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Abstract

Safety is a critical element in the air transport industry. Although fatal air accidents are rare compared to other transport industries, the rapid growth in air travel demands has resulted in a growing aviation risk exposure and new challenges in the aviation sector. Although the issue of airline safety is of serious public concern, notably few studies have investigated the safety efficiency of airlines. This paper aims to propose a novel hybrid method using fuzzy data envelopment analysis (DEA) and fuzzy multi-attribute decision making (F-MADM) for ranking the airlines' safety. In this study, fuzzy DEA is utilised to calculate criteria weights, in contrast to the conventional approach of using DEA for measuring the efficiency of alternatives. A ranking of each airline (DMU) on the basis of obtained weights is then assessed using MADM methods. Six MADM methods including Fuzzy SAW, Fuzzy TOPSIS, Fuzzy VIKOR, ARAS-F, COPRAS-F and Fuzzy MULTIMOORA are implemented to rank the alternatives, and finally, the results are compounded with the utility interval technique. This new hybrid method can efficiently overcome the pitfalls of traditional hybrid DEA-MADM models. The method proposed in this study is used to evaluate the safety levels of seven Iranian airlines and to select the safest one.

Keywords	Airline Safety; Fuzzy DEA; Fuzzy MADM; Utility interval.

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A Novel Hybrid Fuzzy DEA-Fuzzy MADM Method for Airlines Safety Evaluation

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Abstract

Safety is a critical element in the air transport industry. Although fatal air accidents are rare compared to other transport industries, the rapid growth in air travel demands has resulted in a growing aviation risk exposure and new challenges in the aviation sector. Although the issue of airline safety is of serious public concern, notably few studies have investigated the safety efficiency of airlines. This paper aims to propose a novel hybrid method using fuzzy data envelopment analysis (DEA) and fuzzy multi-attribute decision making (F-MADM) for ranking the airlines' safety. In this study, fuzzy DEA is utilised to calculate criteria weights, in contrast to the conventional approach of using DEA for measuring the efficiency of alternatives. A ranking of each airline (DMU) on the basis of obtained weights is then assessed using MADM methods. Six MADM methods including Fuzzy SAW, Fuzzy TOPSIS, Fuzzy VIKOR, ARAS-F, COPRAS-F and Fuzzy MULTIMOORA are implemented to rank the alternatives, and finally, the results are compounded with the utility interval technique. This new hybrid method can efficiently overcome the pitfalls of traditional hybrid DEA-MADM models. The method proposed in this study is used to evaluate the safety levels of seven Iranian airlines and to select the safest one.

Keywords: Airline Safety; Fuzzy DEA; Fuzzy MADM; Utility interval.

1. Introduction

Safety has always been a key factor in the airline industry that leads to an airline's survival, reputation, international prestige, and passengers' confidence (Chang and Yeh, 2004; Cui and Li, 2015). Therefore, the continuous improvement in air safety has been a critical undertaking for the airline industry (Chen and Chen, 2012).

Due to the importance of having an acceptable air safety record for each airline, improving safety has been the top priority for this industry (Hsu et al., 2010; Liou et al., 2008); and as such, the aviation industry must make efforts to establish and implent high safety standards to reduce accident and incident rates (Liao, 2015).

The first thing that is required to manage airline safety is an evaluation mechanism for measuring the overall safety which could assist managers in comparing safety efficiency among airlines and to analyse changes in airline safety performance over time (Chang and Yeh, 2004; Deng et al., 2007). Concerning the vital role of safety levels in airlines, different approaches, e.g. statistical modeling, trend extrapolation, Bayesian belief networks, data envelopment analysis and multi-attribute decision making, have been proposed to evaluate airline safety efficiency and performance (Cui and Li, 2015). With consideration of the literature, multi-attribute decision making (MADM) methods are more strongly underlined by researchers than other methods due to their relative characteristics. However, there are a number of concerns regarding the use of

MADM type methods that we try to address in this research. Below, these concerns and potential solutions are addressed:

First concern: MADM suggests choosing the best alternative from a finite set of decision alternatives regarding multiple (and usually conflicting) attributes/criteria. The weights of these attributes play a very significant role in the process of decision-making. Therefore, how to determine the weights of attributes is crucial to MADM (Wang and Luo, 2010).

Based on literature, criteria importance weights can be calculated based on two categories: subjective methods and objective methods (Wang and Lee, 2009). While subjective methods determine weights solely based on the preference or judgments of decision-makers, objective methods determine the weights of attributes using objective decision matrix information and mathematical models (such as entropy, CCSD[1], QSPM[2]). Therefore, weights are not affected by the subjective judgment or intuition of the decision-maker, especially with regard to any lack of knowledge and experience.

Regarding this issue, data envelopment analysis (DEA) can be considered to be a powerful method for calculating objective weights. DEA is a mathematical programming approach in evaluating the efficiency scores of a set of decision-making units (DMUs). This evaluation approach is based on identifying the optimal weights of several DMU's criteria that are categorized as inputs and outputs. The input and output weights related to each DMU defers from other DMUs, and reveals the strength and weakness points of each DMU. These weights are assigned by DEA to make each DMU look as efficient as possible. DMUs select smaller and larger values for their weak and strength points respectively. Here, we consider the weight of inputs and outputs as the indicator of criteria importance.

Both DEA and MADM are used in this area, but both have limitations (Sinuany-Stern et al., 2000). Previous studies have tried to combine these two powerful methods simultaneously, calculating weights with MADM methods and then using DEA for reassuring a DMU's efficiency (Abdollahi et al., 2015). However, in this paper, DEA as an objective method is used to weight the criteria (contrary to previous researches in this area), and the alternatives are ranked by MADM methods. We show that this combination takes the best of each model, by avoiding pitfalls that could occur.

Second Concern: Regarding the literature, different MADM methods such as analytic hierarchy process (AHP), technique for order preference by similarity to ideal solution (TOPSIS), analytical network process (ANP), and decision making trial and evaluation (DEMATEL), are used for evaluating safety levels. It should be noted that results may differ depending on the use of different MADM methods (Antucheviciene et al., 2011). Therefore, finding the appropriate MADM method is very significant in the performance evaluation. The use of a single prioritization method cannot ensure the best result; besides, such a result would not be robust (Akhavan et al., 2015). In this respect, some studies have applied the combination of

[1] CCSD: Correlation coefficient (CC) and standard deviation (SD)

[2] QSPM: Quantitative Strategic Planning Matrix

different MADM methods with voting approaches such as the Borda and Copeland (Favardin et al., 2002); and have found this grouping to be a more efficient technique in enhancing the precision of the final decision. We use the utility interval aggregation method that was proposed by Wang et al. (2005), as a systematic and logical scientific procedure that can help decision-makers to achieve the optimum ranking of alternatives.

Third Concern: Problems regarding safety evaluation have uncertainty at different levels such as information shortage, the indistinctive situation of the environment, and undefined variables which can lead to an unclear future state of the system. Moreover, using linguistics measures such as low, medium, and high are often employed in order to judge an event especially about privacy issues.

Because of the imprecision and vagueness inherent in the subjective assessment by the experts for safety evaluation (Deng et al., 2007), and considering the vital point that safety data is either not available or secure for many of the airlines (Oster et al., 2013), we use the fuzzy set theory. So, all models used in this article are fully fuzzy.

In light of the aforementioned concerns, in the first step, the fuzzy weight for each criterion is obtained using fully fuzzy DEA. In the second step, ranking of the safest airlines using obtained weights from the previous step is applied. Given that we require a robust decision-making method to select the safest airline, we propose to rank them using six MADM methods: Fuzzy SAW[3], Fuzzy TOPSIS, Fuzzy VIKOR[4], ARAS-F[5], COPRAS-F[6] and Fuzzy MULTIMOORA. Then, the utility interval technique is applied to combine the ranking results of these methods. Weighted utility intervals are computed by constructing a correlation matrix between the ranking methods.

The overall contribution of this study is fourfold: (1) Using a fuzzy DEA-based objective weighting method instead of directly implementing experts' idea; (2) Using novel fuzzy DEA modeling for calculating the weights of criteria instead of estimating the alternative efficiency; (3) Using the utility interval technique to consolidate six different MADM rankings and to select the best answer; (4) Using a MADM-DEA combination for evaluating airline safety efficiency.

The remainder of this paper is organized as follows: In section 2, a brief review of the literature on airline safety evaluation is presented. Section 3 presents the mathematical details of the hybrid approach proposed in this study. In section 4, we use the proposed method to rank and analyze the airlines, and finally, conclusions and future research directions are discussed in the last section.

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[3] Simple additive weighting (SAW)
[4] VIšekriterijumsko kOmpromisno rangiranje (VIKOR)
[5] Fuzzy additive ratio assessment (ARAS-F)
[6] Fuzzy complex proportional assessment (COPRAS_F)

2. Literature survey

Whenever an accident occurs in one country's aviation industry, it draws considerable attention from the government and public, and normally the airline's reputation and international prestige dramatically declines as a result (Liao, 2015).

In the past several years, airline safety has been an essential and popular research topic. Cui and Li (2015) classified this research into two categories: (1) Evaluation of civil aviation safety; (2) Analysis of factors influencing civil aviation safety (Cui and Li, 2015).

Several scientific methods have been applied to evaluate airline safety, for example, statistical modeling based on the Poisson process (Janic, 2000), Trend extrapolation models (Li et al., 2009), Bayesian belief networks (BBN) (Brooker, 2011), Data envelopment analysis (DEA) (Cui and Li, 2015), and multi-attribute decision making (MADM) methods (Chang and Yeh, 2004; Deng et al., 2007; Hsu et al., 2010; Liou et al., 2007; Liou et al., 2008).

We use a combination of Fuzzy DEA and Fuzzy MADM methods for measuring the safety efficiency of airlines in this article. In the next section, a brief review of the literature on MADM, DEA and their combination applications in the airline safety field are discussed.

