Title: Fully Online Clustering of Evolving Data Streams into Arbitrarily Shaped Clusters

Keywords: clustering; online; evolving; arbitrary shape; micro-cluster; atmospheric science

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Sample cover letter for submission of a paper to an SPIE journal

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Dear Professor Pedrycz,

We wish to submit a new manuscript entitled “Fully Online Clustering of Evolving Data Streams into Arbitrarily Shaped Clusters” for consideration by Information Sciences.

We confirm that this work is original and has not been published elsewhere nor is it currently under consideration for publication elsewhere.

In this paper, we report on a new technique for fully online clustering of data into arbitrarily shaped clusters. This is significant because alternative techniques are hybrid online/offline systems providing incremental updates on the clustering results whereas our technique provides continuous access to the clusters. The paper should be of interest to readers in the areas of mining of data streams or monitoring of online systems in many fields, although our interest lies primarily in atmospheric science.

Please address all correspondence concerning this manuscript to me at r.hyde1@lancaster.ac.uk.

Thank you for your consideration of this manuscript.

Sincerely,

Richard Hyde
Fully Online Clustering of Evolving Data Streams into Arbitrarily Shaped Clusters

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Abstract

In recent times there has been an increase in data availability in continuous data streams and clustering of this data has many advantages in data analysis. It is often the case that these data streams are not stationary, but evolve over time, and also that the clusters are not regular shapes but form arbitrary shapes in the data space. Previous techniques for clustering such data streams are either hybrid online / offline methods, windowed offline methods, or find only hyper-elliptical clusters.

In this paper we present a fully online technique for clustering evolving data streams into arbitrary shaped clusters. It is a two stage technique that is accurate, robust to noise, computationally and memory efficient, with a low time penalty as the number of data dimensions increases. The first stage of the technique produces micro-clusters and the second stage combines these micro-clusters into macro-clusters. Dimensional stability and high speed is achieved through keeping the calculations both simple and minimal using hyper-spherical micro-clusters. By maintaining a graph structure, where the micro-clusters are the nodes and the edges are its pairs with intersecting micro-clusters, we minimise the calculations required for macro-cluster maintenance. The micro-clusters themselves are described in such a way that there is no calculation required for the core and shell regions and no separate definition of outer micro-
We demonstrate the ability of the proposed technique to join and separate macro-clusters as they evolve in a fully online manner. There are no other fully online techniques that the authors are aware of and so we compare the technique with popular online / offline hybrid alternatives for accuracy, purity and speed. The technique is then applied to real atmospheric science data streams and used to discover short term, long term and seasonal drift and the effects on anomaly detection. As well as having favourable computational characteristics, the technique can add analytic value over hyper-elliptical methods by characterising the cluster hyper-shape using Euclidean or fractal shape factors. Because the technique records macro-clusters as graphs, further analytic value accrues from characterising the order, degree, and completeness of the cluster-graphs as they evolve over time.

**Keywords:** online, evolving, clustering, micro-cluster, arbitrary shape

1. Introduction

Recent technological advances in many disciplines have seen an increase in the amount of data being provided in continuous streams of data, i.e. 'on-line data'. These data streams range from machine condition monitoring and atmospheric science data to social media analysis. The analysis and clustering of data streams has become increasingly important [1]. However, condition monitoring can suffer from sensor drift due to ageing, temperature fluctuations, modifications or upgrades to machine components, changes in load or type of use. Environmental monitoring will also be affected by sensor drift, but also seasonal variations and secular trends due to technological, socio-economic or climate change. While seasonality and other cyclic periodicities can be moved relatively easily off-line, any attempt to do this online renders the analysis vulnerable to aliasing changing seasonal cycles into secular changes. Other problem datasets are short-term but high-dimension and rapidly changing: chemical batch processors [2], environmental mesocosms [3], or ecological manipulation experiments.
Social media analysis will be affected by the inevitable changes in peoples’ taste, population changes and many other influences. In examples such as these the assumption of a stationary data environment is invalid and techniques for data analysis need to be capable of coping with evolving data streams. It is often the case in such data, particularly that incorporating spatial or relational information, that clusters of related data will not be hyper-elliptical and will fall into arbitrarily shaped groupings. The cases for arbitrary shaped clusters are well established and found in many sources [5, 6, 7]. Specifically a case such as that shown in [3] demonstrates the need for evolving clusters of arbitrary shapes - as the nature of the landscape changes over time, so must the clusters.

The ability to adapt our analytic to these secular (non-periodic) changes requires not only a method of reducing the importance of old data but also a way to divide previously singular clusters of data into multiple clusters. With the previously available techniques discussed in section [2] this is achieved, not by dividing the clusters in an online manner, but rather by re-clustering using an offline clustering technique on demand. With ever-increasing data sets, i.e. 'Big Data', the need to discard or archive the data after processing once becomes necessary for both computational and memory efficiency.

The technique presented in this paper has two distinct stages. The first creates micro-clusters when data samples occur in un-clustered data space. The radius of the micro-clusters, $r_0$, is fixed and should be as small as is practical. In this newly proposed method we use a simple linear ageing process which reduces the 'Energy' of a micro-cluster and allows unused micro-clusters to die out completely. Alternative ageing techniques could be used including those exponential types that leave micro-clusters present, with insignificant Energy, but allow them to be 're-born' and become relevant in the future with further data. The micro-cluster Energy is renewed every time they receive new data. When no data is received the micro-clusters lose some Energy, gradually fading out. If no data is received for a long time the micro-cluster Energy will reach zero and they are are no longer recorded.
The second stage searches for overlapping micro-clusters. The micro-clusters are defined as having a kernel region $< 0.5r_0$ and a shell region $> 0.5r_0$. By only connecting those micro-clusters whose kernel regions overlap into another micro-cluster shell we automatically determine edge micro-clusters. Micro-clusters which do not have at least the user-specified local density, i.e. the minimum number of samples within the radius, remain as separate outlier micro-clusters. Each macro-cluster consists of the graph of intersecting micro-clusters; the adjacency relations for each micro-cluster are stored as a property of that micro-cluster. For convenience, we call micro-clusters in adjacency relations (i.e. intersecting micro-clusters) Siblings. Those micro-clusters with no Siblings define graphs of order 1 without edges (i.e. intersections) and constitute a macro-cluster graph by themselves. Using this graph structure reduces the calculations required to separate clusters if a cluster dies and breaks a chain graph resulting in two groups of micro-clusters no longer being connected.

