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R Rothschild, John S Heywood and Kristen Monaco

The Department of Economics
Lancaster University Management School
Lancaster LA1 4YX
UK

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R. Rothschild[a*], John S. Heywood[bc], Kristen Monaco[d]

[a]*Department of Economics, Lancaster University, Lancaster LA1 4YX, UK*
[b]*Department of Economics, University of Wisconsin-Milwaukee, Milwaukee, WI 53201, USA*
[c]*Department of Commerce, University of Birmingham, Birmingham B15 2TT, UK*
[d]*Department of Economics, California State University-Long Beach Long Beach, CA 90840 USA*

ABSTRACT

This paper explores the concept of *cooperative* merger between two of three entrants that arrive sequentially in a spatial market and practise discriminatory pricing. In this framework, in contrast to much of the theoretical literature, the so-called 'merger paradox' can be comprehensively overturned. We compare our results with those arising when one firm strategically locates two *plants*. Although this second problem is superficially similar to the first, the underlying behavior and implications differ in crucial respects. The welfare consequences of all of our results are demonstrated.

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Keywords: Spatial price discrimination; sequential entry; cooperative merger; merger paradox; multi-plant location.

* Corresponding Author: R.Rothschild@lancaster.ac.uk; Tel: +44 1524 594217; FAX: +44 1524 594244.

1. Introduction

In recent years economists have become increasingly interested in the theoretical and empirical issues that arise when firms merge. The theoretical interest derives largely from the work of Salant et al (1983), who demonstrated the existence of a so-called 'merger paradox'. This paradox arises when merger leaves the participants worse off, and the excluded firms better off, than they are in the pre-merger state. This result, although pervasive in the theoretical literature [see Pepall et al (1999)], sits uncomfortably alongside the observation that, in practice, resistance to merger is often widespread amongst those rivals not invited to participate [see White (1988)].

Several attempts have been made by theorists to identify conditions under which the paradox is overturned. While Salant et al adopt a conventional Cournot-Nash framework in order to obtain their results, other writers such as Gupta et al (1997), Rothschild et al (2000) and Heywood et al (2001) have considered the question within a 'spatial' framework, in which firms, having determined in advance their shares in the gains from merger, make simultaneous *noncooperative* equilibrium location choices and practise price discrimination. Yet, even in this context, the range within which the paradox is overturned is extremely small.

This paper explores the question again, but within a context where firms can *cooperate* in a limited fashion. We retain the assumption that individual firms profit maximize when locating prior to merger. However, instead of assigning an exogenous profit share to each participant (which ultimately influences its location) as in earlier work, we assume that the participants may enter merger agreements that help determine locations and allow side-payments. These agreements must be in the interest of each participant and be incentive compatible at each stage of the game. We show that such a *cooperative merger* may benefit the participants, as it encourages the parties to locate strategically against the excluded firm. Indeed, these locations can result in harm to the excluded firm. The framework that we use is a spatial model in which firms enter the market *sequentially.*[1] Thus, within this framework, we show that when firms locate with the prospect of co-operative merger in mind, there exist conditions under which the merger paradox is comprehensively overturned, and others under which it is partially overturned. The resulting outcomes can usefully be compared in terms of efficiency, as measured by the associated transport costs.

In identifying the circumstances under which the merger paradox can be partially or fully overturned, the paper highlights certain necessary conditions on firms' *cooperative behavior* and their position in an *entry sequence* which must be satisfied. We do not argue that these conditions will always be fulfilled in practice. However, since all earlier work on the merger paradox is based upon *noncooperative* behavior, the analysis of the role of cooperation amongst the merger participants - a natural practical consequence of merger - considerably extends our understanding of the phenomenon.

In the second part of the paper, the results obtained for the case of cooperative merger are compared with those for an apparently similar type of problem: the case where one of *two* firms in the market controls two *plants* which it locates either simultaneously or sequentially in order to maximize their joint-profits. As we shall show, although the structure of the latter problem is superficially similar to the former, the strategy of the two-plant firm against its rival is fundamentally different in both its character and its consequences from that employed by a pair of firms that cooperate under the prospect of merger.

From an empirical and policy perspective, the differences between cooperation and joint-maximization are important. One context within which this fact is of clear empirical relevance is the wave of mergers amongst large supermarket chains in the United Kingdom. Such rationalization is occurring on a large scale, but so too is the process of market entry by individual

[1] The assumption of sequential entry into such markets is not in itself new [see, for example, Rothschild (1976), Hay (1976) Prescott and Visscher (1977) and, relatively recently, Gupta (1992)], but the question of merger has remained neglected.

chains through a process of store proliferation, itself a common phenomenon in this branch of retailing.[2] Clearly, in the retail food industry at least, both merged and multi-plant entities possess degrees of market power which can be used to their advantage. While our results confirm this, they also demonstrate that the nature and extent of the power which can be exercised under the alternative arrangements are different, that these differences are to some degree a reflection of the precise combinations of firms (or plants) which exist, and that the order of entry into the market is itself important. All of these aspects of firms' behavior can be observed in practice. From the point of view of policy, therefore, models which address these questions have considerable potential value.

