Generalised t-V model in one dimension

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1 Introduction

The generalised t-V model [2] of fermions distributed on a chain of L sites:

\[ \hat{H} = -t \sum_{i=1}^{L} \left( \hat{c}_{i}^{\dagger} \hat{c}_{i+1} + \text{h.c.} \right) + \sum_{i=1}^{L} \sum_{m=1}^{P} U_m \hat{n}_i \hat{n}_{i+m} \]

Kinetic energy, i.e. the hopping term, is much smaller than the potential: \( t \ll U_m \).

Potential energy makes sure the particles are never closer than \( p \) sites; otherwise energy cost is \( U_m \). Example for \( p = 2 \):

\[ E_{\text{pot}} = 0 \]
\[ E_{\text{pot}} = U_2 \]
\[ E_{\text{pot}} = U_3 \]
\[ U_m > U_{m+1} \]

Depending on fermion density \( Q = N/L \) we have different phases:

- Critical density \( Q_c = \frac{A}{p+1} q = 1, \ldots, p \):
  - Mott insulator
  - Simple unperturbed ground state

- Away from critical density:
  - Luttinger liquid
  - Highly degenerate ground state of \( \hat{H}_0 \).

Using SCM for near-critical densities, the Hamiltonian is small enough to calculate approximate solution to a very high precision.

Example: \( p = 3, Q = 1/4 \), step 2nd in SCM:

\[ \beta = \left( \begin{array}{cccc}
-\sqrt{2} t & U_2 & -\sqrt{3} t & -2t \\
-\sqrt{2} t & U_2 & -\sqrt{3} t & 0 \\
-\sqrt{3} t & U_3 & -2t & 0 \\
-2t & 0 & 0 & 2U_3
\end{array} \right) \]

This simple 5x5 Hamiltonian gives the ground state energy of the system up to order \((t/U_3)^5\).

2 The Objective

Spacing: \( p > 1 \)
- Non-integrable
- Solved only in the first order perturbation [1,2].
- Using strong coupling expansion, we will try to approximate the analytical solutions to a very high order.

Spacing: \( p = 1 \)
- Integrable
- Solved by using Bethe ansatz approach [3].

The method starts similarly to the perturbation theory. Assume:

\[ \hat{H} = \hat{H}_0 + \lambda \hat{V} \]

where \( \lambda \ll 1 \), so we can treat \( \hat{V} \) as perturbation. Eigenstates \( |\alpha_\lambda \rangle \) of \( \hat{H}_0 \) are known. Now, we want to create a new truncated basis of \( \hat{H} \) using \( |\alpha_\lambda \rangle \):

- Include in your basis the desired subspace of unperturbed states that you want to approximate.
- They are of step 0 in SCM.
- Example: using state |1111⟩.

III Separate

- Act with \( \hat{V} \) on states from previous SCE step (“n”), creating set of states \( S \).
- States in \( S \) are linear combinations of the unperturbed Hamiltonian eigenstates.
- Example: |1111⟩ + |1111⟩ + |1111⟩ + |1111⟩.
- Separate every state in \( S \) according to their unperturbed energy.
- Example: |1111⟩ + |1111⟩ + |1111⟩ + |1111⟩.

IV Orthonormalise

- Orthoromalise the states in set \( S \), so they would be orthonormal to each other and the basis.
- Include them in the basis.
- They are of step 0 + 1 in SCE.
- Example: The basis is now: |1111⟩, |1111⟩, |1111⟩, |1111⟩ + |1111⟩ + |1111⟩.
- Repeat from II until you achieve desired SCE step.

All the information about the desired states (e.g. ground states) will be encoded in the truncated \( \hat{H} \) in the new basis [4,5].

3 Strong Coupling Expansion

I First states

II Act with \( \hat{V} \)

III Separate

IV Orthonormalise

V Repeat

With every SCE step we are increasing the accuracy by two orders in \( \lambda \).

4 Using SCE on the model

Leading order of \( \delta/\mu \):

\[ \left( \frac{t}{U} \right)^{12} \]

Obtained accuracy was \( O(\epsilon^6) + O(\epsilon^8) \).

Summary:
- High precision results for both integrable and non-integrable models in Mott insulating phases.
- Results are fully consistent with other works [1,3].

Further work:
- More observables
- Time dependence
- Temperature dependence

5 Results

Ground state energy:

\[ E_0 = -\frac{1}{2} \frac{\delta}{\mu} t^2 - \frac{1}{4} \left( \frac{\delta}{\mu} \right)^2 t^3 - \frac{1}{2} \frac{\delta}{\mu} t^2 \left( \frac{\delta}{\mu} \right)^2 t^4 + \left( \frac{16}{\mu^2} \right) \left( \frac{\delta}{\mu} \right)^2 \left( \frac{\delta}{\mu} \right)^4 \]

Current density:

\[ \frac{\delta}{\mu} = \frac{\delta}{\mu} t^2 + L \left( \frac{\delta}{\mu} \right) t^4 + L \left( \frac{\delta}{\mu} \right)^2 t^6 \]

Density-density correlations:

:\( \langle \hat{n}_i \hat{n}_{i+s} \rangle \) were also obtained. Leading order is cyclic in \( \delta \), which is consistent with expectations.

6 Conclusions & Outlook

- Densities not exactly equal to \( Q_C \)
- Phase transition investigation
- More observables
- Temperature dependence

References