We call this technique Clustering of Evolving Data-streams into Arbitrary Shapes (CEDAS). We demonstrate the efficacy of CEDAS by testing the algorithm for speed and dimensional effects on artificial data sets. Application of CEDAS to the KDDCup99 data stream set is used to compare cluster purity with DenStream and MR-Stream and also to demonstrate the ability of CEDAS to deal with Big Data, adapt to evolving data streams and detect internet intrusion attacks with high accuracy. We then apply the algorithm to real-world London Air Quality [2] atmospheric monitoring to demonstrate how, by varying the micro-cluster decay time, we can differentiate between short term and long term secular change to discover temporally local anomalies and extremes of the overall distribution.

The rest of the paper is structured as follows: Section 2 provides a review of the current state of the art. Section 3 describes the principles and methodology behind the CEDAS algorithm and provides a description of the pseudo-code given in Appendix A. Section 4 describes the data sets and the methodology of their use throughout the analysis parts of the paper. Section 5 provides analysis of the proposed algorithm and comparisons to alternative techniques in separate.
subsections, each with their own discussion. The findings are summarised in section 6 and finally we consider some directions for future work in Section 7.

2. State of the Art

Alternative online data stream clustering techniques such as ELM [8], DEC [9] provide online clustering of data streams. Both of these techniques operate on data streams and provide clustering results online but are limited to hyper-ellipsoidal cluster shapes. The basis for ELM is to store the local mean as a cluster centre and to adjust the cluster centre and radii as more data arrives. DEC maintains a list of core and non-core clusters defined by the weight of the cluster. The weight decays over time or is increased as new data samples join the cluster. In this way, core clusters may decay to non-core, non-core clusters may disappear or increase their weight to become core clusters or new, non-core, clusters may be created. In both techniques the clusters that are created are hype-ellipsoidal. In the case of concave cluster shapes DEC may create many smaller hyper-ellipsoidal clusters or one large cluster encapsulating all the data depending on the user parameter values.

Other existing data stream clustering methods such as Chameleon [10], DBScan [11] and SPARCL [12] are all techniques for clustering arbitrary shapes offline. Sparcl utilises a two-layer approach whereby k-means [13] clustering is used to create a large number of micro-cluster centres in the first layer. These micro-cluster centres are then further clustered using a hierarchical approach. Chameleon and DBScan are techniques that successfully cluster arbitrary shapes, however, both work offline and therefore require the full data set. An incremental version of DBScan [14] was proposed which allows for incremental modification of the clusters. However, after each increment the micro-clusters re-built and so require the full data from each increment to be available.

A method known as DenStream was proposed in [14] based on the CluStream [15]. A set of core- and potential-micro-clusters are maintained online. Each micro-cluster is created from a stored set of data with a decaying weight. By
decaying the data samples those with a weight below a user-specified threshold are discarded and the memory requirement is limited somewhat although this loses potentially useful micro-clusters. The technique has an initialisation phase, using DBScan, to create an initial set of micro-clusters. Additionally, while the micro-clusters are maintained in an on-line fashion the process of combining the micro-clusters into final clusters is an off-line approach carried out on demand. DenStream is capable of finding arbitrarily shaped clusters as it’s 2nd stage clustering is based on DBScan whereas CluStream, with it’s 2nd stage based on k-means [16], finds hyper-elliptical clusters.

Two developments of DenStream known as SDStream [17] and rDenStream [18] improve on the basic DenStream algorithm. SDStream is based on SWCLustering Algorithm described in [19] and is an offline approach, repeated temporarily at incremental time windows. The authors claim improved quality of clusters over CluStream [15] however, it remains an incremental offline approach requiring storage of past data. rDenstream is a three stage clustering technique also based on DenStream. In rDenStream any discarded clusters are retained in memory and may be re-introduced to improve the clustering at a later time. Processing of this discarded data requires additional processing time and memory allocation. The 2nd stage of these techniques uses DBScan which has been demonstrated have an order of $D^4$ time penalty, where $D$ is the number of dimensions [20]. DBScan becomes impractical for big data and high dimensions and as a result techniques based around DBScan may also suffer from this limitation. The combined online-offline approaches limit this time penalty by only applying the offline DBScan macro-clustering function ‘on demand’ at reduced frequency.

Grid-based technique MR-Stream [21], divides the data space into a tree of grids of decreasing size (increasing resolution). MR-Stream is a combined on- and offline technique with the cluster grid updated online and regular second stage clustering of the grid combined with a tree pruning algorithm. In the extreme case of densely populated data space the offline components must visit each grid space, $2^{DH}$ where $D$ is the number of dimensions and $H$ the
granularity, or resolution, of the clusters.

A recently introduced technique, CODAS [20] demonstrated a new approach to clustering of continuous data streams into arbitrary shaped clusters. CODAS is a two stage technique with a micro-clustering first stage. The micro-clusters are designated as having an inner ‘kernel’ and outer ‘shell’ region. This removes the need for classifying micro-clusters as ‘edge’ or ‘non-edge’ and simplifies the micro-cluster joining calculations. As a result the technique has been demonstrated to be dimensionally stable with a time penalty in the order of $D^{100}$, where $D$ is the number of dimensions. Although CODAS is an online technique it does not allow for the clusters to evolve. That is to say that clusters, once formed, will remain. This means that in cases where the data stream evolves, i.e. forms different clusters at different times, CODAS does not update to remove the old micro-clusters. As a fully online technique, no data is stored and so it is not possible to use techniques such as windowing, or ageing of data to overcome this limitation. The technique presented in this paper builds on the underlying techniques of CODAS but stores the micro-cluster adjacency information to allow the macro-clusters to evolve and to provide additional information on the structure of the cluster-graph.
3. The Proposed Approach

Traditional offline clustering techniques for arbitrary shapes may categorize data samples as 'core' or 'non-core'. However, this requires storage of the data samples and ever-increasing storage capacity which is prohibitive for online clustering. CEDAS stores only the information related to the micro-clusters and a graph structure recording the micro-cluster connections.

The following terminology is defined for the CEDAS approach:

1. Cluster Graph: the structure that defines which micro-clusters join to form which macro-clusters. This is stored by recording the intersects of each micro-cluster in 'Edge', together with the appropriate macro-cluster assignation in 'Macro'.

