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The Gains from the Division of Labour and Comparative Advantage

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The gains from the division of labour and comparative advantage

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Abstract

This paper develops a model of international trade based on comparative advantage and the division of labour. Comparative advantage in intermediate goods determines the extent of the division of labour, while the division of labour and comparative advantage in final goods lead to gains from trade. Labour is used to produce traded intermediate inputs which are used in the production of traded final goods; therefore trade is both inter- and intra-industry in nature. Large countries export a smaller share of final goods and a larger share of intermediate goods than small countries. These predictions find supportive evidence in the data.

JEL Classification: F11.
Keywords: Division of labour; Comparative advantage; gains from trade; intermediate goods trade.

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1 Introduction

The third paragraph of the first chapter of Adam Smith’s *The Wealth of Nations* (Smith, 1776) contains the famous passage in which he describes the impact of the division of labour on productivity in a pin factory. To paraphrase Smith, one worker, working on his own, could produce at most 20 pins in a day. Ten workers, dividing up the tasks of producing pins, could produce 48,000 pins in a day. Hence, the gain to this group of workers from the division of labour in this example is 24,000%. One implication of this is that international trade, by enabling greater levels of specialisation, should result in productivity gains.

This paper develops a model of international trade in which the gains from the division of labour play a central role. As in Adam Smith’s example, the more the production process can be divided into discrete stages, the larger will be the final output. Ricardo’s (1817) comparative advantage also plays an important role, by determining the patterns of specialisation across countries and pinning down the number of stages in the production process. When international trade is allowed, large countries gain more from comparative advantage than from the division of labour, while the opposite is true for small countries. In the model, countries specialise in different subsets of intermediate goods, then trade both intermediate and final goods. Countries will engage in intra-industry trade in intermediate goods, as well as inter-industry trade in final goods. Hence the model also develops the foundations for a model of intra-industry trade based on perfect competition; see also Davis (1995) for a very different formulation.

A key testable prediction of the model is that, provided the gains from the division of labour are not too large, country size is positively associated with the share of consumption goods in its exports, and is negatively associated with the share of intermediate goods in its exports. Using data from the UN Comtrade database, we find some evidence which supports these predictions of the model. This work is broadly related to the empirical literature on trade in intermediate goods and services. For instance, Miroudot et al (2009) and Sturgeon and Memedovic (2010) show that intermediate inputs represent over half of total goods trade, but that this fraction has actually decreased since the 1960s.
There has been a recent resurgence of interest in models of international trade based on the division of labour. A large portion of this literature revolves around models based on external scale economies, for instance Grossman and Rossi-Hansberg (2010) and Ethier and Ruffin (2009). Choi and Yu (2003) survey the earlier literature on international trade under external scale economies, while Wong (2001) offers an alternative treatment. More closely related to the present paper is Chaney and Ossa (2013) who extend the new trade model of Krugman (1979) to allow for multiple production stages.

Also closely related to the present paper are Ethier (1979, 1982). The nature of the division of labour in this paper is similar to that in Ethier (1979, 1982). The main difference is that here, we microfound the division of labour as in Ethier (1982), but the production of intermediate inputs is perfectly competitive. Indeed, where Ethier (1982) has two sources of scale economies (internal to the firm, and due to the division of labour) and one source of comparative advantage (factor endowment differences across countries), in the present paper, there are two sources of comparative advantage (between intermediate goods, and between final goods), and one source of scale economies (the division of labour).

The next section presents the main features of the model. Section 3 outlines the autarkic equilibrium, while Section 4 considers the implications of international trade. Section 5 discusses the trade patterns that arise in the model and provides some supportive empirical evidence. Section 6 concludes.

