Timetabling the major English cricket fixtures

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Abstract

This paper describes a sports timetabling problem that needs to be solved every year. Emphasis is placed on the structural complexity and the difficulty of defining and formulating the problem, taking into account all of the important requirements and preferences of the large number of stakeholders involved. The steps involved in practical implementation are described, and the benefits discussed. One key point made is the value of experience and deep immersion into the problem context on the part of the OR analyst.

Key words

Sports, cricket, timetabling, scheduling, multiobjective, metaheuristics, problem formulation, messy problems

Introduction

Every year the England and Wales Cricket Board (ECB) must timetable all the major cricket fixtures in England (plus a few in Wales, Scotland, Ireland and the Netherlands), apart from a small number of international matches which are timetabled by the International Cricket Council. This is a major undertaking which currently involves 21 clubs (mostly representing counties) playing about 420 matches in three overlapping competitions, as well as another 70 or so matches involving women’s teams, touring teams, ‘A’ teams, Under 19 teams and university teams. Some of these matches will require four days, others only a single day or just an evening.

Over the years the structure of the competitions has changed substantially. No year is exactly like the last, with the speed of change increasing in recent years. This is likely to continue as the competitive environment changes, as marketers and broadcasters continue to try to maximise their commercial returns and as the cricket authorities continue to strive to produce competitions and formats which best serve the interest of the sport.

The ECB has several stakeholders to consider when timetabling the fixtures, including clubs, spectators, administrators, sponsors, broadcasters and the England national team. It is impossible to give all of these exactly what they want, so the problem inevitably involves compromise, with the aim of giving a good enough outcome for all stakeholders.

For the past 19 years this has been achieved using a semi-automated computer system which has always been successful in producing timetables that are at least satisfactory for everyone. Recently the use of the system has been extended to undertake “what if” exercises for different structures that the ECB are considering. Without such help it could be dangerous for the ECB to decide upon significant changes, since there would always be the danger of unexpected negative consequences, or even of the problem becoming infeasible. All runs are made by me, the OR analyst and system creator, rather than by the ECB themselves, as the levels of complexity are so high.
Some generic fixture timetabling approaches have been developed and reported in the academic literature. However, these methods are not applicable to complex situations such as this. Normally they consider just one or maybe two objectives – typically the number of “breaks”, i.e. runs of home matches or runs without a home match, and/or distance travelled. While these are important considerations here also, there are so many further complexities that these approaches are not useful in this case.

Other practical sports timetabling implementations have been reported; these all show that, for the most part, a new approach has to be taken for every new problem. Nothing has been produced which is easily transportable from one context to the next except for the simplest of cases.

A good summary of both theoretical and practical studies is given in Kendall et al. (2009).

This particular problem, in its incarnation from the early 1990s, was reported and discussed in Wright (1994), but the situation is now far more complex in many ways, which are elaborated upon later in this paper.

### The 2010 season’s structure

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(adapted from the diagram produced by the ECB)

This was in fact version 56F of the structure. Some previous versions were quite similar, but others were very different. The ECB were very keen to explore a wide variety of different ideas concerning the structure, and to determine what could or could not be achieved.

The diagram shows a timeline from the start of April until the end of September for the possible “slots” into which matches can be placed. This has been modified slightly so as to accommodate some requests or requirements, and it would have been possible to make other minor changes if needed. Along the top are days of the week and dates, and on the right hand side are the competitions involved.
The squares plainly shaded represent possible time-slots for the County Championship (CC) matches. In this competition there are two divisions with nine county teams in each, with promotion and relegation between the divisions at the end of every season. The competition is double round-robin, i.e. every team plays against every other team in the same division twice, once at home and once away. The matches last up to four days of six hours per day, with each team having two innings. Points are awarded for winning, for tying or for drawing, and there are also bonus points for batting and bowling performances in each team’s first innings.

The number of main CC slots (the top line for each month) is thus 21, with allowable shifts of the starting day by one or two days represented in the second line for May, July and August, and by the third line in August. In theory it would appear to be possible to timetable all of these matches in 18 slots, with eight matches in each slot (four in each division). However, there are various reasons why this is impossible or undesirable. There are a number of other matches against touring teams, already timetabled, which involve some of the counties, who are thus unavailable at these times. There may be requests from a county to miss a particular slot, or adjust its starting date, or requests from the ECB for some slots to have fewer than eight matches. But the main reason is the overlap between the competitions, in particular when televised matches are involved; this is discussed further below.

