

LANCASTER
UNIVERSITY
Management
School



Economics Working Paper Series

2014/020

Infrequent fiscal stabilization

Yuting Bai and Tatiana Kirsanova

The Department of Economics
Lancaster University Management School
Lancaster LA1 4YX
UK

© Authors

All rights reserved. Short sections of text, not to exceed two paragraphs, may be quoted without explicit permission, provided that full acknowledgement is given.

LUMS home page: <http://www.lums.lancs.ac.uk/>

Infrequent fiscal stabilization

Yuting Bai*
University of Lancaster

Tatiana Kirsanova†
University of Glasgow

July 2014

Abstract

This paper studies discretionary non-cooperative monetary and fiscal policy stabilization in a New Keynesian model, where the fiscal policymaker uses a distortionary tax as the policy instrument and operates with long periods between optimal time-consistent adjustments of the instrument. We demonstrate that longer fiscal cycles result in stronger complementarities between the optimal actions of the monetary and fiscal policymakers. When the fiscal cycle is not very long, the complementarities lead to expectation traps. However, with a sufficiently long fiscal cycle – one year in our model – no learnable time-consistent equilibrium exists. Constraining the fiscal policymaker in its actions may help to avoid these adverse effects.

Key Words: Monetary and Fiscal Policy Interactions, Distortionary Taxes, Discretionary policy, LQ RE models

JEL References: E31, E52, E58, E61, C61

*Address: Economics, Lancaster University Management school, Lancaster, LA1 4YX; e-mail y.bai@lancaster.ac.uk.

†Address: Economics, Main Building, University of Glasgow, Glasgow G12 8QQ; e-mail tatiana.kirsanova@glasgow.ac.uk

1 Introduction

Fiscal and monetary policies face different institutional restrictions and operate at different frequencies. Monetary policy maker sets interest rate every month, and the decision process can arguably be described as (constrained) optimization with the clear aim to stabilize the short run fluctuations.¹ In contrast, fiscal decisions are often taken annually, and the policy of contemporary fiscal authorities can rarely be described as aiming to stabilize the economy in the short run. This situation is likely to change, however, if fiscal policy is given a more active short run stabilization role: not only the fiscal policy becomes more focussed on stabilization, but also the decision process becomes more regular. This paper contributes to the discussion on the institutional design of stabilizing fiscal policy, which operates at lower frequency than monetary policy does, uses distortionary taxes as a policy instrument, and acts without implementation lags.

This institutional design has important implications for the dynamics of the economy. With longer fiscal cycle the optimal fiscal adjustments are bigger; they impact more on monetary policy maker and escalate the conflict between the two authorities in case the fiscal policymaker uses distortionary taxes. Indeed, optimal actions of the monetary and the fiscal policy makers are dynamic complements in the sense of Cooper and John (1988): higher tax rate, set by the fiscal policy maker in response to a higher debt level, generates the cost-push inflation, which increases the marginal return to a monetary policy decision to raise the interest rate and contribute to the debt accumulation. In standard quarterly models this reinforcement mechanism is weak; we demonstrate that it is greatly amplified if discretionary fiscal policy operates only infrequently.

We show that the gain from monetary and fiscal policy stabilization of macroeconomic fluctuations can be greatly overestimated, if it is evaluated using models with frequent fiscal policy stabilization. We can fail to account for arising expectations traps (King and Wolman, 2004) with implications of excessive volatility of welfare-relevant economic variables; we can also fail to realize the necessity to constrain the fiscal policymaker, as time-consistent policy may not exist.

We study interactions of monetary and fiscal policies in the Blanchard and Kahn (1980) class of infinite horizon non-singular discrete-time linear dynamic models that is typically used to study aggregate fluctuations in macroeconomics. We use the standard New Keynesian model with monopolistic competition and sticky prices to demonstrate the results. The economy is controlled by monetary and fiscal policy makers which act non-cooperatively at different frequencies. The monetary policy maker optimizes every period while the fiscal policy maker optimizes less frequently, choosing the distortionary tax rate once every several periods. After the tax rate is chosen, it stays at this level until the next fiscal optimization. The fiscal policy maker has an intra-period leadership: the monetary policy maker observes fiscal policy at every period, and the fiscal policy maker knows that the monetary policy maker optimizes every period and takes into account his reaction function when formulating policy.

More specifically, we demonstrate the existence of expectations traps in the case of longer fiscal cycles; although we also find that these traps are unlikely to present a problem for a policymaking. Following Dennis and Kirsanova (2012) we investigate stability properties of these equilibria and find that the Pareto-preferred equilibrium is selected in all cases we study. More importantly, we demonstrate that welfare-preferred discretionary equilibria may not exist, once the fiscal cycle

¹There is an extensive literature on the subject, see e.g. King (1997), Svensson (2010).

is sufficiently long – one year in our model – and the reinforcement mechanism between the optimal actions of the two policymakers is particularly strong. We argue that fiscal policymaker, who optimizes only infrequently and optimally chooses to make large corrections, needs to be made constrained in its actions. We use a number of policy scenarios to illustrate our findings, they include stabilizing debt faster than socially optimal, and infrequent fiscal optimization of a constrained fiscal policymaker.

The paper is organized as follows. In the next Section we present a model of monetary and fiscal policy interactions. Section 3 presents the general framework with infrequent stabilization. Section 5 discusses policy implications in three special cases: quarterly, biannual and annual fiscal stabilization. Section 6 concludes.

2 The Model

We consider the now-mainstream macro policy model, discussed in Woodford (2003), modified to take account of the effects of fiscal policy.² It is a closed economy model with two policymakers, the fiscal and monetary authorities. Fiscal policy is assumed to support monetary policy in stabilization of the economy around the non-stochastic steady state.

The economy consists of a representative infinitely-lived household, a representative firm that produces the final good, a continuum of intermediate goods-producing firms and a monetary and fiscal authority. The intermediate goods-producing firms act under monopolistic competition and produce according to a production function that depends only on labor. Goods are combined via a Dixit and Stiglitz (1977) technology to produce aggregate output. Firms set their prices subject to a Calvo (1983) price rigidity. Households choose their consumption and leisure and can transfer income through time through their holdings of government bonds. We assume that the fiscal authority faces a stream of exogenous public consumption. These expenditures are financed by levying income taxes³ and by issuing one-period risk-free nominal bonds.

We assume that all public debt consists of riskless one-period bonds. The nominal value \mathfrak{B}_t of end-of-period public debt then evolves according to the following law of motion:

$$\mathfrak{B}_t = (1 + i_{t-1}) \mathfrak{B}_{t-1} + P_t G_t - \tau_t P_t Y_t, \quad (1)$$

where τ_t is the share of national product Y_t that is collected by the government in period t , and government purchases G_t are treated as exogenously given and time-invariant. P_t is aggregate price level and i_t is interest rate on bonds. The national income identity yields

$$Y_t = C_t + G_t, \quad (2)$$

where C_t is private consumption. For analytical convenience we introduce $B_t = (1+i_{t-1})\mathfrak{B}_{t-1}/P_{t-1}$ which is a measure of the real value of debt observed at the beginning of period t , so that (1) becomes

$$B_{t+1} = (1 + i_t) \left(B_t \frac{P_{t-1}}{P_t} - \tau_t Y_t + G_t \right). \quad (3)$$

²See e.g. Benigno and Woodford (2003).

³We could use distortionary consumption taxes to finance the deficit. The transmission mechanism would be the same.

The first-order approximation of (3) about the non-stochastic zero-inflation and zero-debt steady state yields

$$b_{t+1} = \frac{1}{\beta} \left(b_t + \left(1 - \frac{C}{Y} \right) g_t - \tau (\tau_t + y_t) \right),$$

where $b_t = \frac{B_t}{Y}$, $c_t = \ln\left(\frac{C_t}{C}\right)$, $\tau_t = \ln\left(\frac{\tau_t}{\tau}\right)$, $g_t = \ln\left(\frac{G_t}{G}\right)$, $y_t = \ln\left(\frac{Y_t}{Y}\right)$ and letters without time subscript denote steady state values of corresponding variables in zero inflation steady state. The private sector's discount factor $\beta = 1/(1+i)$. We have assumed $B = 0$ in order to make the presentation of the model particularly simple. This assumption results in no first-order effects of the interest rate and inflation on debt, so that the final version of the linearized debt accumulation equation can be written as

$$b_{t+1} = \frac{1}{\beta} (b_t + (1-\tau)(1-\theta)g_t - \tau\theta c_t - \tau\tau_t), \quad (4)$$

where we used the linearized (2) to substitute out output and denoted $\theta = C/Y$.⁴

The derivation of the appropriate Phillips curve that describes Calvo-type price-setting decisions of monopolistically competitive firms is standard (Benigno and Woodford, 2003, Sec. A.5) and marginal cost is a function of output and taxes. A log-linearization of the aggregate supply relationship around the zero-inflation steady state yields the following New Keynesian Phillips curve

$$\pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa \left(\left(\frac{1}{\sigma} + \frac{\theta}{\psi} \right) c_t + \frac{(1-\theta)}{\psi} g_t + \frac{\tau}{(1-\tau)} \tau_t \right) + \eta_t, \quad (5)$$

where $\kappa = \frac{(1-\gamma\beta)(1-\gamma)}{\gamma} \frac{\psi}{(\psi+\epsilon)}$ is the slope of Phillips curve. Parameter γ is Calvo parameter, parameter ψ is Frisch elasticity of labour supply, σ is elasticity of intertemporal substitution and parameter ϵ is the elasticity of substitution between differentiated goods. Cost push shock η_t follows an autoregressive process.

The social loss is defined by the quadratic loss function⁵

$$L = \frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \lambda c_t^2). \quad (6)$$

while the monetary and the fiscal policy makers can have different policy objectives, $L^J = \frac{1}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t Q^J (\pi_t, c_t, \tau_t, g_t, b_t)$, $J \in \{M, F\}$. Each policy maker knows the laws of motion (4)-(5) of the aggregate economy and takes them into account when formulating policy. The following assumption follows Clarida, Gali, and Gertler (1999) and substantially simplifies the exposition of the model.

⁴Because we work with one-period debt only, its proportion in the total stock of debt is not very large. We discuss implications of this assumption for policy in Section 3.

⁵The criterion is derived under the assumption of steady state labour subsidy. Here parameter λ is a function of model parameters, $\lambda = \theta\kappa/\epsilon$, and ϵ is the elasticity of substitution between any pair of monopolistically produced goods.

Assumption 1 (policy instruments) *The monetary policymaker chooses consumption c_t and then, conditional on subsequent optimal evolution of c_t and π_t , decides on the value of interest rate that achieves the desired c_t and π_t . The fiscal policy maker uses the tax rate τ_t as policy instrument and keeps government spending constant $g_t = 0$.*

Apart from making the exposition clear, keeping fiscal spending constant allows us to focus on the particular transmission mechanism of monetary and fiscal policy.

Despite the simplicity of the model, finding time-consistent optimal policy is not trivial. Of course, the economy can be completely insulated against shocks if the two policy instruments are adjusted to offset the effect of shocks on inflation and debt. However, such policy would be time-inconsistent as it would need to offset the effect of expectations $\mathbb{E}_t \pi_{t+1}$ on current inflation. In what follows we assume that both policy makers and the private sector know that the decision making is sequential and a different policymaker may be in the office in future periods. We refer to this policy as policy under discretion. Formally, we make the following assumption.

