1. Introduction

[2] The primary acceleration mechanism in the upward current region of the aurora is parallel electric fields. At Earth, this has been inferred by sounding rocket observations in auroral regions [Evans, 1974; Mozer and Kletzing, 1998] and directly observed by satellites [Mozer and Kletzing, 1998; Ergun et al., 1998]. The relationship between parallel electric fields and field-aligned current was theoretically explored by Knight [1973] for the Earth case. Knight [1973] found that parallel electric fields, and hence field-aligned potentials are related to currents carried by electrons that move against the mirror force in the Earth’s magnetosphere. The enhanced current results in strong parallel electric fields which drive electron beams that excite atmospheric emissions, including observable auroral emissions. Ergun et al. [2000] modeled the potential structure in the upward current region using a steady state 1-D spatial 2-D velocity space code. Their analysis validated the Knight [1973] current-voltage relation at Earth.

[3] However, the Knight [1973] analysis is for a system in which the motion of particles along the magnetic field line is dictated by the magnetic mirror force. At Earth, the plasma density distribution along the field line is determined by gravitational forces, and thus monotonically decreases toward the equatorial plane. However for a rapidly rotating system, such as Jupiter or Saturn with rotational periods of ~9.8 h and ~10.6 h, respectively, centrifugal forces are significant and affect the plasma density distribution along the field. Heavy ions are confined to the equator causing a minimum in the plasma density at high latitudes, roughly ~2–3 planetary radii from the planet’s center. The ions are more strongly confined than are the electrons, resulting in an ambipolar electric field. The subsequent potential structure along the field line is not monotonic between the ionosphere and the magnetosphere, violating one of the assumptions of the Knight [1973] analysis.

[4] Bostrom [2003] investigated the current-voltage relation for a magnetic flux tube under general conditions, deriving the relationship using kinetic “orbital motion” theory. The analysis found that the total voltage drop along a flux tube is uniquely determined by the current density as long as particles do not mirror due to local maximums in the effective field-aligned potential (i.e., the Davisson condition is satisfied). In the case where particles are mirrored by a local maximum in the effective potential, the solution method must take into account space charge effects and the Poisson equation to determine the potential structure along the flux tube, and the relationship between the total voltage drop and current is nonunique. The Bostrom [2003] analysis looked at a broad range of voltage structures for the latter case mentioned above, including piecewise solutions where the maximum in effective potential does not occur at the plasma sheet, which is similar to the potential structure along the Io flux tube. However, ambipolar field effects were not accounted for and a solely analytic approach was applied to these solutions. We use both an analytic and Vlasov approach to investigate the nature of the current-voltage relation in this regime, focusing on the Jovian system.

[5] Jupiter’s aurorae have been observed in the ultraviolet, X ray, and infrared wavelengths. In addition to the main and secondary auroral ovals, these images reveal low-latitude auroral “spots” at the footprints of the Jovian satellites, the brightest of which is associated with Io [Clarke et al.,...
The main emission at Io’s footprint is accompanied by an extended “wake” emission downstream of Io. The main emission at Io’s footprint is accompanied by an extended “wake” emission downstream of Io. The main emission at Io’s footprint is accompanied by an extended “wake” emission downstream of Io. The main emission at Io’s footprint is accompanied by an extended “wake” emission downstream of Io. The main emission at Io’s footprint is accompanied by an extended “wake” emission downstream of Io.

2. Model Description

We use the same steady state kinetic Vlasov code used by Ergun et al. [2000] and Su et al. [2003] which is one-dimensional in space and two-dimensional in velocity space, to determine a large-scale, self-consistent solution of auroral spectra of the 1200–1700 Å region, Dols et al. [2000] suggested that the electron population responsible for the excitation of the Io footprint has a mean energy of about 60 keV. This mean energy is comparable to that of electrons beams (~75 keV) created by repeated Fermi acceleration in parallel electric fields generated by Alfvén waves [Crary, 1997]. Clarke et al. [2002] reported that 20° downstream of Io’s footprint the brightness of the wake emissions is on the order of tens of kilorayleigh (kR) based on UV observations by HST. Previous studies [Gérard et al., 2002; Grodent and Gérard, 2001; Waite et al., 1983; Gerard and Singh, 1982] indicated that the efficiency of the electron energy conversion is close to 10 kR erg cm s for primary energies between 10 and 100 keV. In particular, Gérard et al. [2002] suggested that the mean energy for the electrons 20° downstream of Io’s footprint is ~30 keV.