The paper is organized as follows. Section 2 describes the basic spatial framework in which the subsequent analysis is set. As a benchmark for our later results, we provide in this section a statement of an earlier formulation of sequential entry due to Gupta (1992), but where no merger occurs. Section 3 deals with the problem of cooperative merger under sequential entry. Section 4 analyzes and contrasts the 'two-plant' problem. Section 5 concludes.

2. The framework

We consider a market of unit length on which the distribution of consumers is uniform. A consumer located at x_i will buy one unit of the good from the firm that offers the lowest delivered price, $p(x_i)$. Each consumer has a reservation price, r, which is sufficiently high to ensure that all consumers are served.

Without loss of generality, production costs are assumed zero. The cost of transporting one unit of the good to a consumer at x_i is denoted $|x_i - L_i|t$, where L_i is the location of firm i and t is transport cost.

As a benchmark against which to evaluate the outcomes in the different

[2] An interesting variation on this empirical theme arises when merger occurs as an alternative to setting up multiple plants simply because zoning or other restrictions forbid the latter as a form of market entry.

cases, we use the result due to Gupta (1992), in which entry is sequential but no merger occurs. In her analysis, the market consists of three firms, 1, 2 and 3. An equilibrium location is denoted by L_i^j, where j is the order of entry into the market and i is the equilibrium position measured from the left (left, center or right). Thus L_2^3 indicates that in equilibrium the third entrant has the center position.

Each firm chooses its location so as to maximize its profit as given by the difference between the price, set by adjacent rivals' cost functions, and its own delivered cost. The permanent locations are made at the time of entry and in anticipation of the subsequent entrants' reaction functions. The profits to be maximized are

$$\pi_1^1 = \int_0^a (L_2^3 - x)t\,dx - 0.5t[(L_1^1)^2 + (a - L_1^1)^2] \tag{1}$$

$$\pi_2^3 =$$

$$\int_a^b (x - L_1^1)t\,dx + \int_b^c (L_3^2 - x)t\,dx - 0.5t[(L_2^3 - a)^2 + (c - L_2^3)^2] \tag{2}$$

and

$$\pi_3^2 = \int_c^1 (x - L_2^3)t\,dx - 0.5t[(L_3^2 - c)^2 + (1 - L_3^2)^2] \tag{3}$$

where $a = 0.5(L_1^1 + L_2^3)$, $b = 0.5(L_1^1 + L_3^2)$, and $c = 0.5(L_2^3 + L_3^2)$

Gupta's analysis yields the following equilibrium locations: $L_1^1 = 0.275$, $L_2^3 = 0.5$ and $L_3^2 = 0.725$. These in turn yield $\pi_1^1 = 0.074t$, $\pi_2^3 = 0.025t$ and $\pi_3^2 = 0.074t$. Total transport costs are $0.1009t$. This equilibrium is depicted in Figure 1.

Figure 1

3. Merger between pairs of firms.

In this section we retain the assumption that the industry contains three firms. This canonical case allows two firms to merge and one to be the excluded rival. We consider all merger combinations: $\{1, 3\}$, $\{1, 2\}$ and $\{2, 3\}$.

We assume that, in adopting cooperative strategic behavior against the excluded firm, each participant in the merger seeks the largest possible profit for itself, given its position in the entry sequence and subject to the requirement that, once all firms have located, neither it nor its partner could increase its profits by relocating. Thus, each participant seeks for itself the largest profit consistent with the need to maintain the cooperation which facilitates such profits.[3]

The general principle of location under cooperative merger is therefore as follows. Firms decide whether or not a merger should take place. If it does not, then locations and profit are as in Gupta (1992). If it does take place, the essence of the cooperative merger is an agreement on the position of the second entering partner among the three firms (left, center or right) in return for a specified side payment from the first partner. As in spatial models of non-cooperative merger, the location decisions are made prior to the merger but in anticipation of the merger. In contrast to models of non-cooperative merger, there is no exogenously given parameter determining how incremental profits from merger are shared between the partners. Instead, each partner maximizes its profit prior to merger (subject to fulfilling the agreement) as this sets the highest reservation takeover price at the time of merger. Given this maximizing behaviour, the side payment is the minimum required to increase the second entrant's profits to whichever is the larger of (a) its 'no-merger' (ie Gupta) profits, or (b) the largest profits it could obtain by reneging on its commitment to the merger when selecting its position.[4]

[3]The rationale for this assumption about participants' objectives derives from the observation that, in expectation of merging, firms will undertake actions to increase their value, and that in consequence each party will attempt to secure for itself the best possible outcome from the process. At the same time, however, each is aware that cooperation against excluded firms is a necessary step towards attaining this objective, and that partners in the merger must be given the incentive to participate in the process.

