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The Central Bank Inflation Bias in the Presence of Asymmetric Preferences and Non-Normal Shocks

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Abstract

We investigate the nature of the inflation bias in a model that exhibits asymmetries in preferences and non–normality in shocks but simplifies to the classic Barro-Gordon problem as a special case. The inflation bias is shown to depend on the trade-off between preference, structural and the scale and shape parameters of the model.

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1. Introduction

The standard assumptions in the first analyses of independent central banks are that that they target output above the natural rate, the central banks preferences are quadratic and that the supply curve or Phillips curve is linear. These assumptions generate an inflationary bias. (see e.g. Svensson (1997)). However all of these assumptions are questionable. First prominent Central bankers argue that they do not target output above the natural rate. For example, Vickers (1998, p369) writes "There is a large literature on inflation bias but it simply is not applicable to the Monetary Policy Committee. We have no desire to spring inflation surprises to try to bump output above its natural rate (whatever that may be)". Whilst Blinder (1998, p43) argues that policy makers at the Fed do not try to systematically maintain employment above the natural level. As a matter of fact he personally felt duty bound to pick monetary policy so as to hit the natural rate when in office. In addition The Governor of the Bank of England, King (1996), in a theoretical model, assumes that the central bank has no desire to have unemployment rates below (or output rates above) their natural-rate values.

Second there seems no good reason to assume that either output or inflation preferences are necessarily symmetric. For example Cuckierman (2002) writes "While casual observation suggests that policymakers dislike employment below the normal level, it does not support the notion that, given inflation, they also dislike employment above the normal level. Given inflation, some politicians probably even *like* positive output gaps on the view that the higher output is, the better it is. As a matter of fact, it is quite likely that the quadratic function on the output gap, so often used in the academic literature, was chosen mainly for analytical convenience rather than for descriptive realism". Similar observations can be made concerning inflation. The ECB target of 2% inflation is explicitly asymmetric and Nobay and Peel (2003) suggest that the Bank of England had an asymmetric target at least in its first few years of formulation. Finally empirical evidence suggests the supply curve is nonlinear. See for example Clarke, Laxton and Rose (1996), Eisner (1997) and Stiglitz (1984).

It is now recognized that an inflation bias -positive or negative, can arise from either nonlinear preferences of the central bank over either or both of inflation and output or a nonlinear supply or Phillips curve[3] even when the central bank targets the natural rate. (see e.g. Bean (1996), Tambakis (1999), Cuckierman (2002), Nobay and Peel (2000), (2003)). A number of authors have examined time series or cross section data to determine whether there is a significant inflation bias. Ruge-Murcia (2004) assumes inflation preferences are asymmetric so that any bias will depend on the variance of inflation. (see Nobay and Peel (2003)). For the inflation targeting counties he studies he finds some support for the proposition that average inflation is

[3] There is a growing number of papers that report empirical evidence of non linear Taylor rules which are consistent with one or both of nonlinear preferences and a nonlinear Phillips curve. (see e.g Osborne(2005),. Surico (2004),Dolado et al (2004), Dolado et al(2005), Taylor and Davradakis (2006), Martin and Milas,(2007) and Cukierman and Muscatelli (2007)).

negatively related to the variance of inflation. In contrast Gerlach and Cuckierman (2003) assume asymmetry in the output objective and report a positive relationship between average inflation and output variability.

Our purpose in this note is to characterize the nature of the inflation bias that can occur in a one-period Barro-Gordon problem under a fairly general specification of the structural equations and additionally that the random shock to supply can be non-normally distributed. In order to maintain some tractability we assume that the central banks preferences and the Phillips curve take the Linex form so that quadratic preferences as well as a linear supply curve are nested.

Random disturbances are assumed to be drawn from a skew-normal distribution[4] so that the standard normal density is obtained as a special case. This set up allows us to consider the nature of the inflation bias in a more general setting than in previous analyses.

The rest of the paper is structured as follows. In Section 2 we set out our analysis and discuss some of the salient implications relative to the assumption of quadratic loss and linear Phillips curve. In Section 3 we present our theoretical results. Concluding comments are offered in the final section.