Assumption 2 (policy) *Monetary and fiscal policy mix satisfies the following assumptions.*

- (i) *Monetary and fiscal authorities act non-cooperatively.*
- (ii) *Both authorities are assumed to optimize sequentially under time-consistency constraint.*
- (iii) *The monetary policy maker optimizes every period, but the fiscal policy maker optimizes once every N periods, $N \geq 1$.*
- (iv) *The fiscal authority has intra-period leadership.*

The assumption of fiscal intra-period leadership is motivated by the observation that the monetary policy reaction function is much more transparent and predictable, so the fiscal policy maker is able to take it into account when formulating policy.⁶ Using the interest rate as an instrument implies that consumption and price-setting decisions are made simultaneously, while in this model they are consecutive decisions taken by relevant agents. This makes no difference for our results.

The assumption of time-consistency prevents the complete and instantaneous stabilization of the economy. Moreover, the relatively large adjustments of infrequent fiscal policy may create more difficulties for monetary policy to offset the effect of disturbances on the economy. Smooth stabilization may not be possible any more.

The infrequency of fiscal decisions can be interpreted as fiscal commitment to the policy of fixed tax rates in all periods between the optimization. Such policymaking, however, remains sequential, without the ability to manipulate the expectations of the private sector beyond the periods between fiscal reoptimizations.

Assumption 3 (policy objectives) *Both policy makers are benevolent.*

Different objectives of the two policy makers are likely to result in a conflict between the policy makers as one policy maker tries to ‘undo’ the perceived harm done by the other.⁷ We

⁶Simultaneous moves of the two policy makers could be another possibility. Empirical evidence (Fragetta and Kirsanova, 2010) suggests that in countries without fiscal decentralization, like the UK, the regime of fiscal leadership is the most relevant.

⁷See e. g. Dixit and Lambertini (2003), Lambertini (2006).

shall demonstrate that a similar conflict exists even if both policy makers are benevolent but operate at different frequencies. The assumption of different frequencies also makes the leadership structure important. If both policy makers are benevolent and face identical constraints, then the intra-period leadership does not play any role. In our case the policymakers face different constraints, so the leadership does matter. In this paper we have chosen to study fiscal leadership as arguably most empirically relevant.

Finally, we make the assumption which is crucial for clear exposition without the loss of generality.

Assumption 4 *The model is perfect-foresight deterministic.*

If the stochastic model is linear-quadratic (LQ) then the stochastic component of the solution can be obtained in the unique way once the deterministic component is known.⁸ We are interested in issues of existence and uniqueness of the time-consistent policy and these properties are unaffected by the introduction of stochastic components in the LQ framework.

To summarize, the law of motion of the deterministic economy can be written as:

$$\pi_t = \beta\pi_{t+1} + \varkappa c_t + \nu\tau_t, \quad (7)$$

$$b_{t+1} = \frac{1}{\beta}(b_t - \tau\theta c_t - \tau\tau_t), \quad (8)$$

and the initial state \bar{b} is known to all agents, and coefficients $\varkappa = \kappa\left(\frac{1}{\sigma} + \frac{\theta}{\psi}\right)$, $\nu = \kappa\frac{\tau}{(1-\tau)}$. Debt b_t is the only endogenous predetermined state variable. The objectives of each policy maker coincide and are given by formula (6).

3 Discretionary Equilibrium

In this Section we define discretionary equilibrium in which the monetary policy maker reoptimizes every period while the fiscal policy maker decides once every N periods, $N \geq 1$. We refer to the period between fiscal reoptimizations as the fiscal cycle. We denote the set of numbers p congruent to a modulo N as $[p]_N$. There are exactly N different sets $[p]_N$. We shall identify these sets with the corresponding residue: $[p]_N = p$, so p denotes the time period after the latest fiscal reoptimization. Both the monetary and fiscal policy makers optimize in period $0 = [0]_N$. Only the monetary policy maker optimizes in periods $[1]_N, \dots, [N-1]_N$, which are labelled $p = 1, \dots, N-1$. The timing of events is illustrated in Figure 1.

Suppose the monetary and fiscal policy maker both optimize at period t . Because of the LQ nature of the problem we guess and verify later that the private sector's reaction function is a linear function of the state:

$$\pi_{t+p} = \pi_b^p b_{t+p}, \quad p = 0, \dots, N-1. \quad (9)$$

Use (9) for $p+1$ and substitute (7) for the appropriate period to obtain:

$$\pi_{t+p} = \pi_b^{p+1} b_{t+p} + \left(\varkappa - \pi_b^{p+1} \tau \theta\right) c_{t+p} + \left(\nu - \pi_b^{p+1} \tau\right) \tau_{t+p} \quad (10)$$

⁸See Anderson, Hansen, McGrattan, and Sargent (1996).

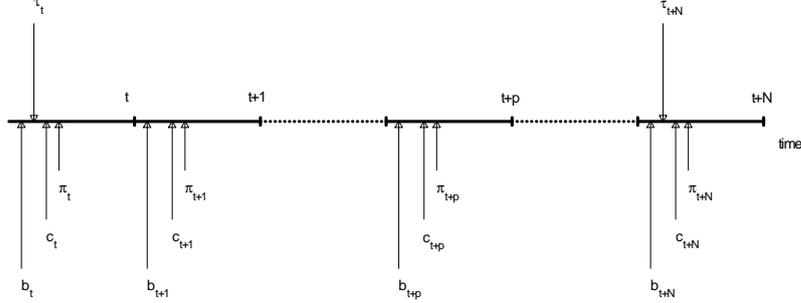


Figure 1: Timing of events

The private sector observes policy and the state, and takes into account the ‘instantaneous’ influence of the policy choice, measured by $(\varkappa - \pi_b^{p+1}\tau\theta)$ and $(\nu - \pi_b^{p+1}\tau)$.

The monetary policy maker’s problem in period $p = 0, \dots, N - 1$ can be described by the following Bellman equation, where the value function depends on the number of periods passed since the last fiscal optimization. Assuming the quadratic form for the appropriate value function we can write the Bellman equation for the monetary policy maker in period p :

$$S^p b_{t+p}^2 = \min_{c_{t+p}} \left(\left(\pi_b^{p+1} b_{t+p} + (\varkappa - \pi_b^{p+1}\tau\theta) c_{t+p} + (\nu - \pi_b^{p+1}\tau) \tau_{t+p} \right)^2 + \lambda c_{t+p}^2 + \beta S^{p+1} \left(\frac{1}{\beta} (b_{t+p} - \tau\theta c_{t+p} - \tau\tau_{t+p}) \right)^2 \right), \quad (11)$$

where we substituted constraints (8) and (10) written for the appropriate period.

Minimization with respect c_{t+p}^p yields the following monetary policy reaction function:

$$c_{t+p} = c_b^p b_{t+p} + c_\tau^p \tau_{t+p} \quad (12)$$

where

$$c_b^p = - \frac{(\varkappa - \tau\theta\pi_b^{p+1})\pi_b^{p+1} - \frac{\tau\theta}{\beta} S^{p+1}}{(\varkappa - \tau\theta\pi_b^{p+1})^2 + \lambda + \frac{\tau^2\theta^2}{\beta} S^{p+1}}, \quad (13)$$

$$c_\tau^p = - \frac{(\varkappa - \tau\theta\pi_b^{p+1})(\nu - \tau\pi_b^{p+1}) + \frac{\tau^2\theta}{\beta} S^{p+1}}{(\varkappa - \tau\theta\pi_b^{p+1})^2 + \lambda + \frac{\tau^2\theta^2}{\beta} S^{p+1}}, \quad (14)$$

and $p = 1, \dots, N - 1$. The monetary policy maker observes fiscal policy, and takes into account its ‘instantaneous’ influence, measured by c_τ^p .

The fiscal policy maker only optimizes in periods $[0]_N$. Suppose the optimization happens at time t . The Bellman equation which describes the fiscal policy decision can be written as:

$$Vb_t^2 = \min_{\tau_t} \left(\sum_{p=0}^{N-1} \beta^p (\pi_{t+p}^2 + \lambda c_{t+p}^2) + \beta^N Vb_{t+N}^2 \right) \quad (15)$$

where constraints (8), (10), and (12) are applied in any period $p = 0, \dots, N - 1$, because the state in period $[N]_N \equiv [0]_N$ depends on fiscal policy in all intermediate periods, τ_{t+p} , $p = 0, \dots, N - 1$.

We assume that the fiscal policy maker, when chooses τ_t also sets τ_{t+p} $p = 1, \dots, N - 1$ such that

$$\tau_{t+p} = \tau_t. \quad (16)$$

This policy has the following representation

$$\tau_{t+p} = \tau_b^p b_{t+p}, \quad p = 1, \dots, N - 1. \quad (17)$$

Indeed, take (17) one period forward and use (16) to obtain

$$\begin{aligned} \tau_{t+p+1} &= \tau_b^{p+1} b_{t+p+1} = \tau_b^{p+1} \frac{1}{\beta} (1 - \tau \theta c_b^p - \tau (1 + \theta c_\tau^p) \tau_b^p) b_{t+p} \\ &= \tau_b^p b_{t+p} \end{aligned}$$

from where

$$\tau_b^{p+1} = \frac{\beta \tau_b^p}{1 - \tau \theta c_b^p - \tau (1 + \theta c_\tau^p) \tau_b^p}, \quad p = 0, \dots, N - 2. \quad (18)$$

The complete set of constraints can be written as

$$\begin{aligned} \pi_{t+p} &= \left(\pi_b^{p+1} + \left(\varkappa - \pi_b^{p+1} \tau \theta \right) c_b^p \right) b_{t+p} + \left(\left(\varkappa - \pi_b^{p+1} \tau \theta \right) c_\tau^p + \nu - \tau \pi_b^{p+1} \right) \tau_{t+p} \\ &= \Pi_b^p b_{t+p} + \Pi_\tau^p \tau_{t+p} = \dots = \Pi_b^{p,0} b_t + \Pi_\tau^{p,0} \tau_t \end{aligned} \quad (19)$$

$$c_{t+p} = c_b^p b_{t+p} + c_\tau^p \tau_{t+p} = \dots = \mathcal{C}_b^{p,0} b_t + \mathcal{C}_\tau^{p,0} \tau_t \quad (20)$$

$$b_{t+p+1} = \frac{1}{\beta} \left((1 - \tau \theta c_b^p) b_{t+p} - \tau (\theta c_\tau^p + 1) \tau_t \right) = \dots = \mathcal{B}_b^{p,0} b_t + \mathcal{B}_\tau^{p,0} \tau_t \quad (21)$$

where the coefficients with superscript 0 are obtained by the recursive substitution τ_{t+k} , $k = 1..p$, and $p = 0, \dots, N - 1$.