[4]Takeover prices or negotiated side payments above the minima may further reallocate the final profits between partners but are unnecessary for us to address further as they do not change location decisions. The only further requirement is that these payments not

The advantage of the cooperative merger for each participant is that profit increases by removing the risk that its eventual merger partner will choose a position 'unfavorable' to itself. It will become apparent that cooperative merger is not tantamount to 'joint-profit maximization'.[5] There is no detailed coordination of exact locations, only a 'trade' of position (left, center or right) for a side-payment. Moreover, as stressed, cooperative merger contrasts in an obvious way with that in earlier models of merger under spatial price discrimination [see Rothschild et al (2000) and Heywood et al (2001)]. In those models the parties, given their agreed shares in the eventual increment from merger, locate *simultaneously and noncooperatively* to maximize their profits.

The game consists of three stages. In the first stage the three firms locate sequentially, with the parties to the merger adopting their cooperative locations and setting the potential side payment; in the second stage firms decide whether to merge and, if they do, the side payment is made; in the third stage the market clears.

Merger between Firms 1 *and* 3*:*

When cooperative merger takes place between Firms 1 and 3, the merging entity will act as a Stackelberg leader vis-a-vis Firm 2, forcing the latter as far as possible towards its market endpoint. In doing this, however, the parties to the merger must ensure that, given L_1^1 and the expected location of Firm 3, Firm 2 will not prefer to locate left of Firm 1. Subject to this constraint, Firm 1 can maximize the extent to which 2 is forced towards its endpoint, only by using Firm 3, when it enters, as a 'buffer' between itself and Firm 2. For this purpose Firm 3 *must* locate in the interval between Firms 1 and 2, and, in terms of the merger agreement, at the midpoint.

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be so large that the profits of the first merging partner drop below its 'no merger' level or what it would receive by reneging on the commitment to the merger.

[5] In section 4 we contrast this case with the 'two-plant' problem, in which behavior is manifestly joint-profit maximizing.

If this outcome occurs, then $L_1^1 < L_2^3 < L_3^2$, and the profits functions (absent merger) are those given in (1), (2) and (3).[6] The incremental profits from merger are then

$$\pi^{m^*} = \int_0^a [(L_3^2 - x) - (L_2^3 - x)]t\,dx + \int_a^b [(L_3^2 - x) - (x - L_1^1)]t\,dx$$

To see how these locations will emerge, consider the following. Given Firm 1's location, 2 chooses L_3^2 in the expectation that 3 will select $L_2^3 = 0.5(L_1^1 + L_3^2)$. In this case, Firm 2's reaction function is $L_3^2 = (3L_1^1 + 4)/7$. These two values can be substituted into (3), and the expression set equal to $(L_1^1)^2/3t$, the maximum profits Firm 2 can obtain at a location to the left of Firm 1.[7] This ensures that 2 could not do better by locating in that interval. Solving yields $L_1^1 = 0.395$. This is therefore the farthest right that Firm 1 can move without rendering its left flank vulnerable to Firm 2. Given $L_1^1 = 0.395$, appropriate substitution in the foregoing expressions for L_2^3 and L_3^2 yields $L_2^3 = 0.568$ and $L_3^2 = 0.741$.

Further substitution, in π^{m^*}, yields incremental profits from the merger of $0.091t$. Firm 3 must now receive as a minimum side payment the amount necessary to discourage *it* from violating the merger agreement and locating to the left of L_1^1. At such a location it would obtain $\pi_1^3 = 0.052t$, an amount which exceeds its profits of $0.025t$ as the third entrant in Gupta's 'no-merger' state. The minimum side payment to Firm 3 is therefore equal to the maximum profits obtainable by 3 left of Firm 1, minus its profits at L_2^3, or $0.052t - 0.015t = 0.037t$. If Firm 1 makes no more than this payment out of π^{m^*}, then, at most, $\pi_1^1 = 0.129t$ and $\pi_2^3 = 0.052t$, at least.[8] Moreover $\pi_3^2 = 0.052t$, and total transport costs are $0.1265t$.

[6] We note that a, b and c are as defined for those three functions.

[7] More precisely, the profits of Firm 2 in this 'short-side' of L^1 are maximized at a location one-third of the way along the interval from the endpoint.

[8] As already assumed, however, a bargain over the increment from merger can further increase the profits of 3 and reduce those of 1.

