

## Preheating in generalized Einstein theories

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We study the preheating scenario in generalized Einstein theories, considering a class of such theories which are conformally equivalent to those of an extra field with a modified potential in the Einstein frame. Resonant creation of bosons from an oscillating inflaton has been studied before in the context of general relativity taking also into account the effect of metric perturbations in linearized gravity. As a natural generalization we include the dilatonic–Brans-Dicke field without any potential of its own and in particular we study the linear theory of perturbations including the metric perturbations in the longitudinal gauge. We show that there is an amplification of the perturbations in the dilaton–Brans-Dicke field on super-horizon scales ( $k \rightarrow 0$ ) due to the fluctuations in the metric, thus leading to an oscillating Newton’s constant with very high frequency within the horizon and with growing amplitude outside the horizon. We briefly mention the entropy perturbations generated by such fluctuations and also the possibility to excite the Kaluza-Klein modes in the theories where the dilatonic–Brans-Dicke field is interpreted as a homogeneous field appearing due to the dimensional reduction from higher-dimensional theories. [S0556-2821(99)01820-2]

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### I. INTRODUCTION

One of the most important epochs in the history of the inflationary Universe is the transition from almost de Sitter expansion to the radiation dominated universe. Until a few years ago proper understanding of this phenomenon was not well established. Recent developments in the theory of resonant particle creation of bosons [1] as well as fermions [2] due to coherent oscillations of the Bose-condensate inflatons may explain satisfactorily the emergence of the radiation era from the ultracold inflationary universe [1]. Although such phenomena are hard to reproduce in a laboratory they are the simplest manifestation of a slowly varying scalar field which rolls down the potential and oscillates as a coherent source at each and every space-time region. Apart from reheating the universe to the temperature ambient for the production of light nuclei this phenomena has multifaceted consequences. It is also a well known mechanism to create a nonequilibrium environment which can be exploited for the generation of net baryon antibaryon asymmetry required for baryogenesis [3]. If there exist general chiral fields, then, in particular, breaking of parity invariance could also lead to different production of left and right fermions. If the rotational invariance is broken explicitly by an axial background then there would be anisotropic distribution of fermions [4]. The same phenomena is also responsible for the generation of primordial magnetic field [5,6] in the context of string cosmology, where the dynamical dilaton field plays the role of an oscillating background.

The preheating scenario has been studied so far in the context of general theory of relativity. It has been well established from the present observations that this theory of gravitation is the correct description of space-time geometry at scales larger than 1 cm. There exists a class of deviant theories which are scalar-tensor gravity theories, known as generalized Einstein theories (GET) of which the Jordan-Brans-Dicke (JBD) theory [7,8] is the simplest and best-studied generalization of general relativity. This theory leads

to variations in the Newtonian gravitation ‘‘constant’’  $G$ , and introduces a new coupling constant  $\omega$ , with general relativity recovered in the limit  $1/\omega \rightarrow 0$ . The constraint on  $\omega$  based on timing experiments using the Viking space probe suggests that it must exceed 500 [9]. The JBD theory also mimics the effective Lagrangian derived from low energy scale of the string theory where the Brans-Dicke (BD) field is called Dilaton and the coupling constant  $\omega$  takes the negative value of  $-1$  [10]. It also represents the  $(4+D)$ -dimensional Kaluza-Klein theories with an inflaton field which has mainly two subclasses out of which we shall consider the one where the inflaton is introduced in an effective four-dimensional theory. In this case the BD-dilaton field plays the role of homogeneous scalar field in 4 dimensions and is related to the size of the compactification. Most of these models are conformally equivalent and can be recast in the form of Einstein gravity theory [11]. The only difference is that the scaling of the fields and their corresponding couplings will be different for different interpretations of BD-dilaton field.

In this paper we shall consider the general relativity limit of the JBD theory as well as other variant theories such as string and Kaluza-Klein and for the sake of consistency we just use one representative of the coupling constant  $\gamma$ , which takes different values accordingly. We must say that there is an obvious advantage in all these theories that they are well constrained at the present day and the evolution of the BD-dilaton field is roughly constant after a period of 60  $e$  foldings of inflation. Here we shall not consider the potential in the dilatonic-BD sector and we assume that the evolution of the dilaton-BD field during the oscillatory phase is solely due to its coupling with the inflaton in the Einstein frame. We assume the quadratic potential  $V(\phi) = \frac{1}{2}m^2\phi^2$  for the inflaton field. It is to be noticed that during the oscillatory phase the field  $\phi$  decreases in the same way as the density of non-relativistic particles of mass  $m$ :  $\rho_\phi = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}m^2\phi^2 \approx a^{-3}$ , provided the coupling  $\gamma$  is very small as in the case of JBD. For simplicity we discuss the physics of weak coupling and

at the end we comment on the strong coupling limits which we get numerically. In the weak limit the coherent oscillations of the homogeneous scalar field correspond to a matter dominated equation of state with vanishing pressure. This suggests that the BD field should evolve according to a well known solution during the dust era [12].