Substitute these constraints into the Bellman equation (15) and differentiate with respect to τ_t to yield:

$$\tau_t = - \frac{\sum_{p=0}^{N-1} \beta^p \left(\Pi_\tau^{p,0} \Pi_b^{p,0} + \lambda \mathcal{C}_\tau^{p,0} \mathcal{C}_b^{p,0} \right) + \beta^N \mathcal{B}_\tau^{N,0} V \mathcal{B}_b^{N,0}}{\sum_{p=0}^{N-1} \beta^p \left(\left(\Pi_\tau^{p,0} \right)^2 + \lambda \left(\mathcal{C}_\tau^{p,0} \right)^2 \right) + \beta^N V \left(\mathcal{B}_\tau^{N,0} \right)^2} b_t \quad (22)$$

From (10), (12) and (17) it follows

$$\pi_b^p = \pi_b^{p+1} + \left(\varkappa - \tau\theta\pi_b^{p+1} \right) c_b^p + \left(\left(\varkappa - \tau\theta\pi_b^{p+1} \right) c_\tau^p + \left(\nu - \tau\pi_b^{p+1} \right) \right) \tau_b^p \quad (23)$$

which determines the time-consistent reaction of the private sector in (10).

The resulting transition of the economy for $p = 0, \dots, N - 1$ can be written as:

$$\begin{aligned} c_{t+p} &= C_b^p b_{t+p} \\ \pi_{t+p} &= \pi_b^p b_{t+p} \\ b_{t+p+1} &= B_b^p b_{t+p} \end{aligned} \quad (24)$$

where

$$\begin{aligned} B_b^p &= \frac{1}{\beta} \left(1 - \tau\theta c_b^p - \tau(\theta c_\tau^p + 1) \tau_b^p \right) \\ C_b^p &= c_b^p + c_\tau^p \tau_b^p \end{aligned}$$

Substitute them into (11) and (15) to yield

$$S^p = \left(\pi_b^p \right)^2 + \lambda \left(C_b^p \right)^2 + \beta S^{p+1} \left(B_b^p \right)^2, \quad p = 0, \dots, N - 1 \quad (25)$$

and

$$V = \sum_{p=0}^{N-1} \beta^p \left(\left(\pi_b^p \right)^2 + \lambda \left(C_b^p \right)^2 \right) \prod_{j=0}^p \left(B_b^{j-1} \right)^2 + \beta^N V \prod_{p=0}^{N-1} \left(B_b^p \right)^2 \quad (26)$$

It follows that $V = S^0 = S^N$: in periods when both benevolent policy makers reoptimize their value functions are the same.

Proposition 1 *Given Assumptions 2-4 the stationary discretionary equilibrium with intra-period fiscal leadership can be described by the set of coefficients $V \cup \{\pi_b^p, c_b^p, c_\tau^p, \tau_b^p, S^p\}_{p=0}^{N-1}$.*

Proof. For a given $b_0 = \bar{b}$, each trajectory $\{b_t, \pi_t, c_t, \tau_t\}_{t=0}^\infty$ which solves the system of first order conditions (8), (10), (12), and (17) we can uniquely map into the set of coefficients $V \cup \{\pi_b^p, c_b^p, c_\tau^p, \tau_b^p, S^p\}_{p=0}^{N-1}$, satisfying (13), (14), (22), (23), (25) and (26). Conversely, if the set of coefficients $V \cup \{\pi_b^p, c_b^p, c_\tau^p, \tau_b^p, S^p\}_{p=0}^{N-1}$ solves (13), (14), (22), (23), (25) and (26) we can uniquely map it into the trajectory $\{b_t, \pi_t, c_t, \tau_t\}_{t=0}^\infty$, solving system (8), (10), (12) for given $b_0 = \bar{b}$. ■

4 Calibration

The model is highly stylized and involves relatively few parameters. Table 1 reports the baseline calibration of parameters. Calibration of β , γ and θ is relatively straightforward, they correspond to the most frequently estimated values of the steady state annual interest rate of 4%, the average frequency of price changes of one year, and consumption to output share of 75%. Estimation and the consequent calibration of the remaining three parameters varies across studies.

Table 1: Calibration

Parameters	calibration		
		base	range
Discount factor	β	0.99	–
Calvo parameter	γ	0.75	–
Consumption to output ratio	θ	0.75	–
Intertemporal elasticity	σ	0.3	[0.1, 1.3]
Frisch elasticity of labour supply	ψ	3.0	[0.3,4]
Elasticity of substitution between goods	ϵ	11.0	[4,11]

Estimates of the Frisch elasticity of labour supply ψ vary widely, depending on whether macro- or micro-evidence is used. Peterman (2012) reports values of $\psi \in [2.9, 3.1]$ from the empirical work which matches volatilities of aggregate worked hours and of wages. This range is consistent with values used by macroeconomists to calibrate general equilibrium models but greater than the estimates of $\psi \in [0.3, 0.8]$ which are obtained in microeconomic studies even if decisions on labour participation are taken into account, see Chetty, Guren, Manoli, and Weber (2011). The main source of this difference lies in the heterogeneity of the workforce’s reservation wages. When a larger proportion of the workforce’s reservation wage is about the market wage, a small change in the market wage leads to a large change in the labour force participation, see Chang and Kim (2005) and Gourio and Noulal (2009). However the density of marginal workers can only be observed at the macro-level; the effect is larger in countries with higher involuntary unemployment which leads to higher aggregate elasticity of labour supply. This effect is not identified at the micro-level where a small change in the market rate often does not lead to a noticeable change in the participation status of an individual. So we consider values between 0.3 and 4 plausible for ψ .

Similarly, estimates of the intertemporal elasticity σ vary depending on the wealth of the representative households and the proportion of nondurable goods in their consumption bundle, see Atkeson and Ogaki (1996), Rotemberg and Woodford (1997). The empirical evidence for σ is quite far-ranging from near 0.1 reported in e.g. Hall (1988) and Campbell and Mankiw (1989), to above 1 reported in e.g. Rotemberg and Woodford (1997). Attanasio and Weber (1993, 1995) find that the estimate of σ increases from 0.3 for the aggregate data to 0.8 for cohort data, suggesting that the aggregation, which is implicit in the macro data, may cause a significant downward shift in the estimate of σ .

The elasticity of substitution between goods, ϵ , determines the monopolistic mark up. Chari et al. (2000) argue for a markup of 11% for the macroeconomy as a whole. Rotemberg and Woodford (1997) obtain elasticity of substitution 7.88, corresponding to a markup of 14.5%. Different industries have different markups, Berry, Levinsohn, and Pakes (1995) and Nevo (2001) report mark ups of 27-45% for automobiles and branded cereals industries.

We use base line values of parameters as reported in the first column in Table 1, but we also investigate the robustness of our results to the range of alternative calibrations given in the second column in Table 1.

5 Policy Interactions

In this section we study how an increase in the length of the fiscal cycle affects the economy under discretionary policy. Dynamic complementarities play a crucial role in shaping the dynamics of the economy once the fiscal cycle becomes longer.

We start with the known case of frequent monetary and fiscal policy stabilization.⁹ We use this example to discuss the transmission mechanisms of monetary and fiscal policy interactions.

We continue with the case of biannual fiscal optimization, which enables us to demonstrate how the dynamic complementarity between the optimal actions of *consequent* monetary policy makers within the fiscal cycle results in multiple discretionary equilibria and potential expectation traps. We also demonstrate that the agents are likely to coordinate on the best equilibrium.

These two cases help us to investigate the more complex case of annual fiscal optimization, which is arguably the most empirically relevant case. We demonstrate how the dynamic complementarity between the optimal actions of monetary and fiscal policymakers leads to expectation traps. Although we demonstrate that in this case the coordination problem is likely to be resolved as well, as all agents are more likely to coordinate on the best equilibrium, we also show that the existence of these equilibria is very sensitive to the parameterization of the model and to the length of fiscal cycle. We argue that actions of the fiscal policymaker should be restricted to some extent, as this ensures the existence of good equilibrium outcome for a wide range of parameterization of the model and policy scenarios, as well as for longer fiscal cycle.

5.1 Quarterly Fiscal Stabilization

In the standard case of frequent stabilization both policy makers operate at the same quarterly frequency. The model is simple enough to prove the following proposition.

Proposition 2 *If $0 < \beta < 1$, $\tau > 0$, $\lambda > 0$ then a stationary discretionary equilibrium exists and unique.*

Proof. The system of first order conditions (13), (14), (22), (23), (25) and (26) can be written as follows (where we omit the index p):

$$c_b = -\frac{(\varkappa - \pi_b \tau \theta) \pi_b - \frac{\tau \theta}{\beta} V}{(\varkappa - \tau \theta \pi_b)^2 + \lambda + \frac{\tau^2 \theta^2}{\beta} V} \quad (27)$$

$$c_\tau = -\frac{(\varkappa - \pi_b \tau \theta) (\nu - \pi_b \tau) + \frac{\tau^2 \theta}{\beta} V}{(\varkappa - \tau \theta \pi_b)^2 + \lambda + \frac{\tau^2 \theta^2}{\beta} V} \quad (28)$$

$$\tau_b = -\frac{((\varkappa - \tau \theta \pi_b) c_\tau + \nu - \tau \pi_b) (\pi_b + (\varkappa - \tau \theta \pi_b) c_b) + \lambda c_\tau c_b}{((\varkappa - \tau \theta \pi_b) c_\tau + \nu - \tau \pi_b)^2 + \lambda c_\tau^2 + \frac{\tau^2}{\beta} (1 + \theta c_\tau)^2 V} \quad (29)$$

$$-\frac{1}{\beta} \frac{\tau (1 + \theta c_\tau) (1 - \tau \theta c_b) V}{((\varkappa - \tau \theta \pi_b) c_\tau + \nu - \tau \pi_b)^2 + \lambda c_\tau^2 + \frac{\tau^2}{\beta} (1 + \theta c_\tau)^2 V}$$

⁹See Blake and Kirsanova (2011) for a general form solution to this class of problems.

$$\pi_b = (\pi_b + (\varkappa - \pi_b \tau \theta) c_b + ((\varkappa - \pi_b \tau \theta) c_\tau + (\nu - \pi_b \tau)) \tau_b) \quad (30)$$

$$V = \pi_b^2 + \lambda (c_b + c_\tau \tau_b)^2 + \frac{1}{\beta} V (1 - \tau \theta c_b - \tau (1 + \theta c_\tau) \tau_b)^2 \quad (31)$$

Introduce new variable, $C_b = c_b + c_\tau \tau_b$. Using several substitutions we transform the system of first order conditions (27)-(31) into the system of two equations in $\{C_b, \tau_b + \theta C_b\}$:

$$C_b(C_b, \tau_b + \theta C_b) = C_b + \frac{\nu(\varkappa - \theta\nu)}{\lambda\tau(\tau_b + \theta C_b) + (\varkappa - \theta\nu)^2} (\tau_b + \theta C_b) = 0 \quad (32)$$

$$\tau_b(C_b, \tau_b + \theta C_b) = (\tau_b + \theta C_b)^2 - \frac{\lambda - (\varkappa - \theta\nu)^2}{\lambda\tau} (\tau_b + \theta C_b) - \frac{(1 - \beta)(\varkappa - \theta\nu)^2}{\lambda\tau^2} = 0 \quad (33)$$

Equation (33) only depends on $z = \tau_b + \theta C_b$ and always has exactly one positive solution as the free term is negative.