2. Local density: the number of samples per micro-cluster

3. Macro-cluster: a cluster consisting of a number of intersecting micro-clusters.


5. Outlier-micro-cluster: a micro-cluster with local density below the threshold.

6. Sample: any data point in \( n \) dimensions.

7. Threshold: the minimum number of samples within the micro-cluster radius of any sample to form a micro-cluster.

In general CEDAS is a data-driven approach to divide the data space into kernel and shell regions based on a user defined radius, \( r_0 \). Each micro-cluster consists of a shell annulus region between radii \( \frac{r_0}{\sqrt{2}} \) and \( r_0 \) and a kernel region being \( < \frac{r_0}{\sqrt{2}} \). Any micro-cluster above a given density threshold is considered for macro-cluster membership. Micro-clusters with kernel regions that intersect another micro-cluster shell region form macro-clusters. Micro-clusters above
the threshold but with no intersections are also considered to be macro-clusters. Shell regions are considered to be edges of macro-clusters.

New data from the data stream will fall in to one of 3 regions:

1. empty space, where it will form a new, outlier-micro-cluster

2. a micro-cluster shell region, where it will be assigned to the cluster, the cluster count updated and the micro-cluster centre recursively updated to the mean of its samples.

3. a micro-cluster kernel region, where it will be assigned to the micro-cluster and the cluster count updated

The micro-cluster that has been modified, or created, by this process is then checked to see if the local density is above the threshold. If this is the case then this micro-cluster is checked for new intersections with other micro-clusters. If new intersections have been made then all the micro-clusters are linked and assigned to the same macro-cluster. This ensures that all linked micro-clusters have the same macro-cluster reference and maintains arbitrarily shaped data space regions of macro-clusters in a fully online manner.

With this approach at any given time a data sample can be checked for its macro-cluster membership, any new sample is immediately clustered and outliers are identified as members of outlier micro-clusters.

Figure 1 shows a subset of a plot of test data. Figure 1(a) shows the kernel and shell radii of the micro-clusters. Where the kernel radius of any cluster intersects a shell radius of any other the clusters combine as shown in Figure 1(b). Here the micro-clusters with intersecting kernel and shell regions have combined to form the red shaded cluster region whereas the micro-cluster with the non-intersecting kernel remains separate as the grey region.

3.1. CEDAS Algorithm

This sub-section describes in detail the steps shown in the pseudo-code given in Appendix A. There are 4 distinct steps for the full algorithm including initialisation:
1. Initialization

2. Update Micro-Clusters

3. Kill Clusters

4. Update Cluster Graph

3.1.1. Initialization

This creates a structure to store the information related to each micro-cluster and takes place with the first data sample. The 'centre' and 'radius' define the region in data space covered by the micro-cluster. 'Count' stores the total number of data samples that have been allocated to the micro-cluster. The value of 'Count' is recorded to allow recursive updates to the micro-cluster centre. 'Macro' is a reference to the macro-cluster to which this micro-cluster belongs. The value of 'Macro' is the same for all micro-clusters in the 'Sibling' list. 'Energy' is a value used to determine the length of time since a micro-cluster received new data. The decay algorithm reduces this value and is discussed later. 'Siblings' is a list of intersecting micro-clusters, if a micro-cluster has no Sibling list then it is a macro-cluster itself. In graph theory terminology the micro-cluster number paired with each intersect constitutes an 'edge' of the form \{µC_c, µC_I\}, where the first term is the current micro-cluster and the second term is the intersecting micro-cluster.

3.1.2. Update Cluster

This part of the algorithm updates the micro-clusters.

In step 10, a flag is set to record whether any micro-cluster positions are changed or a new micro-cluster is created. If none are changed then the later section of the algorithm that updates the cluster graph is not required to run.

Step 11 finds the distance to the nearest micro-cluster centre. If this distance is less than \( r_0 \) the new sample is within a micro-cluster and the (Energy) of that micro-cluster is re-set to 1 and the number of samples that have been placed within that micro-cluster increased by 1.
Step 16 checks if the sample is in the micro-cluster kernel region. If not then the micro-cluster centre is updated recursively in step 18 to the mean of all the included samples to date and the index $u$ of the modified cluster is set.

Steps 20-27 create a new micro-cluster if the sample did not lie in any current micro-clusters.

### 3.1.3. Kill Cluster

Step 28 decays the Energy of each micro-cluster. In the example here we use a simple linear decay but any function could be applied such as the exponential strategy in [14]. Note that with an exponential strategy the Energy of a cluster will never reach zero and some other pruning function will be required. Step 29 lists any clusters that have an Energy less than zero and step 30 checks if any further work is required and returns if no clusters have died.

Steps 33-37 remove all dead clusters and also deletes any references to the dead micro-cluster from all the micro-cluster Sibling lists. Micro-clusters are referred to by their index and some may have been removed from the middle of the graph. This means all the references in any Sibling lists that refer to a higher micro-cluster than the one removed must be decremented by 1. This results in an updated cluster graph.

### 3.1.4. Update Cluster Graph

This section is only required to run if either:

1. a new cluster has been created
2. a cluster centre location has been modified
3. a cluster has died and been removed

Step 41 calculates the distance from the updated cluster centre $C_u$ to the centre of all other micro-clusters $C_i$. Step 42 checks if the central kernel of either micro-cluster intersects the other micro-cluster and stores the intersects in the set $\{j\}$. Step 43 stores all the new intersects along with all the current Siblings of the updated micro-cluster. Step 44 finds the minimum macro-cluster.
number from all the intersecting micro-clusters, Step 45 assigns that number to the modified micro-cluster and step 46 updates the macro-cluster number of the intersecting micro-clusters.

Steps 48-58 reassign the macro-cluster numbers. This is not essential to the algorithm, but without these steps the macro-cluster numbers would be non-consecutive and would increase indefinitely.

4. Test Data

To analyse the performance of the proposed the CEDAS technique the following data sets were used:

4.1. Spiral High Dimensional Data

This dataset comprises three helical data streams, two of which join mid-way through the test while the other stays separate. These data streams are moved through a range of multiple dimensions to examine the time variance of the analysis with higher dimensional data. The data was analysed using CEDAS with a range of values for Decay and settings of InitialRadius = 0.05 and MinimumClusterSize = 4. The data set is then moved into higher dimensional data space by adding additional dimensional data coordinates. By projecting the data back into 3 dimensions the clustered data can be plotted and the results of cluster membership checked while increasing the complexity of the clustering calculations.