Apart from the study of nonperturbative creation of bosons and fermions during preheating, metric perturbations have also been studied extensively in Refs. [13] and [14]. It has been shown that the metric perturbations also grow during preheating in the multiscalar case due to the enhancement of the entropy perturbations. It is also possible to amplify the superhorizon modes causally. Such amplification requires the linear theory of gravitational perturbations to be supplanted by nonlinear perturbation theory. In Ref. [13] the authors have discussed the two fields case in particular where the first one ( $\phi$ ) represents the inflaton and the second ( $\sigma$ ) represents the newly created bosons with an interaction with the inflaton of the form  $\frac{1}{2}g\phi^2\sigma^2$ . Perturbations in  $\sigma$  get amplified along with the metric perturbations. They have argued that all the fields would be possibly amplified except the inflaton which is subdued by transferring its energy to the other fields. Following their claim it is also possible to amplify the other fields such as dilaton-BD field as they are also coupled to the inflaton and can be cause of some concern. In the concluding section we devote ourselves to the discussion of these issues.

## II. THE EQUATIONS

In JBD theory the action in the Jordan frame [7]

$$S = \int \left[ \frac{\Phi_{\text{BD}}}{16\pi} \hat{R} + \frac{\omega^2}{16\pi\Phi_{\text{BD}}} \hat{g}^{\mu\nu} \partial_\mu \Phi_{\text{BD}} \partial_\nu \Phi_{\text{BD}} + \frac{1}{2} \hat{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] \sqrt{-\hat{g}} d^4x, \quad (1)$$

is transformed into Einstein frame

$$S = \int \left[ \frac{1}{2\kappa^2} R + \frac{1}{2} g^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - U(\chi) + \frac{1}{2} e^{-\gamma\kappa\chi} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - e^{-2\gamma\kappa\chi} V(\phi) \right] \sqrt{-g} d^4x, \quad (2)$$

through the conformal transformation

$$g_{\mu\nu} = \Omega^2 \hat{g}_{\mu\nu}, \quad \Omega^2 \equiv \frac{\kappa^2}{8\pi} \Phi_{\text{BD}} \equiv e^{(\kappa\chi/\sqrt{\omega+3/2})}, \quad (3)$$

where  $\kappa^2 = 8\pi G$ ,  $\gamma$  is a constant related to  $\omega$ .  $\chi$  and  $\phi$  are the BD and inflaton fields respectively. For our concern  $\gamma = 1/\sqrt{\omega+3/2}$  and  $U(\chi) = 0$  (see, however, the discussion below and on Sec. III). Observational constraints give  $\omega$

$> 500$  so that  $\gamma < 0.09 \ll 1$ . In the dimensionally reduced theories the original action is different from Eq. (1) but it is still possible to conformally transform it to Eq. (2) [11]. In this case  $\chi$  and the coupling  $\gamma$  are given by

$$\chi = \left[ \frac{2(D+2)}{D} \right]^{1/2} \ln \Phi_{\text{BD}}, \quad (4)$$

$$\gamma = \left[ \frac{2D}{D+2} \right]^{1/2}, \quad (5)$$

where  $\Phi_{\text{BD}} \equiv (b/b_0)^{D/2}$ ,  $b$  is the radius of compactification, and  $b_0$  its present value. Notice that in the Kaluza-Klein case it is possible to make  $U(\chi) = 0$  only when the extra dimensions are compactified on a torus which has zero curvature. Compactifying the extra dimensions on a sphere will give rise to a mass to the dilaton corresponding to the curvature of the sphere. In Sec. III we shall come back to this point again and discuss what should we expect if such a term is also included in the Lagrangian. The number of extra dimensions is  $D$  and  $\gamma$  can at most take a value  $\sqrt{2}$ . In the superstring case  $\gamma$  is exactly  $\sqrt{2}$  [15]. As we have already mentioned during radiation and matter domination  $\chi$  has to be roughly constant in order to comply with the observational data [16] and to reproduce the correct value of the gravitational constant today. In most of the models the evolution of the extra dimensions is also treated to be constant during the radiation era [17].