The unique positive root satisfies $\frac{1}{\beta} |1 - \tau(\theta C_b + \tau_b)| < 1$, (or equivalently $\frac{1-\beta}{\tau} < \tau_b + \theta C_b < \frac{1+\beta}{\tau}$) so that the equilibrium is stationary. To see this, note that if z_+ is the positive root, then $z_- = -\left((1 - \beta)(\varkappa - \theta\nu)^2\right) / (\lambda\tau^2 z_+)$, and $\partial(z_+ + z_-) / \partial z_+ = 1 + (1 - \beta)(\varkappa - \theta\nu)^2 / (\lambda\tau^2 z_+^2) > 0$. (i) We show that $\tau_b + \theta C_b > (1 - \beta) / \tau$. Indeed, suppose $\tilde{z}_+ = (1 - \beta) / \tau$. \tilde{z}_+ is not the positive root to (33); if it was the positive root then the negative root would be $\tilde{z}_- = -(\varkappa - \theta\nu)^2 / (\lambda\tau)$ and their sum should have been equal to the negative linear coefficient, but $\tilde{z}_+ + \tilde{z}_- < (\lambda - (\varkappa - \theta\nu)^2) / (\lambda\tau)$. Moreover, any $\tilde{z}_+ < (1 - \beta) / \tau$ is not a root of (33), because $\partial(z_+ + z_-) / \partial z_+ > 0$. (ii) We show that $\tau_b + \theta C_b < (1 + \beta) / \tau$. Indeed, suppose $\tilde{z}_+ = (1 + \beta) / \tau$. \tilde{z}_+ is not the positive root to (33); if it was the positive root then the negative root would be $\tilde{z}_- = -(1 - \beta)(\varkappa - \theta\nu)^2 / (\lambda\tau(1 + \beta))$ and their sum $\tilde{z}_+ + \tilde{z}_- > (\lambda - (\varkappa - \theta\nu)^2) / (\lambda\tau)$. Moreover, any $\tilde{z}_+ > (1 + \beta) / \tau$ is not a root of (33), because $\partial(z_+ + z_-) / \partial z_+ > 0$. ■

Panel I in Figure 2 presents constraints (32)-(33) in $\{C_b, \tau_b + \theta C_b\}$ space. Solution to equation (33) is plotted with the dashed line, and solution to equation (32) is plotted with solid line. The unique equilibrium is labelled A in Panel I in Figure 2 and its characteristics are given in Table 2.

Equilibrium A is IE-stable under all types of learning discussed in Section ??, and we report this in Table 2.

It is easy to see that because equilibrium A is stationary, i.e. $\frac{1}{\beta} |1 - \tau(\tau_b + \theta C_b)| < 1$, then equation (??) implies that the fix-point of \mathbb{T}_{PS} is locally stable under the PS-learning.

IE-stability under the JF-learning plays an important role in the analysis of cases with longer fiscal cycle. Using the fact that all equilibria are IE-stable under the L-learning, and replicating the steps of the revision process of all agents who are learning, helps us to discover RE equilibria in this and more complex cases with longer fiscal cycle. We illustrate this process in Panel II of Figure 2. Suppose the fiscal policy maker considers implementing policy τ_b , which is not necessarily optimal. In response to this policy the followers learn their optimal response $\{C_b, S, \pi_b\}$. Their learning problem is equivalent to the joint learning in the single-policymaker setting, which is discussed in details in Blake and Kirsanova (2012) and Dennis and Kirsanova (2012). If $\tau_b = 0$ then the fiscal policymaker does not respond to debt and there is unique set $\{C_b, S, \pi_b\}^P$ which describes the case in which the monetary policymaker and the private sector coordinate on the reaction so that in response to higher debt the monetary policymaker generates high demand and

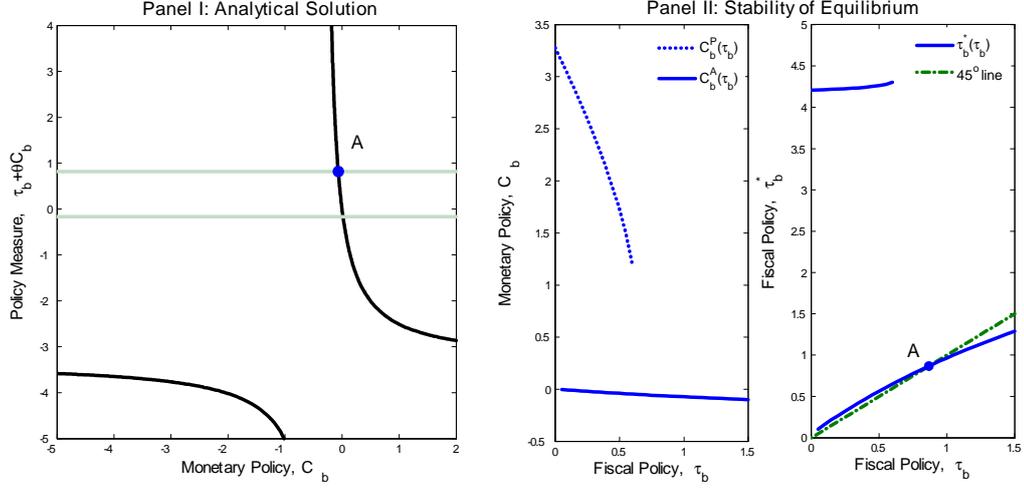


Figure 2: Unique Equilibrium in Frequent Fiscal Optimization Model

Table 2: Characteristics of Equilibria

	Eq. A	Eq. B
<i>Frequent Fiscal Stabilization</i>		
Fiscal Policy $[\tau_b]$	[0.8671]	–
Monetary Policy $[C_b]$	[–0.0657]	–
Private Sector $[\pi_b]$	[0.0048]	–
Normalized Loss L	1.0000	–
IE-stability [PS,JF,J]	[Y,Y,Y]	–
<i>Biannual Fiscal Stabilization</i>		
Fiscal Policy $[\tau_b^0; \tau_b^1]$	[0.79; 0.96]	[3.57; 34.72]
Monetary Policy $[C_b^0; C_b^1]$	$– [6.41; 6.87] \times 10^{-2}$	$[2.72; 48.36] \times 10^{-2}$
Private Sector $[\pi_b^0; \pi_b^1]$	$[4.7; 5.0] \times 10^{-3}$	$[23.8; 0.7] \times 10^{-3}$
Normalized Loss L	0.9956	18.8413
IE-stability [PS,JF,J]	[Y,Y,Y]	[Y,N,N]
<i>Annual Fiscal Stabilization</i>		
Fiscal Policy $[\tau_b^0; \tau_b^1; \tau_b^2; \tau_b^3]$	[0.71; 0.84; 1.04; 1.36]	[1.23; 1.72; 2.87; 9.02]
Monetary Policy $[C_b^0; C_b^1; C_b^2; C_b^3]$	$– [6.2; 7.0; 7.9; 8.3] \times 10^{-2}$	$– [0.10; 0.13; 0.17; 0.35]$
Private Sector $[\pi_b^0; \pi_b^1; \pi_b^2; \pi_b^3]$	$[4.6; 5.2; 5.7; 6.0] \times 10^{-3}$	$– [0.8; 1.0; 1.4; 2.3] \times 10^{-2}$
Normalized Loss L	1.0256	2.0068
IE-stability [PS,JF,J]	[Y,Y,Y]	[Y,Y,N]

accommodates high inflation so that debt is quickly steered back to its equilibrium level. The corresponding positive C_b^P is plotted in the left chart in Panel II. If $\tau_b > 0$ and sufficiently large then the process of debt stabilization is tightly controlled by fiscal policy and in response to higher debt the monetary policymaker and the private sector coordinate on the response $\{C_b, S, \pi_b\}^A$ in which the demand is lowered and inflation is not accommodated. The corresponding positive C_b^A is plotted in the left chart in Panel II.¹⁰ If τ_b is moderate, both types of responses of the monetary policymaker and the private sector exist, as shown in the left chart in Panel II. In response to each $\{C_b, S, \pi_b\}^j, j \in \{A, P\}$ the fiscal policymaker can learn its optimal response $\tau_b^{*j}, j \in \{A, P\}$. For each set $\{C_b, S, \pi_b\}$ the response τ_b^* is unique if it exists, see Section ???. Therefore, for an initial guess τ_b we find the update in the revision process of the fiscal policymaker $\tau_b^*(\tau_b)$. We plot $\tau_b^* = \tau_b^*(\tau_b)$ for a range of initial guesses and for each equilibrium reaction $\{C_b, S, \pi_b\}^j, j \in \{A, P\}$ in the right chart in Panel II in Figure 2 with the solid line. By construction, all points of intersection of this line with the 45° line are the points of discretionary equilibria which are IE-stable under the JF-learning by construction.

The result of Proposition 2 on the uniqueness of the equilibrium is not obvious if the model has dynamic complementarities between action of the economic agents (Cooper and John, 1988). Optimal actions of the monetary authority and of the aggregated private sector can be dynamic complements. Suppose the reaction of fiscal policy is given and fixed at τ_b^* . For a given reaction of the private sector π_b in $\pi_t = \pi_b b_t$ the monetary policy finds the optimal response by solving the corresponding Bellman equation, taking into account its intra-period leadership. If π_b is sufficiently high (low) then in response to higher-than-steady-state debt the monetary policymaker optimally raises demand. Greater tax base leads to higher tax collection and reduces the level of debt towards the steady state. Inflation starts moving back to the steady state. We plot this U-shaped optimal reaction function $C_b = C_b(\pi_b)$ in the left hand side chart of Panel I in Figure 3 with the solid line. In its turn, the optimal reaction of the private sector $\pi_b = \pi_b(C_b)$ is increasing in C_b . If the debt is higher than its steady state level and the monetary policymaker generates higher demand, the total effect of the higher demand on marginal costs is always positive, as the tax rate is fixed to τ_b^* . We plot the positively sloped reaction function $\pi_b = \pi_b(C_b)$ in the left hand side chart of Panel I in Figure 3 with the dashed line. Both lines are positively sloped in the area with relatively large π_b , and this can result in multiplicity of partial equilibria, i.e. in multiplicity of optimal responses of the monetary authorities and the private sector. Indeed, if we reduce (e.g. halve) the fiscal feedback τ_b then there are three points of intersection of optimal reactions of monetary authorities and the private sector, see the right hand side chart. The case in the left hand side chart in Panel I corresponds to multiple discretionary equilibria discussed in Blake and Kirsanova (2012) where the fiscal feedback on debt of *non-strategic* fiscal policy was relatively small to guarantee multiplicity of equilibria.

Optimal actions of fiscal policy and the private sector can be dynamic complements too. Fixing the monetary policy reaction to the optimal level $C_b^* < 0$ produces reactions of the fiscal authority $\tau_b = \tau_b(\pi_b)$ and of the private sector $\pi_b = \pi_b(\tau_b)$ plotted in Panel II in Figure 3. The reaction of the private sector is positively sloped as higher tax rate set by the fiscal authority in response to higher debt τ_b always results in higher prices set by firms. The reaction curve if the

¹⁰There is close resemblance between these two partial equilibria and ‘active’ and ‘passive’ monetary policy described in Leeper (1991).