2.1 Airline safety measurement using MADM

Multi-attribute decision making (MADM) is regarded as a practical approach for ranking a finite number of alternatives involving multiple conflicting criteria (Hwang and Yoon, 2012). With respect to the importance of decision-making in usual human tasks and MADM efficiency, different methods are developed and used in many research areas (Tzeng and Huang, 2011a; Zavadskas et al., 2014). Since different dimensions and measures are used for evaluating safety levels (Chang and Yeh, 2004), MADM is a popular method for measuring airline safety performance. MADM methods such as AHP (Chen and Chen, 2012; Yang and Deyi, 2000); Fuzzy TOPSIS (Deng et al., 2007); DEMATEL (Liou et al., 2008); and DANP (Hybrid DEMATEL and ANP) (Hsu et al., 2010; Liou et al., 2007), were used for this purpose. Keshavarz Ghorabaee et al. (2017), integrated a simulation-based assignment approach with a hybrid decision-making approach, to evaluate the performance of five various airline centres based on twenty-eight predefined criteria and the ideas of fifty-eight experts. They employed a combination of TOPSIS, COPRAS, WASPAS[7] and EDAS[8] methods to prioritize alternatives based on a predefined simulation process. As these researches show, in most cases, criteria weights are calculated based on subjective judgments. In fact, weights determined by subjective approaches reflect the subjective judgment and intuition of the decision-maker. Therefore, the lack of knowledge and experience of decision-makers causes a more negative impact

[7] Weighted Aggregated Sum Product Assessment (WASPAS)
[8] Evaluation Based on Distance from Average Solution (EDAS)

on the rankings of alternatives (Ma et al., 1999). This is the main reason for using weighted approaches to tackle the MADM weakness in the ranking of airlines' safety problem.

2.2 Airline safety efficiency using DEA

Since the original DEA study by Charnes et al. (1978), there has been rapid and continuous growth in this field, and previous studies show that the DEA method appears to be a reliable and powerful tool in many management science fields (Emrouznejad et al., 2008).

As Cui and Li (2017b) mentioned, many researchers have applied DEA to evaluate airline efficiency. According to the increase in the number of researches using standard DEA models to measure the airline efficiency, in recent years, researchers have focused on the use of extended DEA models such as the dynamic DEA models (Cui and Li, 2017a; Cui et al., 2018; Cui et al., 2016; Li et al., 2016), network DEA (Cui and Li, 2017b; Li et al., 2015; Lozano and Gutiérrez, 2014; Mallikarjun, 2015), etc.

On the other hand, contemporary research in airline efficiency models encompasses several applications of the DEA method in this field such as: the technical efficiency of airlines (Barros et al., 2013; Choi, 2017), the operational efficiency of airlines (Merkert and Hensher, 2011; Tavassoli et al., 2014), overall efficiency of airport companies (Liu, 2017), airline energy efficiency (Cui et al., 2016; Xu and Cui, 2017), airline service quality (Choi et al., 2015; Pandey, 2016), airline profitability efficiency (Lee and Johnson, 2012), airline production and marketing efficiency (Lu et al., 2012), airport safety management systems' performance (Chang et al., 2015), air traffic management safety evaluation (Di Gravio et al., 2015), and airline strategic alliances performance (Kottas and Madas, 2018; Min and Joo, 2016).

However, the number of articles that have addressed the issue of airline safety efficiency is not high; a few studies have studied civil aviation safety efficiency (Cui and Li, 2015), and there is no consensus on a precise definition of this concept. In 2015, Cui and Li proposed a definition of civil aviation safety efficiency for the first time. Based on their findings, "Civil aviation safety efficiency is defined to evaluate the effects of safety inputs believed to be vital to the safety performance of airline companies." They applied DEA and the Malmquist index to calculate the civil aviation safety efficiencies of ten Chinese airline companies (Cui and Li, 2015).

2.3 DEA/MADM combination

Literature reveals that MADM and DEA were entirely separate until 1988 (Adler et al., 2002), after that, some researchers tried to use DEA and MADM as an integrated model to complement each other. Some of the potential pitfalls of these methods are listed below:

- DEA in its original version, classifies DMUs into two efficient and inefficient categories by evaluating sets of multiple criteria (inputs and outputs); thus it does not perform a ranking of DMUs (Sinuany-Stern et al., 2000). This means that it would be difficult for a decision-maker to select one DMU if there was more than one efficient unit (Sarkis, 1999). Therefore, during the last decade there have been attempts to fully rank units in the context of DEA (Sinuany-Stern et al., 2000).

- In MADM methods, weights of criteria are often carried out subjectively by the decision-maker (s). In these cases, the decision-maker may be suffering from stress or get confused (especially if the number of criteria is high) and so this may lead to inconsistency (Ma et al., 1999).
- In some MADM methods which use pairwise comparisons (like AHP or ANP), the number of alternatives is practically limited because of the number of pairwise comparisons that need to be made. This is despite the fact that DEA can easily handle hundreds of alternatives (if the data is available) (Sarkis, 1999).

Considering the aforementioned pitfalls, hybrid DEA-MADM methods have been developed. In most of these studies, firstly, the researcher has used the MADM method for calculating a criterion’s weight. Then, they integrate managerial preferences into the DEA model using the calculated weights as a restriction of DEA weights (Abdollahi et al., 2015; Rezaeisaray et al., 2016; Sarkis, 1999; Shang and Sueyoshi, 1995; Tavana et al., 2015).

However, some researchers have used the DEA model as the first step, to construct the pairwise comparison matrix between DMUs and then used AHP or ANP for ranking DMUs (Alem et al., 2013; Sinuany-Stern et al., 2000).

In this article, we calculate the fuzzy weights of safety efficiency criteria (inputs and outputs); utilizing the fuzzy DEA method based on a novel mathematical modeling. After that, we use these fuzzy weights and implement a combination of six MADM ranking methods (Fuzzy SAW, Fuzzy TOPSIS, Fuzzy VIKOR, ARAS-F, COPRAS-F and Fuzzy MULTIMOORA) with utility interval to evaluate the airlines.

3. Methodology

This article aims to provide a combination method for ranking airlines based on their safety efficiency and selecting the safest one. As described later, both DEA and MADM methods have limitations; nevertheless, an integrated model combines the best for both models, which lessens the pitfalls of each one.

As Figure 1 shows, the implementation procedure of the proposed methodology comprises three phases: Weighting Criteria, Fuzzy MADM’s Ranking, and compound the results.

In the first phase, appropriate inputs and outputs are selected, and fuzzy data is prepared. Then the weight of each criterion is calculated by implementing a novel method using the Fuzzy DEA approach.

In the second phase, airlines will be ranked based on their safety efficiency in the inputs and outputs, and using six Fuzzy MADM methods. Finally, in the third phase, the ‘efficient airline in safety’ field is selected by using the utility interval technique.

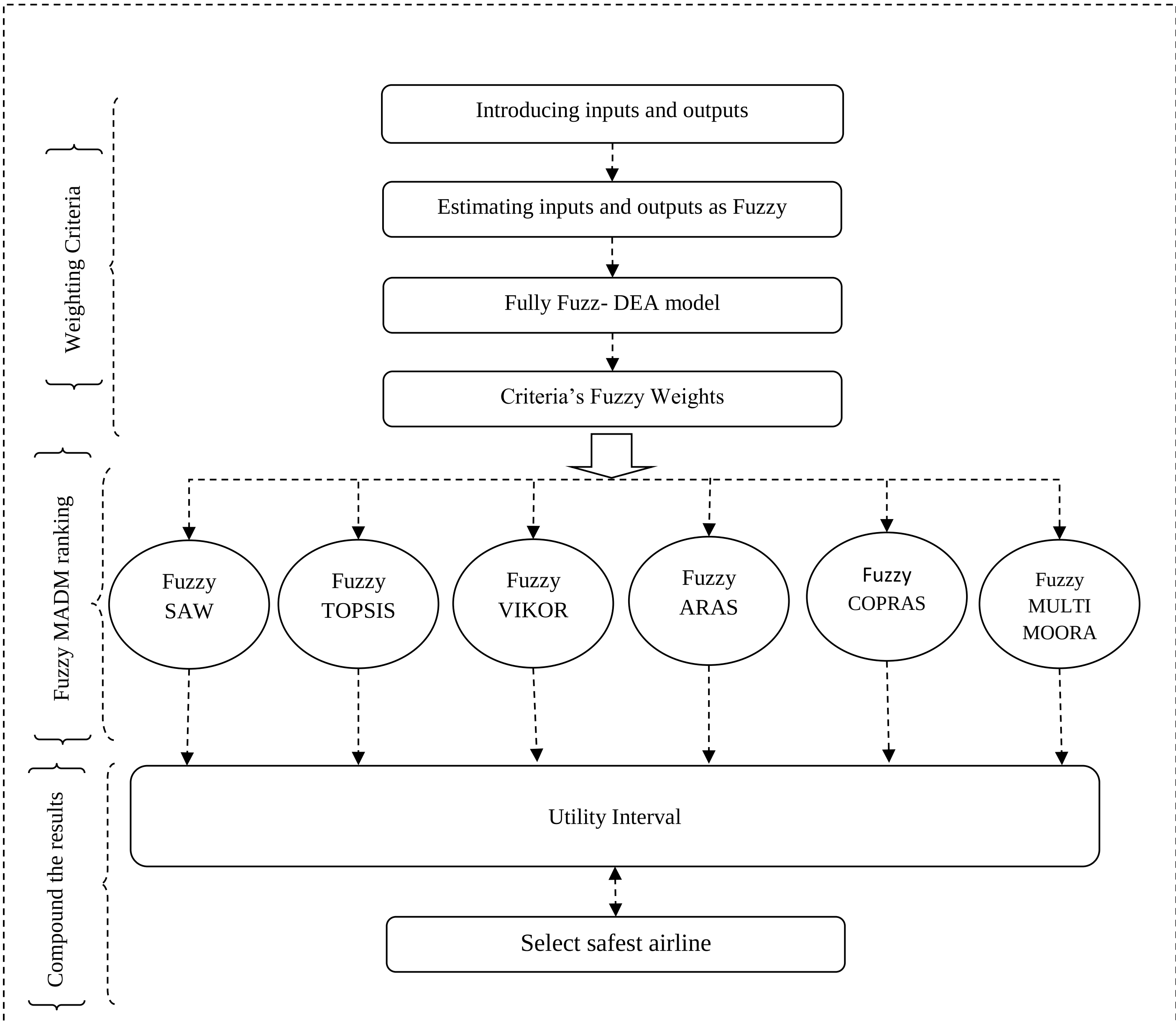

Figure 1. The procedure of the proposed methodology

3.1 Fuzzy Data Envelopment Analysis Method

The usual DEA model requires crisp and measurable data. In real situation, crisp data might not be available for all problems, therefore, the uncertainty of data can be characterized by fuzzy sets. The fuzzy data envelopment analysis (FDEA) is an extended form of the standard DEA, which copes with uncertain data. In fuzzy models, the efficiency scores of DMUs are fuzzy efficiency values or an interval of efficiencies (Alem et al., 2013). Usually, the FDEA method is used when the inputs and outputs of the model have uncertainty or vagueness (Hatami-Marbini et al., 2011; Kao and Liu, 2000). Considering this vital point that the safety data is not available for many of the airlines or is unsecure, we use the fuzzy set theory in this article.