2 The model

There are two countries, $j = H, F$ for Home and Foreign. Labour is the only factor of production, and the two countries have labour endowments $L_j$. All markets are perfectly competitive. There are two final consumption goods, 1 and 2. Consumer utility is identical across countries and takes a CES form:

$$U = C_1^\theta + C_2^\theta \quad 0 < \theta < 1$$

(1)

Where $C$ denotes consumption of a good. Final goods are produced with intermediate inputs, and assembly of final goods is assumed to be costless. Suppose there is a large (infinite) number of possible intermediate inputs. Each country produces a number of intermediate inputs $n_j$, which is assumed to be small relative to the number of possible intermediate
inputs. As a result, the intermediate inputs produced in one country are different from those produced in the other country. All intermediate inputs are used in fixed proportions. Hence let the production functions of the final goods in the two countries be:

\[
Q_{1H} = \gamma (n_H + n_F)^{\beta + 1}x \\
Q_{2H} = (n_H + n_F)^{\beta + 1}x \\
Q_{1F} = (n_H + n_F)^{\beta + 1}x \\
Q_{2F} = \gamma (n_H + n_F)^{\beta + 1}x
\] (2) (3) (4) (5)

Output of each final good depends on the number of Home and Foreign produced inputs, \(n_H\) and \(n_F\), and the quantity of each intermediate input, \(x\), which is assumed to be the same across final goods. \(\gamma > 1\) indicates that Home has a comparative technological advantage in final good 1, while Foreign has a comparative technological advantage in final good 2. It will be shown below that in free trade Home will specialise in good 1 and Foreign in good 2. \(\beta > 0\) measures the payoff from the division of labour. The larger the number of intermediate inputs \(n\), the greater the division of labour, and the larger the output of the final good, analogously to Smith’s pin factory example.

Since all inputs are used in fixed proportions in the production of the final goods, output of each intermediate good must also be the same. Each country has a comparative technological advantage in a subset \(rL_j\) of intermediate goods, \(k_j = 1, ..., rL_j\). Suppose that these \(rL_j\) goods are different between the two countries, and let the production technology of intermediate inputs be:

\[
q_{k_j} = l_{k_j} \\
q_i = q_{k_j} = \alpha l_i = l_{k_j} \quad i \notin k_j \quad \alpha < 1
\] (6) (7)

That \(\alpha < 1\) signifies that the country has a comparative disadvantage in these goods. We make the following assumption throughout the paper:

**Assumption 1:** \(\left(\frac{rL_j + m}{rL_j}\right)^\beta \left[\frac{\alpha(rL_j + m)}{\alpha rL_j + m}\right] < 1\), for any \(m > 0\).

Assumption 1 holds when \(\alpha\) and \(\beta\) are sufficiently small. Appendix A shows that if Assumption 1 holds, we have:

**Proposition 1:** The optimal number of intermediate inputs is \(n_j = rL_j\).
Hence, all the intermediate inputs produced are the ones in which a country has a technological advantage in. As a result, the number of intermediate inputs (hence the extent of the division of labour) depends on the size of the market, as in Adam Smith’s example.

3 Autarky

Consider first the case where the two countries do not trade with each other. Here we analyse the Home country; the solution for the Foreign country is analogous. In this case, Foreign-produced intermediates are not available for use in the production of Home-produced final goods, and all Home-produced intermediates are used at Home, equally split between the two final goods, so the production functions in Home are (making use of $n_j = rL_j$):

\[ Q_{1H} = \gamma n_H^{\beta + 1} \left( \frac{q_H}{2} \right) = \gamma (r L_H)^{\beta + 1} \left( \frac{q_H}{2} \right) \quad (8) \]
\[ Q_{2H} = n_H^{\beta + 1} \left( \frac{q_H}{2} \right) = (r L_H)^{\beta + 1} \left( \frac{q_H}{2} \right) \quad (9) \]

Since all intermediate inputs produced have the same technology, and since output of each intermediate good is the same, the labour used in each intermediate input is also the same. Hence $q_{k_H} = l_{k_H} = L_H/n_H = 1/r$. The size of the labour force influences only the number of intermediate goods, not the output of each intermediate good. This result of the model is similar to Krugman (1980), in which changing labour endowments results in a different number of varieties produced, but not the scale of production of each variety.\(^2\)