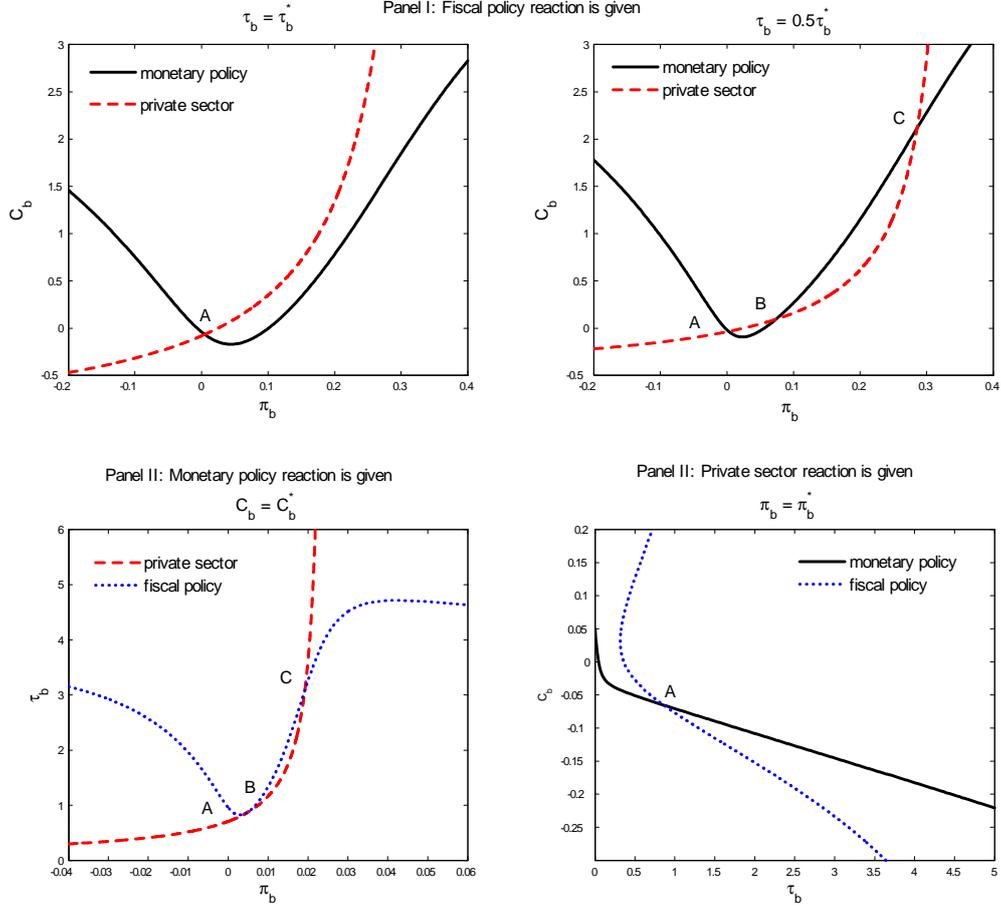


Figure 3: Dynamic complementarities between agents' actions

fiscal policymaker is also positively sloped but only for moderately positive response of inflation to debt, π_b . To understand this suppose the debt is higher than its steady state level. Because $C_b^* < 0$ the demand is automatically reduced and so are marginal costs. Lower demand also contributes to faster debt accumulation. If the response of inflation to debt is only moderately positive, then the optimal response of taxes to debt rises with stronger response of inflation. This will keep debt under control, and will not compromise inflation stabilization. As a result, the reaction curves of the private sector and the fiscal policymaker are both positively sloped, but only in a relatively narrow area of responses of the private sector. For our baseline calibration, and given C_b^* , there are three jointly optimal discretionary responses of fiscal policymaker and the private sector. However, only one of them results in discretionary equilibrium in the model, as the other two partial equilibria require different optimal policy response once the monetary policy becomes strategic.

Finally, and most importantly for our study, optimal actions of the monetary and the fiscal policymakers can also be dynamic complements. Higher tax rate, set by the fiscal policy maker in response to a high debt level, τ_b , generates greater cost-push inflation, which increases the marginal return to a monetary policy decision to reduce demand and contribute to the debt accumulation. The monetary policy reaction function $C_b = C_b(\tau_b)$ is negatively sloped, see Panel III in Figure 3. Conversely, a reduction in response of demand to debt, C_b , makes it optimal to raise taxes in order to prevent too fast accumulation of debt. As a result, the fiscal policy reaction function $\tau_b = \tau_b(C_b)$ is also negatively sloped in wide area, see Panel III in Figure 3.

The presence of dynamic complementarities is a necessary condition for the multiplicity of discretionary equilibria, see King and Wolman (2004) and Blake and Kirsanova (2012). However this condition is not sufficient and, as we argue next, the interaction of the two mechanisms in the model with frequent fiscal optimization results in the uniqueness of the equilibrium.

First, the complementarity between optimal decisions of the private sector and of the monetary policy maker may result in multiplicity only if fiscal policy *optimally* responds to debt only weakly, see Panel I, the right hand chart. The optimal fiscal response τ_b^* even to the weak initial guess τ_b is strong enough to rule out the equilibrium with passive monetary policy.

Second, although the optimal decision of the fiscal policy maker is increasing in the optimal decision of the monetary policy maker, this dynamic complementarity between optimal decisions of the two policy makers in case of frequent fiscal optimization is not strong enough to create the multiplicity.

The following two cases demonstrate how the longer fiscal cycle increases the strength of dynamic complementarities in the model and how this shapes the optimal outcome of monetary and fiscal policy interactions.

5.2 Biannual Fiscal Stabilization

Suppose that both policy makers optimize in even periods, and we index all such periods with index 0. Only the monetary policy maker optimizes in odd periods, we index such periods with index 1. To save on notation we use the period index $p \in \{0, 1\}$ and use $-p$ to indicate odd periods if $p = 0$, and even periods if $p = 1$.

Despite we cannot prove analytically the existence and multiplicity of equilibria, we can find *all* discretionary equilibria numerically.

Proposition 3 *For the base line calibration of the model two discretionary equilibria exist.*

Proof. The first order conditions derived in Section 3 can be written as

$$\tau_b^0 = -\frac{\Pi_\tau^0 \Pi_b^0 + \lambda c_\tau^0 c_b^0 + \beta \Pi_\tau^{1,0} \Pi_b^{1,0} + \beta \lambda \mathcal{C}_\tau^{1,0} \mathcal{C}_b^{1,0} + \beta^2 \mathcal{B}_\tau^{2,0} \mathcal{B}_b^{2,0} V}{(\Pi_\tau^0)^2 + \lambda (c_\tau^0)^2 + \beta (\Pi_\tau^{1,0})^2 + \beta \lambda (\mathcal{C}_\tau^{1,0})^2 + \beta^2 V (\mathcal{B}_\tau^{2,0})^2} \quad (34)$$

$$\tau_b^1 = \frac{\beta \tau_b^0}{1 - \tau ((1 + \theta c_\tau^0) \tau_b^0 + \theta c_b^0)} \quad (35)$$

$$c_b^p = -\frac{(\varkappa - \tau \theta \pi_b^{-p}) \pi_b^{-p} - \frac{\tau \theta}{\beta} S_{-p}}{(\varkappa - \tau \theta \pi_b^{-p})^2 + \lambda + \frac{\tau^2 \theta^2}{\beta} S_{-p}} \quad (36)$$

$$c_\tau^p = -\frac{(\varkappa - \tau \theta \pi_b^{-p}) (\nu - \tau \pi_b^{-p}) + \frac{\tau^2 \theta}{\beta} S_{-p}}{(\varkappa - \tau \theta \pi_b^{-p})^2 + \lambda + \frac{\tau^2 \theta^2}{\beta} S_{-p}} \quad (37)$$

$$S_0 = V \quad (38)$$

$$S_1 = (\pi_b^1)^2 + \lambda (C_b^1)^2 + \beta S_0 (B_b^1)^2 \quad (39)$$

$$V = (\pi_b^0)^2 + \lambda (C_b^0)^2 + \beta ((\pi_b^1)^2 + \lambda (C_b^1)^2) (B_b^0)^2 + \beta^2 V (B_b^0)^2 (B_b^1)^2 \quad (40)$$

After multiple substitutions the system of first order conditions (34)-(40) can be reduced to the polynomial system of two equations $C_b^0(C_b^0, \tau_b^0) = 0$ and $\tau_b^0(C_b^0, \tau_b^0) = 0$, although at the expense of much complexity. We plot solutions to these equations in Panel I in Figure 4. The curves intersect in two points with $V = S_0 > 0, S_1 > 0$, and $|B_b^0 B_b^1| < 1$. We label these points of intersection as equilibria A and B . ■

Multiplicity of discretionary equilibria implies that following a disturbance, for example a higher initial debt level, the economy can follow one of multiple paths, each of which satisfies conditions of optimality and time-consistency. Each of these paths is associated with different monetary and fiscal policies; see Figure 5 which plots two different adjustment paths following the same initial increase in the debt level. For comparison, the Figure also includes responses in case of frequent fiscal stabilization.

Suppose the level of debt is above the steady state and fiscal policy raises the tax rate for two periods. Following the high marginal cost inflation will rise and stay above the steady state for these two periods. The monetary policy maker will find it optimal to intervene. The monetary policy maker at time 0 takes into account monetary policy in period 1. There is a dynamic complementarity between the actions of the two consequent monetary policy makers within the fiscal cycle: the deeper is the future cut in demand, the bigger payoff the current monetary policy maker gets from engineering high demand today. A high demand today results in optimal reduction of demand in the future, within the same fiscal cycle. Two point-in-time equilibria arise. In the first such equilibrium, the period-0 monetary policy maker will keep the current demand low and the period-1 monetary policy maker does not generate a big cut in demand. In the second equilibrium, the period-0 monetary policy maker stimulates high demand in anticipation that the period-1 monetary policy maker will implement a cut in demand. The fiscal policy maker when choosing policy in period 0, perceives the both possibilities. The optimal

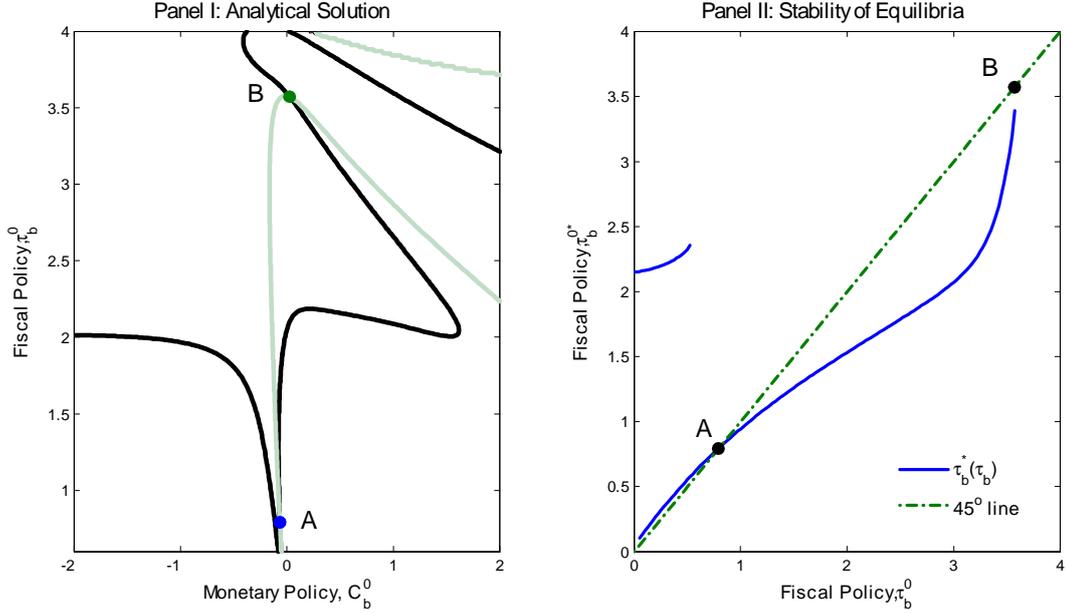


Figure 4: Discretionary Equilibria in the Biannual Fiscal Stabilization Model

fiscal response in the first point-in-time equilibrium response is to raise the tax rate less than in the second equilibrium. The strong response of the tax rate in the second equilibrium generates a ‘zig-zag’ pattern of adjustment: with low two-period-average demand, the increase in the tax rate generates substantial fall in the stock of debt so that the second half year cycle ‘mirrors’ the first half year one, but with the opposite sign. Figure 5 also demonstrates that in equilibrium *A* the paths of all variables ‘approximate’ the optimal paths of the corresponding variables under frequent optimization, and we shall call equilibrium *A* ‘approximating’. We call equilibrium *B* ‘zig-zag’.

