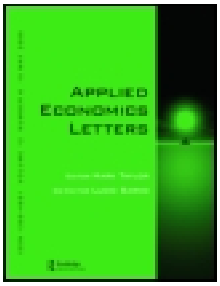

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Empirical pricing kernels obtained from the UK index options market

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Empirical pricing kernels obtained from the UK index options market

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Empirical pricing kernels for the UK equity market are derived as the ratio between risk-neutral densities, inferred from FTSE 100 index options, and historical real-world densities, estimated from time series of the index. The kernels thus obtained are almost compatible with a risk averse representative agent, unlike similar estimates for the US market.

I. Introduction

The pricing kernel assumes a central role in asset pricing literature, as it succinctly summarizes investors' risk and time preferences. With a correctly identified pricing kernel, asset pricing becomes a straightforward discounting of future payoffs by the kernel.

In this article, we estimate empirical pricing kernels from the options market, as option prices have been shown to contain incremental information in forecasting future volatilities and price distributions compared to the time series of asset prices.[1] We express the empirical pricing kernels as the ratio between risk-neutral densities (RND), inferred from FTSE 100 index options, and historical densities obtained from time series of the index, averaged across time to minimize the impact of measurement errors.

In particular, we assume that RND follow certain distributions. They can be either a mixture of two lognormal densities (MLN), a generalized beta distribution of the second kind (GB2), or a flexible spline function, all of which are easy to estimate and able to capture the stylized facts of negative skewness and excess kurtosis that are associated with index distributions.

Using a sample of 126 months from July 1993 to December 2003, we find that the average empirical pricing kernel for the UK equity market is generally downward sloping and does not exhibit the puzzling hump shape documented by Jackwerth (2000), Brown and Jackwerth (2002), and Rosenberg and Engle (2002). This work is also related to Brennan *et al.* (2006), which adopts the traditional asset pricing approach by identifying state variables that fully describe the investment opportunities and specifying flexible functional forms for the pricing kernels.

The rest of the article is organized as follows. Section II introduces the risk-neutral and historical densities and the estimation procedures. Section III

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[1] A vast literature has documented that option-implied volatility provides better forecasts of future volatilities than realized volatility. Poon and Granger (2003) and Taylor (2005) provide recent survey evidence. In terms of density forecasts, Liu *et al.* (2007) demonstrates that distributions from option prices are better forecasts than those obtained from asset price histories.

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discusses data. Section IV presents empirical results and describes the empirical pricing kernels for the UK. Finally, Section V concludes.

II. Risk-Neutral and Historical Densities

Mixture of lognormal densities (MLN)

Following Ritchey (1990) and Melick and Thomas (1997), the RND of the asset price when options expire can be defined as a MLN. The MLN densities are flexible and easy to estimate, with the possibility of attaching an economic interpretation to the parameters when the component densities are determined by specific states of the world when the options expire. The MLN density function in this study is the following weighted average of two lognormal densities g_{LN},

$$g_{\mathrm{MLN}}(x|\theta) = w g_{\mathrm{LN}}(x|F_1, \sigma_1, T) + (1-w) g_{\mathrm{LN}}(x|F_2, \sigma_2, T) \quad (1)$$

with

$$g_{\mathrm{LN}}(x|F,\sigma,T) = \frac{1}{x\sigma\sqrt{2\pi T}} \exp\left[-\frac{1}{2}\left(\frac{\log F - \sigma^2 T/2}{\sigma\sqrt{T}}\right)\right] \quad (2)$$

The parameter vector is $\theta = (F_1, F_2, \sigma_1, \sigma_2, w)$, with $0 \le w \le 1$ and F_1, F_2, σ_1, $\sigma_2 > 0$. The parameters F_1, σ_1 and w denote the mean, volatility, and weight of the first lognormal density, while F_2, σ_2 and $1-w$ are the mean, volatility, and weight of the second lognormal density.

The density is risk-neutral when its expectation equals the current futures price F, i.e. when $wF_1 + (1-w)\ F_2 = F$. The theoretical Euruopean option pricing formula is then simply the weighted average of two option prices given by the Black (1976) formula, denoted by $c_B(\cdot)$,

$$c(X|\theta, r, T) = w c_B(F_{1,} T, X, r, \sigma_1) + (1-w) c_B(F_{2,} T, X, r, \sigma_2) \quad (3)$$

Generalized beta distribution (GB2)

The generalized beta distribution of the second kind (GB2) was first proposed by Bookstaber and McDonald (1987) and utilized by Anagnou-Basioudis *et al.* (2005). The GB2 density incorporates four positive parameters $\theta = (a, b, p, q)$ that permits general combinations of the mean, variance, skewness and kurtosis of a positive random variables. The GB2 density function is defined as,

$$g_{\mathrm{GB2}}(x|a,b,p,q) = \frac{a x^{ap-1}}{b^{ap} B(p,q)[1+(x/b)^a]^{p+q}} \quad (4)$$

with $B(j,k) = \Gamma(j)\Gamma(k)/\Gamma(j+k)$. The density is risk-neutral when

$$F = \frac{bB(p+(1/a), q-(1/a))}{B(p,q)} \quad (5)$$

The parameter b is seen to be a scale parameter, while the product of a and q determines the maximum number of moments and hence the asymptotic shape of the right tail.

