Inelastic electron-electron scattering in a quantum dot broadens the single-particle excitation levels by an amount $\hbar \Gamma_{\text{in}}$. This broadening vanishes at low excitation energies $e$ and remains less than the mean level spacing $\Delta$ as long as $e$ is below the Thouless energy. Early Coulomb-blockade experiments by Sivan et al. agreed with this theoretical prediction, but recent experiments by Folk et al. were interpreted as inconsistent with it.

Inelastic scattering can be detected by the broadening of the single-particle density of states, as was done by Sivan et al. (Ref. 3) Folk et al. instead, used the temperature dependence of the height of the Coulomb-blockade peaks in the conductance. For fully elastic and chaotic scattering the average height is increased by a temperature-independent factor of 4/3 upon application of a magnetic field. Folk et al. measured a suppression of this enhancement factor when the thermal energy $kT$ became larger than $\Delta$. They concluded from this strong temperature dependence that the dephasing rate in quantum dots is larger than $\Delta/h$ at excitation energies well below the Thouless energy, in apparent contradiction with the theoretical expectation. However, in the absence of a quantitative prediction for the temperature dependence of the Coulomb-blockade peak height, it is difficult to decide whether the observed temperature dependence is actually stronger than expected.

What we will do here is use the semiclassical theory of the Coulomb blockade to obtain the temperature dependence in the regime $\Gamma_{\text{el}} \ll \Gamma_{\text{in}}$, with $\Gamma_{\text{el}}$ the mean (elastic) tunnel rate into the quantum dot. We call this the regime of strong inelastic scattering, where ‘strong’ means strong enough to thermalize the distribution of the electrons among the levels in the quantum dot. Both $\Gamma_{\text{el}}$ and $\Gamma_{\text{in}}$ should be less than $kT$, so that we are allowed to use rate equations based on sequential tunneling. The condition for the Coulomb blockade is $\Gamma_{\text{el}} \approx \Delta/h$ and $kT \approx e^2/C$, with $C$ being the capacitance of the quantum dot. We find that the experimental temperature dependence is actually much weaker than predicted by the theory for strong inelastic scattering. Therefore, $\Gamma_{\text{in}} \approx \Gamma_{\text{el}} \approx \Delta/h$ and there is no disagreement between the experimental data of Ref. 4 and the theoretical expectation of a low-energy suppression of inelastic electron-electron scattering in quantum dots.9

The starting point of our analysis is a pair of expressions from Ref. 8 for the $N$th conductance peak in the two cases of purely elastic scattering ($G_{\text{el}}$) and strong inelastic scattering ($G_{\text{in}}$):

$$G_{\text{el}} = \frac{e^2}{kT} P_{\text{eq}}(N, \Gamma_{\text{el}}) \left( \frac{\Gamma_{\text{el}}^{1/2}}{\Gamma_{\text{el}}^{1/2} + \Gamma_{\text{in}}^{1/2}} \right)^N,$$ (1)

$$G_{\text{in}} = \frac{e^2}{kT} P_{\text{eq}}(N, \Gamma_{\text{in}}^{1/2}) \left( \frac{\Gamma_{\text{el}}^{1/2}(\Gamma_{\text{in}}^{1/2})^N}{(\Gamma_{\text{el}}^{1/2} + \Gamma_{\text{in}}^{1/2})^N} \right).$$ (2)

The spectral average of the elastic tunnel rate $\Gamma_{\text{el}}^{1/2}$ into the left or right reservoir is defined by

$$\langle \Gamma_{\text{el}}^{1/2} \rangle_N = \sum_p \Gamma_{\text{el}}^{1/2} \left( 1 - F_{\text{eq}}(E_p, N) \right) f(E_p - \mu).$$ (3)

The equilibrium distributions $P_{\text{eq}}(N)$ and $F_{\text{eq}}(E_p, N)$ give, respectively, the $a$ priori probability to find $N$ electrons in the quantum dot and the conditional probability to find level $p$ occupied by one of the $N$ electrons. (These functions are obtained from the Gibbs distribution in the canonical ensemble.) The function $f(E_p - \mu)$ is the Fermi-Dirac distribution, with $\mu$ an externally tunable parameter that depends linearly on the gate voltage.

If $\Gamma_{\text{in}} \ll \Gamma_{\text{el}}$ one may neglect inelastic scattering and use Eq. (1), while if $\Gamma_{\text{el}} \ll \Gamma_{\text{in}}$ one should use Eq. (2). The key difference between the two equations is that for $G_{\text{el}}$ the fraction $\Gamma_{\text{el}}^{1/2} / (\Gamma_{\text{el}}^{1/2} + \Gamma_{\text{in}}^{1/2})$ as a whole is averaged over the spectrum, while for $G_{\text{in}}$ the numerator and denominator are averaged separately. Since the spectral average extends over about $kT/\Delta$ levels, the difference between $G_{\text{el}}$ and $G_{\text{in}}$ vanishes if $kT$ becomes less than $\Delta$.

In a chaotic quantum dot, the tunnel rates $\Gamma_{\text{el}}$ and $\Gamma_{\text{in}}$ fluctuate independently according to the Porter-Thomas distribution $P(\Gamma) \propto \Gamma^{\beta/2-1} \exp(-\beta\Gamma^2/2)$. (We assume tunneling through two equivalent single-channel point contacts, with energy-independent mean tunnel rate $\Gamma_{\text{el}}$.) The index $\beta=1/2$ in the presence (absence) of a time-reversal-symmetry breaking magnetic field. The mean height $G_{\text{el}}^{\text{max}}$ of the Coulomb-blockade peak for elastic scattering increases upon breaking time-reversal symmetry, by a temperature-
independent factor of 4/3.\textsuperscript{5,6} Inelastic scattering introduces a temperature dependence, which we can study using Eq. (2).