Wanke et al. (2016) used the FDEA method to assess the productive efficiency of Nigerian airlines for the first time for reasons such as the difficulty of obtaining reliable data sources and the uncertainty in the conversion of domestic currency into US Dollars due to internal financial crisis and currency board controls. It is worthy to mention that, although part of the input and output data of Iran's airlines are published in the form of yearbooks, this information has a degree of uncertainty and some errors, as Soltanzadeh and Omrani (2018) have pointed out. Also, providing accurate data sometimes requires a lot of time and cost. Therefore, even in the preparation of yearbooks, there will be differences between actual data and recorded data. Furthermore, Olfat et al. (2016) have emphasized that the calculation of some indicators requires the opinion of experts and stakeholders in the form of a questionnaire. Since the analysis and extraction of scores from such questionnaires is sometimes more complicated, it will cause more uncertainty in the calculated data. All of the above has led us to use fuzzy DEA in our research.

The implementation of DEA with fuzzy sets is commonly used in studies; more than 4000 research articles have been published in international journals and book chapters (Emrouznejad et al., 2008). The fuzzy DEA models can show practically real problems; however, there are various models of fuzzy DEA for evaluating DMUs. These models are classified into four groups; the defuzzification approach, the fuzzy ranking approach, the tolerance approach, and the α-level-based approaches (Lertworasirikul et al., 2003).

Here, we introduce a novel method to identify the weight of each criterion based on introducing a fuzzy ranking approach DEA model.

Consider a set of n DMUs to be evaluated, DMU_j, $j=1,...,n$, each one consumes m semi-positive inputs $\boldsymbol{x}_j=(x_{1j},...,x_{mj})\geq \mathbf{0}_m$ and $\boldsymbol{x}_j\neq \mathbf{0}_m$ to produce s semi-positive outputs $\boldsymbol{y}_j=(y_{1j},...,y_{sj})\geq \mathbf{0}_s$ and $\boldsymbol{y}_j\neq \mathbf{0}_s$. The following well-known CCR model (Charnes et al., 1978) evaluates the unit in question, DMU_o, $o\in\{1,...,n\}$:

$$\begin{array}{lll}
\max & \sum_{r=1}^{s}\mu_r y_{ro} & \\
s.t. & & \\
& \sum_{i=1}^{m}\upsilon_i x_{io}=1 & \\
& \sum_{r=1}^{s}\mu_r y_{rj}-\sum_{i=1}^{m}\upsilon_i x_{ij}\leq 0 & j=1,...,n \\
& \upsilon_i\geq\varepsilon & i=1,...,m \\
& \mu_r\geq\varepsilon & r=1,...,s
\end{array} \tag{1}$$

where υ_i and μ_r are *ith* input and *rth* output weights respectively, and ε is a positive infinitesimal number to avoid weights getting a zero value. Using model (1), the underevaluated DMU_o determines the best weights for its criteria. The objective function of model (1) is to obtain a number between zero and one; the one score is obtained with regards to an efficient DMU. The input and output weights' related to each DMU defers from other DMUs, and reveals the points of strength and weakness of the DMU. DMUs select smaller and larger values for their weak and strong points, respectively. Here, we consider the weights of inputs and outputs as the indicator of the importance of criteria. We evaluate all DMUs and average all gained importance to identify the expected value of criteria importance according to all DMUs.

Now, consider inputs and outputs criteria as fuzzy triangular numbers as well as the weights to deal with such fuzzy criteria as shown below:

$$\begin{array}{l} \tilde{x}_{ij} = (x_{ij}^l, x_{ij}^m, x_{ij}^u), \quad \tilde{y}_{rj} = (y_{rj}^l, y_{rj}^m, y_{rj}^u), \\ \tilde{\upsilon}_i = (\upsilon_i^l, \upsilon_i^m, \upsilon_i^u), \quad \tilde{\mu}_r = (\mu_r^l, \mu_r^m, \mu_r^u) \qquad\qquad \forall j, i, r \end{array} \tag{2}$$

Applying the above assumptions in model (1) leads to the following fuzzy model:

$$\begin{array}{ll} \max \ \sum_{r=1}^{s} \tilde{\mu}_r \otimes \tilde{y}_{ro} & \\ s.t. & \\ \sum_{i=1}^{m} \tilde{\upsilon}_i \otimes \tilde{x}_{io} = \tilde{1} & \\ \sum_{r=1}^{s} \tilde{\mu}_r \otimes \tilde{y}_{rj} - \sum_{i=1}^{m} \tilde{\upsilon}_i \otimes \tilde{x}_{ij} \leq \tilde{0} & j = 1, \ldots, n \\ \tilde{\upsilon}_i \geq \varepsilon \tilde{1} & i = 1, \ldots, m \\ \tilde{\mu}_r \geq \varepsilon \tilde{1} & r = 1, \ldots, s \end{array} \tag{3}$$

Model (3) is a fully fuzzy mathematical programming model which not only shows that the data is fuzzy, but also that decision variables are fuzzy, see (Kumar et al., 2011; Puri and Yadav, 2016) for more details about approaches for solving fully fuzzy DEA models. Here, $\tilde{1}$ and $\tilde{0}$ are the fuzzy forms of 1 and 0 respectively, and $\otimes$ is the sign of the fuzzy numbers multiplication which is defined below:

Definition 1. Let $\tilde{A} = (a^l, a^m, a^u)$ and $\tilde{B} = (b^l, b^m, b^u)$ be fuzzy triangular numbers $a^l \geq 0$ and $b^l \geq 0$; multiplication of these fuzzy numbers is defined in equation (4):

$$\tilde{A} \otimes \tilde{B} = (a^l b^l, a^m b^m, a^u b^u) \tag{4}$$

In order to convert model (3) to a deterministic and solvable model, applying a ranking fuzzy number function $T(.)$ is required:

$$
\begin{aligned}
&\max\ \sum_{r=1}^{s} T(\tilde{\mu}_r \otimes \tilde{y}_{ro}) \\
&s.t. \\
&\sum_{i=1}^{m} T(\tilde{\upsilon}_i \otimes \tilde{x}_{io}) = T(\tilde{1}) \\
&\sum_{r=1}^{s} T(\tilde{\mu}_r \otimes \tilde{y}_{rj}) - \sum_{i=1}^{m} T(\tilde{\upsilon}_i \otimes \tilde{x}_{ij}) \le T(\tilde{0}) \quad && j = 1,\ldots,n \\
&T(\tilde{\upsilon}_i) \ge \varepsilon T(\tilde{1}) && i = 1,\ldots,m \\
&T(\tilde{\mu}_r) \ge \varepsilon T(\tilde{1}) && r = 1,\ldots,s
\end{aligned}
\tag{5}
$$

In this paper, we exert the fuzzy ranking function $T(.)$ that Yager (1981) introduced in equation (6):

$$
T(a^l, a^m, a^u) = 0.5(a^m + 0.5(a^l + a^u)) \tag{6}
$$

Therefore, the solvable model related to (5), and according to the multiplication definition and fuzzy ranking function, can be obtained below:

$$
\begin{aligned}
&\max\ 0.5\sum_{r=1}^{s} (\mu_r^m y_{ro}^m + 0.5(\mu_r^l y_{ro}^l + \mu_r^u y_{ro}^u)) \\
&s.t. \\
&0.5\sum_{i=1}^{m} (\upsilon_i^m x_{io}^m + 0.5(\upsilon_i^l x_{io}^l + \upsilon_i^u x_{io}^u)) = 1 \\
&\sum_{r=1}^{s} (\mu_r^m y_{rj}^m + 0.5(\mu_r^l y_{rj}^l + \mu_r^u y_{rj}^u)) - \sum_{i=1}^{m} (\upsilon_i^m x_{ij}^m + 0.5(\upsilon_i^l x_{ij}^l + \upsilon_i^u x_{ij}^u)) \le 0 \quad && j = 1,\ldots,n \\
&\varepsilon \le \upsilon_i^l \le \upsilon_i^m && i = 1,\ldots,m \\
&\upsilon_i^m \le \upsilon_i^u && i = 1,\ldots,m \\
&\varepsilon \le \mu_r^l \le \mu_r^m && r = 1,\ldots,s \\
&\mu_r^m \le \mu_r^u && r = 1,\ldots,s
\end{aligned}
\tag{7}
$$

Model (7) is a linear programming model that can be solved using ordinary LP software. The objective value of model (7) in optimality is the efficiency of DMU_o. Also, $\tilde{\upsilon}_i^* = (\upsilon_i^{l*}, \upsilon_i^{m*}, \upsilon_i^{u*})$ and $\tilde{\mu}_r^* = (\mu_r^{l*}, \mu_r^{m*}, \mu_r^{u*})$ are the optimal important criteria in the assessment of DMU_o.

The criteria determined in the previous stage and the fuzzy weights taken from the Fuzzy-DEA method are used as inputs for six different MADM methods (Fuzzy SAW, Fuzzy TOPSIS, Fuzzy VIKOR, ARAS-F, COPRAS-F and Fuzzy MULTIMOORA), and the Utility interval method is utilized to select the most efficient airlines according to safety.

4. Case study

Aviation safety is the number one priority in the aviation industry; it is not just a local issue but a subject of international concern. This is why airline safety is fast becoming increasingly complex and controversial. The fact that developing countries have intermittent safety records has been proven in aviation safety research (Oster et al., 2013), and it is obvious that Iran's airlines have never been excluded from this rule. Unfortunately, in recent years, Iran's airline industry has not held a good position in safety records. The International Civil Aviation Organization (ICAO) statistics show that Iranian airlines experienced more than ten fatal accidents between 1995 and 2014. More than 1000 people were injured or died in these accidents; this is very high compared with world standards (from ICAO safety report and Accident statistics).

For this reason, we decided to measure safety efficiency in Iranian airlines and select the most efficient one. We used data from the seven Iranian airports from 2011-2016 to evaluate airline safety efficiency using the following steps:

4.1 Determining the inputs and outputs

According to the existing literature and using the fuzzy Delphi method, this paper constructs a civil aviation safety system index of inputs and outputs.