Despite the clearly increased inflation volatility, the loss in the approximating equilibrium is slightly lower than it is in the unique equilibrium under frequent optimization, see Appendix ???. This is mainly due to faster stabilization of the economy in this equilibrium. The two-period tax rate increase predominantly determines the two-period speed of debt adjustment $|B_b^0 B_b^1|_A = 0.64 < 0.96 = |B_b|^2$. This welfare gain of faster stabilization is slightly higher than the welfare loss of higher volatility. The loss in the ‘zig-zag’ equilibrium is much higher than in the ‘approximating’ equilibrium. Not only it generates the relatively slow speed of adjustment, as $|B_b^0 B_b^1|_B = 0.79 > |B_b^0 B_b^1|_A$, but it also induces very high volatility of economic valuables.

Finally, equilibrium *A* is IE-stable under all types of learning we consider in this paper. Equilibrium *B* is not IE-stable under both JF- and J-learning. Panel II in Figure 4 illustrates this.

To summarize, the main conclusion from the example of biannual fiscal stabilization is the demonstration of the existence of the approximating equilibrium. Although the other equilibrium exists, the approximating equilibrium delivers the best possible outcome under the infrequent

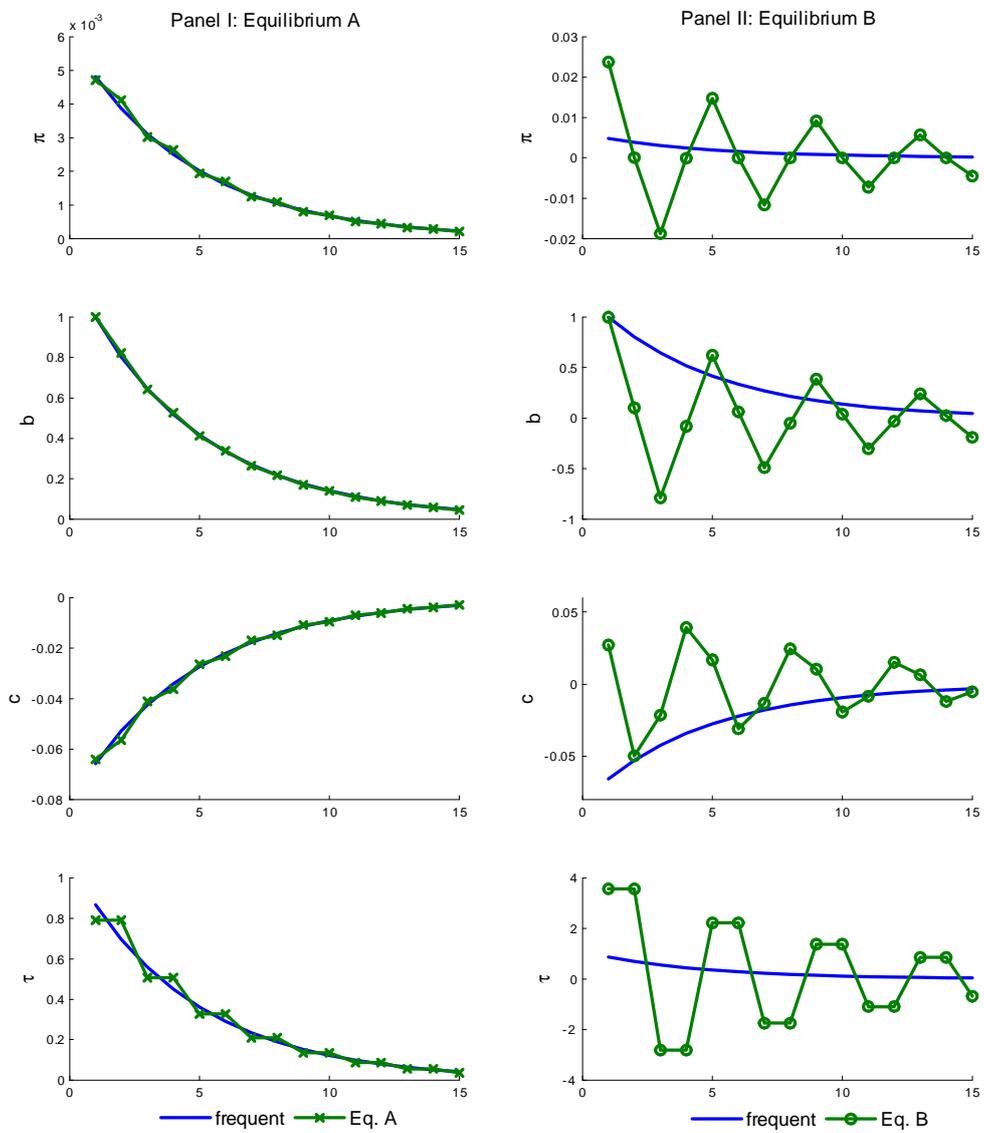


Figure 5: Impulse responses and counterfactual simulations. Fiscal policy optimizes every other period.

discretionary fiscal stabilization, and is also the only equilibrium which is IE-stable under all types of learning we study in this paper. In this equilibrium the monetary policy maker can offset most adverse effects of fiscal infrequency on welfare-related macroeconomic variables. In the next example we argue that we should not take this result for granted once the fiscal cycle becomes longer.

5.3 Annual Fiscal Stabilization

5.3.1 Multiplicity of Discretionary Policy Equilibria

Building on results in the previous section we present the third example of infrequent fiscal stabilization. Arguably, this is the most empirically relevant setup in which the monetary policy maker reoptimizes every quarter, but the fiscal policy maker reoptimizes only at the beginning of every four quarters.

In this model we are unable to present the system of first order conditions as a system of two polynomial equations and use the graphical method of finding solutions. We have to resort to numerical methods and the stability properties to find discretionary equilibria of interest.

Proposition 4 *If the monetary policymaker takes decisions quarterly and the fiscal policymaker optimizes annually then for the base line calibration of the model there are two discretionary equilibria which are IE-stable under the JF-learning.*

Proof. The proof relies on the use of numerical methods. As discussed in Section 5.1 we search for equilibria which are IE-stable under the JF-learning by replicating the steps of the revision process of all agents who are learning. For every, not necessarily optimal τ_b^0 we find all $\lim_{k \rightarrow \infty} \mathbb{T}_{JF}^k(\bar{x}) = x^*$ where \bar{x} is an initial guess of $x = (\pi, c, S)$, as explained in Section ??.¹¹ For every x^* we find $\tau_b^{0*} = \lim_{k \rightarrow \infty} \mathbb{T}_L^k(\bar{\tau}_b^0)$ for the initial guess $\bar{\tau}_b^0$. We, therefore obtain the mapping $\tau_b^{0*} = \tau_b^{0*}(\tau_b^0)$ which is plotted in Panel I in Figure 6 with the solid line. The curve $\tau_b^{0*} = \tau_b^{0*}(\tau_b^0)$ intersects the 45° line in two points, labelled *A* and *B*. By construction, these points are the points of discretionary equilibria which are IE-stable under the JF-learning. With further increase in τ_b^0 no further equilibria were discovered.¹² ■

The dynamic complementarity between the optimal actions of the two policy makers is responsible for the multiplicity of equilibria. An optimal response of monetary policy reinforces the action of fiscal policy: higher levels of taxation have a cost-push effect and so the optimal monetary response is to reduce demand and the tax base. Smaller tax base requires a higher tax rate to ensure the desired speed of debt stabilization. Both policy makers can coordinate on either slow or fast correction of the level of debt towards the target. Figure 7 illustrates these interactions. Consider equilibrium *A*. Suppose the initial debt is higher than in the steady state and the tax rate is kept high for four periods. This implies a steep reduction in debt. The effect of future high tax rates and high marginal cost creates expectations of high future inflation. If monetary policy does not offset the effect of fiscal ‘infrequency’ then debt and consumption adjust in a linear way

¹¹The limit is computed numerically with tolerance $|x_{k+1} - x_k| < 10^{-13}$.

¹²In the area of discontinuity in Panel I the time-consistent representation of the fixed tax rate policy requires infinitely large feedback on debt in the last quarter τ_b^3 . No discretionary equilibria exist there.

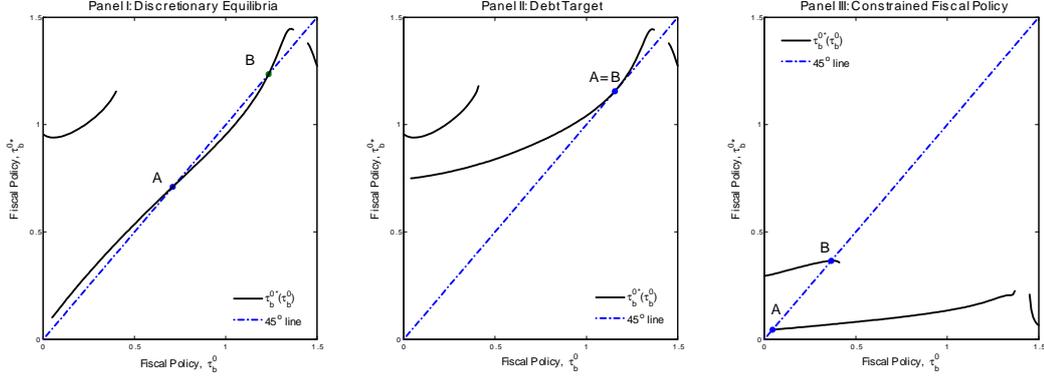


Figure 6: Discretionary Equilibria

between the periods of fiscal optimization. The effect of lower consumption is smaller than the effect of higher tax rate and inflation stays above the frequent optimization solution benchmark. The tax rate remains high for the four periods and, by the end of the fourth period, it is much higher than it would be if optimization happened every period. The tax correction in the fifth period brings inflation down. Figure 7 demonstrates that the ability of the optimal monetary policy to reduce inflation volatility is limited. Indeed, it is clear from the picture that consumption should go down first and then up in the first four periods if the inflation humps in first two periods to be eliminated. Such stabilization results in sub-optimally high volatility of consumption. In what follows we call equilibrium *A* ‘slow approximating’. This equilibrium is IE-stable under all types of learning we consider in this paper, $\lim_{n \rightarrow \infty} \mathbb{T}_j^n(\tau_b^0) = \tau_b^{0A}$, $j \in \{PS, JF, J\}$

Under discretionary equilibrium *B* the tax rate is initially kept above the frequent-optimization benchmark. This generates a much steeper reduction of the level of debt than is observed in the slow approximating equilibrium *A*. The higher tax rate results in a higher level of inflation and lower consumption. We call equilibrium *B* ‘fast approximating’. This equilibrium is not IE-stable under the J-learning but is stable under the private sector learning and the JF-learning. The IE-stability under the JF-learning allowed us to locate it, see Figure 6.