Figure 2 shows the equilibrium locations for this case.

Figure 2

Proposition 1. *When cooperative merger takes place between entrants* 1 *and* 3, *the parties to the merger obtain higher profits, and the excluded firm lower profits, than in the no-merger state. The merger paradox is overturned, but cost efficiency decreases relative to the no-merger state.*

In this case, merger is profitable for the participants, harmful to the excluded firm and a source of cost inefficiency. Consequently, while such a merger might be expected to occur, it could well encounter resistance from outsiders and should therefore attract the attention of antitrust authorities.

Merger between Firms 1 *and* 2*:*

In this case, Firms 1 and 2 must ensure that Firm 3 does not locate between them, since this would eliminate any gain from the merger. Given this requirement, the optimal locations are $L_1^1 = 0.2754$ and $L_2^2 = 0.7246$. These locations yield to the merger participants the largest profits they can obtain, subject to the constraint that Firm 3 will find it most profitable to locate in one of the equal-sized intervals to the left of L_1^1 or right of L_2^2.

The reasoning is as follows. If Firm 3 locates between 1 and 2, it will always choose the midpoint of that interval in order to maximize its profits. When the two parties to the merger are at 0.2754 and 0.7246, respectively, it is easy to show by appropriate substitution in the foregoing profits expressions that Firm 3's profits at a location halfway between them are $0.0252t$. Conversely, if 3 locates to the left of 1 or the right of 2, its profit maximizing location will, as in the previous case, be at one-third of either interval away from the relevant endpoint. Simple calculation shows that its profits in either of these events are then given by $(L_1^1)^2/3t = 0.0253t > 0.0252t$. Clearly, therefore, Firm 3 will not locate between 1 and 2, given locations $L_1^1 = 0.2754$ and $L_2^2 = 0.7246$, but it would do so if the interval between the two firms were any larger.

Since Firm 3 is clearly indifferent between locations to the left of 1 or the right of 2, we suppose without loss of generality that $L_3^3 = 0.908$.[9] Then, given $L_1^1 < L_2^2 < L_3^3$, profits are

$$\pi_1^1 = \int_0^a (L_2^2 - x)t\,dx - 0.5t[(L_1^1)^2 + (a - L_1^1)^2] + \pi^{m^{**}} \tag{4}$$

$\pi_2^2 =$

$$\int_a^b (x - L_1^1)t\,dx + \int_b^c (L_3^3 - x)t\,dx - 0.5t[(L_2^2 - a)^2 + (c - L_2^2)^2] \tag{5}$$

and

$$\pi_3^3 = \int_c^1 (x - L_2^2)t\,dx - 0.5t[(L_3^3 - c)^2 + (1 - L_3^3)^2] \tag{6}$$

where $a = 0.5(L_1^1 + L_2^2)$, $b = 0.5(L_1^1 + L_3^3)$, $c = 0.5(L_2^2 + L_3^3)$ and the increment from merger is

$$\pi^{m^{**}} = \int_0^a [(L_3^3 - x) - (L_2^2 - x)]t\,dx + \int_a^b [(L_3^3 - x) - (x - L_1^1)]t\,dx$$

Given the firms' locations, profits from merger are found by appropriate substitution in $\pi^{m^{**}}$ to obtain $0.1001t$. Firm 2's profits net of any share in the increment from merger are $\pi_2^2 = 0.0412t$, which is more than the $(L_1^1)^2/3t = 0.0253t$ it would get if it violated the agreement at the time of entry and located to the left of L_1^1, but less than it would obtain in the 'no-merger' state. This amount is the $\pi_3^2 = 0.074t$ identified by Gupta (1992). The side payment to 2 must therefore be at least $0.074t - 0.0412t = 0.0328t$ in order to ensure its adherence to the agreement. If 1 makes no more than this side payment it obtains $\pi_1^1 = 0.2415t$.[10] Finally, $\pi_3^3 = 0.0253t > 0.025t$, the profits to Firm 3 in the no-merger state. Total transport costs are $0.1010t$. Thus, we have

[9] A symmetrical argument to the following applies if $L_1^3 = 0.092$.

[10] It is interesting to note here that the necessary side payment to 2 to encourage it to locate at 0.7246, is sufficiently small that even if there were no incremental profit from merger firm 1 would emerge with higher profits than it would absent the cooperative agreement.

Proposition 2: *When cooperative merger takes place between entrants* 1 *and* 2*, all firms in the market obtain at least their profits in the no-merger state. Thus, the free-rider problem associated with the merger paradox may be eliminated, but the excluded firm continues to benefit from the merger. Moreover, cost efficiency is fractionally lower than in the no-merger state.*

Merger between Firms 2 *and* 3*:*

In this case, L_1^1 is constrained by the condition that, given L_3^2, Firm 3 should not locate left of L_1^1. As a profitable alternative to merger Firm 2 might wish to force such an outcome. This is possible if L_1^1 is 'too large'.

