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Is the Big Rip unreachable?

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ABSTRACT

I investigate the repercussions of particle production when the Universe is dominated by a hypothetical phantom substance. I show that backreaction due to particle production prevents the density from shooting to infinity at a Big Rip, but instead forces it to stabilise at a large constant value. Afterwards there is a period of de-Sitter inflation. I speculate that this might lead to a cyclic Universe.

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A phantom substance is defined as a fluid which violates the null energy condition because its pressure is $p < -\rho$, where ρ is its density. Such a hypothetical substance is also called exotic matter, and it is necessary for keeping a wormhole traversable. In cosmology, if the Universe content is dominated by phantom density, then the latter increases and at some finite time $t_{\rm max}$ it becomes infinite, in a singularity called the Big Rip. This is because the Universe undergoes super-inflation where the accelerated expansion gives rise to an event horizon, whose dimensions shrink to zero at $t_{\rm max}$, ripping apart all structures, from galaxies to atoms.

But is it so? So far, most of the studies of the dynamics of the Universe when undergoing accelerated expansion have ignored particle production due to the event horizon, because it is usually negligible, in inflation for example. In this work, it is argued that particle production cannot be ignored when the accelerated expansion is driven by phantom density. Backreaction due to particle production renders the Big Rip singularity unreachable.

We work in Einstein gravity only. In the following, we use natural units, where $c = \hbar = 1$ and Newton's gravitational constant is $8\pi G = m_P^{-2}$, with $m_P = 2.43 \times 10^{18}$ GeV being the reduced Planck mass. For simplicity, isotropy and spatial flatness is assumed throughout.

The existence of an event horizon during accelerated expansion results in particle production of all light (meaning with mass smaller than the Hubble rate $m < H$) non-conformally invariant fields (e.g. a light scalar field, such as the inflaton). During quasi-de Sitter inflation, particle production results in the gravitational generation of density of the order of thermal density with temperature the Hawking temperature of de Sitter space $T = H/2\pi$ [1–7][1]

$$\rho_{\rm gr} = q\frac{\pi^2}{30} g_*^{\rm gr}\left(\frac{H}{2\pi}\right)^4 , \tag{1}$$

where $g_*^{\rm gr}$ is the number of effective relativistic degrees of freedom which undergo particle production (i.e. are light and non-conformally invariant) and $q \sim 1$ is some constant factor due to the fact that the resulting $\rho_{\rm gr}$ is not actually thermal.

Thus, the Friedmann equation obtains an additional term of the form

$$H^2 = \frac{8\pi G}{3}\rho + CH^4 , \tag{2}$$

where ρ is the density of the substance which causes the accelerated expansion and, in view of Eq. (1), we have

$$CH^4 = \frac{8\pi G}{3}\rho_{\rm gr} \Rightarrow C = \frac{qG}{180\pi}\, g_*^{\rm gr} . \tag{3}$$

Strictly speaking, the above are true for $\rho, H \simeq$ constant, which results in quasi-de Sitter inflation. In this case, the CH^4 contribution in the Friedmann equation (2) is negligible and is usually ignored (but see Ref. [9]). However, one expects the same phenomenon to take place in any kind of accelerated expansion, since gravitational particle production occurs whenever there is an event horizon (as with black hole radiation, for example). Thus, qualitatively, the same source term CH^4 would appear in the Friedmann equation as H sets the scale of the accelerated expansion and

[1] Note that, particle production is a non-equilibrium effect and in an isotropic universe its classical counterpart is bulk viscosity [8].

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$C \propto g_*^{\rm gr}$ as in Eq. (3).[2] This term would become important if the cause of accelerated expansion is a phantom substance.[3]

If ρ is the density of a phantom substance then the Friedmann equation (ignoring the extra term CH^4, because it can be negligible at first) results in $\dot\rho > 0$ and $\dot H > 0$. This results in super-inflation which leads to the Big Rip singularity when $\rho \to \infty$ in finite time. However, the growth of H means that the CH^4 in the Friedmann equation (2) will eventually become important, because it increases faster than the H^2 term. This may halt the growth of ρ and prevent the Big Rip from happening [12,13].