4.2. Mackey-Glass Data Streams

3D data stream consisting of 2 Mackey-Glass time series presented as a data stream. The data streams are solutions of the Mackey-Glass non-linear time delay differential equation [22, 23], shown in equation [1]

$$\frac{dx(t)}{dt} = \frac{ax(t - \tau)}{1 + x(t - \tau)^{10} - bx(t)}$$

(1)

The equation is solved numerically at discrete time steps using the 4th order Runge-Kutta method using different values for a and b to create x and y values.
and provide a data stream of 40,020 samples. For each time step 10 random data samples were created around the core value. The data streams are shown in Figure 2a. It can be seen that early in the data the values of both data streams are coincident. They later separate and come together at various times. We would expect that ‘recent’ data will produce a changing number of macro-clusters and an online, evolving clustering technique will detect these changes. A further data set was created by adding additional data of random noise samples at every 5th time step creating a dataset of 44,022 samples. This is used to test the robustness of CEDAS to detecting the clusters in a noisy environment. The data can be seen in Figure 2b. By presenting the data sequentially we create a continually evolving data stream rather than a data stream of static values with sporadic variation, such as the KDDCup data set below, which tests the ability of the algorithm to add, merge and separate macro-clusters.

The data was presented to the CEDAS algorithm 1 sample at a time to imitate an online data stream and the results plotted at each time step to create a video of the results. The CEDAS algorithm was used to detect and report in the plot title the following information:

1. Definite Clusters: these were defined as clusters containing > 15 data samples and > 1 micro-cluster

2. Outlier Clusters: these were defined as containing > 15 data samples all
3. Last Change: the time period at which the last change in the number of Definite Clusters occurred. This information was recorded to allow the state at that time to be reproduced.

4.3. KDDCup99 Data Stream Set

To further test the CEDAS algorithm in a different environment the KDD-Cup99 [24] dataset was used as a data stream presented to the algorithm sequentially. The dataset consists of approximately 5 million samples in the full data set, 500,000 samples in the 10% reduced set, simulating network intrusion attacks on a military installation. The dataset has 42 features and information to classify the data into 22 attack types in addition to the normal network traffic. This data is used to determine the cluster purity and memory use for comparison with alternative techniques and also to validate the clustering results in relation to the number of attack types which occur in a time period.

The dataset is loaded in chunks, to avoid memory issues of holding the data, and passed to the CEDAS algorithm sequentially to mimic an online data stream. To compare the results with alternative techniques the cluster purity is measured at set intervals of 1,000 data samples. The cluster purity measurements have also been placed in to groups of 25 to allow direct comparisons to the results in Wan et al [21]. In addition the number of clusters at these time steps is also plotted to compare the number of known attacks within the time period with the number of clusters created by CEDAS.

4.4. London Air Quality Data

We then apply the technique to data from the Kings College London Air Quality Website [2]. The data is from one site, Westminster Marylebone, in 2 dimensions, labelled $NO_x$, $PM_{10}$. Here and throughout, $NO_x$ is defined as the reactive oxides of Nitrogen, primarily $NO$ and $NO_2$, and $PM_{10}$ is defined as the mass concentration of microscopic airborne particles with aerodynamic diameter of $10\mu m$ or above. The data, which is recorded operationally to monitor breaches contained in 1 micro-cluster
of air pollution legislation and to inform the public of adverse air pollution conditions, is captured at 15 minute intervals and ranges from 1\textsuperscript{st} January 2010 to 30\textsuperscript{th} December 2014 for a total of 87,600 samples. This data is used to test CEDAS ability to differentiate short and long term anomalies and follow the temporal drift of real data.

To allow for clustering to take place the data is normalised to a suitable range relative to the micro-cluster radius, $r_0$. Here the range was based on the data available in the dataset and scaled to $0 - 1$. The data had an actual range from $min = 7.200$ to $max = 1,447$, ppbv (parts-per-billion by volume) for $NO_x$ and $min = -0.9$, $max = 422.8$ ($\mu g m^{-3}$) for $PM_{10}$ and so predicted ranges of 0 to 1500 and 0 to 200 respectively were used. The scaling introduced by this normalisation has an effect on the local density, joining and separation of micro-clusters and so expert knowledge is required to find suitable values for scientific research involving the cluster results.

For this real data test CEDAS was run with different decay times across the different sets of data from London Air Quality kept by Kings College London collected between 2010-2014. The data was presented to the CEDAS algorithm sequentially, in $NO_x$, $PM_{10}$ pairs, to mimic on-line data streams. The micro-clusters were plotted and the transparency of the micro-clusters set according to the value of the Energy in each. In this way if anomalous data appears for a short period of time the cluster adjusts, but it fades over the subsequent time period providing an online visualisation of the Energy of the micro-clusters. This provides a clear visual indication of CEDAS adapting to the changes in the data stream and following long term and short term drift. By using different decay times we demonstrate the different clusters that are created and discuss how this can be useful to investigate different time periods for drift and shift.

5. Results and Discussion

This section contains 3 separate subsections. Within each Subsection are the results and a discussion of the implication of these results. Subsection 5.1
uses the Mackey-Glass data streams described above to validate the ability of
CEDAS to accurately deal with data drift, cluster separation, cluster merging
and noise over time. Subsection 5.2 compares CEDAS with alternative
techniques CluStream, DenStream and MR-Stream with respect to data dimen-
sionality, complexity, processing speed, cluster quality and memory efficiency.
Subsection 5.3 applies the CEDAS algorithm to a real data stream from the
London Air Quality monitoring system to demonstrate how evolving clustering
can aid data mining of data streams containing short term drift, long term drift
and short and long term anomalies.

5.1. CEDAS Functionality with Cluster Separation, Cluster Merging, Drift and
Noise

To validate the correct functionality of CEDAS the algorithm was applied to
the Mackey-Glass data streams using \( \text{Decay} = 1,000 \) samples, \( \text{Radius} = 0.05 \)
and \( \text{MinimumClusterSize} = 12 \).

5.1.1. Cluster Separation and Merging

Using the clean Mackey-Glass data stream The sample number at which
a change in the number of macro-clusters was detected was stored. After the
analysis data was plotted with data from each time period coloured differently.
This is shown in Figure 3a.

As expected, after the initial settling period (red), it is seen that at each
colour change the number of clusters in the data contained in the preceding
decay period has changed. For example, in the green period the data was
contained in a single cluster. At the time the colour changes to black, the data
in the previous 1,000 samples had just separated to 2 separate clusters. When
the colour changes to magenta, the previous 1,000 samples created 1 cluster.