Substituting into the production functions (8) and (9) gives:

\[ Q_{1H} = (r L_H)^{\beta} \gamma \left( \frac{L_H}{2} \right) \quad Q_{2H} = (r L_H)^{\beta} \left( \frac{L_H}{2} \right) \quad (10) \]

Since there is no international trade, Home consumers can only consume Home-produced output. Therefore, Home’s consumer’s utility under autarky is:

\[ U^H_A = (r L_H)^{\beta} 2^{-\theta} [\gamma^\theta + 1] \quad (11) \]

Utility is increasing in the size of the Home labour force $L_H$, the parameter $r$ indicating the number of comparative advantage sectors, the gain from the division of labour $\beta$, the higher

\(^2\) Indeed, another possible way of setting up the model would be to specify the production technology of intermediate inputs as in Krugman (1980); as noted in the Introduction, this would yield the model in Ethier (1982). The present formulation highlights one additional result of the model, which is that it enables us to generate intra-industry trade without recourse to imperfect competition.
the degree of substitutability between final goods in consumption $\theta$, and the larger the technology parameter $\gamma$.

4 International trade

When international trade is allowed, both intermediate inputs and final goods can be freely traded across countries. Proposition 2 (proved in Appendix B) shows that both countries are always specialised in their comparative advantage final goods in free trade:

**Proposition 2:** In free trade, Home is specialised in final good 1 and Foreign is specialised in final good 2.

Making use of the results in the previous sections and solving for the production functions (2) and (5) gives:

$$Q_{1H} = Q_{2F} = \left( \frac{\gamma}{2\tau} \right) \left( rL_H + rL_F \right)^{\beta + 1}$$

(12)

Production of each final good uses intermediate goods produced in both countries, and consumers wish to consume both final goods. If trade in intermediate goods is defined to be intra-industry trade, while trade in final goods is inter-industry, then the model predicts both inter- and intra-industry trade.

Since preferences are homothetic and identical across countries, each country will consume a fraction of the total output of each final good which is proportional to its relative size. Hence, the Home consumer’s utility under free trade is:

$$U_{HT} = 2^{1-\theta} \gamma^\theta (rL_H + rL_F)^{\beta \theta}$$

(13)

Define the gains from trade as the ratio between free trade (13) and autarkic utility (11). The gains from trade are:

$$G_H = \frac{U_{HT}}{U_A} = \left[ \frac{2^\theta \gamma^\theta + 1}{\gamma^\theta + 1} \right] \left( \frac{L_H + L_F}{L_H} \right)^{\beta \theta} > 1$$

(14)

Hence there are gains from trade. The following comparative statics results can be shown:

$$\frac{dG_H}{dL_H} < 0 \quad \frac{dG_H}{dL_F} > 0 \quad \frac{dG_H}{d\beta} > 0$$

(15)

$$\frac{dG_H}{d\gamma} > 0 \quad \frac{dG_H}{d\theta} > 0$$

(16)
As might be expected, the gains from trade increase the smaller is the country, or the larger is the trading partner. The larger the gains from the division of labour $\beta$ or the larger the comparative technological advantage in the final good $\gamma$, the larger the gains from trade. Similarly, the larger the degree of substitutability between final goods $\theta$, the larger the gains from trade, since the easier it becomes for consumers to substitute across goods depending on price.

The comparative advantage in intermediate goods parameter $\alpha$ and the number of comparative advantage intermediate sectors $r$ do not play a role in the gains from trade. This is because, from Assumption 1 and Proposition 1, countries only produce the intermediate goods in which they have a comparative advantage, and are equally productive in these goods. Instead, the role of comparative advantage in intermediate goods is to determine the number of intermediate goods produced in each country; if there is no comparative advantage in intermediate goods, the number of intermediate goods produced would be indeterminate. Thus the model also presents a new role for comparative advantage in models of international trade.

