Implications of Lump Sum Costs for Empirical Design in Corporate Finance Research

Thesis submitted to Lancaster University in fulfilment of the requirements of the degree of Doctor of Philosophy in Finance

by

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Abstract

This thesis consists of two self-contained studies on the implications of lump sum costs for empirical design in the context of corporate finance. The first study analyzes the dynamics of cash holding management and empirically determines the optimal policy for cash reserves. The framework stipulates that cash management is associated with the following actions: 1) allow cash level to freely float within the range bounded by two barriers; 2) refinance back to the target value immediately when cash level hits either barrier. The endogenous pattern identified under this framework facilitates understanding of the dynamics of cash holdings since it allows to estimate both the triggers of cash adjustments, as well as the target of each component of the policy. Further, this empirical application emphasizes the importance of the adjustment cost

setting which refers to the interpretation of cost types for cash refinancing (fixed or fractional). In our model, we allow both fixed and fractional cash adjustment cost, which allows to identify a number of dynamic aspects such as target and thresholds in liquidity management. Also, this study enriches the existing studies of determinants of cash holdings by demonstrating novel effects of covariates on target cash holdings such as the negative impact of cash flow (profitability) much larger than previously estimated effects of industry risk. These findings differ from those in existing studies, either in signs or in magnitude, but are fully consistent with the underlying theory. Overall, the presented research quantifies cash holding management in a dynamic double-barrier model, allows to estimate the trigger of cash refinancing, and hence enhances our understanding of determinants of cash holding policy.

The second study investigates the stickiness in credit rating. The existing literature on credit ratings typically assumes on accurate match between credit quality and agency ratings. This assumption ignores the agencies' trade-off between the reputation among investors and the revenue from issuers when updating credit ratings. Our model controls for the adjustment cost for rating agencies, and hence explains the stickiness embedded in rating assignments. Presented tests empirically demonstrate the existence of the stickiness and its significant impact. This is the first study to explicitly model decision (partial) irreversibility in credit rating research. This paper offers therefore a different explanation of the observed rating deterioration to the upgrades becoming increasingly difficult. Lastly, this study personalizes the standard ordered-probit estimation to allow for stickiness (path-dependence). Our estimation identifies upper and lower threshold groups in which the credit quality does not match assigned ratings, and calculates the likelihood specifically based on their features.

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Declaration of Authorship

I hereby declare that this thesis is my own work, and has not been submitted in substantially the same form for the award of a higher degree elsewhere.

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Chapter 1

Introduction

Many activities in the broad area of corporate finance are associated with some kinds of adjustment costs. Specific assumptions regarding the nature of adjustment costs lead to particular predictions about corporate behavior and the way it can be modeled. A double-barrier model typically arises in the presence of lumpy (non-convex) adjustment costs and exploits the upper and lower thresholds to quantify dynamic decision making. The model predicts that a variable freely floats within range bounded by the two thresholds, and is adjusted back to target level once the variable hits either threshold. This infrequent adjustment pattern is therefore optimal as a result of lumpy adjustment costs. Applications of the double-barrier model allows to study several interesting phenomena such as the trigger of adjustment. Each of the two following chapters of this thesis investigates a separate area of corporate finance with a focus on adjustment dynamics. Chapter 2 is an empirical application of the double-barrier model to the optimal cash holding management. Chapter 3 investigates the existence and effects of stickiness in credit ratings. The dynamic adjustment cost setting in the credit rating area is reflected as the tolerance of deviation from the "target", which results in delayed rating migrations.

In Chapter 2 we develop an empirical double-barrier model to show that modeling infrequent refinancing explicitly leads to new results that complement those in empirical cash management literature. As opposed to the empirical findings based on a static model, we find that cash flow (profitability) reduces the cash distribution target, and that firms retain cash for future cash outflows such as capital expenditure and acquisition. Further, our double-barrier model also focuses on the trigger of refinancing by estimating the upper and lower thresholds. It suggests that a complete policy considers multiple channels that firms can exploit to adjust cash holdings. Moreover, our work reconciles the part of the puzzles in cash holding area caused by infrequent refinancing. Our empirical findings complement the existing studies of determinants of cash holdings by investigating the effects of inertia. Refinancing decisions appear therefore key understand the motive of cash management policy.

Early cash policy studies concentrate on either finding the optimal cash level by weighting the costs and benefits or identifying determinants of actual cash balances. Previous literature identified the positive effect of cash flow (profitability) on cash holdings (Opler et al. 1999, Han & Qiu 2007, Chen et al. 2012), which is inconsistent with the precautionary motive for holding cash. Moreover, empirical studies (Opler et al. 1999, Qiu & Wan 2015, Subramaniam et al. 2011, Chen et al. 2012, Yan 2006) typically estimate the model using observed cash holdings by treating deviations from the true target as a simple error term. However, we show that the deviation may also contain information about firms cash policy since it is a strategic choice rather than a simple residual component. In general, trade-off model studied in the literature focuses on the benefits and costs of cash level itself but ignores the cost of cash refinancing. It is the presence of costs that leads to infrequent refinancing, which we attempt to model empirically in this thesis.

Theoretical developments in cash policy models demonstrate the potential of the double-barrier setting to study the dynamic aspects in cash management under lumpy refinancing costs. The existing literature generally focuses on two aspects of cash policy. It either looks at the value of cash holdings (Nikolov & Whited 2014, Chi & Su 2015) and derives the optimal policy based on the value of marginal cash holdings. Alternatively, it explicitly derives the double-barrier policy as the optimal one given the costly external financing (Whited 2006, Riddick & Whited 2009, Bolton et al. 2011, 2013). For example, Bolton et al. (2011) derive the optimal inaction, payout, and liquidation regions in a frictional market, which results in obtaining the boundaries between adjacent regions that determine triggers of refinancing. In Chapter 2, we also investigate the dynamic cash management policy, which is closely related to the second strand of the theoretical studies mentioned above. The inaction range between barriers captures the effect of inertia, and hence leads to optimal refinancing decisions.

In the model, we allow firms to have some tolerance of a deviation of cash holdings from the optimal level, so that they refinance only when their cash holding level reaches either an upper or a lower threshold. The rationale behind this behavior is that a large amount of refinancing enjoys the economies of scale, which spreads out the fixed component of the refinancing cost. Thresholds serve as refinancing triggers because they are the boundaries beyond which the benefits of refinancing exceed the costs. Moreover, the infrequent refinancing results in only a fraction of observations to lie near the target cash balance. We therefore also identify observations that are likely to lie near (endogenous) triggers, those correspond to observations for which variations in cash balances next period are large enough. We assume large refinancing to be deliberate because it is less likely that managers implement such influential adjustments without careful consideration. Similarly, Hovakimian et al. (2001) and Danis et al. (2014) state that large changes in variables of inherent seem to be deliberate actions. Although these studies focus on capital structure, this view can also apply to the cash holding area due to the similar nature of its dynamics.

Our work contributes to the existing literature by providing a direct comparison between the static and dynamic models. Static models suggest a positive correlation between cash flow (profitability) and cash holdings, while our results argue that this observed positive correlation actually confounds the effect on targets and other observations, and that the correlation between cash flow and cash target is actually negative. This negative correlation is consistent with predictions from dynamic theoretical models (Whited 2006, Riddick & Whited 2009). Moreover, we find that cash holdings targets increase with future cash outflows but decrease with current outflows. Future cash outflows represent extra demand for cash, and hence firms stockpile cash to meet this expected demand. Inversely, current cash outflows reduce anticipated cash demand since it has already been met. Further, compared to other dynamic studies such as Bolton et al. (2011), we provide a more detailed model by relaxing the assumption of fixed threshold, and by controlling for macro economic variables (e.g. GDP growth, recession, cost of carry).

Chapter 3 proposes a stickiness-based model that attempts to explain the observed deterioration in credit ratings. The model focuses on the mechanism of rating process behind the behavior of rating agencies. The results demonstrate the existence of stickiness in credit ratings, and its significant impact on rating migrations. Our results shed light on the debate on whether the downward trend in credit ratings is due to the deteriorating credit quality or to tightening of the rating standards. Controlling for the stickiness, we observe that firms' credit quality actually improves during that period. We also document asymmetry in rating migrations. Upgrades become increasingly difficult while the downgrade standard remains largely unchanged. This mechanism offsets the improvement in credit quality, and hence leads to the perceived "deterioration".

The phenomenon of perceived rating deterioration triggered a recent wave of research (Blume et al. 1998, Jorion et al. 2009, Alp 2013). The rating event involves two main participants; firms that are rated and rating agencies. Hence, possible causes of the deterioration may come from either of the two sides, which are the quality of the borrowing and the stringency of rating standard. Studies such as Blume et al. (1998) analyze the causes of deterioration by matching ratings to firm characteristics. They construct an empirical model attempting to control for all other covariates and isolate the effect of rating standards by focusing on year dummies. The study claims that the continuously strengthened standard indeed contributes to the observed deterioration. This finding leads to a number of following questions since it remains silent on how the other source of the observed deterioration, the quality of firms' borrowing, changes.

High persistence of credit ratings implies certain tolerance for the deviation of the actual credit quality from the nominal rating range, and empirical evidence prove the correlation between the magnitude of this deviation and rating migrations (Altman & Rijken 2004, Mora 2006, Posch 2011). Empirical results indicate that rating migrations are triggered when the borrowers' actual credit quality exceeds the nominal quality of their current ratings by 1.25 notches.[1] Mora (2006) provides more direct evidence about the rating drifts mechanism, and states that rating changes when the divergence between actual quality and assigned ratings is sufficiently large. Posch (2011) further measures the amount of tolerance by extending the frictionless model to allow non-constant thresholds, and shows that default probability has to change by at least two notches before rating agencies react.

The stickiness framework proposed in this research also receives theoretical support from the structure of the agency rating market. Cheng & Neamtiu (2009) emphasize the lack of timeliness and increasing regulatory pressure in agency ratings. There is evidence in the existing literature explaining the origin of stickiness (Altman & Rijken 2004, Posch 2011, Löffler 2004, Altman & Kao 1992, Lando & Skødeberg 2002, Löffler 2005). In general, agencies have incentives to make credit ratings sticky. Löffler (2005) documents that agencies attempt to avoid rating reversal after a migration, and hence contributes to the stability (stickiness). Moreover, Jeon & Lovo (2013) introduce the notion of 'reputation build-up', which suggests that frequent rating adjustments harm the profitability of the agencies by weakening their reputation. More precisely, Bolton et al. (2012) elaborate on the "rating shopping" phenomenon

[1] Notch refers to the minimum rating category in agency rating system. For example, in $S\&P$ rating, rating AAA is one notch above rating $AA+$.

according to which agencies attract business by enhancing the stability, because issuers can shop in the market for the best ratings they can get. The issuer-paid pattern indeed results in extra prudence for agencies in updating ratings, which is detrimental to rating accuracy (Xia 2014, Xia & Strobl 2012). Xia (2014) finds that introducing investor-paid rating agencies (e.g. Egan-Jones Rating Company) improves the accuracy and timeliness above most of the tradition issuer-paid ratings (e.g. S&P's rating).

The reminder of the thesis is organized as follows. Chapter 2 empirically analyzes the optimal cash management policy considering adjustment costs. Chapter 3 investigates the existence and impact of the stickiness in credit ratings, and hence proposes the new explanation of the observed rating deterioration. Chapter 4 presents the main conclusions of the thesis.

Chapter 2

An (S, s) Model of Corporate Cash Holdings

2.1 Introduction

Cash, as the most liquid asset, has been attracting considerable attention from the academia. Cash (or, more generally, liquid assets) management serves firm operations as a core activity intertwining with investing and financing decisions (Graham & Harvey 2001, Lins et al. 2010, Campello et al. 2011). Much effort has been allocated so far to the understanding of cash levels. However, the mechanism behind the decision process of cash holding management has been largely unexplained, at least empirically. Our study attempts to contribute to the literature by analyzing the dynamic cash policy in the double-barrier framework. This framework takes into account two aspects of cash management: 1) allowing cash level to freely float within the range bounded by the upper and lower thresholds; 2) refinancing back to the target value immediately when the cash level hits either barrier. A more traditional view of cash policy is that cash management involves a series of discrete decisions, which assume some optimal level and result in firms remaining around that level. Although conceptually straightforward, this static framework leads to a mixed empirical evidence, which creates some puzzles. For instance, the positive effect of cash flow on cash holdings documented in empirical studies (Opler et al. 1999, Han & Qiu 2007, Chen et al. 2012) contradicts with the precautionary motive to hold cash.[1] The positive correlation between risk measures (e.g. cash flow sensitivity, R&D investment) and cash holdings (Kaplan & Zingales 1997, Bates et al. 2009, Kusnadi & Wei 2011) is often cited as the evidence supporting the argument that cash holding is the reserve prepared for future uncertainty. However, the dynamic theoretical cash model predicts negative sensitivity of cash holdings to cash flow controlling for external financing cost (Riddick & Whited 2009). Many empirical findings contradict this theory by suggesting that cash flow level, which secures firm operation and reduces risk, increases cash holdings.

Early cash policy studies concentrate on identifying determinants of corporate cash holdings. The determinant studies based on trade-off model (Opler et al. 1999, Qiu & Wan 2015, Subramaniam et al. 2011, Chen et al. 2012, Yan 2006) estimate the model based on observed cash holdings by treating deviations from the true target as simple error term. However, we show that the deviation may contain some information about firms' cash policy since its is a strategic choice rather than the residual component. Consequently, the trade-off model focuses on the benefits and

[1]Firms hold cash to prepare for future adverse shocks. Precautionary motive is summarized in Bates et al. (2009) in great detail.

costs of liquidity but ignore the cost of cash refinancing. It is the lumpiness in refinancing decisions that reveal the presence of those (fixed) costs and lead to a number of dynamic aspects in corporate cash management.

The speed of adjustment studies attempt to overcome the issues caused by the inertia in adjusting cash levels to the target. As previously discussed, the adjustment speed studies employ the partial adjustment model (Dittmar & Duchin 2010, Gao et al. 2013, Graham & Leary 2015) and reveal firms' willingness to eliminate the deviation from the optimal level. Although these studies capture some dynamic aspects such as that the refinancing policy aims at offsetting the deviation, the underlying assumption of the static trade-off framework as the infrequent refinancing phenomenon is not explained.

The theoretical developments in cash policy models demonstrate the potential of the double-barrier setting to capture dynamic aspects of cash management including infrequent adjustments. Existing literature can be divided into two strands of theoretical cash policy studies. The first group focuses on the value of cash holdings (Nikolov & Whited 2014, Chi & Su 2015) and derives the optimal policy based on the variations in the marginal cash value. The second group explicitly derives the double-barrier policy as the optimal one, given the costly external financing (Whited 2006, Riddick & Whited 2009, Cunha et al. 2011, Bolton et al. 2011, 2013). For example, Bolton et al. (2011) derive the optimal inaction, payout, and liquidation regions in frictional market, and predict that the boundaries between adjacent regions triggers refinancing. This second strand of studies form our theoretical basis. The inaction range between barriers captures the effect of inertia, and hence, leads to optimal refinancing decisions.

A number of studies theoretically demonstrate the intertwining relationships between cash policy and other policies (Gamba & Triantis 2008, Belhaj 2010, Bolton et al. 2011, 2013, Mahmudi & Pavlin 2013, Nikolov & Whited 2014). Nikolov & Whited (2014) develop a dynamic model of firm investment and financing to interpret the way agency problems affect corporate cash holdings. Bolton et al. (2013) predict that firms cut investment and payout in bad times and issue equity in good times even without urgent demand for funds. These findings underpin the point that investment and financing activities are possible channels of cash refinancing. Further, the close link between cash holding refinancing and other policy decisions predicted by dynamic theoretical models also receives empirical support from a number of studies (Denis & Sibilkov 2009, McLean 2011, Lee & Suh 2011, Brown & Petersen 2011, Brisker et al. 2013, Gao et al. 2013, Harford et al. 2014). Denis & Sibilkov (2009) find that both the cash level and value increase with investment level, especially for financially constrained firms. Harford et al. (2014) indicate that cash holdings relax firms' financial policy by mitigating the debt refinancing risk caused by the shorter maturity debt. The empirical findings further confirm the existence of an interaction between cash management, investment, and financing policy. All in all, both theoretical and empirical evidence imply that there is scope on investigation of cash policy by focusing on refinancing events related to investment and financing activities.

Our work is also related to studies of determinants of cash holdings that also focus on cash value. Papers that attempt to identify the determinants of cash holdings (Opler et al. 1999, Yan 2006, Bates et al. 2009, Subramaniam et al. 2011, Chen et al.

2012, Qiu & Wan 2015) provide us with inspiration in terms of model specification. We follow the basic empirical specification in Opler et al. (1999) and Bates et al. (2009) but further control for macro-economic effects. Furthermore, our paper differs from these studies because the double-barrier model captures the endogenous pattern of cash management. Moreover, our study reveals the real effect of cash management (e.g. cash demand, financial flexibility, financing friction), and hence complements the cash value studies (Jensen 1986, Dittmar et al. 2003, Pinkowitz et al. 2006, Dittmar & Mahrt-Smith 2007, Tong 2011) in terms of understanding the channels through which cash policy affects cash value. Lastly, although our model captures certain dynamic aspects of cash refinancing, we cannot reconcile all puzzles in the area of cash holdings. For example, our model cannot explain the high cash holdings puzzle (Pinkowitz et al. 2012) because it is more likely to be caused by parallel shift in demand expectation instead of cash management inertia.[2] In other words, our study attempts to reconcile the puzzles brought only by ignoring lumpiness in cash refinancing.

As indicated by Bolton et al. (2011), empirical studies on cash policy are based on continuous adjustment assumption. In contrast, we move one step further to formulate the dynamic cash policy under infrequent refinancing framework. The static trade-off model ignores that costly cash refinancing defers firms' cash injection and payout decisions, and hence the observed cash level may deviate from the target. The presence of the lump sum costs leads to infrequent refinancing, which allows the cash level to deviate from the optimal one. It challenges the assumption in the static trade-off model that all cash holding observations come from the same

[2] Pinkowitz et al. (2012) observe the phenomenon that the cash holdings in US firms is significantly higher during the post-crisis period than other time, and summarize it as high cash holding puzzle.

distribution. To tackle this ambiguity caused by the specific refinancing pattern, we address the question "how does infrequent refinancing affect cash policy", rather than studying the optimality based on cash holding targets as in static trade-off studies. The perspective of refinancing leads to the dynamic double-barrier model, and the theoretical development (Baumol 1952, Miller & Orr 1966, Bolton et al. 2011, 2013) has derived the endogenous refinancing decisions based on this double-barrier policy.

In a double-barrier model, firms choose discrete and lumpy cash refinancing policy due to the fixed (lumpy) cost. In other words, firms have some tolerance of the deviation of cash level from the optimum of each adjustment, and refinance only when cash balance hits either the upper or lower threshold. The rationale behind such a pattern is that lumpy refinancing enjoys the economies of scale, reducing the associated fixed cost. Thresholds serve as refinancing triggers because they are the boundaries beyond which the benefits of refinancing exceed the costs. Moreover, the infrequent refinancing makes only part of the observed cash levels satisfy the conditions of targets. More precisely, we isolate those cash holding variations due to firms' deliberate adjustment from those natural variations due to firm operation. The basic principle of the isolation is to set certain cut-off levels in order to filter out variations which are large enough. We assume large refinancing to be deliberate because it is less likely that managers implement such influential adjustments without careful consideration. Similarly, Hovakimian et al. (2001) and Danis et al. (2014) state that large deviations seem to be deliberate actions because managers will not tolerate such a dominant change being suboptimal. Although these studies are about capital structure, such statements can also apply to the cash holding area due to a similar dynamics.

We need to clarify that our model is not specified to measure the exogenous variations in cash holdings explained by independent variables, let alone to find a causal effect. We aim at testing the existence of the endogenous pattern predicted by double-barrier model in cash holding management, that is, cash refinancing is triggered by cash holding level being too high or too low. Empirical evidence consistent with the model prediction supports the double-barrier policy as a suitable choice for cash refinancing behavior (Bolton et al. 2011, Dittmar & Mahrt-Smith 2007, Tong 2011). The existence of refinancing cost (e.g. debt or equity issuance cost, adverse selection cost) further strengthens the link between theory and empirical test in our analysis.

To the best of our knowledge, we are the first to empirically investigate the endogenous patterns in cash holding management. Furthermore, we use a different estimation strategy compared to existing cash policy studies (Dittmar & Duchin 2010, Gao et al. 2013, Graham & Leary 2015). For example, Graham & Leary (2015) tests the possible cash policy shift based on variations in determinant coefficients, while, we attempt to provide a detailed analysis of the policy itself. Also, all of the three studies above referred to employ the partial adjustment model, which ignores refinancing triggers. We estimate the dynamic double-barrier policy model, which contains a complete set of cash dynamic aspects including targets and thresholds.

Our work contribute to the existing literature by providing a direct comparison between the static and dynamic models. The static model suggests a positive correlation between cash flow and cash holdings, while our results indicate that this observed positive correlation actually mixes the effects of targets and thresholds, and that the correlation between cash flow and cash target is actually negative. This negative correlation is consistent with the predictions from the dynamic theoretical model (Whited 2006, Riddick & Whited 2009, Korteweg & Strebulaev 2013). Moreover, we find that cash holdings targets increase with future cash outflows but decrease with current cash outflows. Further, compared to other dynamic studies such as Bolton et al. (2011), we provide a more detailed model by relaxing the fixed thresholds assumption and controlling for macro economic variables (e.g. GDP growth, recession, cost of carry). Lastly, our model quantitatively defines the high and low of cash holding levels from the perspective of firms' perception by estimating thresholds based on covariates.

The reminder of this chapter is organized as follows. Section 2.2 develops and explains the empirical model setting. Section 2.3 discusses the sample and identification strategy. Section 2.4 provides an analysis of the empirical results. Section 2.6 concludes the main findings. The discussion of more technical aspects is relegated to appendices.

2.2 The Model

In this section, we first discuss the assumptions regarding the firms and the financial market. Then, we describe the model setup and explain the origin of the model features. Finally, we explain the empirical setting.

2.2.1 Model Assumptions

The theoretical model is based on the following four assumptions:

1. ***Going Concern***: The firm continues to operate for a non-predefined period.

2. ***Brownian Motion***: Cash holding ratio ($c_t = \frac{C_t}{K}$, c_t denotes the cash ratio, C_t represents cash holding value, and K refers to the value of total assets) of the firm, follows an arithmetic Brownian motion:

$$dc_t = \mu dt + \sigma dz, \tag{2.1}$$

   in which μ and σ are the exogenous drift rate and the standard deviation of the stochastic process, respectively, and dz is an increment of the standard Wiener process.

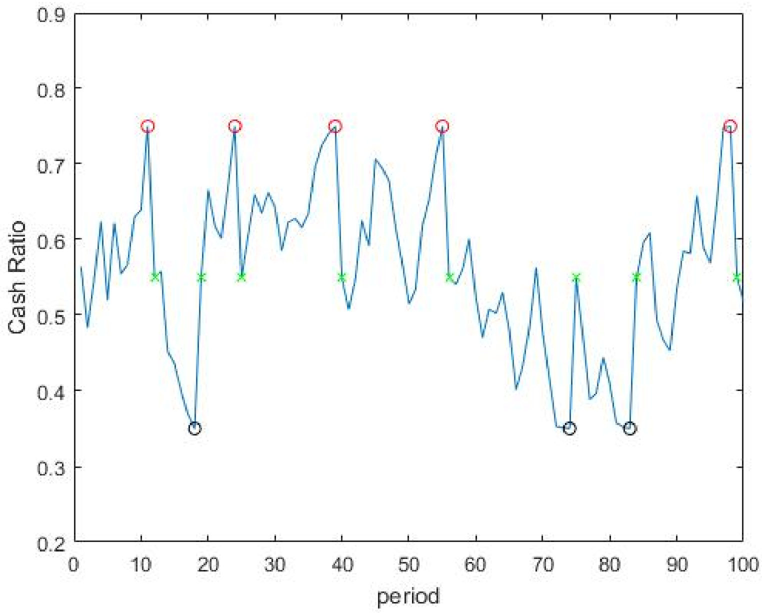
3. ***Market Frictions***: Any deliberate attempts to change cash level (upward or downward) incur transaction costs including both fixed (k_u for the upward and k_d for the downward) and proportional (depending on the amount of adjustment) components.

4. ***Interest Rate***: The exogenously determined interest rate r is the same for borrowing and lending in the capital market. Besides, investors have a subjective discount rate, denoted by δ.

The market assumptions are the ones that distinguish our work from the static trade-off model studies. The static trade-off model assumes a frictionless market, and the benefits and costs of cash holding itself determine the optimal level. However, we argue that firms' behavior is determined by the benefits and costs of refinancing, and the cash level is the cumulative result of a series of refinancing decisions. If the frictionless market assumption holds, any deviation from the optimal cash holding level will reduce the benefits and increase the costs of cash holdings and hence a continuous adjustment will be the best policy. Our model would lead to the same conclusion in the frictionless market. This assumption would suggest zero cost of refinancing, and hence any deviation from the optimal level would trigger adjustment. The benefits of going back to the optimal level must exceed the cost, and this is the theoretical basis of the partial adjustment model (Dittmar & Duchin 2010, Gao et al. 2013, Graham & Leary 2015). However, financial markets in the real world are not frictionless. Refinancing frictions make continuous adjustment suboptimal, because only the deviations which are large enough can be associated with benefits that offset and even exceed the cost of refinancing under this circumstances. Also, the fixed cost component in cash refinancing would lead to an infinite cost following a continuous adjustment policy. In other words, firms must wait until the deviation from the optimal level of ash balance becomes large enough to refinance. This behavior pattern naturally leads to an inaction range around the optimal cash level within which deviation will not be eliminated instantaneously. The double-barrier model attempts to capture this pattern.

The presented model assumes that cash refinancing involves both a fixed and a proportional cost. This assumption attempts to mimic the cost structure of cash injection or distribution faced by firms in the real world. For example, firms need to

FIGURE 2.1: Simulated Cash Ratio Path



This figure illustrates the double barrier cash management policy. Cash ratio, denoted by c_t, itself follows Geometric Brownian Motion with drift rate 0.05 and standard deviation 0.1 ($dc_t = 0.05 * c_t * dt + 0.1 * c_t * dW_t$). The upper and lower thresholds are set at 0.75 and 0.35, respectively. Once reaching the upper threshold as marked by red circles or lower threshold marked by black circles, cash ratio will be reset to optimal distribution target (0.55) next period as denoted by green crosses.

pay fixed brokerage fees in raising money and accessing capital market (Opler et al. 1999), the proportional cost is also possible when firms liquidate assets, cut dividends, or give up investment opportunities. Barclay & Smith (1988) discuss the components of the cost of cash payout, which also includes a fixed part (e.g. hiring investment banks, paying legal and accounting fees of repurchase registration) and a proportional part (e.g. underwriting expenses). This cost structure makes it worth not to immediately eliminate any deviation from the target so that infrequent and lump-sum refinancing becomes the optimal choice.

Figure 2.1 intuitively illustrates the simulated path of cash holdings within the cost structure framework discussed above. The double-barrier policy sets upper and lower thresholds, denoted by red and black circles, respectively. Cash holding itself follows the geometric Brownian motion. The impulse control implied by this policy resets cash holding level to targets once the cash balance hits either threshold. As discussed in Dixit (1993), the resetting threshold is associated with a cash holding level where marginal benefits equals marginal costs.

2.2.2 Empirical Setting

It is assumed that investors decide to establish a firm by providing initial funds with a cash ratio c_0 at $t = 0$. Cash ratio c_t follows the arithmetic Brownian motion as in equation (2.1). The firm is never liquidated, as it is always sufficiently profitable in expectation terms, but requires injection of additional funds if the cash level is too low (hits the lower threshold C_l). Similarly, it distributes extra cash if the level is sufficiently high (hits the upper threshold C_u). Net cash distribution is the source of value that investors derive from owing the firm. This double-barrier setting is commonly seen in existing literature (Bolton et al. 2011, Cunha et al. 2011). When cash ratio hits either threshold level, the firm refinances and sets cash to the target level (C^*).

Our theoretical setting categorizes cash ratio observations into four groups. The first three are upper threshold C_u, distribution target C^*, and lower threshold C_l. If an observation does not belong to any of the three groups, it means that it lies in the inaction range. We specify the following empirical model to suit the dynamic feature following Korteweg & Strebulaev (2013):

$$
\begin{aligned}
C_u &= X\beta + \varepsilon^* + e^{X\theta_u + \varepsilon_u}, \\
C^* &= X\beta + \varepsilon^*, \\
C_l &= X\beta + \varepsilon^* - e^{X\theta_l + \varepsilon_l}.
\end{aligned} \tag{2.2}
$$

in which X is the vector of explanatory variables, β is the coefficient vector for the target equation, and ε^* is the error term in target. We use the exponential term to capture the gap between thresholds and targets. Parameters θ_u and θ_l are coefficients determining the upper and lower exponential terms, respectively. Parameters ε_u and ε_l are the error terms in the upper and lower exponential gaps, respectively. Our setting enables the estimation under dynamic framework and allows different error terms for each group.

Our model relies on the accurate identification of cash refinancing events. Existing literature suggest external financing as the possible channel of cash injection or distribution (Nikolov & Whited 2014, Chi & Su 2015, Denis & Sibilkov 2009, McLean 2011, Lee & Suh 2011, Brown & Petersen 2011, Brisker et al. 2013, Gao et al. 2013, Harford et al. 2014) . Hence, we use net financing cash flow to indicate refinancing events. More precisely, we set cut-off levels of -10% and 10% to isolate refinancing which is large enough.[3] This kind of identification has already been applied in empirical estimation of double-barrier mode (Korteweg & Strebulaev 2013, Danis et al. 2014).

[3]Our robustness tests have also estimated the model based on 5% and 15% cut-offs, and the results are consistent.

Table 2.1: Categorization by Year

Year	Thre-U	Target-U	Target-L	Thre-L	Ordinary	Sum
1987	32	44	133	37	299	545
1988	151	307	522	241	1527	2748
1989	186	301	568	190	1619	2864
1990	165	354	450	184	1687	2840
1991	158	308	504	212	1780	2962
1992	143	278	634	272	1808	3135
1993	144	269	826	280	1815	3334
1994	138	269	829	366	1796	3398
1995	167	267	1031	359	1811	3635
1996	164	335	1145	303	1840	3787
1997	168	322	1045	338	1820	3693
1998	156	313	958	274	1737	3438
1999	154	311	797	256	1733	3251
2000	214	299	735	175	1666	3089
2001	200	383	484	149	1821	3037
2002	128	379	345	219	1849	2920
2003	156	277	480	237	1731	2881
2004	162	247	540	198	1643	2790
2005	140	292	531	212	1540	2715
2006	166	261	543	202	1451	2623
2007	176	302	481	155	1390	2504
2008	206	345	329	94	1464	2438
2009	148	355	216	123	1485	2327
2010	127	265	273	164	1405	2234
2011	139	230	330	140	1329	2168
2012	118	253	339	126	1307	2143
2013	110	231	334	170	1282	2127
2014	122	216	413	122	1206	2079
2015	0	239	323	0	1307	1869
Sum	4238	8252	16138	5798	45148	79574

Note: This table presents our categorization by year. Our categorization method defines fcf as the amount of adjustment. We identify cash distribution when fcf exceed 10% and cash injection when fcf is below -10%. The cash holding observation at the year end when there is injection or distribution is target. Then, the remaining observations which are not targets are categorized into thresholds and ordinary group. We define upper (lower) threshold as those ones before distribution (injection), and the non-identified ones belongs to the ordinary group.

The last step in order to apply empirical double-barrier model is to correctly categorize observations into the four groups, namely, upper threshold, target, lower threshold, and ordinary observations. Our categorization strategy is based on the identification of refinancing events. We regard pre-refinancing observations as the basis of thresholds, and treat post-refinancing observations as targets. More precisely, suppose endogenous cash distribution happens in period t, the observed cash ratio at the end of this period (c_t, post-refinancing) will enter the target group (C^*). The sum of last period observation and operating cash flow ($c_{t-1} + ocf_t$, pre-refinancing) forms the upper threshold (C_u), because this is the amount which invokes refinancing. Similarly, the cash observation after cash injection belongs to the lower threshold group (C_l). All other observations are categorized as the observations falling within the inaction range ("ordinary observations"). This categorization strategy is intuitive because it directly tracks cash refinancing by answering "what is the exact level that invokes refinancing". Other empirical studies on dynamic policy have already applied this strategy in capital structure area Korteweg & Strebulaev (2013), Danis et al. (2014) and durable goods area (Eberly 1994, Attanasio 2000).

2.3 Data

Our sample is based on COMPUSTAT annual fundamental data covering the period from 1988 to 2016. We delete non-US firms to form the sample focusing on the US market. Then, we exclude utility firms (SIC codes from 4900 to 4999) and financial institutions (SIC codes from 6000 to 6999) since they are subject to specific regulation. Furthermore, we ensure that firms in our sample are active observations by excluding firm-year with total assets less than or equal to zero. We also delete observations which have negative or zero sales, because such abnormal situations may distort their financing policies. Our final dataset contains 79,574 observations from 9,883 unique firms. Table 2.1 lists the number of observations in each group by year following our categorization strategy.

We measure cash holding levels by the standard cash ratio defined as cash and marketable securities divided by total assets. Other main alternative measures of cash holdings include cash to non-cash assets (book value of total asset minus cash), logarithm of cash to non-cash assets, and cash to sales (Bates et al. 2009). Bates, Kahle & Stulz (2009) correctly point out that the cash to non-cash asset ratio used by Opler, Pinkowitz, Stulz & Williamson (1999) leads to the outliers problem resulting from firms which allocate most of their assets in cash account. These firms are important to our model since they represents a specific case in cash management. For example, if a firm allocate 95% of asset in cash, it will be regarded as an outlier measured by cash to non-cash asset ratio. However, there may be some reason for the firm to do so. Further, the logarithm measure used by Foley, Hartzell, Titman & Twite (2007) cannot thoroughly eliminate the outliers problem because the firm recognized as an outlier in cash to non-cash ratio measurement will still be an outlier measured by the logarithm of this ratio, and the cash to sale measure introduces the effect of firm operation and industry environment, which are not the focus of our study.

The selection and construction of explanatory variables in this chapter follow Opler et al. (1999) and Bates et al. (2009) and also incorporates new developments from recent literature. We add the future capital expenditure and acquisition spending to reflect their effect on budget. To control the risk of adding these forward-looking variables, we have also estimated our model without them. Moreover, our robustness tests also considers lagged capital expenditure and lagged acquisitions to control for long lasting effects (McConnell & Muscarella 1985) and deviations of large cash outflows on cash account. The industry standard deviation reflects the variation of cash flows at an industry level, and all other variables are at firm-level. The definitions of explanatory variables are as follows (the number in bracket represents COMPUSTAT item):

1. *Chrt*: Cash ratio, defined as cash and marketable securities (#1) divided by book value of total assets (#6).

2. $Chfl$: Cash flow divided by book value of total assets (#6). The cash flow is defined as earnings before depreciation (#13) minus interest (#15), tax (#16), and common dividend (#21) ($Cash\ flow = EBITDA - Interest - Taxes - common\ dividends$) (Bates et al. 2009). The pecking order theory suggests that firms prefer internal cash flow to outside financing, and hence firms with a higher cash flow generating ability accumulate cash more easily.

3. *Mtb*: Market to book ratio captures investment opportunities. It is defined as the sum of book value of total liabilities ($\#6 - \#60$) and market value of equity ($\#199 * \#25$) divided by book value of total assets (#6) ($(BV\ of\ Liability + MV\ of\ equity)/BV\ of\ total\ assets$). It is costly for firms to miss investment opportunities by having insufficient liquidity, and hence we expect firms with better investment opportunities to hold more cash.

4. *Size*: Firm size, defined as the logarithm of book value of total assets (#6). Large firms usually have better access to financial markets (precautionary), and hence we expect firm size to have negative effect on cash holdings.

5. *Nwc*: Net working capital (#179) minus cash and marketable securities (#1) divided by total assets (#6). Net working capital serves as substitute of cash holdings, and hence we expect it to negatively affect the cash ratio.

6. *Capex*: capital expenditure (#128) divided by total assets (#6). Capital expenditure represents a considerable cash outflow. From the dynamic perspective, we expect current capital expenditure to reduce the (actual) cash ratio, since it is associated with a cash disbursement.

7. *F capex*: future capital expenditure, defined the same as *Capex* but it is one period leading to the dependent variable. We expect future capital expenditure to increase the target cash ratio, because it represents cash outflows to be satisfied.

8. *Acqui*: expenditure on acquisitions (#129) divided by total assets (#6). Similar to capital expenditure, we expect current acquisitions to reduce the cash ratio.

9. *F acqui*: future expenditure on acquisitions, defined the same as *Acqui* but it is one period leading to the dependent variable. We expect future acquisitions to increase the target cash ratio for the same reason as *F capex*.

10. *Rd*: Research and development expense (#46) divided by sales (#12). As common in the literature, if the R&D expense is missing, we replace it with zero. R&D expenditure represents future growth opportunities, and it also

increases financial distress risk. Hence, we expect this variable to be positively related to the level of cash.

11. *Lev*: the sum of long-term debt (#9) and current debt (#34) divided by total assets (#6). Cash and leverage are complementary in financing investment opportunities, and hence we expect negative impact of leverage on cash holdings.

12. *Dummy_div*: Dummy variable indicating whether or not the firm pays common dividend in that year. Paying dividends demonstrates a healthy financial position and access to capital markets, which reduces the demand for cash. We expect this dummy to negatively affect the cash ratio.

13. *Dummy*1990: Dummy variable indicates the period of observation. If the observation is within the period from 1990 to 1999 (inclusive), this dummy variable equals one; otherwise it equals zero. We expect it to have a negative effect because the observed cash ratio should be lower than the ratio resulting from changes in firm characteristics (Opler et al. 1999).[4]

14. *Dummy*2000: Dummy variable equals one if the observation is within the period after 2000 (inclusive) and zero otherwise. We expect a negative effect for the same reason as above.

15. *Indstd*: The standard deviation of cash flow at the industry level, which measures industry risk. The first step is to calculate the standard deviation of cash flow at the firm level. For each observation, we take the standard deviation of

[4] We exploit the decades dummies instead of year dummies in this study, because these dummy variables are to detect whether or not there exists shifts in characteristics rather than trend in cash ratio. Decades dummies are more appropriate since the shifts accumulated in ten years are more clear to be spotted. In comparison, year dummies dilute the possible variations in characteristics.

cash flows in previous ten years, and we ensure that there are at least three cash flow observations in the ten-year period. Secondly, for each year, we take the average of the firm-level standard deviation within each industry (indicated by two-digit SIC code). We use this industry level mean standard deviation to indicate the industry risk level. Previous literature treats risk as the main motive to hold cash (Opler et al. 1999, Almeida et al. 2004, Han & Qiu 2007), and hence we expect this industry risk variable to increase cash holdings.

16. $Interest$: The three-month treasure yield from the Federal Reserve. The interest rate represents an opportunity cost of holding cash, and hence we expect it to reduce the cash reserve.

17. $dGDP$: The real GDP growth rate from the U.S. Bureau of Economic Analysis. Similar to interest rate, we expect the real GDP growth to reduce cash holding level.

18. $RECE$: A dummy variable indicating a recession period. It equals one if the period of an observation is within the recession stage, and equals zero otherwise. The definition of a recession period comes from the National Bureau of Economic Research. We expect that recessions reduce cash holding levels since they weaken the whole economy and reduce the demand for cash.

Table 2.2 describes the basic statistics of the variables, and Table 2.3 presents correlations among the covariates.

Table 2.2: Descriptive Statistics

	Mean	Median	Std.	Min	Max	Prc 1	Prc 99
CHS	0.176	0.089	0.209	0.000	0.869	0.000	0.869
Chfl	0.024	0.067	0.175	-0.820	0.269	-0.820	0.269
qrate	0.034	0.039	0.024	0.000	0.089	0.000	0.081
dGDP	0.027	0.027	0.016	-0.028	0.047	-0.028	0.047
RESS	0.135	0.000	0.342	0.000	1.000	0.000	1.000
Indstd	0.074	0.076	0.025	0.023	0.121	0.025	0.121
Mtb	1.943	1.438	1.541	0.552	9.798	0.552	9.798
Size	5.068	4.976	2.154	0.406	10.187	0.630	10.187
Nwc	0.109	0.093	0.185	-0.355	0.583	-0.355	0.581
Rd	0.152	0.000	0.645	0.000	4.924	0.000	4.924
Lev	0.205	0.178	0.182	0.000	0.695	0.000	0.694
Dvc	0.008	0.000	0.019	0.000	0.120	0.000	0.120
F capex	0.058	0.038	0.063	0.000	0.361	0.000	0.345
F acqui	0.023	0.000	0.060	0.000	0.340	0.000	0.340
Capex	0.061	0.039	0.066	0.001	0.386	0.001	0.358
Acqui	0.024	0.000	0.061	0.000	0.345	0.000	0.345
D90	0.420	0.000	0.494	0.000	1.000	0.000	1.000
D00	0.344	0.000	0.475	0.000	1.000	0.000	1.000
D10	0.159	0.000	0.366	0.000	1.000	0.000	1.000

Note: This table presents the descriptive statistics of cash holding ratio and covariates. The values in this table are based on the sample with both cash ratio and covariates winsorized at top and bottom 1 percentile. CHS refers to the dependent variable cash holding, which is the ratio of cash and short-term investment above total asset. $Chfl$ represents cash flow, and it is calculated as subtracting interest expense, tax payments, and common dividend from *ebitda*, and then divided by total asset. *qrate* is the quarterly interest rate. *dGDP* refers to GDP growth rate based on amount in 2009 value. $RESS$ is a dummy variable indicating recession period (defined by NBER) when it equals 1. $Indstd$ is industry cash flow volatility, which is the industry-wide average of firm volatility of previous 10 years cash flows. *Mtb* is market to book ratio and it is calculated as the sum of book liability and market equity divided by total asset. $Size$ is the logarithm of total asset. Nwc refers to net working capital defined as cash and marketable securities divided by total asset. *Rd* is the research and development expense divided by sale. *Lev* measures leverage calculated as the sum of long-term and current debt divided by total asset. *Dvc* the amount of common dividend payment divided by total asset. *Capex* is capital expenditure divided by total asset. *Acqui* represents acquisition expenditure divided by total asset. $Fcapex$ and $Facqui$ are forward capital expenditure and acquisition, which are *Capex* and *Acqui* in next period, respectively. $D90$ is the dummy variable indicating 1990s. Similarly, $D00$ and $D10$ are dummy variables indicating 2000s and 2010s, respectively.

Table 2.3: Covariates Correlation Coefficients

	Chfl	qrate	dGDP	RESS	Indstd	Mtb	Size	Nwc	Rd	Lev	Dvc	F capex	F acqui	Capex	Acqui	D90	D00	D10
Chfl	1	0.042	0.000	0.001	-0.198	-0.236	0.320	0.224	-0.516	0.054	0.076	0.162	0.089	0.115	0.066	0.021	-0.048	-0.006
qrate		1	0.214	0.032	-0.318	-0.077	-0.283	0.189	-0.081	0.091	0.019	0.176	-0.067	0.183	-0.063	0.017	-0.387	-0.513
dGDP			1	-0.781	0.008	0.033	-0.091	0.040	-0.005	0.021	-0.036	0.065	0.035	0.054	0.040	0.195	-0.262	-0.119
RESS				1	-0.072	-0.052	-0.011	0.014	-0.013	0.009	0.016	-0.019	-0.054	0.004	-0.048	-0.134	0.207	-0.163
Indstd					1	0.233	-0.112	-0.152	0.245	-0.222	-0.121	-0.123	0.034	-0.123	0.034	0.099	0.242	0.040
Mtb						1	-0.128	-0.160	0.270	-0.243	0.046	0.064	0.029	0.040	-0.020	0.044	0.037	0.034
Size							1	-0.122	-0.115	0.182	0.222	0.012	0.112	0.002	0.151	-0.085	0.161	0.230
Nwc								1	-0.181	-0.110	0.021	-0.165	-0.020	-0.190	-0.055	0.048	-0.136	-0.106
Rd									1	-0.141	-0.085	-0.082	-0.048	-0.075	-0.052	-0.005	0.065	0.033
Lev										1	-0.077	0.046	0.004	0.103	0.150	0.008	-0.077	-0.038
Dvc											1	0.014	0.021	-0.002	-0.016	-0.038	-0.058	0.070
F capex												1	-0.085	0.685	-0.066	0.044	-0.131	-0.080
F acqui													1	-0.061	0.201	0.019	0.021	0.043
Capex														1	-0.088	0.030	-0.123	-0.089
Acqui															1	0.021	0.025	0.033
D90																1	-0.417	-0.259
D00																	1	-0.224
D10																		1

Note: This table presents the correlation among explanatory variables, as defined in Table 2.2.

2.4 Empirical results

2.4.1 Cross-category Difference

As we discussed previously, there are differences between predictions of dynamic cash holding models (Gamba & Triantis 2008, Riddick & Whited 2009, Belhaj 2010, Bolton et al. 2011, 2013, Mahmudi & Pavlin 2013, Nikolov & Whited 2014) and the findings of static empirical studies (Opler et al. 1999, Qiu & Wan 2015, Subramaniam et al. 2011, Chen et al. 2012, Yan 2006). Before discussing the main findings from dynamic model, we firstly demonstrate the difference between the static and dynamic models in an intuitive way. We show the regression results for each of the target group, and discuss the differences. More precisely, if cash holdings observations come from the same distribution as argued by the static model, regression results for each group will be very similar. In other words, the static model methodology will lead to a poor description of the determinants of cash holding if the regressions results between groups differ significantly. It is worth noting that the results of the regression by groups are not necessarily identical to the main model predictions (dynamic double-barrier model). Although these regressions in subsamples capture the effect of infrequent cash refinancing, they do not estimate the model simultaneously and, therefore, rely on incomplete information.

Table 2.4 reports the regression results by groups. The first two columns report the regression results for the whole sample, and this replicates the static studies and, in addition, controls for macroeconomic variables ($Interest$, $RECE$, $dGDP$). Column 1 reports the results based on the entire sample. In comparison, results in

Table 2.4: OLS by Category

	Whole1	Whole2	Target	Upper	Lower	Ordinary
Inter	0.270***	0.257***	0.176***	0.369***	0.182***	0.306***
	(0.004)	(0.005)	(0.010)	(0.018)	(0.017)	(0.006)
Chfl	-0.039***	-0.049***	-0.037***	-0.030	-0.096***	-0.108***
	(0.003)	(0.004)	(0.006)	(0.024)	(0.012)	(0.007)
qrate	-0.064**	-0.111**	0.148	-0.039	-0.299*	-0.233***
	(0.026)	(0.046)	(0.093)	(0.172)	(0.160)	(0.056)
dGDP	0.048	-0.004	0.004	-1.244***	0.157	0.026
	(0.043)	(0.063)	(0.128)	(0.235)	(0.228)	(0.076)
RESS	0.003	0.001	0.000	-0.042***	-0.001	0.003
	(0.002)	(0.003)	(0.006)	(0.010)	(0.010)	(0.003)
Indstd	0.856***	0.968***	1.198***	0.462***	0.952***	0.768***
	(0.021)	(0.023)	(0.047)	(0.087)	(0.085)	(0.028)
Mtb	0.019***	0.020***	0.019***	0.022***	0.021***	0.023***
	(0.000)	(0.000)	(0.001)	(0.002)	(0.001)	(0.001)
Size	-0.008***	-0.007***	0.001**	-0.009***	0.004***	-0.011***
	(0.000)	(0.000)	(0.001)	(0.001)	(0.001)	(0.000)
Nwc	-0.276***	-0.280***	-0.263***	-0.308***	-0.265***	-0.284***
	(0.003)	(0.003)	(0.006)	(0.012)	(0.011)	(0.004)
Rd	0.083***	0.080***	0.074***	0.132***	0.080***	0.107***
	(0.001)	(0.001)	(0.001)	(0.009)	(0.003)	(0.002)
Lev	-0.387***	-0.397***	-0.365***	-0.459***	-0.335***	-0.423***
	(0.003)	(0.003)	(0.006)	(0.012)	(0.012)	(0.004)
Dvc	-0.337***	-0.335***	-0.321***	-0.340***	-0.663***	-0.538***
	(0.025)	(0.028)	(0.042)	(0.103)	(0.130)	(0.045)
F capex	0.020**	0.030**	0.171***	0.050	-0.016	-0.059***
	(0.009)	(0.012)	(0.020)	(0.064)	(0.031)	(0.019)
F acqui	0.064***	0.049***	0.086***	0.393***	-0.016	0.147***
	(0.008)	(0.009)	(0.016)	(0.074)	(0.019)	(0.016)
Capex	-0.458***	-0.494***	-0.507***	-0.622***	-0.436***	-0.535***
	(0.009)	(0.011)	(0.018)	(0.057)	(0.035)	(0.018)
Acqui	-0.227***	-0.245***	-0.300***	-0.444***	-0.409***	-0.370***
	(0.008)	(0.009)	(0.013)	(0.053)	(0.041)	(0.017)
D90	-0.027***	-0.016***	-0.001	-0.021**	-0.027***	-0.024***
	(0.002)	(0.002)	(0.005)	(0.009)	(0.008)	(0.003)
D00	-0.021***	-0.014***	-0.008	-0.011	-0.029***	-0.022***
	(0.002)	(0.003)	(0.006)	(0.011)	(0.011)	(0.004)
D10	-0.009***	-0.004	0.010	-0.011	-0.030**	-0.014***
	(0.003)	(0.004)	(0.008)	(0.016)	(0.015)	(0.005)
Obs.	102741	79243	24599	4219	5812	44613
Adj. R	0.490	0.508	0.539	0.482	0.578	0.494

Note: This table presents the OLS regression results by category, and the regressions are based on the sample with both cash ratio and covariates winsorized at top and bottom 1 percentile. The Whole1 column reports regression results based on the entire sample. The Whole 2 column presents results based on whole sample but dropping observations with missing financing cash flow values. Financing cash flow is cash adjustment defined in this research. It is not necessary for traditional OLS analysis but essential in $(S,\ s)$ estimation. In other words, column Whole 2 is based on the same sample as $(S,\ s)$ estimation. Column Target is based on the target category only. The following two columns are OLS results based on upper and lower thresholds, respectively. The last column reports the regression in Ordinary group. Standard errors are in brackets, and $***$, $**$, and $*$ indicate the significance levels of 1%, 5%, and 10%, respectively.

column 2 are based on the sample after dropping observations with missing fcf, and hence this is the sample used in the subsequent $(S,\ s)$ estimation. The third column contains the regression results for the target group only. Differing from the whole sample result, we find that the interest rate positively affects the cash holding target. This suggests that the higher financing cost reduces the magnitude of cash refinancing. Further, we document an inverse size effect to that reported in the literature, indicating that large firms hold less cash in general but have higher targets. The forth column shows regression results for the upper threshold group. Also, there is a set of evidence in contrary to that of the full sample regression. For example, $Chfl$ is insignificant for upper threshold observations, $dGDP$ demonstrates a considerably negative effect, and the impact of $Indstd$ dramatically reduces to almost half of the level of the whole sample results. The next column reports the results for the lower threshold group regression. The size effect here differs from that in the first two columns. The last group summarizes the regression in the group of ordinary observations. The comparison between these categorized regression reveals the specificity of correlations in different groups, and hence it justifies designing on estimation strategy that simultaneously considers the relations in each of the four groups.

2.4.2 Dynamic Targets

Table 2.5 reports the estimation of the $(S,\ s)$ model for cash holding covariates. Compared to standard regressions, this model introduces the upper and lower boundary which fits our theoretical setting of infrequent refinancing. More precisely, the existence of the fixed refinancing cost postpones cash injections and distributions for economic benefits. In other words, firms choose to refinance only when the deviation between the actual cash level and the target becomes sufficiently large, and the boundaries of $(S,\ s)$ model quantify the upper and lower limits of this deviation. Secondly, the target identified in $(S,\ s)$ model, as discussed before, attempts to capture the optimal level of cash that firms try to achieve. In contrast, the dependent variable in a standard regression is simply the observed cash level without categorization, which captures the average cash level instead of the target. Thirdly, standard regressions assume a simple mechanism in cash management that any exogenous variation in covariates leads to a corresponding change of the cash level. This continuously adjustment mechanism ignores the endogenous pattern introduced by infrequent refinancing reflecting the fact that firms have the flexibility to wait or to make deliberate adjustments. Our estimation of the $(S,\ s)$ takes into account the above considerations.