If Firm 1 chooses $L_1^1 = 0.275 + \gamma,\ \gamma > 0$, then Firm 2 can set $L_3^2 = 0.725 - \theta,\ 0 < \theta < \gamma$, and thereby actually encourage Firm 3 to violate the terms of the merger by locating to the left of Firm 1. This effectively destroys the merger, but it gives Firm 2 larger profits than it would obtain if it were to proceed. To prevent such an outcome, which would leave it with profits smaller than those it obtains when merger takes place, Firm 1 will locate no farther right than 0.275. Firm 2 will then choose $L_3^2 = 0.725$, leaving Firm 3 to adhere to the agreement by locating at 0.5. The relative locations in this case are therefore $L_1^1 < L_2^3 < L_3^2$, so that, absent merger, the profits of the firms are symmetrical with those in (1), (2) and (3).[11] Incremental profits are

$$\pi^{m^{***}} = \int_b^c [(x - L_1^1) - (L_3^2 - x)]t\,dx + \int_c^1 [(x - L_1^1) - (x - L_2^3)]t\,dx$$

Since locations are identical to those in Gupta (1992), so too are total transport costs. However, the profits of *both* Firms 2 and 3 can be higher than in the Gupta framework. This is because merger yields a total increment of $0.099t$, which can form the basis of negotiation between the two parties. We therefore have

Proposition 3. *When cooperative merger takes place between entrants* 2 *and* 3*, the parties to the merger both obtain at least the profits available to them*

[11] Here, again, a, b and c are as defined for those functions.

in the no-merger state, while the excluded firm obtains the same profits. The free-rider problem associated with the merger paradox is eliminated. Merger has no impact upon cost efficiency.

4. The 'two-plant' problem.

It is evident from the foregoing analysis that, under *cooperative merger*, the first partner to enter will attempt to locate so that the second will position itself *between it and the excluded firm.*[12] The object is to 'force' the excluded firm as far as possible towards the endpoint, and thereby to make the profits of each partner in the merged entity as large as possible consistent with the constraints imposed by the entry sequence and individual location choices. When (as in the case of a merger between Firms 1 and 3) the two parties do not enter in strict sequence, the excluded firm will choose its location in the expectation that the merged entity will adopt such a 'forcing strategy'.

We turn now to the possibility that a single firm might choose to enter with two *plants*. The game then takes the following (two-stage) form. In the first stage, the managers of the two plants identify the locations which maximize the firm's total profits, recognizing that a rival's plant will also enter; in the second stage, the market clears. In order to directly compare the results with those from the earlier section, we again examine all two-plant combinations: $\{1,3\}, \{1,2\}$ and $\{2,3\}$.

We show that when a two-plant firm enters, irrespective of whether the entry sequence for the plants is strict or interrupted by the arrival of the excluded firm, the locations of each are influenced by a different set of considerations from those which hold under merger. This is because, instead of locating so as to maximize each entrant's individual profit in the expectation of eventual merger - as was the case in the previous section - the two-plant firm simply maximizes its 'joint' or total profits.

[12]We have, however, shown that in the symmetrical case involving merger between Firms 1 and 2, this outcome cannot be ensured by the first partner to enter.

One immediate behavioral difference to which this alternative objective function gives rise is, as we shall show, that the first plant will be located as far as possible *towards the excluded firm*, and the second will locate *between the former and its nearer endpoint*. By this means, the first plant maximizes the size of the market of the combined entity, while the second makes its largest possible contribution to joint profits by maximizing its own surplus on the first plant's 'short side'. Again, as we show, there are obvious constraints on the precise locations of the excluded firm as well as the plants, but the contrast with the location pattern under merger is immediately apparent.

The implications of this phenomenon, in most of the cases hitherto considered, are quite substantial, in terms of firms' locations, profits and our measure of efficiency. In order to show these most clearly, we consider first the case where the plants enter first and third in the sequence, respectively.

One firm controls plants 1 *and* 3 :

We show now that the first plant will locate to 'squeeze' the excluded firm (Firm 2) as far as possible towards the latter's endpoint. It ensures by this means that the second plant will be able to locate in the largest possible interval that the first can guarantee it. This maximizes joint-profits.