Mauk et al. [2002] indicated several similarities between the Earth’s aurora and Io’s auroral emission and tail. Ergun et al. [2002] supported this idea by suggesting that the three types of auroral acceleration regions observed by the Fast Auroral SnapshoT (FAST) at Earth, (Alfvénic acceleration region, a downward (with respect to the planet) current region, and an upward current region) are also active at Jupiter in magnetosphere-ionosphere coupling. Delamere et al. [2003] divided the Jupiter-Io interaction into three phases: (1) initial mass-loading interaction, (2) acceleration of the plasma in the Io wake, and (3) steady state decoupling. Delamere et al. [2003] suggested that the first two phases may induce an Alfvénic disturbance that is related to the bright emissions at Io’s magnetic footprint, whereas the third phase sets up field-aligned currents in the downstream region of Io’s wake.

Su et al. [2003] studied the possibility that the extended tail emissions in Io’s wake are due to electron acceleration in an upward current region between Jupiter and Io. Their analysis used a static kinetic Vlasov code to model electron energy fluxes and potential structures for a variety of Io torus compositions, finding that the current densities at high latitudes were strongly dependent on the hot electron population at the Io torus and the population of light ions (i.e., H+). Ergun et al. [2009] models the steady state current system that develops in the Io wake region, including electric fields, field-aligned currents and subsequent auroral emission. In addition, the analysis investigates the role of parallel electric fields in the transfer of angular momentum from the ionosphere to the magnetosphere. The analysis is sensitive to the relationship between the current density and field-aligned potential throughout the system.

In this paper we explore the nature of the current-voltage relation in the Io wake region for a flux tube which intersects the equatorial plane at 5.9 R. We investigate a range of potential drops between the ionosphere and the magnetosphere to determine how the centrifugal confinement of the magnetospheric population affects the relationship between the current density and field-aligned potential.
parallel electric fields in the Io wake region. We prescribe a magnetic flux tube which runs from Jupiter’s ionosphere to the equatorial plane, and intersects the equatorial plane at Io’s orbital distance of L = 5.9. A set of potential drops are specified between Jupiter and the equatorial plane and subsequently adjusted to give the potential structure along the field line and current density at the ionosphere.

[11] The spatial domain, which is 1-D along the flux tube, is divided into \( N_s = 51 \) grids which are evenly spaced along the magnetic dipole field line. The Vlasov code includes the magnetic mirror effect and both gravitational and centrifugal potentials. The gravitational and centrifugal potentials based on the O\(^+\) species are displayed in Figure 2 (bottom), with dotted and dashed lines, respectively. The solid line represents a combination of gravitational and centrifugal potentials. The horizontal axis represents Jovicentric distance in Jupiter radii (\( R_J \)). The left- and right-hand sides are the ionospheric and magnetospheric boundaries, respectively. The minimum potential is located at \( \sim 2.5 \) \( R_J \) along the flux tube.

[12] Cold ionospheric electrons and ions are prescribed as fluids at Jupiter’s ionospheric boundary, while the IO-generated plasma species are assigned as Maxwellian distributions at the Io boundary. The only exception is the Io-generated electrons which are described as a kappa distribution to include the effects of the hot electron population. The temperature and composition of the plasma species are listed in Table 1 and described in more detail below. The distribution functions are broken into \( N_v \times N_s \) velocity-space elements (\( N_v = 50 \)), each of which is treated like a fluid. The potentials at the ionospheric and magnetospheric boundaries must be held fixed to represent the prescribed net potential drop. An estimated electric potential profile, \( \Phi_e \), initializes the model and then, using the set boundary conditions, the velocity space distributions are calculated along the flux tube.