### A. Equations of motion

We shall perform our calculation in the Einstein frame, Eq. (2), for simplicity. The homogeneous equations of motion for a zero-curvature Friedmann universe are

$$\ddot{\chi} + 3H\dot{\chi} + \frac{\gamma}{2} e^{-\gamma\chi} \dot{\phi}^2 - 2\gamma e^{-2\gamma\chi} V(\phi) = 0, \quad (6)$$

$$\ddot{\phi} + 3H\dot{\phi} - \gamma\dot{\chi}\dot{\phi} + e^{-\gamma\chi} V'(\phi) = 0, \quad (7)$$

$$\frac{1}{3} \left[ \frac{1}{2} \dot{\chi}^2 + \frac{1}{2} e^{-\gamma\chi} \dot{\phi}^2 + e^{-2\gamma\chi} V(\phi) \right] = H^2, \quad (8)$$

with an overdot denoting time derivation and a prime denoting partial derivative with respect to  $\phi$ . We use natural units where  $\kappa^2 = 1$ . We do not pay much attention to the field equations during inflation, since much has been studied in this area [15,18], rather we concentrate on the coherent oscillations of the inflaton. Before we turn to linear perturbation theory we should mention that we have to introduce the bosons resonantly produced from the nonperturbative decay of inflaton field which we shall denote by  $\sigma$ . We consider the coupling of  $\sigma$  particles with the inflaton to be  $g/2\phi^2\sigma^2$ , where  $g$  is the coupling strength. Hence, the old potential is modified during oscillations due to the presence of massless  $\sigma$  particles:

$$V(\phi, \sigma) = \frac{1}{2}m^2\phi^2 + \frac{1}{2}g\phi^2\sigma^2. \quad (9)$$

In the Heisenberg representation the equation of motion for the scalar field  $\sigma$  can be expressed in terms of the temporal part of the mode with comoving momentum  $k$ ,

$$\ddot{\sigma} + 3H\dot{\sigma} - \gamma\dot{\chi}\dot{\sigma} + \left[ \frac{k^2}{a^2(t)} + g\phi^2e^{-\gamma\chi} - \zeta R \right] \sigma = 0, \quad (10)$$

where  $\zeta$  is the nonminimal coupling to the curvature, which we shall not take into account. This is the equation of great importance for the study of resonant creation of  $\sigma$  (details can be seen in Ref. [1]). This particular equation takes the form of the well known Mathieu equation after redefining the field. This equation has very rich properties, and its solutions can fall into two categories, either stable or unstable depending on the choice of two parameters  $[A(k), q]$ , defined later [19].

### B. Linear perturbations

Linear perturbations can be taken into account in a gauge invariant fashion using the longitudinal gauge. We write the perturbed metric as

$$ds^2 = (1 + 2\Phi)dt^2 - a^2(t)(1 - 2\Psi)\delta_{ij}dx^i dx^j, \quad (11)$$

where  $\Phi$  and  $\Psi$  are gauge invariant metric potentials. The Fourier modes satisfy the following equations of motion which are derived perturbing the Einstein's equations [20,21].

$$\Phi = \Psi, \quad (12)$$

$$\dot{\Phi} + H\Phi = \frac{1}{2}[\dot{\chi}\delta\chi + e^{-\gamma\chi}\dot{\phi}\delta\phi + e^{-\gamma\chi}\dot{\sigma}\delta\sigma], \quad (13)$$

$$\begin{aligned} \delta\ddot{\chi} + 3H\delta\dot{\chi} + \left[ \frac{k^2}{a^2} - \frac{\gamma^2}{2}e^{-\gamma\chi}\dot{\phi}^2 + 4\gamma^2e^{-2\gamma\chi}V(\phi, \sigma) \right] \delta\chi \\ + \gamma e^{-\gamma\chi}\dot{\phi}\delta\dot{\phi} - 2\gamma e^{-2\gamma\chi}V(\phi, \sigma)_{,\phi}\delta\phi \\ - 2\gamma e^{-2\gamma\chi}V(\phi, \sigma)_{,\sigma}\delta\sigma \\ = 4\dot{\Phi}\dot{\chi} + 4e^{-2\gamma\chi}V(\phi, \sigma)\Phi, \end{aligned} \quad (14)$$

$$\begin{aligned} \delta\ddot{\phi} + (3H - \gamma\dot{\chi})\delta\dot{\phi} + \left[ \frac{k^2}{a^2} + e^{-\gamma\chi}V(\phi, \sigma)_{,\phi,\phi} \right] \delta\phi \\ + e^{-\gamma\chi}V(\phi, \sigma)_{,\phi,\sigma}\delta\sigma - \gamma\dot{\phi}\delta\dot{\chi} - \gamma e^{-\gamma\chi}V(\phi, \sigma)_{,\phi}\delta\chi \\ = 4\dot{\Phi}\dot{\phi} - 2e^{-\gamma\chi}V(\phi, \sigma)_{,\phi}\Phi, \end{aligned} \quad (15)$$