It is possible to decompose the total gains from trade into the component derived from comparative advantage in final goods production, the component derived from the division of labour, and the component derived from the interaction between comparative advantage and the division of labour. To obtain the gains from trade based on comparative advantage alone, set $\beta = 1$ in the gains from trade equation (14) to obtain:

$$G_{CA} = \frac{2\gamma}{\gamma + 1} \left(\frac{L_H + L_F}{L_H}\right)^\theta$$

(17)

Similarly, set $\gamma = 1$ in equation (14) to obtain the gains from trade based on the division of labour alone:

$$G_{DL} = \left(\frac{L_H + L_F}{L_H}\right)^{\beta\theta}$$

(18)

Then, we get:

$$G_H = G_{CA} \times G_{DL} \times \left(\frac{L_H}{L_H + L_F}\right)^\theta$$

(19)

Where the last term is the interaction between the gains from comparative advantage and the gains from the division of labour; this term is positive but less than 1, suggesting that the total gains from trade are less than the combination of the gains from comparative advantage and
the gains from the division of labour, or that comparative advantage and the division of labour are in some sense substitutes for one another.

It can be shown that:

\[ G_{DL} > G_{CA} \quad \text{if} \quad \left( \frac{L_H + L_F}{L_H} \right)^{(\beta - 1)\theta} > \frac{2\gamma^\theta}{\gamma^\theta + 1} \]  

(20)

Hence we have:

**Proposition 3:** The larger is a country relative to its trading partner, the greater the importance of comparative advantage relative to the division of labour as a determinant of the gains from trade.

Intuitively, as a country becomes larger relative to its trading partner, it gains less from the increased division of labour resulting from international trade, and also gains less from the fact that its trading partner has a comparative advantage in a different final good. What Proposition 3 shows is that the gains from the division of labour decrease more rapidly than do the gains from comparative advantage. Hence, the primary source of the gains from trade for large countries is comparative advantage, while for small countries it is the division of labour.

5 Trade in intermediate and final goods

As noted in Section 2 above, the two countries are symmetric in every way except one: their size. Similarly, the two final goods and all intermediate goods are also symmetric in every way, and assembly of final goods from intermediate goods is costless. As a result, the total value of intermediate goods output is equal to the total value of final goods output, and the two final goods are produced in equal quantities and have equal prices. However, with homothetic preferences, the larger country will consume a larger fraction of each final good, in direct proportion to the country’s size. As a result, if trade is balanced, the share of the final good in a country’s exports will be negatively related to the country’s size, while the share of intermediate goods will be positively related to the country’s size.

To make things more concrete, the value of Home’s exports of the final good is:
\[
\frac{L_H}{L_H + L_F} P_1 Q_1 = \frac{L_F}{L_H + L_F} P_1 \left( \frac{Y}{2r} \right) \left( rL_H + rL_F \right)^{\beta + 1} = \frac{1}{2} L_F P_1 \gamma (rL_H + rL_F)^\beta
\]  

(21)

Recall that half of each Home-produced intermediate good is used in the production of final good 2, which is produced in Foreign. The value of Home’s exports of intermediate goods is:

\[
\frac{1}{2} p_H q_H n_H = \frac{1}{2} p_H L_H
\]  

(22)

Hence Home’s exports of the final good as a share of Home’s total exports is:

\[
\frac{L_F P_1 \gamma (rL_H + rL_F)^\beta}{L_F \gamma (rL_H + rL_F)^\beta + p_H L_H}
\]  

(23)

Differentiating this expression with respect to \(L_H\) gives the relationship between the share of final goods exports and country size:

\[
\frac{d}{dL_H} = \frac{L_F P_1 \gamma (rL_H + rL_F)^\beta \cdot \left[ (\beta - 1) L_H - L_F \right]}{\left[ L_F \gamma (rL_H + rL_F)^\beta + p_H L_H \right]^2}
\]  

(24)

The sign of this expression depends on the term \( (\beta - 1) L_H - L_F \); if this term is negative, there will be a negative relationship between country size and its share of final goods exports. This will be true provided \(\beta\) (the gains from the division of labour) is not too large. Since trade is assumed to be balanced, this gives:

**Proposition 4:** If \( (\beta - 1) L_H - L_F < 0 \), there is a negative relationship between country size and the share of final goods in its exports, and a positive relationship between country size and the share of intermediate goods in its exports.