Suppose that the first plant enters at the left of the market. If this happens then, as we show below, the excluded firm can be made to locate to the right, and the second plant will locate left of the first. The relative positions of plants 3 and 1, and the excluded firm, are then as in the case we have considered involving merger between 1 and 2. Hence, after appropriate substitutions, profits expressions (4), (5) and (6) can be used, together with the corresponding measure of the increment from merger, $\pi^{m^{**}}$.[13]

Our argument is as follows. The largest market which the two plants can guarantee for themselves is created when plant 1 locates at the midpoint of

[13]This measure is now to be reinterpreted as the increment accruing to the two-plant firm.

the interval. If plant 1 located at any other point, it would be gifting the excluded firm a larger market than the minimum necessary. Thus, substituting 'P', to denote 'plant', for 'L' in our earlier notation, $P_2^1 = 0.5$. Suppose, without loss of generality, that the excluded firm locates right of P_2^1.[14] The profit maximizing location for this firm is then at $L_3^2 = 1-(1-P_2^1)/3 = 0.833$. If the second plant locates symmetrically with the excluded firm, at $P_1^3 = 0.166$, it maximizes the 'surplus' of the excluded firm's delivered price over its own cost, and thus its contribution to joint-profits. Then $\pi_1^3 + \pi_2^1 = 0.277t$ and $\pi_3^2 = 0.0833t$. The location configuration yields to the two-plant firm its largest attainable profits.[15] Total transport costs are $0.0833t$. Figure 3 shows the equilibrium locations for this case, which can be compared directly with those for the $\{1,3\}$ merger depicted in Figure 2.

Figure 3.

Proposition 4. *When a firm enters with plants* 1 *and* 3, *all firms' locations are comprehensively different from those when Firms* 1 *and* 3 *engage in cooperative merger. Moreover, the profits of the two-plant firm exceed the combined profit for Firms* 1 *and* 3 *under merger* ($0.277t > 0.181t$), *while those of the excluded firm are larger than in the face of such merger* ($0.0833t > 0.052t$). *Finally, total transport costs here are lower than under merger* ($0.0833t < 0.1265t$), *and the minimum attainable in a market which contains three producers, regardless of the form and extent of their cooperation.*

One firm controls plants 1 *and* 2 :

In this case, the outcome for the two-plant firm is exactly the same as it is for the merged entity made up of Firms 1 and 2. In other words, merger

[14] The excluded firm is clearly indifferent between the intervals created by the first plant's choice.

[15] This is easily confirmed by summing the profits terms of the two plants, differentiating and setting equal to zero. Given $L_3^2 = 1 - [(1 - P_2^1)/3]$, and $P_1^3 = P_2^1/3$, this yields $P_2^1 = (3L_3^2)/5 = 0.5$, as asserted.

maximizes the participating firms' profits in the same way as do the locations chosen for the two plants.

The reason for this is that, under merger, and given $L_1^1 = 0.2754$ and $L_2^2 = 0.7246$, the locations of both parties are such that the excluded firm is forced into the smallest market interval which can be created by the choices of the merged firms. Since this is, in effect,[16] tantamount to locations by the two plants such that the excluded firm is forced as far to the right (resp. left) as possible, the result is equivalent to joint-profit maximization by the two-plant firm.

Proposition 5. *When a firm enters with plants* 1 *and* 2, *all firms' locations are identical to those when firms* 1 *and* 2 *engage in cooperative merger. Consequently, the profits of the two-plant firm are the same as the combined profits for Firms* 1 *and* 2 *under merger* $(0.3155t)$, *while the profits of the excluded firm* $(0.0253t)$, *as well as total transport costs* $(0.1010t)$, *are also the same as under merger.*

One firm controls plants 2 *and* 3 :

In this case, the first firm to enter (the excluded firm) can locate in Stackelberg fashion in relation to plant 2. As when merger occurs between 2 and 3, in choosing its location the excluded firm should ensure that the two plants will both be located to one of its sides, rather than to each side.[17]

Suppose therefore that the excluded firm locates from the left of the market, and so as to force both plants to locate to its right.[18] Then the profits expressions in (1), (2) and (3) can be used, after appropriate substitutions, together with the measure of the corresponding increment from merger,

[16] Given the symmetry of the merged firms' locations.

[17] If the latter outcome were, for some reason, unavoidable, the optimal choice for the excluded firm would be the market midpoint, but its profits, once the two plants had located, would then be as low as $0.025t$.

[18] We establish below precisely how this can be achieved.

$\pi^{m^{***}}$, which is added to the sum of the profits terms in (2) and (3). Computation is simplified by the fact that, given any P_2^2, the optimum location for the second plant is $P_3^2 = 1 - [1 - P_2^2]/3$. Given P_3^2 defined in this way, the two plants maximize their joint-profits by differentiating the sum of their profits expressions (including the increment from merger) with respect to P_2^2, setting equal to zero, and solving to obtain the following reaction function in $L^1 : P_2^2 = (9L^1 + 10)/19$.