The squares with a diagonal stripe represent one-day matches in the 40-over league (40L). In this league there are currently three groups with seven teams in each group. These are the eighteen county teams plus, in 2010, Scotland, the Netherlands and a team known as the Unicorns, selected from players in regional leagues. The groups were formed at random and were played as a double round robin. Matches last for about six hours and points are awarded for a win, a tie or a prematurely abandoned match (usually because of bad weather). If the match has come fairly close to a conclusion then the winners and losers are determined by the “Duckworth/Lewis” method (Duckworth & Lewis, 1998). There are also semi-finals and a final (not shown in the diagram) on predetermined dates.

The squares with a dotty background represent “Twenty-twenty” (T20) matches. In 2010 there were two regional groups of nine teams each, with matches again played as a double round robin. These are short matches (lasting under three hours, usually taking place in the evening) for which each team bats for 20 overs only. Points are awarded for wins, ties and abandoned matches, and again the Duckworth/Lewis method is used where appropriate. These also have quarter-finals, semi-finals and a final, not shown in the diagram, on predetermined dates.

If a square contains a “T” in the above diagram, it means that the ECB has agreed with a TV company that a match will be available for television on that date. The vast majority of televised matches are shown by one broadcaster, but there is another regional broadcaster that shows a few matches from its region; this explains why some squares have “TT” in them.

The nature of the competitions and their interest among spectators, sponsors, televisers and players varies markedly between competitions, which has a major effect on the timetabling requirements.

The County Championship is the most traditional form of the domestic game, and does not attract large crowds. It is seen by the ECB mainly as a means of preparing players for Test cricket (Test matches are international matches lasting up to five days, and these have a very high profile, especially within England). There are some local rivalries which bring in more spectators (for
example Lancashire against Yorkshire) and in the big cities (especially in London, which hosts two teams) some matches may be used as an opportunity for corporate hospitality which can substantially swell the ranks.

The 40-over league has been around in some form or other since 1969, though with changes of format along the way. This brings in larger crowds than the county championship, especially on Sundays and in some cases on Friday evenings.

However, the most popular format among spectators (though not among traditionalists!) is twenty-twenty. This has brought in new audiences many of whom would not consider attending any games of the longer formats. This format was introduced to the professional game in 2003 and has since swept the world, including at international level. For example, it has been reliably estimated (British Broadcasting Corporation, 2008) that the TV audience for the twenty-twenty World Cup final between India and Pakistan in 2007 may have exceeded 1.4 billion, i.e. more than 20% of the world’s population. While domestic competitions are obviously far less attractive than this, many counties fill their grounds for these matches. For this reason the number of professional twenty-twenty matches played in England has increased markedly since then, from 97 in 2003 to 151 in 2010.

Constraints and preferences

The first stage of the timetabling process is to ask the teams what their constraints and preferences are, using a questionnaire devised by the ECB. Some of the counties’ requests are quite straightforward to deal with. For example, most teams make requests for home matches on particular dates (often relating to “festivals” that many counties like to have at venues other than their normal grounds). Others will be unable to play home matches because of ground unavailability (often due to preparation for, or the playing of, an international match, but also because of other events such as pop concerts and ground redevelopment). There may also be clashes with other important events (such as Rugby League matches or even a Balloon Festival). Counties also make requests about preferred opponents, or preferred days of the week for one-day matches (and very occasionally for CC matches also). While some of these requests are clearly less important than others and there can be no guarantee that they will all be granted, they are mostly quite easy to understand and to prioritise.

However, there is scope on the questionnaire for other requests, and some counties make a lot of use of this, especially regarding the 40L and T20 matches. Some examples from 2010 are given in the Appendix. These can sometimes be difficult to interpret and to prioritise, especially since some counties make far more requests than others.

An extra complication for 2010 was that Scotland and Netherlands were potentially involved in two other competitions during the season and were therefore particularly restrictive as to when they could play their 40L matches. New 40L slots had to be created for them on May 30th, May 31st, July 31st and August 1st, interfering with the timetabling of the CC competition.

Other requests come from the ECB and from the sponsors. Often these concern specific time-slots. For example, there should be as many matches as possible in the final round of each competition; for other reasons this was also required in 2010 for July 20th to 23rd or 21st to 24th, as preparation for an upcoming Test match, and for July 4th. Also requested were:
- As many “local derbies” as possible on June 18th;
- No more than six matches in the first CC slot;
- If possible, no team to play in more than one CC slot in late June or early July;
- A good geographical spread of smaller grounds for the opening 40L slot;
- TV matches on Saturdays in May to be movable to the following day in case the TV company decides it doesn’t want them after all.