5.1.2. The Effects of Noise

To test the effects of noise on CEDAS the Mackey-Glass dataset is used
with a random noise sample added every 5 data samples as described above.
The random nature of the noise will have some effect on the initial positions
Figure 3: CEDAS Auto Change Detection, changes in colour represent changes in the number of clusters. The changes detected without noise are also detected with noise with the additional changes caused by temporary separate micro-clusters before they rejoin the main clusters.

of micro-clusters if the noise falls within them. This increases the likelihood of an initial micro-cluster separating from the main macro-cluster group. If this occurs then the number of macro-clusters may change briefly. This would give the appearance of false positives when compared with the results from dataset without noise. These additional clusters are in fact present at that time and it is accepted that the noise has in fact changed the clustering.

The results are shown in Figures 3a and 3b. In Figure 3b it can be seen that c, e, f, g, h, o are triggered by the noise and could easily be discounted based on the number of samples, if required. The trigger points without noise can be matched to those with noise as shown in table 1. These are discussed in the following sub-sections.

5.1.3. False Positives

With any online technique, apparent changes at some point in time may turn out to be irrelevant at a later time. An example of such soon-to-be-irrelevant data anomalies are those that result from the added noise. Rather than calling these 'false positives', they could be considered as 'temporary or short-term true positives'. In the event these are caused by temporary misplacement of micro-clusters caused by noise, which are rapidly re-absorbed into the macro-cluster, then these addition clusters will have an unusually short lifespan, i.e. considerably shorter than the set decay period. In this way any triggers that are within a user-defined short time span from a previous trigger could be discounted.
Table 1: Matching Additional 'Noisy' Trigger points in brackets with 'No Noise' trigger points.

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<th>With Noise</th>
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<td>a</td>
<td>a</td>
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<tr>
<td>1</td>
<td>b</td>
<td>b (c)</td>
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<tr>
<td>3</td>
<td>c</td>
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</table>

if required. However, this is not always desirable, as even short term anomalies may be of interest. They may, for example, indicate the start a general drift in the data.

5.1.4. False Negatives

With appropriate settings for decay time and micro-cluster radius, false negatives do not occur. It must be remembered that a different decay time will create different times for cluster separation. This is not indicative of false negatives, but rather a deliberate function of the technique to consider clusters based on data within a defined time frame.

5.1.5. True Positives

As we have demonstrated here, all changes to clusters are correctly detected. With the noisy dataset, although we have some temporary true positives, as
discussed above, CEDAS has successfully detected the same true positives as with the clean dataset as shown in table 1.

5.1.6. True Negatives

If we consider the definition of a ’true negative’ to be that ’no changes in macro-clusters are detected when there are none’ then this occurs with every sample that does not create new clusters.

5.2. Comparisons With Alternative Techniques

5.2.1. Speed and Dimensionality

By utilising hyper-spheres for micro-clusters the cluster joining technique checking for micro-cluster overlap is much simpler than, e.g. hyper-ellipsoidal micro-clusters. Micro-clusters are joined if the edge of the core hypersphere intersects another hyper-sphere shell. This requires only a comparison between the euclidean distance between cluster centres and the sum of the micro-cluster radii. Therefore, the only calculation that is dimensionally dependent is the euclidean distance with complexity $O(D)$ where $D$ is the number of dimensions. The relationship between the number of data dimensions and processing time per sample is linear.

Figure 4: Plot of mean processing time per sample in seconds for varying data dimensionality. Each line represents the processing time for different decay periods which create a proportional increase in micro-clusters.
With each new data sample being assigned to a single micro-cluster it is only necessary to check the intersections for that micro-cluster and then only if the micro-cluster centre has been modified, or a new micro-cluster has been created. This further reduces the required number of calculations. The radii of the micro-clusters is constant and so we only need to compare the euclidean distance between the changed micro-cluster and all others with $1.5r_0$.

The relationship between the number of data dimensions, decay period and calculation time is plotted in Figure 4. Using Matlab’s curve fitting toolbox the relationship between the data dimensions and time per sample was tested for different decay times. In the case of an evolving data stream with continuous drift the decay time is also proportional to the number of micro-clusters. To investigate the relationship between decay time, and so the number of micro-clusters, and run time the coefficients of the $'x'$ term in the linear equations are plotted in Figure 5 and show an approximately linear relationship. This demonstrates that CEDAS has a linear time penalty with both the number of dimensions and the number of micro-clusters.

By comparison, Figure 6 shows the relationship between processing time and dimensionality with the same data set for both DenStream [14] and Clustream [15]. We used the Massive Online Analysis [26] implementation running on R3.2.2 in RStudio 0.98.1102 analysing the same spiral high dimensional dataset as for CEDAS. CluStream was also limited to a maximum of 100 micro-clusters.
For both of these techniques, two tests were run using a decay time of 1,000 samples:

1. Both DenStream and CluStream without carrying out the 2nd stage re-clustering until the end of the data stream.

2. We approximated a fully online technique by carrying out the 2nd stage clustering technique at frequent intervals - every 100 samples for DenStream and every 10 samples for CluStream.

For the DenStream 2nd stage re-clustering DBScan [11] was used as implemented in the 'R' package by Hahsler [27] to allow for arbitrary shaped macro-clusters to form in a similar manner to CEDAS. The results shown in Figures 6a and 6b are for test 1 and the results shown in Figure 6c and 6d are for test 2. Without 2nd stage re-clustering both DenStream and CluStream are faster than CEDAS for low dimensionality data. The break even point is approximately 12D for CluStream and 220D for DenStream. When the second stage re-clustering of the micro-clusters is done frequently enough to approximate fully on-line analysis there is significant time penalty for both DenStream and CluStream. In both cases CEDAS is noticeably faster than both DenStream and CluStream and suffers significantly less time penalty for increasing data dimensionality.

