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OPTIMAL DISTRICTING WITH ENDOGENOUS PARTY PLATFORMS

Emanuele Bracco*

Abstract

This paper proposes a theory of socially optimal districting in a legislative-election model with endogenous party platforms. We generalize the model of Coate and Knight (2007), allowing parties to strategically condition their platforms on the districting. The socially optimal districting reflects the ideological leaning of the population, so that parties internalize voters' preferences in their policy platforms. The optimal seat–vote curve is unbiased when voters are risk-neutral, and—contrary to previous findings—biased against the largest partisan group when voters are risk-averse. The model is then calibrated by an econometric analysis of the elections of U.S. State legislators during the 1990s.

JEL Classification: D70, D72, H7

Keywords: Districting, seat–vote curve, social planner, policy-motivated parties, office-motivated parties.

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1 Introduction

The districting process has been the object of increasing attention from politicians and scholars of both economics and political science. The way district boundaries are drawn can have a substantial effect on the implemented policy, influencing parties' decisions on their proposed platforms and determining the relation between citizens' voting behavior and the composition of the legislature. The classic Downsian benchmark suggesting that voters' preferences, and in particular those of the median voter, determine the policy outcome can be easily altered to someone's favor through strategic reapportionment. Over time, increased technological possibilities and the availability of more precise data on voters has greatly enhanced the potential of strategic reapportionment and its effects.

The existing literature has mainly concentrated on strategic redistricting, or gerrymandering. This is the process of re-drawing the district lines in order to maximize the chances of victory of one party, or insure re-election to incumbent representatives. Optimal districting has instead been neglected until very recently.

This paper contributes to this new strand of literature analyzing the districting from a normative perspective. In particular we focus on the case in which parties strategically choose their policy platforms vis-à-vis the districting. In addition, an empirical analysis of the potential welfare gain from implementing the socially optimal districting is performed with data of State Lower House elections of 28 U.S. States from 1992 to 2000.[1]

Our paper generalizes and expands Coate and Knight's (2007) analysis on socially optimal districting. Their paper describes a polity in which two parties propose exogenous policy platforms, and then bargain in the legislature over the policy to implement: the implemented policy is mechanically chosen as the average between the two parties' exogenous policy platforms, weighted by the seat-share of each party in the legislature. In this framework they define which is the optimal seat–vote curve. This also means that the implemented policy is smoothly related with the legislature composition. The seat–vote curve is a relation $S(V)$ which links the aggregate share of votes for the Democratic Party to its seat share in the legislature. There is a large empirical literature (surveyed in the next section) that attempts to estimate this function, and in particular its bias and responsiveness. An electoral system which gives rise to a biased seat–vote curve is one which awards more (or less) than half of the seats to the party that receives exactly half of the votes ($S(1/2) \neq 1/2$). Responsiveness indicates how the seat share awarded to a party changes with the variation of its vote share ($\Delta S/\Delta V$).

Their finding is that the socially optimal seat–vote curve is biased in favor of the party with a larger partisan base. Therefore the social planner needs to let the implemented policy mirror the policy preferred (on average) by the population. In the primary scenario, in which voters are risk-averse, the optimal policy leans towards the party with a wider partisan base. In a setup in which the implemented policy changes smoothly at the variations of the legislature composition, in order to reach the social optimum, the seat–vote curve needs therefore to reward the larger

[1] The dataset, compiled by Ansolabehere and Snyder (2002), has been kindly provided by Brian Knight, to whom we are greatly indebted. This dateset has been used for the empirical analysis in Coate and Knight (2007).

party, i.e. it needs to be biased in its favor.[2]

Our setup instead allows parties to strategically choose the policy platforms vis-à-vis the districting drawn by the social planner. The legislature then chooses by majority voting the policy to implement.[3] Parties compete to win the median district, i.e. the district that awards them the majority in the legislature, and the social planner anticipates their equilibrium policy platforms and the electoral behavior of voters in order for the expected implemented policy to match the social optimum.

The parties' Nash equilibrium strategy is to "follow" the districting (i.e. the median district's) ideological leaning, each shifting its policy platform leftwards (rightwards) if the median district contains more (less) Democrats than Republicans. The social planner therefore needs to design a median district that mirrors the population's ideological composition, so that both parties' policy platforms are endogenously dragged towards the social optimum.

This cancels the need for a seat–vote curve which is biased in favor of the majority party. There is thus no need to artificially increase the electoral weight of the votes obtained by the larger party. On the contrary, the optimal seat–vote curve has a bias in the opposite direction of Coate and Knight's (2007): a negative bias, that counterbalances the weight of the larger partisan group.

This counterintuitive finding depends on the electoral outcome under socially optimal districting. First of all, electoral competition leads parties to tie in the election for the median representative, and therefore to expect that each carry 50% of the districts. At the same time, at the national level, the party with a larger partisan base still obtains an absolute majority of votes. Such an electoral result—a majority of votes, but only half of the seats—is coherent, as anticipated, with a negatively biased seat–vote curve.

If we were to follow the prescription of Coate and Knight (2007), designing a system with a positive bias, we would need to create a median district heavily slanted in favor of the majority party, which would in turn push the expected implemented policy heavily towards the majority party's bliss point, away from the social optimum.

This paper is organized as follows: Section 2.2 surveys the related literature, Section 2.3 describes the theoretical model, in Section 2.4 we solve the model, Section 2.5 contains the econometric analysis, and Section 2.6 concludes.

2 Related Literature

The welfare and policy effects of districting are relatively unexplored topics in the existing literature. The two seminal papers on these topics are Besley and Preston (2007) and Coate and Knight (2007).

The latter has already been extensively discussed in the Introduction. The former investigates specifically the effect of districting on the policies proposed by

[2]The optimal bias is instead going to be zero in the special case in which voters are risk-neutral, which is not analyzed by Coate and Knight (2007). In the Appendix a more detailed analysis of Coate and Knight (2007) in case of risk-neutral players is given.

[3]For completeness and as a robustness check the legislative bargaining solution à la Coate and Knight (2007) is also analyzed.

parties, i.e. how parties which endogenously set their policy platforms take the districting into consideration in their decision-making. In Besley and Preston's (2007) model, parties strategically propose a nationwide platform after having observed the districting, and the implemented policy is chosen by the party which obtains a parliamentary majority. Their finding is that, as long as the policy space is linear, both parties' proposed policies will be dragged towards the same side towards which the districting is biased. If, instead, the policy space is multi-dimensional, the effect of the districting on parties' policy platform cannot be determined.

Most of the existing work on redistricting focuses on efficient partisan gerrymandering, i.e. how a party which is in control of the districting process can maximize its seat share or the probability of obtaining a working majority, or bipartisan gerrymandering, whose aim is to guarantee the re-election of incumbents. Owen and Grofman (1988), Sherstyuk (1998), and Friedman and Holden (2008) are the most representative examples.

Shotts (2002), Epstein and O'Hallaran (2004), Gilligan and Matsusaka (2006), and Gul and Pesendorfer (2010) give insight also into the policy and welfare consequences of redistricting. The last of these analyzes the effect of gerrymandering on parties' policy platforms at the U.S. federal level, where each party (through State governors or legislators) has power over a number of federal congressional districts. Their finding is that the party which has the power to gerrymander the majority of districts will stick to its preferred (and extreme) policy, while the "minority" party will try to gain votes by proposing a centrist policy.

Gilligan and Matsusaka (2006) also focus on the effect of partisan gerrymandering on policy platforms with a median-voter approach. Their paper compares the median voter's preferred policy with that of the median representative (i.e. the median voter of the median district). Their approach to the welfare analysis is done through analyzing how the maximum distance between these two policies varies with the number of voters and the number of elected representatives. Our approach differs from theirs in that we focus on the efficient policy outcome, i.e. the welfare maximizing (average) preferred policy, rather than the policy which has the approval of a majority of citizens, i.e. the median citizen's preferred policy.

Shotts (2002) and Epstein and O'Hallaran (2004) analyze minorities' representation and their welfare. In particular they analyze the effects of the existence of majority-minority districts, i.e. districts in which the majority of the population belongs to a minority (typically, African-Americans or Latinos). The former is a theoretical model that looks into the effects of the existence of these kinds of districts on the median legislator's preferred policy, in an institutional setup that mirrors the Federal House of Representatives (each party gerrymanders a share of the districts). The latter explores whether the interests of minorities (namely, African-American voters) can be better represented through majority-minority districts electing black Representatives (descriptive representation), or through districts in which the black constituents manage to substantially influence the elected officer's actions (substantive representation).

Our work also relates to the wider electoral-competition literature, and more generally to the political economy literature on the optimal design of political institutions. In particular our results can be easily related to those obtained by Wittman (1983), Calvert (1985), Callander (2008) and Bernhardt et al. (2009).

All these papers describe electoral models which extend the classic Downsian approach. Calvert's paper contains, as does this paper, simultaneously policy- and office-motivated parties. In both Calvert's and in the present paper the classic Downsian result of policy convergence to the median voter's position no longer holds. The combined effect of uncertainty, policy motivation and ideological preferences generates an outcome in which the proposed policies do not completely converge. Both in the present and in Callander's (2008) paper, as long as there is sufficient ideological heterogeneity among candidates, the more office-motivated candidates propose policies nearer to the efficient one. By contrast, Bernhardt et al. (2009) analyzes the welfare effect of policy divergence, as opposed to the classic Downsian convergence result, when voters' preferences are hit by some shocks of known distribution. Its findings are that a moderate policy divergence could be welfare enhancing, working as a sort of insurance for risk-averse voters against the uncertainty of the position of the optimal policy.

Within the wider literature on the optimal design of political institution we can cite Lizzeri and Persico (2001) on electoral systems, Lockwood (2002) on the distribution of tasks across tiers of government, Prat (2002) and Coate (2004) on campaign finance, Persson et al. (2000) on the pros and cons of parliamentary rather than presidential systems. All these papers overlook the effects of districting on electoral outcomes and welfare.

This paper's empirical analysis can be related to the long-standing literature which tries to estimate the seat–vote curve. The seminal paper of this literature is Kendall and Stewart (1950), which used British data and a log-odds functional form, and stated the famous "Cube Law," referring to the responsiveness of the British electoral system. Gelman and King (1994), among others, developed a statistical methodology to estimate the seat–vote curve and to predict the effect of majority-minority districts, also taking into account how the composition of districts affects the electoral outcome. Coate and Knight (2007) developed an econometric technique in order to estimate, through Monte Carlo simulations, the compositions of an electoral district (i.e. how many partisan Democrats, partisan Republicans and Independents there are in each district) through the electoral and demographic data at the district level and through opinion polls at the State level. The empirical analysis of this paper is largely in debt to their analysis, and is performed on the same dataset.

3 The Model

There exists a continuum of voters of mass 1 with Euclidean preferences defined over the policy space $[0, 1]$. Letting i be the bliss point of a given voter, and x the implemented policy, his or her utility is defined as:

$$u_i = \mathcal{L}(i, x) \tag{1}$$

where $\mathcal{L}$ is a generic symmetric loss function on the distance $|i - x|$.

Voters are characterized by their preferred policy, and are of three types: Democrats, Republicans, and Independents, also referred to as swing voters, of mass respectively $\overline{\pi}_D$, $\overline{\pi}_R$, and $\overline{\pi}_I$, with $\overline{\pi}_D + \overline{\pi}_R + \overline{\pi}_I = 1$. The Democrats' ideal policy is 0 and the Republicans' is 1. The social planner decides the districting ζ, i.e. how voters

are spread along a continuum of districts j. In district j there is a share π_{Dj} of Democrats, a share π_{Rj} of Republicans, and a share π_{Ij}, with $\pi_{Dj} + \pi_{Rj} + \pi_{Ij} = 1$.

Following Coate (2004), swing voters' bliss points ω are uniformly distributed as $U[m - \tau, m + \tau]$, with cumulative distribution function $F(\omega) = (\omega - m + \tau)/(2\tau)$, where m is the median swing voter. The median swing voter's bliss point is the realization of a random variable $m \sim U[1/2-\epsilon, 1/2+\epsilon]$, with cumulative distribution function $H(m) = (m - 1/2 + \epsilon)/(2\epsilon)$. The two distributions can be thought of as an idiosyncratic and an aggregate shock hitting independent voters' preferences.

It must be noted that swing voters are assumed to be on average (and *ex-ante*) centrist; the median swing voter's preferred policy is expected to be equidistant from the extremes of the policy line ($E[m] = 1/2$). This is a standard assumption that allows us to have a clear cut distinction between partisans and Independents, and captures the *ex-ante* uncertainty on independent voters' preferences. These two distributions are common knowledge, but the realization of m remains unknown to players.