The first column in Table 2.5 presents the estimation for the target. The most striking evidence occurs in the effect of $Chfl$. In our dynamic model, $Chfl$ has a negative effect on the target (-0.060) which is significant at the 1% level. This finding is very different from the predictions' and evidence in the existing literature, which

Table 2.5: $(S,\ s)$ Model Estimates

	β		θ_u		θ_l	
	Parameter	M.E.	Parameter	M.E.	Parameter	M.E.
Inter	0.217***	-	-0.271***	-	-0.974***	-
	(0.010)		(0.055)		(0.039)	
Chfl	-0.060***	-0.060	-0.697***	-0.385	-0.380***	0.073
	(0.006)		(0.058)		(0.026)	
qrate	-0.140	-0.140	-0.699	-0.466	0.072	-0.165
	(0.091)		(0.532)		(0.365)	
dGDP	-0.167	-0.167	-0.262	-0.289	-0.412	-0.022
	(0.126)		(0.717)		(0.509)	
RESS	-0.012**	-0.012	-0.029	-0.025	0.001	-0.012
	(0.005)		(0.031)		(0.022)	
Indstd	1.098***	1.098	-0.734***	0.756	0.344*	0.977
	(0.047)		(0.270)		(0.190)	
Mtb	0.020***	0.020	0.004	0.022	0.020***	0.013
	(0.001)		(0.006)		(0.002)	
Size	-0.002***	-0.002	-0.037***	-0.020	-0.001	-0.002
	(0.001)		(0.004)		(0.003)	
Nwc	-0.236***	-0.236	-0.306***	-0.378	-0.203***	-0.164
	(0.006)		(0.036)		(0.025)	
Rd	0.079***	0.079	0.124***	0.137	0.043***	0.064
	(0.001)		(0.020)		(0.005)	
Lev	-0.371***	-0.371	-0.619***	-0.660	-0.396***	-0.232
	(0.006)		(0.037)		(0.027)	
Dvc	-0.471***	-0.471	-2.702***	-1.731	0.856***	-0.772
	(0.043)		(0.340)		(0.248)	
F capex	0.294***	0.294	-0.704***	-0.034	0.950***	-0.040
	(0.020)		(0.184)		(0.073)	
F acqui	0.342***	0.342	0.415**	0.535	1.348***	-0.132
	(0.016)		(0.173)		(0.051)	
Capex	-0.487***	-0.487	-0.148	-0.556	-0.631***	-0.265
	(0.018)		(0.167)		(0.079)	
Acqui	-0.290***	-0.290	-0.487***	-0.517	-0.549***	-0.097
	(0.013)		(0.157)		(0.086)	
D90	-0.011**	-0.011	-0.049*	0.000	-0.028	0.000
	(0.005)		(0.027)		(0.019)	
D00	-0.029***	-0.029	-0.018	0.000	-0.079***	0.000
	(0.006)		(0.034)		(0.024)	
D10	-0.016**	-0.016	-0.012	0.000	-0.040	0.000
	(0.008)		(0.048)		(0.033)	
	Parameter		Value			
σ_*	-1.780***		0.169			

	(0.001)	
σ_u	-1.679***	0.187
	(0.011)	
σ_l	-1.433***	0.239
	(0.038)	
ρ_{*u}	0.351***	0.380
	(0.022)	
ρ_{*l}	0.517***	0.535
	(0.019)	

Note: This table reports the main estimation of $(S,\ s)$ model previously specified:

$$\begin{aligned} C_u &= X\beta + \varepsilon^* + e^{X\theta_u + \varepsilon_u}, \\ C^* &= X\beta + \varepsilon^*, \\ C_l &= X\beta + \varepsilon^* - e^{X\theta_l + \varepsilon_l}. \end{aligned} \tag{2.3}$$

There are 79,574 observations from 9883 unique firms. The coefficients β, θ_u, and θ_l estimated in table corresponds to the ones in this empirical model. The marginal effect is formed as the exact amount of effect covariates demonstrated on thresholds. Besides, our maximum likelihood method allows the flexibility to measure the standard deviation (σ_*, σ_u, and σ_l) of error terms (ε^*, ε_u, and ε_l). Moreover, parameter ρ_{*u} measures the correlation between ε^* and ε_u, and ρ_{*l} measures the correlation between ε^* and ε_l. Our estimation has transformed the standard deviation and correlation variables to ensure they suit constraints (standard deviation are positive, and correlation variables are between -1 and 1). Numbers in the parameter column report the estimation of the transformed variables, and numbers in the value column present the value of the variables ($\sigma_\cdot = exp(\cdot)$ and $\rho_\cdot = erf(\cdot)$). Standard errors are in brackets, and $***$, $**$, and $*$ indicate the significance levels of 1%, 5%, and 10%, respectively.

predicts a positive correlation between cash flow and cash holdings (Opler et al. 1999, Han & Qiu 2007, Chen et al. 2012). For example, Opler et al. (1999) find that cash holdings increase by 0.162 for each unit of cash flow increment.[5] Bates et al. (2009) find positive and significant correlation between changes in cash flow and in cash holdings. The logic suggested by this paper is simple: since cash flow is as inflow to cash reserves, higher cash flow leads to higher cash holdings. However, our finding of a negative effect of cash flow is consistent with the prediction of a dynamic theoretical

[5]Opler et al. (1999) define cash holding as the natural log of cash over assets. Although different from our setting, the natural logarithm will not affect the interpretation of the positive sign.

model (Riddick & Whited 2009). While the documented positive effect in a standard regression actually says that the observed cash holdings increase along with cash flow, the negative effect in the $(S,\ s)$ model reflects the correlation with the cash holding target. The key difference is that observed cash level is not necessary the optimal level (target) for firms. The negative effect predicted by our $(S,\ s)$ model supports the view that cash flow reduces the need to hold cash, which is also consistent with the precaution motive, since cash flow makes the firm more safe.

As the counter-party of the inflow effect (represented by possible cash flow), the outflow effect under the dynamic framework also provides contradictory evidence to the results prevalent in static studies. Since an immediate variation in cash holdings is the only channel responding to covariate changes in the simple cash management mechanism implied by a standard regression, this mechanism naturally predicts that outflows reduce cash holdings, and findings in the empirical work that uses static models support this prediction. *Capex* and *Acqui* are the main proxies of cash outflows in Bates et al. (2009), and both the covariates demonstrate considerable negative effect on observed cash holdings (-0.308 and -0.170, respectively). Capital expenditure effect in (Opler et al. 1999) is positive (0.485), but Bates et al. (2009) criticize that this positive sign is sensitive to their definition of cash holding.[6] Riddick & Whited (2009) alternatively explain that productivity shocks make firms invest more and save less cash, but either this interpretation or the previous one (outflows consume cash holdings) confirms the assumption that outflows directly reduce cash holdings.

[6]Opler et al. (1999) use the logarithm of cash over assets as the proxy of cash holdings. Table 3 in Bates et al. (2009) clearly presents that the sign of capital expenditure turns from positive to negative as shifting the definition of cash holding from logarithm to ratio.

However, from the perspective of dynamic cash management, we argue that the exact demand for cash holdings captured by cash target should increase with higher cash outflows. The positive influence of future capital expenditure $F\,capex$ (0.294) and future acquisitions $F\,acqui$ (0.342) supports out argument. Our model uses capital expenditure and acquisitions in next year as a proxy of future values. Strategical decisions, as budgets of capital expenditure and acquisition spending, are made after careful consideration. The implementation is not immediate but lasts for a period as a process. Hence, the effect of these outflows on cash holding takes place at the time when decisions are made but before the implementation process begins. In other words, we match dependent variable in period t with covariates capital expenditure and acquisitions in period $t+1$ ($F\,capex$ and $F\,acqui$). Observations of the two covariates (at then end of year $t+1$) summarize the implementation process, and observation of the dependent variable (at the beginning of year $t+1$) has absorbed the variation of cash demand before the implementation process.[7] The positive coefficients reveal the expansion of cash demand brought by future outflows. Moreover, we also include simultaneous outflows ($capex_t$ and $acqui_t$) in control variables, and document the significantly negative impact (-0.487 and -0.290, respectively). Although the signs are negative, the interpretation is totally different under the framework of dynamic setting due to the dependent variable. These negative impact suggests that the completed implementation process of capital expenditure and acquisition releases the need of cash holdings. Overall, firms need to prepare cash holdings for future outflows, and this extra cash demand disappears when the outflows take place.

[7]The subscript t means the end of year t by definition, which is also the beginning of year $t+1$.

Another result of the dynamic model worth reflecting on is the correlation between cash holdings and *Indstd*. Although it has the same positive sign as in the standard regression results (Opler et al. 1999, Bates et al. 2009), the magnitude of the impact is much larger than that in standard models. As shown in Table 2.5, the dynamic model predicts that the real target of cash holdings soars 110% for each unit of additional industry volatility, but the magnitude of this impact in Bates et al. (2009) is less that 50%. This striking magnitude difference is likely to be driven by the use of cash holding target as dependent variable in the dynamic refinancing setting. More precisely, the large volatility impact in dynamic model concerns the exact demand for cash holdings rather than the observed levels, and it suggests that the cash holding demand is very sensitive to the industry risk level. According to our theoretical setting, refinancing happens only when cash holding level hits either boundary. For observations whose cash holding level have been moved out of the inaction range by variations in industry cash flow volatility,[8] refinancing happens and results in the observed cash holding reflect the target. However, for the remaining observations, for which it still stays within the inaction range,[9] the observed cash holding levels cannot mirror the real target, since industry volatility is insufficient to trigger refinancing. The second group of observations is likely to drive down the magnitude of the effect if the analysis is based on the static model (that pools all observations into a single category). This is indeed the case as shown in Table 2.4. The impact of industry risk in the whole sample is 0.856, but the magnitude dramatically increases to 1.198 once

[8]The inaction range refers to the area between the upper and lower boundaries. In our setting, firms allow cash holdings level freely float in this range without intervention. Hence, refinancing will not happen when cash holding locates within this range, and occurs immediately once it leaves the inaction range.

[9]The inaction range is not fixed since thresholds are also affected by covariates as discussed in next section. The inaction range here naturally refers to the one after shocks in industry volatility.

we use only the target group. Although regressions by group ignore the interaction, this magnitude jump confirms our interpretation and emphasizes the importance of infrequent refinancing.

The first column in Table 2.5 reports a number of control variables whose signs and magnitude are consistent with the standard regression results as in Bates et al. (2009). For example, the dynamic model results also shows positive effect of Mtb on cash target (0.020). Existing literature interprets the positive effect of Mtb on cash holdings as cash is worth more in firms with better investment opportunities (Bates et al. 2009, Dittmar & Mahrt-Smith 2007). We interpret that as investment opportunities increasing the demand for cash in order to avoid missing projects. Moreover, we also find that Nwc negatively affects cash target (-0.236), because Nwc serves as a substitute of cash holdings, reducing cash demand (Opler et al. 1999, Bates et al. 2009, Dittmar & Mahrt-Smith 2007). Furthermore, our results document the negative coefficient for leverage (-0.371), which suggests that the tax saving invoked by interest payments reduces the demand for cash holdings. Debt financing has the tax shield advantage, which reduces possible cash outflow and hence decreases the demand for cash.

2.4.3 Refinancing Thresholds

The trigger of refinancing is another important component in the double-barrier policy, and we model it by quantifying firms' perception of cash holding levels (too high or too low) through thresholds. According to our empirical setting, the double-barrier policy measures the upper (lower) threshold by adding (subtracting) an exponential term to (from) the target. Our analysis is based on the estimation of coefficient vectors θ_k, $k \in \{u, l\}$ within the exponential terms (θ_u and θ_l). The sign of a coefficient in θ_k directly affects the width of the zone between target and the corresponding threshold. A positive sign indicates the increment of the width. However, the exponential transition does not make the level of the threshold immediately known. Therefore, we calculate the marginal effects (ME columns in Table 2.5). The marginal effect is formed as the exact amount of effect covariates demonstrated on thresholds.[10]

Our previous analysis of the target cash levels reveals the variation in real demand of cash holdings. In comparison, the discussion of thresholds concentrates on firms' tolerance of the deviation of cash holdings from the target. The real demand and thresholds jointly offer a complete dynamic policy of cash holding management under the infrequent refinancing framework. We quantify the tolerance for deviation by the width between target and thresholds captured by the exponential term in the empirical setting. In general, low tolerance (narrow zone) implies more active management represented by more frequent refinancing and implies a lower perceived fixed cost of refinancing.

$Chfl$ considerably reduces the deviation tolerance for both upper zone and the lower zone.[11] As shown in Table 2.5, $Chfl$ has a significant negative effect on both exponential terms. It suggests that firms with higher cash flow will make

[10]The precise calculation of a marginal effect is based on the prediction in the empirical setting. Firstly, set all dummy variables at 0, and fix the remaining covariates at their median values. Take cash flow (Chfl) as an example, which is an non-dummy variable. Then, calculate the thresholds $thre_+$ from the empirical model by adding 1% to Chfl median value and keep the other variables at 0 or median values. Similarly, calculate $thre_-$ based on covariates set with Chfl at median value minus 1% and the other covariate values unchanged. The marginal effect of cash flow is $\frac{thre_+ - thre_-}{2\%}$. The marginal effect of dummy variables follows the same method but simple measures the change from 0 to 1 while keeping the other covariates fixed.

[11]We define the area between target and upper threshold as the upper zone, and the area between target and lower threshold as lower zone.

more frequent distributions and injections. The narrowing zones are not surprising because higher cash flow makes the firm "more safe", and hence reduces its refinancing cost. As predicted by our theoretical setting, reducing refinancing cost pulls cash management mechanism closer to the continuously refinancing policy.[12] However, we need to mention that the zones affect future refinancing frequency because current deviation cannot be restricted by the new zones. What determines whether or not there is refinancing are the levels of thresholds after variation, as reflected in the marginal effect. The marginal effects in Table 2.5 indicate that the upper threshold declines by 38.5% for each unit increment in cash flow, which suggests that cash distribution becomes more likely as cash flow (profitability) increases. Further, the lower threshold increases by 7.3% which suggests that the cash injections become more likely as well. However, this is a joint effect of reducing the target and shrinking the lower inaction zone. The narrowing width suggests that future cash injections are generally smaller in magnitude due to the reduced financing cost that results from strong profitability.

Unsurprisingly, the cash outflow covariates affect the policy in an inverse way compared to that of cash inflow. The width of both upper and lower zones enlarges when there will be acquisition spending in future. The refinancing cost increases under the pressure of planned cash outflows,[13] which reduces the frequency of future distributions and injections. In other words, firms with an acquisition plan wait longer to adjust cash holdings. The higher refinancing cost makes small adjustment suboptimal, and hence these firms tolerate larger deviation to realize the economies

[12]When the cost decreases to zero, continuous adjustment becomes optimal.

[13]This is because certain future cash outflows increases the probability to drain cash, and hence reduces firms' credit quality.

of scale. Moreover, the upper threshold increases by 53.5% for each additional unit of $Facqui$. This increment makes conditions that lead to cash distributions more frequent, and hence complements the firms' strategy to stockpile cash holding for future acquisitions. The lower threshold decreases by 13.2%. This is because the enlargement of lower zone dominates the demand increment. In other words, although the firm needs to accumulate cash holdings, investors refuse to inject cash due to their concern about future acquisitions. The widening lower zone and decreasing lower threshold indicated by future capital expenditure support our argument.

Another significant covariate in a cash holding policy is industry risk level, denoted by $Indstd$. Unlike the patterns exhibited by cash inflow and outflow, which simultaneously enlarge or shrink both zones together, industry volatility reduces the upper zone but increases the lower zone. However, this is not unexpected considering the extensive impact of industry risk on cash policy. The widening lower zone reflects the frequency of cash injection drops. The risk level increases firms' refinancing cost and makes the firm less attractive. In contrast, the reduction in the upper zone suggests investors' tolerance of firms' extra cash cash holding reduces. Although upper and lower zones evolve in opposite directions, both movements are in line with the argument that firms speed up cash holding accumulation in risky environment. Moreover, industry volatility demonstrates the strongest impact on thresholds levels. Each additional unit of industry volatility causes the upper and lower thresholds to increase by 75.6% and 97.7%, respectively.

In addition to the existing literature, we include macro-economic covariates such as quarterly interest rate, GDP growth rate, and economy recession indicator defined by NBER (represented by covariates qrate, dGDP, and RESS in Table 2.5, respectively). Although the correlation between observed cash holdings and macro economic variables may be relatively weak, we study the macro variables since our research focuses on the complete cash management policy. The policy our model attempts to establish considers the endogenous pattern of firms' choice. In other words, firms' optimal actions related to cash management depend on their perception whether or not current cash holdings are too high or too low. It is therefore necessary to consider the impact of a broader economy on this perception. The interest rate measures the cost of holding cash, which reduces incentives to hold cash in hand. Accordingly, firms choose to make frequent cash injections instead of keeping cash reserves. The GDP growth rate measures the opportunity cost of holding cash, and hence, it has a similar impact to the interest rate in both the sign and magnitude.

2.5 Robustness Analysis

This section tests the robustness of our main results from four perspectives; the lumpy refinancing framework, the definition of refinancing events, the sensitivity with respect to the cut-off selection, and the definition of variables. Research argues that introducing lumpy refinancing depicts an accurate picture of firms' refinancing behavior (e.g. firms reduce the cash target when inflow is strong and store more cash for possible cash outflows). This differs from the lessons based on the traditional model (Riddick & Whited 2009). However, our estimation is based on a specifically prepared dataset. More precisely, our method categorizes observations into four groups, among which, only the target group can be directly compared to the traditional OLS estimation.

It is naturally to question whether or not our main findings are based on the way our dataset is prepared rather than the lumpy refinancing framework the firms are postulated to operate within. Hence, we implement the first alternative estimation to check the robustness of our results by reducing the difference in dataset between our model and the standard ones. Specifically, we apply the same categorizing procedure as previously but reduce the cut-off level to 1%. It is worth noting that this estimation is not going to test the effect of varying cut-off level but to enlarge the sample size of the target group, which makes the dataset in our model as close to the one in OLS regressions as possible. Based on this setting, the target group now contains 67,398 observations outside of the entire sample size 79,243. Hence, the dataset under 1% cut-off overlaps in 85% with the one in OLS. Therefore,the major difference between the two models is explicitly incorporating refinancing infrequency or not. As reported in Table 2.6, the signs are consistent with the ones in our model, and hence our previous arguments are supported.

The definition of our refinancing is a sufficient magnitude of net financing cash flow (one exceeds the cut-off level). It is also possible that our main findings are driven by this definition. In order to check the robustness of our results, we firstly extract the core deliberate components from fcf, and define refinancing as the sum of net debt issuance ($\#111 - \#114$) and net equity issuance ($\#108 - \#115$) minus common dividend payment ($\#21$). Results based on this definition and the same cut-off level (10%) are presented in Table 2.7, which reports evidence largely consistent with our main results. Moreover, the adopted definition of cash adjustment ignores the investing cash flow, denoted by icf. One might argue that investing cash flow should also be included, because it also provides an opportunity to adjust the cash balance.

For example, firms may choose to liquidate assets to inject cash, or spend more to reduce cash holdings. In response, we argue that only some parts of the entire icf can constitute channels to deliberately adjust cash holdings, since the major components $Capex$ and $Acqui$ are previously determined in our setting. These latter components are deducted to simulate relevant components of icf. We therefore re-estimate our model based on refinancing definitions including the observed investing cash flow ($fcf+icf$) or including the simulated investing cash flow ($fcf+icf+Capex+Acqui$) and 10% cut-off level. The results of the former estimation are reported in Table 2.8, in which one of our main findings, the positive effect of outflows, disappears. This is not surprising since this refinancing definition includes not necessarily deliberate components. In comparison, the results of an estimation after excluding the effect of those components ($fcf + icf + Capex + Acqui$) in Table 2.9 are consistent with our main results.

Next, we check the robustness of our results to the level in defining refinancing events. Firstly, we decrease the cut-off level to 5% and present results in Table 2.10. The results are consistent with our main findings in terms of signs, but the magnitude of regression coefficients is generally smaller in this 5% cut-off setting. For example, the effect of $Indstd$ in the target group decreases to 0.798 from 1.098, and the effect of $F\ capex$ and $F\ acqui$ decrease to 0.142 (from 0.294) and 0.162 (from 0.342), respectively. The small cut-off level allows for a more minor adjustment to cash holdings, which drives down the impact magnitude. Furthermore, we increase the cut-off to 15%, and present the results in Table 2.11. The results are still consistent in terms of signs, but the magnitude now enlarges. These evidence clearly indicates that our main findings are not driven by the cut-off level.

Researchers may criticize that our argument about the large effect of industry cash flow volatility is due to the noisy proxy of average standard deviation. To release this concern, we estimate our model using the predicted industry-wide cash flow standard deviation, denoted by $Indstd_{GARCH}$, as the new proxy. The prediction of this proxy is based on Pooled Panel GARCH(1,1) model following Cermeño & Grier (2006) and Drakos & Goulas (2006).[14] The magnitude of industry risk effect dramatically decreases after exploiting this new proxy as in Table (2.12). The effect on target decreases from 1.098 from our main model to 0.179. However, the industry risk effect in OLS decreases from around 50% to 0.045 after shifting to this new proxy. Our argument that OLS method underestimate the effect of industry risk still holds. Secondly, we estimate our model using market-based cash ratio (cash and marketable securities divided by the sum of book value liability and market value equity) as the dependent variable, and report the results in Table (2.13). The magnitude of coefficients decreases in general, but the main findings (e.g. negative cash flow, positive future outflows, and influential industry risk) do not change. Thirdly, we have added a dummy variable indicating missing *R&D* in Table (2.14), which does not change our estimation significantly. Lastly, we estimate our main without forward-looking variables (*F capex* and *F acqui*) in Table (2.15), and our main findings still hold. w-

[14] There are two steps to construct the variable $Indstd_{GARCH}$. Firstly, estimate the Pooled Panel GARCH model

$$\begin{aligned} y_{i,t} &= \mu_i + b_1 y_{i,t-1} + \varepsilon_{i,t}, \\ \sigma^2_{i,t} &= \alpha_i + c_2\sigma^2_{i,t-1} + c_3\varepsilon^2_{i,t-1}, \end{aligned}$$

in which subscript i indicate a specific section (industry), $y_{i,t}$ represent the average cash flow (defined as before) in year t for industry i, $\sigma_{i,t}$ is the standard deviation of error term $\varepsilon_{i,t}$, μ_i and α_i are industry fixed effects for average cash flow and variance. Coefficients b_1, c_2, c_3, and the industry fixed effects are parameters to be estimated. There are 1,653 observations from 57 industries (categorized by two-digit SIC code) for the sample period 1988 to 2016. The GMM estimation gives the results: $b_1 = 0.580$, $c_2 = 0.002$, $c_3 = 0.002$. Secondly, predict $\sigma_{i,t}$ and merge it back to our main dataset as $Indstd_{GARCH}$.

Table 2.6: Robustness for $(S,\ s)$ Model Estimates: 1% Cut-off

	β		θ_u		θ_l	
	Parameter	M.E.	Parameter	M.E.	Parameter	M.E.
Inter	0.174***	-	-0.081	-	-1.279***	-
	(0.007)		(0.126)		(0.086)	
Chfl	-0.056***	-0.056	-0.824***	-0.323	-0.326***	0.032
	(0.006)		(0.122)		(0.059)	
qrate	0.374***	0.374	-1.182	-0.008	-0.302	0.456
	(0.065)		(1.132)		(0.796)	
dGDP	0.257***	0.257	0.113	0.293	-1.076	0.549
	(0.090)		(1.502)		(1.077)	
RESS	0.016***	0.016	-0.034	0.005	-0.005	0.017
	(0.004)		(0.063)		(0.047)	
Indstd	0.508***	0.508	-2.758***	-0.384	5.026***	-0.857
	(0.033)		(0.631)		(0.506)	
Mtb	0.026***	0.026	-0.013	0.022	0.007	0.024
	(0.001)		(0.014)		(0.006)	
Size	-0.005***	-0.005	-0.137***	-0.049	-0.027***	0.002
	(0.000)		(0.008)		(0.006)	
Nwc	-0.239***	-0.239	-0.253***	-0.321	-0.327***	-0.151
	(0.005)		(0.082)		(0.056)	
Rd	0.086***	0.086	0.172***	0.142	-0.009	0.089
	(0.001)		(0.042)		(0.013)	
Lev	-0.342***	-0.342	-0.349***	-0.455	-0.678***	-0.158
	(0.005)		(0.085)		(0.072)	
Dvc	-0.238***	-0.238	-1.312	-0.663	-1.436	0.151
	(0.038)		(1.199)		(1.300)	
F capex	0.022	0.022	-0.521	-0.147	0.577***	-0.135
	(0.017)		(0.414)		(0.181)	
F acqui	0.066***	0.066	0.800**	0.324	0.998***	-0.205
	(0.013)		(0.406)		(0.134)	
Capex	-0.374***	-0.374	0.263	-0.289	-0.821***	-0.151
	(0.016)		(0.411)		(0.210)	
Acqui	-0.211***	-0.211	-0.519	-0.379	-0.577***	-0.055
	(0.012)		(0.371)		(0.213)	
D90	0.011***	0.011	-0.238***	0.000	-0.098**	0.000
	(0.003)		(0.060)		(0.041)	
D00	0.027***	0.027	-0.017	0.000	-0.202***	0.000
	(0.004)		(0.076)		(0.055)	
D10	0.052***	0.052	-0.001	0.000	-0.121*	0.000
	(0.006)		(0.102)		(0.073)	
	Parameter		Value			
σ_*	-1.623***		0.197			

	(0.001)	
σ_u	-0.370***	0.691
	(0.027)	
σ_l	-1.135***	0.322
	(0.045)	
ρ_{*u}	-0.177***	-0.198
	(0.048)	
ρ_{*l}	0.300***	0.329
	(0.026)	

Note: This robustness test is to verify that the findings in Table 2.5 are driven by introducing infrequency rather than a dataset customized by our categorization. There are 79,574 observations from 9883 unique firms. Again, the traditional OLS models assume immediate refinancing for any cash holding deviation from the target. This test set the low cut-off of refinancing definition (1%) under the infrequent adjustment framework, which is as close as possible to continuously refinancing setting. In simple words, when the amount of fcf (either positive or negative) exceeds 1% of total assets, it is regarded as refinancing. This table reports the estimation of $(S,\ s)$ model previously specified:

$$\begin{aligned} C_u &= X\beta + \varepsilon^* + e^{X\theta_u + \varepsilon_u}, \\ C^* &= X\beta + \varepsilon^*, \\ C_l &= X\beta + \varepsilon^* - e^{X\theta_l + \varepsilon_l}. \end{aligned} \tag{2.4}$$

The coefficients β, θ_u, and θ_l estimated in table corresponds to the ones in this empirical model. The marginal effect is formed as the exact amount of effect covariates demonstrated on thresholds. Besides, our maximum likelihood method allows the flexibility to measure the standard deviation (σ_*, σ_u, and σ_l) of error terms (ε^*, ε_u, and ε_l). Moreover, parameter ρ_{*u} measures the correlation between ε^* and ε_u, and ρ_{*l} measures the correlation between ε^* and ε_l. Our estimation has transformed the standard deviation and correlation variables to ensure they suit constraints (standard deviation are positive, and correlation variables are between -1 and 1). Numbers in the parameter column report the estimation of the transformed variables, and numbers in the value column present the value of the variables ($\sigma. = exp(\cdot)$ and $\rho. = erf(\cdot)$). Standard errors are in brackets, and $***$, $**$, and $*$ indicate the significance levels of 1%, 5%, and 10%, respectively.

e further decompose cash outflow variables into lagged component and annual change component ($\Delta Capex$ and $\Delta Acqui$), and both component negatively affect cash holding target as in Table (2.16).

Table 2.7: Robustness for (S, s) Model Estimates: Net Cash Injection

	β		θ_u		θ_l	
	Parameter	M.E.	Parameter	M.E.	Parameter	M.E.
Inter	0.255***	-	-0.354***	-	-0.906***	-
	(0.008)		(0.047)		(0.033)	
Chfl	-0.042***	-0.042	-0.718***	-0.368	-0.351***	0.084
	(0.007)		(0.057)		(0.025)	
qrate	0.130**	0.130	-0.472	-0.084	0.304	0.021
	(0.060)		(0.347)		(0.229)	
dGDP	0.108	0.108	-0.306	-0.031	0.371	-0.024
	(0.101)		(0.571)		(0.378)	
RESS	0.005	0.005	-0.007	0.002	0.011	0.001
	(0.005)		(0.028)		(0.020)	
Indstd	0.787***	0.787	-0.256	0.671	0.057	0.766
	(0.049)		(0.272)		(0.188)	
Mtb	0.019***	0.019	0.003	0.020	0.019***	0.012
	(0.001)		(0.005)		(0.002)	
Size	-0.006***	-0.006	-0.037***	-0.022	-0.012***	-0.001
	(0.001)		(0.003)		(0.003)	
Nwc	-0.237***	-0.237	-0.282***	-0.364	-0.270***	-0.140
	(0.006)		(0.035)		(0.024)	
Rd	0.082***	0.082	0.170***	0.159	0.036***	0.070
	(0.002)		(0.015)		(0.006)	
Lev	-0.359***	-0.359	-0.560***	-0.613	-0.360***	-0.230
	(0.006)		(0.036)		(0.025)	
Dvc	-0.513***	-0.513	-2.863***	-1.813	0.794***	-0.797
	(0.043)		(0.319)		(0.259)	
F capex	0.225***	0.225	-0.520***	-0.011	0.698***	-0.025
	(0.018)		(0.158)		(0.064)	
F acqui	0.344***	0.344	0.056	0.370	1.226***	-0.095
	(0.017)		(0.162)		(0.051)	
Capex	-0.474***	-0.474	-0.318**	-0.619	-0.672***	-0.234
	(0.016)		(0.145)		(0.069)	
Acqui	-0.312***	-0.312	-0.314**	-0.455	-0.589***	-0.101
	(0.013)		(0.152)		(0.085)	
D90	-0.025***	-0.025	-0.010	0.000	-0.008	0.000
	(0.004)		(0.021)		(0.013)	
D00	-0.032***	-0.032	0.001	0.000	-0.034*	0.000
	(0.005)		(0.027)		(0.018)	
D10	-0.010	-0.010	0.016	0.000	0.004	0.000
	(0.007)		(0.037)		(0.025)	
	Parameter		Value			
σ_*	-1.777***		0.169			

	(0.001)	
σ_u	-2.148***	0.117
	(0.045)	
σ_l	-1.321***	0.267
	(0.037)	
ρ_{*u}	0.588***	0.595
	(0.067)	
ρ_{*l}	0.567***	0.577
	(0.019)	

Note: This robustness test alternates the definition of cash refinancing from fcf as in main model (Table 2.5) to net cash injection. Net cash injection is defined as the sum of debt issuance and equity issuance minuses the sum of debt reduction, equity repurchase, and common dividend payment. There are 79,574 observations from 9883 unique firms. The cut-off level is still 10%. This table reports the estimation of $(S,\ s)$ model previously specified:

$$\begin{aligned} C_u &= X\beta + \varepsilon^* + e^{X\theta_u + \varepsilon_u}, \\ C^* &= X\beta + \varepsilon^*, \\ C_l &= X\beta + \varepsilon^* - e^{X\theta_l + \varepsilon_l}. \end{aligned} \tag{2.5}$$

The coefficients β, θ_u, and θ_l estimated in table corresponds to the ones in this empirical model. The marginal effect is formed as the exact amount of effect covariates demonstrated on thresholds. Besides, our maximum likelihood method allows the flexibility to measure the standard deviation (σ_*, σ_u, and σ_l) of error terms (ε^*, ε_u, and ε_l). Moreover, parameter ρ_{*u} measures the correlation between ε^* and ε_u, and ρ_{*l} measures the correlation between ε^* and ε_l. Our estimation has transformed the standard deviation and correlation variables to ensure they suit constraints (standard deviation are positive, and correlation variables are between -1 and 1). Numbers in the parameter column report the estimation of the transformed variables, and numbers in the value column present the value of the variables ($\sigma. = exp(\cdot)$ and $\rho. = erf(\cdot)$). Standard errors are in brackets, and $***$, $**$, and $*$ indicate the significance levels of 1%, 5%, and 10%, respectively.

Table 2.8: Robustness for $(S,\ s)$ Model Estimates: Net Financing and Investing Cash Flows

	β		θ_u		θ_l	
	Parameter	M.E.	Parameter	M.E.	Parameter	M.E.
Inter	0.223***	-	-0.614***	-	-0.929***	-
	(0.008)		(0.043)		(0.062)	
Chfl	-0.098***	-0.098	-0.450***	-0.276	-0.296***	0.019
	(0.006)		(0.047)		(0.044)	
qrate	0.098	0.098	-2.540***	-0.907	1.204**	-0.378
	(0.080)		(0.385)		(0.584)	
dGDP	0.213*	0.213	-0.443	0.038	1.874**	-0.528
	(0.109)		(0.522)		(0.823)	
RESS	0.005	0.005	-0.027	-0.005	0.138***	-0.053
	(0.005)		(0.022)		(0.035)	
Indstd	1.135***	1.135	-0.616***	0.891	-0.601*	1.372
	(0.040)		(0.190)		(0.309)	
Mtb	0.018***	0.018	0.025***	0.028	-0.005	0.020
	(0.001)		(0.004)		(0.004)	
Size	-0.010***	-0.010	-0.020***	-0.018	0.004	-0.011
	(0.001)		(0.003)		(0.004)	
Nwc	-0.322***	-0.322	0.003	-0.321	-0.270***	-0.216
	(0.006)		(0.027)		(0.037)	
Rd	0.070***	0.070	0.051***	0.090	-0.012	0.075
	(0.001)		(0.016)		(0.010)	
Lev	-0.433***	-0.433	-0.029	-0.444	-0.080*	-0.402
	(0.006)		(0.029)		(0.043)	
Dvc	-0.384***	-0.384	-0.918***	-0.747	2.483***	-1.366
	(0.041)		(0.274)		(0.661)	
F capex	-0.045**	-0.045	-0.967***	-0.427	0.421***	-0.211
	(0.019)		(0.114)		(0.157)	
F acqui	-0.019	-0.019	-0.959***	-0.398	0.569***	-0.244
	(0.015)		(0.082)		(0.149)	
Capex	-0.476***	-0.476	0.018	-0.469	-0.835***	-0.146
	(0.018)		(0.111)		(0.151)	
Acqui	-0.300***	-0.300	0.355***	-0.160	-0.475***	-0.112
	(0.016)		(0.072)		(0.142)	
D90	0.013***	0.013	-0.185***	0.000	0.109***	0.000
	(0.004)		(0.019)		(0.032)	
D00	0.014***	0.014	-0.209***	0.000	0.101**	0.000
	(0.005)		(0.024)		(0.040)	
D10	0.034***	0.034	-0.283***	0.000	0.253***	0.000
	(0.007)		(0.033)		(0.054)	
	Parameter		Value			

σ_*	-1.713***	0.180
	(0.001)	
σ_u	-2.330***	0.097
	(0.044)	
σ_l	-2.110***	0.121
	(0.141)	
ρ_{*u}	-0.677***	-0.662
	(0.041)	
ρ_{*l}	-0.447***	-0.472
	(0.056)	

Note: This robustness check tests the suspicion that investment activity also serves as a channel of cash refinancing, and hence defines cash refinancing as the sum of fcf and icf. The cut-off level is still 10%. There are 79,574 observations from 9883 unique firms. This table reports the estimation of $(S,\ s)$ model previously specified:

$$\begin{aligned} C_u &= X\beta + \varepsilon^* + e^{X\theta_u + \varepsilon_u}, \\ C^* &= X\beta + \varepsilon^*, \\ C_l &= X\beta + \varepsilon^* - e^{X\theta_l + \varepsilon_l}. \end{aligned} \tag{2.6}$$

The coefficients β, θ_u, and θ_l estimated in table corresponds to the ones in this empirical model. The marginal effect is formed as the exact amount of effect covariates demonstrated on thresholds. Besides, our maximum likelihood method allows the flexibility to measure the standard deviation (σ_*, σ_u, and σ_l) of error terms (ε^*, ε_u, and ε_l). Moreover, parameter ρ_{*u} measures the correlation between ε^* and ε_u, and ρ_{*l} measures the correlation between ε^* and ε_l. Our estimation has transformed the standard deviation and correlation variables to ensure they suit constraints (standard deviation are positive, and correlation variables are between -1 and 1). Numbers in the parameter column report the estimation of the transformed variables, and numbers in the value column present the value of the variables ($\sigma. = exp(\cdot)$ and $\rho. = erf(\cdot)$). Standard errors are in brackets, and $***$, $**$, and $*$ indicate the significance levels of 1%, 5%, and 10%, respectively.

Table 2.9: Robustness for $(S,\ s)$ Model Estimates: Net Financing and Investing Cash Flows without *Capex* and *Acqui*

	β		θ_u		θ_l	
	Parameter	M.E.	Parameter	M.E.	Parameter	M.E.
Inter	0.299***	-	-0.600***	-	-0.497***	-
	(0.010)		(0.054)		(0.037)	
Chfl	-0.060***	-0.060	-0.631***	-0.300	-0.228***	0.041
	(0.006)		(0.059)		(0.032)	
qrate	-0.099	-0.099	-1.847***	-0.801	-1.649***	0.630
	(0.096)		(0.501)		(0.340)	
dGDP	0.079	0.079	-0.661	-0.173	-0.997**	0.520
	(0.132)		(0.690)		(0.472)	
RESS	-0.008	-0.008	-0.031	-0.020	-0.049**	0.013
	(0.006)		(0.029)		(0.021)	
Indstd	0.717***	0.717	-0.098	0.680	-1.768***	1.498
	(0.050)		(0.265)		(0.177)	
Mtb	0.012***	0.012	0.013***	0.017	-0.003	0.014
	(0.001)		(0.005)		(0.003)	
Size	0.000	0.000	-0.033***	-0.013	0.008***	-0.003
	(0.001)		(0.003)		(0.002)	
Nwc	-0.306***	-0.306	-0.050	-0.325	-0.302***	-0.172
	(0.007)		(0.035)		(0.024)	
Rd	0.072***	0.072	0.105***	0.113	0.004	0.071
	(0.001)		(0.020)		(0.009)	
Lev	-0.434***	-0.434	-0.315***	-0.554	-0.380***	-0.266
	(0.007)		(0.038)		(0.025)	
Dvc	-0.714***	-0.714	-0.979***	-1.086	-0.033	-0.699
	(0.048)		(0.295)		(0.218)	
F capex	0.231***	0.231	-0.044	0.214	0.562***	-0.017
	(0.021)		(0.195)		(0.079)	
F acqui	0.361***	0.361	0.478***	0.542	1.029***	-0.094
	(0.018)		(0.182)		(0.054)	
Capex	-0.592***	-0.592	-0.110	-0.633	-0.623***	-0.316
	(0.019)		(0.176)		(0.085)	
Acqui	-0.425***	-0.425	-0.242	-0.517	-0.481***	-0.212
	(0.014)		(0.157)		(0.091)	
D90	-0.013**	-0.013	-0.042	0.000	-0.075***	0.000
	(0.005)		(0.027)		(0.017)	
D00	-0.016**	-0.016	-0.080**	0.000	-0.129***	0.000
	(0.006)		(0.034)		(0.022)	
D10	-0.015*	-0.015	-0.069	0.000	-0.167***	0.000
	(0.009)		(0.045)		(0.030)	
	Parameter		Value			

σ_*	-1.679***	0.187
	(0.001)	
σ_u	-2.470***	0.085
	(0.073)	
σ_l	-2.335***	0.097
	(0.127)	
ρ_{*u}	-0.230***	-0.255
	(0.050)	
ρ_{*l}	0.679***	0.663
	(0.056)	

Note: Based on the results in last robustness check (Table 2.8), this test further considers the deliberate component in icf. More precise, since *Capex* and *Acqui* are strategic decisions, they are previously determined components in icf. Hence, it is not possible to liquidate or invest in these terms to adjust cash holdings. This test exclude their effect by defining cash refinancing and the sum of fcf, icf, *Capex*, and *Acqui*. The icf is the net amount after deducting *Capex* and *Acqui*, and hence the sum of three terms is the part of investing cash flow which can be adjusted for cash holding purpose. The cut-off level is still 10%. There are 79,574 observations from 9883 unique firms. This table reports the estimation of $(S,\ s)$ model previously specified:

$$\begin{aligned} C_u &= X\beta + \varepsilon^* + e^{X\theta_u + \varepsilon_u}, \\ C^* &= X\beta + \varepsilon^*, \\ C_l &= X\beta + \varepsilon^* - e^{X\theta_l + \varepsilon_l}. \end{aligned} \tag{2.7}$$

The coefficients β, θ_u, and θ_l estimated in table corresponds to the ones in this empirical model. The marginal effect is formed as the exact amount of effect covariates demonstrated on thresholds. Besides, our maximum likelihood method allows the flexibility to measure the standard deviation (σ_*, σ_u, and σ_l) of error terms (ε^*, ε_u, and ε_l). Moreover, parameter ρ_{*u} measures the correlation between ε^* and ε_u, and ρ_{*l} measures the correlation between ε^* and ε_l. Our estimation has transformed the standard deviation and correlation variables to ensure they suit constraints (standard deviation are positive, and correlation variables are between -1 and 1). Numbers in the parameter column report the estimation of the transformed variables, and numbers in the value column present the value of the variables ($\sigma. = exp(\cdot)$ and $\rho. = erf(\cdot)$). Standard errors are in brackets, and $***$, $**$, and $*$ indicate the significance levels of 1%, 5%, and 10%, respectively.

TABLE 2.10: Robustness for $(S,\ s)$ Model Estimates: 5% Cut-off

	β		θ_u		θ_l	
	Parameter	M.E.	Parameter	M.E.	Parameter	M.E.
Inter	0.200***	-	-0.201***	-	-0.213***	-
	(0.008)		(0.056)		(0.048)	
Chfl	-0.052***	-0.052	-0.501***	-0.260	-0.470***	0.141
	(0.006)		(0.060)		(0.031)	
qrate	0.178**	0.178	-1.894***	-0.606	-1.527***	0.805
	(0.077)		(0.546)		(0.420)	
dGDP	0.066	0.066	0.927	0.450	-3.126***	1.351
	(0.106)		(0.711)		(0.572)	
RESS	0.006	0.006	0.015	0.012	-0.086***	0.040
	(0.005)		(0.030)		(0.025)	
Indstd	0.798***	0.798	-1.773***	0.064	-2.477***	1.816
	(0.039)		(0.269)		(0.217)	
Mtb	0.023***	0.023	0.001	0.023	-0.006	0.025
	(0.001)		(0.006)		(0.003)	
Size	-0.006***	-0.006	-0.070***	-0.035	-0.032***	0.007
	(0.001)		(0.003)		(0.003)	
Nwc	-0.242***	-0.242	-0.345***	-0.385	-0.359***	-0.094
	(0.005)		(0.037)		(0.029)	
Rd	0.082***	0.082	0.366***	0.234	0.031***	0.069
	(0.001)		(0.025)		(0.007)	
Lev	-0.368***	-0.368	-0.292***	-0.489	-0.581***	-0.130
	(0.005)		(0.038)		(0.033)	
Dvc	-0.338***	-0.338	-0.919**	-0.718	1.006***	-0.751
	(0.040)		(0.408)		(0.380)	
F capex	0.142***	0.142	-0.666***	-0.134	-0.101	0.183
	(0.019)		(0.195)		(0.102)	
F acqui	0.162***	0.162	0.637***	0.425	0.957***	-0.232
	(0.015)		(0.172)		(0.064)	
Capex	-0.430***	-0.430	-0.431**	-0.608	-0.134	-0.375
	(0.017)		(0.179)		(0.101)	
Acqui	-0.220***	-0.220	-0.311*	-0.349	-0.406***	-0.053
	(0.013)		(0.166)		(0.117)	
D90	0.000	0.000	-0.268***	0.000	-0.177***	0.000
	(0.004)		(0.027)		(0.021)	
D00	-0.002	-0.002	-0.197***	0.000	-0.242***	0.000
	(0.005)		(0.035)		(0.027)	
D10	0.020***	0.020	-0.283***	0.000	-0.293***	0.000
	(0.007)		(0.048)		(0.038)	
	Parameter		Value			
σ_*	-1.684***		0.186			

	(0.001)	
σ_u	-1.037***	0.354
	(0.011)	
σ_l	-1.600***	0.202
	(0.047)	
ρ_{*u}	-0.223***	-0.247
	(0.018)	
ρ_{*l}	0.379***	0.408
	(0.020)	

Note: This robustness estimation tests whether or not the findings of main model in Table 2.5 are due to a selective cut-off level. This test keeps the refinancing definition of fcf as in the main model, but reduces the cut-off level to 5%. There are 79,574 observations from 9883 unique firms. This table reports the estimation of $(S,\ s)$ model previously specified:

$$\begin{aligned} C_u &= X\beta + \varepsilon^* + e^{X\theta_u + \varepsilon_u}, \\ C^* &= X\beta + \varepsilon^*, \\ C_l &= X\beta + \varepsilon^* - e^{X\theta_l + \varepsilon_l}. \end{aligned} \tag{2.8}$$

The coefficients β, θ_u, and θ_l estimated in table corresponds to the ones in this empirical model. The marginal effect is formed as the exact amount of effect covariates demonstrated on thresholds. Besides, our maximum likelihood method allows the flexibility to measure the standard deviation (σ_*, σ_u, and σ_l) of error terms (ε^*, ε_u, and ε_l). Moreover, parameter ρ_{*u} measures the correlation between ε^* and ε_u, and ρ_{*l} measures the correlation between ε^* and ε_l. Our estimation has transformed the standard deviation and correlation variables to ensure they suit constraints (standard deviation are positive, and correlation variables are between -1 and 1). Numbers in the parameter column report the estimation of the transformed variables, and numbers in the value column present the value of the variables ($\sigma. = exp(\cdot)$ and $\rho. = erf(\cdot)$). Standard errors are in brackets, and $***$, $**$, and $*$ indicate the significance levels of 1%, 5%, and 10%, respectively.

Table 2.11: Robustness for $(S,\ s)$ Model Estimates: 15% Cut-off

	β		θ_u		θ_l	
	Parameter	M.E.	Parameter	M.E.	Parameter	M.E.
Inter	0.202***	-	-0.337***	-	-0.941***	-
	(0.013)		(0.065)		(0.041)	
Chfl	-0.040***	-0.040	-0.607***	-0.322	-0.297***	0.081
	(0.008)		(0.063)		(0.026)	
qrate	0.218*	0.218	-0.705	-0.109	1.169***	-0.261
	(0.124)		(0.626)		(0.377)	
dGDP	0.070	0.070	-1.146	-0.461	0.326	-0.064
	(0.173)		(0.844)		(0.524)	
RESS	0.003	0.003	-0.063*	-0.025	0.046**	-0.016
	(0.008)		(0.036)		(0.023)	
Indstd	1.134***	1.134	0.020	1.143	0.155	1.070
	(0.064)		(0.319)		(0.197)	
Mtb	0.018***	0.018	-0.005	0.016	0.012***	0.013
	(0.001)		(0.006)		(0.002)	
Size	0.004***	0.004	-0.040***	-0.015	0.015***	-0.002
	(0.001)		(0.004)		(0.003)	
Nwc	-0.260***	-0.260	-0.232***	-0.368	-0.232***	-0.165
	(0.008)		(0.042)		(0.026)	
Rd	0.075***	0.075	0.130***	0.135	0.026***	0.064
	(0.002)		(0.023)		(0.005)	
Lev	-0.402***	-0.402	-0.441***	-0.607	-0.370***	-0.250
	(0.008)		(0.042)		(0.027)	
Dvc	-0.750***	-0.750	-1.373***	-1.386	-0.015	-0.744
	(0.057)		(0.315)		(0.237)	
F capex	0.315***	0.315	-0.806***	-0.058	0.886***	-0.048
	(0.025)		(0.195)		(0.071)	
F acqui	0.410***	0.410	0.079	0.447	1.143***	-0.058
	(0.021)		(0.200)		(0.051)	
Capex	-0.573***	-0.573	0.190	-0.485	-0.847***	-0.227
	(0.022)		(0.177)		(0.076)	
Acqui	-0.415***	-0.415	0.027	-0.402	-0.717***	-0.121
	(0.016)		(0.163)		(0.079)	
D90	0.002	0.002	-0.011	0.000	0.015	0.000
	(0.007)		(0.032)		(0.020)	
D00	-0.013	-0.013	0.018	0.000	-0.013	0.000
	(0.008)		(0.040)		(0.025)	
D10	0.012	0.012	0.021	0.000	0.053	0.000
	(0.011)		(0.056)		(0.034)	
	Parameter		Value			
σ_*	-1.677***		0.187			

	(0.001)	
σ_u	-1.166***	0.311
	(0.034)	
σ_l	-1.314***	0.269
	(0.033)	
ρ_{*u}	-0.192***	-0.215
	(0.031)	
ρ_{*l}	0.633***	0.629
	(0.017)	

Note: This robustness estimation tests whether or not the findings of main model in Table 2.5 are due to a selective cut-off level. This test keeps the refinancing definition of fcf as in the main model, but increases the cut-off level to 15%. There are 79,574 observations from 9883 unique firms. This table reports the estimation of $(S,\ s)$ model previously specified:

$$\begin{aligned} C_u &= X\beta + \varepsilon^* + e^{X\theta_u + \varepsilon_u}, \\ C^* &= X\beta + \varepsilon^*, \\ C_l &= X\beta + \varepsilon^* - e^{X\theta_l + \varepsilon_l}. \end{aligned} \tag{2.9}$$

The coefficients β, θ_u, and θ_l estimated in table corresponds to the ones in this empirical model. The marginal effect is formed as the exact amount of effect covariates demonstrated on thresholds. Besides, our maximum likelihood method allows the flexibility to measure the standard deviation (σ_*, σ_u, and σ_l) of error terms (ε^*, ε_u, and ε_l). Moreover, parameter ρ_{*u} measures the correlation between ε^* and ε_u, and ρ_{*l} measures the correlation between ε^* and ε_l. Our estimation has transformed the standard deviation and correlation variables to ensure they suit constraints (standard deviation are positive, and correlation variables are between -1 and 1). Numbers in the parameter column report the estimation of the transformed variables, and numbers in the value column present the value of the variables ($\sigma. = exp(\cdot)$ and $\rho. = erf(\cdot)$). Standard errors are in brackets, and $***$, $**$, and $*$ indicate the significance levels of 1%, 5%, and 10%, respectively.