The theoretical option pricing formula depends on the cumulative distribution function (c.d.f.) of the GB2 density, denoted G_{GB2}, which is a function of the c.d.f. of the beta distribution, denoted G_β,

$$\begin{aligned} G_{\mathrm{GB2}}(x|a,b,p,q) &= G_{\mathrm{GB2}}((x/b)^a|1,1,p,q) \\ &= G_\beta(y(x,a,b)|p,q) \end{aligned} \quad (6)$$

with $y(x,a,b) = (x/b)^a/(1+(x/b)^a)$. If the density is risk-neutral, so that (5) applies, the European call option prices are given by

$$\begin{aligned} c(X|\theta) &= e^{-rT}\int_X^\infty (x-X) g_{\mathrm{GB2}}(x|a,b,p,q)\mathrm{d}x \\ &= Fe^{-rT}\left[1 - G_\beta\left(y(X,a,b)|p+\frac{1}{a}, q-\frac{1}{a}\right)\right] \\ &\quad - Xe^{-rT}\left[1 - G_\beta(y(X,a,b)|p,q)\right] \end{aligned} \quad (7)$$

Flexible densities

To make sure that the parametric densities discussed above are not inferior to more flexible density curves, we also infer RND defined by spline functions, as estimated by Bliss and Panigirtzoglou (2004). We apply their methodology to obtain implied volatilities, denoted by $\sigma(\Delta|\theta)$, which are a function of option delta, Δ, and a parameter vector θ. Numerical methods then give call prices c as functions of strikes X and hence define the RND as $g(X) = e^{rT}\partial^2 c/\partial X^2$. For options on futures, Δ is defined as a function of the Black-Scholes at-the-money volatility σ_A

$$\begin{aligned} \Delta(X) &= e^{-rT} N(d_1(X)) \\ d_1(X) &= \frac{\log(F/X) + \sigma_A^2 T/2}{\sigma_A\sqrt{T}} \end{aligned} \quad (8)$$

The spline function $\sigma(\Delta|\theta)$ is defined over $0 \le \Delta \le \exp(-rT)$. It is composed of linear pieces and cubic polynomials, defined on intervals determined by knot points $\Delta_1 < \Delta_2 < \cdots < \Delta_N$. Each cubic is defined over an interval from Δ_i to Δ_{i+1}, while the function is linear for $\Delta \le \Delta_1$ and $\Delta \ge \Delta_N$. The coefficients of the lines and cubics are constrained by the requirement that $\sigma(\Delta|\theta)$ and its first two derivatives are continuous functions. The spline function has N free parameters (Lange, 1998, p. 104) and there is a unique spline with the required properties that passes through a given set of N points (Δ_i, σ_i). Taking the knot points as given, the parameter vector is the corresponding set of implied volatilities $\theta = (\sigma_1, \ldots, \sigma_N)$.

Estimation of the RND parameters

The RND parameter vector θ is estimated once a month with 4-weeks to maturity so that the densities are nonoverlapping for each of the three density functions. For the MLN and GB2 densities, θ is obtained by minimizing the following average squared difference between observed market call prices and theoretical option prices:

$$\frac{1}{N}\sum_{i=1}^{N}(c_{\text{market}}(X_i) - c(X_i|\theta))^2 \tag{9}$$

with

$$c(X_i|\theta) = e^{-rT}\int_{X_i}^{\infty}(x - X_i)g(x|\theta)\mathrm{d}x, 1 \leq i \leq N$$

In these equations, N is the number of European option prices used for a particular day, $g(x|\theta)$ is a parametric density function, and $c(X|\theta)$ is the associated theoretical option pricing formula, given by either Equation (3) or (7).