2. Analysis under Non-Normality and Asymmetric Preferences

The Phillips curve is assumed to take the form assumed in Nobay and Peel (2000)

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[4] The density function of the Skew-Normal distribution for the noise v is given by

$$pdf(v) = 2\phi(v)\Phi(s\,v)$$

where $\varphi(\ .\ )$ and $\Phi(\ .\ )$ denote the density and cumulative density respectively of a standard normal variable. The Skew-Normal accommodates a rich variety of skewness and kurtosis patterns as the shape parameter s varies and converges to the Normal as $s \to 0$. See e.g. Arnold and Lin (2004).

Formally, the first four moments of the standard Skew-Normal variable v_π are given by

$$E(v) = \delta\sqrt{\frac{2}{\pi}},\ Var(v) = 1 - \frac{2}{\pi}\delta^2,\ Skew = \frac{4-\pi}{2}\left(\frac{E(v)^3}{Var(v)^{\frac{3}{2}}}\right),\ Kurt = 2(\pi - 3)\left(\frac{E(v)^4}{Var(v)^2}\right)$$

where $\delta = \dfrac{s}{\sqrt{1+s^2}}$

As the shape parameter s tends to zero, the moments of the Skew Normal reduce to those of the Standard Normal. More generally, the moment generating admits a closed form expression, that is

$$E(t\,v) = 2\exp\left(\frac{t^2}{2}\right)\Phi(t\ \ \delta)$$

which can be interpreted as twice the moment generating function of the standard normal, times a shape parameter-related adjustment. Clearly, as the shape parameter s tends to zero, the moment generating function reduces to that of a standard normal variable.

$$\pi - \pi^e = \frac{1}{g}\left[\exp(\kappa g(y - y_n)) - 1\right] + \sigma_\pi v_\pi \tag{1}$$

where, π is inflation, π^e is the rational expectation of inflation formed before realization of shocks at time t, y is real output, y_n is the normal or natural rate of output, g, κ are constants and $v_\pi \sim SN(s_\pi)$ is distributed as a Skew-Normal random variable with shape parameter s_π. This non linear Linex form has a number of convenient properties apart from providing analytic tractability. First, in (1) $\kappa > 0$ implies a convex Phillips curve, perhaps the case best supported by empirical evidence (see e.g. Clarke, Laxton and Rose (1996), though $\kappa < 0$, a concave Phillips curve, has some theoretical and empirical support (see e.g. Stiglitz (1984), Eisner (1997). Second, as $g \to 0$ the above relationship (1) reduces to the linear Phillips relationship of the form

$$\pi - \pi^e = \kappa(y - y_n) + \sigma_\pi v_\pi \tag{2}$$

From inspection of (1) the reduced form for output is given by

$$y = f(z) + \sigma_y v_y \tag{3}$$

where $f(z)$ is a conditionally known deterministic function of a set of instruments z and $v_y \sim SN(s_y)$ is distributed as a Skew-Normal random variable with shape parameter s_y. We note that in the case where output is assumed to be distributed conditionally normal we can obtain, by taking the rational expectation of equation (1), that $E_{t-1}y = y_n - \frac{\kappa g}{2}(\text{var}\, y)$, where var y is the variance of y. We observe that output can have a mean higher or lower than the natural rate depending on the curvature of the Phillips curve. (see also see Bean (1996)). In the case of a linear Phillips curve, from (3), $E_{t-1}y = f(z) = y_n$.

Inflation and output preferences are assumed to be given by the Linex form as in Nobay and Peel (2003).

$$L_y(y - y^*; \gamma) = \frac{1}{\gamma^2}\left[\exp[\gamma(y - y^*)] - \gamma(y - y^*) - 1\right]$$
$$L_\pi(\pi - \pi^*; \alpha) = \frac{1}{\alpha^2}\left[\exp[\alpha(\pi - \pi^*)] - \alpha(\pi - \pi^*) - 1\right]$$

where we denote the inflation and output targets by π^* and y^* respectively.