In the operating process, we consider three different input categories:

- ***Labor input****:* Defined as the number of staff (including safety officers and maintenance personnel) deployed in the pre-flight, flying and post-flight stages. Previous studies have used labor input in other airline efficiency fields (Cui and Li, 2015).
- ***Capital input****:* Defined as the fixed assets input on safety (includes new investments in safety control, safety maintenance and safety communication equipment, as well as in other safety hardware). Previous studies used capital input in other airline efficiency fields (Barros et al., 2013; Cui and Li, 2015; Lu et al., 2012; Tavassoli et al., 2014).
- ***Costs****:* Cui and Li (2015) named this input as 'fund input' and defined it as 'the investments in safety software, safety staff, safety technology import, safety operation procedure import, the upgrade of safety control systems, the introducing of safety talents, the training of safety staff, and other investments in safety software and safety staff' (Cui and Li, 2015). Previous studies used cost input in other airline efficiency fields (Barros et al., 2013; Chiou and Chen, 2006; Lu et al., 2012; Merkert and Hensher, 2011). In addition, investments in the research and development of safety technology are intended in this category.

On the other hand, we consider two different output categories:

- ***Financial performance****:* Defined as the ratio of the net profit to the revenue of the airport (annually). Cui and Li named this output as 'net profit rate'. Previous studies used the financial output in other airline efficiency fields (Barros et al., 2013; Cui and Li, 2015; Lu et al., 2012).
- ***Service performance****:* Defined as the rate of the number of safe flights to the total airline flight number per year. Cui and Li defined this output as 'the percent of the passenger turnover volume without accidents or incidents to the total passenger turnover volume'(Cui and Li, 2015).

Table 1. Inputs and Outputs of the fuzzy model

Inputs	Outputs
• Labor Inputs (LI) • Capital Inputs (CI) • Cost (C)	• Financial Performance (FP) • Safety performance (SP)

Labor Inputs (LI), Capital Inputs (CI), and Cost (C), are classified as inputs for DMUs whilst Financial Performance (FP), and Safety performance (SP), are classified as outputs (see Table 1). These inputs and outputs will be used in the proposed Fuzzy DEA as criteria used to determine the weights of each one.

4.2 Preparing the required fuzzy data

As discussed later, we consider each airline safety's criteria as a DMU and use the fully fuzzy DEA method for calculating the weights of each one.

As shown in the previous stage, relevant safety measurement criteria are identified by referring to the literature review and using the fuzzy Delphi method. In this stage, we prepare the required fuzzy data. A group of ten experts in the airline and transportation field from Iranian airline management; including technicians, engineers and managerial officers, evaluate our criteria regarding the alternative airlines. The majority voting system is used to aggregate their responses, and put into the fuzzy scale as shown in Table 2. In the other word, experts' justification has considered the airlines' performance from 2011 to 2016 and evaluated them year by year. We then average the evaluation and scale it within the Table 2 ranges.

Table 2. Linguistic variables scale for fuzzy data

Very Poor (VP)	Poor (P)	Fair (F)	Good (G)	Very Good (VG)	Excellent (E)
(0,0.0135,0.05)	(0.05,0.15,0.3)	(0.3,0.45,0.6)	(0.6,0.75,0.8)	(0.8,0.95,1)	(1,1,1)

According to these linguistic variables, the fuzzified data for the seven Iranian airlines are as follows (see Table 3):

Table 3. Linguistic variables

DMUs	AIRLINES						
	A1	A2	A3	A4	A5	A6	A7
D1	P	F	G	G	F	G	P
D2	P	P	E	P	P	P	F
D3	P	P	P	P	F	F	P
D4	P	P	VG	VG	P	F	P
D5	F	F	F	F	P	F	F

In this table, DMUs are defined as: D1= Labor Inputs (LI); D2= Capital Inputs (CI); D3= Cost (C); D4= Financial Performance (FP) and D5= Safety Performance (SP).

Then in Table 4, the lower bounds (lb), middle bounds (mb), and upper bounds (ub) data for each airline (A1- A7) are shown, giving the values for the linguistic variables' scales.

Table 4. Amounts of lb, mb, and ub

DMUs	AIRLINES						
	A1	A2	A3	A4	A5	A6	A7
D1	(0.05,0.15,0.3)	(0.3,0.45,0.6)	(0.6,0.75,0.8)	(0.6,0.75,0.8)	(0.3,0.45,0.6)	(0.6,0.75,0.8)	(0.05,0.15,0.3)
D2	(0.05,0.15,0.3)	(0.05,0.15,0.3)	(1,1,1)	(0.05,0.15,0.3)	(0.05,0.15,0.3)	(0.05,0.15,0.3)	(0.3,0.45,0.6)
D3	(0.05,0.15,0.3)	(0.05,0.15,0.3)	(0.05,0.15,0.3)	(0.05,0.15,0.3)	(0.3,0.45,0.6)	(0.3,0.45,0.6)	(0.05,0.15,0.3)
D4	(0.05,0.15,0.3)	(0.05,0.15,0.3)	(0.8,0.95,1)	(0.8,0.95,1)	(0.05,0.15,0.3)	(0.3,0.45,0.6)	(0.05,0.15,0.3)
D5	(0.3,0.45,0.6)	(0.3,0.45,0.6)	(0.3,0.45,0.6)	(0.3,0.45,0.6)	(0.05,0.15,0.3)	(0.3,0.45,0.6)	(0.3,0.45,0.6)

(For example, Poor (P) equals (0.05, 0.15, 0.3).

4.3 Calculation criteria's fuzzy weights

As discussed in the methodology section, after determining the airlines' safety efficiency criteria (the inputs and outputs) and preparing the fuzzy data for seven Iranian airlines, the fully fuzzy DEA method is used to calculate the fuzzy weight for each of the five efficiency criteria.

In this step, the optimal fuzzy weights of each criterion (D1-D5) are obtained by Eq. (7). As discussed before, the input and output weights' related to each DMU defers from six other DMUs. The results of model (7) for all airlines are gathered in Table 5.

Table 5. Computational results of DEA

DMU	D1			D2			D3		
	L	M	U	L	M	U	L	M	U
A1	0.000010	0.000015	0.000015	0.000010	0.000010	0.000010	0.000010	0.000010	13.333269
A2	0.000014	0.000014	0.000014	0.000010	0.000010	0.000010	0.000010	0.000010	13.333219
A3	0.000014	0.000014	0.000014	0.000010	0.000010	0.000010	0.000010	0.000010	13.333057
A4	0.000014	0.000014	0.000014	0.000010	0.000010	0.000010	0.000010	0.000010	13.333169
A5	0.000014	0.000014	0.000014	6.153781	6.153781	6.153781	0.000010	0.000010	0.000010
A6	0.000014	0.000014	0.000014	6.153758	6.153758	6.153758	0.000010	0.000010	0.000010
A7	6.153809	6.153809	6.153809	0.000010	0.000010	0.000010	0.000010	0.000010	0.000010
Ave.	0.879127	0.879127	0.879127	1.758227	1.758227	1.758227	0.000010	0.000010	7.618963

DMU	D4			D5		
	L	M	U	L	M	U
A1	0.000010	0.000010	0.000010	0.648536	0.648536	0.648536
A2	0.000010	0.000010	0.000010	0.648536	0.648536	0.648536
A3	0.000010	0.000010	0.000010	0.648536	0.648536	0.648536
A4	0.000010	0.000010	0.000010	0.648536	0.648536	0.648536
A5	0.000010	0.000010	0.000010	0.648536	0.648536	0.648536
A6	0.000010	0.000010	0.000010	0.648536	0.648536	0.648536
A7	4.539695	4.539695	4.539695	0.648536	0.648536	0.648536
Ave.	0.648536	0.648536	0.648536	0.000010	1.523800	3.678336

After evaluating all DMUs' scores, we average all gained weights to identify the expected value of each criteria importance (weight). Therefore, amounts in the last row in Table 5, which are the average of the columns, present an expected value for the weights of criteria. These fuzzy weights are summarized in Table 6.

Table 6. Final weights of criteria

	L	M	U
D1	0.879127	0.879127	0.879127
D2	1.758227	1.758227	1.758227
D3	0.00001	0.00001	7.618963
D4	0.648536	0.648536	0.648536
D5	0.00001	1.5238	3.678336

It is worth noting that the optimal value of model (7), which indicates the efficiency of each airline, is the estimate unity for all except A5 (see Table 7). This shows that DEA does not perform a full ranking based on efficiencies, and it would be difficult for a decision-maker to select one DMU if there are more than one efficient units. In other words, for a small sample of DMUs, the method fails to discriminate, and all DMUs are efficient, therefore we are unable to rank them according to the efficiency scores of the airlines.

Table 7. Efficiency score of airlines using model (7)

Airlines	A1	A2	A3	A4	A5	A6	A7
Efficiency score	1	1	1	1	0.5	1	1

By applying the DEA models for calculating the efficiency of DMUs, a number of them may have an equal efficiency score of one. Some ranking algorithms such as super efficiency approach known as AP approach (Andersen and Petersen, 1993), the common weights approach (Friedman and Sinuany-Stern, 1997), Benchmarking approach (Torgersen et al., 1996), inefficient frontier approach (Yamada, 1994), the norm approach (Jahanshahloo et al., 2004), virtual DMU approach (Wang and Luo, 2006), and DM interference approach (Wang et al., 2009) have been proposed to rank best performers. These methods would fail if data have certain structures. For example, infeasibility of the AP model, the similar result of the common weight model for all candidates, and inefficiency of norm method for ranking non-extreme DMUs are some issues (Aldamak and Zolfaghari, 2017).

To evaluate the result of these ranking models in our case study, we applied the super efficiency approach (AP) for ranking the candidate airlines. Super efficiency approach has been established where the DMU under assessment is excluded in order to improve the ranking (Aldamak and Zolfaghari, 2017). Table 8 indicates the result of the AP approach between 7 DMUs in the problem, and 4 DMUs obtain full efficient rank (equal to 1). Both Table 7 and 8 demonstrate that applying efficiency values, which many types of research rely on, is not always applicable. This is why we proposed a new and innovative approach that is a combination of DEA and MADM approaches.

Table 8. Efficiency score of airlines using super efficiency approach

Airlines	A1	A2	A3	A4	A5	A6	A7
Efficiency	1.88	1	1	3.69	0.5	1	1

4.4 Ranking airlines Using Fuzzy MADM Methods

As described later, we use six different Fuzzy MADM methods including Fuzzy SAW, Fuzzy TOPSIS, Fuzzy VIKOR, ARAS-F, COPRAS-F and Fuzzy MULTIMOORA, for ranking the seven Iranian airlines.

4.4.1 Fuzzy SAW Results

Churchman and Ackoff (1954) utilized the SAW method to cope with a portfolio selection problem. The SAW method is probably the best-known and most widely used MADM method because of its simplicity. In this paper, we use the Fuzzy SAW method according to Chou et al. (2008), to aggregate the fuzzy preferred ratings and to rank the airlines (method detail in Appendix A.1).