Table 2.12: Robustness for $(S,\ s)$ Model Estimates: GARCH Indstd

	OLS	β		θ_u		θ_l	
		Parameter	M.E.	Parameter	M.E.	Parameter	M.E.
Inter	0.348***	0.134***	-	0.140***	-	-0.402***	-
	(0.005)	(0.034)		(0.029)		(0.035)	
Chfl	-0.274***	-0.421***	-0.421	0.105***	-0.344	-0.005	-0.418
	(0.004)	(0.021)		(0.034)		(0.023)	
qrate	-0.130***	-0.054	-0.054	-0.844**	-0.671	0.061	-0.090
	(0.049)	(0.290)		(0.345)		(0.287)	
dGDP	0.192***	0.031	0.031	2.173***	1.618	0.529	-0.288
	(0.068)	(0.402)		(0.504)		(0.400)	
RESS	0.003	-0.027	-0.027	0.087***	0.039	-0.004	-0.025
	(0.003)	(0.017)		(0.021)		(0.017)	
$\text{Indstd}_{\text{GARCH}}$	0.045***	0.179***	0.179	-0.601***	-0.260	-0.015	0.188
	(0.005)	(0.028)		(0.038)		(0.029)	
Mtb	0.013***	0.015***	0.015	-0.026***	-0.004	-0.036***	0.037
	(0.000)	(0.001)		(0.003)		(0.002)	
Size	-0.008***	0.004**	0.004	-0.035***	-0.021	-0.008***	0.009
	(0.000)	(0.002)		(0.001)		(0.003)	
Nwc	-0.304***	-0.099***	-0.099	-0.369***	-0.368	-0.051*	-0.068
	(0.003)	(0.019)		(0.017)		(0.027)	
Rd	0.000***	0.000	0.000	0.046***	0.034	-0.007***	0.004
	(0.000)	(0.000)		(0.004)		(0.000)	
Lev	-0.463***	-0.366***	-0.366	-0.519***	-0.745	-0.143***	-0.280
	(0.003)	(0.018)		(0.026)		(0.020)	
Dvc	0.063***	-0.213***	-0.213	-0.605***	-0.655	0.436***	-0.476
	(0.016)	(0.067)		(0.160)		(0.096)	
F capex	0.023**	0.363***	0.363	-0.287***	0.154	0.231***	0.224
	(0.011)	(0.058)		(0.098)		(0.066)	
F acqui	0.054***	0.857***	0.857	-0.301**	0.637	0.714***	0.426
	(0.008)	(0.089)		(0.121)		(0.104)	
Capex	-0.445***	-0.167***	-0.167	-0.374***	-0.440	-0.201***	-0.046
	(0.011)	(0.050)		(0.086)		(0.063)	
Acqui	-0.185***	-0.301***	-0.301	0.073	-0.248	-0.121***	-0.228
	(0.008)	(0.037)		(0.070)		(0.046)	
D90	-0.007***	0.116***	0.116	-0.268***	0.000	0.006	0.000
	(0.003)	(0.017)		(0.013)		(0.020)	
D00	0.003	0.069***	0.069	-0.221***	0.000	-0.040*	0.000
	(0.003)	(0.020)		(0.019)		(0.024)	
D10	0.007	0.077***	0.077	-0.201***	0.000	-0.006	0.000
	(0.004)	(0.028)		(0.029)		(0.030)	
		Parameter		Value			
σ_*		-0.565***		0.569			

	(0.008)	
σ_u	-0.943***	0.390
	(0.026)	
σ_l	-0.893***	0.410
	(0.049)	
ρ_{*u}	-1.468***	-0.962
	(0.021)	
ρ_{*l}	1.648***	0.980
	(0.034)	

Note: This robustness estimation tests whether or not the argument of industry volatility from main model in Table 2.5 is due to a noisy proxy (industry average standard deviation of cash flow). The $Indstd_{GARCH}$ variable in this estimation is the predicted industry cash flow volatility by GARCH(1,1) model. There are 79,360 observations from 9865 unique firms. The fist column presents the OLS results based on the whole sample, and the rest columns report the estimation of $(S,\ s)$ model previously specified:

$$\begin{aligned}
&C_u = X\beta + \varepsilon^* + e^{X\theta_u + \varepsilon_u}, \\
&C^* = X\beta + \varepsilon^*, \\
&C_l = X\beta + \varepsilon^* - e^{X\theta_l + \varepsilon_l}.
\end{aligned} \tag{2.10}$$

The coefficients β, θ_u, and θ_l estimated in table corresponds to the ones in this empirical model. The marginal effect is formed as the exact amount of effect covariates demonstrated on thresholds. Besides, our maximum likelihood method allows the flexibility to measure the standard deviation (σ_*, σ_u, and σ_l) of error terms (ε^*, ε_u, and ε_l). Moreover, parameter ρ_{*u} measures the correlation between ε^* and ε_u, and ρ_{*l} measures the correlation between ε^* and ε_l. Our estimation has transformed the standard deviation and correlation variables to ensure they suit constraints (standard deviation are positive, and correlation variables are between -1 and 1). Numbers in the parameter column report the estimation of the transformed variables, and numbers in the value column present the value of the variables ($\sigma. = exp(\cdot)$ and $\rho. = erf(\cdot)$). Standard errors are in brackets, and $***$, $**$, and $*$ indicate the significance levels of 1%, 5%, and 10%, respectively.

Table 2.13: Robustness for $(S,\ s)$ Model Estimates: Market-based Cash Ratio

	β		θ_u		θ_l	
	Parameter	M.E.	Parameter	M.E.	Parameter	M.E.
Inter	0.132***	-	-0.247***	-	-1.370***	-
	(0.014)		(0.083)		(0.036)	
Chfl	-0.143***	-0.143	-1.018***	-0.542	-0.115***	-0.112
	(0.010)		(0.092)		(0.027)	
qrate	0.008	0.008	-0.902	-0.346	0.509	-0.127
	(0.165)		(0.791)		(0.368)	
dGDP	-0.144	-0.144	0.488	0.048	0.672	-0.323
	(0.204)		(1.563)		(0.478)	
RESS	-0.011	-0.011	0.049	0.008	0.021	-0.017
	(0.008)		(0.040)		(0.020)	
Indstd	0.421***	0.421	0.156	0.482	-0.534***	0.563
	(0.080)		(0.225)		(0.184)	
Mtb	-0.004***	-0.004	-0.162***	-0.067	-0.007***	-0.002
	(0.001)		(0.019)		(0.002)	
Size	-0.001	-0.001	-0.035***	-0.014	0.007***	-0.003
	(0.001)		(0.012)		(0.002)	
Nwc	-0.044***	-0.044	-0.361***	-0.186	0.127***	-0.078
	(0.009)		(0.075)		(0.023)	
Rd	0.000	0.000	0.065***	0.026	-0.007***	0.002
	(0.000)		(0.012)		(0.000)	
Lev	-0.182***	-0.182	-0.863***	-0.520	-0.073**	-0.162
	(0.018)		(0.033)		(0.037)	
Dvc	-0.053*	-0.053	-1.079***	-0.476	0.166	-0.097
	(0.032)		(0.314)		(0.106)	
F capex	0.263***	0.263	-0.080	0.232	0.750***	0.063
	(0.026)		(0.848)		(0.062)	
F acqui	0.587***	0.587	-0.370*	0.442	1.384***	0.219
	(0.020)		(0.218)		(0.038)	
Capex	-0.118***	-0.118	-0.276	-0.227	-0.187***	-0.068
	(0.022)		(0.697)		(0.063)	
Acqui	-0.153***	-0.153	-0.135	-0.206	-0.273***	-0.081
	(0.019)		(0.116)		(0.057)	
D90	-0.009	-0.009	0.020	0.000	0.005	0.000
	(0.007)		(0.033)		(0.017)	
D00	-0.014	-0.014	0.015	0.000	-0.003	0.000
	(0.009)		(0.031)		(0.022)	
D10	-0.003	-0.003	0.038	0.000	0.045	0.000
	(0.014)		(0.041)		(0.031)	
	Parameter		Value			
σ_*	-1.306***		0.271			

	(0.000)	
σ_u	-0.908***	0.403
	(0.300)	
σ_l	-0.846***	0.429
	(0.007)	
ρ_{*u}	-0.846***	-0.769
	(0.123)	
ρ_{*l}	1.423***	0.956
	(0.005)	

Note: This robustness estimation tests the definition of cash, the dependent variable in estimation. Our main model (Table 2.5) and all other tests define cash as cash and marketable securities divided by book value of total asset. This robustness test define cash as cash and marketable securities divided by market value of total asset. There are 79,574 observations from 9883 unique firms. This table reports the estimation of $(S,\ s)$ model previously specified:

$$\begin{aligned} C_u &= X\beta + \varepsilon^* + e^{X\theta_u+\varepsilon_u}, \\ C^* &= X\beta + \varepsilon^*, \\ C_l &= X\beta + \varepsilon^* - e^{X\theta_l+\varepsilon_l}. \end{aligned} \tag{2.11}$$

The coefficients β, θ_u, and θ_l estimated in table corresponds to the ones in this empirical model. The marginal effect is formed as the exact amount of effect covariates demonstrated on thresholds. Besides, our maximum likelihood method allows the flexibility to measure the standard deviation (σ_*, σ_u, and σ_l) of error terms (ε^*, ε_u, and ε_l). Moreover, parameter ρ_{*u} measures the correlation between ε^* and ε_u, and ρ_{*l} measures the correlation between ε^* and ε_l. Our estimation has transformed the standard deviation and correlation variables to ensure they suit constraints (standard deviation are positive, and correlation variables are between -1 and 1). Numbers in the parameter column report the estimation of the transformed variables, and numbers in the value column present the value of the variables ($\sigma. = exp(\cdot)$ and $\rho. = erf(\cdot)$). Standard errors are in brackets, and $***$, $**$, and $*$ indicate the significance levels of 1%, 5%, and 10%, respectively.

Table 2.14: Robustness for $(S,\ s)$ Model Estimates: Missing RD

	β		θ_u		θ_l	
	Parameter	M.E.	Parameter	M.E.	Parameter	M.E.
Inter	0.297***	-	-0.306***	-	0.049	-
	(0.015)		(0.053)		(0.132)	
Chfl	-0.248***	-0.248	-0.552***	-0.481	0.124*	-0.328
	(0.010)		(0.052)		(0.065)	
qrate	0.168	0.168	-0.389	0.004	-0.498	0.489
	(0.138)		(0.469)		(0.385)	
dGDP	0.076	0.076	-0.608	-0.181	-2.033***	1.389
	(0.194)		(0.671)		(0.645)	
RESS	0.001	0.001	-0.094***	-0.037	-0.095***	0.059
	(0.009)		(0.030)		(0.035)	
Indstd	0.630***	0.630	-0.053	0.608	-1.254***	1.440
	(0.075)		(0.258)		(0.235)	
Mtb	0.008***	0.008	-0.037***	-0.008	-0.053***	0.042
	(0.001)		(0.005)		(0.006)	
Size	0.001	0.001	-0.084***	-0.034	-0.018***	0.013
	(0.001)		(0.004)		(0.003)	
Nwc	-0.295***	-0.295	-0.111***	-0.342	-0.438***	-0.012
	(0.009)		(0.032)		(0.043)	
Rd	0.000*	0.000	0.051***	0.021	-0.007***	0.005
	(0.000)		(0.006)		(0.000)	
Drd	-0.077***	0.077	-0.061***	0.090	-0.140***	0.276
	(0.004)		(0.013)		(0.015)	
Lev	-0.408***	-0.408	-0.264***	-0.520	-0.473***	-0.103
	(0.010)		(0.035)		(0.029)	
Dvc	0.097***	0.097	-0.838***	-0.257	0.856***	-0.456
	(0.031)		(0.284)		(0.175)	
F capex	0.195***	0.195	-0.069	0.166	0.026	0.178
	(0.026)		(0.140)		(0.125)	
F acqui	0.201***	0.201	0.766***	0.525	0.255	0.037
	(0.036)		(0.142)		(0.209)	
Capex	-0.350***	-0.350	0.532***	-0.126	-0.845***	0.195
	(0.028)		(0.128)		(0.147)	
Acqui	-0.253***	-0.253	-0.311**	-0.384	-0.232***	-0.103
	(0.017)		(0.129)		(0.080)	
D90	0.008	0.008	-0.038	0.000	-0.258***	0.000
	(0.007)		(0.025)		(0.035)	
D00	0.005	0.005	0.094***	0.000	-0.288***	0.000
	(0.009)		(0.031)		(0.041)	
D10	0.021	0.021	0.161***	0.000	-0.261***	0.000
	(0.013)		(0.043)		(0.048)	

	Parameter	Value
σ_*	-1.359***	0.257
	(0.004)	
σ_u	-1.472***	0.229
	(0.032)	
σ_l	-1.489***	0.226
	(0.061)	
ρ_{*u}	-0.658***	-0.648
	(0.026)	
ρ_{*l}	0.857***	0.774
	(0.085)	

Note: This robustness estimation add a dummy variable, Drd, to indicate missing RD observations to release the concern of the RD setting of main model in Table 2.5, which replace RD value as 0 if missing. There are 79,574 observations from 9883 unique firms. This table reports the estimation of $(S,\ s)$ model previously specified:

$$\begin{aligned} C_u &= X\beta + \varepsilon^* + e^{X\theta_u + \varepsilon_u}, \\ C^* &= X\beta + \varepsilon^*, \\ C_l &= X\beta + \varepsilon^* - e^{X\theta_l + \varepsilon_l}. \end{aligned} \tag{2.12}$$

The coefficients β, θ_u, and θ_l estimated in table corresponds to the ones in this empirical model. The marginal effect is formed as the exact amount of effect covariates demonstrated on thresholds. Besides, our maximum likelihood method allows the flexibility to measure the standard deviation (σ_*, σ_u, and σ_l) of error terms (ε^*, ε_u, and ε_l). Moreover, parameter ρ_{*u} measures the correlation between ε^* and ε_u, and ρ_{*l} measures the correlation between ε^* and ε_l. Our estimation has transformed the standard deviation and correlation variables to ensure they suit constraints (standard deviation are positive, and correlation variables are between -1 and 1). Numbers in the parameter column report the estimation of the transformed variables, and numbers in the value column present the value of the variables ($\sigma. = exp(\cdot)$ and $\rho. = erf(\cdot)$). Standard errors are in brackets, and $***$, $**$, and $*$ indicate the significance levels of 1%, 5%, and 10%, respectively.

Table 2.15: Robustness for $(S,\ s)$ Model Estimates: No Future Outflows

	β		θ_u		θ_l	
	Parameter	M.E.	Parameter	M.E.	Parameter	M.E.
Inter	0.229***	-	-0.187***	-	-0.801***	-
	(0.010)		(0.053)		(0.037)	
Chfl	-0.271***	-0.271	-0.490***	-0.495	-0.115***	-0.223
	(0.006)		(0.056)		(0.024)	
qrate	-0.133	-0.133	-0.543	-0.381	-0.013	-0.128
	(0.101)		(0.501)		(0.348)	
dGDP	-0.020	-0.020	-1.786***	-0.838	0.207	-0.106
	(0.141)		(0.689)		(0.495)	
RESS	-0.012**	-0.012	-0.077***	-0.046	0.022	-0.021
	(0.006)		(0.029)		(0.020)	
Indstd	1.609***	1.609	-2.498***	0.466	0.591***	1.363
	(0.050)		(0.258)		(0.180)	
Mtb	0.009***	0.009	0.029***	0.022	-0.012***	0.014
	(0.000)		(0.004)		(0.003)	
Size	0.003***	0.003	-0.051***	-0.021	0.006***	0.000
	(0.001)		(0.003)		(0.002)	
Nwc	-0.264***	-0.264	-0.237***	-0.373	-0.237***	-0.166
	(0.007)		(0.033)		(0.023)	
Rd	0.000***	0.000	0.069***	0.032	-0.002***	0.001
	(0.000)		(0.006)		(0.000)	
Lev	-0.439***	-0.439	-0.432***	-0.637	-0.452***	-0.251
	(0.006)		(0.035)		(0.025)	
Dvc	0.096***	0.096	-0.712***	-0.230	1.173***	-0.393
	(0.022)		(0.273)		(0.141)	
Capex	-0.320***	-0.320	-0.317***	-0.465	-0.275***	-0.205
	(0.013)		(0.122)		(0.054)	
Acqui	-0.227***	-0.227	-0.349**	-0.387	-0.187**	-0.149
	(0.012)		(0.144)		(0.077)	
D90	-0.011**	-0.011	-0.012	0.000	0.000	0.000
	(0.005)		(0.026)		(0.018)	
D00	-0.034***	-0.034	0.041	0.000	-0.050**	0.000
	(0.007)		(0.033)		(0.024)	
D10	-0.016*	-0.016	0.007	0.000	0.005	0.000
	(0.009)		(0.045)		(0.032)	
	Parameter		Value			
σ_*	-1.678***		0.187			
	(0.001)					
σ_u	-2.194***		0.111			
	(0.093)					
σ_l	-2.410***		0.090			

	(0.003)	
ρ_{*u}	0.451***	0.476
	(0.046)	
ρ_{*l}	0.923***	0.808
	(0.014)	

Note: This robustness estimation drops the forward-looking variables F capex and F acqui of main model in Table 2.5. There are 79,574 observations from 9883 unique firms. This table reports the estimation of $(S,\ s)$ model previously specified:

$$\begin{aligned} C_u &= X\beta + \varepsilon^* + e^{X\theta_u + \varepsilon_u}, \\ C^* &= X\beta + \varepsilon^*, \\ C_l &= X\beta + \varepsilon^* - e^{X\theta_l + \varepsilon_l}. \end{aligned} \tag{2.13}$$

The coefficients β, θ_u, and θ_l estimated in table corresponds to the ones in this empirical model. The marginal effect is formed as the exact amount of effect covariates demonstrated on thresholds. Besides, our maximum likelihood method allows the flexibility to measure the standard deviation (σ_*, σ_u, and σ_l) of error terms (ε^*, ε_u, and ε_l). Moreover, parameter ρ_{*u} measures the correlation between ε^* and ε_u, and ρ_{*l} measures the correlation between ε^* and ε_l. Our estimation has transformed the standard deviation and correlation variables to ensure they suit constraints (standard deviation are positive, and correlation variables are between -1 and 1). Numbers in the parameter column report the estimation of the transformed variables, and numbers in the value column present the value of the variables ($\sigma. = exp(\cdot)$ and $\rho. = erf(\cdot)$). Standard errors are in brackets, and $***$, $**$, and $*$ indicate the significance levels of 1%, 5%, and 10%, respectively.

Table 2.16: Robustness for $(S,\ s)$ Model Estimates: Δ Outflows

	β		θ_u		θ_l	
	Parameter	M.E.	Parameter	M.E.	Parameter	M.E.
Inter	0.278***	-	-0.502***	-	-0.450***	-
	(0.011)		(0.052)		(0.040)	
Chfl	-0.276***	-0.276	-0.342***	-0.423	-0.218***	-0.175
	(0.007)		(0.054)		(0.028)	
qrate	-0.436***	-0.436	-0.571	-0.680	-4.627***	1.718
	(0.111)		(0.498)		(0.389)	
dGDP	0.094	0.094	-0.651	-0.185	0.561	-0.168
	(0.153)		(0.686)		(0.523)	
RESS	-0.017***	-0.017	-0.026	-0.028	-0.056**	0.008
	(0.007)		(0.029)		(0.023)	
Indstd	1.250***	1.250	0.510**	1.468	-0.046	1.271
	(0.055)		(0.252)		(0.188)	
Mtb	0.009***	0.009	0.016***	0.016	-0.004	0.011
	(0.000)		(0.004)		(0.003)	
Size	0.002**	0.002	-0.049***	-0.019	0.007***	-0.002
	(0.001)		(0.003)		(0.002)	
Nwc	-0.280***	-0.280	-0.226***	-0.377	-0.217***	-0.179
	(0.007)		(0.033)		(0.024)	
Rd	0.000***	0.000	0.079***	0.034	-0.003***	0.001
	(0.000)		(0.007)		(0.000)	
Lev	-0.450***	-0.450	-0.338***	-0.594	-0.418***	-0.255
	(0.007)		(0.035)		(0.026)	
Dvc	0.048*	0.048	-0.313	-0.087	1.194***	-0.508
	(0.025)		(0.272)		(0.155)	
Lag capex	-0.344***	-0.344	-0.420***	-0.523	-0.205***	-0.248
	(0.016)		(0.120)		(0.068)	
Lag acqui	-0.316***	-0.316	-0.506***	-0.533	-0.146	-0.248
	(0.018)		(0.159)		(0.090)	
ΔCapex	-0.286***	-0.286	0.094	-0.245	-0.277***	-0.156
	(0.019)		(0.137)		(0.079)	
ΔAcqui	-0.204***	-0.204	-0.487***	-0.412	-0.113	-0.151
	(0.013)		(0.139)		(0.078)	
D90	-0.009	-0.009	-0.040	0.000	-0.133***	0.000
	(0.006)		(0.026)		(0.020)	
D00	-0.036***	-0.036	0.006	0.000	-0.268***	0.000
	(0.007)		(0.033)		(0.026)	
D10	-0.028***	-0.028	0.005	0.000	-0.341***	0.000
	(0.010)		(0.045)		(0.035)	
	Parameter		Value			
σ_*	-1.614***		0.199			

	(0.001)	
σ_u	-1.760***	0.172
	(0.063)	
σ_l	-1.836***	0.159
	(0.063)	
ρ_{*u}	-0.131***	-0.147
	(0.030)	
ρ_{*l}	0.739***	0.704
	(0.036)	

Note: This robustness estimation decomposes outflow variables Capex and Acqui of main model in Table 2.5 into a lagged componenet and a change component. The change component, denoted by ΔCapex or ΔAcqui, is defined as the difference between the outflow variable values in current year and in previous year.There are 79,574 observations from 9883 unique firms. This table reports the estimation of $(S,\ s)$ model previously specified:

$$\begin{aligned} C_u &= X\beta + \varepsilon^* + e^{X\theta_u + \varepsilon_u}, \\ C^* &= X\beta + \varepsilon^*, \\ C_l &= X\beta + \varepsilon^* - e^{X\theta_l + \varepsilon_l}. \end{aligned} \tag{2.14}$$

The coefficients β, θ_u, and θ_l estimated in table corresponds to the ones in this empirical model. The marginal effect is formed as the exact amount of effect covariates demonstrated on thresholds. Besides, our maximum likelihood method allows the flexibility to measure the standard deviation (σ_*, σ_u, and σ_l) of error terms (ε^*, ε_u, and ε_l). Moreover, parameter ρ_{*u} measures the correlation between ε^* and ε_u, and ρ_{*l} measures the correlation between ε^* and ε_l. Our estimation has transformed the standard deviation and correlation variables to ensure they suit constraints (standard deviation are positive, and correlation variables are between -1 and 1). Numbers in the parameter column report the estimation of the transformed variables, and numbers in the value column present the value of the variables ($\sigma. = exp(\cdot)$ and $\rho. = erf(\cdot)$). Standard errors are in brackets, and $***$, $**$, and $*$ indicate the significance levels of 1%, 5%, and 10%, respectively.

2.6 Conclusions

In this chapter, we develop and estimate a double-barrier policy model of corporate cash holdings. This dynamic policy incorporates firms' flexibility to decide whether or not respond to shocks in covariates, as well as to the shocks to the cash balance itself. The main difference between our model and traditional studies is the following key assumption. We assume the existence of a lumpy cost of cash refinancing, which makes continuous adjustment suboptimal. Existing empirical work implicitly assumes continuous refinancing pattern by exploiting a static OLS regression. In contrast, refinancing is infrequent which is caused by the assumption of refinancing costs. This means that firms allow cash holding levels to freely evolve within a specific range and refinance only when the cash balance hits an upper or a lower threshold. From this perspective, observed cash levels belong to one of four groups, the upper threshold, the lower threshold, the target, and ordinary observations. The dynamic model we propose clearly categorizes these groups, and the simultaneous estimation takes into account the interaction between these aspects.

This model provides new evidence beyond that possible with more standard, static models. Our results indicate that cash inflows (profitability) reduces corporate demand for cash, which is contrary to the positive correlation between cash holdings and profitability reported in other studies. The observed cash holdings may be affected by the direct injection from cash flow, but the target shows an inverse impact if it is clearly isolated. Accordingly, future outflows (lower profitability) positively affect cash holding demand, since firms prepare funds for future expenditure. Although the sign of the industry risk in our model is the same as in standard regression results, its magnitude is considerably larger. The demand variation is not fully reflected in realized cash holdings due to the infrequency of refinancing choice. The double barrier model further allows us to study refinancing triggers. We find that inflows reduce the width of the inaction range, which implies more frequent refinancing. This is because cash inflows indicates healthy financial position (profitability) as well as credit quality, which reduces the cost of refinancing. Inversely, future outflows reduce refinancing frequency. Industry risk enlarges the lower zone but shrinks the upper zone. This suggests less frequent injection but more often distribution in future.

Appendix 2.1 Theoretical Cash Model Under Double-barrier Framework

This theoretical (S, s) model derives the location of thresholds and targets given known geometric Brownian motion process that cash holdings follow and adjustment costs. The relationships revealed confirm the rationale to apply the (S, s) type empirical model to real data of cash holdings. The model is built based on a single firm situation. To simplify the case, we assume cash is the only asset of the firm, denoted by c_t, and follows geometric Brownian motion, but cash itself does not generate any interest.

$$dc = \mu dt + \sigma dw, \tag{2.15}$$

in which dw follows a Winner process. The drift rate μ determines the trend of one step movement within each time unit, while variance rate σ represents the noise term of the movement. Both μ and σ are known parameters which are exogenously determined. The firm has investors and access to capital markets. The firm will distribute cash to investors when it is too high, and will borrow from capital market if cash level is too low. Since adjustments in either side invoke costs immediate adjustment is not optimal. In other words, the firm will allow cash to freely fluctuate without interruption, and adjust immediately when it reaches either boundary (denoted by C_u for the upper barrier and by C_l for the lower barrier) of this inaction range. Considering that the costs of downward and upward adjustments may be different, we separate the target into two, denoted by C^{++} and C^{+},[15] respectively. The general framework of our model is then explained by three parts: 1) the process of c_t is not interrupted within the area bounded by C_u and C_l; 2) when cash balance reaches C_u, the firm cut it back to C^{++} by distributing cash back to investors; 3) the firm increases cash balance to C^{+} whenever c goes to C_l by external financing such as debt issue and equity issue. The derivation and solution of this model follows Dixit (1993).

Firm value

This part derives the firm value (denoted by $FV(c_t)$) under the theoretical model framework. The cash management policy described by the (S, s) model is actually a process reseting the problem. Precisely, when cash balance level meets the condition by reaching either C_u or C_l, the firm reset the position of cash holdings to C^{++} or C^{+}. The resetting means to let the process continue from a different level by paying costs. In other words, the process cannot be retested if there is no thresholds C_u and C_l. In this further simplified situation, the value of the firm is

$$FV(c_t) = 0, \tag{2.16}$$

because cash is the only asset and no cash raising or distribution will occur. Since cash level can never reach any threshold, no cash injection or distribution will occur, which means the firm will have no interaction with investors. Then we regard the firm

[15]There are two targets in this theoretical model, because this is a more general setting which releases all restrictions on refinancing cost components. When we set the fractional costs of cash injection and distribution to be equal, the two targets overlap and become one target. Our empirical analysis adopts this one-target setting, and it is a special case in this theoretical framework.

value under (S, s) model as the value without barrier plus the effect of barriers. The barriers incur cash payout denoted by $f(c_t)$. The current cash payout is observable, but payout in future is unknown. When $f(c_t) > 0$, we identify cash injection through external financing. When $f(c_t) < 0$, we identify cash distribution to investors. For most of the cases, we expect $f(c_t) = 0$, which implies that the cash process is within inaction range.

We use $F(c_t)$ to represent the present value of total future effect of barriers. To demonstrate the value of decision is each interval, we separate the value of current payout and the discounted future barrier effect. Assume current cash level is c:

$$F(c) = f(c)dt + e^{-\rho dt}E[F(c + dc)]. \tag{2.17}$$

The first component $f(c)dt$ is the payout during the sufficiently small interval dt. The second term $e^{-\rho dt}E[F(c+dc)]$ is the present value of firm value at the end of the short interval dt. This term consist of two steps: (1) $E[F(c + dc)$ is the expected barrier effect at the end of dt; (2) the discounting factor $e^{-\rho dt}$ further discounts it back to current time point. This component $e^{-\rho dt}E[F(c + dc)]$ contains the recursive nature, and the inter-temporal decomposition provides the possibility to value the decision making flexibility in cash balance.

For further derivation, we apply the Taylor series expansion to the discounting factor:

$$e^{-\rho dt} = \Sigma_{n=0}^{\infty} \frac{(-\rho dt)^n}{n!} = 1 + \frac{(-\rho dt)^1}{1} + \frac{(-\rho dt)^2}{2 \times 1} + \frac{(-\rho dt)^3}{3 \times 2 \times 1} + \cdots.$$

According to our setting, dt is a quite short time interval, and hence we regard any dt terms with order higher than 1 as negligible components. Then, only the first two terms of the Taylor series expansion count:

$$e^{-\rho dt} = 1 - \rho dt. \tag{2.18}$$

Substitute the $e^{-\rho dt}$ in equation (9) with equation (10):

$$\begin{aligned}
F(c) &= f(c)dt + (1 - \rho dt)E[F(c + dc)] \\
&= f(c)dt + (1 - \rho dt)\{F(c) + [E[F(c + dc)] - F(c)]\} \\
&= f(c)dt + (1 - \rho dt)\{F(c) + E[dF]\} \\
&= f(c)dt + F(c) - \rho dtF(c) + E[dF] - \rho dtE[dF].
\end{aligned}$$

The product of $dtdF$ is also negligible, and rearrangement leads to:

$$\rho dtF(c) = f(c)dt + E[dF]. \tag{2.19}$$

For investors, the firm value increase consists of cash distribution and expected future cash balance (the only asset) drift. Since c_t follows the geometric Brownian motion with mean μ and variance σ^2, Itô's Lemma:

$$dF = \left[\mu F'(c_t) + \frac{1}{2}\sigma^2 F''(c_t)\right] dt + F'(c_t)\sigma dw.$$

Then, we take the expectation so that $E(dw) = 0$:

$$E[dF] = \mu F'(c_t)dt + \frac{1}{2}\sigma^2 F''(c_t)dt. \tag{2.20}$$

Then, substitute the $E[dF]$ in equation (2.18) with equation (2.17) and cancel the dt term:

$$\frac{1}{2}\sigma^2 F''(c) + \mu F'(c) - \rho F(c) + f(c) = 0. \tag{2.21}$$

Our target is to derive the effect of barrier, and hence we choose the normal situation when $f(c) = 0$ which means no payout occurs.

$$\frac{1}{2}\sigma^2 F''(c) + \mu F'(c) - \rho F(c) = 0. \tag{2.22}$$

This is a typical second-order ordinary differential equation, and its standard solution is in the form:

$$F(c) = Ae^{xc},$$

in which A and x are the unknown coefficients. Then, $F'(c) = x \times Ae^{xc}$, $F''(c) = x^2 \times Ae^{xc}$, substitute them into equation (2.20) leads to:

$$Ae^{xc} * (\frac{1}{2}\sigma^2 x^2 + \mu x - \rho) = 0.$$

We set $A \neq 0$ to keep the meaning of our solution. Obvisously, $e^{xc} > 0$, and hence:

$$\frac{1}{2}\sigma^2 x^2 + \mu x - \rho = 0.$$

There are two possible solutions $x_1 = -\alpha$ and $x_2 = \beta$ which make two possible solution term $Ae^{-\alpha c}$ and $Ae^{\beta c}$ independent. Thus we write the effect of barriers as:

$$F(c) = Ae^{-\alpha c} + Be^{\beta c}.$$

The total firm value under (S, s) policy will be

$$FV(c_t) = Ae^{-\alpha c_t} + Be^{\beta c_t}. \tag{2.23}$$

Value matching and smooth pasting

With the help of equation (2.21) we can specify the value matching relationship. In reality, if the firm adjusts cash level downward, it may distribute extra dividend, repurchase shares, or repay debt before expiration. If the cash level is too low, the firm should inject cash through raising debt, issuing equity, or liquidating asset. These activities incur two types of costs: 1) lumpy cost depending on whether or not the adjustment happens, denoted by K_d for downward adjustment and by K_u for upward adjustment. This lumpy value mainly reflects the jump of cash holding levels, because it is the cost to reset the cash holdings process to any different levels and to continue. For example, in reality, it corresponds to the costs to get the permission to liquidate asset and to sell equity, costs of hosting auctions, *etc.* 2) Proportional cost with rates m_d and m_u for downward and upward adjustments is a linear component depending on the amount of the adjustment. For example, the service charge of

IPO,[16] value loss in asset liquidation, tax expense of repurchase, *etc.* The existence of costs make continuous adjustment suboptimal. In other words, the firm will not immediately adjust once cash deviates from the target level. It suggests that the firm will allow cash holdings to freely float within a range without interrupting and will make adjustment only when the cash level go out of this inaction range.

We firstly consider a downward adjustment. Suppose at future time t, c_t reaches the upper threshold C_u, and the firm makes immediate adjustment to decrease the cash balance to the related optimal level C^{++} at the cost of $K_d + m_d(C_u - C^{++})$. Hence, the value matching for downward adjustment is:

$$FV(C^{++}) = FV(C_u) + K_d + m_d(C_u - C^{++}). \tag{2.24}$$

Similarly, the value matching for the upward adjustment is:

$$FV(C^{+}) = FV(C_l) + K_u + m_u(C^{+} - C_l). \tag{2.25}$$

We analytically solve A and B, the parameters in equation (2.23), as:

$$\begin{aligned} A &= \frac{[K_d + m_d*(C_u - C^{++})]*(e^{\beta*C^{+}} - e^{\beta*C_l}) - (e^{\beta*C^{++}} - e^{\beta*C_u})*[K_u + m_u(C^{+} - C_l)]}{(e^{-\alpha*C^{++}} - e^{-\alpha*C_u})*(e^{\beta*C^{+}} - e^{\beta*C_l}) - (e^{\beta*C^{++}} - e^{\beta*C_u})*(e^{-\alpha*C^{+}} - e^{-\alpha*C_l})} \\ B &= \frac{(e^{-\alpha*C^{++}} - e^{-\alpha*C_u})*[K_u + m_u(C^{+} - C_l)] - [K_d + m_d*(C_u - C^{++})]*(e^{-\alpha*C^{+}} - e^{-\alpha*C_l})}{(e^{-\alpha*C^{++}} - e^{-\alpha*C_u})*(e^{\beta*C^{+}} - e^{\beta*C_l}) - (e^{\beta*C^{++}} - e^{\beta*C_u})*(e^{-\alpha*C^{+}} - e^{-\alpha*C_l})} \end{aligned} \tag{2.26}$$

As we stated previously, the adjustment does not stop the process of c_t but continues the process at a different level at the expense of the cost. Hence, there

[16] Usually in percentage of the capital raised.

should be smooth pasting relationships for the downward adjustment and upward adjustment, respectively:

$$FV'(C_u) = FV'(C^{++}) = m_u, \tag{2.27}$$

$$FV'(C_l) = FV'(C^{+}) = -m_d. \tag{2.28}$$

Overall, equations (2.22) to (2.25) enable us to solve the model as demonstrated in the following numerical example.

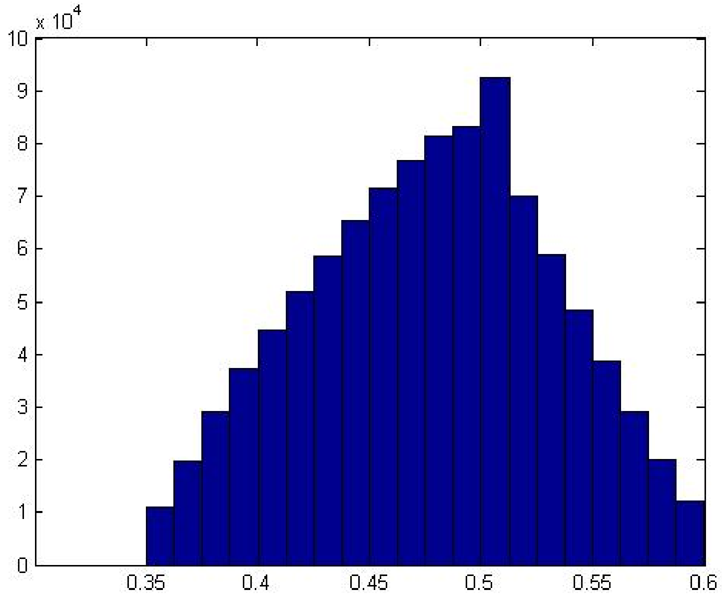
Stationary distribution

Our model assumes cash holdings follow Geometric Brownian Motion (GBM). If we do not apply the double-barrier policy discussed before, the cash holdings observations will not converge to a single distribution for two reasons. Firstly, each observation will have its own mean and standard deviation

$$\begin{aligned} mean &= c_0 + \mu * t, \\ std. &= \sigma * \sqrt{t}. \end{aligned} \tag{2.29}$$

due to the trend of GBM. Secondly, after a sufficiently long period (approaches infinity), the observations will not have a finite mean or standard deviation. However, our resetting policy restricts the GBM with the range between thresholds, which may lead to a long-run stationary distribution as depicted by figure 2.2 is higher than that

FIGURE 2.2: Stationary distribution of cash holdings with low drift



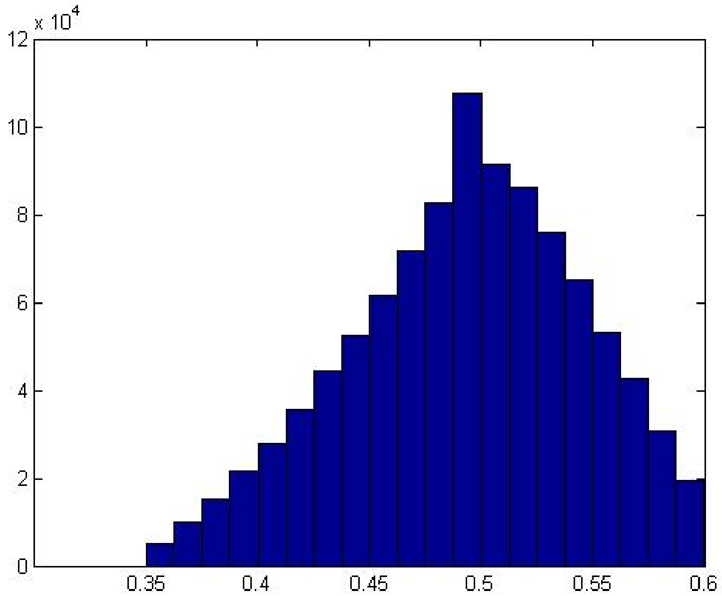
The drift means step length of variation in cash holdings without refinancing. Cash holdings is assumed to follow Geometric Brownian Motion between thresholds.

in figure 2.3. Our example sets a target of 0.5, and the lower and upper thresholds are 0.35 and 0.6, respectively. The drift refers to the step length of variation in cash holdings without refinancing. No matter the drift is high or low, target is the level which is the most likely to occur because of our resetting policy. Under high drift setting, refinancing will become more frequent due to the large step length, and hence the peak in figure 2.3 is higher than that in figure 2.2.

To derive the stationary density function, we apply these further assumptions: 1) the upper barrier is b, target is zero, and the lower barrier is $-b$; 2) at each position, cash balance level has 0.5 probability to move upward by ΔH,[17] and has the probability of 0.5 to move downward by ΔH; 3) there are m steps from the upper threshold to target, and $m = \frac{b}{\Delta H}$, and hence at any time cash balance must be at any one of these steps (the same situation also applies to the lower gap); 4) at the p-

[17]We define step length as unit, and hence $\Delta H = 1$.

FIGURE 2.3: Stationary distribution of cash holdings with high drift



The drift means step length of variation in cash holdings without refinancing. Cash holdings is assumed to follow Geometric Brownian Motion between thresholds.

osition $m-1$, if the cash balance moves upward, it will be adjusted back to the target once touching the threshold, and hence under equilibrium cash level has zero probability to stay at either threshold. Suppose there are plenty of particles following the same policy of cash balance (either moves upward or downward with equal probability, and go back to target once touching thresholds), and the system approaches equilibrium condition after a sufficiently long time. The equilibrium means that each particle continues to move. The number of particles remains stable at each position and at any future time, because the inflow to this position equals outflow from this position. The stationary probability of a position is the fraction of the number of particles at this position over total particles. We use P_i to denote the stationary probability at position i. Consider the first position below upper threshold P_{m-1}, the only source of particle inflow is from the next position P_{m-2}, because from the upper side particles move back to target once reaching the upper threshold. Hence:

$$P_{m-1} = \frac{1}{2} P_{m-2}. \tag{2.30}$$

The part of particles at the position $m - 2$ moving upward is the only inflow of particles at the position $m-1$. With respect to position $m-2$, the inflows can come from either downward moving particles from $m-1$ or upward moving particles from $m-3$. That is:

$$P_{m-2} = \frac{1}{2} P_{m-1} + \frac{1}{2} P_{m-3}. \tag{2.31}$$

From the equations, we know that $P_{m-2} = 2P_{m-1}$ and $P_{m-3} = 3P_{m-1}$. If the process continues, we can derive that $P_{m-n} = nP_{m-1}$ with $0 \leq n < m$. The positions below target have the same argument, which leads to $P_{-m+n} = nP_{-m+1}$ with $0 \leq n < m$. According to our assumption, the probability at either threshold is zero $P_{-m} = P_m = 0$. $m - 1$ is the next step adjacent to the upper threshold, and hence the two positions are pretty close. Similarly, position $-m+1$ is quite close to the lower threshold. Then, we can reasonably assume that $P_{-m+1} = P_{m-1} = K$, and K is a constant. The particles inflow of target position is different, because the particles moving upward (downward) from position $m-1$ $(-m+1)$ also move to target. Hence it has four sources:

$$\begin{aligned} P_0 &= \frac{1}{2} P_1 + \frac{1}{2} P_{-1} + \frac{1}{2} P_{m-1} + \frac{1}{2} P_{-m+1}, \\ &= \frac{1}{2} * (m-1) * K + \frac{1}{2} * (m-1) * K + \frac{1}{2} K + \frac{1}{2} K, \\ &= mK. \end{aligned} \tag{2.32}$$

Under the equilibrium condition of our policy, the cash balance must be at any one position among positions from $m-1$ to $-m+1$, which means the sum of probability of all positions must equal to one:

$$P_{m-1} + P_{m-2} + ... + P_1 + P_0 + P_{-1} + ... + P_{-m+2} + P_{-m+1} = 1. \tag{2.33}$$

That is

$$K + 2K + ... + (m-1)K + mK + (m-1)K + ... + 2k + K = 1.$$

Hence we have $K = \frac{1}{m^2}$. Then we can derive the stationary density function when cash balance is c between b and $-b$[18]:

$$P(c) = P_{\frac{c}{\Delta H}} = (\frac{b}{\Delta H} - \frac{|c|}{\Delta H}) * K.$$

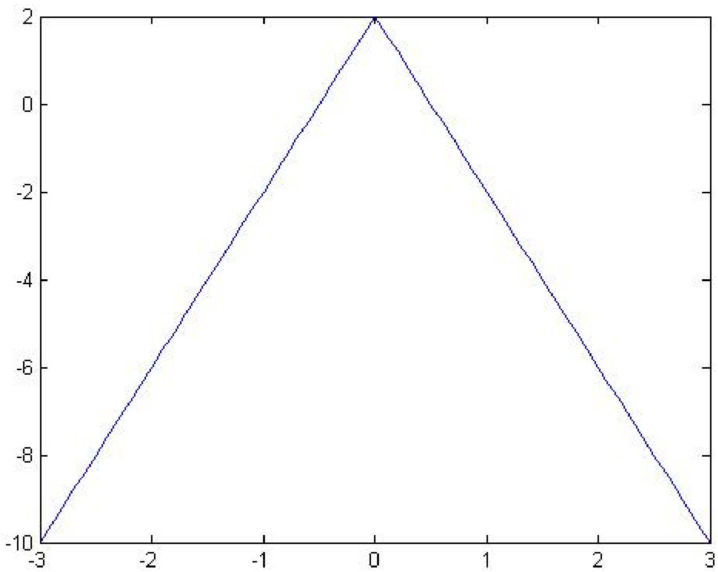
Simplify the equation:

$$P(c) = \frac{b - |c|}{b^2}. \tag{2.34}$$

Figure 2.4 depicts the distribution based on this derived density function.

[18]Cash balance cannot be negative in real case. Our assumptions here is just that target is right at the middle between the two thresholds. The negative lower threshold is for mathematic simplicity, and we can make it consistent with the reality by shifting the whole system upward.

FIGURE 2.4: Stationary distribution of cash holdings based on derived density function



This figure depicts the probability density function derived for the stationary distribution.

Numerical example

The numerical example results are demonstrated in Table 2.17. We studied three different situations, economy expansion, normal time, and economy recession, and we report the thresholds and targets in three panels. The results intuitively proves that it is reasonable to assume that thresholds and targets of our policy are determined by market conditions and firm features as in our model setting. Panel A discusses the expansion situation of economy. The market interest rate ρ is high (15%) and the risk level measure by σ^2 is low (0.9). We further assume the project of the firm reflects average return level, hence the increasing trend μ is as high as ρ. The fixed distribution cost is 0.5, and the proportional cost rate is 0.1. The fixed and proportional cost for upward refinancing are 1 and 0.1, respectively. Based on these information, we get the optimized upper threshold being 13.48, lower threshold 0.791.

The related targets are

Table 2.17: Numerical example

Panel A: Economy expansion						
Setting	ρ	μ	σ^2	$-\alpha$	β	
	0.150	0.150	0.900	-0.768	0.434	
Cost	K_d	m_d	K_u	m_u		
	0.5	0.1	1.0	0.1		
	C_u	C^{++}	C^{+}	C_l	A	B
Initial Value	13	7	4.5	0.5	-1	0.1
	C_u	C^{++}	C^{+}	C_l	A	B
Optimization	13.479	5.208	3.676	0.791	-1.424	0.004
Panel B: Normal situation						
Setting	ρ	μ	σ^2	$-\alpha$	β	
	0.100	0.100	1.200	-0.449	0.310	
Cost	K_d	m_d	K_u	m_u		
	2.0	0.1	3.0	0.2		
	C_u	C^{++}	C^{+}	C_l	A	B
Initial Value	15	9	9	1	3	2
	C_u	C^{++}	C^{+}	C_l	A	B
Optimization	16.075	7.797	5.874	2.343	-4.785	0.010
Panel C: Economy reccession						
Setting	ρ	μ	σ^2	$-\alpha$	β	
	0.050	0.050	3.000	-0.200	0.167	
Cost	K_d	m_d	K_u	m_u		
	5.0	0.3	9.0	0.5		
	C_u	C^{++}	C^{+}	C_l	A	B
Initial Value	17	10	7	2	4.4	1
	C_u	C^{++}	C^{+}	C_l	A	B
Optimization	48.667	19.5221	9.9728	3.5787	-5.4198	0.0041

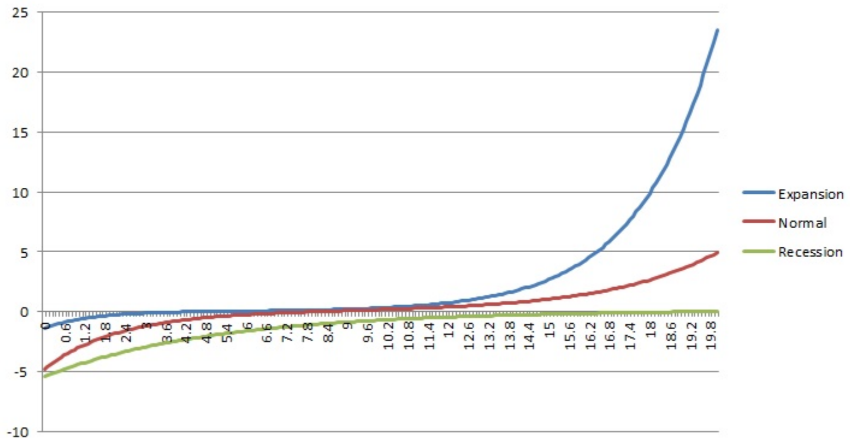
Note: This table presents the numerical examples of our model solution in expansion, normal situation, and recession. Normal situation is the base case, and we increase (decrease) the return of projects but decrease (increase) the refinancing cost to simulate the expansion (recession).

5.21 and 3.68. The normal situation documented in Panel B has interest rate ρ equal to 10%, and the risk level increases to 1.2. The costs also incline compared with expansion time. The optimized thresholds are 16.08 and 7.80 for upper and lower cases, and the related targets are 5.87 and 2.34, respectively. In recession time, as in Panel C, the market interest rate further decrease to 5%, and the risk level raises to 3. The refinancing costs also experience a great increase. This situation leads the upper and lower thresholds to be 48.68 and 3.58, and the related targets to be 19.52 and 9.97.

The comparison among the three scenarios reveals that firms raise both targets and both thresholds in response to recession, and loosen these restrictions when the economy becomes better. Compared Panel C to Panel B, either threshold or target inclines. The high lower threshold makes the upward refinancing more frequent, and the related target implies the amount of upward refinancing also enlarges. On the other side, the upper threshold (48.67) leads to less frequent cash distribution, and the high target implies that the amount of distribution when it happens shrinks. The comparison reveals the firm's attempt to retain cash when the economy experiences depression. However, the firm lowers down the lower threshold to 0.791 when the economy booms because it is not expensive to raise cash from market when needed. The distribution amount is measured by the gap between upper threshold and upper target, and the small gap suggests that it benefits investors if the firm keeps cash in economy expansion because of the fast increasing trend.

Figure 2.5 depicts the relationships between firm value and current cash holding level for each of the three scenarios. As in previous derivation, firm value is the present value of all expected cash distributions or injections in future. In general, firm value increases with current cash level and there are three stages for any scenario. The fi-

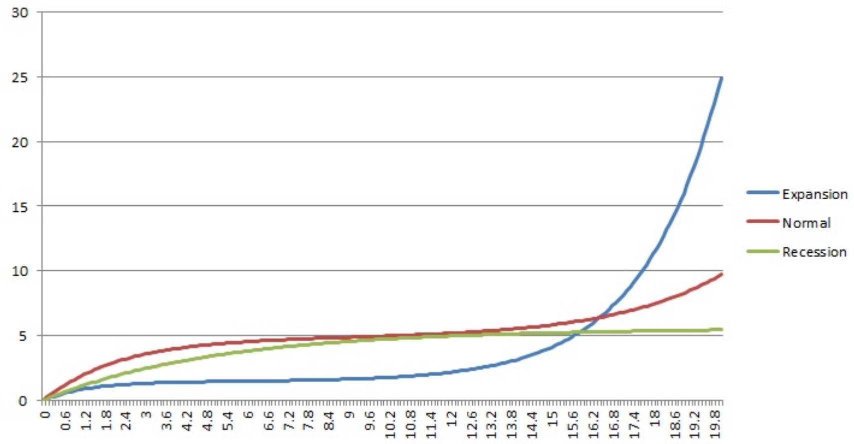
FIGURE 2.5: Firm value variation with current cash position



This figure depicts relationships between firm value and cash holding level. To keep consistent with our numerical example, we demonstrate the relationship in expansion, normal situation, and recession, respectively.

rst stage is when the cash level is low, and the firm value is negative. The negative firm value means the cash injection from investors to the firm is more than the cash distribution from firm to investors. The second stage is approximately the range bounded by the upper and lower thresholds, during which firm value increases slowly with cash level. Since cash value is within the inaction range, an additional increase in cash makes distribution more likely bu cannot incurs distribution. In other words, increase in cash within this range increases only the probability of cash distribution, and hence the present value of true distribution will be affected largely. The third stage is after the upper threshold, and firm value increases rapidly with cash level. Since it is higher than the upper bound, and increase invokes additional distribution immediately. The firm value is lowest at any cash position for the recession economy, this is because the firm is experiencing low growth rate and high financing costs, which makes it less likely to distribute and more likely to get from investors. At the early stage, expansion economy has lower firm value than the normal situation because the target of upward refinancing is higher in expanding economy. In other

FIGURE 2.6: Cash value variation with current cash position



This figure depicts the variations of cash value along with cash holding levels. To keep consistent with our numerical example, we demonstrate the relationship in expansion, normal situation, and recession, respectively.

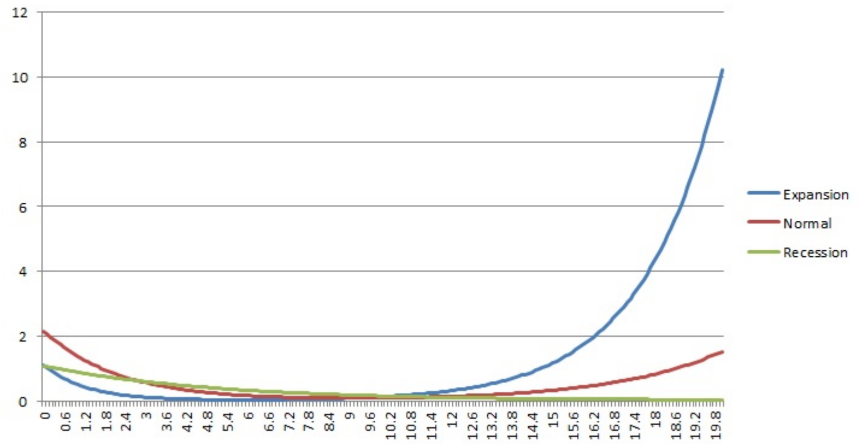
words, firm requires investors to inject more in expansion to take advantage of the growth. In high cash level stage, the expanding economy results in the highest firm value due to the high growth rate.

The value of cash holdings is measured by the gap between current firm value and the firm value with zero cash:

$$CV = FV(c_t) - FV(0) = Ae^{-\alpha c_t} + Be^{\beta c_t} - (A + B). \tag{2.35}$$

The value of cash follows the same shape with the firm value shape, and it is increasing monotonically. The speed of value increment is slow in the middle stage and fast at the beginning and end. Cash worth more when the economy expands compared with normal and recession situations. At the beginning, cash injected establishes the firm, and its value origins from the fact that firms begins to have the potential to earn profits for investors. Then, when cash level enters into the inaction range, extra cash will not increase cash value too much because increment in cash holdings cannot af-

FIGURE 2.7: Marginal value of cash holdings



This figure depicts the variations of marginal cash value along with cash holding levels. To keep consistent with our numerical example, we demonstrate the relationship in expansion, normal situation, and recession, respectively.

fect cash injection or distribution decisions. Finally, when cash level goes above the upper threshold, additional cash incurs distribution immediately, which leads to the stead increase cash value in this range. The stead increase in cash value occurs earlier in expanding economy than in normal and recession of economy, because the upper threshold in expansion in relatively lower.