To summarize, in case of the annual fiscal cycle there are at least two discretionary policy equilibria. Only two equilibria are IE-stable under the JF-learning. Their existence is a result of the strong dynamic complementarity between optimal actions under the two policies, monetary and fiscal, given that fiscal policy uses distortionary taxes as the policy instrument. However, their existence is likely to be non-robust to the model specification; this is suggested by Panel I in Figure 6. Indeed, equilibria *A* and *B* are located on the same curve $\tau_b^{0*}(\tau_b^0)$, which may not intersect the 45-degree line at all. In the next section we discuss why this may occur.

5.3.2 Existence of Approximating Policy Equilibrium

We argue that the existence of the approximating equilibrium is not robust to changes in model calibration and to policy scenarios. To communicate the argument we present several examples.

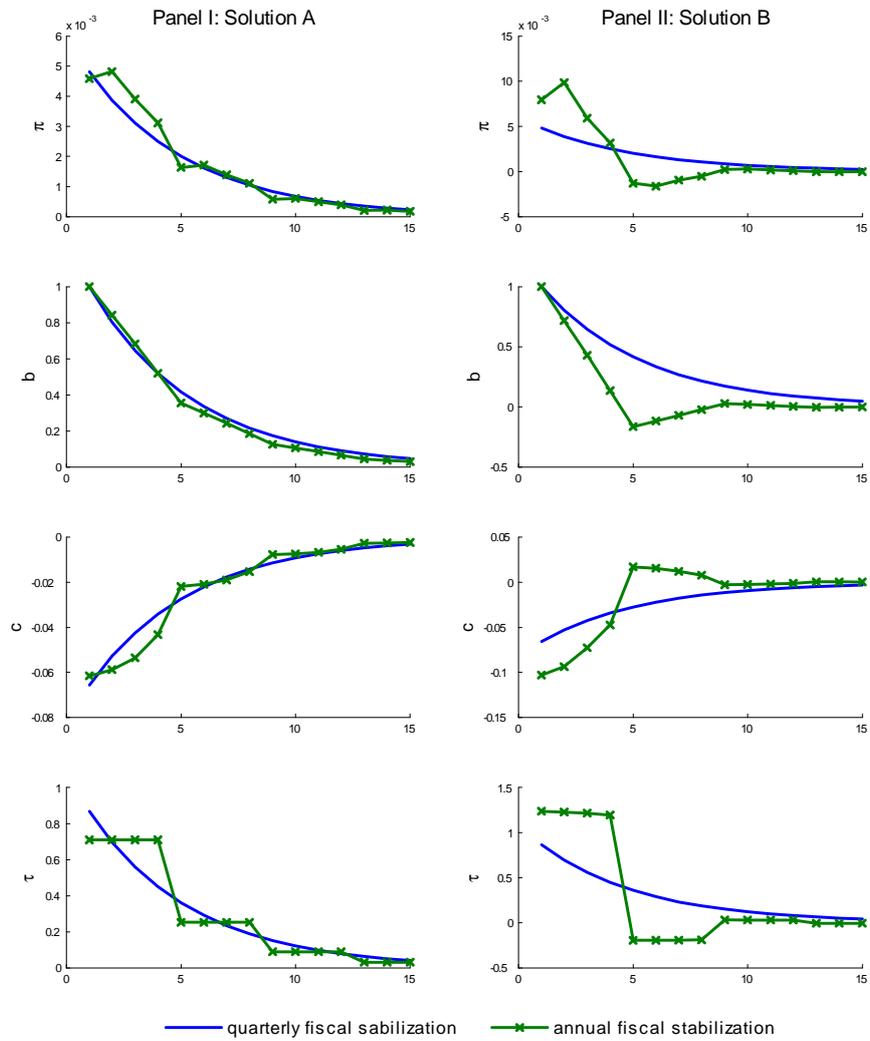


Figure 7: Impulse responses and counterfactual simulations. Fiscal policy optimizes once a year.

Fast Stabilization of Debt Consider a policy scenario in which the fiscal policy maker is assigned an additional target to stabilize debt faster than socially optimal, such as the new European Fiscal Treaty. Suppose the monetary policy maker is benevolent, but the fiscal authority's objective function is modified to include a debt target

$$L^F = \frac{1}{2} \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \lambda c_t^2 + \mu_b b_t^2).$$

If the fiscal policy maker is benevolent, as studied above, then $\mu_b = 0$.

The strength of the dynamic complementarity depends on the calibration of μ . Both approximating equilibria do not exist if $\mu_b > 0$ and is sufficiently high.¹³ In order to understand this result, consider the familiar scenario of high initial debt. Suppose that both policy makers are benevolent and we are in the slow approximating equilibrium A , see the left panel in Figure 7. If we impose a debt target for fiscal policy, i.e. start increasing $\mu_b \geq 0$, the fiscal policy maker will try to speed up the debt stabilization with an increase in the tax rate relative to the benchmark case of $\mu_b = 0$. The cost-push effect will increase inflation more and so the monetary policy maker will choose to engineer a bigger fall in consumption. This, of course, will slow down the speed of debt stabilization and require an even higher tax rate. The process converges: each additional reduction in demand requires a smaller increase in the tax rate. Equilibrium exists, and in this equilibrium the debt is reduced faster than is plotted in the left panel in Figure 7.

This contrasts with the effect of introducing the debt target in equilibrium B . Suppose debt is higher than in the steady state by one unit, policy makers are benevolent and we are in the fast approximating equilibrium B , see the right panel in Figure 7. Note that in this equilibrium debt is stabilized with an observed overshooting after the first year. If we impose a debt target, i.e. start increasing $\mu_b \geq 0$, then raising the tax rate in the first several periods becomes counterproductive. If the tax rate is raised higher than in the $\mu_b = 0$ case, this results in even bigger overshooting of debt, which works towards *destabilizing* the debt. In order to ensure faster debt convergence the tax rate has to rise less and monetary policy has to engineer a smaller fall in consumption. The fiscal policy maker anticipates that demand will not respond much and will lower the tax rate. This process converges: each additional reduction in the size of demand cut requires a smaller reduction in the size of the tax rate increase.

To summarize, with the increasing weight on the debt target equilibria A and B move towards each other so that the dynamics of the economy in equilibria A and B become similar. The dynamic of the economy in response to the higher debt level in the limiting case $A = B$ is plotted in Figure 8. For comparison we also plot the result of frequent stabilization without the debt target.

If the debt target becomes even stronger, then no approximating equilibrium exists. Any proposed increase of the tax rate τ_b^0 results in a strong optimal IE-stable under the JF-learning response of the other agents within the fiscal cycle. To counteract the perceived response requires the bigger initial rise $\tau_b^{0*} = \tau_b^{0*}(\tau_b^0) > \tau_b^0$. We can summarize this outcome in the form of the following proposition.

Proposition 5 *For the base line calibration of the model and with sufficiently high weight on the debt target of fiscal authorities the approximating discretionary equilibrium does not exist.*

¹³For the benchmark calibration of model parameters this threshold value of $\mu = 0.0003$.

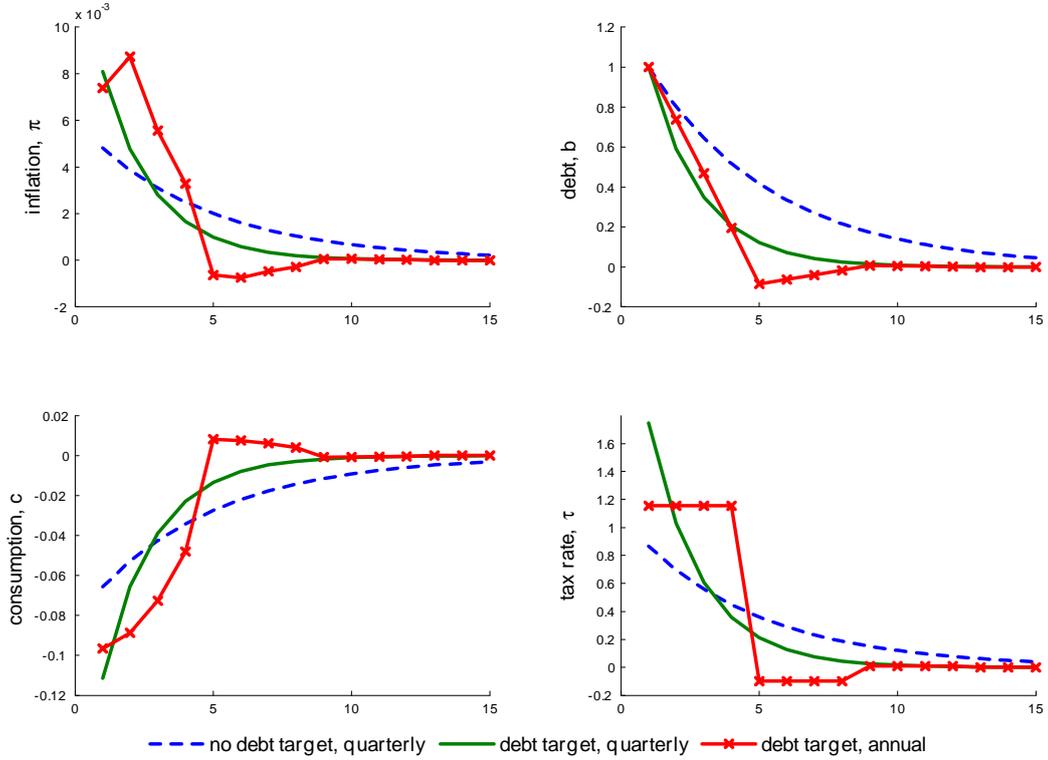


Figure 8: The Limiting Case of the Unique Approximating Equilibrium in the Annual Fiscal Optimization Model

Proof. The proof is numerical. Our iterative approach finds two equilibria under the base line calibration with $\mu_b = 0$. With an increase in parameter μ_b the two equilibria eventually coincide and disappear as $\tau_b^{0*} = \tau_b^{0*}(\tau_b^0)$ does not intersect the 45° degree line. ■

This result does not imply that there is no discretionary equilibrium if equilibria A and B do not exist. Yet another equilibrium might exist. In particular, Panel II in Figure 6 and its similarity with Panel II in Figure 4 suggests that ‘zig-zag’ equilibrium might exist.¹⁴ Strategic complementarity between the actions of subsequent monetary policy makers may lead to a zig-zag adjustment of demand within the fiscal cycle. These adjustments might be ‘fine tuned’ such that the annual average magnitude of them is not large enough to provoke the destabilizing increase in the tax rate. However, such equilibrium is not IE-stable under the JF- and J-learning. Moreover, it is difficult to call such equilibrium ‘approximating’.