We take this prediction of the model to data for all available countries from the UN Comtrade database, using data for 2010. We make use of the Broad Economic Categories (BEC) classification which divides industries into capital goods, intermediate goods, consumption goods, and “unclassified” (see United Nations (2002) for details of the classification). For our analysis, we drop the “unclassified” category before calculating the share of each type of good in total exports\(^3\). Our sample consists of 134 countries, and in the sample, the share of consumption goods in total exports is 26.3%, while the share of intermediate goods is 65.6%, and the share of capital goods is 8.1%. We obtain GDP in real PPP and real US dollar terms, population and land area from the World Development Indicators of the World Bank. Figure 1 shows a scatterplot of the consumption goods share of total exports and GDP in PPP terms; there is a clear negative relationship between the two variables, as predicted by Proposition 4 (corr = -0.185 with a p-value of 0.037).

\(^3\) Including the “unclassified” category leads to similar results to those reported below.
Table 1 reports the results of a regression analysis of the relationship between the consumption share of exports and country size. Country size is measured using one of the four measures above: GDP in real PPP and real US dollar terms, population, and land area, all in natural logs. A series of bivariate regressions is reported with these four measures in Panel A of Table 1, with heteroskedastic-robust standard errors. All the size measures are negatively and significantly related to the consumption share of exports.

It is possible that the relationship between country size and the consumption share of exports is different for different groups of countries. Panel B of Table 1 reports results of the same regressions, this time including a dummy for the OECD and an interaction term between this dummy and the size measure. The OECD dummy and the interaction term are never significantly related to the consumption share of exports, and inclusion of these variables does not change the negative relationship between country size and the consumption share of exports. Finally, Panel C of Table 1 reports results including continent dummies. Once again this does not change the negative relationship between country size and the consumption share of exports.

Of course, what Table 1 shows is that country size and the consumption share of exports are negatively related; it does not imply that one causes the other, or indeed that the model proposed in this paper is the “true” explanation for the observed patterns in the data. What it does suggest, however, is that the model’s predictions are at least consistent with the empirical evidence.

6 Conclusions

This paper develops a model of international trade based on the division of labour and comparative advantage. The extent of the division of labour is determined by comparative advantage in intermediate goods, whereas the gains from international trade arise from the division of the production process into increasing numbers of stages and from comparative advantage in final goods. It is shown that large countries gain more from comparative advantage than from the division of labour, whereas the opposite is true for small countries. Trade in this model is both inter- and intra-industry in nature – countries exchange
intermediate inputs which are used in the production of final goods, which are then traded with each other. Hence the model presented here also provides the foundations for a model of intra-industry trade based on perfect competition. In addition, the model predicts that, if the gains from the division of labour are not too large, then larger countries will have a smaller share of consumption goods in their exports, and a larger share of intermediate goods. These predictions find supportive evidence from the UN Comtrade database.

Appendix A: When countries always produce $n_j = rL_j$ intermediates

This involves comparing the output of the final good that results from producing $n_j = rL_j$ intermediate inputs as in the text, with the output that results from producing fewer or more inputs. The proof is shown for good 1 in the Home country in the case of autarky; the proof for the other cases follows analogously.

Output when Home produces $n_H = rL_H$ intermediate inputs is (as in the text):

$$Q_{1H}^A = (rL_H)^\beta \gamma \left(\frac{L_H}{2}\right)$$

(A1)

Output when Home produces $rL_H - 1$ intermediate inputs is:

$$\widetilde{Q}_{1H}^A = (rL_H - 1)^\beta \gamma \left(\frac{L_H}{2}\right)$$

(A2)

Clearly $\widetilde{Q}_{1H}^A < Q_{1H}^A$, so it is never optimal to produce fewer than $rL_H$ intermediate inputs.