The estimates of θ for the flexible densities are obtained by minimizing a function that combines two criteria, namely the accuracy and the smoothness of the fitted spline function $\sigma(\Delta|\theta)$. From N market prices, we derive implied volatilities and hence co-ordinates $(\Delta_i, \sigma_{\text{market}}\ (X_i))$. Then for a set of weights wi, we select $\sigma_1, \ldots, \sigma_n$ to minimize

$$\eta\sum_{i=1}^{m}w_i(\sigma_{\text{market}}(X_i) - \sigma_i)^2 + (1-\eta)\times\int_{\Delta_1}^{\Delta_M}\sigma''(\Delta|\sigma_1, \ldots, \sigma_m)^2\mathrm{d}\Delta. \tag{10}$$

There is a straightforward solution to this optimization problem (Lange, 1998, p. 111). Implementation requires making a subjective choice for the parameter η, that controls the trade-off between accuracy and smoothness. Following Bliss and Panigirtzoglou (2002, 2004), appropriate weights are proportional to option vega, so we use $w_i = \exp\left(-d_1(X_i)^2/2/\sqrt{(2\pi)}\right)$ with $d_1(X_i)$ given by Equation (8).

Historical densities

ARCH models for daily index returns are estimated and simulated to provide historical real-world densities. The simulated ARCH models must accommodate the stylized facts documented in the literature, including a time-varying conditional mean, a persistent conditional volatility and an asymmetric response of volatility to positive and negative returns. We choose the GJR-GARCH(1,1)-MA(1)-M specification, following Engle and Ng (1993) and Glosten *et al.* (1993). The conditional mean μ_t and the conditional variance h_t of the daily index return r_t are as follows,

$$\begin{aligned} h_t &= \omega + \beta h_{t-1} + (\alpha_1 + \alpha_2 D_{t-1})(r_{t-1} - \mu_{t-1})^2 \\ \mu &= \zeta + \lambda(h_{t-1})1/2 + \Theta(r_{t-1} - \mu_{t-1}) \\ D_t &= 1 \quad \text{if} \quad r_t \leq \mu_t \\ D_t &= 0 \quad \text{otherwise} \end{aligned} \tag{11}$$

Ten years of daily index returns prior to each estimation date t_i are used to estimate the ARCH parameters $\theta = (\omega, \beta, \alpha_1, \alpha_2, \xi, \lambda, \Theta)$, by maximizing the quasi-log-likelihood function, which assumes the conditional distributions are normal. These estimates are consistent even when the normality assumption is false (Bollerslev and Wooldridge, 1992).

The parameters obtained from the returns information up to selected times t_i are used to simulate the ARCH equations for 4-week periods that end on option expiry dates. A large number, M, of simulations of the final asset level $S_{T,i}$ are obtained for each month i. The historical real-world density $\tilde{g}_i$ is then the smooth function obtained by using the Gaussian kernel with bandwidth $H = 0.9\psi/\sqrt[5]{M}$ and with ψ the standard deviation of the simulated final levels. We have set M equal to 100 000. Our formula for the bandwidth H is recommended by Silverman (1992, p. 48) and used by Rosenberg and Engle (2002).

III. Data

The futures and options contracts are written on the FTSE 100 index and they are traded at the London International Financial Futures and Options Exchange (LIFFE). Futures and options have the same expiry month and share a common expiry time, 10:30 on the third Friday. European options can then be valued by assuming that they are written on the futures contract, and hence spot levels of the index are not needed.

Daily settlement values for futures and options prices with 4-week to maturity are obtained from LIFFE, for 126 consecutive expiry months from July 1993 to December 2003 inclusive. The call and put implieds for the same contract are almost identical, as should be expected from put-call parity. We average the call and put implieds and use them to calculate European call prices for each contract.

On average, 37 exercise prices are available for each month. The exercise prices are always separated by 50 index points. Table 1 provides summary information about the option prices.

There are 12 expiry dates per annum for the options but the futures contracts are traded for only one expiration date each quarter. Synthetic futures prices must be calculated for the remaining 8 months. Fair futures prices, F, are the future value of the current spot prices S minus the present value of dividends expected during the life of the futures contract

$$F = e^{rT}(S - \mathrm{PV(dividends)}) \qquad (12)$$

We have obtained actual dividend payments for the 100 component companies of the index from DataStream, and computed the present value of dividends by assuming that future expected dividends can be approximated by realized dividend payments.

Risk-free interest rates are collected from DataStream. We prefer the London Eurocurrency rate to the UK treasury bill rate, because the Eurocurrency rate is a market rate accessible to AA corporate borrowers.

Figure 1 shows typical estimates of the RND, which exhibit marked negative skewness that has already been extensively documented. The MLN and GB2 densities are almost identical, while the spline density differs in the left tail. The historical real-world density is very different from the risk-neutral ones, with much less skewness and kurtosis.