The Linex form nests the quadratic form as a special case. As $\gamma, \alpha \to 0$ we obtain quadratic output and inflation preferences. For a non-zero γ, α the costs to undershooting (overshooting) the inflation or output targets are non-symmetric. Cukierman (2002) states the case for an asymmetric output objective. He writes " But it is hard to see why CBs, social planners, or political authorities would consider, *given inflation*, a positive output gap of a given magnitude to be equivalent to a negative output gap of the same magnitude. A negative output gap means that employment is below the normal level, whereas a positive output gap means employment is above the normal level. While casual observation suggests that policymakers dislike employment below the normal level, it does not support the notion that, given inflation, they also dislike employment above the normal level".

Nobay and Peel suggest inflation preferences may be asymmetric. They note that the costs of inflation beneath target maybe lower ceteris paribus, than the costs of exceeding target. For instance the ECB target is 2.5% or less.

The policy maker's optimization problem is

$$\min_{y} E\left[L\left(\pi-\pi^{*}, y-y^{*};\alpha,\gamma,\lambda\right)\right] = E\left[L_y\left(y-y^{*};\gamma\right)\right] + \lambda E\left[L_\pi\left(\pi-\pi^{*};\alpha\right)\right] \qquad (4)$$

As is standard, e.g. Svensson (1997), inflation is assumed to be set by the authorities after observation of the supply shock.[5]

3. Theoretical Results

Closed form solutions do not exist for the "general case " set out above. However we can elucidate the key implications by considering a number of special cases which we set out as propositions. We first consider the case of a linear Phillips curve so that $f(z) = y_n$. We also assume that the authorities target the natural so that any inflation bias is not a resultant of targeting a higher level of output than the natural rate. For simplicity we set $y^* = y_n = 0$.

In Proposition 1 we set out the general form of optimal policy rule under Linex Preferences and Skew Normal shocks for both inflation and output.

Proposition 1. In the presence of Linex preferences and skew-normal shocks for both inflation and output, the optimal policy rule under the linear Phillips relationship of equation (2), is

$$\pi^{e} = \pi^{*} + \frac{1}{\alpha}\ln\left[1+\frac{\alpha}{\lambda\gamma\kappa}\left[1-mgf\left(v_y;\gamma\sigma_y\right)\right]\right] - \frac{1}{\alpha}\ln\left[mgf\left(v_y;\alpha\kappa\sigma_y\right)\right] - \frac{1}{\alpha}\ln\left[mgf\left(v_\pi;\alpha\sigma_\pi\right)\right]$$

[5] Although we are interested in the inflation bias and the policy maker's choice variable is the inflation rate, the first-order condition for a minimum may be expressed as a function of either inflation or output by applying the chain rule.

where

$$mgf\left(v_y;\gamma\sigma_y\right)=2\exp\left(\frac{\left(\gamma\sigma_y\right)^2}{2}\right)\Phi\left(\delta_y\gamma\sigma_y\right)$$

$$mgf\left(v_y;\alpha\kappa\sigma_y\right)=2\exp\left(\frac{\left(\alpha\kappa\sigma_y\right)^2}{2}\right)\Phi\left(\delta_y\alpha\kappa\sigma_y\right)$$

$$mgf\left(v_\pi;\alpha\sigma_\pi\right)=2\exp\left(\frac{\left(\alpha\sigma_\pi\right)^2}{2}\right)\Phi\left(\delta_\pi\alpha\sigma_\pi\right)$$

and $\delta_y=\frac{s_y}{\sqrt{1+s_y^2}},\delta_\pi=\frac{s_\pi}{\sqrt{1+s_\pi^2}}$ and Φ( ) denotes the cumulative standard normal density.

Proof: See Appendix.

We observe that the inflation bias depends on both of the variances of inflation and output. Since Normality is a special case of Skew Normality and symmetric quadratic preferences is a special case of Linex, a number of interesting combinations are nested as special cases of our Proposition (available on request). In the following Corollary 1 we provide two such cases to indicate the impact and the trade-off between preference and density asymmetries in the context of a linear Phillips curve.

Corollary 1. The optimal policy rule of Proposition 1 in the case (1a) symmetric quadratic inflation loss preferences 1(b) normal shock innovations and symmetric preferences for both variables, is given by the respective equations

$$(1a)\qquad \pi^e=\pi^*-\sqrt{\frac{2}{\pi}}\left(\delta_y\sigma_y\kappa-\delta_\pi\sigma_\pi\right)$$

$$(1b)\qquad \pi^e=\pi^*$$

Proof: See Appendix.