Based on Table 9, the total fuzzy scores for each airline are derived by multiplying the fuzzy rating matrix to their respective weight vectors. Then a crisp value for each total score is computed using a defuzzification method, and a final ranking is attained.

Table 9. Fuzzy and crisp total scores and final ranking resulting from the Fuzzy SAW method

	Fuzzy SAW				
	Total Scores			Crisp Value	Rank
	L	M	u		
Airline 1	0.199	4.218	95.858	26.12342981	1
Airline 2	0.161	3.203	90.284	24.21268583	2
Airline 3	0.076	1.036	79.698	20.46163219	5
Airline 4	0.162	3.107	89.865	24.06041817	3
Airline 5	0.161	3.044	25.317	7.891623457	7
Airline 6	0.155	3.040	25.378	7.903350285	6
Airline 7	0.138	2.593	86.940	23.06628639	4

4.4.2 Fuzzy TOPSIS Results

TOPSIS is an approach to identify an alternative which is closest to the ideal solution and farthest from the worst (negative) ideal solution in a multi-dimensional computing space. Fuzzy TOPSIS is applied as discussed in Appendix A.2 based on Büyüközkan and Çifçi (2012). In this method, the negative and positive ideal solutions (*D*+, *D*–) are calculated, and the ranking score (Cl) is obtained. The results of the Fuzzy TOPSIS method are summarized in Table 10.

According to the results, Airline 1, which has the highest-ranking score (Cl), is selected as the safest airline.

Table 10. Ideal solution and the rank of each alternative resulting from the Fuzzy TOPSIS method

Airline	Fuzzy TOPSIS			
	D+	D-	Cl	RANK
Airline 1	0.38433481	4.720110122	0.924705856	1
Airline 2	0.5763916	4.580551745	0.888229992	2
Airline 3	1.379538304	3.943743419	0.740848151	7
Airline 4	0.720602525	4.475499171	0.861318626	5
Airline 5	0.5763916	4.580551714	0.888229991	3
Airline 6	0.720602525	4.47549914	0.861318625	6
Airline 7	0.650824813	4.510453532	0.8739024	4

4.4.3 Fuzzy VIKOR Results

The VIKOR method is introduced as an applicable technique to deal with multi-criteria decision making problems which have incommensurable units and conflicting criteria (Opricovic and Tzeng, 2004). In this method, the separation $\widetilde{S_j}$ of alternative A_j from the fuzzy best value f_i^*; the separation $\widetilde{R_j}$ of alternative A_j from the fuzzy worst value f_i^0, and the final score ($\widetilde{Q_j}$), are determined based on the Fuzzy VIKOR method, which is explained in Appendix A.3.

According to the values of $\widetilde{Q_j}$ *and* Q_j shown in Table 11, the final ranking of these seven airlines can be presented.

Table 11. Values of $\boldsymbol{\widetilde{Q_j}}$, $\boldsymbol{Q_j}$ and the ranking of each alternative resulting from the Fuzzy VIKOR method

	Fuzzy VIKOR Method				
	$\widetilde{Q_j}$			Q_j	Rank
	L	M	u		
Airline 1	0	0	0	0	1
Airline 2	0.00173314	0.020720735	0.030329656	0.018376067	3
Airline 3	0.019478795	0.170066493	0.200444564	0.140014086	5
Airline 4	0.001668016	0.019683729	0.019838218	0.015218423	2
Airline 5	0.001733194	0.088599027	0.918833092	0.274441085	7
Airline 6	0.003686799	0.03979692	0.748341219	0.207905464	6
Airline 7	0.002736495	0.033319539	0.055397711	0.031193321	4

4.4.4 ARAS-F Results

ARAS (Additive Ratio Assessment) was introduced and developed by Zavadskas and Turskis (2010). In addition, the same authors developed the ARAS-F method to solve different problems in the fuzzy environment (Turskis and Zavadskas, 2010). In this method, the value of the optimality function (S) and the utility degree (K) are determined based on the ARAS-F method as explained in Appendix A.4. The value of the optimality function (S) in the ARAS-F model for each criterion is determined in Table 12. Then, the centre-of-area algorithm is used for the defuzzification of ARAS-F, and the utility degree of an alternative (k_i) is calculated in Table 13.

Table 12. The value of the optimality function resulting from the ARAS-F method

	ARAS_F Method		
	L	M	u
Alternative 0	0.006	0.175	1.376
Airline 1	0.004	0.148	1.329
Airline 2	0.003	0.119	1.181
Airline 3	0.003	0.093	1.003
Airline 4	0.005	0.141	1.214
Airline 5	0.003	0.090	0.528
Airline 6	0.004	0.124	0.690
Airline 7	0.003	0.110	1.144

The second column of Table 13 is a crisp value of the optimality function (S_i), the third column represents the utility degree of each alternative (k_i), and the last column is the ranking of the alternative.
The safest airline according to the calculation results is Airline 1. This means that the best alternative is the first one, and the worst alternative is Airline 5.

Table 13. The optimality function and the degree of an alternative value resulting from the ARAS-F method

Airline	ARAS_F Method		
	s_i	k_i	Rank
Airline 1	0.406920339	0.939360132	1
Airline 2	0.35547457	0.82059953	3
Airline 3	0.298045656	0.688027065	5
Airline 4	0.375142289	0.866001711	2
Airline 5	0.177809039	0.410465407	7
Airline 6	0.235125554	0.542778403	6
Airline 7	0.341577005	0.788517529	4

4.4.5 COPRAS-F Results

The COPRAS method, introduced by Zavadskas and Kaklauskas (1996) is a compromising MCDM technique which aims to find a solution regarding the positive and negative ideal solutions. Based on Appendix A.5, in the current case, calculations are made following Zavadskas and Antucheviciene (2007). In this method, the weighted normalized value (pi) is calculated, and the relative significance or priority value (Qi) for each airline is computed (see Table 14). Finally, the utility degree (Ni) for each alternative is calculated, and the complete ranking of airlines is obtained.

Table 14. Relative weight, utility degree and rank of each alternative resulting from the COPRAS-F method

	COPRAS_F Method					
	P_i			Q_i	N_i	Rank
	L	M	u			
Airline 1	0.000	0.065	199.877	50.00196746	100	1
Airline 2	0.000	0.063	85.357	21.37059994	42.73951811	2
Airline 3	0.003	0.095	11.637	2.957323321	5.914413914	7
Airline 4	0.003	0.098	50.766	12.74100319	25.48100372	4
Airline 5	0.000	0.029	85.175	21.30832731	42.61497775	3
Airline 6	0.001	0.075	50.728	12.71965675	25.43831252	5
Airline 7	0.000	0.060	43.272	10.84807569	21.69529769	6

Table 14 shows that Airline 1 has the higher relative significance value and that Airline 3 has the lowest safety score.

4.4.6 Fuzzy MULTIMOORA Results

The Multi-Objective Optimization by Ratio Analysis (MOORA) was introduced by Brauers and Zavadskas (2006). Subsequently, these authors further developed the method (Brauers and Zavadskas, 2010) and presented the MULTIMOORA (MOORA plus the full multiplicative form).

The method consists of three parts: the ratio system, the reference point approach, and the full multiplicative form. The final normalized score and rank of each airline are shown in Tables 15-17. Calculations of the Fuzzy MULTIMOORA method are made following the steps in Appendix A.6 based on Liu et al. (2014).

Table 15. The normalized assessment of alternatives and rank of each alternative resulting from the Fuzzy Ratio System

	Fuzzy Ratio System				
	y_i^*			BNP	Rank
	L	M	u		
Airline 1	-0.001499538	0.050224477	-0.326525567	-0.092600209	2
Airline 2	-0.004230772	0.017570933	-0.374321689	-0.120327176	3
Airline 3	-0.027262638	-0.180635909	-0.600820738	-0.269573095	5
Airline 4	-0.001353612	0.050323997	-0.322415095	-0.091148236	1
Airline 5	-0.004230855	-0.045793263	-1.24129228	-0.430438799	7
Airline 6	-0.005456754	0.00944429	-1.013369266	-0.336460577	6
Airline 7	-0.008317703	-0.031290784	-0.445842271	-0.161816919	4

Table 16. The normalized maximum distance from the reference point and rank of each alternative resulting from the fuzzy reference point method

	Fuzzy reference point				
	max distance from reference point			BNP	Rank
	L	M	u		
Airline 1	0.006	0.065	0.084	0.051777308	2
Airline 2	0.006	0.065	0.084	0.051777308	2
Airline 3	0.026	0.231	0.278	0.178424859	4
Airline 4	0.006	0.065	0.08	0.050325335	1
Airline 5	0.006	0.065	0.643	0.238215488	6
Airline 6	0.006	0.065	0.643	0.238133673	5
Airline 7	0.007	0.082	0.119	0.06921671	3

Table 17. The normalized overall utility and rank of each alternative resulting from the fuzzy full multiplicative method

	Fuzzy full multiplicative form				
	$\widetilde{U_\iota}$			BNP	Rank
	L	M	u		
Airline 1	4.67E-09	6.08E+06	2.24E+12	7.46E+11	1
Airline 2	2.33E-09	2.03E+06	3.73E+11	1.24E+11	3
Airline 3	8.41E-09	1.15E+06	3.11E+10	1.04E+10	5
Airline 4	2.80E-08	7.70E+06	6.21E+11	2.07E+11	2
Airline 5	1.95E-10	2.25E+05	3.11E+10	1.04E+10	5
Airline 6	5.25E-09	1.22E+06	6.21E+10	2.07E+10	4
Airline 7	2.33E-09	2.03E+06	3.73E+11	1.24E+11	3

The theory of dominance was applied when summarizing the ranks provided by different parts of MULTIMOORA–FG. As shown in Table 18, the first airline (A1) is the safest one.

Table 18. Ranking of the airlines according to MULTIMOORA–FG

	MULTIMOORA–FG			
	The fuzzy Ratio System	The fuzzy Reference Point	The fuzzy Full Multiplicative Form	MULTIMOORA-FG (Final rank)
Airline 1	3	2	3	3
Airline 2	5	4	5	4
Airline 3	1	1	2	1
Airline 4	7	6	5	7
Airline 5	6	5	4	6
Airline 6	4	3	3	4
Airline 7	2	2	1	2

4.5 Compounding the results

As discussed in the previous sections, it should be noted that the application of various MADM methods can yield different results (Antucheviciene et al., 2011). Therefore, selecting the appropriate MADM method is very significant in the decision-making process.