The solution to Eq. (2) is

$$H^2 = \frac{1}{2C}\left(1 \pm \sqrt{1 - \frac{32\pi GC}{3}\rho}\right). \tag{4}$$

For small density, the above equation results in either $H^2 = 1/C =$ constant or $H^2 = (8\pi G/3)\rho$, which is the usual Friedmann equation. The latter solution corresponds to the negative sign in the brackets. However, the above also shows that there is a *maximum* possible value of ρ, which is $\rho_{\rm max} = 3/32\pi GC$, in which case there is a maximum value of the Hubble rate $H_{\rm max} = 1/\sqrt{2C}$. This means that the Big Rip is unreachable, prevented by the backreaction of the gravitational particle production.[4]

Using Eq. (3), we can obtain an estimate of the maximum density

$$\rho_{\rm max} = \frac{3}{32\pi GC} = \frac{135}{8qG^2 g_*^{\rm gr}} = \frac{3H_{\rm max}^2}{16\pi G}$$
$$\Rightarrow\ \rho_{\rm max}^{1/4} = \sqrt{6\pi}\left(\frac{30}{qg_*^{\rm gr}}\right)^{1/4} m_P\,. \tag{5}$$

Thus, $\rho_{\rm max}^{1/4}$ can be smaller than the Planck scale only when $g_*^{\rm gr}$ is very large. Considering string theory, we can reduce $\rho_{\rm max}^{1/4}$ to the string scale $\sim 10^{17}$ GeV by considering $g_*^{\rm gr} \sim 10^5$ or so. We assume that $\rho_{\rm max}^{1/4} < m_P$ such that quantum gravity considerations can be ignored.[5]

Exactly how the density evolves can be revealed by the study of the continuity equation

$$\dot\rho + 3(1+w)H\rho = 0\,, \tag{6}$$

where $w = p/\rho < -1$ is the barotropic parameter of the phantom substance. For simplicity, we consider $w =$ constant. Using Eq. (4) with the negative sign in the brackets, Eq. (6) can be analytically solved to give

$$\frac{1}{\sqrt2}\ln\left(\frac{\sqrt2+\sqrt{2C}H}{\sqrt2-\sqrt{2C}H}\,\frac{\sqrt2-1}{\sqrt2+1}\right) - 2\left(1-\frac{1}{\sqrt{2C}H}\right)$$
$$= \frac{3|1+w|}{\sqrt{2C}}(t_{\rm max}-t)\,, \tag{7}$$

where $t_{\rm max}$ corresponds to the time when $\rho = \rho_{\rm max}$. Note that Eq. (4) (with the negative sign in the brackets) implies that $\sqrt{2C}\,H = (1-\sqrt{1-u})^{1/2}$, where we have defined $u \equiv \rho/\rho_{\rm max}$, i.e. $u(t_{\rm max}) = 1$.

When $H \ll H_{\rm max} = 1/\sqrt{2C}$, the last term in the left-hand-side of the above dominates and we find $H^{-1} = \frac32|1+w|(t_{\rm max}-t)$ as with standard phantom dark energy, only the time $t_{\rm max}$ does not denote the Big Rip, i.e. $\rho(t_{\rm max}) \nrightarrow \infty$, but instead we have $\rho(t_{\rm max}) = \rho_{\rm max}$ which is finite and given by Eq. (5).

The solution in Eq. (7) only applies for $t \le t_{\rm max}$. At $t_{\rm max}$ the solution becomes zero.[6] We can investigate what happens when $t > t_{\rm max}$ by considering the continuity equation (6), which gives

$$\dot u = 3|1+w|Hu\ \Rightarrow\ \dot u_{\rm max} = \frac{3|1+w|}{\sqrt{2C}} > 0\,, \tag{8}$$

where $\dot u_{\rm max} \equiv \dot u(t_{\rm max})$ and we used that $H_{\rm max} = 1/\sqrt{2C}$ and $u(t_{\rm max}) = 1$. Thus, there is a tendency for the density to become larger than $\rho_{\rm max}$, which, however, is forbidden by Eq. (4). This is really because Eq. (2) is not valid any more. Indeed, for $\rho > \rho_{\rm max}$, the density of the gravitationally produced particles would be larger than the phantom density itself, which cannot happen (where is the energy coming from?).

This suggests that the continuity equation needs augmenting, since the removal of energy by gravitational particle production is not considered in Eq. (8); it is implicitly assumed negligible. To take this into account we introduce a negative source term on the right-hand-side of Eq. (6), which becomes (see also Ref. [15])

$$\dot\rho - 3|1+w|H\rho = -3|1+w|H\rho_{\rm gr} = -\frac{9|1+w|C}{8\pi G}H^5\,. \tag{9}$$

The form of this term is given by $\delta\rho/\delta t$, where $\delta\rho = -\rho_{\rm gr} \propto H^4$ as in Eq. (1) and $\delta t \sim H^{-1}$ is the Hubble time. This is because the relativistic particles produced gravitationally in a Hubble time are diluted by the accelerated expansion and replenished by the particles produced in the following Hubble time, meaning that $\delta\rho \propto H^4$ is produced gravitationally per Hubble time $\delta t \sim H^{-1}$. The precise value is $\delta t = H^{-1}/3|1+w|$, which results in $\dot u_{\rm max} = 0$.