[13] The model solves Poisson’s equation along the field line, calculating the error at each spatial step which is defined as

\[
\xi(s) = \nabla^2 \Phi(s) + \frac{e}{e_0} [n_i(s) - n_e(s)]
\]

where \( n_i(s) \) and \( n_e(s) \) are the ion and electron densities at each step as calculated from the distribution functions. We iteratively adjust the electric potential, \( \Phi_e \), to minimize the error \( \xi(s) \), yielding a steady state solution along the field line. The spatial size of the grid (\( ds \sim 10000 \) km) is much larger than the Debye length (\( \lambda_D < 1 \) km); hence the first term on the right-hand side of the equation is negligible, essentially resulting in a quasi-neutral solution. The quasi-static model enforces adiabatic evolution without velocity-space diffusion.

[14] The Vlasov code, as applied to Earth (i.e., including gravitational forces, but excluding centrifugal forces) [Ergun et al., 2000], was validated by the linear approximation to the Knight current-voltage relation [Knight, 1973; Lyons, 1980]:

\[
j = K \Phi, K = \frac{e^2 n}{\sqrt{2\pi m_e T_e}}
\]

where \( j \) and \( \Phi \) are the field-aligned current density and field-aligned potential. \( T_e, m_e, n, \) and \( e \) are the electron temperature, mass, density, and charge. The current-voltage relation of Knight [1973] is linear in the range \( 1 \ll e\Phi/k_B T_e \ll R_M \) where \( R_M \) is the mirror ratio (\( \sim 400 \) for the Io flux tube). However, when the centrifugal confinement of particles to the equatorial plane and hence enhanced density at the equator is included, the current-voltage relation of Knight [1973] no longer applies (see section 3) and a new current-voltage relation is found.

3. Vlasov Solutions

3.1. Boundary Conditions

[15] The solution of the static Vlasov code is driven by boundary conditions. We choose a canonical plasma environment consistent with observations to represent the plasma conditions in the Io wake. The canonical values at the Io wake are used to initialize the Vlasov code which then calculates the current-voltage relationship. The numerical relation is then compared to the analytic model.

[16] Cold electrons and protons are introduced at Jupiter’s ionospheric boundary [Strobel and Atreya, 1983], while \( O^+ \), \( S^+ \), and electrons are assigned at the Io boundary. The ion composition and temperature are based on results published by Bagenal [1994] and Cray et al. [1998], with the simplification of omitting \( S^{++} \) and the minor species of \( O^{++} \) and \( S^{+++} \). Oxygen ions are chosen to represent both
Table 1. Compositions and Temperatures of the Ion and Electron Species for All Cases

<table>
<thead>
<tr>
<th>Species</th>
<th>Density</th>
<th>Temperature</th>
<th>Boundary</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ionospheric H⁺</td>
<td>$2 \times 10^7$ cm⁻³</td>
<td>0.31 eV</td>
<td>left (Jupiter)</td>
</tr>
<tr>
<td>Ionospheric e⁻</td>
<td>$2 \times 10^7$ cm⁻³</td>
<td>0.31 eV</td>
<td>left (Jupiter)</td>
</tr>
<tr>
<td>Io O⁺</td>
<td>1750 cm⁻³</td>
<td>$\parallel = 35$ eV, $\perp = 70$ eV</td>
<td>right (Io torus)</td>
</tr>
<tr>
<td>Io S⁺</td>
<td>250 cm⁻³</td>
<td>50 eV</td>
<td>right (Io torus)</td>
</tr>
<tr>
<td>Io e⁻</td>
<td>2000 cm⁻³</td>
<td>5 eV, $\kappa = 3$</td>
<td>right (Io torus)</td>
</tr>
</tbody>
</table>

mass/charge = 16 ion species (O⁺ and S⁺⁺). The average temperatures of O⁺ and S⁺ are assumed to be 50 eV. In this paper, O⁺ ions are assigned a temperature anisotropy of 2 (i.e., $T_\parallel = 35$ and $T_\perp = 70$). The number of heavy ion species has been limited in the model to minimize computer processing time. The electron temperature and distribution are based on Cassini UVIS observations of EUV emissions from the Io torus [Steffl et al., 2004]. The magnetospheric electron distribution is a $\kappa$ distribution with a core temperature of 5 eV and $\kappa = 3$.