$$\begin{aligned} \delta\ddot{\sigma} + (3H - \gamma\dot{\chi})\delta\dot{\sigma} + \left[ \frac{k^2}{a^2} + g\phi^2e^{-\gamma\chi} \right] \delta\sigma \\ + 2g\sigma\phi e^{-\gamma\chi}\delta\phi - \gamma\dot{\sigma}\delta\dot{\chi} - \gamma g\phi^2\sigma e^{-\gamma\chi}\delta\chi \\ = 4\dot{\Phi}\dot{\sigma} - 2g\phi^2\sigma e^{-\gamma\chi}\Phi, \end{aligned} \quad (16)$$

where  $\delta\phi$ ,  $\delta\chi$ , and  $\delta\sigma$  are gauge invariant perturbations in the respective fields. At this point the evolution equations for the background are not determined by Eqs. (6)–(8); rather they are modified by the presence of  $\sigma$  particles. For the Friedmann equation [Eq. (8)] we now have

$$\frac{1}{3} \left[ \frac{1}{2}\dot{\chi}^2 + \frac{1}{2}e^{-\gamma\chi}\dot{\phi}^2 + \frac{1}{2}e^{-\gamma\chi}\dot{\sigma}^2 + e^{-2\gamma\chi}V(\phi, \sigma) \right] = H^2. \quad (17)$$

Similarly, the homogeneous equations for  $\chi$  and  $\phi$ , Eqs. (6) and (7) are to be modified accordingly with  $V(\phi)$  replaced by  $V(\phi, \sigma)$  and  $V'(\phi)$  replaced by  $V(\phi, \sigma)_{,\phi}$ . It is worth mentioning that in the absence of  $\sigma$  field and the metric perturbation it is possible to express the perturbation in the BD field in a simpler form. Let us further note that in an expanding universe without BD field the behavior of the inflaton field is  $\phi \approx \sin(m_\phi t)/m_\phi t$  and the scale factor goes as  $a(t) \approx a_0(t/t_0)^{2/3}$ . In the presence of the BD field this is modified to  $a(t) \approx a_0(t/t_0)^{(2\omega+3)/(3\omega+4)}$ , such that in the large  $\omega$  limit it reduces to the previous one. We expect similar modification in the behavior of an inflaton field. For our calculation's sake we can assume that the modified evolution of the inflaton for  $\omega > 500$  and  $\gamma < 0.09$  is

$$\phi \approx [1 + \mathcal{O}(\gamma)] \frac{\sin(m_\phi t)}{m_\phi t}. \quad (18)$$

For our purpose the exact numerical expression for  $\mathcal{O}(\gamma)$  does not matter because we are retaining the lowest order in  $\gamma$ . Simplifying the perturbed BD field Eq. (14) with the help of Eq. (18) and noting that  $\delta\chi$  can be rescaled by introducing  $u = a^{3/2}(t)\delta\chi(t)$  and  $z = m_\phi t$ , we get

$$\frac{d^2u}{dz^2} + \left[ \frac{k^2}{a^2m_\phi^2} + \frac{3}{4}\frac{\gamma^2}{z^2} - \frac{5}{4}\frac{\gamma^2}{z^2}\cos(2z) \right] u = 0. \quad (19)$$

The above equation resembles the Mathieu equation [19]:

$$x'' + [A(k) - 2q \cos 2z]x = 0 \quad (20)$$

with

$$A(k) = \frac{k^2}{a^2m_\phi^2} + \frac{3}{4}\frac{\gamma^2}{z^2}, \quad (21)$$

$$q = \frac{5}{8}\frac{\gamma^2}{z^2}. \quad (22)$$

Since  $q \ll 1$  we are never in the broad resonance regime which is essential to amplify the modes during the oscilla-

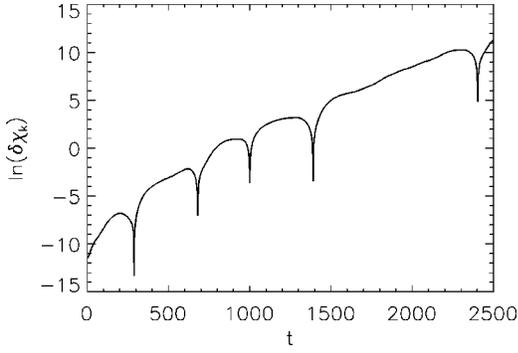


FIG. 1. The evolution of  $\ln \delta\chi$  for  $g(\phi_0^2/m_\phi^2)=9\times 10^3$ ,  $k=0$ , and  $\gamma=1.22$  corresponds to  $D=6$  extra compactified dimensions.

tions of the scalar field in presence of expansion. In this case we are always in the narrow resonance regime [1].