To avoid the possibility of a swing voter being more extremist than party members, the following assumption is made:

Assumption 1. $\epsilon + \tau \leq 1/2$

Parties share the bliss point with their members, and in addition receive a positive utility windfall Ω in case of victory. Parties are therefore simultaneously office- and policy-motivated. The Democratic (Republican) Party strategically chooses its policy platform d (r) vis-à-vis the districting ζ in order to maximize its expected utility. The Democratic and Republican Parties' Nash equilibrium strategies are found by solving the following utility maximization problems:

$$\max_d E[U^D(d(\zeta), r(\zeta), \Omega)] \tag{2}$$

$$\max_r E[U^R(d(\zeta), r(\zeta), \Omega)] \tag{3}$$

Expanding the expected utilities:

$$E[U^D] = p\ [\Omega - \mathcal{L}(0, d(\zeta)] + (1-p)\ \mathcal{L}(0, r(\zeta))$$
$$E[U^R] = p\ \mathcal{L}(1, d(\zeta) + (1-p)\ [\Omega + \mathcal{L}(0, r(\zeta)]$$

where p is the Democratic Party's probability of winning. We will see how this probability depends on the districting ζ both directly and indirectly through parties' policies $\{\{d(\zeta), r(\zeta)\}$. Given that the expected utilities are continuous and defined over a compact set, existence of a Nash equilibrium is not an issue.[4] In each district, parties field candidates that run on their Nash equilibrium policy platforms $\{d^*(\zeta), r^*(\zeta)\}$, and commit to implementing their policy platform in case of victory. The issue of the credibility of the parties' commitment to the publicly announced policy platform is beyond the scope of this paper; nevertheless we would assume

[4]The existence of multiple equilibria is also excluded. As we will see in the next section, in the linear case uniqueness is implied by the fact that the equilibrium sees parties playing strictly dominant strategies. The risk-averse case is instead solved numerically, and neither in our simulations nor through graphical analysis did we find evidence of the existence of any other equilibrium than the one described in Section 2.4.2.

that parties, following the standard citizen-candidate models (Osborne and Slivinski [1996], Besley and Coate [1997]), would commit to a particular policy through fielding a candidate whose preferences corresponded to the party's policy choice. Alternatively, we can think of the proposed platforms as the result of a repeated game with voters as in Alesina (1988).

In each district voters vote sincerely for the candidate with the policy that is nearer to their ideal point. The tie-breaking rule is a random draw. The candidate with the most votes is elected as a Representative. The Representatives then meet in the legislature and decide through majority voting (again with a random draw as a tie-breaking rule) the policy to implement between d and r.

The districting that maximizes the social welfare, defined as the sum of the utility of all voters, is found through solving the social planner's maximization problem:

$$\max_{\zeta} W(\zeta) = \max_{\zeta} \sum_{i} E[\ U^i(d^*(\zeta), r^*(\zeta))\ ] \text{ s.t. } \zeta \in Z \tag{4}$$

with Z being the set of feasible districting.

Abstracting from geographical constraints, a districting ζ is feasible if the average share of voters of each type across districts matches the population's shares, i.e.:

$$\int_j \pi_{zj} = \overline{\pi}_z \text{ for } \forall z \in \{D, R, I\}$$

and each district contains an equal mass of voters, i.e.:

$$\pi_{Dj} + \pi_{Rj} + \pi_{Ij} = 1, \quad \forall j$$

This is the equivalent of having each voter belonging to one and only one district, and each district containing the same number of voters.

The final step of our analysis is to define the correspondence between the optimal districting ζ^* and the seat–vote curve $S(V, \zeta)$ generated by it. Summing up, the timing of the game is the following:

1. A districting plan ζ is implemented.
2. Parties commit to policies $\{d(\zeta), r(\zeta)\} \in [0, 1]$.
3. The position of the median voter m is drawn from its distribution.
4. Citizens vote and in each district and the candidates with the most votes are elected.
5. The elected legislature votes to decide between the two policies and the one that obtains a majority is implemented.

The equilibrium concept is the subgame-perfect Nash equilbrium.

4 Theoretical Results

In this section we will solve the model, obtaining the equilibrium policy platforms and the socially optimal districting. For simplicity of exposition we will to start from the special case in which voters and parties are risk-neutral.

4.1 Case 1: Risk-Neutral Voters

When voters and parties are risk-neutral, the utilities of parties and of a voter i are

$$U^D = \Omega - d, \quad U^R = -(1-d), \quad U_i = -|i-d|$$

if the Democratic Party wins, and

$$U^D = -r, \quad U^R = \Omega - (1-r), \quad U_i = -|i-r|$$

if the Republican Party wins.

We focus our attention on a single district j, which contains a share π_{Ij} of Independents, and a share π_{Dj} of Democrats.

In order to rule out a scenario in which the Republican Party's policy platform is more liberal than the Democratic Party's, and to keep the equilibrium policy platforms and the probability of winning for any party within the interior of the unit interval, one assumption is made:

Assumption 2. $\Omega < 2\epsilon$

In district j there will be a swing voter whose preferred policy is the midpoint between d and r who is indifferent between voting Democrat or Republican. The Democratic Party therefore carries the district if the partisan Democrats and the share of swing voters to the left of the indifferent voter add up to more than a 50%-share:

$$\pi_{Dj} + \pi_{Ij}\ F(\frac{d+r}{2}) > \frac{1}{2} \tag{5}$$

Using the formula for $F(\cdot)$, we can rearrange the expression bringing on the left-hand side the parameters related to the composition of the district:

$$\frac{2\pi_{Dj} + \pi_{Ij} - 1}{\pi_{Ij}} > 1 - 2\frac{\frac{r+d}{2} - m + \tau}{2\tau}$$

Moreover, given that $\pi_{Rj} = (1 - \pi_{Dj} - \pi_{Ij})$, we can rewrite this expression as

$$\frac{\pi_{Dj} - \pi_{Rj}}{\pi_{Ij}} > 1 - 2\frac{\frac{r+d}{2} - m + \tau}{2\tau} \tag{6}$$

and define the variable κ_j as the partisan slant or "Democratic leaning" of a district:

$$\kappa_j \equiv \frac{\pi_{Dj} - \pi_{Rj}}{\pi_{Ij}}$$

Each district j's behavior is therefore described by the value of its partisan slant in favor of the Democrats κ_j. The Democratic Party carries this generic district if κ_j is greater than the right hand side of (6).

This variable assumes the value zero if the district has the same proportion of partisans of each ideology ($\pi_{Dj} = \pi_{Rj}$) and its absolute value is inversely proportional to the amount of Independents. In addition through simple calculations we

can see how a district with $\kappa_j > 1$ is a safe Democratic district, i.e. a district in which the Democratic Party always obtains at least 50% of the votes. Similarly, a district with $\kappa_j < -1$ is a safe Republican district.

Ordering the districts according to their partisan slants κ_j, we can observe how the only district that matters in order to determine the implemented policy is the median one, that is the one determining the parliamentary majority. This means that given a certain median district's partisan slant $\tilde{\kappa}_M$, all the feasible districts $\zeta \in Z$ such that $\kappa_M = \tilde{\kappa}_M$ are equivalent from the point of view of parties and the social planner. They elicit the same strategic behavior from parties, and therefore give the same prescriptions to the social planner for the social optimum to be reached.

Starting from this consideration we can find a closed form solution for the probability of winning of the Democratic Party.

Lemma 1. *The probability* p *that the Democratic Party wins the election, conditional on parties' policy platforms* {d, r} *is*

$$p(d,r) = \frac{1}{2} + \frac{r+d}{4\epsilon} - \frac{1}{4\epsilon} + \frac{\tau}{2\epsilon}\kappa_M \tag{7}$$

where κ_M is the value of partisan slant of the median district in favor of the Democrats.

Proof. *See Appendix.* ■

As anticipated, the Democratic Party's probability of winning depends on the districting per se (κ_M) and on the parties' policy platforms. It increases the more the Democratic policy is to the right (in order to get more votes in the center), the more the Republican policy is extremist, and the more the median district is leaning towards the Democratic Party.[5] These effects are all larger the smaller is the aggregate uncertainty (ϵ).

Given the voting behavior just stated, we can solve the problem set up in (2)–(3) for this specific linear case:

$$d^* = \arg\max_d \{p(d,r^*)\ (\Omega - d) + [1 - p(d,r^*)]\ (-r^*)\} \tag{8}$$

$$r^* = \arg\max_r \{p(d^*,r)\ (d^* - 1) + [1 - p(d^*,r)]\ (\Omega - r + 1)\} \tag{9}$$

where, from Lemma 1, the probability of winning for the Democratic Party $p(d,r)$ depends on the policy proposals of the two parties. Thus, through simple first order conditions, we can find the Nash equilibrium (and dominant strategy) policy platforms given the districting.

[5] In our analysis we shall restrict our attention to the value of κ_M within the interval $[-1, 1]$. Technically, this insures our being in an interior solution for what concerns the policy platforms and the probability of winning. Moreover, as already stated, we know that any κ_j outside that range would generate a safe seat, i.e. a seat in which one of the two parties wins with probability 1. Allowing the median district to be a safe seat implies including in our analysis the case in which one party wins a majority of seats with probability 1, i.e. there is no meaningful electoral competition

Proposition 1. *The Democratic and Republican Parties equilibrium policy platforms are respectively:*

$$d^*(\kappa_M) = \frac{1}{2} - \epsilon + \frac{1}{2}\Omega - \tau\kappa_M \tag{10}$$

$$r^*(\kappa_M) = \frac{1}{2} + \epsilon - \frac{1}{2}\Omega - \tau\kappa_M \tag{11}$$

The equilibrium probability of winning therefore becomes

$$p^* = p(d^*, r^*) = \frac{1}{2}$$

Consistently with Besley and Preston (2007), the districting, represented by the leaning of the median district κ_M, drags the proposed policies towards the side towards which the districting is leaning. The office-motivation variable Ω drives the policies towards the center, in search of the vote of Independents.

This result relies on the symmetry of the loss function that enters into the parties' utilities. Because of this, the districting affects both parties' platforms in exactly the same way.

The next step is to calculate the social welfare function as defined in (4) for this specific case. Given a specific median voter's realization $\tilde{m}$, and an implemented policy t, the social welfare is:

$$w(t, \tilde{m}) = \overline{\pi}_D \ \mathcal{L}(0, t) + \overline{\pi}_R \ \mathcal{L}(1, t) + \overline{\pi}_I \int_{\tilde{m}-\tau}^{\tilde{m}+\tau} \mathcal{L}(x, t) \ f(x)dx \tag{12}$$

where $\overline{\pi}_I$, $\overline{\pi}_D$, and $\overline{\pi}_R$ are the average the share of Independents, Democrats and Republicans at the national level and $\mathcal{L}$ is the loss function already introduced. From the proof of Lemma 1 (in the Appendix) it is also known that the median district (and with it the general election) will be won by the Democratic Party as long as the position of the median voter $\tilde{m}$ falls to the left of a threshold level $\bar{m}$. More precisely the Democratic Party wins when

$$\tilde{m} < \frac{d + r}{2} + \tau\kappa_M \equiv \bar{m}$$

We can therefore write the social welfare function as

$$W(\kappa_M) = \int_{\frac{1}{2}-\epsilon}^{\bar{m}(\kappa_M)} w(d^*(\kappa_M), m) \ h(m)dm + \int_{\bar{m}(\kappa_M)}^{\frac{1}{2}+\epsilon} w(r(\kappa_M)^*, m) \ h(m)dm \tag{13}$$

where $d^*(\kappa_M)$ and $r^*(\kappa_M)$ are the equilibrium values of d and r.

The socially optimal districting, or the socially optimal median district (which is the same), is identified as the κ_M that maximizes (13). The social planner's aim is to design a median district such that the expected implemented policy $[p^* \ d^* + (1 - p^*) \ r^*]$ is equal to the efficient policy, i.e. to the average preferred policy. We define therefore κ_M^* as the socially optimal median district.

First a technical condition is stated:

Condition 1.

$$\theta \equiv \frac{\overline{\pi}_D - \overline{\pi}_R}{\overline{\pi}_I} \in [a, b]$$

with

$$a = \max[-1 + \frac{\Omega}{2\tau}, -1 + \frac{2\epsilon - \Omega}{2\tau}]$$
$$b = \min[1 - \frac{\Omega}{2\tau}, 1 - \frac{2\epsilon - \Omega}{2\tau}]$$

Proposition 2. *To implement a welfare-maximizing districting, the median district must have a partisan slant κ_M such that*

$$\kappa_M^* = \frac{\overline{\pi}_D - \overline{\pi}_R}{\overline{\pi}_I} \tag{14}$$

as long as Condition 1 holds. When Condition 1 does not hold, the socially optimal partisan slant κ_M satisfies the following inequality:

$$\kappa_M^* \geq \frac{\overline{\pi}_D - \overline{\pi}_R}{\overline{\pi}_I} \quad \textit{if} \;\; \theta \in [b, 1] \tag{15}$$

$$\kappa_M^* \leq \frac{\overline{\pi}_D - \overline{\pi}_R}{\overline{\pi}_I} \quad \textit{if} \;\; \theta \in [-1, a] \tag{16}$$

Proof. *See Appendix.* ■

The first part of the Proposition shows that the welfare maximizing districting implies a (socially optimal) median district which is slanted in the same direction as the overall population leaning. More specifically, the median district's partisan slant κ_M must exactly reflect the population composition. The median district could be a perfect microcosm of the whole community.