5.2.2. Speed and Cluster Quality

The KDDCup99 [24] data stream is a popular dataset for testing evolving clustering algorithms such as eClass [28] and it is used here to allow direct comparisons with D-Stream and MR-Stream purity results presented by Wan et al [21]. Two sets of results are presented. The first is the same analysis used by Wan et al. of creating 500 time intervals spaced at 1K samples and placing these into groups of 25 and taking the mean cluster purity over these groups of 25. Taking the mean of a set of results can disguise individual poor results and so the cluster purity for CEDAS at each of the 500 time intervals is also provided. These results are shown in Figure 7.
(a) Without Re-Clustering Low Dimensionality
(b) Without Reclustering High Dimensionality

c) With Re-Clustering, Low Dimensionality  
(d) With Re-Clustering High Dimensionality

Figure 6: Typical analysis time per sample for DenStream, CluStream and CEDAS across various dimensional data.  a) and b) show CluStream and DenStream without 2nd stage re-clustering until the end of the data stream.  c) and d) show DenStream and CluStream with frequent 2nd stage re-clustering. In all plots CEDAS is shown in green.
It should be noted that purity alone, as defined by equation 2, may be a poor measure by itself.

\[
purity = \frac{\sum_{i=1}^{n} \left| \frac{C_{d}^{i}}{C_{i}} \right|}{K} \times 100\% \tag{2}
\]

\[
accuracy = \frac{\sum_{i=1}^{n} \left| C_{d}^{i} \right|}{\sum_{i=1}^{n} \left| C_{i} \right|} \times 100\% \tag{3}
\]

Here \( C_{i} \) is the number of samples in a cluster, \( C_{d}^{i} \) is the number of these samples assigned to the dominant class and \( N \) is the number of clusters. In cases where a high number of samples are contained in one cluster with low purity, yet few samples are contained in a high number of clusters with high purity the result is a high mean purity even though most samples are incorrectly assigned. Equally, the reverse is true when few clusters are present, if 99% of the data is correctly assigned in one cluster and two sample are contained in a second, one of which is mis-assigned the mean purity looks poor. In Wan et al. the relevance of this measure is further reduced by taking the mean of these means and so the purity measure is included here for comparison to Wan et al. only and not to attach any particular significance to the result. The cluster accuracy measure as defined in equation 3 is presented in Figure 7d which is a measure of the number of samples that have been correctly assigned to the dominant class.

By using both the purity and accuracy measures the quality of the clustering can be stated with greater confidence.

The results of the quality analysis are shown in Figure 7. Although the purity at time period 145 is 73%, the mean over the 25 time periods this is 96%. Using the two time periods selected by Wan et al, 27 and 52, we see that the CEDAS purity was 96% and 99.85% compared with MR-Stream at 97.5% and 92% respectively. It is interesting to note that at time periods 26 and 28 CEDAS purity is 100% suggesting that CEDAS adapts quickly to this variation. Using the 25 time periods measure favoured by Wan et al. we see that CEDAS mean purity exceeds that of MR-Stream. When considering the accuracy measure we note that at the time periods 27 and 52 the accuracy measurements are
98.5% and 99.98% respectively. This indicates that nearly all the samples are correctly assigned to the dominant clusters, but the purity is reduced due to few incorrectly assigned samples in clusters with few members. The accuracy of CEDAS remains close to 100% at all times except for 3 single occasions where it drops to around 90% and 2 at around 95%.

Figure 7a shows the results provide by Wan et al. for the mean purity over 25 time steps for both D-Stream and MR-Stream.

Having established via the purity and accuracy measures that the clusters are meaningful it is useful to see if they demonstrate any results of interest. To do this the number of clusters in a time period are compared with the number...
Figure 8: Plot of the number of classes of attack and the number of cluster found by CEDAS in each time period.

of classes given in the data. The plot of these is given in Figure 8 where it can be seen that each time there is a rise in the number of classes, i.e. attacks, the number of clusters also rises. Given that these clusters have high purity, and the accuracy of clustering is also high, these additional clusters must contain attack vectors unique to each type of attack. There are 50 time periods with attack vectors present and these are detected 100%. As discussed above, occasional separated micro-clusters are a feature of evolving techniques and providing they are short-lived and re-absorbed into the main clusters they can be ignored with reasonable confidence. When the number grows beyond 1 sample per cluster, however, they may be indicative of possible attacks. Thus with a threshold of 1 we have 20 false positives. However increasing the threshold to 2 to allow for occasional separated micro-clusters reduces this Figure to 4, and a threshold of 3 reduces this to a single instance. This compares favourably with a mean number of clusters per attack of 8.2.

5.2.3. Memory Efficiency

To demonstrate the efficient memory use of CEDAS, we compare the storage required by MR-Stream and DenStream with that required by CEDAS when clustering the KDDCup99 datastream. The results presented by Wan et al. for MR-Stream are shown in Figure 9a and, when the data stream is evolving and has variety, we see that MR-Stream reaches Figures in the thousands of nodes with a peak approaching 12,000. By contrast, the number of micro-clusters required by DenStream and CEDAS for the same data stream are shown in
Figure 9: Plot of the number of nodes or micro-clusters, which equates to memory use, for MR-Stream, DenStream and CEDAS. CluStream is not shown as it uses a number of micro-clusters set by the user.

Figure 9b: DenStream has a mean value of 181 and maximum of 839 whereas CEDAS has a mean of 20 and peaks at 137. This demonstrates the significant memory saving of micro-clusters over grid based techniques. Even allowing for the CEDAS cluster description consisting of 5 values there is significant saving over MR-Stream.

5.3. Application to Real Atmospheric Data Streams

This section describes how anomaly detection differs between long-, medium and short-term analysis and how CEDAS copes with such variation. To demonstrate this we are define 'Short Term' as being 7 days and 'Medium Term' as being 28 days and 'Long Term' as being one year. The data used for clustering is $NO_2$ with $PM_{10}$ from the London Air Quality database for the Westminster Marylebone monitoring station. Data samples are taken every 15 minutes covering an overall period of 5 years.

In Subsection 5.3.1 we consider how the use of a short decay period can reveal short term data drift that would be disguised in medium term decay periods. Subsection 5.3.2 describes the use of medium term decay periods to investigate possible seasonal variations. Finally, in Subsection 5.3.3 demonstrates how medium term decay periods can be used to investigate long term variations.
In this paper we only provide visual indications of how CEDAS reacts to the evolving data stream. Numerical analysis of the clustering results is discussed in Section 5 Conclusions.

Videos of the CEDAS cluster analysis can be found in the supplementary material.

5.3.1. Short Term Drift and Anomalies

Using a decay period equivalent to 7 days of data we can detect the changes in $NO_x$ and $PM_{10}$ over time. Sample plots are shown in Figure 11 (a)-(c) showing the cluster analysis at 3 different dates for the preceding 7 days. The data for the preceding 28 day period, for the same dates, is shown in Figures 11 (d)-(f).