However, the use of a single MADM method for prioritization cannot ensure a robust approach (Akhavan et al., 2015). When the differences between the alternatives are inherently close together or when the number of alternatives increases, the necessity for a robust aggregation method particularly increases (Varmazyar et al., 2016).

Concerning a shortcoming of usual combination methods (such as averaging function, Borda, Copeland rules, etc.) we use a hybrid approach proposed by Wang et al. (2005). This approach uses a utility interval technique to combine the ranking results of MADM methods. Utility interval provides information on the degree of preference; therefore, it is easier to be understood and accepted.

Ranking results in the second phase (Fuzzy MADM Ranking) show that each of the six ranking methods provides different information on the degrees of preference (see Table 19) and so a robust combination method is required to determine the final decision.

Table 19. The ranking results of MADM methods

Airlines	Fuzzy SAW	Fuzzy TOPSIS	Fuzzy VIKOR	F-ARAS	F-COPRAS	Fuzzy MULTIMOORA
A1	1	1	1	1	1	2
A2	2	2	3	3	2	3
A3	5	7	5	5	7	4
A4	3	5	2	2	4	1
A5	7	3	7	7	3	7
A6	6	6	6	6	5	6
A7	4	4	4	4	6	4

A linear programming (LP) model is first constructed to estimate the interval for each alternative (airline). This model should be solved for each ranking method (Equation 8).

$min/\max u_{i1}$

s.t.

$$u_{ij} - u_{i(j+1)} \geq \varepsilon_{j(j+1)} \quad j=1,2,\dots,n-1 \tag{8}$$

$$\sum_{j=1}^{n} u_{ij} = 1$$

$$u_{ij} \geq 0 \quad j=1,2,\dots,n$$

where u_{ij} is the utility of the j_{th} ranked alternative perceived by the i_{th} ranking method. The objective function calculates the minimum and maximum interval numbers $[u_{ij}^{l}, u_{ij}^{u}]$ for the first-ranked alternative (by each ranking method), and ε is a small positive number.

In the current study, the number of alternatives (j) and the number of ranking methods (i) equal 7 and 6, respectively. To simplify the above LP model, $\varepsilon_{j(j+1)}$ is assumed to be equal to ε. As stated by Wang et al. (2005), ε is ranged as follows:

$$0 \leq \varepsilon \leq \varepsilon_{max} = \frac{2}{n(n-1)} \tag{9}$$

In our article, $\varepsilon_{max} = \frac{1}{21}$ $(n = 7)$; therefore, three sets of evaluation are run for ε = 0, 0.02, 0.04.

Table 20 provides all the utility estimates, which are generated from the rankings indicated in Table 19 and the LP model.

Table 20. Utility interval corresponding to the preference ranking of MADM methods

	Method	A1	A2	A3	A4	A5	A6	A7
ε=0	Fuzzy SAW	[0.1429, 1]	[0, 0.5]	[0, 0.2]	[0, 0.333]	[0, 0.1429]	[0, 0.1667]	[0, 0.25]
	Fuzzy TOPSIS	[0.1429, 1]	[0, 0.5]	[0, 0.1429]	[0, 0.2]	[0, 0.333]	[0, 0.1667]	[0, 0.25]
	Fuzzy VIKOR	[0.1429, 1]	[0, 0.333]	[0, 0.2]	[0, 0.5]	[0, 0.1429]	[0, 0.1667]	[0, 0.25]
	ARAS-F_	[0.1429, 1]	[0, 0.333]	[0, 0.2]	[0, 0.5]	[0, 0.1429]	[0, 0.1667]	[0, 0.25]
	COPRAS-F	[0.1429, 1]	[0, 0.5]	[0, 0.1429]	[0, 0.25]	[0, 0.333]	[0, 0.2]	[0, 0.1667]
	Fuzzy MULTIMOORA	[0, 0.5]	[0, 0.333]	[0, 0.25]	[0.1429, 1]	[0, 0.1429]	[0, 0.1667]	[0, 0.25]
ε=0.02	Fuzzy SAW	[0.2029, 0.7]	[0.1, 0.39]	[0.04, 0.156]	[0.08, 0.2733]	[0, 0.0829]	[0.02, 0.1167]	[0.06, 0.205]
	Fuzzy TOPSIS	[0.2029, 0.7]	[0.1, 0.39]	[0, 0.0829]	[0.04, 0.156]	[0.08, 0.2733]	[0.02, 0.1167]	[0.06, 0.205]
	Fuzzy VIKOR	[0.2029, 0.7]	[0.08, 0.2733]	[0.04, 0.156]	[0.1, 0.39]	[0, 0.0829]	[0.02, 0.1167]	[0.06, 0.205]
	F_ARAS	[0.2029, 0.7]	[0.08, 0.2733]	[0.04, 0.156]	[0.1, 0.39]	[0, 0.0829]	[0.02, 0.1167]	[0.06, 0.205]
	COPRAS-F	[0.2029, 0.7]	[0.1, 0.39]	[0, 0.0829]	[0.06, 0.205]	[0.08, 0.2733]	[0.04, 0.1560]	[0.02, 0.1167]
	Fuzzy MULTIMOORA	[0.1, 0.39]	[0.08, 0.2733]	[0.06, 0.205]	[0.2029, 0.7]	[0, 0.0829]	[0.02, 0.1167]	[0.06, 0.205]
ε=0.04	Fuzzy SAW	[0.2629, 0.4]	[0.2, 0.28]	[0.08, 0.1120]	[0.16, 0.2133]	[0, 0.0229]	[0.04, 0.0667]	[0.12, 0.16]
	Fuzzy TOPSIS	[0.2629, 0.4]	[0.2, 0.28]	[0, 0.0229]	[0.08, 0.1120]	[0.16, 0.2133]	[0.04, 0.0667]	[0.12, 0.16]
	Fuzzy VIKOR	[0.2629, 0.4]	[0.16, 0.2133]	[0.08, 0.1120]	[0.2, 0.28]	[0, 0.0229]	[0.04, 0.0667]	[0.12, 0.16]
	F_ARAS	[0.2629, 0.4]	[0.16, 0.2133]	[0.08, 0.1120]	[0.2, 0.28]	[0, 0.0229]	[0.04, 0.0667]	[0.12, 0.16]
	COPRAS-F	[0.2629, 0.4]	[0.2, 0.28]	[0, 0.0229]	[0.12, 0.16]	[0.16, 0.2133]	[0.08, 0.1120]	[0.04, 0.0667]
	Fuzzy MULTIMOORA	[0.2, 0.28]	[0.16, 0.2133]	[0.12, 0.16]	[0.2629, 0.4]	[0, 0.0229]	[0.04, 0.0667]	[0.12, 0.16]

The aggregated utility (weighted average utility) of each alternative can be calculated as follows:

$$u_j^L = \sum_{j=1}^{m} w_i u_{ij}^L \qquad j = 1,2,\ldots,n \tag{10}$$

$$u_j^U = \sum_{j=1}^{m} w_i u_{ij}^U \qquad j = 1,2,\ldots,n \tag{11}$$

where, w_i (i = 1, ..., m) is the relative weight of the i_{th} ranking method. The related weights are computed by developing the correlation matrix between the ranking methods indicated in Table 21. The normalized sum of each method's correlation is taken into account as the weight in Eqs. (10) and (11). The weighted average utility intervals of the case in this study for different values of ε are summarized in Table 22.

Table 21. Correlation matrix and weights of each method

	Fuzzy SAW	Fuzzy TOPSIS	Fuzzy VIKOR	F_Copras	F_ARAS	Fuzzy MULTIMOORA
Fuzzy SAW	1.000	0.571	0.964	0.536	0.964	0.875
Fuzzy TOPSIS	0.571	1	0.464	0.893	0.464	0.219
Fuzzy VIKOR	.964	.464	1.000	.464	1.000	.948
F_Copras	.536	.893	.464	1.000	.464	.255

F_ARAS	0.964	0.464	1.000	0.464	1.000	0.948
Fuzzy MULTIMOORA	0.88	0.22	0.95	0.26	0.95	1.00
Sum	4.910	3.611	4.840	3.612	4.840	4.245
Weight	0.188426	0.138575	0.18574	0.138614	0.18574	0.162906

Table 22. The weighted average utility interval for ε=0, 0.02, 0.04

	A1	A2	A3	A4	A5	A6	A7
ε=0	[0.1196, 0.9185]	[0, 0.4188]	[0, 0.1896]	[0.0233, 0.4620]	[0, 0.2046]	[0, 0.1729]	[0, 0.2345]
ε=0.02	[0.1861, 0.6495]	[0.0903, 0.3332]	[0.0303, 0.1403]	[0.0972, 0.3517]	[0.0259, 0.1446]	[0.0237, 0.1240]	[0.0526, 0.1886]
ε=0.04	[0.2526, 0.3805]	[0.1805, 0.2475]	[0.0606, 0.0909]	[0.1712, 0.2414]	[0.0519, 0.0846]	[0.0474, 0.0751]	[0.1051, 0.1427]

The degrees of preferences among the alternatives are calculated based on Eq. (12). Further details about the ranking process can be found in Wang et al. (2005).

$$P_{ij} = P(u_i > u_j) = \frac{\max\left(0, u_i^U - u_j^L\right) - \max\left(0, u_i^L - u_j^U\right)}{\left(u_i^U - u_i^L\right) + \left(u_j^U - u_j^L\right)} \qquad i, j = 1, \dots, n; i \neq j \tag{12}$$

Matrix of the degrees of preference (PD) is shown in Table 23 for ε=0.

Table 23. Matrix of the degrees of preference (PD) for ε=0

-	0.754291	0.929186	0.723335	0.915296	0.945153	0.888814
0.245709	-	0.688363	0.461224	0.6718	0.707791	0.641053
0.070814	0.311637	-	0.264682	0.480974	0.523034	0.447064
0.276665	0.538776	0.735318	-	0.718172	0.755396	0.686275
0.084704	0.3282	0.519026	0.281828	-	0.541987	0.465953
0.054847	0.292209	0.476966	0.244604	0.458013	-	0.424399
0.111186	0.358947	0.552936	0.313725	0.534047	0.575601	-

In the next step, we calculate the matrix of preference relation.

$$m_{ij} = \begin{cases} 1 & if\ p_{ij} > 0.5 + \bar{\delta} \\ 0 & if\ p_{ij} \leq 0.5 + \bar{\delta} \end{cases} \tag{13}$$

The threshold set ($\bar{\boldsymbol{\delta}}$) by the decision-maker would be zero because there is no specific requirement. The matrix of preference relation is shown in Table 24 (with respect to the data provided in Table 23).