In case the distribution of ideologies in the population is particularly unbalanced (if Condition 1 does not hold, as in the second part of the Proposition), the socially optimal districting will imply a median district that is more "extremist" than the population as a whole. This depends primarily on technical reasons, which are thoroughly explained in the proof of Proposition 2.

Finally, even if we chose to follow the long tradition of abstracting from geographical constraints, some words are still needed on the feasibility of the socially optimal districting. Our simple conclusion is that the optimality condition on the median district imposed by Proposition 2 is always feasible. The most intuitive example for this would be to have the so-called "uniform districting," i.e. having all the districts with the exact same composition, which is of course the general population's composition as well. A more formal discussion of feasibility constraints is given in the Appendix.

The Seat–Vote Curve

The next step is to find the seat–vote curve that is consistent with the socially optimal districting. The seat–vote curve is a mathematical relation $S(V)$ that links the number of votes obtained by a party at the national level with the amount of seats won in the legislature.

In our model the policy to be implemented is decided by majority voting in the legislature, i.e. by the median legislator. Consequently, from the welfare point of view, whether a party has a slim or an overwhelming majority in the legislature is

irrelevant: the implemented policy would in fact be the same. In order to reach the welfare optimum it is required that the two policy platforms and the composition of the legislature are such that the expected implemented policy matches the socially optimal policy. For this reason it is structurally impossible to find a unique seat–vote curve that would maximize social welfare.

It is possible, however, to characterize the voting behavior of the median district in equilibrium. More precisely, we are going to investigate which party carries the median district (i.e. obtains a majority in the legislature) when both parties obtain half of the votes at the national level. In this way we can pin down the position of the seat–vote curve in one point at least ($V = 1/2$), and from that determine the direction of the bias.

The bias of the seat–vote curve is defined as

$$\alpha = S(1/2) - \frac{1}{2}$$

The value of the bias α represents how many seats over the cut-off of 50% the Democratic Party obtains in correspondence to 50% of the votes.

When each party obtains 50% of votes at the national level, the party which wins in the median district enjoys a bias in its favor, having obtained an absolute majority of seats with only half of the votes. Taking the Democratic Party as a reference, if the median district were carried by the Democratic Party, this would mean that the seat–vote curve is positively biased ($\alpha > 0$); if the median district were carried by the Republicans, then of course the bias would be negative ($\alpha < 0$).

A further definition is introduced: we define a bias as "reinforcing" if it favors the party with the largest partisan base, and "counterbalancing" if it favors the smallest partisan group, i.e.

$$\textit{reinforcing}: \text{ if} \begin{cases} \alpha > 0 \text{ and } \overline{\pi}_D > \overline{\pi}_R \\ \alpha < 0 \text{ and } \overline{\pi}_D < \overline{\pi}_R \end{cases}$$

$$\textit{counterbalancing}: \text{ if} \begin{cases} \alpha > 0 \text{ and } \overline{\pi}_D < \overline{\pi}_R \\ \alpha < 0 \text{ and } \overline{\pi}_D > \overline{\pi}_R \end{cases}$$

Lemma 2. *The seat–vote curve has a positive bias $\alpha > 0$ if the median district is more slanted towards the Democrats than the whole population, a negative bias $\alpha < 0$ if the median district is more slanted towards the Republican than the whole population, and no bias $\alpha = 0$ if the median district's partisan slant mirrors the population's partisan slant, i.e.:*

$$\kappa_M > \frac{\overline{\pi}_D - \overline{\pi}_R}{\overline{\pi}_I} \Rightarrow \alpha > 0$$

$$\kappa_M < \frac{\overline{\pi}_D - \overline{\pi}_R}{\overline{\pi}_I} \Rightarrow \alpha < 0$$

$$\kappa_M = \frac{\overline{\pi}_D - \overline{\pi}_R}{\overline{\pi}_I} \Rightarrow \alpha = 0$$

Proof. *See Appendix.* ■

This leads us to one of the major findings of this paper:

Proposition 3. *The seat–vote curve is unbiased ($\alpha = 0$) if the socially optimal districting is implemented and Condition 1 holds.*

If Condition 1 does not hold, the optimal seat–vote curve will be biased in favor of the party most represented in the population (reinforcing bias).

Proof. *See Appendix.* ■

If the population is ideologically "balanced enough" (i.e., Condition 1 holds), the socially optimal seat–vote curve must not be biased: no party should get any advantage from the districting. When the population composition is very extremist (i.e., Condition 1 does not hold), the bias will be reinforcing, i.e. it will favor the party that has the larger popular support.

The intuition of this can be easily explained. From Proposition 2 we know that as long as Condition 1 holds, in equilibrium the median district will be a microcosm of the whole population with

$$\kappa_M^* = \frac{\overline{\pi}_D - \overline{\pi}_R}{\overline{\pi}_I}$$

therefore its voting behavior will exactly reflect the national voting pattern.

If the parties, at the national level, each obtain half of the vote, the same will happen in the median district, with each party expecting to obtain a majority in the legislature with 50% probability. This implies an unbiased seat–vote curve.

In equilibrium, parties condition their platforms on the districting, following its ideological leaning; in turn, the social planner draws the districting in a way that reflects the population's average ideology. In this way, the social planner forces parties to "internalize" the population preferences, dragging both platforms towards the social optimum. The expected winning policy is already consistent with the social optimum, and there is no need to give further advantage to any party through a biased seat–vote curve.

This first result of our paper, for the special case in which players are risk-neutral, is in line with what would have happened to the results of Coate and Knight (2007), had they considered the risk-neutral case.

4.2 Case 2: Risk-Averse voters

We now analyze the more general case in which parties and citizens are risk-averse. We assume that preferences are symmetric around the voter's bliss point, and strictly concave. The form of the loss-function becomes: $\mathcal{L}(i, x) = -|i - x|^\rho$ (with $\rho > 1$) and citizen i's utility if policy x is implemented becomes:

$$U_i = -|i - x|^\rho$$

Analogously, parties' utility functions are, if the Democratic Party wins:

$$U^D = \Omega - d^\rho,\ U^R = -(1 - d)^\rho$$

while if the Republican Party wins:

$$U^D = -r^\rho,\ U^R = \Omega - (1 - r)^\rho$$

The same procedure is followed as in the linear case. Nevertheless, it is not possible to find a closed form solution, not even for specific values of ρ. In order to simplify

the exposition, and rule out the possibility of the Democratic Party's running on a more conservative policy than the Republican Party, Assumption 2 is substituted by the following:

Assumption 3. $\Omega = 0$

The results exposed in this section would still hold in case the office-motivation parameter Ω were small enough. The threshold level, though, is closely dependent on the population composition and on the degree of risk-aversion. For this reason, we chose to focus on the more basic case in which parties are solely office-motivated.

Both parties will therefore choose their policy platforms in order to maximize the following expected utilities:

$$U^D = p(d,r)\ \mathcal{L}(0,d) + [1 - p(d,r)]\ \mathcal{L}(0,r) \tag{17}$$
$$U^R = p(d,r)\ \mathcal{L}(1,d) + [1 - p(d,r)]\ \mathcal{L}(1,r) \tag{18}$$

Each party's expected utility—as represented by (17)–(18)—is single-peaked with respect to each party's own strategic variable.

The pure strategy Nash equilibrium policy platforms $\{d^*(\kappa_M), r^*(\kappa_M)\}$ can be found through the first- and second-order conditions; given Lemma 1 and knowing the equilibrium policy platforms, the equilibrium probability of winning for the Democratic Party $p^*(\kappa_M)$ is then found. With these values in hand, we calculate the social welfare as in (13), which is then maximized with respect to the variable that determines the districting, i.e. the slant of the median district κ_M.

Intuitively, parties will react more substantially to the districting the nearer is their policy platform to their ideal policy. This is given by the fact that with concave utility, the loss increases more than proportionally as one moves away from one's own ideal point. As it is not possible to find a closed form solution for the slant of the socially optimal median district in the case of risk-averse players, the results of these section come from numerical simulations.

Conjecture 1. *When voters are risk-averse, there exist values of the parameters such that*

$$0 < \kappa_M^* < \frac{\overline{\pi}_D - \overline{\pi}_R}{\overline{\pi}_I} \ \ \textit{if} \ \ \overline{\pi}_D > \overline{\pi}_R$$
$$\kappa_M^* > \frac{\overline{\pi}_D - \overline{\pi}_R}{\overline{\pi}_I} > 0 \ \ \textit{if} \ \ \overline{\pi}_D < \overline{\pi}_R$$

There are values of the parameters such that the optimal slant for the median district is smaller in absolute value than in the linear case.[6] In other words, the

[6]Our numerical simulations consistently gave us this result as long as $\min[\overline{\pi}_D, \overline{\pi}_R] > 15\%$. This condition is met by all 48 contiguous States in the USA, according to the already cited NYT-CBS annual polls and *a fortiori* in all the States in our empirical analysis. The unicity of the pure-strategy Nash equilibrium has also been confirmed. The simulations are performed focusing on the cases where $\rho \in [1, 10]$ and $\tau \geq 0.3$. This last restriction allows us to focus on cases in which the swing voters' preferences are sufficiently heterogeneous, which is consistent with anecdotal knowledge and with the findings of our empirical analysis in Section 2.5. These results also hold as long as the office-motivation parameter Ω is small enough. Higher values of the parameter Ω may have two distinct effects. First, it may lead us to

median district needs to be slanted in the same direction of the whole population, but is going to be more centrist than the population as a whole. A sample of the simulation results is reported in Table 1.

The electoral behavior that follows from Conjecture 1 could be summarized as follows. Electoral competition leads parties once more to a draw in the median-representative elections, with each of them expecting to obtain 50% of the seats in the legislature. The median district leans towards the largest party, albeit to a lesser extent than in the linear case. This depends on the fact that the higher is the voters' risk-aversion, the more centrist will be the social optimum: partisan (i.e. extremist) voters' preferences carry more weight, the higher the concavity of the utility function, which forces the social planner to take the minority extremist group into greater consideration.

Still, parties' policy platforms are affected by the districting, and dragged in the same direction in which the general population leans, even if less than in the benchmark linear case. At the national level, with a more centrist indifferent voter, the vote share accruing to each party reflects more closely the population composition: the party with a larger partisan base receives an absolute majority of votes. A positive office-motivation would instead slightly turn the probability of winning in favor of the party with a larger partisan base, that is the one that, having a bliss point nearer to the socially optimal policy, is more keen on moving its policy platform towards it. This feature does not change the direction of the seat–vote curve bias, but reduces its value.

The effect on the seat–vote curve can be summed up in this way:

Conjecture 2. *When voters and parties are risk-averse, there exist values of the parameters such that the socially optimal seat–vote curve's bias is counterbalancing.*

This result is exactly the opposite of Coate and Knight (2007). Their prescription regarding the seat–vote curve is to have a reinforcing bias. If we were to implement that in our setup, the median district would be one which is more extremist than the whole population, and parties will move their policy platforms towards the majority party's bliss point, in a way that would make the expected implemented policy too extremist when compared to the social optimum. Voters' risk-aversion instead makes the social optimum closer to the ideological center, and allowing parties to endogenously condition their policy platforms on the districting implies that the social planner must counterbalance the tendency of parties to move their platforms too decidedly away from the center, following the districting.

4.3 Case 3: Parties with Asymmetric Office-Motivation

In this section we explore how our results change when parties differ in their office-motivation. To that end we perform the same analysis as in Section 4, simply assuming that the office-motivation parameter Ω is not equal across parties.

violate the assumption according to which the Democratic Party cannot run on a more conservative policy than the Republican Party. Secondly, in some cases in which the population's ideological distribution is very unbalanced and the risk-aversion parameter ρ is very high the social planner may need to reward the largest party in order to reach the social optimum, giving us opposite results from the one exposed in Conjecture 1.

We can therefore re-write the parties' expected utility as:

$$E[U^D] = p\ [\Omega_D + \mathcal{L}(0,d)] + (1-p)\ \mathcal{L}(0,r) \tag{19}$$
$$E[U^R] = p\ \mathcal{L}(1,d) + (1-p)\ [\Omega_R + \mathcal{L}(1,d)] \tag{20}$$

where Ω_D (Ω_R) is the Democratic (Republican) Party's office-motivation parameter. In order to rule out the cases in which the Republicans run on a more liberal platform than the Democrats, and to insure that the equilibrium policy platforms and the probability of winning are within the unit interval, Assumption 2 is modified as follows:

Assumption 2′. $(\Omega_D + \Omega_R) < 4\epsilon. \quad |\Omega_D - \Omega_R| < 4\epsilon$

By tracking exactly the same technique as in Case 1, we can see how the probability of winning for the Democratic Party as in Lemma 1 is not affected by the Ωs. The equilibrium policy platforms, nevertheless, are indeed going to be different:

Proposition 1′. *The Democratic and Republican Parties' equilibrium policy platforms are respectively:*

$$d^*(\kappa_M) = \frac{1}{2} - \epsilon + \frac{1}{2}\Omega_D - \tau\kappa_M \tag{21}$$
$$r^*(\kappa_M) = \frac{1}{2} + \epsilon - \frac{1}{2}\Omega_R - \tau\kappa_M \tag{22}$$

As one may have expected the more office-motivated party proposes a more centrist policy, being more willing to go after a higher winning probability to the detriment of its ideological coherence. The equilibrium probability of winning therefore becomes

$$p^* = p(d^*, r^*) = \frac{1}{2} + \frac{1}{8\epsilon}(\Omega_D - \Omega_R)$$

showing an ex-post electoral advantage for the more office-motivated party.