We can see that the 7 day period preceding 24/03/11 is markedly different from the 7 day period preceding 06/02/11. Despite these difference in the 7 day data, by comparing the plots (d)-(f) we can see that overall, for the preceding 28 day periods the spread of data values has been more consistent. The data shown in the black and green clusters of the 7 day analysis in 11 (b) may be considered anomalous for that week, but in Figure 11 (e) we see that it is not unusual over the preceding 28 day period. However, data such as that in the
Figure 11: Sample plots of short term decay periods (a)-(c) and medium term decay periods (d)-(f). The short term variations indicated in (a)-(c) show the data varies over different 7 day periods. The medium term variations in (d)-(f) show that the data over the 28 day periods is more consistent and disguises the 7-day variation.

yellow and magenta cluster of (b) is seen to still be anomalous over the 28 day period, Figure 11 (e), where the clusters are now coloured khaki and blue.

This demonstrates that, by selecting suitable decay periods, the clustering results from the proposed algorithm provides relevant analysis of how data behaves over different time periods and how CEDAS can follow these changes in a fully online manner.

5.3.2. Medium Term Drift and Anomalies

The plots in Figure 12 are the cluster results for a 28 day decay period taken at different dates throughout the year. Over the 5 year period of the data streams this approximate pattern is repeated each year. The primary variation is not in the maximum, minimum or range of either $NO_x$ or $PM_{10}$ but rather in the range of the $PM_{10} : NO_x$ ratio. This is particularly noticeable when comparing, e.g. March and July where at any given value of $NO_x$ the range of
PM_{10} values is greater in March. Anomalous data can still be seen in March indicating that some unusual events are present.

This demonstrates the ability of CEDAS to follow such seasonal drifts, if they exist, and find data that is anomalous within that local time frame.

5.3.3. Long Term Drift and Anomalies

For long term changes, i.e. changes across years, the data could be analysed in multiple ways. For example, the data could be clustered on the full 365 day decay period. However, as we have already indicated in the Subsection 5.3.2 there are variations within that year which may be hidden in the way described
in Subsection 5.3.1 With this information it is reasonable to consider an analysis of 28 day decay periods, at the same date, for subsequent years. Examples of these cluster results are provided in Figure 13 and shows the results for data of the 28 days preceding 01/04 for the years 2010-2015.

The shape of the main cluster can be seen to vary between years indicating the changes in data values. Anomalies are indicated and are for the particular year under consideration. In all cases we see some relatively minor anomalies with values that are slightly different form the main cluster. These could be symptomatic of the data undergoing normal drift and changes. March 2012, however, shows some more extreme anomalies, shown in blue and green, with

Figure 13: Plots of CEDAS clustering with a 28 day decay time showing variation of the data for March over a 5 year period.
particularly high $PM_{10}$ values. Thus we see that these anomalies detected in March 2012 were not measured in any other year. This demonstrates how CEDAS may be used to analyse yearly changes, i.e. long term shifts in data and find anomalies independent of drift.

6. Summary and Conclusions

A new, fully online clustering technique for clustering data into arbitrarily shaped clusters is proposed. In Section 5, the algorithm has been described. The technique has been applied to the various data sets described in Section 4 and the results presented and discussed. In this section the results are summarised together with appropriate conclusions.

6.1. Technique Validity

Section 5.1 demonstrates the ability of CEDAS to accurately divide and merge evolving data streams where appropriate demonstrating the validity of the technique. The proposed algorithm is also shown to be robust to noise.

6.2. Cluster Quality

In Section 5.2.2 we compared the proposed algorithm to ClusStream, DenStream and MR-Stream and demonstrated that in the tested scenarios CEDAS performed as well as, or better, than all three alternatives. Including the additional accuracy measure provides evidence that the mean cluster purity measure is, in the case of CEDAS, a fair measure of the cluster quality.

6.3. Computational Efficiency

When working with stable data-streams with few micro-clusters, DenStream and CluStream approach the speed of the newly proposed technique. However, when the data stream evolves more rapidly, or there are a higher number of micro-clusters, the offline portion of combined online/ offline techniques becomes a limiting factor and CEDAS becomes significantly faster. In the case
of low dimensionality and where the second stage, offline, technique is not re-
quired often then DenStream and CluStream may also be faster. However, this
precludes these techniques from being considered as fully online. If excessive
periods of time are allowed between second stage clustering important clusters
and their information may go unnoticed. By being fully online the proposed
technique will not suffer from this limitation. It should also be noted that
CluStream finds only hyper-elliptical and not arbitrarily shaped clusters.

6.4. Memory Efficiency

In general, the similarities in the micro-cluster stage means there is a sim-
ilarity between memory use for CEDAS, DenStream and ClusStream. The for
micro-clusters of a similar size the number will be similar for each technique.
MR-Stream is highly memory intensive, not only does it store data for all the
cluster nodes, but also for those nodes on the higher plane. MR-Stream claims
to use this information to reduce the calculations required for the second stage
clustering. However, in the case of a highly populated data space this will re-
sults in an increase in memory storage and calculations as a high proportion
of the nodes and their parents need to be stored and visited during the second
stage clustering. In an effort to reduce the memory requirements MR-Stream
prunes nodes with a low density, however this implies a possible loss of data
without regard to it’s relevance to the current state.

6.5. Dimensionality

The proposed algorithm has a linear complexity and time penalty relative
to the number of data dimensions. DenStream and CluStream have a similar
linear complexity and time penalty, however, it is shown in section 5.2 that the
penalty is lower for CEDAS. MR-Stream has penalty of \(n^{DH}\) for dense data
space rendering it more suitable to low dimensional sparse data, particularly
when considered with the memory requirements.
6.6. Decay Time and the Number of Micro-Clusters

The proposed algorithm has a linear time penalty related to the number of micro-clusters. This is common to all two stage clustering techniques, including those alternatives discussed previously. In cases where the data is fairly static in the data space this has little relevance, however, if the data sample are continuously drifting through the data space there is a direct relationship between the speed of drift and the number of micro-clusters. Thus, in the worst case, there is a linear relationship between the decay time and the processing time. In practice data streams that drift at such a high rate are likely to be rare and may require a different type of analysis in any case. In the case of fairly static data the number of micro-cluster will vary little and no time penalty results from an increase in decay time.

6.7. Anomalies, Drift and Time

Sections 5.2.2 and 5.3 discuss the ability of the proposed algorithm to cope with drift and anomaly detection in real data streams. In both these sections CEDAS proved capable of accurately detecting anomalies within the defined time periods demonstrating possible applications in network security and atmospheric science research. The results in section 5.2.2 demonstrate how CEDAS could be used to automate detection across multiple dimensions that cannot be easily visualised, whereas Section 5.3 presents a visualization for primary interpretation by the user. In Section 7 some possible techniques are briefly discussed to place a numerical measure of the cluster information.