The existing algorithms for finding solutions are not suited to obtaining *all* possible equilibria in a complex case with many states. We could only do this for the quarterly and the biannual models. However, Figure 6 makes it clear that the approximating equilibrium will disappear if the

¹⁴We cannot use the ‘continuity’ argument as the reaction function of an agent is described by a rational function, not by a polynomial function.

debt target is sufficiently strong, rather than we suddenly became unable to locate it numerically. We can be reasonably sure that if an additional equilibrium exists in the annual optimization model, this equilibrium will not be IE-stable under the JF- and J-learning, it will also generate a very low level of social welfare because of the high volatility in macroeconomic variables.

Calibration of the Model The existence of the approximating equilibrium is also sensitive to the calibration of the model. Calibrations of the model which result in stronger reactions of monetary and fiscal policies are likely to lead to non-existence of the approximating equilibrium.

Both approximating equilibria do not exist if the Frisch elasticity labour supply ψ is reduced to 2.3, or if the baseline value for the elasticity of intertemporal substitution σ is only slightly increased to 0.35, or if the elasticity of substitution between goods ϵ is reduced to 5.7 which corresponds to an increase of the mark up to 21%.

All these threshold values of parameters are completely plausible and are within the range of estimates which are often obtained in empirical studies of aggregated data, as we argue in Section 4.

High Level of Debt The main case discussed in Section 2 assumes zero level of the steady state debt. This assumption reflects the relatively small proportion of the short-term debt in a typical developed economy, and allows us to present many results in an analytical way. However, high and persistent level of debt is not uncommon.

We can rewrite our model in the more general form, using interest rate as the monetary policy instrument, and retaining the possibility to study implications of higher level of steady state debt. We can demonstrate that the size of the steady state level of debt affects the strength of the dynamic complementarity. The effect of the nominal interest rate and inflation on the process of debt accumulation rises linearly with the steady state level of debt. In response to high inflation the optimal monetary policy will raise interest rate; both the high (real) interest rate and the consequently low tax base increase the rate of debt accumulation, and this effect is stronger with higher steady state level of debt.

The numerical analysis of this scenario produces diagrams that are remarkably similar to the case of the debt target. If the steady state debt to output ratio reaches approximately 0.25 – which corresponds to short-term debt to annual output ratio of 0.07 – then discretionary equilibria A and B coincide. With higher debt to output ratio the approximating equilibrium does not exist.

Frequency of Fiscal Optimization The strength of the dynamic complementarity depends on the frequency of fiscal optimization. The longer the period between the reoptimizations the longer the tax rate remains fixed, and the stronger action of monetary policy is required in order to offset the adverse effect on inflation when the tax is adjusted. The approximating discretionary equilibrium may not exist.

Constraining the Fiscal Policymaker In order to preserve the approximating equilibrium the strength of the complementarity should be reduced. One way to achieve this is to constrain

the fiscal policymaker by imposing penalty μ_τ on the excessive movement of fiscal instrument

$$L^F = \frac{1}{2} \sum_{t=0}^{\infty} \beta^t (\pi_t^2 + \lambda c_t^2 + \mu_\tau \tau_t^2).$$

If μ_τ is not too large then this policy results in the unique IE-stable under the JF-learning equilibrium. However, if μ_τ is sufficiently large then the complementarity between the monetary policymaker and the private sector's actions leads to multiplicity, very similar to the result in Dennis and Kirsanova (2012). Panel III in Figure 6 demonstrates the outcome when $\mu_\tau = 0.1$. There are two stationary equilibria, both of which are IE-stable under all types of learning which we consider. If fiscal policy does not react to debt sufficiently strongly – in this case because it is constrained – then the agents can either coordinate on equilibrium *A* in which the private sector does not expect the monetary policymaker reacts to debt but stabilized inflation and the monetary policymaker validates these expectations, or they can coordinate on equilibrium *B* in which the private sector expects the monetary policymaker accommodates inflationary shocks but ensures fast stabilization of domestic debt and the monetary policymaker validates these expectations.

6 Conclusion

In this paper we study the implications of infrequent discretionary fiscal optimization for the stabilization of the economy, assuming dynamic interactions of monetary and fiscal policymakers where both policymakers are benevolent and the fiscal policymaker uses distortionary taxes to stabilize the economy. We demonstrate the presence of dynamic complementarity between the optimal monetary and fiscal policies. A higher tax rate, which is required to stabilize higher debt, will have a cost-push effect. The optimal monetary policy response to this will generate a reduction in demand and in the tax base, and faster debt accumulation. Anticipating this, the fiscal authorities will wish to raise tax rates further.

If both policies operate with the same frequency, this reinforcement mechanism is weak and does not lead to significant adverse effects. However, with longer fiscal cycle, the effect of this mechanism is greatly amplified. If the length of fiscal cycle is not too long then expectation traps arise. With more periods between fiscal reoptimizations and with stronger reinforcement mechanism an (IE-stable) discretionary equilibrium may not exist.

We demonstrate the latter outcome for many practical scenarios. We argue, therefore, that the fiscal policymaker who reoptimizes only infrequently should be constrained. As one of possible ways to constrain the policymaker we demonstrate that a moderate penalty on variability of the fiscal instrument can be useful in reducing the degree of dynamic complementarity between the actions of the two policymakers.

References

- Anderson, E. W., L. P. Hansen, E. R. McGrattan, and T. J. Sargent (1996). Mechanics of Forming and Estimating Dynamic Linear Economies. In H.M.Amman, D.A.Kendrick, and J.Rust (Eds.), *Handbook of Computational Economics*, pp. 171–252. Elsevier.

- Atkeson, A. and M. Ogaki (1996). Wealth-varying intertemporal elasticities of substitution: Evidence from panel and aggregate data. *Journal of Monetary Economics* 38(3), 507–534.
- Attanasio, O. P. and G. Weber (1993). Consumption Growth, the Interest Rate and Aggregation. *Review of Economic Studies* 60(3), 631–649.
- Attanasio, O. P. and G. Weber (1995). Is Consumption Growth Consistent with Intertemporal Optimization? Evidence from the Consumer Expenditure Survey. *Journal of Political Economy* 103(6), 1121–1157.
- Benigno, P. and M. Woodford (2003). Optimal Monetary and Fiscal Policy: A Linear-Quadratic Approach. In *NBER Macroeconomics Annual 2003*, pp. 271–333.
- Berry, S., J. Levinsohn, and A. Pakes (1995). Automobile Prices in Market Equilibrium. *Econometrica* 63(4), 841–890.
- Blake, A. P. and T. Kirsanova (2011). Inflation-Conservatism and Monetary-Fiscal Policy Interactions. *International Journal of Central Banking* 7(2), 41–83.
- Blake, A. P. and T. Kirsanova (2012). Discretionary Policy and Multiple Equilibria in LQ RE Models. *Review of Economic Studies* 79, 1309–1339.
- Blanchard, O. and C. Kahn (1980). The Solution of Linear Difference Models Under Rational Expectations. *Econometrica* 48, 1305–1311.
- Calvo, G. (1983). Staggered Prices in a Utility-Maximising Framework. *Journal of Monetary Economics* 12, 383–398.
- Campbell, J. Y. and N. G. Mankiw (1989). Consumption, Income and Interest Rates: Reinterpreting the Time Series Evidence. In *NBER Macroeconomics Annual*, pp. 185–246.
- Chang, Y. and S.-B. Kim (2005). On the Aggregate Labor Supply. *Federal Reserve Bank of Richmond Economic Quarterly* 91(1), 21–37.
- Chari, V. V., P. J. Kehoe, and E. R. McGrattan (2000). Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem? *Econometrica* 68(5), 1151–1180.
- Chetty, R., A. Guren, D. Manoli, and A. Weber (2011). Are Micro and Macro Labor Supply Elasticities Consistent? A Review of Evidence on the Intensive and Extensive Margins. *American Economic Review Papers and Proceedings* 101(2-3), 471–475.
- Clarida, R. H., J. Gali, and M. Gertler (1999). The Science of Monetary Policy: A New Keynesian Perspective. *Journal of Economic Literature* 37, 1661–1707.
- Cooper, R. and A. John (1988). Coordination Failures in Keynesian Models. *Quarterly Journal of Economics* 103(3), 441–463.

- Dennis, R. and T. Kirsanova (2012). Expectations Traps and Coordination Failures with Discretionary Policymaking. Federal Reserve Bank of San Francisco Working paper 10-2, available at SSRN: <http://ssrn.com/abstract=1664924>.
- Dixit, A. and L. Lambertini (2003). Interactions of Commitment and Discretion in Monetary and Fiscal Policies. *American Economic Review* 93, 1522–1542.
- Dixit, A. and J. Stiglitz (1977). Monopolistic Competition and Optimum product Diversity. *American Economic Review* 67, 297–308.
- Fragetta, M. and T. Kirsanova (2010). Strategic Monetary and Fiscal Policy Interactions: An Empirical Investigation. *European Economic Review* 54(7), 855–879.
- Gourio, F. and P.-A. Noulal (2009). The Marginal Worker and The Aggregate Elasticity of Labor Supply. Boston University, mimeo.
- Hall, R. E. (1988). Intertemporal Substitution in Consumption. *Journal of Political Economy* 96(2), 339–357.
- King, M. (1997). Changes in UK Monetary Policy: Rules and Discretion in Practice. *Journal of Monetary Economics* 39, 81–97.
- King, R. and A. Wolman (2004). Monetary Discretion, Pricing Complementarity, and Dynamic Multiple Equilibria. *Quarterly Journal of Economics* 119(4), 1513–1553.
- Lambertini, L. (2006). Monetary-Fiscal Interactions with Conservative Central Bank. *Scottish Journal of Political Economy* 53, 90–128.
- Leeper, E. M. (1991). Equilibria Under ‘Active’ and ‘Passive’ Monetary and Fiscal Policies. *Journal of Monetary Economics* 27, 129–147.
- Nevo, A. (2001). Measuring Market Power in the Ready-to-Eat Cereal Industry. *Econometrica* 69(2), 307–342.
- Peterman, W. B. (2012). Reconciling Micro and Macro Estimates of the Frisch Labor Supply Elasticity. The Federal Reserve Board Finance and Economics Discussion Paper 2012-75.
- Rotemberg, J. J. and M. Woodford (1997). An Optimization-based Econometric Framework for the Evaluation of Monetary Policy. In *NBER Macroeconomics Annual*, pp. 297–344.
- Svensson, L. (2010). Inflation targeting. In *Handbook of Monetary Economics*, Volume 3. Elsevier.
- Woodford, M. (2003). *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton, NJ.: Princeton University Press.