Marginal value of the firm measures the value of each additional unit cash:

$$MVC = -\alpha * Ae^{-\alpha c_t} + \beta * Be^{\beta c_t}. \tag{2.36}$$

Based on equation (2.24), we replace A and B with their analytic solutions:

$$
\begin{aligned}
MVC = & -\alpha * \frac{\begin{array}{c}[K_d + m_d * (C_u - C^{++})] * (e^{\beta * C^{+}} - e^{\beta * C_l}) \\ -\,(e^{\beta * C^{++}} - e^{\beta * C_u}) * [K_u + m_u(C^{+} - C_l)]\end{array}}{\begin{array}{c}(e^{-\alpha * C^{++}} - e^{-\alpha * C_u}) * (e^{\beta * C^{+}} - e^{\beta * C_l}) \\ -\,(e^{\beta * C^{++}} - e^{\beta * C_u}) * (e^{-\alpha * C^{+}} - e^{-\alpha * C_l})\end{array}} e^{-\alpha c_t} \\
& + \beta * \frac{\begin{array}{c}(e^{-\alpha * C^{++}} - e^{-\alpha * C_u}) * [K_u + m_u(C^{+} - C_l)] \\ -\,[K_d + m_d * (C_u - C^{++})] * (e^{-\alpha * C^{+}} - e^{-\alpha * C_l})\end{array}}{\begin{array}{c}(e^{-\alpha * C^{++}} - e^{-\alpha * C_u}) * (e^{\beta * C^{+}} - e^{\beta * C_l}) \\ -\,(e^{\beta * C^{++}} - e^{\beta * C_u}) * (e^{-\alpha * C^{+}} - e^{-\alpha * C_l})\end{array}} e^{\beta c_t}.
\end{aligned}
\tag{2.37}
$$

At the initial stage, cash is below the lower threshold, which means cash injected invokes the firm and requires more invested. Hence, the marginal value of cash decreases. Then the marginal value of cash stays near zero within the inaction range, because the money increment within this area has almost zero effect on the decisions of injection or distribution. The marginal value of cash increases at a high speed after the inaction range. Expanding economy has the earliest occurrence of zero marginal value range and the earliest steady increasing. Economy in recession will have the longest zero marginal value range because it has the largest gap between refinancing thresholds as illustrated by the numerical example.

Appendix 2.2 Likelihood Function

This appendix presents the likelihood function in our estimation. In order to study the trigger of cash refinancing as well as the target, our research exploits identification strategy to categorize the observations into four groups, namely upper threshold, target, lower threshold, and ordinary observation, and calculate the loglikelihood for each group specifically. In the following presentation, $\phi(\cdot)$ and $\Phi(\cdot)$ denote the probability density function and cumulative function of normal distribution, respectively; ε^* represents the error term for the true target; ε_u and ε_l denote the error terms in the upper and lower gaps; y refers to the observed cash holding level; x is the explanatory variable set; β is the coefficient vector for the target; θ_u and θ_l represent the coefficient sets for determinants of the upper and lower exponential terms, respectively; σ^*, σ_u and σ_l are standard deviations for error terms ε^*, ε_u, and ε_l, respectively; ρ_1 is the correlation coefficient between ε^* and ε_u; ρ_4 is the correlation coefficient between ε^* and ε_l; θ_{ML} represents the vector of all parameters to be estimated $\theta_{ML} = [\beta, \theta_u, \theta_l, \sigma^*, \sigma_u, \sigma_l, \rho_1, \rho_4]$.

$Case\ 1:\ Upper\ Threshold$ When a firm makes no cash refinancing now but adjusts cash downward next period, the observation in current period falls into this group. The cash value observations in this group hence reflects the upper thresholds of cash holdings because they have exceeded the upper thresholds and invoked future cash distributions. As suggested by our empirical setting, equation (2.2), the error in the upper exponential term is conditional on the error term for the true target

$\varepsilon_u|\varepsilon^* = log(y_1 - x_1\beta - \varepsilon^*) - x_1\theta_u$ with $\varepsilon^* < y_1 - x_1\beta$. The likelihood is:

$$p_1 = \int_{-\inf}^{y_1 - x_1\beta} \phi(\varepsilon^*, 0, \sigma^*) \cdot \phi(log(y_1 - x_1\beta - \varepsilon^*) - x_1\theta_u, \frac{\sigma_u\rho_1}{\sigma^*} \cdot \varepsilon^*, \sigma_u\sqrt{1-\rho_1^2}) d\varepsilon^* \quad (2.38)$$

Case 2 : *Target* When a firm makes cash refinancing now, either injection or distribution, the observation in current period falls into this group. Our research assumes adjustments bring cash holding level back to target level, and hence the observations in this group are the true target with error term. The likelihood is:

$$p_2 = \phi(y_2 - x_2\beta, 0, \sigma^*) \quad (2.39)$$

Case 3 : *Lower Threshold* When a firm makes no cash refinancing now but adjusts cash upward next period, the observation in current period falls into this group. In contrary to *Case* 1, the observations in this group convey information about the lower thresholds of cash holdings. As suggested by our empirical setting, equation (2.2), the error in the lower exponential term is $\varepsilon_l|\varepsilon^* = log(-y_4 + x_4\beta + \varepsilon^*) - x_4\theta_l$ with $\varepsilon^* > y_4 - x_4\beta$. The likelihood is:

$$p_4 = \int_{y_4 - x_4\beta}^{\inf} \phi(\varepsilon^*, 0, \sigma^*) \cdot \phi(log(-y_4 + x_4\beta + \varepsilon^*) - x_4\theta_l, \frac{\sigma_l\rho_4}{\sigma^*} \cdot \varepsilon^*, \sigma_l\sqrt{1-\rho_4^2}) d\varepsilon^* \quad (2.40)$$

Case 4 : *Ordinary Observation* When a firm makes no cash refinancing in this and next periods, the observation in current period is regarded as the ordinary one since it is neither threshold nor target. Since these observations do not cause any future refinancing, they are within the range bounded by the upper and lower threshold. Hence, $\varepsilon_u|\varepsilon^* > log(y_0 - x_0\beta - \varepsilon^*) - x_0\theta_u$ and $\varepsilon_l|\varepsilon^* > log(-y_0 + x_0\beta + \varepsilon^*) - x_0\theta_l$. Then, we calculate the likelihood by part:

$$
\begin{aligned}
p_{01} &= \int_{-\inf}^{y_0 - x_0\beta} \phi(\varepsilon^*, 0, \sigma^*) \cdot (1 - \Phi(log(y_0 + x_0\beta - \varepsilon^*) - x_0\theta_u, \frac{\sigma_u \rho_1}{\sigma^*} \cdot \varepsilon^*, \sigma_u\sqrt{1-\rho_1^2}))d\varepsilon^* \\
p_{02} &= \int_{y_0 - x_0\beta}^{\inf} \phi(\varepsilon^*, 0, \sigma^*) \cdot (1 - \Phi(log(-y_0 + x_0\beta + \varepsilon^*) - x_0\theta_l, \frac{\sigma_l \rho_4}{\sigma^*} \cdot \varepsilon^*, \sigma_l\sqrt{1-\rho_4^2}))d\varepsilon^* \\
p_0 &= p_{01} + p_{02}
\end{aligned}
\tag{2.41}
$$

Then, take the logarithm of the probability to back out the log-likelihood:

$$
l = log(p_1) + log(p_2) + log(p_4) + log(p_0) \tag{2.42}
$$

Our calculation of standard error is based on the Hessian Matrix: $Var(\hat{\theta_{ML}}) = [-H(\hat{\theta_{ML}})]^{-1}$, in which Hessian represents the second order derivative matrix $H(\hat{\theta_{ML}}) = \frac{\partial^2 l}{\partial\theta_i \partial\theta_j}$. More precise, the Hessian matrix in our case is:

$$
H = \begin{bmatrix}
\frac{\partial^2 l}{\partial\beta\partial\beta} & \frac{\partial^2 l}{\partial\beta\partial\theta_u} & \frac{\partial^2 l}{\partial\beta\partial\theta_l} & \frac{\partial^2 l}{\partial\beta\partial\theta_u} & \frac{\partial^2 l}{\partial\beta\partial\sigma^*} & \frac{\partial^2 l}{\partial\beta\partial\sigma_u} & \frac{\partial^2 l}{\partial\beta\partial\sigma_l} & \frac{\partial^2 l}{\partial\beta\partial\rho_1} & \frac{\partial^2 l}{\partial\beta\partial\rho_2} \\
\frac{\partial^2 l}{\partial\theta_u\partial\beta} & \frac{\partial^2 l}{\partial\theta_u\partial\theta_u} & \frac{\partial^2 l}{\partial\theta_u\partial\theta_l} & \frac{\partial^2 l}{\partial\theta_u\partial\theta_u} & \frac{\partial^2 l}{\partial\theta_u\partial\sigma^*} & \frac{\partial^2 l}{\partial\theta_u\partial\sigma_u} & \frac{\partial^2 l}{\partial\theta_u\partial\sigma_l} & \frac{\partial^2 l}{\partial\theta_u\partial\rho_1} & \frac{\partial^2 l}{\partial\theta_u\partial\rho_2} \\
\frac{\partial^2 l}{\partial\theta_l\partial\beta} & \frac{\partial^2 l}{\partial\theta_l\partial\theta_u} & \frac{\partial^2 l}{\partial\theta_l\partial\theta_l} & \frac{\partial^2 l}{\partial\theta_l\partial\theta_u} & \frac{\partial^2 l}{\partial\theta_l\partial\sigma^*} & \frac{\partial^2 l}{\partial\theta_l\partial\sigma_u} & \frac{\partial^2 l}{\partial\theta_l\partial\sigma_l} & \frac{\partial^2 l}{\partial\theta_l\partial\rho_1} & \frac{\partial^2 l}{\partial\theta_l\partial\rho_2} \\
\frac{\partial^2 l}{\partial\sigma^*\partial\beta} & \frac{\partial^2 l}{\partial\sigma^*\partial\theta_u} & \frac{\partial^2 l}{\partial\sigma^*\partial\theta_l} & \frac{\partial^2 l}{\partial\sigma^*\partial\theta_u} & \frac{\partial^2 l}{\partial\sigma^*\partial\sigma^*} & \frac{\partial^2 l}{\partial\sigma^*\partial\sigma_u} & \frac{\partial^2 l}{\partial\sigma^*\partial\sigma_l} & \frac{\partial^2 l}{\partial\sigma^*\partial\rho_1} & \frac{\partial^2 l}{\partial\sigma^*\partial\rho_2} \\
\frac{\partial^2 l}{\partial\sigma_u\partial\beta} & \frac{\partial^2 l}{\partial\sigma_u\partial\theta_u} & \frac{\partial^2 l}{\partial\sigma_u\partial\theta_l} & \frac{\partial^2 l}{\partial\sigma_u\partial\theta_u} & \frac{\partial^2 l}{\partial\sigma_u\partial\sigma^*} & \frac{\partial^2 l}{\partial\sigma_u\partial\sigma_u} & \frac{\partial^2 l}{\partial\sigma_u\partial\sigma_l} & \frac{\partial^2 l}{\partial\sigma_u\partial\rho_1} & \frac{\partial^2 l}{\partial\sigma_u\partial\rho_2} \\
\frac{\partial^2 l}{\partial\sigma_l\partial\beta} & \frac{\partial^2 l}{\partial\sigma_l\partial\theta_u} & \frac{\partial^2 l}{\partial\sigma_l\partial\theta_l} & \frac{\partial^2 l}{\partial\sigma_l\partial\theta_u} & \frac{\partial^2 l}{\partial\sigma_l\partial\sigma^*} & \frac{\partial^2 l}{\partial\sigma_l\partial\sigma_u} & \frac{\partial^2 l}{\partial\sigma_l\partial\sigma_l} & \frac{\partial^2 l}{\partial\sigma_l\partial\rho_1} & \frac{\partial^2 l}{\partial\sigma_l\partial\rho_2} \\
\frac{\partial^2 l}{\partial\rho_1\partial\beta} & \frac{\partial^2 l}{\partial\rho_1\partial\theta_u} & \frac{\partial^2 l}{\partial\rho_1\partial\theta_l} & \frac{\partial^2 l}{\partial\rho_1\partial\theta_u} & \frac{\partial^2 l}{\partial\rho_1\partial\sigma^*} & \frac{\partial^2 l}{\partial\rho_1\partial\sigma_u} & \frac{\partial^2 l}{\partial\rho_1\partial\sigma_l} & \frac{\partial^2 l}{\partial\rho_1\partial\rho_1} & \frac{\partial^2 l}{\partial\rho_1\partial\rho_2} \\
\frac{\partial^2 l}{\partial\rho_4\partial\beta} & \frac{\partial^2 l}{\partial\rho_4\partial\theta_u} & \frac{\partial^2 l}{\partial\rho_4\partial\theta_l} & \frac{\partial^2 l}{\partial\rho_4\partial\theta_u} & \frac{\partial^2 l}{\partial\rho_4\partial\sigma^*} & \frac{\partial^2 l}{\partial\rho_4\partial\sigma_u} & \frac{\partial^2 l}{\partial\rho_4\partial\sigma_l} & \frac{\partial^2 l}{\partial\rho_4\partial\rho_1} & \frac{\partial^2 l}{\partial\rho_4\partial\rho_2}
\end{bmatrix},
$$

and more details about the calculation of the Hessian matrix is in the attached MATLAB program.

Appendix 2.3 MATLAB Program

```
%              Title: Maximum Likelihood Esitmation of (S, s)
%              Author: Qifan Zhai
%              Date: 2017-04-12

function [beta_ss,se_ss,result,hessian,c_ini]=mle_20170412_fminunc_g_h(yche,x,id)

% In the original data, id=2 indicates target after distribution and id=3 indicates target after injection
id(id==3)=2; % Merge the two targets
% id=1: upper threshold; id=2: target; id=4: lower threshold; id=0: ordinary

y1=yche;x1=x;y1(id~=1,:)=[];x1(id~=1,:)=[];y2=yche;x2=x;y2(id~=2,:)=[];x2(id~=2,:)=[];y4=yche;x4=x;y4(id~=4,:)=[];x4(id~=4,:)=[];y0=yche;x0=x;y0(id~=0,:)=[];x0(id~=0,:)=[];n1=length(y1);n2=length(y2);n4=length(y4);n0=length(y0);
[~,nvar]=size(x1);

 c_beta=(x2'*x2)\(x2'*y2)*(rand-0.5)*2;c_theta_u=(x1'*x1)\(x1'*log(abs(y1-x1*c_beta)))*(rand-0.5)*2;c_theta_l=(x4'*x4)\(x4'*log(abs(x4*c_beta-y4)))*(rand-0.5)*2;c_cov=rand(5,1)*2-1;c_ini=[c_beta;c_theta_u;c_theta_l;c_cov];

 tic;options = optimoptions(@fminunc,'Algorithm','trust-region','SpecifyObjectiveGradient',true,'HessianFcn','objective','MaxIter',4000000,'MaxFunEvals',10000000,'display','Iter','TolX',10^-6,'TolFun',10^-6);
 [beta_ss,~,~,~,~,hessian]=fminunc(@(ini)loglike_g_h(y1,x1,y2,x2,y4,x4,y0,x0,nvar,n1,n2,n4,n0,ini),c_ini,options);toc;

 se_ss=sqrt(diag(inv(hessian)));
 result=[c_ini,beta_ss,se_ss];
end

function [l,g,h]=loglike_g_h(y1,x1,y2,x2,y4,x4,y0,x0,nvar,n1,n2,n4,n0,ini)
r1=ini(3*nvar+4,1);r4=ini(3*nvar+5,1);
```

```
b=ini(1:nvar,1);tu=ini(nvar+1:2*nvar,1);tl=ini(2*nvar+1:3*nvar,1);sig2=exp(ini(3*nvar+1,1));sig_u=exp(ini(3*nvar+2,1)); sig_l=exp(ini(3*nvar+3,1)); rho1=erf(r1);rho4=erf(r4);
m1=(sig_u/sig2)*rho1;sig1=sig_u*sqrt(1-rho1^2);m4=(sig_l/sig2)*rho4;sig4=sig_l*sqrt(1-rho4^2);

lb=(10^(-3));ub=1-10^(-3);
% Notation: Cu=1; C*=2; Cl=4; C_ordi=0
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% 1. Upper Threshold
funode1=@(t1,y)normpdf(log(t1)+y1-x1*b,0,sig2).*normpdf(log(-log(t1))-x1*tu,m1*(log(t1)+y1-x1*b),sig1)./t1;[~,result1]=ode45(funode1,[lb 0.6 ub],zeros(n1,1));prob1ode=result1(3,:)';
l1=sum(log(prob1ode));

part1_funode1=@(t1,y)(exp(-(log(t1)+y1-x1*b).^2/(2*sig2^2)-(log(-log(t1))-x1*tu-m1*(log(t1)+y1-x1*b)).^2/(2*sig1^2))./t1).*(log(t1)+y1-x1*b);
part2_funode1=@(t1,y)(exp(-(log(t1)+y1-x1*b).^2/(2*sig2^2)-(log(-log(t1))-x1*tu-m1*(log(t1)+y1-x1*b)).^2/(2*sig1^2))./t1).*(log(-log(t1))-x1*tu-m1*(log(t1)+y1-x1*b));
part3_funode1=@(t1,y)(exp(-(log(t1)+y1-x1*b).^2/(2*sig2^2)-(log(-log(t1))-x1*tu-m1*(log(t1)+y1-x1*b)).^2/(2*sig1^2))./t1).*((log(t1)+y1-x1*b).*(log(-log(t1))-x1*tu-m1*(log(t1)+y1-x1*b)));
part4_funode1=@(t1,y)(exp(-(log(t1)+y1-x1*b).^2/(2*sig2^2)-(log(-log(t1))-x1*tu-m1*(log(t1)+y1-x1*b)).^2/(2*sig1^2))./t1).*((log(t1)+y1-x1*b).^2);
part5_funode1=@(t1,y)(exp(-(log(t1)+y1-x1*b).^2/(2*sig2^2)-(log(-log(t1))-x1*tu-m1*(log(t1)+y1-x1*b)).^2/(2*sig1^2))./t1).*((log(-log(t1))-x1*tu-m1*(log(t1)+y1-x1*b)).^2);

part6_funode1=@(t1,y)(exp(-(log(t1)+y1-x1*b).^2/(2*sig2^2)-(log(-log(t1))-x1*tu-m1*(log(t1)+y1-x1*b)).^2/(2*sig1^2))./t1).*(((log(t1)+y1-x1*b).^2).*(log(-log(t1))-x1*tu-m1*(log(t1)+y1-x1*b)));
part7_funode1=@(t1,y)(exp(-(log(t1)+y1-x1*b).^2/(2*sig2^2)-(log(-log(t1))-x1*tu-m1*(log(t1)+y1-x1*b)).^2/(2*sig1^2))./t1).*((log(t1)+y1-x1*b).*((log(-log(t1))-x1*tu-m1*(log(t1)+y1-x1*b)).^2));
```

```
part8_funode1=@(t1,y)(exp(-(log(t1)+y1-x1*b).^2/(2*sig2^2)-(log(-log(t1))-x1*tu-m1*(log(t1)+y1-x1*b)).^2/(2*sig1^2))./t1).*((log(t1)+y1-x1*b).^3);
part9_funode1=@(t1,y)(exp(-(log(t1)+y1-x1*b).^2/(2*sig2^2)-(log(-log(t1))-x1*tu-m1*(log(t1)+y1-x1*b)).^2/(2*sig1^2))./t1).*((log(-log(t1))-x1*tu-m1*(log(t1)+y1-x1*b)).^3);

part10_funode1=@(t1,y)(exp(-(log(t1)+y1-x1*b).^2/(2*sig2^2)-(log(-log(t1))-x1*tu-m1*(log(t1)+y1-x1*b)).^2/(2*sig1^2))./t1).*(((log(t1)+y1-x1*b).^2).*((log(-log(t1))-x1*tu-m1*(log(t1)+y1-x1*b)).^2));
part11_funode1=@(t1,y)(exp(-(log(t1)+y1-x1*b).^2/(2*sig2^2)-(log(-log(t1))-x1*tu-m1*(log(t1)+y1-x1*b)).^2/(2*sig1^2))./t1).*(((log(t1)+y1-x1*b).^3).*(log(-log(t1))-x1*tu-m1*(log(t1)+y1-x1*b)));
part12_funode1=@(t1,y)(exp(-(log(t1)+y1-x1*b).^2/(2*sig2^2)-(log(-log(t1))-x1*tu-m1*(log(t1)+y1-x1*b)).^2/(2*sig1^2))./t1).*((log(t1)+y1-x1*b).*((log(-log(t1))-x1*tu-m1*(log(t1)+y1-x1*b)).^3));
part13_funode1=@(t1,y)(exp(-(log(t1)+y1-x1*b).^2/(2*sig2^2)-(log(-log(t1))-x1*tu-m1*(log(t1)+y1-x1*b)).^2/(2*sig1^2))./t1).*((log(t1)+y1-x1*b).^4);
part14_funode1=@(t1,y)(exp(-(log(t1)+y1-x1*b).^2/(2*sig2^2)-(log(-log(t1))-x1*tu-m1*(log(t1)+y1-x1*b)).^2/(2*sig1^2))./t1).*((log(-log(t1))-x1*tu-m1*(log(t1)+y1-x1*b)).^4);

[~,part1_result1]=ode45(part1_funode1,[lb 0.6 ub],zeros(n1,1)); part1=part1_result1(3,:)'/(2*pi*sig2*sig1);
[~,part2_result1]=ode45(part2_funode1,[lb 0.6 ub],zeros(n1,1)); part2=part2_result1(3,:)'/(2*pi*sig2*sig1);
[~,part3_result1]=ode45(part3_funode1,[lb 0.6 ub],zeros(n1,1)); part3=part3_result1(3,:)'/(2*pi*sig2*sig1);
[~,part4_result1]=ode45(part4_funode1,[lb 0.6 ub],zeros(n1,1)); part4=part4_result1(3,:)'/(2*pi*sig2*sig1);
[~,part5_result1]=ode45(part5_funode1,[lb 0.6 ub],zeros(n1,1)); part5=part5_result1(3,:)'/(2*pi*sig2*sig1);

[~,part6_result1]=ode45(part6_funode1,[lb 0.6 ub],zeros(n1,1)); part6=part6_result1(3,:)'/(2*pi*sig2*sig1);
[~,part7_result1]=ode45(part7_funode1,[lb 0.6 ub],zeros(n1,1)); part7=part7_result1(3,:)'/(2*pi*sig2*sig1);
```

```
[~,part8_result1]=ode45(part8_funode1,[lb 0.6 ub],zeros(n1,1)); part8=part8_result1
    (3,:)'/(2*pi*sig2*sig1);
[~,part9_result1]=ode45(part9_funode1,[lb 0.6 ub],zeros(n1,1)); part9=part9_result1
    (3,:)'/(2*pi*sig2*sig1);

[~,part10_result1]=ode45(part10_funode1,[lb 0.6 ub],zeros(n1,1)); part10=
    part10_result1(3,:)'/(2*pi*sig2*sig1);
[~,part11_result1]=ode45(part11_funode1,[lb 0.6 ub],zeros(n1,1)); part11=
    part11_result1(3,:)'/(2*pi*sig2*sig1);
[~,part12_result1]=ode45(part12_funode1,[lb 0.6 ub],zeros(n1,1)); part12=
    part12_result1(3,:)'/(2*pi*sig2*sig1);
[~,part13_result1]=ode45(part13_funode1,[lb 0.6 ub],zeros(n1,1)); part13=
    part13_result1(3,:)'/(2*pi*sig2*sig1);
[~,part14_result1]=ode45(part14_funode1,[lb 0.6 ub],zeros(n1,1)); part14=
    part14_result1(3,:)'/(2*pi*sig2*sig1);

deno=repmat(prob1ode,1,nvar);
deno2=repmat(prob1ode.^2,1,nvar);
coef1=rho1/(1-rho1^2);coef2=sig_u*rho1/(sqrt(1-rho1^2)*sig1^3);coef3=m1/(rho1*sig1
    ^2);coef4=(1+rho1^2)/(sig2*sig_u*(1-rho1^2)^2);coef5=(1+3*rho1^2)/((sig_u^2)*(1-
    rho1^2)^3);% sig_u/(sig2*sig1*sig1)+2*m1*sig_u*rho1/(sqrt(1-rho1^2)*sig1^3);%

p_b=repmat(part1,1,nvar).*x1/(sig2^2)-m1*repmat(part2,1,nvar).*x1/(sig1^2);% p_b
p_tu=repmat(part2,1,nvar).*x1/(sig1^2); % p_tu
p_sig2=(part4/(sig2^3)-m1*part3/(sig2*sig1^2)-prob1ode/sig2)*sig2; % p_sig2
p_sigu=(part3*m1/(sig_u*sig1^2)+part5/(sig_u*sig1^2)-prob1ode/sig_u)*sig_u;
p_rho1=(part3*m1/(rho1*sig1^2)-part5*coef1/(sig1^2)+prob1ode*coef1)*exp(-r1^2)*2/
    sqrt(pi);
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Gradient Components
g1b=sum(p_b./deno,1)';g1tu=sum(p_tu./deno,1)';g1sig2=sum(p_sig2./prob1ode);g1sigu=
    sum(p_sigu./prob1ode);g1rho1=sum(p_rho1./prob1ode);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Common secondary derivative terms
part1_sig2=part8/(sig2^3)-part6*m1/(sig2*sig1*sig1)-part1/sig2;
```

```
part1_sigu=part6*m1/(sig_u*sig1*sig1)+part7/(sig_u*sig1*sig1)-part1/sig_u;
part1_rho1=part6*coef3-part7*coef2+part1*coef1;

part2_sig2=part6/(sig2^3)-part7*m1/(sig2*sig1*sig1)+part1*m1/sig2-part2/sig2;
part2_sigu=part7*m1/(sig_u*sig1*sig1)+part9/(sig_u*sig1*sig1)-part1*m1/sig_u-part2/sig_u;
part2_rho1=part7*coef3-part9*coef2-part1*m1/rho1+part2*coef1;

part3_sig2=(part11/(sig2^2)-part10*m1/(sig1^2)+part4*m1-part3)/sig2;
part3_sigu=(part10*m1/(sig1^2)+part12/(sig1^2)-part4*m1-part3)/sig_u;
part3_rho1=part10*coef3-part12*coef2-part4*m1/rho1+part3*coef1;

part4_sig2=part13/(sig2^3)-part11*m1/(sig1*sig1*sig2)-part4/sig2;
part4_sigu=(part11*m1/(sig1^2)+part10/(sig1^2)-part4)/sig_u;
part4_rho1=part11*coef3-part10*coef2+part4*coef1;

part5_sigu=part12*m1/(sig_u*sig1^2)+part14/(sig_u*sig1^2)-part5/sig_u-part3*2*rho1/sig2;
part5_rho1=part12*coef3-part14*coef2-part3*2*m1/rho1+part5*coef1;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Hessian

p_b_b=part4/(sig2^4)-part3*2*m1/(sig2*sig2*sig1*sig1)+part5*m1*m1/(sig1^4);

h1bb_part1=p_b'*(-p_b./deno2);
h1bb_part2=x1'*((repmat(p_b_b,1,nvar).*x1)./deno)-x1'*x1*(1/(sig2^2)+m1*m1/(sig1^2));
h1bb=h1bb_part1+h1bb_part2;

p_b_tu=part3/(sig1^2*sig2^2)-part5*m1/(sig1^4);
h1btu_part1=p_b'*(-p_tu./deno2);
h1btu_part2=x1'*((repmat(p_b_tu,1,nvar).*x1)./deno)+x1'*x1*m1/(sig1^2);
h1btu=h1btu_part1+h1btu_part2;

```

```
p_b_sig2_prep=part1_sig2/(sig2^2)-2*part1/(sig2^3)+part2*m1/(sig2*sig1*sig1)-
    part2_sig2*m1/(sig1^2);p_b_sig2=repmat(p_b_sig2_prep,1,nvar).*x1;
h1bsig2_part1=p_b'*(-p_sig2./deno2(:,1));
h1bsig2_part2=sum(p_b_sig2./deno,1)';
h1bsig2=h1bsig2_part1+h1bsig2_part2*sig2;

p_b_sigu_prep=part1_sigu/(sig2^2)-part2_sigu*m1/(sig1^2)+part2*m1/(sig1*sig1*sig_u);
    p_b_sigu=repmat(p_b_sigu_prep,1,nvar).*x1;
h1bsigu_part1=p_b'*(-p_sigu./deno2(:,1));
h1bsigu_part2=sum(p_b_sigu./deno,1)';
h1bsigu=h1bsigu_part1+h1bsigu_part2*sig_u;

p_b_rho1_prep=part1_rho1/(sig2^2)-part2_rho1*m1/(sig1^2)-part2*coef4;p_b_rho1=repmat
    (p_b_rho1_prep,1,nvar).*x1;
h1brho1_part1=p_b'*(-p_rho1./deno2(:,1));
h1brho1_part2=sum(p_b_rho1./deno,1)';
h1brho1=h1brho1_part1+h1brho1_part2*exp(-r1^2)*2/sqrt(pi);

% h_tu series
h1tub=h1btu';

p_tu_tu=repmat(part5,1,nvar).*x1/(sig1^2);
h1tutu_part1=p_tu'*(-p_tu./deno2);
h1tutu_part2=x1'*(p_tu_tu./deno)-x1'*x1;
h1tutu=h1tutu_part1+h1tutu_part2/(sig1^2);

p_tu_sig2=repmat(part2_sig2/(sig1^2),1,nvar).*x1;
h1tusig2_part1=p_tu'*(-p_sig2./deno2(:,1));
h1tusig2_part2=sum(p_tu_sig2./deno,1)';
h1tusig2=h1tusig2_part1+h1tusig2_part2*sig2;

p_tu_sigu_prep=part2_sigu/(sig1^2)-part2*2/(sig_u*sig1^2);p_tu_sigu=repmat(
    p_tu_sigu_prep,1,nvar).*x1;
h1tusigu_part1=p_tu'*(-p_sigu./deno2(:,1));
h1tusigu_part2=sum(p_tu_sigu./deno,1)';
h1tusigu=h1tusigu_part1+h1tusigu_part2*sig_u;
```

```

p_tu_rho1_prep=part2_rho1/(sig1^2)+part2*2*sig_u*rho1/((sig1^3)*sqrt(1-rho1^2)); p_tu_rho1=repmat(p_tu_rho1_prep,1,nvar).*x1;
h1turho1_part1=p_tu'*(-p_rho1./deno2(:,1));
h1turho1_part2=sum(p_tu_rho1./deno,1)';
h1turho1=h1turho1_part1+h1turho1_part2*exp(-r1^2)*2/sqrt(pi);

% h1sig2 series

h1sig2b=h1bsig2';
h1sig2tu=h1tusig2';

p_sig2_sig2_prep=-3*part4/(sig2^4)+part4_sig2/(sig2^3)+2*part3*m1/(sig2*sig2*sig1*sig1)-part3_sig2*m1/(sig2*sig1*sig1)+prob1ode/(sig2^2)-p_sig2/(sig2^2);
h1sig2sig2_part1=p_sig2'*(-p_sig2./deno2(:,1));
h1sig2sig2_part2=sum(p_sig2_sig2_prep./deno(:,1));
h1sig2sig2=(h1sig2sig2_part1/sig2+h1sig2sig2_part2*sig2)*sig2+g1sig2;

p_sig2_sigu_prep=part4_sigu/(sig2^3)-(part3_sigu/sig_u-part3/(sig_u^2))*coef1/(sig2^2)-p_sigu/(sig_u*sig2);
h1sig2sigu_part1=p_sig2'*(-p_sigu./deno2(:,1));
h1sig2sigu_part2=sum(p_sig2_sigu_prep./deno(:,1));
h1sig2sigu=(h1sig2sigu_part1/sig2+h1sig2sigu_part2*sig_u)*sig2;

p_sig2_rho1_prep=part4_rho1/(sig2^3)-part3*coef4/sig2-part3_rho1*coef1/(sig2*sig2*sig_u)-(p_rho1/(exp(-r1^2)*2/sqrt(pi)))/sig2;
h1sig2rho1_part1=p_sig2'*(-p_rho1./deno2(:,1));
h1sig2rho1_part2=sum(p_sig2_rho1_prep./deno(:,1));
h1sig2rho1=(h1sig2rho1_part1/sig2+h1sig2rho1_part2*(exp(-r1^2)*2/sqrt(pi)))*sig2;

% h1sigu series
h1sigub=h1bsigu';
h1sigutu=h1tusigu';
h1sigusig2=h1sig2sigu';

```

```
p_sigu_sigu_prep=part3_sigu*m1/(sig_u*sig1^2)-part3*2*coef1/(sig2*sig_u^3)+part5_sigu/(sig_u*sig1*sig1)-part5*3/(sig_u*sig_u*sig1*sig1)-p_sigu/(sig_u^2)+prob1ode/(sig_u^2);
h1sigusigu_part1=p_sigu'*(-p_sigu./deno2(:,1));
h1sigusigu_part2=sum(p_sigu_sigu_prep./deno(:,1));
h1sigusigu=(h1sigusigu_part1/sig_u+h1sigusigu_part2*sig_u)*sig_u+g1sigu;

p_sigu_rho1_prep=part3_rho1*coef1/(sig2*sig_u^2)+part3*coef4/sig_u+part5_rho1/((1-rho1^2)*sig_u^3)+part5*2*rho1/((sig_u^3)*(1-rho1^2)^2)-p_rho1/(sig_u*(exp(-r1^2)*2/sqrt(pi)));
h1sigurho1_part1=p_sigu'*(-p_rho1./deno2(:,1));
h1sigurho1_part2=sum(p_sigu_rho1_prep./deno(:,1));
h1sigurho1=(h1sigurho1_part1/sig_u+h1sigurho1_part2*(exp(-r1^2)*2/sqrt(pi)))*sig_u;

% h1rho1 series

h1rho1b=h1brho1';
h1rho1tu=h1turho1';
h1rho1sig2=h1sig2rho1';
h1rho1sigu=h1sigurho1';

p_rho1_rho1_prep=part3_rho1*coef3+part3*2*rho1/(sig2*sig_u*(1-rho1^2)^2)-(part5_rho1*coef1/(sig1^2)+part5*coef5)+p_rho1*coef1/(exp(-r1^2)*2/sqrt(pi))+prob1ode*(1+rho1^2)/(1-rho1^2)^2;
h1rho1rho1_part1=p_rho1'*(-p_rho1./deno2(:,1));
h1rho1rho1_part2=sum(p_rho1_rho1_prep./deno(:,1));
h1rho1rho1=(h1rho1rho1_part1/(exp(-r1^2)*2/sqrt(pi))+h1rho1rho1_part2*(exp(-r1^2)*2/sqrt(pi)))*(exp(-r1^2)*2/sqrt(pi))-2*g1rho1*(exp(-r1^2)*2/sqrt(pi))*r1/(exp(-r1^2)*2/sqrt(pi));


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% 2. Target
l2=-n2*log(sig2)-(y2-x2*b)'*(y2-x2*b)/(2*sig2^2);
% Gradient
g2b=x2'*(y2-x2*b)/(sig2^2);
```

```
g2sig2=-n2/sig2+(y2-x2*b)'*(y2-x2*b)/(sig2^3);
% Hessian
h2bb=-x2'*x2/(sig2^2);h2bsig2=-2*x2'*(y2-x2*b)/(sig2^3);h2sig2b=h2bsig2';h2sig2sig2=n2/sig2^2-3*(y2-x2*b)'*(y2-x2*b)/(sig2^4);


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% 3. Lower
funode4=@(t4,y)normpdf(-log(t4)+y4-x4*b,0,sig2).*normpdf(log(-log(t4))-x4*tl,m4*(-log(t4)+y4-x4*b),sig4)./t4;[~,result4]=ode45(funode4,[lb 0.6 ub],zeros(n4,1));prob4ode=result4(3,:)';
l4=sum(log(prob4ode));

tic;
part1_funode4=@(t4,y)(exp(-(-log(t4)+y4-x4*b).^2/(2*sig2^2)-(log(-log(t4))-x4*tl-m4*(-log(t4)+y4-x4*b)).^2/(2*sig4^2))./t4).*(-log(t4)+y4-x4*b);
part2_funode4=@(t4,y)(exp(-(-log(t4)+y4-x4*b).^2/(2*sig2^2)-(log(-log(t4))-x4*tl-m4*(-log(t4)+y4-x4*b)).^2/(2*sig4^2))./t4).*(log(-log(t4))-x4*tl-m4*(-log(t4)+y4-x4*b));
part3_funode4=@(t4,y)(exp(-(-log(t4)+y4-x4*b).^2/(2*sig2^2)-(log(-log(t4))-x4*tl-m4*(-log(t4)+y4-x4*b)).^2/(2*sig4^2))./t4).*((-log(t4)+y4-x4*b).*(log(-log(t4))-x4*tl-m4*(-log(t4)+y4-x4*b)));
part4_funode4=@(t4,y)(exp(-(-log(t4)+y4-x4*b).^2/(2*sig2^2)-(log(-log(t4))-x4*tl-m4*(-log(t4)+y4-x4*b)).^2/(2*sig4^2))./t4).*((-log(t4)+y4-x4*b).^2);
part5_funode4=@(t4,y)(exp(-(-log(t4)+y4-x4*b).^2/(2*sig2^2)-(log(-log(t4))-x4*tl-m4*(-log(t4)+y4-x4*b)).^2/(2*sig4^2))./t4).*((log(-log(t4))-x4*tl-m4*(-log(t4)+y4-x4*b)).^2);

part6_funode4=@(t4,y)(exp(-(-log(t4)+y4-x4*b).^2/(2*sig2^2)-(log(-log(t4))-x4*tl-m4*(-log(t4)+y4-x4*b)).^2/(2*sig4^2))./t4).*(((-log(t4)+y4-x4*b).^2).*(log(-log(t4))-x4*tl-m4*(-log(t4)+y4-x4*b)));
part7_funode4=@(t4,y)(exp(-(-log(t4)+y4-x4*b).^2/(2*sig2^2)-(log(-log(t4))-x4*tl-m4*(-log(t4)+y4-x4*b)).^2/(2*sig4^2))./t4).*((-log(t4)+y4-x4*b).*((log(-log(t4))-x4*tl-m4*(-log(t4)+y4-x4*b)).^2));
part8_funode4=@(t4,y)(exp(-(-log(t4)+y4-x4*b).^2/(2*sig2^2)-(log(-log(t4))-x4*tl-m4*(-log(t4)+y4-x4*b)).^2/(2*sig4^2))./t4).*((-log(t4)+y4-x4*b).^3);
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part9_funode4=@(t4,y)(exp(-(-log(t4)+y4-x4*b).^2/(2*sig2^2)-(log(-log(t4))-x4*tl-m4
    *(-log(t4)+y4-x4*b)).^2/(2*sig4^2))./t4).*((log(-log(t4))-x4*tl-m4*(-log(t4)+y4-
    x4*b)).^3);

part10_funode4=@(t4,y)(exp(-(-log(t4)+y4-x4*b).^2/(2*sig2^2)-(log(-log(t4))-x4*tl-m4
    *(-log(t4)+y4-x4*b)).^2/(2*sig4^2))./t4).*(((-log(t4)+y4-x4*b).^2).*((log(-log(t4
    ))-x4*tl-m4*(-log(t4)+y4-x4*b)).^2));
part11_funode4=@(t4,y)(exp(-(-log(t4)+y4-x4*b).^2/(2*sig2^2)-(log(-log(t4))-x4*tl-m4
    *(-log(t4)+y4-x4*b)).^2/(2*sig4^2))./t4).*(((-log(t4)+y4-x4*b).^3).*(log(-log(t4)
    )-x4*tl-m4*(-log(t4)+y4-x4*b)));
part12_funode4=@(t4,y)(exp(-(-log(t4)+y4-x4*b).^2/(2*sig2^2)-(log(-log(t4))-x4*tl-m4
    *(-log(t4)+y4-x4*b)).^2/(2*sig4^2))./t4).*((-log(t4)+y4-x4*b).*((log(-log(t4))-x4
    *tl-m4*(-log(t4)+y4-x4*b)).^3));
part13_funode4=@(t4,y)(exp(-(-log(t4)+y4-x4*b).^2/(2*sig2^2)-(log(-log(t4))-x4*tl-m4
    *(-log(t4)+y4-x4*b)).^2/(2*sig4^2))./t4).*((-log(t4)+y4-x4*b).^4);
part14_funode4=@(t4,y)(exp(-(-log(t4)+y4-x4*b).^2/(2*sig2^2)-(log(-log(t4))-x4*tl-m4
    *(-log(t4)+y4-x4*b)).^2/(2*sig4^2))./t4).*((log(-log(t4))-x4*tl-m4*(-log(t4)+y4-
    x4*b)).^4);

[~,part1_result4]=ode45(part1_funode4,[lb 0.6 ub],zeros(n4,1)); part1=part1_result4
    (3,:)'/(2*pi*sig2*sig4);
[~,part2_result4]=ode45(part2_funode4,[lb 0.6 ub],zeros(n4,1)); part2=part2_result4
    (3,:)'/(2*pi*sig2*sig4);
[~,part3_result4]=ode45(part3_funode4,[lb 0.6 ub],zeros(n4,1)); part3=part3_result4
    (3,:)'/(2*pi*sig2*sig4);
[~,part4_result4]=ode45(part4_funode4,[lb 0.6 ub],zeros(n4,1)); part4=part4_result4
    (3,:)'/(2*pi*sig2*sig4);
[~,part5_result4]=ode45(part5_funode4,[lb 0.6 ub],zeros(n4,1)); part5=part5_result4
    (3,:)'/(2*pi*sig2*sig4);

[~,part6_result4]=ode45(part6_funode4,[lb 0.6 ub],zeros(n4,1)); part6=part6_result4
    (3,:)'/(2*pi*sig2*sig4);
[~,part7_result4]=ode45(part7_funode4,[lb 0.6 ub],zeros(n4,1)); part7=part7_result4
    (3,:)'/(2*pi*sig2*sig4);
[~,part8_result4]=ode45(part8_funode4,[lb 0.6 ub],zeros(n4,1)); part8=part8_result4
    (3,:)'/(2*pi*sig2*sig4);
```

```
[~,part9_result4]=ode45(part9_funode4,[lb 0.6 ub],zeros(n4,1)); part9=part9_result4(3,:)'/(2*pi*sig2*sig4);

[~,part10_result4]=ode45(part10_funode4,[lb 0.6 ub],zeros(n4,1)); part10=part10_result4(3,:)'/(2*pi*sig2*sig4);
[~,part11_result4]=ode45(part11_funode4,[lb 0.6 ub],zeros(n4,1)); part11=part11_result4(3,:)'/(2*pi*sig2*sig4);
[~,part12_result4]=ode45(part12_funode4,[lb 0.6 ub],zeros(n4,1)); part12=part12_result4(3,:)'/(2*pi*sig2*sig4);
[~,part13_result4]=ode45(part13_funode4,[lb 0.6 ub],zeros(n4,1)); part13=part13_result4(3,:)'/(2*pi*sig2*sig4);
[~,part14_result4]=ode45(part14_funode4,[lb 0.6 ub],zeros(n4,1)); part14=part14_result4(3,:)'/(2*pi*sig2*sig4);

deno=repmat(prob4ode,1,nvar);
deno2=repmat(prob4ode.^2,1,nvar);
coef1=rho4/(1-rho4^2);coef2=sig_l*rho4/(sqrt(1-rho4^2)*sig4^3);coef3=m4/(rho4*sig4^2);coef4=(1+rho4^2)/(sig2*sig_l*(1-rho4^2)^2);coef5=(1+3*rho4^2)/((sig_l^2)*(1-rho4^2)^3);% sig_l/(sig2*sig4*sig4)+2*m4*sig_l*rho4/(sqrt(1-rho4^2)*sig4^3);%

p_b=repmat(part1,1,nvar).*x4/(sig2^2)-m4*repmat(part2,1,nvar).*x4/(sig4^2);
p_tl=repmat(part2,1,nvar).*x4/(sig4^2);
p_sig2=(part4/(sig2^3)-m4*part3/(sig2*sig4^2)-prob4ode/sig2)*sig2;
p_sigl=(part3*m4/(sig_l*sig4^2)+part5/(sig_l*sig4^2)-prob4ode/sig_l)*sig_l;
p_rho4=(part3*m4/(rho4*sig4^2)-part5*coef1/(sig4^2)+prob4ode*coef1)*exp(-r4^2)*2/sqrt(pi);


%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Gradient Components
g4b=sum(p_b./deno,1)';g4tl=sum(p_tl./deno,1)';g4sig2=sum(p_sig2./prob4ode);g4sigl=sum(p_sigl./prob4ode);g4rho4=sum(p_rho4./prob4ode);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Common secondary derivative terms
part1_sig2=part8/(sig2^3)-part6*m4/(sig2*sig4*sig4)-part1/sig2;
```

```
part1_sigl=part6*m4/(sig_l*sig4*sig4)+part7/(sig_l*sig4*sig4)-part1/sig_l;
part1_rho4=part6*coef3-part7*coef2+part1*coef1;

part2_sig2=part6/(sig2^3)-part7*m4/(sig2*sig4*sig4)+part1*m4/sig2-part2/sig2;
part2_sigl=part7*m4/(sig_l*sig4*sig4)+part9/(sig_l*sig4*sig4)-part1*m4/sig_l-part2/
    sig_l;
part2_rho4=part7*coef3-part9*coef2-part1*m4/rho4+part2*coef1;

part3_sig2=(part11/(sig2^2)-part10*m4/(sig4^2)+part4*m4-part3)/sig2;
part3_sigl=(part10*m4/(sig4^2)+part12/(sig4^2)-part4*m4-part3)/sig_l;
part3_rho4=part10*coef3-part12*coef2-part4*m4/rho4+part3*coef1;

part4_sig2=part13/(sig2^3)-part11*m4/(sig4*sig4*sig2)-part4/sig2;
part4_sigl=(part11*m4/(sig4^2)+part10/(sig4^2)-part4)/sig_l;
part4_rho4=part11*coef3-part10*coef2+part4*coef1;

part5_sigl=part12*m4/(sig_l*sig4^2)+part14/(sig_l*sig4^2)-part5/sig_l-part3*2*rho4/
    sig2;
part5_rho4=part12*coef3-part14*coef2-part3*2*m4/rho4+part5*coef1;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

% Hessian

p_b_b=part4/(sig2^4)-part3*2*m4/(sig2*sig2*sig4*sig4)+part5*m4*m4/(sig4^4);
h4bb_part1=p_b'*(-p_b./deno2);
h4bb_part2=x4'*((repmat(p_b_b,1,nvar).*x4)./deno)-x4'*x4*(1/(sig2^2)+m4*m4/(sig4^2))
    ;
h4bb=h4bb_part1+h4bb_part2;

p_b_tl=part3/(sig4^2*sig2^2)-part5*m4/(sig4^4);
h4btl_part1=p_b'*(-p_tl./deno2);
h4btl_part2=x4'*((repmat(p_b_tl,1,nvar).*x4)./deno)+x4'*x4*m4/(sig4^2);
h4btl=h4btl_part1+h4btl_part2;

p_b_sig2_prep=part1_sig2/(sig2^2)-2*part1/(sig2^3)+part2*m4/(sig2*sig4*sig4)-
    part2_sig2*m4/(sig4^2);p_b_sig2=repmat(p_b_sig2_prep,1,nvar).*x4;
```

```
h4bsig2_part1=p_b'*(-p_sig2./deno2(:,1));
h4bsig2_part2=sum(p_b_sig2./deno,1)';
h4bsig2=h4bsig2_part1+h4bsig2_part2*sig2;

p_b_sigl_prep=part1_sigl/(sig2^2)-part2_sigl*m4/(sig4^2)+part2*m4/(sig4*sig4*sig_l);
    p_b_sigl=repmat(p_b_sigl_prep,1,nvar).*x4;
h4bsigl_part1=p_b'*(-p_sigl./deno2(:,1));
h4bsigl_part2=sum(p_b_sigl./deno,1)';
h4bsigl=h4bsigl_part1+h4bsigl_part2*sig_l;

p_b_rho4_prep=part1_rho4/(sig2^2)-part2_rho4*m4/(sig4^2)-part2*coef4;p_b_rho4=repmat
    (p_b_rho4_prep,1,nvar).*x4;
h4brho4_part1=p_b'*(-p_rho4./deno2(:,1));
h4brho4_part2=sum(p_b_rho4./deno,1)';
h4brho4=h4brho4_part1+h4brho4_part2*exp(-r4^2)*2/sqrt(pi);

% h_tl series
h4tlb=h4btl';

p_tl_tl=repmat(part5,1,nvar).*x4/(sig4^2);
h4tltl_part1=p_tl'*(-p_tl./deno2);
h4tltl_part2=x4'*(p_tl_tl./deno)-x4'*x4;
h4tltl=h4tltl_part1+h4tltl_part2/(sig4^2);

p_tl_sig2=repmat(part2_sig2/(sig4^2),1,nvar).*x4;
h4tlsig2_part1=p_tl'*(-p_sig2./deno2(:,1));
h4tlsig2_part2=sum(p_tl_sig2./deno,1)';
h4tlsig2=h4tlsig2_part1+h4tlsig2_part2*sig2;

p_tl_sigl_prep=part2_sigl/(sig4^2)-part2*2/(sig_l*sig4^2);p_tl_sigl=repmat(
    p_tl_sigl_prep,1,nvar).*x4;
h4tlsigl_part1=p_tl'*(-p_sigl./deno2(:,1));
h4tlsigl_part2=sum(p_tl_sigl./deno,1)';
h4tlsigl=h4tlsigl_part1+h4tlsigl_part2*sig_l;
```

```
p_tl_rho4_prep=part2_rho4/(sig4^2)+part2*2*sig_l*rho4/((sig4^3)*sqrt(1-rho4^2));
    p_tl_rho4=repmat(p_tl_rho4_prep,1,nvar).*x4;
h4tlrho4_part1=p_tl'*(-p_rho4./deno2(:,1));
h4tlrho4_part2=sum(p_tl_rho4./deno,1)';
h4tlrho4=h4tlrho4_part1+h4tlrho4_part2*exp(-r4^2)*2/sqrt(pi);

% h4sig2 series

h4sig2b=h4bsig2';
h4sig2tl=h4tlsig2';

p_sig2_sig2_prep=-3*part4/(sig2^4)+part4_sig2/(sig2^3)+2*part3*m4/(sig2*sig2*sig4*
    sig4)-part3_sig2*m4/(sig2*sig4*sig4)+prob4ode/(sig2^2)-p_sig2/(sig2^2);
h4sig2sig2_part1=p_sig2'*(-p_sig2./deno2(:,1));
h4sig2sig2_part2=sum(p_sig2_sig2_prep./deno(:,1));
h4sig2sig2=(h4sig2sig2_part1/sig2+h4sig2sig2_part2*sig2)*sig2+g4sig2;

p_sig2_sigl_prep=part4_sigl/(sig2^3)-(part3_sigl/sig_l-part3/(sig_l^2))*coef1/(sig2
    ^2)-p_sigl/(sig_l*sig2);
h4sig2sigl_part1=p_sig2'*(-p_sigl./deno2(:,1));
h4sig2sigl_part2=sum(p_sig2_sigl_prep./deno(:,1));
h4sig2sigl=(h4sig2sigl_part1/sig2+h4sig2sigl_part2*sig_l)*sig2;

p_sig2_rho4_prep=part4_rho4/(sig2^3)-part3*coef4/sig2-part3_rho4*coef1/(sig2*sig2*
    sig_l)-(p_rho4/(exp(-r4^2)*2/sqrt(pi)))/sig2;
h4sig2rho4_part1=p_sig2'*(-p_rho4./deno2(:,1));
h4sig2rho4_part2=sum(p_sig2_rho4_prep./deno(:,1));
h4sig2rho4=(h4sig2rho4_part1/sig2+h4sig2rho4_part2*(exp(-r4^2)*2/sqrt(pi)))*sig2;

% h4sigl series
h4siglb=h4bsigl';
h4sigltl=h4tlsigl';
h4siglsig2=h4sig2sigl';