Our main result concerning the composition of the socially optimal median district hinges on the following modified condition:

Condition 1′.

$$\theta' \equiv \frac{\overline{\pi}_D - \overline{\pi}_R}{\overline{\pi}_I} + \frac{1}{16\epsilon\tau}(\Omega_D^2 - \Omega_R^2) \in [a, b]$$

with

$$a = \max[-1 + \frac{\Omega_D}{2\tau}, -1 + \frac{4\epsilon - \Omega_D - \Omega_R}{4\tau}]$$
$$b = \min[1 - \frac{\Omega_R}{2\tau}, 1 - \frac{4\epsilon - \Omega_D - \Omega_R}{4\tau}]$$

which helps us to rule out corner solutions.

Proposition 2′. *To implement a welfare-maximizing districting, the median district must have a skewness κ_M such that*

$$\kappa_M^* = \frac{\overline{\pi}_D - \overline{\pi}_R}{\overline{\pi}_I} + \frac{1}{16\epsilon\tau}(\Omega_D^2 - \Omega_R^2) \tag{23}$$

as long as Condition 1′ holds. When Condition 1′ does not hold instead, the socially optimal skewness κ_M satisfies the following inequality:

$$\kappa_M^* \geq \frac{\overline{\pi}_D - \overline{\pi}_R}{\overline{\pi}_I} + \frac{1}{16\epsilon\tau}(\Omega_D^2 - \Omega_R^2) \quad \textit{if} \;\; \theta' \in [b, 1] \tag{24}$$

$$\kappa_M^* \leq \frac{\overline{\pi}_D - \overline{\pi}_R}{\overline{\pi}_I} + \frac{1}{16\epsilon\tau}(\Omega_D^2 - \Omega_R^2) \quad \textit{if} \; \theta' \in [-1, a] \tag{25}$$

Proof. *See Appendix.* ■

Intuitively (focusing on the main case, when Condition 1′ holds) the social planner ought to shape the median district in a way that mirrors the population composition (first member on the right-hand side of equation [23]), but should also add a further "slant" in favor of the most office-motivated party (second member on the right-hand side of equation [23]), which is the party that is more willing to pander to the Independents' preferences. In a sense the social planner should favor the less ideological party. When turning to the seat–vote curve composition, these modified findings lead us to a richer result, which we can describe in the following Proposition:

Proposition 3′. *When players are risk-neutral and parties are not equally office-motivated ($\Omega_D \neq \Omega_R$), the seat–vote curve has a reinforcing bias in favor of the more office-motivated player. If the distribution of ideologies in the population is particularly unbalanced (Condition 1′ does not hold), then the optimal seat–vote curve will include a bias in favor of the party most represented in the population.*
Proof. *See Appendix.* ■

As long as Condition 1′ holds, when the socially optimal district is implemented, the bias α is going to be in favor of the more office-motivated party, i.e. it will be positive if the Democratic Party is more office-motivated ($\Omega_D > \Omega_R$) and negative (i.e. in favor of the Republicans) if the Republican Party is more office-motivated ($\Omega_D < \Omega_R$). When the population composition is very extremist (if Condition 1′ does not hold), the optimal bias will be further shifted in favor of the largest partisan group.

The intuitive reason why the social planner should award an advantage (bias in its favor) to the more office-motivated party is straightforward. In this way the social planner rewards the fact that the more office-motivated party proposes a policy platform nearer to the social optimum, because it will be more interested in winning the independent votes and increasing its probability to win.

A further feature of having asymmetric parties must be underlined, especially in light of the empirical evidence on elections. In Case 1 (risk-neutral players, symmetric parties) the social planner sets up a median district which is a microcosm of the whole nation (Proposition 2), and the optimal seat–vote curve stemming from it is unbiased (Proposition 3). This also implies that both at the national level and in the median district each party expects to receive exactly 50% of votes. Allowing parties to be asymmetrically office-motivated instead creates a more realistic electoral outcome. As pointed out above, the more office-motivated party enjoys a higher probability of winning than its opposition, and receives (on average) an absolute majority of votes both in the median district and at the national level. This mirrors more closely the majority of elections, in which there is an *ex-ante* frontrunner, but no party is 100% sure of winning on the day before the elections.

Having our theoretical model encompass this (very likely) situation will be of great help when calibrating the theoretical model on actual electoral data.

Case 4: Legislative Bargaining à la Coate and Knight (2007)

In this section, in order to have a closer comparison with Coate and Knight (2007), we check if our results in Cases 1 and 2 are dependent on the particular way the legislature decides on the implemented policy (majority voting) or if our results are robust to the adoption of the legislative bargaining solution that Coate and Knight (2007) assumed. In their model, parties represent the two extreme policy platforms (0 and 1), and the legislature implements a policy which is the average of the two parties' platforms weighted by their seat shares. In this section we keep our base case characteristics of endogenous party platform, but the policy to implement will be chosen through a bargaining process à la Coate and Knight (2007) and not though majority voting.

In order to compare more closely our result with the ones of Coate and Knight (2007), the analysis of risk-averse voters is performed assuming quadratic utility functions, and it is assumed that parties are solely policy motivated ($\Omega = 0$). The latter assumption is coherent with a polity in which there is no explicit ruling party, and where the implemented policy is the fruit of bargaining between the two parliamentary delegations.

This setup implies a particularly highly sophisticated social planner, who must anticipate the parties' endogenous policy proposals, their effect on voting in each district, on the composition of the legislature, and the consequent effect on the "bargaining" in the legislature. As in Coate and Knight (2007), we assume that voters vote "naively" for the party proposing the policy nearer to their ideal point, not taking into considerations forms of strategic voting in which citizens anticipate the bargaining in the legislature. The assumption that each party proposes a single nation-wide policy before the elections is also maintained, and Representatives will stand for that policy, and bargain from that position.

The timing of the game could be therefore be described as follows:

1. The districting is implemented.
2. Parties propose their policies $\{d, r\}$.
3. Voters vote in each district for the party whose policy is closer to their bliss point.
4. The legislature is elected.
5. Bargaining: the implemented policy $x = S\ d + (1 - S)\ r$ is the average between the proposed ones, weighted by the seats of each party, where S is the seat share won by Democrats.

For the sake of brevity, we are going to analyze the case of risk-neutral players (linear utilities) simultaneously with the risk-averse case. We define parties' preferences as:

$$U^D = \mathcal{L}(0, x),\ U^R = \mathcal{L}(1, x) \tag{26}$$

which in case of risk-neutrality becomes

$$U^D = -[S\ d + (1-S)\ r],\ U^R = -[1 - (S\ d + (1-S)\ r)] \tag{27}$$

which in the case of risk-averse players becomes

$$U^D = -[S\ d + (1-S)\ r]^2,\ U^R = -[1 - (S\ d + (1-S)\ r)]^2 \tag{28}$$

where S is the seat share obtained by the Democratic Party. The social planner, instead of picking the leaning of the median district—which would be meaningless in this setup—picks the welfare-maximizing bias and responsiveness of the seat–vote curve. In order to more easily compare our results to the ones of Coate and Knight (2007), we assume a linear functional form:

$$S = \frac{1}{2} + a + b\ \left(V - \frac{1}{2}\right) \tag{29}$$

where V is the quantity of nation-wide votes obtained by the Democratic Party, a the bias (how many more seats than 50% the Democratic Party would get with 50% of the votes) and b the responsiveness of the curve (how much the seat share would vary with a 1% variation in the vote percentage).

The amount of votes V the Democratic Party can expect to have is therefore:

$$V = \overline{\pi}_D + \frac{\overline{\pi}_I}{2} + \frac{\overline{\pi}_I}{2\tau}\left(\frac{r+d}{2} - \frac{1}{2}\right) \tag{30}$$

The Democratic Party will obtain the votes of all its partisans ($\overline{\pi}_D$) and of all the swing voters who are expected to be on the left hand side of the indifferent voter i, with ideology $(d+r)/2$.

Plugging this expression for Democratic votes (30) into the seat–vote function (29), and then plugging the resulting expression for S into the parties' utilities (26), we can maximize each party's utility function with respect to its policy proposal and obtain parties' reaction function and the equilibrium policy platforms.

Proposition 4. *If the implemented policy is decided through parliamentary bargaining, and the seat–vote curve is as in (29), the interior solution for parties' equilibrium policies is:*

$$d^* = \frac{1}{2} - \frac{\tau}{\overline{\pi}_I}(\overline{\pi}_D - \overline{\pi}_R) - \frac{\tau(2a+1)}{b\overline{\pi}_I} \tag{31}$$

$$r^* = \frac{1}{2} - \frac{\tau}{\overline{\pi}_I}(\overline{\pi}_D - \overline{\pi}_R) - \frac{\tau(2a-1)}{b\overline{\pi}_I} \tag{32}$$

independently of voters' attitude towards risk.

Proof. *See Appendix.* ■

The proposed policies keep some of the characteristics present in the "majority voting" base case. Policies will be more liberal if there are more Democratic than Republican partisans in the population. Again, the districting will equally affect the decisions of both parties: if the districting favors the Democrats ($a > 0$), both policies will be dragged towards the left.

Corollary 1. *In the setting of Proposition 4, the Democratic Party will obtain on average exactly 50% of seats (independently of voters' attitude towards risk), and the expected policy implemented by the parliament will be*

$$x^* = \frac{1}{2} - \frac{\tau}{\overline{\pi}_I}(\overline{\pi}_D - \overline{\pi}_R) - \frac{2a\tau}{b\overline{\pi}_I} \tag{33}$$

Proof. Plugging (31)–(32) into (30), and then back again into (29), we obtain the equilibrium expected share of seats of the Democratic Party in the legislature ($S^* = 1/2$), and the "bargained" policy as in (33). ■.

We can see how the implemented policy x^* maintains the now well-known characteristics: more liberal if there are more liberals than conservatives, and more liberal if the districting favors the Democrats. Moreover the elections, as in the absence of bargaining, will produce on average an absolutely balanced legislature.

Now that we know the equilibrium implemented policy (33), we can plug it in to the social welfare function as in (13). The social planner will decide the seat–vote curve (i.e. a and b) such that social welfare is maximized.

Proposition 5. *If voters are risk-neutral, the social planner should implement a seat–vote curve such that $a^* = 0$, while the optimal value of the responsiveness b is indeterminate. If voters are risk-averse, the optimal bias and responsiveness must be such that*

$$a^* = -\frac{(\overline{\pi}_D - \overline{\pi}_R)(2\tau - \overline{\pi}_I)}{4\tau} \cdot b^*$$

Proof. *See Appendix.* ■

Comparing Proposition 3 and Conjecture 2 with Proposition 5, it is clear that our findings are robust with respect to the way the legislature decides on the policy to implement: whether majority voting or legislative bargaining is used, the social planner should implement an unbiased seat–vote curve if voters are risk-neutral, and one biased against the largest partisan group if voters are risk-averse.[7]

We can now look a bit more deeply into the results of Proposition 5. To avoid confusion, we add the subscript l as in *linear* to indicate the risk-neutral case results, and q as in *quadratic* to indicate the risk-averse case results. In the case of risk-neutrality, the equilibrium policy platforms will be:

$$d_l^* = \frac{1}{2} - \frac{\tau}{\overline{\pi}_I}(\overline{\pi}_D - \overline{\pi}_R) - \frac{\tau}{b\,\overline{\pi}_I} \tag{34}$$

$$r_l^* = \frac{1}{2} - \frac{\tau}{\overline{\pi}_I}(\overline{\pi}_D - \overline{\pi}_R) + \frac{\tau}{b\,\overline{\pi}_I} \tag{35}$$

$$x_l^* = \frac{1}{2} - \frac{\tau}{\overline{\pi}_I}(\overline{\pi}_D - \overline{\pi}_R) \tag{36}$$

[7] The bias of the optimal seat–vote curve in the case of risk-averse voters is negative as long as $\overline{\pi}_I < 2\tau$. This is always the case according to the empirical simulations reported in next section.

while in the case of risk-aversion among voters:

$$d_q^* = \frac{1}{2} - \frac{1}{2}(\overline{\pi}_D - \overline{\pi}_R) - \frac{\tau}{b\ \overline{\pi}_I} \tag{37}$$

$$r_q^* = \frac{1}{2} - \frac{1}{2}(\overline{\pi}_D - \overline{\pi}_R) + \frac{\tau}{b\ \overline{\pi}_I} \tag{38}$$

$$x_q^* = \frac{1}{2} - \frac{1}{2}(\overline{\pi}_D - \overline{\pi}_R) \tag{39}$$

In the case of risk-neutral voters, assuming a linear seat–vote curve leads us to have an indeterminate responsiveness. Any responsiveness—i.e. any mean-preserving increase in the variance of the value of the implemented policy—will not have any effect on social welfare, given that both the seat–vote curve and the utility functions are linear. In the case of risk-aversion, optimal bias and responsiveness are inversely related. In this case the social planner has to choose an optimal bias–responsiveness bundle such that on average the implemented policy is equal to the socially optimal one, and the standard deviation of the implemented policy is minimized. Observing Propositions 4 and 5 we can see how a larger bias (and smaller responsiveness) increases the distance between the party platforms, while increasing the responsiveness b increases the steepness of the seat–vote curve, and with that the variance of the expected implemented policy. It means that the social planner has to choose between a more "stable" (low b) seat–vote curve, that nevertheless allows parties to be more extremist, or constrain parties to more centrist policies (low a), which in turn binds the social planner to a more responsive (and therefore risky) seat–vote curve.