6.8. General Conclusions

Clustering of Evolving Data Streams into Arbitrary Shapes has been demonstrated to be a robust and accurate technique with linear complexity across both data stream size and data stream dimensionality. It is a fully online technique providing constant and immediate access to the clustering results. The techniques has been applied to real life datasets and shown to produce useful incites into evolving data streams.
7. Future Work

The work presented in this paper demonstrates the ability of the proposed technique to accurately cluster data from evolving data streams, however, no analysis of these clusters is considered. Proposed future work could consider quantitative methods for measuring the clustering results. Well established shape factors such as circularity, solidity or waviness etc may provide insight into the changing relationship between clusters over time. Macro-clusters are agglomerations of micro-clusters which suggests fractal analysis [29]. Providing some measure of the location, spread, size and shape of the macro-clusters can provide information towards a quantitative assessment of the similarity and connection between the internal cluster space and difference measure to other macro-clusters.

Appendix A. Pseudo Code

- $C_i^\mu$ - micro-cluster $i$'s data structure containing:
  - $C_i^\mu(Centre)$ - vector $\in \mathbb{R}$ with length = number of dimensions, micro-cluster $i$'s centre co-ordinates
  - $C_i^\mu(Count)$ - integer, number of data samples that have been assigned to micro-cluster $i$
  - $C_i^\mu(Macro)$ - integer, micro-cluster $i$'s macro-cluster membership
  - $C_i^\mu(Energy)$ - Energy $\in \mathbb{R}$, current value of assigned to micro-cluster $i$
  - $C_i^\mu(Siblings)$ - vector of integers, list of 'Sibling' micro-clusters linked to micro-cluster $i$

- $C^\mu$ for all the above, but without subscript refers to all micro-clusters

- $d_i$ - $\in \mathbb{R}$, distances from new data sample to the micro-cluster centre $i$

- $d_{min}$ - $\in \mathbb{R}$, distance to the nearest micro-cluster centre
\{D\} - vector of integers, set of indices of dead micro-clusters. (For the decay process described here this is a vector of length 1).

Decay - \(\in \mathbb{R}\), rate at which \(C_1^\mu(\text{Energy})\) is reduced

\(G\) - temporary variable for re-assigning macro-cluster numbers

\(i\) - integer, index value

\(N_c\) - integer, number of micro-clusters

\(r_0\) - \(\in \mathbb{R}\), micro-cluster radius, user input

\(x\) - vector \(\in \mathbb{R}\) with length = number of dimensions, current data sample

\(u\) - integer, index of updated or created micro-cluster

**Input:** \(x, R_0\)

**Initialization:**

1: while \(x \neq \{\}\) do
2: if \(C^\mu = \emptyset\) then
3: \(C_1^\mu(\text{Centre}) = S\)
4: \(C_1^\mu(\text{Count}) = 1\)
5: \(C_1^\mu(\text{Macro}) = 1\)
6: \(C_1^\mu(\text{Energy}) = 1\)
7: \(C_1^\mu(\text{Sibling}) = 1\)
8: \(N_c = N_c + 1\)
9: \(u = N_c\)
10: end if

**Update Micro-Cluster:**

11: \(u = 0\)
12: \(d_{\text{min}} = \|x - C_i^\mu(\text{Centre})\|_{\text{min}}\)
13: if \(d_{\text{min}} < r_0\) then
14: \(i = \arg\min_{j=1}^{K} \{d_j\}\)
15: \(C_i^\mu(\text{Energy}) = 1\)
16: \[ C_i^{\mu}(\text{Count}) = C_i^{\mu}(\text{Count}) + 1 \]
17: if \( d_i(\text{min}) > \frac{R_0}{2} \) then
18: \( u = i \)
19: \[ C_{\mu} = \frac{(C_{\mu}(\text{Count}) - 1) \times C_{\mu} + S}{C_{\mu}(\text{Count})} \]
20: end if
21: else
22: \( C_i^{\mu}(\text{Centre}) = x \)
23: \( C_i^{\mu}(\text{Count}) = 1 \)
24: \( C_i^{\mu}(\text{Macro}) = 1 \)
25: \( C_i^{\mu}(\text{Energy}) = 1 \)
26: \( C_i^{\mu}(\text{Sibling}) = 1 \)
27: \( N_c = N_c + 1 \)
28: end if

Kill Clusters:
29: \( C_i(\text{Energy}) = C_i(\text{Energy}) - \text{Decay} \)
30: \( \{D\} = \text{find}(C_i(\text{Energy}) < 0) \)
31: if \( \{D\} = \emptyset \) then
32: return
33: else
34: for all \( D_i \):
35: delete \( C_{D_i}^{\mu} \)
36: delete \( C(\text{Sibling}) = D_i \)
37: \( C_i^{\mu}(\text{Sibling}) > D_i = C_i(\text{Sibling}) - 1 \)
38: \( N_c = N_c - 1 \)
39: end if
40: if \( u \neq 0 \) then
41: \( d_{ui} = ||C_{\mu}^{\mu}(\text{Centre}) - C_i^{\mu}(\text{Centre})|| \)
42: \( \{j\} = \text{find}(D_{uj} < (1.5 \times r_0)) \)
43: \( C_{\mu}^{\mu}(\text{Sibling}) = C_{\mu}^{\mu}(\text{Sibling}) \cup \{j\} \)
44: \( G = \min\{C_{C_{\mu}(\text{Sibling})}(\text{Macro})\} \)
45: \( C_{\mu}^{\mu}(\text{Macro}) = G \)
46: \( C_{\mu}^{\mu}(\text{Sibling}) (\text{Macro}) = G \)

47: \textbf{end if} \\

Housekeeping: Reassign Macro Cluster Numbers

48: \( C^{\mu}(\text{Macro}) = 0 \)

49: \( G = 0 \)

50: \textbf{for} \( i = 1 \) to \( N_{c} \) \textbf{do}

51: \textbf{if} \( C_{i}(\text{Macro}) = 0 \) \textbf{then}

52: \( G = G + 1 \)

53: \( C_{i}^{\mu}(\text{Macro}) = G \)

54: \( C_{i}^{\mu}(\text{Sibling}) = G \)

55: \textbf{else}

56: \( C_{i}^{\mu}(\text{Sibling}) = C_{i}^{\mu}(\text{Macro}) \)

57: \textbf{end if}

58: \textbf{end for}

59: \textbf{end while}

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Supplementary material for on-line publication only
Click here to download Supplementary material for on-line publication only: CEDAS_Marylebone_NOx-PM10-7d-Supplementary.