During preheating the resonant modes should grow as  $\delta\chi_k \propto \exp(\mu_k t)$ , where  $\mu_k$  is the Floquet index and its analytical estimation has been done in Ref. [1] for the bosonic fields,  $\mu_k \approx \ln(1+2e^{-\pi(k/ag\phi_0)^2} - Q)$ , where  $Q$  is contributed by the initial and final quantum states of the created bosons, which in the case of preheating are not thermal but have the characteristics of a squeezed state [1]. As it has been pointed out in Ref. [13], the resonant production of newly created bosons grows for  $k=0$ , so the super-Hubble modes are present even in the absence of metric perturbation. In this case, however, the BD modes are not amplified. We recognize that in the Einstein equations as we perturb the matter sector we automatically perturb the gravitational sector, and gravity being a source of negative heat capacity it can only transfer energy to the metric perturbation which acts back to the matter sector enhancing the resulting perturbation in the matter fields. Energetically it is easier to transfer energy to the lowest modes so most of the energy is transferred to  $\Phi_{k=0}$  and that is the reason why  $k=0$  is favored.

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### C. Numerical result

Our analytical approximation breaks down as we increase the numerical value for the coupling constant  $\gamma$  and the perturbation equations become intractable as we introduce the metric perturbations. Hence we solve the system of 14 first order differential equations numerically. Here while solving the homogeneous equation for  $\sigma$  we consider only the zero mode contribution, hence Eq. (10) can be treated as a homogeneous background equation for the linear perturbation in  $\sigma$  with  $k=0$ . By doing so we neglect the zero mode contribution from the metric potential and including the zero mode metric contribution can only lead to enhancement in the amplification in the matter and the BD-dilaton sector as discussed in Ref. [13]. For our numerical calculation we have assumed that the perturbation in  $\phi$  and in  $\chi$  contains the generic, scale invariant spectrum produced during inflation. We set  $\Phi_k(t_0)=10^{-5}$  and we also consider the same initial conditions for the fluctuations in the BD-dilaton and the fluctuations in the matter fields  $\delta\chi_{k=0}=\delta\phi_{k=0}=\delta\sigma_{k=0}=10^{-5}$

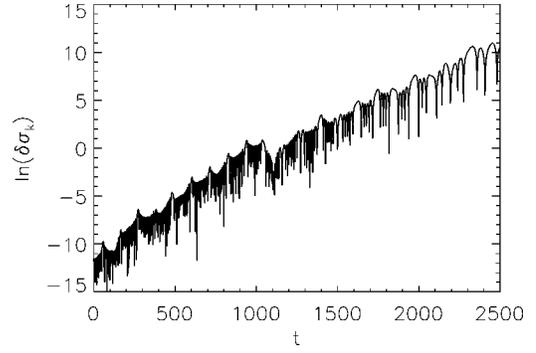


FIG. 2. The evolution of  $\ln \delta\sigma$  for  $g(\phi_0^2/m_\phi^2)=9\times 10^3$ ,  $k=0$  and  $\gamma=1.22$  corresponds to  $D=6$  extra compactified dimensions.

while the corresponding time derivatives are set to zero.<sup>1</sup> The initial condition for  $\sigma$  is that of a plane wave solution and the BD-dilaton field  $\chi$  is assumed to be very small  $\ll 1$  to match the present strength of the gravitational constant (see Figs. 1 and 3).

We have plotted the fluctuations in the BD-dilaton field and the perturbations in the  $\sigma$  field for  $k=0$  (see Figs. 2 and 4). In the weak limit for  $\gamma=0.09$ , the perturbations do not grow much and they saturate much earlier in comparison with the strong coupling limit, such as  $\gamma=1.22$ , which, in the case of a Kaluza-Klein theory corresponds to six extra dimensions compactified to a six-dimensional torus. In the strong limit case the nonlinearity is achieved as soon as the metric perturbation  $\Phi=1$ . It is obvious that for Kaluza-Klein theories and for string motivated theories the linear theory soon becomes invalid unless the backreaction is taken into account. It is important to notice that our  $q$  parameter is not the same as in general relativity. Instead of defining the  $q$  parameter to be  $q=g(\phi_0^2/m_\phi^2)$ , where  $\phi_0$  is the initial amplitude of the inflaton field, which is  $0.08M_{\text{Pl}}$  in our case, in presence of the BD field  $q$  is modified by an exponential factor  $q=g(\phi_0^2/m_\phi^2)e^{-\gamma\chi}$ . The negative sign in the exponent drags down the  $q$  parameter. For the figures  $g(\phi_0^2/m_\phi^2)=9\times 10^3$ . There is some difference between the perturbations in  $\sigma$  and  $\chi$  as visible from the plots (see Figs. 1–4). It is important to note that  $\chi$  has as such no potential with minima unlike  $\phi$  and  $\sigma$  fields which oscillate around the minima of their respective potentials and for the modes outside the horizon the perturbed  $\chi$  field almost stops oscillating and starts growing exponentially. The increment in the perturbations in the strong coupling limit can be understood qualitatively. Let us first see how the scale factor behaves in these two cases. As we have already discussed the oscillations in the inflaton field on average correspond to zero pressure. This suggests that the BD-dilaton field on average evolves similar to the matter dominated era, and in presence of the BD-dilaton  $a(t) \approx a_0(t/t_0)^{(2\omega+3)/(3\omega+4)}$ . For large  $\omega$  and small  $\gamma$ , the growth in  $a(t)$  is roughly the same as in general relativity,