The findings of the cases analyzed are summed up in Table I. We can see how the socially optimal seat–vote curve should be unbiased no matter what the constitution (parliamentary bargaining with exogenous or endogenous policy platforms, or majority voting with endogenous policy platforms) if voters and parties are risk-neutral, and parties are equally office-motivated. It should instead be biased against the largest party when voters and parties are risk-averse, while the "reinforcing" bias found by Coate and Knight (2007) is socially optimal only in the concurrence of exogenous party platforms, risk-aversion, and parliamentary bargaining.

5 An Empirical Analysis

In this section we estimate the possible welfare gain from implementing the socially optimal districting when voters are risk-neutral. The aim of this operation is to compare the result of our model to the one obtained by Coate and Knight (2007), and more generally to have a quantitative estimation of what is the welfare loss of not laying down the socially optimal districting. We will see that our empirical analysis is coherent with the theoretical prediction of the model as long as parties are asymmetrically office-motivated. To perform this analysis we need to calculate the welfare under the optimal districting W^{opt} and under the actual districting W^{act}. Both of these measures are calculated using the formula as in (13). Given that the value of the social welfare is within the $[-1, 0]$ interval, we define the welfare gain

as:[8]

$$W^{gain} = \frac{(1 + W^{opt}) - (1 + W^{act})}{1 + W^{act}} \tag{40}$$

The dataset has been kindly provided by Brian Knight, and it is the same data on which Coate and Knight (2007) performed their empirical analysis. It contains the electoral results of the elections of State Lower Houses, for the 5 electoral rounds held after the redistricting that followed the 1990 Census (1992, 1994, 1996, 1998 and 2000), together with demographic data on each district's population (e.g. average household income, percentage of African-Americans, elderly people, college graduates, share of urban, suburban and rural population). We also use State-level polling data from the New York Times-CBS survey,[9] in which a representative sample of voters is annually asked to self-identify as Democrat, Republican or independent. For the purpose of this paper the 1991–2000 average level of self-identification as Democrat, Republican, or independent in each of the 28 States is used.

From the raw data just described, we are in possession only of the shares of Democrats ($\overline{\pi}_D$), Republicans ($\overline{\pi}_R$) and Independents ($\overline{\pi}_I$) for each State. We instead need to estimate the uncertainty parameters (ϵ, τ), to identify the median district, and to calculate the Democratic skewness κ_M^{act} and each parties' office-motivation parameter $\{\Omega_D, \Omega_R\}$. Once we have been able to do so, we can calculate through our theoretical findings the optimal Democratic slant of the median district, the policy platforms of the two parties according to both the actual and optimal districting, and finally the welfare and the welfare gain.

5.1 Empirical Approach

In this section we explain the technique through which it is possible to estimate the composition of each district in terms of partisan Democrats and Republicans and Independents, and the uncertainty parameters.

First of all, we need to make more explicit the theoretical relation between the policy platforms proposed by the two parties, the composition of each district (and in particular the median district) and the amount of votes obtained by the Democratic Party. From Proposition 1′ we know that the theoretical equilibrium policies proposed by the two parties are:

$$d^* = \frac{1}{2} - \epsilon + \frac{1}{2}\Omega_D - \tau\kappa_M$$
$$r^* = \frac{1}{2} + \epsilon - \frac{1}{2}\Omega_R - \tau\kappa_M$$

[8] Coate and Knight (2007) defined voters' utility as

$$U_i = \gamma + \beta\ \mathcal{L}(i, x)$$

assuming $\beta/\gamma \geq 1$; we instead assumed a simpler functional form: $U_i = \mathcal{L}(i, x)$. According to their (more general) utility's functional form the value of welfare gain we find through (40) would be the *maximum* welfare gain. Choosing to use our simpler functional form, implicitly implies assuming that $\beta/\gamma = 1$. Our results about the welfare gain are perfectly comparable with the one by Coate and Knight (2007). In fact in their paper they mostly refer to the maximimum welfare gains, which correspond to the case when $\beta/\gamma = 1$.

[9] These data are available at web site http://php.indiana.edu/ wright1/cbs7603_pct.zip. Of this annual survey we used data from 1991 to 2000.

The share of votes V_j earned by the Democratic Party in a representative district j is

$$V_j = \pi_{Dj} + \frac{\pi_{Ij}}{2\tau}(\frac{r^* + d^*}{2} + m - \tau) \tag{41}$$

Substituting in d^* and r^* we obtain

$$V_j = \pi_{Dj} + \frac{\pi_{Ij}}{2\tau}\left[\frac{1}{4}(\Omega_D - \Omega_R) + \tau\ (1 - \kappa_M) + (\frac{1}{2} - m)\right] \tag{42}$$

where π_{Dj} is the share of Democrats in district j, π_{Ij} is the share of Independents in district j, κ_M is the Democratic leaning of the median district, m is the ideology of the median swing voter, which is the result of the realization of a random variable with mean 1/2.

It can be observed that while each Democratic partisan votes for the Democratic Party, only a share of the Independents does. The amount of Independents voting Democratic depends on three factors, represented within the square brackets in (42): first of all, a higher office-motivation leads the Democratic Party's policy more to the center, increasing its share of votes, secondly the policy platforms are affected by the districting κ_M (a more Democratic leaning districting drags the platforms leftwards, decreasing the Democratic vote share), and finally the share of votes hinges on the realization of the random shock hitting m.

The only parameter of (42) that can be directly found in the data is V_j: of each district we have at most five observations of the share of votes obtained by the Democratic Party in each election round. All other parameters need to be estimated.

Knowing the distribution of m, from (41) we can compute the mean $E[V_j]$ and standard deviation σ_{V_j} of the Democratic share of votes, for a given district j:

$$E[V_j] = \pi_{Dj} + \frac{\pi_{Ij}}{2\tau}\left[\frac{1}{4}\ \Omega_{DR} + \tau\ (1 - \kappa_M)\right], \quad \sigma_{V_j} = \frac{\pi_{Ij}\epsilon}{2\sqrt{3}\ \tau} \tag{43}$$

where $\Omega_{DR} = (\Omega_D - \Omega_R)$. Analogous arguments can be applied to obtain the mean and standard deviation of the Democratic votes at the State level:

$$E[V] = \overline{\pi}_D + \frac{\overline{\pi}_I}{2\tau}\left[\frac{1}{4}\Omega_{DR} + \tau\ (1 - \kappa_M)\right], \quad \sigma_V = \frac{\overline{\pi}_I\epsilon}{2\sqrt{3}\ \tau} \tag{44}$$

With a sufficiently long panel, the estimation of moments (mean and variance of votes to the Democratic Party in each district and overall in each State) would simply consist of reporting the sample average and standard error. In the U.S. though, redistricting happens every ten years, and therefore for each district our database contains at most five observations. Moreover there is a non-negligible number of uncontested districts, i.e. districts in which in all the five election rounds only one candidate ran. We therefore track the methodology already used in Coate and Knight (2007), calculating the districts' moments as a function of the demographic

characteristics of each district, using a bootstrapping technique over the following random effect model with heteroscedasticity:

$$V_{jt} = X_j'\theta_1 + \sigma_1\xi_j + u_{jt} \tag{45}$$

$$ln(u_{jt}^2) = X_j'\theta_2 + \sigma_2\nu_j + ln(\omega_t^2) \tag{46}$$

where V_{jt} is the share of votes for the Democratic Party in district j at the election round t, X_j is the matrix of controls, including the demographic data on the district and the State-level polls, θ_1 and θ_2 are the parameters to be estimated, and ξ_j and ν_j are the district-specific random effects, which are assumed to be normally distributed and with standard error σ_ξ and σ_ν, respectively. The last terms in the two equation are the usual white noise. The estimation technique is a standard two-step approach: first the percentage of votes to the Democratic Party (V_{jt}) are regressed on the district characteristics (X_j). Then the log of the square residuals of the first-step regression is regressed on the same set of variables. This two-step regression is performed on the whole dataset, containing 28 States. State-specific dummies are added. In this way we allow districts with the same demographic characteristics, but belonging to different States to have different voting behavior.

Nevertheless these regressions do not allow us to know the district-specific moments. Two sources of uncertainty are to be controlled: first of all we have the estimated and not the true values of the parameters of the two regressions $(\theta_1, \theta_2, \sigma_1, \sigma_2)$; secondly, we do not observe the random effects (ξ_j, ν_j). That is why a bootstrapping technique is used: for each replication $r = 1, 2, ..., 100$ a sample of size N is drawn with replacement from the dataset of N districts, where N is the number of districts with at least one contested election. Then the estimation with the standard two-step approach is performed for each of the 100 samples. In this way we obtain the entire distribution of the parameter estimates $(\theta_1^r, \theta_2^r, \sigma_1^r, \sigma_2^r)_{r=1}^{r=100}$. Nevertheless to learn the district-specific mean and standard error we need a further step in order to take the random effects into account. We therefore draw from a standard normal distribution the random effects (ξ_j^r, ν_j^r) for each replication r and each district j. Thus we can calculate the district-specific mean and variance:

$$E[V_j]^r = X_j'\theta_1^r + \sigma_1^r\xi_j^r \tag{47}$$

$$\sigma_j^r = \sqrt{exp(X_j'\theta_2^r + \sigma_2^r\nu_j^r)} \tag{48}$$

We are then able to calculate the State-specific moments averaging across districts within a State, and use the average estimated parameters from the bootstrap to calculate the simulated mean and variance of the voting distribution of uncontested districts.

5.2 Calibration of the Theoretical Model

After having performed these regressions, we can then proceed to the calibration of our theoretical results.

In this section we will indicate with a hat the parameters obtained from our empirical simulations, and with a tilde the ones that come from calibrating the

empirical findings into our theoretical model. In line with Coate and Knight (2007), we assume that

$$\epsilon + \tau = \frac{1}{2} \tag{49}$$

Let's recall (44), which describes the standard deviation of the votes for the Democratic Party in a particular State.

$$\widehat{\sigma}_V = \frac{\overline{\pi}_I \epsilon}{2\sqrt{3}\tau} \tag{50}$$

As our notation suggests, σ_V is obtained from the bootstrap procedure just described, $\overline{\pi}_I$ is instead assumed to be equal to the data from the NYT-CBS polls. From (49) and (50) we can therefore obtain the State-level indices of aggregate and idiosyncratic uncertainty ($\widetilde{\epsilon}$ and $\widetilde{\tau}$). Consequently we can calculate the district-specific share of Independents:

$$\widetilde{\pi}_{Ij} = \frac{2\sqrt{3}\ \widetilde{\tau}\widehat{\sigma}_j^r}{\widetilde{\epsilon}} \tag{51}$$

Lastly, in the next paragraphs we will identify the median district for each State, and calculate the value of its Democratic slant κ_M together with the office-motivation parameters.

In order to do this, we must first recall the formula to calculate the Democratic slant of a district:

$$\kappa_j = \frac{\pi_{Dj} - \pi_{Rj}}{\pi_{Ij}} \tag{52}$$

Plugging the expression for π_{Dj} from (50)—and knowing that $\pi_{Dj} + \pi_{Rj} + \pi_{Ij} = 1$—into (52), we obtain

$$\kappa_j = \frac{2\widehat{E[V_j]} - 1}{\pi_{Ij}} - \frac{\Omega_{DR}}{4\tau} + \kappa_M \tag{53}$$

Even if κ_M and the office-motivation parameter are still unknown, we are still able to order all the districts according to the value of $(2\widehat{E[V_j]} - 1)/(\pi_{Ij})$ State by State, and pinpoint the median district. For States with an even number of districts we arbitrarily chose to pinpoint the $\left(\frac{N}{2} + 1\right)$-th district as the median.