Combining Eqs. (4) and (9) we may write the continuity equation as

$$\dot u = 6|1+w|H_{\rm max}\sqrt{1-u}\,(1-\sqrt{1-u})^{3/2}\,, \tag{10}$$

where $H_{\rm max} = 1/\sqrt{2C}$. From the above it is evident that $\dot u_{\rm max} = 0$ as expected. Also note that, in the limit $u \ll 1$ the above becomes

$$\dot u \simeq \tfrac32|1+w|\sqrt2 H_{\rm max}u^{3/2} \simeq 3|1+w|Hu\,, \tag{11}$$

which is the usual continuity equation (cf. Eq. (8)) and we considered that $H = H_{\rm max}(1-\sqrt{1-u})^{1/2} \simeq H_{\rm max}u^{1/2}/\sqrt2$ in this limit. Note that in the limit $u \ll 1$ we recover the Friedmann equation as expected because $H^2 \simeq \frac12 H_{\rm max}^2 u = 8\pi G\rho/3$, where $H_{\rm max} = 1/\sqrt{2C}$ and $\rho_{\rm max}$ is given by Eq. (5).

[2] In general, in accelerated expansion the Hawking temperature is $T = \frac12|3w+1|(H/2\pi)$ [10]. The numerical factor $\frac12|3w+1|$ can be incorporated into q in Eq. (1).

[3] In principle, other terms involving the derivatives of the Hubble rate, such as $H\ddot H$, $H^2\dot H$ or $(\dot H)^2$ may also be important, additionally to the H^4 term considered here. Such terms are negligible in quasi-de Sitter inflation because $|\dot H| \ll H^2$, but this would not necessarily be so for a phantom substance. However, in Ref. [11] it is shown that the density of such derivative terms is proportional to α, where the latter is the coefficient of R^2 in a modified gravity Lagrangian $\mathcal{L} = \frac12 m_P^2 R + \alpha R^2$. In this work we consider Einstein gravity only, which means $\alpha = 0$ and these terms are absent.

[4] A similar result was obtained in Ref. [14], where the effect of the conformal anomaly was taken into account, which can also produce a contribution $\delta\rho \propto H^4$.

[5] In a quantum gravity setup, the H^4 correction in the Friedmann equation (2) might be the leading order to terms proportional to H^6 or H^8 for example. Such terms would grow even faster than the H^4. However, we expect such terms to be Planck suppressed such that they always remain subdominant because $H^{4+n}/m_P^n < H^4$ for all $n \ge 1$ since $H \le H_{\rm max} \ll m_P$ because $\rho_{\rm max}^{1/4} < m_P$.

[6] In view of Eqs. (1) and (5), we have $\rho_{\rm gr} \le \rho \Leftrightarrow 8\pi G\rho/3 \ge CH^4$. This means that we cannot connect with the positive branch of H^2 in Eq. (4), which ranges between $1/\sqrt{2C}$ and $1/\sqrt{C}$, because this requires $8\pi G/3\rho < CH^4$.

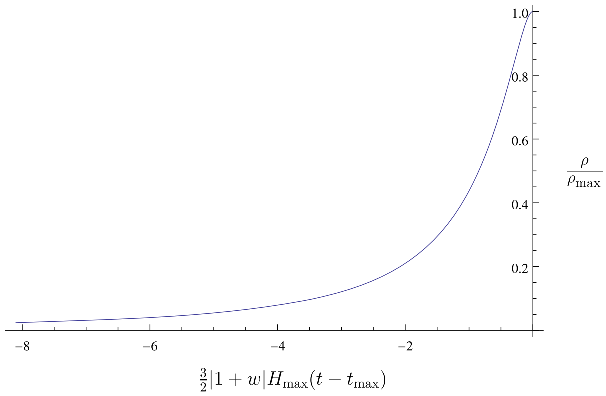

Fig. 1. Evolution of $u = \rho/\rho_{\max}$ as time approaches $t_{\max}$. The Big Rip is avoided and the density approaches a constant value $\rho_{\max}$.

The solution to Eq. (10) is

$$\frac{\rho}{\rho_{\max}} = u = 1 - \left\{1 - \left[\tfrac{3}{2}|1+w|H_{\max}(t_{\max}-t)+1\right]^{-2}\right\}^{2}. \quad (12)$$

If we consider the limit $t \ll t_{\max}$ and the condition $H_{\max}t_{\max} \gg 1$ we find that the above asymptotes to the value $\rho \to 1/6(1+w)^2 t_{\max}^2$. This is the same value one obtains in the limit $t \ll t_{\max}$ for standard phantom dark energy, for which $\rho = 1/6(1+w)^2(t_{\max}-t)^2$.