From 1(a) we note that the inflation bias depends on the variances of inflation and output and also skewness. In particular, observe that the inflation bias will be positive if and only if $\delta_\pi\sigma_\pi>\kappa\sigma_y\delta_y$. This contrasts with the standard case of quadratic preferences 1(b) where there is no bias given that the central bank targets the natural rate.

We now turn our attention to the impact of preference and density asymmetries in the presence of a non-linear Phillips curve of Linex form.

Proposition 2. In the presence of Linex preferences and skew-normal shocks for both inflation and output, the optimal policy rule under the non-linear Phillips relationship of equation (1) does not admit a closed form expression.

Proof: See Appendix

Although the assumptions of Proposition 2 do not lead to a closed-form solution to the optimal policy problem, it is possible to achieve a result by allowing simpler inflation preferences. We provide such an analysis in Proposition 3.

Proposition 3. In the presence of Linex preferences for output, symmetric quadratic preferences for inflation and skew-normal shocks for both inflation and output, the optimal policy rule under the non-linear Phillips relationship of equation (1) is given by

$$\pi^e = \pi^* + \frac{1 - mgf\left(v_y;\gamma\sigma_y\right)}{\lambda\gamma\kappa\, mgf\left(v_y;\kappa g\,\sigma_y\right)} + \frac{1}{g}\left[1 - \exp\left[3\frac{\left(\kappa g\sigma_y\right)^2}{2}\right]\frac{\Phi\left(\delta_y 2\kappa g\sigma_y\right)}{\Phi\left(\delta_y \kappa g\sigma_y\right)}\right] - \sigma_\pi\delta_\pi\sqrt{\frac{2}{\pi}}$$

where

$$\delta_\pi = \frac{s_\pi}{\sqrt{1+s_\pi^2}}$$

$$mgf\left(v_y;\gamma\sigma_y\right) = 2\exp\left(\frac{\left(\gamma\sigma_y\right)^2}{2}\right)\Phi\left(\delta_y\gamma\sigma_y\right)$$

$$mgf\left(v_y;\kappa g\,\sigma_y\right) = 2\exp\left(\frac{\left(\kappa g\sigma_y\right)^2}{2}\right)\Phi\left(\delta_y\kappa g\sigma_y\right)$$

and Φ( ) denotes the cumulative standard normal density.

Proof: See Appendix

We observe a complex form of the inflation bias. As in Proposition 1 above a number of interesting combinations are nested as special cases of our Proposition 3. In the following Corollary 3 we provide three such cases (others available on request) which indicate the impact and the trade-off between preference and density asymmetries in the context of a non-linear Phillips curve.

Corollary 3. 3(a) symmetric quadratic output loss preferences, or (3b) linear Phillips, (3c) normal shock innovations and symmetric preferences

$$(3a)\quad \pi^e = \pi^* - \frac{\sqrt{\frac{2}{\pi}}\delta_y \sigma_y}{\lambda\kappa\, mgf\left(v_y ; \kappa g \sigma_y\right)} + \frac{1}{g}\left[1 - \exp\left[3\frac{\left(\kappa g \sigma_y\right)^2}{2}\right]\frac{\Phi\left(\delta_y 2\kappa g \sigma_y\right)}{\Phi\left(\delta_y \kappa g \sigma_y\right)}\right] - \sigma_\pi \delta_\pi \sqrt{\frac{2}{\pi}}$$

$$(3b)\quad \pi^e = \pi^* + \frac{1 - mgf\left(v_y ; \gamma\sigma_y\right)}{\lambda\gamma\kappa} - \sqrt{\frac{2}{\pi}}\kappa\delta_y \sigma_y - \sigma_\pi \delta_\pi \sqrt{\frac{2}{\pi}}$$

$$(3c)\quad \pi^e = \pi^*$$

Proof: See Appendix

4. Conclusion

In order to demonstrate how non-normality of errors and non linear structural equations impact on the inflation bias we have investigated the properties of a model that exhibits the Barro-Gordon problem as a special case.