```

```
p_sigl_sigl_prep=part3_sigl*m4/(sig_l*sig4^2)-part3*2*coef1/(sig2*sig_l^3)+
    part5_sigl/(sig_l*sig4*sig4)-part5*3/(sig_l*sig_l*sig4*sig4)-p_sigl/(sig_l^2)+
    prob4ode/(sig_l^2);
h4siglsigl_part1=p_sigl'*(-p_sigl./deno2(:,1));
h4siglsigl_part2=sum(p_sigl_sigl_prep./deno(:,1));
h4siglsigl=(h4siglsigl_part1/sig_l+h4siglsigl_part2*sig_l)*sig_l+g4sigl;

p_sigl_rho4_prep=part3_rho4*coef1/(sig2*sig_l^2)+part3*coef4/sig_l+part5_rho4/((1-
    rho4^2)*sig_l^3)+part5*2*rho4/((sig_l^3)*(1-rho4^2)^2)-p_rho4/(sig_l*(exp(-r4^2)
    *2/sqrt(pi)));
h4siglrho4_part1=p_sigl'*(-p_rho4./deno2(:,1));
h4siglrho4_part2=sum(p_sigl_rho4_prep./deno(:,1));
h4siglrho4=(h4siglrho4_part1/sig_l+h4siglrho4_part2*(exp(-r4^2)*2/sqrt(pi)))*sig_l;

% h4rho4 series

h4rho4b=h4brho4';
h4rho4tl=h4tlrho4';
h4rho4sig2=h4sig2rho4';
h4rho4sigl=h4siglrho4';

p_rho4_rho4_prep=part3_rho4*coef3+part3*2*rho4/(sig2*sig_l*(1-rho4^2)^2)-(part5_rho4
    *coef1/(sig4^2)+part5*coef5)+p_rho4*coef1/(exp(-r4^2)*2/sqrt(pi))+prob4ode*(1+
    rho4^2)/(1-rho4^2)^2;
h4rho4rho4_part1=p_rho4'*(-p_rho4./deno2(:,1));
h4rho4rho4_part2=sum(p_rho4_rho4_prep./deno(:,1));
h4rho4rho4=(h4rho4rho4_part1/(exp(-r4^2)*2/sqrt(pi))+h4rho4rho4_part2*(exp(-r4^2)*2/
    sqrt(pi)))*(exp(-r4^2)*2/sqrt(pi))-2*g4rho4*(exp(-r4^2)*2/sqrt(pi))*r4/(exp(-r4
    ^2)*2/sqrt(pi));
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% 4. Ordinary Observation
tic;
% upper part
funode01=@(t01,y)normpdf(log(t01)+y0-x0*b,0,sig2).*(1-normcdf(log(-log(t01))-x0*tu,
    m1*(log(t01)+y0-x0*b),sig1))./t01;
[~,result01]=ode45(funode01,[lb 0.6 ub],zeros(n0,1));prob01ode=result01(3,:)';
```

```
% lower part
funode02=@(t02,y)normpdf(-log(t02)+y0-x0*b,0,sig2).*(1-normcdf(log(-log(t02))-x0*tl,m4*(-log(t02)+y0-x0*b),sig4))./t02;
[~,result02]=ode45(funode02,[lb 0.6 ub],zeros(n0,1));prob02ode=result02(3,:)';
prob0ode=prob01ode+prob02ode;
l0=sum(log(prob0ode));

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% function 01
part1_funode01=@(t01,y)normpdf(log(t01)+y0-x0*b,0,sig2).*(log(t01)+y0-x0*b)./t01;
part2_funode01=@(t01,y)normpdf(log(t01)+y0-x0*b,0,sig2).*normcdf(log(-log(t01))-x0*tu,m1*(log(t01)+y0-x0*b),sig1).*(log(t01)+y0-x0*b)./t01;
part3_funode01=@(t01,y)normpdf(log(t01)+y0-x0*b,0,sig2).*normpdf(log(-log(t01))-x0*tu,m1*(log(t01)+y0-x0*b),sig1)./t01;
part4_funode01=@(t01,y)normpdf(log(t01)+y0-x0*b,0,sig2).*((log(t01)+y0-x0*b).^2)./t01;
part5_funode01=@(t01,y)normpdf(log(t01)+y0-x0*b,0,sig2)./t01;
part6_funode01=@(t01,y)normpdf(log(t01)+y0-x0*b,0,sig2).*normcdf(log(-log(t01))-x0*tu,m1*(log(t01)+y0-x0*b),sig1).*((log(t01)+y0-x0*b).^2)./t01;
part7_funode01=@(t01,y)normpdf(log(t01)+y0-x0*b,0,sig2).*normcdf(log(-log(t01))-x0*tu,m1*(log(t01)+y0-x0*b),sig1)./t01;
part8_funode01=@(t01,y)normpdf(log(t01)+y0-x0*b,0,sig2).*normpdf(log(-log(t01))-x0*tu,m1*(log(t01)+y0-x0*b),sig1).*(log(t01)+y0-x0*b)./t01;
part9_funode01=@(t01,y)normpdf(log(t01)+y0-x0*b,0,sig2).*normpdf(log(-log(t01))-x0*tu,m1*(log(t01)+y0-x0*b),sig1).*(log(-log(t01))-x0*tu-m1*(log(t01)+y0-x0*b))./t01;
part10_funode01=@(t01,y)normpdf(log(t01)+y0-x0*b,0,sig2).*((log(t01)+y0-x0*b).^3)./t01;
part11_funode01=@(t01,y)normpdf(log(t01)+y0-x0*b,0,sig2).*normcdf(log(-log(t01))-x0*tu,m1*(log(t01)+y0-x0*b),sig1).*((log(t01)+y0-x0*b).^3)./t01;
part12_funode01=@(t01,y)normpdf(log(t01)+y0-x0*b,0,sig2).*normpdf(log(-log(t01))-x0*tu,m1*(log(t01)+y0-x0*b),sig1).*((log(t01)+y0-x0*b).^2)./t01;
part13_funode01=@(t01,y)normpdf(log(t01)+y0-x0*b,0,sig2).*normpdf(log(-log(t01))-x0*tu,m1*(log(t01)+y0-x0*b),sig1).*(log(t01)+y0-x0*b).*(log(-log(t01))-x0*tu-m1*(log(t01)+y0-x0*b))./t01;
```

```
part14_funode01=@(t01,y)normpdf(log(t01)+y0-x0*b,0,sig2).*((log(t01)+y0-x0*b).^4)./
    t01;
part15_funode01=@(t01,y)normpdf(log(t01)+y0-x0*b,0,sig2).*normcdf(log(-log(t01))-x0*
    tu,m1*(log(t01)+y0-x0*b),sig1).*((log(t01)+y0-x0*b).^4)./t01;
part16_funode01=@(t01,y)normpdf(log(t01)+y0-x0*b,0,sig2).*normpdf(log(-log(t01))-x0*
    tu,m1*(log(t01)+y0-x0*b),sig1).*((log(t01)+y0-x0*b).^3)./t01;

part18_funode01=@(t01,y)normpdf(log(t01)+y0-x0*b,0,sig2).*normpdf(log(-log(t01))-x0*
    tu,m1*(log(t01)+y0-x0*b),sig1).*((log(t01)+y0-x0*b).^2).*(log(-log(t01))-x0*tu-m1
    *(log(t01)+y0-x0*b))./t01;
part19_funode01=@(t01,y)normpdf(log(t01)+y0-x0*b,0,sig2).*normpdf(log(-log(t01))-x0*
    tu,m1*(log(t01)+y0-x0*b),sig1).*(log(t01)+y0-x0*b).*((log(-log(t01))-x0*tu-m1*(
    log(t01)+y0-x0*b)).^2)./t01;
part20_funode01=@(t01,y)normpdf(log(t01)+y0-x0*b,0,sig2).*normpdf(log(-log(t01))-x0*
    tu,m1*(log(t01)+y0-x0*b),sig1).*((log(-log(t01))-x0*tu-m1*(log(t01)+y0-x0*b)).^2)
    ./t01;
part21_funode01=@(t01,y)normpdf(log(t01)+y0-x0*b,0,sig2).*normpdf(log(-log(t01))-x0*
    tu,m1*(log(t01)+y0-x0*b),sig1).*((log(-log(t01))-x0*tu-m1*(log(t01)+y0-x0*b)).^3)
    ./t01;

[~,part1_result01]=ode45(part1_funode01,[lb 0.6 ub],zeros(n0,1));part1=
    part1_result01(3,:)';
[~,part2_result01]=ode45(part2_funode01,[lb 0.6 ub],zeros(n0,1));part2=
    part2_result01(3,:)';
[~,part3_result01]=ode45(part3_funode01,[lb 0.6 ub],zeros(n0,1));part3=
    part3_result01(3,:)';
[~,part4_result01]=ode45(part4_funode01,[lb 0.6 ub],zeros(n0,1));part4=
    part4_result01(3,:)';
[~,part5_result01]=ode45(part5_funode01,[lb 0.6 ub],zeros(n0,1));part5=
    part5_result01(3,:)';
[~,part6_result01]=ode45(part6_funode01,[lb 0.6 ub],zeros(n0,1));part6=
    part6_result01(3,:)';
[~,part7_result01]=ode45(part7_funode01,[lb 0.6 ub],zeros(n0,1));part7=
    part7_result01(3,:)';
[~,part8_result01]=ode45(part8_funode01,[lb 0.6 ub],zeros(n0,1));part8=
    part8_result01(3,:)';
```

```
[~,part9_result01]=ode45(part9_funode01,[lb 0.6 ub],zeros(n0,1));part9=
    part9_result01(3,:)';
[~,part10_result01]=ode45(part10_funode01,[lb 0.6 ub],zeros(n0,1));part10=
    part10_result01(3,:)';
[~,part11_result01]=ode45(part11_funode01,[lb 0.6 ub],zeros(n0,1));part11=
    part11_result01(3,:)';
[~,part12_result01]=ode45(part12_funode01,[lb 0.6 ub],zeros(n0,1));part12=
    part12_result01(3,:)';
[~,part13_result01]=ode45(part13_funode01,[lb 0.6 ub],zeros(n0,1));part13=
    part13_result01(3,:)';
[~,part14_result01]=ode45(part14_funode01,[lb 0.6 ub],zeros(n0,1));part14=
    part14_result01(3,:)';
[~,part15_result01]=ode45(part15_funode01,[lb 0.6 ub],zeros(n0,1));part15=
    part15_result01(3,:)';
[~,part16_result01]=ode45(part16_funode01,[lb 0.6 ub],zeros(n0,1));part16=
    part16_result01(3,:)';

part17=part16;
[~,part18_result01]=ode45(part18_funode01,[lb 0.6 ub],zeros(n0,1));part18=
    part18_result01(3,:)';
[~,part19_result01]=ode45(part19_funode01,[lb 0.6 ub],zeros(n0,1));part19=
    part19_result01(3,:)';
[~,part20_result01]=ode45(part20_funode01,[lb 0.6 ub],zeros(n0,1));part20=
    part20_result01(3,:)';
[~,part21_result01]=ode45(part21_funode01,[lb 0.6 ub],zeros(n0,1));part21=
    part21_result01(3,:)';

p01_b=part1/(sig2^2)-part2/(sig2^2)-part3*m1;
p01_tu=part3;
p01_sig2=part4/(sig2^3)-part5/sig2-part6/(sig2^3)+part7/sig2-part8*m1/sig2;
p01_sigu=part8*m1/sig_u+part9/sig_u;
p01_rho1=part8*m1/rho1-part9*rho1/(1-rho1^2);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
part1_b=repmat(part4/(sig2^2)-part5,1,nvar).*x0;
part1_sig2=part10/(sig2^3)-part1/sig2;
```

```

part2_b=repmat(part6/(sig2^2)+part8*m1-part7,1,nvar).*x0;
part2_tu=-repmat(part8,1,nvar).*x0;
part2_sig2=part11/(sig2^3)-part2/sig2+part12*m1/sig2;
part2_sigu=-part12*m1/sig_u-part13/sig_u;
part2_rho1=-part12*m1/rho1+part13*rho1/(1-rho1^2);

part3_b=repmat(part8/(sig2^2)-part9*m1/(sig1^2),1,nvar).*x0;
part3_tu=repmat(part9/(sig1^2),1,nvar).*x0;
part3_sig2=part12/(sig2^3)-part3/sig2-part13*m1/(sig2*sig1*sig1);
part3_sigu=part13*m1/(sig_u*sig1*sig1)+part20/(sig_u*sig1*sig1)-part3/sig_u;
part3_rho1=part13*m1/(rho1*sig1*sig1)-part20*rho1/((1-rho1^2)*sig1*sig1)+part3*rho1
    /(1-rho1^2);

part4_sig2=part14/(sig2^3)-part4/sig2;
part5_sig2=part4/(sig2^3)-part5/sig2;
part6_sig2=part15/(sig2^3)-part6/sig2+part16*m1/sig2;
part7_sig2=part6/(sig2^3)-part7/sig2+part8*m1/sig2;
part8_sig2=part17/(sig2^3)-part8/sig2-part18*m1/(sig2*sig1*sig1);

part6_sigu=-part16*m1/sig_u-part18/sig_u;
part7_sigu=-part8*m1/sig_u-part9/sig_u;
part8_sigu=part18*m1/(sig_u*(sig1^2))+part19/(sig_u*(sig1^2))-part8/sig_u;

part6_rho1=-part16*m1/rho1+part18*rho1/(1-rho1^2);
part7_rho1=-part8*m1/rho1+part9*rho1/(1-rho1^2);
part8_rho1=part18*m1/(rho1*(sig1^2))-part19*rho1/((1-rho1^2)*(sig1^2))+part8*rho1
    /(1-rho1^2);

part9_sigu=part19*m1/(sig_u*sig1*sig1)+part21/(sig_u*sig1*sig1)-part9/sig_u-part8*m1
    /sig_u;
part9_rho1=part19*m1/(rho1*sig1*sig1)-part21*rho1/((1-rho1^2)*sig1*sig1)+part9*rho1
    /(1-rho1^2)-part8*m1/rho1;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Hessian Component
```

```
p01_bb=part1_b/(sig2^2)-part2_b/(sig2^2)-part3_b*m1;
p01_btu=-part2_tu/(sig2^2)-part3_tu*m1;
p01_bsig2=part1_sig2/(sig2^2)-part2_sig2/(sig2^2)-part3_sig2*m1-(2*part1/(sig2^3)-2*part2/(sig2^3)-part3*m1/sig2);
p01_bsigu=-part2_sigu/(sig2^2)-(part3_sigu*m1+part3*m1/sig_u);
p01_brho1=-part2_rho1/(sig2^2)-(part3_rho1*m1+part3*m1/rho1);

p01_tutu=part3_tu;
p01_tusig2=part3_sig2;
p01_tusigu=part3_sigu;
p01_turho1=part3_rho1;

p01_sig2sig2=-3*part4/(sig2^4)+part5/(sig2^2)+3*part6/(sig2^4)-part7/(sig2^2)+2*part8*m1/(sig2^2)+(part4_sig2/(sig2^3)-part5_sig2/sig2-part6_sig2/(sig2^3)+part7_sig2/sig2-part8_sig2*m1/sig2);
p01_sig2sigu=-part6_sigu/(sig2^3)+part7_sigu/sig2-(part8_sigu*m1/sig2+part8*m1/(sig2*sig_u));
p01_sig2rho1=-part6_rho1/(sig2^3)+part7_rho1/sig2-(part8_rho1*m1/sig2+part8*sig_u/(sig2^2));

p01_sigusigu=part8_sigu*rho1/sig2+(part9_sigu/sig_u-part9/(sig_u^2));
p01_sigurho1=part8_rho1*rho1/sig2+part8/sig2+part9_rho1/sig_u;

p01_rho1rho1=part8_rho1*sig_u/sig2-(part9_rho1*rho1/(1-rho1^2)+part9*(1+rho1^2)/((1-rho1^2)^2));
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% function 02

part1_funode02=@(t02,y)normpdf(-log(t02)+y0-x0*b,0,sig2).*(-log(t02)+y0-x0*b)./t02;
part2_funode02=@(t02,y)normpdf(-log(t02)+y0-x0*b,0,sig2).*normcdf(log(-log(t02))-x0*tl,m4*(-log(t02)+y0-x0*b),sig4).*(-log(t02)+y0-x0*b)./t02;
part3_funode02=@(t02,y)normpdf(-log(t02)+y0-x0*b,0,sig2).*normpdf(log(-log(t02))-x0*tl,m4*(-log(t02)+y0-x0*b),sig4)./t02;
part4_funode02=@(t02,y)normpdf(-log(t02)+y0-x0*b,0,sig2).*((-log(t02)+y0-x0*b).^2)./t02;
part5_funode02=@(t02,y)normpdf(-log(t02)+y0-x0*b,0,sig2)./t02;
```

```
part6_funode02=@(t02,y)normpdf(-log(t02)+y0-x0*b,0,sig2).*normcdf(log(-log(t02))-x0*tl,m4*(-log(t02)+y0-x0*b),sig4).*((-log(t02)+y0-x0*b).^2)./t02;
part7_funode02=@(t02,y)normpdf(-log(t02)+y0-x0*b,0,sig2).*normcdf(log(-log(t02))-x0*tl,m4*(-log(t02)+y0-x0*b),sig4)./t02;
part8_funode02=@(t02,y)normpdf(-log(t02)+y0-x0*b,0,sig2).*normpdf(log(-log(t02))-x0*tl,m4*(-log(t02)+y0-x0*b),sig4).*(-log(t02)+y0-x0*b)./t02;
part9_funode02=@(t02,y)normpdf(-log(t02)+y0-x0*b,0,sig2).*normpdf(log(-log(t02))-x0*tl,m4*(-log(t02)+y0-x0*b),sig4).*(log(-log(t02))-x0*tl-m4*(-log(t02)+y0-x0*b))./t02;
part10_funode02=@(t02,y)normpdf(-log(t02)+y0-x0*b,0,sig2).*((-log(t02)+y0-x0*b).^3)./t02;
part11_funode02=@(t02,y)normpdf(-log(t02)+y0-x0*b,0,sig2).*normcdf(log(-log(t02))-x0*tl,m4*(-log(t02)+y0-x0*b),sig4).*((-log(t02)+y0-x0*b).^3)./t02;
part12_funode02=@(t02,y)normpdf(-log(t02)+y0-x0*b,0,sig2).*normpdf(log(-log(t02))-x0*tl,m4*(-log(t02)+y0-x0*b),sig4).*((-log(t02)+y0-x0*b).^2)./t02;
part13_funode02=@(t02,y)normpdf(-log(t02)+y0-x0*b,0,sig2).*normpdf(log(-log(t02))-x0*tl,m4*(-log(t02)+y0-x0*b),sig4).*(-log(t02)+y0-x0*b).*(log(-log(t02))-x0*tl-m4*(-log(t02)+y0-x0*b))./t02;
part14_funode02=@(t02,y)normpdf(-log(t02)+y0-x0*b,0,sig2).*((-log(t02)+y0-x0*b).^4)./t02;
part15_funode02=@(t02,y)normpdf(-log(t02)+y0-x0*b,0,sig2).*normcdf(log(-log(t02))-x0*tl,m4*(-log(t02)+y0-x0*b),sig4).*((-log(t02)+y0-x0*b).^4)./t02;
part16_funode02=@(t02,y)normpdf(-log(t02)+y0-x0*b,0,sig2).*normpdf(log(-log(t02))-x0*tl,m4*(-log(t02)+y0-x0*b),sig4).*((-log(t02)+y0-x0*b).^3)./t02;
part18_funode02=@(t02,y)normpdf(-log(t02)+y0-x0*b,0,sig2).*normpdf(log(-log(t02))-x0*tl,m4*(-log(t02)+y0-x0*b),sig4).*((-log(t02)+y0-x0*b).^2).*(log(-log(t02))-x0*tl-m4*(-log(t02)+y0-x0*b))./t02;
part19_funode02=@(t02,y)normpdf(-log(t02)+y0-x0*b,0,sig2).*normpdf(log(-log(t02))-x0*tl,m4*(-log(t02)+y0-x0*b),sig4).*(-log(t02)+y0-x0*b).*((log(-log(t02))-x0*tl-m4*(-log(t02)+y0-x0*b)).^2)./t02;
part20_funode02=@(t02,y)normpdf(-log(t02)+y0-x0*b,0,sig2).*normpdf(log(-log(t02))-x0*tl,m4*(-log(t02)+y0-x0*b),sig4).*((log(-log(t02))-x0*tl-m4*(-log(t02)+y0-x0*b)).^2)./t02;
part21_funode02=@(t02,y)normpdf(-log(t02)+y0-x0*b,0,sig2).*normpdf(log(-log(t02))-x0*tl,m4*(-log(t02)+y0-x0*b),sig4).*((log(-log(t02))-x0*tl-m4*(-log(t02)+y0-x0*b)).^3)./t02;
```

```
[~,part1_result02]=ode45(part1_funode02,[lb 0.6 ub],zeros(n0,1));part1=part1_result02(3,:)';
[~,part2_result02]=ode45(part2_funode02,[lb 0.6 ub],zeros(n0,1));part2=part2_result02(3,:)';
[~,part3_result02]=ode45(part3_funode02,[lb 0.6 ub],zeros(n0,1));part3=part3_result02(3,:)';
[~,part4_result02]=ode45(part4_funode02,[lb 0.6 ub],zeros(n0,1));part4=part4_result02(3,:)';
[~,part5_result02]=ode45(part5_funode02,[lb 0.6 ub],zeros(n0,1));part5=part5_result02(3,:)';
[~,part6_result02]=ode45(part6_funode02,[lb 0.6 ub],zeros(n0,1));part6=part6_result02(3,:)';
[~,part7_result02]=ode45(part7_funode02,[lb 0.6 ub],zeros(n0,1));part7=part7_result02(3,:)';
[~,part8_result02]=ode45(part8_funode02,[lb 0.6 ub],zeros(n0,1));part8=part8_result02(3,:)';
[~,part9_result02]=ode45(part9_funode02,[lb 0.6 ub],zeros(n0,1));part9=part9_result02(3,:)';
[~,part10_result02]=ode45(part10_funode02,[lb 0.6 ub],zeros(n0,1));part10=part10_result02(3,:)';
[~,part11_result02]=ode45(part11_funode02,[lb 0.6 ub],zeros(n0,1));part11=part11_result02(3,:)';
[~,part12_result02]=ode45(part12_funode02,[lb 0.6 ub],zeros(n0,1));part12=part12_result02(3,:)';
[~,part13_result02]=ode45(part13_funode02,[lb 0.6 ub],zeros(n0,1));part13=part13_result02(3,:)';
[~,part14_result02]=ode45(part14_funode02,[lb 0.6 ub],zeros(n0,1));part14=part14_result02(3,:)';
[~,part15_result02]=ode45(part15_funode02,[lb 0.6 ub],zeros(n0,1));part15=part15_result02(3,:)';
[~,part16_result02]=ode45(part16_funode02,[lb 0.6 ub],zeros(n0,1));part16=part16_result02(3,:)';

part17=part16;
```

```
[~,part18_result02]=ode45(part18_funode02,[lb 0.6 ub],zeros(n0,1));part18=
    part18_result02(3,:)';
[~,part19_result02]=ode45(part19_funode02,[lb 0.6 ub],zeros(n0,1));part19=
    part19_result02(3,:)';
[~,part20_result02]=ode45(part20_funode02,[lb 0.6 ub],zeros(n0,1));part20=
    part20_result02(3,:)';
[~,part21_result02]=ode45(part21_funode02,[lb 0.6 ub],zeros(n0,1));part21=
    part21_result02(3,:)';

p02_b=part1/(sig2^2)-part2/(sig2^2)-part3*m4;
p02_tl=part3;
p02_sig2=part4/(sig2^3)-part5/sig2-part6/(sig2^3)+part7/sig2-part8*m4/sig2;
p02_sigl=part8*m4/sig_l+part9/sig_l;
p02_rho4=part8*m4/rho4-part9*rho4/(1-rho4^2);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
part1_b=repmat(part4/(sig2^2)-part5,1,nvar).*x0;
part1_sig2=part10/(sig2^3)-part1/sig2;

part2_b=repmat(part6/(sig2^2)+part8*m4-part7,1,nvar).*x0;
part2_tl=-repmat(part8,1,nvar).*x0;
part2_sig2=part11/(sig2^3)-part2/sig2+part12*m4/sig2;
part2_sigl=-part12*m4/sig_l-part13/sig_l;
part2_rho4=-part12*m4/rho4+part13*rho4/(1-rho4^2);

part3_b=repmat(part8/(sig2^2)-part9*m4/(sig4^2),1,nvar).*x0;
part3_tl=repmat(part9/(sig4^2),1,nvar).*x0;
part3_sig2=part12/(sig2^3)-part3/sig2-part13*m4/(sig2*sig4*sig4);
part3_sigl=part13*m4/(sig_l*sig4*sig4)+part20/(sig_l*sig4*sig4)-part3/sig_l;
part3_rho4=part13*m4/(rho4*sig4*sig4)-part20*rho4/((1-rho4^2)*sig4*sig4)+part3*rho4
    /(1-rho4^2);

part4_sig2=part14/(sig2^3)-part4/sig2;
part5_sig2=part4/(sig2^3)-part5/sig2;
part6_sig2=part15/(sig2^3)-part6/sig2+part16*m4/sig2;
part7_sig2=part6/(sig2^3)-part7/sig2+part8*m4/sig2;
```

```
part8_sig2=part17/(sig2^3)-part8/sig2-part18*m4/(sig2*sig4*sig4);

part6_sigl=-part16*m4/sig_l-part18/sig_l;
part7_sigl=-part8*m4/sig_l-part9/sig_l;
part8_sigl=part18*m4/(sig_l*(sig4^2))+part19/(sig_l*(sig4^2))-part8/sig_l;

part6_rho4=-part16*m4/rho4+part18*rho4/(1-rho4^2);
part7_rho4=-part8*m4/rho4+part9*rho4/(1-rho4^2);
part8_rho4=part18*m4/(rho4*(sig4^2))-part19*rho4/((1-rho4^2)*(sig4^2))+part8*rho4
    /(1-rho4^2);

part9_sigl=part19*m4/(sig_l*sig4*sig4)+part21/(sig_l*sig4*sig4)-part9/sig_l-part8*m4
    /sig_l;
part9_rho4=part19*m4/(rho4*sig4*sig4)-part21*rho4/((1-rho4^2)*sig4*sig4)+part9*rho4
    /(1-rho4^2)-part8*m4/rho4;

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Hessian Components
p02_bb=part1_b/(sig2^2)-part2_b/(sig2^2)-part3_b*m4;
p02_btl=-part2_tl/(sig2^2)-part3_tl*m4;
p02_bsig2=part1_sig2/(sig2^2)-part2_sig2/(sig2^2)-part3_sig2*m4-(2*part1/(sig2^3)-2*
    part2/(sig2^3)-part3*m4/sig2);
p02_bsigl=-part2_sigl/(sig2^2)-(part3_sigl*m4+part3*m4/sig_l);
p02_brho4=-part2_rho4/(sig2^2)-(part3_rho4*m4+part3*m4/rho4);

p02_tltl=part3_tl;
p02_tlsig2=part3_sig2;
p02_tlsigl=part3_sigl;
p02_tlrho4=part3_rho4;

p02_sig2sig2=-3*part4/(sig2^4)+part5/(sig2^2)+3*part6/(sig2^4)-part7/(sig2^2)+2*
    part8*m4/(sig2^2)+(part4_sig2/(sig2^3)-part5_sig2/sig2-part6_sig2/(sig2^3)+
    part7_sig2/sig2-part8_sig2*m4/sig2);
p02_sig2sigl=-part6_sigl/(sig2^3)+part7_sigl/sig2-(part8_sigl*m4/sig2+part8*m4/(sig2
    *sig_l));
```

```
p02_sig2rho4=-part6_rho4/(sig2^3)+part7_rho4/sig2-(part8_rho4*m4/sig2+part8*sig_l/(sig2^2));

p02_siglsigl=part8_sigl*rho4/sig2+(part9_sigl/sig_l-part9/(sig_l^2));
p02_siglrho4=part8_rho4*rho4/sig2+part8/sig2+part9_rho4/sig_l;

p02_rho4rho4=part8_rho4*sig_l/sig2-(part9_rho4*rho4/(1-rho4^2)+part9*(1+rho4^2)/((1-rho4^2)^2));
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
p0_b=p01_b+p02_b;
p0_tu=p01_tu;
p0_tl=p02_tl;
p0_sig2=p01_sig2+p02_sig2;
p0_sigu=p01_sigu;
p0_sigl=p02_sigl;
p0_rho1=p01_rho1;
p0_rho4=p02_rho4;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Gradient
g0b=x0'*(p0_b./prob0ode);
g0tu=x0'*(p0_tu./prob0ode);
g0tl=x0'*(p0_tl./prob0ode);
g0sig2=sum(p0_sig2./prob0ode);
g0sigu=sum(p0_sigu./prob0ode);
g0sigl=sum(p0_sigl./prob0ode);
g0rho1=sum(p0_rho1./prob0ode);
g0rho4=sum(p0_rho4./prob0ode);

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Hessian
dn1=prob0ode;
dn12=prob0ode.^2;
dnn=repmat(prob0ode,1,nvar);
dnn2=dnn.^2;

znn=zeros(nvar,nvar);zn1=zeros(nvar,1);
```

```
% derivative of 1/p0
rp0_b=-(repmat(p0_b,1,nvar).*x0)./dnn2;
rp0_tu=-(repmat(p0_tu,1,nvar).*x0)./dnn2;
rp0_tl=-(repmat(p0_tl,1,nvar).*x0)./dnn2;
rp0_sig2=-p0_sig2*sig2./dn12;
rp0_sigu=-p0_sigu*sig_u./dn12;
rp0_sigl=-p0_sigl*sig_l./dn12;
rp0_rho1=-p0_rho1*(exp(-r1^2)*2/sqrt(pi))./dn12;
rp0_rho4=-p0_rho4*(exp(-r4^2)*2/sqrt(pi))./dn12;
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% h0b series    p02_brho4
h0bb_part1=(repmat(p0_b,1,nvar).*x0)'*rp0_b;
h0bb_part2=x0'*((p01_bb+p02_bb)./dnn);
h0bb=h0bb_part1+h0bb_part2;

h0btu_part1=(repmat(p0_b,1,nvar).*x0)'*rp0_tu;
h0btu_part2=x0'*(p01_btu./dnn);
h0btu=h0btu_part1+h0btu_part2;

h0btl_part1=(repmat(p0_b,1,nvar).*x0)'*rp0_tl;
h0btl_part2=x0'*(p02_btl./dnn);
h0btl=h0btl_part1+h0btl_part2;

h0bsig2_part1=(repmat(p0_b,1,nvar).*x0)'*rp0_sig2;
h0bsig2_part2=x0'*((p01_bsig2+p02_bsig2)./dn1);
h0bsig2=h0bsig2_part1+h0bsig2_part2*sig2;

h0bsigu_part1=(repmat(p0_b,1,nvar).*x0)'*rp0_sigu;
h0bsigu_part2=x0'*(p01_bsigu./dn1);
h0bsigu=h0bsigu_part1+h0bsigu_part2*sig_u;

h0bsigl_part1=(repmat(p0_b,1,nvar).*x0)'*rp0_sigl;
h0bsigl_part2=x0'*(p02_bsigl./dn1);
h0bsigl=h0bsigl_part1+h0bsigl_part2*sig_l;
```

```
h0brho1_part1=(repmat(p0_b,1,nvar).*x0)'*rp0_rho1;
h0brho1_part2=x0'*(p01_brho1./dn1);
h0brho1=h0brho1_part1+h0brho1_part2*(exp(-r1^2)*2/sqrt(pi));

h0brho4_part1=(repmat(p0_b,1,nvar).*x0)'*rp0_rho4;
h0brho4_part2=x0'*(p02_brho4./dn1);
h0brho4=h0brho4_part1+h0brho4_part2*(exp(-r4^2)*2/sqrt(pi));

% h0tu series
h0tub=h0btu';

h0tutu_part1=(repmat(p0_tu,1,nvar).*x0)'*rp0_tu;
h0tutu_part2=x0'*(p01_tutu./dnn);
h0tutu=h0tutu_part1+h0tutu_part2;

h0tutl=(repmat(p0_tu,1,nvar).*x0)'*rp0_tl;

h0tusig2_part1=(repmat(p0_tu,1,nvar).*x0)'*rp0_sig2;
h0tusig2_part2=x0'*(p01_tusig2./dn1);
h0tusig2=h0tusig2_part1+h0tusig2_part2*sig2;

h0tusigu_part1=(repmat(p0_tu,1,nvar).*x0)'*rp0_sigu;
h0tusigu_part2=x0'*(p01_tusigu./dn1);
h0tusigu=h0tusigu_part1+h0tusigu_part2*sig_u;

h0tusigl=(repmat(p0_tu,1,nvar).*x0)'*rp0_sigl;

h0turho1_part1=(repmat(p0_tu,1,nvar).*x0)'*rp0_rho1;
h0turho1_part2=x0'*(p01_turho1./dn1);
h0turho1=h0turho1_part1+h0turho1_part2*(exp(-r1^2)*2/sqrt(pi));

h0turho4=(repmat(p0_tu,1,nvar).*x0)'*rp0_rho4;

% h0tl series
h0tlb=h0btl';

```

```
h0tltu=h0tutl';

h0tltl_part1=(repmat(p0_tl,1,nvar).*x0)'*rp0_tl;
h0tltl_part2=x0'*(p02_tltl./dnn);
h0tltl=h0tltl_part1+h0tltl_part2;

h0tlsig2_part1=(repmat(p0_tl,1,nvar).*x0)'*rp0_sig2;
h0tlsig2_part2=x0'*(p02_tlsig2./dn1);
h0tlsig2=h0tlsig2_part1+h0tlsig2_part2*sig2;

h0tlsigu=(repmat(p0_tl,1,nvar).*x0)'*rp0_sigu;

h0tlsigl_part1=(repmat(p0_tl,1,nvar).*x0)'*rp0_sigl;
h0tlsigl_part2=x0'*(p02_tlsigl./dn1);
h0tlsigl=h0tlsigl_part1+h0tlsigl_part2*sig_l;

h0tlrho1=(repmat(p0_tl,1,nvar).*x0)'*rp0_rho1;

h0tlrho4_part1=(repmat(p0_tl,1,nvar).*x0)'*rp0_rho4;
h0tlrho4_part2=x0'*(p02_tlrho4./dn1);
h0tlrho4=h0tlrho4_part1+h0tlrho4_part2*(exp(-r4^2)*2/sqrt(pi));

% h0sig2 series
h0sig2b=h0bsig2';

h0sig2tu=h0tusig2';

h0sig2tl=h0tlsig2';

h0sig2sig2_part1=p0_sig2'*rp0_sig2;
h0sig2sig2_part2=sum((p01_sig2sig2+p02_sig2sig2)./dn1);
h0sig2sig2=(h0sig2sig2_part1+h0sig2sig2_part2*sig2)*sig2+g0sig2*sig2;

h0sig2sigu_part1=p0_sig2'*rp0_sigu;
h0sig2sigu_part2=sum((p01_sig2sigu)./dn1);
h0sig2sigu=(h0sig2sigu_part1+h0sig2sigu_part2*sig_u)*sig2;
```

```
h0sig2sigl_part1=p0_sig2'*rp0_sigl;
h0sig2sigl_part2=sum((p02_sig2sigl)./dn1);
h0sig2sigl=(h0sig2sigl_part1+h0sig2sigl_part2*sig_l)*sig2;

h0sig2rho1_part1=p0_sig2'*rp0_rho1;
h0sig2rho1_part2=sum((p01_sig2rho1)./dn1);
h0sig2rho1=(h0sig2rho1_part1+h0sig2rho1_part2*(exp(-r1^2)*2/sqrt(pi)))*sig2;

h0sig2rho4_part1=p0_sig2'*rp0_rho4;
h0sig2rho4_part2=sum((p02_sig2rho4)./dn1);
h0sig2rho4=(h0sig2rho4_part1+h0sig2rho4_part2*(exp(-r4^2)*2/sqrt(pi)))*sig2;

% h0sigu series

h0sigub=h0bsigu';

h0sigutu=h0tusigu';

h0sigutl=h0tlsigu';

h0sigusig2=h0sig2sigu';

h0sigusigu_part1=p0_sigu'*rp0_sigu;
h0sigusigu_part2=sum((p01_sigusigu)./dn1);
h0sigusigu=(h0sigusigu_part1+h0sigusigu_part2*sig_u)*sig_u+g0sigu*sig_u;

h0sigusigl=p0_sigu'*rp0_sigl*sig_u;

h0sigurho1_part1=p0_sigu'*rp0_rho1;
h0sigurho1_part2=sum((p01_sigurho1)./dn1);
h0sigurho1=(h0sigurho1_part1+h0sigurho1_part2*(exp(-r1^2)*2/sqrt(pi)))*sig_u;

h0sigurho4=p0_sigu'*rp0_rho4*sig_u;

% h0sigl series
```

```
h0siglb=h0bsigl';

h0sigltu=h0tusigl';

h0sigltl=h0tlsigl';

h0siglsig2=h0sig2sigl';

h0siglsigu=h0sigusigl';

h0siglsigl_part1=p0_sigl'*rp0_sigl;
h0siglsigl_part2=sum((p02_siglsigl)./dn1);
h0siglsigl=(h0siglsigl_part1+h0siglsigl_part2*sig_l)*sig_l+g0sigl*sig_l;

h0siglrho1=p0_sigl'*rp0_rho1*sig_l;

h0siglrho4_part1=p0_sigl'*rp0_rho4;
h0siglrho4_part2=sum((p02_siglrho4)./dn1);
h0siglrho4=(h0siglrho4_part1+h0siglrho4_part2*(exp(-r4^2)*2/sqrt(pi)))*sig_l;

% h0rho1 series

h0rho1b=h0brho1';

h0rho1tu=h0turho1';

h0rho1tl=h0tlrho1';

h0rho1sig2=h0sig2rho1';

h0rho1sigu=h0sigurho1';

h0rho1sigl=h0siglrho1';

h0rho1rho1_part1=p0_rho1'*rp0_rho1;
```

```
h0rho1rho1_part2=sum((p01_rho1rho1)./dn1);
h0rho1rho1=(h0rho1rho1_part1+h0rho1rho1_part2*(exp(-r1^2)*2/sqrt(pi)))*(exp(-r1^2)
    *2/sqrt(pi))-g0rho1*(2*r1*exp(-r1^2)*2/sqrt(pi));

h0rho1rho4=p0_rho1'*rp0_rho4*(exp(-r1^2)*2/sqrt(pi));

% h0rho4 series

h0rho4b=h0brho4';

h0rho4tu=h0turho4';

h0rho4tl=h0tlrho4';

h0rho4sig2=h0sig2rho4';

h0rho4sigu=h0sigurho4';

h0rho4sigl=h0siglrho4';

h0rho4rho1=h0rho1rho4';

h0rho4rho4_part1=p0_rho4'*rp0_rho4;
h0rho4rho4_part2=sum((p02_rho4rho4)./dn1);
h0rho4rho4=(h0rho4rho4_part1+h0rho4rho4_part2*(exp(-r4^2)*2/sqrt(pi)))*(exp(-r4^2)
    *2/sqrt(pi))-g0rho4*(2*r4*exp(-r4^2)*2/sqrt(pi));

h0=[h0bb,h0btu,h0btl,h0bsig2,h0bsigu,h0bsigl,h0brho1,h0brho4;...
    h0tub,h0tutu,h0tutl,h0tusig2,h0tusigu,h0tusigl,h0turho1,h0turho4;...
    h0tlb,h0tltu,h0tltl,h0tlsig2,h0tlsigu,h0tlsigl,h0tlrho1,h0tlrho4;...
    h0sig2b,h0sig2tu,h0sig2tl,h0sig2sig2,h0sig2sigu,h0sig2sigl,h0sig2rho1,h0sig2rho4
    ;...
    h0sigub,h0sigutu,h0sigutl,h0sigusig2,h0sigusigu,h0sigusigl,h0sigurho1,h0sigurho4
    ;...
    h0siglb,h0sigltu,h0sigltl,h0siglsig2,h0siglsigu,h0siglsigl,h0siglrho1,h0siglrho4
    ;...
```

```
    h0rho1b,h0rho1tu,h0rho1tl,h0rho1sig2,h0rho1sigu,h0rho1sigl,h0rho1rho1,h0rho1rho4
    ;...
    h0rho4b,h0rho4tu,h0rho4tl,h0rho4sig2,h0rho4sigu,h0rho4sigl,h0rho4rho1,h0rho4rho4
    ];

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% Log likelihood
l=-l1-l2-l4-l0;

% Gradient
gb=g1b+g2b+g4b;
gsig2=g1sig2+g2sig2+g4sig2;
g124=[gb;g1tu;g4tl;gsig2;g1sigu;g4sigl;g1rho1;g4rho4];
g0=[g0b;g0tu;g0tl;g0sig2*sig2;g0sigu*sig_u;g0sigl*sig_l;g0rho1*(exp(-r1^2)*2/sqrt(pi
    ));g0rho4*(exp(-r4^2)*2/sqrt(pi))];
g=-(g124+g0);
% Hassian
hbb=h1bb+h2bb+h4bb;hbsig2=h1bsig2+h2bsig2+h4bsig2;hsig2b=hbsig2';hsig2sig2=
    h1sig2sig2+h2sig2sig2+h4sig2sig2;
znn=zeros(nvar,nvar);zn1=zeros(nvar,1);z1n=zeros(1,nvar);
h124=[hbb,h1btu,h4btl,hbsig2,h1bsigu,h4bsigl,h1brho1,h4brho4;h1tub,h1tutu,znn,
    h1tusig2,h1tusigu,zn1,h1turho1,zn1;h4tlb,znn,h4tltl,h4tlsig2,zn1,h4tlsigl,zn1,
    h4tlrho4;hsig2b,h1sig2tu,h4sig2tl,hsig2sig2,h1sig2sigu,h4sig2sigl,h1sig2rho1,
    h4sig2rho4;h1sigub,h1sigutu,z1n,h1sigusig2,h1sigusigu,0,h1sigurho1,0;h4siglb,z1n,
    h4sigltl,h4siglsig2,0,h4siglsigl,0,h4siglrho4;h1rho1b,h1rho1tu,z1n,h1rho1sig2,
    h1rho1sigu,0,h1rho1rho1,0;h4rho4b,z1n,h4rho4tl,h4rho4sig2,0,h4rho4sigl,0,
    h4rho4rho4];
h=-(h0+h124);
% The accuracy of Gradient and Hassian are confirmed by tests in numerical way.
end
```

Chapter 3

Estimating Determinants of Corporate Credit Ratings when Ratings are Sticky

3.1 Introduction

Credit rating, as a crucial indicator of firms' risk level, assists investors in decision making, and even becomes the standard for the national capital assessment after the *New Basel Accord* in 1999. The credit rating in the US market has deteriorated in the past few decades. More precisely, rating downgrades have dominated the migration trend in these decades, which is supported by empirical evidence. For instance, studies reveal that rating migrations tend to move in the same direction, and the consecutive downgrades are more highly-correlated (Altman & Kao 1992, Lando & Skødeberg 2002, Du 2003).[1] In other words, rating downgrades become the dominating trend in both frequency and persistence. Unsurprisingly, the observed rating levels demonstrate downward momentum, and the average rating dropped by three notches between 1985 and 2009 Baghai et al. (2014).

The phenomenon of rating deterioration has led to debate whether it is a result of a decreasing credit quality or the tightening of rating standard (Blume et al. 1998, Jorion et al. 2009, Alp 2013). A rating event involves two main participants, which are firms to be evaluated and rating agencies. Hence, possible causes of the deterioration may come from either of the two sides, which are the quality of the borrower and the stringency of rating standard. Studies, such as Blume et al. (1998), Jorion et al. (2009), and Alp (2013), focus on the cause of deterioration by matching ratings to firm characteristics. They use empirical models attempting to control for all relevant covariates and leave the effect of the (time-varying) rating standards to year dummies. Their general conclusion is that the continuous tightening of rating standards indeed contributes to the observed rating deterioration. While offering valuable insights, this line of research cannot put a definitive end to the debate, since it does not address the question of the evolution of the quality of firms' borrowing.

This chapter attempts to fill the void in the literature by focusing on the mechanism of rating migrations. The migrations are informative because they consider the most recent updates of rating agencies, and hence involve the trigger and motivation of agencies' decisions. To directly assess the interaction between credit quality and varying standards, it is necessary to filter out the effect of each component. This

[1]Rating downgrades tend to be followed by downgrades, and upgrades tend to be followed by further upgrades.

cannot be done without reflecting in the empirical design the mechanism of how ratings are assigned and updated. The empirical work in rating area, largely agrees on two attributes of credit rating, namely, the high stability and the slow respondance. Empirical results suggest that ratings demonstrate considerably higher stability compared with credit quality in current economic conditions (Kealhofer et al. 1998, Carey & Hrycay 2001, Löffler 2004). In other words, it is not unusual that the credit quality changes but the ratings do not. Moreover, credit ratings are also characterized by their slow respondence. For example, during the Asian crisis in 1997, rating agencies made no downgrades in 1996 and even the first half year of 1997. Designed as a forward-looking indicator, the slow reaction of credit rating events investors' trust. These two features imply that the credit ratings are sticky.

This research proposes a stickiness-based model of rating process, and explores the impact of stickiness on rating migrations.[2] Differing from existing empirical studies (Blume et al. 1998, Jorion et al. 2009, Alp 2013), the model has embedded stickiness in the empirical design and the estimation. The basic assumption in current empirical research is that firms' credit quality matches its rating. More precisely, rating agencies apply immediate rating update to eliminate any deviations outside of its current rating, resulting in ratings being accurate at all times. However, this research challenges the assumption of accurate and contemporaneous rating. We argue that credit ratings demonstrate the high-stability and the slow-respondance features, which makes credit rating a lagged indicator of credit quality. In other words, these features of credit rating introduce some tolerance for deviations of credit quality

[2]Here, we define stickiness as the deviation of credit quality from the nominal range of its rating. Suppose the nominal Z-score range of rating AA is 5 to 10. The stickiness refers to the situation when credit quality moves to 4 or 11 but the rating still remains of AA.

outside of the ratings' nominal range in the same way in which lump sum cost affect optimal control policies. The tolerance may be small, but it represents a different mechanism of rating process and leads to a specific definition of stickiness. More precisely, the immediate migration assumed in most other studies interprets credit rating as a continuous spectrum with a single boundary between each two adjacent categories. However, the stickiness-based model presented here assumes that adjacent rating categories overlap to a certain extent, similar to the hysteresis effect.[3] The overlapping area belongs to both categories, and hence it reflects the credit quality entering next rating range but the deviation not being sufficient to invoke rating migration, which is our definition of stickiness. Compared to the traditional view (which implies zero lump sum cost of rating adjustment), our model allows for two boundaries between adjacent rating categories to overlap. Entering the overlapping area solely cannot invoke rating changes, and rating migration will be triggered only when the deviation is large enough.

Our first aim is to empirically demonstrate the existence of stickiness in credit ratings, which is the theoretical basis of the mechanism this chapter proposes. This demonstration is important for a number of reasons. First, the proposed mechanism must fit the observed phenomenon in market, which is the high stability and slow respondence. Second, whether or not stickiness exists determines whether it is appropriate to use the standard estimation models such as linear regression and ordered logit. Earlier empirical studies also question the reliability of credit rating as a contemporaneous measure of credit quality (Altman 1998, Becker & Milbourn 2008, Hull

[3]For example, suppose ratings A and B are next to each other. Traditional view assumes their nominal Z-score ranges are 10 to 6, and 5 to 1, respectively. However, our model under the stickiness framework assumes the ranges of ratings A and B are 10 to 5 and 6 to 1, respectively.

et al. 2004, Norden & Weber 2004), and hence the existence of stickiness deserves more careful studying. Our findings strongly support the presence of rating stickiness by showing that the introduction of stickiness in the estimation absorbs the decreasing trend of yearly intercept, and that the terms of explanatory variables interacting with migration dummies are statistically significant. Further, we decompose the stickiness into the t-dimensional one and the z-dimensional one. The t-dimensional stickiness measures agencies' delay in implementing rating migrations. In other words, the length of period between credit quality moving outside of the nominal range and the migration. In comparison, the z-dimensional stickiness measures agencies' tolerance of the deviation of credit quality from the target range of its rating. We find that the z-stickiness dominates and explains most of the effect.

Our results directly address the debate about the interaction of credit quality deterioration and rating standards. The stickiness-based framework allows to isolate the effect of rating standards, and such a filtering reveals the variation of credit quality during our sample period. Although the observed rating levels deteriorate during 1985 to 2015, the credit quality actually improves with an increment of 0.573 measured in Z-score. Moreover, our findings confirm the contribution of more stringent standards to the deterioration, in line with other studies (Blume et al. 1998, Jorion et al. 2009, Alp 2013). However, the unique feature of our model is that it reveals the asymmetry between upgrade and downgrade decisions. More precisely, the rating standards for upgrades strengthens, but the the standard for downgrades does not exhibit a clear trend in how stringent it is. Further tests confirm the asymmetry. The frequency and magnitude of upgrades drop dramatically, but this is not the case for downgrades.

This asymmetry further leads to the observed deterioration from the perspective of stickiness-based mechanism.

Finally, our study also contributes to the literature that investigates determinants of credit ratings (Horrigan 1966, Pogue & Soldofsky 1969, Kamstra et al. 2001). Although rating agencies are thought to apply complex and proprietary approaches (such as grid method) to assign ratings, academic interest in estimating determinants of ratings and to establish the link between ratings and firm characteristics is high.[4] This chapter introduces stickiness and hence provides a different perspective on the understanding of the rating process. The comparison between our results and those in the existing literature can further demonstrate the effect of stickiness. For instance, the existing literature documents a decreasing trend in year intercepts to support the tightening rating standard argument. When we control in estimation for stickiness, this decreasing trend disappears. Moreover, different from static methods applied in earlier studies, we have adjusted the ordered probit model to fit the stickiness feature and to incorporate the dynamic aspect of the rating mechanism. This adjusted model nests the ordered probit, and it can empirically detect the existence of stickiness. Further tests in Appendix 3.6 provide further support that results estimated by the adjusted model converge to that by the standard ordered probit when the stickiness goes to zero.

Section 3.2 summarizes existing literature to provide the background. Section 3.3 explains the model and the estimation. Section 3.4 describes the data. Section

[4]Moody's (2018) illustrates the application of grid method in rating assignment. They firstly identify grid factors, which are dimensions agencies consider to evaluate credit quality (e.g. financial policy, leverage and coverage), and estimate a quality score for each factor. Then, calculate the weighted average value, denoted by x, as the indicator of the overall credit quality. Finally, transforms the numeric quality score x into ratings (e.g. assign Aaa if $x < 1.5$).

3.5 presents main findings, and Section 3.6 concludes.

3.2 Literature Review

Empirical work has provided plenty of evidence on high stability of credit ratings (Kealhofer et al. 1998, Carey & Hrycay 2001, Löffler 2004). Usually, the stability reflects in the situation that rating updates fall behind variation of credit quality, and hence detrimentally affect the accuracy of ratings. Ellis (1997), using survey data of 200 CFOs and 400 institutional fixed income investors, reports that 70% of the interviewees believe that ratings should improve to reflect recent positive changes in credit quality. Kealhofer et al. (1998) create a rating of current credit quality based on default probability suggested by Merton (1974). Compared with this current-condition rating, the agency rating demonstrates a much higher probability of staying in the same rating category (around 90%). Carey & Hrycay (2001) allocate issuers into rating categories following banks' internal rating methodology. This rating grade reflects borrowers point-in-time credit quality as this method fulfills the frequent information update needed for banks' monitoring and risk-control purposes. This current-condition rating indicates that 40%-50% observations remain unchanged over one-year horizon, while the remaining rate in agency rating is 80%-90%, indicating that agency ratings are twice more stable than current-condition ratings.

The high stability of credit ratings has created tolerance for the deviation of actual quality from the nominal rating range, and empirical evidence demonstrates the correlation between the magnitude of this deviation and the probability of rating migrations (Altman & Rijken 2004, Mora 2006, Posch 2011). These empirical results show that rating migrations are triggered when the borrowers' actual credit quality exceeds the nominal quality of their current ratings by 1.25 notches. Mora (2006) provides more direct evidence about the rating drifts mechanism, and states that rating changes when the divergence between actual quality and assigned ratings is sufficiently large. Posch (2011) further measures the amount of tolerance (inertia) by extending the model with frictions to allow for non-constant thresholds, and shows that default probability has to change by at least two notches before rating agencies react.[5]

The stickiness proposed in this research also receives theoretical support from the structure of agency rating market. Cheng & Neamtiu (2009) emphasize the lack of timeliness and increasing regulatory pressure in agency ratings, which implies that the accuracy of the ratings deserves more careful investigation. There is evidence from existing literature explaining the origins of stickiness. In general, agencies have incentives to make credit rating sticky considering their profitability and reputation. Löffler (2005) documents that agencies attempt to avoid rating reversal after a migration, which contributes to the stability which causes stickiness.[6] Moreover, Jeon & Lovo (2013) introduce the 'reputation build-up' which suggests that frequent rating adjustments harm the profitability of the agencies by weakening their reputation to potential issuers. More precisely, Bolton et al. (2012) elaborate on the "rating

[5]Default probability is the indicator of credit quality in Posch (2011), and it is the basis of rating assignment. For example, if the default probability p is within the first notch ($0 < p \leqslant 10\%$), this observation will be assigned the best rating AAA.