Table 24. Matrix of preference relation (M_{pl}) for ε=0

$$\begin{vmatrix} 0 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{vmatrix}$$

Finally, we calculate the sum of the elements of each row in the above matrix of preference relation and generate the final aggregated ranking. The ranked results under different values of ε are presented in Table 25.

It is clear that under all assumptions of ε, Airline 1 (A1) is significantly superior to the other airlines, but the ranking of the second and third airlines changes from A4 to A2 when ε takes the maximum value ($\varepsilon = 0.04$). This problem also occurs between A3 and A5. It is clear that under the assumption of weak orders ($\varepsilon = 0$ and 0.02), Airline 1 (A1) is slightly superior to the others, but under the strict order ($\varepsilon = 0.04$), A1 is far superior.

On the other hand, when ε takes the maximum value ($\varepsilon = 0.04$), then A1 is superior to the second airline (A2) with 100% confidence. It is recommended to take the maximum value of ε into consideration since the dominant relation between alternatives can be determined to the best degree of preference.

Table 25. The aggregated corresponding rankings

	Final Ranking
ε=0	$A_1 >^{0.7233} A_4 >^{0.5388} A_2 >^{0.6411} A_7 >^{0.5340} A_5 >^{0.5190} A_3 >^{0.5230} A_6$
ε=0.02	$A_1 >^{0.7693} A_4 >^{0.5255} A_2 >^{0.7406} A_7 >^{0.6435} A_3 >^{0.5002} A_5 >^{0.5521} A_6$
ε=0.04	$A_1 >^{1} A_2 >^{0.5561} A_4 >^{1} A_7 >^{1} A_3 >^{0.6190} A_5 >^{0.6159} A_6$

A schematic comparison between different MADM methods and aggregated ranking results is illustrated in Figure 2. It is inferred that where the variation in rankings increases, i.e. A4 and A5, the aggregated ranking results converges to one of them.

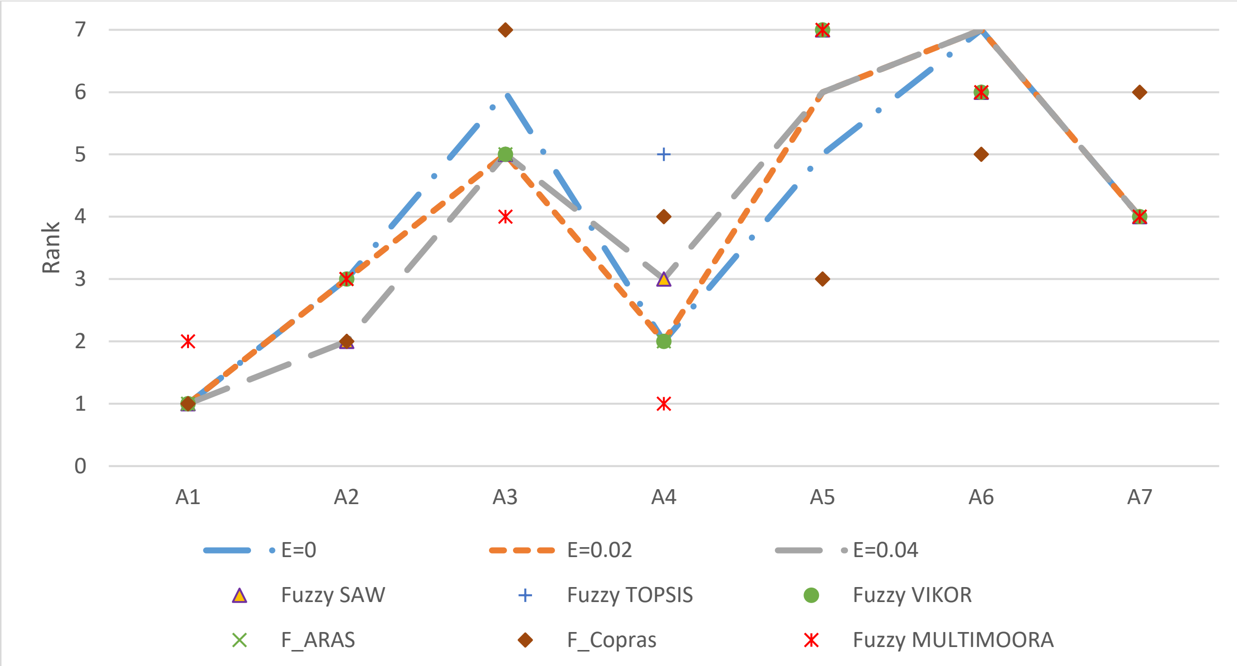

Figure 2. Ranking obtained using different MADM methods and aggregated ranking results

5. Conclusion and further work

The air transport industry plays a major role in the world economy. One of the key elements to maintaining the vitality of civil aviation is to ensure safe, secure and efficient operations at the global, regional and national levels. Measuring airline safety efficiency can help managers make suitable decisions based on comparisons between efficient and non-efficient airlines. Based on these facts, we decided to compare the safety efficiency for seven Iranian airlines. As there are different criteria for measuring airlines' safety which have different weights, MADM methods are more strongly underlined by researchers than other methods due to their relative characteristics. However, there are a number of concerns about using MADM methods that we addressed the main items of in this research.

Due to the vagueness of the information and a higher aviation risk in Iranian airlines, this paper proposes a novel hybrid method using Fuzzy DEA and Fuzzy MADM for measuring the safety efficiency of seven Iranian airlines. Unlike previous methods, we used Fuzzy DEA for calculating a criterion's weight, then the ranking of each airline based on obtained weights was determined using MADM methods.

Considering our proposed approach, the results show that different MADM methods often produce different outcomes for ranking a set of alternative decisions and this may confuse decision-makers. This is especially difficult when alternatives are very similar to each other, or the number of alternatives increases. Thus, we used the utility interval aggregation method to fill the gap and to assist decision-makers in making robust decisions.

The results show that the maximum value of a discriminatory factor could determine the best dominant relations between alternatives.

Some recommendations for future research are as follows:

- Researchers could use Interval DEA-Interval MADM methods
- Methods such as DEMATEL and ANP can help in defining better networks between criteria
- Other MADM methods may have different results, and they can develop the proposed approach.

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Appendix:

Appendix A.1. Fuzzy SAW

After calculating the relative weights and the performance score of each criterion with respect to each alternative, we can use the FSAW method to aggregate the fuzzy preferred ratings to rank the order of alternatives. The procedure of SAW for FMADM can be summarized below (Tzeng and Huang, 2011b):

Step 1: Calculate the relative fuzzy weight $\widetilde{w_j}$ of the j[th] attribute.

Step 2: Obtain the fuzzy decision matrix whose elements are composed of a set of fuzzy comparable ratings $\widetilde{r_{ij}}(x)$ for the j[th] attribute, with respect to the i[th] alternative.

Step 3: Synthesize the fuzzy value $\widetilde{u_i}(x)$ for the i[th] alternative, which is a summation of multiplying the relative fuzzy weight $\widetilde{w_j}$ and $\widetilde{r_{ij}}(x)$ as follows: $\widetilde{u_i}(x) = \sum_j \widetilde{w_j} * \widetilde{r_{ij}}(x)$.

Step 4: Compute a crisp value for each total score using a defuzzification method and select the alternative(s) with the maximum total score.

Appendix A.2. Fuzzy TOPSIS

A brief description of the Fuzzy TOPSIS steps is shown here (Büyüközkan and Çifçi, 2012):

Step 1: Establish a fuzzy decision matrix for evaluation of the alternatives. With m alternatives and n criteria, the fuzzy MADM problem can be expressed as:

$$\widetilde{D} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \dots & C_n \end{array} \\ \begin{bmatrix} \tilde{x}_{11} & \tilde{x}_{12} & \dots & \tilde{x}_{1n} \\ \vdots & \ddots & & \\ \vdots & & \ddots & \\ \tilde{x}_{m1} & \tilde{x}_{m2} & \dots & \tilde{x}_{mn} \end{bmatrix} \end{array}$$

$\widetilde{D}$ represents the fuzzy decision matrix with alternatives A and criteria C; and can be seen with linguistic and fuzzy terms.

Step 2: Normalize the decision matrix. Normalized fuzzy decision matrix $\tilde{R}$ is calculated as:

$$\tilde{R} = [\widetilde{r_{ij}}]_{m*n}, i = 1,2, \dots, m; j = 1,2, \dots, n,$$

$$\widetilde{r_{ij}} = (\frac{a_{ij}}{c_j^+}, \frac{b_{ij}}{c_j^+}, \frac{c_{ij}}{c_j^+})$$

where $C_j^+ = max_i\ C_{ij}$.

Step 3: Compute weighted decision matrix. Weighted normalized fuzzy decision matrix is computed below:

$$\tilde{v}_{ij} = \tilde{r}_{ij} \otimes \widetilde{w}_j$$

where $\widetilde{w_j}$is the fuzzy weight for the criterion j and $\tilde{v} = [\tilde{v}_{ij}]_{m*n}, i = 1,2, \dots, m;\ j = 1,2, \dots, n.$

Step 4: Calculate the distances from the positive and negative ideal points. Since the fuzzy triangular numbers are included in the [0,1] range, the positive and negative ideal reference points (FPIRP, FNIRP) are as follows:

$$A^+ = \{\tilde{v}_1^+, \tilde{v}_2^+, \dots, \tilde{v}_n^+\}, A^- = \{\tilde{v}_1^-, \tilde{v}_2^-, \dots, \tilde{v}_n^-\}$$

where $\tilde{v}_{ij}^+ = (1,1,1)\ and\ \tilde{v}_{ij}^- = (0,0,0)$.

The next step is to calculate the distance of alternatives from FPIRP and FNIRP.

$$d_i^+ = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^+), i = 1,2, \dots, m; j = 1,2, \dots, n,$$

$$d_i^- = \sum_{j=1}^{n} d(\tilde{v}_{ij}, \tilde{v}_j^-), i = 1,2, \dots, m; j = 1,2, \dots, n,$$

$$d(\tilde{A}, \tilde{B}) = \sqrt{\frac{1}{3}[(a_1 - b_1)^2 + (a_2 - b_2)^2 + (a_3 - b_3)^2]}$$

Step 5: Rank the alternatives.

Appendix A.3. Fuzzy VIKOR

The fuzzy VIKOR method has been developed to determine the compromise solution of the fuzzy multi-criteria problem. Suppose there are n alternatives to be evaluated with respect to m criteria. A brief description of Fuzzy VIKOR steps below (Chang, 2014):

Step 1. Identify appropriate linguistic variables.

Step 2. Identify the key evaluation criteria.