We assume the authorities target the natural rate, as they profess to do. Not surprisingly the model exhibits an inflation bias that depends in a complex manner on the various parameters of model. The bias will not be zero if either preferences or the Phillips curve are nonlinear. Of particular interest is that the bias depends on the scale and shape parameters of the model. In this context, Goodhart's (2001) comments on the Bank of England Monetary Policy Committee are interesting. He writes: "But in either case the existence of a skew would affect our decision on the appropriate interest rate. Unlike uncertainty and variance, skew and risk mapped directly into the interest rate decision." Our model captures this observation.

The results appear to have implications for the measure of inflation that policy makers target. If the chosen measure exhibits excess skew, say the overall CPI, as opposed to, say, a trimmed mean that does not, then we would expect, ceteris paribus, more evidence of bias in the former case than in the latter. This type of prediction might form the basis for an empirical test in future analysis.

References

Arnold, B.C. and Lin, G.D., (2004), "Characterizations of the skew-normal and the generalized chi distributions", *Sankhyã: Indian Journal of Statistics*, Vol. 66(4), pp. 593–606

Barro, R., and D. Gordon (1983). "Rules, discretion, and reputation in a model of monetary policy." *Journal of Monetary Economics*, 12: 101–121.

Bean, C. (1996). “The convex Phillips curve and macroeconomic policymaking under uncertainty.” Working Paper, London School of Economics.

Blinder, A., (1997), “What Central Bankers Could Learn from Academics and Vice Versa”, *Journal of Economic Perspectives*, Vol. 11, pp. 3-19

Christodoulakis, G. A., (2006), “Generalised Rational Bias in Financial Forecasts”., *Annals of Finance*, Vol. 2, pp. 397-405

Clark, P., D. Laxton, and D. Rose (1996). “Asymmetry in the U.S. output-inflation nexus.” *IMF Staff Papers*, 43: 216–251.

Cukierman, A. (2002), “Are Contemporary Central Banks Transparent About Economic Models and Objectives and What Difference Does It Make?”, *The Federal Reserve Bank of St Louis*, pp 15-35.

Cukierman, A. and Gerlach, S., (2003), “The inflation bias revisited: theory and some international evidence”, *The Manchester School*, Vol. 71, pp. 473-565.

Cukierman A. and Muscatelli, V.A. (2007) “Non Linear Taylor Rules and Asymmetric Preferences in Central Banking - Evidence from the UK and the US”. CEPR discussion paper DP6428

Dolado, J., Dolores, R. M and Ruge-Murcia, F.R. (2004).” Nonlinear monetary policy rules: Some new evidence for the U.S.” Studies in Nonlinear Dynamics and Econometrics 8, 3

Dolado, J., Dolores, R. M and Ruge-Murcia, F.R. (2005).”Are monetary-policy reaction functions asymmetric?: The role of nonlinearity in the Phillips curve “. European Economic Review 49, 485-503

Eisner, R. (1997). “A new view of the NAIRU.” In P. Davidson and J. Kregel (eds.), *Improving the Global Economy:Keynesianism and the Growth in Output and Employment*. Cheltenham U.K.: Edward Elgar.,

Goodhart, C. A. E. (2001). “Monetary transmission lags and the formulation of the policy decision on interest rates”. Federal Reserve Bank of St. Louis 3, 165–182.

King, M (1996), “How Should Central Banks Reduce Inflation? Conceptual Issues”, in Achieving Price Stability, *Federal Reserve Bank of Kansas City,* Symposium Series, pp 53-91.

Martin, C. And Milas, C..(2007) “Testing the Opportunistic Approach to Monetary Policy." Keele Discussion Paper Series

Nobay, A. R. and D. A. Peel, (2000), “Optimal Monetary Policy with a Non-Linear Phillips Curve”. *Economics Letters*, Vol. 67, pp. 159-164

Nobay, A. R. and D. A. Peel, (2003), Optimal Discretionary Monetary Policy in a Model of Asymmetric Central Bank Preferences, *Economic Journal*, Vol. 113, pp. 657-665

Osborn, D.R., Kim, D.H. and Sensier,M. (2005). "Nonlinearity in the Fed's monetary policy rule," Journal of Applied Econometrics 20(5), 621-639.