[6]It is worth to clarify that stability does not equal to stickiness. Stability refers to the observed fact that rating does not change. This feature causes the phenomenon that rating tends to stay even when credit quality has changed. Stickiness refers to this phenomenon that rating does not change when it should.

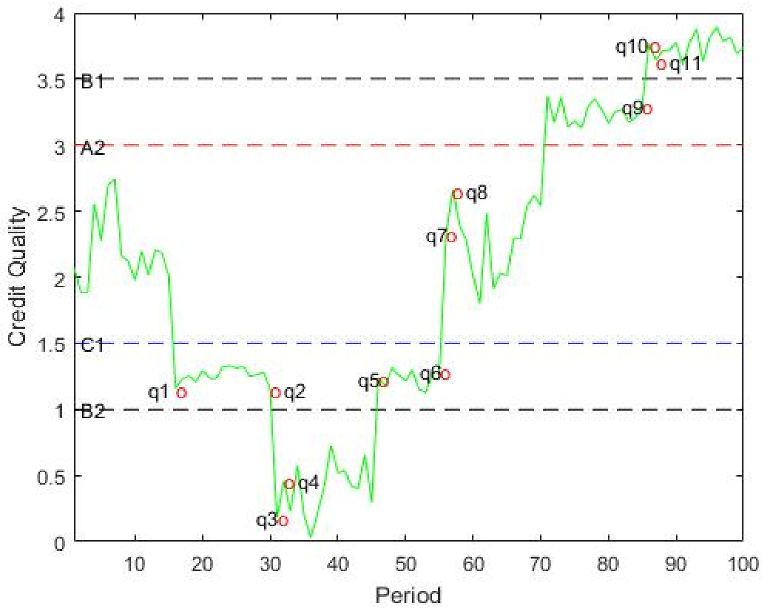
shopping" phenomenon according to which agencies attract business by enhancing the stability as issuers can shop in the market for the best ratings they can get. The issuer-paid pattern indeed results in extra cautiousness for agencies to update ratings which is detrimental to rating accuracy (Xia 2014). Xia (2014) find that introducing investor-paid rating agencies (e.g. Egan-Jones Rating Company) improves the accuracy and timeliness over the traditional issuer-paid ratings (e.g. S&P's rating).

A number of studies investigates the changes in rating standards (Blume et al. 1998, Jorion et al. 2009, Alp 2013, Baghai et al. 2014). Blume et al. (1998) is one of the the early studies that attempts to explain the reported declining credit quality using accounting ratios and market information. Their study finds that the credit rating deterioration is not fully explained by changes in credit quality, and that it is caused at least partly by the increasingly stringent standard. In other words, rating agencies become more and more conservative in issuing higher ratings. Alp (2013) quantifies this effect, showing that tightened standard leads to 1.5 notches drop in ratings from 2002 to 2007. Baghai et al. (2014) find the drop to be 3 notches from 1985 to 2009. Alp (2013) also finds that the the tightening standards pattern applies to investment grade bonds but the speculative grade bonds reflect loosening standards. This is consistent with Jorion et al. (2009) who study the origins of the standards tightening. This study finds that the accounting quality affects the rating standard tightening for the investment grade issuers. After incorporating changes in accounting quality, the tightening pattern in rating standards disappears. Baghai et al. (2014) study the impact of tightening rating standards on firms' behavior, and show that firms affected more by the tightening standards (measured by the difference between the actual rating and the predicted rating assuming constant standards) issue less debt, hold more cash, experience lower growth, and are less likely to access debt market. These studies rely on the ordered probit model, and this static model is subject to the criticism of neglecting ratings' time-varying nature and the effect of omitted variables (Blume et al. 1998, Shumway 2001, Du 2003). Shumway (2001) criticizes the static model for the timeliness bias. Most of the models use rolling-average covariates, which fail to reflect the true level corresponding to the observed rating if there is rating migration just after an observation. Therefore, it is essential to integrate the dynamics of the rating mechanism to explain this phenomenon. This study considers the mechanism by isolating observations whose credit quality moves outside of the ratings' nominal range. This isolation leads to three categories (exceeding the upper threshold, exceeding the lower threshold, and normal "inaction" observations), for which the likelihood function is separately formulated.

3.3 The Model

Rating agencies are careful in choosing the migration frequency since immediate updates harm reputation (Jeon & Lovo 2013), while no updates lead to the criticism from investors (Ellis 1997). The optimal policy for agencies is to wait until the credit quality gap from nominal range becomes sufficiently large, and then implement updates to eliminate the gap. This reaction pattern leads to stickiness in credit ratings. The stickiness-based model proposed here differs from the common approach in the literature on rating mechanism mainly in two aspects. Typically, researchers interpret rating categories as a spectrum of credit quality. In simple words, single boundary separates two adjacent rating categories. However, our model absorbs more dynamics

FIGURE 3.1: Credit Rating Mechanism



The mechanism of rating migrations under stickiness framework. This example contains 100 observations from one firm within consecutive 100 period. The vertical axis indicates credit quality of the firm, and the horizontal axis represents period. There are three rating categories, A, B, and C, in which A indicates the best credit quality and C indicates the worst. The nominal quality range of rating A is the area from line $A2$ and above; the quality range for rating B is the area between lines $B1$ and $B2$; and the range for rating C is the area below line $C1$. The green line presents the path of the firm's credit quality movements, and demonstrates the mechanism of rating migration. A migration is triggered by credit quality crossing the boundaries of its nominal range. For instance, dots $q2$, $q3$, and $q4$ depict the process of a downgrade migration. When credit quality drops from $q2$ to $q3$, it moves outside of rating B's range and this magnitude of deviation exceeds agency's tolerance. A downgrade decision is made at $q3$ but implemented at $q4$ to fit the slow-respondance feature. Inversely, dots $q9$, $q10$, and $q11$ describe a rating upgrade process. Credit quality crosses $B1$, the upper boundary of rating B, to $q10$, and the rating upgrade is observed next period at $q11$.

for the agency, by allowing for an overlap between two neighboring ratings. The overlapping area reflects agencies' tolerance of credit quality deviation, and hence represents the stickiness. Figure 3.1 demonstrates the mechanism of rating migrations under the stickiness framework. There are three rating categories, namely A, B, and C, in which A indicates the best credit quality and C indicates the worst. The nominal quality range of rating A is the area from line $A2$ and above; the quality range for rating B is the area between lines $B1$ and $B2$; and the range for rating C is the area below line $C1$. The area between lines $B1$ and $A2$ is the overlapping range of ratings A and B. It reflects the stickiness by allowing credit quality deviation from either upper (rating A) or lower (rating B) position without invoking migrations.

Secondly, the standard approach simply assumes a perfect match between credit quality and ratings. More precisely, the location of credit quality itself determines its rating. Under this concept, agencies has no freedom but to immediately close any deviation in credit quality by updating. However, this setting contradicts with the stability and slow respondence features of credit ratings. Our model deals with this challenge by forming a rating migration mechanism which is determined by both its current rating and the deviation outside this range. Migrations are invoked by credit quality hitting either upper or lower boundary, but differing from the traditional view, the upper threshold of rating B is not necessary the lower threshold of rating A (suppose ratings A and B are adjacent, and A indicates better quality). As in Figure 3.1, the green line presents the path of the firm's credit quality movements, and demonstrates the mechanism of rating migrations. A migration is triggered by credit quality crossing the boundaries of its nominal range. For instance, points $q2$, $q3$, and $q4$ indicate different credit quality levels, depicting the process of a downgrade migration. Credit quality enters the range of rating C at $q1$, but this movement will not cause downgrade since it has not touched the lower threshold of rating B (the line $B2$). When credit quality drops from $q2$ to $q3$, it moves outside of rating B's range and this magnitude of deviation exceeds agency's tolerance. A downgrade decision is made at $q3$ but implemented at $q4$ to fit the slow-respondance feature. Inversely, points $q9$, $q10$, and $q11$ describe a rating upgrade process. Credit quality crosses $B1$, the upper boundary of rating B, to $q10$, and the rating upgrade is observed next period at $q11$.

In general, our model includes three groups of observations, namely those exceeding the upper threshold, the lower threshold, and the observations which have their ratings correctly matched with credit quality. The estimation of this model accordingly requires an accurate identification for data in each of the three groups. Our identification methodology concentrates on rating migrations. Rating upgrade (downgrade) is triggered by credit quality breaching the upper (lower) threshold of its previous rating range. Hence, the observation before an upgrade (downgrade) is informative for the upper (lower) threshold. For example, there is an upgrade in Figure 3.1 depicted by points $q9$, $q10$, and $q11$. The upgrade happens at $q11$, which suggests that the credit quality at $q10$ exceeds the upper threshold (i.e. $Z_{q10} \geqslant B1$).[7] Hence, observation $q10$ falls into the group of upper threshold. Similarly, $q3$ can be an example of lower threshold identification through downgrade (i.e $Z_{q10} \leqslant B2$).

Further, we use the Z-score to represent the credit quality (Z_{it})

$$Z_{it} = \beta X_{it} + \varepsilon_{it}, \tag{3.1}$$

in which β is the coefficient set, matrix X_{it} contains firm characteristics as covariates, and ε_{it} represents the normally-distributed error term. The Z-score serves as a linking function which transforms firm characteristics to ratings through categorization. Our data contains five rating levels with rating 5 the best quality and rating 1 the worst.[8]

[7]The upgrade action happens one period behind the breach because of the time agencies need to collect and interpret information. This setting fits the slow-respondance feature of credit rating.

[8]Details are given in Section 3.4.

R^*_{it} in equation (3.2) represents observations that are not thresholds. In other words, the credit quality and rating levels are correctly matched for these observations. Each rating level R_i is quantified by the nominal Z-score range with upper and lower boundaries U_i and L_i, respectively.[9] Hence,

$$R^*_{it} = \begin{cases} 5 \quad if\ Z_{it} \in [L_5, \infty) \\ 4 \quad if\ Z_{it} \in [L_4, U_4] \\ 3 \quad if\ Z_{it} \in [L_3, U_3] \\ 2 \quad if\ Z_{it} \in [L_2, U_2] \\ 1 \quad if\ Z_{it} \in (-\infty, U_1] \end{cases} \tag{3.2}$$

the credit quality, denoted by Z_{it} is within the nominal range for each rating level. This part is exactly the same as the ordered probit model because of the same assumption.[10]

$$R^u_{it} = \begin{cases} 4 \quad if\ Z_{it} \geqslant U_4 \\ 3 \quad if\ Z_{it} \geqslant U_3 \\ 2 \quad if\ Z_{it} \geqslant U_2 \\ 1 \quad if\ Z_{it} \geqslant U_1 \end{cases} \tag{3.3}$$

[9]Obviously, rating level 5 indicates the best quality and hence no further upgrade available. Its upper threshold is infinity. On the other hand, the worst rating level 1 has minus infinity as the lower threshold.

[10]The same assumption refers to the accurate match between rating and quality.

R^u_{it} in equation (3.3) is the rating level of an observation in upper threshold group. As stated before, credit quality and ratings are not matched in this group because quality exceeds upper threshold ($Z_{it} \geqslant U_{R_{it}}$) and is the trigger of upgrade.

$$R^l_{it} = \begin{cases} 5 \quad if\, Z_{it} \leqslant L_5 \\ 4 \quad if\, Z_{it} \leqslant L_4 \\ 3 \quad if\, Z_{it} \leqslant L_3 \\ 2 \quad if\, Z_{it} \leqslant L_2 \end{cases} \tag{3.4}$$

Lastly, R^l_{it} in equation set (3.4) means the rating for lower threshold observations, and the trigger of downgrade requires $Z_{it} \leqslant L_{R_{it}}$. We estimate the three parts jointly through the Maximum Likelihood method.

3.4 Data

Our sample contains 1,488 US firms from 1985 to 2014, which leads to 20,557 observations overall. The *S&P* ratings have been obtained from the Compustat Ratings File. This sample excludes observations with negative or zero total assets, financial firms (SIC code 6000-6999), and quasi-governmental enterprises (SIC 9000 and above). Missing explanatory values reduce the sample to 20,557 firm-year observations from 1,488 unique firms for the full sample analysis. We merged ratings based on the original S&P categories: our rating A includes S&P ratings from AAA to AA;

TABLE 3.1: Number of Companies by Year and S&P Rating Category

Year	A	B	C	D	E	Total	A	B	C	D	E
1985	75	164	142	154	159	694	10.8%	23.6%	20.5%	22.2%	22.9%
1986	73	154	159	194	247	827	8.8%	18.6%	19.2%	23.5%	29.9%
1987	69	137	170	230	284	890	7.8%	15.4%	19.1%	25.8%	31.9%
1988	66	135	170	200	242	813	8.1%	16.6%	20.9%	24.6%	29.8%
1989	62	135	173	187	208	765	8.1%	17.6%	22.6%	24.4%	27.2%
1990	57	137	164	177	154	689	8.3%	19.9%	23.8%	25.7%	22.4%
1991	58	149	165	161	116	649	8.9%	23.0%	25.4%	24.8%	17.9%
1992	52	152	182	196	121	703	7.4%	21.6%	25.9%	27.9%	17.2%
1993	50	159	190	222	123	744	6.7%	21.4%	25.5%	29.8%	16.5%
1994	43	146	191	230	113	723	5.9%	20.2%	26.4%	31.8%	15.6%
1995	41	158	182	216	113	710	5.8%	22.3%	25.6%	30.4%	15.9%
1996	41	152	194	234	115	736	5.6%	20.7%	26.4%	31.8%	15.6%
1997	41	147	221	238	103	750	5.5%	19.6%	29.5%	31.7%	13.7%
1998	34	160	224	265	110	793	4.3%	20.2%	28.2%	33.4%	13.9%
1999	29	143	222	260	94	748	3.9%	19.1%	29.7%	34.8%	12.6%
2000	25	130	214	233	107	709	3.5%	18.3%	30.2%	32.9%	15.1%
2001	20	119	207	235	104	685	2.9%	17.4%	30.2%	34.3%	15.2%
2002	24	113	211	256	98	702	3.4%	16.1%	30.1%	36.5%	14.0%
2003	21	108	217	253	92	691	3.0%	15.6%	31.4%	36.6%	13.3%
2004	21	101	217	259	103	701	3.0%	14.4%	31.0%	36.9%	14.7%
2005	25	94	229	231	99	678	3.7%	13.9%	33.8%	34.1%	14.6%
2006	25	89	217	239	92	662	3.8%	13.4%	32.8%	36.1%	13.9%
2007	25	90	183	226	98	622	4.0%	14.5%	29.4%	36.3%	15.8%
2008	20	87	183	204	112	606	3.3%	14.4%	30.2%	33.7%	18.5%
2009	19	82	179	186	113	579	3.3%	14.2%	30.9%	32.1%	19.5%
2010	17	77	185	183	96	558	3.0%	13.8%	33.2%	32.8%	17.2%
2011	16	74	187	192	82	551	2.9%	13.4%	33.9%	34.8%	14.9%
2012	16	70	197	177	85	545	2.9%	12.8%	36.1%	32.5%	15.6%
2013	17	73	198	165	70	523	3.3%	14.0%	37.9%	31.5%	13.4%
2014	17	73	200	155	66	511	3.3%	14.3%	39.1%	30.3%	12.9%
Total	1099	3608	5773	6358	3719	20557	5.3%	17.6%	28.1%	30.9%	18.1%

The distribution of ratings for our sample firms over time. The sample contains 1,488 firms from 1985 to 2014, with 20,557 observations overall. The ratings have been obtained from the Compustat Ratings File. We merged ratings based on the original S&P categories: A includes S&P ratings from AAA to AA; rating B includes S&P ratings from $AA-$ to A; rating C includes S&P ratings from $A-$ to BBB; rating D includes S&P ratings from $BBB-$ to $BB-$; and rating E includes S&P ratings $CCC+$ and below.

Table 3.2: Number of Cutoff Identified by Rating Category

	Upper	Middle	Lower	Total
A	0	1001	98	1099
B	23	3272	313	3608
C	159	5207	407	5773
D	329	5632	397	6358
E	338	3381	0	3719
Total	849	18493	1215	20557

Cutoff observations identified for the Adjusted Ordered Probit estimation. Our identification relies on the observed rating migration. More precise, suppose for a firm in two consecutive years t and $t+1$, if there is an upgrade happens in year $t+1$ (eg. $Rating_t = C$ and $Rating_{t+1} = B$), the credit quality in t is regarded as exceeding the upper cutoff of its original rating, and hence observation in t is upper cutoff observation. Inversely, we identify lower cutoff observation based on downgrade. The observations not related to any rating migration are defined as middle category observations.

our rating B includes S&P ratings from $AA-$ to A; our rating C includes S&P ratings from $A-$ to BBB; our rating D includes S&P ratings from $BBB-$ to $BB-$; and our rating D includes S&P ratings $CCC+$ and below.[11] Therefore, this sample includes the full spectrum of S&P rating categories with our ratings from A to C corresponds to the investment grade, and the remaining being the speculative grade. Table 3.1 presents the distribution of ratings in the sample. We identify 849 and 1,215 observations to form the upper threshold and lower threshold groups, respectively. Table 3.2 provides details about the threshold categories.

The selection of explanatory variables follows existing literature (Blume et al. 1998, Jorion et al. 2009, Alp 2013). *Intcov* measures interest coverage calculated by *ebitda* divided by interest expense (*ebitda*/*xint*). Variables $k1$ to $k4$ indicate different

[11] Our merging strategy differs from that in other studies (Blume et al. 1998, Jorion et al. 2009, Alp 2013, Baghai et al. 2014) in terms of the width of each category and the coverage of ratings. Our categorization strategy identifies upper and lower thresholds based on rating migrations. We merge credit ratings to ensure that there are sufficient number of rating migrations for each category in each year.

levels of $Intcov$. We do this to capture the non-linearity of interest coverage effect of credit rating, following Blume et al. (1998). $k1$ indicates $Intcov$ range from 0 to 5 (e.g. an observation with $Intcov = 3$ will have $k1 = 3$, $k2 = 0$, $k3 = 0$, $k4 = 0$). $k2$ indicates $Intcov$ range from 5 to 10 (e.g. an observation with $Intcov = 7$ will have $k1 = 5$, $k2 = 2$, $k3 = 0$, $k4 = 0$). $k3$ indicates a range from 10 to 20, and $k4$ indicates $Intcov$ above 20. Vol is the volatility of profit, calculated as the standard deviation of the last 5 years of $ebitda/sale$. $Tlev$ refers to total leverage measured by total debt divided by total asset ($\frac{dlc+dltt}{at}$). $Rent$ is the rent expense divided by total asset ($xrent/at$). Tan refers tangibility, measured by property, plant and equipment divided by total asset (ppe/at). Dni is a dummy variable which equals to one when net income is negative. $Ddiv$ is a dummy variable which equals to one when a firm pays dividend in a given period. Rd is the research and development expense divided by total asset (xrd/at). Rd is set to zero when the expense is missing. Mtb is the market to book ratio measured by total asset minus book value of equity plus market value of equity and then divided by book value of assets ($(at - bv + mv)/at$). The market value of equity is the product of year-end price and number of shares outstanding ($prcc_f * csho$). Book value of equity is shareholders' equity minus preferred stock liquidating value plus deferred taxes and investment tax credit ($seq - pstkl + txditc$). The deferred tax credit $txditc$ is set to zero if missing. Equity (seq) will be replaced by either common equity plus preferred stock at par value ($ceq + pstk$) or total asset minus total liability ($at - lt$) if missing. Preferred stock liquidating value $prstkl$ will be replaced by either redemption value $pstkrv$ or par value $pstk$ if missing. Firm size ($Size$) is the logarithm of total asset. $Beta$ and $Rmse$ measure systematic risk and

Table 3.3: Descriptive Statistics

	Mean	Median	Std.	10th	90th
Intcov	9.848	5.350	15.221	1.536	19.548
k1	4.044	5.000	1.356	1.801	5.000
k2	1.896	0.544	2.172	0.000	5.000
k3	1.615	0.000	3.326	0.000	10.000
k4	2.491	0.000	10.667	0.000	0.628
Vol	0.035	0.020	0.050	0.006	0.071
Tlev	0.333	0.312	0.180	0.129	0.560
Rent	0.023	0.013	0.031	0.003	0.052
Tan	0.389	0.344	0.236	0.104	0.743
Dni	0.196	0.000	0.397	0.000	1.000
Ddiv	0.671	1.000	0.470	0.000	1.000
Rd	0.016	0.000	0.031	0.000	0.053
Mtb	1.496	1.275	0.682	0.933	2.346
Size	7.942	7.904	1.614	5.848	10.101
Beta	0.952	0.958	2.012	0.231	1.943
Rmse	0.068	0.020	0.167	0.010	0.083

Descriptive statistics of covariates for the whole sample. All continuous variables winsorized at top and bottom 1 percentile. $Intcov$ measures interest coverage calculated by *ebitda* divided by interest expense ($ebitda/xint$). $k1$ measures the $Intcov$ range from 0 to 5 (e.g. an observation with $Intcov = 3$ will have $k1 = 3$, $k2 = 0$, $k3 = 0$, $k4 = 0$). $k2$ measures the range from 5 to 10 (e.g. an observation with $Intcov = 7$ will have $k1 = 5$, $k2 = 2$, $k3 = 0$, $k4 = 0$). $k3$ measures the range from 10 to 20, and $k4$ measures the range above 20. Vol is the volatility of current and past four year profits ($ebitda/sale$). $Tlev$is leverage measured by debt divided by total asset ($\frac{dlc+dltt}{at}$). $Rent$ is the rent expense divided by total asset ($xrent/at$). Tan refers tangibility, measured by property, plant and equipment divided by total asset (ppe/at). *Dni* is the dummy variable which equals one when net income is negative. *Ddiv* is the dummy variable which equals one when firms pay dividend. *Rd* is research and development expense divided by total asset (xrd/at). *Rd* is set to zero when the expense is missing. Mtb is the market to book ratio measured by total asset minus book equity plus market equity and then divided by book assets ($(at - bv + mv)/at$). The market value of equity is the product of year-end price and number of shares outstanding ($prcc_f * csho$).[12] Equity *seq* will be replaced by either common equity plus preferred stock par value ($ceq+pstk$) or total asset minus total liability ($at - lt$) if missing. Preferred stock liquidating value $prstkl$ will be replaced by either redemption value *pstkrv* or par value *pstk* if missing. Firm size $Size$ is the logarithm of total asset. *Beta* and *Rmse* measure Systematic Risk and Idiosyncratic Risk, respectively. They are estimated in market model regressions of a firms daily stock returns on the CRSP value-weighted index return. The regressions are adjusted for nonsynchronous trading effects using the Dimson (1979) procedure with

[12]Book equity is shareholders' equity on balance sheet minus preferred stock plus deferred taxes and investment tax credit ($seq - pstkl + txditc$). The deferred tax credit *txditc* is set to zero if missing.

one leading and one lagging value of the market return. One firm-year observations of *Beta* and *Rmse* are computed from one regression using firm-specific daily stock returns from one calendar year.

idiosyncratic risk, respectively.[13] They are estimated in market model regressions of a firms daily stock returns on the CRSP value-weighted index return. The regressions are adjusted for nonsynchronous trading effects using the Dimson (1979) procedure with one leading and one lagging value of the market return. One firm-year observations of *Beta* and *Rmse* are computed from one regression using firm-specific daily stock returns from one calendar year. All continuous variables are winsorized at top and bottom 1 percentile. Table 3.3 reports the descriptive statistics of these variables.

3.5 Empirical Results

This section presents the chapter's main findings. Firstly, we test the existence of stickiness and explore its effect on credit ratings. Subsequently, empirical results show that credit quality actually improves though the rating levels deteriorate. Lastly, we document the asymmetry in rating standard stringency, which contributes to explaining the rating deterioration phenomenon.

3.5.1 Stickiness Effect

This section presents the estimation of both ordered probit and adjusted ordered probit models given in equations (3.2), (3.3), and (3.4). A comparison of the results

[13] *Beta* is the coefficient in the market model estimation, and *Rmse* is the standard error.

of the two models demonstrates the difference in coefficients with and without considering stickiness, and hence explores the effect of stickiness. Table 3.4 presents the main estimation. The ordered probit regression neglects stickiness, as in Models 1 and 5, and provides the basis of comparison in the analysis. Our adjusted ordered probit regression, as in Models 2 and 6, considers both time-series (t) and credit quality tolerance (z) stickiness. Moreover, we use lagged ratings as dependent variable in the ordered probit regression, as in Models 3 and 7, to separate the effect of t-stickiness. We measure the z-stickiness by adjusted ordered probit model with the new categorizing assumption of immediate rating adjustment, as in Models 4 and 8. The Fama-MacBeth models takes the decades average (85-94, 95-04, and 05-14) of coefficients for adjusted ordered regressions controlling for all stickiness based on every two years of data. Values in parentheses present the standard error for each estimation. Standard errors for adjusted ordered models are calculated using bootstrapping method, and those for Fama-MacBeth models use Delta method. Panel A presents estimation for explanatory variables, Panel B provides the estimation of cut-offs, and Panel C lists year dummy intercepts.

Model 1 in Table 3.4 summarizes the estimation of ordered probit, which attempts to replicate the findings in existing literature without considering stickiness. The estimated effects of covariates are consistent with prior studies (Blume et al. 1998, Jorion et al. 2009, Alp 2013) and our expectations. Consistent with Blume et al. (1998), firms with better ability to pay back borrowings receive higher ratings as indicated by the positive signs of $Intcov$, Tan, $Ddiv$, and $Size$. Interest coverage is a direct measure of the ability to bear credit, and tangible assets can be used as collateral to reduce debt holders' risk. Paying dividends indicates healthy financial

TABLE 3.4: Regressions Estimating the Determinants of Credit Ratings

Panel A: Explanatory Variable Coefficients

	Ordered Model 1	Adj. Model 2	Ordered Model 3	Adj. Model 4	Ordered Model 5	Adj. Model 6	Ordered Model 7	Adj. Model 8	FM(85-94) Model 9	FM(95-04) Model 10	FM(05-14) Model 11
Stickiness		t, z	t	z		t, z	t	z	t, z	t, z	t, z
Intcov	0.012	-0.001	0.012	0.001							
	(0.001)	(0.001)	(0.001)	(0.001)							
k1					0.306	0.047	0.315	0.097	0.022	0.115	0.016
					(0.010)	(0.013)	(0.010)	(0.016)	(0.021)	(0.025)	(0.035)
k2					0.078	0.001	0.082	0.012	0.001	0.006	0.014
					(0.006)	(0.009)	(0.006)	(0.011)	(0.018)	(0.017)	(0.019)
k3					0.026	-0.003	0.027	0.002	0.014	-0.005	-0.003
					(0.004)	(0.005)	(0.004)	(0.006)	(0.015)	(0.010)	(0.011)
k4					0.003	-0.001	0.004	0.000	-0.006	-0.002	0.002
					(0.001)	(0.001)	(0.001)	(0.001)	(0.005)	(0.002)	(0.002)
Vol	-3.318	-2.124	-6.493	-2.404	-5.410	-1.958	-4.864	-2.056	-2.479	-1.495	-2.780
	(0.212)	(0.303)	(0.222)	(0.337)	(0.230)	(0.316)	(0.223)	(0.345)	(0.547)	(0.520)	(0.747)
Tlev	-2.038	-1.229	-1.922	-1.561	-1.162	-1.121	-0.809	-1.309	-1.381	-0.674	-1.287
	(0.058)	(0.084)	(0.058)	(0.108)	(0.065)	(0.096)	(0.064)	(0.108)	(0.150)	(0.160)	(0.228)
Rent	-2.830	-4.747	-5.824	-4.803	-7.975	-4.821	-7.253	-5.013	-6.185	-3.370	-7.623
	(0.316)	(0.407)	(0.289)	(0.475)	(0.303)	(0.431)	(0.298)	(0.475)	(0.753)	(0.673)	(1.080)
Tan	0.830	0.377	0.814	0.428	0.859	0.366	0.752	0.413	0.204	0.514	0.421
	(0.039)	(0.056)	(0.038)	(0.065)	(0.039)	(0.059)	(0.039)	(0.064)	(0.099)	(0.111)	(0.131)
Dni	-0.651	-0.514	-0.398	-0.725	-0.418	-0.488	-0.210	-0.675	-0.448	-0.489	-0.668
	(0.024)	(0.033)	(0.023)	(0.036)	(0.025)	(0.033)	(0.024)	(0.042)	(0.060)	(0.061)	(0.094)
Ddiv	1.169	0.103	1.123	0.207	0.995	0.096	1.041	0.192	0.141	0.063	0.226

	(0.022)	(0.029)	(0.021)	(0.035)	(0.022)	(0.029)	(0.022)	(0.033)	(0.057)	(0.053)	(0.068)
Rd	0.061	-1.523	0.567	-1.938	-0.341	-1.673	-0.229	-2.275	-1.659	-3.098	0.627
	(0.308)	(0.440)	(0.307)	(0.553)	(0.314)	(0.446)	(0.311)	(0.526)	(0.760)	(0.818)	(1.206)
Mtb	0.517	0.411	0.430	0.434	0.387	0.405	0.305	0.413	0.436	0.424	0.407
	(0.014)	(0.021)	(0.014)	(0.026)	(0.015)	(0.023)	(0.015)	(0.023)	(0.049)	(0.037)	(0.059)
Size	0.465	0.133	0.419	0.152	0.442	0.138	0.439	0.162	0.147	0.123	0.203
	(0.007)	(0.010)	(0.007)	(0.012)	(0.007)	(0.010)	(0.007)	(0.013)	(0.019)	(0.020)	(0.024)
Beta	-0.032	-0.019	-0.035	-0.006	-0.036	-0.019	-0.035	-0.006	-0.021	-0.022	-0.067
	(0.004)	(0.005)	(0.004)	(0.006)	(0.004)	(0.005)	(0.004)	(0.007)	(0.008)	(0.010)	(0.025)
Rmse	-0.006	0.003	-0.387	0.011	-0.196	0.026	-0.169	0.060	0.112	0.047	-0.436
	(0.057)	(0.076)	(0.055)	(0.077)	(0.057)	(0.077)	(0.056)	(0.071)	(0.115)	(0.111)	(0.213)
Pseudo R	0.334	0.089	0.323	0.139	0.371	0.090	0.353	0.143	-	-	-
Nobs.	20557	20557	20557	20557	20557	20557	20557	20557	-	-	-

Panel B: Cutoffs

	Ordered	Adj.	Ordered	Adj.	Ordered	Adj.	Ordered	Adj.	FM(85-94)	FM(95-04)	FM(05-14)
	Model 1	Model 2	Model 3	Model 4	Model 5	Model 6	Model 7	Model 8	Model 9	Model 10	Model 11
U1/C12	2.489	1.844	1.701	1.784	2.903	2.046	3.120	2.210	1.795	2.515	2.624
	(0.073)	(0.106)	(0.072)	(0.099)	(0.085)	(0.116)	(0.084)	(0.129)	(0.193)	(0.204)	(0.275)
U2/C23	4.233	2.673	3.403	2.685	4.815	2.913	4.957	3.183	2.640	3.510	3.688
	(0.076)	(0.111)	(0.075)	(0.107)	(0.089)	(0.125)	(0.088)	(0.137)	(0.218)	(0.214)	(0.297)
U3/C34	5.574	3.335	4.696	3.352	6.236	3.593	6.322	3.885	3.148	4.238	4.791
	(0.078)	(0.117)	(0.077)	(0.115)	(0.092)	(0.132)	(0.091)	(0.149)	(0.241)	(0.287)	(0.511)
U4/C45	6.983	4.247	6.058	4.309	7.686	4.510	7.725	4.846	4.256	5.965	6.758
	(0.083)	(0.156)	(0.081)	(0.154)	(0.096)	(0.160)	(0.095)	(0.171)	(0.501)	(0.384)	(0.519)
L2		-0.812		-1.302		-0.580		-0.836	-0.813	-0.193	-0.105
		(0.106)		(0.108)		(0.122)		(0.136)	(0.214)	(0.207)	(0.290)
L3		-0.274		-0.619		-0.017		-0.095	-0.300	0.479	0.631

		(0.112)		(0.111)		(0.128)		(0.146)	(0.224)	(0.225)	(0.311)
L4		0.130		-0.141		0.395		0.412	0.078	0.995	1.083
		(0.117)		(0.119)		(0.138)		(0.154)	(0.235)	(0.234)	(0.333)
L5		0.470		0.235		0.733		0.797	0.428	1.401	1.078
		(0.137)		(0.143)		(0.151)		(0.171)	(0.274)	(0.441)	(0.615)

Panel C: Year Dummy Intercepts

	Ordered Model 1	Adj. Model 2	Ordered Model 3	Adj. Model 4	Ordered Model 5	Adj. Model 6	Ordered Model 7	Adj. Model 8	FM(85-94) Model 9	FM(95-04) Model 10	FM(05-14) Model 11
D1986	0.038	0.031	-0.098	-0.377	-0.327	0.024	-0.167	-0.393			
	(0.063)	(0.096)	(0.059)	(0.076)	(0.060)	(0.087)	(0.059)	(0.076)			
D1987	-0.122	0.053	-0.243	-0.342	-0.445	0.049	-0.337	-0.350			
	(0.060)	(0.090)	(0.058)	(0.065)	(0.059)	(0.090)	(0.059)	(0.074)			
D1988	-0.067	0.093	-0.228	-0.205	-0.394	0.086	-0.302	-0.219			
	(0.061)	(0.098)	(0.059)	(0.074)	(0.060)	(0.091)	(0.060)	(0.079)			
D1989	-0.013	0.086	-0.224	-0.122	-0.389	0.081	-0.348	-0.132			
	(0.062)	(0.095)	(0.060)	(0.070)	(0.061)	(0.088)	(0.061)	(0.074)			
D1990	0.089	0.209	-0.135	-0.165	-0.266	0.209	-0.181	-0.161			
	(0.063)	(0.097)	(0.061)	(0.078)	(0.062)	(0.101)	(0.062)	(0.073)			
D1991	0.108	0.221	-0.150	-0.041	-0.183	0.229	-0.134	-0.027			
	(0.064)	(0.096)	(0.062)	(0.085)	(0.063)	(0.094)	(0.063)	(0.081)			
D1992	0.080	0.362	-0.215	-0.003	-0.195	0.373	-0.169	0.027			
	(0.062)	(0.096)	(0.061)	(0.078)	(0.062)	(0.093)	(0.061)	(0.076)			
D1993	0.011	0.214	-0.302	-0.028	-0.336	0.220	-0.317	-0.020			
	(0.062)	(0.090)	(0.060)	(0.080)	(0.061)	(0.091)	(0.061)	(0.079)			
D1994	-0.061	0.202	-0.306	-0.146	-0.414	0.203	-0.351	-0.144			
	(0.062)	(0.094)	(0.060)	(0.078)	(0.061)	(0.093)	(0.061)	(0.068)			
D1995	-0.140	0.137	-0.360	-0.198	-0.528	0.131	-0.440	-0.209			

	(0.062)	(0.089)	(0.061)	(0.080)	(0.062)	(0.089)	(0.062)	(0.078)
D1996	-0.269	0.198	-0.487	-0.236	-0.685	0.187	-0.619	-0.261
	(0.062)	(0.092)	(0.060)	(0.081)	(0.061)	(0.094)	(0.061)	(0.079)
D1997	-0.290	-0.001	-0.539	-0.150	-0.735	-0.017	-0.704	-0.186
	(0.062)	(0.098)	(0.060)	(0.065)	(0.061)	(0.095)	(0.061)	(0.075)
D1998	-0.279	0.010	-0.486	-0.238	-0.749	-0.011	-0.690	-0.284
	(0.061)	(0.091)	(0.060)	(0.084)	(0.061)	(0.097)	(0.060)	(0.082)
D1999	-0.370	-0.171	-0.532	-0.345	-0.824	-0.196	-0.729	-0.394
	(0.062)	(0.105)	(0.060)	(0.076)	(0.062)	(0.095)	(0.061)	(0.077)
D2000	-0.534	-0.376	-0.618	-0.629	-0.967	-0.398	-0.810	-0.677
	(0.063)	(0.099)	(0.061)	(0.077)	(0.063)	(0.093)	(0.062)	(0.076)
D2001	-0.528	-0.292	-0.632	-0.567	-0.950	-0.313	-0.817	-0.612
	(0.064)	(0.100)	(0.062)	(0.084)	(0.063)	(0.099)	(0.063)	(0.082)
D2002	-0.433	-0.154	-0.548	-0.507	-0.852	-0.176	-0.723	-0.553
	(0.063)	(0.100)	(0.062)	(0.078)	(0.063)	(0.101)	(0.062)	(0.075)
D2003	-0.653	-0.180	-0.743	-0.698	-1.036	-0.193	-0.865	-0.725
	(0.064)	(0.091)	(0.062)	(0.083)	(0.063)	(0.093)	(0.063)	(0.082)
D2004	-0.909	-0.299	-1.036	-0.633	-1.319	-0.315	-1.187	-0.667
	(0.064)	(0.098)	(0.062)	(0.082)	(0.063)	(0.089)	(0.063)	(0.081)
D2005	-1.059	-0.392	-1.171	-0.775	-1.509	-0.413	-1.352	-0.820
	(0.064)	(0.092)	(0.063)	(0.079)	(0.064)	(0.098)	(0.064)	(0.089)
D2006	-1.203	-0.437	-1.309	-0.850	-1.721	-0.464	-1.542	-0.910
	(0.065)	(0.108)	(0.063)	(0.081)	(0.065)	(0.101)	(0.064)	(0.083)
D2007	-1.273	-0.575	-1.432	-0.739	-1.800	-0.605	-1.674	-0.810
	(0.066)	(0.100)	(0.064)	(0.087)	(0.066)	(0.100)	(0.065)	(0.090)
D2008	-1.084	-0.310	-1.192	-0.723	-1.647	-0.345	-1.529	-0.806
	(0.066)	(0.093)	(0.065)	(0.089)	(0.067)	(0.095)	(0.066)	(0.090)
D2009	-1.089	0.003	-1.175	-0.662	-1.629	-0.030	-1.509	-0.738

	(0.067)	(0.099)	(0.066)	(0.087)	(0.067)	(0.104)	(0.067)	(0.097)
D2010	-1.236	-0.164	-1.353	-0.631	-1.735	-0.191	-1.604	-0.690
	(0.068)	(0.097)	(0.067)	(0.095)	(0.068)	(0.099)	(0.068)	(0.090)
D2011	-1.252	-0.206	-1.427	-0.500	-1.759	-0.233	-1.702	-0.565
	(0.068)	(0.096)	(0.067)	(0.083)	(0.068)	(0.096)	(0.068)	(0.086)
D2012	-1.283	-0.051	-1.428	-0.607	-1.822	-0.084	-1.765	-0.684
	(0.069)	(0.101)	(0.067)	(0.090)	(0.069)	(0.099)	(0.068)	(0.092)
D2013	-1.365	-0.197	-1.524	-0.509	-1.909	-0.232	-1.847	-0.586
	(0.069)	(0.099)	(0.068)	(0.090)	(0.070)	(0.096)	(0.069)	(0.092)
D2014	-1.374	-0.395	-1.522	-0.619	-1.938	-0.431	-1.876	-0.709
	(0.070)	(0.106)	(0.069)	(0.089)	(0.070)	(0.096)	(0.070)	(0.094)

There are 20,557 observations from 1,488 unique firms in the full sample. Models 1, 3, 5, and 7 use ordered probit regression which assumes no stickiness. The other results are estimated by our adjusted model controlling for stickiness. Row "Stickiness" indicates different levels of stickiness considered in our model, $z-dimensional$ stickiness, $t-dimensional$ stickiness, or both. The Fama-MacBeth coefficients are decade average of regression coefficients by Adjusted model. Panel A presents estimation for explanatory variables, Panel B provides the estimation of cut-offs, and Panel C lists year dummy intercepts if applicable. We report standard error in parenthesis.

condition and considerable profitability. Moreover, potential growth opportunities also improve credit ratings consistent with Alp (2013). (The letter is captured by the positive coefficients of Mtb and Rd.) Lastly, risk factors lead to a more conservative assessment from the rating agencies. Cash flow uncertainty, denoted by Vol, and systematic risk level, represented by $Beta$ significantly reduce firms' credit quality and drive down the ratings. Credit risk measured by total leverage $Tlev$ demonstrates a similar effect.

Model 2 in Table 3.4 introduces stickiness by estimating the adjusted ordered probit model described by equations (3.2), (3.3), and (3.4). As explained previously, this estimation controls stickiness from both time-series (t) and tolerance of credit quality deviation (z). The introduction of stickiness does not change the coefficients in large magnitude compared to Model 1. Further, we decompose the stickiness into the t-dimensional and z-dimensional ones in Models 3 and 4, respectively. The t-dimensional stickiness assumes zero tolerance of agencies regarding credit quality deviation from its nominal range of the ratings, but assumes action a delay in taking by one period. In other words, current ratings match lagged firm features, and ordered probit model is appropriate with all covariates lagged by one period as in Model 3. Model 4 measures the z-dimensional stickiness which assumes agencies allow some deviation in credit quality but react immediately when the deviation exceeds the tolerance.[14] The coefficients of control variables in Model 3 and Model 4 do not deviate from that in the full stickiness results in Model 2.

[14]For example, in the downgrade situation depicted by points $q2$, $q3$, and $q4$ in Figure 3.1, credit quality breaches the threshold at $q3$. The z-dimensional stickiness assumes rating changes at $q3$ to reflect the immediate reaction. In comparison, the full stickiness assumes rating updates happens at $q4$ to reflect the slow-respondance.

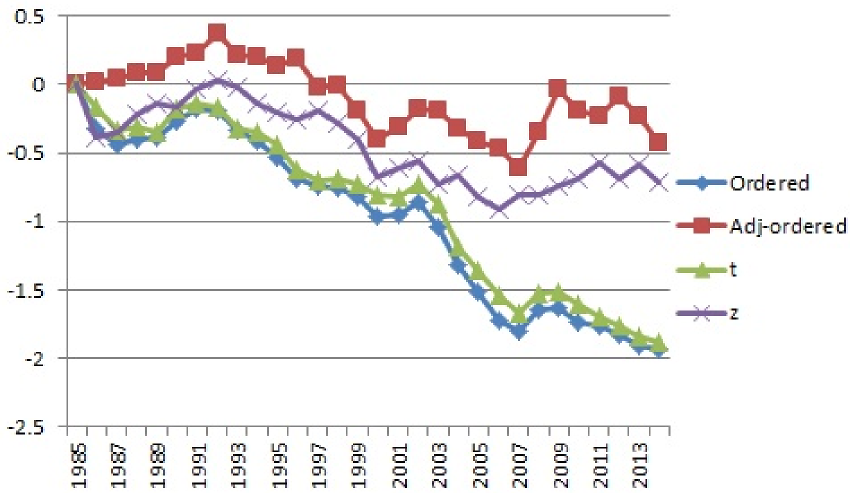
Moreover, Blume et al. (1998) expressed concerns about a non-linear relationship between interest coverage and ratings. Following their method, we decompose the *Intcov* into four components according to its magnitude as discussed in Subsection 3.4. We replicate Models 1 to 4 with *Intcov* replaced by the components, and Models 5 to 8 summarize this replication. This decomposition reveals that the correlation between components of interest coverage and ratings weakens along the magnitude of interest coverage. As in Model 5, one unit of increment in the low range of interest coverage, the $k1$, brings 0.306 extra credit quality measured by Z-score, but this effect shrinks to 0.003 as the range shifts to $k4$.[15] A similar effect also appears in other studies (Blume et al. 1998, Alp 2013).

The key novel evidence from adjusted ordered probit model is on the stringency of rating standard over the analyzed period. Existing empirical studies (Blume et al. 1998, Alp 2013) consider the declining year dummy intercepts (as in Panel B of Table 3.4) to be the indicators of rating standard becoming more strict. Those studies find that the intercepts indeed move downward, and hence claim that rating standards are strengthening. (More precisely, the decreasing year dummy intercepts implies that a firm whose all characteristics remain unchanged will receive a lower rating by just stepping into the next year.[16]) Figure 3.2 plots the year intercepts of Models 5 to 8 (Panel B, Table 3.4). The downward moving intercepts of the ordered probit model confirm the pattern in Blume et al. (1998). However, this trend disappears in the adjusted model which controls for rating stickiness. In addition, rating stickiness is

[15] As discussed in previous section, $k1$ measures the *Intcov* range from 0 to 5, and $k4$ measures the range above 20.

[16] A lower rating refers to a worse rating. This paper exploits the same rating denotement as in (Blume et al. 1998, Jorion et al. 2009, Alp 2013), which means that a better rating matches a higher number.

Figure 3.2: Plot of the Estimates of the Year Dummy Intercepts



Year dummy intercepts estimated from ordered probit and adjusted ordered probit models in Table 3.4, based on a panel dataset containing 20,557 firm-year observations from 1985 to 2014. Rating stickiness is further separated into the t-dimensional and z-dimensional ones. The t stickiness refers to the delay of rating adjustment in time series. It is measured by the year intercepts from Ordered Probit model with lagged rating being the dependent variable. The z stickiness measures the tolerance of credit quality deviation, and the year intercepts are from adjusted ordered Probit with different categorization method. The difference in categorization refers to neglecting the time-series delay of rating adjustment. More precise, our main adjusted ordered probit model assumes the rating migration at time t is caused by the breaching of rating threshold at time $t-1$. However, the z stickiness model assumes the rating migration at time t happens because of the breaching at the time t as well. Hence, it is an assumption of immediate adjustment.

further separated into the t-dimensional and z-dimensional ones. The t-stickiness refers to the delay of rating adjustment in time series. It is measured by the year intercepts from ordered probit model with lagged rating being the dependent variable. The z-stickiness measures the tolerance of credit quality deviation, and the year intercepts are from the adjusted ordered probit with a different categorization method. The difference in categorization refers to ignoring the time-series delay of rating adjustment.

The disappearing downward pattern in year intercepts suggests that these dummy variables may contain more information than rating standards. For example, Du

(2003) challenges the conclusion of tightening rating standards in Blume et al. (1998) and provides an alternative explanation. Du (2003) interprets the decreasing year dummy coefficients as the outcome of the situation that new bonds are mainly issued by low quality firms. From the perspective of stickiness, changes in year dummies reflect both the standard effect and the unabsorbed impact from rating mechanism. In other words, the model setting and estimation of existing studies are in static way, which ignores agencies' freedom in timing the rating migrations. Compared to the stickiness-based model, the estimation of ordered probit model forces the threshold observations, either upper or lower, to be withinits current rating, but the credit quality of these observations actually deviates into the ranges of other ratings. For example, dot $q10$ in Figure 3.1 is an upper threshold observation whose credit quality enters range of rating A but its current rating is still B since migration happens in next period. Static estimation will force $q10$ to fit the range of rating B, which leads to the negative year intercepts. Inversely, the fitting of the ignored lower threshold observations tend to have positive year intercepts. The downward trend implies asymmetric influence between the upper and lower threshold groups, and it seems that the effect from the upper threshold observations strongly affect the estimation results.

We further demonstrate the impact of stickiness through rating migrations (Table 3.5). The first three columns present the true migration observations from perspectives of an upgrade, no migration, and a downgrade. The next three columns summarize the predicted migrations without considering stickiness, and the last three columns state the prediction under the stickiness framework. The inferred migrations for non-stickiness and stickiness predictions are based on the same credit quality, and

Table 3.5: Predicted Rating Migrations

	TRUE			Adj-No Stickiness			Adj-Stickiness		
	Up.	Stay	Down.	Up.	Stay	Down.	Up.	Stay	Down.
1986	18	755	54	81	673	73	0	822	5
1987	23	823	44	104	717	69	0	839	51
1988	30	744	39	93	639	81	0	765	48
1989	30	704	31	70	618	77	0	719	46
1990	21	636	32	120	531	38	0	652	37
1991	25	598	26	100	477	72	0	634	15
1992	28	648	27	166	480	57	1	683	19
1993	41	674	29	89	519	136	0	728	16
1994	22	678	23	92	548	83	1	704	18
1995	24	658	28	71	560	79	0	696	14
1996	31	678	27	128	566	42	0	723	13
1997	35	695	20	37	546	167	0	733	17
1998	35	720	38	90	574	129	0	765	28
1999	23	688	37	48	523	177	0	711	37
2000	14	647	48	37	482	190	0	652	57
2001	19	610	56	66	523	96	0	639	46
2002	9	645	48	100	528	74	0	647	55
2003	13	625	53	123	516	52	0	667	24
2004	18	651	32	63	556	82	0	678	23
2005	18	623	37	32	542	104	0	664	14
2006	19	603	40	47	554	61	0	645	17
2007	27	563	32	35	468	119	0	598	24
2008	10	559	37	83	456	67	0	574	32
2009	13	533	33	251	303	25	0	561	18
2010	20	516	22	42	427	89	0	552	6
2011	26	510	15	34	451	66	0	548	3
2012	16	512	17	140	378	27	0	538	7
2013	19	497	7	34	416	73	0	521	2
2014	16	484	11	13	373	125	0	507	4
Total	643	18277	943	2389	14944	2530	2	19165	696

Predicted rating migrations. The first three columns report the true number of upgrade observations, unchanged observations, and downgrade observations. The next three columns reports the predicted migration volume assuming no stickiness. The last three columns present the predicted migration numbers with stickiness. The credit quality is calculated using the regression results in Model 6 in Table 3.4.

the only difference is the model embedded.[17] As shown in the columns that correspond to non-stickiness, this model tends to overpredict the frequency of both upgrades and downgrades. The total number of migrations predicted is almost three times the number of the actual rating adjustments. In comparison, the prediction from stickiness framework is close to the actual value in magnitude. Moreover, the stickiness framework may reveal the asymmetry of the impact between upper and lower thresholds, since upgrades very infrequent during this period.

3.5.2 Credit Quality

The stickiness considered in the presented model absorbs effects of the rating migration process, and hence leaves the coefficients directly determined by credit quality. We further decompose the variation of credit quality during the deterioration period, and find out that the credit quality of firms actually improves. Table 3.6 summarizes the variation of predicted average level of credit quality between the last six-year period (2009-2014) average level and the first six-year period (1985-1990). The contribution of each variable is the product of increment in period average and the corresponding coefficients. Column 1 reports the contribution of variables to the credit quality variation. Overall, credit quality, measured by Z-score, increases by 0.573, while firm size contributes the most (0.312). Further, we separate the full sample by rating levels and firm size, and find that investment grade rated and large firms experience more credit quality improvement compared to speculative rated and small firms.

[17]The credit quality is the predicted Z-score based on Model 6 in Table 3.4.