Step 3. Determine the fuzzy importance weights of evaluation criteria.

Step 4. Construct the decision matrix.

Step 5. Identify the fuzzy best value f_i^* and fuzzy worst value f_i^0 of all criteria.

The fuzzy best values $f_i^* = (l_i^*, m_i^*, u_i^*)$ and $f_i^0 = (l_i^0, m_i^0, u_i^0)$ are determined respectively as:

$\tilde{f}_i^* = \underset{j}{\text{Max}}\,\tilde{x}_{ij}\;, \tilde{f}_i^\circ = \underset{j}{\text{Min}}\,\tilde{x}_{ij}\,, \quad for\;\; i\; \in B$

$\tilde{f}_i^* = \underset{j}{\text{Min}}\,\tilde{x}_{ij}\;, \tilde{f}_i^\circ = \underset{j}{\text{Max}}\,\tilde{x}_{ij}\,, \quad for\;\; i\; \in C$

where B is associated with benefit criteria, and C is related to cost criteria.

Step 6. Compute the normalized fuzzy difference $\tilde{d}_{ij}$

The fuzzy difference $\tilde{d}_{ij}$ between $\tilde{x}_{ij}$ and the fuzzy best value f_i^* (or fuzzy worst value f_i^0) can be obtained as:

$\tilde{d}_{ij} = (\tilde{f}_i^* - \tilde{x}_{ij})/\,(u_i^* - l_i^\circ) \quad for\;\; i \in B$

$\tilde{d}_{ij} = (\tilde{x}_{ij} - \tilde{f}_i^*)/\,(u_i^\circ - l_i^*) \quad for\;\; i \in C$

Step 7. Compute the values $\tilde{S}_j$ and $\tilde{R}_j$.

This step is to measure the separation $\tilde{S}_j$ of alternative A_j from the fuzzy best value f_i^*, as well as to measure the separation $\tilde{R}_j$ of alternative A_j, from the fuzzy worst value f_i^0. These values can be measured as:

$$\tilde{S}_j = \sum_{i=1}^{n} (\widetilde{w}_i \otimes \tilde{d}_{ij})$$

$$\tilde{R}_j = \underset{i}{\text{Max}}\;(\widetilde{w}_i \otimes \tilde{d}_{ij})$$

where $\tilde{S}_j = (S_j^l, S_j^m, S_j^u)$ is a fuzzy weighted sum referring to the separation measure of A_j from the fuzzy best value; similarly, $\tilde{R}_j = \left(R_j^l, R_j^m, R_j^u\right)$ is a fuzzy operator MAX denoting the separation measure of A_j from the fuzzy worst value, and W_i is the importance weight of criterion C_i.

Step 8. Compute the value $\tilde{Q}_j$.

The value $\tilde{Q}_j = (Q_j^l, Q_j^m, Q_j^u)$ can be calculated as:

$$\tilde{Q}_j = v\left(\tilde{S}_j - \tilde{S}^*\right)/\,(S^{\circ u} - S^{*l}) \oplus (1-v)\left(\tilde{R}_j - \tilde{R}^*\right)/(R^{\circ u} - R^{*l})$$

where $\tilde{S}^* = Min_j\tilde{S}_j$, $S^{\circ u} = Max_j S_j^u$, $\tilde{R}^* = Min_j\tilde{R}_j$, $R^{\circ u} = Max_j R_j^u$ and v ($v = \frac{n+1}{2}$) is taken as a weight for the maximum utility, whereas the $1 - v$ is the weight of the individual loss. The best values of S and R respectively, are denoted by $\tilde{S}^*\; and\; \tilde{R}^*$.

Step 9. Rank the alternatives.

Rank the alternatives by sorting the values S, R and Q into ascending order (after the defuzzification, $\tilde{S}_j\,, \tilde{R}_j\; and\; \tilde{Q}_j$ are converted into the crisp numbers S_j, R_j and Q_j). The index Q_i implies the separation measure of Ai from the best alternative, i.e. the smaller the value Q, the better the alternative is.

Appendix A.4. ARAS-F

The ARAS-F method is implemented as described below:

1. Obtain the normalized weighted matrix $(\hat{P}_{ij}^s)$ from DEA.

2. The following task is to determine the values of the optimality function:

$H_i^s = \sum_{j=1}^{n} \hat{P}_{ij}^s \qquad \forall\, i = 0,1,\dots,m \quad S \in (B,M,T),$

where $H_i^s = (H_i^B\ ,\ H_i^M\ ,\ H_i^T\ )$ is the value of the optimality function of the i -th alternative.

3. Obtain the H_0^s which is calculated as follows; where x_{0j}^S is the optimal value of the j-th criterion, but if the optimal value of the j-th criterion is unknown, then:

$x_{0j}^S = \max_i x_{ij}^S \quad if \quad \max_i x_{ij}^S \quad preferable \qquad \forall\, j = 1,\dots,m \quad S \in (B,M,T),$

$x_{0j}^S = \min_i x_{ij}^S \quad if \quad \min_i x_{ij}^S \quad preferable \qquad \forall\, j = 1,\dots,m \quad S \in (B,M,T),$

$H_0^s = \sum_{j=1}^{n} x_{0j}^S \times NW_j^S \qquad \forall\, S \in (B,M,T),$

And [x_{01}^S , … , x_{0j}^S , … , x_{0n}^S] is the projection weight of optimal values.

4. As far as the optimality function (H_i^s) has a direct and proportional relationship with the value of x_{0j}^S, the greater value of the optimality function illustrates a more effective alternative. The priorities of alternatives can be determined according to the value of H_i^s. There are several methods for defuzzification; the centre-of-area is the most practical and simple to apply as given below (Akhavan et al., 2015):

$H_i = \frac{1}{4}\left(H_i^B + 2\, H_i^M + H_i^T\right)\ \forall\, i = 0,1,\dots,m\ .$

where H_i is the composite score of alternative i. The value of H_i can be positive, negative or zero.

5. The degree of alternative utility is determined by a comparison of the variant, which is analyzed with the most ideal one (H_0). The equation used for the calculation of the utility degree of an alternative (K_i) is given below:

$K_i = \frac{H_i}{H_0} \quad \forall\, i = 0,1,\dots,m$

where: H_i and H_0 are the optimal criterion values, obtained from formula (11). The calculated values K_i are in the interval $[0,1]$ and can be ordered in an increasing sequence; this is the preferred order of precedence. The complex relative efficiency of the reasonable alternative can be determined according to the utility function values.

Appendix A.5. COPRAS-F

In the COPRAS method, the judgement is made according to the utility degree (N_i) which is calculated for each alternative on the basis of the relative significance or priority value (Q_i). In this paper, the COPRAS method is implemented according to (Zavadskas and Antucheviciene, 2007) as described below:

1. Calculation of B_i^s for each positive criterion (criteria that is preferred to be maximized) regarding the normalized-weighted matrix from DEA ($\hat{P}_{ij}^s$)

$B_i^s = \sum_{j=1}^{k} \hat{P}_{ij}^s \qquad \forall\, i = 1,\dots,m \quad S \in (B,M,T)$

2. Similar to that above, the summation of negative criterion (criteria that is preferred to be minimized) namely C_i^s is being calculated here. It is clear that smaller values of C_i^s are more preferable.

$$C_i^s = \sum_{j=K+1}^{n} \hat{P}_{ij}^s \qquad \forall\, i = 1, \ldots, m \quad S \in (B, M, T)$$

3. The relative weight of each alternative is then calculated.

$$Q_i^s = B_i^s + \frac{\sum_{i=1}^{n} C_i^s}{C_i^s \sum_{i=1}^{n} \frac{1}{C_i^s}} \quad \forall\, i = 1, \ldots, m \quad S \in (B, M, T)$$

4. We have considered a centre-of-area for defuzzification of Q_i^s. In this stage, each alternative is represented by receiving a crisp number, i.e. Q_i in which the larger values of these are more preferable.

5. Determination of the optimality criterion:

$$Q_{max} = \max_i \,(Q_i)$$

6. Calculation of the utility degree of each alternative as given below:

$$N_i = \frac{Q_i}{Q_{max}} \times 100$$

N_i can be ordered in an increasing sequence; this is the preferred order of precedence.

Appendix A.6. Fuzzy MULTIMOORA

The MULTIMOORA method begins with the decision matrix X, where its elements x_{ij} denote the values of the ith alternative on the jth criterion (objective), i=1,2,…,m and j=1,2,…,n. The method consists of three parts: the ratio system, the reference point approach and the full multiplicative form.

a. The ratio system

The ratio system employs the vector data normalization by comparing an alternative of a criterion to all values of that criterion.

$$x_{ij}^* = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}$$

where x_{ij}^* represents the normalized value of the ith alternative on the jth criterion. These normalized values are added (if the desirable value of criterion is the maximum) or subtracted (if the desirable value is the minimum). Thus, the summarizing index of each alternative is derived in this way:

$$y_i^* = \sum_{j=1}^{g} x_{ij}^* - \sum_{j=g+1}^{n} x_{ij}^*,$$

where g= 1,2,…, n denotes the number of criteria to be maximized and y_i^* is the normalized assessment of the ith alternative with respect to all criteria. The ranking of alternatives is then given according to every ratio: the higher the index, the higher the rank.

b. The reference point approach

The reference point approach is based on the ratio system. The maximal objective reference point (MORP) is found according to the ratios obtained in the previous steps. The jth coordinate of the reference point can be described as $r_j = max_i \; x_{ij}^*$ in the case of maximization. Every coordinate of this vector represents the maximum or minimum of certain criterion. Then, every element of the normalized decision matrix is re-calculated, and the final rank is given according to the deviation from the reference point and the min–max Metric of Tchebycheff:

$$min_i \; \{max_j \; |r_j - x_{ij}^*|\}$$

c. The full multiplicative form

Brauers and Zavadskas (2011) updated the MOORA by using the full multiplicative form method, which embodies maximization as well as the minimization of a purely multiplicative utility function. The overall utility of the ith alternative can be expressed as a dimensionless number by the following relation:

$$U_i' = \frac{A_i}{B_i},$$

where $A_i = \prod_{j=1}^{g} x_{ij}$ denotes the product of criteria of the ith alternative to be maximized with g=1,2,…,n being the number of criteria to be maximized, and where $B_i = \prod_{j=g+1}^{n} x_{ij}$ denotes the product of criteria of the ith alternative to be minimized with n-g being the number of criteria to be minimized.

d. The dominance theory

Brauers and Zavadskas (2011) developed the theory of dominance to summarize the three-rank lists provided by different parts of MULTIMOORA into a single list.

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