<sup>1</sup>We have chosen our initial conditions to be scale-invariant, i.e., independent of  $k$ . There are, however, other subtleties involved in deciding the initial conditions. For a recent discussion we refer to Ref. [13].

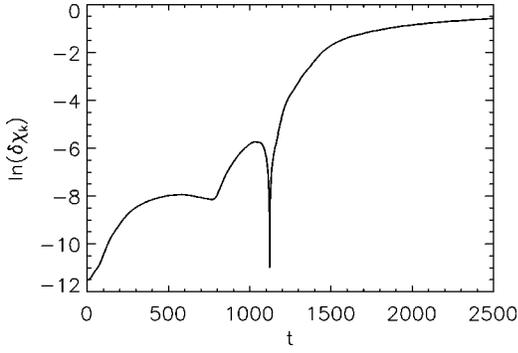


FIG. 3. The evolution of  $\ln \delta\chi$  for  $g(\phi_0^2/m_\phi^2)=9\times 10^3$ ,  $k=0$  and  $\gamma=0.09$  corresponds to  $\omega=500$  in JBD theory.

but for small  $\omega$  and  $\gamma$  close to  $\sqrt{2}$ , the departure of the behavior of the inflaton oscillations from general relativity is quite significant. Since  $\phi \sim 1/t \sim 1/a^{(3\omega+4)/(2\omega+3)}$ , for small  $\omega$ , the amplitude of the inflaton decreases slowly compared to the general relativity limit, therefore leading to a remarkable growth in the production of  $\sigma$  particles.

#### D. Perturbing Newton's constant

In [22] the authors have considered the oscillations in the Newton's constant by introducing a potential for the BD field explicitly and concluded that  $G$  oscillates about its mean value and for reasonable values of the mass of the BD field ( $m \geq 1$  GeV), the oscillations have very high frequency  $\nu^{-1} \ll 1$  s, compared to the Hubble expansion. In their case, however, the amplitude of the oscillations is exponentially small, and the oscillation energy is dissipated through Hubble redshift. Considering our scenario, where we do not have to invoke the potential for the BD explicitly, the perturbations in the BD field will cause the oscillations in the Newton's constant in the Jordan frame even if the modes are well within the horizon. Such oscillations will not only have a temporal but also a spatial variation. As we approach to  $k=0$  mode, gradually the amplitude of such oscillations also increases exponentially in time and causes a large variation in the Newton's constant. Such oscillations are certainly permissible but whether they would withstand all known tests from cosmology and general relativity is not known at the moment. It is expected that the classical fluctuations will lead to nonlinearity after some time and the linear theory of perturbation will break down when  $\Phi=1$ . Such provocative interpretation of the Newton's constant and the nondecayable property of the BD field could be the source of dark matter in our universe. Had we introduced a potential with a minimum for the BD-dilaton field, superimposed on the exponential growth, we should also see small oscillations with frequency and amplitude depending on the exact form of the potential.

#### E. Entropy perturbations

During inflation the presence of BD field renders the three-curvature of comoving hypersurfaces in terms of Bardeen's gauge invariant quantity  $\zeta$  time varying on super-horizon scales, thus producing not only adiabatic but also isocurvature perturbations [18,23]. It has been estimated in

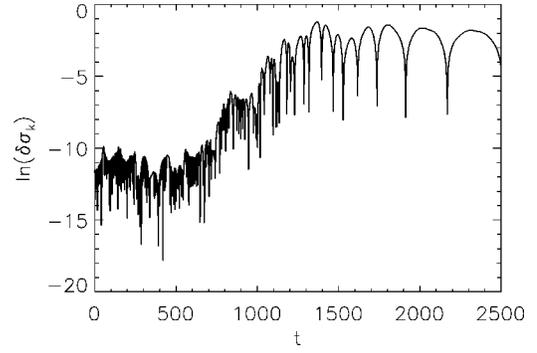


FIG. 4. The evolution of  $\ln \delta\sigma$  for  $g(\phi_0^2/m_\phi^2)=9\times 10^3$ ,  $k=0$  and  $\gamma=0.09$  corresponds to  $\omega=500$  in JBD theory.