Table 3.6: Determinants of Deviation in Credit Quality

	Full	Invest.	Specu.	Diff.	Large	Small	Diff.
k1	0.016	0.022	0.008	0.015	0.018	0.013	0.005
	(0.001)	(0.001)	(0.000)		(0.001)	(0.001)	
k2	0.002	0.001	0.002	-0.001	0.002	0.001	0.000
	(0.000)	(0.000)	(0.000)		(0.000)	(0.000)	
k3	0.026	0.014	0.047	-0.032	0.028	0.024	0.004
	(0.001)	(0.001)	(0.002)		(0.002)	(0.002)	
k4	-0.027	-0.013	-0.053	0.040	-0.025	-0.029	0.003
	(0.002)	(0.002)	(0.004)		(0.002)	(0.003)	
Vol	0.020	0.036	0.001	0.035	0.027	0.014	0.013
	(0.003)	(0.005)	(0.002)		(0.004)	(0.004)	
Tlev	0.091	0.146	0.015	0.131	0.105	0.077	0.028
	(0.006)	(0.008)	(0.006)		(0.008)	(0.008)	
Rent	0.040	0.034	0.054	-0.021	0.038	0.041	-0.002
	(0.004)	(0.006)	(0.004)		(0.006)	(0.005)	
Tan	-0.017	-0.009	-0.029	0.021	-0.018	-0.016	-0.002
	(0.001)	(0.001)	(0.002)		(0.002)	(0.002)	
Dni	0.026	0.039	0.012	0.027	0.027	0.025	0.003
	(0.004)	(0.006)	(0.003)		(0.005)	(0.005)	
Ddiv	0.007	0.014	0.000	0.014	0.009	0.004	0.005
	(0.001)	(0.002)	(0.001)		(0.002)	(0.002)	
Rd	0.005	0.008	0.000	0.007	0.003	0.008	-0.005
	(0.001)	(0.001)	(0.002)		(0.002)	(0.002)	
Mtb	0.087	0.040	0.173	-0.133	0.095	0.080	0.014
	(0.006)	(0.006)	(0.011)		(0.008)	(0.008)	
Size	0.312	0.334	0.295	0.039	0.319	0.304	0.015
	(0.005)	(0.005)	(0.007)		(0.007)	(0.007)	
Beta	-0.007	-0.011	0.000	-0.011	-0.008	-0.006	-0.003
	(0.001)	(0.001)	(0.001)		(0.001)	(0.001)	
Rmse	-0.007	-0.011	-0.001	-0.010	-0.008	-0.007	-0.001
	(0.000)	(0.001)	(0.000)		(0.001)	(0.001)	
Total	0.573	0.646	0.525	0.121	0.612	0.535	0.077

Variation of predicted credit quality between the last six-year period (2009-2014) average level and the first six-year period (1985-1990) average level. The Z-score following Model 9 of Table 3.4 serves the proxy of credit quality: Z-score$= 0.023*k1+0.001*k2+0.014*k3-0.006*k4-2.479*Vol-1.381*Tlev-6.185*Rent+0.204*tan-0.448*Dni+0.141*Ddiv-1.659*Rd+0.436*Mtb+0.147*Size-0.021*Beta+0.112*Rmse$. The contribution of each variable is the product of increment in period average and the corresponding coefficients.[18] The standard errors in parentheses are calculated using Delta method.

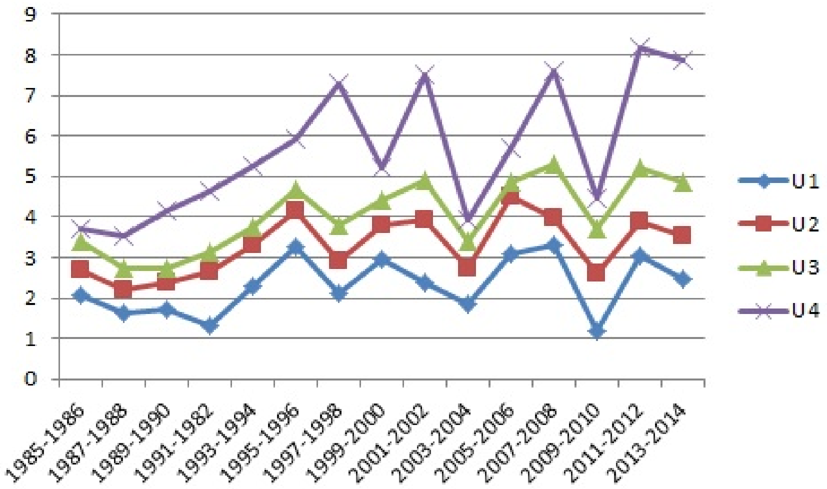
3.5.3 Asymmetry

Since credit quality improves during this period, the deterioration in ratings is concluded to be caused by a tightening rating standard. Unlike in previous empirical studies (Blume et al. 1998, Jorion et al. 2009, Alp 2013), our model allows us to separate the standards for upgrade and downgrade decisions, by isolating upper and lower thresholds from their combined (average) effect. Figure 3.3 shows the upper threshold movements from 1985 to 2014 for every two years. Upper thresholds represent cut-off levels for ratings, for which every crossing by credit quality causes an upgrade to the adjacent rating above. As shown in Figure 3.3, upper thresholds generally rise for each rating category, which suggests that upgrade migration becomes increasingly difficult. For example, the threshold $U4$ has experienced a dramatic increase during this period, which implies that upgrade to the best rating category (level 5) became increasingly difficult.[19] Moreover, Figure 3.4 plots the lower cut-off variation in the same format. Lower thresholds exhibit substantial variation around their original levels, but no visible trend is present. The latter implies no tightening on relaxation of standards for downgrades. Overall, our results suggest that the standard stringency

[18]The average Vol in period 2009 to 2017 is 0.033, and the average level in period 1985 to 1990 is 0.041. Then, the contribution of Vol to credit quality variation is 0.020 (which is $(0.033-0.041)*(-2.479)$).

[19]Threshold $U4$ refers to the upper cut-off for rating level 4. The firm will receive upgrade to rating level 5 once its credit quality crosses this boundary, and level 5 is the best rating in this study.

Figure 3.3: Plot of the Estimates of the Upper Cut-off for Rating Categories

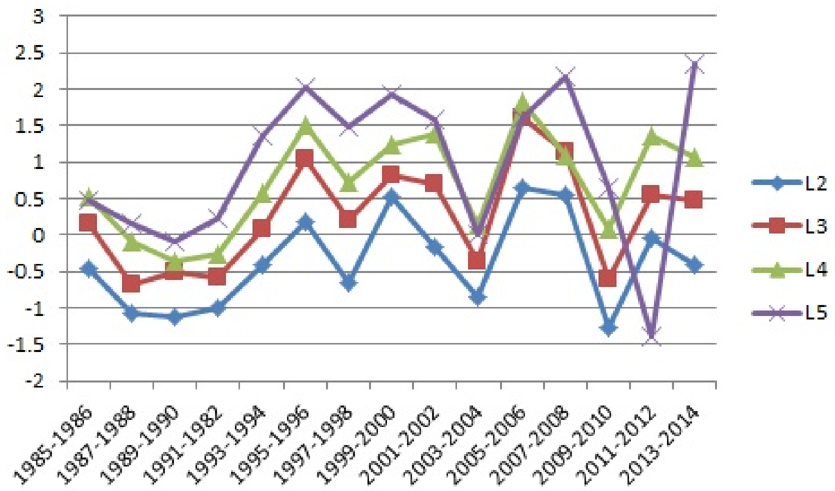


The stickiness in rating behavior makes the separation of cutoff between adjacent ratings. For example, suppose two rating categories A and B are next to each other (A indicates better quality), the stickiness argues that the lower cutoff of A is not necessarily being the upper cutoff of B. Upgrade happens only when one's credit quality reaches the upper threshold of its current rating. The plots are based on the estimates of upper cutoff from Adjusted Ordered Probit model for every two-year period. Overall, there are five rating categories, but the upper cutoff for the highest rated category is infinity (no further upgrade available).

for upgrade decisions goes up but for downgrade decisions keeps flat. We consider this kind of asymmetry to be the main cause of the deterioration in credit ratings.

The asymmetry may result from the partial release of information as suggested in other empirical studies (Altman & Kao 1992, Lando & Skødeberg 2002). Altman & Kao (1992) create a measure defined by the frequency of subsequent migrations in the same direction (i.e. upgrade followed by upgrade, downgrade followed by downgrade) divided by the frequency of subsequent migrations in the opposite directions (i.e. upgrade followed by downgrade, downgrade followed by upgrade). This statistic is larger than 1, which means that rating migrations tend to be followed by migrations in the same direction. The correlation between consecutive downgrades is higher than the one between consecutive upgrades. Lando & Skødeberg (2002) confirm this migr-

FIGURE 3.4: Plot of the Estimates of the Lower Cut-off for Rating Categories



The stickiness in rating behavior makes the separation of cutoff between adjacent ratings. For example, suppose two rating categories A and B are next to each other (A indicates better quality), the stickiness argues that the lower cutoff of A is not necessarily being the upper cutoff of B. Downgrade happens only when one's credit quality reaches the lower threshold of its current rating. The plots are based on the estimates of lower cutoff from Adjusted Ordered Probit model for every two-year period. Overall, there are five rating categories, but the lower cutoff for the worst rated category is negative infinity (no further downgrade available).

ation correlation pattern using semi-parametric regression based on continuous observations. Löffler (2005) interprets this phenomenon, that agencies respond partially to a piece of information and "dole out the bad news in small doses rather than savaging the bond issuer - who is, after all, their customer - all in one go" (Economist 1997). This is also consistent with the partial adjustment pattern documented by Altman & Rijken (2004). From this perspective, rating agencies respond to a piece of information in more than one period, and hence, breaks a big rating migration decision down to several small migrations. This responding pattern also contributes to the stickiness in credit ratings.

Further, we study the determinants of rating migrations to investigate the role this asymmetry in credit rating deterioration. Table 3.7 reports the coefficients of m-

Table 3.7: Regressions of Migration Determinants

	Model 1				Model 2			
	Oprobit	ME1	ME2	ME3	Oprobit	ME1	ME2	ME3
k1	-0.042	0.004	-0.001	-0.003	-0.031	0.003	-0.001	-0.002
	(0.014)	(0.001)	(0.000)	(0.001)	(0.015)	(0.001)	(0.000)	(0.001)
k2	-0.015	0.001	0.000	-0.001	-0.011	0.001	0.000	-0.001
	(0.010)	(0.001)	(0.000)	(0.001)	(0.011)	(0.001)	(0.000)	(0.001)
k3	-0.006	0.001	0.000	0.000	-0.005	0.000	0.000	0.000
	(0.006)	(0.001)	(0.000)	(0.000)	(0.007)	(0.001)	(0.000)	(0.000)
k4	-0.002	0.000	0.000	0.000	-0.003	0.000	0.000	0.000
	(0.001)	(0.000)	(0.000)	(0.000)	(0.002)	(0.000)	(0.000)	(0.000)
Vol	-0.038	0.003	-0.001	-0.002	-0.479	0.043	-0.011	-0.032
	(0.271)	(0.023)	(0.006)	(0.016)	(0.320)	(0.028)	(0.007)	(0.021)
Tlev	-0.728	0.060	-0.016	-0.044	-0.838	0.074	-0.019	-0.056
	(0.089)	(0.007)	(0.003)	(0.005)	(0.100)	(0.009)	(0.003)	(0.007)
Rrent	-1.451	0.121	-0.033	-0.088	-0.492	0.044	-0.011	-0.033
	(0.407)	(0.034)	(0.010)	(0.025)	(0.670)	(0.060)	(0.015)	(0.045)
Tan	0.107	-0.009	0.002	0.006	-0.024	0.002	-0.001	-0.002
	(0.057)	(0.005)	(0.001)	(0.003)	(0.108)	(0.010)	(0.002)	(0.007)
Dni	-0.511	0.056	-0.033	-0.023	-0.512	0.037	0.002	-0.039
	(0.034)	(0.005)	(0.004)	(0.001)	(0.036)	(0.002)	(0.002)	(0.003)
Ddiv	-0.275	0.021	-0.003	-0.018	-0.270	0.018	0.007	-0.024
	(0.031)	(0.002)	(0.001)	(0.002)	(0.037)	(0.002)	(0.003)	(0.004)
Rd	-1.224	0.102	-0.027	-0.074	-2.436	0.218	-0.054	-0.165
	(0.444)	(0.037)	(0.011)	(0.027)	(0.757)	(0.069)	(0.018)	(0.052)
Mtb	0.235	-0.020	0.005	0.014	0.290	-0.025	0.006	0.019
	(0.022)	(0.002)	(0.001)	(0.001)	(0.025)	(0.002)	(0.001)	(0.002)
Size	-0.030	0.002	-0.001	-0.002	-0.052	0.005	-0.001	-0.003
	(0.010)	(0.001)	(0.000)	(0.001)	(0.013)	(0.001)	(0.000)	(0.001)
Beta	0.007	-0.001	0.000	0.000	0.006	-0.001	0.000	0.000
	(0.006)	(0.001)	(0.000)	(0.000)	(0.006)	(0.001)	(0.000)	(0.000)
Rmse	0.142	-0.012	0.003	0.009	0.139	-0.012	0.003	0.009
	(0.080)	(0.007)	(0.002)	(0.005)	(0.088)	(0.008)	(0.002)	(0.006)
D1986	-0.277	0.029	-0.016	-0.013	-0.276	0.024	-0.005	-0.019
	(0.092)	(0.012)	(0.008)	(0.003)	(0.093)	(0.008)	(0.002)	(0.006)
D1987	-0.204	0.020	-0.010	-0.010	-0.211	0.019	-0.004	-0.014
	(0.091)	(0.010)	(0.007)	(0.004)	(0.092)	(0.008)	(0.002)	(0.006)
D1988	-0.087	0.008	-0.003	-0.005	-0.085	0.008	-0.002	-0.006
	(0.092)	(0.009)	(0.004)	(0.005)	(0.094)	(0.008)	(0.002)	(0.006)
D1989	-0.013	0.001	0.000	-0.001	-0.013	0.001	0.000	-0.001
	(0.094)	(0.008)	(0.002)	(0.006)	(0.096)	(0.009)	(0.002)	(0.006)
D1990	-0.105	0.010	-0.004	-0.006	-0.096	0.009	-0.002	-0.006
	(0.096)	(0.009)	(0.005)	(0.005)	(0.098)	(0.009)	(0.002)	(0.007)

D1991	-0.034	0.003	-0.001	-0.002	-0.040	0.004	-0.001	-0.003
	(0.097)	(0.009)	(0.003)	(0.006)	(0.100)	(0.009)	(0.002)	(0.007)
D1992	0.011	-0.001	0.000	0.001	0.014	-0.001	0.000	0.001
	(0.096)	(0.008)	(0.002)	(0.006)	(0.098)	(0.009)	(0.002)	(0.007)
D1993	0.065	-0.005	0.001	0.004	0.066	-0.006	0.002	0.004
	(0.094)	(0.007)	(0.001)	(0.006)	(0.096)	(0.009)	(0.002)	(0.006)
D1994	-0.056	0.005	-0.002	-0.003	-0.068	0.006	-0.001	-0.005
	(0.096)	(0.009)	(0.003)	(0.005)	(0.098)	(0.009)	(0.002)	(0.007)
D1995	-0.095	0.009	-0.003	-0.005	-0.107	0.009	-0.002	-0.007
	(0.096)	(0.009)	(0.004)	(0.005)	(0.098)	(0.009)	(0.002)	(0.007)
D1996	-0.040	0.003	-0.001	-0.002	-0.061	0.005	-0.001	-0.004
	(0.095)	(0.008)	(0.003)	(0.005)	(0.098)	(0.009)	(0.002)	(0.007)
D1997	0.049	-0.004	0.001	0.003	0.021	-0.002	0.000	0.001
	(0.095)	(0.007)	(0.001)	(0.006)	(0.098)	(0.009)	(0.002)	(0.007)
D1998	-0.059	0.005	-0.002	-0.003	-0.074	0.007	-0.002	-0.005
	(0.094)	(0.009)	(0.003)	(0.005)	(0.096)	(0.009)	(0.002)	(0.007)
D1999	-0.140	0.013	-0.006	-0.007	-0.159	0.014	-0.003	-0.011
	(0.095)	(0.010)	(0.005)	(0.004)	(0.098)	(0.009)	(0.002)	(0.007)
D2000	-0.334	0.036	-0.021	-0.015	-0.349	0.030	-0.007	-0.024
	(0.096)	(0.013)	(0.010)	(0.003)	(0.099)	(0.008)	(0.002)	(0.007)
D2001	-0.305	0.033	-0.018	-0.014	-0.316	0.028	-0.006	-0.022
	(0.096)	(0.013)	(0.009)	(0.003)	(0.099)	(0.009)	(0.002)	(0.007)
D2002	-0.283	0.030	-0.016	-0.013	-0.285	0.025	-0.006	-0.019
	(0.096)	(0.012)	(0.009)	(0.003)	(0.100)	(0.009)	(0.002)	(0.007)
D2003	-0.387	0.044	-0.027	-0.017	-0.390	0.034	-0.007	-0.027
	(0.096)	(0.014)	(0.012)	(0.003)	(0.099)	(0.008)	(0.002)	(0.007)
D2004	-0.202	0.020	-0.010	-0.010	-0.215	0.019	-0.004	-0.015
	(0.097)	(0.011)	(0.007)	(0.004)	(0.101)	(0.009)	(0.002)	(0.007)
D2005	-0.244	0.025	-0.013	-0.012	-0.244	0.021	-0.005	-0.017
	(0.098)	(0.012)	(0.008)	(0.004)	(0.102)	(0.009)	(0.002)	(0.007)
D2006	-0.272	0.028	-0.015	-0.013	-0.295	0.026	-0.006	-0.020
	(0.099)	(0.012)	(0.009)	(0.004)	(0.102)	(0.009)	(0.002)	(0.007)
D2007	-0.098	0.009	-0.003	-0.005	-0.116	0.010	-0.002	-0.008
	(0.100)	(0.010)	(0.005)	(0.005)	(0.104)	(0.009)	(0.002)	(0.007)
D2008	-0.187	0.018	-0.009	-0.010	-0.185	0.016	-0.004	-0.012
	(0.101)	(0.011)	(0.007)	(0.004)	(0.104)	(0.009)	(0.002)	(0.007)
D2009	-0.130	0.012	-0.005	-0.007	-0.135	0.012	-0.003	-0.009
	(0.102)	(0.010)	(0.006)	(0.005)	(0.106)	(0.009)	(0.002)	(0.007)
D2010	-0.038	0.003	-0.001	-0.002	-0.037	0.003	-0.001	-0.002
	(0.104)	(0.009)	(0.003)	(0.006)	(0.108)	(0.010)	(0.002)	(0.007)
D2011	0.132	-0.010	0.001	0.009	0.142	-0.013	0.003	0.009
	(0.104)	(0.007)	(0.001)	(0.008)	(0.108)	(0.010)	(0.003)	(0.007)
D2012	0.009	-0.001	0.000	0.001	0.013	-0.001	0.000	0.001
	(0.105)	(0.009)	(0.002)	(0.007)	(0.109)	(0.010)	(0.002)	(0.007)

D2013	0.131	-0.010	0.001	0.009	0.128	-0.003	0.001	0.002
	(0.107)	(0.007)	(0.001)	(0.008)	(0.111)	(-0.001)	(0.000)	(0.001)
D2014	0.042	-0.003	0.001	0.003	0.029	-0.003	0.001	0.002
	(0.108)	(0.008)	(0.001)	(0.007)	(0.112)	(0.010)	(0.003)	(0.008)
Cut1	-2.506				-2.880			
	(0.124)				(0.255)			
Cut2	1.206				0.899			
	(0.122)				(0.253)			
Pseudo R	0.0511				0.0708			
Ind. FE	No				Yes			

Determinants of rating migrations under Ordered Probit model. The dependent variable is 1 if downgrade happens at that firm-year observation; it is 2 if rating remains; and it is 3 if upgrade happens. The first and fifth columns present the estimation, and the other columns provide marginal effect. The marginal effect for continuous variables refers to the slope of a specific probability[20] at the mean level of that variable. For dummy variables, marginal effect measures the probability deviation due to changes of that variable from 0 to 1. Model 2 controls for industry fixed effects.

igration determinants under the ordered probit model.[21] The dependent variable is rating migrations: it takes the value 1 if downgrade happens at that firm-year observation; 2 if rating remains; and 3 if upgrade happens. The first and fifth columns present the estimation, and the other columns report marginal effects. Most of the year intercepts in Model 1 demonstrates negative signs, which suggests that the probability of being a downgrade observation increases with time, holding all other control variables the same. This is consistent with the observed asymmetry that upgrades become increasingly difficult. Marginal effects reported in columns 2 to 4 confirm the asymmetry. $ME1$ reports the probability of being a downgrade observation, and most of the year dummies show positive contribution. Inversely, year dummy impact

[20] For "ME1" columns, the probability refers to the probability of being downgrade observations. For "ME2" columns, the probability refers to the probability of being observations with remaining ratings. For "ME3" columns, the probability refers to the probability of being upgrade observations.

[21] The adjusted ordered probit model is not appropriate in migration studies since it ignores stickiness embedded in decisions to upgrade or downgrade.

in $ME3$ has a negative sign, which means the probability to be an upgrade observations decreases. This pattern does not change after controlling for industry fixed effects as in Model 2. This evidence supports that asymmetry exists between upgrade and downgrade decision standards, and hence contributes to the rating deterioration.

3.5.4 Robustness Analysis

To check the robustness of our results, we perform a number of additional tests. To further support the claim about the existence of stickiness, we provide evidence from the interaction analysis and lag rating analysis. Table 3.8 summarizes the interaction analysis, in which the specification follows Model 5 in Table 3.4, but adding interaction terms. One fundamental assumption of stickiness is that agencies update ratings after credit quality breaches a threshold. One way to test this conjecture is to demonstrate the existence of a relationship between covariates and ratings at the breaching point, the period right before a rating migration, differs from the one in the other periods. In line with our expectations, all interactions terms in Panel A are highly significant, suggesting that the threshold observations statistically differ from normal observations. Panel B controls lagged credit rating in interaction terms, and the highly significant effects strongly confirms our argument. Moreover, we argue that past and current locations on credit quality spectrum jointly determine the rating migration behavior and, hence, the observed ratings. In other words, rating in last period has to be an influential factor under the stickiness framework. Therefore, we use the lagged rating analysis to show this (Table 3.9). The lagged credit rating variable, denoted by $LagY$, is highly significant in all models, consistent with our initial assumption. We further apply the lag rating analysis for each year data, and plot the coefficients and t-statistics of $LagY$ in Figure 3.7. The coefficients spike around 3 and t-statistics range between 24 and 6. These highly significant coefficients of $LagY$ further support our claim.

To investigate whether our results are caused by shifts in relationships of firm characteristics rather than rating stickiness, we apply Fama-MacBeth estimation for every ten years data as in Models 9 to 11 in Table 3.4. We firstly estimate regressions using adjusted ordered probit on every two-year data controlling for both z-dimensional and t-dimensional stickiness. Then, we take the average of regression coefficients for every decade (85-94, 95-04, and 05-14) to calculate the Fama-MacBeth coefficients. The comparison between these three models reveals that the relations between credit rating and firm characteristics do not shift during our sample period. Further, we redo the credit quality contribution decomposition but varying the relationship (regression coefficients) applied. The original contribution analysis reported in Table 3.6 is based on the relation in decade from 1985 to 1994. Then, we replicate this analysis by using 1995-2004 relation and 2005-2014 relation, with Table 3.10 and Table 3.11 summarizing the results, respectively. Our main results indicate that credit quality does not change. Further, we also plot the average predicted credit quality controlling for lagged rating in Figure 3.8.[22] This figure suggests a slight improvement in credit quality during our sample period.

Furthermore, we re-run our tests within investment grade and speculative grade

[22]The prediction is based on the relationship in each year regression controlling for lagged rating. The relationships are estimated in lagged rating analysis in Figure 3.7.

subsamples. Table 3.12 replicates Table 3.4 within subsamples of investment grade observations and speculative grade observations. "Investment grade" refers to BBB- or above under S&P credit rating framework. The subsample includes ratings A, B, and C in our merged ratings, which totally contains 10,480 observations. In comparison, observations with S&P ratings below BBB- fall into the speculative grade, which covers ratings D and E in our merged category and totally include 10,077 observations. Figure 3.5 and Figure 3.6 plot the year intercepts for the investment grade and speculative grade subsamples, respectively. Our main results in Table 3.4 indicate that the downward trend of year intercepts in the ordered probit model disappears after controlling for stickiness. This pattern also exists in the investment grade subsample, but is less clear in the speculative grade subsample. Further, Table 3.13 shows the goodness of fit through predicted ratings. Panel A presents the predicted ratings without considering stickiness in rating assignment, and Panel B reports the predictions within stickiness framework. Prediction with stickiness tends to overvalue ratings, while predicted ratings without stickiness spikes around the actual one. Moreover, we have controlled missing *R&D* dummy variable and replicates regressions of Model 5 to 8 in Table 3.4 as shown in Table 3.14 to test the sensitivity of our *R&D* setting (replace missing values with zero). The relationships estimated do not significantly change. Figure 3.9 plot the year dummy intercepts in Table 3.14 regressions. The pattern of diminishing downward trend as stickiness being controlled still exists. Lastly, our argument about asymmetry between upgrade and downgrade rating standard are based on Figure 3.3 and 3.4, which are estimated based on two-year moving window. We replicate these estimations based on expanding window as shown in Figure 3.10 and 3.11. The asymmetry still exists. The downward sloping lower cut-off in Figure 3.11 even suggests downgrade becomes increasingly easy, which further enhances our explanation to the credit rating deterioration.

3.6 Conclusion

The stickiness-based model used in this study facilitates the understanding of the observed general deterioration on credit ratings. The contribution of this study is that it embeds rating mechanism, with its inertia equivalent to the presence of lump sum costs, into the model setting and estimation. The model therefore enables us to separate the effects of credit quality variation and changes in rating standards. Our findings shed light on the debate whether the downward trend in credit ratings is due to the deteriorating credit quality or the tightening of rating standard. The results support the existence of stickiness in credit ratings, and demonstrate its significant impact on rating migrations. After introducing stickiness, the downward trend of year dummy intercepts disappears and the interaction terms are statistically significant. These findings strongly underpin the existence of stickiness. After controlling for stickiness, we find that firms' credit quality actually improves during that period. We also document asymmetry in rating migrations. Upgrades become increasingly difficult while downgrade standard appear to remain the same. This mechanism offsets the slight improvement in credit quality, and hence leads to the preserved credit quality "deterioration" (i.e. based on ratings). Also, our study contributes to the literature on rating determinants. Controlling for the rating mechanism allows us to more precisely measure the effects of the explanatory factors affecting credit ratings.

Table 3.8: Interaction Analysis

Panel A: Interaction without Lagged Rating	Model 1	Model 2	Model 3	Model 4		Model 5		
Dependent	y	y	lag y	y			y	
Interac.					M		U	D
Inter M					-0.235			
					(0.267)			
Inter U	1.314	1.522	1.128				-2.126	
	(0.045)	(0.049)	(0.044)				(0.488)	
Inter D	2.087	2.487	1.774					2.352
	(0.056)	(0.060)	(0.055)					(0.347)
k1	0.312	0.363	4.253	0.308	0.059	0.319	0.123	-0.108
	(0.010)	(0.012)	(0.095)	(0.011)	(0.031)	(0.011)	(0.058)	(0.040)
k2	0.075	0.135	6.166	0.073	0.045	0.075	0.022	0.000
	(0.007)	(0.007)	(0.100)	(0.007)	(0.022)	(0.007)	(0.038)	(0.028)
k3	0.025	0.024	7.578	0.023	0.022	0.024	-0.014	0.022
	(0.004)	(0.004)	(0.103)	(0.004)	(0.014)	(0.004)	(0.024)	(0.018)
k4	0.003	0.005	9.019	0.002	0.008	0.003	0.024	-0.008
	(0.001)	(0.001)	(0.107)	(0.001)	(0.004)	(0.001)	(0.007)	(0.005)
Vol	-5.332	-3.959	0.313	-5.536	3.201	-5.739	2.055	3.572
	(0.233)	(0.275)	(0.010)	(0.250)	(0.672)	(0.252)	(1.278)	(0.876)
Tlev	-1.392	-1.866	0.080	-1.413	1.698	-1.464	1.985	0.004
	(0.066)	(0.079)	(0.006)	(0.070)	(0.209)	(0.071)	(0.387)	(0.275)
Rrent	-9.271	-8.289	0.026	-9.111	4.489	-9.475	0.443	0.399
	(0.310)	(0.529)	(0.004)	(0.325)	(0.962)	(0.329)	(2.085)	(1.204)
Tan	0.859	0.961	0.003	0.842	-0.211	0.873	-0.249	-0.197
	(0.040)	(0.079)	(0.001)	(0.041)	(0.131)	(0.042)	(0.220)	(0.173)
Dni	-0.490	-0.392	-4.841	-0.496	0.531	-0.515	-0.097	0.143
	(0.025)	(0.028)	(0.225)	(0.027)	(0.074)	(0.027)	(0.164)	(0.091)
Ddiv	0.981	0.830	-1.004	0.955	0.304	0.995	0.148	-0.292
	(0.022)	(0.027)	(0.065)	(0.023)	(0.068)	(0.024)	(0.115)	(0.093)
Rxrd	-1.637	-0.043	-8.082	-1.648	0.921	-1.715	0.265	-0.612
	(0.319)	(0.569)	(0.302)	(0.336)	(1.029)	(0.338)	(1.790)	(1.337)
Mtb	0.461	0.480	0.770	0.456	-0.465	0.472	-0.165	-0.038
	(0.015)	(0.018)	(0.039)	(0.016)	(0.049)	(0.016)	(0.074)	(0.073)
Size	0.459	0.629	-0.259	0.454	-0.050	0.469	-0.015	-0.113
	(0.007)	(0.010)	(0.025)	(0.007)	(0.022)	(0.008)	(0.039)	(0.029)
Beta	-0.038	-0.026	1.026	-0.035	-0.011	-0.037	-0.011	-0.051
	(0.005)	(0.005)	(0.022)	(0.005)	(0.017)	(0.005)	(0.030)	(0.025)
Rmse	-0.171	-0.120	-1.087	-0.141	-0.212	-0.149	0.233	-0.389
	(0.058)	(0.067)	(0.314)	(0.060)	(0.202)	(0.060)	(0.384)	(0.259)
Cut1	4.293	6.461	0.362	2.957		3.035		
	(0.097)	(0.204)	(0.015)	(0.091)		(0.091)		

Cut2	6.314	8.920	0.449	4.907		5.085		
	(0.102)	(0.209)	(0.007)	(0.094)		(0.095)		
Cut3	7.798	10.738	-0.037	6.342		6.573		
	(0.105)	(0.213)	(0.004)	(0.097)		(0.098)		
Cut4	9.300	12.553	-0.208	7.803		8.080		
	(0.110)	(0.216)	(0.057)	(0.101)		(0.103)		
Pseudo R	0.395	0.491	0.371	0.378		0.400		
Ind. FE	No	Yes	No	No		No		
Year FE	Yes	Yes	Yes	Yes		Yes		

Panel B: Interaction with Lagged Rating

	Model 1	Model 2	Model 3		Model 4		
Dependent	y	y	y		y		
Inter M				-0.414			
				(0.406)			
Inter U	-0.840	-1.048				-1.727	
	(0.067)	(0.070)				(0.736)	
Inter D	0.589	0.777					1.117
	(0.051)	(0.053)					(0.525)
lag y	2.936	2.867	2.920	0.342	2.947	-0.035	0.019
	(0.024)	(0.026)	(0.025)	(0.053)	(0.025)	(0.111)	(0.076)
k1	0.085	0.135	0.080	0.049	0.083	0.145	-0.013
	(0.015)	(0.017)	(0.016)	(0.048)	(0.016)	(0.089)	(0.061)
k2	0.010	0.042	0.009	-0.005	0.009	0.000	-0.009
	(0.010)	(0.011)	(0.010)	(0.033)	(0.010)	(0.058)	(0.042)
k3	0.006	0.008	0.003	0.030	0.003	0.022	0.028
	(0.006)	(0.006)	(0.006)	(0.022)	(0.006)	(0.037)	(0.028)
k4	-0.001	-0.001	-0.001	0.003	-0.001	0.006	-0.002
	(0.001)	(0.002)	(0.001)	(0.006)	(0.001)	(0.011)	(0.007)
Vol	-2.568	-2.701	-2.570	0.907	-2.627	-3.325	1.798
	(0.330)	(0.385)	(0.350)	(1.003)	(0.353)	(1.902)	(1.296)
Tlev	-1.554	-1.952	-1.629	1.722	-1.665	1.574	0.769
	(0.097)	(0.113)	(0.102)	(0.315)	(0.103)	(0.584)	(0.411)
Rrent	-5.766	-4.855	-5.878	5.761	-6.048	2.276	1.951
	(0.452)	(0.751)	(0.473)	(1.444)	(0.478)	(3.054)	(1.821)
Tan	0.513	0.517	0.513	-0.388	0.525	-0.098	-0.350
	(0.058)	(0.114)	(0.061)	(0.200)	(0.062)	(0.326)	(0.261)
Dni	-0.705	-0.680	-0.711	0.287	-0.728	-0.603	0.274
	(0.037)	(0.039)	(0.039)	(0.110)	(0.039)	(0.224)	(0.135)
Ddiv	0.192	0.165	0.191	-0.088	0.201	0.218	-0.222
	(0.033)	(0.039)	(0.035)	(0.110)	(0.035)	(0.181)	(0.145)
Rxrd	-1.973	-2.690	-1.884	-0.661	-1.944	-0.433	-1.689
	(0.466)	(0.807)	(0.490)	(1.526)	(0.494)	(2.710)	(1.936)
Mtb	0.415	0.484	0.423	-0.395	0.432	-0.156	-0.128

	(0.022)	(0.026)	(0.023)	(0.075)	(0.023)	(0.112)	(0.108)
Size	0.169	0.250	0.170	-0.095	0.173	0.059	-0.067
	(0.011)	(0.015)	(0.011)	(0.036)	(0.011)	(0.062)	(0.047)
Beta	-0.012	-0.009	-0.011	0.014	-0.012	-0.026	-0.008
	(0.007)	(0.007)	(0.007)	(0.027)	(0.007)	(0.047)	(0.038)
Rmse	0.041	0.051	0.069	-0.336	0.068	0.324	-0.548
	(0.086)	(0.099)	(0.090)	(0.308)	(0.091)	(0.573)	(0.395)
Cut1	4.976	5.592	4.935		4.975		
	(0.128)	(0.282)	(0.135)		(0.136)		
Cut2	8.798	9.658	8.717		8.838		
	(0.140)	(0.291)	(0.146)		(0.148)		
Cut3	12.365	13.386	12.254		12.409		
	(0.154)	(0.300)	(0.159)		(0.161)		
Cut4	15.968	17.124	15.836		16.020		
	(0.172)	(0.312)	(0.176)		(0.179)		
Pseudo R	0.791	0.804	0.789			0.793	
Ind. FE	No	Yes	No			No	
Year FE	Yes	Yes	Yes			Yes	

Stickiness in credit ratings through interaction analysis using ordered probit model. The specification generally follows Model 5 in Table 3.4 but adds interaction terms. The dependent variable is observed credit ratings or lagged ratings as presented. Inter M refers to the dummy variable with value 1 indicating rating migration (either upgrade or downgrade). Similarly, Inter U and Inter D refer to dummies specifically indicating upgrade and downgrade, respectively. Model 4 includes terms interacting with migration dummies, and model 5 further replaces the migration interaction terms with upgrade and downgrade interaction terms. Panel A report the results without lagged rating interactions, and Panel B controls these interaction terms.

Table 3.9: Lag Rating Analysis: Real Data

	Model 1	Model 2	Model 3	Model 4
Lag Y	2.916	2.900	2.905	2.870
	(0.023)	(0.024)	(0.023)	(0.024)
Intcov	-0.001	0.002		
	(0.001)	(0.001)		
k1			0.045	0.087
			(0.015)	(0.015)
k2			-0.002	0.014
			(0.009)	(0.010)
k3			0.000	0.008
			(0.006)	(0.006)
k4			-0.001	0.000
			(0.001)	(0.001)
Vol	-2.901	-2.683	-2.750	-2.378
	(0.327)	(0.331)	(0.332)	(0.336)
Tlev	-1.495	-1.613	-1.394	-1.360
	(0.088)	(0.090)	(0.096)	(0.098)
Rent	-4.410	-5.023	-4.460	-5.245
	(0.437)	(0.445)	(0.439)	(0.449)
Tan	0.605	0.471	0.600	0.462
	(0.057)	(0.058)	(0.057)	(0.059)
Dni	-0.660	-0.704	-0.637	-0.663
	(0.035)	(0.036)	(0.036)	(0.037)
Ddiv	0.189	0.196	0.183	0.185
	(0.033)	(0.033)	(0.033)	(0.033)
Rd	-0.489	-1.588	-0.555	-1.876
	(0.457)	(0.469)	(0.458)	(0.472)
Mtb	0.371	0.392	0.363	0.368
	(0.021)	(0.021)	(0.021)	(0.022)
Size	0.079	0.145	0.079	0.153
	(0.009)	(0.011)	(0.009)	(0.011)
Beta	-0.010	-0.008	-0.010	-0.008
	(0.007)	(0.007)	(0.007)	(0.007)
Rmse	0.035	0.006	0.060	0.049
	(0.086)	(0.087)	(0.087)	(0.088)
Cut1	4.538	4.919	4.709	5.273
	(0.098)	(0.113)	(0.111)	(0.128)
Cut2	8.135	8.573	8.321	8.959
	(0.108)	(0.125)	(0.123)	(0.141)
Cut3	11.524	12.029	11.706	12.416
	(0.124)	(0.141)	(0.137)	(0.155)
Cut4	14.951	15.547	15.120	15.919

	(0.145)	(0.162)	(0.155)	(0.174)
Pseudo R	0.769	0.774	0.769	0.775
Year Dummy	No	Yes	No	Yes

Ordered probit regression results controlling from lagged rating. *LagY* refers to the rating lagged by one year, and all other variables are defined as in Table 3.3. There are 19,069 observations after creating lag rating variable. Model 1 exploit interest coverage, and Model 2 further controls year fixed effect based on Model 1. Model 3 decomposes interest coverage into four components, and Model 4 further controls year dummies. Standard error is in the parentheses.

Table 3.10: Determinants of Deviation in Credit Quality based on 95-04 Relationship

	Full	Invest.	Specu.	Diff.	Large	Small	Diff.
k1	0.080	0.112	0.039	0.074	0.093	0.068	0.024
	(0.003)	(0.005)	(0.002)		(0.005)	(0.005)	
k2	0.008	0.007	0.011	-0.005	0.009	0.007	0.002
	(0.000)	(0.000)	(0.000)		(0.000)	(0.000)	
k3	-0.010	-0.006	-0.018	0.013	-0.011	-0.009	-0.002
	(0.000)	(0.000)	(0.001)		(0.001)	(0.001)	
k4	-0.009	-0.004	-0.017	0.013	-0.008	-0.009	0.001
	(0.001)	(0.001)	(0.001)		(0.001)	(0.001)	
Vol	0.012	0.022	0.000	0.021	0.016	0.008	0.008
	(0.002)	(0.003)	(0.001)		(0.002)	(0.003)	
Tlev	0.045	0.072	0.008	0.065	0.052	0.038	0.014
	(0.003)	(0.004)	(0.003)		(0.004)	(0.004)	
Rent	0.022	0.018	0.030	-0.011	0.021	0.022	-0.001
	(0.002)	(0.003)	(0.002)		(0.003)	(0.003)	
Tan	-0.042	-0.022	-0.073	0.052	-0.045	-0.039	-0.005
	(0.003)	(0.004)	(0.005)		(0.004)	(0.004)	
Dni	0.028	0.043	0.013	0.030	0.030	0.027	0.003
	(0.004)	(0.006)	(0.003)		(0.006)	(0.006)	
Ddiv	0.003	0.006	0.000	0.007	0.004	0.002	0.002
	(0.001)	(0.001)	(0.000)		(0.001)	(0.001)	
Rd	0.010	0.014	0.001	0.014	0.005	0.015	-0.010
	(0.002)	(0.002)	(0.004)		(0.003)	(0.003)	
Mtb	0.085	0.039	0.168	-0.129	0.092	0.078	0.014
	(0.006)	(0.006)	(0.011)		(0.008)	(0.008)	
Size	0.257	0.276	0.244	0.032	0.264	0.251	0.012
	(0.004)	(0.004)	(0.006)		(0.006)	(0.006)	
Beta	-0.007	-0.012	0.000	-0.012	-0.009	-0.006	-0.003
	(0.001)	(0.001)	(0.001)		(0.001)	(0.001)	
Rmse	-0.003	-0.004	0.000	-0.004	-0.003	-0.003	0.000
	(0.000)	(0.000)	(0.000)		(0.000)	(0.000)	
Total	0.479	0.563	0.404	0.159	0.509	0.451	0.058

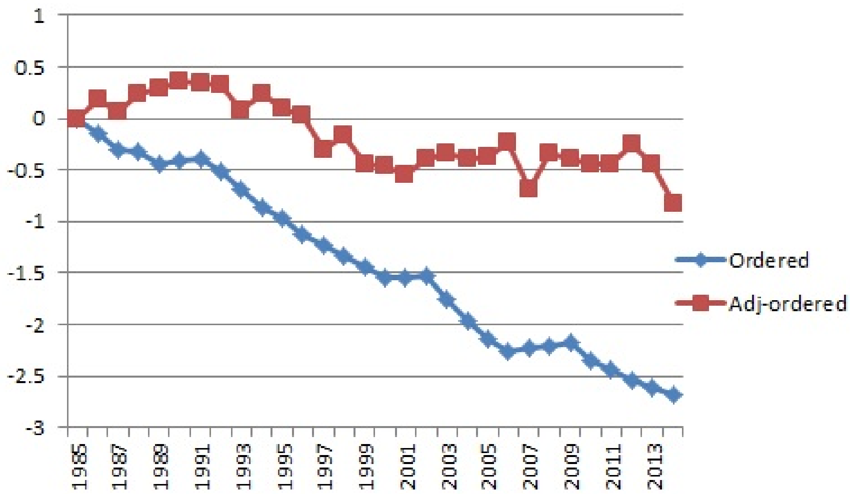
Variation of predicted credit quality between the last six-year period (2009-2014) average level and the first six-year period (1985-1990) average level. The Z-score following Model 10 of Table 3.4 serves the proxy of credit quality: Z-score$= 0.115 * k1 + 0.006 * k2 - 0.005 * k3 - 0.002 * k4 - 1.495 * Vol - 0.674 * Tlev - 3.370 * Rent + 0.514 * tan - 0.489 * Dni + 0.063 * Ddiv - 3.098 * Rd + 0.424 * Mtb + 0.123 * Size - 0.022 * Beta + 0.047 * Rmse$. This relation is the average coefficients of the first five sub-period Adjusted-Ordered Probit estimation, covering 1995 to 2004. The contribution of each variable is the product of increment in period average and the corresponding coefficients.

TABLE 3.11: Determinants of Deviation in Credit Quality based on 05-14 Relationship

	Full	Invest.	Specu.	Diff.	Large	Small	Diff.
k1	0.010	0.015	0.005	0.010	0.012	0.009	0.003
	(0.000)	(0.001)	(0.000)		(0.001)	(0.001)	
k2	0.019	0.016	0.026	-0.011	0.021	0.017	0.004
	(0.001)	(0.001)	(0.001)		(0.001)	(0.001)	
k3	-0.005	-0.003	-0.009	0.006	-0.005	-0.004	-0.001
	(0.000)	(0.000)	(0.000)		(0.000)	(0.000)	
k4	0.008	0.004	0.016	-0.012	0.008	0.009	-0.001
	(0.001)	(0.001)	(0.001)		(0.001)	(0.001)	
Vol	0.023	0.040	0.001	0.039	0.030	0.016	0.015
	(0.003)	(0.005)	(0.002)		(0.005)	(0.005)	
Tlev	0.085	0.137	0.014	0.122	0.098	0.072	0.026
	(0.005)	(0.007)	(0.006)		(0.007)	(0.007)	
Rent	0.049	0.042	0.067	-0.025	0.047	0.050	-0.003
	(0.005)	(0.007)	(0.005)		(0.007)	(0.007)	
Tan	-0.035	-0.018	-0.060	0.042	-0.037	-0.032	-0.004
	(0.002)	(0.003)	(0.004)		(0.003)	(0.003)	
Dni	0.039	0.059	0.018	0.041	0.041	0.037	0.004
	(0.006)	(0.008)	(0.005)		(0.008)	(0.008)	
Ddiv	0.011	0.023	0.000	0.023	0.015	0.007	0.008
	(0.002)	(0.003)	(0.001)		(0.003)	(0.003)	
Rd	-0.002	-0.003	0.000	-0.003	-0.001	-0.003	0.002
	(0.000)	(0.001)	(0.001)		(0.001)	(0.001)	
Mtb	0.081	0.037	0.162	-0.124	0.088	0.075	0.013
	(0.005)	(0.006)	(0.010)		(0.008)	(0.008)	
Size	0.427	0.458	0.405	0.054	0.438	0.417	0.021
	(0.007)	(0.007)	(0.009)		(0.009)	(0.009)	
Beta	-0.023	-0.037	0.000	-0.037	-0.028	-0.018	-0.009
	(0.003)	(0.004)	(0.002)		(0.004)	(0.004)	
Rmse	0.028	0.044	0.004	0.041	0.029	0.027	0.003
	(0.002)	(0.002)	(0.001)		(0.002)	(0.002)	
Total	0.716	0.815	0.649	0.166	0.757	0.677	0.080

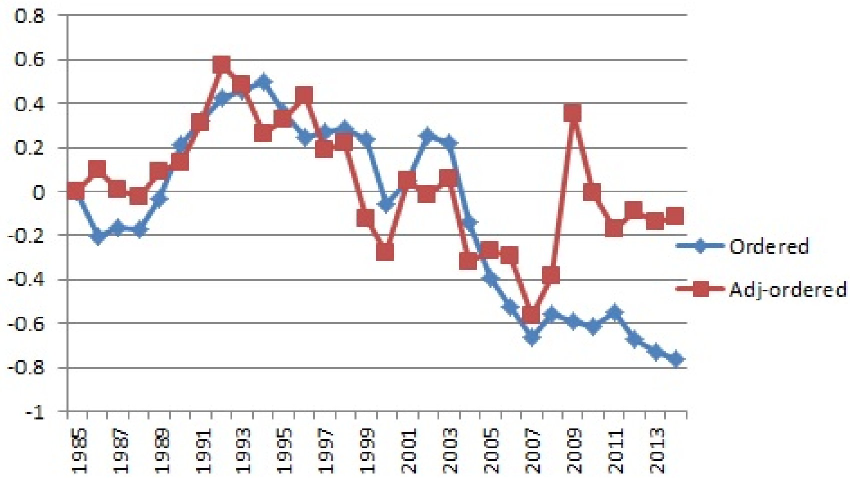
Variation of predicted credit quality between the last six-year period (2009-2014) average level and the first six-year period (1985-1990) average level. The Z-score following Model 11 of Table 3.4 serves the proxy of credit quality: Z-score= $0.016 * k1 + 0.014 * k2 - 0.003*k3+0.002*k4-2.780*Vol-1.287*Tlev-7.623*Rent+0.421*tan-0.668*Dni+0.226*Ddiv+0.627*Rd+0.407*Mtb+0.203*Size-0.067*Beta-0.436*Rmse$. This relation is the average coefficients of the first five sub-period Adjusted-Ordered Probit estimation, covering 2005 to 2014. The contribution of each variable is the product of increment in period average and the corresponding coefficients.

FIGURE 3.5: Investment: Plot of the Estimates of the Year Dummy Intercepts



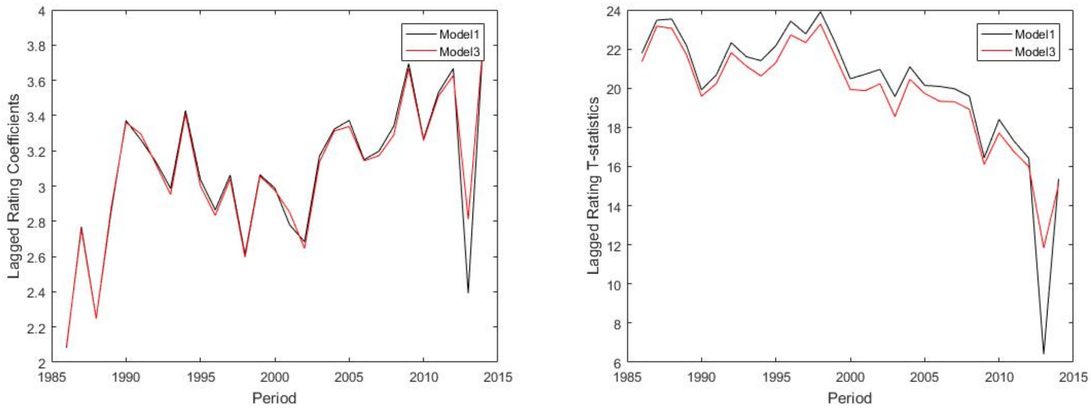
Year dummy intercepts estimated from Ordered Probit and adjusted ordered probit models of investment grade subsample in Table 3.12.

FIGURE 3.6: Speculative: Plot of the Estimates of the Year Dummy Intercepts



Year dummy intercepts estimated from Ordered Probit and adjusted ordered probit models of speculative grade subsample in Table 3.12.

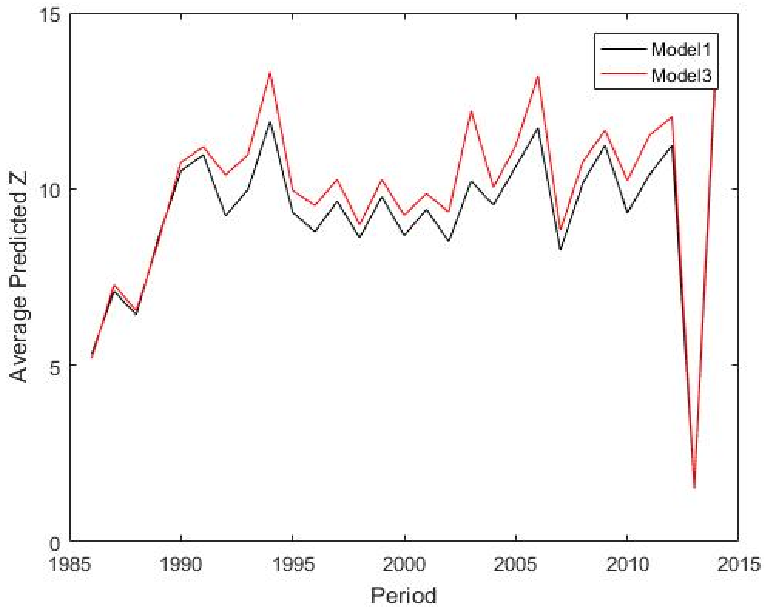
FIGURE 3.7: Lag Rating Analysis: Real Data



(A) Lag Rating Coefficients (B) Lag Rating T-statistics

Further tests of lag rating analysis in addition to Table 3.9. We apply Model 1 and Model 3 regression coefficients in Table 3.9 for each year data, and plot the coefficients and T-statistics of the *LagY* variable.

FIGURE 3.8: Plot of the Predicted Average Credit Quality



The average predicted credit quality (Z-score) for each year from Model 1 and 3 tests as in Figure 3.7.