The ECB also had suggestions about possible dates and venues for some other matches as yet untimetabled, e.g. women’s matches and Under-19 matches. These preferences had to be borne in mind.

Over and above this are preferences that remain unsaid, but are still important. This is one area where detailed experience is vital. For example, some teams play at more than one ground, and it may be important to ensure that they play against their chief rivals at their largest ground; some counties prefer their important local derbies not to be played early or late in the season; consecutive home matches are unpopular (unless requested), especially for T20 matches; most counties prefer Fridays or Sundays for 40L and T20 home matches, with Mondays and Saturdays usually being the least popular days; no county likes to have more than five consecutive days of cricket; etc.

Following on from reading and interpreting the explicit requests and considering all the unspoken requests, I make a preliminary analysis which leads to several conversations between me and the ECB. Sometimes this concerns infeasibilities; for example, in 2010 there was one particular day on which more than half of the clubs in a 40L division wanted a home match, which is clearly impossible, so discussions had to be undertaken before deciding who was going to be disappointed. In addition, there are usually points of clarification to be discussed concerning the precise meaning of the clubs’ requests and decisions to be made as to their relative importance, and, in addition, further requests often come in as the process is under way. While the main focus is on the 420 matches in the three main competitions, the requirements of the other 70 matches need also to be considered.

Other requests for information, clarification, suggestions etc. involve further communication over a period of a few weeks; I have in my email folder well over 200 emails from my main contact at the ECB concerning the timetabling of the 2010 season, from original ideas about the structure to the final agreement of the detailed timetable.

**Television**

Before the main timetabling procedure can get under way, it is necessary to specify matches for TV that satisfy the broadcasters, since they are inevitably the most powerful people involved, bringing more money into English cricket than any other source. The specification of precise TV matches is usually not very important for slots where there will be plenty of choice, for which the broadcasters can make their decisions during the course of the season, but there are many days (notably Mondays and Saturdays, but also Tuesdays, Wednesdays and Thursdays for 40L matches) where there will probably only be one match available in each TV slot, so this has to be selected carefully. The TV company wants to have a good spread of grounds, but is also interested in the away teams since they like to televise every county a roughly equal number of times over the season. Travel is of some importance to them, but not overwhelmingly so, since they have the option of using two completely separate TV crews if matches on consecutive days are a long way apart geographically.
This is complicated by the fact that the midweek matches to be televised must normally take place in the evenings, which for all parts of the season except June and the first half of July means that floodlights must be available. Some counties have their own floodlights, but many do not. There is a set of mobile lights that can be used, but this is expensive to hire, to set up and to move around the country. Some counties would prefer not to play any of their home matches under lights, but in this they are often overruled by the ECB because it would make it impossible for them to negotiate a set of TV matches that the TV company would accept.

Even more tricky is making sure that the knock-on effects on the remaining timetable are not unacceptable. For example, the midweek matches in July and August often occur during a CC slot, meaning that the counties involved cannot have a match in that CC slot (though in some cases they may be able to use a “shifted” slot, as discussed earlier). This can have a major effect, occasionally even on the feasibility of the problem (there can be a danger of there simply not being enough possible slots for a team), but frequently on a team’s home/away balance of CC matches, which is an important consideration. Thus preliminary analysis is essential before selecting possible TV matches, including a series of “what if” runs of the model, so as to ensure that a good schedule can still be produced for all stakeholders.

Objectives

When the TV matches have finally been agreed (which can take a lot of time and consultation), the rest of the timetable for the three main competitions is addressed. The problem is set up as an optimisation model with large numbers of objectives and constraints.

Some objectives are relatively easy to formulate. For example, it is agreed that the two matches within a double round robin involving any pair of teams should not be too close together. If they are closer than a certain threshold, a penalty cost is applied which is larger the closer together the matches are. Similarly, if the number of matches in a slot is less than the number required, there is a penalty cost proportional to the square of the shortfall. There are also penalty costs for teams not having their requests met, with weights related to the perceived importance of these requests.

However, other objectives are less straightforward. For example, travel is clearly an important consideration, yet the issue is not just about overall travel distance, time or cost. A journey of 200 miles with a full day in which to undertake it would not be problematic; however, a journey of only 120 miles to be undertaken between matches on consecutive days certainly would be. In some cases, this could mean ending one match as late as 10 o’clock in the evening and starting the next one as early as ten thirty in the morning. Thus the travel-related element of the cost needs to be closely related to the conditions under which the journey will be made.