From Lemma 1 and Proposition 1 we know that where parties are equally office-motivated, each party will win with probability 50%, i.e. the expected amount of votes for the Democratic Party in the median district will be exactly 50%. Let's use the expression for the share of votes to the Democratic Party in a district, see (43), and apply it to the median district:

$$\widehat{E[V_M]} = \pi_{DM} + \frac{\widetilde{\pi}_{IM}}{2\widetilde{\tau}} \left[\widetilde{\tau}\ (1 - \kappa_M) + \frac{1}{4}(\widetilde{\Omega}_{DR})\right] = \frac{1}{2} + \frac{\widetilde{\pi}_{IM}}{8\widetilde{\tau}}(\widetilde{\Omega}_{DR}) \tag{54}$$

From our simulations we can directly obtain $\widehat{E[V_M]}$, while $\widetilde{\pi}_{IM}$ and $\widetilde{\tau}$ have been calibrated from (50) and (51). Consequently, from (54) we are able to extract a value for $\widetilde{\Omega}_{DR}$, the calibrated value of the extra office-motivation of the Democratic Party. It is not however going to be possible to disentangle the effects of each party's office-motivation. This, together with the requirement that office-motivation parameters be strictly positive, leads us to assume the following:

Assumption 4. $\widetilde{\Omega}_D = \max\{\widetilde{\Omega}_{DR}, 0\}$. $\widetilde{\Omega}_R = -\min\{\widetilde{\Omega}_{DR}, 0\}$.

Next, we are able to calculate the actual slant of the median district κ_M^{act}. To do so we need to go back to (44), which describes the voteshare of the Democratic Party at the national level. We report it below, indicating with a tilde the parameters already found through the calibration, and with a bar the values taken directly from our raw data:

$$\widehat{E[V]} = \overline{\pi}_D + \frac{1}{2}\overline{\pi}_I \ (1 - \kappa_M^{act}) + \frac{\overline{\pi}_I}{8\widetilde{\tau}} \ (\widetilde{\Omega}_D - \widetilde{\Omega}_R)$$

and obtain a value for the actual partisan slant κ_M^{act} of the median district.

With the calibrated values of these parameters in hand, we can calculate the welfare under the actual districting (through calculating the endogenous actual policy platforms as in Proposition $1'$) and the welfare under the socially optimal districting (through calculating the socially optimal median district's partisan slant as in Proposition $2'$ and the endogenous optimal policy platforms as in Proposition $1'$). This work is performed separately for each of the 28 States in our dataset.

In Table III we report some findings from both the empirical simulation and the model's calibration. In particular it is worth underlining how six out of 28 States (Mississippi, Montana, New Mexico, New York, Rhode Island, Utah) violate Assumption $2'$. In these States the values of the calibrated office-motivation parameters ($\widetilde{\Omega}_D$ and $\widetilde{\Omega}_R$) do not fulfil our model's assumptions, and therefore are discarded from our analysis. This rests on the fact that the data suggests that in these States one of the two parties enjoys a large *ex-ante* electoral advantage. In States where the data (and anecdotal knowledge as well) show a very strong predominance of one of the two parties there is *de facto* no electoral competition for a legislative majority, and therefore our model fails to be a good predictor of parties' behaviors.

In Table IV the results from the welfare analysis are spelled out. Unsurprisingly, allowing parties to strategically choose their platforms makes parties more willing to appeal to independent voters' preferences, however the actual districting may have been designed. Consequently, the welfare gain from implementing the socially optimal districting is quite small, mostly below 1%, even smaller than the one found by Coate and Knight (2007), where parties were constrained to run on extremist platforms. Our empirical finding therefore strongly reinforces Coate and Knight (2007) when they state that a shift towards a districting properly laid down by a benevolent social planner would not significantly increase the overall population's welfare.

6 Concluding remarks

We analyzed the socially optimal districting in a framework in which parties endogenously choose nationwide policy platforms vis-à-vis the districting. Parties react strategically to the districting, presenting policy platforms that will be more liberal the more the the districting leans towards the Democrats. We investigated what the districting should be in order to reach the welfare optimum. Secondly, we analyzed which seat–vote curve is consistent with the social optimum.

Our analysis shows that the social planner should design the districting so that the median district's composition reflects the population's ideological leaning, in

order to bring the expected implemented policy towards the welfare maximizing policy.

The consequent seat–vote curve that stems from this finding is unbiased in the special case in which players are risk neutral (linear preferences). In this case, the optimal median district must exactly match the general population composition, both parties win with equal probability by proposing policies skewed towards the larger partisan group in the population, i.e. equidistant from the welfare maximizing policy, and they both expect to get exactly 50% of votes at the national level. A tied election would therefore give way to the implementation of a policy which is more liberal if the population leans on the left, and more conservative if the population leans on the right, matching perfectly the welfare maximizing policy.

In the more interesting scenario in which voters and parties are risk-averse, the optimal seat–vote curve instead needs to be biased *against* the largest partisan group. In this case the social planner ought to draw a median district which is more centrist than the population, albeit still leaning in the same direction. Consequently the policy proposals will be—even if less decidedly than in the risk-neutral case—leaning towards the same side as the population (and the districting). The party with a larger partisan base will get more than half of the votes and will expect to win exactly half of the seats. For this reason the seat–vote curve will show a counterbalancing bias, i.e. a bias *against* the larger party in the population.

The results regarding the bias of the seat–vote curve are very robust: even in those cases where the party caucuses in the legislature bargain over the policy to implement (as in Coate and Knight, 2007), the bias remains null if voters are risk-neutral and counterbalancing if voters are risk-averse.

These results strongly contradict Coate and Knight's (2007) prescriptions regarding the socially optimal districting. In their seminal paper they modeled a polity in which parties' platform are exogenous, but the implemented policy is the result of a "bargaining" process in which the chosen policy is the average of the exogenous party platforms weighted by their seat share, i.e., the policy which maximizes the legislators' joint utility. In such a setup, the optimal seat–vote curve is unbiased when voters are risk-neutral, but requires a reinforcing bias when voters are risk-averse. This is because of the joint effect of the bargaining process in the legislature and the exogeneity of parties' policy platforms. The implemented policy moves quite smoothly with changes in electoral results, therefore the social planner has to give a seat advantage to the party that proposes the policy closer to the socially optimal one, so that the average legislator's preferred policy matches the average voter's.

An empirical calibration of the base model (majority voting and risk-neutral agents) is performed on data from the U.S. State Lower Houses' elections of 28 States for 5 electoral rounds (1992 to 2000). It appears that implementing the socially optimal districting would indeed increase the social welfare, but the size of this effect is even smaller than the one found by Coate and Knight (2007). Letting parties condition their policy platforms on the districting naturally constrains them to take swing voters' preferences into greater account. This in turn makes the size of the potential welfare gain from implementing the optimal districting very small: in the majority of States it remains below 1%.

Further research is needed in order to analyze in greater depth the political econ-

omy of districting and its policy and welfare consequences. The effects of migration across districts, the microfoundation of the process through which the legislature decides the implemented policy and the political-economy motivations that lead politicians to devolve or not the power of redistricting to independent commissions have yet to be analyzed.

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Appendix

Coate and Knight (2007) with risk-neutral voters

From Coate and Knight (2007) we know that the share of votes obtained by the Democratic Party is

$$V = \overline{\pi}_D + \frac{\overline{\pi}_I}{2\tau}\left(\frac{1}{2} - m + \tau\right) \tag{55}$$

We can rearrange this expression and obtain

$$m = \frac{1}{2} + \tau\left(\frac{\overline{\pi}_I + 2\overline{\pi}_D - 2V}{\overline{\pi}_I}\right) \tag{56}$$

The implemented policy is the weighted average of the Democratic Party's policy (0) and the Republican Party's policy (1), weighted by the seat share S. The policy is therefore $1 - S$. Consequently we can write a social welfare function W

$$W = -\left[\overline{\pi}_D(1-S) + \overline{\pi}_R\ S + \frac{\overline{\pi}_I}{2\tau}\int_{m-\tau}^{m+\tau} |(1-S) - x| dx\right] \tag{57}$$

that could also be transformed in this way:

$$W = -\left[\overline{\pi}_D(1-S) + \overline{\pi}_R\ S + \frac{\overline{\pi}_I}{2\tau}\int_{m-\tau}^{1-S} ((1-S) - x)\ dx + \int_{1-S}^{m+\tau} (x - (1-S))\ dx\right]$$

We can then maximize W with respect to S through simple first order conditions:

$$\frac{\partial W}{\partial S} = \overline{\pi}_D - \overline{\pi}_D + \frac{\overline{\pi}_I}{\tau}(1 - S - m) = 0 \tag{58}$$

Substituting into (58) the value of m as in (56), and solving for S we obtain the seat–vote curve:

$$S = \frac{1}{2} + \frac{2\tau}{\overline{\pi}_I}\left(V - \frac{1}{2}\right) \tag{59}$$

which is unbiased.

Proof of Lemma 1

The Democratic Party wins if it carries the median district, i.e.,

$$\pi_{IM}[F(\frac{d+r}{2})] + \pi_{DM} > \frac{1}{2} \tag{60}$$

Rearranging,

$$\kappa_M > 1 - 2F(\frac{r+d}{2}) \tag{61}$$

Swing voters are uniformly distributed around the median m as in $\omega \sim U[m-\tau, m+\tau]$, therefore

$$F(x) = \frac{x - m + \tau}{2\tau}$$

Substituting in (61):

$$\kappa_M > 1 - (\frac{\frac{r+d}{2} - m + \tau}{2\tau}) \tag{62}$$

Rearranging for m,

$$m < \frac{r+d}{2} + \tau\kappa_M \tag{63}$$

The median swing voter's bliss point m is distributed as in $m \sim U[1/2-\epsilon, 1/2+\epsilon]$, therefore

$$H(m) = \frac{m - \frac{1}{2} + \epsilon}{2\epsilon} \tag{64}$$

From (63) and (64), we can obtain the probability p for the Democratic Party to win:

$$p = H(\frac{r+d}{2} + \tau\kappa_M)$$
$$p = \frac{1}{2} + \frac{r+d}{4\epsilon} - \frac{1}{4\epsilon} + \frac{\tau\kappa_M}{2\epsilon}$$

□

Proof of Propositions 2 and 2′

The results of Propositions 2 and 2′ hinge on the fact that in equilibrium the endogenous policies fall within the following intervals:

$$d^* \in [\bar{m} - \tau, 1/2 + \tau - \epsilon], \text{ and } r^* \in [1/2 - \tau + \epsilon, \bar{m} + \tau] \tag{65}$$

If this is the case, getting rid of the modulus operator, the social welfare function as in (13) can be rewritten as

$$\begin{aligned} W(\kappa_M) = \int_{\frac{1}{2}-\epsilon}^{\bar{m}} \Big[\overline{\pi}_D \ (0 - d^*) + \overline{\pi}_R \ (d^* - 1) + \\ + \overline{\pi}_I \int_{m-\tau}^{d^*} [-(d^* - x)] \ f(x)dx \ + \overline{\pi}_I \int_{d^*}^{m+\tau} [-(x - d^*)] \ f(x)dx \Big] \ h(m)dm + \\ + \int_{\bar{m}}^{\frac{1}{2}+\epsilon} \Big[\overline{\pi}_D \ (0 - r^*) + \overline{\pi}_R \ + (r^* - 1) + \\ + \overline{\pi}_I \int_{m-\tau}^{r^*} [-(r^* - x)] \ f(x)dx \ + \int_{r^*}^{m+\tau} [-(x - r^*)] \ f(x)dx \Big] \ h(m)dm \\ \text{with } \bar{m} = \frac{d^* + r^*}{2} + \tau\kappa_M \end{aligned} \tag{66}$$

The second order condition show us that the function is always concave ($\frac{\partial^2 W}{\partial \kappa_M^2} = -\tau$), therefore the first-order conditions are necessary and sufficient.

Solving the first-order condition,

$$\arg\max_{\kappa_M} W = \frac{\overline{\pi}_D - \overline{\pi}_R}{\overline{\pi}_I} + \frac{1}{16\epsilon\tau}(\Omega_D^2 - \Omega_R^2)$$

From these results, simple algebra shows us that in equilibrium $d^*(\kappa_M^*)$ and $r^*(\kappa_M^*)$ belong to the intervals as in (65) as long as Condition 1—Condition 1′ in case of Proposition 2′—holds.

In the cases when Condition 1 (Condition 1′ in case of Proposition 2′) does not hold, the social welfare function is no longer equal to the one in (66). This is due to the fact that for some realizations of m, all the independent voters lie on the same side of the winning policy. This implies that the sign of some of the arguments of the absolute values in the loss functions changes with respect to (66).

As an example, here is the proof for one of the cases one could analyze:

Special Case: $\theta \in [b, 1]$ and $\Omega_D = \Omega_R = \Omega$

If $\theta \in [b, 1]$, simple algebra shows us that $d^* < \bar{m} - \tau$. Therefore the social welfare function is equal to

$$\begin{aligned} W(\kappa_M)' = \int_{\frac{1}{2}-\epsilon}^{d^*+\tau} \Big[\overline{\pi}_D\ (0 - d^*) + \overline{\pi}_R\ (d^* - 1) + \\ + \overline{\pi}_I \int_{m-\tau}^{d^*} [-(d^* - x)]\ f(x)dx\ + \overline{\pi}_I \int_{d^*}^{m+\tau} [-(x - d^*)]\ f(x)dx\Big]\ h(m)dm + \\ + \int_{d^*+\tau}^{\bar{m}} \Big[\overline{\pi}_D\ (0 - d^*) + \overline{\pi}_R\ + (d^* - 1) + \\ + \overline{\pi}_I \int_{m-\tau}^{m+\tau} [-(x - d^*)]\ f(x)dx\Big]\ h(m)dm + \\ + \int_{\bar{m}}^{\frac{1}{2}+\epsilon} \Big[\overline{\pi}_D\ (0 - r^*) + \overline{\pi}_R\ + (r^* - 1) + \\ + \overline{\pi}_I \int_{m-\tau}^{r^*} [-(r^* - x)]\ f(x)dx\ + \int_{r^*}^{m+\tau} [-(x - r^*)]\ f(x)dx\Big]\ h(m)dm \end{aligned} \tag{67}$$

Notice the difference between the first two lines of (66) and the first four of (67). Again, the second-order conditions on (67) show that the function is always concave.