In addition to the plasma boundary conditions, the boundary condition of the electric potential is an important parameter in determining the current density throughout the system. Auroral emissions are a result of the high-energy precipitating electrons. We look at potential drops between the ionosphere and the magnetosphere which range from 100 V to 30 kV, consistent with the Gérard et al. [2002] analysis. The current density is subsequently determined by the Vlasov code. In the following sections, the value of the current density stated is that at the base of the flux tube at the ionosphere.

3.2. Solutions

In order to determine the relationship between the current density and potential, the composition and temperature are held constant for all runs so that any changes in current density are a pure reflection of the variation in potential drop between the ionosphere and magnetosphere. Table 1 shows the composition and species temperatures for all runs. For an exploration on how varying the composition and temperature of the ionospheric and magnetospheric populations affects the current density, the reader is referred to Su et al. [2003].

Figure 3 displays the densities and potential structure for a potential drop of 30 kV between the ionosphere and magnetosphere. The thin solid, dashed-dot, and dashed-dot-dot lines represent the magnetospheric electron, O⁺, and S⁺ densities, respectively. The bold solid and dashed lines represent the ionospheric H⁺ and electron densities. The horizontal axis again represents the Jovicentric distance along the flux tube from the ionosphere to the magnetosphere. The S⁺ density falls rapidly due to the centrifugal forces imposed. The O⁺ population then declines with the electron population, maintaining quasi-neutrality. The potential drop occurs at $\sim 2.2 \, R_J$ which coincides with the minimum in the sum of the gravitational and centrifugal potentials (Figure 2). The ionospheric electron and ion levels are nearly identical below the potential drop, as the contribution from the high-energy tail of the magnetospheric electron population is negligible.

Figure 3 (middle) displays the electric potential profile along the flux tube. A sharp potential jump occurs at $\sim 2.2 \, R_J$. The width of the potential drop is much narrower than the resolution of the grid size, resulting in a sharp discontinuity. Below and above the large potential drop, ambipolar electric fields are set up to maintain quasi-neutrality corresponding to an ambipolar potential structure (Figure 3, bottom) of order tens to a hundred volts between the ionosphere and magnetosphere. The ambipolar potential near the equatorial plane depends on the centrifugal confinement of the ions and hence the rotation rate of the plasma. It is insensitive to the total potential drop prescribed to the system. Near the sharp potential drop, the gravitational potential overcomes the centrifugal potential (Figure 2). The ambipolar potential near the sharp drop can vary by $\sim 20$ V depending on the prescribed potential difference between the ionosphere and magnetosphere. This is caused by variations in the ionospheric H⁺ density at high latitudes.

As the potential drop increases (from Case 1 to Case 10), the ionospheric electrons and protons become more confined to the jovian atmosphere. For example, when $\Phi = 100 \, V$, the H⁺ population extends $\sim 3.5 \, R_J$ along the flux tube as the tail end of the Maxwellian distribution can overcome the potential drop, traveling farther along the flux tube. The variation in the ionospheric H⁺ density along the flux tube contributes to the small differences in the ambipolar potential above the sharp potential drop discussed above. The larger the potential drop prescribed, the more effective the evacuation of the auroral cavity.

