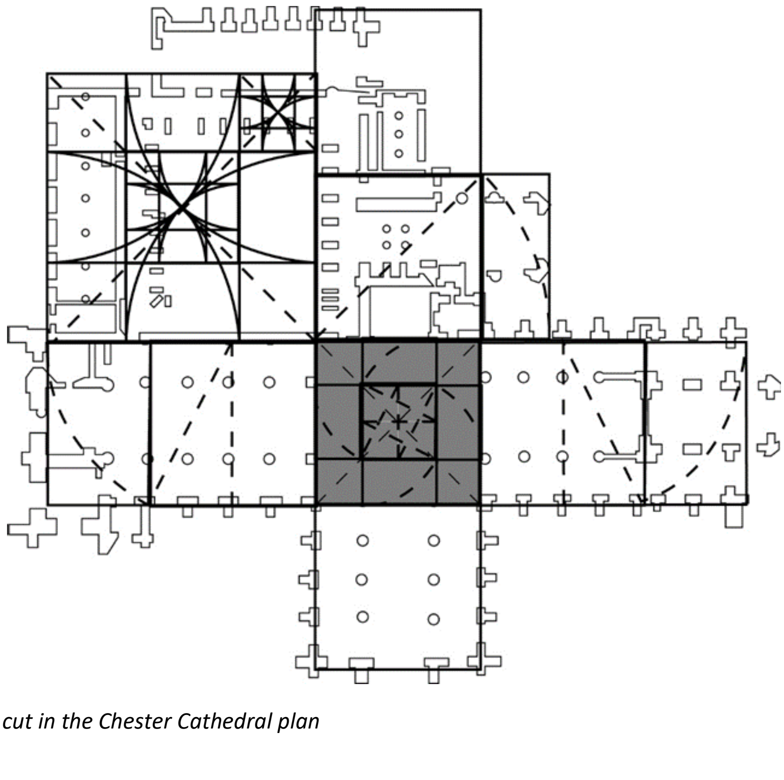

Figure 16. The sacred cut in the Chester Cathedral plan

When *ad quadratum* figures are drawn based on the basic central square (S1 in Figure 17), the third square (S3 in Figure 17) marks the overlap of the cross. Square 4 (S4) goes through the key stones in east, west and south directions. Square 5 (S5) marks the north edge of the North Transept; Square 6 goes through the south end of the South Transept.

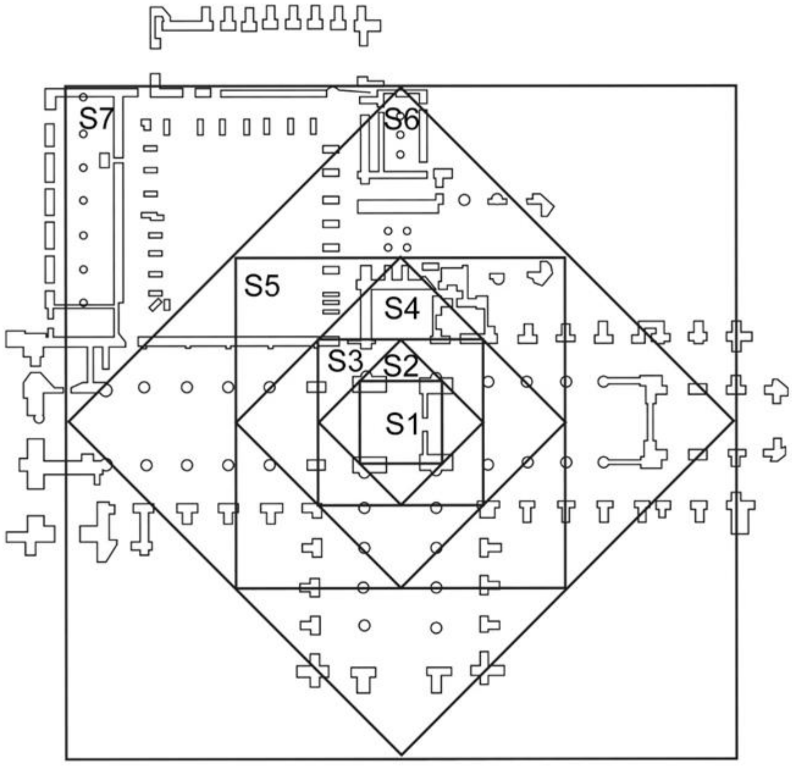

Figure 17. The ad quadratum in Chester Cathedral plan

Root rectangles (square to root-five rectangles) can be drawn based on the length of the cathedral plan (Figure 18). The square marks the aisle of the south transept; the root-two rectangle defines the boundary of the south transept; the root-four rectangle marks the west end of the nave and the root- five rectangle defines the west end of the whole plan. Similarly, root rectangles also mark the key points vertically. When root rectangles are drawn based on the distance between the south transept and the refectory (shown in Figure 19), the square marks the north edge of the nave; the root-two rectangle marks the north edge of the Vestibule. The root-three and the root-four rectangle define the boundary of the refectory.