The evolution of the density ρ until the time $t_{\max}$ is shown in Fig. 1. As we approach $t_{\max}$, instead of shooting to infinity, the backreaction due to the particle production forces the growth of ρ to be halted and the latter gently reaches $\rho_{\max}$ at $t = t_{\max}$. What happens afterwards? Well, the constant values $\rho = \rho_{\max}$ (given by Eq. (5)) and $H = H_{\max} = 1/\sqrt{2C}$ are solutions to Eqs. (2) and (10) so that the density and the Hubble parameter remain constant. Thus, the Universe undergoes de-Sitter inflation, even though it is filled at equal parts by a phantom substance with $w < -1$ and radiation due to particle production which is constantly diluted and replenished so that it has a constant net density $\rho_{\rm gr} \sim H_{\max}^4$.

We can also envisage an interesting hypothetical scenario, where, due to an unknown process, the phantom substance drastically decays into a minute residual density with value $\lesssim (10^{-3}\,\text{eV})^4$. Its decay products reheat the Universe and the hot big bang ensues. Eventually the phantom density takes over once more, originally as dark energy,[7] but its density soon increases rapidly up to the scale of grand unification $\gtrsim (10^{16}\,\text{GeV})^4$ or so, such that in gives rise to another boot of de-Sitter inflation.[8] We thus might have a cyclic Universe, which is schematically shown in Fig. 2.[9] If the scenario can be embedded in string theory [20], then one might wonder if decompactification can happen when the density grows comparable to the string scale. Afterwards the extra dimensions might compactify at a different Calabi–Yau, implying that the laws of physics might be different in every cycle. This might lead to a multiverse in time and not in space.

For simplicity, we have assumed isotropy throughout our considerations. In the presence of anisotropy the picture could be substantially altered. In many cyclic models, for example the Mixmaster universe in Ref. [21], the cumulative anisotropic effects grow and become overwhelming, possibly destabilising the cyclic behaviour. In contrast to the Mixmaster scenario however (and other similar cyclic models, e.g. Ref. [22]), the cyclic universe considered here does not involve a contracting phase (which would enlarge the anisotropy). The total density of the Universe is growing and falling but the expansion never halts or reverses itself. Thus, in this case, any existing anisotropy might remain negligible, although this needs to be investigated.

We have discussed the repercussions of particle production when the Universe is dominated by a hypothetical phantom substance. We have argued that backreaction due to particle production prevents the density from shooting to infinity at a Big Rip, but instead forces it to stabilise at a large constant value, which could be near the string scale or the scale of grand unification, which is a little lower. After assuming its constant maximum value there is a period of de-Sitter inflation. This might lead to a cyclic Universe, provided some unknown mechanism terminates inflation and drastically reduces the phantom density, giving rise to the thermal bath of the hot big bang.

We have considered Einstein gravity and ignored quantum gravity corrections taking the maximum density to remain sub-Planckian.[10] We have not introduced a specific model of phantom dark energy,[11] in order to emphasise that our treatment and results are generic and due to the existence of an event horizon in accelerated expansion, which leads to particle production as in black holes. In that sense, our finding that the Big Rip is unreachable, seems unavoidable.

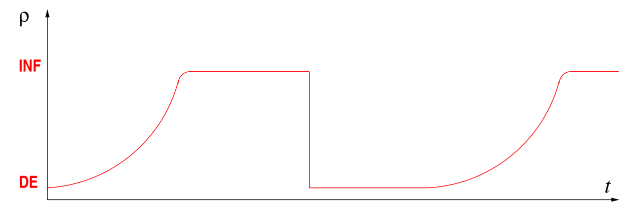

Fig. 2. Schematic evolution of the phantom density ρ in a hypothetical cyclic scenario. ρ increases with time approximating a constant $\rho_{\max}$, which once assumed leads to a period of inflation. Then, some unknown process reduces drastically the phantom density such that it becomes dark energy, until another cycle begins.

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[7] Note that the Planck satellite observations favour phantom dark energy [16].

[8] Of course, we know that the observed red spectrum of curvature perturbations demands that $\dot{\rho}$ is not zero, but slightly negative during inflation. One might hypothesise that the drastic decay of ρ at the end of inflation is preceded by some greatly suppressed decay process during inflation. An example of this possibility is studied in Ref. [17].

[9] This is similar to Ref. [18] although in those works w is taken to vary periodically in time. For a cyclic Universe due to phantom dark energy in the context of braneworlds see Ref. [19].

[10] For avoiding the Big Rip singularity in modified gravity see Ref. [23].

[11] We only took $w = \text{constant} < -1$ for simplicity.

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