The potential drops and corresponding current densities are summarized in Table 2 and displayed in Figure 4. The Vlasov solution results are displayed in the insert with a dashed line and crosses. It is clear that the current density increases with increasing potential; however, the linear growth regime exists only from $\sim 300 \, V$ to $\sim 9 \, kV$. At that point the current density plateaus, growing only slightly with increased potential drops. The solid line is the current-voltage relation corresponding to our analytic expression discussed below, and the dot-dashed line displays the current densities given by the Knight [1973] current-voltage relation. It is clear from Figure 4 that the Knight [1973] current-voltage relation does not apply. The theoretical aspects of this are discussed further in section 4.

4. Analytical Relation

Knight [1973] derives a current-voltage relation as a steady state solution for a Maxwellian plasma with a monotonic potential between the ionosphere and plasma sheet in which the plasma density is also monotonic and only affected by mirror forces. The electron population is described by a Maxwellian and the distribution function at the magnetosphere is integrated over velocity space such that only electrons which are not trapped by magnetic mirror forces contribute to the field-aligned currents.

At Jupiter, the heavy ions are confined to the equatorial plane due to large centrifugal forces. The electrons are less confined by centrifugal forces, but their mobility is impeded by an ambipolar electric field which develops to maintain quasi-neutrality along the magnetic field. The ionospheric populations are bound to the ionosphere by gravitational forces. This confinement causes a
density depletion of current carriers at high latitudes, resulting in a ‘high-latitude current choke’ which limits the field-aligned currents that can flow through the system. The density distribution is nonmonotonic along the flux tube, as shown in Figure 3. In addition, the ambipolar potential increases the magnitude of the potential near the magnetosphere, resulting in a nonmonotonic potential structure between the ionosphere and plasma sheet.

The critical limitation of the field-aligned current flow occurs at the point of minimum density along the flux tube and it is at this location that a significant field-aligned potential develops. In this case, the current-voltage relation resulting from the ‘high-latitude current choke’ is similar to that derived by Paschmann et al. [2003, chapter 3, equation (3.37)] as

$$j = j_s + j_x(R_x - 1) \left(1 - e^{-\frac{x}{T_x(R_x - 1)}}\right)$$  \hspace{1cm} (3)

where $j_s$ is the electron thermal current which is defined as $j_s = en_x \sqrt{\frac{2}{m_e}}$, $R_x$ is the magnetic mirror ratio defined at the top of the acceleration region, $T_x$ is the electron temperature (expressed in units of energy), $n_x$ is the electron density at the top of the acceleration region and $e$ is the fundamental charge. The resulting current-voltage relation resembles the Earth formulation derived by Knight [1973]; however, there is one key difference. The ‘choke’ relation is based on the plasma parameters and mirror ratio at high latitudes. The electron population at high latitudes is the energetic tail of the magnetospheric population which is not as constrained by ambipolar forces. In addition, the location of the field-aligned potentials, which coincides with the position of lowest density along the flux tube, defines the magnetic mirror ratio. If the current density is below the electron thermal current, current can flow freely along the flux tube and we assume $F = 0$.

Figure 4 shows the ‘high-latitude current choke’ current-voltage relationship (solid line) and the current-voltage relationship derived by Knight [1973] (dot-dashed), with the results from the Vlasov solutions (dashed line with crosses) compared to the ‘high-latitude current choke’ current-voltage relation (solid line with stars) in the insert. The current density from the Knight [1973] current-voltage relation saturates when the field-aligned potential reaches ~400 kV, as opposed to ~10 kV with the ‘high-latitude current choke’ current-voltage relation. The saturated current density (~400 $\mu$A/m$^2$) found by the Knight [1973] relationship is nearly 50 times greater than that found with the ‘high-latitude current choke’ (~7.5 $\mu$A/m$^2$), grossly over-estimating the field-aligned currents that flow through the Io flux tube.