The next step is to identify L^1. Given any location choice by the first entrant, the 'profits' of the two plants, if they were to locate to the left and right of L^1 would be $(L^1)^2/3t$ and $(1 - L^1)^2/3t$, respectively. The excluded firm must therefore ensure that the joint-profits of the two plants, if they both locate to its right, must be at least the sum of their individual profits to either side of $L^1 : [2L^1(L^1 - 1) + 1]/3t$.

In order to identify the L^1 which maximizes π_1, subject to the need to protect the excluded firm's left flank, we substitute the two plants' reaction function into the sum of (2), (3) and $\pi^{m^{***}}$, to obtain joint profits for the two-plant firm of $[7(L^1)^2 - 2L^1 + 1]/19t$. This profit is at least as large as that obtainable by the two plants if they locate to either side of L^1 iff $[7(L^1)^2 - 2L^1 + 1]/19t \geq [2L^1(L^1 - 1) + 1]/3t$, or $L^1 \leq 0.245$. If $L_1^1 = 0.245$, then $P_2^1 = 0.64$ and $P_3^2 = 0.88$. Hence

Proposition 6. *When a firm enters with plants* 2 *and* 3*, all firms' locations are comprehensively different from those when Firms* 2 *and* 3 *engage in cooperative merger. Moreover, the profits of the two plant firm are larger than the combined profits for Firms* 2 *and* 3 *under merger* $(0.210t > 0.198t)$*, while those of the excluded firm are larger than in the face of such merger* $(0.1368t > 0.74t)$*. Finally, total transport costs are higher here than under merger* $(0.1050t > 0.1009t)$.

Table 1 summarizes and compares the central results for the *cooperative merger* and *two-plant* cases offered in the two foregoing sections.

Table 1

5. Concluding comments.

This paper shows first that when cooperative merger occurs in a market under conditions of spatial price discrimination and sequential entry, the merger paradox may be overturned. However, the extent to which this occurs depends upon the particular combination which makes up the merger. Specifically, we have observed that merger between the first and third entrants comprehensively overturns the paradox, but decreases cost efficiency relative to the no-merger state. By contrast, while merger between the first and second entrants eliminates the free-rider aspect of the paradox, the excluded firm continues to benefit and cost efficiency decreases. Finally, merger between the second and third entrants yields gains to the parties involved without affecting in any way the excluded firm's profits, or cost efficiency.[19]

The paper also shows that, in two out of three of the cases considered, the cooperative merger problem yields a strikingly different solution to the 'two-plant' problem. Only in one case are they analytically indistinguishable. We offer an explanation for this general result in terms of the preferred location strategies of firms in the two contexts. A strong result which emerges from the comparison of the two problems is that the latter will generate combined profits for the two-plants, as well as profits for the excluded firm, that are at least as high as those obtainable under merger. Moreover, transport costs in two of the cases considered are either less than or equal to those which arise under merger.

From a policy perspective the results of our comparative analysis are worthy of note. They suggest, for example, that an anti-trust authority can, and should, draw different inferences from instances of cooperative merger to those it draws from location by a multi-plant firm. While the two may seem

[19] Given these sharp results, a natural question is whether mergers might in some sense be *endogenous*. It is clear that the profits of the individual participants are substantially greater in some mergers than in others, and it is to be expected that this will determine the mergers which ultimately occur. This question is explored in depth in the *noncooperative* merger framework in Heywood et al (2002).

superficially similar they can, and often will, yield fundamentally different results in terms of both profits and social welfare. From an empirical point of view, this is important, because the fact that entry is sequential is no bar to 'pre-play' communication between firms. Consequently, when opportunities exist for such behavior, the distinction between cooperative merger and joint-profit maximization is of more than purely theoretical interest.

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Figure 1: Equilibrium Locations Under Sequential Entry