Ref. [18] that at the end of inflation when  $\chi$  is very close to zero the metric perturbation is dominated by the adiabatic contribution and at the time of horizon crossing the fractional energy density fluctuations in  $\chi$  are much smaller than the adiabatic ones. This suggests that the isocurvature fluctuations in the JBD model are negligible. If we assume that the BD field is a candidate for the cold dark matter then one can estimate its energy density from the exact solution of the BD field during the dust era, which is a good approximation during the oscillatory phase when the average pressure becomes zero and energy density falls as  $1/t$ . Since

$$\rho_\chi = \frac{1}{2} \dot{\chi}^2 \cong \gamma^2 \rho, \quad (23)$$

where  $\rho$  is the post inflationary energy density of the universe, it is obvious that the energy density of the BD field is negligible when compared to the total. Usually for multiple fields the difference between the relative perturbations are defined as entropy perturbation  $S_{1,2}$  [24] by

$$S_{1,2} = \frac{\delta\rho_1}{\rho_1 + p_1} - \frac{\delta\rho_2}{\rho_2 + p_2}, \quad (24)$$

where  $\rho_1, \rho_2, p_1, p_2$  are respectively the energy densities and pressures. For nonadiabatic initial conditions  $\delta\chi/\dot{\chi} \neq \delta\phi/\dot{\phi} \neq \delta\sigma/\dot{\sigma}$ ,  $S_{\chi,\phi}$ ,  $S_{\chi,\sigma}$ , and  $S_{\sigma,\phi}$  would not vanish, thus producing entropy perturbation inside as well as outside the horizon. Thus for superhorizon modes isocurvature fluctuations would evolve though adiabatic fluctuation would have frozen. The growth in the isocurvature perturbation outside the horizon gives the tilt in the spectrum, which is usually blueshifted and roughly estimated by the ratio of effective mass square of the cold dark matter (CDM) field and square of the Hubble expansion. If  $\chi$  is treated as a CDM field then the effective mass is roughly  $\gamma^2 V(\phi, \sigma)$ , the main contribution coming from the large coupling  $0.5g\phi^2\sigma^2$ , but the coupling is subdued by the presence of  $\gamma^2$ , provided we are in a weak limit JBD theory. In string motivated and in Kaluza-Klein theories the effective mass increases a lot. During preheating the Hubble expansion is very small compared to the effective mass and thus results in an extreme blue tilt specially in string motivated theories.

### F. Shaking the Kaluza-Klein modes

As we have described, depending on the numerical value of the coupling constant  $\gamma$ , the role of  $\chi$  also changes, and in fact it covers a wide range of theories from effective action for superstring models to JBD theories. If  $\Phi_{\text{BD}}$  is interpreted as a homogeneous field which appears as an effective mass term for the Kaluza-Klein modes [25] in the effective four-dimensional Kaluza-Klein action defined as  $M_l^2(\Phi_{\text{BD}})$  then, as mentioned before, the BD-dilaton can be related to the radius of the compactified  $D$  dimensions if the compactification is done on a  $D$  dimensional torus or on sphere. In terms of  $\chi$ :

$$\chi = \chi_0 \ln \left[ \frac{b}{b_0} \right], \quad (25)$$

$$\chi_0 = \sqrt{\frac{D(D+2)}{2}}. \quad (26)$$

If the extra dimensions are compactified on a sphere then the mass  $M_l(\chi)$  of the four-dimensional Kaluza-Klein field  $\phi_{lm}$  in terms of  $\chi$

$$M_l^2(\chi) = \frac{l(l+D-1)}{b_0^2} e^{-(D+2)\chi/\chi_0}. \quad (27)$$

In the usual case,  $\chi$  is assumed to be very close to zero after inflation and its variation is assumed to be negligible in radiation and matter dominated eras. That leads to very high mass of these modes, very close to Planck scale and proportional to the inverse square of the radius of the extra dimensions. During reheating the perturbation in  $\chi$  grows exponentially in time and that causes fluctuations in the mass of the Kaluza-Klein modes. The perturbations effectively give rise to the Casimir force as a fluctuating boundary condition for the  $D$ -dimensional sphere. This in fact can stabilize the potential for the homogeneous scalar field, in our notation  $\Phi_{\text{BD}}$ . Though we have neglected such potentials in our analysis since we have concentrated on the compactification on a torus, in this case the Kaluza Klein mass scale is still inversely proportional to the square of the size of the compactification radius. If the compactification is done on a sphere and if a potential term for the dilaton is taken into account then the effect of resonance will be even more pronounced. However, the potential lacks a local minima and in order to stabilize it one needs to invoke the Casimir effects at a one-loop level. The issue raises a few questions such as particle creation in the external dimensions due to fluctuating boundary and their stability requires a detailed study.