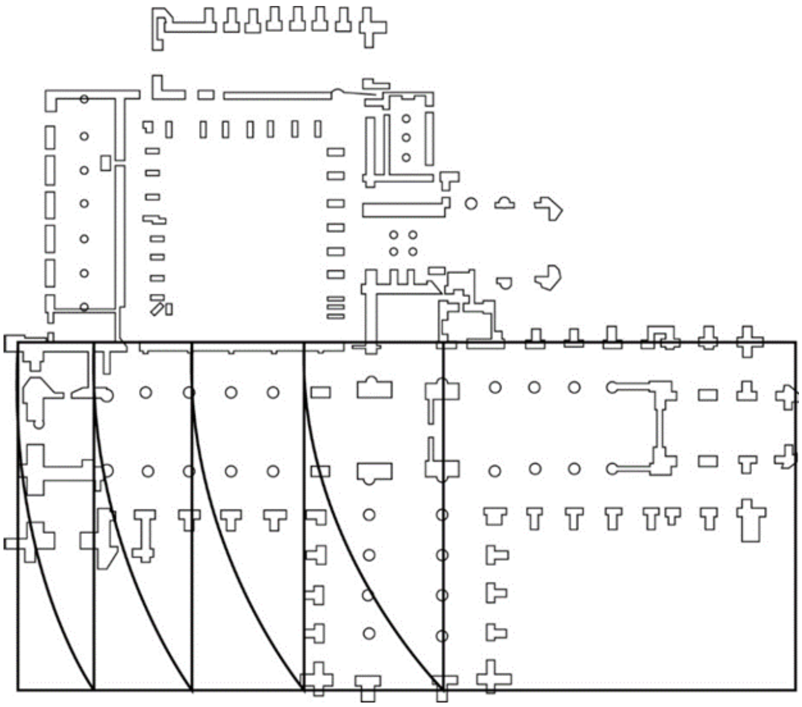

Figure18. The root rectangles in Chester Cathedral plan (no. 1)

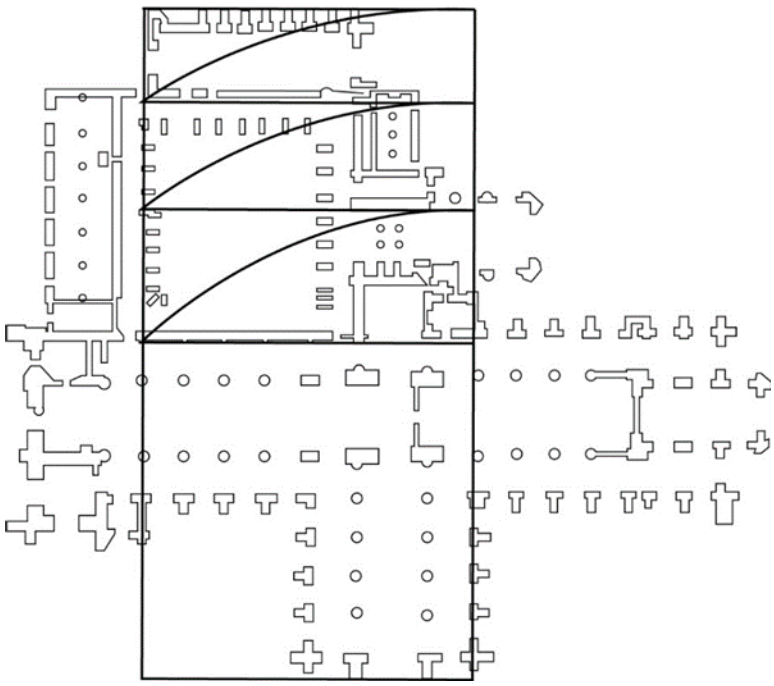

Figure 19. The root rectangles in Chester Cathedral plan (no. 2)

5. Results

After applying the same methods to the twenty selected cathedral plans, it is interesting to notice that although the appearances and results are different from cathedral to cathedral, all the plans can be constructed based on a square and its derivative constructions (found by applying simple geometric operations). The constructions of golden-section rectangles; the use of modular systems; sacred cut as well as root rectangles (root-two to root-five rectangles) were found frequently as well.

6. Conclusion

Cathedrals as religious architecture, have been studied by scholars worldwide. Their construction, especially their floor plans, reveal various geometric features. This paper reviewed the possible geometrical methods which may have been applied in cathedral design. It was found that often cathedral floor plans were based on square-related constructions. Furthermore, the combinational use of the golden-section rectangle; modular systems; sacred cut as well as root rectangles (root-two to root-five rectangles) were found in most of the selected cases.

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Chaoran Wang: lecturer in Design at Lancaster University (UK). Research interests: Structure and form in visual arts, design and architecture. In particular, the geometric structures underpinning two-dimensional imagery.

Michael Hann: Chair of Design Theory at the University of Leeds (UK). Interested in the geometry of design. Key publications include: *Structure and Form in Design* (2012, Berg), *Symbol, Pattern and Symmetry* (Bloomsbury, 2013*), Stripes, Grids and Checks* (Bloomsbury, 2015).