Table 1. Summary statistics for the dataset of FTSE 100 index option prices.
The average number of option prices is 37 per month for the 126 expiry months from July 1993 to December 2003 inclusive. The moneyness of a call option is defined by $X/F-1$, with $\boldsymbol{X}$ the exercise and $\boldsymbol{F}$ the futures price

Moneyness	No. of options	Percentage (%)
≤ -0.20	19	0.4
$(-0.20, -0.10]$	392	8.3
$(-0.10, -0.03]$	849	18.0
$(-0.03, 0.03]$	702	14.9
$(0.03, 0.10]$	731	15.5
$(0.10, 0.20]$	818	17.3
$(0.20, 0.30]$	572	12.1
$(0.30, 0.40]$	303	6.4
>0.40	335	7.1
Totals	4721	100.0

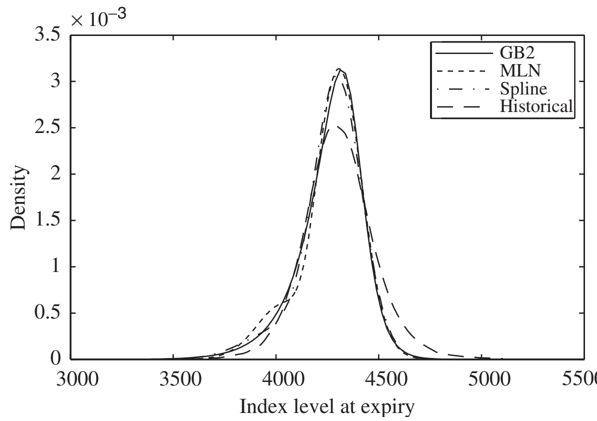

Fig. 1. Three risk-neutral densities and the historical density on March 21, 1997

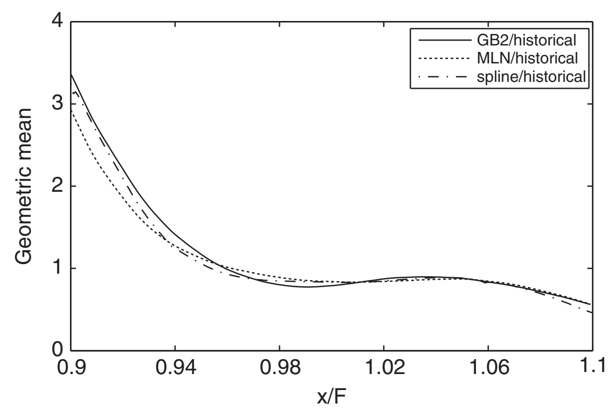

Fig. 2. Empirical pricing kernels, as geometric averages across all expiry months

IV. Empirical Pricing Kernels

Empirical pricing kernels are estimated by using RNDs obtained from the options market and historical real-world densities from index returns. Three empirical pricing kernels $M(x) = e^{-rT}g(x)/\tilde{g}(x)$ are constructed for each option expiry date, with $g(x)$ either the MLN, GB2 or spline RND and $\tilde{g}(x)$ the historical density from GARCH simulations. The geometric mean of each set of kernels is computed, across expiry dates, to reduce the impact of the noise created first by fitting the RNDs and second, by using different sources of information to find g and $\tilde{g}$. We plot the geometric means of the three sets of ratios $g(yF)/\tilde{g}(yF)$ against the moneyness variable $y = x/F$ in Fig. 2. All three graphed kernels are generally decreasing functions of x/F, although they are almost flat between 0.98 and 1.05.

None of our empirical pricing kernels for the UK equity market resembles those of Ait-Sahalia and Lo (2000), Jackwerth (2000), Brown and Jackwerth (2002) and Rosenberg and Engle (2002) for the US market. These researchers estimate very clear hump-shaped kernels, using S&P 500 data that ends in 1995, which challenges economic theory and indicates that the representative agent has a risk-seeking utility function in some wealth region. The risk-seeking range obtained by Jackwerth (2000) is $0.96 \leq x/F \leq 1.01$, while Rosenberg and Engle (2002) obtain $0.96 \leq x/F \leq 1.02$. Brennan *et al*. (2006) also use FTSE 100 index options data to estimate the option pricing kernel, approximated by a three-term Chebyshev polynomial whose state variables are the real interest rate, the maximum Sharpe ratio, and volatility. The pricing kernel is upward sloping in the region $1.03 \leq x/F \leq 1.05$.

V. Conclusion

Using a data set from July 1993 to December 2003, we derive three series of RND, namely the mixture of two lognormal densities, the generalized beta distribution, and a flexible spline distribution from FTSE 100 index options data. We also fit GARCH models to the time series of FTSE 100 index returns and simulate historical real-world densities. The empirical pricing kernel obtained from the two sets of densities is broadly downward sloping and therefore consistent with economic theories.

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