If the Kaluza-Klein modes are excited their excitations may also give rise to some undesirable particles such as the pyrgon [26], whose energy density decreases as  $a^{-3}$ , giving during the radiation era  $\rho_{\text{pyrgon}}/\rho_{\text{rad}} \sim a$ . Their stringent requirement to decay [26] can enhance their lifetime and they can be a candidate for dark matter. Overproduction of such particles could have overcome the critical density, but their number density could have been diluted by invoking a sec-

ondary phase of inflation. Such a phase is also desirable to smoothen the large superhorizon resonances as mentioned in Ref. [13].

### III. CONCLUSIONS

We have studied the preheating scenario in the context of scalar-tensor theories. We have discussed the linear perturbation theory and showed that the perturbations in BD-dilaton field grow outside the horizon. The interpretation of such modes is not very clear at the moment but one may hope that by including backreaction in a consistent way may solve this problem. Nevertheless, including perturbations in the BD-dilaton field introduces new physics well within the horizon such as temporal and spatial variation in Newton's constant. Such perturbations could play an important role during structure formation and it is important to see whether such variations in  $G$  could be detected or not. In the strong coupling regime fluctuations in  $\chi$  field grow faster than the fluctuations in the case of weak coupling (JBD theory) for the zero mode and can also excite the Kaluza-Klein modes by lowering the mass of  $\phi_{l,m}$ . Such excitations can give rise to highly noninteracting quanta which can be a very good candidate for the present dark matter.

We have also discussed the entropy perturbations during reheating which are solely due to the fact that there is more than one scalar field and the initial conditions in the relative density fluctuations in the respective components are nonadiabatic. In general relativity the reason behind such super amplification in the fluctuations is due to the enhancement in the entropy perturbations outside the horizon. In our case such amplification is even stronger especially in the strong coupling case because we have an extra field which contributes its fluctuations to the entropy perturbation. Isocurvature fluctuations will be generated with a large tilt in string motivated and Kaluza-Klein theories provided BD-dilaton is treated as a CDM field.

Here we must point out that we have intentionally neglected the potential term coming from the dilaton sector. Such a scheme is possible provided the extra dimensions are compactified on a torus and not on a sphere. Compactifying on a sphere gives rise to a term proportional to the curvature of the sphere and such a term acts as a potential for the dilaton. However, as we noted before, the curvature term lacks the global minima in the direction of the dilaton field and one has to invoke the first order Casimir corrections at a one-loop level to give rise to a global minimum. Parametric excitations of the Kaluza-Klein modes in such a potential have already been discussed in Ref. [25] but the author has not taken the metric perturbations into account. Inclusion of the dilaton potential may even enhance the metric perturbation as it acts as an extra source term for Eq. (15). Concrete predictions requires a detailed study and it is worth investigating this point as a separate issue. We must mention that recently there has been an intense discussion upon the initial conditions for the perturbations in the matter and in the BD-dilaton fields. We have chosen our initial conditions to be scale invariant. There have been other proposals such as the commonly used quantum plane wave initial conditions for

the fluctuations in the fields. This, however, favors small  $k$  modes more than the large  $k$  modes. Another attractive proposal can be evolving the BD-dilaton from 60  $e$  foldings before the end of inflation when  $aH \ll k$  until the horizon crossing and then subsequently evolving the fields during the oscillatory phase. These are open issues which require further investigation. Our analysis was mainly restricted to the  $k \rightarrow 0$  mode but we should also expect the same physical consequences for other modes excited during preheating although with a much smaller magnitude.

The issue of backreaction becomes important after a few inflaton oscillations and the subject becomes almost intractable as we increase the number of fields gradually. Backreaction has been considered before by many authors [1], but in our situation we have to worry about the contribution coming from the BD-dilaton sector also. A self-consistent study of the inflaton and the  $\sigma$  particles should take into account the effective change in the mass of the inflaton but in this case such corrections will be larger due to the contribution from the BD-dilaton sector. Similarly, the BD-dilaton will also acquire an effective mass. To make a complete picture one must answer all these relevant issues and we

leave them for future investigation. We also point it out that our analysis has ignored the contribution coming from  $\Phi_{k=0}$  to the background evolution equations. Inclusion of such terms will certainly help to amplify the fluctuations even faster and we can expect the fluctuations to become nonlinear before. The aspects of nonlinearity in the gravitational sector is least understood and the further evolution of the nonlinear modes will become important leaving an imprint on the cosmic microwave background radiation which will be the ultimate verification for the validity of such claims.

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