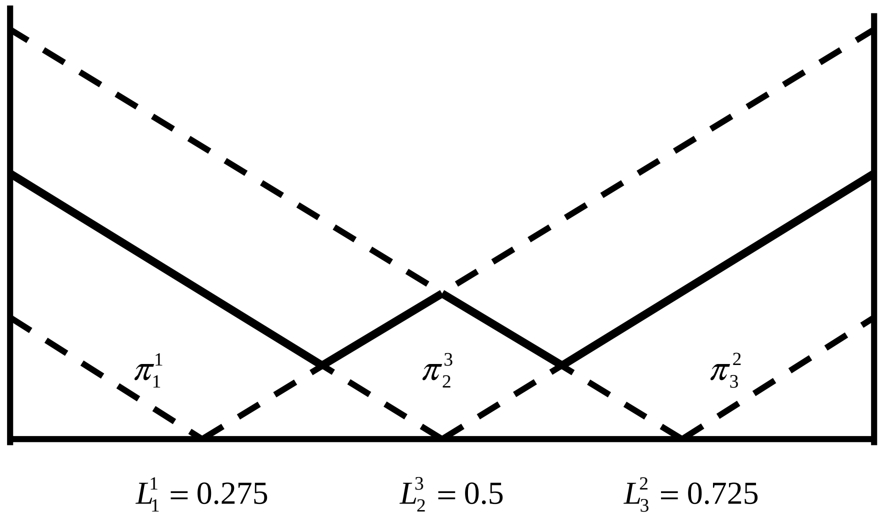
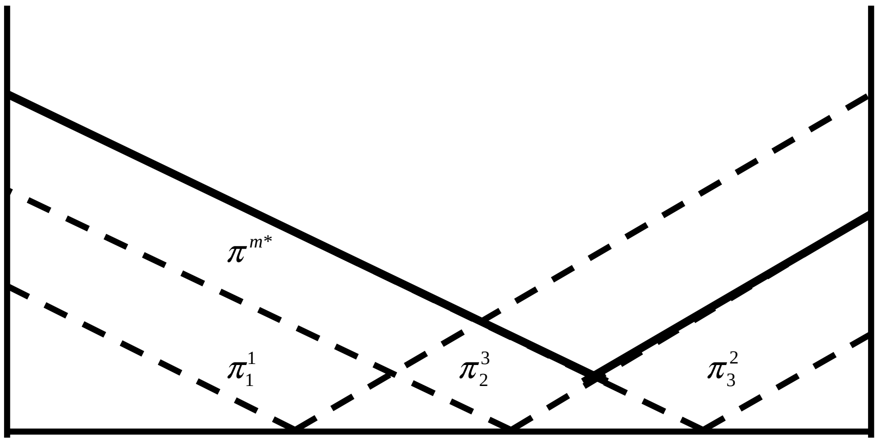


Figure 2: Equilibrium Locations When Firms 1 and 3 Merge



$L_1^1 = 0.395$ $L_2^3 = 0.568$ $L_3^2 = 0.741$

Figure 3: Equilibrium Locations When One Firm Controls Plants 1 and 3

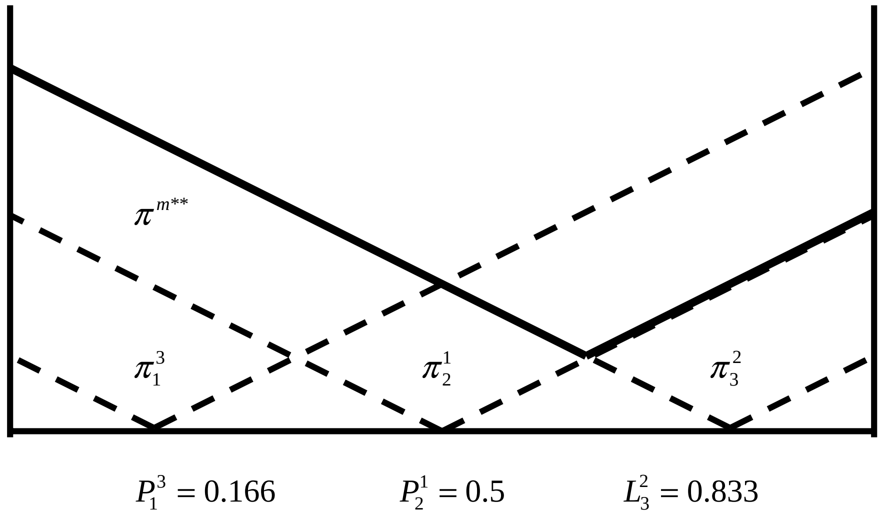


Table 1: Comparison of Locations and Profits

	Entrant 1		Entrant 2		Entrant 3	
	Location	*Profit*	*Location*	*Profit*	*Location*	*Profit*
No Merger (Gupta)	.275	.074t	.725	.074t	.500	.025t
Merger of Entrants 1 and 3	.395	.129t	.741	.052t	.568	.052t
Merger of Entrants 1 and 2	.2754	.2415t	.7246	.074t	.908	.0253t
Merger of Entrants 2 and 3	.275	.074t	.725	.173t	.500	.025t
Joint Maximization Entrants 1 and 3	.500	.1365t	.833	.0833t	.167	.1365t
Joint Maximization Entrants 1 and 2	.2754	.158t	.7246	.158t	.908	.0253t
Joint Maximization Entrants 2 and 3	.245	.1368t	.640	.105t	.880	.105t

Notes: In all merger cases the profits shown assume only the minimum side payment has been made to the second merging partner and that the first partner retains the balance of the increment from merger. In all joint maximization cases the profits shown simply divide the total earned equally between the two plants.