Output when Home produces $rL_H + m$ intermediate inputs is more complicated, since Home has a comparative disadvantage in any intermediate inputs in excess of $rL_H$. Labour market clearing implies $L_H = rL_H l_{kH} + \frac{m}{\alpha} l_{kH}$, hence:

$$\widetilde{Q}_{1H}^A = (rL_H + m)^\beta \left[\frac{\alpha(rL_H+m)}{\alpha rL_H+m}\right] \gamma \left(\frac{L_H}{2}\right)$$

(A3)

Now, $Q_{1H}^A > \widetilde{Q}_{1H}^A$ if $(\frac{rL_H+m}{rL_H})^\beta \left[\frac{\alpha(rL_H+m)}{\alpha rL_H+m}\right] < 1$, which is stated as Assumption 1 in the text. This will be true provided $\alpha$ and $\beta$ are sufficiently small.
Appendix B: Proof that both countries always specialise in free trade

The proof involves comparing the no-trade relative prices of the final goods in the two countries with the free trade relative price. From the consumer’s first order condition we have:

\[ \frac{p_1}{p_2} = \left( \frac{c_1}{c_2} \right)^{\theta - 1} \]  

(B1)

The no-trade relative price in Home and Foreign is:

\[ \left( \frac{p_1}{p_2} \right)_H = \gamma^{\theta - 1} \quad \left( \frac{p_1}{p_2} \right)_F = \gamma^{1-\theta} \]  

(B2)

While the free trade relative price is:

\[ \left( \frac{p_1}{p_2} \right)_{FT} = 1 \]  

(B3)

The following relationships always hold if \( \gamma > 1 \):

\[ \left( \frac{p_1}{p_2} \right)_F \left( \frac{p_1}{p_2} \right)_{FT} > \left( \frac{p_1}{p_2} \right)_H \]  

(B4)

That is, the free trade relative price always lies strictly between the no-trade relative prices in the two countries. Hence profit maximisation by firms will ensure that in free trade the Home country specialises in good 1 while the Foreign country specialises in good 2.
References


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Table 1: The relationship between the consumption share of exports and country size.
Dependent variable = consumption share of exports

### Panel A: Basic regressions

<table>
<thead>
<tr>
<th>Size variable</th>
<th>ln(GDP, constant PPP)</th>
<th>ln(GDP, constant US$)</th>
<th>ln(pop)</th>
<th>ln(land area)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Country size</td>
<td>-0.017</td>
<td>-0.014</td>
<td>-0.026</td>
<td>-0.025</td>
</tr>
<tr>
<td></td>
<td>(0.007)**</td>
<td>(0.006)**</td>
<td>(0.011)**</td>
<td>(0.008)*****</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.04</td>
<td>0.03</td>
<td>0.07</td>
<td>0.09</td>
</tr>
<tr>
<td>$N$</td>
<td>127</td>
<td>128</td>
<td>134</td>
<td>134</td>
</tr>
</tbody>
</table>

### Panel B: OECD dummy

| Country size                  | -0.020                | -0.018                | -0.027  | -0.026        |
|                               | (0.009)**             | (0.009)**             | (0.009)**| (0.009)****   |
| OECD = 1                      | 0.106                 | 0.151                 | -0.166  | -0.147        |
|                               | (0.379)               | (0.343)               | (0.295) | (0.167)       |
| OECD * Size                   | -0.003                | -0.004                | 0.010   | 0.011         |
|                               | (0.014)               | (0.013)               | (0.018) | (0.013)       |
| $R^2$                         | 0.04                  | 0.04                  | 0.07    | 0.09          |
| $N$                           | 127                   | 128                   | 134     | 134           |

### Panel C: Continent dummies

| Country size                  | -0.018                | -0.017                | -0.021  | -0.021        |
|                               | (0.007)**             | (0.006)**             | (0.012)*| (0.010)****   |
| $R^2$                         | 0.11                  | 0.11                  | 0.10    | 0.12          |
| $N$                           | 127                   | 128                   | 134     | 134           |

Notes: Figures in parentheses are heteroskedastic-robust standard errors. *** Significant at 1%; ** significant at 5%; * significant at 10%. All results include an unreported constant. Estimation method is OLS.
Figure 1: Scatterplot of consumption goods as a share of total exports against GDP.