An even more complex requirement is that a team’s home matches should be well spread out, and any blank slots should also be spread out. This applies to each competition separately, since the audiences are usually very different. Everyone involved agrees that this is extremely important, yet there is no precise definition as to what it means. The approach taken is very similar to that reported in Wright (1994), and involves large numbers of weights, to be chosen with great care. For example, the cost applied to having the first two CC matches at home is greater than the cost applied to having two consecutive CC home matches in July, since no team likes to have a lot of home cricket in April; the weather is usually colder and the attendances are always smaller.
All the objectives are then combined using weights into a single complex cost function. There are two reasons for this: first that the number of objectives is large, and pure multi-objective Pareto methods have generally been found to be effective only for two-objective problems; and secondly that the ECB is not interested in evaluating several alternatives, but simply wants to be presented with one good solution. The values of the weights are determined largely from experience, based on what is most important and what is most difficult to achieve, but may need to be changed as the solution procedure progresses, as detailed below.

**Solution technique**

The solution procedure has changed slightly over the years, but has always been some kind of metaheuristic approach. The current method involves a variety of simulated annealing which uses a modified acceptance criterion depending on the effect not only of the overall change in cost but also the change in the individual subcosts, since this is a multi-objective problem. See Wright (2001) for a precise definition and explanation of this method.

Selection of neighbourhoods and perturbations is an important issue. Originally simple moves and swaps were tried, but this produced neighbourhoods which were far too small to be effective. Thus a number of more complex perturbations were included, including some based on Kempe Chains. Some of these features have not changed significantly since the 1994 paper (Wright, 1994), but then there was no overlap between the competitions; now that there is a substantial and increasing amount of overlap every year, there is a need for some perturbations that involve matches from more than one competition.

I make several runs of the system on my laptop, with a typical run taking anything up to 8 hours, with very slow cooling between a sufficiently high starting temperature to ensure sufficient diversification and a sufficiently low temperature to ensure sufficient intensification, though this is always followed by a straightforward local improvement from the best solution found to ensure that the final solution produced is a local optimum.

However, this cannot be a completely automatic process; interaction between myself as the OR analyst and the computer is necessary for a really good solution to be produced. This is partly because the weightings for the objectives have to be determined to some extent by trial and error; since the nature of the problem changes from year to year, the importance of various factors will change. For example, with the 40L competition having such a large gap in 2010, it is especially important for the number of games played by each team at the halfway stage to be as nearly equal as possible, and for no team to have played more than three home games in the first stage (if there is to be an imbalance of home matches, teams generally prefer more of them to be in July or August, *ceteris paribus*).

Moreover, even without such a structural change, it is never absolutely obvious from one year to the next which will be the hardest criteria to satisfy, and thus what problem areas are likely to remain even after a long run of the system. I have tried using automatic weight-changing methods (see for example Parr and Thompson, 2003) but they have so far proved ineffective for this problem. There is at present no alternative to a detailed inspection of the output followed by manual changes to one or more parameters. Clearly, in order to do this, I must draw on my experience concerning both what is likely to be achievable and what is likely to be acceptable. If I had not been involved over...
many years, starting from when the problem was much simpler, the timetables produced would inevitably not be as acceptable in practice as they are now.

After several runs a timetable is reached which appears to be as satisfactory as can be achieved, though of course there is no pretence to optimality. The proposed timetable is given to the ECB contact, who usually asks for one or two minor changes, and further runs are made to try to accommodate these if possible. Then the timetable is passed on to the counties, and again there will be a few requests for changes, some of which may be reasonable and others not, and of the reasonable ones some will be achievable without causing knock-on problems but others not. Eventually, the final timetable is accepted and published, and while one or two stakeholders may end up not entirely satisfied with what they have been given, the overall satisfaction levels are high.

**Contrast with original problem**

During the 1990s, the structure had different quirks (for example, sometimes a four-day match would be paused on a Saturday night, a Sunday match would be played and then the original match would resume on Monday), and there were some changes between years, but many of the current complications were absent.

For instance, for many years there were just two competitions to be timetabled, with more regular and clearly-defined slots, and a team’s opponents and venues were the same for each competition (i.e. if it had a home match against team X in one competition, it also had a home match against team X in the other competition). Therefore nearly every match could be linked together with its partner in the other competition, meaning that both competitions could be timetabled together. There were no days on which matches from both competitions needed to be scheduled; the TV requirements and preferences were much simpler; no cricket was played under floodlights; and the stakeholders were far less demanding!

Thus the OR intervention has been built up slowly from a relatively simple starting position to the current exceptionally complex one.