The ‘high-latitude current choke’ current-voltage relation reproduces the Vlasov solution results fairly well (Figure 4, insert), with the linear region of the current-voltage relation restricted to a potential range of ~200 V to

\[\text{Figure 3. Densities and potential structure for Case 10. (top) The final density structure along the flux tube for the five species modeled in the Vlasov solution. An auroral cavity forms at a distance of ~2.2 R$_J$ Jovicentric where the mirror ratio is ~8. (middle) The large-scale potential structure along the field line with a sharp drop at ~2.2 R$_J$. (bottom) The ambipolar structure that develops. The solid line is the ambipolar structure at the ionosphere edge of the potential drop, while the dashed line is the ambipolar structure on the magnetospheric side of the potential drop.}\]
10 kV. The current density saturates at a value equal to the mirror ratio times the electron thermal current, and is therefore limited by the location of the acceleration region. To determine the high-latitude electron properties, we modeled the magnetospheric electron distribution used in the Vlasov solutions \( \kappa = 3 \), as two Maxwellians distributions. The high-latitude electron population can be described as a Maxwellian where \( T_{eH} = 300 \text{ eV} \) and \( n_{eH} = 2 \text{ c m}^{-3} \), or .1\% of the electron density. This is consistent with that measured by the plasma science experiment on Voyager [Sittler and Strobel, 1987]. \( R_s = 8 \) to represent the formation of the auroral cavity at \( \sim 2 R_p \). The analytic expression and Vlasov solution are not identical as the Kappa distribution used in the Vlasov solution allows for a broader spread of energies and densities in the high-latitude electron population. However, the shape and range of current densities can be reproduced with equation (3). The Vlasov results are well approximated by the ‘high-latitude current choke’ current-voltage relation, which accounts for the development of the auroral cavity at high latitudes.

The current-voltage relation given by equation (3) is consistent with that given by Boström [2003]. It is important to note that neither our analysis nor the Boström [2003] analysis account for the effect of trapped electrons in the auroral cavity or wave diffusion, which could further reduce the current density at the ionosphere for a given field-aligned potential.

5. Discussion and Conclusions

We have presented a ‘high-latitude current choke’ current-voltage relation which describes the interaction of the current and field-aligned potential in a centrifugally confined plasma. We used a 1-D spatial, 2-D velocity static Vlasov code to validate this analytic formulation. The Vlasov solutions show:

1. The centrifugal forces, which result from a rapidly rotating system, confine heavy ions to the equatorial plane which in turn restrict the motion of the electrons along the flux tube as an ambipolar electric field is set up. The ambipolar potential results in a nonmonotonic potential structure between the ionosphere and magnetosphere, contrary to the assumptions of the Knight [1973] analysis.

2. The Knight [1973] current-voltage relation does not apply for a centrifugally confined plasma as it does not account for density depletions along the field line unrelated

![Figure 4. Current-voltage relations as derived by Knight [1973] (dot-dashed line) and the ‘high-latitude current choke’ relation (solid line). The current-voltage relationship derived by Knight [1973] saturates at a larger current density (~50 times larger) than the ‘high-latitude current choke’ relationship. The insert shows the current-voltage relation as determined by the Vlasov solution (dashed line, crosses) versus the analytic ‘high-latitude current choke’ current-voltage relation (solid line, stars).](image-url)
to mirror forces. The density structure along the flux tube for a centrifugally confined plasma is not monotonic.

32. The resulting current-voltage relationship is nonlinear and governed by the system parameters at high latitudes where the flux tube density is at a minimum. Hence, our analytic expression depends on the high-latitude electron density, electron temperature, and the mirror ratio at the top of the acceleration.

33. The maximum field-aligned current density at the ionosphere is much less than that derived by the Knight [1973] analysis when the effects of the 'high-latitude current choke' are included. As a consequence, the impact of field-aligned potentials in the decoupling process for a system in which the centrifugal confinement of plasma is important need to be reexamined.

34. A sharp potential drop occurs where the sum of the gravitational and centrifugal potentials are at a minimum. This minimum is located at \(~2.5\) R\(_J\). Jovicentric for a flux tube which intersects the equatorial plane at 5.9 R\(_J\).

35. Acknowledgments. Wolfgang Baumjohann thanks Robert Lysak and another reviewer for their assistance in evaluating this paper.

References