The case we are analyzing is one in which $\overline{\pi}_D > \overline{\pi}_R$, therefore we need to show that

$$\arg\max_{\kappa_M} W \leq \arg\max_{\kappa_M} W' \tag{68}$$

The two welfare functions are twice continuously differentiable, strictly concave in κ_M. They are in fact simple parabolas. Therefore,

$$\begin{aligned} \frac{\partial W}{\partial \kappa_M} < 0, \quad \forall \kappa_M > \arg\max_{\kappa_M} W \\ \frac{\partial W'}{\partial \kappa_M} > 0, \quad \forall \kappa_M < \arg\max_{\kappa_M} \quad W' \end{aligned}$$

Let

$$\tilde{\kappa}_M \equiv \{\kappa_M |\ \frac{\partial W}{\kappa_M} - \frac{\partial W'}{\kappa_M} = 0\} \tag{69}$$

then

$$\arg\max_{\kappa_M} W \leq \tilde{\kappa}_M \leq \arg\max_{\kappa_M} W' \tag{70}$$

It can be calculated that

$$\tilde{\kappa}_M = 1 + \frac{\Omega - 2\epsilon}{2\tau}$$

therefore we need to show that

$$1 + \frac{\Omega - 2\epsilon}{2\tau} \geq \frac{\overline{\pi}_D - \overline{\pi}_R}{\overline{\pi}_I} \tag{71}$$

Considering that we are analyzing the case in which $\frac{\overline{\pi}_D - \overline{\pi}_R}{\overline{\pi}_I} \in [b, 1]$, expression (71) can be rewritten as

$$1 + \frac{\Omega - 2\epsilon}{2\tau} \geq \min\left[1 - \frac{\Omega}{2\tau}, 1 + \frac{\Omega - 2\epsilon}{2\tau}\right]$$

which is always true.

□

Proof of Lemmas 2 and 2′, and Propositions 3 and 3′

Lemma 2′ and Proposition 3′ are generalizations of Lemma 2 and Proposition 3. We proceed therefore just with the proof of the more general ones.

First, let's analyze for which realization of m the Democratic Party obtains exactly 50% of votes under the socially optimal districting. The votes the Democratic Party is going to obtain at the national level are

$$V(m) = \overline{\pi}_D + \frac{\overline{\pi}_I}{2\tau}(\frac{r+d}{2} - m + \tau) \tag{72}$$

while the votes obtained by the Democratic Party in the median district are

$$V_M(m) = \pi_{DM} + \frac{\pi_{IM}}{2\tau}(\frac{r+d}{2} - m + \tau) \tag{73}$$

Let's re-define in a more convenient way the skewness of the socially optimal median district:

$$\kappa_M^* = \frac{\overline{\pi}_D - \overline{\pi}_R}{\overline{\pi}_I} + \lambda \tag{74}$$

From Proposition 2′, we know that λ is going to be equal to zero when Condition 1′ holds and the parties are equally office-motivated ($\Omega_D = \Omega_R$). This can be summed up in this way:

$$\text{Condition } 1' \text{ does not hold, } \Omega_D = \Omega_R, \begin{cases} \theta \in [-1, a] & \rightarrow \quad \lambda < 0 \\ \theta \in [b, 1] & \rightarrow \quad \lambda > 0 \end{cases}$$

$$\text{Condition } 1' \text{ holds, } \begin{cases} \Omega_D > \Omega_R \rightarrow & \lambda > 0 \\ \Omega_D < \Omega_R \rightarrow & \lambda < 0 \end{cases}$$

Plugging in the equilibrium value of the policy platforms $\{d^*(\kappa_M^*), r^*(\kappa_M^*)\}$ from Proposition 1′ and the welfare maximizing value of the socially optimal districting as in (74),

$$V(m) = \frac{1}{2} + \overline{\pi}_I \; [-\frac{1}{2\tau} \; (m - \frac{1}{2}) - \frac{\lambda}{2}] \tag{75}$$

and we can obtain the realization of m, which we are going to call $\tilde{m}$, such that $V = 1/2$:

$$\tilde{m} = \frac{1}{2} + \frac{\Omega_D - \Omega_R}{4} - \tau\lambda \tag{76}$$

Now we can verify which party carries the median district when $m = \tilde{m}$, (i.e., $V_M(\tilde{m}) \lesseqgtr 1/2$) and enjoys a seat–vote curve bias in its favor.

If V_M is the quantity of votes for the Democratic Party in the median district,

$$V_M(\tilde{m}) = \frac{1}{2} + \frac{\pi_{Im}}{2}\lambda \tag{77}$$

This means that the Democratic Party will carry the district as long as $\lambda > 0$, the Republican Party will carry it if $\lambda < 0$, and elections will result in a tie if $\lambda = 0$. This proves Lemmas 2 and 2′ and Propositions 3 and 3′. □

Proof of Proposition 4

We define the bargained policy as

$$x = S\ d + (1 - S)\ r$$

Therefore the utility functions can be rewritten as

$$U^D = -x^q \tag{78}$$

$$U^R = -(1 - x)^q \tag{79}$$

where $q = 1$ in the risk-neutral case and $q = 2$ in the risk-averse case. We have already defined the seat–vote curve as

$$S = \frac{1}{2} + a + b\ \left(V - \frac{1}{2}\right) \tag{80}$$

Finally the votes obtained by the Democratic Party can be written as

$$V = \overline{\pi}_D + \frac{\overline{\pi}_I}{2} + \frac{\overline{\pi}_I}{2\tau}\left(\frac{r + d}{2} - \frac{1}{2}\right) \tag{81}$$

Substituting (81) into (80), and then (80) into (78) and (79), we obtain the fully explicit (and concave) utility function of each party, depending on the fundamentals of the economy and on the two platforms. The optimal Nash equilibrium platforms are derived from these simple maximization problems:

$$d^* = \arg\max_d \{U^D(d, r^*)\} \tag{82}$$

$$r^* = \arg\max_r \{U^R(d^*, r)\} \tag{83}$$

Given that we restrict our attention to the interior solutions, whatever the value of q the best-response and the equilibrium platforms are going to be the same. Analyzing the best response of the Democratic Party:

$$\frac{\partial UD^D}{\partial d} = -q\ x^{q-1} \cdot \frac{\partial x}{\partial d} = 0 \tag{84}$$

which, $\forall q \geq 1$, can be simplified as

$$\frac{\partial x}{\partial d} = 0 \tag{85}$$

which is the first-order condition for the linear case. The two first-order conditions for the two parties are orthogonal and give us Proposition 4. □

Feasibility: an Intuitive Argument

As already argued in the body of the paper, districts with the same median-district partisan slant κ_M are strategically equivalent, i.e., elicit the exact same strategic behavior from voters, parties and social planner and generate the same social welfare. Consequently there are very many different specific districtings that fulfil the optimality conditions.

This section gives an intuitive argument on why the feasibility constraints are unlikely to be binding. We impose specific and stringent assumptions on the districting, and find sufficient conditions for feasibility, and argue that these sufficient conditions are very unlikely to be binding. *A fortiori* in a general case, feasibility is hardly going to be an issue.

When voters are risk-neutral and parties are symmetric (see Proposition 2), the optimal median district has the same partisan slant as the whole population. Consequently a districting consistent with optimality is always feasible (one trivial example being uniform districting, in which all the districts have the exact same composition, which obviously is also the general population composition).

In case the parties are not symmetric, and/or voters are risk-averse instead of neutral, feasibility remains an issue.

Let's assume

Assumption 5. $\kappa_j = c \quad \forall j < M; \quad \kappa_j = C \quad \forall j > M; \quad \pi_{Ij} = \overline{\pi}_I \quad \forall j$

where j is the index referring to districts, ordered by κ_j, and M is the index given to the median district. With this assumption we restrict our attention to districtings in which all districts to the left of the median are uniform among themselves, and so are all the districts above the median. Obviously, just by the very definition of median, it is also true that $\kappa_c \leq \kappa_M \leq \kappa_C$. Moreover, again just to simplify the analysis, it is assumed that all districts contains the same amount of independents.

We also assume

Assumption 6. $\overline{\pi}_D \geq \overline{\pi}_R$

and then generalize our results to encompass the opposite case as well.

Let's find out which is the "most extreme" districting that is feasible under Assumption 5, i.e. so as to maximize $(\kappa_C - \kappa_c)$. We can re-distribute voters across districts so to have this "most extreme feasible districting" in the following way: one can move all Republican voters that belong to districts on the right-hand-side of the median, and move them on the left-hand side (substituting them with Democratic voters who belonged to districts on the left-hand side). The resulting districting is

the following:

$$\pi_{Dc} = \overline{\pi}_D - \overline{\pi}_R; \quad \pi_{Rc} = 2\overline{\pi}_R;$$
$$\pi_{DC} = \overline{\pi}_D + \overline{\pi}_R; \quad \pi_{RC} = \overline{\pi}_R - \overline{\pi}_R = 0;$$
$$\pi_{Ic} = \pi_{IC} = \pi_{IM} = \overline{\pi}_I$$

and the consequent partisan slants are

$$\kappa_c = \frac{\overline{\pi}_D - 3\overline{\pi}_R}{\overline{\pi}_I}, \quad \kappa_C = \frac{\overline{\pi}_D + \overline{\pi}_R}{\overline{\pi}_I}, \quad \kappa_M \in [\kappa_c, \kappa_C]$$

More generally, lifting Assumption 6, the most extremist feasible districting given Assumption 5 is such that

$$\kappa_M \in \left[\frac{\overline{\pi}_D - 3\overline{\pi}_R}{\overline{\pi}_I}, \frac{\overline{\pi}_D + \overline{\pi}_R}{\overline{\pi}_I}\right] \text{ if } \quad \overline{\pi}_D \geq \overline{\pi}_R;$$
$$\kappa_M \in \left[-\frac{\overline{\pi}_D + \overline{\pi}_R}{\overline{\pi}_I}, \frac{3\overline{\pi}_D - \overline{\pi}_R}{\overline{\pi}_I}\right] \text{ if } \quad \overline{\pi}_D < \overline{\pi}_R$$

Applying these formulae to the NYT-CBS poll data as in Table III, we can see how the feasible interval for κ_M is almost always a superset of the interval $[-1, 1]$. The only exeption is Rhode Island in which the feasible interval is $\kappa_M \in [-0.91, 1.44]$. This is largely due to the fact that in Rhode Island a very large share of the population declares itself as independent, and in Assumption 5 we constrained the social planner to uniformly distribute the independents across districts. Lifting that part of the assumption would allow Rhode Island's social planner to implement any district in the $[-1, 1]$ interval.

This simple exercise shows how only under very peculiar circumstances the social planner may not be able to draw a districting in which the median district partisan slant is any value within $[-1, 1]$.

Proof of Proposition 5

In the linear case,

$$W(a,b) = -\{\overline{\pi}_D\ x^* + \overline{\pi}_R\ (1 - x^*) +$$
$$+\frac{\overline{\pi}_I}{4\epsilon\tau}\int_{\frac{1}{2}-\epsilon}^{\frac{1}{2}+\epsilon}\int_{m-\tau}^{x^*}(x^* - y)\ dy + \int_{x^*}^{m+\tau}(y - x^*)dy\ dm\}$$
$$\text{where } \ x^* = \frac{1}{2} - \frac{\tau}{\overline{\pi}_I}(\overline{\pi}_D - \overline{\pi}_R) - \frac{2a\tau}{b\overline{\pi}_I}$$

In the quadratic case,

$$W(a,b) = -\{\overline{\pi}_D\ x^{*2} + \overline{\pi}_R\ (1 - x^*)^2 + \frac{\overline{\pi}_I}{4\epsilon\tau}\int_{\frac{1}{2}-\epsilon}^{\frac{1}{2}+\epsilon}\int_{m-\tau}^{m+\tau}(x^* - y)^2 dy\ dm\}$$
$$\text{where } \ x^* = \frac{1}{2} - \frac{\tau}{\overline{\pi}_I}(\overline{\pi}_D - \overline{\pi}_R) - \frac{2a\tau}{b\overline{\pi}_I}$$

First of all, the second-order condition with respect to x^* confirms that the function is strictly concave, therefore the first-order conditions are necessary and sufficient. The maximum is such that

$$\arg\max_{a,b} W(a,b) = \left\{ (a,b) \quad | \quad \frac{\partial W}{\partial a} = 0 \quad \text{and} \quad \frac{\partial W}{\partial b} = 0 \right\}$$

Solving this, we find that in the linear case $\frac{\partial W}{\partial a} = 0$ for $a = 0$, while $\frac{\partial W}{\partial b}|_{a=0} = 0$, leaving the value of b^* in the linear case indeterminate.