**Benefits of the system**

This system has several benefits:

- The what-if runs enable the ECB to modify the structure every year, trying out all sorts of different structures to satisfy fast-changing commercial imperatives;
- New innovations can be introduced with confidence – for example, no detailed study needed to be undertaken into the feasibility of including twenty-twenty cricket during the counties’ summer schedule, since the ECB had confidence (after consulting me) that the scheduling system would be able to cope.
- The ECB can give priority to developmental considerations concerning all levels of professional cricket;
- Counties and other teams get what suits them best commercially; for example, they are able to schedule other events (such as concerts) at their grounds in the confidence that this will not upset the cricket schedule, and host their festivals at the ideal times;
- The TV company knows that it will be given a schedule of matches for TV that fits in very closely with all of their requirements and preferences;
• Sponsors can be sure that the competitions will be well structured and stand a good chance of reaching a climax in the final slot;
• The service given to the TV companies and sponsors gives the ECB a strong negotiating hand;
• Spectators get a good balance of home matches to watch.

Lessons for other types of OR implementation

It has been emphasised that the system could not be as effective as it is if I had not built up experience over many years. A new consultancy company coming in afresh would find it very difficult to come up with outcomes which were anywhere near as satisfactory.

This goes back to the fundamental early principles of OR; as Russ Ackoff et al. (1962) said, “The more of the context of a problem that a scientist can comprehend, the greater are his chances of finding a truly adequate solution”. This is especially true for such a messy problem as this, with so many considerations and stakeholders; the OR analyst must become immersed.

Another lesson is the crucial importance of capturing all of the important considerations, requests and preferences from the start when formulating a model, since it will almost certainly be too difficult to include them at a later stage. Simple models may be appropriate in some contexts, but sometimes simplicity equates to inadequacy.

References


Appendix

This is a sample of some of the “other” questionnaire requests from the counties. Names of teams have been replaced by asterisks. It can be seen that most of these concern 40L matches and especially T20 matches, reflecting most clubs’ current priorities.

• At least 4 40L home games to be played on a Sunday afternoon and one floodlit game on a Thursday or Friday.
• All T20 home games to be under floodlight with Fridays the first choice, followed by Thursdays then Wednesdays. If we have to play a daytime game, we prefer to play on a Saturday rather than a Sunday. We would also request no more than one T20 (home) game per week.

• Twenty / 20 Cricket – We would request a minimum of six home games on Friday nights and Sunday afternoons (preferably a split of three Fridays and three Sundays). Forty Over Cricket. – We would hope for a minimum of four home games on Sundays, preferably five.

• TWENTY 20: as many Friday night matches as possible; no matches on a Saturday; no back to back home matches.

• We request two T20 matches on Tuesdays, Thursdays, Fridays and Sundays but not back-to-back matches. As we have not requested six Friday / Sunday T20 matches we would request five 40-over Sunday slots and a Thursday evening floodlit match.

• We don't think we will be able to play home matches on Saturdays due to lack of suitable grounds and even for away games it may prove difficult with premier league players contracted to their clubs. I don't see our home grounds being suitable for floodlit matches really and would favour a start to the season on 2 May.

• We … do not wish to have more than two home games in the month (April).

• Please can our Twenty20 matches be as evenly spread as possible? Happy with a Saturday if required.

• We are very keen to play as many (40L) matches as possible, when we are away from home, in batches to save on travelling costs. Also August weather has not been good in ***** of late for home matches.

• Our preferred days of the week are Thursday evenings followed by Fridays as the next option.

• ***** and ***** festivals should be at different times. This would assist both counties in ensuring that their festivals are well supported at these grounds.

• It would be helpful if a match at ***** could be delayed until the 24th or even 25th August to provide extra breathing space (after a Test match).

• We request that our home T20 game against local neighbours ***** takes place on a Friday evening with a strong preference for a mid July or late June slot (ideally Friday 16th July). If this was not possible our second preference would be for the game to be played on either a Saturday or Sunday. We would also request the maximum number of Friday night slots for home T20 games in the remaining schedule. Ideally no more than 2 home games in any week.

• On Sunday 4th July we would like a Home T20 v a non-prime (i.e. not ******, ******) to allow a Countywide Schools Competition final to be hosted pre a T20. We would also request a Thursday night home T20 against ***** with order of preference 17th June, 10th June, 24thJune. T20 home fixtures are more attractive on a Thursday evening, especially against the more popular locally based teams (*******, *****). This is due to possible clashes with (rugby league) fixtures below and corporate hospitality opportunities.