Ruge-Murcia, J., (2004), “The Inflation Bias When the Central Banker Targets the Natural Rate of Unemployment”, *European Economic Review,* Vol. 48, pp. 91-107.

Stiglitz, J.E, (1984) ``Price Rigidities and Market Structure", *American Economic Review*74.pp350-55.

Surico, P.2004. "Inflation Targeting and Nonlinear Policy Rules: The Case of Asymmetric Preferences," CESifo Working Paper Series

Svensson, L. (1997), “Optimal Inflation Targets, `Conservative' Central Banks, and Linear Inflation Contracts”, *American Economic Review*, Vol. 87, pp 98--114.

Tambakis, D.N. (1999) “ Monetary Policy with a Nonlinear Phillips Curve and Asymmetric Loss”,*Studies in Nonlinear Dynamics and Econometrics*, 1999, 3(4): 223–237.

Taylor, M.P. and Davradakis, E. (2006) "Interest Rate Setting and Inflation Targeting: Evidence of a Nonlinear Taylor Rule for the United Kingdom", *Studies in Nonlinear Dynamics & Econometrics*: 4,1. 1359-1359

Vickers J. (1998), “Inflation Targeting in Practice: The UK Experience”, *Bank of England Quarterly Bulletin*, Vol. 38, No. 4, pp. 368-375.

Walsh, C., (2002) “Are contemporary central banks transparent about economic models and objectives and what difference does it make? – commentary”, *The Federal Reserve Bank of St Louis*, pp 37-46.

Appendix

Proof of Proposition 1.

Assuming that relationship (2) holds, we differentiate (4) with respect to y to obtain

$$\frac{1}{\gamma}\left[\exp\left[\gamma\left(y-y^*\right)\right]-1\right]+\frac{\kappa\lambda}{\alpha}\left[\exp\left[\alpha\left(\pi-\pi^*\right)\right]-1\right]=0$$

taking expectations we obtain

$$\frac{1}{\gamma}\left[mgf\,(y;\gamma)\exp\left(-\gamma y^*\right)-1\right]+\frac{\kappa\lambda}{\alpha}\left[E\left[\exp\left(\alpha\left(\pi-\pi^*\right)\right)\right]-1\right]=0$$

Substituting for π from equation (2) and for y using equation (3) and using Proposition 1 of Arnold and Lin (2004) we obtain

$$\frac{1}{\gamma}\left[\exp(\gamma f\,(z))mgf\left(v_y;\gamma\sigma_y\right)\exp\left(-\gamma y^*\right)-1\right]$$
$$\times\frac{\kappa\lambda}{\alpha}\left[\exp\left(\alpha\left(\pi^e-\pi^*\right)\right)\exp(\alpha\kappa f\,(z))mgf\left(v_y;\alpha\kappa\sigma_y\right)\exp\left(-\alpha\kappa y_n\right)mgf\left(v_\pi;\alpha\sigma_\pi\right)-1\right]=0$$

where

$$mgf\left(v_y;\gamma\sigma_y\right)=2\exp\left(\frac{\left(\gamma\sigma_y\right)^2}{2}\right)\Phi\left(\frac{s_y}{\sqrt{1+s_y^2}}\gamma\sigma_y\right)$$

$$mgf\left(v_y;\alpha\kappa\sigma_y\right)=2\exp\left(\frac{\left(\alpha\kappa\sigma_y\right)^2}{2}\right)\Phi\left(\frac{s_y}{\sqrt{1+s_y^2}}\alpha\kappa\sigma_y\right)$$

$$mgf\left(v_\pi;\alpha\sigma_\pi\right)=2\exp\left(\frac{\left(\alpha\sigma_\pi\right)^2}{2}\right)\Phi\left(\frac{s_\pi}{\sqrt{1+s_\pi^2}}\alpha\sigma_\pi\right)$$

Solving with respect to π^e we obtain the result. ■

Proof of Corollary 1. As shape parameters $s_\pi \to 0$ and $s_y \to 0$ the moment generating functions reduce to normality

$$mgf\left(v_y;\gamma\sigma_y\right)=\exp\left(\frac{\left(\gamma\sigma_y\right)^2}{2}\right)$$

$$mgf\left(v_y;a\varkappa\sigma_y\right)=\exp\left(\frac{\left(a\varkappa\sigma_y\right)^2}{2}\right)$$

$$mgf\left(v_\pi;a\sigma_\pi\right)=\exp\left(\frac{\left(a\sigma_\pi\right)^2}{2}\right)$$

Substituting to Proposition 1 we obtain (1a) and (1b).
■

Proof of Proposition 2.