Table 3.12: Subsample Analysis

Panle A: Coefficients				
	Investment		Speculative	
	Model 1	Model 2	Model 3	Model 4
k1	0.398	0.100	0.292	0.059
	(0.028)	(0.045)	(0.015)	(0.020)
k2	0.097	-0.011	0.018	0.004
	(0.010)	(0.017)	(0.015)	(0.020)
k3	0.070	0.016	-0.024	-0.021
	(0.005)	(0.009)	(0.011)	(0.014)
k4	0.006	0.001	0.010	-0.003
	(0.001)	(0.002)	(0.003)	(0.004)
Vol	-10.011	-2.740	-1.958	-1.332
	(0.714)	(1.147)	(0.272)	(0.364)
Tlev	-0.541	-1.284	-0.888	-1.035
	(0.149)	(0.253)	(0.092)	(0.123)
Rrent	-14.343	-7.472	-4.149	-3.421
	(0.759)	(1.213)	(0.412)	(0.549)
Tan	0.869	0.342	0.097	-0.001
	(0.066)	(0.110)	(0.073)	(0.098)
Dni	-0.426	-0.476	-0.390	-0.509
	(0.060)	(0.103)	(0.035)	(0.046)
Ddiv	0.830	0.213	0.656	0.030
	(0.060)	(0.096)	(0.034)	(0.045)
Rxrd	1.404	-0.125	-5.208	-3.397
	(0.511)	(0.856)	(0.531)	(0.714)
Mtb	0.328	0.354	0.324	0.448
	(0.022)	(0.037)	(0.033)	(0.044)
Size	0.366	0.123	0.349	0.142
	(0.011)	(0.019)	(0.014)	(0.019)
Beta	-0.075	-0.010	-0.011	-0.022
	(0.011)	(0.019)	(0.006)	(0.008)
Rmse	0.245	0.573	0.035	0.059
	(0.143)	(0.225)	(0.077)	(0.105)
U1/Cut1	5.303	3.615	2.945	2.201
	(0.190)	(0.305)	(0.149)	(0.198)
U2/Cut2	6.984	4.602		
	(0.196)	(0.327)		
L1		0.443		-0.439
		(0.310)		(0.205)
L2		0.881		
		(0.325)		
Pseudo R	0.2705	0.0828	0.3294	0.1204

Obs.	10480	10480	10077	10077
Panle B: Time Dummies				
	Investment		Speculative	
	Model 1	Model 2	Model 3	Model 4
D1986	-0.150	0.187	-0.202	0.094
	(0.087)	(0.137)	(0.107)	(0.143)
D1987	-0.303	0.065	-0.167	0.009
	(0.088)	(0.138)	(0.104)	(0.141)
D1988	-0.315	0.239	-0.173	-0.026
	(0.089)	(0.142)	(0.108)	(0.146)
D1989	-0.436	0.291	-0.032	0.093
	(0.089)	(0.143)	(0.111)	(0.152)
D1990	-0.415	0.367	0.209	0.134
	(0.090)	(0.144)	(0.116)	(0.158)
D1991	-0.388	0.347	0.317	0.313
	(0.089)	(0.146)	(0.122)	(0.165)
D1992	-0.509	0.325	0.422	0.571
	(0.089)	(0.144)	(0.118)	(0.159)
D1993	-0.687	0.075	0.460	0.486
	(0.088)	(0.142)	(0.117)	(0.160)
D1994	-0.868	0.232	0.497	0.264
	(0.090)	(0.143)	(0.118)	(0.159)
D1995	-0.969	0.092	0.364	0.324
	(0.089)	(0.144)	(0.120)	(0.162)
D1996	-1.133	0.032	0.248	0.437
	(0.090)	(0.145)	(0.117)	(0.162)
D1997	-1.237	-0.298	0.272	0.184
	(0.090)	(0.146)	(0.119)	(0.159)
D1998	-1.329	-0.171	0.287	0.220
	(0.091)	(0.146)	(0.116)	(0.157)
D1999	-1.441	-0.444	0.237	-0.127
	(0.094)	(0.154)	(0.118)	(0.152)
D2000	-1.545	-0.458	-0.059	-0.276
	(0.097)	(0.154)	(0.118)	(0.155)
D2001	-1.537	-0.541	0.047	0.052
	(0.099)	(0.162)	(0.119)	(0.160)
D2002	-1.520	-0.392	0.251	-0.013
	(0.099)	(0.162)	(0.119)	(0.156)
D2003	-1.751	-0.347	0.219	0.053
	(0.099)	(0.162)	(0.122)	(0.162)
D2004	-1.955	-0.388	-0.142	-0.316
	(0.101)	(0.163)	(0.118)	(0.156)
D2005	-2.143	-0.368	-0.395	-0.271
	(0.101)	(0.162)	(0.122)	(0.163)

D2006	-2.253	-0.229	-0.527	-0.297
	(0.102)	(0.163)	(0.123)	(0.165)
D2007	-2.231	-0.692	-0.665	-0.568
	(0.104)	(0.173)	(0.124)	(0.163)
D2008	-2.203	-0.344	-0.559	-0.382
	(0.105)	(0.177)	(0.124)	(0.165)
D2009	-2.174	-0.395	-0.588	0.350
	(0.107)	(0.177)	(0.125)	(0.173)
D2010	-2.342	-0.437	-0.617	-0.007
	(0.109)	(0.181)	(0.129)	(0.176)
D2011	-2.429	-0.449	-0.548	-0.169
	(0.110)	(0.183)	(0.131)	(0.177)
D2012	-2.540	-0.252	-0.669	-0.090
	(0.111)	(0.180)	(0.132)	(0.183)
D2013	-2.604	-0.442	-0.728	-0.137
	(0.111)	(0.185)	(0.138)	(0.189)
D2014	-2.684	-0.826	-0.766	-0.117
	(0.111)	(0.185)	(0.141)	(0.195)

Main comparison of Table 3.4 between estimations of ordered probit and adjusted ordered probit within subsamples of investment grade observations and speculative grade observations. Investment grade refers to BBB- or above under S&P credit rating framework. It includes ratings A, B, and C, in our merged ratings, which totally contains 10,480 observations. In comparison, observations with S&P ratings below BBB- falls into the speculative grade, which covers ratings D and E in our merged category and totally includes 10,077 observations.

Table 3.13: Predicted Ratings from Adjusted Ordered Model

Panel A: Predicted Rating without Stickiness						
			TRUE			
Predicted	A	B	C	D	E	Total
A	152	101	23	15	3	294
B	595	1058	474	111	5	2243
C	323	1975	3174	1393	163	7028
D	27	466	1989	3546	1210	7238
E	2	8	113	1293	2338	3754
Total	1099	3608	5773	6358	3719	20557
Panel B: Predicted Rating with Stickiness						
			TRUE			
Predicted	A	B	C	D	E	Total
A	1095	3547	5061	3298	762	13763
B	2	42	429	1582	787	2842
C	2	2	248	1101	1117	2470
D	0	17	35	317	844	1213
E	0	0	0	60	209	269
Total	1099	3608	5773	6358	3719	20557

A measure of goodness of fit for our models is to use this panel about predicted ratings. Each column presents the spectrum of predicted ratings for each true rating category. For example, the number at row 1 column 2 in Panel A is 101, which means there are 101 observations with true rating B but predicted as A. This prediction is based on the estimation of Model 6 in Table 3.4. Panel A and B are based on the same predicted credit quality but different rating assignment method. Panel A presents the prediction without considering stickiness, which means immediate rating adjustment and single set of rating boundaries. Since our adjusted ordered model predicts separate rating standards for upgrade and downgrade, we take the average between the two sets to simulate the single stream of standard. For example, U_1 is the upper boundary of rating 1, L_2 is the lower boundary of rating 2, and hence the simulated single boundary between ratings 1 and 2 is $\frac{(U_1+L_2)}{2}$. The ratings in Panel A are predicted by assigning credit quality values to each category under this standard. Prediction in Panle B considers stickiness, which applies separated standards for upgrade and downgrade as in Model 6. More precise, we firstly assign ratings for the first observation of the firm based on credit quality and the set of downgrade standard to capture the initial rating inflation problem due to the issuer-paid payment structure of rating agencies, or the so-called "rating shopping" problem.

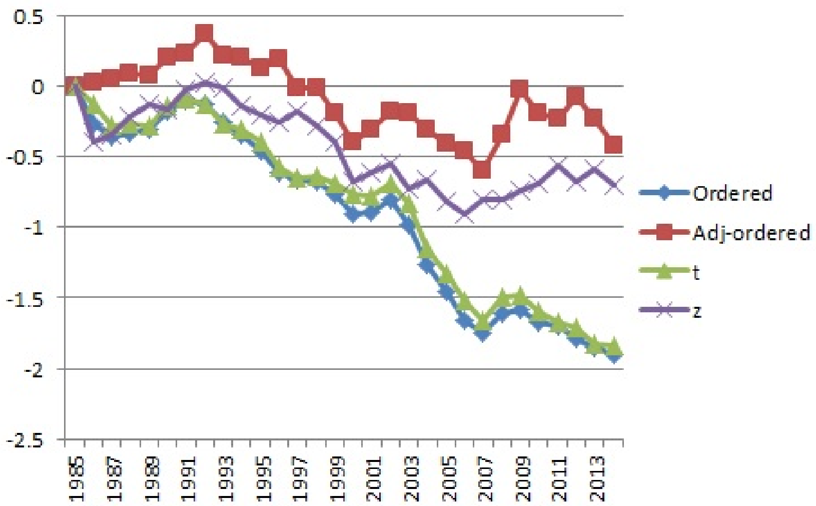
Table 3.14: Regression with Missing R&D Dummy

	Ordered	Adj.	Ordered	Adj.
	Model 1	Model 2	Model 3	Model 4
Stickiness		t,z	t	z
k1	0.313	0.046	0.316	0.097
	(0.010)	(0.014)	(0.010)	(0.016)
k2	0.079	0.000	0.083	0.012
	(0.006)	(0.009)	(0.006)	(0.010)
k3	0.026	-0.003	0.027	0.002
	(0.004)	(0.006)	(0.004)	(0.006)
k4	0.003	-0.001	0.004	0.000
	(0.001)	(0.001)	(0.001)	(0.001)
Vol	-5.065	-1.933	-4.696	-2.041
	(0.230)	(0.303)	(0.223)	(0.333)
Tlev	-1.184	-1.116	-0.851	-1.306
	(0.065)	(0.095)	(0.064)	(0.108)
Rrent	-8.552	-4.723	-7.603	-4.957
	(0.310)	(0.435)	(0.305)	(0.486)
Tan	0.799	0.381	0.721	0.422
	(0.040)	(0.065)	(0.040)	(0.060)
Dni	-0.409	-0.489	-0.200	-0.676
	(0.025)	(0.035)	(0.024)	(0.041)
Ddiv	0.992	0.094	1.038	0.191
	(0.022)	(0.031)	(0.022)	(0.034)
Rxrd	-1.022	-1.962	-0.642	-2.430
	(0.363)	(0.543)	(0.358)	(0.600)
Drxrd	0.034	-0.035	0.035	-0.019
	(0.022)	(0.033)	(0.022)	(0.035)
Mtb	0.394	0.405	0.311	0.413
	(0.015)	(0.021)	(0.015)	(0.026)
Size	0.445	0.136	0.440	0.161
	(0.007)	(0.011)	(0.007)	(0.012)
Beta	-0.036	-0.019	-0.036	-0.006
	(0.004)	(0.006)	(0.004)	(0.005)
Rmse	-0.171	0.025	-0.212	0.059
	(0.057)	(0.073)	(0.056)	(0.082)
U1/C12	3.006	2.021	3.165	2.196
	(0.060)	(0.095)	(0.059)	(0.073)
U2/C23	4.923	2.886	5.006	3.169
	(0.059)	(0.086)	(0.058)	(0.062)
U3/C34	6.345	3.565	6.372	3.871
	(0.060)	(0.094)	(0.060)	(0.079)
U4/C45	7.792	4.483	7.774	4.832

	(0.061)	(0.093)	(0.061)	(0.078)
L2		-0.606		-0.850
		(0.093)		(0.075)
L3		-0.045		-0.110
		(0.105)		(0.083)
L4		0.367		0.397
		(0.094)		(0.076)
L5		0.706		0.782
		(0.096)		(0.079)
Pseudo R	0.371	0.090	0.354	0.143
Nobs	20557	20557	20557	20557

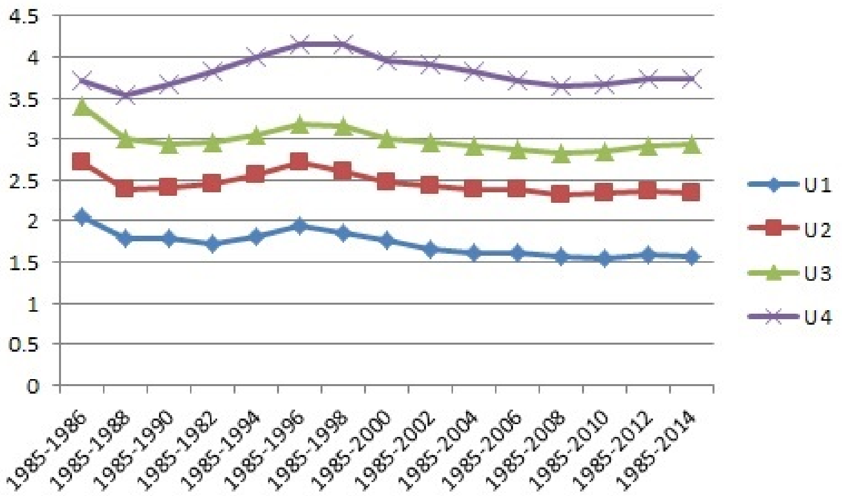
This table replicates Model 5 to 8 in Table 3.4 with additional dummy variable *Drxrd* indicating missing *R&D* observations. There are 20,557 observations from 1,488 unique firms in the full sample. Row "Stickiness" indicates different levels of stickiness considered in our model, z-dimensional stickiness, t-dimensional stickiness, or both. We report standard error in parenthesis.

FIGURE 3.9: Plot of the Estimates of the Year Dummy Intercepts



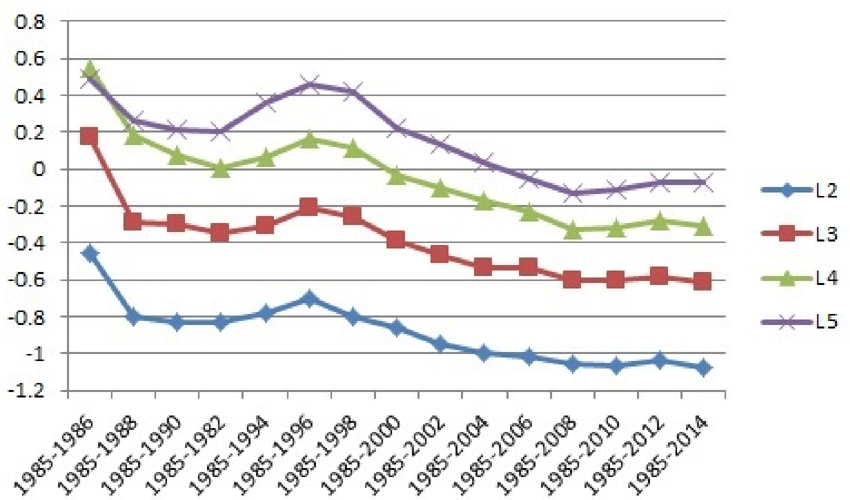
Year dummy intercepts estimated from ordered probit and adjusted ordered probit models in Table 3.14, based on a panel dataset containing 20,557 firm-year observations from 1985 to 2014. This figure replicates Figure 3.2 with missing *R&D* dummy under control. Rating stickiness is further separated into the t-dimensional and z-dimensional ones. The t stickiness refers to the delay of rating adjustment in time series. It is measured by the year intercepts from Ordered Probit model with lagged rating being the dependent variable. The z stickiness measures the tolerance of credit quality deviation, and the year intercepts are from adjusted ordered Probit with different categorization method. The difference in categorization refers to neglecting the time-series delay of rating adjustment. More precise, our main adjusted ordered probit model assumes the rating migration at time t is caused by the breaching of rating threshold at time $t-1$. However, the z stickiness model assumes the rating migration at time t happens because of the breaching at the time t as well. Hence, it is an assumption of immediate adjustment.

Figure 3.10: Plot of the Estimates of the Upper Cut-off: Expanding Window



The plots are based on the estimates of upper cutoff from Adjusted Ordered Probit model for two-year expanding window. For example, the first estimation is based on the sub-sample 1985 to 1986, and the second estimation is based on the sub-sample 1985-1988. The following estimations follows the same expanding pattern.).

Figure 3.11: Plot of the Estimates of the Lower Cut-off: Expanding Window



The plots are based on the estimates of lower cutoff from Adjusted Ordered Probit model for two-year expanding window. For example, the first estimation is based on the sub-sample 1985 to 1986, and the second estimation is based on the sub-sample 1985-1988. The following estimations follows the same expanding pattern.

Appendix 3.1 Credit Rating Simulation

This simulation concentrates on releasing the two main concerns of our credit rating study: 1) the reliability of our adjusted ordered probit model; 2) the existence of $z - stickiness$ impact. This experiment simulates data for 300 firms in 50 period[23]. For simplification, there is only one explanatory variable x from unstable distribution $x_{it} \in D[\mu_i(t),\ \sigma_i(t)]$, in which both μ and σ are decreasing function of period t. Then, the credit quality (the basis of rating and unobserved in real world) is determined by:

$$z_{it} = bx_{it} + \epsilon_{it}, \tag{3.5}$$

in which the error term ϵ_{it} is normally distributed with zero mean and standard deviation equal to 1. In most of the time, we set the true coefficient of b to be a random number 0.5426, and this setting leads to a decreasing average credit quality. In some other cases requiring increasing credit quality, we set b to -0.5426.

There are two rating mechanisms under consideration, namely the simple rating (no stickiness, denoted by r_1) and sticky rating (with stickiness, denoted by r_2). Each mechanism includes two category: $r = 2$ indicates the best credit quality, and $r = 1$ indicates the remaining. The simple rating mechanism r_1 set the median level of z_{it} (0.768) as the cutoff level ($S = 0.768$), and any observations with credit quality higher than S will be assigned the best rating $r_1 = 2$ and the rest receives $r_1 =$

[23]Actually, we simulate 52 period data for each firm but drop the first and last observations, which make 50 observations available for each firm. The initial rating under stickiness framework comes from a slightly different mechanism. It has no past rating, and hence there is no way to determine migration (i.e. upgrade, downgrade, and stay) which is a crucial factor in stickiness study. We also drop the last observation to keep the comparability to data within t-stickiness introduced in possible further test.

1. This assignment ignores stickiness and hence is consistent with the assumptions of Ordered-probit estimation. Secondly, the sticky rating assignment requires the separate invoking thresholds for upgrade and downgrade decisions. When credit quality exceeds the upper threshold U, that observation will be assigned rating $r_2 = 2$. Inversely, the observation receives rating $r_2 = 1$ if its credit quality locates below the lower threshold L. When the credit quality falls between U and L, the observation keeps its rating in last period.

Estimation Methods

We simulate rating data in order to test the performance of our new estimation methodology of the adjusted ordered probit model. There is one firm with 1,000 period in this simple simulation. Rating simulation follows two steps: 1) simulate credit quality, and 2) assign ratings. Firstly, create four normally distributed random variables (x_1 to x_4) with different mean and standard deviation. Randomly select the true coefficients as $b_1 = 0.543$, $b_2 = -0.958$, $b_3 = 0.267$, and $b_4 = 498$. Assume the z score ($z = b_1 * x_1 + b_2 * x_2 + b_3 * x_3 + b_4 * x_4$) captures credit quality. Secondly, select two set of rating standards for upgrade and downgrade. For simplification, use the 40th (-0.583) and 80th (0.758) percentiles as the upgrade standard, and 20th (-1.387) and 60th (0.077) percentiles as the downgrade standard. Further, assume the first rating in this simulation is B, since we assume current rating is determined by credit quality in last period. If the credit quality at time t reaches either upgrade or downgrade boundary of its current rating (determined by last period quality), an rating migration to a suitable position happens next period. Moreover, the observation before a migr-

Table 3.15: Simulation: Estimation Comparison

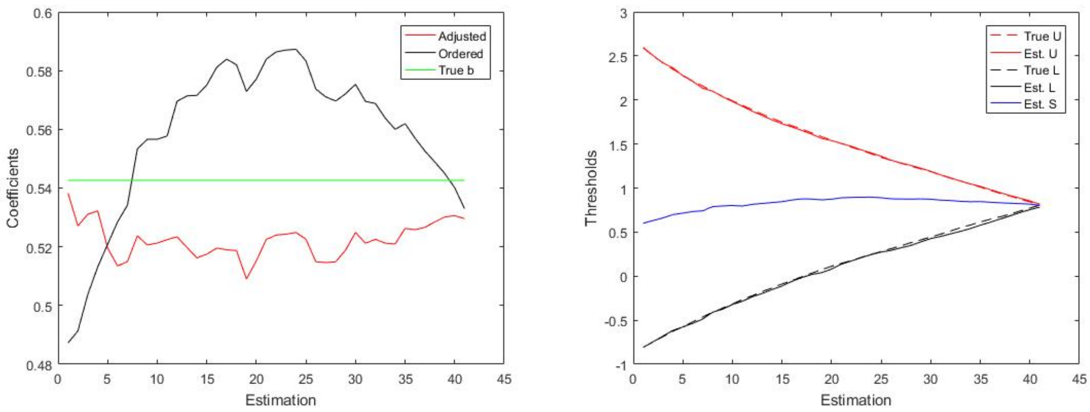
	Simple Rating r_1		Sticky Rating r_2	
	Model 1	Model 2	Model 1	Model 2
Estimation	Ordered	Adjusted	Ordered	Adjusted
Stickiness	NO	z	NO	z
b=0.543	0.538	0.536	0.600	0.544
	(0.009)	(0.010)	(0.010)	(0.010)
S=0.768	0.775		0.863	
	(0.017)		(0.018)	
U=1.107		0.781		1.143
		(0.019)		(0.021)
L=0.433		0.763		0.448
		(0.023)		(0.023)
Pseudo R	0.214	0.201	0.248	0.206

Estimations based on simulated data. The ordered probit model (Model 1) ignores stickiness, while the adjusted ordered probit model (Model 2) controls z-stickiness. The first two columns apply Model 1 and Model 2 to simple rating r_1 which contains no stickiness in rating assignment; and the last two columns apply the two estimation methods to sticky rating data. Standard error is in the parentheses.

ation is regarded as threshold. For example, if an observation carries rating B at time t, and upgrade (downgrade) happens at time $t+1$, the observation at t is informative about the upper (lower) boundary of rating B. As summarized in Table 3.15, the estimated coefficients from the adjusted ordered probit model are much more closer to the true values.

Moreover, the Adjusted model is designed as a more general model than the ordered probit one. In order to test the reliability of the Adjusted model on the magnitude of stickiness, we exploit a set of estimations in narrowing stickiness environment. There are 41 estimations with the $z - stickiness$ decreasing to 0. More precise, the U and L are set at 90th and 10th percentiles of credit quality in the first estimation. Then U (L) decreases (increases) by 1 percentile until the median value,

Figure 3.12: Z-stickiness Only Estimation



(a) Coefficients (b) Thresholds

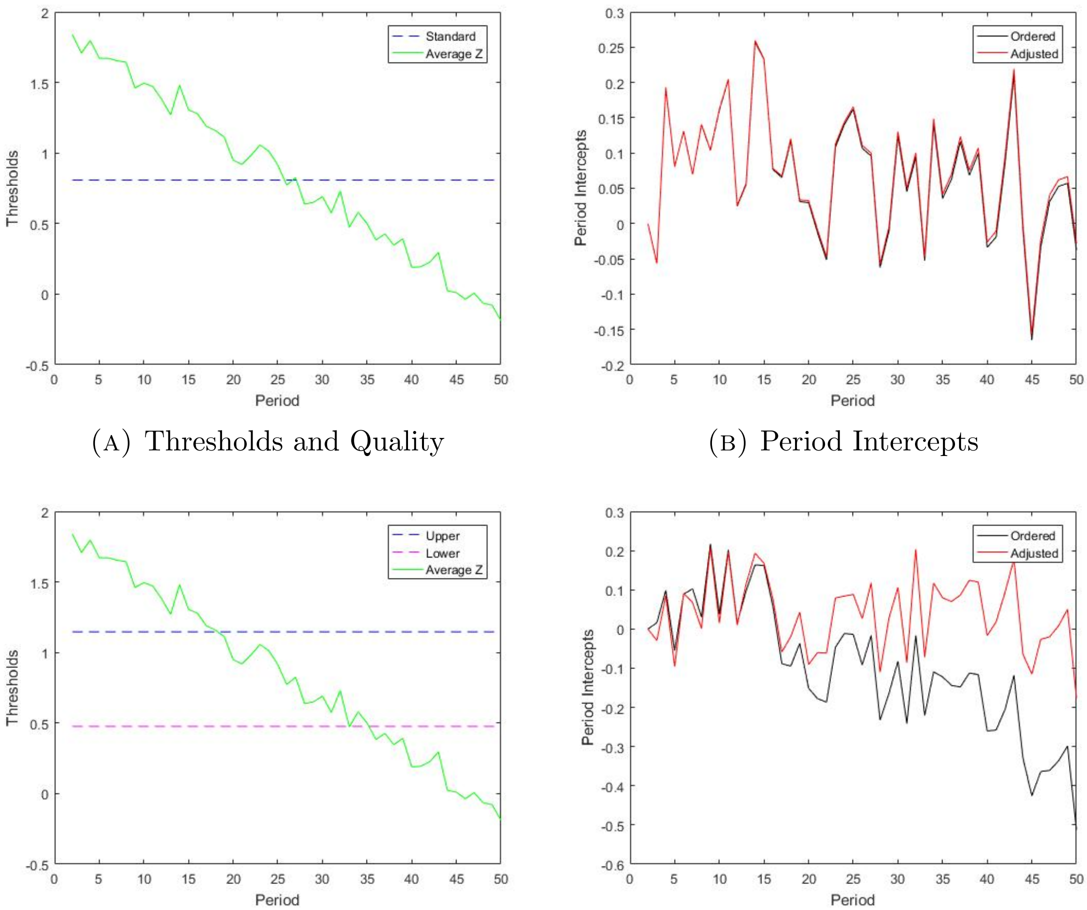
The estimated x coefficients and thresholds of ordered probit model and the adjusted model. There are 41 estimations with the z-stickiness decreasing to 0. More precise, the U and L are set at 90th and 10th percentiles of credit quality in the first estimation. Then U (L) decreases (increases) by 1 percentile until the median value, at which U and L overlap and the rating assignment becomes the simple one.

at which U and L overlap and the rating assignment becomes the simple one. As plotted in Figure 3.12, the estimated x coefficients from the Adjusted model are more closer to the true value in most of the time, while the accuracy in ordered probit model results demonstrates sort of dependence on the stickiness level. Also, the estimated U and L thresholds from the Adjusted model strictly follows the true values. Unsurprisingly, the two estimation models converge at the end of the test when stickiness reduces to zero.

Stickiness Impact

Usually, the year dummy intercepts in ordered probit model is interpreted as the indicators of rating stringency. The main evidence of increasingly stringent rating standard argued in existing literature is the decreasing year dummy coefficients. However,

FIGURE 3.13: Plot of Period Intercepts: Flat Standard



(A) Thresholds and Quality

(B) Period Intercepts

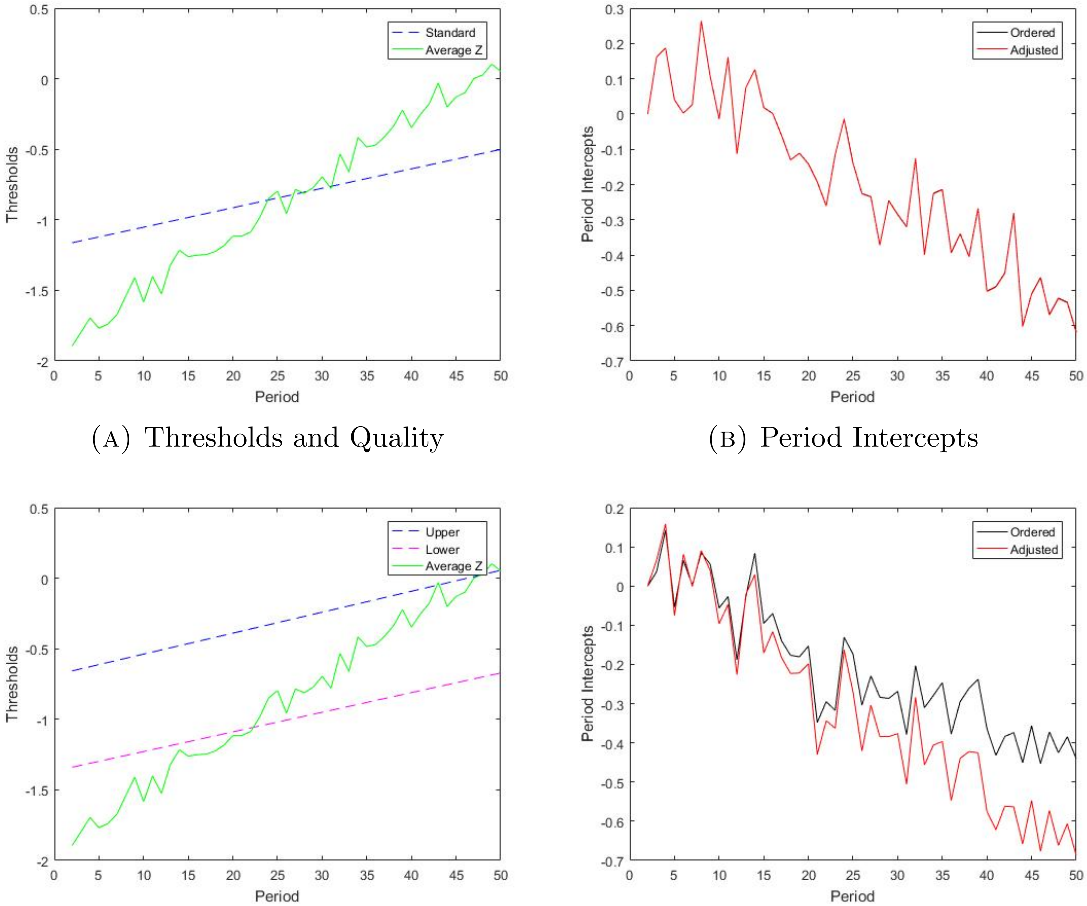
(C) Thresholds and Quality

(D) Period Intercepts

Period intercepts from ordered probit model and the adjusted model. The simple rating framework results are in Figures (a) and (b); and Figures (c) and (d) plot the results when introducing z-stickiness. In this case, both the single standard S and the double thresholds U and L are flat.

the year dummies in ordered probit model may not be accurate stringency indicators when stickiness exists. As shown in Figure 3.13, we set the rating standard flat for both simple and sticky ratings, which means the rating stringency does not vary in this case. The simple rating contains no stickiness, and hence the year dummy intercepts spike around zero. Unsurprisingly, the period intercepts for the ordered and the adjusted models almost overlap in part (b). In contrast, the ordered model period intercepts demonstrate downward trend by merely introducing stickiness (part (c) and (d)), which fails to reflect the flatness of the double barriers U and L. The p-

FIGURE 3.14: Plot of Period Intercepts: Increasing Standard



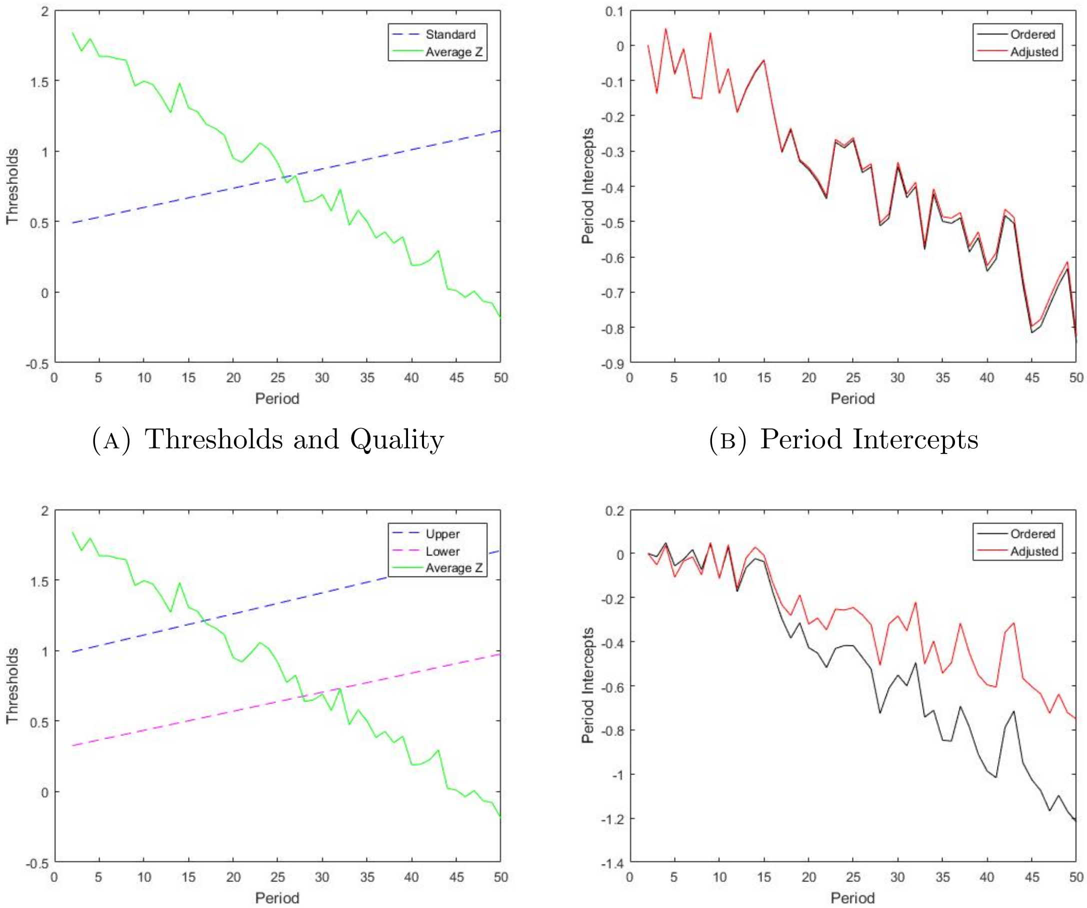
(A) Thresholds and Quality (B) Period Intercepts

(C) Thresholds and Quality (D) Period Intercepts

Period intercepts of from ordered probit model and the adjusted model. The simple rating framework results are in Figures (a) and (b); and Figures (c) and (d) plot the results when introducing z-stickiness. In this case, both the single standard S and the double thresholds U and L are increasing.

eriod dummies in the adjusted model keep at the zero level. Hence, this divergence documents the impact of stickiness. Similar pattern also appears when rating standards are increasing and decreasing as in Figure 3.14 and Figure 3.15. We have also investigated (but not reported) the situations when credit quality are increasing, and this phenomenon still exists. Moreover, the lagged rating analysis in Table 3.16 presents the stickiness impact from the position dependence perspective. In general, lagged rating is insignificant when rating is assigned without stickiness, but highly significant when stickiness exists.

FIGURE 3.15: Plot of Period Intercepts: Decreasing Standard



(A) Thresholds and Quality

(B) Period Intercepts

(C) Thresholds and Quality

(D) Period Intercepts

Period intercepts of from ordered probit model and the adjusted model. The simple rating framework results are in parts (a) and (b); and parts (c) and (d) plot the results when introducing z-stickiness. In this case, both the single standard S and the double thresholds U and L are decreasing.

Table 3.16: Simulation: Lag Rating Analysis

Panel A: Decreasing Credit Quality				
	Simple		Sticky	
	Model 1	Model 2	Model 1	Model 2
Lr	0.146	0.034	0.830	0.715
	(0.024)	(0.026)	(0.025)	(0.027)
x	0.604	0.520	0.603	0.528
	(0.010)	(0.012)	(0.011)	(0.013)
Cut	1.135	0.482	2.345	1.769
	(0.038)	(0.116)	(0.041)	(0.117)
Pseudo R	0.266	0.276	0.356	0.365
Year Dummy	No	Yes	No	Yes
Panel B: Increasing Credit Quality				
	Simple		Sticky	
	Model 1	Model 2	Model 1	Model 2
Lr	-0.040	0.004	0.640	0.701
	(0.023)	(0.023)	(0.023)	(0.024)
x	-0.450	-0.554	-0.455	-0.561
	(0.009)	(0.012)	(0.009)	(0.012)
Cut	-0.734	-1.026	0.447	0.066
	(0.040)	(0.102)	(0.039)	(0.107)
Pseudo R	0.168	0.180	0.222	0.234
Year Dummy	No	Yes	No	Yes

Estimation with lagged rating as an additional explanatory variable based on simulated data. All estimations in this table are using ordered probit model since the lagged rating variable attempts to capture the position persistence. Model 1 is the basic specification, while Model 2 further controls for the period dummy. Panel A summarizes the case when credit quality is increasing, and Panel B depicts the case when credit quality is decreasing. Standard error is in the parentheses.

Appendix 3.2 Likelihood Function

Our estimation differs from the traditional ordered probit model from two aspects: 1) categorization of observations; 2) separation of cut-offs. The likelihood function reflects these features. In following presentation, $\Phi(\cdot, 0, 1)$ denotes the cumulative function of standard normal distribution, x refers the explanatory variable set for that group, β is the coefficient vector to be estimated, r refers credit rating observed (ranges from 1 to 5 with increasing credit quality in this research), u_r refers to the upper cut-off of rating r, and l_r refers to the lower cut-off of rating r accordingly.

$Case\ 1:\ Upper\ Threshold$ When a firm has experienced rating upgrade in next period, its observation in current period falls into the upper threshold group. Its credit quality has exceeded the upper cut-off of its current rating now, which leads to the rating upgrade in next period. Hence, observations in this group contains the information of upper cut-offs. More precise, the upper cut-offs are the lowest possible level of the credit quality conveyed by observations in this group, and hence the probability of being in such a situation is

$$p_1 = 1 - \Phi(u_r - x \cdot \beta, 0, 1). \tag{3.6}$$

$Case\ 2:\ Ordinary\ Observation$ When a firm has its credit rating stayed the same in next period (no upgrade or downgrade), its observation in current period is within the ordinary group. The credit quality of observations in this group match their ratings. In other words, the credit quality is correctly bounded by the upper and lower cut-offs of their observed credit ratings in this group, and hence the probability in this situation is

$$p_2 = \Phi(u_r - x \cdot \beta, 0, 1) - \Phi(l_r - x \cdot \beta, 0, 1). \tag{3.7}$$

$Case\ 3:\ \ Lower\ Threshold$ In contrary to the upper cut-off group, the lower cut-off group contains observations which has been downgraded in next period. These observations are informative for the lower cut-off s of credit ratings. In detail, the credit quality is below the lower cut-offs of their ratings for observations in this group, and the probability is

$$p_3 = \Phi(l_r - x \cdot \beta, 0, 1). \tag{3.8}$$

Overall, the loglikelihood function is

$$loglike = log(p_1) + log(p_2) + log(p_3). \tag{3.9}$$

Our calculation of standard error is based on the Hessian Matrix:

$$H = \begin{bmatrix} \frac{\partial^2 loglike}{\partial\beta\partial\beta} & \frac{\partial^2 loglike}{\partial\beta\partial u} & \frac{\partial^2 loglike}{\partial\beta\partial l} \\ \frac{\partial^2 loglike}{\partial u\partial\beta} & \frac{\partial^2 loglike}{\partial u\partial u} & \frac{\partial^2 loglike}{\partial u\partial l} \\ \frac{\partial^2 loglike}{\partial l\partial\beta} & \frac{\partial^2 loglike}{\partial l\partial u} & \frac{\partial^2 loglike}{\partial l\partial l} \end{bmatrix}.$$

The covariance matrix is hence $Var(\hat{\theta_{ML}}) = [-H(\hat{\theta_{ML}})]^{-1}$, in which θ_{ML} is the vector contains all variables to be estimated and it includes β, u_r, and l_r in this case.

More details about the calculation of the Hessian matrix is in the attached MATLAB program.

Appendix 3.3 MATLAB Program

```
function [beta1,se1,results1,pseudo_r]=adj_order_probit_fix_musig(yche,xx,id)
nvar=length(xx(1,:));
nr=max(yche); % ensure rating from 1 to nr, indicating quality (AAA=nr)
n=length(yche);

% for subsample
  id2=id;id2(yche==min(yche) & id==3)=2;id2(yche==max(yche) & id==1)=2;id=id2;


y1=yche;x1=xx;y1(id~=1,:)=[];x1(id~=1,:)=[];% id=1: upper threshold
y2=yche;x2=xx;y2(id~=2,:)=[];x2(id~=2,:)=[];% id=2: observation
y3=yche;x3=xx;y3(id~=3,:)=[];x3(id~=3,:)=[];% id=3: lower threshold

nn=nvar+nr*2-2;

c_initial_coeff = (xx'*xx)\(xx'*yche)*(rand-0.5)*2;
c_initial_shreshold_u = (1:2:2*(nr-1))';
c_initial_shreshold_l = c_initial_shreshold_u-1;
 c_ini1 = [c_initial_coeff;c_initial_shreshold_u;c_initial_shreshold_l];


tic;options = optimoptions(@fminunc,'Algorithm','trust-region','
    SpecifyObjectiveGradient',true,'HessianFcn','objective','MaxIter',4000000,'
    MaxFunEvals',10000000,'display','Iter','TolX',10^-6,'TolFun',10^-6);

 x10=y1-y1+1;x20=y2-y2+1;x30=y3-y3+1;nvar2=1;c_ini0 = [rand(1,1);
    c_initial_shreshold_u;c_initial_shreshold_l];
 [beta0,loglike0,~,~,~,~]=fminunc(@(ini)loglike_g_h(y1,x10,y2,x20,y3,x30,nr,nvar2,
    ini),c_ini0,options);toc;

 display('stage2')
   c_ini1 = [c_initial_coeff;beta0(2:end,1)];
[beta1,loglike,~,~,~,hessian]=fminunc(@(ini)loglike_g_h(y1,x1,y2,x2,y3,x3,nr,nvar,
    ini),c_ini1,options);toc;
```

```


 hinv=pinv(hessian);
 se1=sqrt(diag(hinv));t_stat1=beta1./se1;df1=n-nn;p_value1=(1-tcdf(abs(t_stat1),df1))*2;
 results1=[c_ini1,beta1,se1,t_stat1,p_value1];

 pseudo_r=1-loglike/loglike0;
  disp('Pseudo R2')
  disp(pseudo_r)

end

function [l,g,h]=loglike_g_h(y1,x1,y2,x2,y3,x3,nr,nvar,ini)

b=ini(1:nvar,1);upper=[ini(nvar+1:nvar+nr-1,1);inf];lower=[-inf;ini(nvar+nr:nvar+nr+nr-2,1)];mu=0;sig=1;

% Part I: Log Likelihood
% id=1: upper threshold
p1=1-normcdf(upper(y1)-x1*b,mu,sig);p1(y1==nr)=1;
l1=sum(log(p1));

% id=2: ordinary observation
pu2=normcdf(upper(y2)-x2*b,mu,sig);
pl2=normcdf(lower(y2)-x2*b,mu,sig);
l2=sum(log(pu2-pl2));

% id=3: lower threshold
p3=normcdf(lower(y3)-x3*b,mu,sig);p3(y3==1)=1;
l3=sum(log(p3));

ll=-l1-l2-l3;

m=10^0;

```

```
l=ll*m;

% Part II: Gradient
normpdf1=normpdf(upper(y1)-x1*b,mu,sig);
normpdf2u=normpdf(upper(y2)-x2*b,mu,sig);normpdf2l=normpdf(lower(y2)-x2*b,mu,sig);
normpdf3=normpdf(lower(y3)-x3*b,mu,sig);

% b
g1b=x1'*(normpdf1./p1); % id=1: upper threshold
g2b=-x2'*((normpdf2u-normpdf2l)./(pu2-pl2)); % id=2: traditional observation
g3b=-x3'*(normpdf3./p3); % id=3: lower threshold
gb=g1b+g2b+g3b;

% u
g1u=-normpdf1./p1;
g2u=normpdf2u./(pu2-pl2);
gu_=[g1u;g2u];yu_=[y1;y2];gu=zeros(nr-1,1);

% l
g2l=-normpdf2l./(pu2-pl2);
g3l=normpdf3./p3;
gl_=[g2l;g3l];yl_=[y2;y3];gl=zeros(nr-1,1);

for i=1:nr-1
    ggu=gu_;
    ggu(yu_~=i,:)=[];
    gu(i)=sum(ggu);
    ggl=gl_;
    ggl(yl_~=i+1,:)=[];
    gl(i)=sum(ggl);
end



g=-[gb;gu;gl]*m;

```

```
ggu2=-normpdf2u.*(upper(y2)-x2*b-mu)/sig;ggu2(y2==nr)=0;
ggl2=-normpdf2l.*(lower(y2)-x2*b-mu)/sig;ggl2(y2==1)=0;

% Part III: Hessian

% hbb

h1bb_part1=repmat(normpdf1./p1,1,nvar).*x1;
h1bb_part2=repmat((normpdf1.*(upper(y1)-x1*b-mu))./p1,1,nvar).*x1;
h1bb=-h1bb_part1'*h1bb_part1+h1bb_part2'*x1/(sig^2);

h2bb_part1=repmat((normpdf2u-normpdf2l)./(pu2-pl2),1,nvar).*x2;
h2bb_part2_fun=-sig*(ggu2-ggl2)./(pu2-pl2);
h2bb_part2=repmat(h2bb_part2_fun,1,nvar).*x2;
h2bb=-h2bb_part1'*h2bb_part1-h2bb_part2'*x2/(sig^2);

h3bb_part1=repmat(normpdf3./p3,1,nvar).*x3;
h3bb_part2=repmat((normpdf3.*(lower(y3)-x3*b-mu))./p3,1,nvar).*x3;
h3bb=-h3bb_part1'*h3bb_part1-h3bb_part2'*x3/(sig^2);

hbb=h1bb+h2bb+h3bb;

% hbu

h1bu_part1_fun=(normpdf1./p1).^2;
h1bu_part1=repmat(h1bu_part1_fun,1,nvar).*x1;
h1bu_part2_fun=-(normpdf1./p1).*(upper(y1)-x1*b-mu);
h1bu_part2=repmat(h1bu_part2_fun,1,nvar).*x1/(sig^2);
h1bu=h1bu_part1+h1bu_part2;

h2bu_part1_fun=(normpdf2u.*(normpdf2u-normpdf2l)./(pu2-pl2))./(pu2-pl2);
h2bu_part1=repmat(h2bu_part1_fun,1,nvar).*x2;
h2bu_part2_fun=normpdf2u.*(upper(y2)-x2*b-mu)./(pu2-pl2);
h2bu_part2=repmat(h2bu_part2_fun,1,nvar).*x2/(sig^2);
h2bu=h2bu_part1+h2bu_part2;
```

```
hbu_raw=[h1bu;h2bu];yu_=[y1;y2];hbu=zeros(nvar,nr-1);

% hbl

h2bl_part1_fun=normpdf2l.*(normpdf2u-normpdf2l)./((pu2-pl2).^2);
h2bl_part1=-repmat(h2bl_part1_fun,1,nvar).*x2;
h2bl_part2_fun=normpdf2l.*(lower(y2)-x2*b-mu)./(pu2-pl2);
h2bl_part2=-repmat(h2bl_part2_fun,1,nvar).*x2/(sig^2);
h2bl=h2bl_part1+h2bl_part2;

h3bl_part1_fun=(normpdf3./p3).^2;
h3bl_part1=repmat(h3bl_part1_fun,1,nvar).*x3;
h3bl_part2_fun=normpdf3.*(lower(y3)-x3*b-mu)./p3;
h3bl_part2=repmat(h3bl_part2_fun,1,nvar).*x3/(sig^2);
h3bl=h3bl_part1+h3bl_part2;

hbl_raw=[h2bl;h3bl];yl_=[y2;y3];hbl=zeros(nvar,nr-1);


for i=1:nr-1
    hbu_=hbu_raw;
    hbu_(yu_~=i,:)=[];
    hbu(:,i)=sum(hbu_,1)';
    hbl_=hbl_raw;
    hbl_(yl_~=i+1,:)=[];
    hbl(:,i)=sum(hbl_,1)';
end


% hub
hub=hbu';

% huu

h1uu_part1=-(normpdf1./p1).^2;
h1uu_part2=(normpdf1.*(upper(y1)-x1*b-mu)./p1)/(sig^2);
```

```
h1uu=h1uu_part1+h1uu_part2;

% ggu2=-normpdf2u.*(upper(y2)-x2*b-mu)/sig;ggu2(y2==nr)=0;

h2uu_part1=-(normpdf2u./(pu2-pl2)).^2;
h2uu_part2=(ggu2./(pu2-pl2))/sig;
h2uu=h2uu_part1+h2uu_part2;

huu_raw=[h1uu;h2uu];

% hul

hul_raw=normpdf2l.*normpdf2u./((pu2-pl2).^2);


% hll

h2ll_part1=-(normpdf2l./(pu2-pl2)).^2;
h2ll_part2=normpdf2l.*(lower(y2)-x2*b-mu)./(pu2-pl2)/(sig^2);
h2ll=h2ll_part1+h2ll_part2;

h3ll=-(normpdf3./p3).^2-(normpdf3.*(lower(y3)-x3*b-mu)./p3)/(sig^2);

hll_raw=[h2ll;h3ll];


yu_=[y1;y2];y22=y2;yl_=[y2;y3];

% huu hul humu husig
% hll hlmu hlsig

huu=zeros(nr-1,nr-1);hll=huu;hul=huu;

for i=1:nr-1
    huu_=huu_raw;
    huu_(yu_~=i,:)=[];
```

```
    huu(i,i)=sum(huu_);
    hul_=hul_raw;
    hul_(y22~=i+1,:)=[];
    hul(nr-i,i)=sum(hul_);
    hll_=hll_raw;
    hll_(yl_~=i+1,:)=[];
    hll(i,i)=sum(hll_);
end

% hlb
hlb=hbl';

% hlu
hlu=hul';

% hll hlmu hlsig

h=-[hbb,hbu,hbl;hub,huu,hul;hlb,hlu,hll]*m;
% The accuracy of Gradient and Hassian are confirmed by tests in numerical way.
end
```

Chapter 4

Conclusions

Adjustment cost is involved into almost every aspects in financing area but currently receive little consideration in academic studies. Different types of adjustment introduce new dynamics in finance area such that the existence of targets and thresholds. This thesis provides two possible application of the model derived within a framework with dynamic adjustment cost, one investigating the optimal cash holding policy and providing evidence on the stickiness of credit rating.

The first study is motivated by the existence of adjustment cost in firms' cash policy. Expensive cash injection and withdraw make immediate adjustment sub-optimal. The consideration of these cost in our model leads to a better understanding of firms' cash refinancing behavior, and contributes to the comprehension of other aspects in corporate finance given the intertwined cash policy with other crucial decisions, such as financial and investment policy (Bolton et al. 2011).

Our study develops and estimates a double-barrier model to describe firms' cash holding management behavior. As oppose to the traditional studies, our model assumes infrequent refinancing and allow cash holding levels to freely evolve within a range. The estimation of the model yields some interesting results. The most important one suggests that cash inflow reduces firms demand of cash holding, which is contrary to the positive correlation in the literature. The observed cash holding may be affected by the direct injection from cash flow, but the real target shows endogenously inverse impact if clearly isolated. Accordingly, future outflows positively affects cash holding demand, since firms prepare funds for future expenditure. Although the sign of industry risk impact in our model is the same as in standard regression results, the magnitude is much larger. The demand variation does not fully reflect in realized cash holdings due to the infrequency of refinancing choice. Besides, the double barrier model further allows us to study the trigger of refinancing. We find that inflows shrink the width of inaction range which implies more frequent refinancing, and future outflows affect inversely. Industry risk enlarges the lower zone but shrinks the upper zone. It suggests less frequent injection but more often distribution in future, which provides additional channels to stockpile cash holdings.

The second study attempts to investigate the credit rating deterioration in the US in the recent decades. In order to do this, establish an unified framework of rating agency decision making. Our model interprets the dynamic in rating agencies' decision making process from two aspects, the quantification of firms' credit quality and the tolerance of credit quality deviation from the ratings' nominal range. Our model integrates the two aspects by introducing stickiness into estimation, and makes it possible to infer rating agencies' decision making. Our first goal is to quantify the effect of each covariate on the observed rating deterioration. Our second goal is to reconcile stylized facts in existing empirical studies (e.g. slow respondence, asymmetric migration, pro-cyclical or countercyclical, etc.) by revealing their origin. Unlike existing literature, our model assumes the existence of costs for agencies to adjust rating, which causes stickiness in credit rating.

The study investigates the stickiness in credit ratings, and contributes to existing literature in a number of ways. First, this paper provides direct empirical evidence about the existence of rating stickiness. The concept of stickiness integrates the concern of slow response (Altman & Rijken 2004, Löffler 2004) and the reversal avoidance motivation (Löffler 2005) of credit rating agency. Our results prove stickiness by showing that the introduction of stickiness in estimation absorbs the decreasing trend of of yearly intercept, and that the terms of explanatory variables interacting with migration dummies are statistically significant. Secondly, this study investigates the effects of stickiness in credit ratings. The consideration of stickiness predicts better rating categories but less rating migrations. Thirdly, this research provides explanation of credit rating deterioration from a stickiness perspective. Under the stickiness framework, our study demonstrates that the absolute credit quality surprisingly increases, the downgrade standard keeps at the similar level with slight ascending, but the raising upper threshold makes upgrades very difficult. In addition, this paper contributes to our comprehension of rating agencies' behavioral pattern. Agencies wait until the deviation of credit quality from its current rating range becomes sufficient and then decides migration. Finally, the estimation applied in this study enriches the method of determinants study in credit ratings. Our estimation method is a more general one, and it considers the stickiness in credit rating. This feature enables it to estimate the credit rating determination within the dynamic framework.

This thesis provides some interesting insights to decision makers that are involved in designing, implementing and assessing firm policies. Chapter 2 tests the existence of the endogenous pattern predicted by double-barrier model in cash holding management, that is cash refinancing is invoked by cash holding level being too high or too low. Inversely, the empirical evidence consistent with the model prediction supports the double-barrier policy as a suitable choice for cash refinancing behavior. The existence of refinancing cost (e.g. debt or equity issuance cost, opportunity cost, etc) further strengthens the link between theory and empirical test in our research. Chapter 3 attempts to establish an unified framework of rating agency decision making. The aggregate dynamic behavior model of rating agencies unifies the economic determinants study into the endogenous behavior pattern, and hence it is possible to estimate the effect of a variable on agencies' rating migration decision. Also, this study reconciles some stylized facts in existing empirical studies (e.g. slow respondence, asymmetric migration, pro-cyclical or countercyclical, etc.) by revealing their origin.

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