Qualitatively, the effect of inelastic scattering on the 4/3-enhancement factor can be understood as follows. The spectral average \( \langle \cdots \rangle_{P} \), defined precisely in Eq. (3), can be approximated by an average over \( kT/\Delta \) levels around the Fermi energy in the quantum dot containing \( N \) electrons. If \( kT \gg \Delta \) the spectral average becomes equivalent to an ensemble average. The ensemble averages of \( \Gamma_{\ell}^{l} \) and \( \Gamma_{r}^{r} \) are both equal to the \( \beta \)-independent value \( \Gamma_{el} \), so the peak height (2) for strong inelastic scattering simplifies to \( G_{in} = \frac{1}{2} \Gamma_{el} (e^{2}/kT) P_{eq}(N) \) — independent of whether time-reversal symmetry is broken or not. This explains why the enhancement factor drops from 4/3 to 1 as \( kT \) becomes larger than \( \Delta \) in the case of strong inelastic scattering.

For a quantitative comparison, we have plotted in Fig. 1 the temperature dependence of the parameter

\[
\alpha = 1 - \frac{G_{in}^{\max}(\beta = 1)}{G_{in}^{\max}(\beta = 2)},
\]

(4)

which drops from 1/4 to 0 as \( kT \) becomes larger than \( \Delta \). The solid curve is for equally-spaced spin-degenerate levels \( (E_{2p} = E_{2p-1} = p\Delta, \Gamma_{l}^{l} = \Gamma_{r}^{r} = \Gamma_{2p} = \Gamma_{2p-1}) \). Because the spin degeneracy might be lifted spontaneously,\textsuperscript{10} we also show for comparison the case of equally spaced nondegenerate levels \( (E_{p} = p\Delta/2, \text{ all } \Gamma_{p} \text{'s independent}) \). In either case \( \Delta \) is defined as the mean level spacing of a single spin degree of freedom.

We see that the temperature dependence is stronger for non-degenerate levels. An even stronger temperature dependence (not shown) is found if, instead of equally spaced levels, we use a Wigner-Dyson distribution. The data points are the experimental results of Folk \textit{et al.}\textsuperscript{4} for GaAs quantum dots of four different areas. The values of \( \Delta \) used are those given in Ref. 4, estimated from the area \( A \) and the two-dimensional density of states (\( \Delta = 2\pi\hbar^{2}/mA \), with \( m \) the effective mass of the electrons). There is therefore no adjustable parameter in the comparison between theory and experiment.

It is clear from Fig. 1 that the experimental temperature dependence is much weaker than the theoretical prediction, regardless of whether we include spin degeneracy or not. We have found that the theory would fit the data within the error bars if we would rescale \( kT/\Delta \) by a factor of 3 (with spin degeneracy) or a factor of 5 (without spin degeneracy). Such a large factor is beyond the experimental uncertainty in level spacing or temperature. We conclude that the inelastic scattering rate is well below \( \Gamma_{el} \) and \( \Delta/\hbar \) for a range of energies within \( kT \). One possible explanation of the deviation of our theoretical curves from the experimental data would be that only the high-lying levels have equilibrated, while the low-lying levels have not. Such an explanation would be consistent with the scenario put forward in Ref. 2, according to which the discreteness of the spectrum prevents the low-lying levels to equilibrate on an arbitrarily long time scale.

We conclude with two suggestions for future research on this topic. From the theoretical side, it would be useful to generalize Ref. 8 to an arbitrary ratio of \( \Gamma_{el} \) and \( \Gamma_{in} \) [going beyond the two limits of large and small \( \Gamma_{el}/\Gamma_{in} \) given in Eqs. (1) and (2)]. From the experimental side, it would be of interest to compare data for the temperature dependence of \( \alpha \) for different values of \( \Gamma_{el} \), that is to say, for different heights of the tunnel barriers separating the quantum dot from the electron reservoirs. We would expect the data points in Fig. 1 to approach the theoretical curves as the tunnel barriers are made higher and higher, giving more precise information on the rate of inelastic scattering.

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\begin{thebibliography}{9}
\bibitem{7} Folk \textit{et al.} (Ref. 4) interpret their data in terms of the dephasing rate \( \Gamma_{\phi} \), but we would argue that their experiment is more sensitive to the inelastic scattering rate \( \Gamma_{in} \) than to \( \Gamma_{\phi} \). It is inelastic scattering that destroys the \( \frac{1}{3} \)-enhancement of the conductance peaks — not dephasing. Indeed, the theories of Refs. 4–6 use a model of “sequential tunneling,” in which there is no phase coherence at all between the electron entering and leaving the quantum dot. It is irrelevant for the \( \frac{1}{3} \)-enhancement whether
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such phase coherence exists or not. One can conclude from the experimental data that $\Gamma_{\alpha, \beta} \approx \Gamma_\phi, \Gamma_\phi \approx \Delta/\hbar$, but the relative magnitude of $\Gamma_\phi$ and $\Gamma_\phi$ remains undetermined.


9 A similar analysis of the experiment has been proposed independently by P. W. Brouwer (private communication).