In the quadratic case the solution leaves one free variable, i.e., is such that

$$a^* = -\frac{(\overline{\pi}_D - \overline{\pi}_R)(2\tau - \overline{\pi}_I)}{4\tau} \cdot b^*$$

□

	$\overline{\pi}_D$	κ_M^*—Linear Case	κ_M^*—Quadratic Case	Is $\lvert\kappa_M^*\rvert$ smaller in the quadratic case?
$\overline{\pi}_R$=0.2	0.01	-0.241	-0.238	Y
	0.05	-0.2	-0.1875	Y
	0.1	-0.143	-0.128	Y
	0.15	-0.0769	-0.0625	Y
	0.2	0	0	Y
	0.25	0.0909	0.0625	Y
	0.3	0.2	0.125	Y
	0.35	0.333	0.1875	Y
	0.4	0.5	0.25	Y
	0.45	0.714	0.312	Y
	0.49	0.935	0.3625	Y
$\overline{\pi}_R$=0.3	0.01	-0.42	-0.3625	Y
	0.05	-0.385	-0.3125	Y
	0.1	-0.333	-0.25	Y
	0.15	-0.273	-0.1875	Y
	0.2	-0.2	-0.125	Y
	0.25	-0.111	-0.0625	Y
	0.3	0	0	Y
	0.35	0.143	0.0625	Y
	0.4	0.333	0.125	Y
	0.45	0.6	0.1875	Y
	0.49	0.905	0.2375	Y
$\overline{\pi}_R$=0.4	0.01	-0.661	-0.4875	Y
	0.05	-0.636	-0.438	Y
	0.1	-0.6	-0.37	Y
	0.15	-0.555	-0.3125	Y
	0.2	-0.5	-0.25	Y
	0.25	-0.429	-0.42	Y
	0.3	-0.333	-0.125	Y
	0.35	-0.2	-0.0625	Y
	0.4	0	0	Y
	0.45	0.333	0.0625	Y
	0.49	0.818	0.1125	Y

Table 1: Sample of Simulation Results. Verification that the median district's partisan slant is smaller in absolute value in the risk-averse case, for some values of the population composition, and given $\rho = 2$, $\Omega = 0$, $\epsilon = 0.1$, $\tau = 0.4$.

	Risk-neutral Voters	Risk-averse Voters
Exogenous Platform + Parliamentary Bargaining	No bias	Reinforcing bias Small responsiveness
Endogenous Platforms + Majority Voting	No bias	Counterbalancing Bias
Endogenous Platform + Parliamentary Bargaining	No bias	Counterbalancing Bias

Table 2: Socially optimal seat–vote curve bias according to the constitution and players' risk-aversion.

State	$\overline{\pi}_D$	$\overline{\pi}_R$	$\overline{\pi}_I$	$\widehat{E[V]}_m$	$\widehat{\sigma}_{Vm}$	$\widehat{E[V]}$	$\widehat{\sigma}_V$	$\widetilde{\epsilon}$	$\widetilde{\tau}$	$\widetilde{\pi}_{Im}$	$\widetilde{\Omega}_{DR}$	$\widetilde{\Omega}_D$	$\widetilde{\Omega}_R$	$\widetilde{\Omega}_D + \widetilde{\Omega}_R < 4\epsilon$	$\lvert\widetilde{\Omega}_D - \widetilde{\Omega}_R\rvert < 4\epsilon$	A1′
AL	36.77%	31.44%	31.79%	51.971%	0.0173	58.212%	0.0171	0.0784	0.4216	32.297%	0.2059	0.2059	0	Y	Y	Y
CA	40.11%	34.92%	24.97%	53.258%	0.0236	55.734%	0.0257	0.1313	0.3687	22.984%	0.4181	0.4181	0	Y	Y	Y
CO	28.84%	32.88%	38.28%	46.170%	0.0285	47.280%	0.0241	0.0895	0.4105	45.222%	-0.2782	0	0.2782	Y	Y	Y
CT	31.77%	27.07%	41.16%	51.522%	0.0553	54.880%	0.0384	0.1222	0.3778	59.172%	0.0777	0.0777	0	Y	Y	Y
DE	37.41%	28.82%	33.77%	47.128%	0.0304	48.596%	0.0311	0.1209	0.3791	33.073%	-0.2634	0	0.2634	Y	Y	Y
FL	37.03%	35.43%	27.54%	47.474%	0.0258	50.250%	0.0226	0.1107	0.3893	31.447%	-0.2501	0	0.2501	Y	Y	Y
IA	31.00%	29.69%	39.31%	48.595%	0.0303	49.433%	0.0281	0.0993	0.4007	42.285%	-0.1065	0	0.1065	Y	Y	Y
IL	36.53%	29.16%	34.31%	51.301%	0.0353	55.871%	0.0269	0.1069	0.3931	44.982%	0.0910	0.0910	0	Y	Y	Y
KS	28.18%	40.56%	31.26%	44.447%	0.0333	46.076%	0.0302	0.1254	0.3746	34.427%	-0.4834	0	0.4834	Y	Y	Y
KY	46.69%	31.43%	21.88%	51.813%	0.0200	53.962%	0.0191	0.1161	0.3839	22.904%	0.2431	0.2431	0	Y	Y	Y
ME	25.66%	29.50%	44.84%	51.888%	0.0392	52.178%	0.0343	0.1048	0.3952	51.157%	0.1167	0.1167	0	Y	Y	Y
MI	33.86%	30.35%	35.79%	47.853%	0.0252	53.533%	0.0225	0.0895	0.4105	40.062%	-0.1760	0	0.1760	Y	Y	Y
MO	35.20%	28.32%	36.48%	51.363%	0.0342	54.623%	0.0275	0.1036	0.3964	45.398%	0.0952	0.0952	0	Y	Y	Y
MS	39.78%	35.25%	24.97%	58.288%	0.0193	62.297%	0.0178	0.0989	0.4011	27.060%	0.9829	0.9829	0	N	N	N
MT	29.62%	33.97%	36.41%	44.496%	0.0245	45.003%	0.0246	0.0948	0.4052	36.359%	-0.4908	0	0.4908	N	N	N
NM	37.08%	35.58%	27.34%	54.838%	0.0208	54.527%	0.0197	0.0999	0.4001	28.887%	0.5360	0.5360	0	N	N	N
NV	32.29%	39.61%	28.10%	50.760%	0.0318	53.048%	0.0261	0.1218	0.3782	34.218%	0.0672	0.0672	0	Y	Y	Y
NY	40.23%	29.46%	30.31%	55.201%	0.0201	63.226%	0.0221	0.1009	0.3991	27.530%	0.6032	0.6032	0	N	N	N
OH	35.62%	32.17%	32.21%	45.180%	0.0360	50.607%	0.0287	0.1181	0.3819	40.317%	-0.3653	0	0.3653	Y	Y	Y
OK	48.33%	33.52%	18.15%	51.227%	0.0251	53.117%	0.0268	0.1692	0.3308	16.961%	0.1913	0.1913	0	Y	Y	Y
OR	36.56%	33.82%	29.62%	48.958%	0.0340	50.124%	0.0301	0.1303	0.3697	33.384%	-0.0923	0	0.0923	Y	Y	Y
PA	40.12%	36.08%	23.80%	47.958%	0.0341	52.118%	0.0278	0.1440	0.3560	29.181%	-0.1993	0	0.1993	Y	Y	Y
RI	30.76%	17.13%	52.11%	60.129%	0.0252	62.914%	0.0236	0.0678	0.4322	55.726%	0.6284	0.6284	0	N	N	N
SC	33.28%	35.65%	31.07%	48.111%	0.0172	50.852%	0.0172	0.0803	0.4197	31.088%	-0.2040	0	0.2040	Y	Y	Y
TN	35.62%	31.64%	32.74%	48.448%	0.0218	53.715%	0.0227	0.0969	0.4031	31.467%	-0.1591	0	0.1591	Y	Y	Y
UT	21.59%	48.12%	30.29%	41.152%	0.0251	41.992%	0.0245	0.1094	0.3906	31.067%	-0.8901	0	0.8901	N	N	N
VA	32.18%	33.66%	34.16%	48.689%	0.0246	51.139%	0.0264	0.1057	0.3943	31.829%	-0.1299	0	0.1299	Y	Y	Y
WI	32.88%	30.19%	36.93%	47.786%	0.0253	50.647%	0.0247	0.0941	0.4059	37.880%	-0.1898	0.0000	0.1898	Y	Y	Y

Table 3: Median district figures for each of the 28 States: Does Assumption 1′ hold?

State	$\overline{\pi}_D$	$\overline{\pi}_R$	$\widetilde{\epsilon}$	$\widetilde{\Omega}_D$	$\widetilde{\Omega}_R$	$\widetilde{\kappa}^{act}$	$\widetilde{d}^{act}$	$\widetilde{r}^{act}$	$\widetilde{\kappa}^{opt}$	$\widetilde{d}^{opt}$	$\widetilde{r}^{opt}$	Act. Welfare	Opt. Welfare	Welfare Gain
AL	36.77%	31.44%	0.07842	0.205857	0	-0.22692	0.620172	0.674083	0.247776	0.420051	0.473963	-0.42175	-0.40665	2.6113%
CA	40.11%	34.92%	0.131261	0.418104	0	0.032026	0.565982	0.619451	0.433583	0.417913	0.471382	-0.42822	-0.4208	1.2983%
CO	28.84%	32.88%	0.089481	0	0.27816	-0.13282	0.465042	0.504925	-0.23718	0.507887	0.547769	-0.38814	-0.38729	0.1399%
CT	31.77%	27.07%	0.122238	0.07773	0	-0.07151	0.44364	0.649252	0.122366	0.370401	0.576013	-0.3755	-0.37258	0.4679%
DE	37.41%	28.82%	0.120863	0	0.263417	0.163802	0.317034	0.427051	0.159727	0.318579	0.428596	-0.39206	-0.39206	0.0002%
FL	37.03%	35.43%	0.110731	0	0.250141	-0.12072	0.43626	0.532652	-0.03263	0.40197	0.498362	-0.41686	-0.41644	0.0713%
IA	31.00%	29.69%	0.099341	0	0.106497	-0.00426	0.402363	0.547798	0.015515	0.394442	0.539877	-0.38287	-0.38284	0.0050%
IL	36.53%	29.16%	0.106893	0.090991	0	-0.06954	0.465941	0.634232	0.227121	0.34932	0.517611	-0.39958	-0.39364	0.9885%
KS	28.18%	40.56%	0.125448	0	0.483357	-0.46759	0.549687	0.558905	-0.7068	0.639286	0.648504	-0.39858	-0.39523	0.5570%
KY	46.69%	31.43%	0.116132	0.243075	0	0.493605	0.315927	0.426653	0.780278	0.205882	0.316608	-0.4162	-0.41275	0.5912%
ME	25.66%	29.50%	0.104822	0.116693	0	-0.10897	0.496586	0.647884	-0.06509	0.479247	0.630545	-0.36484	-0.36467	0.0269%
MI	33.86%	30.35%	0.08954	0	0.175995	-0.20656	0.495244	0.586326	0.045398	0.391826	0.482908	-0.39896	-0.3943	0.7758%
MO	35.20%	28.32%	0.103578	0.095182	0	-0.00481	0.445921	0.605485	0.202386	0.363783	0.523347	-0.39126	-0.38815	0.5099%
NV	32.29%	39.61%	0.121768	0.06718	0	-0.433	0.575598	0.785544	-0.25437	0.508035	0.71798	-0.41192	-0.41023	0.2883%
OH	35.62%	32.17%	0.118057	0	0.365287	-0.16971	0.446762	0.500232	-0.07784	0.411675	0.465144	-0.42216	-0.4013	3.6097%
OK	48.33%	33.52%	0.169241	0.191348	0	0.617096	0.222323	0.46513	0.856858	0.14302	0.385827	-0.42216	-0.42043	0.2986%
OR	36.56%	33.82%	0.130258	0	0.092325	0.021713	0.361714	0.576067	0.081444	0.339629	0.553983	-0.40769	-0.4075	0.0330%
PA	40.12%	36.08%	0.144019	0	0.199265	-0.14814	0.408716	0.597121	0.121342	0.312786	0.501191	-0.42615	-0.42307	0.5361%
SC	33.28%	35.65%	0.080314	0	0.203974	-0.25261	0.525701	0.584342	-0.15343	0.484076	0.542718	-0.41058	-0.40993	0.1088%
TN	35.62%	31.64%	0.096931	0	0.159081	-0.20402	0.485302	0.599623	0.081081	0.370388	0.484709	-0.40717	-0.40181	0.9047%
VA	32.18%	33.66%	0.105704	0	0.1299	-0.19237	0.470146	0.616604	-0.06863	0.421357	0.567814	-0.39813	-0.3971	0.1713%
WI	32.88%	30.19%	0.094073	0	0.189796	-0.07909	0.438033	0.531281	0.013883	0.400292	0.493539	-0.39115	-0.3905	0.1064%

Table 4: Welfare Gain