Assuming that relationship (1) holds, we differentiate (4) with respect to *y* to obtain

$$\frac{1}{\gamma}\left[\exp\left[\gamma\left(y-y^*\right)\right]-1\right]+\frac{\kappa\lambda}{\alpha}\exp\left[\kappa g\left(y-y_n\right)\right]\left[\exp\left[\alpha\left(\pi-\pi^*\right)\right]-1\right]=0$$

Substituting for π from (1) and taking expectations we cannot arrive to a closed form expression since the term

$$\exp\left[\alpha\left(\pi-\pi^*\right)\right]$$

involves two nested exponential functions. ■

Proof of Proposition 3.

As $\alpha\to 0$ the Linex function for inflation reduces to a quadratic loss of the form

$$L_\pi\left(\pi-\pi^*;\alpha\right)=\frac{1}{2}\left(\pi-\pi^*\right)^2$$

Then, differentiating (4) using equation (1), we obtain the first order condition

$$\frac{1}{\gamma}\left[\exp\left[\gamma\left(y-y^*\right)\right]-1\right]+\kappa\lambda\left(\pi-\pi^*\right)\exp\left(\kappa g\left(y-y_n\right)\right)=0$$

and substituting for π from (1) we obtain

$$\frac{1}{\gamma}\left[\exp(\gamma y)\exp\left(-\gamma y^*\right)-1\right]$$

$$+\kappa\lambda\exp(\kappa g y)\exp(-\kappa g y_n)\left[\left(\pi^e-\pi^*\right)+\frac{1}{g}(\exp(\kappa g y)\exp(-\kappa g y_n)-1)+\sigma_\pi v_\pi\right]=0$$

Then, substituting for y from equation (3) and taking expectations using Proposition 1 of Arnold and Lin (2004), we obtain

$$\frac{1}{\gamma}\left[mgf\left(v_y;\gamma\sigma_y\right)-1\right]+\kappa\lambda\, mgf\left(v_y;\kappa g\sigma_y\right)\left(\pi^e-\pi^*\right)+\frac{\kappa\lambda}{g}\, mgf\left(v_y;2\kappa g\sigma_y\right)$$

$$-\frac{\kappa\lambda}{g}\, mgf\left(v_y;\kappa g\sigma_y\right)+\kappa\lambda\, mgf\left(v_y;\kappa g\sigma_y\right)\sigma_\pi E(v_\pi)=0$$

Solving with respect to π^e we obtain the result. ■

Proof of Corollary 3.

Allowing $s_\pi \to 0$ the term $\sigma_\pi\delta_\pi\sqrt{\frac{2}{\pi}}$ vanishes which proves (3a). Allowing $s_y \to 0$

$$mgf\left(v_y;\gamma\sigma_y\right)=\exp\left(\frac{\left(\gamma\sigma_y\right)^2}{2}\right)$$

$$mgf\left(v_y;a\kappa\sigma_y\right)=\exp\left(\frac{\left(g\kappa\sigma_y\right)^2}{2}\right)$$

Substituting to Proposition 3 we obtain result (3b). Taking the limit as $\gamma \to 0$, the term

$$\lim_{\gamma\to 0}\ \frac{1-mgf\left(v_y;\gamma\sigma_y\right)}{\lambda\gamma\kappa\, mgf\left(v_y;\kappa\, g\,\sigma_y\right)}$$

$$=-\frac{\sqrt{\frac{2}{\pi}}\delta_y\sigma_y}{\lambda\kappa\, mgf\left(v_y;\kappa g\sigma_y\right)}$$

which proves (3c). ■